Effects of Wind

1 Equations of Motion

An aircraft flies in air. So its speed is also measured with respect to the air. In case of winds, this velocity is not equal to the velocity of the aircraft with respect to the ground. Instead, this velocity can be expressed as

$$\mathbf{V_g} = \mathbf{V} + \mathbf{V_w}, \tag{1.1}$$

where $\mathbf{V_w}$ is the velocity of the wind with respect to the ground. We usually assume this velocity is directed parallel to the ground, in such a way that the aircraft experiences headwind. Therefore

$$\mathbf{V_w} = \begin{bmatrix} V_w & 0 & 0 \end{bmatrix} E_g. \tag{1.2}$$

To derive the equations of motion, we look at the acceleration of the aircraft, which is

$$\mathbf{a} = \dot{\mathbf{V}}_\mathbf{g} = \dot{\mathbf{V}} + \dot{\mathbf{V}}_\mathbf{w} = \begin{bmatrix} \dot{V} & 0 & 0 \end{bmatrix} E_a + \begin{bmatrix} V & 0 & 0 \end{bmatrix} \dot{E}_a + \begin{bmatrix} -\dot{V}_w & 0 & 0 \end{bmatrix} E_g. \tag{1.3}$$

Note that we have not written the term involving $\dot{E}_g$. This is because E_g doesn't change, so $\dot{E}_g = 0$. If we assume the aircraft is in a symmetric flight, we can rewrite the acceleration as

$$\mathbf{a} = \begin{bmatrix} \dot{V} - \dot{V}_w \cos\gamma & 0 & -V\dot{\gamma} - \dot{V}_w \sin\gamma \end{bmatrix} E_a. \tag{1.4}$$

Using Newton's second law this time results in

$$T - D - W\sin\gamma = \frac{W}{g}\left(\dot{V} - \dot{V}_w \cos\gamma\right), \tag{1.5}$$

$$L - W\cos\gamma = \frac{W}{g}\left(V\dot{\gamma} + \dot{V}_w \sin\gamma\right). \tag{1.6}$$

These are the equations of motion if the airplane experiences headwind. But not only the equations of motion are slightly different. Also the kinematics equations have changed. They now are

$$\dot{H} = V\sin\gamma, \qquad \text{and} \qquad \dot{s} = V\cos\gamma - V_w. \tag{1.7}$$

2 Taking Off and Landing

When the airplane is in the air, it's relatively simple to calculate the effects of it. But what if the aircraft is near the ground? We first consider the take-off and then look at the landing.

During take-off the aircraft needs to reach the lift-off speed V_{lof} to be able to lift off from the ground. This speed is measured with respect to the wind. The speed of the aircraft is, however, measured with respect to the ground. So there is a difference. Let's suppose $V_{lof} = 100kts$. If there is a headwind of $20kts$, then the aircraft only needs to have a velocity of $V = 80kts$ with respect to the ground to take off. If, however, the wind comes from the back of the aircraft, it needs a velocity of $V = 120kts$. So if you take off with headwind, you need a much lower velocity, and thus a much shorter runway. Therefore its preferable to take off with headwinds.

The situation is virtually the same for landings. If you land with headwinds, you have a much lower velocity with respect to the ground, and therefore it's much easier to come to a full stop. If you land with the wind blowing in your back, then you need a much greater distance to come to a complete stop.

3 Wind Gradients

When climbing or descending, the situation is approximately the same. It's better to climb/descend with headwinds. To show this, we derive an equation which involves the **wind gradient** dV_w/dH. First we rewrite $\dot{V}$ and $\dot{V}_w$ to

$$\dot{V} = \frac{dV}{dt} = \frac{dV}{dH}\frac{dH}{dt} = \frac{dV}{dH}RC \qquad \text{andidentically} \qquad \dot{V}_w = \frac{dV}{dH}RC. \tag{3.1}$$

Substituting this in equation 1.5 will give

$$\frac{W}{g}\left(\frac{dV}{dH}RC - \frac{dV_w}{dH}RC\cos\gamma\right) = T - D - W\sin\gamma. \tag{3.2}$$

Multiplying by V/W will then give

$$\frac{V}{g}RC\left(\frac{dV}{dH} - \frac{dV_w}{dH}\cos\gamma\right) + V\sin\gamma = \frac{TV - DV}{W} = \frac{P_a - P_r}{W} = RC_{st}. \tag{3.3}$$

Rewriting $V\sin\gamma = RC$ and bringing terms within brackets will give

$$RC\left(1 + \frac{V}{g}\frac{dV}{dH} - \frac{V}{g}\frac{dV_w}{dH}\right) = RC_{st}. \tag{3.4}$$

Climbs are usually performed at an approximately constant velocity, so we can assume that $dV/dH = 0$. Now look at the term dV_w/dH. Winds usually blow harder at higher altitudes. In case of headwinds this means that $dV_w/dH > 0$ and thus $RC > RC_{st}$. If the wind is in your back, then $dV_w/dH < 0$ and thus $RC < RC_{st}$. So to climb faster, it is better to have headwinds.

The situation is similar for descending. Since $RD = -RC$ and $RD_{st} = -RD_{st}$ we find

$$RD\left(1 + \frac{V}{g}\frac{dV}{dH} - \frac{V}{g}\frac{dV_w}{dH}\right) = RD_{st}. \tag{3.5}$$

This once more shows that $RD > RD_{st}$ in case you are descending with headwinds.

4 Microbursts

A microburst is a column of descending air, like is displayed in figure 1. As the air comes close to the ground, it diverges in multiple directions. It's similar to the water coming out of a tap. When the flow of water hits the sink, it moves away in all directions.

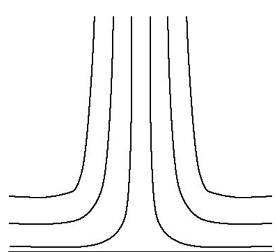

Figure 1: Microburst.

When an aircraft is flying into a microburst, it first experiences a headwind. It therefore slows down a bit with respect to the ground. When it passes the center of the microburst, the direction of the wind suddenly seems to reverse. The aircraft has then lost almost all of its speed, and more or less plummets to the ground. Although microbursts are relatively rare phenomena, they have caused multiple aircrafts to crash.