Problem 3. Mission to Pluto

Consider a satellite mission from the Earth to Pluto. The interplanetary flight from the Earth to Pluto is conducted by means of a Hohmann transfer orbit. The satellite starts from a circular parking orbit at 800 km above the surface of the Earth.

a. What is the average mass density ratio of Pluto compared to Earth?

b. Derive the orbital revolution periods (in years) of the Earth and Pluto around the Sun.

c. What is the apocenter location and what is the pericenter location of the Hohmann transfer orbit?

d. Compute the travel time from the Earth to Pluto.

e. Compute the synodic period for the Pluto-Earth-Sun constellation.

f. How can the travel time to Pluto be reduced significantly?

g. Compute the required velocity change to launch the satellite from its parking orbit around the Earth into the Hohmann transfer orbit.

Problem 4. Perturbing forces/coverage

Perturbing forces, i.e. forces that deviate from the central gravity field term, cause a satellite's orbit around the Earth to deviate from a perfect Keplerian orbit. These perturbing forces can sometimes be advantageous for certain satellite orbits around the Earth.

The argument of perigee of satellites flying in an orbit with *critical inclination* is constant.

a. Compute the *critical inclinations.*

b. Explain how Russia used this *critical inclination* to her advantage when designing the *Molnya* orbits for telecommunication.

A satellite with a nadir looking altimeter is flying in a circular orbit with an inclination of 108 degrees.

c. Compute the percentage of the total surface of the Earth that can never be covered by the altimeter. The Earth is represented by a perfect sphere.

For problems 3 and 4 the following equations may be used without derivation:

$$\Delta\omega = \frac{3}{2}\pi J_2 \left(\frac{R_e}{p}\right)^2 (5\cos^2 i - 1) \ (\textit{per orbital revolution}) \tag{1}$$

$$\Delta\Omega = -3\pi J_2 \left(\frac{R_e}{p}\right)^2 \cos i \ (\textit{per orbital revolution}) \tag{2}$$

Furthermore, the following parameters are given:

- Mean radius of the Earth: $R_e = 6371$ km
- Mean radius of Pluto: $R_{Pluto} = 0.178\ R_e$
- Gravity parameter of the Earth: $\mu_e = 398600$ km^3/s^2
- Gravity parameter of Pluto: $\mu_{Pluto} = 0.00213\ \mu_e$
- Gravity parameter of the Sun: $\mu_{Sun} = 1.3271 \times 10^{11}$ km^3/s^2
- Distance Earth-Sun : $r_e = 1$ AU
- Distance Pluto-Sun: $r_{Pluto} = 39.0$ AU
- 1 AU $= 150 \times 10^6$ km
- Second zonal harmonic coefficient: $J_2 = 1.082627 \times 10^{-3}$

Grading:
- Problem 3: 25 points
- problem 4: 15 points

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a) $$\frac{\rho_{pluto}}{\rho_{earth}} = \frac{M_{pluto}}{M_{earth}} \times \frac{R_{earth}^3}{R_{pluto}^3}$$

$M_{pluto} = 0{,}00213 \cdot M_{earth}$.
$R_{earth} = 6371$ km but
$R_{pluto} = 0{,}178\, R_{earth}$.

$$= \frac{0{,}00213 \cdot M_e}{M_e} \times \frac{R_e^3}{(0{,}178 R_e)^3}$$

$$= 0{,}00213 \times \frac{1}{0{,}178^3} = 0{,}377$$

b) orbital period of earth = 1 year
" " of pluto = $\cancel{T_e \sqrt{\left(\frac{R_{pluto}}{R_{earth}}\right)^3}}$

$r_p = 39.0$ AU
$r_e = 1.0$ AU
$T_e = 1$

$$= T_e \sqrt{\left(\frac{r_p}{r_e}\right)^3}$$

so Orbital period $T_p = 243{,}55$ j

c) Pericentre is the position of the earth at departure
Apocentre is the " " " pluto at arrival

d) $T_{homann} = \frac{1}{2}\sqrt{\left(\frac{r_e + r_p}{2}\right)^3}$

$= \frac{1}{2}\sqrt{\left(\frac{40}{2}\right)^3} = 44{,}72$ jaren.

e) Synodic period $= \dfrac{1}{\frac{1}{T_e} - \frac{1}{T_p}}$ T_e = orbital period of earth, T_p = orbital period of Pluto.

$= \dfrac{1}{\frac{1}{1} - \frac{1}{243.55}} = 1{,}004$ years ($\cancel{3}$b)

f) Travel time can be reduced by swing by's

g) ~~....~~ See summary:

5.3 $^{+5.4}$ Example travel - Sun-Centered Pa

$\Delta V_e = V_{he} - V_{ec} = 11.316 - 7{,}726 = \underline{3{,}590 \text{ km/s}}$

$\Rightarrow$ the answer (although, the method to get the answer)

Problem 4   Perturbing forces/coverage

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a) $\Delta\omega = \frac{3}{2} n J_2 \left(\frac{R_e}{p}\right)^2 (5\cos^2 i - 1) = \underline{\underline{0}}$

To be zero, $5\cos^2 i = 1$

$$\cos^2 i = \frac{1}{5}$$

$$\cos i = \frac{1}{\sqrt{5}}$$

So $i = 63{,}43°$ $i = \arccos \frac{1}{\sqrt{5}}$

or $180 - 63{,}43° = 116{,}56°$

b) The apogee is above Russia, and the apogee is the most far part (the highest point above Russia) and because of that, the velocity is the lowest and so the satellite will stay the longest above Russia.

c) Spherical surface area $= 2\pi(1-\cos\alpha)R^2$

Total surface area $= 4\pi R^2$

$\alpha = (90 - \text{inclinate angle})$

missing Area: $2 \times 2\pi(1-\cos\alpha)R^2$

missing Area $= (1-\cos\alpha) \times 100\% = 4{,}89$

$\alpha = 108 - 90 = 18$