At 33,000 ft: $\rho = 7\ 9656 \times 10^{-4}$ slug/ft^3

At 33,500 ft: $\rho = 7\ 8165 \times 10^{-4}$ slug/ft^3

Hence, the density altitude is

$$33{,}000 + 500 \left(\frac{7.9656 - 7.919}{7\ 9656 - 7.8165}\right) = \boxed{33{,}156 \text{ ft}}$$

4.1 $A_1V_1 = A_2V_2$

Let points 1 and 2 denote the inlet and exit conditions respectively Then,

$$V_2 = V_1 \left(\frac{A_1}{A_2}\right) = (5) \left(\frac{1}{4}\right) = \boxed{1.25 \text{ ft/sec}}$$

4.2 From Bernoulli's equation,

$$p_1 + \rho \frac{V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2}$$

$$p_2 - p_1 = \frac{\rho}{2} (V_1^2 - V_2^2)$$

In consistent units,

$$\rho = \frac{62.4}{32.2} = 1\ 94 \text{ slug/ft}^3$$

Hence,

$$p_2 - p_1 = \frac{1.94}{2} [(5)^2 - (1\ 25)^2]$$

$$p_2 - p_1 = 0.97\ (23\ 4) = \boxed{22.7 \text{ lb/ft}^2}$$

4.3 From Appendix A; at 3000m altitude,

$p_1 = 7.01 \times 10^4$ N/m^2

$\rho = 0.909$ kg/m^3

From Bernoulli's equation,

$$p_2 = p_1 + \frac{\rho}{2}(V_1^2 - V_2^2)$$

$$p_2 = 7.01 \times 10^4 + \frac{0.909}{2}[60^2 - 70^2]$$

$$p_2 = 7.01 \times 10^4 - 0.059 \times 10^4 = \boxed{6.95 \times 10^4 \text{ N/m}^2}$$

4.4 From Bernoulli's equation,

$$p_1 + \frac{\rho}{2}V_1^2 = p_2 + \frac{\rho}{2}V_2^2$$

Also from the incompressible continuity equation

$$V_2 = V_1 (A_1/A_2)$$

Combining,

$$p_1 + \frac{\rho}{2}V_1^2 = p_2 + \frac{\rho}{2}(A_1/A_2)^2$$

$$V_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left[(A_1/A_2)^2 - 1\right]}}$$

At standard sea level, $\rho = 0.002377$ slug/ft^3 Hence,

$$V_1 = \sqrt{\frac{2(80)}{(.002377)[(4)^2 - 1]}} = \boxed{67 \text{ ft/sec}}$$

Note that also $V_1 = 67 \left(\frac{60}{80}\right) = 46$ mi/h (This is approximately the landing speed of World War I vintage aircraft)

4.5 $p_1 + \frac{1}{2}\rho V^2 = p_3 + \frac{1}{2}\rho V^2$

$$V_1^2 = \frac{2(p_3 - p_1)}{\rho} + V_3^2 \qquad (1)$$

$$A_1 V_1 = A_3 V_3, \text{ or } V_3 = \frac{A_1}{A_3} V_1 \qquad (2)$$

Substitute (2) into (1)

$$V_1^2 = \frac{2(p_3 - p_1)}{\rho} + \left(\frac{A_1}{A_3}\right)^2 V_1^2$$

or, $$V_1 = \sqrt{\frac{2(p_3 - p_1)}{\rho\left[1 - \left(\frac{A_1}{A_3}\right)^2\right]}} \qquad (3)$$

Also,

$$A_1 V_1 = A_2 V_2$$

or, $$V_2 = \left(\frac{A_1}{A_2}\right) V_1 \qquad (4)$$

Substitute (3) into (4)

$$V_1 = \frac{A_1}{A_2}\sqrt{\frac{2(p_3 - p_1)}{\rho\left[1 - \left(\frac{A_1}{A_3}\right)^2\right]}}$$

$$V_2 = \frac{3}{15}\sqrt{\frac{2(1.00 - 1.02)\ x\ 10^5}{(1.225)\left[1-\left(\frac{3}{2}\right)^2\right]}}$$

$$V_2 = \boxed{102.22\ \text{m/sec}}$$

Note: It takes a pressure difference of <u>only</u> 0.02 atm to produce such a high velocity.

4.6 $V_1 = 130\ \text{mph} = 130\left(\frac{88}{60}\right) = 190.7\ \text{ft/sec}$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

$$V_2^2 = \frac{2}{\rho}(p_1 - p_2) + V_1^2$$

$$V_2^2 = \frac{2(1760.9 - 1750.0)}{0.0020482} + (190.7)^2$$

$$V_2 = \boxed{216.8\ \text{ft/sec}}$$

4.7 From Bernoulli's equation,

$$p_1 - p_2 = \frac{\rho}{2}(V_2^2 - V_1^2)$$

And from the incompressible continuity equation,

$$V_2 = V_1 (A_1/A_2)$$

Combining:

$$p_1 - p_2 = \frac{\rho}{2} V_1^2 [(A_1/A_2)^2 - 1]$$

Hence, the maximum pressure difference will occur when simultaneously:

1. V_1 is maximum

2. ρ is maximum i.e sea level

The design maximum velocity is 90 m/sec, and $\rho = 1.225$ kg/m^3 at sea level. Hence,

$$p_1 - p_2 = \frac{1.225}{2}(90)^2\left[(1.3)^2 - 1\right] = \boxed{3423 \text{ N/m}^2}$$

Please note: In reality the airplane will most likely exceed 90 m/sec in a dive, so the airspeed indicator should be designed for a maximum velocity somewhat above 90 m/sec.

