Helicopters

1 Actuator Disk Theory

In a helicopter, lift isn't created by the wings, but by rotating blades. This enables helicopters to hover. In case of hovering flight, the thrust produced by the rotor blades T must be equal to the weight W.

But how can we calculate this thrust? One way to do this is by using the **actuator disk theory**, which has already been treated in the Aerodynamics A course. We will briefly repeat that theory here. Suppose we have a set of rotating blades, called the **actuator disk**. We can consider 4 points now. Point 0 is infinitely far to the left of the disk, while point 1 is just a little bit to the left of the disk. Identically, point 2 is just to the right of the disk and point 3 is infinitely far to the right.

If we assume the velocity is evenly distributed over the actuator disk, and that the flow is incompressible and irrotational, we can derive a few simple relations. Let's define the **induced velocity** V_i as $V_i = V_1 - V_0$. We can now show that $V_2 = V_1$ and also $V_3 - V_2 = V_i$. So therefore the total velocity increment of the air is $V_3 - V_0 = 2V_i$. We can also show that the thrust now is

$$T = 2mV_i = 2\rho\pi r^2 V_i, \tag{1.1}$$

where m is the mass flow of air that goes through the disk and r is the actuator disk radius.

2 Helicopter Power

How much power would the engine need to provide to hover? In an ideal situation the power would be the difference between the rate of power that comes out of the system, and the rate of power that goes in. This would make the so-called **ideal power**

$$P_{id} = e_{out} - e_{in} = \frac{1}{2}m\left(V_0 + 2V_i\right)^2 - \frac{1}{2}mV_0^2 = 2mV_i\left(V_0 + V_i\right) = T\left(V_0 + V_i\right). \tag{2.1}$$

In a hovering flight, the initial velocity V_0, being the velocity far away from the disk, is 0. So we also know that $P_{id} = TV_i = WV_i$. If we apply $T = W$ to the last equation of the previous paragraph, we can also find that

$$V_i = \sqrt{\frac{W}{2\rho\pi r^2}}. \tag{2.2}$$

The ideal power of the helicopter now becomes

$$P_{id} = WV_i = W\sqrt{\frac{W}{2\rho\pi r^2}}. \tag{2.3}$$

So to reduce the ideal power, we need to decrease our weight, increase the rotor blade size or simply fly at a lower altitude. However, the ideal power isn't the actual power that is needed from the engine. There are always losses. These losses are all combined in one sign. This is the so-called **figure of merit** M, defined as

$$M = \frac{P_{id}}{P_{hov}}, \tag{2.4}$$

where P_{hov} is the actual power that is necessary to hover. For most modern helicopters the figure of merit is about $M = 0.7$.

3 Blade Element Theory

In the **blade element theory** we look at an infinitely small piece of a rotor blade and derive the lift dL and drag dD acting on that piece. Using those forces we can integrate to find the thrust and drag of the full rotor blades.

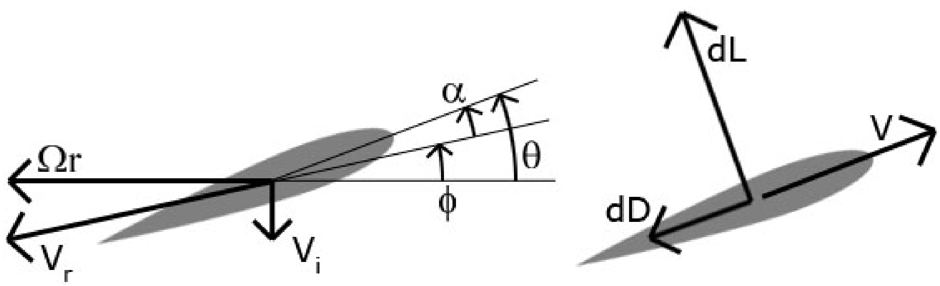

Figure 1: Motions and forces in the blade element theory.

