2 Of particular interest is that the exit velocity is increased by a very small amount, namely by only 61 m/sec, although the exit Mach number has been doubled. The higher Mach number of 20 is achieved not by a large increase in exit velocity by rather by a large decrease in the speed of sound at the exit. This is characteristic of most conventional hypersonic wind tunnels -- the higher Mach numbers are not associated with corresponding increases in the test section flow velocities

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5.1 Assume the moment is governed by

$$M = f(V_\infty, \rho_\infty, S, \mu_\infty, a_\infty)$$

More specifically:

$$M = Z\, V_\infty^{\,a}\, \rho_\infty^{\,b}\, S^d\, a_\infty^{\,e}\, \mu_\infty^{\,f}$$

Equating the dimensions of mass, m, length, ℓ, and time t, and considering Z dimensionless,

$$\frac{m\ell^2}{t^2} = \left(\frac{\ell}{t}\right)^a \left(\frac{m}{\ell^3}\right)^b \left(\ell^2\right)^d \left(\frac{\ell}{t}\right)^e \left(\frac{m}{\ell^t}\right)^f$$

$1 = b + f$ (For mass)

$2 = a - 3b + 2d + e - f$ (For length)

$-2 = -a-e-f$ (for time)

Solving a, b, and d in terms of e and f,

$b = 1 - f$

and, $a = 2 - e - f$

and, $2 = 2 - e - f - 3 + 3f + 2d + e - f$

or $0 = -3 + f + 2d$

$$d = \frac{3-f}{2}$$

Hence,

$$M = Z\, V_\infty^{2-e-f}\, \rho_\infty^{1-f}\, S^{(3-f)/2}\, a_\infty^{e}\, \mu_\infty^{f}$$

$$= Z\, V_\infty^2\, \rho_\infty\, S S^{1/2} \left(\frac{a_\infty}{V_\infty}\right)^e \left(\frac{\mu_\infty}{V_\infty \rho_\infty S^{1/2}}\right)^f$$

Note that $S^{1/2}$ is a characteristic length; denote it by the chord, c

$$M = \rho_\infty\, V_\infty^2\, S\, c\, Z \left(\frac{a_\infty}{V_\infty}\right)^e \left(\frac{\mu_\infty}{V_\infty \rho_\infty c}\right)^f$$

However, $a_\infty / V_\infty = 1/M_\infty$

and $\dfrac{\mu_\infty}{V_\infty \rho_\infty c} = \dfrac{1}{Re}$

Let

$$Z \left(\frac{1}{M_\infty}\right)^e \left(\frac{1}{Re}\right)^f = \frac{c_m}{2}$$

where c_m is the moment coefficient Then, as was to be derived, we have

$$M = \frac{1}{2} \rho_\infty\, V_\infty^2\, c\, c_m$$

or, $M = q_\infty\, S\, c\, c_m$

5.2 From Appendix D, at 5° angle of attack,

$c_\ell = 0.67$

$c_{m_{c/4}} = -0.025$

(Note: Two sets of lift and moment coefficient data are given for the NACA 1412 airfoil -- with and without flap deflection. Make certain to read the code properly, and use only the unflapped data, as given above. Also, note that the scale for $c_{m_{c/4}}$ is different than that for c_ℓ -- be careful in reading the data.)

With regard to c_d, first check the Reynolds number,

$$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(0.002377)(100)(3)}{(3.7373 \times 10^{-7})}$$

$$Re = 1.9 \times 10^6$$

In the airfoil data, the closest Re is 3×10^6. Use c_d for this value.

$c_d = 0.007$ (for $c_\ell = 0.67$)

The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(100)^2 = 11.9 \text{ lb/ft}^2$$

The area per unit span is $S = 1(c) = (1)(3) = 3 \text{ ft}^2$

Hence, per unit span,

$$L = q_\infty S c_\ell = (11.9)(3)(0.67) = \boxed{23.9 \text{ lb}}$$

$$D = q_\infty S c_d = (11.9)(3)(0.007) = \boxed{0.25 \text{ lb}}$$

$$M_{c/4} = q_\infty S c\, c_{m_{c/4}} = (11.9)(3)(3)(-0.025) = \boxed{-2.68 \text{ ft.lb}}$$

5.3 $\rho_\infty = \dfrac{p_\infty}{RT_\infty} = \dfrac{(1.01 \times 10^5)}{(287)(303)} = 1.61 \text{ kg/m}^3$

