AE1102 (Introduction to Aerospace II) Space exercises

Please note: This document provides detailed solutions to the problems that are stated in the lecture slides by prof. Ambrosius during course year 2010-2011. Since some problems were quite hard to solve without any guidance, I created this document. Referenced page numbers in this document may change over the years, due to changes that will be made by the lecturers. However, I expect the slides to remain in use for several years.

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Section 1: Orbital Mechanics

Questions from file: AE1102_space_slides_1-3.pdf

Page 29

Compute the value of g0.

$$g = \frac{\mu}{r^2} \Rightarrow g_0 = \frac{GM_{Earth}}{\left(R_{Earth} + h\right)^2} = \frac{398601 \cdot 10^9}{\left(6378 \cdot 10^3 + 0\right)^2} = 9,80ms^{-2}$$

Compute the velocity of a satellite at 800 km altitude (Envisat).

$$V_{orbit} = R_E \sqrt{\frac{g_0}{R_E + h_{orbit}}} = 6378 \cdot 10^3 \sqrt{\frac{9,80}{\left(6378 + 800\right) \cdot 10^3}} = 7,45 \cdot 10^3 ms^{-1}$$

Compute the velocity of the Moon around the Earth.

$$V_{orbit} = R_E \sqrt{\frac{g_0}{R_E + h_{orbit}}} = 6378 \cdot 10^3 \sqrt{\frac{9,80}{\left(6378 + 384000\right) \cdot 10^3}} = 1,01 \cdot 10^3 ms^{-1}$$

Compute the orbital height of a geostationary satellite.

$$T = \frac{2\pi\left(R_E + h\right)}{V_{orbit}} = \frac{2\pi\left(R_E + h\right)}{R_E\sqrt{\frac{g_0}{R_E + h}}} \Rightarrow T \cdot R_E \cdot \sqrt{\frac{g_0}{R_E + h}} = 2\pi\left(R_E + h\right) \Rightarrow$$

$$T^2 R_E^2 \frac{g_0}{R_E + h} = 4\pi^2\left(R_E + h\right)^2 \Rightarrow \frac{T^2 R_E^2 g_0}{4\pi^2} = \left(R_E + h\right)^3 \Rightarrow \sqrt[3]{\frac{T^2 R_E^2 g_0}{4\pi^2}} = R_E + h \Rightarrow$$

$$h = \sqrt[3]{\frac{T^2 R_E^2 g_0}{4\pi^2}} - R_E = \sqrt[3]{\frac{\left(\left(23 \cdot 60 + 56\right)60\right)^2 3678000^2 \cdot 9,80}{4\pi^2}} = 35787 \cdot 10^3 m$$

Note: T is in seconds!

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What is the velocity of the payload at 90 degrees?

$$V = at = 0,005 \cdot 79,3 = 0,4ms^{-1}$$

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Is the orbital velocity of a spacecraft dependent of its mass or not?

No, orbital velocity is a function of height: $V_{orbit} = R_E \sqrt{\frac{g_0}{R_E + h_{orbit}}}$.

What happens to a free-floating astronaut, when he swings a hammer and hits the nail?

The astronaut will experience a reaction force equal to the force that he exerts on the nail, causing him to accelerate away from the nail according to $F = ma$.

What happens to a free-floating astronaut when he torques a bolt?

The astronaut will accelerate in a direction perpendicular to the wrench that he is using.

What is the motion of an object within the ISS if it remains untouched?

None with respect to the position of the ISS, in other words, equal to motion of the ISS itself.

Why is it possible to move heavy objects near the ISS with small force?

Because there is no gravity, the product mg is zero. In other words, objects have no weight.

Why is it possible to dock a 120 ton Space Shuttle with a 330 ton ISS?

I don’t really get the question. Anyway, the weights are not relevant.

Questions from file: AE1102_space_slides_5-6.pdf

Page 14

Consider the Earth. What is the radial acceleration?

