Intro to AE

AE1110-I

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Preface

So this is a Summary of basically everything you'll need to get through Intro I. Obviously I'm not going to every bit of information from the lectures and the book in here, so if you want further clarification I recommend looking back at the slides. These notes go in the same order as the lectures so you should have no problem finding further information. The latest version of these notes is always available on my website, (alanrh.com) along with other resources that I find useful. If you find any mistakes and you need anything corrected shoot me an email, but please don't email me if you want further explanation because I can't promise individual help to everyone. Also, I'd like to thank the lecturers for uploading their slides so that I could shamelessly steal their diagrams and graphs because I am not smart enough to draw good enough charts.

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Contents

List of Symbols

γ	Climb angle	deg
λ_h	Horizontal tail taper ratio	-
$\Lambda_{0.25h}$	Horizontal tail quarter chord sweep	deg
$\Re$	Universal gas constant	J/mol*K
ρ	Density	kg/m^3
R	Specific gas constant for Air	J/kg*K
a	Speed of sound	m/s
b	Wing span	m
c_r	Chord length at root	m
c_t	Chord length at tip	m
$C_{m_{ac}}$	Pitching moment around the aerodynamic centre	-

1 Fundamentals

1.1. Aerostatic Equations

The Aerostatic equations basically deal with things that simply float, rather than fly through the air. It's kinda in the name: Aero*static*. The first order of business if to derive the relations between pressure and density from our secondary school physics equations.

$$pV = n\Re T$$
$$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$$
$$p * \frac{m}{\rho} = n\Re T$$
$$n = \frac{m}{M}$$
$$p * \frac{m}{\rho} = \frac{m}{M}\Re T$$
$$\frac{p}{\rho} = \frac{\Re}{M}T$$
$$p = \rho\frac{\Re}{M}T$$

$$p = \rho RT \tag{1.1}$$

What you see here is the most basic formula for aerostatic equations. You'll use this to Calculate pressure or density, whichever you have.

1.1.1. Balloons

This is the point where we start worrying about actually calculating things. First off; how do balloons float? It's just the Archimedes principle. They displace a certain volume of air, and replace it with an equal volume of a different gas that weighs less. The difference in weight between these volumes is experienced as a Lift.

$$L = F_{gravity\ air} - F_{gravity\ gas}$$

But we know that Force is mass times acceleration so we can substitute this in:

$$L = m_{air}g - m_{gas}g$$

Mass is simply the density of a gas multiplied by a volume *(which is equal for both quantities remember?)*:

$$L = \rho_{air}Vg - \rho_{gas}Vg$$
$$L = Vg\ (\rho_{air} - \rho_{gas})$$
$$L = \rho_{air}Vg\ (1 - \frac{\rho_{gas}}{\rho_{air}})$$

Here's the point where we must look back to equation 1.1. Or rather, the more general version of it. We rearange it to find a general solution for density (ρ).

$$\rho = \frac{PM}{\Re T}$$

With this equation for density we can substitute it in and get the general equation for aerostatic lift.

$$L = \rho_{air} V g \ (1 - \frac{P_{gas} M_{gas} \Re T_{air}}{P_{air} M_{air} \Re T_{gas}})$$

Notice how □is present on both the top and bottom of the fraction. Well, that's a universal constant, so cancel that out and you get:

$$L = \rho_{air} V g \ (1 - \frac{P_{gas} M_{gas} T_{air}}{P_{air} M_{air} T_{gas}}) \tag{1.2}$$

Lighter Gas

To find the lift generated by a balloon filled with a light gas, like helium, take a look at equation 1.2. The pressure of the air around the balloon is equal to that of the gas. and the temperature is also the same.

$$L = \rho_{air} V g \ (1 - \frac{M_{gas}}{M_{air}}) \tag{1.3}$$

A light gas balloon's lift is a function of the ration of molar masses. For helium $M = 4$ and for air $M \approx 28.97$ depending on the air sample.

Hot Air

In a hot air balloon, the molar masses, and the pressures are the same, but, the temperature is not (who would have guessed). Thus, the equation becomes:

$$L = \rho_{air} V g \ (1 - \frac{T_{air}}{T_{gas}})$$

But we can reformat this into:

$$L = \rho_{air} V g \ (\frac{\Delta T}{T + \Delta T}) \tag{1.4}$$

1.2. The International Standard Atmosphere

In order to compare various flight characteristics we need a common atmosphere to measure performance, because a plan can fly very differently in cold, windy air, compared to hot, still air. This is why we have the *ISA*. It's not a perfect analogue of the real world, but that's the point! We just use it as a baseline for comparison, and we're not gonna complicate it too much, because; again, we're engineers and we're lazy.

1.2.1. Altitudes

So, there are a hand-full of ways of defining altitude. Firstly there's the *Absolute Altitude*, this is the distance between you and the centre of the planet, but we mainly use that for space-flight and orbital mechanics, so, you'll deal with that next quarter. Next there's the *Geometric Altitude*. This is essentially, just the height you are above the ground, equal to the absolute altitude, minus the radius of the earth.

Then we encounter the *Geopotential Altitude*. This is a real fun one because it makes our lives much easier. So, as you should know as you move farther away from the centre of the earth, the force of gravity weakens in relation to the inverse square law. But, that's an effort to compute, so the geopotential altitude pretends that the potential energy per unit mass increases linearly.

$$E = m * g * h \tag{1.5}$$

This is the simple equation for potential energy that you'll have learned in secondary school. But when the value of g decreases as you increase in altitude, it causes problems for us. So what if we want a constant g_0? In that case, to keep the same energy calculated, we'll have to change either the value of Mass (difficult) or the value for altitude (easy). Therefore:

$$E = m * g * h = m * g_0 * h_{geopotential} \tag{1.6}$$

Now, the Geopotential altitude is so useful for us in this course, that we assume this is what we mean by altitude. so Absolute altitude is written as h_a, Geometric altitude is written as h_g, and the Geopotential altitude is written as h. To calculate one based on the other:

$$h_a = h_g + Radius_{earth} \tag{1.7}$$

$$h = (\frac{Re}{Re + h_g}) * h_g \tag{1.8}$$

1.2.2. Hydrostatic Equation

So, air is squishy (or compressible if you'd prefer), so the pressure you feel is due to the fluid (air) above you. To calculate the pressure change in a sample of air we have to consider how that comes to be.

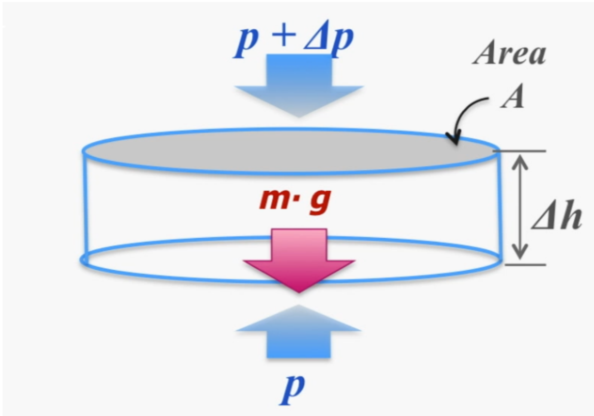


Figure 1.1: *A sample of air with a difference in pressure*

We assume the atmosphere is in equilibrium (it isn't but that's too complicated to deal with), and so in equilibrium, the sum of the forces acting on this air sample sum to zero.

$$m * g + (p + \Delta p) * A = p * A$$
$$\rho * V * g + p * A + \Delta p * A = p * A$$
$$\rho * \Delta h * A * g + p * A + \Delta p * A = p * A$$
$$\Delta p = -\rho * \Delta h * g$$

But, this only works for big steps, and when it comes to accuracy, and integration, we need to take small steps.

$$dp = -\rho * g * dh \tag{1.9}$$

1.2.3. Toussaint's Formula

Very simple concept this time, as you go up in altitude, the change in temperature is *basically* linear within different layers in the atmosphere, so we can write a simple function to find the temperature at a new altitude within a layer, based on the stating temperature (T_0) and the lapse rate (a). And due to the aerostatic equation 1.1, we can use this temperature to find other atmospheric characteristics.

$$T_1 = T_0 + a(h_1 - h_0) \tag{1.10}$$

$$a = \frac{dT}{dh} \tag{1.11}$$

1.2.4. The ISA equations

Wonderful, you've made it this far, now let's work out how to actually calculate ISA values. (Warning, this is mostly maths). Recall 1.9:

$$dp = -\rho * g * dh$$

$$dp = -\frac{p}{RT} * g * dh$$

$$\frac{1}{p}dp = -\frac{g}{RT} * dh$$

But wait, remember how $a = \frac{dT}{dh}$? That thus means $dh = \frac{dT}{a}$

$$\frac{1}{p}dp = -\frac{g}{RT} * \frac{dT}{a}$$

$$\int_{p_0}^{p_1} \frac{1}{p}dp = -\frac{g}{aR}\int_{T_0}^{T_1} \frac{1}{T}dT$$

$$\ln p_1 - \ln p_0 = -\frac{g}{aR}\ln T_1 - \ln T_0$$

$$\frac{p_1}{p_0} = e^{-\frac{g}{aR}\ln T_1 - \ln T_0}$$

$$\frac{p_1}{p_0} = \frac{T_1}{T_0}^{-\frac{g}{aR}} \tag{1.12}$$

But what if a is 0? You can't divide by zero! In that case, just jump back a few steps. Notice below how g,R,and T are all constant? Well that kinda makes the integration even easier.

$$\frac{1}{p}dp = -\frac{g}{RT} * dh$$

$$\int_{p_0}^{p_1} \frac{1}{p}dp = -\frac{g}{RT}\int_{h_0}^{h_1} dh$$

$$\frac{p_1}{p_0} = e^{-\frac{g}{RT}(h_1 - h_0)} \tag{1.13}$$

1.3. Lift and Drag

What's the point of studying all about air if you're not going to learn about how aeroplanes actually fly? Nothing. Anyway, there are 4 main forces that act on a plane, well, actually there are only 3, but we separate out the aerodynamic force into it's components to get:

- **Lift (L)**: The upwards force generated (mostly) by the wings.
- **Weight (W)**: Caused by the empty weight, the fuel, and the payload.
- **Thrust (T)**: The horizontal force generated by the engines.
- **Drag (D)**: The resistance to motion as you move through the air.

In stable flight, these balance out to be:

$$L = W \tag{1.14}$$

$$T = D \tag{1.15}$$

1.3.1. Lift

Lift is given by the equation:

$$L = C_L \frac{1}{2} \rho V^2 S \tag{1.16}$$

- C_L: The coefficient of lift is defined by:
  - The angle of attack (α)
  - The shape of the wing
  - A whole bunch of other design factors that we hide behind a coefficient.
- ρ: The density is a function of altitude and temperature [$\frac{kg}{m^3}$]
- **V**: The velocity is something we design for [$\frac{m}{s}$]
- **S**: The surface area of the *wing*, is also a design parameter [m^2]

When we account for units we get Lift in: $\frac{kg*m}{s^2}$, or more simply put, Newtons [N]

1.3.2. Bernoulli

What's up with that $\frac{1}{2}\rho V^2$ thing? That's what we call dynamic pressure, and it's how we generate lift. according to Bernoulli and his magical experiments, total pressure in a streamline is always constant (kinda). So, when the dynamic pressure over the top of the wing is higher, then the static pressure on the top of the wind is lower. Thus the static pressure on the lower part of the wing is higher, and this pushes the aircraft up, and this is what we call Lift. Which is why Lift is a function of the dynamic pressure.

1.3.3. Drag

Yeah, Drag is simple to understand, as you move through air, the air pushes back on you. Simple. And it's so simple that the equation for drag is the same as the equation for lift (1.16), except with a different coefficient, because why bother with something new?

$$D = C_D \frac{1}{2} \rho V^2 S \tag{1.17}$$

So, that equation is pretty simple to understand, but let's take a closer look at the coefficent there. It's made up of two components, C_{D_0}, the 'zero lift drag' and, C_{D_i}, the induced drag due to lift. C_{D_i} is made up of:

$$C_{D_i} = \frac{C_L^2}{\pi A e}$$

Where A is the Aspect ratio of the wing,($A = \frac{span}{chord} = \frac{surface\ area^2}{chord}$), and e is the *Oswald Efficiency Factor* (measure characteristic of an aircraft). Thus giving us the useful equation to find the coefficient of drag:

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e} \tag{1.18}$$

Let's dive a little deeper into drag though. It's a result of moving through the air, but, more specifically it comes from 3 main sources:

- Friction Drag: caused by the friction as the air moves along the aircraft
- Pressure Drag: induced by lift
- Wave Drag: caused by going super/trans-sonic

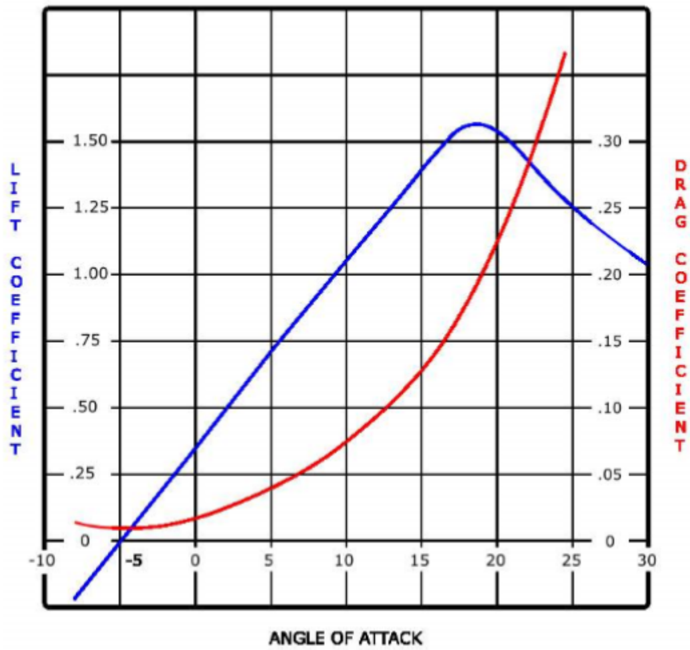


Figure 1.2: *Coefficients vs Angle of Attack*

See, in this chart you can see how drag increases with an increases with an angle of attack, but so does lift. This is very simple to understand really. As you increase your angle of attack, you push more air down, generating more lift, (until you stall). But at the same time you're also hitting the oncoming air with a larger cross section, you can't just slide through the air like a knife. We can plot the C_D verses C_L to find the *Drag Polar*, shown below.

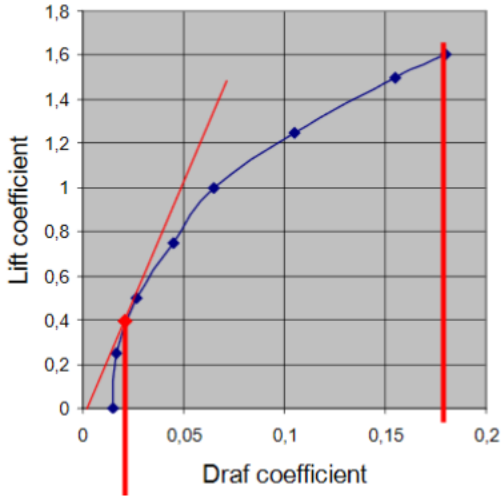


Figure 1.3: *Coefficient of Drag vs Coefficient of Lift*

1.3.4. The Drag Polar

That's right, I misled you! Drag is actually somewhat complex, ha HA! So, You can see that the coefficient of lift and drag change non-linearly. So, yeah flying at different speeds means we get different efficiencies. *Side note:* Lift/Drag is the same as C_L/C_D because if you can remember the equations for lift and drag (1.16, 1.17) the only differences between the two are the coefficients, so when we divide the two, all other variables cancel each other out.

Anyway, we want to fly at the highest possible $\frac{L}{D}$ value, this is the most efficient way to fly, but not the fastest. It shows us how much energy we lose to drag when we move the plane forward. So at higher values of $\frac{C_L}{C_D}$, we can dedicate the fuel to staying in the air, rather than dealing with drag. This is such an important design characteristic that it also defines the glide ratio.

1.3.5. Glide Ratio

The Glide Ratio is another simple concept, how far forward can you glide for a specific drop in altitude. The Glide angle is shown as γ in Figure (1.4). If we remember the equations for stable flight (1.14, 1.15), we can thus derive the equations for gliding flight.

$$L = W_1 = W\cos\gamma$$

$$D = W_2 = W\sin\gamma$$

We can combine these equations to get:

$$\frac{L}{D} = \frac{\cos\gamma}{\sin\gamma} = \frac{1}{\tan\gamma} \tag{1.19}$$

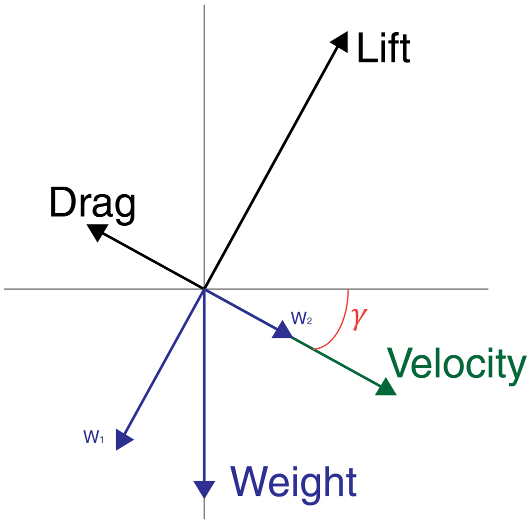


Figure 1.4: *Forces Acting on a Gliding Aircraft*

γ defines the glide ratio. Or more specifically the tan of γ defines the glide ratio. When you have a small glide angle, you travel farther for a smaller drop in altitude. (*Duh.*) So, naturally, if we want to maximise how far we can glide, we this have to minimise γ, and maximise $\frac{L}{D}$.

1.3.6. Reynolds Number

Okay, so, you don't REALLY need to know much about viscosity and such, but like, it's handy? So, when you have bigger wings, then the relative size of a molecule of air (not a thing, but shut up, imagine it is), is much smaller. But when you have tiny wings, the molecules are bigger! And air feels thicker. This is a similar principle to what makes insects move through water differently to fish. To fish, water is a thin liquid that you just swim through, but a tiny insect doesn't really swim though the water, it instead pushes the water bit by bit behind it in order to move forward.

You don't need to know any of this, but this is why we need to account for these variables. Aerodynamicists will test for characteristics of a wing, or aircraft at various Reynolds numbers. You'll need to calculate this value to be able to read the data charts given to you.

$$Re = \frac{\rho V c}{\mu} = \frac{V c}{\nu} \tag{1.20}$$

Where V is the velocity of the air, c is the length of the wing chord, μ is the Dynamic Viscosity, and ν is the Kinematic Viscosity.You'll do more of this in Aerodynamics, this is all you need to know for the fundamentals.

1.3.7. Wing Anatomy

A diagram is going to describe this way better than I ever could so, look at this diagram:

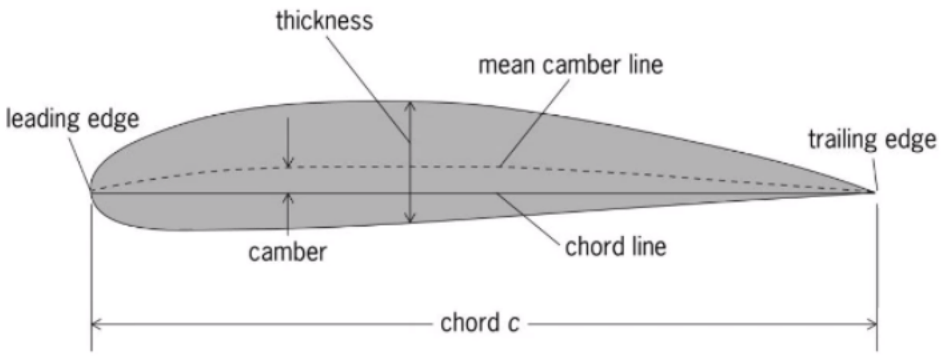


Figure 1.5: *Parts of an Aerofoil*

NACA Numbers NACA numbers are very often used to describe aerofoils. They come in either 4 number, or 5 number formats. The 4 Number formats mean this;

A:B:C

Where A is the % camber at, 10B% along the chord, with a maximum c% thickness to chord ratio. That's not super clear, so take **NACA 2412** as an example. This has 2% camber at 40% along the chord, with a 12% thickness to chord ratio.

The 5 number formats are similar except they're kinda dumb.

A:B:C

Where A is the design lift C_l, divided by 0.15? I dunno either. Americans can be weird. With max thickness B/2% along the chord, with a maximum c% thickness to chord ratio. For example take **NACA 23012** as an example. This gives us a design lift coefficient of $C_l = (0.15 * 2) = 0.3$. A max thickness at $\frac{30}{2} = 15\%$ along the chord, and a 12% thickness to chord ratio.

Wing Geometry right, so here's something you already know, an aerofoil isn't the only element of a wing, you also have tapers and taper ratio ($\frac{C_t}{C_r}$), and span and such. Basically: here's another diagram.

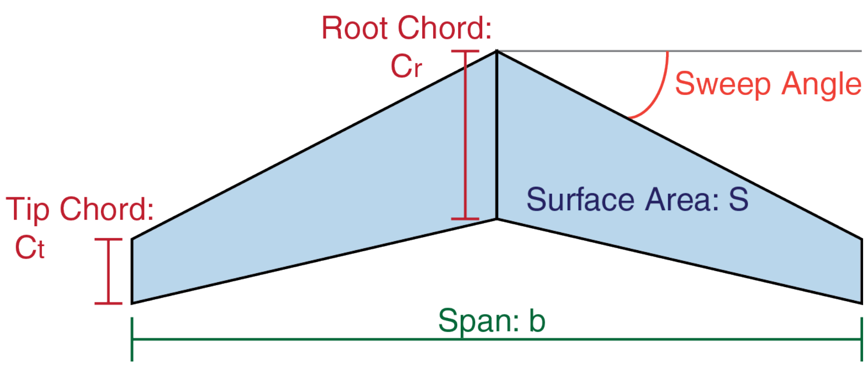


Figure 1.6: *Parts of a Wing*

1.4. Stability and Control

1.4.1. Control

You might remember this lecture very vividly as *that one lecture that made your brain leak out of your skull*. That's normal. This was the the moment (no pun intended) where most people in first year realised that aerospace is not the easiest degree you can do.

Controlling a plane is hard. The original pioneers knew this, and they knew it well because so many people crashed and died in gliders and early planes. We learn from the deaths of others in order to save lives. In the beginning the Wright brothers controlled their planes by bending the wings (*this was called Wing-Warping*) to generate more lift on one side of the plane allowing them to turn. Meanwhile in Europe, the pioneers were busy using rigid wings that couldn't be easily bent. They were so focused on being stable in flight that they weren't able to control where they were going particularly well. This was until 1908 when Léon Lavavasseur put the first ailerons on the Antoinette IV. Nowadays we use ailerons all the time, because thicker wings generate more lift, and thicker wings are hard to bend.

An aircraft has 6 degrees of freedom, Movement in the (x,Y,Z) directions and rotations about each axis, (Roll, Pitch, Yaw) respectively. But even though an aircraft has these 6 degrees of freedom, it only has 4 degrees it can control. But with integration over time, it can move to any position in space by using rotation and thrust. A plane uses:

- **Ailerons** to control **Roll** about the X axis ϕ
- **Elevators** to control **Pitch** about the Y axis θ
- **A Rudder** to control **Yaw** about the Z axis Ψ
- **Throttle** to control **Speed** along the X axis

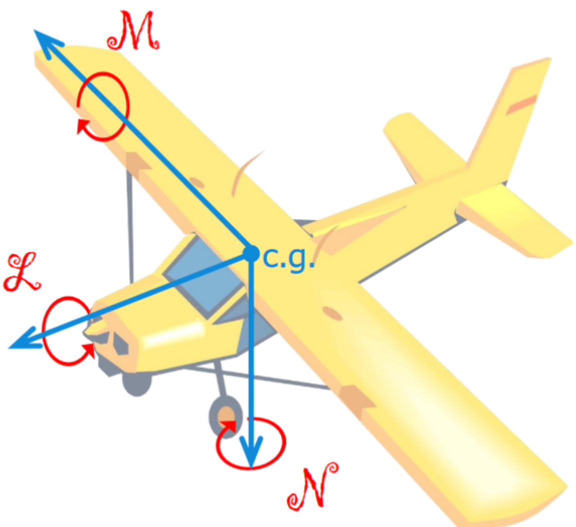


Figure 1.7: *The Axes of a Plane*

In the above diagram you'll notice some odd things: Firstly, make note that the moments about each axis are written as *L,M,N* (but squiggly) about the X,Y,Z axes. And then you'll notice that the Z axis is pointing down? The only reason for that is to make a pitching up moment (*M*) be positive. Also, remember that a Negative deflection angle, will result in a positive response from the aircraft.

There isn't much to explain about these next diagrams, just be familiar with the diagrams and the symbols used in them.

