Aircraft Performance 2 - Formula Overview

Cruise Flight

Equations of motion

$$\frac{W}{g}\frac{dV}{dt} = T - D - W\sin\gamma \tag{1}$$

$$\frac{W}{g}V\frac{d\gamma}{dt} = L - W\cos\gamma \tag{2}$$

Kinematics equations

$$\frac{ds}{dt} = V\cos\gamma, \qquad \frac{dH}{dt} = V\sin\gamma \tag{3}$$

Efficiency of jet aircrafts

$$\eta_{tot} = \frac{P_{out}}{P_{in}} = \frac{TV}{H\frac{F}{g}} = \frac{V}{c_T}\frac{g}{H} \tag{4}$$

Efficiency of propeller aircrafts

$$\eta_{tot} = \frac{P_{out}}{P_{in}} = \frac{TV}{H\frac{F}{g}} = \frac{\eta_p}{c_p}\frac{g}{H} \tag{5}$$

Range

Range

$$R = \frac{V}{c_T}\frac{C_L}{C_D}\ln\left(\frac{W_0}{W_1}\right) = \frac{H}{g}\eta_{tot}\frac{C_L}{C_D}\ln\left(\frac{W_0}{W_1}\right) \tag{6}$$

Lift coefficient for maximum range

$$C_L = \sqrt{\frac{1}{3}C_{D_0}\pi Ae} \tag{7}$$

Density for maximum range at speed limit

$$\rho_{opt} = \frac{W}{S}\frac{2}{V_{lim}^2}\frac{1}{C_L} \tag{8}$$

Take-Off

Equations of motion

$$\frac{W}{g}\frac{dV}{dt} = T - D - D_g = T - D - \mu(W - L) \tag{9}$$

Ground drag

$$D_g = \mu N = \mu(W - L) \tag{10}$$

Mean acceleration (a at $\frac{1}{2}\sqrt{2}V_{lof}$)

$$\bar{a} = \frac{g}{W}\left(\bar{T} - \bar{D} - \bar{D}_g\right) \tag{11}$$

Ground phase distance

$$s_{ground} = \frac{1}{2\bar{a}}V_{lof}^2 \tag{12}$$

Air phase distance

$$\gamma_{scr}s_{air} = \frac{V_{scr}^2 - V_{lof}^2}{2g} + H_{scr} \tag{13}$$

Climbing Flight

Steady rate Of climb

$$RC_{st} = \frac{T - D}{W}V = \frac{Pa - Pr}{W} = P_s \tag{14}$$

Kinetic correction factor

$$\frac{RC_{st}}{RC} = 1 + \frac{1}{2g}\frac{d(V^2)}{dH} \tag{15}$$

Energy height

$$E_h = H + \frac{1}{2g}V^2 \tag{16}$$

Time derivative of energy height

$$\frac{dE_h}{dt} = \frac{T - D}{W}V = P_s \tag{17}$$

Climb Time

Climb time to given height

$$t_{climb} = \int_0^{t_{climb}} dt = \int_0^H \frac{1}{RC}dH \tag{18}$$

Climb time to given energy height

$$t_{climb} = \int_0^{t_{climb}} dt = \int_0^{H_e} \frac{1}{P_s}dH_e \tag{19}$$

Lift coefficient for minimal climb time

$$C_L = \sqrt{3C_{D_0}\pi Ae} \tag{20}$$

Flight speed for minimal climb time

$$V = \sqrt{\frac{2}{\rho}\frac{1}{C_L}\frac{W}{S}}, \qquad V_e = \sqrt{\frac{2}{\rho_0}\frac{1}{C_L}\frac{W}{S}} \tag{21}$$

Landing

Energy landing equation

$$\frac{V_t^2}{2g} - \left(\frac{V_a^2}{2g} + H_{scr}\right) = -\frac{1}{2}s_{air}\left(\bar{\gamma}_a + \left(\frac{C_D}{C_L}\right)_t\right) \tag{22}$$

Break distance

$$s_{ground} = \int_{V_t}^{0} \frac{V\,dV}{a} = -\frac{V_t^2}{2\bar{a}} \tag{23}$$

Mean acceleration

$$\bar{a} = \frac{g}{W}\left(\bar{D} + \bar{D}_g + \bar{T}_{rev}\right) \tag{24}$$

Aircraft Systems

Air path system

$$E_a = [\mu]\,[\gamma]\,[\chi]\,E_e \tag{25}$$

Moving earth system

$$E_e = [\chi]^T\,[\gamma]^T\,[\mu]^T\,E_a \tag{26}$$

General Equations of Motion

$$m\dot{V} = T - D - W\sin\gamma \tag{27}$$

$$mV\dot{\chi}\cos\gamma = L\sin\mu \tag{28}$$

$$mV\dot{\gamma} = L\cos\mu - W\cos\gamma \tag{29}$$

Effects of Wind

Equations of motion

$$T - D - W\sin\gamma = \frac{W}{g}\left(\dot{V} - \dot{V}_w\cos\gamma\right) \tag{30}$$

$$L - W\cos\gamma = \frac{W}{g}\left(V\dot{\gamma} + \dot{V}_w\sin\gamma\right) \tag{31}$$

Kinematics equations

$$\dot{H} = V\sin\gamma, \qquad \dot{s} = V\cos\gamma - V_w \tag{32}$$

Wind gradients

$$RC\left(1 + \frac{V}{g}\frac{dV}{dH} - \frac{V}{g}\frac{dV_w}{dH}\right) = RC_{st} \tag{33}$$

Turning Flight

Turn radius

$$R = \frac{V^2}{g\sqrt{n^2-1}} \tag{34}$$

Turn velocity

$$V = \sqrt{\frac{2}{\rho}\frac{1}{C_L}\frac{nW}{S}} \tag{35}$$

Turn power

$$P_r = DV = \sqrt{\frac{2}{\rho}\frac{C_D^2}{C_L^3}\frac{(nW)^3}{S}} \tag{36}$$

Helicopters

Thrust

$$T = 2mV_i = 2\rho\pi r^2 V_i \tag{37}$$

Ideal hover power

$$P_{id} = WV_i = W\sqrt{\frac{W}{2\rho\pi r^2}} \tag{38}$$

Hover power

$$P_{hov} = \frac{\bar{c_{d_p}}\sigma}{8}\rho\left(\Omega R\right)^3\pi R^2 \tag{39}$$

Rotor solidity

$$\sigma = \frac{nRc}{\pi R^2} \tag{40}$$

Flight power

$$P_p = P_{hov}\left(1 + \mu^2\right) \tag{41}$$

Advance ratio

$$\mu = \frac{V\cos\alpha_d}{\Omega R} \tag{42}$$