4.8 The isentropic relations are

$$\frac{p_e}{p_o} = \left(\frac{\rho_e}{\rho_o}\right)^{\gamma} = \left(\frac{T_e}{T_o}\right)^{\frac{\gamma}{\gamma-1}}$$

Hence,

$$T_e = T_o\left(\frac{p_e}{p_o}\right)^{\frac{\gamma-1}{\gamma}} = (300)\left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = \boxed{155\text{K}}$$

From the equation of state:

$$\rho_o = \frac{p_o}{RT_o} = \frac{(10)(1.01\text{x}10^5)}{(287)(300)} = 11.73 \text{ kg/m}^3$$

Thus,

$$\rho_e = \rho_o\left(\frac{p_e}{p_o}\right)^{\frac{1}{\gamma}} = 11.73\left(\frac{1}{10}\right)^{\frac{1}{1.4}} = \boxed{2.26 \text{ kg/m}^3}$$

As a check on the results, apply the equation of state at the exit.

$p_e = \rho_e R T_e$?

$1.01 \times 10^5 = (2.26)(287)(155)$

$1.01 \times 10^5 = 1.01 \times 10^5$ It checks!

4.9 Since the velocity is essentially zero in the reservoir, the energy equation written between the reservoir and the exit is

$$h_o = h_e + \frac{V_e^2}{2}$$

or, $V_e^2 = 2\,(h_o - h_e)$ (1)

However, $h = c_pT$ Thus Eq. (1) becomes

$$V_e^2 = 2\,c_p\,(T_o - T_e)$$

$$V_e^2 = 2\,c_p\,T_o\left(1 - \frac{T_e}{T_o}\right) \tag{2}$$

However, the flow is isentropic, hence

$$\frac{T_e}{T_o} = \left(\frac{p_e}{p_o}\right)^{\frac{\gamma-1}{\gamma}} \tag{3}$$

Substitute (3) into (1)

$$V_e = \sqrt{2\,c_pT_o\left[1 - \left(\frac{p_e}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{4}$$

This is the desired result Note from Eq. (4) that V_e increases as T_o increases, and as p_e/p_o decreases Equation (4) is a useful formula for rocket engine performance analysis

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4.10 The flow velocity is certainly large enough that the flow must be treated as compressible. From the energy equation,

$$c_p\,T_1 + \frac{V_1^2}{2} = c_p\,T_2\,\frac{V_2^2}{2} \qquad (1)$$

At a standard altitude of 5 km, from Appendix A,

$$p_1 = 5.4 \times 10^4\ \text{N/m}^2$$

$$T_1 = 255.7\ \text{K}$$

Also, for air, $c_p = 1005$ joule/(kg)(K). Hence, from Eq. (1) above,

$$T_2 = T_1 + \frac{V_1^2 - V_2^2}{2\ c_p}$$

$$T_2 = 255.7 + \frac{(270)^2 - (330)^2}{2(1005)}$$

$$T_2 = 255.7 - 17.9 = 237.8\text{K}$$

Since the flow is also isentropic,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Thus,

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}} = 5.4 \times 10^4 \left(\frac{237.8}{255.7}\right)^{\frac{1.4}{1.4-1}}$$

$$p_2 = \boxed{4.19 \times 10^4\ \text{N/m}^2}$$

Please note: This problem and problem 4.3 ask the same question. However, the flow velocities in the present problem require a compressible analysis. Make certain to examine the solutions of both problems 4.10 and 4.3 in order to contrast compressible versus incompressible analyses.

4.11 From the energy equation

$$c_p\, T_o = c_p\, T_e + \frac{V_e^2}{2}$$

or, $$T_e = T_o - \frac{V_e^2}{2\, c_p}$$

$$T_e = 1000 - \frac{1500^2}{2(6000)} = 812\ 5R$$

In the reservoir, the density is

$$\rho_o = \frac{p_o}{RT_o} = \frac{(7)(2116)}{(1716)(1000)} = 0\ 0086\ \text{slug/ft}^3$$

From the isentropic relation,

$$\frac{\rho_e}{\rho_o} = \left(\frac{T_e}{T_o}\right)^{\frac{1}{\gamma-1}}$$

$$\rho_e = 0\ 0086 \left(\frac{812.5}{1000}\right)^{\frac{1}{1\ 4-1}} = 0\ 0051\ \text{slug/ft}^3$$

From the continuity equation,

$$\dot{m} = \rho_e A_e V_e$$

Thus, $$A_e = \frac{\dot{m}}{\rho_e V_e}$$

In consistent units,

$$\dot{m} = \frac{1.5}{32\ 2} = 0\ 047\ \text{slug/sec.}$$

Hence,

$$A_e = \frac{\dot{m}}{\rho_e V_e} = \frac{0.047}{(0\ 0051)(1500)} = \boxed{0.0061\ ft^2}$$

