Measurement Devices

1 Airspeed Indication

An airspeed indicator doesn't indicate the **true airspeed** (TAS), but measures the difference between the total and the static pressure. For low velocities this is the dynamic pressure $q = \frac{1}{2}\rho V^2$. So for low velocities this works well. However, for high velocities ($M > 0.3$) compressibility effects need to be taken into account, and this equation can not be applied anymore. Instead, we use thermodynamics to derive an alternative equation for the velocity. We start by noting the following:

$$c_pT + \frac{1}{2}V^2 = c_pT_t \quad \Rightarrow \quad \frac{T_t}{T} = 1 + \frac{V^2}{2c_pT} = 1 + \frac{V^2\rho R}{2c_pp} = 1 + V^2\frac{\rho}{p}\frac{c_p - c_v}{2c_p} = 1 + V^2\frac{\rho}{p}\frac{\gamma-1}{2\gamma} \tag{1}$$

Now we can use the formula of Poisson to get:

$$\frac{p_t}{p} = \left(\frac{T_t}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + V^2\frac{\rho}{p}\frac{\gamma-1}{2\gamma}\right)^{\frac{\gamma}{\gamma-1}} \quad \Rightarrow \quad \left(\frac{p_t}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1 = V^2\frac{\rho}{p}\frac{\gamma-1}{2\gamma} \tag{2}$$

Solving for V gives:

$$V = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p}{\rho}\left(\left(\frac{p_t}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)} = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p}{\rho}\left(\left(\frac{q_c}{p}+1\right)^{\frac{\gamma-1}{\gamma}} - 1\right)} \tag{3}$$

where the pressure difference q_c is defined as $q_c = p_t - p$. So for low speeds $q_c = q$. The last equation indicates that the velocity is a function of q_c, p and T. q_c can be measured, but p and T are very difficult to measure accurately. Therefore the **calibrated airspeed** (CAS) V_c $[m/s]$ is introduced. The indicator assumes that the airplane is flying at 0 m altitude in **International Standard Atmosphere** (ISA), such that $p = p_0$ and $T = T_0$. Thus:

$$V_c = \sqrt{\frac{2\gamma}{\gamma-1}RT_0\left(\left(\frac{q_c}{p_0}+1\right)^{\frac{\gamma-1}{\gamma}} - 1\right)} \tag{4}$$

For $M << 1$ the calibrated airspeed is (approximately) equal to the equivalent airspeed (but not the true airspeed!). And finally there is one more airspeed, the **indicated airspeed** (IAS). This airspeed is equal to the calibrated airspeed, accept for (usually minor) errors caused by the airspeed indicator. However, if, for example, the dial of the airspeed indicator gets stuck, the indicated airspeed is definitely not equal to the calibrated airspeed.

2 International Standard Atmosphere (ISA)

The hydrostatic equation is:

$$\frac{dp}{dh} = -\rho g \tag{5}$$

Since g isn't constant at high altitudes, we define the geopotential height H $[m]$ such that $g\,dh = g_0dH$ where g_0 $[m/s^2]$ is the gravitational acceleration at zero altitude. So this gives:

$$\frac{dp}{dH} = -\rho g_0 \tag{6}$$

Combining this with the gas law, we have three unknowns and two equations. Therefore we assume that for the troposphere $T = T_0 + \lambda H$ with $T_0 = 288.15$ K and $\lambda = -0.0065$ K/m. If we now combine this assumption with equation 6 and with the gas law, we get:

$$\frac{dp}{p} = -\frac{g_0\,dH}{R(T+\lambda H)} \tag{7}$$

Integrating and working out the result gives:

$$\frac{p}{p_0} = \left(1 + \frac{\gamma H}{T_0}\right)^{-\frac{g_0}{\lambda R}} = \left(\frac{T}{T_0}\right)^{-\frac{g_0}{\lambda R}} \qquad \Leftrightarrow \qquad \frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{-\frac{g_0}{\lambda R}-1} \tag{8}$$

Solving for H gives an equation which is used in altimeters to find the **pressure altitude**:

$$H = \left(\left(\frac{p}{p_0}\right)^{-\frac{\lambda R}{g_0}} - 1\right)\frac{T_0}{\lambda} \tag{9}$$

where p can be found by measuring the pressure difference between the static pressure and the pressure of a vacuum. In this way a pilot can find out how high he approximately is.

If every pilot would use the same reference values p_0 and T_0, there is sufficient data to avoid collisions in the air. However, when an airplane has its reference pressure set at ISA-pressure, and if the real pressure is different, he will get wrong readings. Since a pilot would like to know whether his plane is 5 meters or 5 kilometers above the ground, there is a system which gets rid of these disadvantages.

When an airplane lifts off, the pilot asks the control tower for the **QNH** pressure, which is the actual pressure around the airfield. He sets the reference pressure of his altimeter to that value, so he knows his actual altitude quite accurately. When he gets above a certain **transition altitude** (which can differ per region, but is usually a few thousand feet), he sets the reference pressure of his altimeter to the ISA-pressure, such that all airplanes flying in that region have the same reference pressure. When the aircraft gets close to his arrival destination, the pilot asks for the local QNH. When the aircraft passes the local transition altitude, the pilot adjusts the reference pressure of his altimeter to the local QNH. He then has the right altitude reading on his altimeter, so he can land his airplane safely.

Next to an airspeed indicator and an altimeter, a pilot usually also has a thermometer, measuring the temperature outside the airplane.