Landing

1 The Procedure

It's nice to get an airplane up in the air. But what goes up must come down, and preferably as smooth as possible. Therefore a landing is a part of every flight.

Before an airplane can land, it first must descend. It does this with a certain **descend angle** $\bar{\gamma}$, defined as $\bar{\gamma} = -\gamma$, as is be shown in figure 1. This angle is usually $\bar{\gamma} = 3°$. The **approach velocity** should minimally be at $V_a = 1.3V_{min}$, in case of any unexpected events. The flight path and approach velocity are controlled by the pilot. If the airplane is going too fast, the pilot will pull up the nose of the aircraft. If the descend angle is wrong, the pilot will change the power setting of the engine.

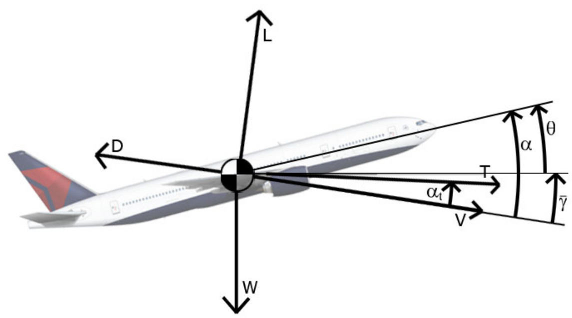

Figure 1: Forces acting on the aircraft during descend.

The official landing procedure starts when the aircraft reaches the screen height, which is about 50 feet for landings. When the aircraft almost hits the ground, the pilot will turn off the power on the engines and the aircraft will **flare**. The flare is the maneuver in which the pilot pulls up the nose of the aircraft, decreasing the rate of descent drastically.

When the aircraft makes a **touchdown** on the ground, it should have a **touchdown velocity** of about $V_t = 1.15V_{min}$. After that it will hit the brakes. Depending on the aircraft, several other things to slow down the aircraft can be used. The engines can switch to reverse thrust. Also spoilers can be used to reduce lift. Both mechanisms will be discussed later in this chapter.

2 Energy Landing Equation

Just like in the landing, we would like to know the distance an aircraft needs to land. First we look at the part of the landing in which the aircraft is still in the air. This part starts when the aircraft is at the screen height. To find an equation for the distance, we first have to look at the equation of motion. For a landing aircraft, this is

$$\frac{W}{g}\frac{dV}{dt} = T - D + W \sin\bar{\gamma}. \tag{2.1}$$

Let's divide this equation by the weight first. If we then multiply it by dt and multiply again $V = ds/dt$ we will get

$$\frac{1}{2g}d(V^2) = \frac{T - D}{W}ds + \sin\bar{\gamma}\, ds = \frac{T - D}{W}ds - dH. \tag{2.2}$$

Now we can integrate from the screen height (with height H_{scr} and approach velocity V_a) to the ground (with height 0 and touchdown velocity V_t). The result will be

$$\frac{V_t^2}{2g} - \left(\frac{V_a^2}{2g} + H_{scr}\right) = \frac{\bar{T} - \bar{D}}{W} s_{air} \tag{2.3}$$

Just like we did in previous parts, we have used the mean thrust $\bar{T}$ and drag $\bar{D}$. This time we assume that the value of $\bar{T} - \bar{D}$ is the average of the value at screen height $(\bar{T} - \bar{D})_{scr}$ and the value at touchdown $(\bar{T} - \bar{D})_t$. At screen height the aircraft is in a steady descent, so $(\bar{T} - \bar{D})_{scr} = W\gamma = -W\bar{\gamma}_a$, where $\bar{\gamma}_a$ is the approach angle. At touchdown the pilot will have put out the engines, so $T_t = 0$. Therefore $(\bar{T} - \bar{D})_t = -D_t = -W\left(\frac{C_D}{C_L}\right)_t$. Combining all the data gives

$$\frac{V_t^2}{2g} - \left(\frac{V_a^2}{2g} + H_{scr}\right) = \frac{\bar{T} - \bar{D}}{W} s_{air} = -\frac{1}{2} s_{air} \left(\bar{\gamma}_a + \left(\frac{C_D}{C_L}\right)_t\right). \tag{2.4}$$

This equation is called the **energy landing equation**. This is because once more energy is involved. The left hand side of the equation is proportional to the change in energy, while the middle side is related to the work done on the aircraft. Using the energy landing equation, the value of s_{air} can be determined.