4.12 $V_1 = 1500\ mph = 1500 \left(\frac{88}{60}\right) = 2200\ ft/sec$

$$C_p T_1 + \frac{V_1^2}{2} C_p T_2 \frac{V_2^2}{2}$$

$$V_2^2 = 2\ C_p (T_1 - T_2) + V_1^2$$

$$V_2^2 = 2\ (6000)(389\ 99 - 793\ 32) + (2200)^2$$

$$V_2 = \boxed{6.3\ ft/sec}$$

Note: This is a very small velocity compared to the initial freestream velocity of 2200 ft/sec At the point in question, the velocity is very near zero, and hence the point is nearly a stagnation point.

4.13 At the inlet, the mass flow of air is

$$\dot{m}_{air} = \rho AV = (3\ 6391 \times 10^{-4})(20)(2200) = 16\ 0/slug/sec$$

$$\dot{m}_{fuel} = (0\ 05)(16\ 01) = 0\ 8\ slug/sec$$

Total mass flow at exit = 16 01 + 0 8 = $\boxed{16.81\ slug/sec}$

4.14 From problem 4 11,

$$V_e = 1500\ ft/sec$$

$$T_e = 812\ 5R$$

Hence,

$$a_e = \sqrt{\gamma R T_e} = \sqrt{(1.4)(1716)(812.5)}$$

$$= 1397 \text{ ft/sec}$$

Thus, $M_e = \dfrac{V_e}{a_e} = \dfrac{1500}{1397} = \boxed{1.07}$

Note that the nozzle of problem 4.11 is just barely supersonic.

4.15 From Appendix A,

$$T_\infty = 216.66\text{K}$$

Hence,

$$a_\infty = \sqrt{\gamma R T}$$

$$= \sqrt{(1.4)(287)(216.66)} = 295 \text{ m/sec}$$

Thus, $M_\infty = \dfrac{V_\infty}{a_\infty} = \dfrac{250}{295} = \boxed{0.847}$

4.16 At standard sea level, $T_\infty = 518.69\text{R}$

$$a_\infty = \sqrt{\gamma R T} = \sqrt{(1.4)(1716)(518.69)} = 1116 \text{ ft/sec}$$

$$V_\infty = M_\infty a_\infty = (3)(1116) = 3348 \text{ ft/sec}$$

Since 60 mi/hr - 88 ft/sec., then

$$V_\infty = 3348\,(60/88) = \boxed{2283 \text{ mi/h}}$$

4.17 $V = 2200$ ft/sec

$$a = \sqrt{\gamma RT} = \sqrt{(1.4)(1716)(389.99)} = 967.94 \text{ ft/sec}$$

$$M = \frac{V}{a} = \frac{2200}{967.94} = \boxed{2.27}$$

4.18 The test section density is

$$\rho = \frac{p}{RT} = \frac{1.01 \times 10^5}{(287)(300)} = 1.173 \text{ kg/m}^3$$

Since the flow is low speed, consider it to be incompressible, i.e., with the above density throughout.

$$p_1 - p_2 = \frac{\rho}{2} V_2^2 [1 - (A_2/A_1)^2] \qquad (1)$$

In terms of the manometer reading,

$$p_1 - p_2 = \omega \Delta h \qquad (2)$$

where $\omega = 1.33 \times 10^5$ N/m^3 for mercury.

Thus, combining Eqs. (1) and (2),

$$\Delta h = \frac{\rho}{2\omega} V_2^2 [1 - (A_2/A_1)^2]$$

$$= \frac{1.173}{(2)(1.33 \times 10^5)} (80)^2 [1 - (1/20)^2]$$

$$\Delta h = 0.028 \text{m} = \boxed{2.8 \text{ cm}}$$

4.19 $V_2 = 200 \text{ mph} = 300 \left(\frac{88}{60}\right) = 293.3$ ft/sec

(a) $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$

$A_1 V_1 = A_2 V_2$

$V_1 = \frac{A_2}{A_1} V_2$

$p_1 + \frac{1}{2}\rho \left(\frac{A_2}{A_1}\right)^2 V_2^2 = p_2 + \frac{1}{2}\rho V_2^2$

$p_1 - p_2 = \frac{1}{2}\rho \left[1 - \left(\frac{A_2}{A_1}\right)^2\right] V_2^2$

$p_1 - p_2 = \frac{0.002377}{2}\left[1 - \left(\frac{4}{20}\right)^2\right](293.3)^2$

$p_1 - p_2 = \boxed{98.15\ \text{lb/ft}^2}$

(b) $p_1 + \frac{1}{2}\rho V_1^2 = p_3 + \frac{1}{2}\rho V_3^3$

$A_1 V_1 = A_2 V_2 \quad : \quad V_1 = \frac{A_2}{A_1} V_2$

$A_2 V_2 = A_3 V_3 \quad : \quad V_3 = \frac{A_2}{A_3} V_2$

$p_1 - p_3 = \frac{1}{2}\rho \left[\left(\frac{A_2}{A_3}\right)^2 - \left(\frac{A_2}{A_1}\right)^2\right] V_2^2$

$p_1 - p_3 = \frac{0.002377}{2}\left[\left(\frac{4}{18}\right)^2 - \left(\frac{4}{20}\right)^2\right](293.3)^2$

$p_1 - p_3 = \boxed{0.959\ \text{lb/ft}^2}$

Note: By the addition of a diffuser, the required pressure difference was reduced by an order of magnitude Since it costs money to produce a pressure difference (say by running compresors or vacuum pumps), then a diffuser, the purpose of which is to improve the aerodynamic efficiency, allows the wind tunnel to be operated more economically.

