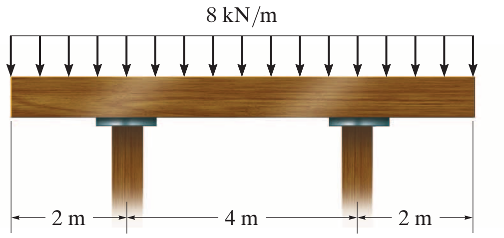
11–1. The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 6.5$ MPa and an allowable shear stress of $\tau_{allow} = 500$ kPa. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of 1.25.



$$I_x = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$Q_{max} = \bar{y}'A' = (0.3125b)(0.625b)(b) = 0.1953125b^3$$

Assume bending moment controls:

$$M_{max} = 16 \text{ kN}\cdot\text{m}$$

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$6.5(10^6) = \frac{16(10^3)(0.625b)}{0.16276b^4}$$

$b = 0.21143 \text{ m} = 211 \text{ mm}$ **Ans.**

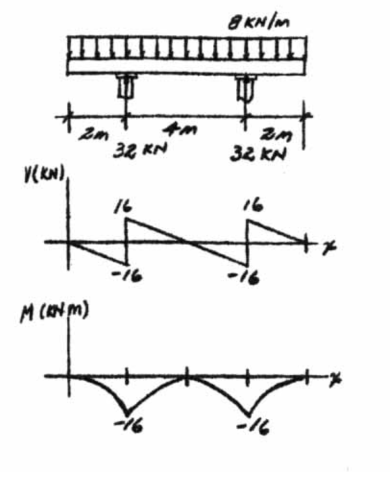
$h = 1.25b = 264 \text{ mm}$ **Ans.**

Check shear:

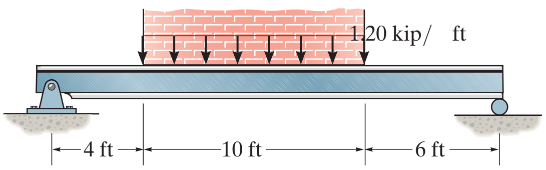
$$Q_{max} = 1.846159(10^{-3}) \text{ m}^3$$

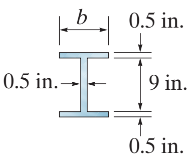
$$I = 0.325248(10^{-3}) \text{ m}^4$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{16(10^3)(1.846159)(10^{-3})}{0.325248(10^{-3})(0.21143)} = 429 \text{ kPa} < 500 \text{ kPa, OK}$$



11–2. The brick wall exerts a uniform distributed load of 1.20 kip/ft on the beam. If the allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi, select the lightest wide-flange section with the shortest depth from Appendix B that will safely support the load.





Bending Stress: From the moment diagram, $M_{max} = 44.55$ kip · ft. Assuming bending controls the design and applying the flexure formula.

$$S_{req\,d} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{44.55\,(12)}{22} = 24.3\ \text{in}^3$$

Two choices of wide flange section having the weight 22 lb/ft can be made. They are W12 × 22 and W14 × 22. However, W12 × 22 is the shortest.

Select W12 × 22 $\left(S_x = 25.4\ \text{in}^3,\ d = 12.31\ \text{in.},\ t_w = 0.260\ \text{in.}\right)$

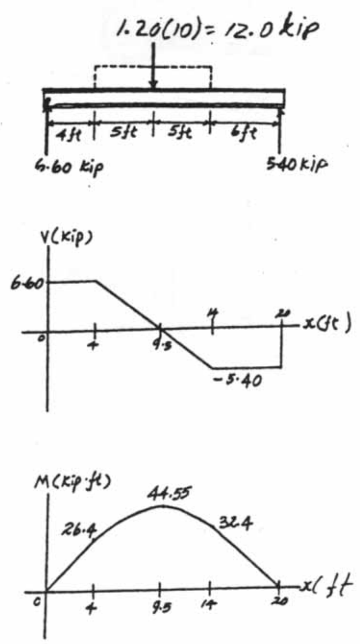
Shear Stress: Provide a shear stress check using $\tau = \dfrac{V}{t_w\,d}$ for the W12 × 22 wide - flange section. From the shear diagram, $V_{max} = 6.60$ kip.

$$\tau_{max} = \frac{V_{max}}{t_w\,d}$$

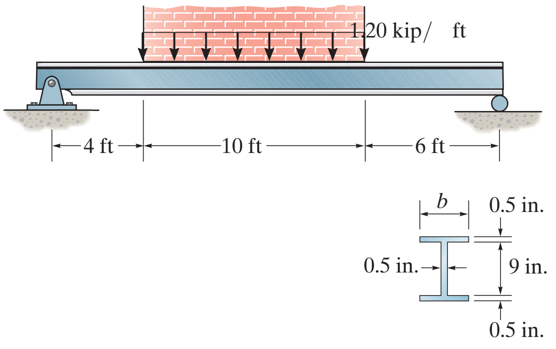
$$= \frac{6.60}{0.260(12.31)}$$

$$= 2.06\ \text{ksi} < \tau_{allow} = 12\ \text{ksi}\ (O.K!)$$

Hence, **Use** W12 × 22 **Ans.**



11–3. The brick wall exerts a uniform distributed load of 1.20 kip/ft on the beam. If the allowable bending stress is $\sigma_{allow} = 22$ ksi, determine the required width b of the flange to the nearest $\frac{1}{4}$ in.



Section Property:

$$I = \frac{1}{12}(b)\left(10^3\right) - \frac{1}{12}(b - 0.5)\left(9^3\right) = 22.583b + 30.375$$

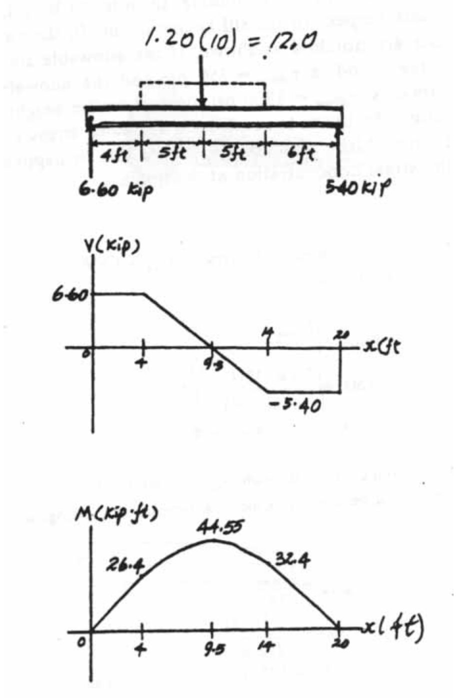
Bending Stress: From the moment diagram, $M_{max} = 44.55$ kip · ft.

$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

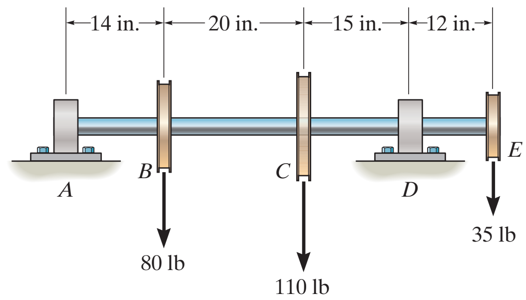
$$22 = \frac{44.55(12)(5)}{22.583b + 30.375}$$

$$b = 4.04 \text{ in.}$$

Use $b = 4.25$ in. **Ans.**



***11–4.** Draw the shear and moment diagrams for the shaft, and determine its required diameter to the nearest $\frac{1}{4}$ in. if $\sigma_{allow} = 7$ ksi and $\tau_{allow} = 3$ ksi. The bearings at A and D exert only vertical reactions on the shaft. The loading is applied to the pulleys at B, C, and E.



$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

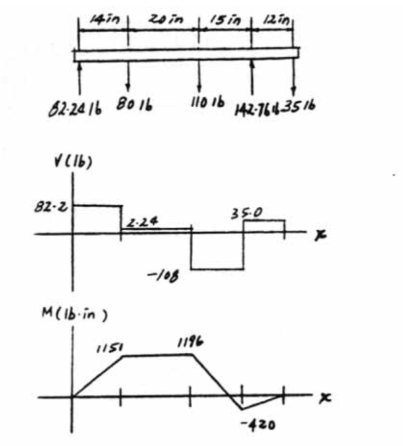
$$7(10^3) = \frac{1196\,c}{\frac{\pi}{4}c^4}; \qquad c = 0.601 \text{ in.}$$

$$d = 2c = 1.20 \text{ in.}$$

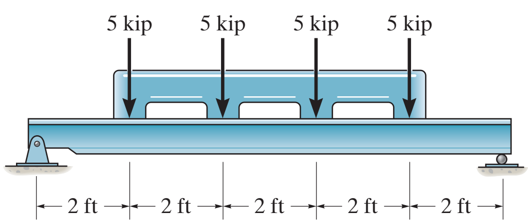
Use $d = 1.25$ in. **Ans.**

Check shear:

$$\tau_{max} = \frac{V_{max}\,Q}{It} = \frac{108\left(\frac{4(0.625)}{3\pi}\right)(\pi)\left(\frac{0.625^2}{2}\right)}{\frac{\pi}{4}(0.625)^4\,(1.25)} = 117 \text{ psi} < 3 \text{ ksi OK}$$



•11–5. Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the machine loading shown. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.



Bending Stress: From the moment diagram, $M_{max} = 30.0$ kip·ft. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{30.0(12)}{24} = 15.0 \text{ in}^3$$

Select W12 × 16 $\left(S_x = 17.1 \text{ in}^3,\ d = 11.99 \text{ in.},\ t_w = 0.220 \text{ in.}\right)$

Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w\,d}$ for the W12 × 16 wide-flange section. From the shear diagram, $V_{max} = 10.0$ kip

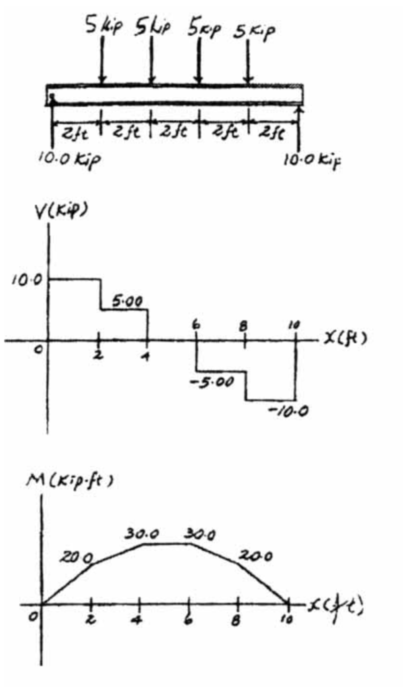
$$\tau_{max} = \frac{V_{max}}{t_w\,d}$$

$$= \frac{10.0}{0.220(11.99)}$$

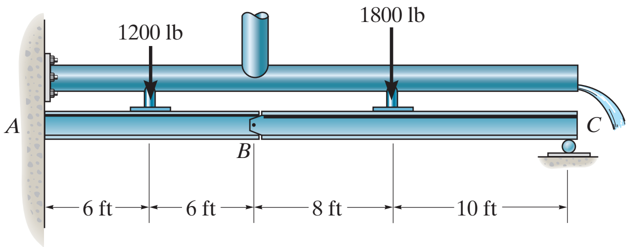
$$= 3.79 \text{ ksi} < \tau_{allow} = 14 \text{ ksi } (O.K!)$$

Hence,

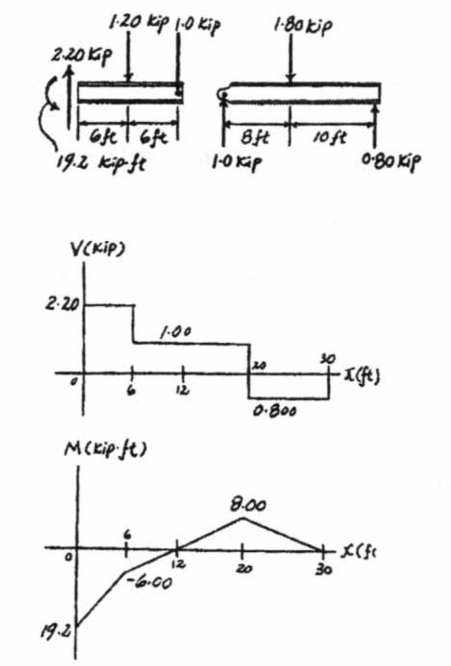
Use W12 × 16 **Ans.**



11–6. The compound beam is made from two sections, which are pinned together at B. Use Appendix B and select the lightest-weight wide-flange beam that would be safe for each section if the allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi. The beam supports a pipe loading of 1200 lb and 1800 lb as shown.



Bending Stress: From the moment diagram, $M_{max} = 19.2$ kip·ft for member AB. Assuming bending controls the design, applying the flexure formula.



$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{19.2(12)}{24} = 9.60 \text{ in}^3$$

Select W10 × 12 $\left(S_x = 10.9 \text{ in}^3, d = 9.87 \text{ in.}, t_w = 0.19 \text{ in.}\right)$

For member BC, $M_{max} = 8.00$ kip·ft.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{8.00(12)}{24} = 4.00 \text{ in}^3$$

Select W6 × 9 $\left(S_x = 5.56 \text{ in}^3, d = 5.90 \text{ in.}, t_w = 0.17 \text{ in.}\right)$

Shear Stress: Provide a shear stress check using $\tau = \dfrac{V}{t_w d}$ for the W10 × 12 wide-flange section for member AB. From the shear diagram, $V_{max} = 2.20$ kip.

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{2.20}{0.19(9.87)}$$

$$= 1.17 \text{ ksi} < \tau_{allow} = 14 \text{ ksi } (O.K!)$$

Use W10 × 12 **Ans.**

For member BC (W6 × 9), $V_{max} = 1.00$ kip.

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

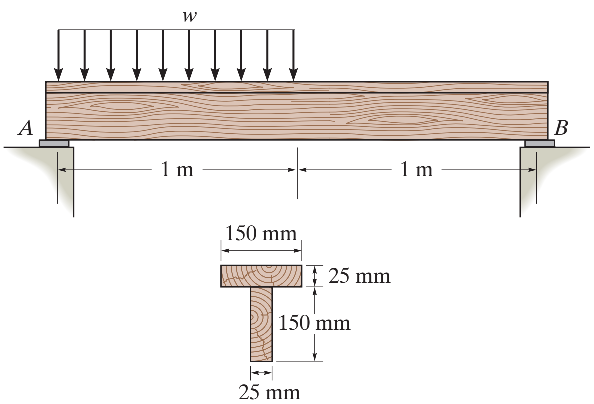
$$= \frac{1.00}{0.17(5.90)}$$

$$= 0.997 \text{ ksi} < \tau_{allow} = 14 \text{ ksi } (O.K!)$$

Hence,

Use W6 × 9 **Ans.**

11–7. If the bearing pads at A and B support only vertical forces, determine the greatest magnitude of the uniform distributed loading w that can be applied to the beam. $\sigma_{\text{allow}} = 15$ MPa, $\tau_{\text{allow}} = 1.5$ MPa.



The location of c, Fig. b, is

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.1625(0.025)(0.15) + 0.075(0.15)(0.025)}{0.025(0.15) + 0.15(0.025)}$$

$$= 0.11875 \text{ m}$$

$$I = \frac{1}{12}(0.025)(0.15^3) + (0.025)(0.15)(0.04375)^2$$

$$+ \frac{1}{12}(0.15)(0.025^3) + 0.15(0.025)(0.04375)^2$$

$$= 21.58203125(10^{-6}) \text{ m}^4$$

Referring to Fig. b,

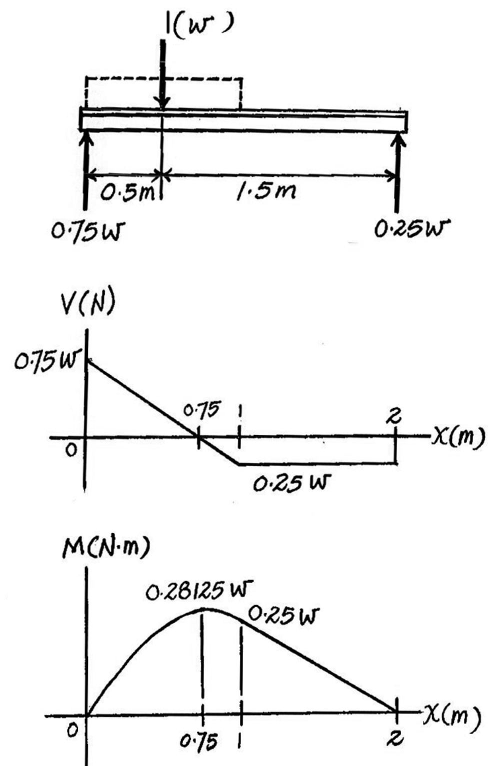
$$Q_{\text{max}} = \bar{y}'A' = 0.059375\,(0.11875)(0.025)$$

$$= 0.176295313(10^{-4}) \text{ m}^3$$

Referring to the moment diagram, $M_{\text{max}} = 0.28125\,w$. Applying the Flexure formula with $C = \bar{y} = 0.11875$ m,

$$\sigma_{\text{allow}} = \frac{M_{\text{max}}\,c}{I}; \qquad 15(10^6) = \frac{0.28125w(0.11875)}{21.582(10^{-6})}$$

$$W = 9.693(10^3) \text{ N/m}$$



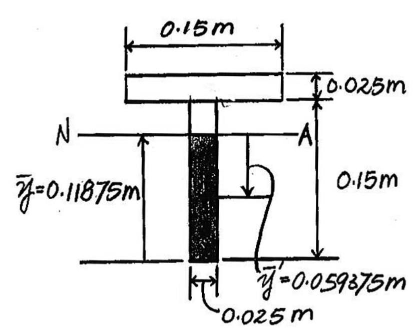
(a)

Referring to shear diagram, Fig. a, $V_{\text{max}} = 0.75\,w$.

$$\tau_{\text{allow}} = \frac{V_{\text{allow}}\,Q_{\text{max}}}{It}; \qquad 1.5(10^6) = \frac{0.75w\left[0.17627(10^{-3})\right]}{21.582(10^{-6})(0.025)}$$

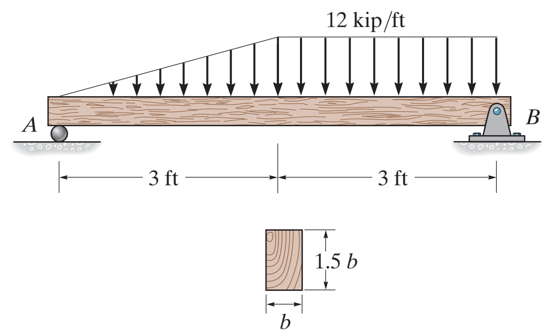
$$W = 6.122(10^3) \text{ N/m}$$

$$= 6.12 \text{ kN/m } (\textit{Control!}) \qquad \textbf{Ans.}$$



(b)

***11–8.** The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 1.20$ ksi and an allowable shear stress of $\tau_{allow} = 100$ psi. Determine its smallest dimensions to the nearest $\frac{1}{8}$ in. if it is rectangular and has a height-to-width ratio of 1.5.



