

Delft University of Technology  
Faculty Aerospace Engineering

# Space Engineering & Technology I

## Space Missions & Systems

### AE1-801

First semester, first part

### In-class problems, example problems, test and exam problems

Issue 4.1

#### NOTE

This collection of problems **only** covers the part **Space Missions & Systems** of the course Space Engineering and Technology I.

Problems of the part **Orbital Dynamics** can be found on the Blackboard site of the course.

Delft, September 2005

Lecturer:

Ir. R.J. Hamann

System Integration

Design & Analysis of Space Systems

e-mail: R.J.Hamann@lr.tudelft.nl

# Space Engineering & Technology I

## Space Missions & Systems

*In-class problems, example problems, test and exam problems*

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### LEARNING GOALS AE1-801/SMS

To be able

- to place Space Flight in its general context
- to place major space development steps in time and to understand their mutual relation
- to distinguish space markets
- to summarise the major classes of space applications and their main characteristics
- to understand the characteristics of the space environment and their impact on spacecraft characteristics
- to identify the impact of major mission parameters on the space vehicle concept
- to understand the break-down of space mission and space vehicle and the main functions of these components

### READING MATERIAL

<i>Subject SET I, SMS</i>	<i>Spacecraft Systems Engineering [Fortescue]</i>	<i>Space Mission Analysis and Design [Larson]</i>
Introduction	--	--
Space and Space Flight	<b>1</b>	<b>8.1, 8.1.4</b>
Mission Break-down and Spacecraft Break-down	--	<b>1.2 (pp. 7-12), 10 pp. (301-304)</b>
Space Flight History	<b>1</b>	--
Launch Vehicles	<b>7.3, 7.3.1, 7.3.2, 7.4, 7.5, 7.6, 7.7</b>	--
The Moon Program	--	--
Space Stations	--	--
Space Markets and Missions	--	<b>1.2, 1.3, 9.1</b>
Spacecraft: Scientific Research	<b>8.8, 11.8, 18.1, 18.10.2, 18.10.3</b>	<b>7.4, 9, 10.2</b>
Spacecraft: Earth Observation	<b>18.10.4, 19.3.3, 19.3.4</b>	<b>7.4, 9.4</b>
Spacecraft: Communication	<b>12.1, 18.10.1, 19.3.2</b>	<b>13, 13.1, 13.1.1</b>
Spacecraft: Navigation	<b>18.10.5</b>	<b>11.7, 11.7.2</b>
The Space Mission	--	--
The Space Lecture Series	--	--

### EXAMINATION

The regular exam on the material of Space Engineering & Technology I (ae1-801) is scheduled after the second semester (June) together with the material treated in the part "Orbit Dynamics" of that course:

- The part Space Missions & Systems (Hamann) is weighted 25%, equivalent to 45 minutes of exam time,
- The part Orbit Dynamics (Ambrosius/Visser) is weighted 75%, equivalent to 2 hours and 15 minutes exam time.

A second exam opportunity is offered in August.

The part "Space Missions & Systems" also examined after the first period of the first

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semester (October/November): a light test of two hours duration. Bonus points are awarded for the regular exam in **June** only:

- Grade for the test 10: 20% bonus in June,
- Grade 8 and 9: 15 % bonus,
- Grade 6 and 7: 10% bonus,
- Grade 4 and 5: 5% bonus,
- Grade 3 and lower: no bonus.

The exam problems for “Space Missions & Systems” are based on in-class problems and example problems.

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## IN-CLASS AND EXAMPLE PROBLEMS

### *Space and Space Flight*

#### In-class problems

#### Heat Balance

Radiation heat balance of a flat surface with area A, front side looking at the sun and cold space, rear side perfectly isolated

Heat balance

$$\alpha AS = \varepsilon A \sigma (T^4 - T_{sp}^4)$$

where

$\alpha$  = solar absorptance (fraction of solar radiation absorbed; metal 0.1, black paint 0.9, white paint 0.4)

$\varepsilon$  = (infrared) emissivity (fraction of black body radiation emitted; metal 0.04, black or white paint 0.9, glass 0.9)

$\sigma$  = Stefan-Boltzmann constant ( $56.7051 \times 10^{-9} \text{ W}/(\text{m}^2\text{K}^4)$ )

$T$  = surface temperature in degree Kelvin

$T_{sp}$  = temperature of cold space

Resulting surface temperature:

metal 498 K

black paint 396 K

rear side metalized glass mirror (second surface mirror) 228K

*(glass may be replaced by plastic for a modest degradation in performance, but a large gain in cost)*

If you use a number of layers of highly reflecting material the absorptance and emittance can effectively be reduced by a factor of 50 to 100 >> thermal blankets.

Using the different material properties the inside of the spacecraft can be kept between -10 and +40 C, the normal operating range of electronic equipment

#### Robot arm motion in the absence of gravity

#### EXAM AUGUST 2005 (15 minutes)

A robot arm of 10 m length ( $l$ ) has to move a payload of 6000 kg mass. The actuator at the root of the arm exercises 300 Nm torque. How fast has it moved the payload over 180 degrees?

Force and acceleration at the end of the arm:

$$F = \frac{\text{Torque}}{l} = \frac{300}{10} = 30 \text{ N}$$

$$a = \frac{F}{\text{payload mass}} = \frac{30}{6000} = 0.005 \text{ m/s}^2$$

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A 90 degree arc with full acceleration/deceleration is covered in

$$t = \sqrt{2 s / a} = \sqrt{2 l (\pi/2) / a} =$$

$$\sqrt{(10 \pi / 0.005)} = 79.3 \text{ s}$$

180 degrees is covered in less than 160 seconds.

#### Example Problems

- 1-1 A Space Mission can be defined, designed and developed as an isolated item. Do you agree or disagree with this statement? Motivate your answer and illustrate it with an example.
- 1-2 List four or five typical characteristics of space, that can be exploited by space missions
- 1-3 Calculate the temperature of a flat surface with area A, with its front side looking to the sun and cold space and its rear side looking to cold space, both sides with the same optical properties. Do this for a metal surface, for a black painted surface and for a mirror surface.

### ***Mission Break-down and Spacecraft Break-down***

#### Example Problems

- 1-4 List the spacecraft subsystems and their functions.
- 1-5 List the mission segments according to [Larson]. Is this breakdown always used?

### ***Space Flight History***

#### Example Problems

- 2-1 Which were the driving forces for the rapid development in space technology
- in the period between 1940 and 1955?
  - in the period between 1955 and 1970?

### ***Launch Vehicles***

#### In-class problem

**Why do rockets have stages?**

***How fast can a rocket go?***

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Tsiolkowsky:  $V_e = c \ln \frac{M_0}{M_e}$  (for derivation see ae1-019, part Introduction Space

Technology)

where

$V_e$  final velocity in zero-gravity

$c$  nozzle exhaust velocity

$M_0$  Start mass of rocket

$M_e$  End mass of rocket

This is neglecting gravity and atmospheric drag; which will be addressed in detail in the course Space Engineering & Technology I, part Mission Analysis I)

Practical values

$c$  3000 m/s (will be addressed in lecture ae2-s02).

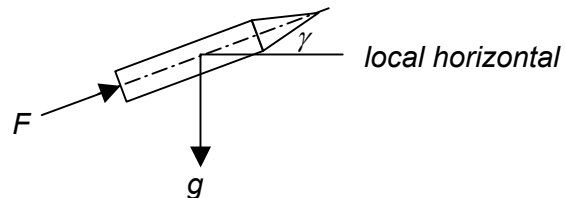
$\frac{M_0}{M_e}$  5 to 6, i.e. 80 to 85 % of rocket mass are propellants (will be addressed in lecture ae2-s02).

So final velocity is limited to:

$$V_e = 3000 \ln 6 = 5375 \text{ m/s}$$

When the rocket lifts off, it is in a vertical position and gravity is pulling it down; when it achieves its final (circular) orbit, the velocity vector is purely radial and the gravity force is compensated by the centrifugal force. It can be derived, that in the presence of a gravity field the equation of Tsiolkowsky is modified into:

$$V_e = c \ln \frac{M_0}{M_e} - \int_0^{t_b} g(t) \cdot \sin(\gamma(t)) \cdot dt$$



where  $g(t)$  is the acceleration of gravity at time  $t$ ,  $\gamma(t)$  the angle relative to the local horizontal and  $t_b$  is the burning time of the rocket engine.

The gravity term is equal to  $g_0$  (the acceleration of gravity at 0 m altitude) at lift off and equal to zero ( $\sin(\gamma) = 0$ ), when the circular orbit is achieved. In a fair approximation we may assume:

$$[g(t) \cdot \sin(\gamma(t))]_{total \text{ trajectory}} = 0.5 \cdot g_0$$

Assuming a burning time of 5 minutes, the velocity loss due to gravity is then:

$$\Delta V_g = -0.5 \cdot g_0 \cdot t_b = -0.5 \cdot 9.81 \cdot 5 \cdot 60 = 1472 \text{ m/s}$$

In the presence of gravity a single stage rocket can hence obtain a maximum velocity of typically 3900 m/s.

The last velocity loss is due to aerodynamic drag, which is in the order of 300 to 400 m/s, yielding a maximum single stage rocket velocity of 3600 m/s.

### **How fast does a satellite have to go?**

Mean earth radius: 6370 km

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Length of circular orbit at zero altitude:

$$L = 2\pi R = 40020 \text{ km}$$

Orbital period:

90 minutes (Source: lectures ae1-801, part Mission Analysis I)

Orbital velocity:

$$\frac{40020 * 1000}{90 * 60} = 7410 \text{ m/s}$$

This is considerably larger than the 3600 m/s calculated above.

### **Conclusion**

Single stage rocket does not deliver enough velocity to reach low earth orbit, even when gravity and atmospheric drag are neglected. Feasibility of future developments depends on

- achieving higher nozzle exhaust velocities,
- designing lighter launcher structures.

### **Example Problems**

- 2-2 Every launcher project starts developing a completely new concept. Do you agree or disagree with this statement? Motivate your answer and illustrate with an example.
- 2-3 Calculate the theoretical final velocity of the V2 with Tsiolkovski, assuming that the exhaust velocity of the rocket engine is 2250 m/s (ethanol + 25% water, liquid oxygen). Is this sufficient to bring a satellite in a low earth orbit with 90 minutes orbital period? Motivate your answer.
- 2-4 You intend to launch 50 satellites of 3000 kg each into GTO. Candidate launchers are Ariane 5 launcher at a cost of 125 M\$ each and an expected reliability of 0.9 and a Proton D1e at a cost of 60 M\$ and a reliability of 0.85. Which launcher do you select? Motivate your answer with numerical results.
- 2-5 Why does Arianespace serve 60% of the commercial launcher market and what upgrades are planned for Ariane 5 to maintain this position?

## ***The Moon Program***

### **Example Problems**

- 3-1 Describe the sequence of events for the first successful Apollo mission to the Moon and their purpose.

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### **Space Stations**

#### **Example Problems**

- 3-2 List the differences and commonalities between the Spacelab concept and the MIR concept. Address mission, development concept and science in your list.

### **Space Markets and Missions**

#### **Example Problems**

- 4-1 List the steps in Space System development and their definition according to [Larson]
- 4-2 Which are the essential elements in the Mission Need Analysis for a new Space System?
- 4-3 Give two distinct ways to segment the Space Market and list the segments for each of them
- 4-4 Calculate from the spacecraft market turnover in 1996 and the number of launches in that year the average cost of one satellite (assuming a dedicated launch for each satellite)

### **Spacecraft: Scientific Research**

#### **In-class problem**

#### **Where do disturbance forces come from?**

#### ***Aerodynamic disturbance force***

Even in space (in the vicinity of the Earth or other planets up to several thousands of kilometers altitude) there is some atmosphere present, hence an aerodynamic force will be exerted on a satellite. As in "normal" aerodynamics, that force is proportional to the surface area of the satellite, the density of the atmosphere and the square of the velocity of the satellite. It also is a function of the shape of the satellite, but to a far lesser extent than for aircraft or launchers. This force is usually called aerodynamic drag, and works always in the direction opposite to the velocity and always leads to a loss of velocity.