4.20 In the test section

$$\rho = \frac{p}{RT} = \frac{2116}{(1716)(70+460)} = 0.00233 \text{ slug/ft}^3$$

The flow velocity is low enough so that incompressible flow can be assumed Hence, from Bernoulli's equation,

$$p_o = p + \frac{1}{2}\rho V^2$$

$$p_o = 2116 + \frac{1}{2}(0.00233)\,[150\,(88/60)]^2$$

(Remember that 88 ft/sec = 60 mi/h)

$$p_o = 2116 + \frac{1}{2}(0.00233)(220)^2$$

$$p_o = \boxed{2172 \text{ lb/ft}^2}$$

4.21 The altimeter meaasures pressure altitude. Thus, from Appendix B, p = 1572 lb/ft^2. The air density is then

$$\rho = \frac{p}{RT} = \frac{1572}{(1716)(500)} = 0.00183 \text{ slug/ft}^3$$

Hence, from Bernoulli's equation,

$$V_{true} = \sqrt{\frac{2(p_o - p)}{\rho}} = \sqrt{\frac{2(1650-1572)}{0.00183}}$$

$$V_{true} = 292 \text{ ft/sec}$$

The equivalent airspeed is

$$V_e = \sqrt{\frac{2(p_o - p)}{\rho_s}} = \sqrt{\frac{2(1650-1572)}{0.002377}}$$

$$V_e = \boxed{256 \text{ ft/sec}}$$

4.22 The altimeter measures pressure altitude. Thus, from Appendix A, $p = 7.95 \times 10^4$ N/m^2. Hence,

$$\rho = \frac{p}{RT} = \frac{7.95 \times 10^4}{(287)(280)} = 0.989 \text{ kg/m}^3$$

The relation between V_{true} and V_e is

$$V_{true}/V_e = \sqrt{\rho_s / \rho}$$

Hence,

$$V_{true} = 50 \sqrt{(1.225)/0.989} = \boxed{56 \text{ m/sec}}$$

4.23 In the test section,

$$a = \sqrt{\gamma RT} = \sqrt{(1.4)(287)(270)} = 329 \text{ m/sec}$$

$$M = V/a = 250/329 = 0.760$$

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = [1 + 0.2\,(0.760)^2]^{3.5} = 1.47$$

Hence,

$$p_o = 1.47p = 1.47\,(1.01 \text{ x } 10^5) = \boxed{1.48 \text{ x } 10^5 \text{ N/m}^2}$$

4.24 $p = 1.94 \text{ x } 10^4 \text{ N/m}^2$ from Appendix A

$$M_1^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_o}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = \frac{2}{1.4-1}\left[\left(\frac{2.96 \text{ x } 10^4}{1.94 \text{ x } 10^4}\right)^{0.286} - 1\right]$$

$$M_1^2 = 0.642$$

$$M_1 = \boxed{0.801}$$

4.25 $$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_o}{p} = [1 + 0.2\,(0.65)^2]^{3.5} = 1.328$$

$$p = \frac{p_o}{1.328} = \frac{2339}{1.328} = 1761 \text{ lb/ft}^2$$

From Appendix B, this pressure corresponds to a pressure altitude, hence altimeter reading of $\boxed{5000 \text{ ft.}}$

4.26 At standard sea level,

$$T = 518.69R$$

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + 0.2\,(0.96)^2 = 1.184$$

$$T_o = 1.184T = 1.184\,(518.69)$$

$$T_o = 614.3R = 154.3F$$

4.27 $a_1 = \sqrt{\gamma R T_1} = \sqrt{(1.4)(287)(220)} = 297$ m/sec

$M_1 = V_1/a_1 = 596/197 = 2.0$

The flow is supersonic. Hence, the Rayleigh Pitot tube formula must be used.

$$\frac{p_{o_2}}{p_1} = \left[\frac{(\gamma+1)^2 M_1^2}{4\gamma\ M_1^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right]$$

$$\frac{p_{o_2}}{p_1} = \left[\frac{(2.4)^2(2)^2}{4(1.4)(2)^2 - 2(0.4)}\right]^{3.5}\left[\frac{1-1.4+2(1.4)(2)^2}{2.4}\right]$$

$$\frac{p_{o_2}}{p_1} = 5.64$$

$p_1 = 2.65 \times 10^4$ N/m^2 from Appendix A.

