Propulsion and Performance Formulas

Definitions concerning piston engines

P_a = The available engine power. This is the power that does the actual work. ($W = J/s$)

W = Work done by an engine. (J)

Δt = Change in time. (s)

T = Thrust force. (N)

V = Velocity. (m/s)

P_{br} = Break power / Shaft power. So it's the power in the shaft of the piston engine. ($W = J/s$)

Q = Propeller torque. (Nm)

Ω = Rotation speed of the propeller. (rad/s)

η_p = Propeller efficiency. (no unit)

Δp = The pressure difference between to different time periods of the cycle of a four-stroke engine, in which the volume is equal. ($Pa = N/m^2$)

p_e = The effective mean pressure in a piston engine. ($Pa = N/m^2$)

V_{stroke} = The maximum change in volume during one stroke of a piston engine. This is equal to the stroke length times the piston area. (m^3)

V_{total} = The maximum volume change of all the cylinders of an engine. (m^3)

η_{mech} = The mechanical efficiency inside the engine. (no unit)

N = The number of cylinders in a piston engine. (no unit)

n = The number of rotation per second of the crankshaft. (s^{-1})

Formulas and explanations concerning piston engines

Naturally, the work done by an engine is equal to the work done per second. However, power is also force times distance traveled. So:

$$P_a = \frac{W}{\Delta t} = TV \tag{1}$$

But this is not the power the engine really creates (there occur things like friction, drag by the propeller, etc). The power of the engine is actually equal to:

$$P_{br} = Q\Omega \tag{2}$$

The difference between these two powers can be explained by an efficiency below 100%. The efficiency is usually between 75% and 85%. In formula, the efficiency is:

$$\eta_p = \frac{P_a}{P_{br}} = \frac{TV}{Q\Omega} \tag{3}$$

Now assume there is a four-stroke piston engine, and there is a diagram which shows the pressure inside the piston engine given a certain volume, during the four strokes. It can be shown that:

$$W = \int_{V_{min}}^{V_{max}} \Delta p dV = \int_{V_{min}}^{V_{max}} p_e dV \tag{4}$$

The replacement of Δp (which is not constant) for p_e (which is constant) is per definition true. This formula indicates that the work done by a piston engine in a cycle is equal to the area under the V-p diagram.

Because p_e is constant, the following formula applies:

$$W = p_e \int_{V_{min}}^{V_{max}} dV = p_e V_{stroke} \tag{5}$$

The force done by a piston engine depends on the cycles per second, the work per cycle, and the number of cylinders. Naturally, the crankshaft of an engine rotates twice, while only 1 cycle occurs, so the amount of cycles per rotation is $\frac{n}{2}$. This data implicate the following formula:

$$P_{br} = \eta_{mech} N W \frac{n}{2} = \eta_{mech} N p_e V_{stroke} \frac{n}{2} = \eta_{mech} p_e V_{total} \frac{n}{2} \tag{6}$$

Definitions concerning fan/jet engines

λ = The by-pass ratio. Low by-pass engines have their by-pass ratio at about 1 or 2. High by-pass engines (also called fan engines) have their by-pass ratio at about 5 to 8. This is standard for modern commercial aircrafts. High by-pass ratios are usually more efficient. (no unit)

m = The mass flow of air. (kg/s)

m_{cold} = The cold mass flow of air that goes through the engine. This is the part of the airflow that does not pass through the turbine. (kg/s)

m_{hot} = The hot mass flow of air that goes through the engine. This is the part of the airflow that does pass through the turbine. (kg/s)

ΔE_k = The change in kinetic energy per every kilogram air passing through the engine. (J/kg)

V_j = Relative exhaust speed (with respect to the airplane). (m/s)

V_0 = Relative air speed (with respect to the airplane). (m/s)

P_j = Jet power. ($W = J/s$)

P_a = Available engine power. This is the power that does the actual work. ($W = J/s$)

T = Thrust. (N)

η_j = The propulsive efficiency. (no unit)

Formulas and explanations concerning fan/jet engines

The by-pass ratio is per definition equal to the following ratio:

$$\lambda = \frac{m_{cold}}{m_{hot}} \tag{7}$$

Now look at the increase of kinetic energy a kilogram of air gets, because of the propulsion:

$$\Delta E_k = \frac{1}{2} V_j^2 - \frac{1}{2} V_0^2 \tag{8}$$

Now the power of the engine can also be calculated:

$$P_j = m \Delta E_k = \frac{1}{2} m (V_j^2 - V_0^2) \tag{9}$$

The available power, however, is still equal to $P_a = TV_0$. And since the thrust is of course equal to $T = m(V_j - V_0)$, it can be known that:

$$\eta_j = \frac{P_a}{P_J} = \frac{TV_0}{\frac{1}{2}m(V_j^2 - V_0^2)} = \frac{m(V_j - V_0)V_0}{\frac{1}{2}m(V_j - V_0)(V_j + V_0)} = \frac{2}{1 + \frac{V_j}{V_0}} \tag{10}$$

And from this formula, it can be derived that the efficiency is higher if the relative exhaust speed is closer to the relative air speed.