3.1 An examination of the standard temperature distribution through the atmosphere given in Figure 3.3 of the text shows that both 12 km and 18 km are in the same constant temperature region. Hence, the equations that apply are Eqs. (3.9) and (3.10) in the text. Since we are in the same isothermal region with therefore the same base values of p and ρ, these equations can be written as

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1} = e^{-(g_o/RT)(h_2-h_1)}$$

where points 1 and 2 are any two arbitrary points in the region. Hence, with g_o = 9.8 m/sec^2 and R = 287 joule/kgK, and letting points 1 and 2 correspond to 12 km and 18 km altitudes respectively,

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1} = e^{-\frac{9.8}{(287)(216.66)}(6000)} = 0.3884$$

Hence:

$$p_2 = (0.3884)(1.9399 \times 10^4) = \boxed{7.53 \times 10^3 \text{ N/m}^2}$$

$$\rho_2 = (0.3884)(3.1194 \times 10^{-1}) = \boxed{0.121 \text{ kg/m}^3}$$

and of course

$$T_2 = \boxed{216.66\text{K}}$$

These answers check the results listed in Appendix A of the text within round-off error.

3.2 From Appendix A of the text, we see immediately that p = 2.65 x 10^4 N/m^2 corresponds to 10,000 m, or 10 km, in the standard atmosphere. Hence,

$$\boxed{\text{pressure altitude} = 10 \text{ km}}$$

The outside air density is

$$\rho = \frac{p}{RT} = \frac{2.65x10^4}{(287)(220)} = 0\ 419\ kg/m^3$$

From Appendix A, this value of ρ corresponds to 9.88 km in the standard atmosphere.

Hence,

$$\boxed{\text{density altitude} = 9.88\ km}$$

3.3 At 35,000 ft, from Appendix B, we find that $p = 4.99 \times 10^2 = \boxed{499\ lb/ft^2}$.

3.4 From Appendix B in the text,

33,500 ft corresponds to $p = 535.89\ lb/ft^2$

32,000 ft corresponds to $\rho = 8.2704 \times 10^{-4}\ slug/ft^3$

Hence,

$$T = \frac{p}{\rho R} = \frac{535.89}{(8.2704 \times 10^{-4})(1716)} = \boxed{378\ R}$$

3.5 $$\frac{|h - h_G|}{h} = 0.02 = \left|1 - \frac{h_G}{h}\right|$$

From Eq. (3 6), the above equation becomes

$$\left|1 - \left(\frac{r + h_G}{r}\right)\right| = \left|1 - 1 - \frac{h_G}{r}\right| = 0.02$$

$$h_G = 0.02\ r = 0.02\ (6.357 \times 10^6)$$

$$h_G = 1.27 \times 10^5\ m = \boxed{127\ km}$$

3.6 $T = 15 - 0.0065h = 15 - 0.0065(5000) = -17.5°C = 255.5°K$

$$a = \frac{dT}{dh} = -0.0065$$

From Eq (3.12)

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-g_o/aR} = \left(\frac{255.5}{288}\right)^{-(9.8)/(-0.0065)(287)} = 0.533$$

$$p = 0.533\, p_1 = 0.533\,(1.01 \times 10^5) = \boxed{5.38 \times 10^4\ N/m^2}$$

3.7 $$\ell n \frac{p}{p_1} = -\frac{g}{RT}(h - h_1)$$

$$h - h_1 = -\frac{1}{g} RT\ \ell n \frac{p}{p_1} = -\frac{1}{24.9}(4157)(150)\ \ell n\, 0.5$$

Letting $h_1 = 0$ (the surface)

$$h = 17{,}358\ m = \boxed{17.358\ km}$$

3.8 A standard altitude of 25,000 ft falls within the first gradient region in the standard atmosphere. Hence, the variation of pressure and temperature are given by:

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-\frac{g}{aR}} \qquad (1)$$

and

$$T = T_1 + a\,(h - h_1) \qquad (2)$$

Differentiating Eq (1) with respect to time:

$$\frac{1}{p_1}\frac{dp}{dt} = \left(\frac{1}{T_1}\right)^{-\frac{g}{aR}}\left(-\frac{g}{AR}\right) T^{\left(-\frac{g}{aR}-1\right)}\frac{dT}{dt} \qquad (3)$$