The moment of inertia of the beam's cross-section about the neutral axis is $I = \frac{1}{12}(b)(1.5b)^3 = 0.28125b^4$. Referring to the moment diagram, $M_{max} = 45.375 \text{ kip} \cdot \text{ft}$.

$$\sigma_{allow} = \frac{M_{max}\, c}{I}; \qquad 1.2 = \frac{45.375(12)(0.75b)}{0.28125b^4}$$

$$b = 10.66 \text{ in}$$

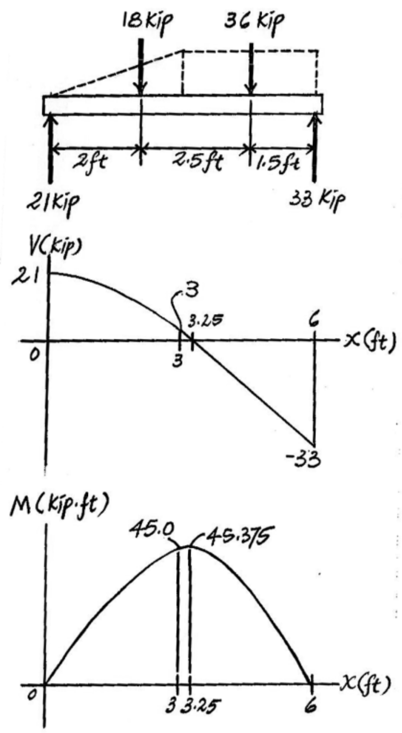
Referring to Fig. *b*, $Q_{max} = \bar{y}'A' = 0.375b\,(0.75b)(b) = 0.28125b^3$. Referring to the shear diagram, Fig. *a*, $V_{max} = 33$ kip.

$$\tau_{max} = \frac{V_{max}\, Q_{max}}{It}; \qquad 100 = \frac{33(10^3)(0.28125b^3)}{0.28125b^4(b)}$$

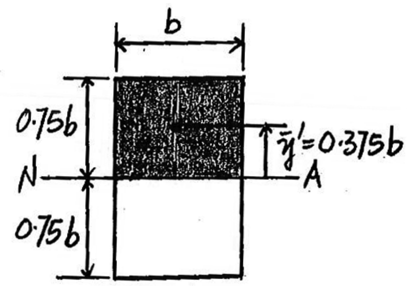
$$b = 18.17 \text{ in } (Control!)$$

Thus, use

$$b = 18\frac{1}{4} \text{ in}$$ **Ans.**

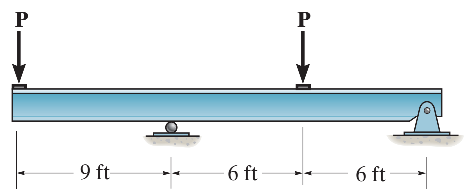


(a)



(b)

•11–9. Select the lightest-weight W12 steel wide-flange beam from Appendix B that will safely support the loading shown, where $P = 6$ kip. The allowable bending stress is $\sigma_{\text{allow}} = 22$ ksi and the allowable shear stress is $\tau_{\text{allow}} = 12$ ksi.



From the Moment Diagram, Fig. a, $M_{\max} = 54$ kip · ft.

$$S_{\text{req'd}} = \frac{M_{\max}}{\sigma_{\text{allow}}}$$

$$= \frac{54(12)}{22}$$

$$= 29.45 \text{ in}^3$$

Select W12 × 26 $\left[S_x = 33.4 \text{ in}^3,\ d = 12.22 \text{ in and } t_w = 0.230 \text{ in.}\right]$

From the shear diagram, Fig. a, $V_{\max} = 7.5$ kip. Provide the shear-stress check for W 12 × 26,

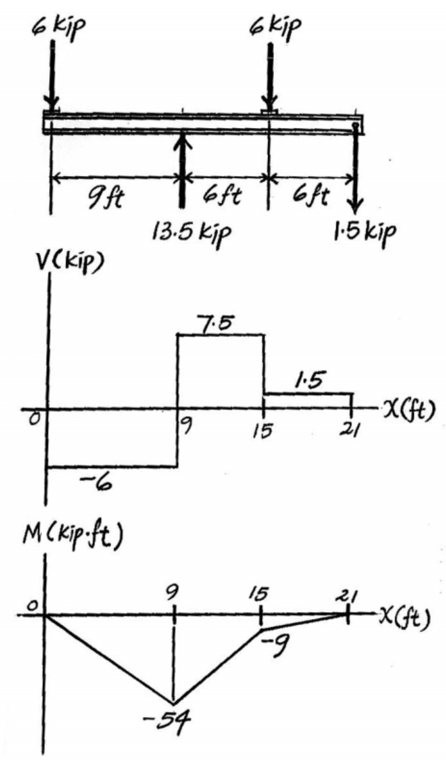
$$\tau_{\max} = \frac{V_{\max}}{t_w\, d}$$

$$= \frac{7.5}{0.230(12.22)}$$

$$= 2.67 \text{ ksi} < \tau_{\text{allow}} = 12 \text{ ksi } (O.K!)$$

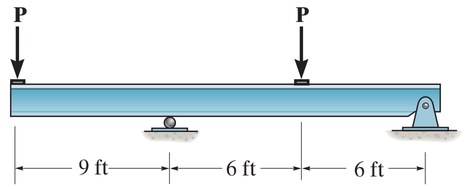
Hence

Use W12 × 26 **Ans.**



(a)

11–10. Select the lightest-weight W14 steel wide-flange beam having the shortest height from Appendix B that will safely support the loading shown, where $P = 12$ kip. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.



From the moment diagram, Fig. a, $M_{max} = 108$ kip · ft.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{108(12)}{22}$$

$$= 58.91 \text{ in}^3$$

Select W14 × 43 $\left[S_x = 62.7 \text{ in}^3, d = 13.66 \text{ in and } t_w = 0.305 \text{ in.}\right]$

From the shear diagram, Fig. a, $V_{max} = 15$ kip. Provide the shear-stress check for W14 × 43,

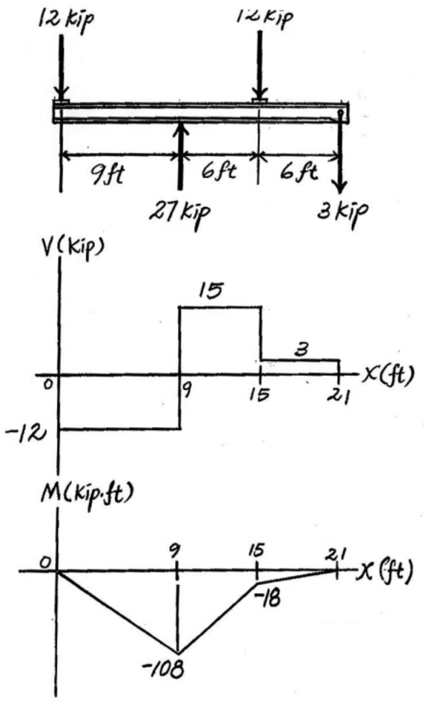
$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{15}{0.305(13.66)}$$

$$= 3.60 \text{ ksi} < \tau_{allow} = 12 \text{ ksi}, (O.K!)$$

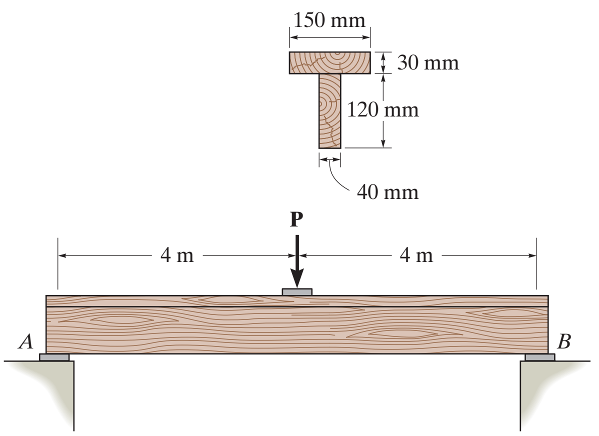
Hence,

Use W14 × 43 **Ans.**



(a)

11–11. The timber beam is to be loaded as shown. If the ends support only vertical forces, determine the greatest magnitude of **P** that can be applied. $\sigma_{allow} = 25$ MPa, $\tau_{allow} = 700$ kPa.



$$\bar{y} = \frac{(0.015)(0.150)(0.03) + (0.09)(0.04)(0.120)}{(0.150)(0.03) + (0.04)(0.120)} = 0.05371 \text{ m}$$

$$I = \frac{1}{12}(0.150)(0.03)^3 + (0.15)(0.03)(0.05371 - 0.015)^2 + \frac{1}{12}(0.04)(0.120)^3 +$$

$$(0.04)(0.120)(0.09 - 0.05371)^2 = 19.162(10^{-6}) \text{ m}^4$$

Maximum moment at center of beam:

$$M_{max} = \frac{P}{2}(4) = 2P$$

$$\sigma = \frac{Mc}{I}; \qquad 25(10^6) = \frac{(2P)(0.15 - 0.05371)}{19.162(10^{-6})}$$

$$P = 2.49 \text{ kN}$$

Maximum shear at end of beam:

$$V_{max} = \frac{P}{2}$$

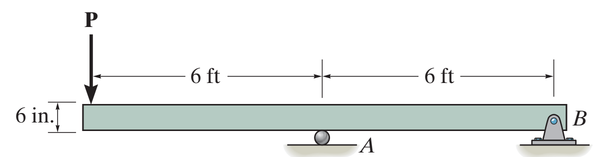
$$\tau = \frac{VQ}{It}; \qquad 700(10^3) = \frac{\frac{P}{2}\left[\frac{1}{2}(0.15 - 0.05371)(0.04)(0.15 - 0.05371)\right]}{19.162(10^{-6})(0.04)}$$

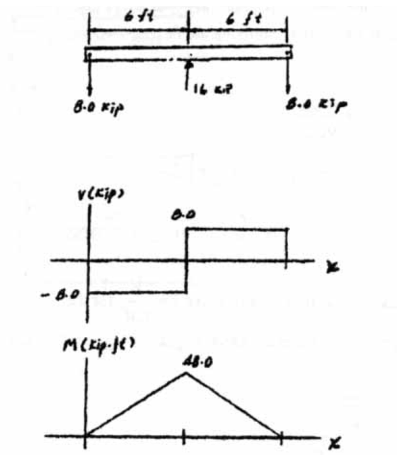
$$P = 5.79 \text{ kN}$$

Thus,

$P = 2.49$ kN **Ans.**

***11–12.** Determine the minimum width of the beam to the nearest $\frac{1}{4}$ in. that will safely support the loading of $P = 8$ kip. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 15$ ksi.





Beam design: Assume moment controls.

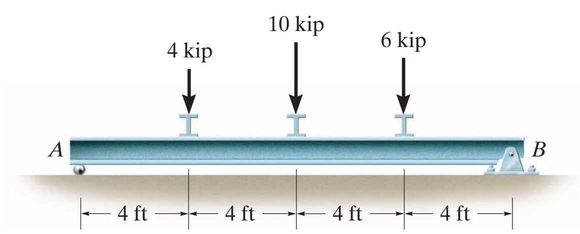
$$\sigma_{allow} = \frac{M\,c}{I}; \qquad 24 = \frac{48.0(12)(3)}{\frac{1}{12}(b)(6^3)}$$

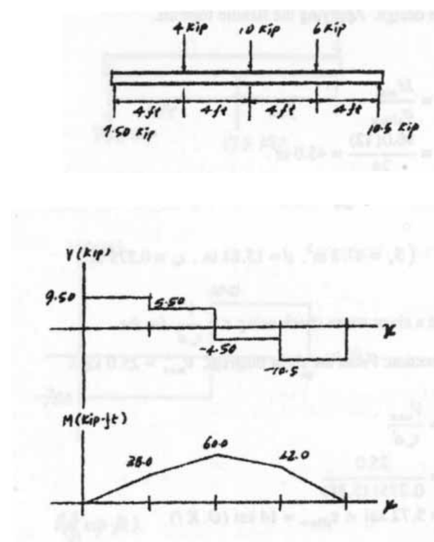
$b = 4$ in. **Ans.**

Check shear:

$$\tau_{max} = \frac{VQ}{I\,t} = \frac{8(1.5)(3)(4)}{\frac{1}{12}(4)(6^3)(4)} = 0.5 \text{ ksi} < 15 \text{ ksi OK}$$

•11–13. Select the shortest and lightest-weight steel wide-flange beam from Appendix B that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.





Beam design: Assume bending moment controls.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{60.0(12)}{22} = 32.73 \text{ in}^3$$

Select a $W\ 12 \times 26$

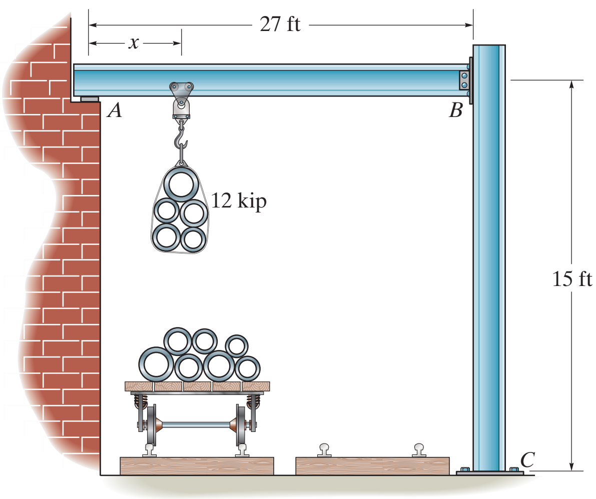
$S_x = 33.4 \text{ in}^3$, $d = 12.22$ in., $t_w = 0.230$ in.

Check shear:

$$\tau_{avg} = \frac{V}{A_{web}} = \frac{10.5}{(12.22)(0.230)} = 3.74 \text{ ksi} < 12 \text{ ksi}$$

Use $W\ 12 \times 26$ **Ans.**

11–14. The beam is used in a railroad yard for loading and unloading cars. If the maximum anticipated hoist load is 12 kip, select the lightest-weight steel wide-flange section from Appendix B that will safely support the loading. The hoist travels along the bottom flange of the beam, $1 \text{ ft} \leq x \leq 25 \text{ ft}$, and has negligible size. Assume the beam is pinned to the column at B and roller supported at A. The allowable bending stress is $\sigma_{\text{allow}} = 24$ ksi and the allowable shear stress is $\tau_{\text{allow}} = 12$ ksi.



Maximum moment occurs when load is in the center of beam.

$$M_{\max} = (6 \text{ kip})(13.5 \text{ ft}) = 81 \text{ lb} \cdot \text{ft}$$

$$\sigma_{\text{allow}} = \frac{M}{S}; \qquad 24 = \frac{81(12)}{S_{\text{req'd}}}$$

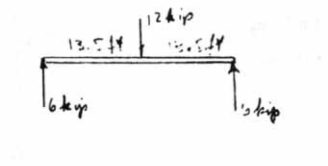
$$S_{\text{req'd}} = 40.5 \text{ in}^3$$

Select a $W\ 14 \times 30$, $S_x = 42.0 \text{ in}^3$, $d = 13.84$ in, $t_w = 0.270$ in.

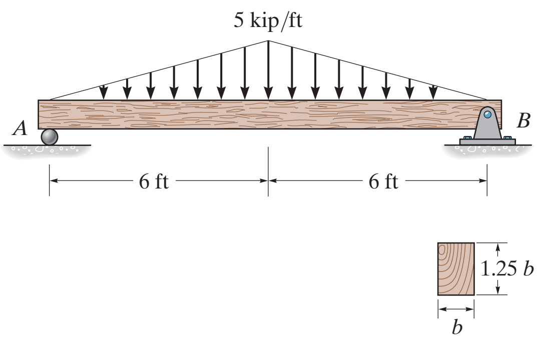
At $x = 1$ ft, $V = 11.56$ kip

$$\tau = \frac{V}{A_{\text{web}}} = \frac{11.36}{(13.84)(0.270)} = 3.09 \text{ ksi} < 12 \text{ ksi}$$

Use W14 × 30 **Ans.**



11–15. The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 960$ psi and an allowable shear stress of $\tau_{allow} = 75$ psi. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of 1.25.



$$I = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$S_{req'd} = \frac{I}{c} = \frac{0.16276b^4}{0.625b} = 0.26042\text{b}^3$$

Assume bending moment controls:

$$M_{max} = 60 \text{ kip} \cdot \text{ft}$$

$$\sigma_{allow} = \frac{M_{max}}{S_{req'd}}$$

$$960 = \frac{60(10^3)(12)}{0.26042\, b^3}$$

$$b = 14.2 \text{ in.}$$

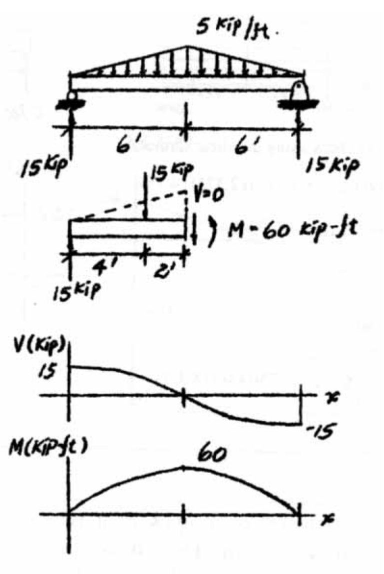
Check shear:

$$\tau_{max} = \frac{1.5V}{A} = \frac{1.5(15)(10^3)}{(14.2)(1.25)(14.2)} = 88.9 \text{ psi} > 75 \text{ psi NO}$$

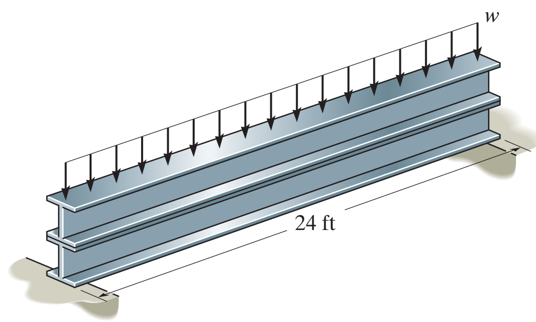
Shear controls:

$$\tau_{allow} = \frac{1.5V}{A} = \frac{1.5(15)(10^3)}{(b)(1.25b)}$$

$$b = 15.5 \text{ in.}$$ **Ans.**



***11–16.** The simply supported beam is composed of two W12 × 22 sections built up as shown. Determine the maximum uniform loading w the beam will support if the allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.

