Take-Off

1 The Procedure

Aircrafts usually fly. But before they can fly, they first have to take off from the ground. There are several moments during the take-off run that deserve some special attention. We now take a close look at the lift-off procedure of a multi-engined jet aircraft. The take-off procedure for other airplanes is similar.

Before take-off, the airplane taxis to the start of the runway. At that moment its velocity is $V_0 = 0$. When clearance is given by the control tower, the pilot gives the engines maximum power. At a certain velocity V_{dec}, called the **decision speed**, the pilot needs to make a decision. It is the last moment at which the airplane can still stop its take-off. In case of an engine failure it might be worth staying on ground.

If the pilot decides to continue the take-off, the airplane will soon reach the **rotation speed** V_r. At this point the pilot pulls the nose of the aircraft up, while the rear wheels still stay on the ground. For a brief moment the aircraft will have a certain constant **rotational speed** q_{rot}, until the desired pitch angle θ_{req} is reached.

A short moment later the **lift-off speed** V_{lof} is reached. This is the speed at which the wheels no longer touch the ground. However, the take-off has not yet been completed. The aircraft still needs to reach the **screen height** H_{scr}. At this height, which is usually 35 or 50 feet, the aircraft is safe from obstacles like trees or buildings. The speed of the aircraft at the screen height must be at least a certain **climb-out speed** (sometimes called **safety speed**) V_{scr} to ensure a safe climb.

2 Decision Speed

All the velocities that were mentioned in the previous paragraph are given to the pilot by the aircraft manual. However, some one needs to write the manual, so there must be a way to determine these velocities. We primarily examine the decision speed.

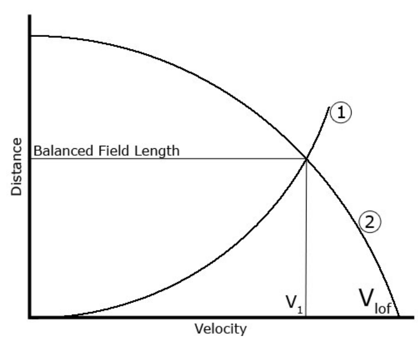

Figure 1: Clarification of decision speed.

Let's take a look at figure 1. Two graphs are displayed. Graph 1 is the distance you need to stop if you apply full brakes at the given velocity. Graph 2 is the distance you need to take off if one engine fails. If engine failure occurs at low velocities, the distance to stop is small. So it is preferable to stop. For high velocities stopping requires to much runway space. It's wiser to take off with the remaining engines.

The intermediate point, where stopping requires as much distance as taking off, is of special interest to us. The velocity at which this occurs is the **decision speed**. At this speed you can still stop the aircraft, but if you continue accelerating, you have to lift off. The distance you need to either stop or to take off is called the **balanced field length**.

Now let's look at the other velocities. The screen height is usually equal to the minimum velocity plus a safety factor. So $V_{scr} = jV_{min}$, where for most aircrafts $j = 1.2$. The rotation speed V_r is then set at the velocity such that V_{scr} is still reached at the screen height if engine failure occurs.

You may also be wondering what the procedure is if you only have one engine on your aircraft. If that engine fails, you should brake if you're able to. If you are not, you should still take off and hope there is an empty field ahead of you in which you can make an emergency landing.

3 Equations of Motion for Ground Phase

To learn more about the ground phase, we look at the forces acting on the aircraft. These forces are shown in figure 2.

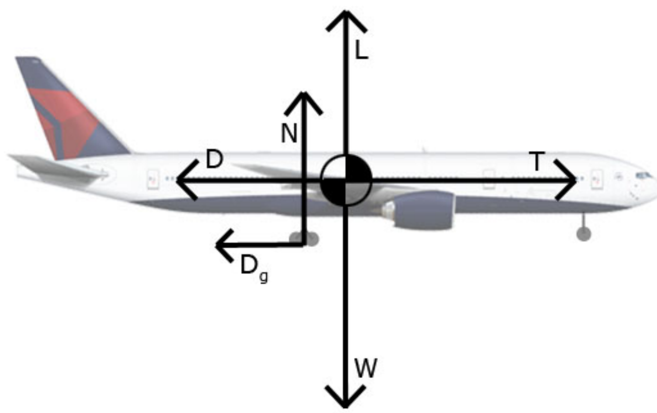

Figure 2: Forces acting on the aircraft during ground run.