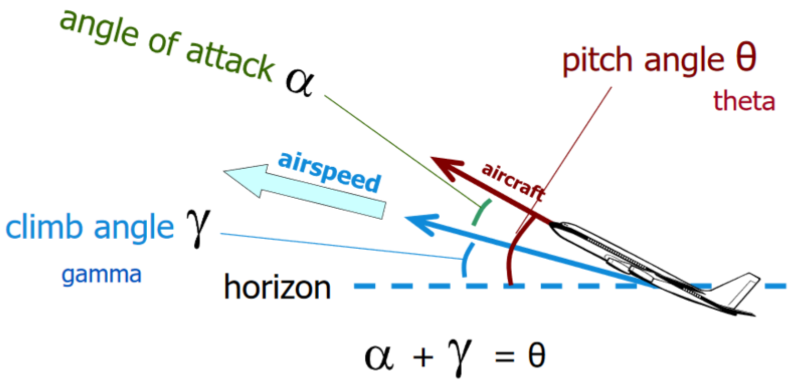


Figure 1.8: *Angles about the Y axis*

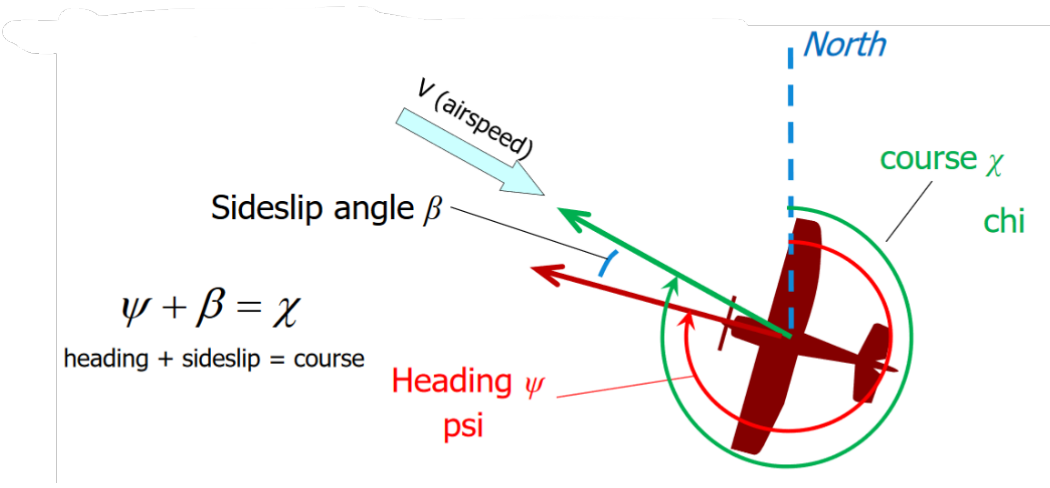


Figure 1.9: *Angles about the Z axis*

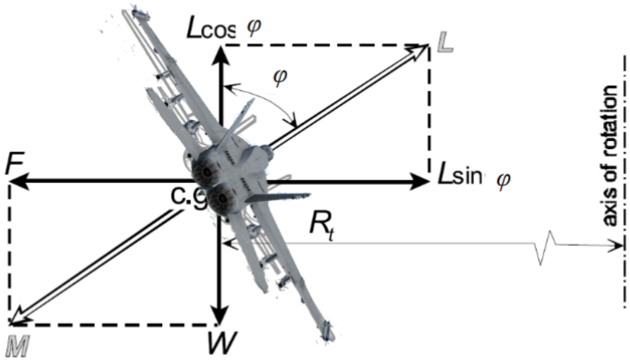


Figure 1.10: *Bank Angles about the X axis*

Figure 1.10 needs a bit of extra examination. You can see the relation between the extra lift you need to generate and the bank angle ϕ. Also pay attention to the load factor: $n = \frac{L}{W} = \frac{1}{\cos\phi}$, if this is too high the wings will snap off. It's not fun. In level stable flight this is 1. Just pay attention to this diagram in particular.

1.4.2. Stability

This is the difficult part of this section, but with practice you can figure this out. We deal with both static and dynamic stability in this course, but we mainly focus on static stability because it's easier to explain and understand. We'll also only really be looking at stability around the Y axis, or longitudinal moments *M*. Thus we'll be looking at the moment coefficient C_m.

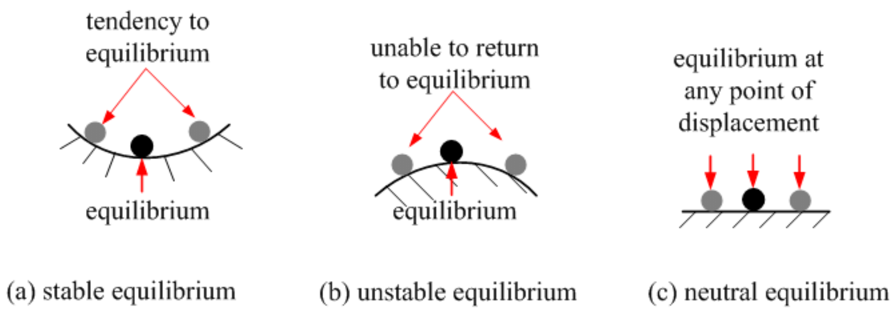


Figure 1.11: *Positive, Negative, and Neutral Stability*

We say that something is positively stable, when it tends to go back to a stable position after something causes it to move. Like in 1.11 (a), if you push the ball up, it will go back to the starting point. We say something is negatively stable, (or simply, unstable) when it continues to deviate when something causes it to move. Think of these with respect to an aircraft. If something causes the plane to pitch up (pilot induced oscillations for example) we don't want the plane to continue pitching up as a result, we want it to pitch down, and return to it's original position. Thus, for a positive change in angle of attack, we need a negative reaction in the moment coefficient, and vice versa.

$$\frac{\Delta C_m}{\Delta \alpha} = \frac{<0}{>0} = \frac{>0}{<0} \Rightarrow <0$$

Over small changes we instead write this in the form $\frac{dC_m}{d\alpha} < 0$. And because this is so often used, we can also write it in the form C_{m_α} If we graph this out we see the relation with C_m and α

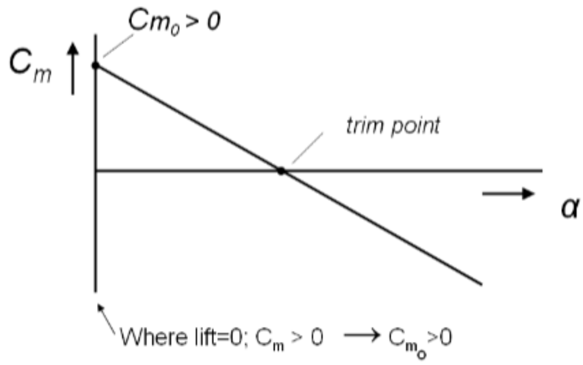


Figure 1.12: *Moment coefficient with relation to angle of attack*

From here we can see the two conditions needed for static stability.

- C_{m_0}> 0; if lift = 0; pitching moment has to be positive (nose up). This means it'll pitch up to increase the angle of attack and generate more lift. This Must be the case because you need to be able to set a trim point for a given angle of attack, but if $C_{m_0} \leq 0$, you're kinda screwed. There won't be a trim point.
- $C_{m_\alpha} < 0$ pitching moment has to become more negative when the angle of attack increases

The trim point is the point where the aircraft is stable, as in, this is where the aircraft will revert to when something causes it to pitch up or down. Or where the moment coefficient is zero for a given angle of attack. You can change this when you fly so that you have the correct angle of attack for sufficient lift.

There are a few definitions we need to clear up before we go into the derivation for static stability. Firstly, The **Aerodynamic Centre** this is the point on the airfoil about which the moment does not change, with a change in angle of attack. we take this to be the point where the wing generates its lift. Obviously the surface of the wing generates lift, not just a point, but we simplify it for the sake of calculation.

Next we have to clear up that the lift of an aircraft L is made up of the components L_{Wb} and L_H, Wb stands wing body, which is the lift generated by the main wing, and H stands for Horizontal stabiliser. This is the tail wing. And because we're talking about stable flight, lift is equal to weight.

$$L = W = L_{Wb} + L_H \tag{1.21}$$

The angle of attack of the tail is a bit different to the wing body. See, the way wings generate lift is they push air downwards, thus, we get a **downwash** after the air passes the main wing. This is not the case for canards because the amount of air they push down is negligible compared to to amount of air the main wing body pushes down.

$$\alpha_H = \alpha - \epsilon + i_H \tag{1.22}$$

i_H is the tail setting, because we can give it a different angle relative to the aircraft body. Thus, the change in α_H is:

$$\frac{d\alpha_H}{d\alpha} = \frac{d}{d\alpha}(\alpha - \epsilon + i_H) = 1 - \frac{d\epsilon}{d\alpha} \tag{1.23}$$

The term $\frac{d\epsilon}{d\alpha}$ is the change in downwash with relation to a change in angle of attack. This is roughly 0.1 for a standard aircraft.

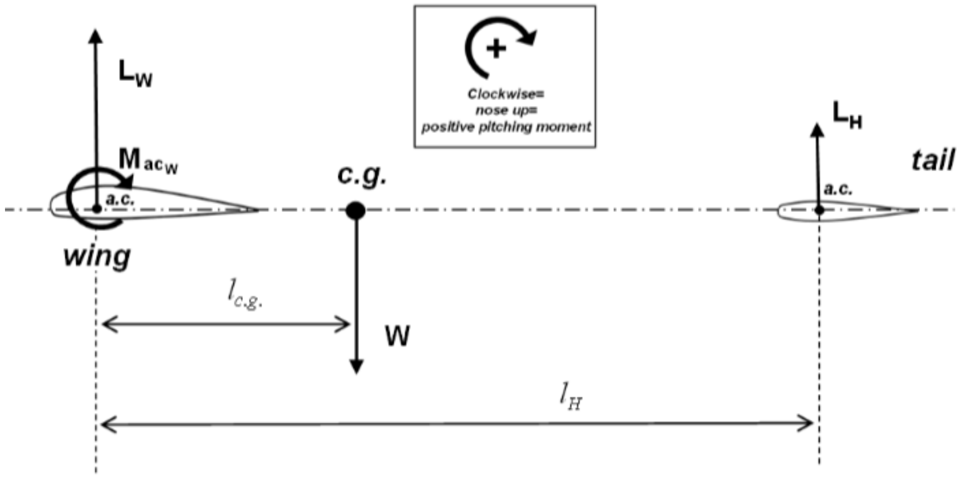


Figure 1.13: *The Forces Causing an Aircraft to Rotate*

As with most bodies, we take the moments about the centre of gravity. Summing up these forces we get:

$$M = \Sigma M = M_{ac_{Wb}} + L_{Wb}(l_{cg}) - L_H(l_H - l_{cg})$$

Recall equation (1.21), and simplify to:

$$M = M_{ac_{Wb}} + L_{Wb}(l_{cg}) + L_H(l_{cg}) - L_H(l_H)$$

$$M = M_{ac_{Wb}} + L(l_{cg}) - L_H(l_H)$$

Now at this point, we want to take out all physical dimensions, to get the coefficients.

$$\frac{M}{\frac{1}{2}\rho V^2 Sc} = \frac{M_{ac_{Wb}}}{\frac{1}{2}\rho V^2 Sc} + \frac{L(l_{cg})}{\frac{1}{2}\rho V^2 Sc} - \frac{L_H(l_H)}{\frac{1}{2}\rho V^2 Sc}$$

We see how many values are crossed out. Recall equation (**??**) for finding lift. The lift equation for the horizontal stabiliser uses S_H that's why we can't cancel it out.

$$C_m = C_{m_{ac_{Wb}}} + C_L\frac{l_{cg}}{c} - C_{L_H}\frac{S_H(l_H)}{Sc}$$

The value at the end, $\frac{S_H(l_h)}{Sc}$ is a special variable called the **tail volume**, and we substitute it for the symbol V_H. Though, it's dimensionless, so don't think too much about the name.

$$C_m = C_{m_{ac_{Wb}}} + C_L\frac{l_{cg}}{c} - C_{L_H}V_H$$

We're deriving this because we want to find $\frac{dC_m}{d\alpha}$. So the next step is to differentiate with respect to α.

$$\frac{C_m}{\alpha} = \frac{C_{m_{ac_{Wb}}}}{d\alpha} + \frac{C_L}{d\alpha}\frac{l_{cg}}{c} - \frac{C_{L_H}}{d\alpha}V_H$$

Remember the definition of the aerodynamic centre? It's the location where the moment about it doesn't change with a change in angle of attack.

$$C_{m_\alpha} = \frac{C_L}{d\alpha}\frac{l_{cg}}{c} - \frac{C_{L_H}}{d\alpha}V_H$$

Let's take a closer look at these factors; $\frac{C_L}{d\alpha}$ is a design characteristic. It's the slope of $C_L - \alpha$ curve. We also have this data for the tail, but instead of α we have α_H but this is fine, so long as we use equation (1.23).

$$C_{m_\alpha} = \frac{C_L}{d\alpha}\frac{l_{cg}}{c} - \frac{C_{L_H}}{d\alpha_H}\frac{d\alpha_H}{d\alpha}V_H$$

$$C_{m_\alpha} = \frac{C_L}{d\alpha}\frac{l_{cg}}{c} - \frac{C_{L_H}}{d\alpha_H}(1 - \frac{d\epsilon}{d\alpha})V_H$$

The values for the change in C_L with respect to angles of attack are a and a_t for the wing and tail respectively.

$$C_{m_\alpha} = a\frac{l_{cg}}{c} - a_t V_H(1 - \frac{d\epsilon}{d\alpha}) \tag{1.24}$$

With the knowledge that for an aircraft to be stable, C_{m_α} must be less that 0; we can Say:

$$0 > a\frac{l_{cg}}{c} - a_t V_H(1 - \frac{d\epsilon}{d\alpha})$$

$$a\frac{l_{cg}}{c} < a_t V_H(1 - \frac{d\epsilon}{d\alpha})$$

From this we can conclude a number of things:

- A larger tail will contribute to static stability
- A longer distance between tail and wing will contribute to stability
- A center of gravity that is just after the wing or even before the wing contributes to stability (forward cg => more stable, aft c.g. less stable)

That was pretty long winded, and it will take a few tries to get this into your head, but here's a quick rundown on how to derive this in an exam:

- Draw the diagram of the wing and tail with the forces and moments acting on it.
- Find the equation for moments about the centre of gravity.
- Make it dimensionless to use the coefficients.
- Differentiate with respect to α
- Substitute in $\frac{d\alpha_H}{d\alpha}$
- Substitute in a and a_t

You should be able to do the rest of the calculations on the day, by yourself. but if you can remember these steps, everything else will come to you.

The Neutral Point The Neutral Point is a location on the aircraft, that if the centre of gravity aligns with it, the plane exhibits neutral stability. $l_{cg} = l_{np} \therefore C_{m_\alpha} = 0$

$$\therefore a\frac{l_{cg}}{c} = a_t V_H (1 - \frac{d\epsilon}{d\alpha})$$

The distance between the neutral point and the centre of gravity is called the static margin, You don't want to end up with the centre of gravity behind the neutral point, so you want a comfortable margin of error.

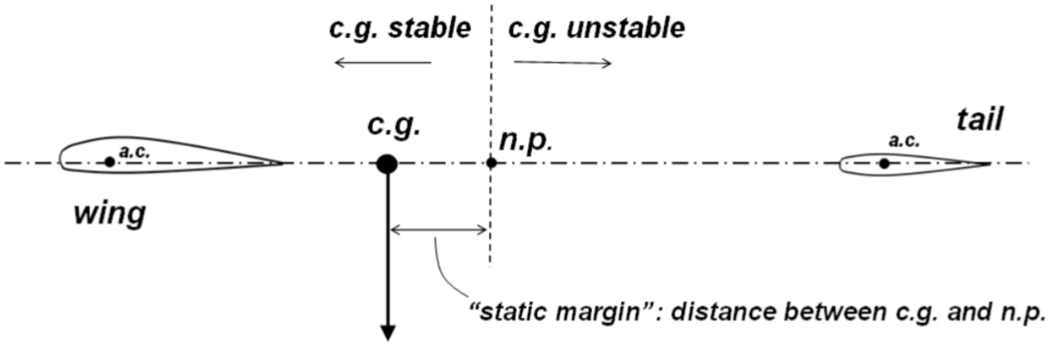


Figure 1.14: *The Neutral Point and Static Margin*

1.5. Propulsion

Engines are hard to make. Obviously. They're so complicated, and they have so many parts in them, but the basic principle of propulsion, is that if you push something back, you'll go forward. Newton's Third Law of motion, basically.

1.5.1. Air Breathing Engines

Recall Newton's **second** of motion; $F = ma$, we use this to calculate the thrust that an engine gives. But, we deal with force over a given time period, For example, in this diagram You see the massflow at the intake is the same as the massflow at the exhaust. The air is simply sped up so that the massflow remains constant.

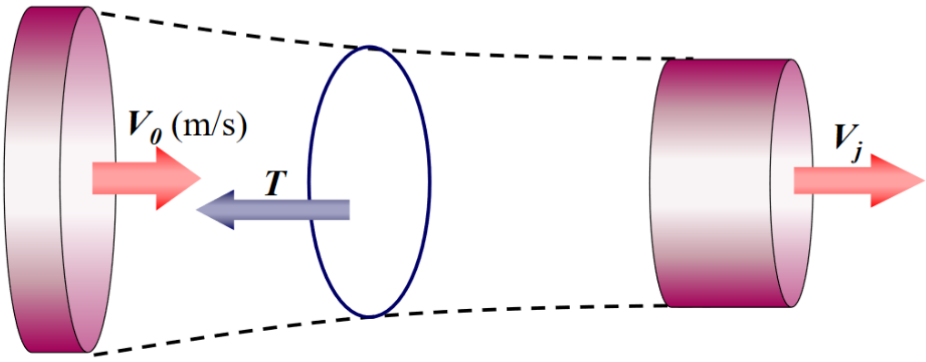


Figure 1.15: *Momentum of an Air Breathing Engine*

Massflow ($\dot{m}$) is an important concept. This is very simply the amount of mass that flows through the engine per unit time. Represented by $[\frac{kg}{s}]$. You can find this with ρAV where A is the Area of the intake and V is the velocity of the incoming air. Check this yourself by making sure the units add up.

$$T = ma$$

$$T = \dot{m}\Delta t\frac{\Delta V}{\Delta t}$$

$$T = \dot{m}\Delta t\frac{V_j - V_0}{\Delta t}$$

$$T = \dot{m}(V_j - V_0) \tag{1.25}$$

1.5.2. Propeller Theory

Propellers are actually much more efficient than turbofan engines, they can convert up to 75% of the input energy to available Power. Which is unheard of for turbofan engines. Take a look at figure (1.16):

In figure (1.16) the thrust is coming out of the page and pointing toward us. R is the radius of blade, S is the sideforce, this is analogous to drag on a stationary wing. ω is the angular velocity of the blade.

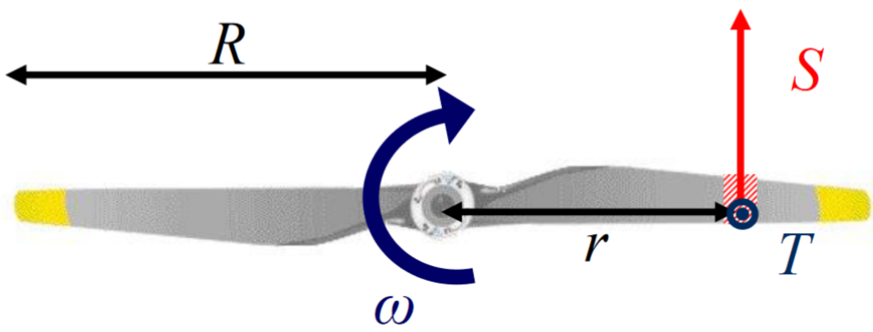


Figure 1.16: *A Propeller is Just a Spinning Wing*

To analyse the propeller, what we do is we take tiny elemental cross sections of the blade, and we see that it's simply an airfoil, at a distance r from the axis of rotation. The elements of the wing farther from the axis are spinning faster than those close to the axis. Recall; $V = r\omega$. We also know that lift generated by a wing is based on the speed of the air flowing over it. Recall (**??**). To combat this, and to make sure an equal amount of lift (or in this case, thrust) is generated across the span of the blade, the elements are given a different angle of attack, and such a different coefficient of lift. This spreads the force across the blade, making it stable. This is why propellers have a large twist.

When talking about efficiency we must first discuss Power. P_a is the power available for Thrust, and P_{br} is the Brake-shaft Power. This is the power it takes to, well, power the engine, confusing sentence, I know, but I hope it makes sense.

- Work done: Force * Distance $W = T\Delta s$
- Break-shaft Power: Total possible power output * Throttle Setting [%]
- Available Power: Work done per unit time $P_a = \frac{T\Delta s}{\Delta t} = TV$
- Efficiency: $\eta = \frac{P_a}{P_{br}}$

1.5.3. Air Breathing Engines

The core concepts behind air breathing engines is pretty similar no matter what you're talking about. The stages are: *Intake, Compression, Combustion, Exhaust*, or as you might know it: *Suck, Squeeze, Bang, Blow* This is the case for piston engines in cars, or gigantic turbofan engines.

We can demonstrate the work done by an engine if we take a piston engine as a particular example.

If we want to find the work done by an engine, we can consider that work done is force by distance. And we know that pressure is force per unit area. Thus we can represent it as follows.

$$W = Fs$$

$$\therefore W = pA\Delta x$$

$$\therefore W = p\Delta v$$

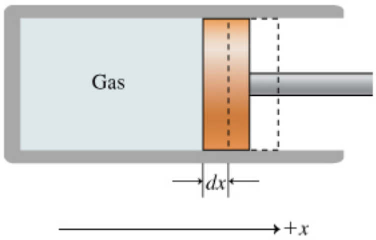


Figure 1.17: *A Piston is a Clear Demonstration of an Air Breathing Engine*

Where v is volume, not velocity. If you graph this, then the work done will be the area under a the graph. In my opinion this next diagram (1.18) is the clearest explanation of how an engine works.

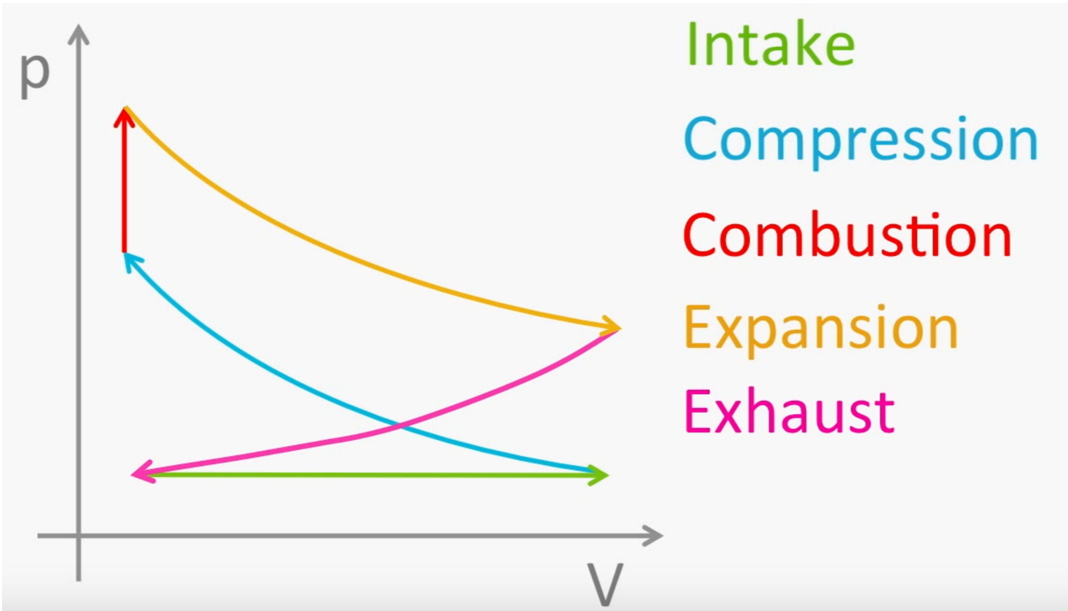


Figure 1.18: *The Work Done is the Area Enclosed by the Graph*

Firstly you intake air, then compress it, Which reduces the volume but increases the pressure. Then you ignite the fuel causing the pressure to increase without an increase in volume. Then the extra pressure pushes the piston back causing the rotor to spin. You then exhaust the air and start the cycle again.

Work done is the area under the graph. Simple statement, but there's more to it. You must do work to make the engine cycle happen, and so the work done to make the prop spin is the total work, minus the input work. And such we see that it is actually the area enclosed by the graph. This is the net mechanical work.

If you wanted to increase the area enclosed by the graph, and thus, do more work, you could do a number of things. You could add an afterburner, which heats up the air in the expansion phase, increasing the pressure again. You could use an inter-cooler, which cools the air and *reduces* the pressure in the compression phase, right before combustion. You can also use a compressor to intake more air letting you fly in less dense air; ie, higher up.

