Non-ideal gas turbines

Previously, we have considered an ideal gas turbine. But we don't live in an ideal world. So now it's time to get rid of most of the simplifying assumptions.

1 Adjustments for the real world

In a non-ideal world, things are often slightly different than in an ideal world. How do we take those differences into account? That's what we'll examine now.

1.1 Specific heat

Previously, we have assumed that the specific heat c_p and the specific heat ratio k were constant. However, they are not. They vary because of three reasons. The temperature changes, the pressure changes and the composition of the gas changes. The latter is caused by adding fuel. The change in c_p and k due to pressure changes is usually negligible. However, temperature and composition change do have an important effect.

How do we cope with this? Well, when performing computer calculations, we can simply make several iterations. But, for hand calculations, this is too much work. Instead, we can take mean values for c_p and k, for every step. Often used values are $c_{p,air} = 1000 J/kg\,K$ and $k_{air} = 1.4$ for the compression stage. Similarly, $c_{p,gas} = 1150 J/kg\,K$ and $k_{gas} = 1.33$ for the expansion stage in the turbine. By doing this, our calculations are quite accurate. But deviations from the real world of up to 5% may still occur.

1.2 Kinetic energy

Previously, we have neglected the kinetic energy of the gas. But the gas often has a non-negligible velocity c. To solve this problem, we use a nice trick. We define the **total enthalpy** h_0, the **total temperature** T_0 and the **total pressure** p_0. The total enthalpy is defined as

$$h_0 = h + \frac{1}{2}c^2, \tag{1.1}$$

where h is the static enthalpy. With static enthalpy, we mean the enthalpy when not taking into account the velocity. We can derive the total temperature, by using

$$h_0 = c_p T_0 = c_p T + \frac{1}{2}c^2, \qquad \text{which gives} \qquad T_0 = T + \frac{c^2}{2c_p}. \tag{1.2}$$

We can find the total pressure p_0 from the isentropic flow relations. The result will be

$$p_0 = p\left(\frac{T_0}{T}\right)^{\frac{k}{k-1}}. \tag{1.3}$$

When using these total values, we don't have to take into account the kinetic energy anymore. That makes life just a bit easier.

2 The isentropic and the polytropic efficiency

There are two very important parameters, that strongly influence the gas turbine properties. They are the isentropic and the polytropic efficiency. Let's examine them.

2.1 Isentropic efficiency

Let's examine the compressor and the turbine. In reality, they don't perform their work isentropically. To see what does happen, we examine the compressor. In the compressor, the gas is compressed. In an ideal (isentropic) case, the enthalpy would rise from h_{02} to h_{03s}. However, in reality, it rises from h_{02} to h_{03}, which is a bigger increase. Similarly, in an ideal (isentropic) turbine, the enthalpy would decrease from h_{04} to h_{0gs}. However, in reality, it decreases from h_{04} to h_{0g}, which is a smaller decrease. Both these changes are visualized in figure 1.

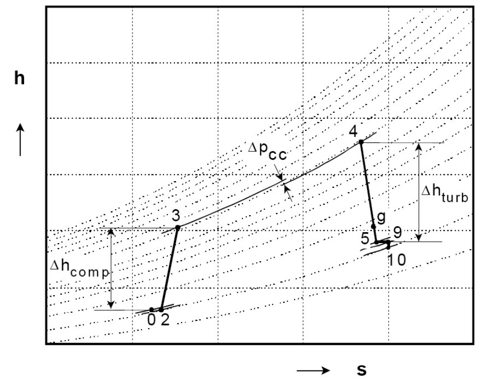

Figure 1: The enthalpy-entropy diagram for a non-ideal cycle.

This effect can be expressed in the **isentropic efficiency**. The efficiencies for compression and expansion are, respectively, given by

$$\eta_{is,c} = \frac{h_{03s} - h_{02}}{h_{03} - h_{02}} = \frac{T_{03s} - T_{02}}{T_{03} - T_{02}} \qquad \text{and} \qquad \eta_{is,t} = \frac{h_{04} - h_{0g}}{h_{04} - h_{0gs}} = \frac{T_{04s} - T_{0g}}{T_{04} - T_{0gs}}. \tag{2.1}$$

You may have trouble remembering which difference goes on top of the fraction, and which one goes below. In that case, just remember that we always have $\eta_{is} \leq 1$.