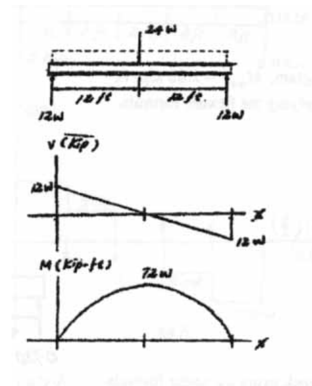


Section properties:

For $W12 \times 22$ ($d = 12.31$ in. $I_x = 156\text{ in}^4$ $t_w = 0.260$ in. $A = 6.48\text{ in}^2$)

$$I = 2\left[156 + 6.48\left(\frac{12.31}{2}\right)^2\right] = 802.98\text{ in}^4$$

$$S = \frac{I}{c} = \frac{802.98}{12.31} = 65.23\text{ in}^3$$



Maximum Loading: Assume moment controls.

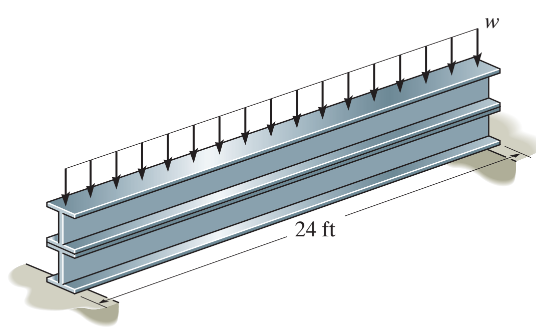
$$M = \sigma_{allow} S (72\,w)(12) = 22(65.23)$$

$$w = 1.66\text{ kip/ft} \qquad \textbf{Ans.}$$

Check Shear: (Neglect area of flanges.)

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{12(1.66)}{2(12.31)(0.26)} = 3.11\text{ ksi} < \tau_{allow} = 14\text{ ksi OK}$$

•11–17. The simply supported beam is composed of two W12 × 22 sections built up as shown. Determine if the beam will safely support a loading of $w = 2$ kip/ft. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.

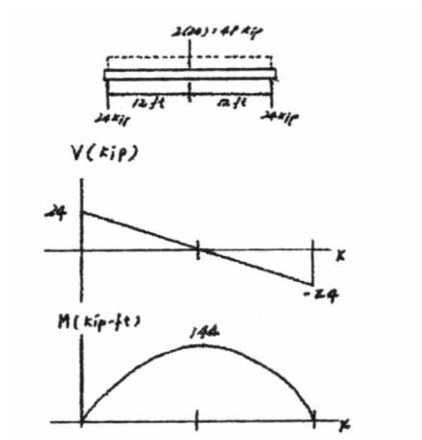


Section properties:

For $W\ 12 \times 22$ ($d = 12.31$ in. $I_x = 156\text{ in}^4$ $t_w = 0.260$ in. $A = 6.48\text{ in}^2$)

$$I = 2[156 + 6.48(6.155^2)] = 802.98\text{ in}^4$$

$$S = \frac{I}{c} = \frac{802.98}{12.31} = 65.23\text{ in}^3$$



Bending stress:

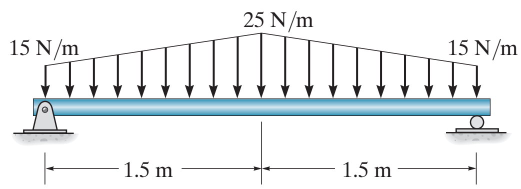
$$\sigma_{max} = \frac{M_{allow}}{S} = \frac{144\,(12)}{65.23} = 26.5\text{ ksi} > \sigma_{allow} = 22\text{ ksi}$$

No, the beam falls due to bending stress criteria. **Ans.**

Check shear: (Neglect area of flanges.)

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{24}{2(12.31)(0.26)} = 3.75\text{ ksi} < \tau_{allow} = 14\text{ ksi OK}$$

11–18. Determine the smallest diameter rod that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 167$ MPa and the allowable shear stress is $\tau_{allow} = 97$ MPa.

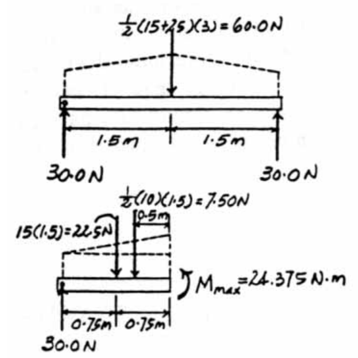


Bending Stress: From the moment diagram, $M_{max} = 24.375\ \text{N} \cdot \text{m}$. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$167\left(10^6\right) = \frac{24.375\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

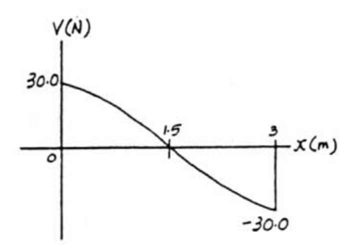
$$d = 0.01141\ \text{m} = 11.4\ \text{mm} \qquad \textbf{Ans.}$$



Shear Stress: Provide a shear stress check using the shear formula with

$$I = \frac{\pi}{4}\left(0.005707^4\right) = 0.8329\left(10^{-9}\right)\ \text{m}^4$$

$$Q_{max} = \frac{4(0.005707)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.005706^2\right)\right] = 0.1239\left(10^{-6}\right)\ \text{m}^3$$

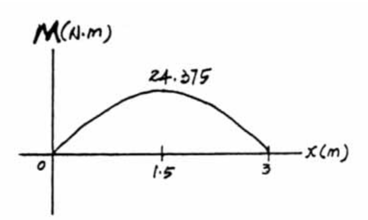


From the shear diagram, $V_{max} = 30.0$ N.

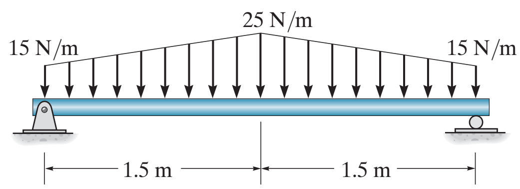
$$\tau_{max} = \frac{V_{max}\, Q_{max}}{It}$$

$$= \frac{30.0\left[0.1239(10^{-6})\right]}{0.8329\ (10^{-9})(0.01141)}$$

$$= 0.391\ \text{MPa} < \tau_{allow} = 97\ \text{MPa}\ (O.K.!)$$



11–19. The pipe has an outer diameter of 15 mm. Determine the smallest inner diameter so that it will safely support the loading shown. The allowable bending stress is $\sigma_{\text{allow}} = 167$ MPa and the allowable shear stress is $\tau_{\text{allow}} = 97$ MPa.



Bending Stress: From the moment diagram, $M_{\max} = 24.375\ \text{N}\cdot\text{m}$. ↗. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{\text{allow}} = \frac{M_{\max}\,c}{I}$$

$$167\left(10^6\right) = \frac{24.375(0.0075)}{\frac{\pi}{4}\left[0.0075^4 - \left(\frac{d_i}{2}\right)^4\right]}$$

$$d_i = 0.01297\ \text{m} = 13.0\ \text{mm} \qquad \textbf{Ans.}$$

Shear Stress: Provide a shear stress check using the shear formula with

$$I = \frac{\pi}{4}\left(0.0075^4 - 0.006486^4\right) = 1.0947\left(10^{-9}\right)\ \text{m}^4$$

$$Q_{\max} = \frac{4(0.0075)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.0075^2\right)\right] - \frac{4(0.006486)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.006486^2\right)\right]$$

$$= 99.306\left(10^{-9}\right)\ \text{m}^3$$

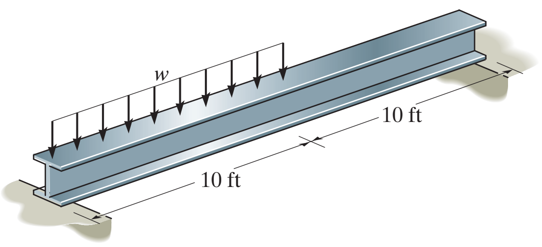
From the shear diagram, $V_{\max} = 30.0$ N. ↗

$$\tau_{\max} = \frac{V_{\max}\,Q_{\max}}{It}$$

$$= \frac{30.0\left[99.306(10^{-9})\right]}{1.0947(10^{-9})(0.015 - 0.01297)}$$

$$= 1.34\ \text{MPa} < \tau_{\text{allow}} = 97\ \text{MPa} \quad (O.K!)$$

***11–20.** Determine the maximum uniform loading w the W12 × 14 beam will support if the allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.



From the moment diagram, Fig. a, $M_{max} = 28.125\,w$. For $W12 \times 14$, $S_x = 14.9\text{ in}^3$, $d = 11.91$ in and $t_w = 0.200$ in.

$$\sigma_{allow} = \frac{M_{max}}{S}$$

$$22 = \frac{28.125\,w\,(12)}{14.9}$$

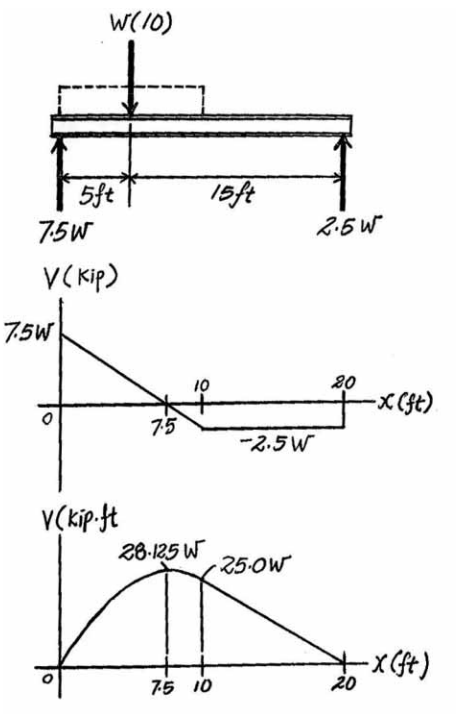
$$w = 0.9712\text{ kip/ft} = 971\text{ lb/ft}$$ **Ans.**

From the shear diagram, Fig. a, $V_{max} = 7.5(0.9712) = 7.284$ kip. Provide a shear stress check on $W12 \times 14$,

$$\tau_{max} = \frac{V_{max}}{t_w\,d}$$

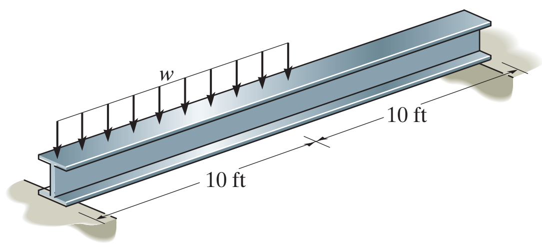
$$= \frac{7.284}{0.200(11.91)}$$

$$= 3.06\text{ ksi} < \tau_{allow} = 12\text{ ksi }(O.K)$$



(a)

•**11–21.** Determine if the W14 × 22 beam will safely support a loading of $w = 1.5$ kip/ft. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.



For $W14 \times 22$, $S_x = 29.0 \text{ in}^3$, $d = 13.74$ in and $t_w = 0.23$ in. From the moment diagram, Fig. a, $M_{max} = 42.1875 \text{ kip} \cdot \text{ft}$.

$$\sigma_{max} = \frac{M_{max}}{S}$$

$$= \frac{42.1875(12)}{29.0}$$

$$= 17.46 \text{ ksi} < \sigma_{allow} = 22 \text{ ksi } (O.K!)$$

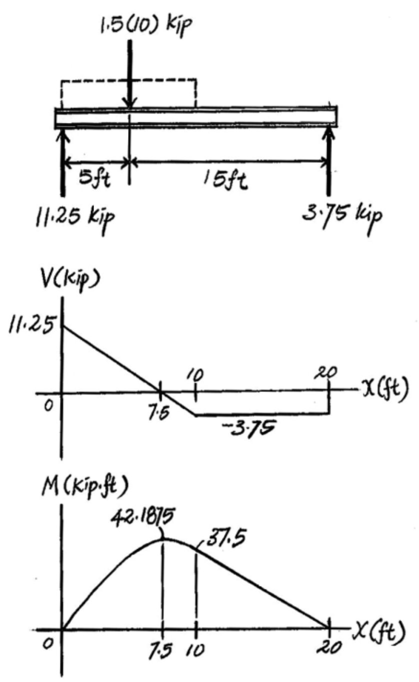
From the shear diagram, Fig. a, $V_{max} = 11.25$ kip.

$$\tau_{max} = \frac{V_{max}}{t_w\, d}$$

$$= \frac{11.25}{0.23(13.74)}$$

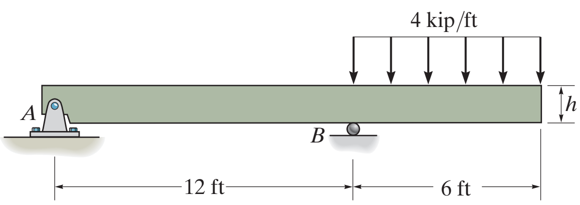
$$= 3.56 \text{ ksi} < \tau_{allow} = 12 \text{ ksi (O.K!)}$$

Based on the investigated results, we conclude that $W14 \times 22$ can safely support the loading.



(a)

11–22. Determine the minimum depth h of the beam to the nearest $\frac{1}{8}$ in. that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 21$ ksi and the allowable shear stress is $\tau_{allow} = 10$ ksi. The beam has a uniform thickness of 3 in.



The section modulus of the rectangular cross-section is

$$S = \frac{I}{C} = \frac{\frac{1}{12}(3)(h^3)}{h/2} = 0.5\,h^2$$

From the moment diagram, $M_{max} = 72$ kip · ft.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$0.5h^2 = \frac{72(12)}{21}$$

$$h = 9.07 \text{ in}$$

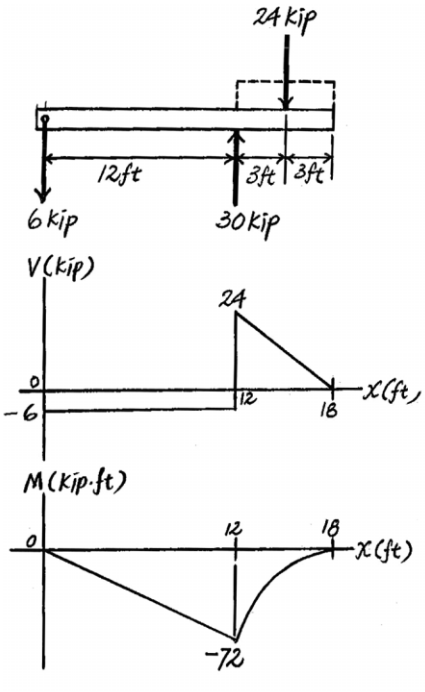
Use $h = 9\frac{1}{8}$ in **Ans.**

From the shear diagram, Fig. a, $V_{max} = 24$ kip. Referring to Fig. b, $Q_{max} = \bar{y}'A' = \left(\frac{9.125}{4}\right)\left(\frac{9.125}{2}\right)(3) = 31.22 \text{ in}^3$ and $I = \frac{1}{12}(3)\left(9.125^3\right) = 189.95 \text{ in}^4$. Provide the shear stress check by applying shear formula,

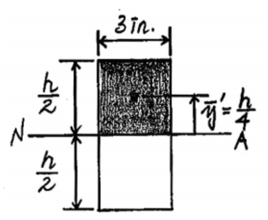
$$\tau_{max} = \frac{V_{max}\,Q_{max}}{It}$$

$$= \frac{24(31.22)}{189.95(3)}$$

$$= 1.315 \text{ ksi} < \tau_{allow} = 10 \text{ ksi } (O.K!)$$

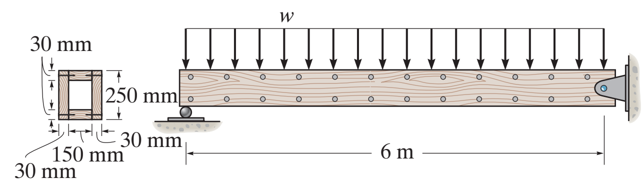


(a)



(b)

11–23. The box beam has an allowable bending stress of $\sigma_{allow} = 10$ MPa and an allowable shear stress of $\tau_{allow} = 775$ kPa. Determine the maximum intensity w of the distributed loading that it can safely support. Also, determine the maximum safe nail spacing for each third of the length of the beam. Each nail can resist a shear force of 200 N.