Hence,

$$p_{o_2} = 5.64\,(2.65 \times 10^4) = \boxed{1.49 \times 10^5 \text{ N/m}^2}$$

4.28 $q = \frac{1}{2}\rho V^2 = \frac{1}{2}\left(\frac{\gamma p}{\gamma p}\right)\rho V^2 = \frac{\gamma}{2} p\left(\frac{\rho}{\gamma p}\right)V^2 = \frac{\gamma}{2}\, p\,\frac{V^2}{a^2}$

Hence:

$$\boxed{q = \frac{\gamma}{2}\, p\, M^2}$$

4.29 $q_\infty = \frac{\gamma}{2} p_\infty M_\infty^2 = 0.7\, p_\infty M_\infty^2$ (1)

Use Appendix A to obtain the values of p_∞ corresponding to the given values of h.

Then use Eq. (1) above to calculate q_∞.

h(km)	60	50	40	30	20
p_∞(N/m^2)	25.6	87.9	299.8	1.19×10^3	5.53×10^3
M	17	9.5	5.5	3	1
q_∞(N/M^2)	5.2×10^3	5.6×10^3	6.3×10^3	7.5×10^3	3.9×10^3

Note that q_∞ progressively increases as the shuttle penetrates deeper into the atmosphere, that it peaks at a slightly supersonic Mach number, and then decreases as the shuttle completes its entry.

4.30 Recall that total pressure is defined as that pressure that would exist if the flow were slowed isentropically to zero velocity. This is a definition; it applies to all flows -- subsonic or supersonic. Hence, Eq. (4.74) applies, no matter whether the flow is subsonic or supersonic.

$$\frac{p_o}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\gamma/(\gamma-1)} = [1 + 0.2\,(2)^2]^{1.4/0.4} = 7.824$$

Hence:

$$p_o = 7.824\, p_\infty = 7.824\,(2116) = \boxed{1.656 \times 10^4 \ \frac{\text{lb}}{\text{ft}^2}}$$

Note that the above value is not the pressure at a stagnation point at the nose of a blunt body, because in slowing to zero velocity, the flow has to go through a shock wave, which is non-isentropic. The stagnation pressure at the nose of a body in a Mach 2 stream is the total pressure behind a normal shock wave, which is lower than the total pressure of the freestream, as calculated above. This stagnation pressure at the nose of a blunt body is given by Eq. (4.79).

$$\frac{p_{o_2}}{p_1} = \left[\frac{(\gamma+1)^2 M_\infty^2}{4\gamma\ M_2^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right] =$$

$$= \left[\frac{(2.4)^2 (2)^2}{4(1.4)(2)^2 - 2(0.4)}\right]^{1.4/0.4} \left[\frac{1-1.4+2(1.4)(2)^2}{2.4}\right] = 5.639$$

Hence,

$$p_{o_2} = 5.639\ p_\infty = 5.639\ (2116) = \boxed{1.193 \times 10^4\ \frac{lb}{ft^2}}$$

If Bernoulli's equation is used, the following wrong result for total pressure is obtained.

$$p_o = p_\infty + q_\infty = p_\infty + \frac{1}{2}\ \rho\ V_\infty^2 = p_\infty + \frac{\gamma}{2}\ p_\infty\ M_\infty^2$$

$$p_o = 2116 + 0.7\ (2116)\ (2)^2 = 0.804 \times 10^4\ \frac{lb}{ft^2}$$

Compared to the correct result of $1.656 \times 10^4\ \frac{lb}{ft^2}$, this leads to an error 51%.

4.31 $$\frac{p_e}{p_o} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{-\gamma}{\gamma-1}}$$

$$p_e = 5(1.01 \times 10^5)\ [1 + 0.2\ (3)^2]^{-3.5}$$

$$p_e = \boxed{1.37 \times 10^4\ N/m^2}$$

$$\frac{T_e}{T_o} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{-1} = [1 + 0.2\ (3)^2]^{-1}$$

$$T_e = (500)(0.357) = \boxed{178.6K}$$

$$\rho_e = \frac{p_e}{RT_e} = \frac{1.37 \times 10^4}{(287)(178.6)} = \boxed{0.267 \text{ kg/m}^3}$$

4.32 $$\frac{p_e}{p_o} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{-\gamma}{\gamma-1}}$$

Hence,

$$M_e^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_e}{p_o}\right)^{\frac{1-\gamma}{\gamma}} - 1\right]$$