By using the isentropic relations, we can rewrite the above equations. We then find that

$$\eta_{is,c} = \frac{\left(\frac{p_{03}}{p_{02}}\right)^{\frac{k_{air}-1}{k_{air}}} - 1}{\frac{T_{03}}{T_{02}-1}} \qquad \text{and} \qquad \eta_{is,t} = \frac{\frac{T_{0g}}{T_{04}-1}}{\left(\frac{p_{0g}}{p_{04}}\right)^{\frac{k_{gas}-1}{k_{gas}}} - 1}. \tag{2.2}$$

The specific work received by the compressor, and delivered by the turbine, is now given by

$$\dot{W}_{s,c} = \frac{c_{p_{air}} T_{02}}{\eta_{is,c}} \left(\left(\frac{p_{03}}{p_{02}}\right)^{\frac{k_{air}-1}{k_{air}}} - 1 \right) \qquad \text{and} \qquad \dot{W}_{s,t} = c_{p_{gas}} T_{04} \eta_{is,t} \left(\left(\frac{p_{0g}}{p_{04}}\right)^{\frac{k_{gas}-1}{k_{gas}}} - 1 \right). \tag{2.3}$$

Note that, through the definition of p_{0g}, these two quantities must be equal to each other. (That is, as long as the mass flow doesn't change, and there are no additional losses when transmitting the work.)

2.2 Polytropic efficiency

Let's examine a compressor with a varying pressure ratio. In this case, it turns out that also the isentropic efficiency varies. That makes it difficult to work with.

To solve this problem, we divide the compression into an infinite number of small steps. All these infinitely small steps have the same isentropic efficiency. This efficiency is known as the **polytropic efficiency**.

The resulting process is also known as a **polytropic** process. This means that there is a **polytropic exponent** n, satisfying

$$\frac{T_0}{T_{0_{initial}}} = \left(\frac{p_0}{p_{0_{initial}}}\right)^{\frac{n_{air}}{n_{air}-1}}. \tag{2.4}$$

The polytropic efficiencies for compression $\eta_{\infty c}$ and expansion $\eta_{\infty t}$ are now given by, respectively,

$$\eta_{\infty c} = \frac{k_{air}-1}{k_{air}}\frac{n_{air}}{n_{air}-1} = \frac{\ln\left(\frac{p_{03}}{p_{02}}\right)^{\frac{k_{air}-1}{k_{air}}}}{\ln\left(\frac{T_{03}}{T_{02}}\right)} \qquad \text{and} \qquad \eta_{\infty t} = \frac{k_{gas}}{k_{gas}-1}\frac{n_{gas}-1}{n_{gas}} = \frac{\ln\left(\frac{T_{0g}}{T_{04}}\right)}{\ln\left(\frac{p_{0g}}{p_{04}}\right)^{\frac{k_{gas}-1}{k_{gas}}}}. \tag{2.5}$$

The full compression/expansion process also has an isentropic efficiency. It is different from the polytropic efficiency. In fact, the relation between the two is given by

$$\eta_{is,c} = \frac{\left(\frac{p_{03}}{p_{02}}\right)^{\frac{k_{air}-1}{k_{air}}}-1}{\left(\frac{p_{03}}{p_{02}}\right)^{\frac{k_{air}-1}{\eta_{\infty c}k_{air}}}-1} \qquad \text{and} \qquad \eta_{is,t} = \frac{\left(\frac{p_{0g}}{p_{04}}\right)^{\eta_{\infty t}\frac{k_{gas}-1}{k_{gas}}}-1}{\left(\frac{p_{0g}}{p_{04}}\right)^{\frac{k_{gas}-1}{k_{gas}}}-1}. \tag{2.6}$$

There are a few important rules to remember. For compression, the polytropic efficiency is higher than the isentropic efficiency. (So, $\eta_{\infty c} > \eta_{is,c}$.) For expansion, the polytropic efficiency is lower than the isentropic efficiency. (So, $\eta_{\infty t} < \eta_{is,t}$.) Finally, if the pressure ratio increases, then the difference between the two efficiencies increases.