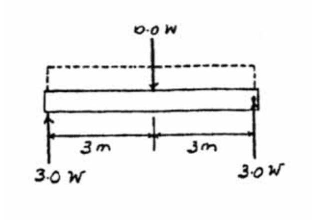
Section Properties:

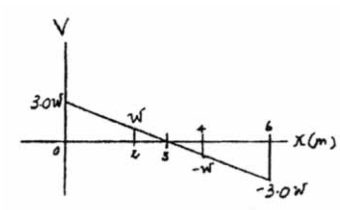
$$I = \frac{1}{12}(0.21)\left(0.25^3\right) - \frac{1}{12}(0.15)\left(0.19^3\right) = 0.1877\left(10^{-3}\right)\text{ m}^4$$

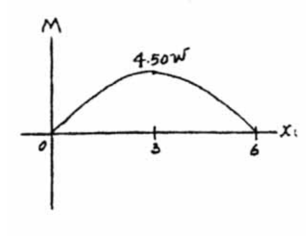
$$Q_A = \overline{y}_1{}'A' = 0.11(0.03)(0.15) = 0.495\left(10^{-3}\right)\text{ m}^3$$

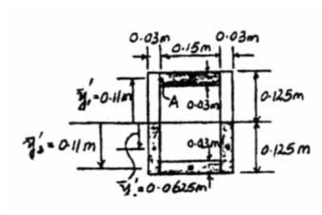
$$Q_{max} = \Sigma\overline{y}'A' = 0.11(0.03)(0.15) + 0.0625(0.125)(0.06)$$

$$= 0.96375\left(10^{-3}\right)\text{ m}^3$$









Bending Stress: From the moment diagram, $M_{max} = 4.50w$. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$10\left(10^6\right) = \frac{4.50w\,(0.125)}{0.1877\,(10^{-3})}$$

$$w = 3336.9\text{ N/m}$$

Shear Stress: Provide a shear stress check using the shear formula. From the shear diagram, $V_{max} = 3.00w = 10.01$ kN.

$$\tau_{max} = \frac{V_{max}\,Q_{max}}{It}$$

$$= \frac{10.01(10^3)\left[0.96375(10^{-3})\right]}{0.1877(10^{-3})(0.06)}$$

$$= 857\text{ kPa} > \tau_{allow} = 775\text{ kPa (\textit{No Good!})}$$

Hence, shear stress controls.

$$\tau_{allow} = \frac{V_{max}\,Q_{max}}{It}$$

$$775\left(10^3\right) = \frac{3.00w\left[0.96375(10^{-3})\right]}{0.1877(10^{-3})(0.06)}$$

$$w = 3018.8\text{ N/m} = 3.02\text{ kN/m} \qquad \textbf{Ans.}$$

Shear Flow: Since there are two rows of nails, the allowable shear flow is $q = \frac{2(200)}{s} = \frac{400}{s}$.

11–23. Continued

For $0 \leq x < 2$ m and $4 \text{ m} < x \leq 6$ m, the design shear force is $V = 3.00w = 9056.3$ N.

$$q = \frac{VQ_A}{I}$$

$$\frac{400}{s} = \frac{9056.3\left[0.495(10^{-3})\right]}{0.1877(10^{-3})}$$

$$s = 0.01675 \text{ m} = 16.7 \text{ mm}$$ **Ans.**

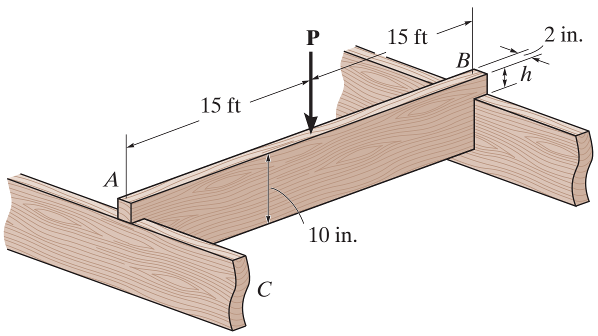
For $2 \text{ m} < x < 4$ m, the design shear force is $V = w = 3018.8$ N.

$$q = \frac{VQ_A}{I}$$

$$\frac{400}{s} = \frac{3018.8\left[0.495(10^{-3})\right]}{0.1877(10^{-3})}$$

$$s = 0.05024 \text{ m} = 50.2 \text{ mm}$$ **Ans.**

***11–24.** The simply supported joist is used in the construction of a floor for a building. In order to keep the floor low with respect to the sill beams C and D, the ends of the joists are notched as shown. If the allowable shear stress for the wood is $\tau_{allow} = 350$ psi and the allowable bending stress is $\sigma_{allow} = 1500$ psi, determine the height h that will cause the beam to reach both allowable stresses at the same time. Also, what load P causes this to happen? Neglect the stress concentration at the notch.



Bending Stress: From the moment diagram, $M_{max} = 7.50P$. Applying the flexure formula.

$$\sigma_{alllow} = \frac{M_{max}\,c}{I}$$

$$1500 = \frac{7.50P(12)(5)}{\frac{1}{12}(2)(10^3)}$$

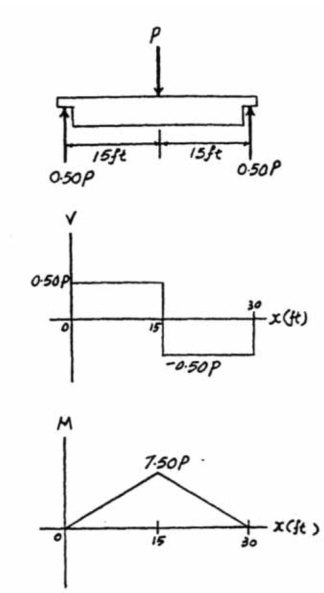
$$P = 555.56 \text{ lb} = 556 \text{ lb} \qquad \textbf{Ans.}$$

Shear Stress: From the shear diagram, $V_{max} = 0.500P = 277.78$ lb. The notch is the critical section. Using the shear formula for a rectangular section.

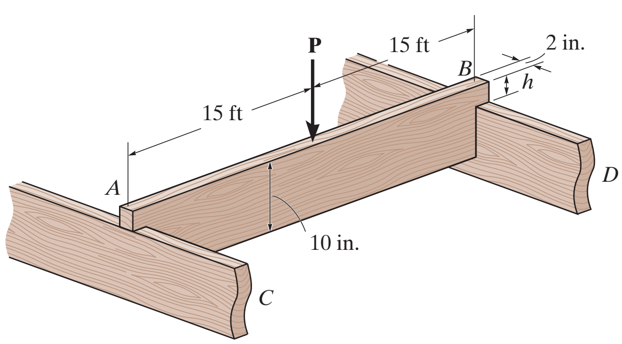
$$\tau_{allow} = \frac{3V_{max}}{2A}$$

$$350 = \frac{3(277.78)}{2(2)\,h}$$

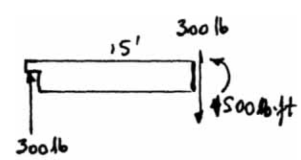
$$h = 0.595 \text{ in.} \qquad \textbf{Ans.}$$



11–25. The simply supported joist is used in the construction of a floor for a building. In order to keep the floor low with respect to the sill beams C and D, the ends of the joists are notched as shown. If the allowable shear stress for the wood is $\tau_{allow} = 350$ psi and the allowable bending stress is $\sigma_{allow} = 1700$ psi, determine the smallest height h so that the beam will support a load of $P = 600$ lb. Also, will the entire joist safely support the load? Neglect the stress concentration at the notch.



The reaction at the support is $\frac{600}{2} = 300$ lb



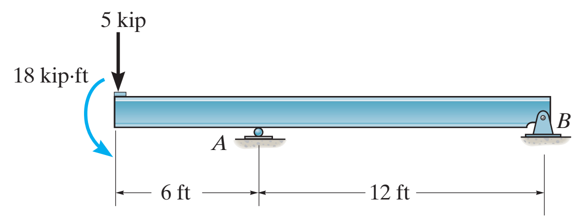
$$\tau_{allow} = \frac{1.5V}{A}; \qquad 350 = \frac{1.5(300)}{(2)(h)}$$

$$h = 0.643 \text{ in.} \qquad \textbf{Ans.}$$

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{4500(12)(5)}{\frac{1}{12}(2)(10)^3} = 1620 \text{ psi} < 1700 \text{ psi OK}$$

Yes, the joist will safely support the load. **Ans.**

11–26. Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the loading shown. The allowable bending stress is $\sigma_{\text{allow}} = 22$ ksi and the allowable shear stress is $\tau_{\text{allow}} = 12$ ksi.



From the moment diagram, Fig. *a*, $M_{\max} = 48$ kip · ft.

$$S_{\text{req'd}} = \frac{M_{\max}}{\sigma_{\text{allow}}}$$

$$= \frac{48(12)}{22}$$

$$= 26.18 \text{ in}^3$$

Select $W\ 14 \times 22\ \left[S_x = 29.0 \text{ in}^3, d = 13.74 \text{ in. and } t_w = 0.230 \text{ in.}\right]$

From the shear diagram, Fig. *a*, $V_{\max} = 5$ kip. Provide the shear stress check for $W\ 14 \times 22$,

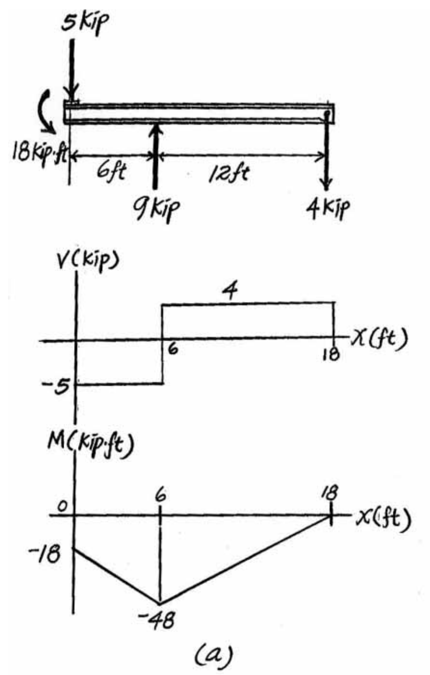
$$\tau_{\max} = \frac{V_{\max}}{t_w d}$$

$$= \frac{5}{0.230(13.74)}$$

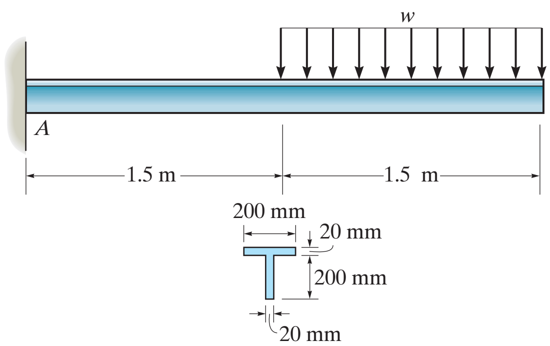
$$= 1.58 \text{ ksi} < \tau_{\text{allow}} = 12 \text{ ksi}, (O.K!)$$

Use W14 × 22 **Ans.**

W12 × 22 would work also.



11–27. The T-beam is made from two plates welded together as shown. Determine the maximum uniform distributed load w that can be safely supported on the beam if the allowable bending stress is $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 70$ MPa.



The neutral axis passes through centroid c of the beam's cross-section. The location of c, Fig. b, is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.21(0.02)(0.2) + 0.1(0.2)(0.02)}{0.02(0.2) + 0.2(0.02)}$$

$$= 0.155 \text{ m}$$

$$I = \frac{1}{12}(0.2)(0.02^3) + 0.2(0.02)(0.055)^2$$

$$+ \frac{1}{12}(0.02)(0.2^3) + 0.02(0.2)(0.055)^2$$

$$= 37.667\,(10^{-6}) \text{ m}^4$$

Referring to Fig. b,

$$Q_{max} = \bar{y}' A' = 0.0775(0.155)(0.02)$$

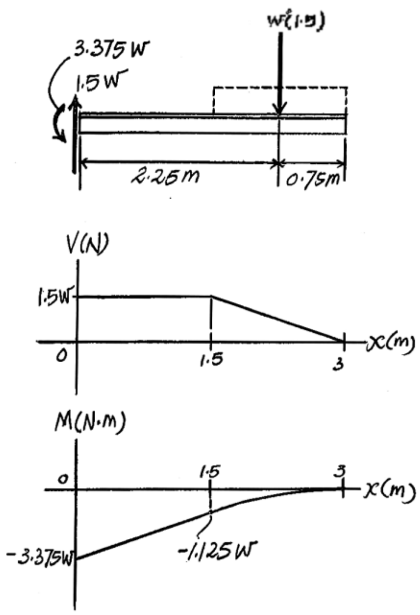
$$= 0.24025(10^{-3}) \text{ m}^3$$

Referring to the moment diagram, $M_{max} = -3.375\, w$. Applying the flexure formula with $C = \bar{y} = 0.155$ m,

$$\sigma_{allow} = \frac{M_{max}\, c}{I}; \qquad 150(10^6) = \frac{3.375\, w\,(0.155)}{37.667(10^{-6})}$$

$$w = 10.80(10^3) \text{ N/m}$$

$$= 10.8 \text{ kN/m } (\textit{Control}!)$$ **Ans.**



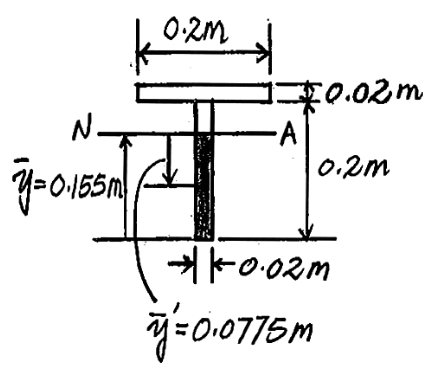
(a)

Referring to the shear diagram, $V_{max} = 1.5w$.

$$\tau_{allow} = \frac{V_{max} Q_{max}}{It}; \qquad 70(10^6) = \frac{1.5\, w\left[0.24025(10^{-3})\right]}{37.667(10^{-6})(0.02)}$$

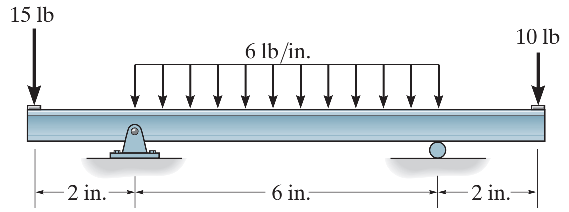
$$w = 146.33(10^3) \text{ N/m}$$

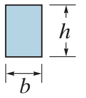
$$= 146 \text{ kN/m}$$



(b)

***11–28.** The beam is made of a ceramic material having an allowable bending stress of $\sigma_{allow} = 735$ psi and an allowable shear stress of $\tau_{allow} = 400$ psi. Determine the width b of the beam if the height $h = 2b$.





Bending Stress: From the moment diagram, $M_{max} = 30.0\ \text{lb} \cdot \text{in}$. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$735 = \frac{30.0\left(\frac{2b}{2}\right)}{\frac{1}{12}(b)(2b)^3}$$

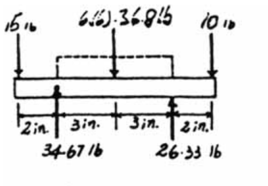
$$b = 0.3941\text{ in.} = 0.394\text{ in.} \qquad \textbf{Ans.}$$

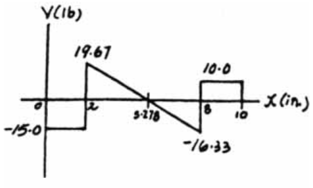
Shear Stress: Provide a shear stress check using the shear formula for a rectangular section. From the shear diagram, $V_{max} = 19.67$ lb.

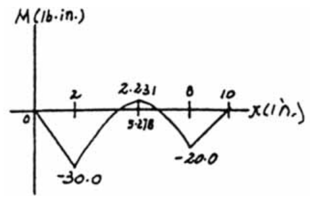
$$\tau_{max} = \frac{3V_{max}}{2A}$$

$$= \frac{3(19.67)}{2(0.3941)(2)(0.3941)}$$

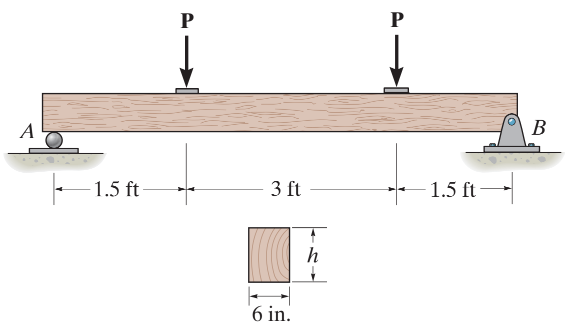
$$= 94.95\text{ psi} < \tau_{allow} = 400\text{ psi } (O.K!)$$







•11–29. The wood beam has a rectangular cross section. Determine its height h so that it simultaneously reaches its allowable bending stress of $\sigma_{allow} = 1.50$ ksi and an allowable shear stress of $\tau_{allow} = 150$ psi. Also, what is the maximum load P that the beam can then support?



The section modulus of the rectangular cross-section about the neutral axis is

$$S = \frac{I}{C} = \frac{\frac{1}{12}(6)\,h^3}{h/2} = h^2$$

From the moment diagram, Fig. a, $M_{max} = 1.5P$.

$$M_{max} = \sigma_{allow}\, S$$

$$1.5P(12) = 1.50(10^3)\,h^2$$

$$P = 83.33h^2 \qquad \textbf{(1)}$$

Referring to Fig. b, $Q_{max} = \bar{y}'A' = \frac{h}{4}\left(\frac{h}{2}\right)(6) = 0.75\,h^2$ and $I = \frac{1}{12}(6)\,h^3 = 0.5h^3$.
From the shear diagram, Fig. a, $V_{max} = P$.