From Appendix D,

$c_\ell = 0.98$

$c_{m_{c/4}} = -0.012$

Checking the Reynolds number, using the viscosity coefficient from the curve given in Chapter 4,

$\mu_\infty = 1.82 \times 10^{-5}$ kg/m sec at T = 303K,

$$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(1.157)(42)(0.3)}{182 \times 10^{-5}} = 8 \times 10^5$$

This Reynolds number is considerably less than the lowest value of 3×10^6 for which data is given for the NACA 23012 airfoil in Appendix D. Hence, we can use this data only to give an educated guess; use

$c_d \approx 0.01$, which is about 10 percent higher than the value of 0.009 given for Re = 3×10^6

The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.161)(42)^2 = 1024 \text{ N/m}^2$$

The area per unit span is S = (1)(0.3) = 0.3 m^2. Hence,

$L = q_\infty S c_\ell = (1024)(0.3)(0.98) = \boxed{301N}$

$D = q_\infty S c_d = (1024)(0.3)(0.01) = \boxed{3.07N}$

$M_{c/4} = q_\infty S c c_m = (1024)(0.3)(0.3)(-0.012) = \boxed{-1.1Nm}$

5.4 From the previous problem, $q_\infty = 1020 \text{ N/m}^2$

$L = q_\infty S c_\ell$

Hence,

$$c_\ell = \frac{L}{q_\infty S}$$

The wing area $S = (2)(0.3) = 0.6 \text{ m}^2$

Hence,

$$c_\ell = \frac{200}{(1024)(0.6)} = 0.33$$

From Appendix D, the angle of attack which corresponds to this lift coefficient is

$$\boxed{\alpha = 2^\circ}$$

5.5 From Appendix D, at $\alpha = 4^\circ$,

$c_\ell = 0.4$

Also, $V_\infty = 120 \left(\frac{88}{60}\right) = 176 \text{ ft/sec}$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(176)^2 = 36.8 \text{ lb/ft}^2$$

$L = q_\infty S c_\ell$

$$S = \frac{L}{q_\infty c_\ell} = \frac{29.5}{(36.8)(0.4)} = \boxed{2 \text{ ft}^2}$$

5.6 $L = q_\infty S c_\ell$

$D = q_\infty S c_d$

Hence,

$$\frac{L}{D} = \frac{q_\infty S\ c_\ell}{q_\infty S\ c_d} = \frac{c_\ell}{c_d}$$

We must tabulate the values of c_ℓ/c_d for various angles of attack, and find where the maximum occurs. For example, from Appendix D, at $\alpha = 0°$,

$$c_\ell = 0.25$$

$$c_d = 0.006$$

Hence

$$\frac{L}{D} = \frac{c_\ell}{c_d} = \frac{0.25}{0.006} = 41.7$$

A tabulation follows

α	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
c_ℓ	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.05	1.15
c_d	0.006	0.006	0.006	0.0065	0.0072	0.0075	0.008	0.0085	0.0095	0.0105
$\frac{c_\ell}{c_d}$	41.7	58.3	75	84.6	90.3	100	106	112	111	110

From the above tabulation,

$$\left(\frac{L}{D}\right)_{max} \approx \boxed{112}$$

5.7 At sea level

$$\rho_\infty = 1.225\ kg/m^3$$

$p_\infty = 1.01 \times 10^5$ N/m^2

Hence,

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(50)^2 = 1531 \text{ N/m}^2$$

From the definition of pressure coefficient,

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{(0.95 - 1.01) \times 10^5}{1531} = \boxed{-3.91}$$

5.8 The speed is low enough that incompressible flow can be assumed From Bernoulli's equation,

$$p + \frac{1}{2}\rho V_\infty^2 = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2 = p_\infty + q_\infty$$

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{q_\infty - \frac{1}{2}\rho V^2}{q_\infty} = 1 - \frac{\frac{1}{2}\rho V^2}{\frac{1}{2}\rho_\infty V_\infty^2}$$

Since $\rho = \rho_\infty$ (constant density)

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(\frac{62}{55}\right)^2 = 1 - 1.27 = \boxed{-0.27}$$

5.9 The flow is low speed, hence assumed to be incompressible From problem 5.8,

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(\frac{195}{160}\right)^2 = \boxed{-0.485}$$

5.10 The speed of sound is

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(510)} = 1107 \text{ ft/sec}$$