1.5.4. Continuous Combustion Engines

Continuous combustion engines don't use pistons or anything of the sort, instead they Just constantly intake air, heat it up, and use this new pressure to exhaust it out, and generate thrust. A **Turboprop** or **Turboshaft** engine have a propeller at the front the is powered by the exhaust, and this can generate more thrust as well. A **Turbofan** is a jet engine with a fan at the front that allows more air to bypass the jet engine, and this allows for a greater massflow, and is thus more efficient.

1.5.5. Jet Efficiency

Jet Power, (P_j) is the net kinetic energy that the jet engine can generate.

$$P_j = \frac{1}{2}\dot{m}V_j^2 - \frac{1}{2}\dot{m}V_0^2$$

If we want to find the efficiency of a jet engine, we must combine some equations from earlier.

$$T = \dot{m}(V_j - V_0)$$

$$\eta = \frac{P_a}{P_j}$$

$$P_a = TV_0$$

Because V_0 is just the true airspeed of the aircraft.

$$\eta = \frac{TV_0}{\frac{1}{2}\dot{m}V_j^2 - \frac{1}{2}\dot{m}V_0^2}$$

$$\eta = \frac{\dot{m}(V_j - V_0)V_0}{\frac{1}{2}\dot{m}(V_j^2 - V_0^2)}$$

$$\eta = \frac{2(V_j - V_0)V_0}{(V_j^2 - V_0^2)}$$

$$\eta = \frac{2V_0}{V_j + V_0}$$

$$\eta = \frac{2}{\frac{v_j}{V_0} + 1} \tag{1.26}$$

So what you can see from this is that a highly efficient engine doesn't speed up the air that much, because we want $\frac{V_j}{V_0}$ to be as small as possible. A more efficient engine speeds up a large amount of air, but only slightly, instead of speeding up a small amount of air greatly. This is why modern engines have such large intakes on them, and why Turbofans have that air bypass section on them.

1.6. Cockpits and Systems

1.6.1. Cockpits

A short chapter this time: Cockpits! A standard cockpit has a whole bunch of analogue instruments, but Airbus introduced the *Glass Cockpit* in the A320, along with Fly-by-Wire. I'm not going to explain all of the instruments because you don't really need to know what they do, but, a passenger cockpit contains the following:

- Circuit breakers
- Overhead control panel
- Mode Control Panel
- Display Control Panel
- Standby instruments
- Primary Flight Display
- Navigation Display
- Upper EICAS (Engine Indicator Crew Alert System)
- Lower EICAS
- Command Display Unit (CDU)
- Throttle levers
- Flaps Speed brake
- Radio Control Panel
- Trim panel
- Printer

Fighter planes contain these as well:

- MFDs (Multi Function Displays)
- UFC (up Front control)
- Wide Angle Collimator HUD
- HMD (Helmet mounted Display)

1.6.2. Systems

This chapter contains a whole bunch of constants and such, but you don't need to remember them, they'll be given to you in the exam if you need them, and in your career you can just google the values anyway. The only thing you might need to remember is the formula for the speed of sound, and Mach number:

$$a = \sqrt{\gamma R T} \tag{1.27}$$

Where $\gamma = 1.4$ for most cases.

$$M = \frac{V_{TAS}}{a} \tag{1.28}$$

Next lets talk about Equivalent airspeed, and True airspeed. Why do we have these anyway? If you consider how a plane flies (**??**) there are a whole bunch of variables to keep in mind. But basically, A plane needs a certain amount of air flowing over the wing to stay in the air. That is a function of both the density of the air, and the velocity in the air around you.

A pilot flying the plane however doesn't want to have to do these calculations in real time, that's a safety issue. They just want to know the minimum speed to fly to not stall. So instead, equivalent airspeed is the speed at which you'd be flying in standard ISA sea level conditions.

$$C_L \frac{1}{2} \rho_0 V_{EAS}^2 S = C_L \frac{1}{2} \rho V_{TAS}^2 S$$

$$\rho_0 V_{EAS}^2 = \rho V_{TAS}^2$$

$$V_{EAS} = \sqrt{\frac{\rho}{\rho_0}} V_{TAS} \tag{1.29}$$

Indicated airspeed is the same as Equivalent airspeed for the purposes of this course, because we can assume there's no indicator errors.

Instruments

The most important instrument to know about it a Pitot tube, This is how we measure, basically anything in a plane.

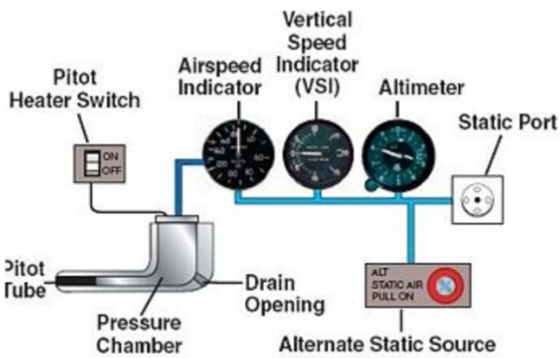


Figure 1.19: *A Pitot Tube, and Static port to get all the readings we need*

The principle behind how a pitot tube, is that you have two ports, one that measures,the total air pressure, and one that measures the static air pressure. Recall Bernoulli from earlier.

$$P_{total} = P_{static} + P_{dynamic}$$

$$P_{total} = P_{static} + \frac{1}{2}\rho V^2$$

With this information it's easy to calculate the speed of the aircraft, and it's all based on the ISA and the reference point that the pilot sets (QNH). The difference between this reference point at the static pressure is the how we find the altitude.

Altitudes

The altitude reading we get from the Pitot tube isn't totally accurate. this is the pressure altitude, and we calibrate it for the sea level when we take off, and then switch to ISA when we get above 4500ft.

Altitudes for commercial flights are measured in flight levels. Which are increments of 100ft, so FL350 would be an altitude of 35,000ft. We start using these altitudes above FL45. This is the "crossover altitude". This has an impact on how cruise flight works too. Below FL45, we fly at a constant CAS (Calibrated Airspeed). And above FL45, we fly at a constant Mach number.

When cruising the plane will take a few steps up in altitude (as the weight reduces due to used fuel), this is because at a lower air density, you have less drag.

1.7. Helicopters

We're not going into nearly as much detail with these vehicles. Helicopters are:

- Creating more noise (internal external)
- Less fuel efficient
- Less environmentally friendly
- More expensive to buy operate
- Less comfortable for passengers
- Harder to fly

But they can also hover, and take off anywhere. Which makes them great for specialised cases like rescue missions etc.

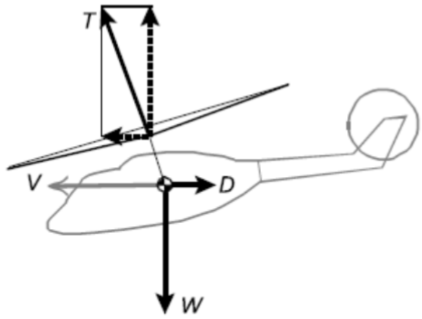


Figure 1.20: *Equilibrium of Forces on a Helicopter: Note how the Forward Thrust and Lift are the same vector.*

A helicopter is controlled using a **Swash Plate** mechanism. This is a plate that lets you control the angle of attack of the rotor blades. The rotors are connected to the top swash plate, and rotate with it. If you lift the bottom swash plate (which doesn't spin, and is connected to the top swash plate via a bearing) you'll in turn rotate the blades, raising their angle of attack. This lets you increase lift without needing to control the speed of the rotor. You can also *tilt* the swash plate, this will mean that the blades have a different angle of attack at different position in the cycle. We call this **Cyclic Pitch**. As opposed to **Collective Pitch** we saw earlier. A helicopter turns by utilising collective pitch to generate different amounts of lift on different sides of the craft, and thus, creating a turning moment. The tail rotor is there to counteract the spin induced by the main rotor.

Autorotation A helicopter is essentially a hunk of metal hovering in the sky. If the rotor fails you can't just glide down to safety like in an airplane. You're somewhat screwed. The last saving grace is autorotation. This is when air flowing up through the blades is enough to make the rotor spin a little and generate some lift, to slow the descent. But for this to kick in you need to either have enough horizontal speed to let the air quickly rush past the blades and cause them to move, or you need to be really high up so that when you fall down you have enough time to gather speed and make the blades spin.

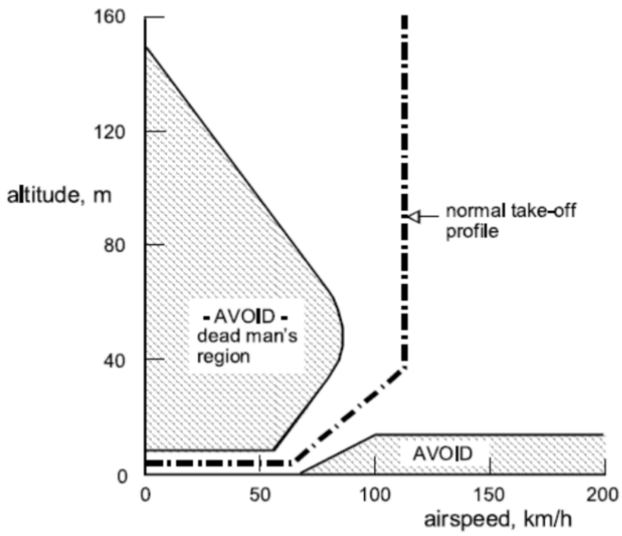


Figure 1.21: *Ideal Helicopter Flight Profile*

2 Structures

Thankfully, the structures section of this course isn't as long as the fundamentals, so this chapter won't be nearly as long. I'm going to assume you've seen the *two* lectures from the course, and as such I won't be going into such detail.

2.1. Weight

The weight of an aircraft is made of three main components:

- Empty Weight
  - Structure
    - Wings
    - Horizontal and Vertical Tail
    - Fuselage
    - Landing Gear
    - Surface Controls
    - Propuslsion System
  - Systems
    - Instruments and Navigation
    - Hydraulics and Pneumatics
    - Electrical System
    - Electronics
    - Furnishings
    - Air conditioning and Anti-Ice
  - Crew
  - Operating Items
- Payload
  - Passengers
  - Cargo
- Fuel

As a formula this is:

$$W_{Tot} = W_{Empty} + W_{Payload} + W_{Fuel} \tag{2.1}$$

The goal when it comes to aircraft design is to **minimise** the Aircraft empty weight. The aircraft designer doesn't have control of the crew or the operating items, that's up to the air-line. Instead we are in charge of reducing the weight of the structure and systems. Or more specifically, the structure is handled by the structural and materials engineer and the systems are handled by the systems designer, because in a fully functioning company the jobs are distributed to more than one person.

The idea behind reducing the aircraft empty weight is that it either allows us to increase our payload and carry more people, or increase our fuel capacity and fly further (Just look at the Airbus A321XLR). Or we can reduce the amount of fuel we need to burn to fly, which is both cheaper and better for the environment. It's mainly because it's cheaper though. Fuel is a majority of the direct operating cost for running an aircraft, Up 70% for wide-bodies!

Why do we need less fuel if the aircraft is lighter. It's because it takes energy to fly. Energy is work, which is the thrust by the distance flown: Recall (1.14) and (1.15)

$$E = T \cdot d$$
$$T = D$$
$$T = D \cdot \frac{W}{L}$$
$$T = \frac{D}{L} \cdot W$$
$$T = \frac{C_D}{C_L} \cdot W$$
$$E \cong= \frac{C_D}{C_L} \cdot W \cdot d$$

2.2. Structure

The functions of a structure are to

- Carry loads
- Protect the payload
- Maintain an aerodynamic shape
- Connect the subsystems

To fulfil this duty the structure must be:

- Strong
- Stiff
- Lightweight
- Durable
- Cost-effective
- Maintainable

Technically speaking the structure doesn't *need* to be maintainable, but it would be a pretty rubbish aircraft if you had to throw it out entirely as soon as a problem presented itself.

2.3. History

This section will be short and sweet. Essentially back in the early 20th Century, Aircraft were made of Steel, wood, and linen, and they had a lot of cables. As things progressed and airplanes became more complex, the structures started involving trusses, spars, and ribs to reinforce the wings and fuselage. These were made of Steel rods and tubes.

In the 1920s we got large planes, with load-carrying wings made of wood, wrapped in a canvas skin to give it an aerodynamic shape. But then in the 1930s we got a major new design with stiffened shell structures made of aluminium. Where the skin both gave an aerodynamic shape, *and* was load bearing.

In the 1940s and 1950s we got pressurised cabins that allowed the planes to fly much higher, faster, and above the weather. This was thanks to improved aluminium alloys. But this presented many design challenges (just look at the safety history of the De Havilland Comet).

The 1980s saw a time of much progress, this was when we first saw composites being used in aircraft manufacturing. Airbus was ,naturally, ahead of the curve when they release the A310 with a composite rudder. In 2005, The Airbus A380 was announced and it used GLARE, a Fibre-Metal laminate developed in TUDelft. In 2009 the Boeing 787 was announced at it was made up of 50% composite materials!

In the modern era, we've now just started experimenting with polymers, and 3D printing to let us manufacture things in new, different ways. The point is, we've come a long way since the wright brothers first flew.

2.4. Trusses

You should be familiar with these from statics, but to recap; trusses are simple light structures that maintain the aerodynamic shape when covered with a skin. You can reinforce a truss with cables to make it hold its shape, but keep in mind that cables can only resist tension loads, so you need cables going in more than one direction to resist compression.

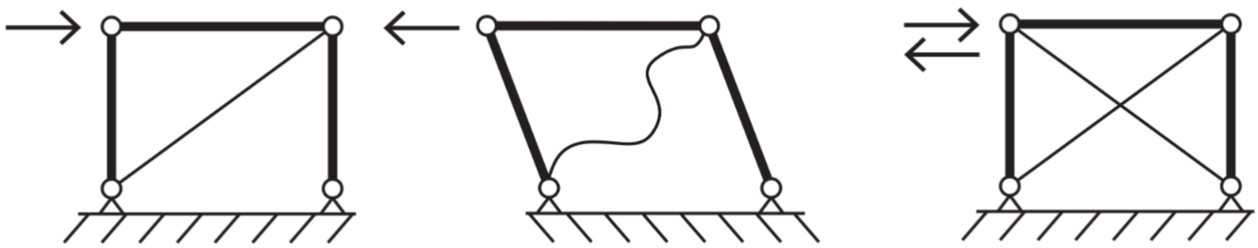


Figure 2.1: *Trusses Reinforced with Cables*

Rods can support both compressive or tensile loads, so we can choose to use just one rod instead of two cables. Or we can choose to use two rods, but sometimes that's not ideal. Rods are heavy, complicate the assembly process and make calculating loads tricky. This also brings up the question of design philosophies,

Fail safe, and **Safe Life** are two different ways to think about designing a structure. Safe Life is a philosophy in which the structure is designed for a the number of flights, landings, or flight hours, during which there is a low probability that the strength will degrade below its design strength. And once this number of flights is reached, the member **must** be replaced.

Fail safe, on the other hand, is a philosophy in which the structure is designed to permit it to retain required residual strength for a period of unrepaired use after failure or partial failure of a principal structural element. A fail safe design requires regular inspection and monitoring in order to detect failure.

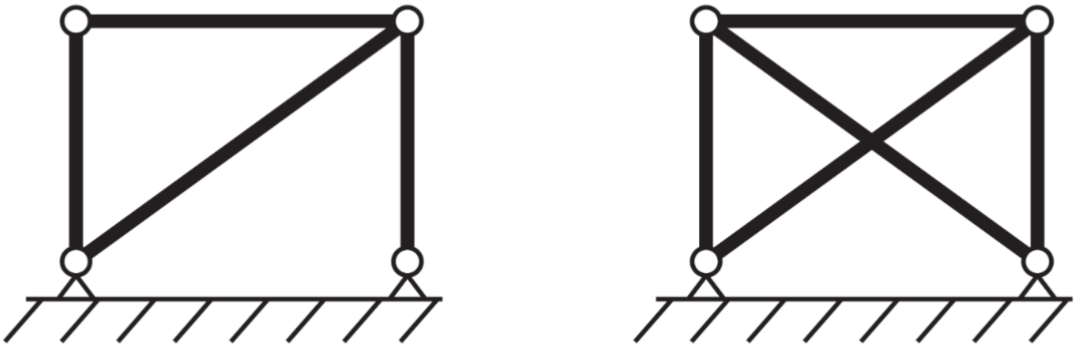


Figure 2.2: *Safe Life (left) vs Fail Safe (right) Designs.*

We can also incorporate the skin into the structure, as it will resist tension in a similar way to cables. Sometimes the skin buckles, but that's not an issue as long as it's *elastic* buckling *Plastic* buckling will cause permanent deformation.

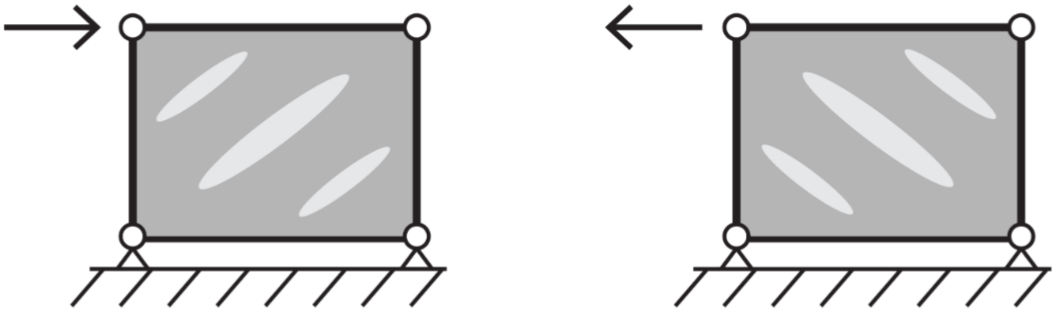


Figure 2.3: *Buckling Occurs in line with the deformations.*

2.5. Anatomy

Airframe The airframe of an aircraft is basically the chassis of the plane. It's the basic structure and doesn't contain any equipment or interior installations.

Primary Structures Primary structures, Sometimes called Principal Structural Elements (PSE), are critical load bearing elements of the aircraft. If a primary structure is critically damaged, the whole aircraft will fail. **Failure of a primary structure is catastrophic.**

Secondary Structures A secondary structure or Non-Principal Structural Element is a structural element of an aircraft or spacecraft that carries only aerodynamic and inertial loads generated on or in the structure. **Failure is not catastrophic.**

Fuselage When it comes to the fuselage, there are four main components you should know about:

- **Frames:** These are the reinforcements that go around the fuselage, like hoops, or window frames.
- **Stringers:** These are the long string-like reinforcements that stretch along the body of the aircraft.
- **Skin:** This is the outermost part of the plane body. It's just like skin on anything else, and it keeps the cabin pressurised.
- **Bulkheads:** These are the very strong "end-caps" on the fuselage that hold in the pressure on either end.

Wings Wings naturally have to be very strong, and to do this there are 4 main reinforcement structures.

- **Wingbox:** This is the central part of the wing, It's a primary structure so it's critical to the integrity of the aircraft.
- **Wing Skin Panels:** Just like on the rest of the aircraft, the skin on the wings also bear a load.
- **Ribs** These are the elements that go perpendicular (mostly) to the length of the wing, they are what maintain the aerodynamic shape. Because we can optimise for weight so much, you'll see that almost every rib in a wing has a different design.
- **Spars:** These are the elements that run along the length of the wing. They're part of the wingbox.

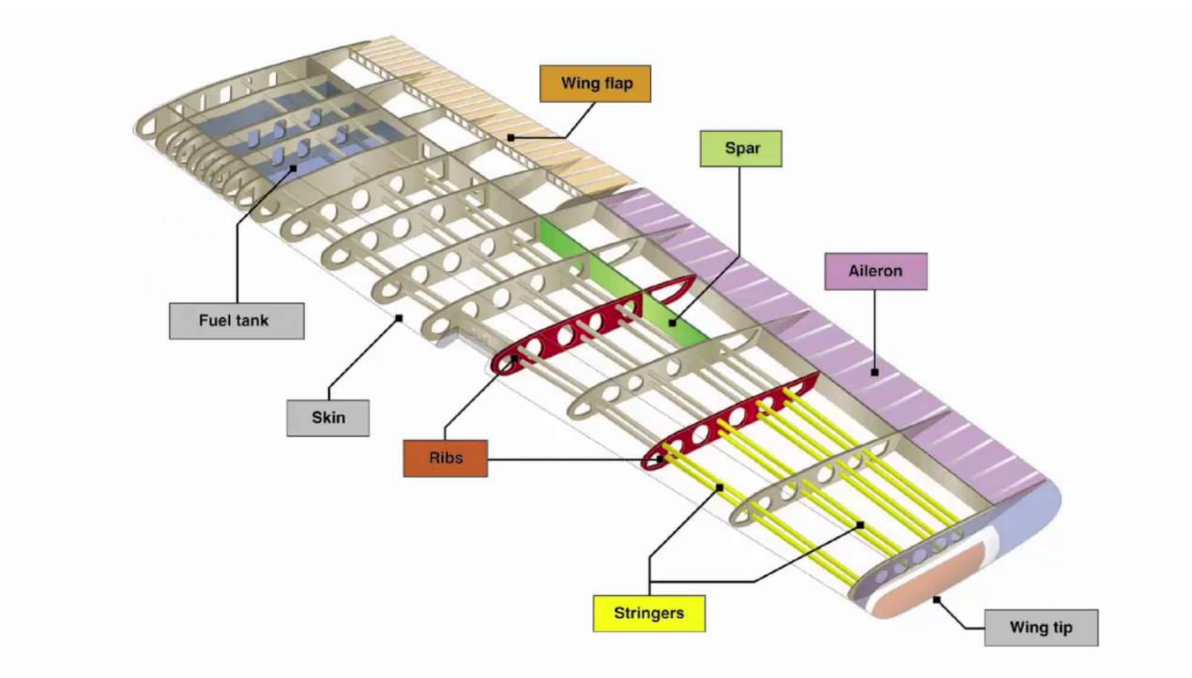


Figure 2.4: *Anatomy of a Wing, Note the Rib Designs*

2.6. External Loads

An important feature in flight that we need to consider is the load factor n, where

$$n = \frac{L}{W} \tag{2.2}$$

This is important, and it's how we describe certain limitations of flight. Or in other words, it helps us describe the flight envelope.

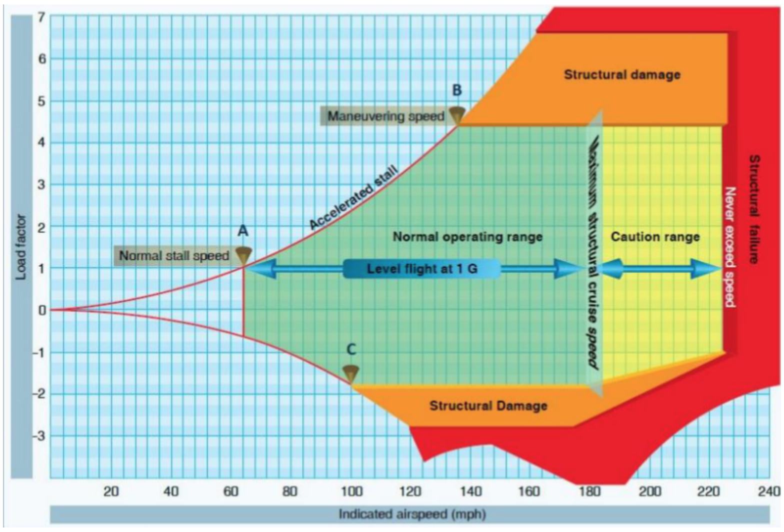


Figure 2.5: *Example of a V-n Diagram, or Flight envelope*

This leads us to define limits. The **Ultimate Load** is the absolute maximum load a plane can withstand, it will fail after 3 seconds at this load. To make sure this *doesn't* happen, we apply a safety factor (usually about 1.5 to 2) and create a limit load. Which the aircraft is likely never going to see, and we say "don't go over this, very dangerous". No damage will happen as a result of reaching the limit load, but we set a limit for safety reasons.

2.7. Material Properties

All materials act like springs to some extent. If you pull on them they will elongate linearly, to a point. This is where we introduce the concepts of **Stress** and **Stain**. Stress is the force applied to a material per unit area. Not a pressure, though it is measured in pascals. Strain is the elongation experience per unit length. It is unitless.

$$\sigma = \frac{F}{A_0} \tag{2.3}$$

Where σ is stress, F is the Load applied, and A_0 is the cross sectional area.

$$\epsilon = \frac{\Delta L}{L_0} \tag{2.4}$$

Where ϵ is strain. L_0 is the initial length, and ΔL is the change in length.

$$E = \frac{\sigma}{\epsilon} \tag{2.5}$$

Where E is Young's modulus, or the slope of the stress-strain curve.

