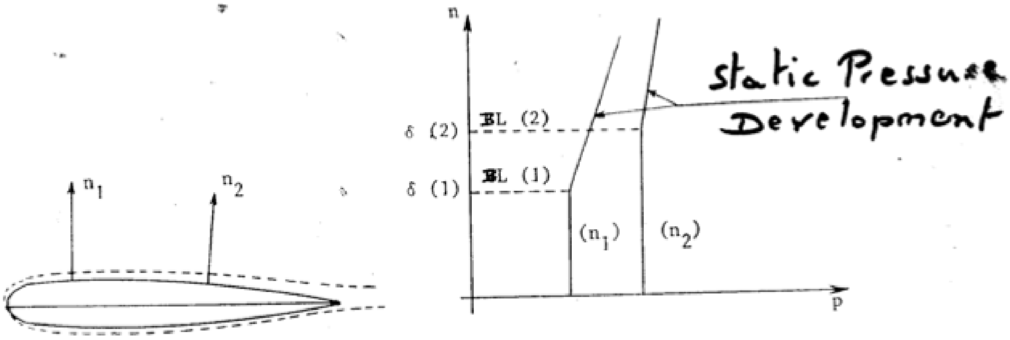
Pressure Coefficient, C_p

How to quantify the Velocity Distribution around an Airfoil

- Static Pressure Distribution

$$\frac{1}{\rho}\overrightarrow{grad\,p} + \overrightarrow{grad\,\frac{v^2}{2}} = 0$$



By Definition:

$$C_p = \frac{P - P_0}{\frac{1}{2}\rho {v_0}^2}$$

Where:

P - Static pressure at the point of interest

P_0 - Free stream static pressure

v_0 - Free stream velocity

ρ - Free stream density

$$q_0 = \frac{1}{2}\rho {v_0}^2$$

$$\therefore \boxed{C_p = \frac{P - P_0}{q_0}}$$

Also:

$$P_0 = R\rho_0 T_0$$

$$v_0 = M_0 a_0 \quad \& \quad {a_0}^2 = \gamma RT$$

$$\therefore q_0 = \frac{1}{2}\rho {v_0}^2 = \frac{1}{2}\rho (M_0 a_0)^2 = \frac{1}{2}{M_0}^2 \gamma R\rho T = \frac{1}{2}{M_0}^2\gamma P_0$$

$$\therefore C_p = \frac{P - P_0}{\frac{1}{2}{M_0}^2\gamma P_0}$$

$$\therefore \boxed{C_p = \frac{\frac{P}{P_0} - 1}{\frac{1}{2}\gamma {M_0}^2}}$$

Incompressible Flow

$\forall M_0$, Bernoulli's Equation is given by:

$$\frac{1}{\rho}dp + v.dv = 0$$

Pressure Coefficient, C_p

If Incompressible Subsonic Flows: $\rho = \rho_0 = \text{constant}$

$$\therefore P + \frac{1}{2}\rho_0 v^2 = P_0 + \frac{1}{2}\rho_0 {v_0}^2$$

$$\therefore P - P_0 = \frac{1}{2}\rho_0({v_0}^2 - v^2)$$

Therefore:

$$C_p = \frac{P - P_0}{\frac{1}{2}\rho_0 {v_0}^2} = \frac{{v_0}^2 - v^2}{{v_0}^2}$$

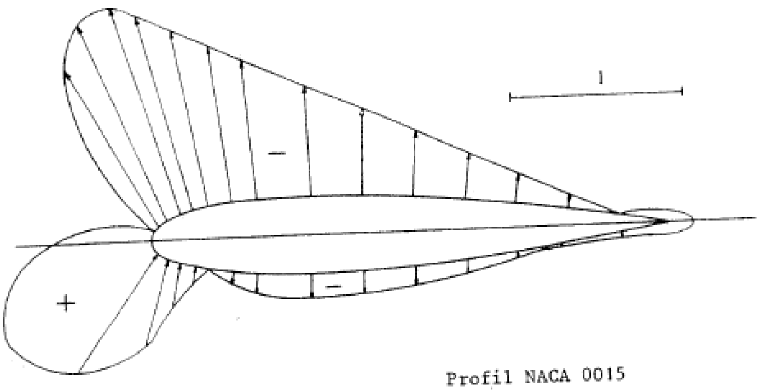
$$\therefore\ C_p = 1 - \left(\frac{v}{v_0}\right)^2$$