The drag can be written as:

$$D = 0.5 C_D A \rho V^2 \cos i$$

where

$C_D$  is the drag coefficient (about equal to 2 for most spacecraft configurations)

$A$  is the surface area

$\rho$  is the air density

$V$  is the flight velocity

$i$  is the angle between velocity and surface normal



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Aerodynamic drag can also be used to "capture" a satellite in an orbit around a planet (aero braking).

### **Solar radiation disturbance force**

A surface (hence also a satellite) experiences a force due to the pressure of the solar radiation. This solar radiation force is proportional to:

- the surface area  $A$
- the solar flux  $S$  in  $W/m^2$
- $(2-\alpha)$ , where  $\alpha$  is the solar absorptance ( $\alpha =$  unity minus the reflection)
- $\cos i$ , where  $i$  is the angle between the sun vector and the surface normal and inversely proportional to the speed of light  $c$

Expressed in a formula

$$F_s = \frac{S}{c} \cdot A \cdot (2 - \alpha) \cdot \cos i$$

So except for the term related to the solar absorptance it is very similar to the aerodynamic drag. The force is always acting in the same direction as the sun vector (vector from the sun to the satellite).

The solar radiation can be used to propel the spacecraft (solar sailing). It is only effective relatively close to the sun, as the solar flux  $S$  decreases with the square of the distance from the sun.

### **Gravity force**

Of course the main force exerted on a body in space is the force of gravity; it keeps the satellite in its orbit around a celestial body. We may calculate the orbital velocity orbital period of a satellite in a circular orbit around the Earth in a relatively simple way.

The centripetal force for a rotational motion is:

$$F_n = a_n \cdot M_{S/C} = \frac{V^2}{R} \cdot M_{S/C} \quad (1)$$

where  $V$  is the circular velocity,  $R$  the radius of the circle and  $M_{S/C}$  the mass of the spacecraft.

The gravity force is given by:

$$F_g = g \cdot M_{S/C}$$

where  $g$  is the acceleration due to gravity.

For a circular orbit centripetal and gravity force are in equilibrium:

$$F_n = F_g; \quad \frac{V^2}{R} \cdot M_{S/C} = g \cdot M_{S/C}; \quad \frac{V^2}{R} = g; \quad V = \sqrt{R \cdot g} \quad (2)$$

Newton's law of gravity states that

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$$F_g = \frac{G \cdot M_{\text{earth}} \cdot M_{S/C}}{R^2} = g \cdot M_{S/C}; \quad G \cdot M_{\text{earth}} = g \cdot R^2$$

where  $G$  is the universal gravity constant and  $M_{\text{earth}}$  the mass of the Earth. Or:

$$g \cdot R^2 = g_0 \cdot R_{\text{earth}}^2; \quad g = g_0 \frac{R_{\text{earth}}^2}{R^2} \quad (3)$$

where  $g_0$  is the acceleration due to gravity at the Earth surface ( $R = R_{\text{earth}}$ ).

Substituting (3) in (2) we obtain:

$$V_{\text{orbit}} = \sqrt{g_0 \cdot R_{\text{orbit}} \cdot \frac{R_{\text{earth}}^2}{R_{\text{orbit}}^2}} = R_{\text{earth}} \cdot \sqrt{g_0} \cdot \frac{1}{\sqrt{R_{\text{orbit}}}}$$

With  $R_{\text{earth}} = 6370$  km and  $g_0 = 9.81$  m/s<sup>2</sup> this becomes:

$$V_{\text{orbit}} = 6.37 \cdot 10^6 \cdot \sqrt{9.81} \cdot \frac{1}{\sqrt{R_{\text{orbit}}}} = \frac{1.995 \cdot 10^7}{\sqrt{R_{\text{orbit}}}}$$

Orbital period can be written as:

$$t_{\text{orbit}} = \frac{2\pi \cdot R_{\text{orbit}}}{V_{\text{orbit}}}$$

For several orbital altitudes this results in:

Orbital altitude (km)	$R_{\text{orbit}}$ (m)	$V_{\text{orbit}}$ (m/s)	$t_{\text{orbit}}$ (minutes)
0	$6.37 \times 10^6$	7905	84.4
200	$6.57 \times 10^6$	7784	88.4
500	$6.87 \times 10^6$	7612	94.5
1000	$7.37 \times 10^6$	7349	105.0
20000	$2.637 \times 10^7$	3888	710.3
35850	$4.222 \times 10^7$	3071	1439.7

Sometimes gravity is used in a very specific way:

- To accelerate a spacecraft when passing a celestial body (swing by),
- To achieve very specific orbits
  - a sun-synchronous orbit (using the fact that the Earth is not spherical)
  - an "orbit" in a libration point (the point where the gravity forces of two celestial bodies are in equilibrium). The satellite maintains then a fixed position relative to those two bodies.

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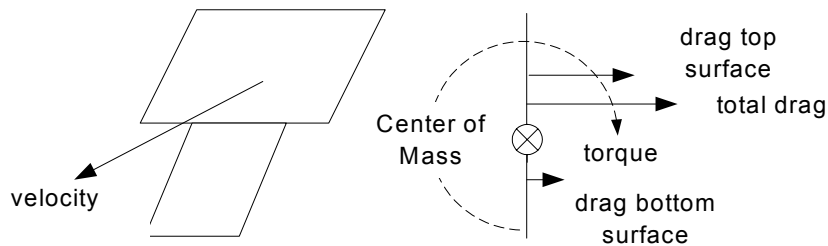
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### Where do disturbance torques come from?

#### **Aerodynamic disturbance torque**

An aerodynamic disturbance torque results, if the aerodynamic force vector does not go through the center of mass of the satellite. An example is given below.

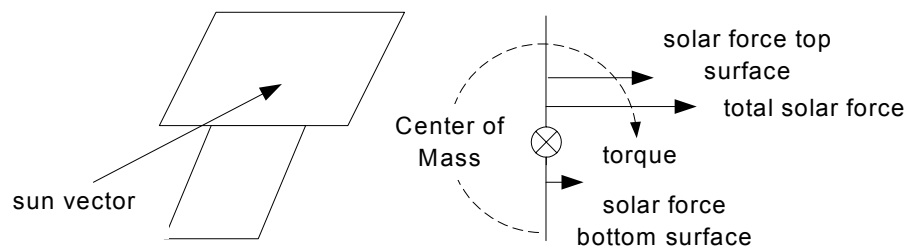


Each of the components of the drag and its distance to the center of mass is individually calculated. The sum is the disturbance torque. The aerodynamic torque can be used to align one

of the satellite axes with the velocity vector (aerodynamic stabilization).

#### **Solar radiation disturbance torque**

A solar radiation disturbance torque results, if the solar radiation force vector does not go through the center of mass of the satellite. An example is given below.



Again, each of the components of the solar force and its distance to the center of mass is individually calculated. The sum is the

disturbance torque. The solar radiation torque can be used to align one of the satellite axes with the sun vector. This is done by manipulating the position and solar absorptance of the satellite surfaces.

#### **Gravity gradient torque**

Let us now consider two equal masses,  $m_1$  and  $m_2$ , connected by a boom. Mass  $m_1$  is further away from the Earth than mass  $m_2$  (see the figure below). From (3) we may derive that mass  $m_1$  and  $m_2$  will experience a force due to the gravity field:

$$F_{g_1} = m_1 \cdot g = m_1 \cdot g_0 \frac{R_{earth}^2}{R_1^2}$$

and

$$F_{g_2} = m_2 \cdot g = m_2 \cdot g_0 \frac{R_{earth}^2}{R_2^2}$$

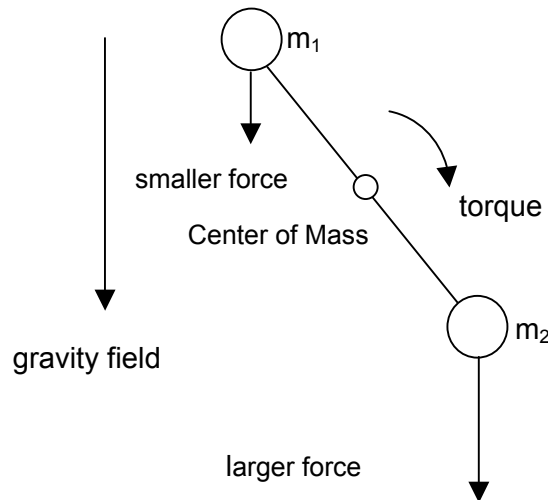
As  $R_1 > R_2$  it follows  $F_{g_1} < F_{g_2}$ .

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For a circular motion  $V$  equals  $\omega R$  and we may rewrite the centripetal force  $F_n$  given in (1) as

$$F_n = a_n \cdot m = \frac{V^2}{R} \cdot m = \frac{\omega^2 R^2}{R} \cdot m = \omega^2 R m$$

The rotational velocity  $\omega$  is equal for both  $m_1$  and  $m_2$ . As  $R_1 > R_2$  it follows  $F_{n1} > F_{n2}$ .

So for  $m_1$  the centripetal force, which is directed away from the Earth, is larger than that acting on  $m_2$  and the gravitational force, which is directed towards the Earth, is smaller than that acting on  $m_2$ . Consequently  $m_2$  will experience a resulting force that pulls it towards the Earth, while  $m_1$  is pulled away from the Earth. As a result the boom, connecting the two masses, will tend to align itself with the gradient of the gravity field. This is a stable equilibrium position. There is another equilibrium position, when the boom is horizontal; this position is, however, unstable.

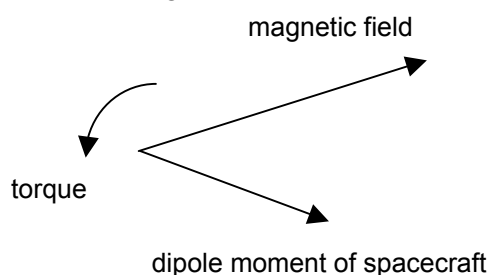
Note that the reasoning also applies, if the masses  $m_1$  and  $m_2$  are not equal.

In a mathematical sense the moment of inertia of the configuration shown in the figure in the plane of the drawing is far larger than that around the axis of symmetry. This causes the gravity gradient torque.

The gravity gradient torque may be used to stabilise a satellite passively. Next to the mass needed to produce the desired mass distribution, no other energy and hardware is required.

### **Magnetic torque**

Every satellite has a small (residual) magnetic dipole moment; you cannot avoid using small amounts of magnetic material. This dipole moment will interact with a magnetic field, as is the Earth magnetic field, as shown in the figure below. The spacecraft acts as the needle of a compass.



