Cruise Flight

1 Equations of Motion

Flying airplanes spend most of the time in cruise flight. It is therefore an important part of the flight. To learn more about the cruise flight, we look at the forces acting on the aircraft during a steady symmetrical flight. These forces, the **thrust** T, the **drag** D, the **weight** W and the **lift** L are shown in figure 1.

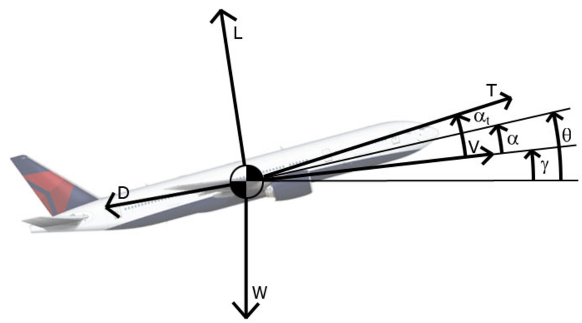

Figure 1: Forces acting on the aircraft.

Per definition the weight points downward, the lift is perpendicular to the velocity vector and the drag is parallel to the velocity vector. We also assume that the thrust points in the direction of the velocity, so the **thrust angle of attack** $\alpha_t = 0$. Other angles are the **flight path angle** γ, the **angle of attack** α and the **pitch angle** θ.

We now use Newton's second law, $\mathbf{F} = m\mathbf{a}$. Looking in the direction parallel to the velocity will give

$$\frac{W}{g}\frac{dV}{dt} = T - D - W \sin\gamma. \tag{1.1}$$

Looking in the direction perpendicular to the velocity will give

$$\frac{W}{g} V \frac{d\gamma}{dt} = L - W \cos\gamma. \tag{1.2}$$

Furthermore, we have two kinematic equations, being

$$\frac{ds}{dt} = V\cos\gamma \qquad \text{and} \qquad \frac{dH}{dt} = V \sin\gamma. \tag{1.3}$$

A cruise flight is a **quasi-stationary flight**, meaning that at any given point you can assume the flight is stationary. However, you can not assume that the entire flight is stationary. In fact, the weight of the aircraft decreases during flight, according to

$$\frac{dW}{dt} = -F(V, H, \Gamma), \tag{1.4}$$

where F is the fuel flow, which depends on the velocity V, the height H and the engine setting Γ. Cruise flights are usually steady horizontal flights. **Steady flight** implies that $dV/dt = 0$. **Horizontal flight** means that $\gamma = 0$. Combining this with the equations of motion gives

$$T = D \qquad \text{and} \qquad L = W. \tag{1.5}$$

2 Cruise Flight Strategy

During the cruise flight you want to use as few fuel as possible. If you want to stay as long in the air as possible, you should maximize the endurance. The **endurance** E is the time which you can stay in the air with a given amount of fuel. To maximize E you should minimize the fuel flow F.

If, however, you want to go as far as possible, you should maximize the range. The **range** R is the distance you can cover with a given amount of fuel. So the distance ds per weight of fuel used dW_f should be maximized. So we need to maximize

$$\frac{ds}{dW_f} = \frac{ds/dt}{dW_f/dt} = \frac{V}{F}. \tag{2.1}$$

For jet engines we can assume that the fuel flow varies linearly with the thrust, so $F = c_T T = c_T D$, where the constant term c_T is the **specific fuel consumption**. Also, from the definition of the lift and drag coefficient follows that $D/C_D = L/C_L = W/C_L$. Combining all this gives

$$\frac{V}{F} = \frac{V}{c_T D} = \frac{V}{c_T W}\frac{C_L}{C_D} = \sqrt{\frac{1}{SW c_T^2}\frac{2}{\rho}\frac{C_L}{C_D^2}}. \tag{2.2}$$

So to increase the range we should decrease ρ. That is simple. We should fly as high as possible.

Now let's consider we're flying at a given altitude (and thus constant density ρ). We need to maximize C_L/C_D^2. Differentiating this fraction by using $C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$ will show that C_L/C_D^2 is minimal if

$$C_L = \sqrt{\frac{1}{3}C_{D_0}\pi A e}. \tag{2.3}$$

So we now have sufficient data to determine our strategy. We should fly as high as possible and choose our V such that V/F is at a maximum.