This creates 2 main results:

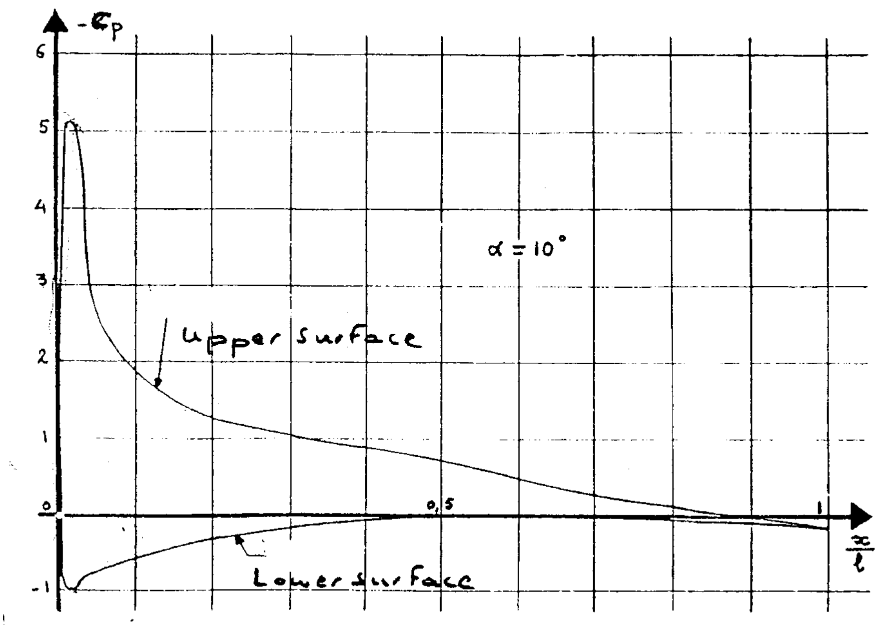
- At the Stagnation Point
  v = 0 => C_p = 1

- v = v_0 => C_p = 0

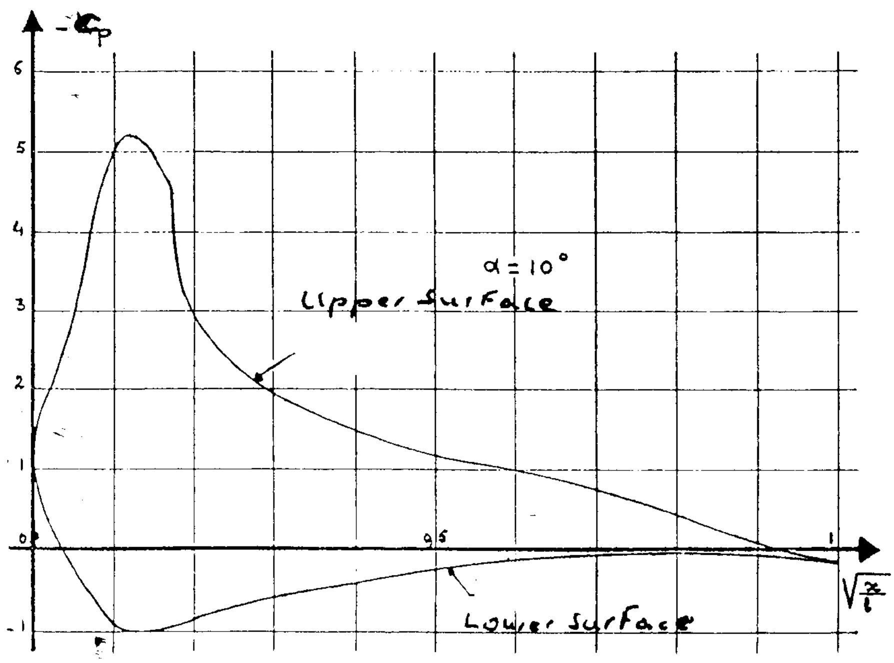
Pressure Distribution Around A Profile



C_p Distributions Along A Profile



Pressure Coefficient, C_p



Boundary Layer Development Along A Profile