$$M_e^2 = 5[(0.2)^{-0.286} - 1] = 2.92$$

$$M_e = 1.71$$

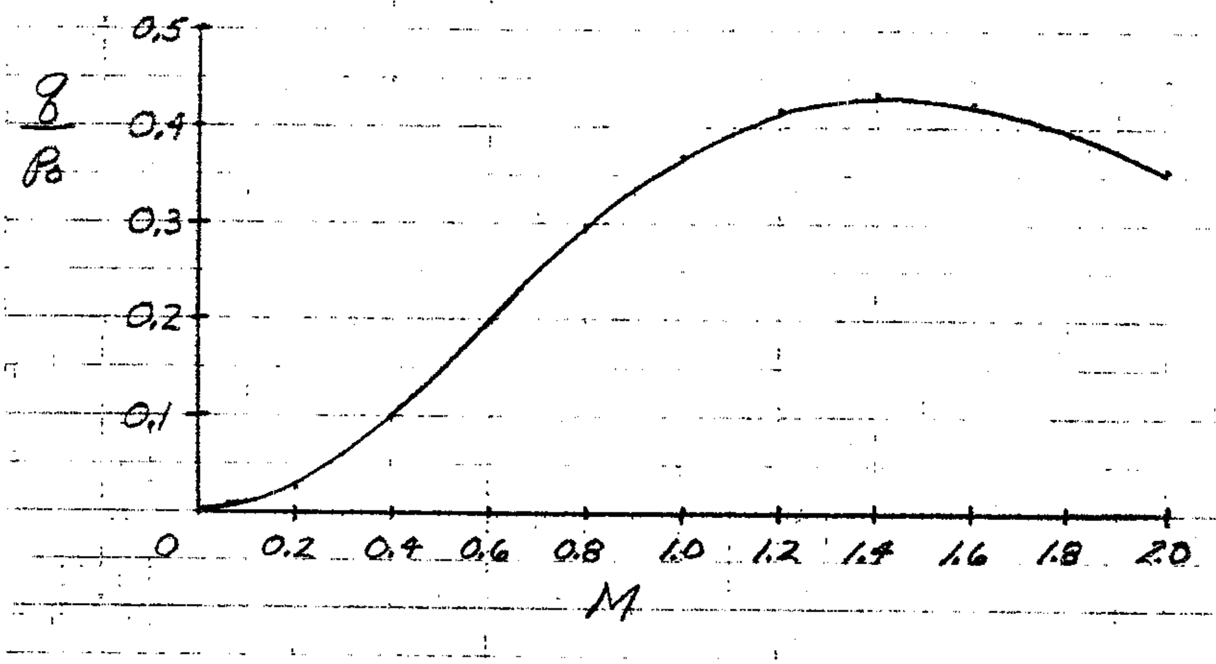
$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma - 1}{\gamma}\right)M^2\right]^{(\gamma+1)/(\gamma-1)}$$

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{(1.71)^2}[(0.833)(1 + 0.2\,(1.71)^2)]^6$$

$$\frac{A_e}{A_t} = \boxed{1.35}$$

4.33 $$\frac{q}{p_o} = \frac{\gamma}{2}\frac{p}{p_o}M^2 = \frac{\gamma}{2}M^2\left[1 + \frac{\gamma - 1}{2}M^2\right]^{-\gamma/(\gamma-1)} = 0.7\,M^2\,(1 + 0.2\,M^2)^{-3.5}$$

M	M^2	q/p_o
0	0	0
0.2	0.04	0.027
0.4	0.16	0.100
0.6	0.36	0.198
0.8	0.64	0.294
1.0	1.0	0.370
1.2	1.44	0.416
1.4	1.96	0.431
1.6	2.56	0.422
1.8	3.24	0.395
2.0	4.00	0.358



Note that the dynamic pressure increases with Mach number for $M < 1.4$ but decreases with Mach number for $M > 1.4$. I.e., in an isentropic nozzle expansion, there is a peak local dynamic pressure which occurs at $M = 1.4$

4.34 First, calculate the value of the Reynolds number.

$$Re_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{(1.225)(200)(3)}{(1.7894 \times 10^{-5})} = 4.10 \times 10^7$$

The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(200)^2 = 2.45 \times 10^4 \text{ N/m}^2$$

Hence,

$$\delta_L = \frac{5.2L}{\sqrt{Re_L}} = \frac{5.2(3)}{\sqrt{4.1 \times 10^7}} = 0.0024\text{m} = \boxed{0.24 \text{ cm}}$$

and

$$C_f = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{4.1 \times 10^7}} = 0.00021$$

The skin friction drag on one side of the plate is:

$$D_f = q_\infty S c_f = (2.45 \times 10^4)(3)(17.5)(0.00021)$$

$$D_f = 270\text{N}$$

The total skin friction drag, accounting for both the top and the bottom of the plate is twice this value, namely

$$\text{Total } D_f = \boxed{540\text{N}}$$

4.35 $$\delta = \frac{0.37L}{(Re_L)^{0.2}} = \frac{0.37(3)}{(4.1 \times 10^7)^{0.2}} = 0.033\text{m} = \boxed{3.3 \text{ cm}}$$

From problem 4.24, we find

$$\delta_{turbulent}/\delta_{laminar} = \frac{3.3}{0.24} = \boxed{13.75}$$

The turbulent boundary layer is more than an order of magnitude thicker than the laminar boundary layer.

$$C_f = \frac{0.074}{(Re_L)^{0.2}} = \frac{0.074}{(4.1 \times 10^7)^{0.2}} = 0.0022$$