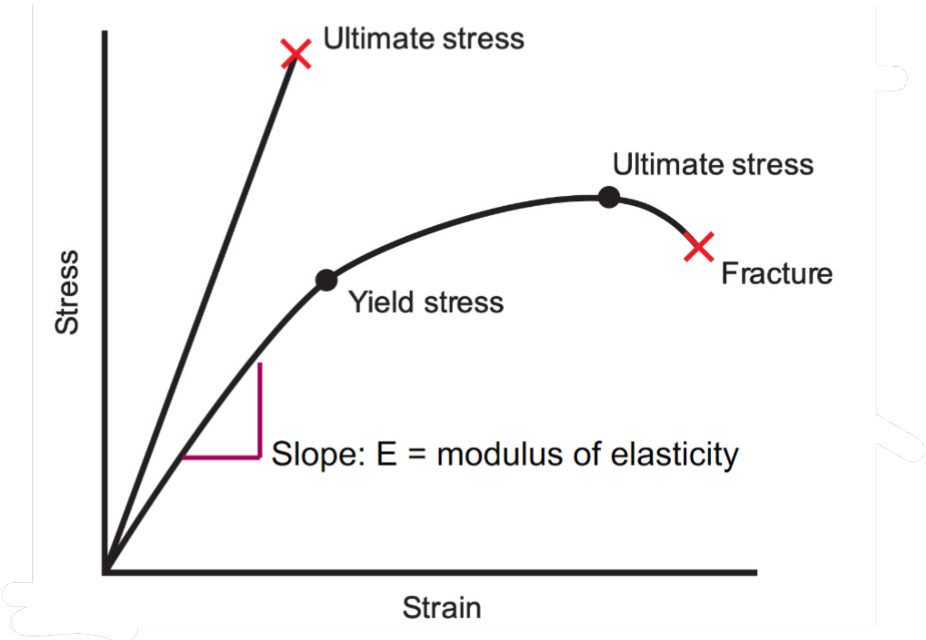


Figure 2.6: *The Stress-Strain Cuve of Brittle (left) and Ductile (right) Materials*

You'll notice in figure (2.6) that a ductile material (like metal) yields and deforms after plastically deforms after that. Whereas a Brittle material (like composites) will just reach a maximum load and then break.Thus when we set our safety limits, we must take these characteristics into account. The loading of a ductile material must not exceed the Yield Stress. And the loading on brittle materials must not exceed a given fraction of the ultimate stress (usually about 30%).

2.8. Fuselage Stresses

When we calculate the maximum stresses that a fuselage can withstand we make a lot of simplifications. Firstly, We assume it's a perfect cylinder. Secondly, we assume it's thin walled, where thickness t is significantly smaller than the length L, and lastly we assume that there are no cutouts like doors or windows.

2.8.1. Calculating Longitudinal Stress

So with the given parameters of ΔP, Radius R, thickness t, we can calculate the stress. Firstly we must define a few things, For starters, The cross sectional area of the fuselage is the thickness, times the circumference.

$$A = t \cdot 2\pi R$$

The pressure is simply the force divided the area. Except the area that the pressure acting on is not just the skin thickness, but it is instead the ends of the fuselage, which have an area of πR^2. Thus:

$$\Delta P = \frac{F}{A} = \frac{F}{\pi R^2} \Rightarrow F = \Delta P \cdot \pi R^2$$

If we recall (2.3) We know that stress is the loading per area, so:

$$\sigma_{long} = \frac{F}{A} = \frac{\Delta P \cdot \pi R^2}{t \cdot 2\pi R} = \frac{\Delta P R}{2t} \tag{2.6}$$

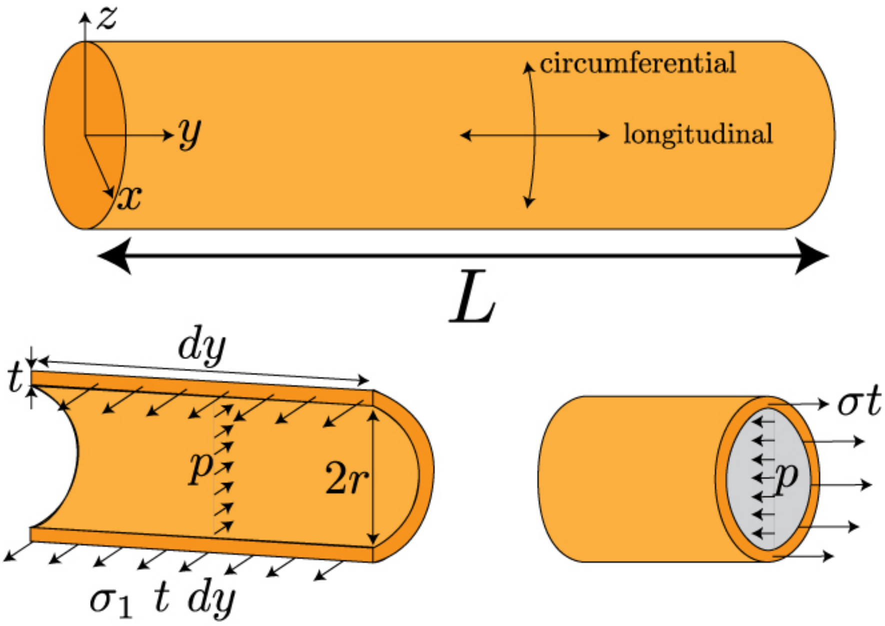


Figure 2.7: *The Hoop Stress (left) and Longitudinal Stress(right) in a Cylinder*

2.8.2. Calculating Hoop Stress

This one is a tad bit more tricky, First of all we imagine that that we're working with only *half* the cylinder. And we can do this because the pressure is equal all throughout the vessel. The We'll take the pressure acting on the flat site of this new semi-cylinder. The area of this is:

$$A_1 = 2RL$$

Then for finding the area acted on by the pressure in the skin we look at the two sides of the fuselage.

$$A_0 = 2tL$$

Similar process to (2.6):

$$\sigma_{hoop} = \frac{F}{A} = \frac{\Delta P \cdot 2RL}{2tL} = \frac{\Delta PR}{t} \tag{2.7}$$

2.9. Materials

2.9.1. Overview

We've dealt with a bunch of information so far, but let's take a step back and examine, what exactly is a material? Well, we don't really have a clear cut definition, but it's basically just: Substances and matter, that have specific properties, and are shapeless. You don't need to know these specific characteristics, but just; intuitively, know what a material is.

With materials, we can make semi finished parts. Consider turning raw aluminium into sheets of metal, with this we can make structural elements, and with the elements we can in turn, make a structure. All pretty simple. Naturally, there are far too many things to cover in this course, so for the purposes of aerospace we stick to a few small categories.

There are 4 main categories of materials: **Polymers, Ceramics, Metals, and Composites**. But we don't worry about ceramics and polymers for aerospace structures because they don't have the right properties.

2.9.2. Metals

Metals are **Isotropic** materials. This means they have the same properties in all directions. They can be strengthened with processes like heat treating. They can be melted down and cast into new shapes, we call this plastic behaviour. They are easy to work with and process. They are often very low cost. And they are able to be reused or recycled.

2.9.3. Polymers

We don't generally use polymers in aerospace structures, but it's good to have an understanding of what they are because plastics are wildly abundant in the world.

Polymers are *macro-molecular* substances. So while a metal like aluminium is just a mass of aluminium atoms, plastics are instead long chains of different atoms bound together into long molecules. Hydrocarbons are what you're probably most familiar with.

Thermoplastics are a type of plastic which exhibit plastic properties (it's a bit confusing, I know). Essentially the monomers (the individual sections that make up the long chains in the polymers) make weak bonds with one another, so you can heat up the plastic and melt it down and reverse the bonding process. You can melt it down and reuse the plastic.

Thermosets are a different type of plastic which cannot be melted down and reformed. The curing process of a thermoset is irreversible because the monomers for very strong bonds with one another.

Polymers are Isotropic, they have low strength and stiffness. They come in so many different forms, they are often extremely cheap, and are very easy to work with. But the properties of plastics depend on the temperature as well. At **low temperatures** plastics are elastic, and brittle. at **medium temperatures** they are rubbery, this is after the Glass Transition Temperature. At **high temperatures** they are viscous and like a liquid. This is after the melting temperature.

2.9.4. Composites

Composites are engineering materials made of two or more distinct and structurally complementary substances with different physical or chemical properties. Combined, they produce structural or functional properties not present in the individual components.

Composites are **anisotropic**. That's because they're often made of fibres going in one direction and a matrix binding them all together, thus the strength along the length fibres will be much more than the strength perpendicular to the fibres.

They are a layered structure, often going in multiple directions to mitigate the anisotropic qualities of a single layer. They're pretty expensive because they're difficult to make and they're a pain to work with.

For the purposes of this course we only deal with continuous fibre composites, we'll go into more detail about the other configurations in Material Science next quarter.

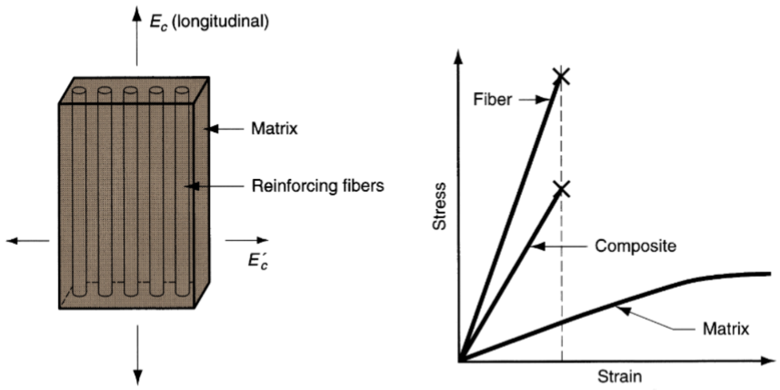


Figure 2.8: *Composite properties are a combination of the properties of the constituent materials*

They are only strong and stiff in tension. And they exhibit anisotropic behaviour. We use multiple layers going perpendicular to one another to give us strength in multiple directions, we call this cross ply. The function of the fibres is to give the composite strength and stiffness, while the function of the resin matrix is to support the fibres, bind them together, and to protect them from damage.

2.10. Estimating Composite Properties

When we design something we always need a starting point to do our calculations. We can calculate the approximate properties of a composite with *the rule of mixtures*. This is where we add the properties of each constituent material, with respect to what fraction of volume they take up. So for Unidirectional fibre composites that would be:

$$\sigma_{composite} = \sigma_{fibres} \cdot f_{fibres} + \sigma_{resin} \cdot f_{resin} \tag{2.8}$$

Note that this is parallel to the fibres, so when we're acting perpendicular to the fibres, the strength of the fibres is 0. And we can also substitute in any property we like, such as E, and it'll give is an adequate, rough estimate.

2.11. The Production Triangle

Because there is more to designing an aircraft than just making something amazing, you also have to consider other parts in the production process. Thus, the load carrying capacity of a structure depends on the **Design**, the **Materials**, and the **Production** techniques used to manufacture it. These are all dependent on each other, and sometimes you cannot afford a certain combination, so you have to go back to the drawing board.

2.12. Fatigue

Fatigue is a damage phenomenon induced by large number of load cycles below ultimate strength. The result is permanent deterioration of material or structure, causing a reduction in load-bearing capacity. It is because of dynamic loading, so a force being applied over and over, but not continuously. If this force applied is lower than the breaking force, it can still break the part after a period of time.

As a part experiences fatigue cycles it goes through stages. First everything is fine; then the Crack Initiation stage, followed by the Crack Growth, then it reaches a critical length and failure occurs. From here we can define two limits. The First being the **visibility limit** when

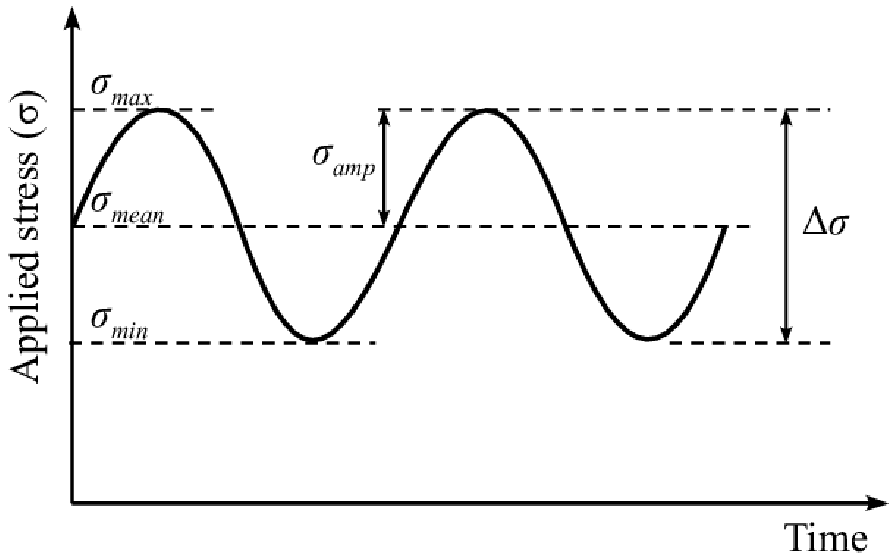


Figure 2.9: *Idealised Fatigue Loading*

we can first detect a crack, and the other is the **criticality limit**, when the part fails. You must inspect the part at least 3 times between the first point it's visible, and when it reaches critical length.

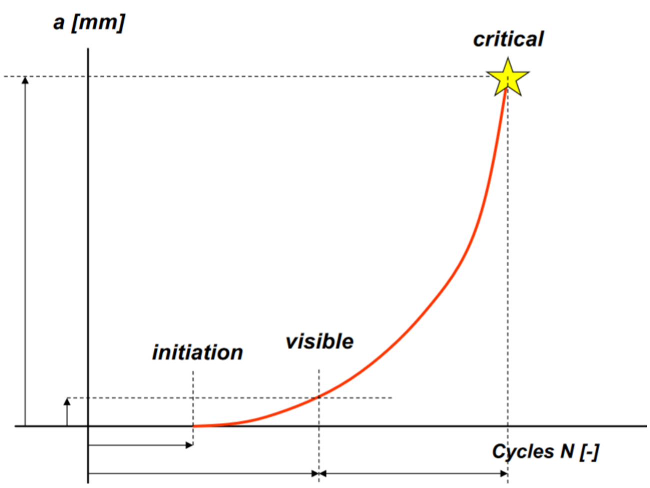


Figure 2.11: *Crack Length (a) compared to Number of Cycles (N)*

2.13. Going Supersonic

2.13.1. Materials

When you go fast, you generate a lot of heat. And aluminium, the metal we use for most planes, has a relatively low melting point, so when you fly extremely fast, you end up weakening the structure significantly. Thus different metals like titanium or specifically chosen steel alloys are used instead. The melting point of titanium is $1941K$ which is why it made up more than 90% of the structure in the SR71 Blackbird.4

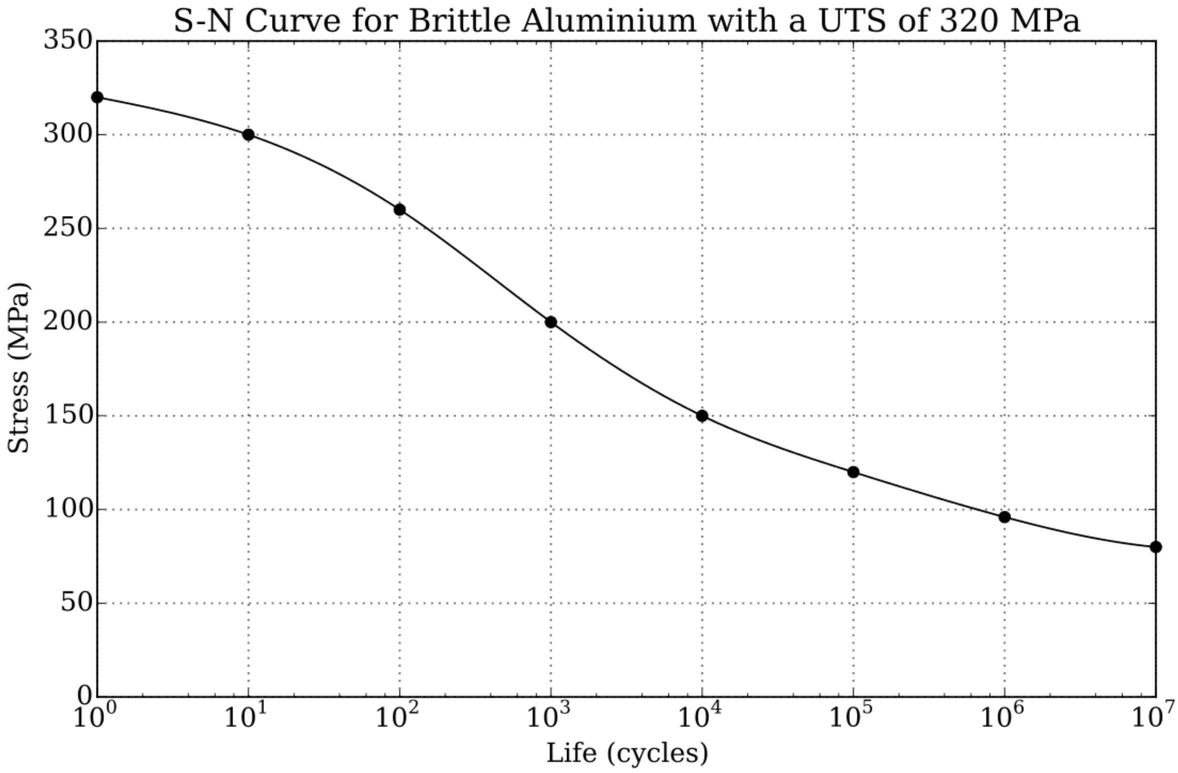


Figure 2.10: *SN Curve: How many cycles a sample can endure at a given stress.*

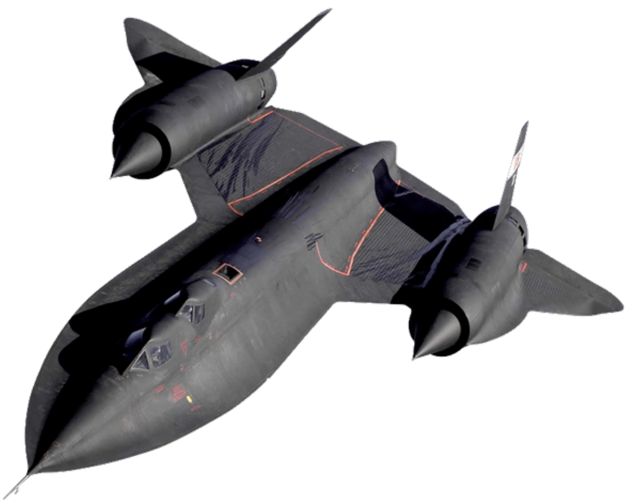


Figure 2.12: *The Lockheed SR71 Blackbird.*

2.13.2. Shock Waves

When a plane goes supersonic, it leaves a cone of waves behind it. This is because the air moves out from the craft at the speed of sound in a sphere. But the plane moves forward in a different speed. From this knowledge we can draw the following diagram:

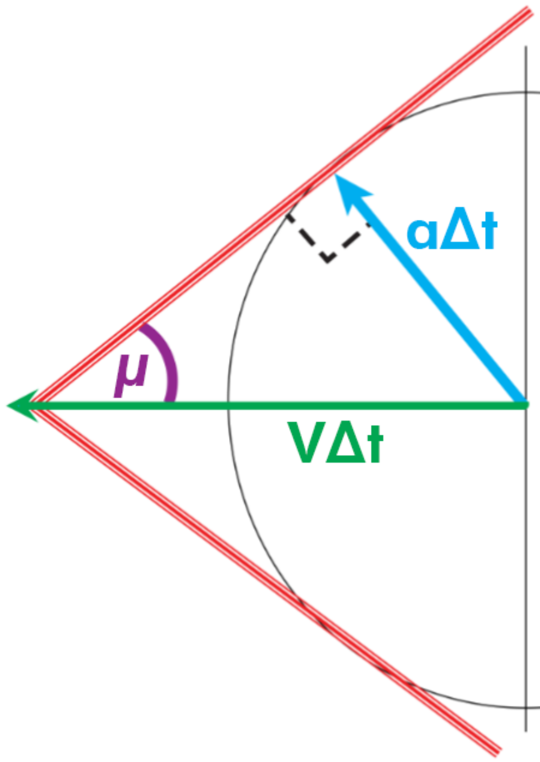


Figure 2.13: *This is what we see when we see a Mach Cone.*

$V\Delta t$ is the distance the plane flies in a given time t, and $a\Delta t$ is the distance sound can travel in the same amount of time. Given that we know the mach number is just the speed of the aircraft in relation to the speed of sound we can use trigonometry to find M from this diagram.

$$\sin\mu = \frac{a\Delta t}{V\Delta t} = \frac{a}{V} = \frac{1}{M}$$

Thus:

$$M = \frac{1}{\sin\mu} \tag{2.9}$$

3

Aerodynamics

Aerodynamics is dealt with in an odd way in this course. There are a lot of derivations but you don't really need to know them all, only some of them. As such, I won't be deriving all of the equations you need. I'll just be explaining them.

3.1. Fundamentals

3.1.1. Fundamental Quantities

- **Pressure** p: Normal force per unit area on a surface.

$$p = \lim_{dA \to 0} \frac{dF}{dA} \tag{3.1}$$

- **Density** ρ: Amount of a substance per unit volume. It is the inverse of the specific volume, which is the volume of 1kg of a substance.

$$\rho = \lim_{dv \to 0} \frac{dm}{dv} \tag{3.2}$$

- **Temperature** T Average kinetic Energy of a gas. The kinetic energy is:

$$KE = \frac{3}{2}kT \tag{3.3}$$

where k is the Boltzmann constant of $1.38x10^{-23}JK^{-1}$

- **Velocity** V (this is a capital V): $\frac{ds}{dt}$ distance traveled by an infinitesimally small element in a fluid, tangent to a streamline, in a given unit of time.

These are the four fundamental quantities that we deal with throughout the aerodynamics section. So we try to relate everything back to these when we define things. We can relate them together with the equation of state:

$$p = \rho RT \tag{3.4}$$

This is the equation of state for a perfect gas, the same as the one we derived in eq(1.1). A perfect gas is a gas in which inter-molecular forces are negligible. But that's often not the case, so to approximate an imperfect gas we can use the Berthelot equation instead:

$$\frac{p}{\rho RT} = 1 + \frac{ap}{T} - \frac{bp}{T^3} \tag{3.5}$$

The difference between these equations lessens as the pressure decreases, or as the temperature increases. This is because the distance between air molecules decreases, and so there are fewer inter-molecular interactions.

The aerodynamic forces that act on a wing (or really anything that moves through the air) are: The sheer stress, or the friction force (τ_w) along the wing, and the pressure force acting all around the wing (p).

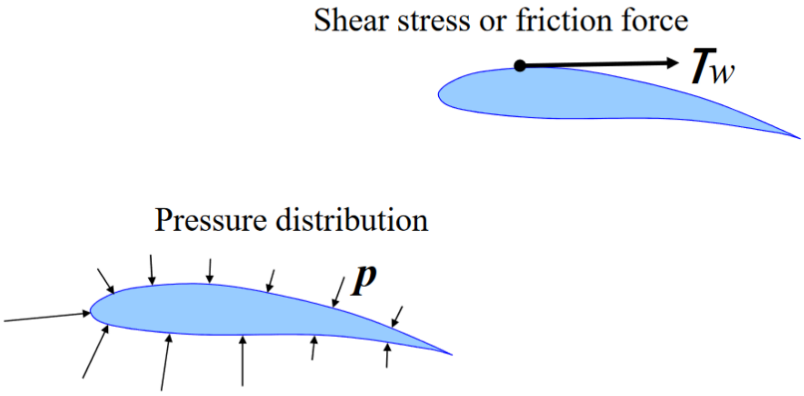


Figure 3.1: *The Aerodynamic Forces that act on a wing.*

3.1.2. Fundamental Equations

We've seen the fundamental quantities of aerodynamics, but what are the ways of using these? We find there are 3 fundamental equations for that:

- **Continuity equation**: Conservation of mass.

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \tag{3.6}$$

- **Momentum (or Euler) equation**: Conservation of momentum

$$dp = -\rho V dV \tag{3.7}$$

- **Bernoulli's law**: for incompressible flow.

