Propulsion

1 Engine Thrust and Heat

Let's look at an engine flying at a velocity V_0 $[m/s]$ (with respect to the air). The air enters the engine with a velocity V_0. Suppose it exits the engine with a velocity V_j $[m/s]$. Also suppose that the mass of the air that goes through the engine every second (the **mass flow**) is m $[kg/s]$. The **thrust** can now be found using the change in momentum:

$$T = I_{out} - I_{in} = m(V_j - V_0) \tag{1}$$

Now suppose the engine uses m_f $[kg/s]$ kilograms of fuel per second. The **fuel flow** F $[N/s]$ is now defined as:

$$F = m_f \cdot g \tag{2}$$

If we know the **heating value** H $[J/kg]$ (the amount of joules in one kilogram of fuel), we can also find the added energy Q $[J/s]$ every second:

$$Q = m_f \cdot H = \frac{F \cdot H}{g} \tag{3}$$

And if we want to find the heat q_{in} $[J/kg]$ added to one kilogram of air, we can calculate it using:

$$q_{in} = \frac{Q}{m} = H\frac{m_f}{m} \tag{4}$$

2 Engine Power and Efficiency

When we know the velocity of the air prior to and after the engine, and if we now the mass flow of the air, we can calculate the energy put into the air every second - the **jet power** P_j $[J/s]$:

$$P_j = \frac{1}{2}mV_j^2 - \frac{1}{2}mV_0^2 = \frac{1}{2}m(V_j^2 - V_0^2) = \frac{1}{2}m(V_j - V_0)(V_j + V_0) = \frac{1}{2}T(V_j + V_0) \tag{5}$$

The **power available** P_a $[J/s]$, which is the work added every second, can be calculated using:

$$P_a = T \cdot V_0 = m_f \cdot V_0(V_j - V_0) \tag{6}$$

Using these equations for the power, we can calculate various efficiencies. The **propulsive efficiency** η_j is defined as:

$$\eta_j = \frac{P_a}{P_j} = \frac{T \cdot V_0}{\frac{1}{2}T(V_j + V_0)} = \frac{2V_0}{V_j + V_0} = \frac{2}{1 + \frac{V_j}{V_0}} \tag{7}$$

The **thermal efficiency** η_{th} is defined as:

$$\eta_{th} = \frac{P_j}{Q} = \frac{\frac{1}{2}m(V_j^2 - V_0^2)}{m_f \cdot H} = \frac{V_j^2 - V_0^2}{2q_{in}} \tag{8}$$

The **total efficiency** η_{tot} is defined as:

$$\eta_{tot} = \frac{P_a}{Q} = \frac{T \cdot V_0}{Q} = \eta_j \cdot \eta_{th} \tag{9}$$

There are multiple ways to increase efficiency. To increase propulsive efficiency, it is necessary to keep V_j close to V_0 (instead of giving a little bit of air a lot of acceleration, give a lot of air a little bit of acceleration). To increase thermal efficiency, the **pressure ratio** ε (the ratio of the pressure before and after the compressor) should be high. So the air should be compressed as much as possible.

Using the equation $Q = m\,q_{in}$, the thrust can also be rewritten as:

$$T = Q\frac{\eta_{tot}}{V_0} = m\,q_{in}\frac{2}{V_0 + V_j}\eta_{th} \tag{10}$$

This equation also shows that if the thrust increases (thus the engine gets a higher RPM and thus a higher compression ratio) also the thermal efficiency increases.

3 The Perfect Jet Engine

The **perfect jet engine** is a jet engine at which most steps are isentropic. Only when the air goes through the combustion chamber, the entropy rises. At that step, the temperature rises isobaric. From this fact the entropy after the combustion chamber can be found (by following the isobar lines in a Mollier diagram, for example). It is also assumed that all the energy taken out by the turbine is put back in the system by the compressor.