1. at sea surface

Note: this question is the same as the first question in this document!

$$g = \frac{\mu}{r^2} \Rightarrow g_0 = \frac{GM_{Earth}}{\left(R_{Earth} + h\right)^2} = \frac{398601 \cdot 10^9}{\left(6378 \cdot 10^3 + 0\right)^2} = 9,80ms^{-2}$$

2. for an earth-observation satellite at 800 km altitude

$$g = \frac{\mu_E}{R^2} = \frac{\mu_E}{\left(R_{Earth} + h\right)^2} = \frac{398601 \cdot 10^9}{\left(6378 \cdot 10^3 + 800 \cdot 10^3\right)^2} = 7,74ms^{-2}$$

3. for a GPS satellite at 20200 km altitude

$$g = \frac{\mu_E}{R^2} = \frac{\mu_E}{\left(R_{Earth} + h\right)^2} = \frac{398601 \cdot 10^9}{\left(6378 \cdot 10^3 + 20200 \cdot 10^3\right)^2} = 0,56ms^{-2}$$

4. for a geostationary satellite at 35800 km altitude

$$g = \frac{\mu_E}{R^2} = \frac{\mu_E}{\left(R_{Earth} + h\right)^2} = \frac{398601 \cdot 10^9}{\left(6378 \cdot 10^3 + 35800 \cdot 10^3\right)^2} = 0,22ms^{-2}$$

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Consider the situation of the Earth, the Sun and a satellite somewhere on the line connecting the two main bodies. Where is the point where the attracting forces of the Earth and the Sun, acting on the satellite, are in equilibrium?

$$F_{Earth} = F_{sun} \Rightarrow G\frac{m_{Earth} \cdot m_{sat}}{r_1^2} = G\frac{m_{sun} \cdot m_{sat}}{r_2^2} \Rightarrow \frac{\mu_{Earth}}{r_1^2} = \frac{\mu_{sun}}{r_2^2} \Rightarrow$$

$$\frac{\mu_{Earth} \cdot r_2^2 - \mu_{sun} \cdot r_1^2}{r_1^2 \cdot r_2^2} = 0$$

r ≠ 0 and r_2 = AU-r_1:

$$\mu_{Earth}\left(AU - r_1\right)^2 - \mu_{sun} r_1^2 = 0 \Rightarrow \mu_{Earth}\left(AU^2 - 2r_1 + r_1^2\right) - \mu_{sun} \cdot r_1^2 = 0$$

$$\left(\mu_{sun} - \mu_{Earth}\right) r_1^2 + 2\mu_{Earth} r_1 - \mu_{Earth} AU^2 = 0 \Leftrightarrow \left\{ax^2 + bx + c = 0\right\}$$

Solving gives one positive answer: $r_1 = 2,58 \cdot 10^8 m$

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Consider an obsolete GEO satellite which is put into a graveyard orbit: 300 km above the standard GEO altitude.
What is the orbital period of this graveyard orbit?

$$T_{graveyard} = 2\pi\sqrt{\frac{(h_{geo}+300)}{\mu_{Earth}}} = 2\pi\sqrt{\frac{(42164,14+300)}{398600,4415}} = 87085,2s$$

Note: Dimensional analysis: $[s] = \sqrt{\frac{[km]^3}{\left[\frac{km^3}{s^2}\right]}} = \sqrt{[s^2]} = [s]$

$$87085,2s = 24h + 11m + 25,2s$$

If this graveyard orbit were to develop from perfectly circular to eccentric, what would be the maximum value of this eccentricitywhen the pericenter of this orbit were to touch the real GEO?

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{(42164,14+300)-42164,14}{(42164,14+300)+42164,14} = 0,003545$$

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Consider a satellite at 800 km altitude above the Earth's surface, with its velocity perpendicular to the radius.
compute the escape velocity

$$V_{esc} = \sqrt{\frac{2\mu}{r_p}} = \sqrt{\frac{2\mu_{Earth}}{r_{Earth}+h}} = \sqrt{\frac{2\cdot 398601}{6378+800}} = 10,538km/s$$

if the satellite would have a velocity 0.2 km/s larger than the escape velocity,
what would the excess velocity V∞ be?

$$V^2 = V_{esc}^2 + V_\infty^2 \Rightarrow V_\infty = \sqrt{V^2 - V_{esc}^2} = \sqrt{10,738^2 - 10,538^2} = 2,063km/s$$

idem, if $V=V_{esc}+1.0$?

$$V^2 = V_{esc}^2 + V_\infty^2 \Rightarrow V_\infty = \sqrt{V^2 - V_{esc}^2} = \sqrt{11,538^2 - 10,538^2} = 4,699km/s$$

Questions from file: AE1102_space_slides_7-8.pdf

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1. How much ΔV would be needed to transfer from a circular parking orbit at 185 km altitude to the pericenter of an orbit with h_p = 185 km and h_a = 35822 km? The orbits are coplanar, i.e. within the same plane.