Note, that there is no resulting force in addition to the torque. You can use this effect to actively (by using switchable magnetic coils or torquers) or passively (by building permanent magnets into the spacecraft) to stabilize a

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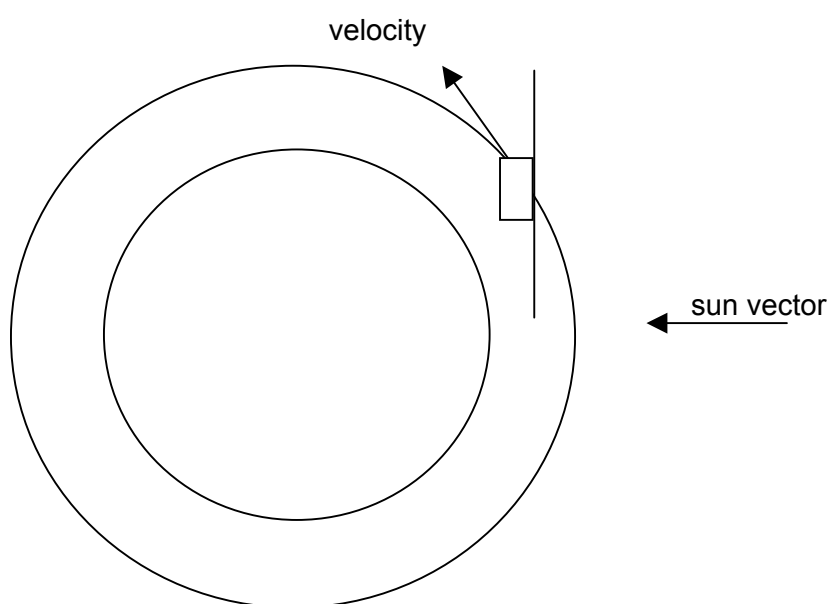
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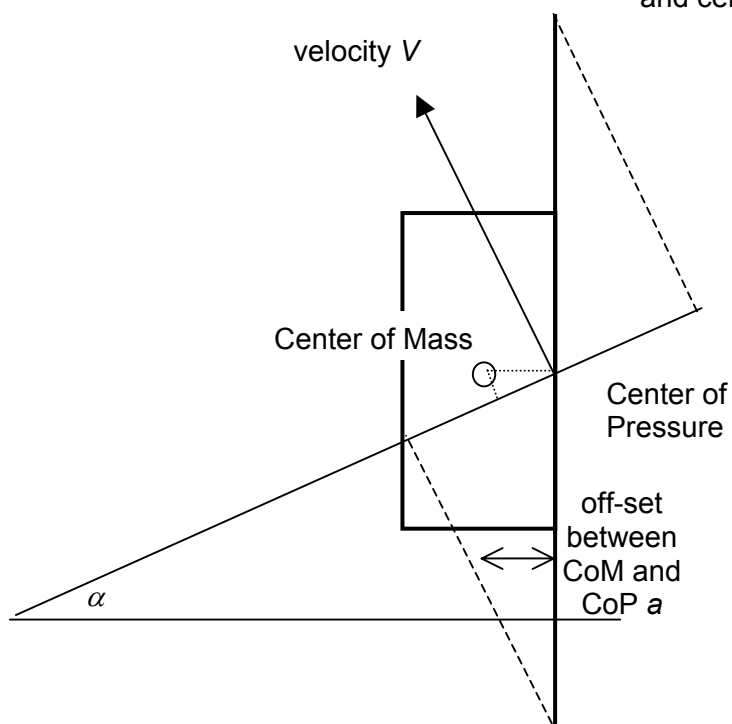
satellite.

### Some disturbance torques are cyclic

Although generally disturbance torques are directed in arbitrary directions, they often show some systematic behavior (pending the orbit and orientation of the satellite). Sometimes they are cyclic, which means that over a certain period of time their integrated value equals zero. So if you manage to store their energy over that period, you do not have to eliminate them by spending "expensive" fuel (mass). An example is given below.



A symmetrical satellite is in a low circular equatorial orbit, the solar panel normal always pointing towards the sun. Assume that the sun vector lies in the orbital plane. The center of mass of the satellite will be located in the spacecraft body (it is much heavier than the solar panels). The center of (aerodynamic) pressure will be located in the middle of the plane through the solar panels, so there will be an off-set between center of mass and center of pressure.



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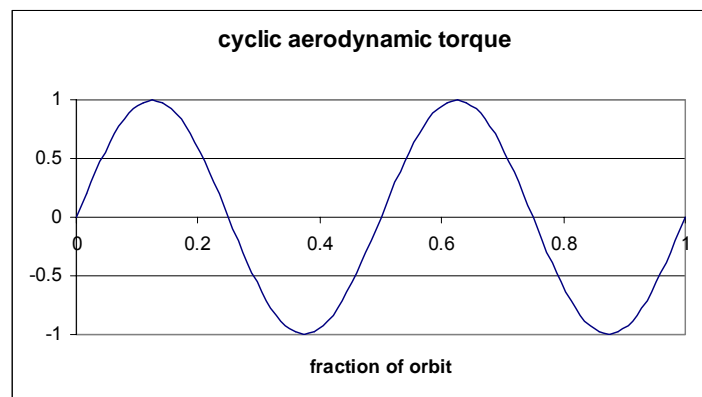
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At  $\alpha = 0$  the satellite velocity vector is normal to the sun vector. If we assume the satellite body to have small dimensions compared to the solar panel area, the aerodynamic drag is then zero.

The projected area of the solar panel on the plane normal to the velocity vector is  $A \sin \alpha$ . The offset of the center of pressure (in the center of the solar panel) and the center of mass is  $a \cos \alpha$ . So the aerodynamic disturbance torque is:

$$T_{aero} = 0.5 \cdot a \cdot C_D \cdot \rho \cdot V^2 \cdot A \sin \alpha \cdot a \cos \alpha = 0.25 \cdot a \cdot C_D \cdot \rho \cdot V^2 \sin(2\alpha)$$

Or the aerodynamic torque varies with twice the orbital frequency. This is shown in the picture at the below.



The vertical axis shows the variable  $\frac{T_{aero}}{0.25 \cdot a \cdot C_D \rho \cdot A \cdot V^2}$

If we now have an actuator that is able to store the angular momentum generated over one quarter of an orbit, we do not have to spend electrical or chemical energy to dump this angular momentum.

### Example problems

- 5-1 Which objects have been observed by ANS and IRAS? Which properties of these objects have been studied? Why have they been studied from space?
- 5-2 What is the minimum time it takes IRAS to map the total sky, taking into account the maximum allowed offset of the instrument axis towards the sun vector and neglecting the yearly variation of the angle between sun vector and equatorial plane?
- 5-3 Where should the IRAS ground station be located to have maximum contact time with the satellite?
- 5-4 Why does an astronomy satellite observing the full sky in a sun-synchronous orbit need to have a telescope field of view of at least  $0.99^\circ$  and a lifetime of at least 6 months?
- 5-5 Calculate the acceleration due to air drag of the LAGEOS satellite assuming a drag coefficient of 2.0, an air density of  $5.4 \times 10^{-18} \text{ kg/m}^3$  and an orbital period of 228.4

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minutes. What is the total velocity change in one year? What is the total error in the distance traveled in that year (both in absolute and relative (%) terms)? Assume that the orbital altitude does not change over time. Is this satellite a stable position reference? Motivate your answer.

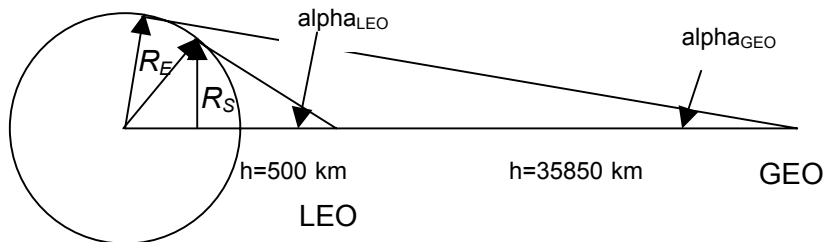
### Spacecraft: Earth Observation

#### In class-problem

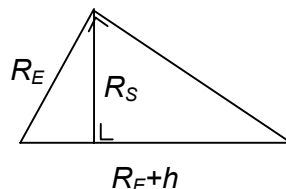
#### How does a satellite see the Earth?

Low Earth orbit; distance to surface	$h_{LEO} = 500 \text{ km}$
Mean earth radius	$R_E = 6370 \text{ km}$
Geostationary orbit; distance to surface	$h_{GEO} = 35850 \text{ km}$

From the figure you may deduce some information about resolution obtained for LEO and GEO orbit in an Earth observation mission.



Or in detail:



The radius of the Earth disc viewed from the satellite can be calculated from:

$$\frac{R_S}{R_E} = \frac{\sqrt{(R_E + h)^2 - R_E^2}}{R_E + h} \quad \leftarrow \text{Pythagoras}$$

Calculating  $R_S$  for LEO and GEO

$$R_{S_{LEO}} = 2386 \text{ km and } R_{S_{GEO}} = 6297 \text{ km}$$

If the total visible area of the Earth disc is viewed from the satellite with a 1024 x 1024 pixel detector (digital camera), the LEO pixel measures  $(2386*2)/1024 = 4.66 \text{ km}$ , the GEO pixel  $(6297*2)/1024 = 12.26 \text{ km}$ . In practice scanning techniques (and better detectors) are used to improve the resolution, but the fact stays, that Earth observation from GEO is at least a factor 3 worse than from LEO.

# Space Engineering & Technology I

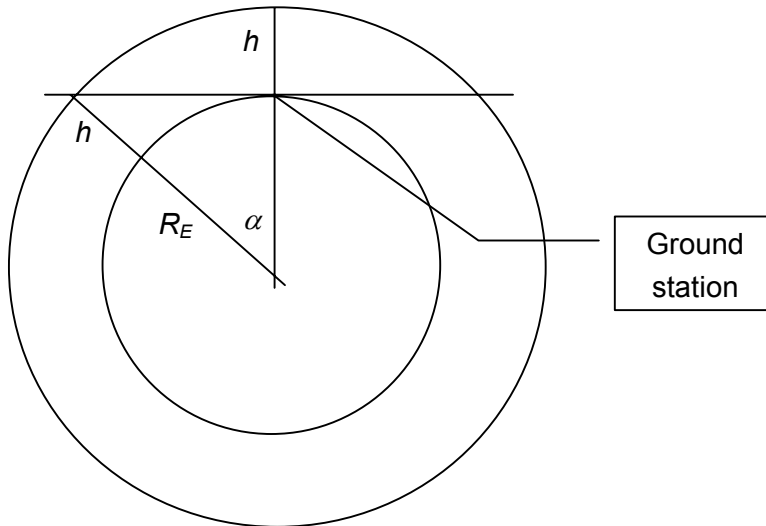
## Space Missions & Systems

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**How does a ground station see the satellite?**



$h$  = orbit altitude = **500 km**

$R_E$  = Earth radius = **6370 km**

Ground station

Calculating the angle  $\alpha$

$$\cos \alpha = \frac{R_E}{R_E + h} = \frac{6370}{6370 + 500} = 0.927$$

$$\alpha = 21.99 \text{ deg}$$

Assuming the orbital period to be equal to 90 minutes, the maximum ground contact time becomes

$$t_{\text{contact}} = \frac{2\alpha}{2\pi} t_{\text{orbit}} = \frac{2 \cdot 21.99}{360} \cdot 90 = 10.99 \text{ minutes}$$

In practice a ground station cannot receive the signal of a satellite, that is less than 10 degrees above the horizon and generally the ground station is not located directly under the flight path of the satellite, so practical contact times are of the order of 6 to 7 minutes for a satellite in a 500 km orbit.

**What does a geostationary orbit mean for data transfer?**

Assume both a geostationary and a low earth orbit satellite observe the same area on the earth surface around a ground station from a **circular, equatorial orbit** and make a "picture" of it once a day.

The LEO satellite "sees" the area and the ground station 11 minutes per orbit, which has a period of 90 minutes (source: lectures Mission Analysis I of ae1-801). The GEO satellite "sees" the area and the ground station continuously.