Hence,

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{700}{1107} = 0.63$$

In problem 5.9, the pressure coefficient at the given point was calculated as -0.485. However, the conditions of problem 5.9 were low speed, hence we identify

$$C_{p_o} = -0.485$$

At the new, higher free stream velocity, the pressure coefficient must be corrected for compressibility. Using the Prandtl-Glauert Rule, the high speed pressure coefficient is

$$C_p = \frac{C_{p_o}}{\sqrt{1 - M_\infty^{\ 2}}} = \frac{-0.485}{\sqrt{1-(0.63)^2}} = \boxed{-0.625}$$

5.11 The formula derived in problem 5.8, namely

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2,$$

utilized Bernoulli's equation in the derivation. Hence, it is <u>not valid</u> for compressible flow. In the present problem, check the Mach number.

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(505)} = 1101 \text{ ft/sec}$$

$$M_\infty = \frac{780}{1101} = 0.708$$

The flow is clearly compressible! To obtain the pressure coefficient, first calculate ρ_∞ from the equation of state.

$$\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{2116}{(1716)(505)} = 0.00244 \text{ slug/ft}^3$$

To find the pressure at the point on the wing where V = 850 ft/sec, first find the temperature from the energy equation

$$c_p\,T + \frac{V^2}{V} = c_p T_\infty + \frac{V_\infty^{\ 2}}{2}$$

$$T = T_\infty + \frac{V_\infty^{\ 2} - V^2}{2c_p}$$

The specific heat at constant pressure for air is

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(1716)}{(14 - 1)} = 6006 \frac{\text{ft lb}}{\text{slug R}}$$

Hence,

$$T = 505 + \frac{780^2 - 850^2}{2(6006)} = 505 - 9\ 5 = 495\ 5\text{R}$$

Assuming isentropic flow

$$\frac{p}{p} = \left(\frac{T}{T_\infty}\right)^{\frac{\gamma}{\gamma-1}}$$

$$p = (2116)\left(\frac{495.5}{505}\right)^{35} = 1980 \text{ lb/ft}^2$$

From the definition of C_p

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}} = \frac{1980 - 2116}{\frac{1}{2}(0\ 00244)(780)^2}$$

$$C_p = \boxed{-0.183}$$

5.12 A velocity of 100 ft/sec is low speed. Hence, the desired pressure coefficient is a low speed value, C_{p_o}.

$$C_p = \frac{C_{p_o}}{\sqrt{1 - M_\infty^2}}$$

From problem 5 11,

$$C_p = -0.183 \text{ and } M_\infty = 0.708. \text{ Thus, } 0.183 = \frac{C_{p_o}}{\sqrt{1-(0.708)^2}}$$

$$C_{p_o} = (-0.183)(0.706) = \boxed{-0.129}$$

5.13 Recall that the airfoil data in Appendix D is for low speeds. Hence, at $\alpha = 4^\circ$, c_{ℓ_o} = 0.58

Thus, from the Prandtl-Glauert rule,

$$c_\ell = \frac{c_{\ell_o}}{\sqrt{1 - M_\infty^2}} = \frac{0.58}{\sqrt{1-(0.8)^2}} = \boxed{0.97}$$

5.14 The lift coefficient measured is the high speed value, c_ℓ. Its low speed counterpart is c_{ℓ_o}, where

$$c_\ell = \frac{c_{\ell_o}}{\sqrt{1 - M_\infty^2}}$$

Hence,

$$c_{\ell_o} = (0.85)\sqrt{1-(0.7)^2} = 0.607$$

For this value, the low speed data in Appendix D yield

$$\boxed{\alpha = 2^{\circ}}$$

5.15 First, obtain a curve of $C_{p,cr}$ versus M_∞ from

$$C_{p,cr} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{2+(\gamma-1)M_\infty^2}{\gamma+1}\right)^{\gamma/(\gamma-1)} - 1\right]$$

Some values are tabulated below for $\gamma = 1.4$.