3 Losses occurring in the gas turbine

In a non-ideal world, losses occur at several places in the gas turbine. There are also several types of losses. We will examine a few.

3.1 Pressure losses

Previously, we have assumed that no pressure losses occurred. This is, of course, not true. Pressure losses occur at several places. First of all, in the combustor. The **combustion chamber pressure loss** is given by $\Delta p_{cc} = p_{03} - p_{04}$. The **combustor pressure loss factor** is now defined as

$$\varepsilon_{cc} = \frac{p_{04}}{p_{03}} = \frac{p_{03} - \Delta p_{cc}}{p_{03}}. \tag{3.1}$$

Pressure losses also occur at the inlet and at the exhaust duct. For industrial gas turbines, these pressure losses are defined as

$$\Delta p_{0_{inlet}} = p_{amb} - p_{01} \qquad \text{and} \qquad \Delta p_{0_{exhaust}} = p_{05} - p_{amb}. \tag{3.2}$$

We will examine these pressure differences for jet engines in a later chapter.

3.2 Mechanical losses

Losses also occur due to internal friction in the system. These **mechanical losses** are joined together in one term, being the **transmission efficiency** η_m. It is given by

$$\eta_m = \frac{\text{turbine power} - \text{mechanical losses}}{\text{turbine power}}. \tag{3.3}$$

3.3 Combustor efficiency

Ideally, we will have a **full combustion** of the fuel in the combustion chamber. In this case, we would get the maximum heat out of it. This maximum heat is called the **lower heating value** LHV of the fuel, also known as the **lower calorific value** LCV. However, in reality, we have an **incomplete combustion**. This results in combustion products like carbon monoxide (CO) and unburned fuel.

Next to this, heat may also escape. To take this into account, the **combustor efficiency** η_{cc} is used. It is defined as

$$\eta_{cc} = \frac{\dot{m}_{air} c_{p_{gas}} \left(T_{04} - T_{03}\right)}{\dot{m}_{fuel} LHV}. \tag{3.4}$$

3.4 Heat exchange

Let's reconsider the heat exchanger. In the previous chapter, we have assumed that, after heat exchange, we had $T_{03.5} = T_{05}$. In other words, the heat of the gas entering the combustor equals the heat of the gas leaving the turbine. In reality, this is of course not the case. We thus have $T_{03.5} < T_{05}$.

To take this effect into account, we use the **heat exchanger effectiveness** E. First, we define the coefficients C_{cold} and C_{hot} as

$$C_{cold} = c_{p_{in,cold}} \dot{m}_{in,cold} \qquad \text{and} \qquad C_{hot} = c_{p_{in,hot}} \dot{m}_{in,hot}. \tag{3.5}$$

The subscript cold stands for the cold flow: the flow leaving the compressor. Similarly, hot stands for the flow leaving the turbine. We also define C_{min} as the lowest of the coefficients C_{cold} and C_{hot}. Now, the heat exchanger effectiveness E is given by

$$E = \frac{C_{cold}}{C_{min}} \frac{T_{0_{out,cold}} - T_{0_{in,cold}}}{T_{0_{in,hot}} - T_{0_{out,cold}}} = \frac{C_{hot}}{C_{min}} \frac{T_{0_{in,hot}} - T_{0_{out,hot}}}{T_{0_{out,hot}} - T_{0_{in,cold}}}. \tag{3.6}$$

The term in the top of the fraction indicates the amount of heat exchanged. The term in the bottom is a measure of the amount heat that can be exchanged.

We can try to simplify the above relation. If the mass flow and the specific heat are constant ($\dot{m}_{cold} = \dot{m}_{hot}$ and $c_{p_{cold}} = c_{p_{hot}}$), then the effectiveness is given by

$$E = \frac{T_{03.5} - T_{03}}{T_{05} - T_{03}}. \tag{3.7}$$