Per 24 hours the ratio of the time available to send the data of the picture to the ground station is:



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part of LEO orbit that ground station is visible      number of passes in LEO orbit over ground station on the equator

$$\frac{t_{contact_{LEO}}}{t_{contact_{GEO}}} = \frac{11}{24 \cdot 60} \cdot \frac{24 \cdot 60}{90} = \frac{11}{90} = 0.12$$

Hence the data rate of the LEO satellite has to be 8.2 times larger to transfer the same amount of data. Note, that in this example the factor 0.12 is also the ratio of the time available to take the picture of the object on the earth surface; the LEO satellite has to "work" 8.2 times as hard as the GEO satellite.

The data rate is proportional to the transmission power (source: Space Engineering & Technology II). However, as the geostationary satellite is further away from the ground station, the GEO satellite needs more power to transmit the same amount of data (see chapter 7 of the lecture notes).

So even with the much smaller data rate, the GEO satellite still needs

distance from Earth      narrower antenna beam

$$\frac{h_{GEO}^2}{h_{LEO}^2} \cdot \frac{t_{contact_{LEO}}}{t_{contact_{GEO}}} \cdot \frac{1}{61.24} = \frac{5141 \cdot 0.12}{61.24} = 10.07$$

times more communication power than the LEO satellite.

A geo-stationary orbit is generally chosen, if the mission needs

- to overview the full earth disc (e.g. meteorological satellite),
- to have continuous contact with a ground station (e.g. communication satellite).

The calculations show that you pay for these "services" in power required for the communication link.

### Example problems

- 6-1 List at least two reasons why you would perform Earth Observation from space
- 6-2 Calculate for the Meteosat satellite (see page 32) how many pixels the visual image of the Earth disc contains (use the VIS resolution), assuming the pixels have equal size in North-South and East-West direction. If we assume that each pixel uses 8 bits, how many bits contains one Earth image? What is the corresponding bit rate for sending that image to a ground station?

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### Spacecraft: Communication

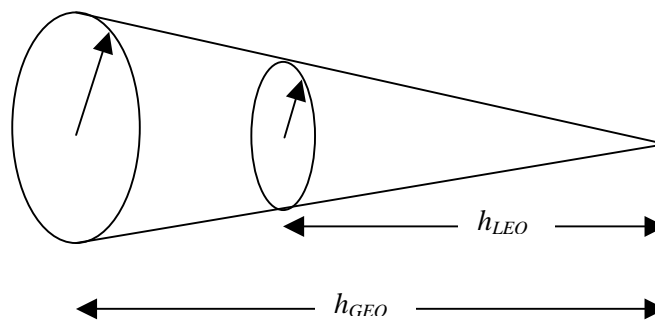
#### In-class problem

#### What does a geostationary orbit mean for communication?

Low Earth orbit; distance to surface  $h_{LEO} = 500 \text{ km}$

Geostationary orbit; distance to surface  $h_{GEO} = 35850 \text{ km}$

For a given antenna and transmitter power, the energy of the signal expands in cone:



The area ratio of the "antenna" pattern at LEO and at GEO is:

$$\frac{A_{LEO}}{A_{GEO}} = \frac{h_{LEO}^2}{h_{GEO}^2} = \frac{500^2}{35850^2} = 0.000195$$

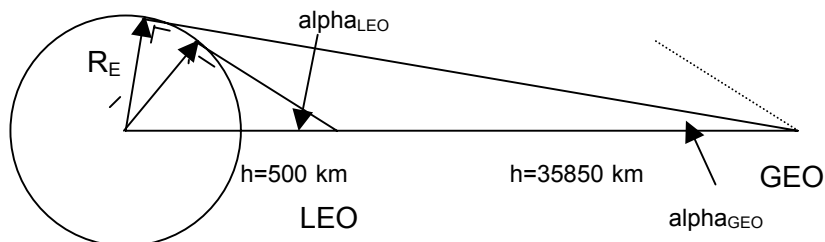
This is also the energy (power) loss the communication link experiences. Somewhere in the communication chain transmitter power - transmitter antenna size - receiver antenna size - receiver power this loss has to be compensated.

Suppose, your hand-held receiver has 0.2 Watt receiver power sufficient to receive the LEO signal, it would need

$$\frac{1}{0.000195} \cdot 0.2 = 5141 \cdot 0.2 = 1028 \text{ Watt}$$

power to receive the GEO signal.

Luckily there is some other factor to consider. We have assumed, that the antenna on the LEO satellite and the GEO satellite is identical, that is it radiates its transmission power in a beam of the same dimensions. That is not necessary, as shown in the following figure.



The angle  $\alpha_{LEO}$  is much larger than the angle  $\alpha_{GEO}$ , and any signal, that misses the Earth is wasted. Hence we can make the antenna angle of the satellite smaller, and concentrate the transmission power in a narrower beam.

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The cone half angle  $\alpha$  can be calculated as

$$\alpha = \arcsin \frac{R_E}{R_E + h}$$

or  $\alpha_{LEO} = 68.01^\circ$  and  $\alpha_{GEO} = 8.68^\circ$ . The transmission energy content of the beam reaching the Earth's surface is inversely proportional to the square of this angle, keeping the transmission power constant. So the strength of the signal from the smaller beam is  $(68.01/8.68)^2 = 61.42$  times larger than that of the wider beam. Note, that you see a larger part of the earth from GEO than from LEO.

The receiver power of your hand-held receiver may hence at least be reduced by the factor **61.42**; the required receiver power is then:

$$1028/61.42 = 16.74 \text{ Watt}$$

This is still a factor 83.7 higher than required for the LEO satellite case.

### Example problems

7-1 List the disadvantages and advantages of each of the following personal communication systems:

- a system using geostationary satellites
- a system like Iridium
- a fully land-based system (GSM)

Address both space and ground segment

7-2 Assuming that

- a mobile telephone user is willing to pay a world-wide charge of 0.1 US\$ per minute for a 99% availability of the service
- the user makes 100 such one minute calls per month
- a MSS constellation with 6 years lifetime costs 4 G\$
- operations of the system cost 200 M\$ per year
- billing services cost 100 \$ per user per year
- investors want to recover their money with 20% annual yield

How many customers does the constellation operator need? Do you think that is economically feasible? Motivate your answer.

Assuming that

- the constellation has 66 satellites
- all calls take place during day time (8 hours)
- the users are uniformly distributed over the Earth's surface
- all mobile calls are to a fixed telephone number
- a peak load of two times the average load (think about the 99% availability)

How many telephone lines should be available per satellite? Do you think this is technically feasible with an Iridium-like constellation?

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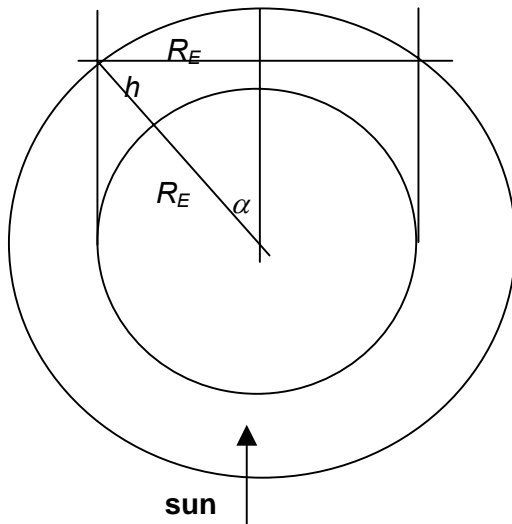
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### Spacecraft: Navigation

#### In-class problem

#### How expensive is eclipse?



$h = \text{orbit altitude} = 20000 \text{ km}$

$R_E = \text{Earth radius} = 6370 \text{ km}$

Calculating the angle  $\alpha$

$$\sin \alpha = \frac{R_E}{R_E + h} = \frac{6370}{6370 + 20000} = 0.242$$

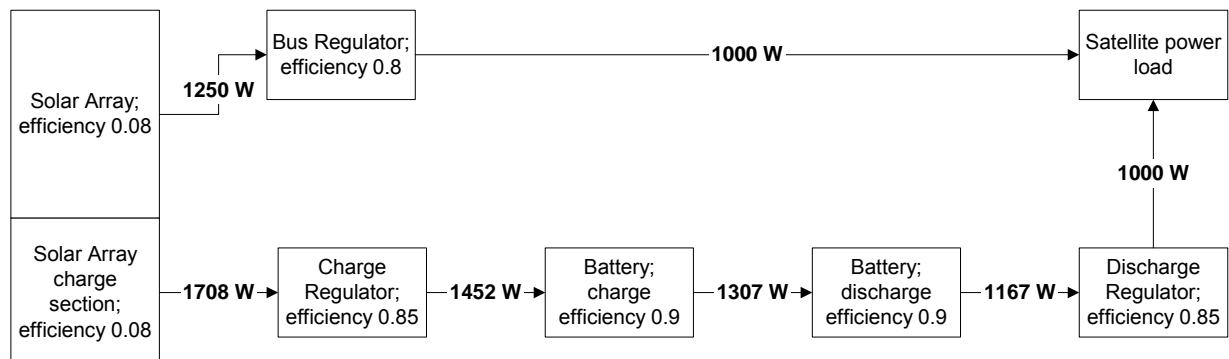
$$\alpha = 14 \text{ deg}$$

Assuming the orbital period to be equal to 720 minutes (12 hours), the time the satellite is in the Earth's shadow (eclipse) becomes

$$t_{\text{eclipse}} = \frac{2\alpha}{2\pi} t_{\text{orbit}} = \frac{2 \cdot 14}{360} \cdot 720 = 56 \text{ minutes}$$

The sunlit period of the orbit is then 664 minutes.

The schematic of the electrical power provision to the satellite can be drawn as:



The charge regulator is used if the solar array voltage is not equal to the satellite bus voltage. The battery charge regulator ensures that the battery charge voltage and current have the correct value to achieve optimum battery charging. The battery discharge regulator ensures that the bus voltage keeps the correct value during eclipse. Battery charge and discharge

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efficiency are related to the internal resistance of the battery.

The schematic represents the worst case situation (low efficiency units and Si solar cells). Using GaAs solar cells and higher efficiency units gives a better picture (more power or a smaller solar array).

Once we know the power needed, we can calculate the solar array area (assuming the sun vector is normal to the solar array) with

$$P_{solar\ array} = S_{sun} A_{solar\ array} \eta_{solar\ array}$$

where

$S$  = solar flux = 1353 W/m<sup>2</sup>

$\eta$  = efficiency

$A$  = area

$P$  = electrical power output

Now we can calculate the total energy needed to provide 1000 W to the satellite both in sunlight and in eclipse

	<i>worst case</i>
energy needed in eclipse	56 minutes 1708 W = 95648 Wmin
battery charge power during sunlight	95648 Wmin/664 minutes = 144.0 W
satellite power during sunlight	1250 W
total solar array delivered power	1394 W
fraction of solar array power needed for eclipse	10.3 %
total solar array area	12.88 m <sup>2</sup>

Note that for a low Earth orbit the fraction of solar array power needed for eclipse may increase to some 45 to 50 %.

### Example problems

8-1 Calculate the required solar array power for a satellite with the following characteristics:

- Equatorial orbit 500 km, orbital period 90 minutes, satellite bus power 200 W
- Efficiencies:
  - solar array 0.15 (GaAs solar cells)
  - bus regulator 0.85
  - battery charge and discharge regulator 0.85
  - battery charge and discharge 0.9
- Solar flux 1353 W/m<sup>2</sup>; solar array normal always pointing to the sun

Draw a diagram of the power subsystem. Calculate the eclipse duration. Determine the total solar array area required and the proportion of that area needed to generate the eclipse power.

8-2 What is the principle used for position determination in the GPS system? How many satellite signals are necessary to determine the position?