M_∞	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$C_{p,cr}$	-3.66	-2.13	-1.29	-0.779	-0.435	-0.188	0

Now, obtain the variation of the minimum pressure coefficient, C_p, with M_∞, where C_{p_o} = -0.90 From the Prandtl-Glauert rule,

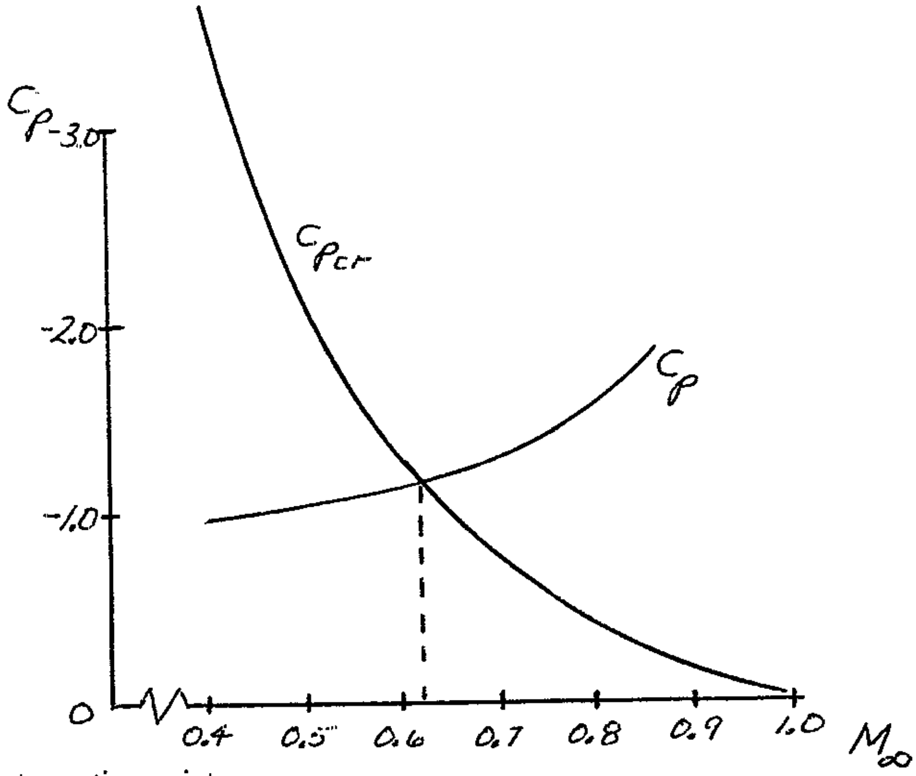
$$C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^2}}$$

$$C_p = \frac{-0.90}{\sqrt{1-M_\infty^2}}$$

Some tabulated values are:

M_∞	0.4	0.5	0.6	0.7	0.8	0.9
C_p	-0.98	-1.04	-1.125	-1.26	-1.5	-2.06

A plot of the two curves is given on the next page.



From the intersection point,

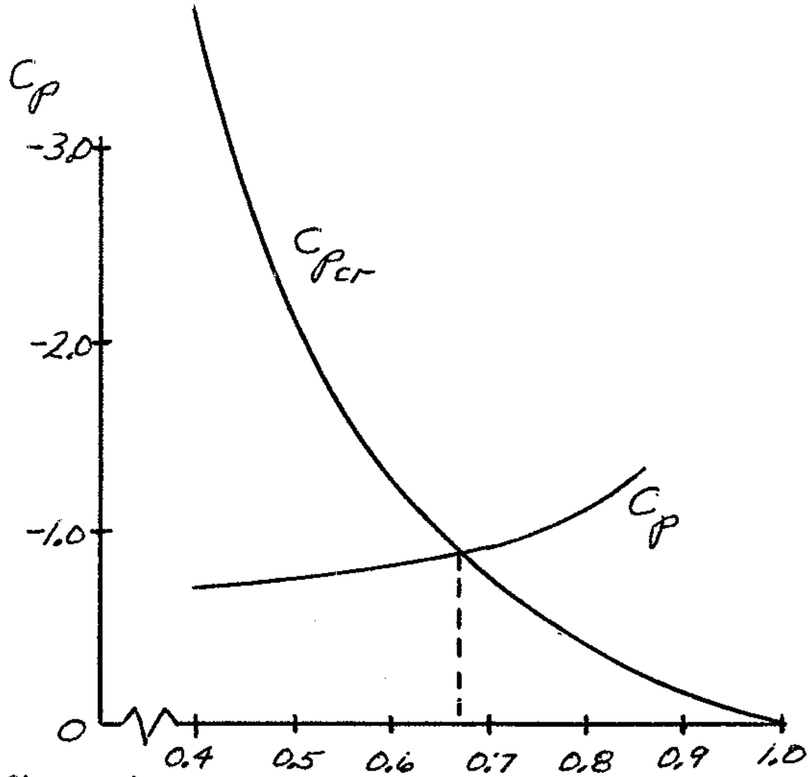
$$M_{cr} = \boxed{0.62}$$

5.16 The curve of $C_{p,cr}$ versus M_∞ has already been obtained in the previous problem; it is a universal curve, and hence can be used for this and all other problems. We simply have to obtain the variation of C_p with M_∞ from the Prandtl-Glauert rule, as follows:

$$C_p = \frac{C_{p_o}}{\sqrt{1 - M_\infty^{\,2}}} = \frac{-0.65}{\sqrt{1 - M_\infty^{\,2}}}$$

M_∞	0.4	0.5	0.6	0.7	0.8	0.9
C_p	-0.71	0.75	-0.81	-0.91	-1.08	-1.49

The results are plotted below.



From the point of intersection,

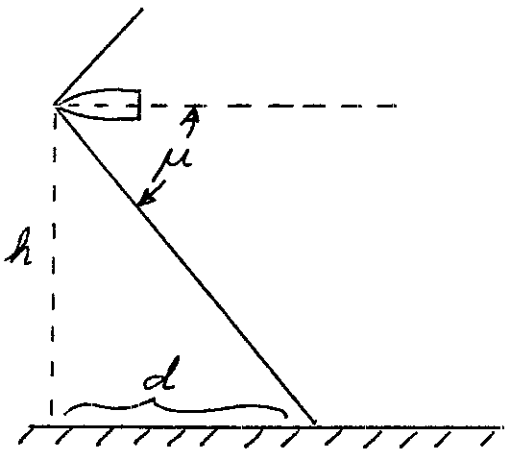
$M_{cr} = \boxed{0.68}$

Please note that, comparing problems 5.15 and 5.16, the critical Mach number for a given airfoil is somewhat dependent on angle of attack for the simple reason that the value of the minimum pressure coefficient is a function of angle of attack. When a critical Mach number is stated for a given airfoil in the literature, it is usually for a small (cruising) angle of attack.

5.17 Mach angle = μ = arc sin (1/M)

μ = arc sin (1/2)= $\boxed{30^\circ}$

5.18



$$\mu = \mathrm{Sin}^{-1}\left(\frac{1}{M}\right) = \mathrm{Sin}^{-1}\left(\frac{1}{2\ 5}\right) = 23\ 6^{\circ}$$

$$d = h/\mathrm{Tan}\mu = \frac{10\mathrm{km}}{0\ 436} = \boxed{22.9\ \mathrm{km}}$$

5.19 At 36,000 ft, from Appendix B,

$T_\infty = 390\ 5^{\circ}R$

$\rho_\infty = 7\ 1 \times 10^{-4}$ slug/ft^3

Hence,

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1\ 4)(1716)(390\ 5)} = 969 \ \ \mathrm{ft/sec}$$

$$V_\infty = a_\infty M_\infty = (969)(2\ 2) = 2132 \ \mathrm{ft/sec}$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^{\ 2} = \frac{1}{2}\ (7\ 1 \times 10^{-4})(2132)^2 = 1614 \ \mathrm{lb/ft^2}$$

In level flight, the airplane's lift must balance its weight, hence

$$L = W = 16{,}000 \ \mathrm{lb}$$

From the definition of lift coefficient,

$$C_L = L/q_\infty S = 16{,}000/(1614)(210) = 0\ 047$$

Assume that all the lift is derived from the wings (this is not really true because the fuselage and horizontal tail also contribute to the airplane lift.) Moreover, assume the wings can be approximated by a thin flat plate Hence, the lift coefficient is given approximately by

$$C_L = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

Solve for α,

$$\alpha = \frac{1}{4} C_L \sqrt{M_\infty^2 - 1} = \frac{1}{4}(0\ 047)\sqrt{(2\ 2)^2 - 1}$$

$\alpha = 0\ 023$ radians (or 1 2 degrees)

The wave drag coefficient is approximated by

$$C_{D_w} = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.023)^2}{\sqrt{(2\ 2)^2 - 1}} = 0\ 00108$$

Hence,

$$D_w = q_\infty\ S\ C_{D_w} = (1614)(210)(0\ 00108)$$

$$D_w = \boxed{366\ \text{lb}}$$

5.20 (a) At 50,000 ft, $\rho_\infty = 3\ 6391 \times 10^{-4}$ slug/ft^3 and $T_\infty = 390°R$ Hence,