Differentiating Eq (2) with respect to time:

$$\frac{dT}{dt} = a\frac{dh}{dt} \qquad (4)$$

Substitute Eq (4) into (3)

$$\frac{dp}{dt} = -\ p_1(T_1)^{\frac{g}{aR}}\left(\frac{g}{R}\right) T^{-\left(\frac{g}{aR}+1\right)}\frac{dh}{dt} \qquad (5)$$

In Eq. (5), dh/dt is the rate-of-climb, given by dh/dt = 500 ft/sec. Also, in the first gradient region, the lapse rate can be calculated from the tabulations in Appendix B. For example, take 0 ft and 10,000 ft, we find

$$a = \frac{T_2 - T_1}{h_2 - h_1} = \frac{483.04 - 518.69}{10{,}000 - 0} = -0.00357\frac{{}^\circ R}{ft}$$

Also from Appendix B, p_1 = 2116.2 lb/ft^2 at sea level, and T = 429.64 °R at 25,000 ft. Thus,

$$\frac{g}{aR} = \frac{32.2}{(-0.00357)(1716)} = -5.256$$

Hence, from Eq (5)

$$\frac{dp}{dt} = -(2116.2)(518.69)^{-5.256}\left(\frac{32.2}{1716}\right)(429.64)^{4.256}(500)$$

$$\frac{dp}{dt} = \boxed{-17.17\frac{lb}{ft^2\ sec}}$$

3.9 From the hydrostatic equation, Eq. (3.2) or (3.3),

$$dp = -\rho \, g_o \, dh$$

or $$\frac{dp}{dt} = -\rho \, g_o \, \frac{dh}{dt}$$

The upward speed of the elevator is dh/dt, which is

$$\frac{dh}{dt} = \frac{dp / dt}{-\rho \, g_o}$$

At sea level, $\rho = 1.225 \text{ kg/m}^3$. Also, a one-percent change in presure per minute starting from sea level is

$$\frac{dp}{dt} = -(1.01 \times 10^5)(0.01) = -1.01 \times 10^3 \text{ N/m}^2 \text{ per minute}$$

Hence

$$\frac{dh}{dt} = \frac{-1.01 \times 10^3}{(1.225)(9.8)} = \boxed{84.1 \text{ meter per minute}}$$

3.10 From Appendix B:

At 35,500 ft: $p = 535.89 \text{ lb/ft}^2$

At 34,000 ft: $p = 523.47 \text{ lb/ft}^2$

For a pressure of 530 lb/ft^2, the pressure altitude is

$$33{,}500 + 500 \left(\frac{535.89 - 530}{535.89 - 523.47} \right) = \boxed{33737 \text{ ft}}$$

The density at the altitude at which the airplane is flying is

$$\rho = \frac{p}{RT} = \frac{530}{(1716)(390)} = 7.919 \times 10^{-4} \text{ slug/ft}^3$$

From Appendix B:

At 33,000 ft: $\rho = 7\ 9656 \times 10^{-4}$ slug/ft^3

At 33,500 ft: $\rho = 7\ 8165 \times 10^{-4}$ slug/ft^3

Hence, the density altitude is

$$33{,}000 + 500\left(\frac{7.9656 - 7.919}{7\ 9656 - 7.8165}\right) = \boxed{33{,}156 \text{ ft}}$$

4.1 $A_1V_1 = A_2V_2$

Let points 1 and 2 denote the inlet and exit conditions respectively Then,

$$V_2 = V_1\left(\frac{A_1}{A_2}\right) = (5)\left(\frac{1}{4}\right) = \boxed{1.25 \text{ ft/sec}}$$

4.2 From Bernoulli's equation,

$$p_1 + \rho\,\frac{V_1^2}{2} = p_2 + \rho\,\frac{V_2^2}{2}$$

$$p_2 - p_1 = \frac{\rho}{2}\,(V_1^2 - V_2^2)$$

In consistent units,

$$\rho = \frac{62.4}{32.2} = 1\ 94 \text{ slug/ft}^3$$

Hence,

$$p_2 - p_1 = \frac{1.94}{2}\,[(5)^2 - (1\ 25)^2]$$

$$p_2 - p_1 = 0.97\ (23\ 4) = \boxed{22.7 \text{ lb/ft}^2}$$