The perfect jet engine consists of 5 parts. So when air goes through the jet engine, it has 6 consecutive states. These states change as follows:

- The inlet (1-2): $w_{in} = q_{in} = 0$ $\quad V_2 \approx 0 \quad \Rightarrow \quad \Delta h = \frac{1}{2}V_1^2$
- The compressor (2-3): $q_{in} = 0$ $\quad V_3 \approx V_2 \approx 0 \quad \Rightarrow \quad \Delta h = w_{in}$
- The combustion chamber (3-4): $w_{in} = 0$ $\quad V_4 \approx V_3 \approx 0 \quad \Rightarrow \quad \Delta h = q_{in}$
- The turbine (4-5): $w_{out} = -w_{in} \quad q_{in} = 0$ $\quad V_5 \approx V_4 \approx 0 \quad \Rightarrow \quad \Delta h = -w_{in}$
- The exhaust (5-6): $w_{in} = q_{in} = 0$ $\quad V_5 \approx 0 \quad \Rightarrow \quad \Delta h = -\frac{1}{2}V_6^2$

Note that the free stream velocity is $V_0 = V_1$ and the exhaust velocity is $V_j = V_6$.

4 The Turbo-Prop Engine

The **turbo-prop engine** is very similar to the jet engine. The turbo-prop is in fact a jet engine with a propeller in front of it. This propeller accelerates air just slightly, giving the engine a higher propulsive efficiency. An indication for this is the by-pass ratio, defined as:

$$\lambda = \frac{M_{cold}}{M_{hot}} \tag{11}$$

where M_{cold} is the cold airflow (passing through the propeller) and M_{hot} is the hot airflow (passing through the compressor). In a turbo-prop engine, there is one additional step. Between the (first) turbine and the exhaust, there is a second turbine. This second turbine is connected to the propeller. However, since the turbo-prop engine isn't a jet engine, there also isn't a jet power. Instead, there is the shaft power (brake power) P_{br} $[J/s]$. The propulsive efficiency η_p and the thermal efficiency η_{th} are now defined as:

$$\eta_p = \frac{P_a}{P_{br}} \qquad \eta_{th} = \frac{P_{br}}{Q} \tag{12}$$

The propeller in a turbo-prop engine doesn't give the air a lot of acceleration. So let's assume that $V = V_j$. The thermal efficiency now is:

$$\eta_{th} = \frac{P_{br}}{Q} = \frac{m(h_5 - h_6)}{Q} = \frac{m(h_5 - h_{exhaust}) - \frac{1}{2}mV^2}{Q} \tag{13}$$

For a jet engine, the thermal efficiency can be expressed as:

$$\eta_{th} = \frac{P_j}{Q} = \frac{\frac{1}{2}mV_j^2 - \frac{1}{2}mV_0^2}{Q} = \frac{m(h_5 - h_{exhaust}) - \frac{1}{2}mV^2}{Q} \tag{14}$$

These equations are equal! (Note that for a jet engine $h_{exhaust} = h_6$ but for a turbo-prop engine $h_{exhaust} = h_7$, due to the extra step in a turbo-prop engine.)

There is another similarity between the turbo-prop engine and the jet engine. For both engines the efficiency decreases if the temperature increases, or if the aircraft flies higher (lower density).

5 Nozzles

When engines fly at (nearly) supersonic speeds, their engines often have a nozzle with a throat. For the sonic regions $V \uparrow$ as $A \downarrow$, where A is the cross-section of the engine. However, for supersonic speed $V \uparrow$ as $A \uparrow$. The throat (where A is smallest) is the point at which the transition from sonic to supersonic velocity occurs. Thus $M_t = 1 \Leftrightarrow V_t = a$ at the throat.

The nozzle should be made in such a way that at the exhaust the pressure in the engine p_e is equal to the outside pressure p_0. If $p_e > p_0$ (the airplane is flying at a height higher than it was designed for) the exhausted air continues to expend and accelerate after it has exited the nozzle, but this acceleration does not contribute to the aircraft thrust. So there is a waste of energy. If $p_e < p_0$ (the airplane is flying at a height lower than it was designed for) the exhausted air causes shock waves, which also mean a waste of energy.