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1\ 4)(1716)(390)} = 968\ \ \text{ft / sec}$$

and $V_\infty = a_\infty M_\infty = (968)(2\ 2) = 2130$ ft/sec

The viscosity coefficient at $T_\infty = 390°R = 216\ 7K$ can be estimated from an extrapolation of the straight line given in Fig 4 30. The slope of this line is

$$\frac{d\mu}{dT} = \frac{(2.12 - 1.54)\ x\ 10^{-5}}{(350 - 250)} = 5.8\ x\ 10^{-8} \frac{kg}{(m)(sec)(K)}$$

Extrapolating from the sea level value of $\mu = 1.7894 \times 10^{-5}$ kg/(m)(sec), we have at T_∞ =

216 7 K.

$$\mu_\infty = 1.7894 \times 10^{-5} - (5.8 \times 10^{-8})\ (288 - 216.7)$$

$$\mu_\infty = 1.37 \times 10^{-5}\ kg/(m)(sec)$$

Converting to english engineering units, using the information in Chapter 4, we have

$$\mu_\infty = \frac{1.37\ x\ 10^{-5}}{1.7894\ x\ 10^{-5}} (3.7373\ x\ 10^{-7} \frac{slug}{ft\ sec}) = 2.86\ x\ 10^{-7} \frac{slug}{ft\ sec}$$

Finally, we can calculate the Reynolds number for the flat plate:

$$Re_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{3.6391\ x\ 10^{-4} (2130)(202)}{2.86\ x\ 10^{-7}} = 5.47 \times 10^8$$

Thus, from Eq (4.100) reduced by 20 percent

$$C_f = (0.8)\ \frac{0.074}{(Re_L)^{0.2}} = (0.8) \frac{0.074}{(5.74\ x\ 10^8)^{0.2}} = 0.00106$$

The wave drag coefficient is estimated from

$$c_{d,w} = \frac{4\alpha^2}{\sqrt{M_\infty^{\ 2} - 1}}$$

where $\alpha = \dfrac{2}{57.3} = 0.035$ rad.

Thus,

$$c_{d,w} = \frac{4(0.035)^2}{\sqrt{(2.2)^2 - 1}} = 0.0025$$

Total drag coefficient = 0 0025 + (2)(0.00106) = $\boxed{0.00462}$

Note: In the above, C_f is multiplied by two, because Eq. (4.100) applied to only one side of the flat plate. In flight, both the top and bottom of the plate will experience skin friction, hence that total skin friction coefficient is 2(0.00106) = 0.00212.

(b) If α is increased to 5 degrees, where α = 5/57.3 - 0.00873 rad, then

$$c_{d,w} = \frac{4(0.0873)^2}{\sqrt{(2.2)^2 - 1}} = 0.01556$$

Total drag coefficient = 0.01556 + 2(0.00106) = $\boxed{0.0177}$

(c) In case (a) where the angle of attack is 2 degrees, the wave drag coefficient (0.0025) and the skin friction drag coefficient acting on both sides of the plate (2 x 0.00106 = 0.00212) are about the same. However, in case (b) where the angle of attack is higher, the wave drag coefficient (0.0177) is about eight times the total skin friction coefficient. This is because, as α increases, the strength of the leading edge shock increases rapidly. In this case, wave drag dominates the overall drag for the plate.

5.21 $$V_\infty = 251 \text{ km/h} = \left(251\frac{\text{km}}{\text{h}}\right)\left(\frac{1\text{h}}{3600\text{sec}}\right)\left(\frac{1000\text{m}}{1\text{km}}\right) = 69.7 \text{ m/sec}$$

$$\rho_\infty = 1.225 \text{ kg/m}^3$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(69.7)^2 = 2976 \text{ N/m}^2$$

$$C_L = \frac{L}{q_\infty S} = \frac{9800}{(2976)(16.2)} = 0.203$$

$$C_{D_i} = \frac{C_L^{\,2}}{\pi e AR} = \frac{(0.203)^2}{\pi(0.62)(7.31)} = 0.002894$$

