23-01-12

zaterdag 26 januari 2013
12:28

+ = vrij zeker goed, +/- = twijfel, - = 'educated guess'

1 D non-symmetric +/-

2 $y = 4x(-t/2) = 12\,\text{sinc}(-3t) = 12\,\text{sinc}(3t)$ $y = 4\,\Pi(t/3)$, C +

3 $x = 4r(t+1) - 8r(t) + 4r(t-2) + 4$

avg. $P = \frac{4^2}{2} = 8\,W$

C +/-

4 C + keep in mind that this one is linear. according to the new definition, it's order would be three!

5 everything from 3-7 Hz. $10\sin^2(4\pi t) = 5 - 5\cos 8\pi t$ $f(-5\cos 8\pi t) = 4\,Hz$
so $y(t) = -5\cos 8\pi t$

B +

6 B +
fold around $f_s/2$

7 highest f in signal is 23 Hz (take fourier like in 14A).

D +

8 B +

9 $\frac{5}{2^{10}} = 4.9\cdot10^{-3}\,V$
max deviation from real value is $2.4\cdot10^{-3}\,V$

A +

10 B + (ϕ is not linear. $\phi(2x)$ will become $\phi(2x\sqrt{2})$)

11 signal power is A^2. noise power is σ^2. $\frac{A^2}{\sigma^2}$, D +

12 $\phi\left(\frac{5}{2.2}\right) = 1.096$

C +/-

13 trigo fourier series: $x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$

this is an even function, so all b_n are 0.

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,dt = 0$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos(n\omega_0 t)\,dt = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\cos\left(\frac{2n\pi t}{T_0}\right)dt$$

$$= \frac{2}{T_0}\int_{-T_0/2}^{0} A\left(1+\frac{4t}{T_0}\right)\cos\left(\frac{2n\pi t}{T_0}\right)dt + \frac{2}{T_0}\int_0^{T_0/2} A\left(1-\frac{4T}{T_0}\right)\cos\left(\frac{2n\pi t}{T_0}\right)dt$$

$$= \frac{2A}{T_0}\int_{-T_0/2}^{0} \frac{4t}{T_0}\cos\frac{2n\pi t}{T_0} + \cos\frac{2n\pi t}{T_0}\,dt + \frac{2A}{T_0}\int_0^{T_0/2}\cos\frac{2n\pi t}{T_0} - \frac{4t}{T_0}\cos\frac{2n\pi t}{T_0}\,dt$$

$$= \frac{2AT_0^2}{4n^2\pi^2T_0}\left[\frac{4}{T_0}\cos\frac{2n\pi t}{T_0} + \frac{4\cdot 2n\pi t}{T_0\cdot T_0}\sin\frac{2n\pi t}{T_0}\right]_{-T_0/2}^{0} + \frac{2A}{T_0}\left[\frac{T_0}{2n\pi}\sin\frac{2n\pi T}{T_0}\right]_{-T_0/2}^{0}$$

$$- \frac{2AT_0^2}{4n^2\pi^2T_0}\left[\frac{4}{T_0}\cos\frac{2n\pi t}{T_0} + \frac{8n\pi t}{T_0^2}\sin\frac{2n\pi t}{T_0}\right]_0^{T_0/2} + \frac{2A}{T_0}\left[\frac{T_0}{2\pi n}\sin\frac{2n\pi t}{T_0}\right]_0^{T_0/2}$$

$$= \frac{2A}{n^2\pi^2}\left(1 - \left(\cos\frac{-2n\pi T_0/2}{T_0} - \frac{2n\pi T_0/2}{T_0}\sin\frac{-2n\pi T_0/2}{T_0}\right)\right) + \frac{A}{n\pi}\left(-\sin\frac{-2n\pi T_0/2}{T_0}\right)$$

$$- \frac{2A}{n^2\pi^2}\left(\left(\cos\frac{2n\pi T_0/2}{T_0} + \frac{2n\pi T_0/2}{T_0}\sin\frac{2n\pi T_0/2}{T_0}\right) - 1\right) + \frac{A}{n\pi}\sin\frac{2n\pi T_0/2}{T_0}$$

$$= \frac{2A}{n^2\pi^2} - \frac{2A}{n^2\pi^2}\cos n\pi - \frac{2A}{n^3\pi^3}\sin n\pi + \frac{A}{n\pi}\sin n\pi - \frac{2A}{n^2\pi^2}\cos n\pi - \frac{2A}{n^3\pi^3}\sin n\pi + \frac{2A}{n^2\pi^2}$$

$$+ \frac{A}{n\pi}\sin n\pi$$

$$= \frac{4A}{n^2\pi^2} - \frac{4A}{n^2\pi^2}\cos n\pi - \frac{4A}{n^3\pi^3}\sin n\pi + \frac{2A}{n\pi}\sin n\pi$$

for n = even: $\frac{4A}{n^2\pi^2} - \frac{4A}{n^2\pi^2} = 0$, so $a_{n,\,n=\text{even}} = 0$.

