Problem 2 (35 points)

This problem (2) is related to the part *Rocket Motion* of the course *ae1-801*.

Many modern launch vehicles are equipped with booster to enhance the performance during the initial portion of the flight when the rocket is still very heavy. The boosters are usually a number of relatively small solid propellant rockets, clustered around the base of a large central core rocket consuming liquid propellants. Depending on the configuration, some of the boosters are ignited simultaneously with the core engine and some may be ignited at a later time. In general, the boosters have a shorter burn time than the core stage.

You are being asked to evaluate the performance of a conceptual new rocket (Fire Flash-6) consisting of a central core and 6 boosters. Three boosters are ignited at the launch platform, simultaneously with the core engine. After burnout they are jettisoned, and at the same time the other three boosters are ignited. They are also jettisoned immediately after burnout. The rocket flies a vertical trajectory.

The following design parameters are available:

Total mass of the central core rocket	Mc_0 = 100,000 kg
Total propellant mass of the central core rocket	Mc_p = 90,000 kg
Thrust of central core engine	Fc = 1000 kN
Specific impulse of the central core engine	Isp_c = 300 s
Total mass of each booster	Mb_0 = 10,000 kg
Total propellant mass of each booster	Mb_p = 9000 kg
Specific impulse of the boosters	Isp_b = 200 s
Thrust-to-weight ratio at liftoff	ψ_0 = 1.2105

As already mentioned, the rocket moves perfectly vertical throughout the flight. In addition you may assume that the flight takes place in a homogeneous gravity field ($g = g_0$ = 9.81 m/s2 = constant) and in vacuum. At the moment of burnout of the central core it has reached an altitude of 358 km above sea level.

To assess the performance you should answer the following questions:

a. Compute the total lift-off mass of the rocket (2 pts)
b. Compute the thrust of a single booster rocket (4 pts)
c. Compute the burn time of the booster rockets (2 pts)
d. Compute the total burn time of the central core rocket (2 pts)
e. List the expression for the consecutive calculation steps that are needed to compute the velocity of the rocket at burnout of the central core (9 pts). *Hint: In one of the steps, set up an equation to compute the effective "average" specific impulse for the combined rocket engines during the boost phase.*
f. Follow the calculations steps you have listed under e) and compute the velocity of the rocket at burnout of the first set of 3 boosters, at burnout of the second set of 3 boosters and at burnout of the central core. (8 pts)
g. List the consecutive calculation steps that are necessary to compute the total duration of the flight after ignition of the engines (t_0) until the core stage crashes on the ground. (4 pts)
h. Follow the calculations steps you have listed under g) and calculate the total flight duration from the moment of ignition of the engines until the core stage crashes on the ground. (4 pts). If you did not succeed in calculating the velocity under question f), assume Ve_2 = 4828.5 m/s.

N.B. Derivations of formulae like Tsiolkowski's equation are not requested, but may be helpful to find the correct expressions.

a) $M_{total} = 6 \cdot M_{b_0} + M_{c_0}$

b) $\psi_0 = \frac{T}{M_0 \cdot g_0}$ given ψ_0 is for 3 boosters + core.

Where $M_0 = M_{total}$

$\Rightarrow \psi_0 = \frac{3F_b + F_c}{M_0 \cdot g_0}$

$F_b = \frac{\psi_0 \cdot M_0 \cdot g_0 - F_c}{3}$

$F_b = 300$ kN

c) $t_{b,b} = \frac{M_{b_p}}{\dot{m}_b}$ $\quad\quad$ $\dot{m}_b = \frac{F_b}{I_{spec,b} \cdot g_0} = 152{,}91$ kg/s

So the burn time of a booster is

$t_{b,b} =$ 58,86 s

d) Use the same method as c)

$t_{b,c} = \frac{M_{c_p}}{\dot{m}c}$ $\quad\quad$ $\dot{m}_c = \frac{F_c}{I_{spec,c} \cdot g_0} = 339{,}789$ kg/s

$t_{b,c} = 264{,}87$ s

little check: 'in general, the boosters have a shorter burn time than the core stage'

so $2 \cdot t_{b,b} \leq t_{b,c}$, because in general you don't throw off the core first and then the boosters.