Using the Vis Viva equation, we see that

$$\Delta V = V_2 - V_1 = \sqrt{\mu\left(\frac{2}{r}-\frac{1}{a}\right)} - \sqrt{\frac{\mu}{r}}$$

This gives

$$\Delta V = \sqrt{\mu\left(\frac{2}{r}-\frac{1}{a}\right)} - \sqrt{\frac{\mu}{r}} = \sqrt{\mu_E\left(\frac{2}{R_E+h_p}-\frac{1}{\frac{1}{2}\left(\left(R_E+h_p\right)+\left(R_E+h_a\right)\right)}\right)} - \sqrt{\frac{\mu_E}{R_E+h_{park}}} =$$
$$\sqrt{\mu_E\left(\frac{2}{6378+185}-\frac{1}{\frac{1}{2}\left(\left(6378+185\right)+\left(6378+35822\right)\right)}\right)} - \sqrt{\frac{398601}{6378+185}} = 2,4596 km/s$$

2. How much ΔV would be needed to change the inclination of an orbit from 29.8° to 0°, if both orbits are circular and at an altitude of 35822 km?

Express ΔV as a function of V and i:

$$\sin\left(\tfrac{1}{2}\Delta i\right) = \frac{\frac{1}{2}\Delta V}{2}$$
$$\tfrac{1}{2}\Delta V = V\sin\left(\tfrac{1}{2}\Delta i\right) \Rightarrow \Delta V = 2V\sin\left(\tfrac{1}{2}\Delta i\right)$$

Remember:

$$V_{orbit} = \sqrt{\frac{\mu}{a}} = \sqrt{\frac{\mu_E}{R_E+h_{orbit}}}$$

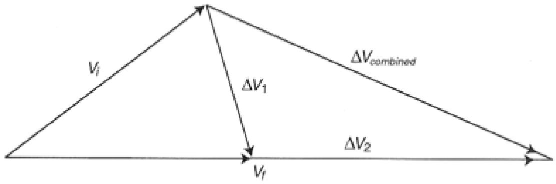
Substitute:

$$\Delta V = 2\sqrt{\frac{\mu_E}{R_E+h_{orbit}}}\sin\left(\tfrac{1}{2}\Delta i\right) = 2\sqrt{\frac{398601}{6378+35822}}\sin\left(\tfrac{1}{2}\cdot 29,8^\circ\right) = 1,581 km/s$$

Note: You could also solve this problem using the cosine rule.

3. How much ΔV would be needed to transfer from a circular parking orbit at 185 km altitude and with an inclination of 29.8°, to the pericenter of an orbit with h_p = 185 km and h_a = 35822 km, but with an inclination of 0° (i.e. in a single, combined maneuver)?

Remember this picture:



$$\Delta V_{combined}^2 = V_i^2 + V_f^2 - 2V_iV_f \cos(\Delta i) \Leftrightarrow c^2 = a^2 + b^2 - 2ab\cos\alpha$$

$$\Delta V_{combined} = \sqrt{V_i^2 + V_f^2 - 2V_iV_f \cos(\Delta i)}$$

From the above formula, we see that we'll have to find the initial and final velocities:

$$V_i = \sqrt{\frac{\mu_E}{R_E + h_{parking}}} = \sqrt{\frac{398601}{6378+185}} = 7,79324 km/s$$

$$V_f = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} = \sqrt{398601\left(\frac{2}{6378+185} - \frac{1}{\frac{1}{2}(2R_E + 185 + 35822)}\right)} = 10,25284 km/s$$

Plug in:

$$\Delta V_{combined} = \sqrt{7,79324^2 + 10,25284^2 - 2\cdot 7,79324 \cdot 10,25284 \cos(29,8°)} = \boxed{5,2136 km/s}$$

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4. How much ΔV would be needed to transfer from a circular parking orbit at 185 km altitude to the pericenter of an orbit with h_p = 185 km and h_a = 35822 km? The orbits are coplanar, i.e. within the same plane.

Equal to question 1.

The method for the other questions is exactly the same, and the answers are available in the slides.