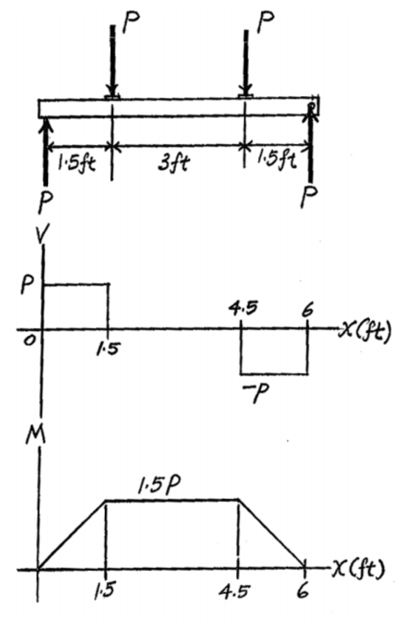
$$\tau_{max} = \frac{V_{max}\,Q_{max}}{It}$$

$$150 = \frac{P\,(0.75\,h^2)}{0.5\,h^3\,(6)}$$

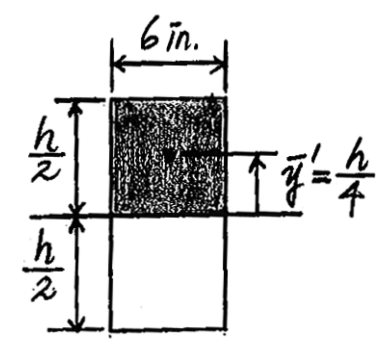
$$P = 600\,h \qquad \textbf{(2)}$$

Solving Eq (1) and (2)

$h = 7.20$ in $\qquad P = 4320$ lb $= 4.32$ kip **Ans.**

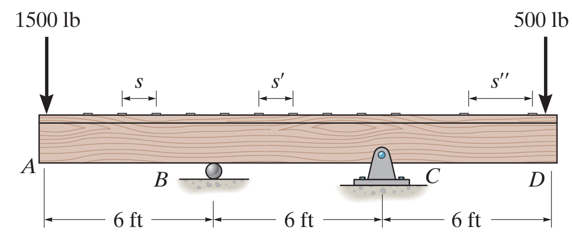


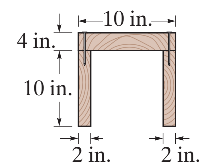
(a)



(b)

11–30. The beam is constructed from three boards as shown. If each nail can support a shear force of 300 lb, determine the maximum allowable spacing of the nails, s, s', s'', for regions AB, BC, and CD respectively. Also, if the allowable bending stress is $\sigma_{allow} = 1.5$ ksi and the allowable shear stress is $\tau_{allow} = 150$ psi, determine if it can safely support the load.





The neutral axis passes through centroid c of the beam's cross-section. The location of c, Fig. b, is

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{12(4)(10) + 2\left[5(10)(2)\right]}{4(10) + 2(10)(2)}$$

$$= 8.50 \text{ in}$$

The moment of inertia of the beam's cross-section about the neutral axis is

$$I = 2\left[\frac{1}{12}(2)(10^3) + 2(10)(3.50)^2\right]$$

$$+ \frac{1}{12}(10)(4^3) + 10(4)(3.50)^2$$

$$= 1366.67 \text{ in}^4$$

Referring to Fig. b,

$$Q_{max} = 2\bar{y}_2' A_2' = 2\left[4.25(8.50)(2)\right] = 144.5 \text{ in}^3$$

$$Q_A = \bar{y}_1' A_1' = 3.50(4)(10) = 140 \text{ in}^3$$

Referring to the moment diagram, Fig. a, $M_{max} = 9000$ lb·ft. Applying flexure formula with $C = \bar{y} = 8.50$ in,

$$\sigma_{max} = \frac{M_{max}\, c}{I}$$

$$= \frac{9000(12)(8.50)}{1366.67}$$

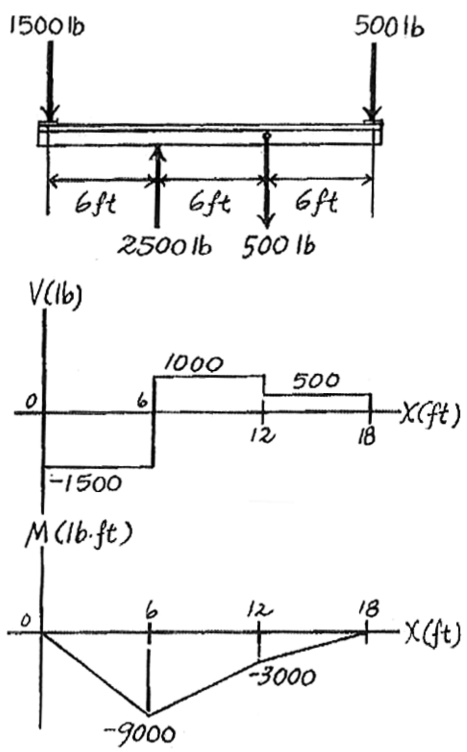
$$= 671.70 \text{ psi} < \sigma_{allow} = 1.50 \text{ ksi } (O.K!)$$

Referring to shear diagram, Fig. a, $V_{max} = 1500$ lb.

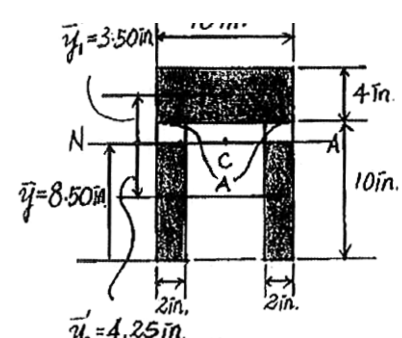
$$\tau_{max} = \frac{V_{max}\, Q_{max}}{It}$$

$$= \frac{1500\,(144.5)}{1366.67\,(4)} = 39.65 \text{ psi} < \tau_{allow} = 150 \text{ psi } (O.K!)$$

$S'' = 11\frac{1}{2}$ in. Yes, it can support the load. **Ans.**



(a)



(b)

11–30. Continued

Since there are two rows of nails, the allowable shear flow is $q_{\text{allow}} = \frac{2F}{S} = \frac{2(300)}{S} = \frac{600}{S}$. For region AB, $V = 1500$ lb. Thus

$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{600}{S} = \frac{1500\,(140)}{1366.67} \qquad S = 3.904\text{ in}$$

Use $S = 3\frac{3}{4}$ in **Ans.**

For region BC, $V = 1000$ lb. Thus

$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{600}{S'} = \frac{1000(140)}{1366.67} \qquad S' = 5.85\text{ in}$$

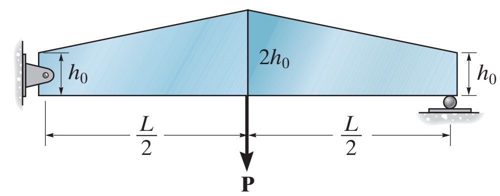
Use $S' = 5\frac{3}{4}$ in **Ans.**

For region CD, $V = 500$ lb. Thus

$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{600}{S''} = \frac{500\,(140)}{1366.67} \qquad S'' = 11.71\text{ in}$$

Use $S'' = 11\frac{1}{2}$ in **Ans.**

11–31. The tapered beam supports a concentrated force **P** at its center. If it is made from a plate that has a constant width b, determine the absolute maximum bending stress in the beam.

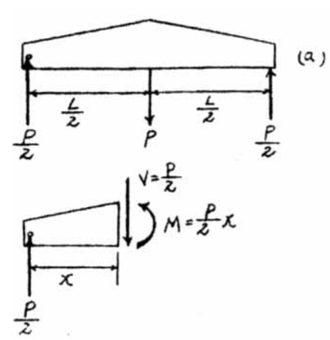


Section Properties:

$$\frac{h - h_0}{x} = \frac{h_0}{\frac{L}{2}} \qquad h = \frac{h_0}{L}(2x + L)$$

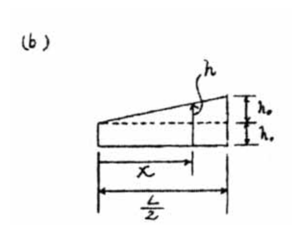
$$I = \frac{1}{12}(b)\left(\frac{h_0^3}{L^3}\right)(2x + L)^3$$

$$S = \frac{\frac{1}{12}(b)\left(\frac{h_0^3}{L^3}\right)(2x + L)^3}{\frac{h_3}{2L}(2x + L)} = \frac{bh_0^2}{6L^2}(2x + L)^2$$



Bending Stress: Applying the flexure formula.

$$\sigma = \frac{M}{S} = \frac{\frac{Px}{2}}{\frac{bh_0^2}{6L^2}(2x + L)^2} = \frac{3PL^2x}{bh_0^2(2x + L)^2} \qquad \textbf{[1]}$$



In order to have the absolute maximum bending stress, $\frac{d\sigma}{dx} = 0.$

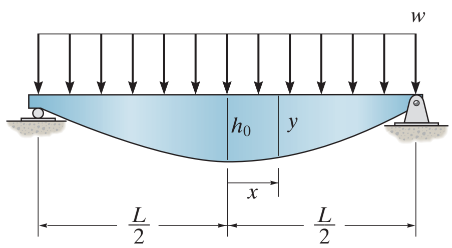
$$\frac{d\sigma}{dx} = \frac{3PL^2}{bh_0^2}\left[\frac{(2x + L)^2(1) - x(2)(2x + L)(2)}{(2x + L)^4}\right] = 0$$

$$x = \frac{L}{2}$$

Substituting $x = \frac{L}{2}$ into Eq. [1] yields

$$\sigma_{\max} = \frac{3PL}{8bh_0^2} \qquad \textbf{Ans.}$$

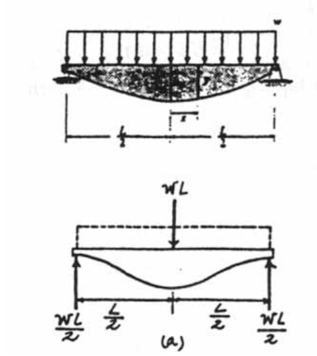
***11–32.** The beam is made from a plate that has a constant thickness b. If it is simply supported and carries a uniform load w, determine the variation of its depth as a function of x so that it maintains a constant maximum bending stress σ_{allow} throughout its length.



Moment Function: As shown on FBD(b).

Section Properties:

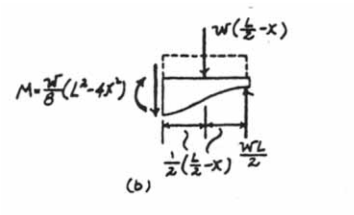
$$I = \frac{1}{12}by^3 \qquad S = \frac{I}{c} = \frac{\frac{1}{12}by^3}{\frac{y}{2}} = \frac{1}{6}by^2$$



Bending Stress: Applying the flexure formula.

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{\frac{w}{8}(L^2 - 4x^2)}{\frac{1}{6}by^2}$$

$$\sigma_{\text{allow}} = \frac{3w(L^2 - 4x^2)}{4by^2} \qquad \textbf{[1]}$$



At $x = 0$, $y = h_0$. From Eq. [1],

$$\sigma_{\text{allow}} = \frac{3wL^2}{4bh_0^2} \qquad \textbf{[2]}$$

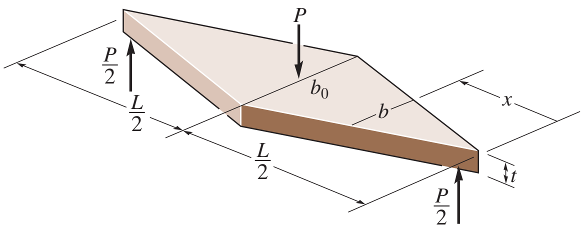
Equating Eq. [1] and [2] yields

$$y^2 = \frac{h_0^2}{L^2}\left(L^2 - 4x^2\right)$$

$$\frac{y^2}{h_0^2} + \frac{4x^2}{L^2} = 1 \qquad \textbf{Ans.}$$

The beam has a **semi-elliptical** shape.

•**11–33.** The beam is made from a plate having a constant thickness t and a width that varies as shown. If it supports a concentrated force **P** at its center, determine the absolute maximum bending stress in the beam and specify its location x, $0 < x < L/2$.



Section properties:

$$\frac{b}{b_0} = \frac{x}{\frac{L}{2}}; \qquad b = \frac{2b_0}{L}x$$

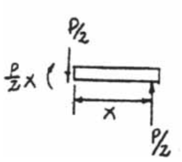
$$I = \frac{1}{12}\left(\frac{2b_0}{L}x\right)t^3 = \frac{b_0 t^3}{6L}x$$

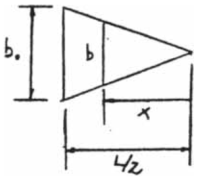
$$S = \frac{I}{c} = \frac{\frac{b_0 t}{6L}x}{\frac{t}{2}} = \frac{b_0 t^2}{3L}x$$

Bending stress:

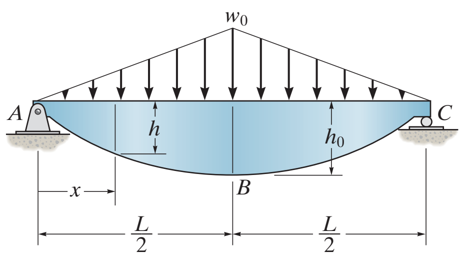
$$\sigma = \frac{M}{S} = \frac{\frac{P}{2}x}{\frac{b_0 t^2}{3L}x} = \frac{3PL}{2b_0 t^2}$$ **Ans.**

The bending stress is independent of x. Therefore, the stress is constant throughout the span. **Ans.**





11–34. The beam is made from a plate that has a constant thickness b. If it is simply supported and carries the distributed loading shown, determine the variation of its depth as a function of x so that it maintains a constant maximum bending stress σ_{allow} throughout its length.



Support Reactions: As shown on the free-body diagram of the entire beam, Fig. *a*.

Moment Function: The distributed load as a function of x is

$$\frac{w}{x} = \frac{w_0}{L/2} \qquad w = \frac{2w_0}{L}x$$

The free-body diagram of the beam's left cut segment is shown in Fig. *b*. Considering the moment equilibrium of this free-body diagram,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M + \frac{1}{2}\left[\frac{2w_0}{L}x\right]x\left(\frac{x}{3}\right) - \frac{1}{4}w_0 L x = 0$$

$$M = \frac{w_0}{12L}\left(3L^2x - 4x^3\right)$$

Section Properties: At position x, the height of the beam's cross section is h. Thus

$$I = \frac{1}{12}bh^3$$

Then

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^2$$

Bending Stress: The maximum bending stress σ_{max} as a function of x can be obtained by applying the flexure formula.

$$\sigma_{max} = \frac{M}{S} = \frac{\frac{w_0}{12L}\left(3L^2x - 4x^3\right)}{\frac{1}{6}bh^2} = \frac{w_0}{2bh^2L}\left(3L^2x - 4x^3\right), \qquad \textbf{(1)}$$

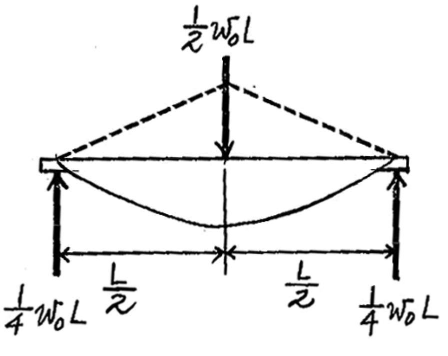
At $x = \frac{L}{2}$, $h = h_0$. From Eq. (1),

$$\sigma_{max} = \frac{w_0L^2}{2bh_0^{\ 2}} \qquad \textbf{(2)}$$

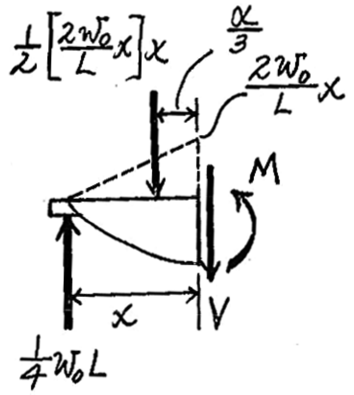
Equating Eqs. (1) and (2),

$$\frac{w_0}{2bh^2L}\left(3L^2x - 4x^3\right) = \frac{w_0L^2}{2bh_0^{\ 2}}$$

$$h = \frac{h_0}{L^{3/2}}\left(3L^2x - 4x^3\right)^{1/2} \qquad \textbf{Ans.}$$

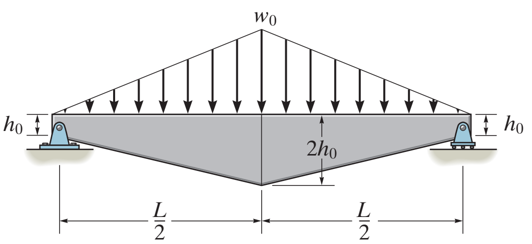


(a)



(b)

11–35. The beam is made from a plate that has a constant thickness b. If it is simply supported and carries the distributed loading shown, determine the maximum bending stress in the beam.



Support Reactions: As shown on the free - body diagram of the entire beam, Fig. a.