But what if the determined velocity V with which we need to fly is beyond a **speed limit** of our aircraft? In that case we should fly at the speed limit, so $V = V_{lim}$. It now is important to minimize the drag D. Minimum drag occurs if C_L/C_D is maximal, resulting in a lift coefficient of $C_L = \sqrt{C_{D_0}\pi A e}$. We also need to choose our height H (and thus the density ρ) such that the drag D is minimal. This occurs if

$$\rho_{opt} = \frac{W}{S}\frac{2}{V_{lim}^2}\frac{1}{C_L}. \tag{2.4}$$

3 Finding the Range

Now we are curious, what will the range be if we fly in this optimal condition? To find an equation for the range, we first express the distance traveled ds as a function of the change in weight dW according to

$$F = -\frac{dW}{dt} = -\frac{dW}{ds}\frac{ds}{dt} = -\frac{dW}{ds}V \qquad \Rightarrow \qquad ds = -\frac{V}{F}dW. \tag{3.1}$$

The range can now be found by integration. To get a meaningful result, we make a change-of-variable according to the above relation. So the range is

$$R = \int_{s_0}^{s_1} ds = \int_{W_0}^{W_1} -\frac{V}{F}dW = -\frac{V}{c_T}\frac{C_L}{C_D}\int_{W_0}^{W_1}\frac{1}{W}dW = \frac{V}{c_T}\frac{C_L}{C_D}\ln\left(\frac{W_0}{W_1}\right), \tag{3.2}$$

where W_0 is the initial mass of the aircraft and W_1 the final mass. The above equation is called the **equation of Breguet**. We will examine it more closely at the end of this chapter.

4 Optimum Cruise Condition

We have determined how to fly as optimal as possible at a given point. But as we continue flying our weight W decreases and we're not flying optimal anymore. What should we do? We can consider several flying strategies and then select the best.

1. Keep constant engine settings Γ and height H. Although the velocity V will increase, the factor V/F will be far below optimal during the rest of the flight.
2. Keep constant velocity V and height H. This time the factor V/F will be better, but it will still be below optimal.
3. Keep constant angle of attack α and height H. Doing this will result in the optimal V/F during flight. However, the velocity V will decrease, which is unwanted if we want to get somewhere fast.
4. Keep constant angle of attack α and velocity V. The value of V/F will still remain optimal. The height at which we are flying increases during flight. This is usually considered the best strategy.

So the conclusion is that it is best to fly at a constant velocity V and angle of attack α with a slightly positive flight path angle γ.

5 Propeller Aircrafts

The previous paragraphs were all about jet aircrafts. Now let's briefly consider propeller aircrafts. There is a fundamental difference between these two aircrafts. For jet aircrafts the fuel consumption is approximately proportional to the thrust. For propeller aircrafts the fuel consumption is more or less proportional to the power P_{br} of the engine. So

$$F = c_p P_{br} = \frac{c_p}{\eta_p} P_a = \frac{c_p}{\eta_p} TV, \tag{5.1}$$

where η_p is the propulsive efficiency. The factor V/F, which is so important for the range, now becomes

$$\frac{V}{F} = \frac{\eta_p}{c_p} \frac{C_L}{C_D} \frac{1}{W}. \tag{5.2}$$

Deriving the equation of Breguet for propeller aircrafts will then give

$$R = \frac{\eta_p}{c_p} \frac{C_L}{C_D} \ln\left(\frac{W_0}{W_1}\right). \tag{5.3}$$

This looks rather similar to the equation of the range for jet aircrafts. This can be illustrated further if we look at the efficiency of both engines. The total efficiency of a propeller aircraft is

$$\eta_{tot} = \frac{P_{out}}{P_{in}} = \frac{TV}{H\frac{F}{g}} = \frac{\eta_p}{c_p} \frac{g}{H}, \tag{5.4}$$

where H is the **specific energy** of the fuel in J/kg. The total efficiency of a jet aircraft is

$$\eta_{tot} = \frac{P_{out}}{P_{in}} = \frac{TV}{H\frac{F}{g}} = \frac{V}{c_T} \frac{g}{H}, \tag{5.5}$$

Filling this in in Breguet's equation will give the same result for both types of aircraft, being

$$R = \frac{H}{g} \eta_{tot} \frac{C_L}{C_D} \ln\left(\frac{W_0}{W_1}\right). \tag{5.6}$$

The factor H/g is a measure of the fuel quality. The factor η_{tot} indicates the quality of the propulsion system. The factor C_L/C_D shows the aerodynamic quality. And finally the fraction W_0/W_1 indicates the quality of the structure of the aircraft. So the conclusion is simple: Well-designed aircrafts fly further with less fuel.