1. Compute the total ΔV needed to go from a circular parking orbit at an altitude of 185 km to the International Space Station, in a circular orbit at 350 km.

$$\Delta V = \Delta V_A + \Delta V_B$$

$$\Delta V_A = V_{transfer,p} - V_{parking}$$

$$\Delta V_B = V_{ISS} - V_{transfer,a}$$

$$V_{transfer,p} = \sqrt{\mu_E\left(\frac{2}{r_{init}} - \frac{1}{a_{transfer}}\right)}$$

$$V_{parking} = \sqrt{\frac{\mu_E}{r_{init}}}$$

$$V_{ISS} = \sqrt{\frac{\mu_E}{r_{final}}}$$

$$V_{transfer,a} = \sqrt{\mu_E\left(\frac{2}{r_{final}} - \frac{1}{a_{transfer}}\right)}$$

$$\Delta V = \left[\sqrt{\mu_E\left(\frac{2}{r_{init}} - \frac{1}{a_{transfer}}\right)} - \sqrt{\frac{\mu_E}{r_{init}}}\right] + \left[\sqrt{\frac{\mu_E}{r_{final}}} - \sqrt{\mu_E\left(\frac{2}{r_{final}} - \frac{1}{a_{transfer}}\right)}\right] =$$

$$\left[\sqrt{\mu_E\left(\frac{2}{6378+185} - \frac{1}{\frac{1}{2}(2\cdot 6378+185+350)}\right)} - \sqrt{\frac{398601}{6378+185}}\right] +$$

$$\left[\sqrt{\frac{398601}{6378+350}} - \sqrt{\mu_E\left(\frac{2}{6378+350} - \frac{1}{\frac{1}{2}(2\cdot 6378+185+350)}\right)}\right] = 0,09615 km/s$$

Question 2 to 4 are solved in a similar fashion.

5. End-of-life disposal: compute the total ΔV needed to go from a circular operational orbit at an altitude of 780 km to an elliptical orbit with pericenter at 200 km and apocenter at 780 km.

Note: from here on, I'll start writing delta V calculations as absolute value, so I don't have to pay attention to which velocity I write down first.

$$\Delta V = \left|V_{operational} - V_{disposal,a}\right|$$

$$V_{operational} = \sqrt{\frac{\mu}{r}}$$

$$V_{disposal,a} = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$\Delta V = \left|\sqrt{\frac{\mu_E}{R_E + h_{operational}}} - \sqrt{\mu_E\left(\frac{2}{R_E + h_{operational}} - \frac{1}{\frac{1}{2}(2R_E + h_p + h_a)}\right)}\right| =$$

$$\left|\sqrt{\frac{398601}{6378+780}} - \sqrt{398601\left(\frac{2}{6378+780} - \frac{1}{\frac{1}{2}(2\cdot 6378+200+780)}\right)}\right| = 0,1592 km/s$$

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Consider a Hohmann transfer from Earth (at 1 AU) to Jupiter (at 5.2 AU). What is:
1. the semi-major axis of the transfer orbit?

$$a = \frac{r_{sun-Earth} + r_{sun-Jupiter}}{2} = \frac{1+5,2}{2} = 3,1AU$$

2. the eccentricity of the transfer orbit?

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{5,2-1}{5,2+1} = 0,6674$$

3. the pericenter velocity in this transfer orbit?

$$V_p = \sqrt{\mu_{sun}\left(\frac{2}{r} - \frac{1}{a}\right)} = \sqrt{1,3271\cdot 10^{11}\left(\frac{2}{149,6\cdot 10^6} - \frac{1}{3,1\cdot 149,6\cdot 10^6}\right)} = 38,5751 km/s$$

4. the apocenter velocity in this transfer orbit?

$$V_a = \sqrt{\mu_{sun}\left(\frac{2}{r} - \frac{1}{a}\right)} = \sqrt{1,3271\cdot 10^{11}\left(\frac{2}{5,2\cdot 149,6\cdot 10^6} - \frac{1}{3,1\cdot 149,6\cdot 10^6}\right)} = 7,4183 km/s$$

5. the travel time?

$T_{transfer} = \pi\sqrt{\frac{a_{transfer}{}^3}{\mu_{sun}}} = \pi\sqrt{\frac{\left(3,1\cdot 149,6\cdot 10^6\right)^3}{1,3271\cdot 10^{11}}} = 86126734 s$, which is approximately 997 days.

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Consider a transfer from Earth (at 1 AU) to Jupiter (at 5.2 AU). Assume the spacecraft leaves the Earth orbit in a direction tangential to the orbit of the Earth itself, with an excess velocity ΔV equal to 11 km/s. (hint: this is not a Hohmann transfer orbit). What is:
1. the velocity of the Earth?

$$V_{Earth} = \sqrt{\frac{\mu_{sun}}{r_{sun-earth}}} = \sqrt{\frac{1,3271\cdot 10^{11}}{1\cdot 149,6\cdot 10^6}} = 29,7842 km/s$$

2. the semi-major axis of the transfer orbit?