Moment Function: The distributed load as a function of x is

$$\frac{w}{x} = \frac{w_0}{L/2}; \qquad w = \frac{2w_0}{L}x$$

The free - body diagram of the beam's left cut segment is shown in Fig. b. Considering the moment equilibrium of this free - body diagram,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M + \frac{1}{2}\left(\frac{2w_0}{L}x\right)x\left(\frac{x}{3}\right) - \frac{w_0 L}{4}x = 0$$

$$M = \frac{w_0}{12L}\left(3L^2x - 4x^3\right)$$

Section Properties: Referring to the geometry shown in Fig. c,

$$\frac{h - h_0}{x} = \frac{h_0}{L/2}; \qquad h = \frac{h_0}{L}(2x + L)$$

At position x, the height of the beam's cross section is h. Thus

$$I = \frac{1}{12}bh^3$$

Then

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^2 = \frac{bh_0^{\,2}}{6L^2}(2x + L)^2$$

Bending Stress: Applying the flexure formula,

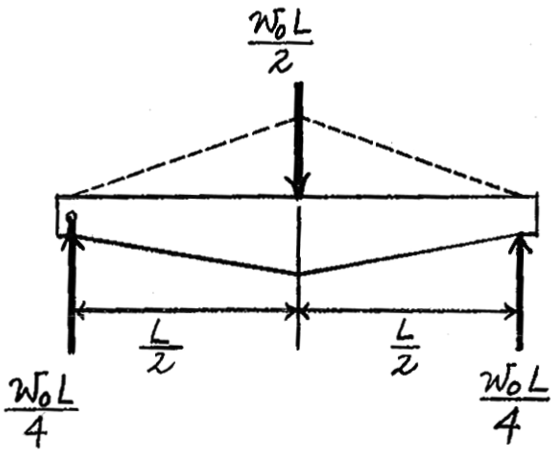
$$\sigma_{\max} = \frac{M}{S} = \frac{\frac{w_0}{12L}\left(3L^2x - 4x^3\right)}{\frac{bh_0^{\,2}}{6L^2}(2x + L)^2}$$

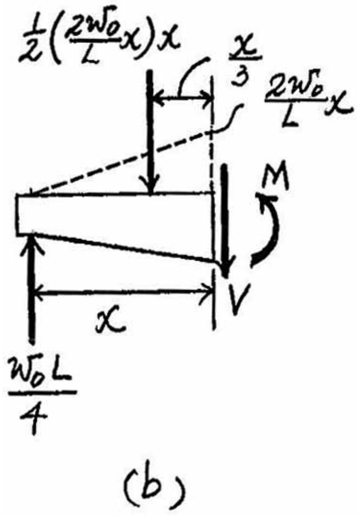
$$\sigma_{\max} = \frac{w_0 L}{2bh_0^{\,2}}\left[\frac{3L^2x - 4x^3}{(2x + L)^2}\right] \qquad \textbf{(1)}$$

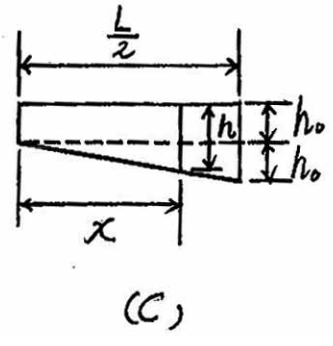
In order to have absolute maximum bending stress, $\frac{d\sigma_{\max}}{dx} = 0$.

$$\frac{d\sigma_{\max}}{dx} = \frac{w_0 L}{2bh_0^{\,2}}\left[\frac{(2x + L)^2\left(3L^2 - 12x^2\right) - \left(3L^2x - 4x^3\right)(2)(2x + L)(2)}{(2x + L)^4}\right] = 0$$

$$\frac{w_0 L}{2bh_0^{\,2}}\left[\frac{3L^3 - 8x^3 - 6L^2x - 12Lx^2}{(2x + L)^3}\right] = 0$$







11–35. Continued

Since $\frac{w_0 L}{2bh_0{}^2} \neq 0$, then

$$3L^3 - 8x^3 - 6L^2x - 12Lx^2 = 0$$

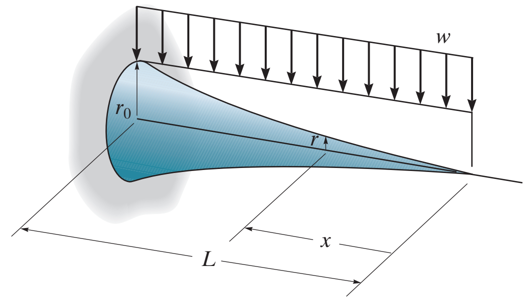
Solving by trial and error,

$$x = 0.2937L = 0.294L$$

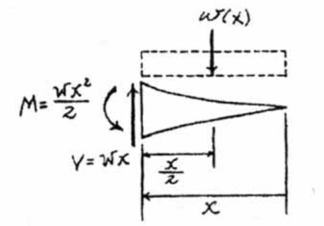
Substituting this result into Eq. (1),

$$\sigma_{\substack{\text{abs}\\\text{max}}} = \frac{0.155 w_0 L^2}{bh_0{}^2} \qquad \textbf{Ans.}$$

***11–36.** Determine the variation of the radius r of the cantilevered beam that supports the uniform distributed load so that it has a constant maximum bending stress $\sigma_{\max}$ throughout its length.



Moment Function: As shown on FBD.



Section Properties:

$$I = \frac{\pi}{4} r^4 \qquad S = \frac{I}{c} = \frac{\frac{\pi}{4} r^4}{r} = \frac{\pi}{4} r^3$$

Bending Stress: Applying the flexure formula.

$$\sigma_{\max} = \frac{M}{S} = \frac{\frac{wx^2}{2}}{\frac{\pi}{4} r^3}$$

$$\sigma_{\max} = \frac{2wx^2}{\pi r^3} \qquad \textbf{[1]}$$

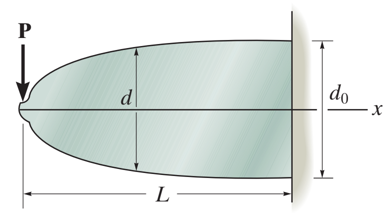
At $x = L, r = r_0$. From Eq. [1],

$$\sigma_{\max} = \frac{2wL^2}{\pi r_0^3} \qquad \textbf{[2]}$$

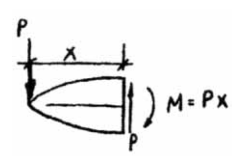
Equating Eq. [1] and [2] yields

$$r^3 = \frac{r_0^3}{L^2} x^2 \qquad \textbf{Ans.}$$

•11–37. Determine the variation in the depth d of a cantilevered beam that supports a concentrated force **P** at its end so that it has a constant maximum bending stress σ_{allow} throughout its length. The beam has a constant width b_0.



Section properties:



$$I = \frac{1}{12}(b_0)(d^3) \qquad S = \frac{I}{c} = \frac{\frac{1}{12}(b_0)(d^3)}{d/2} = \frac{b_0 d^2}{6}$$

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{Px}{b_0 d^2/6} \qquad \textbf{(1)}$$

At $x = L$

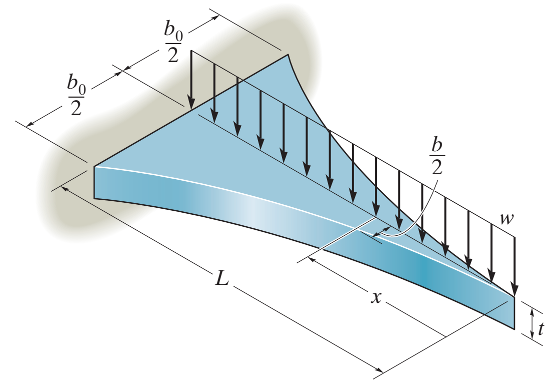
$$\sigma_{\text{allow}} = \frac{PL}{b_0 {d_0}^2/6} \qquad \textbf{(2)}$$

Equate Eqs. (1) and (2):

$$\frac{Px}{b_0 d^2/6} = \frac{PL}{b_0 {d_0}^2/6}$$

$$d^2 = \left(\frac{{d_0}^2}{L}\right)x; \qquad d = d_0\sqrt{\frac{x}{L}} \qquad \textbf{Ans.}$$

11–38. Determine the variation in the width b as a function of x for the cantilevered beam that supports a uniform distributed load along its centerline so that it has the same maximum bending stress σ_{allow} throughout its length. The beam has a constant depth t.



Section properties:

$$I = \frac{1}{12} b\, t^3 \qquad S = \frac{I}{c} = \frac{\frac{1}{12} b\, t^3}{\frac{t}{2}} = \frac{t^2}{6} b$$

Bending stress:

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{\frac{w\,x^2}{2}}{\frac{t^2}{6} b} = \frac{3wx^2}{t^2 b} \qquad \textbf{(1)}$$

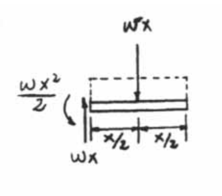
At $x = L, b = b_0$

$$\sigma_{\text{allow}} = \frac{3wL^2}{t^2 b_0} \qquad \textbf{(2)}$$

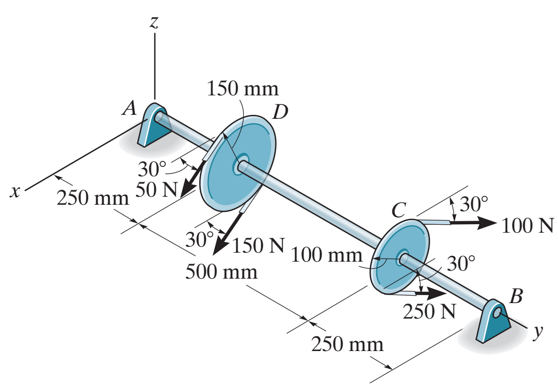
Equating Eqs. (1) and (2) yields:

$$\frac{3wx^2}{t^2 b} = \frac{3wL^2}{t^2 b_0}$$

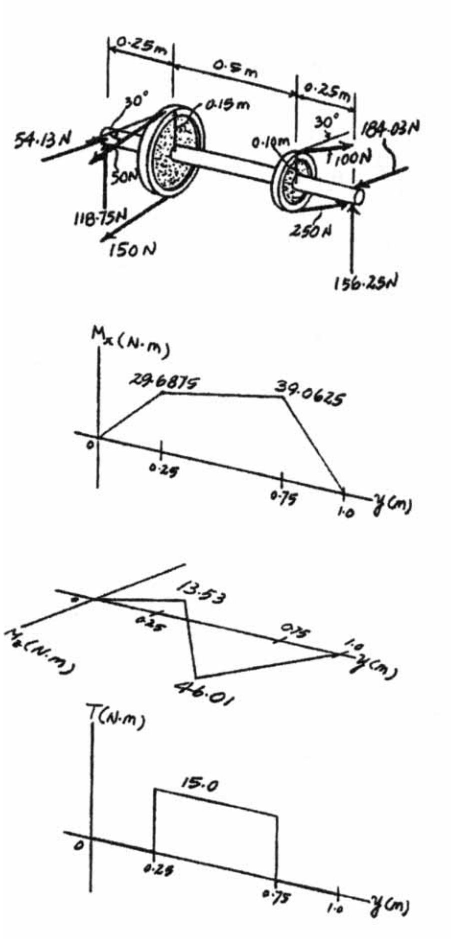
$$b = \frac{b_0}{L^2} x^2 \qquad \textbf{Ans.}$$



11–39. The shaft is supported on journal bearings that do not offer resistance to axial load. If the allowable normal stress for the shaft is $\sigma_{allow} = 80$ MPa, determine to the nearest millimeter the smallest diameter of the shaft that will support the loading. Use the maximum-distortion-energy theory of failure.



Torque and Moment Diagrams: As shown.



In-Plane Principal Stresses: Applying Eq. 9–5 with $\sigma_y = 0, \sigma_x = \frac{Mc}{I} = \frac{4M}{\pi c^3}$, and $\tau_{xy} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2M}{\pi c^3} \pm \sqrt{\left(\frac{2M}{\pi c^3}\right)^2 + \left(\frac{2T}{\pi c^3}\right)^2}$$

$$= \frac{2M}{\pi c^3} \pm \frac{2}{\pi c^3}\sqrt{M^2 + T^2}$$

Maximum Distortion Energy Theory: Let $a = \frac{2M}{\pi c^3}$ and $b = \frac{2}{\pi c^3}\sqrt{M^2 + T^2}$, then $\sigma_1^2 = a^2 + b^2 + 2ab, \sigma_1\sigma_2 = a^2 - b^2, \sigma_2^2 = a^2 + b^2 - 2ab$, and $\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3b^2 + a^2$.

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$

$$3\left(\frac{2}{\pi c^3}\sqrt{M^2 + T^2}\right)^2 + \left(\frac{2M}{\pi c^3}\right)^2 = \sigma_{allow}^2$$

$$c = \left[\frac{4}{\pi^2 \sigma_{allow}^2}\left(4M^2 + 3T^2\right)\right]^{\frac{1}{6}}$$

Shaft Design: By observation, the critical section is located just to the left of gear C, where $M = \sqrt{39.0625^2 + 46.01^2} = 60.354$ N·m and $T = 15.0$ N·m. Using the *maximum distortion energy theory*,

$$c = \left[\frac{4}{\pi^2 \sigma_{allow}^2}\left(4M^2 + 3T^2\right)\right]^{\frac{1}{6}}$$

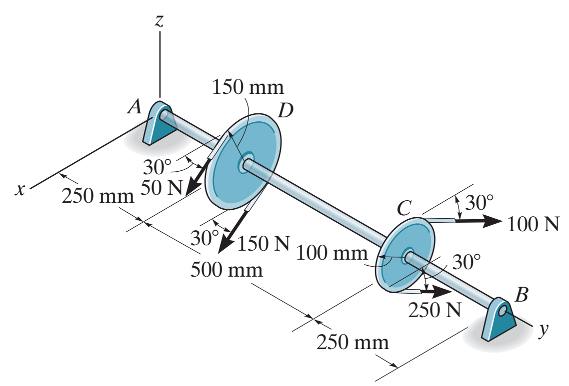
$$= \left\{\frac{4}{\pi^2\,[80(10^6)]^2}\left[4(60.354)^2 + 3(15.0)^2\right]\right\}^{\frac{1}{6}}$$

$$= 0.009942 \text{ m}$$

$$d = 2c = 2(0.009942) = 0.01988 \text{ m} = 19.88 \text{ mm}$$

Use $d = 20$ mm **Ans.**

***11–40.** The shaft is supported on journal bearings that do not offer resistance to axial load. If the allowable shear stress for the shaft is $\tau_{allow} = 35$ MPa, determine to the nearest millimeter the smallest diameter of the shaft that will support the loading. Use the maximum-shear-stress theory of failure.



Shaft Design: By observation, the critical section is located just to the left of gear C, where $M = \sqrt{39.0625^2 + 46.01^2} = 60.354\ \text{N}\cdot\text{m}$ and $T = 15.0\ \text{N}\cdot\text{m}$. Using the *maximum shear stress theory*.

$$c = \left(\frac{2}{\pi\tau_{allow}}\sqrt{M^2 + T^2}\right)^{\frac{1}{3}}$$

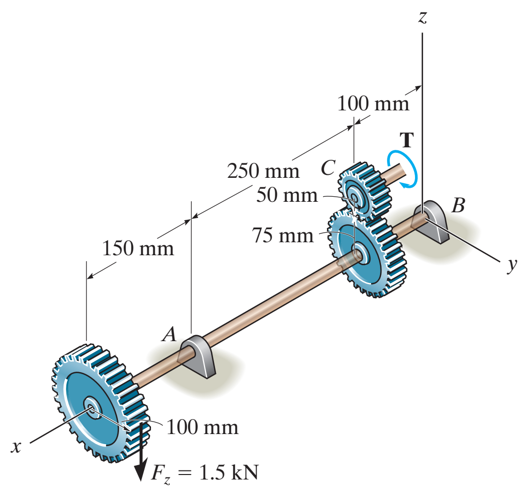
$$= \left[\frac{2}{\pi(35)(10^6)}\sqrt{60.354^2 + 15.0^2}\right]^{\frac{1}{3}}$$

$$= 0.01042\ \text{m}$$

$$d = 2c = 2(0.01042) = 0.02084\ \text{m} = 20.84\ \text{mm}$$

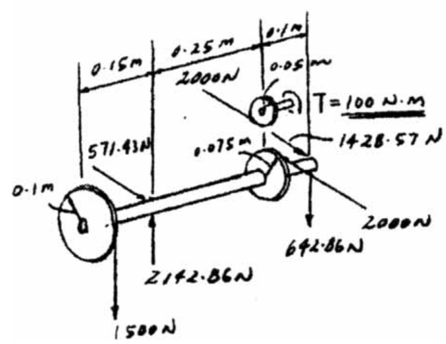
Use $d = 21$ mm **Ans.**

•11–41. The end gear connected to the shaft is subjected to the loading shown. If the bearings at A and B exert only y and z components of force on the shaft, determine the equilibrium torque T at gear C and then determine the smallest diameter of the shaft to the nearest millimeter that will support the loading. Use the maximum-shear-stress theory of failure with $\tau_{\text{allow}} = 60$ MPa.



From the free - body diagrams:

$T = 100\ \text{N} \cdot \text{m}$ **Ans.**



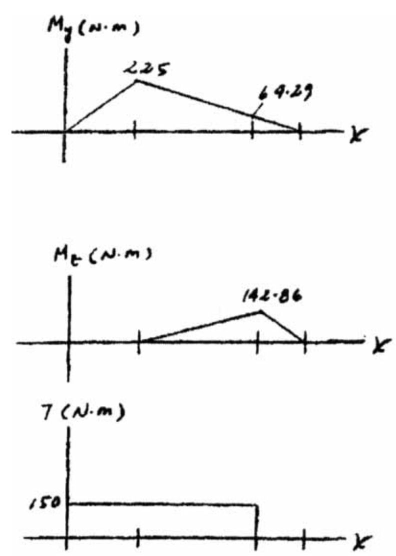
Critical section is at support A.

$$c = \left[\frac{2}{\pi\,\tau_{\text{allow}}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(60)(10^6)}\sqrt{225^2 + 150^2}\right]^{\frac{1}{3}}$$

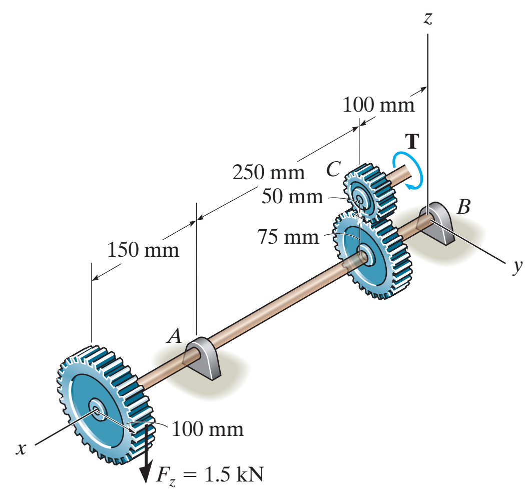
$$= 0.01421\ \text{m}$$

$d = 2c = 0.0284\ \text{m} = 28.4\ \text{mm}$

Use $d = 29$ mm **Ans.**

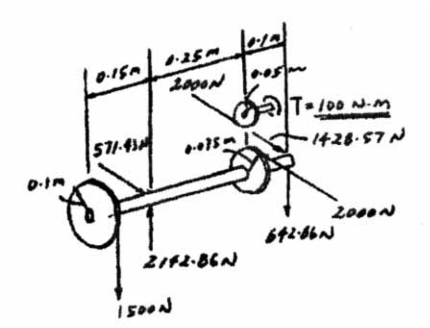


11–42. The end gear connected to the shaft is subjected to the loading shown. If the bearings at A and B exert only y and z components of force on the shaft, determine the equilibrium torque T at gear C and then determine the smallest diameter of the shaft to the nearest millimeter that will support the loading. Use the maximum-distortion-energy theory of failure with $\sigma_{\text{allow}} = 80$ MPa.