for n = odd: $\frac{4A}{n^2\pi^2} + \frac{4A}{n^2\pi^2} = \frac{8A}{n^2\pi^2}$, so $a_{n,\,n=\text{odd}} = \frac{8A}{n^2\pi^2}$

so, substituting $n = 2n-1$ to only use the odd n, $x(t)$ becomes:

so, substituting $n = 2n-1$ to only use the odd n, $x(t)$ becomes:

$$x(t) = \frac{8A}{\pi^2}\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos\left((2n-1)\frac{2\pi t}{T_0}\right)$$

+

14 a $6\,\text{sinc}(3t)\cos^2(12\pi t) = 3\,\text{sinc}(3t)(1+\cos 24\pi t) = 3\,\text{sinc}\,3t + 3\,\text{sinc}\,3t\cos 24\pi t$

$$X(f) = \Pi\left(\frac{f}{3}\right) + \Pi\left(\frac{f}{3}\right) * \left(\tfrac{1}{2}\delta(f-12) + \tfrac{1}{2}\delta(f+12)\right)$$

$$= \Pi\left(\frac{f}{3}\right) + \tfrac{1}{2}\Pi\left(\frac{f+12}{3}\right) + \tfrac{1}{2}\Pi\left(\frac{f-12}{3}\right)$$ +

all other multiplications are 0. in exam, write the full multiplication!

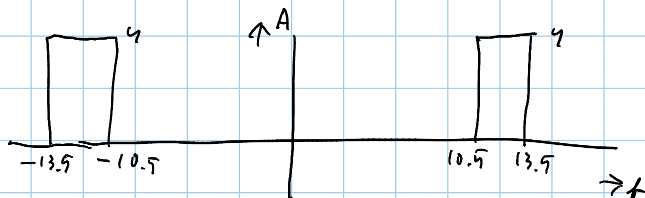
b $$E = \int_{-\infty}^{\infty} |X(f)|^2\,df = \int_{-\infty}^{\infty} \Pi\left(\frac{f}{3}\right)^2 + \tfrac{1}{4}\Pi\left(\frac{f+12}{3}\right)^2 + \tfrac{1}{4}\Pi\left(\frac{f-12}{3}\right)^2 df$$

$$= [f]_{-1\frac{1}{2}}^{1\frac{1}{2}} + \tfrac{1}{2}[f]_{10\frac{1}{2}}^{13\frac{1}{2}} = 4\tfrac{1}{2}\ \text{J}$$ +

c $$Y(f) = 4\Pi\left(\frac{f}{3}\right) * \left(\tfrac{1}{2}\delta(f-12) + \tfrac{1}{2}\delta(f+12)\right) = 2\Pi\left(\frac{f+12}{3}\right) + 2\Pi\left(\frac{f-12}{3}\right)$$

$$Y(f) = X(f)H(f),\quad H(f) = \frac{Y(f)}{X(f)} = \frac{2\Pi\left(\frac{f+12}{3}\right) + 2\Pi\left(\frac{f-12}{3}\right)}{\Pi\left(\frac{f}{3}\right) + \frac{1}{2}\Pi\left(\frac{f+12}{3}\right) + \frac{1}{2}\Pi\left(\frac{f-12}{3}\right)}$$

$$= 4\Pi\left(\frac{f+12}{3}\right) + 4\Pi\left(\frac{f-12}{3}\right)$$



+/-

d $$Y(f)^2 = 4\Pi\left(\frac{f+12}{3}\right)^2 + 4\Pi\left(\frac{f-12}{3}\right)^2 \quad \text{(again, others are zero)}$$

$$\int_{-\infty}^{\infty} 4\Pi\left(\frac{f+12}{3}\right)^2 + 4\Pi\left(\frac{f-12}{3}\right)^2 df = 8[f]_{10\frac{1}{2}}^{13\frac{1}{2}} = 24\ \text{J}$$ +

e yes. the $H(f)$ determined in c only takes in to account the frequencies that can be found in $x(t)$ and $y(t)$. all other frequencies are ignored, because they don't influence $y(t)$. so for the other frequencies, a random frequency response can be chosen.

+

15 a $$P_{Rx} = P_{Tx} G_{at}\left(\frac{\lambda}{4\pi d}\right)^2 G_{ar} = 10\log 250 + 10\log\left(\left(\frac{3\cdot 10^8}{4\pi\cdot 670000\cdot 1090\cdot 10^6}\right)^2\right) + 10\log 10 - 10\log 4 = -121.72\ \text{dBW}$$

b $$P = kTB = 1.38\cdot 10^{-23}\cdot 362\cdot 2\cdot 10^6 = 9.99\cdot 10^{-15}\ \text{W}$$ +

$$10\log(9.9912\cdot 10^{-15}) = -140\ \text{dBW}$$ +

$10 \log(9.99(2 \cdot 10^{-15})) = -140$ dBW

c $\quad SNR = -121.72 - -140 = 18.28$ dB

so, the requirement is just met.