Note: remember the hint: this is not a Hohmann transfer orbit. Instead, we'll have to find the value of the semi-major axis by modifying the vis viva equation:

$$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \Rightarrow -\frac{V^2}{\mu} + \frac{2}{r} = \frac{1}{a} \Rightarrow a = \frac{1}{\frac{2}{r} - \frac{V^2}{\mu}} = \frac{1}{\frac{2}{149,6\cdot 10^6} - \frac{(29,7842+11)}{1,3271\cdot 10^{11}}} =$$

$$1,1935\cdot 10^9 km = 7,98 AU$$

Note: the Earth's velocity was added to the velocity of the spacecraft, which makes sense.

3. the eccentricity of the transfer orbit?

We know that the perihelion radius is 1 AU and we know the length of the semi-major axis. From this follows the aphelion radius and thus the eccentricity:

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{\left(2a - r_p\right) - r_p}{\left(2a - r_p\right) + r_p} = \frac{2 \cdot 7,98 - 2}{2 \cdot 7,98} = 0,8747$$

4. the value for the true anomaly θ when the vehicle leaves the Earth vicinity?

$$\theta = 0°$$

5. the value of θ when the vehicle arrives at the target planet?

We know that the distance to Jupiter measured from the sun equals 5.2 AU. Going back through the material, we find Kepler's first law: $r = \frac{a\left(1-e^2\right)}{1+e\cos\theta}$. We're asked to calculate θ and we've already found values for e, r and a. So here goes:

$$r = \frac{a\left(1-e^2\right)}{1+e\cos\theta} \Rightarrow \theta = \cos^{-1}\left(\frac{\frac{a\left(1-e^2\right)}{r} - 1}{e}\right) = \cos^{-1}\left(\frac{\frac{7,98\left(1-0,8747^2\right)}{5,2} - 1}{0,8747}\right) = 136,98°$$

6. the travel time?

See AE1102_space_slides_5-6.pdf, page 41-42 (and pages before for derivation). We have

$$r = a\left(1 - e\cos E\right) \Rightarrow E = \cos^{-1}\left(\frac{\frac{r}{a} - 1}{-e}\right) = \cos^{-1}\left(\frac{\frac{5,2}{7,98} - 1}{-0,8747}\right) = 1,1612 rad$$, and

$$E - e\sin E = \sqrt{\frac{\mu}{a^3}}\left(t - t_p\right) \Rightarrow t - t_p = \frac{E - e\sin E}{\sqrt{\frac{\mu}{a^3}}} = \frac{1,1612 - 0,8747\sin 1,1612}{\sqrt{\frac{1,3271 \cdot 10^{11}}{\left(7,98 \cdot 149,6 \cdot 10^6\right)^3}}} = 4,0629 \cdot 10^7 s$$

or 470 days.

Section 2: Rocket motion and Launchers

Questions from file: AE1102_space_slides_9-10.pdf

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Consider a vehicle with dry mass (i.e. construction + payload mass) of 1000 kg, and 4000 kg of propellant. The specific impulse is equal to 300 s, and the propellant mass flow is equal to 80 kg/s (and constant).
Questions:
1. What is the thrust?

To get ahead of any confusion: your file might mention a mass flow of 80 kg/s. It should be 50 kg/s.

We have $w = I_{sp} g_0$ and $T = \dot{m} w$. Substitution yields

$$T = \dot{m} I_{sp} g_0 = 50 \cdot 300 \cdot 9{,}81 = 147 kN$$

2. What is the acceleration of the vehicle at the beginning of the thrust interval?

$$T = ma \Rightarrow a = \frac{T}{m} = \frac{147150}{1000 + 4000} = 29{,}43 ms^{-2}$$

3. What is the acceleration of the vehicle at the end of the thrust interval?

$$T = ma \Rightarrow a = \frac{T}{m} = \frac{147150}{1000} = 147{,}15 ms^{-2}$$

4. What is the gain in velocity?

$$\Delta V = I_{sp} g_0 \ln \Lambda = 300 \cdot 9{,}81 \ln\left(\frac{1000 + 4000}{1000}\right) = 4737 ms^{-1}$$

