Climbing Flight

1 Rate of Climb

In the previous chapter we saw the energy equation. If we differentiate it with respect to the time t we will find

$$(T - D)\frac{ds}{dt} = \frac{W}{2g}\frac{d(V^2)}{dH}\frac{dH}{dt} + W\frac{dH}{dt}. \tag{1.1}$$

Note that we have made a change of variable to the height H in the middle term. We did this because we wanted to express things as a function of the **rate of climb**, denoted by $RC = dH/dt$. We can thus rewrite this as

$$\frac{T - D}{W}V = \frac{1}{2g}\frac{d(V^2)}{dH}RC + RC. \tag{1.2}$$

Let's briefly consider stationary flight. In this case $d(V^2) = 0$, so the so-called **stationary rate of climb**, denoted by RC_{st}, will be

$$RC_{st} = \frac{T - D}{W}V = \frac{Pa - Pr}{W} = P_s, \tag{1.3}$$

where P_s is the **specific excess power**. It is the excess power per Newton of aircraft weight. Combining this with the previous equation will

$$\frac{RC_{st}}{RC} = 1 + \frac{1}{2g}\frac{d(V^2)}{dH}. \tag{1.4}$$

The factor on the right hand side of this equation is called the **kinetic correction factor**. The reason for this is in fact quite logical. Your engines produce a certain amount of excess power $(T - D)\,V$. If V is constant, this energy is spend on increasing the potential energy (that is, climbing). If V increases, the kinetic energy increases, so less energy can be spend on increasing the potential energy. A correction factor is then necessary to indicate the difference between the stationary and non-stationary rate of climb.

2 Climb Time

The **climb time** is the time it takes to reach a given height. It can be found by using the definition of the rate of climb. The result will be

$$t_{climb} = \int_0^{t_{climb}} dt = \int_0^H \frac{1}{RC}dH. \tag{2.1}$$

Let's assume we want to minimize climb time for a small propeller aircraft. To do this, we need to maximize the rate of climb. Since a propeller aircraft is a low speed aircraft, the quantity $d(V^2)/2 = V\,dV$ is small enough to be neglected. The kinetic correction factor then becomes 1, so we can assume $RC = RC_{st} = P_s$. So we need to maximize

$$P_s = \frac{Pa - Pr}{W} = \frac{Pa}{W} - \frac{DV}{L} = \frac{Pa}{W} - \frac{C_D V}{C_L} = \frac{Pa}{W} - \sqrt{\frac{2}{\rho}\frac{W}{S}\frac{C_D^2}{C_L^3}}. \tag{2.2}$$

Since propeller aircrafts have a constant power available, the left side of the equation is constant. We therefore need to minimize C_D^2/C_L^3. The resulting lift coefficient will be $C_L = \sqrt{3C_{D_0}\pi Ae}$. The corresponding true airspeed V and equivalent airspeed V_e will be

$$V = \sqrt{\frac{2}{\rho}\frac{1}{C_L}\frac{W}{S}} \qquad \text{and} \qquad V_e = V\sqrt{\frac{\rho}{\rho_0}} = \sqrt{\frac{2}{\rho_0}\frac{1}{C_L}\frac{W}{S}}. \tag{2.3}$$

Here we see that the optimal equivalent airspeed remains constant for different heights. In fact, when pilots in real low-speed aircrafts climb, they usually keep a constant indicated airspeed.

But what about high speed aircrafts? In this case the climb time becomes

$$t_{climb} = \int_0^H \frac{1}{RC} dH. = \int_0^H \frac{1 + \frac{V}{g_0}\frac{dV}{dH}}{RC_{st}} dH. \tag{2.4}$$

To maximize the climb time is very difficult now. If we maximize the rate of climb at one given time, we will get a very low value at another time. It is therefore important to maximize the rate of climb over the entire climb. But how to do that is a question we're currently not able to answer.

3 Zoom Flight and Energy Height

When an aircraft is flying, it has both kinetic energy and potential energy. These energies can be easily converted to each other by initiating a dive, or by pulling the aircraft's nose up. A flight procedure where one type of energy is converted to the other is called a **zoom flight**. To indicate the effect of zoom flights, we look at the **energy** E of the airplane, which stays constant during a zoom flight. This is

$$E = WH + \frac{W}{2g}V^2. \tag{3.1}$$

If we define the **specific energy** E_h as the energy per Newton of weight, we will find

$$E_h = \frac{E}{W} = H + \frac{1}{2g}V^2. \tag{3.2}$$

We see that E_h is more or less similar to the height. In fact, if the airplane converts all its kinetic energy to potential energy, then its height H will be equal to the specific energy E_h of the aircraft. The specific energy E_h is therefore also called the **energy height**. We now look at how this energy height changes in time. This goes according to

$$\frac{dE_h}{dt} = \frac{dH}{dt} + \frac{1}{2g}\frac{d(V^2)}{dt} = V\sin\gamma + \frac{V}{g}\frac{dV}{dt}. \tag{3.3}$$

We can rewrite the equation of motion of the aircraft to

$$\frac{1}{g}\frac{dV}{dt} = \frac{T-D}{W} - \sin\gamma. \tag{3.4}$$

Combining the previous two equations will result in a rather simple relation. This is

$$\frac{dE_h}{dt} = \frac{T-D}{W}V = P_s. \tag{3.5}$$

So the time derivative of the specific energy is the specific excess power. Let's suppose we want to reach a certain energy height with our aircraft. How long would that take us? This is suddenly a rather easy problem. The time it takes will be

$$t_{climb} = \int_0^{H_e} \frac{1}{P_s} dH_e. \tag{3.6}$$

So the conclusion is simple. To minimize the climb time, we should maximize the specific excess power for every H. Especially supersonic aircrafts make good use of this fact. When they are climbing they will, at some point, reach the transonic barrier. At transonic flight speeds the drag D is very high, resulting in a low specific excess power. To minimize this effect, a supersonic aircraft initiates a dive as soon as he reaches the transonic barrier. In this way he spends as few time as possible in the transonic flight regime. So to climb as efficient as possible, a dive is necessary. Although this is rather counterintuitive, it has proven to work in practice.