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \tag{3.8}$$

Let's look at these in a bit more detail. starting with the continuity equation. For this we imagine a steady flow of air, so: $\dot{m}_{in} = \dot{m}_{out}$. So if you imagine air flowing through a tube that widens, you get something like figure (3.2)

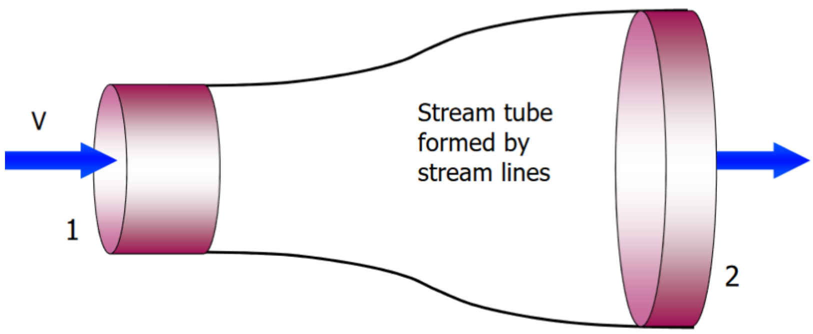


Figure 3.2: *The Conservation of Mass.*

if you can notice, the same amount of air is being pushed out through a larger area, so to compensate, and to make sure the same amount of mass is being pushed out, the speed of the air must decrease. If we are dealing with incompressible flow we can say that $\rho_1 = \rho_2$ and thus AV must be constant.

The Euler Equation comes from Newtons 2^{nd} law of motion. We neglect all of the forces acting on the gas, except for pressure. Thus, we assume that gravity has no effect, we assume that viscosity is negligible (so it's an inviscid flow), and we assume it's a steady flow.

This is a differential equation so in order to use it we have to integrate it from A to B.

$$\int_A^B dp + \rho V dV = 0$$

$$= \int_{p1}^{p2} dp + \int_{V1}^{V2} \rho V dV = 0$$

If we are dealing with incompressible flow, we can say ρ is constant.

$$= p\frac{1}{2}\rho V dV = P_{tot}$$

And here we see the Bernoulli equation for incompressible flow.

3.2. Applying Bernoulli's Law

3.2.1. Coefficient of Pressure

We can look at an airfoil and see how the static pressure p changes as you move along the chord. This happens because the shape of the airfoil makes air speed up as it flows over it.

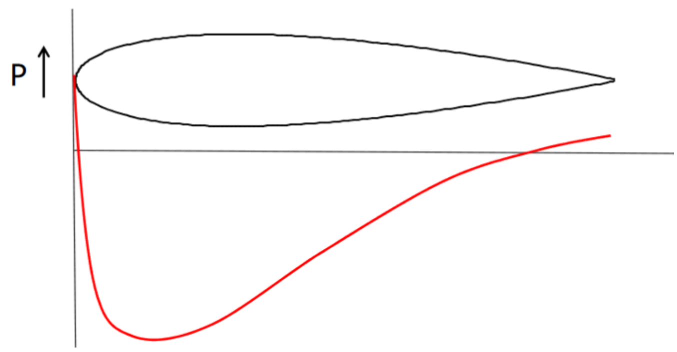


Figure 3.3: *The Static Pressure on an airfoil decreases as the air speeds up.*

We can also plot the different in pressure on the upper and lower surface of an airfoil. We can do this for a number of speeds, giving us a graph that looks like this:

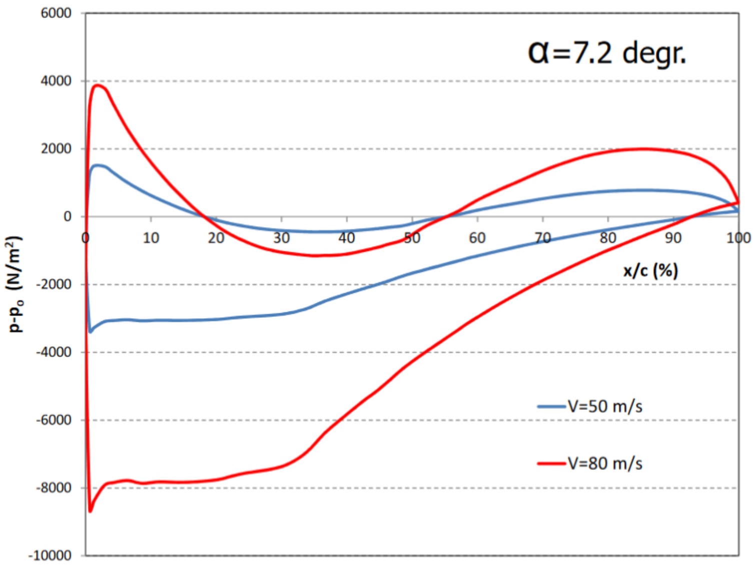


Figure 3.4: *Notice how the shape of the graph is the same despite the different speeds*

What we can do here is introduce a new coefficient, C_p, the coefficient of pressure. This is a unitless value and it shows us the value of the static pressure in relation to the dynamic pressure at a given point along the chord.

$$C_p = \frac{p - p_0}{\frac{1}{2}\rho V_0^2} = \frac{p - p_0}{q_0} \tag{3.9}$$

Note that the subscript "0" means that we are talking about the incoming flow. Now that we have C_p we can redraw our graphs and we find that they collapse into the same shape. This is incredibly useful because it means we don't need to find experimental data for every possible speed at which an aircraft might fly.

We often manipulate the graphs a bit for our convenience. Because the C_p is negative on the upper side, and positive on the lower side, we flip the graph so the upper side of the graph shows us the C_p values for the upper part of the airfoil.

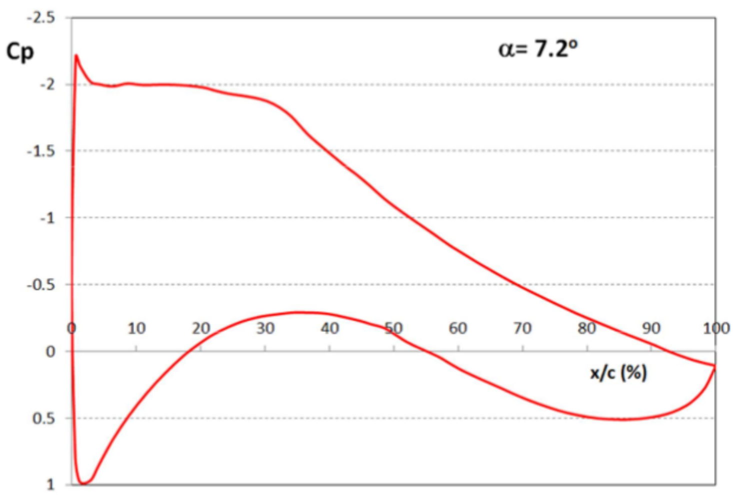


Figure 3.5: *Standard format for Cp Graphs*

3.2.2. Stagnation Point

Bernoulli's law applies along a streamline, so we can look at the different streamlines both over and under an airfoil. But there is a special, unique streamline that ends right on the nose of the airfoil and doesn't flow over or under it. This is the **stagnation point**, where the velocity of the streamline is equal to 0. and with this information we can do some interesting calculations.

$$p_0 + \frac{1}{2}\rho V_0^2 = p_1 + \frac{1}{2}\rho V_1^2$$

$$p_0 + \frac{1}{2}\rho V_0^2 = p_1$$

$$\frac{1}{2}\rho V_0^2 = p_1 - p_0$$

$$C_p = \frac{p_1 - p_0}{\frac{1}{2}\rho V_0^2}$$

3.2.3. Pitot Tubes

Pitot tubes are also an application of Bernoulli's law, but we already covered them in (1.6.2).

3.3. Compressibility

3.3.1. Thermodynamics

If we have a unit mass of a gas and we want to change the amount of energy in it we can either do work or change the amount of heat energy in it. This gives us the first law of thermodynamics: the change in energy in a system is equal to the sum of heat energy added (or taken away), and the work don on (or by) the system.

$$de = \delta q + \delta w \tag{3.10}$$

We also know that work is force by distance ($w = F\Delta x$) and because we are dealing with a gas, we can rewrite this as a function of pressure p; $w = -p\Delta V$. The minus sign appears in this equation because if do work on a gass the volume will decrease. With this information we can rewrite the first law of thermodynamics to include pressure (because pressure is one of the fundamental quantities of a gas, remember?)

$$de = \delta q - pdv \tag{3.11}$$

We're going to introduce the concept of enthalpy here. We don't go into too much detail about what exactly it is; but it's the energy and the pressure times the volume: $h = e + pv$, if we differentiate this, and then combine it with the above equation (3.11) we get:

$$dh = \delta q + vdp \tag{3.12}$$

3.3.2. Specific Heat

Specific heat is a the amount of energy we need to add to a system to get a temperature change of one degree.

$$c = \frac{\delta q}{dT} \tag{3.13}$$

There are two types of processes that we look at in detail in this course, and they both have impacts on the specific heat. We can have a constant volume process, where you add heat to a volume of gas in a rigid container; thus causing the pressure to increase.

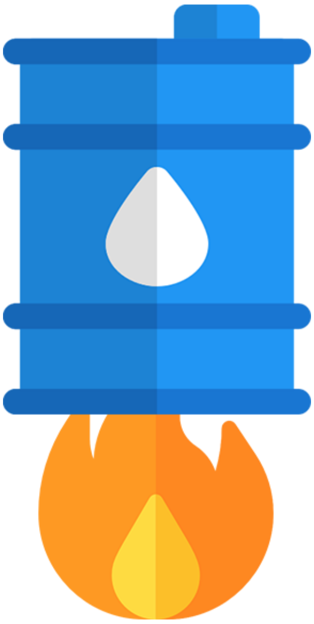


Figure 3.6: *If you heat up a sealed barrel the volume of gas cannot increase.*

We could also have a process where we have a constant pressure and in turn the volume would change, This could happen if you heated up an open barrel, then the pressure would equalise with the atmosphere. Or you could have a system with a piston keeping the pressure constant. Use your imagination.

This gives us specific heats for constant volume processes (c_v) and constant pressure processes (c_p). Note that we're using a lowercase c so this is not the same as the coefficient of pressure. And as usual we want to relate these back to the fundamental quantities.

Recall (3.11). With a constant volume process $dv = 0$, so $\delta q = de$, and as such, $de = c_v dT$. We can integrate this to get the equation:

$$e = c_v T \tag{3.14}$$

Recall (3.12). With a constant pressure process $dp = 0$, so $\delta q = dh$, and as such, $dh = c_p dT$. We can integrate this to get the equation:

$$h = c_p T \tag{3.15}$$

These equations are general equations for a perfect gas. and if we calculate the values of c_v and c_p for air; we find they are:

- $c_v = 720\ [Jkg^{-1}K^{-1}]$
- $c_p = 1008\ [Jkg^{-1}K^{-1}]$

for any process where the temperature is less than $600K$.

3.3.3. Isentropic Flow

I promise we'll get there eventually. But until then we have another concept to introduce; Isentropic flow, which is both:

- Adiabatic process: The net heat energy added is zero.
$$\delta q = 0$$
- Reversible process: No energy is lost to friction, nor is there any dissipative effects.

In an isentropic flow there are several important relationships between pressure, Temperature, and density. You don't need to derive them. If you're curious about how I got to the next steps, go re-watch the second lecture in the aerodynamics block. It covers everything nicely.

If we compare the specific heats; we get the ratio of specific heats γ.

$$\frac{c_p}{c_v} = \gamma \tag{3.16}$$

In air, with processes with a temperature under $600K$ we find $\gamma = 1.4$. We can use this new relationship of specific heats to compute values of temperature, pressure, density, with respect to each other.

$$\frac{p_2}{p_1} = (\frac{T_2}{T_1})^{\frac{\gamma}{\gamma-1}} \tag{3.17}$$

$$\frac{p_2}{p_1} = (\frac{\rho_2}{\rho_1})^{\gamma} \tag{3.18}$$

These are the equations for Isentropic flow, and they are *only* relevant to compressible flow. These are really useful, but we've not yet related the velocity V to the other fundamental quantities. That's why we must now introduce the energy equations.

Once again, the process for arriving here is not mandatory knowledge, and you won't need to derive it on the exam, so I won't cover it here, but basically we're using equations (3.12) and (3.7) and the knowledge that in an adiabatic process $\delta q = 0$.

$$h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2$$

$$c_pT + \frac{1}{2}V^2 = \ const. \tag{3.19}$$

Note that this is *not* a function of density. Don't make that mistake. This is the energy equation for frictionless, adiabatic flow.

3.3.4. Summary of Equations

we've derived a lot of equations today. But as a quick recap of the important ones, we have three for steady frictionless, incompressible flow:

- $A_1V_1 = A_2V_2$
3.6
- $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$
3.8
- $p = \rho RT$
3.4

And for steady, isentropic, compressible flow we have 4 equations:

- $\rho_1A_1V_1 = \rho_2A_2V_2$
3.6
- $\frac{p_2}{p_1} = (\frac{T_2}{T_1})^{\frac{\gamma}{\gamma-1}} = (\frac{\rho_2}{\rho_1})^{\gamma}$
3.17 & 3.17
- $c_pT_1 + \frac{1}{2}V_1^2 + c_pT_2 + \frac{1}{2}V_2^2$
3.19
- $p = \rho RT$
3.4

3.4. Speed of Sound

The speed of sound in air is related to how the pressure changes with respect to the change in density:

$$a = \sqrt{\frac{dp}{d\rho}}$$

or in a more useful format for us:

$$a = \sqrt{\gamma RT} \tag{3.20}$$

The Mach number is the relation between the speed of something moving though the air, and the speed of sound:

$$M = \frac{V}{a} \tag{3.21}$$

There are different ways of describing the speed of something with respect to the Mach number. And with each of these flow states we find distinct characteristics.

- M<1 Subsonic
- M=1 Sonic
- M≈1 Transsonic
- M>1 Supersonic
- M>5 Hypersonic

That's all well and good, but let's go back to the maths. Recalling (3.12), we know that $h = e + pv$ (I'm using v the specific volume), we also know, thanks to (3.4) that $pv = RT$ so we can combine these equations to get:

$$h = e + pv$$

And with the specific heats for constant volume, and constant pressure processes (3.14 & 3.15) we can arrive at:

$$c_p T = c_v T + RT$$

And thus:

$$c_p = \frac{\gamma R}{\gamma - 1} \tag{3.22}$$

3.5. Reservoir Flow

when we analyse a wind tunnel we know for a fact that the velocity of the air in the reservoir is 0. And as such, we can use our useful energy equation (3.19):

$$c_p T_1 + \frac{1}{2} V_1^2 c_p T_0$$

$$\therefore \frac{T_0}{T_1} = 1 + \frac{1}{2} \frac{V_1^2}{c_p T_1}$$

Or thanks to our relation between γ and c_p (3.13) we can substitute out c_p (and skip forward a bit, to get:

$$\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \tag{3.23}$$

This is a very useful equation, that won't be given to you on the day of the exam, so you should make a note of it, and learn it off. We can also relate it back to equations (3.18 & 3.17) to get other relations.

$$\frac{p_0}{p_1} = (1 + \frac{\gamma - 1}{2} M_1^2)^{\frac{\gamma}{\gamma - 1}} \tag{3.24}$$

$$\frac{\rho_0}{\rho_1} = (1 + \frac{\gamma - 1}{2} M_1^2)^{\frac{1}{\gamma - 1}} \tag{3.25}$$

Equation (3.25) gives us an interesting look at compressibility. Notice that for M<0.3, the density of the flow is more than 95% the density of the stagnant air. Thus we say that a flow is incompressible if it's moving at a speed less than M=0.3.

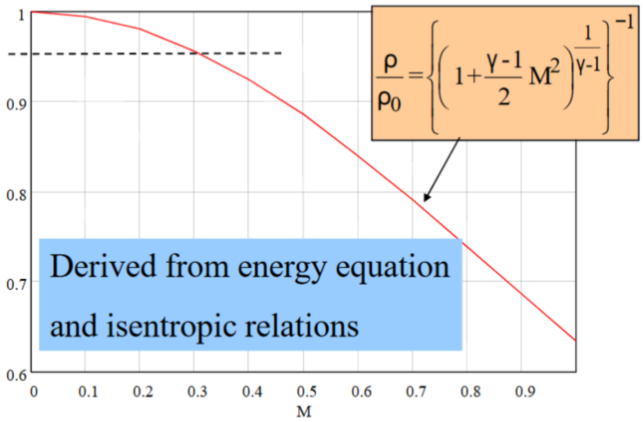


Figure 3.7: *The Density of the Air vs the Mach Number*

3.5.1. Supersonic Wind Tunnels

From our equation (3.6) (ρAV is constant) we can (but I won't) derive an equation that shows us the relation between changing the area of a wind tunnel, and the change in Velocity:

$$\frac{dA}{A} = (M^2 - 1)\frac{dV}{V} \tag{3.26}$$

With this we can see what happens in a wind tunnel where we want to increase the speed. If **M<1** then to increase the velocity we must **decrease the area**. If **M>1** then to increase the velocity we must **Increase the area**.

But then we get a special case when we arrive at M=1. The change in area must be 0 regardless of how the speed of the flow changes. Thus, in a supersonic wind tunnel you will find subsonic flow before the throat. Sonic flow in the throat. and supersonic flow after the throat.

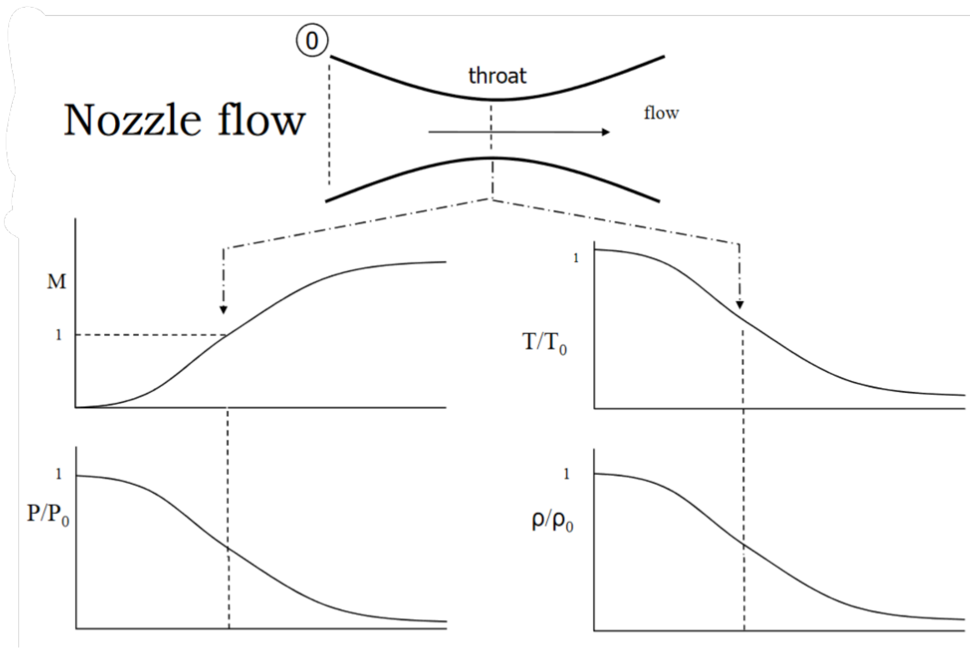


Figure 3.8: *Characteristics of a Supersonic Wind Tunnel, or a Rocket Nozzle*

3.6. Viscous Flow

3.6.1. The Boundary Layer

Up until now we've mainly dealt with inviscid flow, but now it's time to start discussing viscous flow. In real life the flow at the surface sticks to the surface because of friction between the gas and the solid material. Thus right at the velocity.

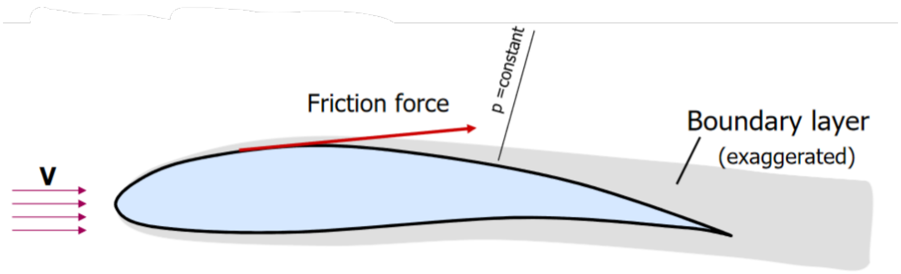


Figure 3.9: *Boundary Layer around an Airfoil*

Around the object flowing through the air we find the boundary layer. Which is where the flow of the air is reduced due to friction. the pressure through the boundary layer is constant perpendicular to the surface. Inside the boundary layer, Bernoulli's law isn't valid, because that requires inviscid flow.

If we zoom in and take a closer look at the boundary layer, we can see the speed of the flow with respect to the distance above the surface. We use the symbol u to show the speed of the flow in the boundary layer, and we use the symbol V to show the flow outside of it, as we are used to doing.

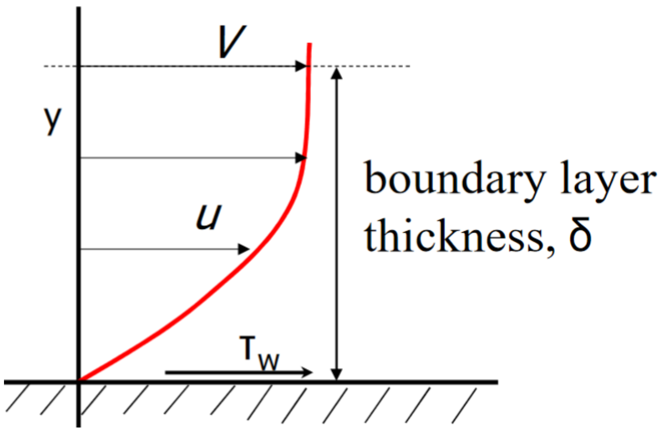


Figure 3.10: *Speed of the Flow in a Boundary Layer*

At the surface the speed of the flow is 0, because of the friction between the air molecules and the surface, but above that, the speed is greater than zero, but still slow because there is some friction between the air molecules themselves. The sheer stress on the surface can be written as:

$$\tau_w = \mu(\frac{du}{dy}) \tag{3.27}$$

3.6.2. Boundary Layer Thickness

Before we start calculating the thickness of a boundary layer, we must first define a few things.First let's deal with the idea of a local Reynolds number. If you recall the equation (1.20) from earlier, you can quite easily see how this is derived.:

$$Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty} = \frac{V_\infty x}{\nu_\infty} \tag{3.28}$$

This shows us the Reynolds number at position x along a flat plate.

Laminar flow is where the streamlines are smooth and regular and a fluid element moves smoothly along a streamline. Whereas **Turbulent flow** is where the streamlines break up and a fluid element moves in a random irregular way. Just go look at YouTube videos demonstrating the different types of flow. It's a good way to spend an afternoon.

Now, if we want to calculate the thickness of the Boundary layer at position x we must consider if it's a laminar or turbulent layer first of all. From laminar boundary layer theory (not going into that here) we can see the thickness is:

$$\delta = \frac{5.2x}{Re_x} \tag{3.29}$$

Thus we can say that the thickness of the boundary layer δ is proportional to $\sqrt{x}$.

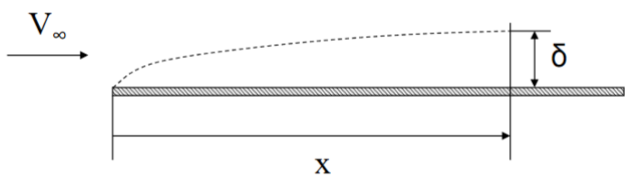


Figure 3.11: *Boundary Layer Thickness* δ

Turbulent boundary layers aren't as easy to model because they are, by definition, chaotic. But from experiments, we've found that the thickness is roughly:

$$\delta = \frac{0.37x}{Re_x^{0.2}} \tag{3.30}$$

3.6.3. Skin Friction Drag

When something flies through the air, the friction between the surface and the air molecules causes drag. We call this, unsurprisingly; skin friction drag. To calculate this force we must look at all of the forces acting on a plate.

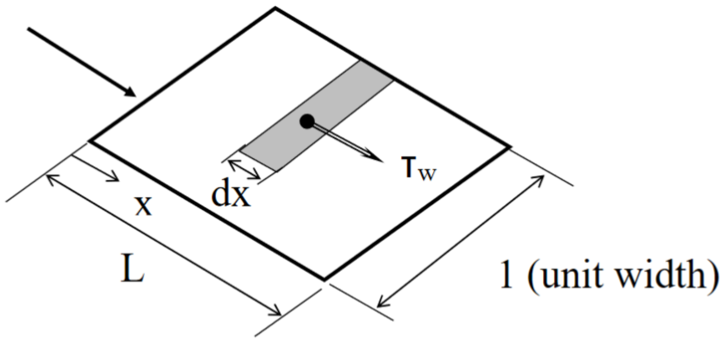


Figure 3.12: *Skin Friction Element on a Plate.*

The total force acting on the plate is the sum of all friction forces, and the pressure forces. But because the pressure is acting on both the top and the bottom of the plate, it cancels out to zero. The friction force on an element on the plate is the sheer stress, times the elemental length dx, times the width of the plate. or in mathematical terms: $\tau_w \cdot x \cdot dx \cdot 1m$. And if we want the total force over the length of the plat we must integrate.

$$D_f = \int_0^L \tau_w dx \tag{3.31}$$

As with most things in the course, we'd like to get a coefficient from this to allow us to calculate more things. We describe the local skin friction coefficient, c_f, as the sheer stress divided by the dynamic pressure:

$$c_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\tau_w}{q_\infty} \tag{3.32}$$

Or with laminar boundary layer theory we can write this in terms of the local Reynolds number:

$$c_{f_x} = \frac{0.664}{\sqrt{Re_x}} \tag{3.33}$$

Looking at this you can see that the skin friction is more prominent at the beginning of the plate rather than at the end. That's because the friction causes the air loses momentum as it flows along the the plate, and thus, the drag is reduced. If we want to use this to calculate the total drag along the plate, we once again integrate and this gives us:

$$D_f = \frac{1.328 q_\infty L}{\sqrt{Re_L}} \tag{3.34}$$

Thus, if we want to find the coefficient of friction over the whole plate we must divide the drag by the dynamic pressure and the surface area:

$$C_f = \frac{D_f}{q_\infty S}$$

$$C_f = \frac{1.328}{\sqrt{Re_L}} \tag{3.35}$$

For a turbulent boundary layer we find that it's either:

$$C_f = \frac{0.074}{Re_L^0.2} \tag{3.36}$$

or

$$C_f = \frac{0.455}{(\log Re_L)^{2.58}} \tag{3.37}$$

We see that the coefficient of friction changes with respect to $L^{\frac{-1}{5}}$ for turbulent flow, whereas it changes with respect to $L^{\frac{-1}{2}}$ for laminar flow. Hence, the friction in a turbulent boundary layer is higher than in laminar flow.