From the free-body diagrams:

$T = 100\ \text{N}\cdot\text{m}$ **Ans.**



Critical section is at support A.

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \dfrac{\sigma_x}{2}, b = \sqrt{\dfrac{\sigma_x^2}{4} + \tau_{xy}^2}$

$$\sigma_1 = a + b, \sigma_2 = a - b$$

Require,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2 a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{\text{allow}}^2$$

$$a^2 + 3b^2 = \sigma_{\text{allow}}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{\text{allow}}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{Mt}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{\text{allow}}^2$$

$$\frac{1}{c^4}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{\text{allow}}^2$$

$$c^4 = \frac{16}{\sigma_{\text{allow}}^2\pi^2}M^2 + \frac{12T^2}{\sigma_{\text{allow}}^2\pi^2}$$

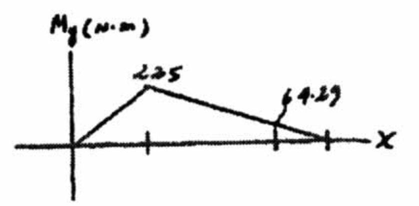
$$c = \left(\frac{4}{\sigma_{\text{allow}}^2\pi^2}(4M^2 + 3T^2)\right)^{\frac{1}{2}}$$

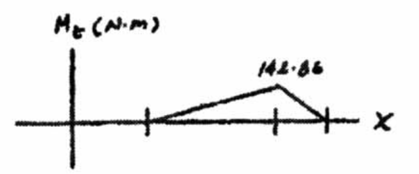
$$= \left[\frac{4}{(80(10^6))^2(\pi)^2}(4(225)^2 + 3(150)^2)\right]^{\frac{1}{2}}$$

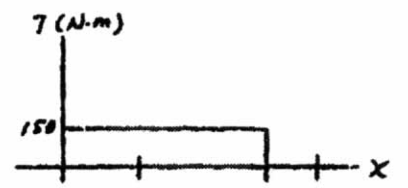
$$= 0.01605 \text{ m}$$

$d = 2c = 0.0321 \text{ m} = 32.1 \text{ mm}$

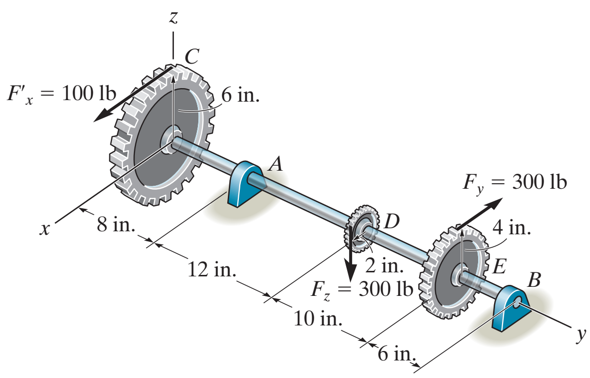
Use $d = 33$ mm **Ans.**







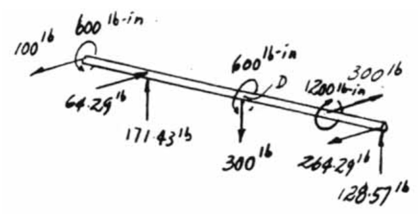
11–43. The shaft is supported by bearings at A and B that exert force components only in the x and z directions on the shaft. If the allowable normal stress for the shaft is $\sigma_{\text{allow}} = 15$ ksi, determine to the nearest $\frac{1}{8}$ in. the smallest diameter of the shaft that will support the loading. Use the maximum-distortion-energy theory of failure.



Critical moment is just to the right of D.

$$M = \sqrt{2057^2 + 1229^2} = 2396 \text{ lb} \cdot \text{in.}$$

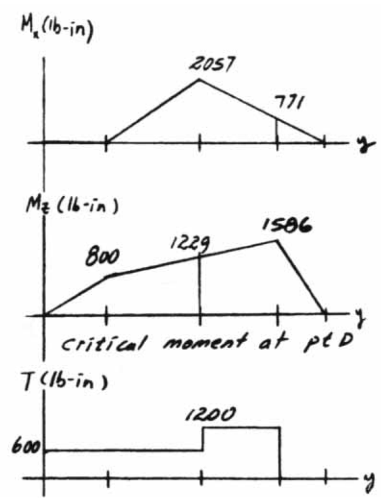
$$T = 1200 \text{ lb} \cdot \text{in.}$$



Both states of stress will yield the same result.

$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Let $\dfrac{\sigma}{2} = A$ and $\sqrt{\dfrac{\sigma^2}{4} + \tau^2} = B$



$$\sigma_a^2 = (A + B)^2, \sigma_b^2 = (A - B)^2$$

$$\sigma_a \sigma_b = (A + B)(A - B) = A^2 - B^2$$

$$\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2 = A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB$$

$$= A^2 + 3B^2 = \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right) = \sigma^2 + 3\tau^2$$

$$\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2 = \sigma_{\text{allow}}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{\text{allow}}^2, \qquad \textbf{(1)}$$

$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

From Eq. (1)

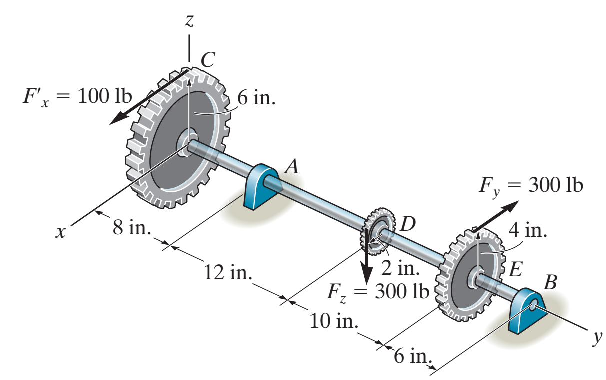
$$\frac{16M^2}{\pi^2 c^6} + \frac{12T^2}{\pi^2 c^6} = \sigma_{\text{allow}}^2$$

$$c = \left(\frac{16M^2 + 12T^2}{\pi^2 \sigma_{\text{allow}}^2}\right)^{1/6} = \left[\frac{16(2396)^2 + 12(1200^2)}{\pi^2((15)(10^3))^2}\right]^{1/6} = 0.605 \text{ in.}$$

$$d = 2c = 1.210 \text{ in.}$$

Use $d = 1\dfrac{1}{4}$ in. **Ans.**

***11–44.** The shaft is supported by bearings at A and B that exert force components only in the x and z directions on the shaft. If the allowable normal stress for the shaft is $\sigma_{allow} = 15$ ksi, determine to the nearest $\frac{1}{8}$ in. the smallest diameter of the shaft that will support the loading. Use the maximum-shear-stress theory of failure. Take $\tau_{allow} = 6$ ksi.



Critical moment is just to the right of D.

$$M = \sqrt{(2057)^2 + (1229)^2} = 2396 \text{ lb} \cdot \text{in.}$$

$$T = 1200 \text{ lb} \cdot \text{in.}$$

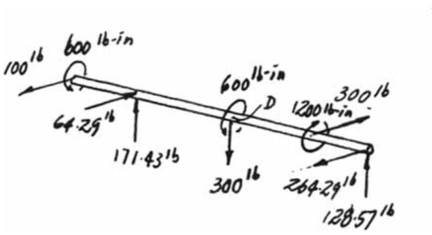
Use Eq. 11-2,

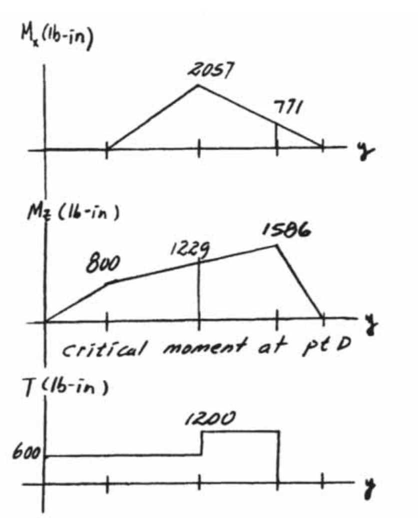
$$c = \left(\frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}\right)^{1/3}$$

$$c = \left(\frac{2}{\pi(6)(10^3)}\sqrt{(2396)^2 + (1200)^2}\right)^{1/3} = 0.6576 \text{ in.}$$

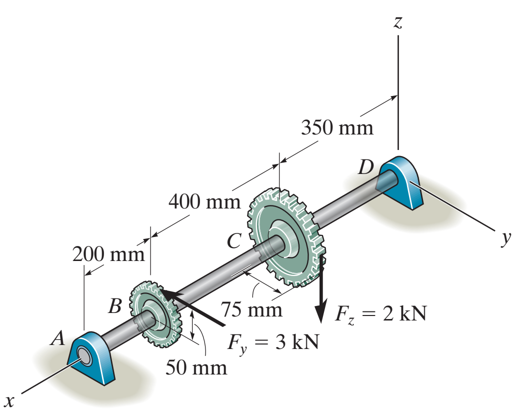
$$d_{req'd} = 2c = 1.315 \text{ in.}$$

Use $d = 1\frac{3}{8}$ in. **Ans.**





•11–45. The bearings at A and D exert only y and z components of force on the shaft. If $\tau_{allow} = 60$ MPa, determine to the nearest millimeter the smallest-diameter shaft that will support the loading. Use the maximum-shear-stress theory of failure.



Critical moment is at point B:

$$M = \sqrt{(473.7)^2 + (147.4)^2} = 496.1\ \text{N}\cdot\text{m}$$

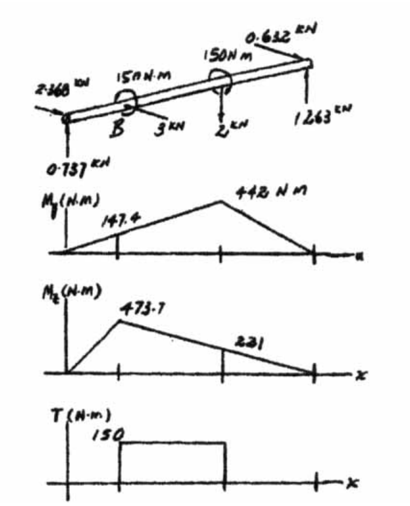
$$T = 150\ \text{N}\cdot\text{m}$$

$$c = \left(\frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}\right)^{1/3} = \left(\frac{2}{\pi(60)(10^6)}\sqrt{496.1^2 + 150^2}\right)^{1/3} = 0.0176\ \text{m}$$

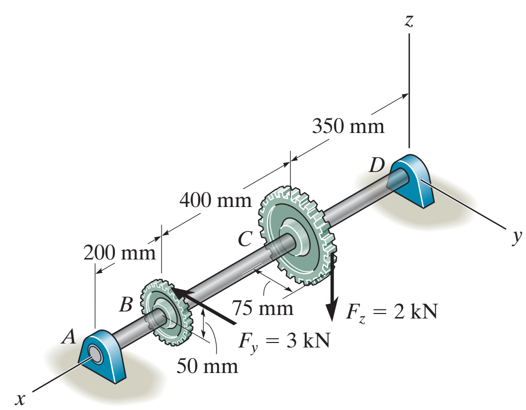
$$c = 0.0176\ \text{m} = 17.6\ \text{mm}$$

$$d = 2c = 35.3\ \text{mm}$$

Use $d = 36$ mm **Ans.**



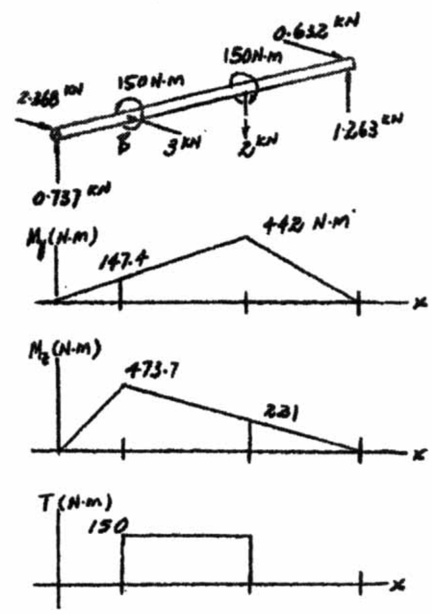
11–46. The bearings at A and D exert only y and z components of force on the shaft. If $\tau_{allow} = 60$ MPa, determine to the nearest millimeter the smallest-diameter shaft that will support the loading. Use the maximum-distortion-energy theory of failure. $\sigma_{allow} = 130$ MPa.



The critical moment is at B.

$$M = \sqrt{(473.7)^2 + (147.4)^2} = 496.1 \text{ N}\cdot\text{m}$$

$$T = 150 \text{ N}\cdot\text{m}$$



Since,

$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Let $\frac{\sigma}{2} = A$ and $\sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = B$

$$\sigma_a^2 = (A + B)^2 \qquad \sigma_b^2 = (A - B)^2$$

$$\sigma_a \sigma_b = (A + B)(A - B)$$

$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB$$

$$= A^2 + 3B^2$$

$$= \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right)$$

$$= \sigma^2 + 3\tau^2$$

$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = \sigma_{allow}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{allow}^2 \qquad \textbf{(1)}$$

$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

From Eq (1)

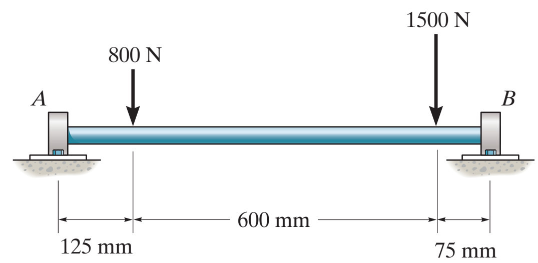
$$\frac{16M^2}{\pi^2 c^4} + \frac{12T^2}{\pi^2 c^4} = \sigma_{allow}^2$$

$$c = \left(\frac{16M^2 + 12T^2}{\pi^2 \sigma_{allow}^2}\right)^{1/6}$$

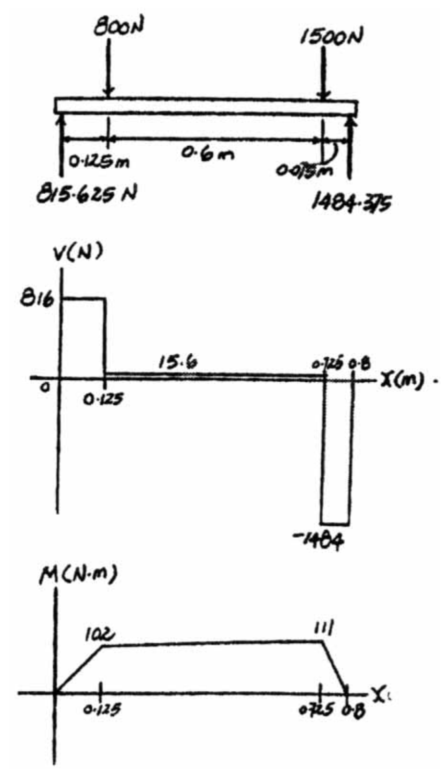
$$= \left(\frac{16(496.1)^2 + 12(150)^2}{\pi^2((130)(10^4))^2}\right)^{1/4} = 0.01712 \text{ m}$$

$$d = 2c = 34.3 \text{ mm} \qquad \textbf{Ans.}$$

11–47. Draw the shear and moment diagrams for the shaft, and then determine its required diameter to the nearest millimeter if $\sigma_{allow} = 140$ MPa and $\tau_{allow} = 80$ MPa. The bearings at A and B exert only vertical reactions on the shaft.



Bending Stress: From the moment diagram, $M_{max} = 111\ \text{N}\cdot\text{m}$. Assume bending controls the design. Applying the flexure formula.



$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$140\left(10^6\right) = \frac{111\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02008 \text{ m} = 20.1 \text{ mm}$$

Use $\qquad d = 21$ mm $\qquad$ **Ans.**

Shear Stress: Provide a shear stress check using the shear formula with

$$I = \frac{\pi}{4}\left(0.0105^4\right) = 9.5466\left(10^{-9}\right)\text{ m}^4$$

$$Q_{max} = \frac{4(0.0105)}{3\pi}\left[\frac{1}{2}(\pi)(0.0105)^2\right] = 0.77175\left(10^{-6}\right)\text{ m}^3$$

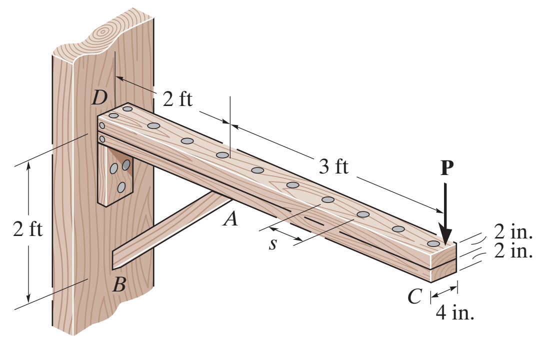
From the shear diagram, $V_{max} = 1484$ N.

$$\tau_{max} = \frac{V_{max}\, Q_{max}}{It}$$

$$= \frac{1484\left[0.77175(10^{-6})\right]}{9.5466(10^{-9})(0.021)}$$

$$= 5.71 \text{ MPa} < \tau_{allow} = 80 \text{ MPa } (O.K!)$$

***11–48.** The overhang beam is constructed using two 2-in. by 4-in. pieces of wood braced as shown. If the allowable bending stress is $\sigma_{allow} = 600$ psi, determine the largest load P that can be applied. Also, determine the associated maximum spacing of nails, s, along the beam section AC if each nail can resist a shear force of 800 lb. Assume the beam is pin-connected at A, B, and D. Neglect the axial force developed in the beam along DA.