$D_i = q_\infty S C_{D_i} = (2976)(16.2)(0.002894) = \boxed{139.5 \text{ N}}$

5.22 $V_\infty = 85.5$ km/h = 23.75 m/sec

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(23.75)^2 = 345 \text{ N/m}^2$$

$$C_L = \frac{L}{q_\infty S} = \frac{9800}{(345)(16.2)} = 1.75$$

$$C_{D_i} = \frac{C_L^2}{\pi e AR} = \frac{(1.75)^2}{\pi(0.62)(7.31)} = 0.215$$

$$D_i = q_\infty S C_{D_i} = (345)(16.2)(0.215) = \boxed{1202 \text{ N}}$$

Note: The induced drag at low speeds, such as near stalling velocity, is considerably larger than at high speeds, near maximum velocity. Compare the results of problems 5.20 and 5.21

5.23 First, obtain the infinite wing lift slope. From Appendix D for a NACA 65-210 airfoil,

$$C_\ell = 1.05 \text{ at } \alpha = 8^\circ$$

$$C_\ell = 0 \text{ at } \alpha_{L=0} = -1.5^\circ$$

Hence,

$$a_o = \frac{1.05 - 0}{8 - (-1.5)} = 0.11 \text{ per degree}$$

The lift slope for the finite wing is

$$a = \frac{a_o}{1 + \frac{57.3\ a_o}{\pi\ e_1 AR}} = \frac{0.11}{1 + \frac{57.3(0.11)}{\pi\ (.9)(5)}} = 0.076 \text{ per degree}$$

At $\alpha = 6^\circ$,

$$C_L = a(\alpha - \alpha_{L=0}) = (0.076)\,[6 - (-1.5)] = \boxed{0.57}$$

The total drag coefficient is

$$C_D = c_d + \frac{C_L^{\,2}}{\pi e AR} = (0.004) + \frac{(0.57)^2}{\pi\ (0.9)(5)}$$

$$C_D = 0.004 + 0.023 = \boxed{0.027}$$

5.24 $q_\infty = \frac{1}{2}\rho_\infty V_\infty^{\,2} = \frac{1}{2}(0.002377)(100)^2 = 11.9 \text{ lb/ft}^2$

at $\alpha = 10^\circ$, L = 17.9 lb Hence

$$C_L = \frac{L}{q_\infty S} = \frac{17.9}{(11.9)(1.5)} = 1.0$$

at $\alpha = -2^\circ$, L = 0 Hence $\alpha_{L=0} = -2^\circ$

$$a = \frac{dC_L}{d\alpha} = \frac{1.0 - 0}{[10 - (-2)]} = 0.083 \text{ per degree}$$

This is the finite wing lift slope

$$a = \frac{a_o}{1 + \frac{57.3\ a_o}{\pi\ eAR}}$$

Solve for a_o

$$a_o = \frac{a}{1 - \frac{57.3\ a}{\pi\ eAR}} = \frac{0.083}{1 + \frac{57.3(0.083)}{\pi\ (0.95)(6)}}$$

$$a_o = \boxed{0.11 \text{ per degree}}$$

5.25 At $\alpha = -1°$, the lift is zero. Hence, the total drag is simply the profile drag.

$$C_D = c_d + \frac{C_L^{\ 2}}{\pi eAR} = c_d + 0 = c_d$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^{\ 2} = \frac{1}{2}(0.002377)(130)^2 = 20.1 \text{ lb/ft}^2$$

Thus, at $\alpha = \alpha_{L=0} = -1°$

$$c_d = \frac{D}{q_\infty S} = \frac{0.181}{(20.1)(1.5)} = 0.006$$

At $\alpha = 2°$, assume that c_d has not materially changed, i.e., the "drag bucket" of the profile drag curve (see Appendix D) extends at least from -1° to 2°, where c_d is essentially constant. Thus, at $\alpha = 2°$,

$$C_L = \frac{L}{q_\infty S} = \frac{5}{(20.1)(1.5)} = 0.166$$

$$C_D = \frac{D}{q_\infty S} = \frac{0.23}{(20.1)(1.5)} = 0.00763$$

However:

$$C_D = c_d + \frac{C_L^{\ 2}}{\pi eAR}$$

$$0.00763 = 0.006 + \frac{(0.166)^2}{\pi\ e(6)} = 0.006 + \frac{0.00146}{e}$$

$e = \boxed{0.90}$

To obtain the lift slope of the airfoil (infinite wing), first calculate the finite wing lift slope.