So let's look at an infinitely small piece of a rotor blade, at a distance r from the center. Such a piece is shown in figure 1. Since the rotor rotates with an angular velocity of Ω, the small piece of rotor blade has a velocity component to the left of Ωr. Due to the induced velocity, there is a downward pointing velocity component of V_i. Together they form the resultant velocity V_r. Since this velocity is not directed horizontally, the air has a certain **inflow angle** ϕ. Also a **pitch angle** θ and an **angle of attack** α can be distinguished.

The small bit of thrust dT acting on the rotor blade part now is

$$dT = dL\cos\phi - dD\sin\phi \approx dL, \tag{3.1}$$

where we have approximated $\cos\phi \approx 1$ and $\sin\phi \approx 0$. The power that is needed to keep the rotor rotating is

$$dP = dD\,\Omega r = (dL\sin\phi + dD\cos\phi)\,\Omega r \approx dT\,V_i + dD\,\Omega r, \tag{3.2}$$

where we this time have approximated $\cos\phi \approx 1$ and $\sin\phi \approx \tan\phi = V_i/\Omega r$. To find the total power that is needed for all the rotor blades, we integrate over one entire rotor blade and multiply by the number of blades n. The power then becomes

$$P = n\int_0^R V_i\,dT + n\int_0^R \Omega r\,dD = TV_i + P_p = P_i + P_p, \tag{3.3}$$

where P_i is the induced component of the power and P_p is the power necessary to compensate for the profile drag of the blades. Also R is the length of the entire rotor blade. The variable P_i is usually approximated using $P_i = kP_{id}$, where the correction factor k is often about 1.2. The power component P_p is a bit more difficult to determine. If c_{d_p} is the profile drag component of the rotor blade piece, we can write

$$P_p = n\int_0^R \Omega r\,dD = n\int_0^R c_{d_p}\frac{1}{2}\rho\,(\Omega r)^3\,cdr = \frac{\bar{c}_{d_p}\sigma}{8}\left(\Omega R^2\right)^3\pi R^2. \tag{3.4}$$

Here the sign $\bar{c}_{d_p}$ indicates the mean drag coefficient for the entire rotor. The factor σ is called the **rotor solidity**, defined as

$$\sigma = \frac{nRc}{\pi R^2}. \tag{3.5}$$

There is a simple physical meaning behind this rotor solidity. It's the part of the rotor disk that is filled with blades. The surface area of the rotor disk is simply πR^2, while the surface area of the blades is nRc.

4 Forward Flight

Helicopters would tend to get boring if they can only hover. They can also fly forward. To let helicopters fly forward, you first have to rotate the rotor disk. When doing this, you have to keep in mind the gyroscopic effect, as the rotor disk is rotating quite rapidly.

The angle with which the rotor disk is tilted forward is called the **disk angle of attack** α_d. Since we're flying forward now, we experience drag. First of all a drag force H_0 acts on the rotor disk. But the rest of the helicopter (the so-called **bus**) is also suspect to drag. This is the so-called **parasite drag** D_{par}. The total power that the engine now needs to provide is

$$kTV_i + P_p + H_0 V + D_{par} V = P_i + P_p + P_{drag} + P_{par}. \tag{4.1}$$

Let's look at the individual power components now. To find an expression for the profile drag P_p we can once more use the blade element theory. But it's a bit more complicated this time. The velocity of a small blade element depends not only on the distance r from the rotor disk center. It also depends on the angle ψ it makes with respect to the direction of the velocity. If a small piece of rotor moves in the same direction as the velocity of the helicopter, than the two velocities need to be added up. So then $V_r = \Omega R + V \cos \alpha_d$. In the opposite case they need to be subtracted. In general we can say that

$$V_r = \Omega R + V \cos \alpha_d \sin \mu. \tag{4.2}$$

What we do now is simply integrate over the entire rotor disk, as if there was a rotor blade at every small angle $d\psi$. We then divide by 2π to get the average profile power for a certain rotor blade. Of course we also multiply by the number of rotor blades n. The profile power then becomes

$$P_p = \frac{n}{2\pi} \int_0^{2\pi} \left(\int_0^R c_{d_p} \frac{1}{2} \rho \left(\Omega r + V \cos \alpha_d \sin \mu\right)^2 c \, \Omega r \, dr \right) d\psi = \frac{\bar{c_{d_p}} \sigma}{8} \rho \left(\Omega R\right)^3 \pi R^2 \left(1 + \mu^2\right). \tag{4.3}$$

