## Helicopters

## 1 The Tail Rotor

Most (traditional) helicopters have a termmain rotor and a **tail rotor**. The main rotor provides the lift. To do that, it is rotating with an angular velocity  $\Omega$  [rad]. (Most helicopters have blades rotating counter-clockwise, when seen from the top, so we will assume the blades are rotating counter-clockwise.) This rotation gives it a velocity, and since the rotor blades have the shape of a wing foil, it causes lift. However, next to lift, it also causes drag. This drag results in a (clockwise) torque. To prevent the rotor from slowing down, the helicopter engine causes an equal but opposite (and thus counter-clockwise) torque. However, Newton's third law implies that every action has an equal and opposite reaction. So if the helicopter causes a counter-clockwise torque on the blades, the blades cause a clockwise torque on the helicopter, which would make it rotate. This is an undesirable effect, and that's why a tail rotor is added.

Suppose a torque Q [Nm] is necessary for the main rotor to have a constant angular velocity. If  $l_t$  [m] is the distance between the main rotor and the tail rotor, and if  $T_t$  [N] is the thrust from the tail rotor, then to prevent the helicopter from rotating, the following condition should be true:

$$Q = l_t \cdot T_t \tag{1}$$

However, tail rotors have a lot of disadvantages. They consume a lot of power, are dangerous, noisy, expensive and under bad wind conditions they give only a marginal control authority. That's why a lot of alternatives for a tail rotor have been introduced over the years.

## 2 Helicopter Performance

A helicopter stays in the air by thrusting air downward. Far above the main rotor the air is still standing still. When the air passes through the main rotor, it has an induced velocity  $V_i$  [m/s]. We can now express the amount of air passing through the rotor blades every second. If the rotor blades have length R [m], then the **mass flow** m [kg/s] is:

$$m = \rho \pi R^2 V_i \tag{2}$$

But when the air has just passed the disk, it is still accelerating. When the air is relatively far below the main rotor, it has a velocity of  $2V_i$ . Using the momentum equation, we can calculate the thrust of the main rotor:

$$T = I_{out} - I_{in} = 2mV_i - 0 = 2mV_i = 2\rho\pi R^2 V_i^2$$
(3)

For a hovering helicopter, the thrust equals the weight (T = W). The **induced power**  $P_i$  [J/s] is equal to the change in kinetic energy of the flow:

$$P_{i} = \frac{1}{2}m(2V_{i})^{2} = 2mV_{i}^{2} = TV_{i} = WV_{i} = W\sqrt{\frac{W}{2\rho\pi R^{2}}}$$
(4)

Next to the induced power, there is also the hover power  $P_{hov}$ , which can be calculated using:

$$P_{hov} = Q\Omega = l_t T_t \Omega \tag{5}$$

The real power necessary for hovering, the hover power, is usually not the same as the ideal induced power. The **Figure of Merit** M is defined as:

$$M = \frac{P_i}{P_{hov}} \tag{6}$$

For the tail rotor there is also a (different) Figure of Merit  $M_t$ .

## 3 Helicopter Control

Helicopters can do more than simple hovering. They can also fly in multiple directions. First of all, helicopters can go up and down. To achieve this either the angular velocity  $\Omega$  or the rotor blade pitch is increased. The latter can be done by using a **swash plate**. This is a plate below the rotor blades. A small bar is connected to the front of every rotor blade. So if the swash plate goes up, the front of the rotor blades go up as well, increasing the pitch.

The swash plate can not only go up/down. It can also rotate slightly. When this happens, one of the rotor blades gets an increased pitch, while the other (being on the opposite side) gets a decreased pitch. This causes a moment, which causes the helicopter to rotate. By rotating the helicopter, the direction of the thrust changes, which causes the helicopter to go forward/backward or left/right. When using this trick, it is important not to forget the gyroscopic effect. This is present since the rotor blades are spinning rather fast.

The rotor blades are generating lift. If they would be entirely fixed to the rotor axis, there would be large bending moments. This is why they are usually able to rotate in multiple directions. The reason why they still point outward is because of the centrifugal effect. This method offers various advantages, but can sometimes be slightly dangerous. One example is that it can cause ground resonance if the helicopter is hovering close to the ground.