$$a = \frac{(0.166 - 0)}{[2 - (-2)]} = 0\ 055 \text{ per degree}$$

$$a_o = \frac{a}{1 - \dfrac{57.3\ a}{\pi\ eAR}} = \frac{0.055}{1 - \dfrac{57.3(0055)}{\pi\ (0\,9)(6)}}$$

$$a_o = \boxed{0\ 068 \text{ per degree}}$$

5.26 $$V_{stall} = \sqrt{\frac{2W}{\rho_\infty S\ C_{L_{max}}}} = \sqrt{\frac{2(7780)}{(1\,225)(16\,6)(2\,1)}}$$

$$V_{stall} = \boxed{19.1 \text{ m/sec} = 68.7 \text{ km/h}}$$

5.27 (a) $$\alpha = \frac{5}{57\,3} = 0\,087 \text{ radians}$$

$$c_\ell = 2\ \pi\ \alpha = 2\ \pi\ (0.087) = \boxed{0.548}$$

(b) Using the Prandtl-Glauert rule,

$$c_\ell = \frac{c_{\ell_o}}{\sqrt{1 - M_\infty^2}} = \frac{0.548}{\sqrt{1 - (0\,7)^2}} = \boxed{0.767}$$

(c) From Eq (5 50)

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.087)}{\sqrt{(2)^2 - 1}} = \boxed{0.2}$$

5.28 For $V_\infty = 21\ 8$ ft/sec at sea level

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(21.8)^2 = 0.565 \text{ lb/ft}^2$$

1 ounce = 1/16 lb = 0.0625 lb.

$$C_L = \frac{L}{q_\infty S} = \frac{0.0625}{(0.565)(1)} = \boxed{0.11}$$

For a flat plate airfoil

$$c_\ell = 2\pi\alpha = 2\pi(3/57.3) = \boxed{0.329}$$

The difference between the higher value predicted by thin airfoil theory and the lower value measured by Cayley is due to the low aspect ratio of Cayley's test wing, and viscous effects at low Reynolds number

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5.29 From Eqs. (5.1) and (5.2), writtten in coefficient form

$$C_L = C_N \cos\alpha - C_A \sin\alpha$$

$$C_D = C_N \sin\alpha + C_A \cos\alpha$$

Hence:

$$C_L = 0.8 \cos 6^\circ - 0.06 \sin 6^\circ = 0.7956 - 0.00627 = \boxed{0.789}$$

$$C_D = 0.8 \sin\alpha + 0.06 \cos\alpha = 0.0836 + 0.0597 = \boxed{0.1433}$$

Note: At the relatively small angles of attack associated with normal airplane flight, C_L and C_N are essentially the same value, as shown in this example.

5.30 First solve for the angle of attack and the profile drag coefficient, which stay the same in this problem.

$$c_L = a\,\alpha = \frac{a_o\,\alpha}{1+57.3\ a_o/(\pi\ e_1 AR)}$$

or, $$\alpha = \frac{C_L}{a_o}[1 + 57\ 3\ a_o/\pi\ e_1\ AR)]$$

$$= \frac{0.35}{0\,11}\ \{1 + 57\ 3\ (0.11)/[\pi\ (0\ 9)(7)]\} = 4.2^\circ$$

The profile drag can be obtained as follows

$$C_D = \frac{C_L}{(C_L / C_D)} = \frac{0.35}{29} = 0\,012$$

$$C_D = c_d + \frac{C_L^{\ 2}}{\pi e AR}$$

or, $$c_d = C_D - \frac{C_L^{\ 2}}{\pi e AR} = 0\ 012 - \frac{(0.35)^2}{\pi\ (9)(7)} = 0.0062$$

Increasing the aspect ratio at the same angle of attack increases C_L and reduces C_D For

AR = 10, we have

$$C_L = a\,\alpha = \frac{a_o\,\alpha}{1+57.3\ a_o/(\pi\ e_1 AR)}$$

$$= \frac{(0.11)(4.2)}{1+57.3\ (0\,11)/[\pi\ (0\,9)(10)]} = 0.3778$$

$$C_D = c_d + \frac{C_L^{\ 2}}{\pi e AR} = 0.062 - \frac{(0.3778)^2}{\pi\ (9)(10)} = 0\,0062\ + 0.005048 = 0\ 112$$

Hence, the new value of L/D is

$$\frac{C_L}{C_D} = \frac{0.3778}{0\,0112} = \boxed{33.7}$$
