Aerodynamics Formulas

Definitions

p = The air pressure. ($Pa = N/m^2$)
ρ = The air density. (kg/m^3)
g = The gravitational constant. (Value at sea level is $9.81 N/kg$) (N/kg)
h = The height above the earth surface. (m)
V = The speed of the airplane relative to the air. (m/s)
p_t = The total pressure. ($Pa = N/m^2$)
p_0 = The static pressure. ($Pa = N/m^2$)
S = The wing surface. (m^2)
L = The lift force. (N)
C_L = The lift coefficient. (no unit)
D = The drag force. (N)
C_D = The drag coefficient. (no unit)

C_{D_i} = The induced drag coefficient. (no unit)
e = The Oswald factor. (Usually has a value between 0.8 and 0.9) (no unit)
A = The aspect ratio. (no unit)
b = The wing span (from left wing tip to right wing tip, so it's not just the length of one wing). (m)
D_i = The induced drag. (no unit)
C_{D_0} = The friction and pressure drag coefficient. (no unit)

M = The Mach number. (no unit)
a = The speed of sound. ($340 m/s$ at sea level) (m/s)
Re = The Reynolds number. (no unit)
L = A characteristic length. Often the length of an object. (m)
μ = The viscosity of the air. (Normal air has viscosity $17.9 \times 10^{-6} kg/(ms)$) ($kg/(ms)$)

W = The weight of the aircraft. (N)
T = The thrust of the aircraft. (N)
L_w = The wing loading. ($Pa = N/m^2$)
n = The load factor. (no unit)

Two-dimensional aerodynamics formulas

The pressure in a certain part of the atmosphere is equal to the weight of the air column on top. The formula describing this statement is known as the hydrostatic equation:

$$dp = -\rho g(dh) \tag{1}$$

An equation which looks a bit like the previous equation, is the Euler equation:

$$dp = -\rho V(dV) \tag{2}$$

So, if we integrate this equation, we find bernoulli's equation:

$$p + \frac{1}{2}\rho V^2 = C \tag{3}$$

Where C is a constant. So $p + \frac{1}{2}\rho V^2$ is constant for any 2 points along a streamline. Using this formula, the airspeed can be calculated:

$$V_0 = \sqrt{2\frac{p_t - p_0}{\rho}} \tag{4}$$

Bernoulli's equation states that $-dp = d(\frac{1}{2}\rho V^2)$. By integrating that over the wing surface, and implementing a constant, the following formula can be found:

$$L = C_L \frac{1}{2}\rho V^2 S \tag{5}$$

Similar to this, also the drag force can be calculated:

$$D = C_D \frac{1}{2}\rho V^2 S \tag{6}$$

Induced Drag

However, the previously discussed formulas work well for two-dimensional cases. In three dimensions there is also another type of drag, called the induced drag. This type of drag also has a coefficient:

$$C_{D_i} = \frac{C_L^2}{\pi A e} \tag{7}$$

But in this case, A is not known yet. A, the aspect ratio, is the relationship between the length and the width of the wing. However, the width of the wing is not constant. So by multiplying the ratio $A = \frac{wingspan}{wingwidth}$ on both sides of the fraction by the wing span, the following formula appears:

$$A = \frac{b^2}{S} \tag{8}$$

Now, using all this data (and the fact that $C_L = \frac{2L}{\rho V^2 S}$, the induced drag can be calculated:

$$D_i = C_{D_i} \frac{1}{2}\rho V^2 S = \frac{2L^2}{\rho S \pi A e}\frac{1}{V^2} \tag{9}$$

So by using the formula:

$$C_D = C_{D_0} + C_{D_i} = C_{D_0} + \frac{C_l^2}{\pi A e} \tag{10}$$

The total drag can be calculated, using equation (6).

Characteristic numbers

There are also a few numbers which characteristic the type of flow. An example is the Mach number, which is calculated using:

$$M = \frac{V}{a} \tag{11}$$

There are different names for different ranges of Mach numbers:

- $M < 0.8$: Subsonic
- $0.8 < M < 1.2$: Transonic
- $1.2 < M < 4$: Supersonic
- $4 < M$: Hypersonic

Next to the Mach number, there is also the Reynolds number:

$$Re = \frac{\rho V L}{\mu} \tag{12}$$

The Reynolds number is an indication if, and where, separation occurs. High Reynolds numbers usually result in a more turbulent flow, while low Reynolds numbers result in a more laminar flow.

Flight types

In a horizontal (no change of height) steady (no roll) straight (no yaw) flight, the following conditions must apply:

$$W = L = C_L \frac{1}{2} \rho V^2 S \tag{13}$$

$$T = D = C_D \frac{1}{2} \rho V^2 S \tag{14}$$

Divide these equations, and you will find that:

$$\frac{W}{T} = \frac{L}{D} = \frac{C_L}{C_D} \tag{15}$$

Also, it is interesting to notice that the minimal speed an airplane can have, can be calculated, if the maximum lift coefficient is known:

$$W = L = C_{L_{max}} \frac{1}{2} \rho V_{min}^2 S \tag{16}$$

There is also a factor called the wing loading. This is equal to:

$$L_w = \frac{W}{S} = C_L \frac{1}{2} \rho V^2 \tag{17}$$

However, when there is no horizontal flight, but if the airplane is climbing, some of the previous formulas don't apply. In this case, a load factor can be introduced. This can be calculated as follows:

$$n = \frac{L}{W} \tag{18}$$

So in a horizontal flight, the load factor is 1.

Since $L_{max} = C_{L_{max}} \frac{1}{2} \rho V^2 S$ and $W = C_{L_{max}} \frac{1}{2} \rho V_{min}^2 S$ it can also be derived that:

$$n_{max} = \frac{L_{max}}{W} = \left(\frac{V}{V_{min}}\right)^2 \tag{19}$$