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### **The Space Mission**

#### **Example problems**

- 9-1 We want to observe the complete radiation spectrum of celestial bodies for long periods of time from space. It seems attractive to go far from the Earth, as
- the Earth itself is a source of radiation (heat)
  - close to Earth the time during which a celestial body (a point in inertial space) can be observed uninterrupted is limited (the Earth passes through the satellite - object line)
  - we have long contact periods with the ground station, so we need less on-board storage of data, less sophisticated autonomy and see “cosmic” events in real time
- But there are disadvantages .....  
List the disadvantages for this concept.

### **ANSWERS TO EXAMPLE PROBLEMS**

Below the (Numerical) answers to some of the example problems are compiled.

#### *Problem 1-3*

	metal	black paint	mirror	
solar absorptance	0.1	0.9	0.1	[-]
emissivity	0.04	0.9	0.9	[-]
Stefan-Boltzmann constant		5.67E-08		[W/(m <sup>2</sup> K <sup>4</sup> )]
Solar flux in space		1400		[W/m <sup>2</sup> ]
Temperature of cold space		4		[K]
Surface Temperature	419	333	192	[K]

#### *Problem 2-3*

$$V_{\max} = 2546 - 387 = 2159 \text{ms}^{-1}$$

$$v_{\text{orbit}} = 7410 \text{ms}^{-1}$$

#### *Problem 2-4*

28 Arianes are selected at a total cost of  $28 \cdot (125 + 800) = 25,900$  M\$. Corresponding figure for the Proton are: 59 Protons at a total cost of  $59 \cdot (60 + 400) = 27,140$  M\$.

#### *Problem 5-2*

To map the total sky requires a  $180 - 60 = 120$  degrees rotation around the Sun. This takes  $120/360 \cdot 365 = 122$  days.

#### *Problem 5-5*

$$a_{\text{drag}} = 1.18 \cdot 10^{-13} \text{m/s}^2$$

Total velocity change in one year:

$$\Delta V = 3.72 \cdot 10^{-6} \text{m/s}$$

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Position error:

$$\Delta s = 58.8 \text{ m}$$

### *Problem 5-5*

Total number of "pixels" is  $5038 \times 5038 = 25.4 \text{ E6}$  pixels

One Earth disc image:  $25.4 \text{ E6} \times 8 = 2.03 \text{ E8}$  bits

Data transfer rate:  $2.03 \text{ E8} / (30 \times 60) = 112.8 \text{ kbps}$ .

### *Problem 7-2*

Number of users  $N = 2.08 \times 10^7$

Number of calls per satellite including reserve for peak load (factor 2):  $2162 \times 2 = 4324$

Number of two-way channels available per Iridium satellite 2800

### *Problem 8-1*

The solar array should be able to deliver 235W to the satellite during sunlight and 342W during ecipse.

$$t_{\text{eclipse}} = 34 \text{ min}$$

Total solar array area:  $2.18 \text{ m}^2$

Fraction of area needed for ecipse: 46.9%

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## TEST AND EXAM PROBLEMS

### TEST OCTOBER 2001

#### Problem 1 (45 minutes)

##### Mission and System Break-down

A mission may be broken down in several segments.

- 1a List the seven mission segments according to the convention used in "Space Mission Analysis & Design" [Larson].
- 1b List the function of each of the mission segments. Split the Space Segment in two parts (Payload and Bus).
- 1c Give the definition of each of the mission segments.
- 1d Is this breakdown always used? Illustrate and motivate your answer.

#### Problem 2 (65 minutes)

##### Technical and Economic Feasibility of a Space-based GSM System

- 2a Assuming that
- a mobile telephone user is willing to pay a world-wide charge of 0.15 US\$ per minute for a 99% availability of the service
  - the user makes 100 such one minute calls per month
  - a MSS constellation with 6 years lifetime costs 4 G\$
  - operations of the system cost 200 M\$ per year
  - billing services cost 100 \$ per user per year (expenses of the mobile telephone operator)
  - investors want to recover their money with 20% annual yield

Calculate income, expenses and investor's profit as a function of the number of customers  $N$ . How many customers does the constellation operator need?

- 2b Do you think that is economically feasible? Motivate your answer.

- 2c Assuming that

- the constellation has 66 satellites
- all calls take place during day time (8 hours)
- the users are uniformly distributed over the Earth's surface
- all mobile calls are to a fixed telephone number
- a peak load of two times the average load (think about the 99% availability)

How many telephone lines should be available per satellite? Hint: calculate first the total average number of telephone calls per day of 8 hours.

- 2d The Iridium constellation has the following characteristics:

- World-wide system for mobile communication
- 2800 two-way channels for speech per satellite
- 66 satellites distributed over 6 orbit planes (constellation)
- Cost (all system): 4 G\$
- Circular sun-synchronous orbit: 785 km
- Lifetime: 5-6 year
- Inter-satellite links

Do you think an Iridium-like constellation is a technically feasible solution for your problem? If not, indicate why. Could you think of measures to improve the situation? Motivate your answer.

*Answers*



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- 2a: Total income: 1080 N \$; Total expenses:  $600 N + 4 \times 10^9 + 1.2 \times 10^9$  \$; Total profit:  $4.8 \times 10^9$  \$; Number of users:  $2.08 \times 10^9$
- 2c: Number of calls (lines) per satellite including reserve for peak load: 4324

### Problem 3 (70 minutes)

#### Launching a Satellite by means of a Single Stage Launcher

A small satellite will be launched in a low, circular orbit with an altitude  $h_E$  of 500 km. The assignment is to determine which characteristics a single stage launcher shall have to enable this. The Earth radius  $R_E$  equals 6370 km.

- 3a Calculate the orbital velocity  $V_{500}$  in that low earth orbit, taking into account that the orbital period equals 94.62 minutes.
- 3b During launch the rocket "loses" velocity due to the gravity forces. This velocity loss is:

$$\Delta V_g = \int_{t=0}^{t=t_b} -g(t) \cdot \sin(\gamma(t)) \cdot dt$$

where  $g(t)$  is the acceleration due to gravity at time  $t$ ,  $\gamma(t)$  is the angle between the velocity vector of the launcher and the horizontal and  $t_b$  is the burning time of the rocket engine. At the start of the launcher  $\gamma$  equals  $90^\circ$ , when the engine stops (at the injection in the circular orbit)  $\gamma$  equals  $0^\circ$ . An acceptable assumption is that during the full flight  $g(t) \cdot \sin(\gamma(t))$  has an average value equal to  $0.5 g_0$ .

Can you list another loss occurring during the launcher ascent?

- 3c Calculate the velocity loss (gravity loss)  $\Delta V_g$  during the launch, assuming a rocket engine burning time  $t_b$  of 5 minutes.
- 3d The loss due to air drag during the flight equals 350 m/s. What is the total velocity increment  $V_{total}$  that the launcher has to give to the satellite?
- 3e The launcher uses as propellants liquid hydrogen and oxygen, one of the most energetic chemical propellant combinations. The combustion products have an exhaust velocity  $w_e$  of 4200 m/s.

Calculate with Tsiolkowski the mass ratio  $M_0/M_e$  of the launcher, required to give the velocity increment calculated in point 3d to the satellite ( $M_0$  is the mass of the launcher at the start,  $M_e$  the mass when all propellant has been used). Is that a realistic (achievable) value? Calculate also  $M_p/M_0$  (the fraction of the total mass, that is used for propellants, where the propellant mass  $M_p = M_0 - M_e$ ).

- 3f Can you describe a method to decrease the required velocity increment?
- 3g The satellite is launched from the equator in exactly westward direction. Has this a positive or negative impact on the required velocity increment to achieve the orbit?
- 3h Calculate the velocity increment  $\Delta V_{eq}$  of the launcher due to the rotation of the earth, when the satellite is launched in the most favorable direction on the equator.
- 3i What is the total velocity increment  $V_{total}$  that the launcher has to impart to the satellite in this case?
- 3j Calculate with Tsiolkowski the mass ratio  $M_0/M_e$  of the launcher required to give the velocity increment to the satellite as calculated in point i. Calculate also  $M_p/M_0$ . Is the difference with the results from point 3e large?
- 3l Which conclusion do you draw from the results of the calculations?

#### Answers

- 3a  $V_{500} = 7603 \text{ m/s}$
- 3c  $\Delta V_g = 1471.5 \text{ m/s}$
- 3d  $V_{total} = 9425 \text{ m/s}$
- 3e  $M_0/M_e = 9.43$ ;  $M_p/M_0 = 0.894$
-

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- 3h  $\Delta V_{eq} = 463 \text{ m/s}$   
3i  $V_{total} = 8963 \text{ m/s}$   
3j  $M_O/M_e = 8.44; M_p/M_O = 0.882$

### EXAM JUNE 2002; JUNE 2005

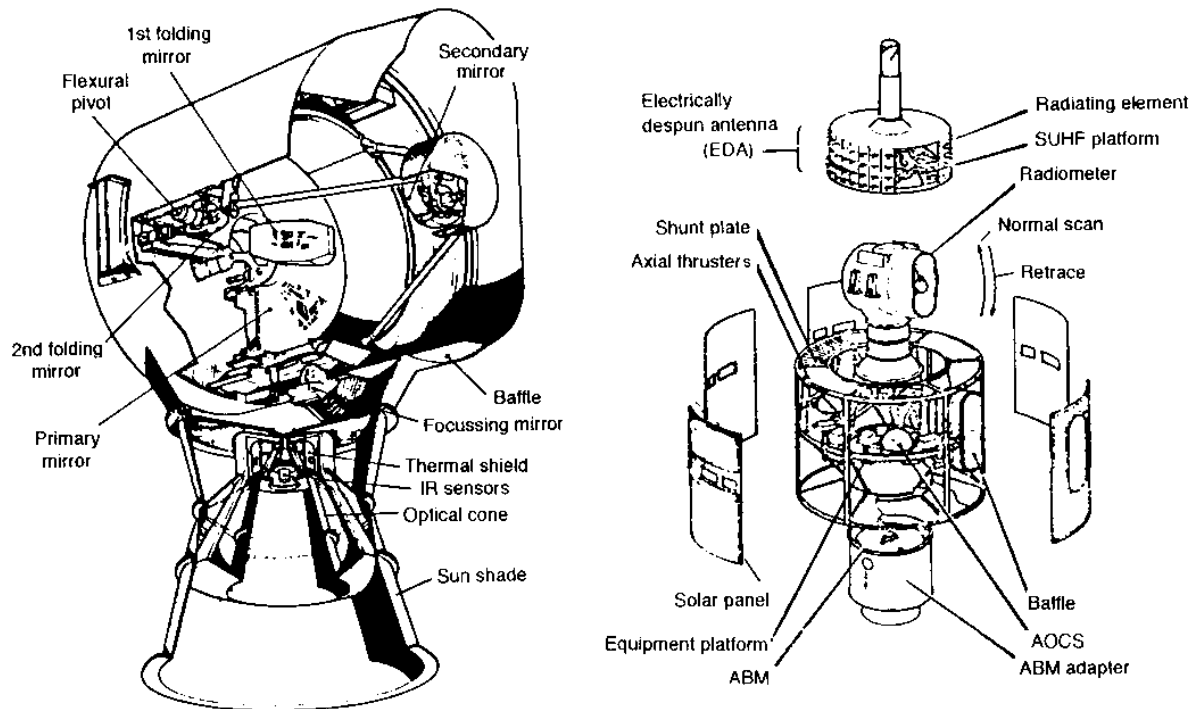
#### Problem 1 (50 minutes)

##### Image Size and Bit Rate of a Weather Satellite

Meteosat is a geo-stationary satellite at an altitude  $h_{GEO}$  of 35850 km, spinning at 100 revolutions per minute; its spin axis normal to the orbit plane. The sensor "scans" the Earth once each revolution (one "line") and builds one Earth image every 30 minutes in visible and IR bands. After each scan (revolution) the sensor mirrors are adjusted, such that the next line is scanned during the next revolution. The satellite sends the images and other data collected from Earth located platforms to Earth. Its mass distribution is:

- 362 kg dry mass,
- 39 kg hydrazine propellant for attitude and orbit control,
- Apogee Boost Motor (ABM) propellant mass of 360 kg.

