Turning Flight

1 Turn Strategy

Flying in a straight line tends to get boring after a while, and it limits your possible destinations. So you need to be able to turn the aircraft. How should we do this? We consider three options

- Change μ. To see the effect, we examine the three-dimensional equations of motion that were derived previously. Let's examine the second equation. Since $\mu \neq 0$ also $\sin\mu \neq 0$ and thus $\dot{\chi} \neq 0$. So we will turn. Now look at the third equation. Since $\cos\mu$ will decrease we find that $\dot{\gamma} < 0$. That means we start plummeting to the ground. Since that is rather unwanted, this option isn't a very good one.
- In addition to changing μ also increase the angle of attack α. If we do this in such a way that $L\cos\mu$ stays constant, we won't drop to the ground. But now look at the first equation of motion. Since the angle of attack α increased also the drag D increased. Therefore $\dot{V} < 0$. This means we will slow down and reduce lift. Of course we can increase α even more, but we can't do this indefinitely. After a while we will stall and fall to the ground. So this isn't a good option either.
- Let's now change μ, increase the angle of attack α and increase the thrust T. If we increase the thrust enough to compensate for the additional drag D, we won't slow down. Therefore we are performing a perfect horizontal turn.

The third method seems to be a good one. But we have already mentioned that the equations of motion don't consider rudder and aileron deflection. In a real airplane we also need to compensate with the rudder to prevent adverse yaw. But if we do that too, we are actually able to perform a perfect horizontal turn.

2 Turn Radius and Load Factor

So we now know how to rotate an aircraft. But how much space do we need? From physics we know that $\mathbf{F_c} = mV^2/R$, where $\mathbf{F_c}$ is the **centrifugal force** and R is the **turn radius**. Since we are considering a horizontal turn, we know that $\gamma = \dot{\gamma} = 0$. The centrifugal force therefore is

$$F_c = L\sin\mu = \frac{W}{g}\frac{V^2}{R}. \tag{2.1}$$

In a horizontal turn the relation $L = W$ doesn't hold anymore. Instead, from the equations of motion we can find that

$$W = L\cos\mu = L\cos\phi, \tag{2.2}$$

where ϕ is a variable which is used to indicate the bank angle in horizontal turns. In this case it's equal to μ. Now we can define the **load factor** n as

$$n = \frac{L}{W} = \frac{1}{\cos\phi} = \frac{V^2}{g}\frac{1}{R\sin\mu}. \tag{2.3}$$

If we now use the fact that $\mu = \phi$ and also the mathematical relation $\sin^2 x + \cos^2 x = 1$ we can express the turn radius in the load factor. The result will be

$$R = \frac{V^2}{g\sqrt{n^2-1}}. \tag{2.4}$$

Note that always $n > 1$. So in every horizontal turn you make you feel slightly heavier.

We would also like to know how long a turn takes. The angular velocity of the turn Ω is simply equal to $\Omega = V/R = \dot{\chi}$. Knowing this, we know that the time it takes to make a complete $360°$ turn is

$$T_{2\pi} = \frac{2\pi}{\Omega} = 2\pi \frac{R}{V} = 2\pi \frac{V}{g\sqrt{n^2 - 1}}. \tag{2.5}$$

So if you want to make a fast turn you actually have to fly as slow as possible. This may seem counterintuitive. To do something fast you need to decrease your speed. But when you're driving a bike or a car you also slow down when making a turn. So it actually makes sense to all of us.

3 Power During a Turn

We know that we need to increase the engine power during a turn. But by how much is this? We know that $L = nW$, so from that we can derive that

$$V = \sqrt{\frac{2}{\rho} \frac{1}{C_L} \frac{nW}{S}}. \tag{3.1}$$

We can also express the drag as

$$D = \frac{C_D}{C_L} nW. \tag{3.2}$$

If we combine these two equations, we can find the power required to be

$$P_r = DV = \sqrt{\frac{2}{\rho} \frac{C_D^2}{C_L^3} \frac{(nW)^3}{S}}. \tag{3.3}$$

So we find that the power required is proportional to $n\sqrt{n}$. So the higher the load factor, the higher the power required. Of course our engine has a maximum power available. So this determines the maximum load factor n_{max}. Using equation 3.2 and the relation $T = D$ we find that

$$n_{max} = \frac{T}{W} \frac{C_L}{C_D}. \tag{3.4}$$

However, for low velocities the maximum load factor isn't bounded by the power available. It is then bounded by the maximum lift coefficient $C_{L_{max}}$ according to

$$n_{max} = \frac{L_{max}}{W} = \frac{\rho V^2 S C_{L_{max}}}{2W}. \tag{3.5}$$

In both cases the maximum velocity in a turn will be

$$V_{n_{max}} = \sqrt{\frac{2}{\rho} \frac{1}{C_L} \frac{n_{max} W}{S}}. \tag{3.6}$$