Introduction to Aerospace Engineering Formulas

1 Aerodynamics Formulas

1.1 Definitions

p = The air pressure. ($Pa = N/m^2$)

ρ = The air density. (kg/m^3)

g = The gravitational constant. (Value at sea level is $9.81 N/kg$) (N/kg)

h = The height above the earth surface. (m)

V = The speed of the airplane relative to the air. (m/s)

p_t = The total pressure. ($Pa = N/m^2$)

p_0 = The static pressure. ($Pa = N/m^2$)

S = The wing surface. (m^2)

L = The lift force. (N)

C_L = The lift coefficient. (no unit)

D = The drag force. (N)

C_D = The drag coefficient. (no unit)

C_{D_i} = The induced drag coefficient. (no unit)

e = The Oswald factor. (Usually has a value between 0.8 and 0.9) (no unit)

A = The aspect ratio. (no unit)

b = The wing span (from left wing tip to right wing tip, so it's not just the length of one wing). (m)

D_i = The induced drag. (no unit)

C_{D_0} = The friction and pressure drag coefficient. (no unit)

M = The Mach number. (no unit)

a = The speed of sound. ($340 m/s$ at sea level) (m/s)

Re = The Reynolds number. (no unit)

L = A characteristic length. Often the length of an object. (m)

μ = The viscosity of the air. (Normal air has viscosity $17.9 \times 10^{-6} kg/(ms)$) ($kg/(ms)$)

W = The weight of the aircraft. (N)

T = The thrust of the aircraft. (N)

L_w = The wing loading. ($Pa = N/m^2$)

n = The load factor. (no unit)

1.2 Two-dimensional aerodynamics formulas

The pressure in a certain part of the atmosphere is equal to the weight of the air column on top. The formula describing this statement is known as the hydrostatic equation:

$$dp = -\rho g(dh) \tag{1.1}$$

An equation which looks a bit like the previous equation, is the Euler equation:

$$dp = -\rho V(dV) \tag{1.2}$$

So, if we integrate this equation, we find bernoulli's equation:

$$p + \frac{1}{2}\rho V^2 = C \tag{1.3}$$

Where C is a constant. So $p + \frac{1}{2}\rho V^2$ is constant for any 2 points along a streamline. Using this formula, the airspeed can be calculated:

$$V_0 = \sqrt{2\frac{p_t - p_0}{\rho}} \tag{1.4}$$

Bernoulli's equation states that $-dp = d(\frac{1}{2}\rho V^2)$. By integrating that over the wing surface, and implementing a constant, the following formula can be found:

$$L = C_L \frac{1}{2}\rho V^2 S \tag{1.5}$$

Similar to this, also the drag force can be calculated:

$$D = C_D \frac{1}{2}\rho V^2 S \tag{1.6}$$

1.3 Induced Drag

However, the previously discussed formulas work well for two-dimensional cases. In three dimensions there is also another type of drag, called the induced drag. This type of drag also has a coefficient:

$$C_{D_i} = \frac{C_L^2}{\pi A e} \tag{1.7}$$

But in this case, A is not known yet. A, the aspect ratio, is the relationship between the length and the width of the wing. However, the width of the wing is not constant. So by multiplying the ratio $A = \frac{wing\ span}{wing\ width}$ on both sides of the fraction by the wing span, the following formula appears:

$$A = \frac{b^2}{S} \tag{1.8}$$

Now, using all this data (and the fact that $C_L = \frac{2L}{\rho V^2 S}$), the induced drag can be calculated:

$$D_i = C_{D_i}\frac{1}{2}\rho V^2 S = \frac{2L^2}{\rho S \pi A e}\frac{1}{V^2} \tag{1.9}$$

So by using the formula:

$$C_D = C_{D_0} + C_{D_i} = C_{D_0} + \frac{C_l^2}{\pi A e} \tag{1.10}$$

The total drag can be calculated, using equation 1.6.

1.4 Characteristic numbers

There are also a few numbers which characteristic the flow type. An example is the Mach number, which is calculated using:

$$M = \frac{V}{a} \tag{1.11}$$

There are different names for different ranges of Mach numbers:

- $M < 0.8$: Subsonic
- $0.8 < M < 1.2$: Transonic
- $1.2 < M < 4$: Supersonic
- $4 < M$: Hypersonic

Next to the Mach number, there is also the Reynolds number:

$$Re = \frac{\rho V L}{\mu} \tag{1.12}$$

The Reynolds number is an indication if, and where, separation occurs. High Reynolds numbers usually result in a more turbulent flow, while low Reynolds numbers result in a more laminar flow.

1.5 Flight types

In a horizontal (no change of height) steady (no roll) straight (no yaw) flight, the following conditions must apply:

$$W = L = C_L \frac{1}{2} \rho V^2 S \tag{1.13}$$

$$T = D = C_D \frac{1}{2} \rho V^2 S \tag{1.14}$$

Divide these equations, and you will find that:

$$\frac{W}{T} = \frac{L}{D} = \frac{C_L}{C_D} \tag{1.15}$$

Also, it is interesting to notice that the minimal speed an airplane can have, can be calculated, if the maximum lift coefficient is known:

$$W = L = C_{L_{max}} \frac{1}{2} \rho V_{min}^2 S \tag{1.16}$$

There is also a factor called the wing loading. This is equal to:

$$L_w = \frac{W}{S} = C_L \frac{1}{2} \rho V^2 \tag{1.17}$$

However, when there is no horizontal flight, but if the airplane is climbing, some of the previous formulas don't apply. In this case, a load factor can be introduced. This can be calculated as follows:

$$n = \frac{L}{W} \tag{1.18}$$

So in a horizontal flight, the load factor is 1.