The resolution of the earth image is 2.5 km in the visual channel (VIS) and 5 km in the infra-red channel (IR)



- 1a Draw a picture of the Earth and Meteosat in its orbit, indicating the Earth radius  $R_{EARTH}$ , the orbit altitude  $h_{GEO}$  and the half cone angle  $\alpha_{GEO}$  the Earth is included in, when seen from the satellite.
- 1b Calculate the diameter of the Earth disc  $D_s$  as seen by Meteosat, assuming the Earth radius  $R_{EARTH}$  equals 6370 km and the geostationary orbit altitude  $h_{GEO}$  equals 35850 km. Calculate also  $\alpha_{GEO}$ .
- 1c What is the most Northern or Southern latitude of the Earth, that is visible from Meteosat?
- 1d How many geostationary satellites would be needed to cover the whole Earth between these latitudes? Motivate your answer.

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- 1e Calculate how many pixels  $N_{E-W}$  the line of the image on the equator of the Earth disc contains in the visual channel (use the VIS resolution).
- 1f What is the time needed to build up one line of the image?
- 1g Assuming the pixels have equal size in North-South and East-West direction and that the image of the earth is a square, calculate how many pixels the image of the Earth disc contains in the visual channel (use the VIS resolution).
- 1h If we assume that each pixel uses 8 bits, how many bits contains one Earth image?
- 1i What is the minimum bit rate for sending one full Earth image to a ground station?
- 1j Each revolution of the satellite one line of the Earth image is scanned. Deduce from the answer on 1g how many lines the Earth image has.
- 1k In how much time is one full image of the Earth disc build up (using the results of 1j)? Is this compatible with the time available to build up one image?
- 1l Assume now that 2 pixels in North-South direction are scanned per scan and that the mirrors are stepping 2 North-South pixels per revolution. How much time is takes now to produce one full Earth image?
- 1m How much time is left after completion of the scan according to 1l? For what purpose this time will be used in your opinion?
- 1n What is the bit rate corresponding to the infra-red channel of Meteosat (use the infra-red channel resolution)?
- 1o Assuming Meteosat has three visual and one infra-red channel: what is the total minimum bit rate of Meteosat?

### Answers

- 1b  $R_s = 6297 \text{ km}$ ;  $D_s = 12594 \text{ km}$
- 1c *Latitude: 81.3 degrees*
- 1d *Number of satellites: 2.21  $\approx$  3; Overlap: 21.3 degrees*
- 1e *Number of pixels on equator: 5038*
- 1f *One line build up in 0.6 seconds*
- 1g *Size of one visual image: 25.4 x 106 pixels*
- 1h *Size of one visual image: 103.2 x 106 bits*
- 1i *Bit rate  $_{min} = 112.9 \text{ kbps}$*
- 1j *Number of lines per image: 5038*
- 1k *Scan time 5038 lines: 3022.8 seconds*
- 1l *Scan time full Earth image 1511.4 seconds*
- 1m *Time left after image scan: 288.6 seconds*
- 1n *Bit rate IR channel: 28.2 kbps*
- 1o *Total bit rate: 366.9 kbps*

## EXAM AUGUST 2002

### Problem 1 (50 minutes)

#### Power Subsystem Design

A satellite is in a circular equatorial orbit of 800 km altitude with a period  $t_{orbit}$  of 100.87 minutes. Its payload observes the Earth by means of a powerful Synthetic Aperture Radar (SAR) over the full orbit. Payload and satellite require 2500 W electrical power ( $P_{PL}$ ) continuously. The power is provided by a solar array, the surface of which is pointed towards the sun in the sunlit part of the orbit.

- 1a The electrical power  $P_{PL}$  of 2500 W is generated by the solar array. However, the voltage output by the solar array is different from the one required by the satellite, hence a bus voltage regulator is used to adapt the voltage. The efficiency  $\eta_{BR}$  of the bus regulator is 90%.

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- The solar array is covered by solar cells. They convert sunlight into electrical power with an efficiency  $\eta_{SA}$  of 8%. (8% of the radiative energy of the Sun is converted into electrical energy). All data are End of Life values.
- Draw a block diagram showing the solar array, the bus regulator and payload/satellite. Note the data given in the text above in the diagram.
- 1b Calculate the electrical power  $P_{BRin}$  at the input of the bus regulator, required to deliver 2500 W to the satellite and payload.
- 1c The minimum solar flux  $S_{SUN}$  equals  $1350 \text{ W/m}^2$ . How much solar array area  $A_{SAnominal}$  is needed to provide the input power of the bus regulator, you have calculated in 1b?
- 1d Calculate the duration of the eclipse  $t_{eclipse}$  (period in the shadow of the Earth) in the 800 km orbit (draw a picture). The radius of the Earth  $R_E$  is 6371 km and the Sun delivers a perfectly parallel beam of light and that the Sun is in the orbit plane. How long is the sunlit phase of the orbit  $t_{sunlit}$ ?
- 1e Compute the energy (in Ws or Joules) required during the sunlit period  $E_{sunlit}$ .
- 1f The energy during eclipse is provided by a battery. The battery has an internal efficiency  $\eta_{bat}$  of 90%, that is 10% of the energy or power is converted into heat, when the battery is charged or discharged. Also, the voltage at the output of the battery is not equal to the voltage needed by the satellite and the payload; a discharge regulator is used to adapt the voltage. The discharge regulator has an efficiency  $\eta_{DR}$  of 85%.
- Draw the block diagram showing the battery, the discharge regulator and the satellite and payload. Note in the diagram all data given above.
- 1g What is the power  $P_{batout}$  required at the battery output to deliver the 2500 W to the payload/satellite?
- 1h And which power losses  $P_{intdischarge}$  occur in the battery during discharge?
- 1i What is the energy  $E_{batdischarge}$  the battery should contain prior to discharge to provide the satellite with power during eclipse, assuming that the battery can be fully discharged?
- 1j The battery has to be charged during the sunlit phase of the orbit. A special charge section of the solar array is used for it, which converts the solar radiation with an efficiency  $\eta_{SA}$  of 8% into electrical power. Current and voltage at the output of the solar array have to be adjusted to values fit to charge the battery. A charge regulator is used for that purpose, which has an efficiency  $\eta_{CR}$  of 80%. During charging the battery converts 10% of the incoming power or energy into heat (power loss).
- Draw a block diagram showing the charge section of the solar array, the charge regulator and the battery. Note down the data known in it.
- 1k Assume, that the battery needs to contain  $7 \times 10^6$  Ws, when it is fully charged. How much energy  $E_{batin}$  needs to be fed into the battery?
- 1l How much power  $P_{batin}$  (Watt) needs to be fed into the battery during the sunlit phase of the orbit?
- 1m How much power  $P_{CRin}$  should the charge section of the solar array deliver to the charge regulator in order to feed the required power into the battery?
- 1n Assuming the solar flux  $S_{SUN}$  is  $1350 \text{ W/m}^2$  and the solar cell efficiency  $\eta_{SA}$  of 8%, what is the area of the charge section  $A_{SAcharge}$  of the solar array?
- 1o What is the total power  $P_{SA}$  the solar array can deliver? What part (in percentage) of it becomes available for satellite and payload?

### Answers

- 1b Regulator input power: 2778 W  
1c Solar array area:  $25.7 \text{ m}^2$
-

# Space Engineering & Technology I

## Space Missions & Systems

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- 1d *Eclipse duration: 35.14 minutes; Sunlit period: 65.73 minutes*
- 1e *Energy required in sunlit phase:  $10.96 \times 10^6$  Ws*
- 1g *Battery output power: 2941 W*
- 1h *Battery power prior to internal loss: 3268 W; Battery internal power losses: 327 W*
- 1i *Energy stored in battery:  $6.89 \times 10^6$  Ws*
- 1k *Energy fed into battery:  $7.8 \times 10^6$  Ws*
- 1l *Power fed into battery during sunlit phase: 1975 W*
- 1m *Charge section power: 2464 W*
- 1n *Charge section area:  $22.8 \text{ m}^2$*
- 1o *Total solar array power : 5669 W; Percentage power available for stallite/payload: 44.1%*

### TEST OCTOBER 2002

#### Problem 1 (55 minutes)

##### Orbit maintenance

An earth observation satellite is in a circular low earth orbit with an altitude of 500 km ( $h$ ) with an orbital period  $t_{orbit}$  of 94.62 minutes. The mission requires, that this altitude is accurately maintained over a mission duration of 5 years. The satellite has a dry mass (without propellants)  $M_e$  of 1000 kg. The satellite carries large antennae to perform its observations and has a large solar array to provide electrical power to the payload; the average surface area of the satellite  $A_{SC}$  equals  $40 \text{ m}^2$ .

- 1a Which three external disturbance forces are exerted on the satellite in its orbit? Which of these disturbance forces can directly and to a large extent influence the altitude of the orbit? Motivate your answer.
- 1b Write down the expression for the force  $D_{atm}$  exerted by the atmosphere on the satellite. Which parameters determine the magnitude of the drag?
- 1c Calculate the velocity  $V_{SC}$  of the satellite in its orbit. The Earth radius  $R_E$  equals 6371 km.
- 1d Calculate the drag force  $D_{atm}$  exerted by the atmosphere on the satellite, assuming that the drag  $C_D$  equals 2.0 and that the maximum density of the atmosphere on an altitude of 500 km  $\rho_{500}$  equals  $2.82 \times 10^{-12} \text{ kg/m}^3$ .
- 1e Compute the acceleration  $a_{SC}$  of the satellite without propellants caused by this force. What is the velocity loss  $\Delta V_{atm}$  of the satellite due to drag during the life time  $t_{mission}$  of 5 years? What will be the consequence of this velocity loss?
- 1f To keep the satellite on an altitude of 500 km the velocity loss  $\Delta V_{atm}$  must be compensated by the orbit correction system. This rocket system uses as a propellant hydrazine, which produces an exhaust velocity  $c$  of 1800 m/s. Compute with Tsiolkowsky what must be the mass ratio  $M_0/M_e$  of the rocket system to compensate the velocity loss. Compute the required value of the wet mass  $M_0$  of the satellite and how much hydrazine  $M_{prop}$  is required.
- 1g Do you think that the acceleration  $a_{SC}$ , that you have computed in 1e, is correct? Is the actual acceleration larger or smaller than this value? What is the consequence for the required quantity of propellant?

##### Answers

- 1c  $D_{SC} = 6.52 \times 10^{-3} \text{ N}$
- 1d  $a_{SC} = 6.52 \times 10^{-6} \text{ m/s}^2$ ;  $\Delta V_{atm} = 1028.8 \text{ m/s}$
- 1f  $M_0 = 1771$ ;  $M_{prop} = 771 \text{ kg}$
- 1g  $a_{SC} = 3.68 \times 10^{-6} \text{ m/s}^2$

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### Problem 2 (30 minutes)

#### Panel surface finish and temperatures

A metal panel with an area  $A$  of  $1\text{m}^2$  is attached to a spacecraft. It is perfectly isolated from heat sources at its back side, while its front side is looking to the Sun and to cold space. Cold space has a temperature  $T_{sp}$  of  $2\text{K}$ . The solar absorptance  $\alpha$  of the metal is  $0.1$  (10% of the energy of the sunlight is absorbed), the infrared emissivity  $\varepsilon$  of the metal is  $0.04$  (4% of the infrared energy of the panel is emitted). The panel must be kept between  $250$  and  $400\text{K}$ .