3 Brake Distance

When the aircraft safely touches down on the ground, how much distance does it need to brake? To find this distance, we once more use the equation $a\,ds = V\,dV$. The **brake distance** then is

$$s_{ground} = \int_{V_t}^{0} \frac{V\,dV}{a} = -\frac{V_t^2}{2\bar{a}}. \tag{3.1}$$

Note that we have again used the mean acceleration. The minus sign on the right hand side is present because this mean acceleration is negative. (The aircraft is slowing down.) In fact, it can be expressed as

$$\bar{a} = -\frac{g}{W}\left(\bar{D} + \bar{D}_g - \bar{T}\right) = -\frac{g}{W}\left(\bar{D} + \bar{D}_g + \bar{T}_{rev}\right), \tag{3.2}$$

where T_{rev} is the magnitude of the **reverse thrust**. Just like during take-off, it is a good approximation to take values of D, D_g and T_{rev} at a velocity of $\frac{1}{2}\sqrt{2}V_t$ as the mean values.

Substituting the previous equation in the one before it will give the brake distance. It will be

$$s_{ground} = \frac{W}{g} \frac{V_t^2}{\bar{D} + \bar{D}_g + \bar{T}_{rev}}. \tag{3.3}$$

4 Braking

For the aircraft to come to a full stop, brakes are used. It is useful to know how those brakes work. That is why the wheels of an aircraft are tested very well before they are used. To do this, engineers put them on a floor which is able to move. Let's suppose the floor will move with a velocity V, as is shown in figure 2. The wheel with radius r will then get an angular velocity ω. If the wheel does not slip, then $V = \omega r$. If the wheel does slip, these two values are not equal. The so-called **slip** is therefore defined as the dimensionless factor

$$\text{slip} = \frac{V - \omega r}{V}. \tag{4.1}$$

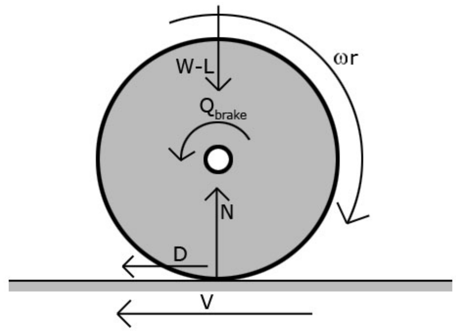

Figure 2: Forces/Moments acting on the wheel of an aircraft.

Now let's examine the forces acting on the wheel. From examining vertical forces we find that $N = W - L$. There is only one horizontal force, being the ground drag. This ground drag is equal to

$$D_g = \mu N = \mu \left(W - L \right), \tag{4.2}$$

where μ is the friction coefficient of the wheel. We want to come to a stop as fast as possible, so we intent to maximize the ground drag. We do this by maximizing the friction coefficient. It turns out that μ highly depends on the slip. For small slip values the friction coefficient still increases. However, at a certain point the coefficient reaches its maximum μ_{max}. If the slip increases even further, μ decreases again. So it's wise to allow some slip, but not too much.

If we now examine the moments acting on the aircraft, we will find that

$$\mu \left(W - L \right) r - Q_{brake} = I \frac{d\omega}{dt}, \tag{4.3}$$

where Q_{brake} is the **brake torque** applied by the braking system. This brake torque can be set. Given a certain brake torque, equilibrium points can be found for ω. Knowing ω, also the slip can be determined. However, some of these equilibrium points are stable, while others are unstable. A **stable** equilibrium point means that ω will return to its original value after small deviations. Stable equilibrium points have a positive value of $d\mu/d(\text{slip})$.

For **unstable** equilibrium points the opposite happens. When small deviations occur, the slip diverges away from the equilibrium point. This could result in a rapid increase in slip and thus a rapid decrease in ground drag. To prevent this, aircrafts are often equipped with an **anti-lock braking system** (ABS). Unstable equilibrium points have a negative value of $d\mu/d(\text{slip})$.

To increase the ground drag, you don't only need a high friction coefficient. You also need a normal force. When the aircraft touches down, the lift is still approximately equal to the weight, so $N = W - L = 0$. To solve this problem, **spoilers** are used. These are small deflectable plates on top of the wing. If they are deployed, they disturb the airflow and decrease the lift enormously.

So the landing procedure can now be determined. As soon as you touch down, deploy the spoilers and brake in such a way that the friction coefficient is as high as possible.