We have seen all forces already in the chapter about cruise, except the **normal force** N and the **gear drag** D_g. But what can we say about this gear drag? We can assume that it is proportional to the normal force N, so

$$D_g = \mu N = \mu(W - L), \tag{3.1}$$

where μ is the friction coefficient. This coefficient depends on the wheels of the aircraft, as well as the environment. If we now look at the forces acting on the aircraft in horizontal direction, we will find

$$\frac{W}{g}\frac{dV}{dt} = T - D - D_g = T - D - \mu(W - L). \tag{3.2}$$

It is important to note the other assumptions we have made. We assume that the runway is horizontal, that there is no wind and once more that the thrust is in the direction of the velocity.

When you apply normal aerodynamics equations to calculate the aerodynamic forces L and D, you should be careful. Since the aircraft is still on the ground, there is the so-called **ground effect**. The air which our wings push down is pushed back up by the ground. This causes the effective angle of attack to increase. The lift therefore goes up. The ground also reflects the vortices our wing creates, partially canceling them. This causes the induced drag to decrease. The ground effect is therefore considered to be a positive effect for airplanes.

4 Take-Off Distance

Now we are curious about the amount of runway the aircraft needs for the entire take-off procedure. We first look at the ground phase. From dynamics we know that $a\,ds = V\,dV$. Rewriting gives

$$s_{ground} = \int_0^{V_{lof}} \frac{V\,dV}{a} = \frac{1}{\bar{a}} \int_0^{V_{lof}} V\,dV = \frac{1}{2\bar{a}} V_{lof}^2. \tag{4.1}$$

Here the quantity $\bar{a}$ is the **mean acceleration**, which is equal to

$$\bar{a} = \frac{g}{W}\left(\bar{T} - \bar{D} - \bar{D}_g\right), \tag{4.2}$$

where $\bar{T}$, $\bar{D}$ and $\bar{D}_g$ are the mean thrust, drag and gear drag, respectively. We use this mean acceleration because it is very hard to express the thrust, drag and gear drag as a function of the velocity during a ground run. In practice, however, the mean acceleration $\bar{a}$ is approximately equal to the acceleration at $\frac{1}{2}\sqrt{2}V_{lof}$. So the thrust, drag and gear drag only have to be calculated at one given velocity to find the take-off distance corresponding to the ground phase.

Now let's consider the airborne part of the take-off. To find this, we first have to rewrite the equation of motion to

$$T - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt} = \frac{W}{g}\frac{dV}{ds}\frac{ds}{dt} = \frac{W}{g}\frac{dV}{ds}V = \frac{W}{2g}\frac{d(V^2)}{ds}. \tag{4.3}$$

Rewriting again will now give the so-called **equation of energy**, being

$$(T - D)ds = \frac{W}{2g}d(V^2) + W\sin\gamma\,ds = \frac{W}{2g}d(V^2) + W dH, \tag{4.4}$$

where the variable H indicates the height. The left side of this equation indicates the power that is put into the system, while the right side shows the change in kinetic and potential energy. To find the distance needed to reach the screen height, we once more integrate. We will find

$$\int_0^{s_{air}} (T - D)ds = \int_{V_{lof}}^{V_{scr}} \frac{W}{2g}d(V^2) + \int_0^{H_{scr}} W dH, \tag{4.5}$$

$$(\bar{T} - \bar{D})s_{air} = \frac{W}{2g}\left(V_{scr} - V_{lof}\right) + W H_{scr}. \tag{4.6}$$

At the screen height we should have reached a steady climb. We therefore may assume that $(\bar{T} - \bar{D}) = (T - D)_{scr}$. Since the speed is constant in a steady climb, we know that $dV_{scr} = 0$ and thus also $dV_{scr}^2 = 0$. This implies that $(T - D)_{scr} - W\sin\gamma_{scr} = 0$, or equivalently $W\gamma_{scr} = (T - D)_{scr} = (\bar{T} - \bar{D})$, where we have used $\sin\gamma_{scr} \approx \gamma_{scr}$. Now we find that

$$\gamma_{scr}s_{air} = \frac{V_{scr}^2 - V_{lof}^2}{2g} + H_{scr}. \tag{4.7}$$

To find the total take-off distance, simply add up the airborne distance s_{air} to the ground run distance s_{ground}.