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Consider a rocket with dry mass (i.e. construction + payload mass) of 1000 kg. It is to take the payload to escape velocity, so ΔV = 11.2 km/s. The specific impulse is equal to 300 s.
Questions:
1. How much propellant mass is required?

$$\Delta V = I_{sp} g_0 \ln \frac{M_{dry} + M_{propellant}}{M_{dry}} = I_{sp} g_0 \ln\left(1 + \frac{M_{propellant}}{M_{dry}}\right) \Rightarrow e^{\left(\frac{\Delta V}{I_{sp} g_0}\right)} = 1 + \frac{M_{propellant}}{M_{dry}} \Rightarrow$$

$$M_{propellant} = M_{dry}\left[e^{\left(\frac{\Delta V}{I_{sp} g_0}\right)} - 1\right] = 1000\left[e^{\left(\frac{11200}{300 \cdot 9{,}81}\right)} - 1\right] = 43954 kg$$

2. If the (constant) thrust were to be delivered over 2 minutes of time, what would be the thrust? What would be the acceleration at burnout? What would be the acceleration at ignition?

Thrust: $\dot{m} = \frac{F}{I_{sp} g_0} \Rightarrow F = \dot{m} I_{sp} g_0 = \frac{43954}{120} \cdot 300 \cdot 9{,}81 = 1{,}078 MN$

$$a_{burnout} = \frac{F}{M_{dry}} = \frac{1,078 \cdot 10^6}{1000} = 1078 ms^{-2}$$

$$a_{ignition} = \frac{F}{M_{dry} + M_{propellant}} = \frac{1,078 \cdot 10^6}{1000 + 43954} = 23,98 ms^{-2}$$

3. If the (constant) thrust were to be delivered over 8 minutes of time, what would be the thrust? What would be the acceleration at burnout? What would be the acceleration at ignition?

Thrust: $\dot{m} = \frac{F}{I_{sp} g_0} \Rightarrow F = \dot{m} I_{sp} g_0 = \frac{43954}{8 \cdot 60} \cdot 300 \cdot 9,81 = 0,2695 MN$

$$a_{burnout} = \frac{F}{M_{dry}} = \frac{0,2695 \cdot 10^6}{1000} = 270 ms^{-2}$$

$$a_{ignition} = \frac{F}{M_{dry} + M_{propellant}} = \frac{0,2695 \cdot 10^6}{1000 + 43954} = 6,00 ms^{-2}$$

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Consider a lander hovering 10 m above the surface of the Moon, in search for a suitable landing spot. Given: μ_{moon} = 4906×109 m^3/s^2, R_{moon} = 1738×10^3 m; dry mass of vehicle (i.e. structure + payload) =500kg; I_{sp}=300s; g_0 =9.81m/s^2.

Questions:

1. Compute the gravitational acceleration that the vehicle experiences.

See AE1102_space_Slides_1-3.pdf, page 27. $g = \frac{\mu_{moon}}{r_{moon}^2} = \frac{4906 \cdot 10^9}{1738 \cdot 10^3} = 1,624 ms^{-2}$

2. Derive an equation for the propellant mass as a function of time.

I first read this question as ‘Derive an equation for the propellant mass at a certain time’. From the answer, I noticed that the question should be interpreted as ‘Derive an equation for the amount of propellant mass that the lander has to have on board as a function of the total hovering time’.

In a vacuum, the speed that the lander would have after time t would be $\Delta V = gt$. Note that this is independent of mass. Since we want ΔV to be zero, we can use Tsiolkovsky’s equation to find the amount of propellant required.

$$gt = I_{sp} g_0 \ln\left(\frac{M_{dry} + M_{propellant}}{M_{dry}}\right) = I_{sp} g_0 \ln\left(1 + \frac{M_{propellant}}{M_{dry}}\right) \Rightarrow e^{\left(\frac{gt}{I_{sp} g_0}\right)} = 1 + \frac{M_{propellant}}{M_{dry}} \Rightarrow$$

$$M_{propellant} = M_{dry}\left(e^{\left(\frac{gt}{I_{sp} g_0}\right)} - 1\right)$$

3. Compute how much propellant would be needed for a maximum hover period of 1 second.

$$500\left(e^{\left(\frac{1,624 \cdot 1}{300 \cdot 9,81}\right)} - 1\right) = 0,276 kg$$

The other two questions are solved in exactly the same way.

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See answers of page 45.