$M_A = M_{max} = 3P$

Section properties:

$$I = \frac{1}{12}(4)(4)^3 = 21.33 \text{ in}^4$$

$$S = \frac{I}{c} = \frac{21.33}{2} = 10.67 \text{ in}^3$$

$$M_{max} = \sigma_{allow}\, S$$

$$3P(12) = 600(10.67)$$

$$P = 177.78 = 178 \text{ lb}$$ **Ans.**

Nail Spacing:

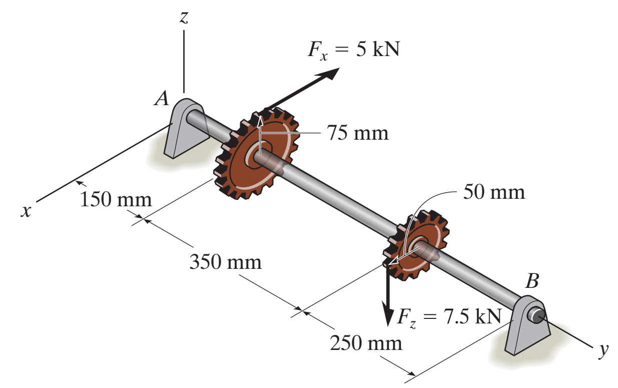
$$V = P = 177.78 \text{ lb}$$

$$Q = (4)(2)(1) = 8 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{177.78(8)}{21.33} = 66.67 \text{ lb/in.}$$

$$S = \frac{800 \text{ lb}}{66.67 \text{ lb/in.}} = 12.0 \text{ in.}$$ **Ans.**

•11–49. The bearings at A and B exert only x and z components of force on the steel shaft. Determine the shaft's diameter to the nearest millimeter so that it can resist the loadings of the gears without exceeding an allowable shear stress of $\tau_{allow} = 80$ MPa. Use the maximum-shear-stress theory of failure.

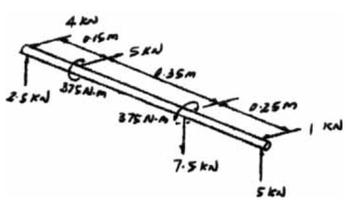


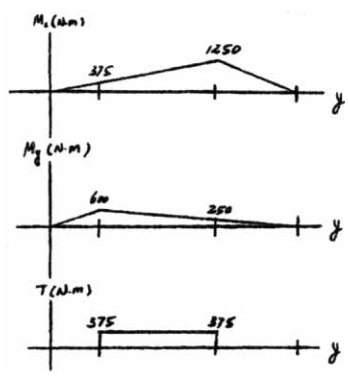
Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75\ \text{N}\cdot\text{m}$

$$c = \left[\frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(80)(10^6)}\sqrt{1274.75^2 + 375^2}\right]^{\frac{1}{3}} = 0.0219\ \text{m}$$

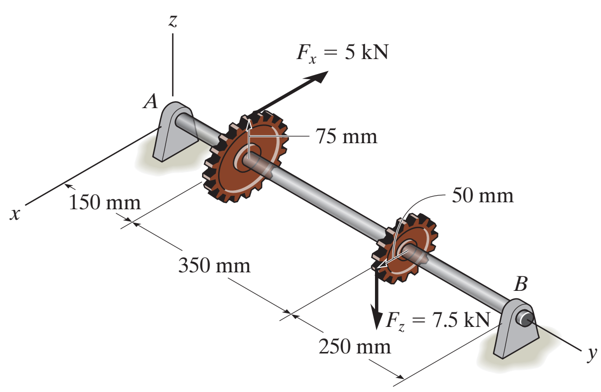
$d = 2c = 0.0439$ m $= 43.9$ mm

Use $d = 44$ mm **Ans.**



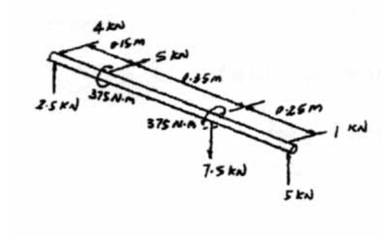


11–50. The bearings at A and B exert only x and z components of force on the steel shaft. Determine the shaft's diameter to the nearest millimeter so that it can resist the loadings of the gears without exceeding an allowable shear stress of $\tau_{\text{allow}} = 80$ MPa. Use the maximum-distortion-energy theory of failure with $\sigma_{\text{allow}} = 200$ MPa.



Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75\ \text{N}\cdot\text{m}$

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$



Let $a = \dfrac{\sigma_x}{2}, b = \sqrt{\dfrac{\sigma_x^2}{4} + \tau_{xy}^2}$

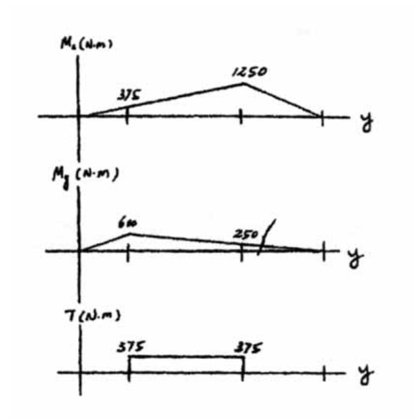
$\sigma_1 = a + b, \qquad \sigma_2 = a - b$

Require,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{\text{allow}}^2$$

$$a^2 + 3b^2 = \sigma_{\text{allow}}^2$$



$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{\text{allow}}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{\text{allow}}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{\text{allow}}^2$$

$$c^6 = \frac{16}{\sigma_{\text{allow}}^2\pi^2}M^2 + \frac{12T^2}{\sigma_{\text{allow}}^2\pi^2}$$

$$c = \left[\frac{4}{\sigma_{\text{allow}}^2\pi^2}(4M^2 + 3T^2)\right]^{\frac{1}{4}}$$

$$= \left[\frac{4}{(200(10^6))^2(\pi)^2}(4(1274.75)^2 + 3(375)^2)\right]^{\frac{1}{4}}$$

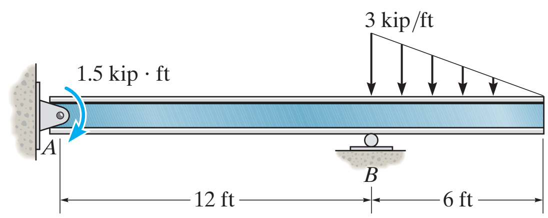
$= 0.0203$ m $= 20.3$ mm

$d = 40.6$ mm **Ans.**

Use

$d = 41$ mm **Ans.**

11–51. Draw the shear and moment diagrams for the beam. Then select the lightest-weight steel wide-flange beam from Appendix B that will safely support the loading. Take $\sigma_{allow} = 22$ ksi, and $\tau_{allow} = 12$ ksi.



Bending Stress: From the moment diagram, $M_{max} = 18.0$ kip · ft. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{18.0(12)}{22} = 9.82 \text{ in}^3$$

Select W10 × 12 $\left(S_x = 10.9 \text{ in}^3, d = 9.87 \text{ in.}, t_w = 0.19 \text{ in.}\right)$

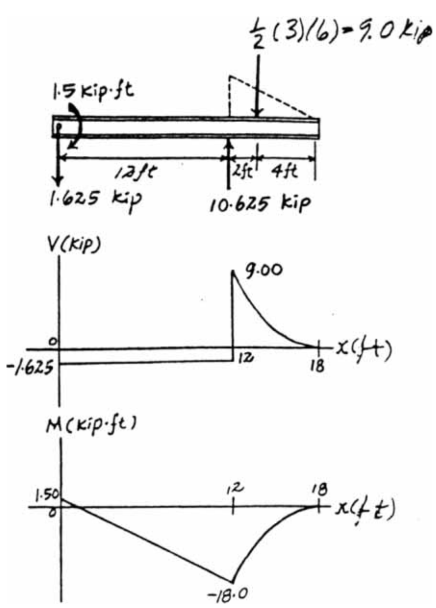
Shear Stress: Provide a shear stress check using $\tau = \dfrac{V}{t_w d}$ for the W10 × 12 wide - flange section. From the shear diagram, $V_{max} = 9.00$ kip

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

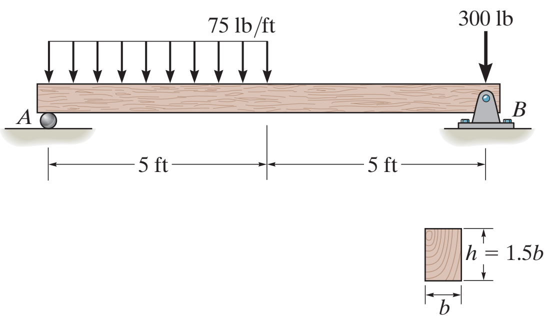
$$= \frac{9.00}{0.19(9.87)}$$

$$= 4.80 \text{ ksi} < \tau_{allow} = 12 \text{ ksi } (O.K!)$$

Hence, **Use** W10 × 12 **Ans.**



***11–52.** The beam is made of cypress having an allowable bending stress of $\sigma_{allow} = 850$ psi and an allowable shear stress of $\tau_{allow} = 80$ psi. Determine the width b of the beam if the height $h = 1.5b$.



$$I_x = \frac{1}{12}(b)(1.5b)^3 = 0.28125\, b^4$$

$$Q_{max} = \bar{y}'A' = (0.375b)\,(0.75b)(b) = 0.28125\, b^3$$

Assume bending controls.

$$M_{max} = 527.34 \text{ lb} \cdot \text{ft}$$

$$\sigma_{allow} = \frac{M_{max}\, c}{I}; \qquad 850 = \frac{527.34(12)(0.75\, b)}{0.28125\, b^4}$$

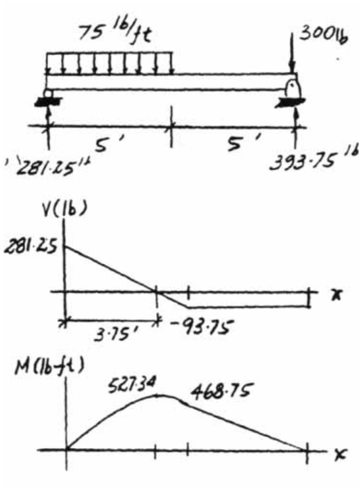
$b = 2.71$ in. **Ans.**

Check shear:

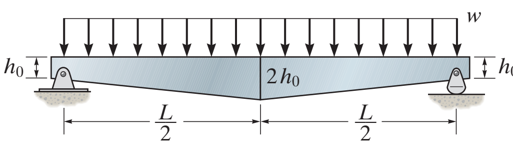
$I = 15.12 \text{ in}^4 \qquad Q_{max} = 5.584 \text{ in}^3$

$$\tau_{max} = \frac{VQ_{max}}{I\,t} = \frac{281.25(5.584)}{15.12(2.71)}$$

$$= 38.36 \text{ psi} < 80 \text{ psi}$$ OK

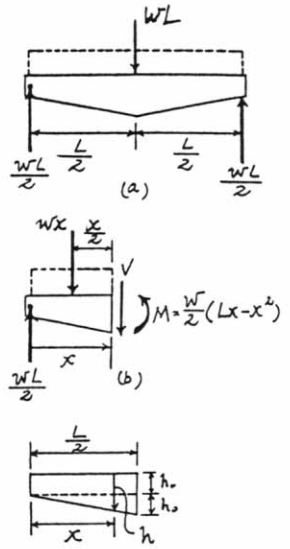


•**11–53.** The tapered beam supports a uniform distributed load w. If it is made from a plate and has a constant width b, determine the absolute maximum bending stress in the beam.



Support Reactions: As shown on FBD(a).

Moment Function: As shown on FBD(b).



Section Properties:

$$\frac{h - h_0}{x} = \frac{h_0}{\frac{L}{2}} \qquad h = \frac{h_0}{L}(2x + L)$$

$$I = \frac{1}{12}(b)\left(\frac{h_0^3}{L^3}\right)(2x + L)^3$$

$$S = \frac{\frac{1}{12}(b)\left(\frac{h_0^3}{L^3}\right)(2x + L)^3}{\frac{h_0}{2L}(2x + L)} = \frac{bh_0^2}{6L^2}(2x + L)^2$$

Bending Stress: Applying the flexure formula.

$$\sigma = \frac{M}{S} = \frac{\frac{w}{2}(Lx - x^2)}{\frac{bh_0^2}{6L^2}(2x + L)^2} = \frac{3wL^2(Lx - x^2)}{bh_0^2(2x + L)^2} \qquad [1]$$

In order to have the absolute maximum bending stress, $\frac{d\sigma}{dx} = 0$.

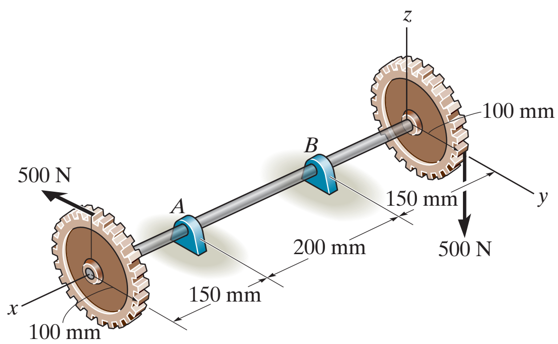
$$\frac{d\sigma}{dx} = \frac{3wL^2}{bh_0^2}\left[\frac{(2x + L)^2(L - 2x) - (Lx - x^2)(2)(2x + L)(2)}{(2x + L)^4}\right] = 0$$

$$x = \frac{L}{4}$$

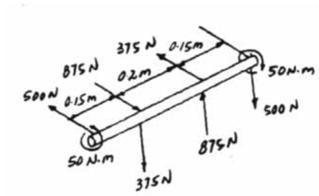
Substituting $x = \frac{L}{4}$ into Eq. [1] yields

$$\sigma_{\max} = \frac{wL^2}{4bh_0^2} \qquad \textbf{Ans.}$$

11–54. The tubular shaft has an inner diameter of 15 mm. Determine to the nearest millimeter its outer diameter if it is subjected to the gear loading. The bearings at A and B exert force components only in the y and z directions on the shaft. Use an allowable shear stress of $\tau_{\text{allow}} = 70$ MPa, and base the design on the maximum-shear-stress theory of failure.



$$I = \frac{\pi}{4}(c^4 - 0.0075^4) \text{ and } J = \frac{\pi}{2}(c^4 - 0.0075^4)$$



$$\tau_{\text{allow}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\text{allow}} = \sqrt{\left(\frac{Mc}{2I}\right)^2 + \left(\frac{Tc}{J}\right)^2}$$

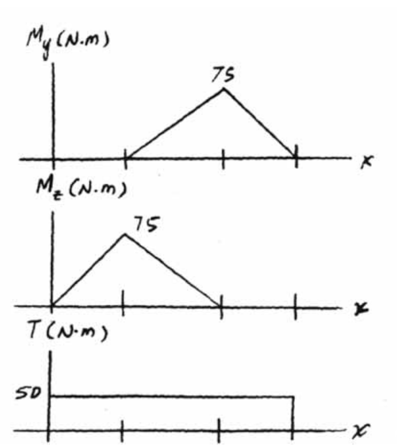
$$\tau_{\text{allow}}^2 = \frac{M^2 c^2}{4I^2} + \frac{T^2 c^2}{J^2}$$

$$\left(\frac{c^4 - 0.0075^4}{c}\right)^2 = \frac{4M^2}{\pi^2} + \frac{4T^2}{\pi^2}$$

$$\frac{c^4 - 0.0075^4}{c} = \frac{2}{\pi\,\tau_{\text{allow}}}\sqrt{M^2 + T^2}$$

$$\frac{c^4 - 0.0075^4}{c} = \frac{2}{\pi(70)(10^6)}\sqrt{75^2 + 50^2}$$

$$c^4 - 0.0075^4 = 0.8198(10^{-6})c$$

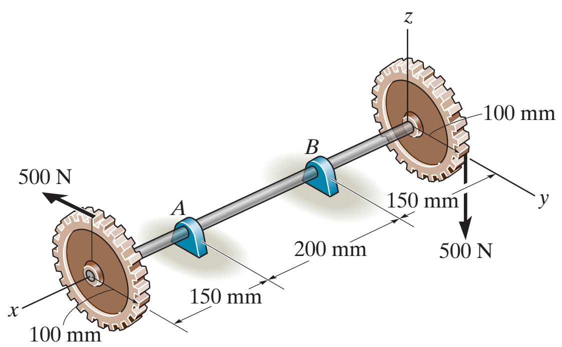


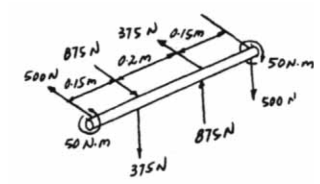
Solving, $c = 0.0103976$ m

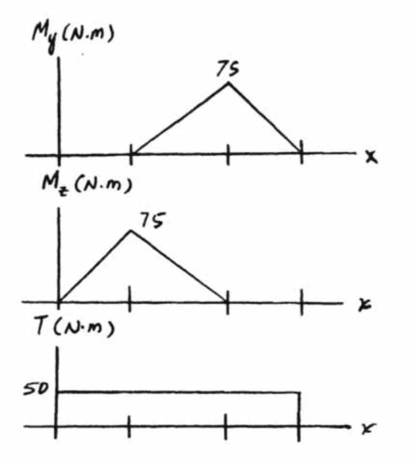
$d = 2c = 0.0207952 \text{ m} = 20.8 \text{ mm}$

Use $d = 21$ mm **Ans.**

11–55. Determine to the nearest millimeter the diameter of the solid shaft if it is subjected to the gear loading. The bearings at A and B exert force components only in the y and z directions on the shaft. Base the design on the maximum-distortion-energy theory of failure with $\sigma_{\text{allow}} = 150$ MPa.







$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \dfrac{\sigma_x}{2}, b = \sqrt{\dfrac{\sigma_x^2}{4} + \tau_{xy}^2}$

$\sigma_1 = a + b, \sigma_2 = a - b$

Require,

$$\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_1^2 = \sigma_{\text{allow}}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{\text{allow}}$$

$$a^2 + 3b^2 = \sigma_{\text{allow}}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{\text{allow}}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{\text{allow}}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{\text{allow}}^2$$

$$c^6 = \frac{16}{\sigma_{\text{allow}}^2 \pi^2} M^2 + \frac{12T^2}{\sigma_{\text{allow}}^2 \pi^2}$$

$$c = \left(\frac{4}{\sigma_{\text{allow}}^2 \pi^2}(4M^2 + 3T^2)\right)^{\frac{1}{4}}$$

$$= \left[\frac{4}{(150(10^6))^2(\pi)^2}(4(75)^2 + 3(50)^2)\right]^{\frac{1}{4}} = 0.009025 \text{ m}$$

$d = 2c = 0.0181$ m

Use $d = 19$ mm **Ans.**