3.7. Transition

After enough resistance and disturbance in a flow, a laminar flow will eventually turn turbulent. This is what we call transition. The graph (3.14) shows the relation for skin friction along **one side** of a flat plate. The Reynolds number increases based on the length of a plate. At a certain point the friction jumps and increases. This is when the flow goes from laminar to turbulent: Transition. With a bigger Reynolds Number a larger portion of the flow has a turbulent Boundary layer.

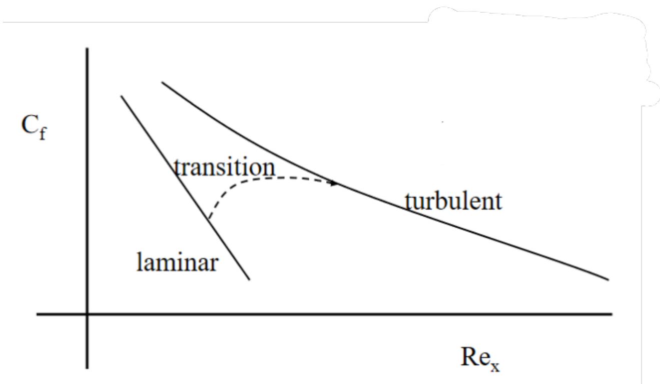


Figure 3.13: *Transition is when a flow goes from laminar to Turbulent*

Sometimes we model transition as happening at a point, other times we look at how transition occurs like in this diagram:

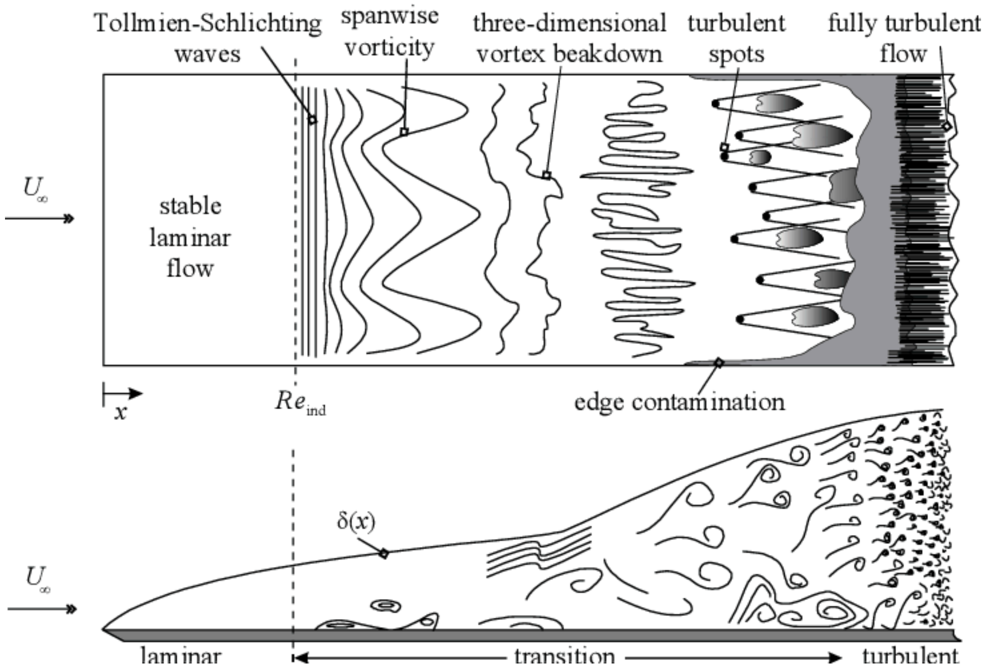


Figure 3.14: *Transition occurring along a flat plate*

The Reynolds number increases along the length of the airfoil, and once it hits a certain value, transition begins. We call this the critical Reynolds number Re_{crit}, or as it is labeled in the diagram,Re_{und}. The Reynolds number occurs at a critical distance along the plate x_{crit}. Though, the process of transition can be sped up with surface roughness or adverse pressure gradients etc.

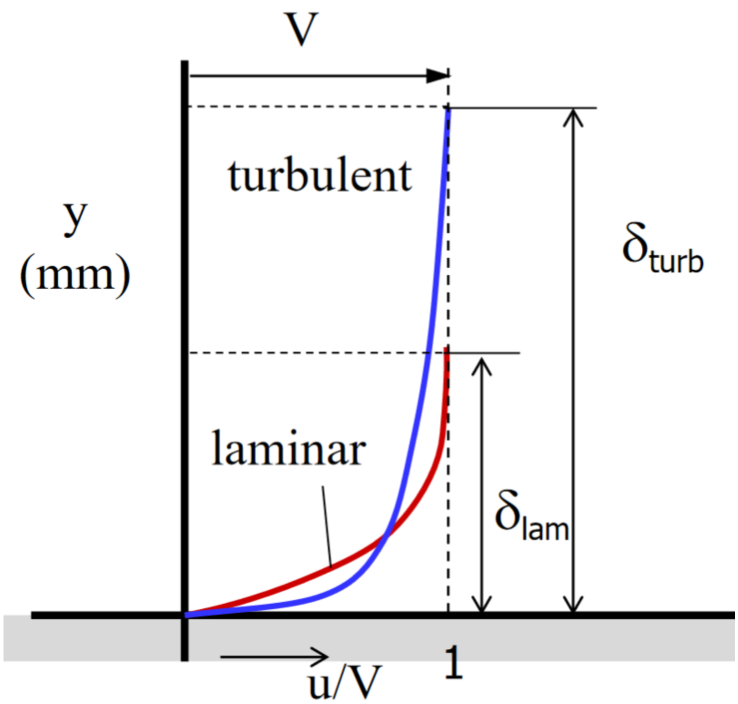


Figure 3.15: *Speed in Laminar and Turbulent boundary layers*

As you can see: the speed near the plate is faster for the turbulent boundary layer therefore the skin friction is higher. So when we transition to a turbulent layer we end up with more skin friction. We can use this fact for our advantage and observe where transition takes place on an airfoil.

To do so, we can coat an airfoil in a luminous oil, and place it in a wind tunnel. The boundary layer will pull this oil along because of the friction between the oil and the air. But as the air slows, it stops being able to pull the oil along. then we hit transition and suddenly the flow becomes turbulent, and the skin friction increases dramatically. At the position where little oil is found, transition is complete.

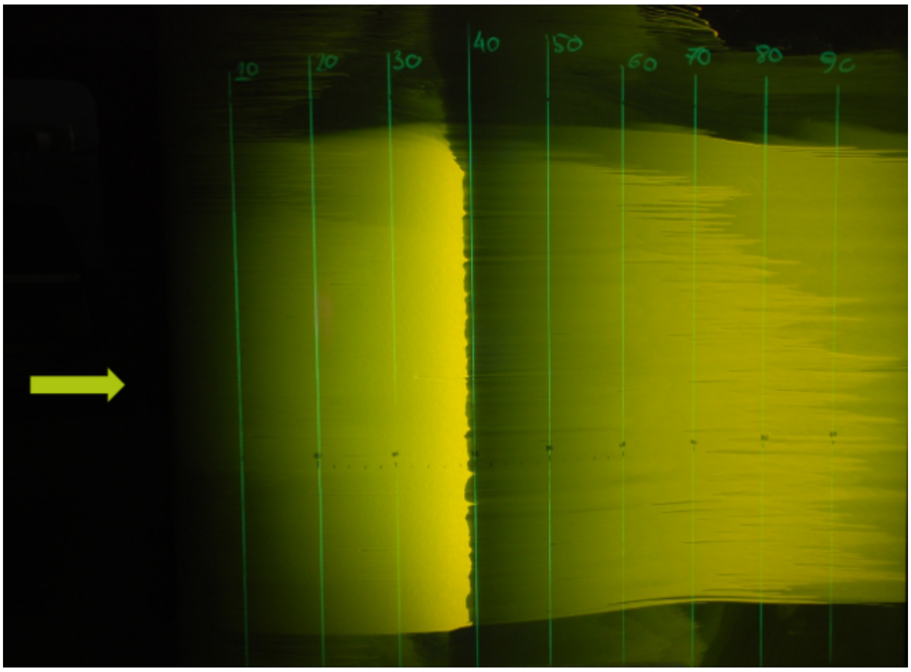


Figure 3.16: *Transition at about 39% the length of the chord*

Increasing the Angle of attack will move the transition point forward. You can detect this with a microphone as well as oil. Turbulent boundary layers are very loud in comparison to laminar flows.

3.8. Pressure Distributions

3.8.1. Adverse Pressure Gradients

Take a look at the diagram (3.5). On the left hand side you can see the coefficient of pressure increase on the top side of the wing (it's going down in the chart, but the Y axis is flipped). This is a pressure gradient. It's where the static pressure increases on the wing. If this gradient is too steep, then we have something called an *Adverse Pressure Gradient*, this is where the air flowing around the wing can't adapt to the change in pressure and has difficulties following the curve of the airfoil.

3.8.2. Transition Points

Transition occurs over a much shorter distance on an airfoil than on a flat plate thus we can often just model it as a transition point rather than a transition region. Remember how an increase in an angle of attack will move the transition point forward? Well we can chart that on a pressure distribution for various angles of attack.

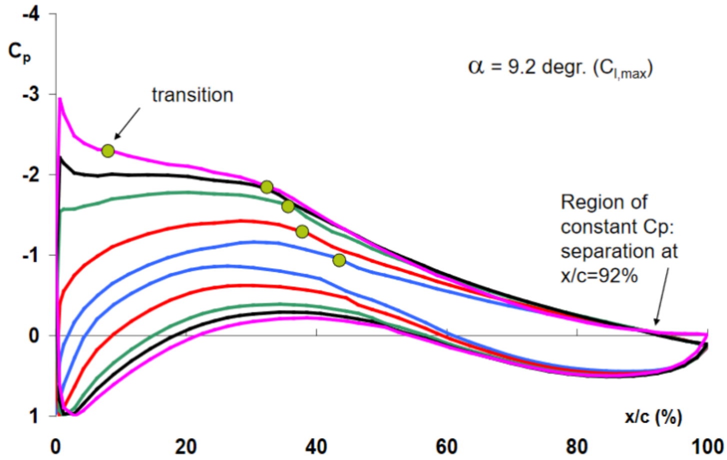


Figure 3.17: *Notice the Transition points for the different Angles*

If you increase angle of attack, you move transition point forward. But, eventually you get a pressure peak right at the beginning of the chord which destabilises the Boundary layer, moving transition point forward a lot. Thus you get a turbulent boundary layer over most of the chord. This layer must follow the contour of the air foil, and that's hard, so separation occurs. Notice that separation occurs at 92% along the chord of the airfoil at an angle of attack of 9.2°. This is because the air has lost all of its velocity near the skin. Continuing to increase the angle of attack even more will just move the separation point forward even more.

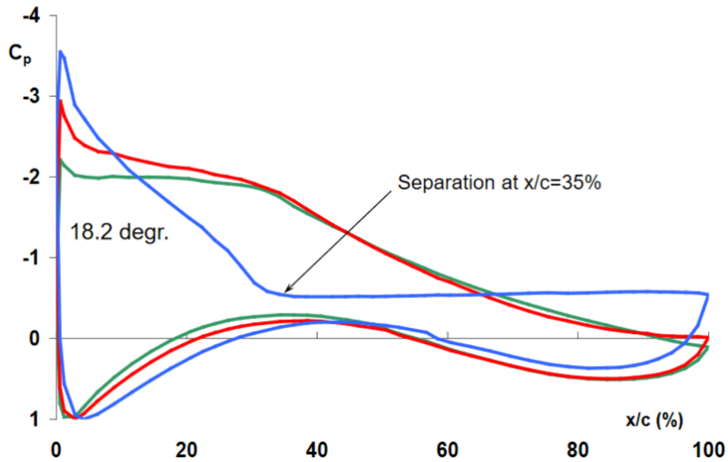


Figure 3.18: *separation occurs much much sooner along the chord*

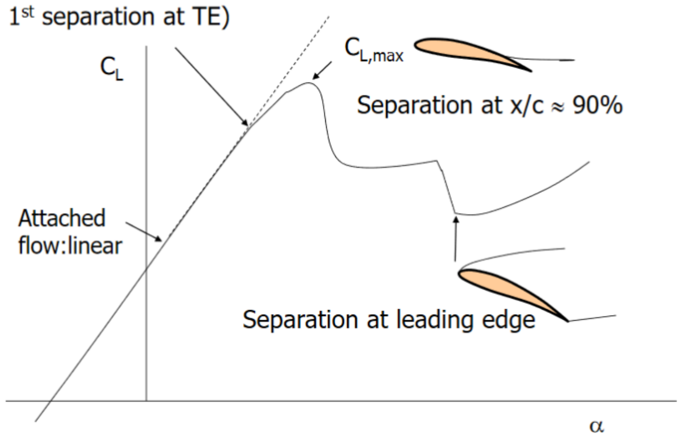


Figure 3.19: *the coefficient of lift isn't directly related to angle of attack*

3.9. Flow Separation

Separation is generally pretty bad for flying because it means we have a loss in lift, we have increased drag, and it results in the generation of unsteady loads. In general we can we experience both laminar and turbulent separation. Laminar separation happens at a low Reynolds number (below 500,000), and Turbulent separation will happen at a higher a higher Reynolds number. If you can recall that the Reynolds number is the *inertia force*s divided by the *viscous forces* this will make sense.

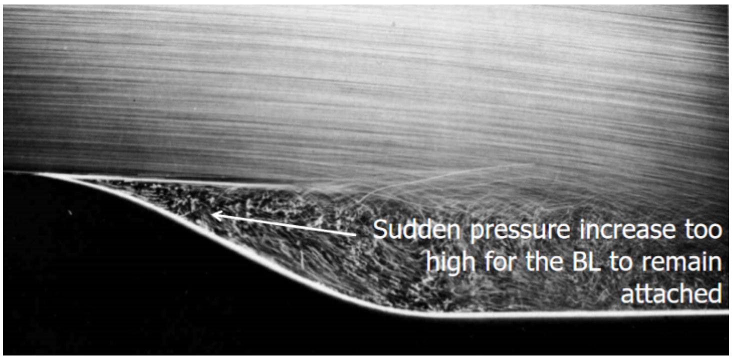


Figure 3.20: *Laminar Separation over an Expansion*

When we calculate the Reynolds number over a body, we use a definite characteristic of it. For an airfoil we use the length L, and for cylinders of spheres we use the diameter d:

$$Re = \frac{\rho_\infty V_\infty d}{\mu_\infty} = \frac{V_\infty d}{\nu_\infty} \tag{3.38}$$

When we have a low Reynolds number we have a more viscous fluid, or rather, the effect of viscosity is higher, and so bluff (blunt) bodies might cause laminar separation and experience a high pressure drag as a result. If you **increase** the Reynolds number, this will in turn reduce the coefficient of friction (C_f) and reduce the boundary layer thickness, as we explored earlier.

Not only that, but if we increase the Reynolds number we will destabilize the laminar boundary layer by increasing the amplitude of the Tollmien-Schlichting waves =>the transition location moves forward towards the leading edge on an object. Thus a turbulent boundary layer will be present over more of the object. Despite this, the lower friction coefficient will have a greater impact so the drag will go down.

We also see that if we increase the Reynolds number the lift slope goes up as a result of a ”decambering” of the airfoil. Due to thick boundary layers the outer flow “sees” an airfoil shape with less curvature. When the boundary layer gets thinner the increased curvature of the flow means a little higher lift. Essentially, the thinner boundary layer means the airfoil can deflect the outer flow more.

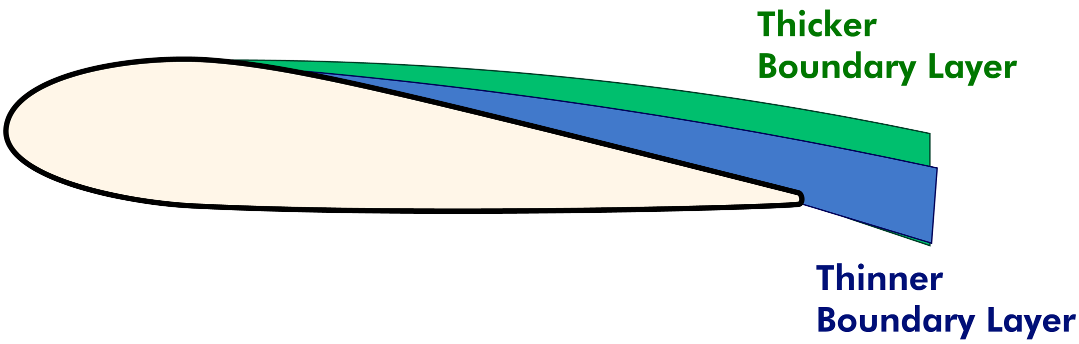


Figure 3.21: *A thinner boundary layer deflects air downwards*

3.9.1. Drag

There are two main parts of a drag coefficient, the friction drag and the pressure drag. The friction drag is caused by a sheer force on the surface of an object. Pressure drag is caused by a difference in static pressure in front of, and behind an object.

$$C_d = C_{d_{pressure}} + C_{d_{friction}} \tag{3.39}$$

Separation drastically increases the pressure drag so we’d much prefer to have a flow go turbulent so we deal with friction drag instead of a laminar separation (and hence pressure drag). It is for this reason that sport balls are seldom smooth. The roughness on a golf ball or tennis ball disturbs the air causing it to go turbulent, reducing the drag due to separation.

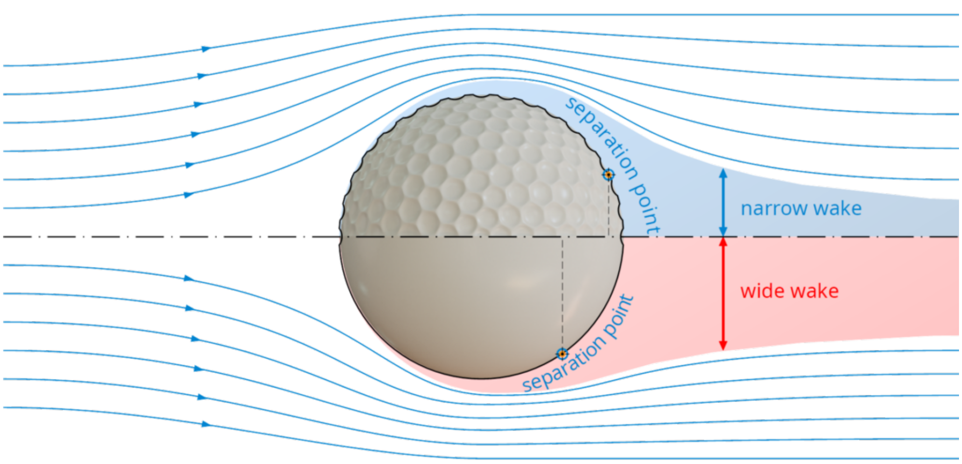


Figure 3.22: *A Golf Ball is Dimpled to Reduce Pressure Drag.*

Turbulent boundary layers have more kinetic energy in the flow near the surface, and as a result of this, flow separation may be postponed. But flow separation can also be postponed on bluff bodies by guiding the airflow to a point. This is why Airplanes narrow to a point at the end of the fuselage rather than just having a flat end cap.

3.10. Airfoil Characteristics

3.10.1. Examining Properties

We're familiar with looking at how the coefficients of drag, lift and change with respect to the angle of attack, but we can also plot the coefficient of lift with respect to the coefficient of drag and get a cool C-shaped graph that allows us to find $\frac{C_l}{C_d}_{max}$ easily, like this:

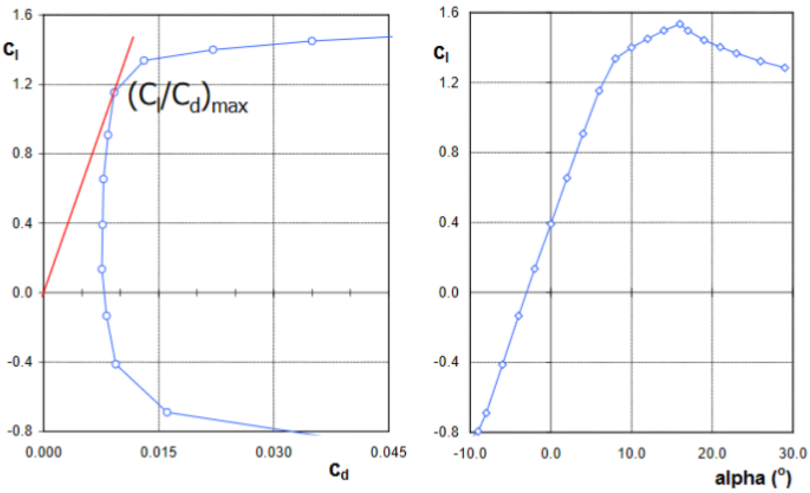


Figure 3.23: *A second way of looking at airfoil characteristics*

3.10.2. Airfoil Designs for Different Uses

When we design an airfoil we can design it to be good in different use-cases. for instance we can design an airfoil that has mostly laminar airflow over it, and we can have an airfoil that has mostly turbulent airflow over it, even under the same conditions.

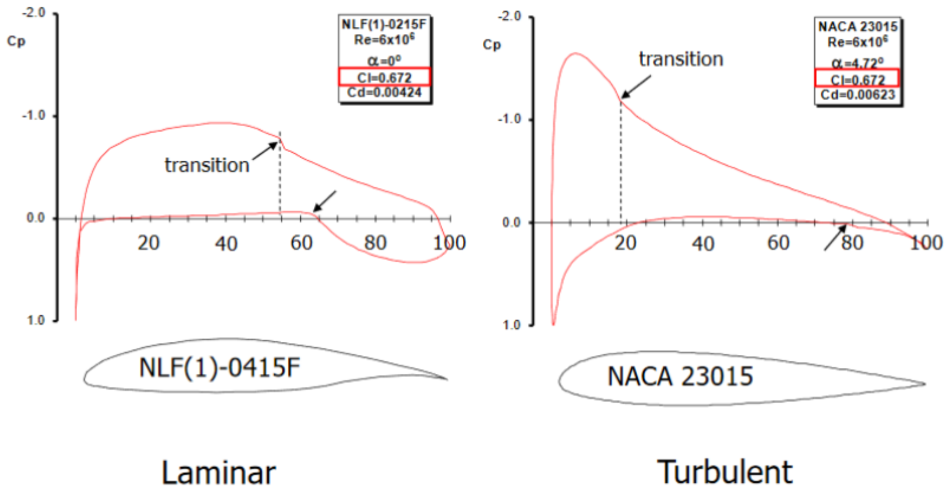


Figure 3.24: *Transition occurs later on a laminar airfoil, but lift is generated more evenly along the airfoil.*

We can design for pretty much anything. If we increase the camber of an airfoil we shove the C_{L_α} curve up, but we don't increase the $C_{L_{max}}$ because separation occurs earlier. You can add flaps to a wing as well and they'll deflect more air down and thus increase the coefficient of lift, and we can even design airfoils that work well in supersonic speeds, but we'll come back to them.