- 2a Calculate the solar radiation (in Watts) absorbed  $Q_{abs}$  by the metal panel, assuming a Solar flux  $S$  of  $1420\text{W/m}^2$ .
- 2b Write down the formula for the heat balance for the metal panel. Does the area of the metal panel influence the temperature the panel will reach?
- 2c Calculate the temperature  $T_{pa}$  of the metal panel. The value of the Stefan-Boltzman constant  $\sigma$  is  $56.7051 \times 10^{-9}\text{W}/(\text{m}^2\text{K}^4)$ . Is this temperature in the range allowed?
- 2d To correct the temperature, you have two different surface finishes at your disposition: black paint and rear side metalized glass mirror (Second Surface Mirror). Optical properties of these materials are:

black paint  $\alpha = 0.9, \varepsilon = 0.9$

rear side metalized glass mirror:  $\alpha = 0.1, \varepsilon = 0.9$

Which surface finish will you use to keep the metal panel within its required temperature range? Which temperature does the panel reach? Why do you not use the other surface finish (motivate with numbers)?

#### Answers

2a  $Q_{abs} = 142\text{W}$

2c  $T_{pa} = 500.2\text{K}$

2d Black paint:  $T_{pa} = 398$ ; Second surface mirror:  $T_{pa} = 230\text{K}$

### Problem 3 (35 minutes)

#### Launcher selection

You are planning to launch 50 satellites of  $3000\text{kg}$  each in Geo-stationary Transfer Orbit (GTO). Your satellites cost  $100\text{M}\$$  per piece. Candidate launch vehicles are the Ariane 5 launcher, costing  $125\text{M}\$$  per piece and having an expected reliability of  $0.9$ , a Long March 2E of  $47\text{M}\$$  per piece and a reliability of  $0.75$  and a Proton D1e launcher of  $60\text{M}\$$  per piece and a reliability of  $0.85$ . Each of these launchers may launch 1 or more satellites at the same time up to the satellite mass in GTO. Performance of the launchers is:

Launcher	Satellite mass in GTO (kg)
Ariane 5	6800
Proton D1e	5500
Long March 2E	3700

- 3.1 Compute for each of the three launchers the cost per kg payload in GTO. Which launcher is the cheapest according to this calculation?
  - 3.2 How many launchers of each kind do you need to launch the 50 satellites, without taking into account the reliability?
  - 3.3 Calculate for each of the three launchers the required number launch vehicles, taking into account the reliability (so also the loss of satellites).
  - 3.4 What is for each of the three launchers the total launch cost, taking into account extra launch vehicles as a consequence of failures to be expected (and also the cost of extra satellites)? Which launcher do you select? Motivate your answer with numerical results.
-

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- 3.5 Can you think of a way to exploit the unused launch capacity of the launchers you have not selected?

#### Answers

- 3.1 *Proton*  
3.2 *Proton/Long March: 50 launchers; Ariane 5: 25 launchers*  
3.3 *Ariane 5: 28 launchers plus 3 extra satellites; Proton: 59 launchers plus 9 extra satellites; Long March: 67 launchers plus 17 extra satellites*  
3.4 *Ariane 5: 4100 M\$; Proton 4440 M\$; Long March 4849 M\$*

### EXAM JUNE 2003

#### Problem 1 (30 minutes)

##### Orbit choice for an astronomy mission

We want to observe the complete radiation spectrum of celestial bodies for long periods of time from space. It seems attractive to go far from the Earth, as

- The Earth itself is a source of radiation (heat and reflected sun light)
- Close to Earth the time, during which a celestial body (a point in inertial space) can be observed uninterruptedly, is limited (the Earth passes through the satellite – object line)
- We have long contact periods with the ground station, so we need less on-board storage of data, less sophisticated autonomy and see “cosmic” events in real-time.

There are however also disadvantages.

#### Questions

List the disadvantages of this concept from the following points of view:

- 1a Space environment
- 1b Launch capacity
- 1c Communication
- 1d Pointing accuracy
- 1e Electrical power
- 1f Orbit and altitude control

Be specific in your answers; give a rationale for each disadvantage you can identify.

#### Problem 2 (15 minutes)

##### Robot arm performance

A robot arm of 10 m length has to move a payload of 6000 kg mass. The actuator at the root of the arm exercises 300 Nm torque.

- 2a How fast has the robot arm moved the payload over 180 degrees?
- 2b To save electrical power the actuator is a small motor with a reduction gearbox (gear ratio 1:400). What is the motor torque?

#### Answers

- 2a  *$F = 30 \text{ N}$ ;  $a = 0.005 \text{ m/s}^2$ ; 180 degrees covered in 159.6 seconds*
- 2b *Required motor torque: 0.75 Nm*

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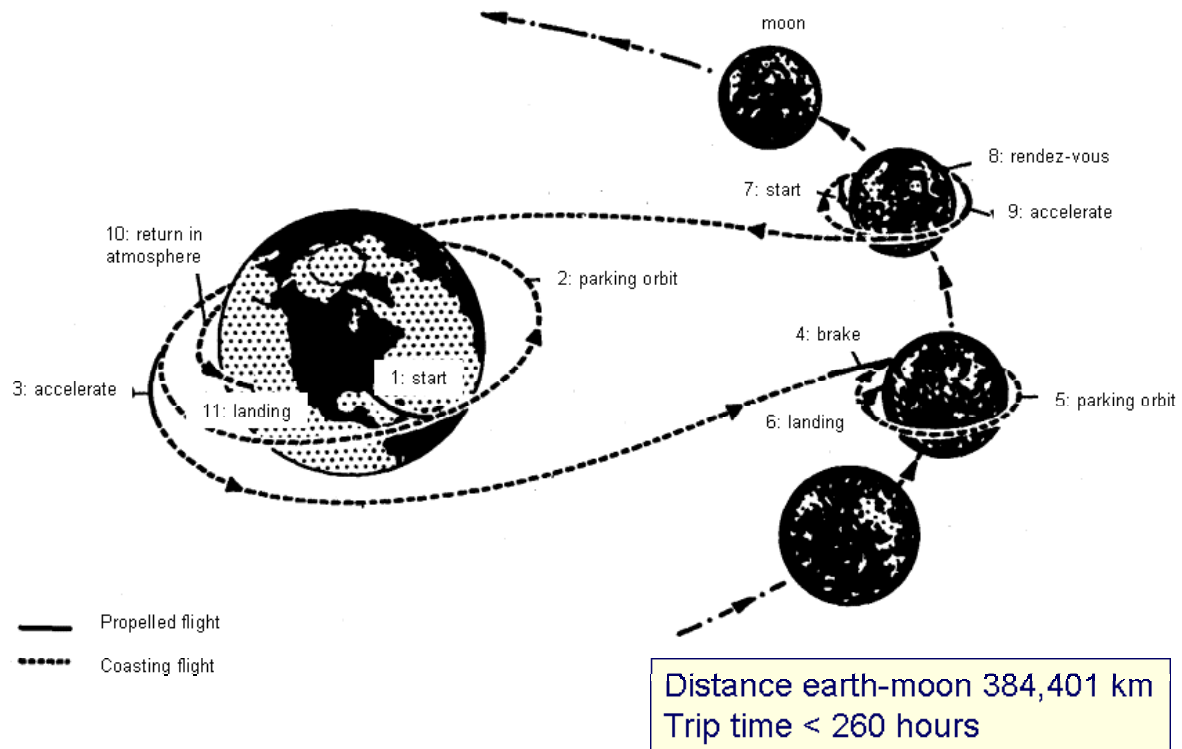
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### EXAM AUGUST 2003

#### Problem 1 (25minutes)

##### Apollo mission sequence

During the first successful Apollo mission, leading to a landing on the Moon, a number of events can be distinguished as shown in the picture below.



#### Questions

- 1a Describe the sequence of events of the Apollo mission and their purpose or function.  
[Example:  
3. accelerate: Get in a transfer orbit to the Moon; approach the Moon]
- 1b How many (rocket) stages can you distinguish in the Saturn V- Apollo Service Module – Lunar Module combination?
- 1c Assume that the Saturn-Apollo combination has six stages and that the typical velocity increment of a rocket stage is 3300 m/s. What is the total delta V that the Apollo mission can achieve? Is this a reasonable figure? Motivate your answer. Think of the escape velocity.

#### Answers

- 1b Six stages
- 1c Total delta V: 19800 m/s used for escape velocity (11200 m/s), atmospheric drag losses (250 m/s); gravity losses during powered flight (1000 m/s), moon decent and moon ascent

#### Problem 2 (20 minutes)

##### Comparison GEO and LEO satellite power subsystem

A geo-stationary satellite GEOSAT orbits the Earth at 35850 km altitude.

- 2a Assuming that the Sun is in the equatorial plane and that the radius of the Earth is



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- 6370 km, what is the duration of eclipse? Illustrate by means of a drawing.
- 2b What is the duration of the sunlit phase of the orbit?
- 2c Calculate the maximum duration of the eclipse for LEOSAT, a satellite in a low, equatorial Earth orbit of 500 km altitude and 90 minutes duration.
- 2d What is the duration of the sunlit phase of this low Earth orbit?
- 2e If both satellites need the same amount of power (1000 Watt), compare qualitatively:
- The size of the battery
  - The size of the solar array.
- Motivate your answer.

#### Answers

- 2a  $t_{\text{eclipse}} = 69.4$  minutes
- 2b Sunlit period: 1371 minutes
- 2c  $t_{\text{eclipse}} = 34$  minutes
- 2d Sunlit period: 56 minutes
- 2e Ratio (size LEOSAT battery/size GEOSAT battery)  $\approx 0.5$ ; ratio (solar array charge section LEOSAT/solar array charge section GEOSAT)  $\approx 12$

### TEST OCTOBER 2003

#### Problem 1 (7 minutes)

##### Space technology development drivers

Which were the driving forces for the rapid developments in space technology

- 1a In the period between 1940-1955?
- 1b In the period between 1955-1970?

Illustrate your answer with some simple examples.

#### Problem 2 (17 minutes)

##### Gravity gradient stabilization

A simple way to control the attitude of a spacecraft is to use the gradient in the gravity field of a celestial body.

- 1a Explain by means of a drawing, how gravity gradient stabilization works.
- 1b How do you achieve that a spacecraft is gravity gradient stabilized (think of the Moments of Inertia)?
- 1c What is the major advantage of gravity gradient stabilization?

#### Problem 3 (12 minutes)

##### IRAS orbit and ground station

The IRAS (Infra-Red Astronomy Satellite) has been designed to produce a map of the infra-red sources in the sky (infra-red sky survey). Therefore the satellite was launched in a sun synchronous orbit exactly over the borderline between the lit and the dark half of the Earth.

- 3a What is approximately the local time in the point on the Earth surface directly under the satellite (nadir)?
- 3b Where should the IRAS ground station be located to have maximum contact time with the satellite? Give a rationale for your answer.

#### Problem 4 (42 minutes)

##### LAGEOS orbit maintenance

LAGEOS is a spherical satellite with a dry mass  $M_e$  of 411 kg and a diameter  $D$  of 0,6 m. Its surface is covered with mirrors. By projecting a pulsed laser on the satellite from the Earth

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and by measuring the running time of the signal, the distance to the satellite can be determined accurately. The circular orbital altitude  $h_{orbit}$  of LAGEOS is 5900 km and the orbital period  $t_{orbit}$  is 228.4 minutes. The Earth radius  $R_E$  is 6370 km.