Since $L_{max} = C_{L_{max}} \frac{1}{2} \rho V^2 S$ and $W = C_{L_{max}} \frac{1}{2} \rho V_{min}^2 S$ it can also be derived that:

$$n_{max} = \frac{L_{max}}{W} = \left(\frac{V}{V_{min}} \right)^2 \tag{1.19}$$

2 Propulsion and Performance Formulas

2.1 Definitions concerning piston engines

P_a = The available engine power. This is the power that does the actual work. ($W = J/s$)

W = Work done by an engine. (J)

Δt = Change in time. (s)

T = Thrust force. (N)

V = Velocity. (m/s)

P_{br} = Break power / Shaft power. So it's the power in the shaft of the piston engine. ($W = J/s$)

Q = Propeller torque. (Nm)

Ω = Rotation speed of the propeller. (rad/s)

η_p = Propeller efficiency. (no unit)

Δp = The pressure difference between to different time periods of the cycle of a four-stroke engine, in which the volume is equal. ($Pa = N/m^2$)

p_e = The effective mean pressure in a piston engine. ($Pa = N/m^2$)

V_{stroke} = The maximum change in volume during one stroke of a piston engine. This is equal to the stroke length times the piston area. (m^3)

V_{total} = The maximum volume change of all the cylinders of an engine. (m^3)

η_{mech} = The mechanical efficiency inside the engine. (no unit)

N = The number of cylinders in a piston engine. (no unit)

n = The number of rotation per second of the crankshaft. (s^{-1})

2.2 Formulas and explanations concerning piston engines

Naturally, the work done by an engine is equal to the work done per second. However, power is also force times distance traveled. So:

$$P_a = \frac{W}{\Delta t} = TV \tag{2.1}$$

But this is not the power the engine really creates. The power of the engine is actually equal to:

$$P_{br} = Q\Omega \tag{2.2}$$

The difference between these two powers can be explained by an efficiency below 100%. The efficiency is usually between 75% and 85%. In formula, the efficiency is:

$$\eta_p = \frac{P_a}{P_{br}} = \frac{TV}{Q\Omega} \tag{2.3}$$

Now assume there is a four-stroke piston engine, and there is a diagram which shows the pressure inside the piston engine given a certain volume, during the four strokes. It can be shown that:

$$W = \int_{V_{min}}^{V_{max}} \Delta p dV = \int_{V_{min}}^{V_{max}} p_e dV \tag{2.4}$$

The replacement of Δp (which is not constant) for p_e (which is constant) is per definition true. This formula indicates that the work done by a piston engine in a cycle is equal to the area under the V-p diagram.

Because p_e is constant, the following formula applies:

$$W = p_e \int_{V_{min}}^{V_{max}} dV = p_e V_{stroke} \tag{2.5}$$

The force done by a piston engine depends on the cycles per second, the work per cycle, and the number of cylinders. Naturally, the crankshaft of an engine rotates twice, while only 1 cycle occurs, so the amount of cycles per rotation is $\frac{n}{2}$. This data implicate the following formula:

$$P_{br} = \eta_{mech} N W \frac{n}{2} = \eta_{mech} N p_e V_{stroke} \frac{n}{2} = \eta_{mech} p_e V_{total} \frac{n}{2} \tag{2.6}$$

2.3 Definitions concerning fan/jet engines

λ = The by-pass ratio. Low by-pass engines have their by-pass ratio at about 1 or 2. High by-pass engines (also called fan engines) have their by-pass ratio at about 5 to 8. This is standard for modern commercial aircrafts. High by-pass ratios are usually more efficient. (no unit)

m = The mass flow of air. (kg/s)

m_{cold} = The cold mass flow of air that goes through the engine. This is the part of the airflow that does not pass through the turbine. (kg/s)

m_{hot} = The hot mass flow of air that goes through the engine. This is the part of the airflow that does pass through the turbine. (kg/s)

ΔE_k = The change in kinetic energy per every kilogram air passing through the engine. (J/kg)

V_j = Relative exhaust speed (with respect to the airplane). (m/s)

V_0 = Relative air speed (with respect to the airplane). (m/s)

P_j = Jet power. ($W = J/s$)

P_a = Available engine power. This is the power that does the actual work. ($W = J/s$)

T = Thrust. (N)

η_j = The propulsive efficiency. (no unit)

2.4 Formulas and explanations concerning fan/jet engines

The by-pass ratio is per definition equal to the following ratio:

$$\lambda = \frac{m_{cold}}{m_{hot}} \tag{2.7}$$

Now look at the increase of kinetic energy a kilogram of air gets, because of the propulsion:

$$\Delta E_k = \frac{1}{2} V_j^2 - \frac{1}{2} V_0^2 \tag{2.8}$$

Now the power of the engine can also be calculated:

$$P_j = m \Delta E_k = \frac{1}{2} m (V_j^2 - V_0^2) \tag{2.9}$$

The available power, however, is still equal to $P_a = T V_0$. And since the thrust is of course equal to $T = m(V_j - V_0)$, it can be known that:

$$\eta_j = \frac{P_a}{P_J} = \frac{T V_0}{\frac{1}{2} m (V_j^2 - V_0^2)} = \frac{m(V_j - V_0) V_0}{\frac{1}{2} m (V_j - V_0)(V_j + V_0)} = \frac{2}{1 + \frac{V_j}{V_0}} \tag{2.10}$$

And from this formula, it can be derived that the efficiency is higher if the relative exhaust speed is closer to the relative air speed.