3.10.3. Obtaining Lift from Pressure Distribution

In class you would have seen the derivation for how we find this, but you don't need to know how to derive it, just know what it means. So firs let's talk about aerodynamic forces. There is an aerodynamic force acting on an airfoil, that's how they fly. We just divide this into the components Lift, and Drag. which are perpendicular to each other. We define lift as being perpendicular to the direction of flight, and Drag is parallel to the direction of flight.

what we can instead do is resolve the aerodynamic force into a normal force N and a tangential force T with respect to the mean chord line. This normal force can be found by integrating the pressure acting on the surfaces of the airfoil.

$$N = \int_{x=0}^{x=c} p_l dx - \int_{x=0}^{x=c} p_u dx \tag{3.40}$$

What this equation shows us is that the normal force acting on the wing is equal to the pressure pushing up on the airfoil, minus the pressure pushing it down. Which is the area enclosed on a pressure distribution graph. But, again, because we are engineers, we want this to be a coefficient. And so, when we get it to be unitless (by diving the normal force by the chord length and the dynamic pressure) we get:

$$C_n = \int_0^1 (C_{p_l} - C_{p_u}) d(\frac{x}{c}) \tag{3.41}$$

This coefficient is the area enclosed by the distribution of pressure coefficients.

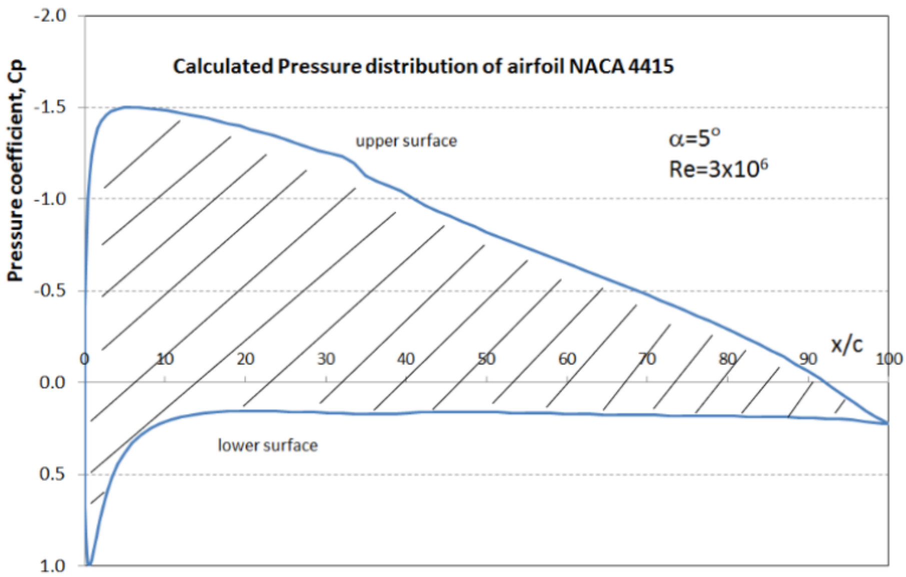


Figure 3.25: *The Enclosed area is how we find C_n*

But, the normal force isn't the same as the lift force? Exactly! It depends on the angle of attack, so to get the lift from the normal force we use:

$$L = N\cos\alpha - T\sin\alpha \tag{3.42}$$

$$D = N\sin\alpha + T\cos\alpha \tag{3.43}$$

the fin thing about this is that if α is less than $5°$ we can basically say that it's irrelevant, and so $L \approx N$ and so:

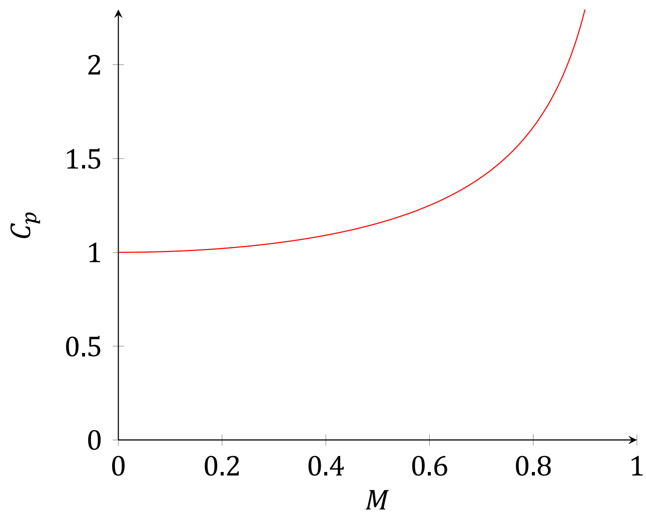
$$C_l \approx C_n = \int_0^1 (C_{p_l} - C_{p_u}) d(\frac{x}{c}) \tag{3.44}$$

3.11. Effect of Compressibility on Lift

Air is compressible, and as a result of that, when it compresses the coefficient of pressure changes. Using the **Prandtl-Glauert Rule** we can get an approximate the C_p at a particular point, but only for infinite airfoils, not finite wings. And it only works when the airflow is below $M = 0.7$.

$$C_p = \frac{C_{p_0}}{\sqrt{1 - M_\infty^2}} \tag{3.45}$$

If we plot this for an increasing mach number we get a chart a bit like this:



You can see that as we move towards $M = 1$ the chart goes a bit crazy, which is why it's only valid for $M < 0.7$. If we use this correction in conjunction with equation (3.44) We get:

$$C_l \approx \int_0^1 \frac{(C_{p_l} - C_{p_u})}{\sqrt{1 - M_\infty^2}} d(\frac{x}{c})$$

Which in turn gives us the useful equation:

$$C_l = \frac{C_{l_0}}{\sqrt{1 - M_\infty^2}} \tag{3.46}$$

3.12. Critical Mach Number

When air flows over an airfoil it speeds up. That's how we generate lift. So when air flows over an airfoil the Mach number increases, reaching at maximum at the peak Mach number,

M_{peak}. The critical Mach number is when the incoming flow is fast enough to make $M_{peak} = 1.00$.

The fastest speed occurs at the lowest point of pressure on an airfoil, $C_{p_{min}}$. Because as the speed increases, so too does the dynamic pressure, and as such, the static pressure decreases. If we chart the critical pressure coefficient (this is a complicated function, but we don't worry about it for now) and the Prandtl-Glauert correction we can find the critical Mach number for an airfoil.

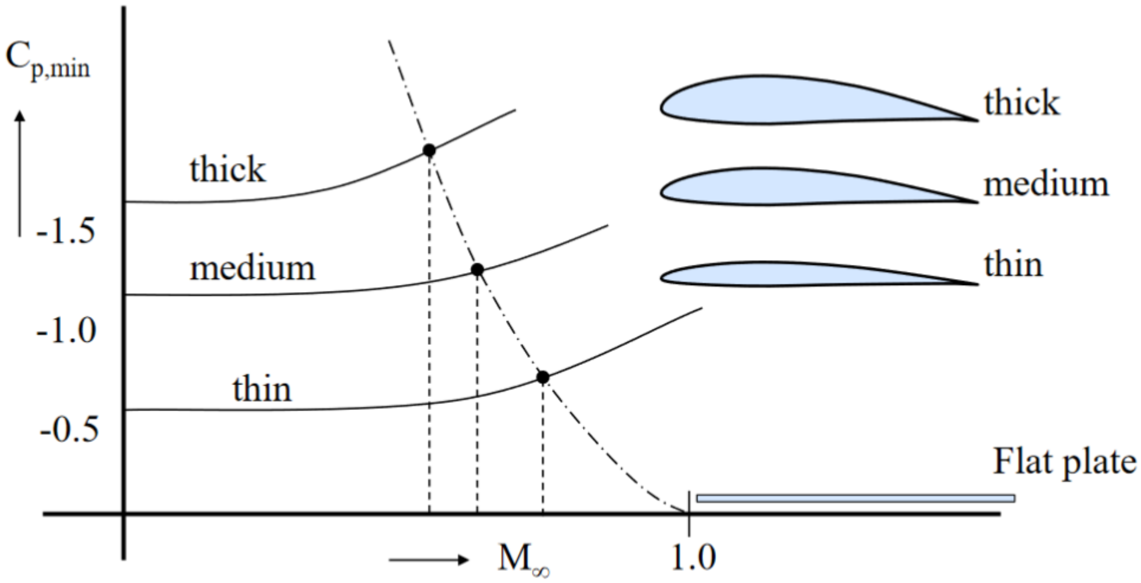


Figure 3.26: *Thicker airfoil reaches critical pressure coefficient at a lower value of M_∞*

3.13. Drag Divergence

3.13.1. Supercritical Airfoils

As you fly faster, the coefficient of drag increases. If you exceed the critical Mach number, you will get a shock wave over the wing and there will be an adverse pressure gradient, and separation will occur, causing lots of drag. So we'd like the either avoid a shock, or at least push it back to the trailing edge of the airfoil so that we don't have separation over the wing. This is where supercritical airfoils come into play.

A supercritical airfoil is one where we design it so we stay close to $C_{p_{crit}}$ over the entire upper surface, to reduce overspeeds that are too high. Because of this, we have to limit how thick the upper surface can be.

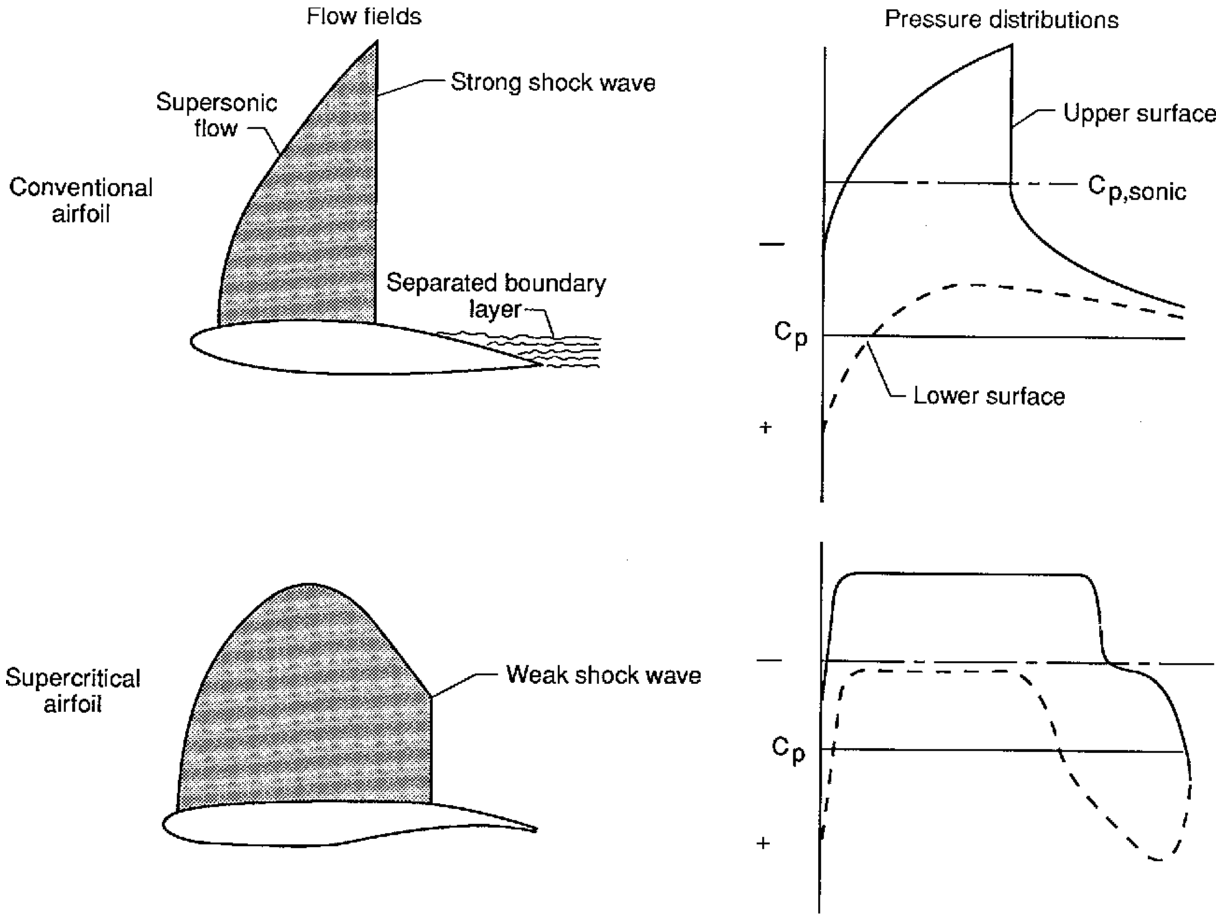


Figure 3.27: *Notice the difference in shock strengths*

3.13.2. Swept Wings

If we know that a thicker airfoil makes air speed up over a wing, what we can do is elongate the wing to change the percentage thickness. One way of doing this is by sweeping the wings back, This means the air travels over the a longer chord, with the same max thickness, but the ratio between the two is lower so we get a higher M_{crit} and M_{DD}

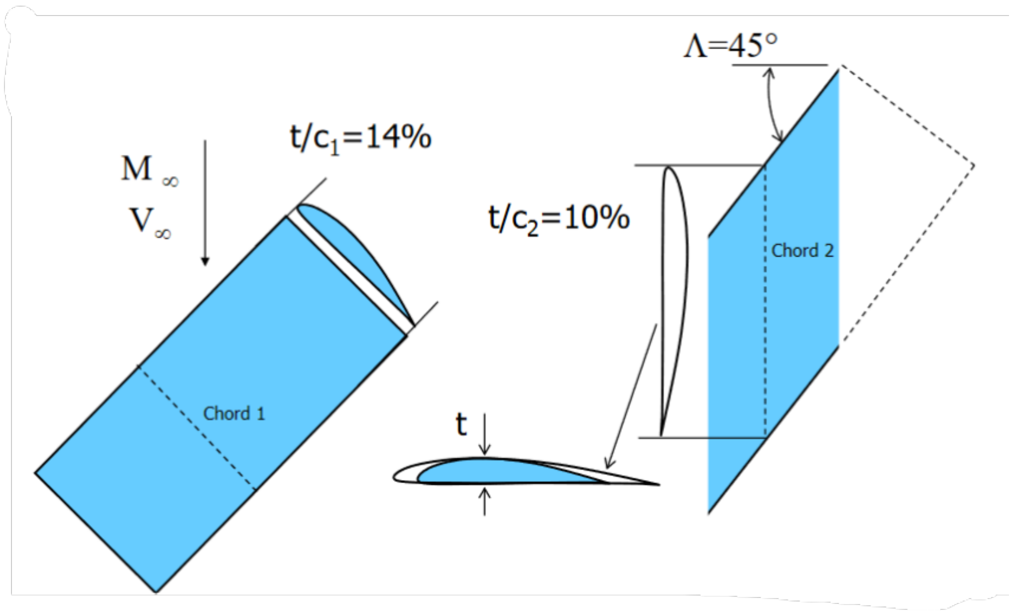


Figure 3.28: *A wing swept at angle* Λ

3.14. Finite Wings

Up until now we've dealt only with infinite airfoils, but that's now how planes work. There is a clear endpoint to any wing because welcome to reality. When we come into the real world we start to notice the presence of tip vorticies. This is because there is a lower pressure above the wing, and a higher pressure below so the air flows around the tip of the wing. Thus at the tip, lift generated is zero.

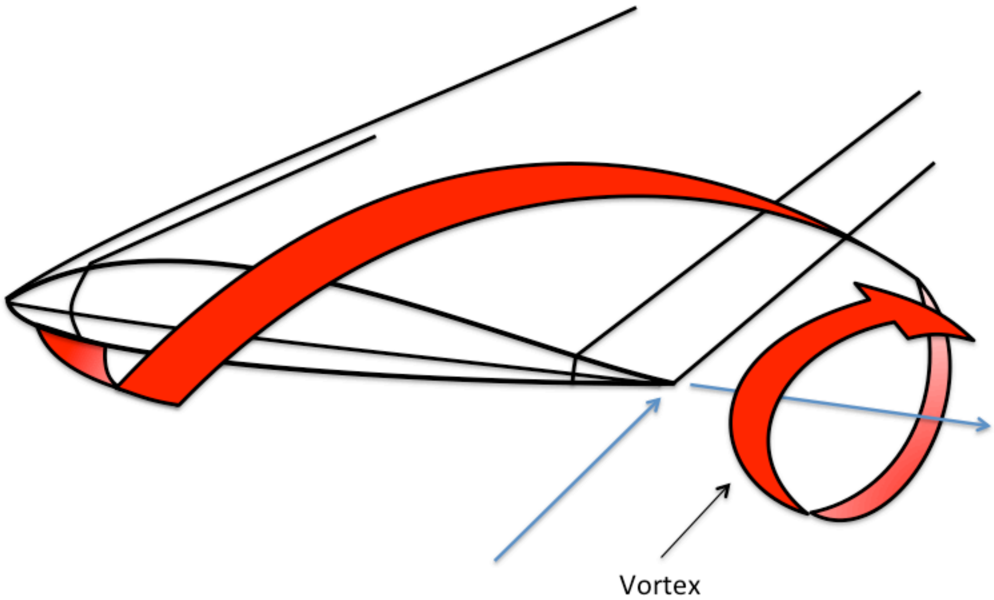
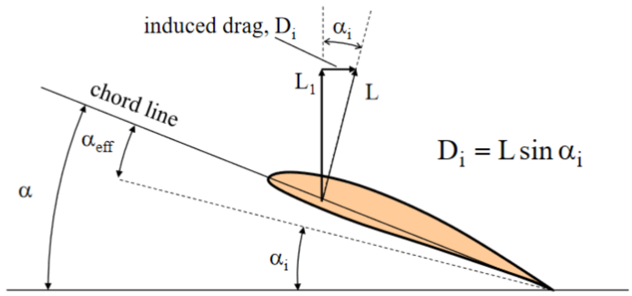


Figure 3.29: *A Tip Vortex in Action*

A trailing tip vortex will cause a downwash w behind the entire wing. Which will mean that the angle of attack α, is actually changed a bit. Thus we get the induced angle of attack α_i. This will mean that the lift is actually rotated back a bit, and the component of this vector in the horizontal direction is called the induced drag.



Since α is small : $\sin \alpha_i \approx \alpha_i$ => $\boxed{D_i = L.\alpha_i}$

$C_{D_i} = C_L \alpha_i$

Figure 3.30: *Induced drag along a wing*

We're going to start working our way to defining what the induced drag coefficient is.

$$C_{D_i} = C_L \alpha_i \tag{3.47}$$

In an ideal circumstance, an aircraft will have an elliptical lift distribution. This is the case on the spitfire from WWII. When this is the case, the induced angle of attack is :

$$\alpha_i = \frac{C_L}{\pi A}$$

where A is the aspect ratio of the wings ($\frac{b^2}{S}$). With this knowledge we can say that the induced drag coefficient of an elliptical wing, is:

$$C_{D_i} = \frac{C_L^2}{\pi A} \tag{3.48}$$

But if we want to find this for something other than an elliptical wing, we include a span efficiency factor e_1, which compares the efficiency of a wing to that of an elliptical wing. This is usually less than 1, and for an elliptical wing, it is 1. Combining these all together we can find the Drag coefficient of a **wing** to be:

$$C_D = C_d + \frac{C_L^2}{\pi A e_1} \tag{3.49}$$

This is for a wing. **A Wing** you hear me? For a complete aircraft we use the Oswald efficiency factor, and we use the zero lift drag C_{D_0}. Thus:

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

3.15. Lift Curve Slope

Because of the downwash acting on a wing, a finite wing will have a lower lift curve slope than an infinite wing. So the effective angle of attack, is the angle of attack minus the induced angle of attack.

$$\alpha_{eff} = \alpha - \alpha_i \tag{3.50}$$

For a wing of a general plane we can write:

$$\alpha_i = \frac{C_L}{\pi A e_1} rads \; = \frac{57.3 C_L}{\pi A e_1} \; degrees \tag{3.51}$$

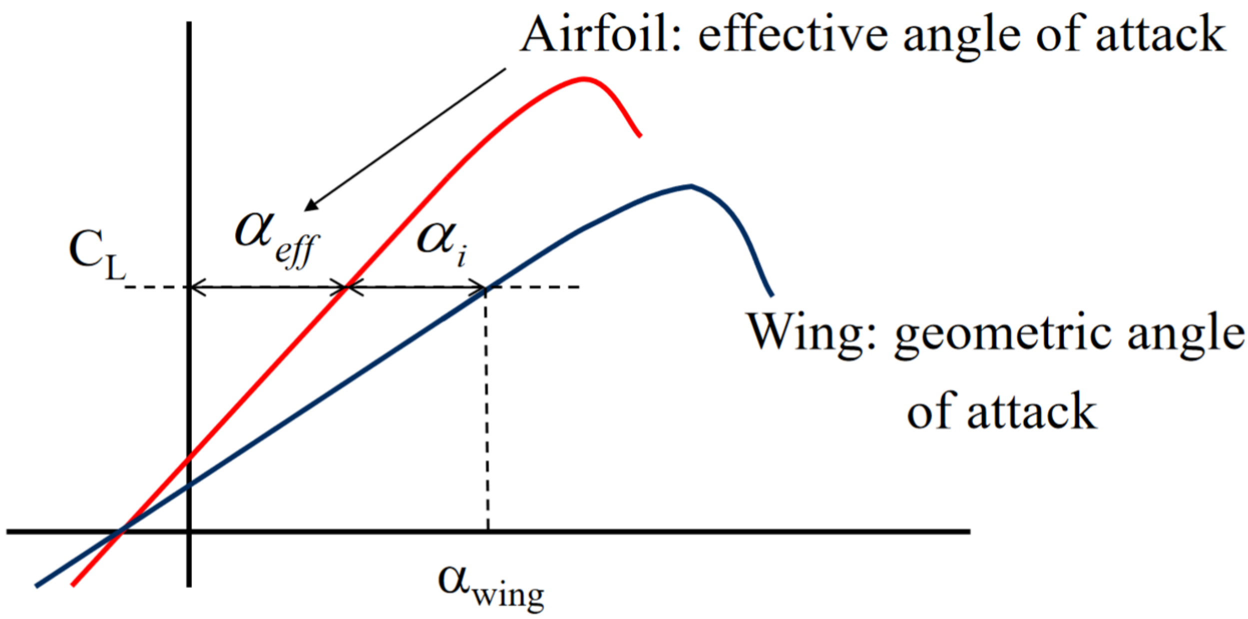


Figure 3.31: *Lift curve, a_0, for an airfoil and, a, for a wing*

From thin airfoil theory we know that $a_0 = 2\pi$ per radian. And $a_0 = \frac{dC_l}{d\alpha}$. From these we can conclude that the lift curve slop for a wing is:

$$a = \frac{dC_L}{d\alpha} = \frac{a_0}{1 + \frac{a_0}{\pi A e_1}} \tag{3.52}$$

This will give us the lift curve for a wing in radians.

Lastly, let's just take a moment to look at the total drag on a wing:

$$C_D = C_d + \frac{C_L^2}{\pi A e_1}$$

But what is contained within that C_d? that's the effects of skin friction, pressure drag, and shockwaves all added together. Not downwash, because this is an infinite wing.

$$C_d = C_{d_f} + C_{d_p} + C_{d_w} \tag{3.53}$$

4

Flight Performance

4.1. Diagrams

Some things you'll need to reproduce an awful lot in the flight mechanics section are diagrams. Specifically the free body diagram, and the kinetic diagram of an aircraft flying through the air. In this course we assume that the earth is flat, and that gravity is constant. This is of course not true, but the effects are negligible and it makes our calculations much easier.

We're going to start with the free body diagram, I've reduced down the aircraft to just its centre of mass because we don't really care about any of the other forces acting on it for now, It also sames you a lot of time in the exam drawing the centre of mass instead of a whole aircraft.

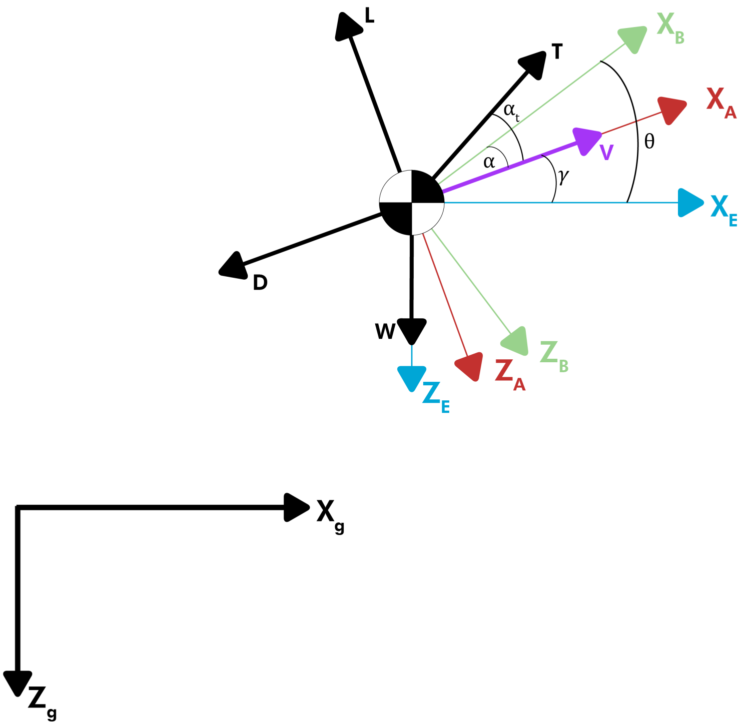


Figure 4.1: *The FBD of an aircraft with all axes*

This Diagram contains the 4 main forces acting on an aircraft, Lift, Drag, Thrust, and weight. and the respective angles. We've also included the axes as they are relevant too. The Moving Earth axis, which is related to the horizon. The Air path axis, which is related to the direction of flight, and the Body axis which is related to the body of the aircraft. And then of course, there is the ground axis, which isn't pinned to the craft.