Note that the profile power can also be written as $P_p = P_{hov} \left(1 + \mu^2\right)$, where the hover power is equal to

$$P_{hov} = \frac{\bar{c_{d_p}} \sigma}{8} \rho \left(\Omega R\right)^3 \pi R^2. \tag{4.4}$$

The sign μ indicates the so-called **advance ratio** and is defined as

$$\mu = \frac{V \cos \alpha_d}{\Omega R}. \tag{4.5}$$

In practice $\mu_{max} \approx 0.3$. If the helicopter goes any faster, two things can occur. It may occur that the blade facing headwind (the so-called **advancing blade** with velocity $\Omega R + V \cos \alpha_d$) experiences a supersonic flow. It is also possible that the so-called **retreating blade** with velocity $\Omega R - V \cos \alpha_d$ actually moves backward. Both situations are not really positive.

The drag power due to H_0 is rather difficult to determine. That's why it is simply approximated by

$$P_{drag} = H_0 V \cos \alpha_d = P_{hov} 4.65 \mu^2. \tag{4.6}$$

The most important contribution to the drag is the parasite drag. Especially for high velocities this drag dominates. The power necessary to counteract this drag is

$$P_{par} = D_{bus} V = C_{D_{bus}} \frac{1}{2} \rho V^3 A_{ref}, \tag{4.7}$$

where $C_{D_{bus}}$ is the drag coefficient of the bus and A_{ref} is a certain reference area.

Now we can draw our conclusions. The values of P_p and P_{drag} are fairly constant with increasing V. The induced power P_p even decreases if we fly forward. The parasite drag, however, strongly increases. To reduce drag it's therefore best to make the helicopter bus as aerodynamic as possible.

5 Autorotation

What happens if the engine of the helicopter fails? It turns out that the helicopter will not fall to the ground. Instead, a phenomenon called **autorotation** occurs. The rotor disk starts rotating itself.

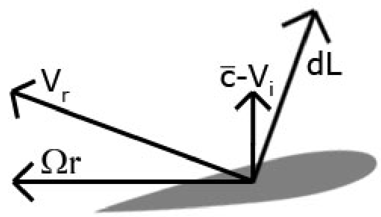

Figure 2: Forces acting on an autorotating rotorblade.

To explain why autorotation occurs, we examine the forces acting on the rotor blades. These forces are shown in figure 2. When the engine fails, the helicopter will start to lose height. It will get a certain descend speed $\bar{c}$. Due to this descend speed, it will seem like the induced velocity now has magnitude $\bar{c} - V_i$ and is pointed upward. Since the induced velocity points upward, the resultant velocity will also be tilted upward. Therefore the lift vector will be tilted in the forward direction. The vertical component of the lift vector still provides thrust, but now the horizontal component powers the rotor blades. The rotor disk rotates on its own.

This however only happens if the pilot takes action. If the engine fails, the descend speed will increase. This means that the resultant velocity vector will start to tilt upward, increasing the angle of attack. If the pilot doesn't do anything, stall will occur and almost all lift will be gone. Therefore the pilot needs to decrease the pitch angle of the rotor blade as soon as engine failure occurs.

But what if the pilot forgets to reduce the pitch? In this case the helicopter is in serious trouble. Since there is stall, the rotor blade hardly has any lift and can't power itself. The only chance the pilot has is to start performing a forward flight with the helicopter, converting potential energy to kinetic energy. When sufficient velocity has been gathered, the pilot can change that kinetic energy into rotational velocity of the rotor blade. If, however, there isn't enough height (and thus not enough potential energy) to reach the right velocity, the pilot will probably not live to learn from his mistake.