- 4a What is the velocity  $V_{orbit}$  of the satellite?
- 4b Calculate the acceleration due to air drag  $a_{drag}$  of the LAGEOS satellite assuming a drag coefficient  $C_D$  of 2.0, an air density  $\rho$  of  $5.4 \times 10^{-18}$  kg/m<sup>3</sup>.
- 4c What is the total velocity change  $\Delta V$  in one year?
- 4d What is the total error in the distance travelled  $\Delta s$  in that year, both in absolute and relative (%) terms? Assume that the orbital altitude does not change over time.
- 4e Assuming that the force executed by solar radiation is negligible, which force can be determined very accurately by means of LAGEOS? Motivate your answer.
- 4f Assuming that you use pressurized nitrogen gas and thrusters with an exhaust velocity  $w_e$  of 80 m/s, how much gas ( $M_p$  in kg) would you need to compensate the velocity loss during that year (use Tsiolkowski)? Does it seem useful to incorporate such an orbit control system in the satellite? Motivate your answer.

#### Answers

- 4a  $V_{orbit} = 5625$  m/s.
- 4b  $a_{drag} = 1.16 \times 10^{-13}$  m/s<sup>2</sup>
- 4c Total velocity change in one year due to air drag:  $3.66 \times 10^{-6}$  m/s
- 4d The absolute total error in distance traveled in one year due to air drag: 57.8 m;  
relative position error  $3.26 \times 10^{-8}$  %
- 4f  $M_p$  0.0000188 kg

#### Problem 5 (35 minutes)

##### Impact of orbit altitude on receiver power

This problem compares the receiver power required for a mobile telephone for a system using satellites in Low Earth Orbit (LEO; orbital altitude  $h_{LEO}$  500 km) or geo-stationary satellites (GEO;  $h_{GEO}$  orbital altitude 35850 km). The Earth radius  $R_E$  is 6370 km.

- 5a Assuming the energy of a radio signal for a given antenna and a given transmitter power expands in a cone, what is the ratio of the energy (power) loss  $P_{LEO}/P_{GEO}$  of the communication link with the LEO satellite and of the GEO satellite?
- 5b Assuming the receiver of your hand-held telephone needs 0.2 W receiver power for the system with LEO satellites, which power  $P_{rec(GEO)}$  would it need for that same system of satellites on geostationary altitude? Is this a reasonable and feasible value?
- 5c What is the minimum required angle of the satellite antenna beam  $\alpha$  when it should cover the whole visible Earth? Calculate this for the LEO case and the GEO case. Draw a picture.
- 5d The antenna beam needed in case of the GEO satellite is narrower than that needed for the LEO case. What is the ratio of the area of the LEO beam and the GEO beam?
- 5e Assuming the beam width of the GEO satellite is adapted to the minimum desired dimension, what becomes now the receiver power  $P_{rec(GEO)}$  required for your hand-held phone for the system with GEO satellites (combine the answer of 5b and 5d)?

#### Answers

- 5a The ratio of the power loss of the communication link at GEO and at LEO altitude is 0.000195
- 5b  $P_{rec(GEO)} = 1028$  W
- 5c  $\alpha_{LEO} = 68.01^\circ$ ,  $\alpha_{GEO} = 8.68^\circ$
- 5d The strength of the signal from the smaller beam is 61.42 times larger than that of the

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wider beam.

5e  $P_{rec(GEO)} = 16.74 \text{ W}$

### EXAM JUNE 2004

#### Problem 1 (45 minutes)

##### Solar sailing to the boundary of the Solar System

You are requested to estimate major system sizing and mission parameters for a mission to the boundaries of our solar system. The propulsive method that will be used for the mission is solar sailing, that is using the pressure exerted by the Sun's radiation. The force  $F_s$  exerted by the radiation can be expressed as:

$$F_s = \frac{S}{c} \cdot A \cdot (2 - \alpha) \cdot \cos i$$

where  $A$  is the surface area of the solar sail,  $S$  is the solar flux in  $\text{W/m}^2$  (equals  $1400 \text{ W/m}^2$  at the Earth),  $\alpha$  is the solar absorptance of the solar sail ( $\alpha = \text{unity} - \text{reflection}$ ),  $i$  is the angle between the sun vector and the surface normal and  $c$  is the speed of light (equals  $3 \times 10^8 \text{ m/s}$ ).

During the trip the spacecraft will not always have "the Sun in the back", so you may assume that the propulsive force is only active during 50% of the travelling time. The remainder of the time the sail will be turned in such a way that no force will be exerted on the spacecraft. You may also assume that during the propulsive phase of the trip the angle  $i$  equals 0 degrees.

1a For which value of the solar absorptance  $\alpha$  is the propulsive force maximum? What kind of material has such properties?

You are now going to estimate the time it takes to travel towards the edge of the solar system. To this purpose you are going to estimate the propulsive force at several locations along the road. The solar absorptance of the solar sail equals 0.1. Remember that the solar flux  $S$  is decreasing with the square of the distance to the Sun. The intermediate points in the trajectory are:

Location	Distance to the Sun (km)
Earth (starting point)	$149.6 \times 10^6$
Mars	$227.9 \times 10^6$
Jupiter	$778.3 \times 10^6$
Uranus	$2870 \times 10^6$
Pluto (end of trip)	$5900 \times 10^6$

1b Calculate for each of these locations the magnitude of the solar flux in  $\text{W/m}^2$ .

Your spacecraft has a mass of 250 kg and the solar sail has an area of  $100 \text{ m}^2$ .

1c Calculate for each of the intermediate points the solar radiation force  $F_s$  and the acceleration of the spacecraft  $a_{s/c}$ . Do not take into account the 50% "efficiency" of the propulsive phase.

You may assume that the average acceleration during each stage of the trip is the average of the acceleration at the beginning of stage and at the end of the stage (e.g. the acceleration during the stage Earth - Mars is  $0.5 \times$  (acceleration at the Earth + acceleration at Mars).

1d Calculate for each of the four stages the average acceleration  $a_{av}$ .

1e Calculate the length (in meters) and duration (in seconds) of the stage Earth - Mars, assuming the propulsive force is acting continuously giving the average acceleration as you have calculated in 1d (so not taking into account, that it is available only 50% of the time) and that you are traveling in a straight line along the planets. Calculate also the velocity your spacecraft has at the end of the stage.

1f Estimate the time it takes to travel from Mars to Jupiter, from Jupiter to Uranus and from Uranus to Pluto, assuming no acceleration by the solar sail (so traveling with the

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velocity as you have calculated in 1e, which you set equal to 625 m/s from now on, and you travel in a straight line). Calculate also what the maximum velocity increment (delta V) due to the solar sail would have been during these stages, using the average accelerations you have calculated in 1d. How does this compare with the velocity you had at Mars? Do you think it is worthwhile to use solar sailing in these later stages?

- 1g What is the total mission duration to travel to Mars including the 50% availability of the propulsive power?
- 1h Do you think that the assumption of 50% availability of propulsive power and the accuracy of the result of the calculations in 1f and 1g is acceptable, that is that the order of magnitude of your estimate is correct? Motivate your answer.

The maximum time for the mission to Mars should be comparable to the one achieved with chemical propulsion: two years.

- 1i How much solar sail area is needed to achieve this mission duration? Try to answer this by using the same assumptions as above and by observing the formulae you have used to calculate the traveling time in question 1d and 1e.

### Answers

2b

Location	$S$ ( $W/m^2$ )
Earth	1400
Mars	603
Jupiter	52
Uranus	4
Pluto	0.9

2c

Location	$F_s$ (N)	$a_{S/C}$ ( $m/s^2$ )
Earth	8.87E-4	3.55E-6
Mars	3.82E-4	1.53E-6
Jupiter	3.28E-5	1.31E-7
Uranus	2.41E-6	9.64E-9
Pluto	5.70E-7	2.28E-9

2d

Stage	Average acceleration $a_{av}$ ( $m/s^2$ )
Earth-Mars	2.54E-6
Mars-Jupiter	8.30E-7
Jupiter-Uranus	7.03E-8
Uranus-Pluto	5.96E-9

2e

Stage	Distance (m)	Travel time (s)	$a_{av}$ ( $m/s^2$ )	$V_{Mars}$ (m/s)
Earth-Mars	7.83E10	2.48E8	2.54E-6	630

2f

Stage	Distance (m)	Travel time (s)	$a_{av}$ ( $m/s^2$ )	Delta V (m/s)
Mars-Jupiter	5.50E11	8.66E8	8.30E-7	718
Jupiter-Uranus	2.09E12	3.29E9	7.03E-8	231
Uranus-Pluto	3.03E12	4.77E9	5.96E-9	29

2g Travel time to Mars 15.7 years.

2i Solar sail area 6200  $m^2$ .

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### EXAM AUGUST 2004

#### Problem 1 (15 minutes)

##### Space Market Segmentation

Several distinct segmentations of the Space Market are used. One of them is a segmentation according to application (where the product or service is for).

1a List the market segments according to this segmentation.

Another segmentation is the one according to the type of customer.

1b Give the two types of customer, that are generally distinguished.

1c Which are the major drivers for each of these types of customer in term of performance and cost?

In a third segmentation the term "Infrastructure" is used.

1d Describe what is contained in that segment.

1e List and describe two of the other three segments that are used in this segmentation.

#### Problem 2 (30 minutes)

##### International Space Station orbit maintenance

The International Space Station (ISS) is traveling in a circular, low Earth orbit at an altitude  $h_{orbit}$  of 407 km. It has an orbital period  $t_{orbit}$  of 93 minutes.

2a Calculate the orbital velocity  $V_{orbit}$  of the ISS in m/s, assuming that the radius of the Earth  $R_{Earth}$  equals 6378 km.

The dimensions of the ISS are 80 x 109 x 44 m and the density of the atmosphere  $\rho_a$  at 407 km altitude varies between 7.3E-13 (solar minimum) and 7.5E-12 kg/m<sup>3</sup> (solar maximum). You may assume that the effective area of the ISS subjected to aerodynamic drag is 10% of the total "frontal" area and that the drag coefficient  $C_D$  equals 2.0.

2b Calculate the maximum and minimum area of the ISS, assuming the velocity vector is always normal to one of the sides of the Space Station "box" and taking into account the 10% effective area.

2c Write down the formula for the aerodynamic drag  $F_D$  of an object in space. Is this expression the same as the one used for aircraft? If so, why do you think that is the case?

2d Calculate the minimum and the maximum value for the atmospheric drag force the ISS is experiencing.

The mass of the International Space Station  $M_{ISS}$  is 455 tonnes.

2e Calculate the maximum and minimum value of the deceleration of the ISS.

2f What is the minimum velocity loss (negative delta V) of the ISS in one orbit? And how large is the maximum velocity loss of the ISS in three months?

2g You are going to correct the orbital altitude for this maximum velocity loss of 66 m/s. Assuming you have propellant on board with an effective rocket exhaust velocity  $w_e$  of 3000 m/s, calculate with Tsiolkowsky the mass ratio of the ISS (that is the ratio between mass before orbit correction  $M_0$  and the mass after completion of the orbit correction  $M_e$ ).

2h If  $M_0$  equals to 455 tonnes, how much propellant  $M_p$  is needed for this orbit correction? If a Progress vehicle can lift 2500 kg propellant to the ISS, what is the maximum frequency of Progress flights during a solar maximum?

#### Answers

2a  $V_{orbit} = 7640$  m/s

2b *Maximum effective area of ISS 872 m<sup>2</sup>; minimum 352 m<sup>2</sup>*

2d *Minimum:  $F_D = 0.015$  N; maximum 0.382 N*

2e *Minimum deceleration: 3.3E-8 m/s<sup>2</sup>; maximum 8.4E-6 m/s<sup>2</sup>*

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- 2f *Minimum velocity loss is per orbit  $1.84E-4$  m/s; maximum velocity loss in three months: 66 m/s*
- 2g  *$M_O/M_e = 1.022$*
- 2h *4 Progress flights per 3 months*