The angle α_t is the Thrust angle of attack, because sometimes this isn't aligned with the aircraft body. The angle γ is the flight path angle.

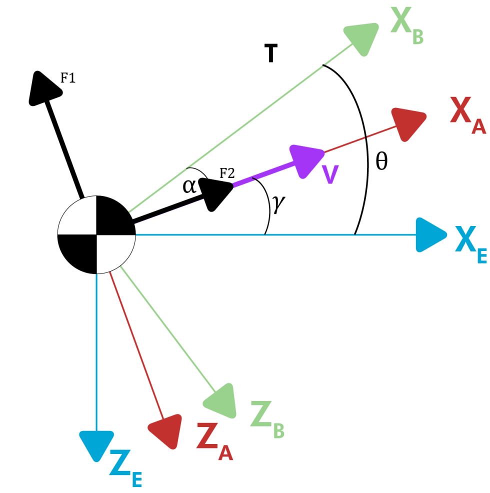


Figure 4.2: *The Kinetic Diagram of an aircraft with all axes*

The kinetic diagram shows us the accelerations of an aircraft. I've labeled them as $F1$ and $F2$ in the diagram but they're usually written as $F1 = mV\frac{d\gamma}{dt}$. This is the acceleration in the vertical direction. And $F2 = m\frac{dV}{dt}$ we can also write the mass as the weight divided by the force of gravity, giving us our equations of motion:

$$\Sigma F_{||V} : T\cos\alpha_t - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt} \tag{4.1}$$

$$\Sigma F_{\perp V} : L - W\cos\gamma + T\sin\alpha_t = \frac{W}{g}V\frac{d\gamma}{dt} \tag{4.2}$$

These come from the standard equation: $F = ma$, and are going to be used throughout the course. But can we solve these equations? Yes, but only if we have at most two unknowns. We can describe the variables in these equations as independent (t), state(γ,V), or other(T,L,W,D,α_t). We want to be able to express everything in terms of the state and independent variables.

4.2. The Drag Polar

From the aerodynamics section you know that the coefficient of drag is:

$C_D = C_{d_0} + \frac{CL^2}{\pi A \Phi}$

Where Φ is the span efficiency factor. But often we want to be more accurate, so instead of a one term drag polar like this, we instead use a *two term drag polar* in the form:

$$C_D = C_{D_0} + k_1 C_L + k_2 C_L^2 \tag{4.3}$$

If we for now assume that Lift is equal to Weight (which is usually the case) we can do some derivations. From the lift equation (**??**) we can find the coefficient of lift to be:

$$C_L = \frac{W}{S}\frac{2}{\rho}\frac{1}{V^2} \tag{4.4}$$

Using this and the drag equation (1.17), and a two term drag polar we find that Drag is a composite function. Where one part of it is dependent on v^2 and another part of it is dependent on $\frac{1}{V^2}$

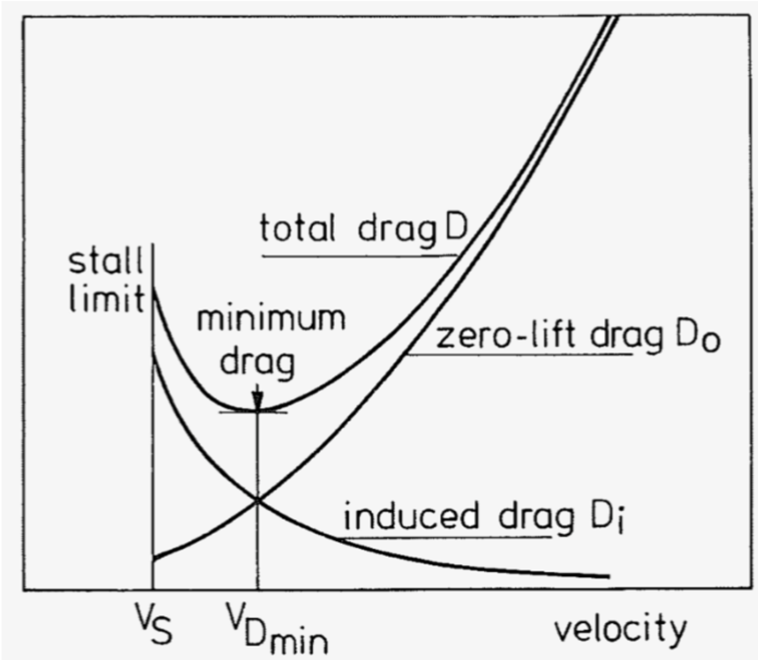


Figure 4.3: *Drag vs Speed*

Interestingly the drag is lowest when you speed up a little bit. Wild isn't it? anyway, with this information you can see that if you fly slowly most of your drag comes from the induced drag, and to minimise this you want a large aspect ratio. If you fly fast, most of the drag is from your zero lift drag, so you want small wings and a low aspect ratio.

4.3. Propulsion

From earlier we know that Thrust is related to the massflow of air, and the change in velocity of the air.

$$T \approx \dot{m}(V_j - V - 0)$$

It's only approximately equal, because we also exhaust the fuel, but that's a negligible amount of mass. Because the environment is a thing we want to discuss propulsive efficiency. Recall that work is force by distance, Power is work per unit time, and jet power is a function of massflow.

$$W = T\Delta x$$

$$P_a = TV$$

$$P_j = \frac{1}{2}\dot{m}(V_j^2 - V_0^2)$$

We'll also introduce a new characteristic H, which is the amount of energy per unit of fuel. If we want to know how much energy we're using we need to use this value, along with the amount of fuel we're using per unit time to get:

$$Q = \dot{m}_f H \tag{4.5}$$

When talking about efficiency we can say that the total efficiency is:

$$\eta_t = \frac{P_a}{Q} \tag{4.6}$$

Or we can rewrite this using Thermal and propulsive efficiency:

$$\eta_t = \eta_{th} \cdot \eta_j \tag{4.7}$$

where thermal efficiency is:

$$\eta_{th} = \frac{P_j}{Q} \tag{4.8}$$

And Propulsive efficiency is:

$$\eta_j = \frac{P_a}{P_j} = \frac{2}{1 + \frac{V_j}{V_0}} \tag{4.9}$$

A jet aircraft has *basically* constant thrust with increasing airspeed, whereas a propeller aircraft, has constant power with increasing airspeed, due to the fact that you can rotate propeller to change the efficiency. We can plot the Thrust and power of jets and propeller aircraft on force and power diagrams and we find something like this:

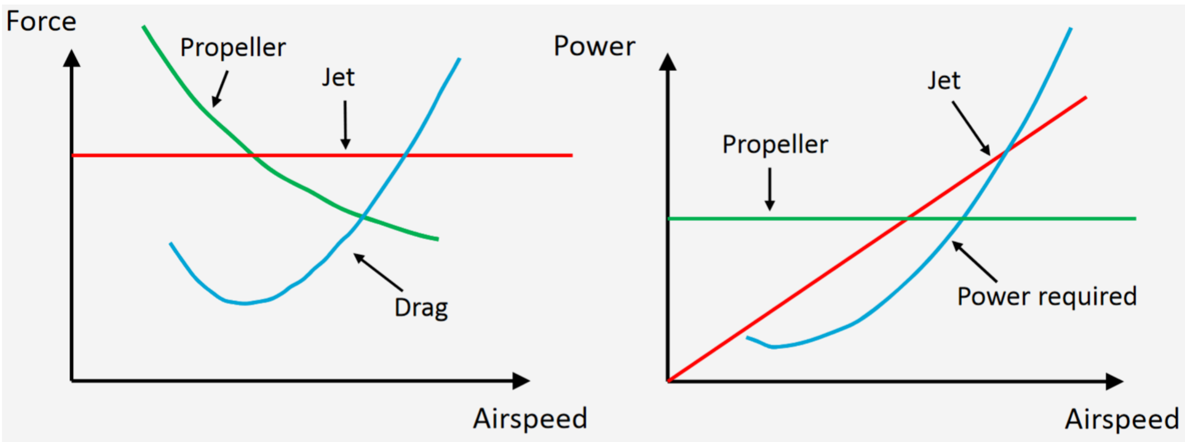


Figure 4.4: *Performance Diagrams*

4.4. Horizontal Flight Performance

Most of a flight is spent in cruise, which is, for the most part, is just horizontal flight. Let's start with our equations (4.1 & 4.2) and make a few assumptions.

$$\Sigma F_{||V} : T\cos\alpha_t - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

$$\Sigma F_{\perp V} : L - W\cos\gamma + T\sin\alpha_t = \frac{W}{g}V\frac{d\gamma}{dt}$$

- **Steady Flight**: We assume that the aircraft is not accelerating and thus: $\frac{dV}{dt} = 0$
- **Horizontal Flight**: The aircraft is not increasing altitude, and so $\gamma = 0$
- **Symmetric:** The aircraft is not turning so $\beta = 0$
- **Thrust is is aligned with the body**: The aircraft isn't a niche design so: $\alpha_t = 0$

From this, we can can reduce our equations down to what we had earlier on in the course (1.14 & 1.15)

$$L = W$$

$$T = D$$

With this information and the lift equation (1.16) we can find the useful equation for speed.

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} \tag{4.10}$$

To find the minimum speed we can fly with this equation, we simply plug in the maximum coefficient of lift. Everything else is a constant. If you try to fly any slower, you won't be able to generate enough lift. To calculate *maximum* speed just use the relations $T = D$ and $P_a = P_r$.

4.4.1. Range

Generally speaking we want to fly as far as possible with the amount of fuel we have. Or in other words, we want to maximise the meters we can fly per kilogram of fuel. To do so we want to optimise the specific range by flying at the point of minimum drag. We'll call F the fuel flow which is the kilograms of fuel used per second. This, the specific range is:

$$\frac{V}{F} = [\frac{ms^{-1}}{kgs^{-1}}] = [\frac{m}{kg}] \tag{4.11}$$

But how do we actually calculate this? Well, the fuel flow is the specific fuel consumption c_p (what, *another* cp? How delightful) combined with the breakshaft power. and we know that the power available is the breakshaft power multiplied by the propulsive efficiency. so:

$$F = c_p\frac{P_a}{\eta_j} \tag{4.12}$$

And because we know that the power available is directly relate to the thrust and velocity, we can now write this as:

$$\frac{V}{F} = \frac{\eta_j}{c_p \cdot D} \tag{4.13}$$

We will assume that the efficiency and the specific fuel consumption are constant (because we are only looking at one instant in time). This clearly shows us that that to maximise our range, we must fly at the point of minimum drag. On our force diagram this is easy to find, but in our power diagram we need to find the tangent with the lowest slope. This is because the slope of the tangent will be $\frac{DV}{V}_{min}$

To actually find this analytically we need to do some maths (scary!)

$$D = D\frac{L}{L}$$

$$D = \frac{D}{L}L$$

$$D = \frac{C_D}{C_L}W$$

$$D_{min} \Rightarrow (\frac{C_D}{C_L})_{min} \Rightarrow (\frac{C_L}{C_D})_{max}$$

To mind a maximum (or minimum) we want to differentiate with respect to C_L and find the point where it is zero. Working with a two term drag polar we can divide by the C_L and then differentiate.

$$\frac{d}{dC_L}\frac{C_{D_0}}{C_L} + k_1 + k_2C_L$$

$$= -\frac{C_{D_0}}{C_L^2} + k_2 = 0$$

$$k_2 = \frac{C_{D_0}}{C_L^2}0$$

$$C_L = \sqrt{\frac{C_{D_0}}{k_2}}$$

If you want to work out what speed you need to fly at for this: use equation (4.10). If you change the weight, the C_L wont change, but the optimal speed to fly at will. As you fly you use up fuel, and so, the weight decreases, and so the optimal speed, in turn decreases.

4.4.2. Endurance

Endurance is when we want to stay flying for the longest amount of time, not that we want to fly the maximum distance. For Maximum endurance we want to use the least amount of fuel per time. And so this requires minimum fuel flow. So with the knowledge that $P_a = P_r$ for steady flight, and with equation (4.12) we can find:

$$F_{min} = (\frac{c_p \cdot P_r}{\eta_j})_{min} = (\frac{c_p \cdot D \cdot V}{\eta_j})_{min} \tag{4.14}$$

Thus, for minimum fuel flow we want to find the minimum power required, or the minimum $C \cdot V$. this is because for the same reason as before c_p and η_j are constant. To find this point, we're going to do more scary maths.

$$P_r = DV$$

$$P_r = D\frac{L}{L}V$$

$$P_r = \frac{D}{L}WV$$

From equation (4.10):

$$P_r = W\sqrt{\frac{C_D^2}{C_L^3}\frac{W^3}{S}\frac{2}{\rho}}$$

Disregarding the constants, we find that for minimum P_r, and maximum endurance: we need:

$$(\frac{C_D^2}{C_L^3})_{min}$$

Just like before when we were calculating the maximum range, we want to differentiate and find when this is equal to zero.

$$\frac{3C_L^2C_D^2 - 2C_DC_L^3\frac{dC_D}{dC_L}}{C_D^4} = 0$$

$$\Rightarrow\ 3C_D - 2C_L\frac{dC_D}{dC_L} = 0$$

$$\Rightarrow\ \frac{3}{2}\frac{C_D}{C_L} = \frac{dC_D}{dC_L}$$

With this information we can use the drag polar to find:

$$-k_2C_L^2 + k_1C_L + 3C_D = 0 \tag{4.15}$$

This is a quadratic equation that you can solve for. You'll get two results for C_L but you need to use your brain to notice that one of them will be bogus and the other one will make sense. As per usual, you can use equation (4.10) to find the optimal speed for maximum endurance.

4.4.3. Speed Stability

We like to make assumptions when we do our calculations for horizontal flight, but there's really no such thing as perfectly steady flight. What about gusts of wind or other disturbances. Well let's take a look at our equation of motion for horizontal flight (4.1):

$$\Sigma F_{||V} : T\cos\alpha_t - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

If we assume horizontal flight, but we don't assume the acceleration is 0, then we find:

$$T - D = \frac{dV}{dt} \tag{4.16}$$

This means that the acceleration you experience is the difference between the thrust and drag.

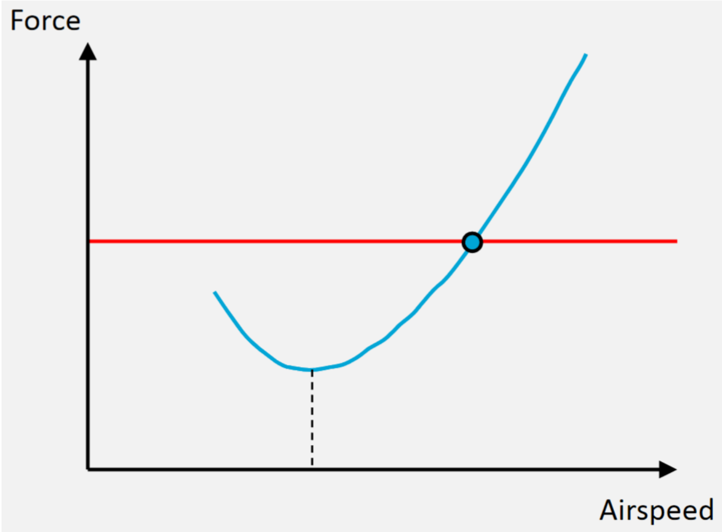


Figure 4.5: *Flying at a particular position.*

The red line shows us the thrust we can fly at (at a given throttle setting) and the blue line shows us the drag for different airspeeds. If a headwind comes along, this will reduce the airspeed, moving the blue dot backward, and it will reduce the drag. In this instance $T - D > 0$ so the aircraft will compensate for slowing down by speeding back uup due to excess speed.

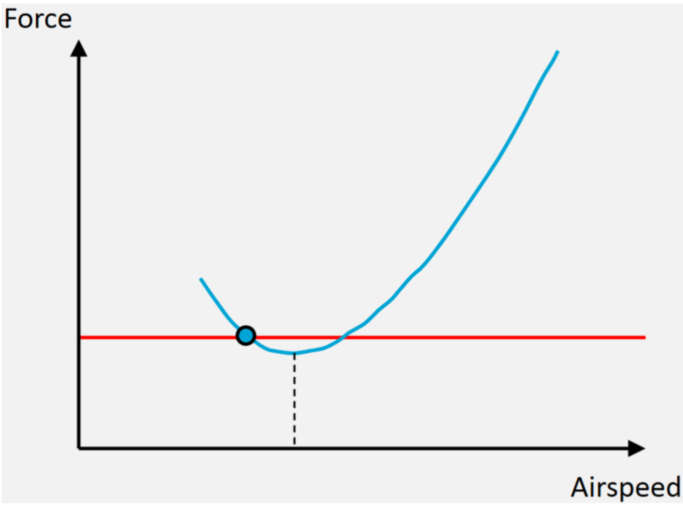


Figure 4.6: *Flying at a different position.*

Consider the case where you're already flying slowly. If a headwind gust disturbs the plane making it slow down, the blue dot will move backwards, but, in this instance, the drag will increase, meaning that $T - D < 0$, and so the plane will continue to slow down more. This is why when you fly on the back side of the power curve you have speed instability, so it's a good idea to avoid there.

4.5. Climbing Flight

4.5.1. Maximum Climb Angle

when we discuss climbing flight we make some assumptions. We assume it's steady and straight flight, but we don't assume it's horizontal (obviously). Thus: $\frac{dV}{dt} = 0$, $\frac{d\gamma}{dt} = 0$, but

$\gamma \neq 0$. Using our equations of motion (4.1 & 4.2) we can derive:

$$\Sigma F_{||V} : T \cos \alpha_t - D - W \sin \gamma = \frac{W}{g} \frac{dV}{dt} :$$

$$T - D = W \sin \gamma \tag{4.17}$$

$$\Sigma F_{\perp V} : L - W \cos \gamma + T \sin \alpha_t = \frac{W}{g} V \frac{d\gamma}{dt} :$$

$$L = W \cos \gamma \tag{4.18}$$

But often we can say that $\cos \gamma$ is 1, and that $\sin \gamma \approx \gamma$ because we seldom exceed 15°. from this we can see that:

$$\sin \gamma = \frac{T - D}{W} \tag{4.19}$$

If we multiply this by the speed, we can see that the rate of climb (ROC) is

$$V \sin \gamma = \frac{P_r - P_a}{W} = ROC \tag{4.20}$$

If we want to find the maximum climb angle, you need to find $(T - D)_{max}$ and for jets, this happens at D_{min} and luckily enough, we already derived that to find the maximum range. $C_{L_{opt}} = \sqrt{\frac{C_{D_0}}{k_2}}$

4.5.2. Gliding flight

When we have gliding flight, we have no thrust, and so to have the minimum descent angle, you want $-D$ to be as large as possible. Which just so happens at the same point of D_{min}. We also use $\overline{\gamma}$ for the descent angle so

$$\sin \overline{\gamma} = \frac{D}{W} \tag{4.21}$$

$$\overline{\gamma} = \arcsin \frac{C_D}{C_L} \tag{4.22}$$

Interestingly if you increase the weight of the aircraft you won't actually change the descent angle, you just make it descend faster.

4.5.3. Maximum Rate of Climb

As we saw earlier, the rate of climb is the excess power over the weight. eq(4.20). So in order to find the Maximum rate of climb we need to find $(P_r - P_a)_{max}$ Which is again, the same derivation we found for getting the maximum endurance. $-k_2 C_L^2 + k_1 C_L + 3C_D = 0$

4.6. Performance Diagrams

4.6.1. Performance Limits

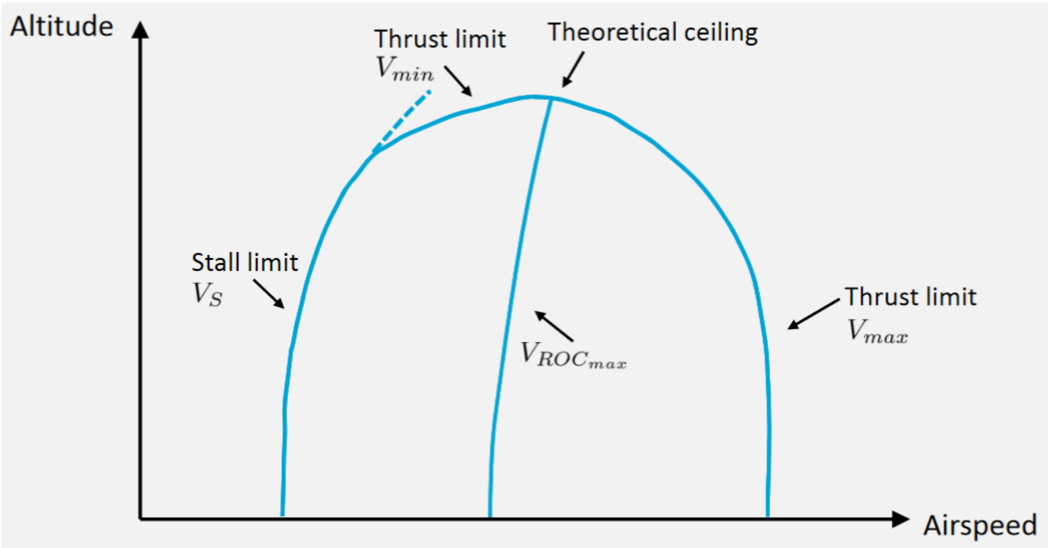


Figure 4.7: *Limits of how an aircraft can fly.*

Aircraft are impacted by flying at different altitudes, and that's how we get diagrams like this. Recall equation (4.10), as we increase our altitude the air density decreases, and so as such, our minimum airspeed must increase to compensate.

As we increase in altitude, we are also limited by thrust because the massflow decreases due to a lower air density. We can increase the difference between the air intake velocity and the exhaust velocity but not enough to compensate.

$$\frac{T}{T_0} = \frac{\rho}{\rho_0} \tag{4.23}$$

we find our theoretical ceiling at the point where $ROC = 0$ or where $P_a = P_r$. We call this the theoretical ceiling because in reality it's really hard to get there, because as you near it the rate of climb slows and it's harder to go higher. So in actuality we have a service ceiling that's a bit lower than that, usually at the point where ROC is ≈ 500 ft/minute.

To flight at a high altitude you need to have a high aspect ratio, a low zero lift drag, a low weight, a high maximum thrust, and a high efficiency factor.

4.6.2. Operational Limits

If you remember from the structures section, we have something called a load factor n which is the lift divided by the weight (2.2). We combine this and the equivalent airspeed (so we don't need a different chart for each altitude)

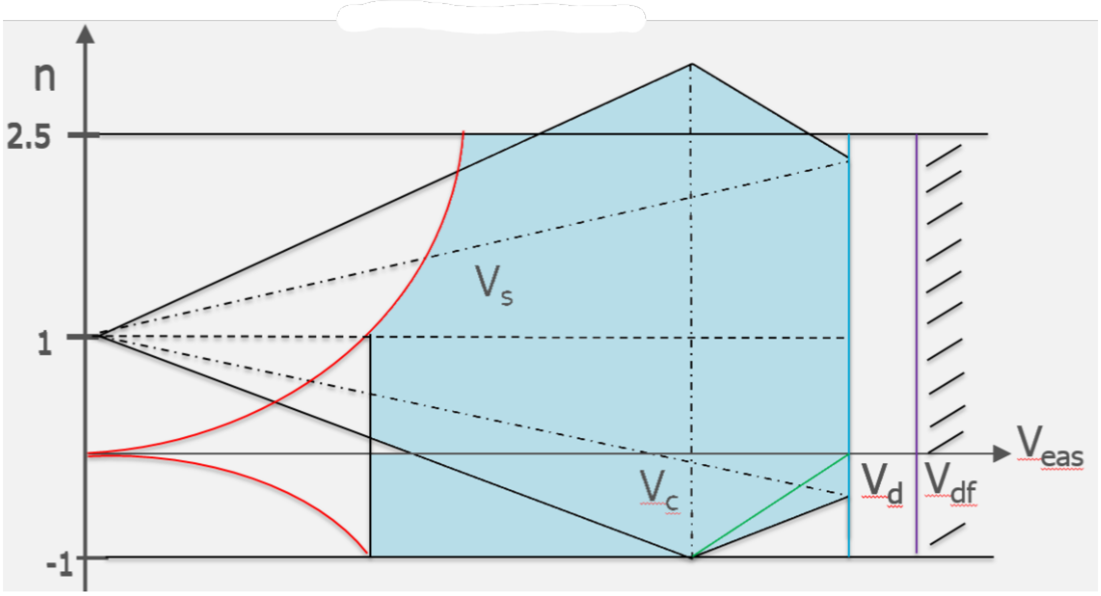


Figure 4.8: *Where an aircraft is allowed to fly.*