The skin friction drag on one side is then

$$D_f = q_\infty S C_f = (2/45 \times 10^4)(3)(17.5)(0.0022)$$

$$D_f = 2830N$$

The total, accounting for both top and bottom is

$$\text{Total } D_f = \boxed{5660N}$$

From problem 4.24, we find

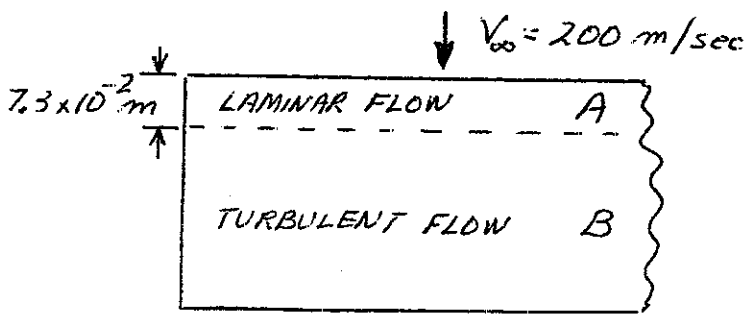
$$\left(D_{f_{turbulent}}\right)/\left(D_{f_{laminar}}\right) = \frac{5660}{540} = \boxed{10.5}$$

The turbulent skin friction drag is an order of magnitude larger than the laminar value.

4.36 $$R_{e_{x_{cr}}} = \frac{\rho_\infty V_\infty x_{cr}}{\mu_\infty}$$

$$x_{cr} = Re_{x_{cr}}\left(\frac{\mu_\infty}{\rho_\infty V_\infty}\right) = \frac{(10^6)(1.789 \times 10^{-5})}{(1.225)(200)}$$

$$x_{cr} = 7.3 \times 10^{-2} m$$



The turbulent drag that would exist over the first 7.3 x 10^{-2}m of chord length from the leading edge (area A) is

$$D_{f_A} = \frac{0.074}{(Re_{cr})^{0.2}} q_\infty S_A \quad \text{(on one side)}$$

$$D_{f_A} = \frac{0.074}{(10^6)^{0.2}} (2.45 \times 10^4)(7.3 \times 10^{-2})(17.5)$$

$$D_{f_A} = 146\text{N} \qquad \text{(on one side)}$$

From problem 4.25, the turbulent drag on one side, assuming both areas A and B to be turbulent, is 2830N. Hence, the turbulent drag on area B alone is:

$$D_{f_B} = 2830 - 146 - 2684\text{N} \qquad \text{(turbulent)}$$

The laminar drag on area A is

$$D_{f_A} = \frac{1.328}{(Re_{cr})^{0.5}} q_\infty S$$

$$D_{f_A} = \frac{1.328}{(10^6)^{0.5}} (2.45 \times 10^4)(7.3 \times 10^{-2})(17.5)$$

$$D_{f_A} = 42\text{N} \qquad \text{(laminar)}$$

Hence, the skin friction drag on one side, assuming area A to be laminar and area B to be turbulent is

$$D_f = D_{f_A}\text{ (laminar)} + D_{f_B}\text{ (turbulent)}$$

$$D_f = 42 + 2684 = 2726\text{N}$$

The total drag, accounting for both sides, is

$$\text{Total } D_f = \boxed{5452\text{N}}$$

Note: By comparing the results of this problem with those of problem 4.25, we see that the flow over the wing is mostly turbulent, which is usually the case for real airplanes in flight.

4.37 The relation between changes in pressure and velocity at a point in an inviscid flow is given by the Euler equation, Eq. (4 8)

$$dp = -\rho\ V\ d\ V$$

Letting s denote distance along the streamline through the point, Eq. (4 8) can be written as

$$\frac{dp}{ds} = -\rho\ V\ \frac{dV}{ds}$$

or,
$$\frac{dp}{ds} = -\rho\ V^2\ \frac{(dV / V)}{ds}$$

(a)
$$\frac{(dV / V)}{ds} = 0\ 02 \text{ per millimeter}$$

Hence,

$$\frac{dp}{ds} = -(1\ 1)(100)^2(0\ 02) = 220\ \frac{N}{m^2} \text{ per millimeter}$$

(b)
$$\frac{dp}{ds} = -(1.1)(1000)^2(0.02) = 22{,}000\ \frac{N}{m^2} \text{ per millimeter}$$

Conclusion: At a point in a high-speed flow, it requires a much larger pressure gradient to achieve a given percentage change in velocity than for a low speed flow, everything else being equal

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4.38 We use the fact that total pressure is constant in an isentropic flow. From Eq. (4 74) applied in the freestream.

$$\frac{p_o}{p_\infty} = \left(1 + \frac{\gamma - 1}{\gamma} M_\infty^{\ 2}\right)^{\frac{\gamma}{\gamma-1}} = [1 + 0\ 2\ (0\ 7)^2]^{3.5} = 1\ 387$$

From Eq (4 74) applied at the point on the wing,

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = [1 + 0.2\ (1.1)^2]^{3\ 5} = 2.135$$

Hence,

$$p = \left[\left(\frac{p_o}{p_\infty}\right) / \left(\frac{p_o}{p}\right)\right] p_\infty = \left(\frac{1.387}{2.135}\right) p_\infty = 0\,65\ p_\infty$$

At a standard altitude of 3 km, from Appendix A, $p_\infty = 7.0121 \times 10^4\ N/m^2$. Hence,

$$p = (0\ 65)(7\ 0121 \times 10^4) = \boxed{4.555 \times 10^4\ N/m^2}$$

4.39 This problem is simply asking what is the equivalent airspeed, as discussed in Section 4 12. Hence,

$$V_e = V\left(\frac{\rho}{\rho_s}\right)^{1/2} = (800)\left(\frac{1.0663 \times 10^{-3}}{2.3769 \times 10^{-3}}\right)^{1/2} = \boxed{535.8\ \text{ft.sec}}$$