So the core has a greater burn time.

e) Here we are going to calculate 3 stages

[1] first stage: core + 3 boosters @ lift-off

[2] second stage: core + 3 boosters + jettisoning the 3 empty boosters

[3] final stage: core + jettisoning the other 3 empty boosters

[1] first stage:

$\dot{m}_{total} = \dot{m}_c + 3 \cdot \dot{m}_b$

$F_{total} = F_c + 3 \cdot F_b$

Calculate $I_{spec,avg}$ $\Rightarrow I_{spec,avg} = \frac{F_{total}}{\dot{m}_{total} \cdot g_0} = \frac{3\dot{m}_b \cdot F_{spec,b} + \dot{m}_c \cdot F_{spec,c}}{3\dot{m}_b + \dot{m}_c}$

$M_{0,1} = M_{total} = 160 \cdot 10^3$ kg

$M_{e,1} = M_{0,1} - 3 \cdot M_{b_p} - \dot{m}_c \cdot t_{b,b}$

The jettisoning of the boosters will be between steps (1) and (2).

This is because jettisoning of boosters isn't used for propulsion.

So there is no momentum change caused by the jettisoning of boosters.

$$\Lambda_1 = \frac{M_{0,1}}{M_{e,1}}$$

$V_{e1} = I_{spec,avg} \cdot g_o \cdot \ln \Lambda_1 - g_o \cdot t_{b,b}$

2 second stage

Use the same $I_{spec,avg}$, because $\dot{m}_{total}$, F_{total} & g_0 don't change.

$M_{0,2} = M_{e,1} - 3 \cdot (M_{b_0} - M_{b_p})$

$3 \cdot (M_{b_0} - M_{b_p})$ refers to jettisoning 3 empty boosters.

$M_{e,2} = M_{0,2} - 3 \cdot M_{b_p} - \dot{m}_c \cdot t_{b,b}$

$$\Lambda_2 = \frac{M_{0,2}}{M_{e,2}}$$

$V_{e2} = V_{e1} + I_{spec,avg} \cdot g_o \cdot \ln \Lambda_2 - g_o \cdot t_{b,b}$

3 final stage

Calculate the remaining time of the core after burnout of 6 boosters.

$t_r = t_c - 2 \cdot t_{b,b}$

$M_{0,3} = M_{e,2} - 3 \cdot (M_{b_0} - M_{b_p})$

$M_{e,3} = M_{c_0} - M_{c_p}$

For check one can use:

$M_{e,3} = M_{0,3} - \dot{m}_c \cdot t_r$

$$\Lambda_3 = \frac{M_{0,3}}{M_{e,3}}$$

$V_{e3} = V_{e2} + I_{spec,c} \cdot g_o \cdot \ln \Lambda_3 - g_o \cdot t_r$

Don't add V_{e1} to V_{e3}, because V_{e1} is already added to V_{e2}.

f)

1 first stage:

$\dot{m}_{total} = 798{,}52$ kg/s

$F_{total} = 1{,}9$ MN

$I_{spec,avg} = 242{,}55$ s

$M_{0,1} = 160 \cdot 10^3$ kg

$M_{e,1} = 113 \cdot 10^3$ kg

$V_{e1} = 250$ m/s

2 second stage:

$M_{0,2} = 110 \cdot 10^3$ kg

$M_{e,2} = 63 \cdot 10^3$ kg

$V_{e2} = 998{,}74$ m/s

3 final stage:

$t_r = 147{,}15$ s

$M_{0,3} = 60 \cdot 10^3$ kg

$M_{e,3} = 10 \cdot 10^3$ kg

check:

$M_{e,3} = 10000{,}04865$ kg

$V_{e3} = 4828{,}34$ m/s

g)

To find the total time, we need to add up the launch time, the coasting time and the drop time. The coasting time can be found using:

$$t_c = \frac{V_{e3}}{g_o}$$

The coasting height is:

$$h_c = \frac{1}{2}\frac{V_{e3}^{\ 2}}{g_o}$$

The maximum height of the rocket then is:

$$h_a = 358 \cdot 10^3 + h_c$$

Now we can find the fall time:

$$h_a = \frac{1}{2} g_o \cdot t^2 \Rightarrow t_{fall} = \sqrt{\frac{2 \cdot h_a}{g_o}}$$

$$t_{total} = t_{b,c} + t_c + t_{fall}$$

h)

use $V_{e3} = 4828{,}5$ m/s as final velocity of core

$t_c = 492{,}20$ s

$h_c = 1188298{,}28$ m

$h_a = 1546298{,}28$ m

$t_{fall} = 561{,}47$ s

$t_{total} = 1318{,}54$ s