4.40 (a) From Eq. (4 88)

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^{\ 2}}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2} M_e^{\ 2}\right)\right]^{\frac{\gamma+1}{\gamma-1}} = \frac{1}{(10)^2}\left\{\frac{2}{2\,4}\left[1 + 0\,2\ (10)^2\right]\right\}^6 = 2\ 87 \times 10^5$$

Hence:

$$\frac{A_e}{A_t} = \sqrt{2.87 \times 10^5} = \boxed{535.9}$$

(b) From Eq. (4.87)

$$\frac{p_o}{p_e} = \left(1 + \frac{\gamma - 1}{2} M_e^{\ 2}\right)^{\frac{\gamma}{\gamma - 1}} = [1 + 0.2\,(10)^2]^{3.5} = 4.244 \times 10^4$$

At a standard altitude of 55 km, p = 48.373 N/m^2. Hence

$$p_o = (4.244 \times 10^4)(48.373) = 2.053 \times 10^6 \text{ N/m}^2 = \boxed{20.3 \text{ atm}}$$

(c) From Eq. (4.85)

$$\frac{T_o}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^{\ 2} = 1 + 0.2\,(10)^2 = 21$$

At a standard altitude of 55 km, T = 275.78 K. Hence,

$$T_o = 275.78\,(21) = \boxed{5791 \text{ K}}$$

Examining the above results, we note that:

1. The required expansion ratio of 535.9 is <u>huge</u>, but is readily manufactured.

2. The required reservoir pressure of 20.3 atm is large, but can be handled by proper design of the reservoir chamber.

3. The required reservoir temperature of 5791 K is tremendously large, especially when you remember that the surface temperature of the sun is about 6000 K. For a continuous flow hypersonic tunnel, such high reservoir tempertures can not be handled. In practice, a reservoir temperature of about half this value or less is employed, with the sacrifice made that "true temperature" simulation in the test stream is not obtained.

4.41 The speed of sound in the test stream is

$$a_e = \sqrt{\gamma \ R \ T_e} = \sqrt{(1.4)(287)(275.78} = 332.9 \ \text{m/sec}$$

Hence,

$$V_e = M_e \, a_e = 10 \ (332.9) = \boxed{3329 \ \text{m/sec}}$$

4.42 (a) From Eq. 4.88, for $M_e = 20$

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^{\,2}}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^{\,2}\right)\right]^{\frac{\gamma+1}{\gamma-1}} = \frac{1}{(20)^2}\left\{\frac{2}{2.4}\left[1+0.2 \ (20)^2\right]\right\}^6 = 2.365 \text{ x } 10^8$$

Hence:

$$\frac{A_e}{A_t} = \boxed{15{,}377}$$

(b) From Eq. (4.85)

$$\frac{T_o}{T_e} = \left(1+\frac{\gamma-1}{2}M_e^{\,2}\right)^{-1} = [1+(0.2)(20)^2]^{-1} = 0.01235$$

Hence,

$$T_e = (5791)(0.01235) = 71.5 \text{ K}$$

$$a_e = \sqrt{\gamma \ R \ T_e} = \sqrt{(1.4)(287)(71.5)} = 169.5 \ \text{m/sec}$$

$$V_e = M_e \, a_e = 20 \ (169.5) = \boxed{3390 \ \text{m/sec}}$$

Comments:

1. To obtain Mach 20, i.e., to double the Mach number in this case, the expansion ratio must be increased by a factor of 15,377/535.9 = 28.7. High hypersonic Mach numbers demand wind tunnels with very large exit-to-throat ratios. In practice, this is usually obtained by designing the nozzle with a small throat area.

2 Of particular interest is that the exit velocity is increased by a very small amount, namely by only 61 m/sec, although the exit Mach number has been doubled. The higher Mach number of 20 is achieved not by a large increase in exit velocity by rather by a large decrease in the speed of sound at the exit. This is characteristic of most conventional hypersonic wind tunnels -- the higher Mach numbers are not associated with corresponding increases in the test section flow velocities

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5.1 Assume the moment is governed by

$$M = f(V_\infty, \rho_\infty, S, \mu_\infty, a_\infty)$$

More specifically:

$$M = Z\, V_\infty^{a}\, \rho_\infty^{b}\, S^{d}\, a_\infty^{e}\, \mu_\infty^{f}$$

Equating the dimensions of mass, m, length, ℓ, and time t, and considering Z dimensionless,

$$\frac{m\ell^2}{t^2} = \left(\frac{\ell}{t}\right)^{a}\left(\frac{m}{\ell^3}\right)^{b}\left(\ell^2\right)^{d}\left(\frac{\ell}{t}\right)^{e}\left(\frac{m}{\ell t}\right)^{f}$$

$1 = b + f$ (For mass)

$2 = a - 3b + 2d + e - f$ (For length)

$-2 = -a-e-f$ (for time)

Solving a, b, and d in terms of e and f,

$b = 1 - f$

and, $a = 2 - e - f$