Compressor and turbines

In this chapter, we will look at the compressor and the turbine. They are both **turbomachinery**: machines that transfer energy from a rotor to a fluid, or the other way around. The working principle of the compressor and the turbine is therefore quite similar.

1 Axial compressors

We will mainly look at axial compressors. This is because they are the most used type of compressors. Also, axial compressors work very similar to axial turbines.

1.1 Euler's equation for turbomachinery

Let's examine a rotor, rotating at a constant angular velocity ω. The initial radius of the rotor is r_1, while the final radius is r_2. A gas passes through the rotor with a constant velocity c. The rotor causes a moment M on the gas. The power needed by the rotor is thus $P = M\omega$.

It would be nice if we can find an expression for this moment M. For that, we first look at the force F acting on the gas. It is given by

$$F = \frac{d(mc)}{dt} = \dot{m}c, \tag{1.1}$$

where we have used the assumption that c stays constant. Only the tangential component F_u contributes to the moment. Every bit of gas contributes to this tangential force. It does this according to

$$dF_u = \dot{m}dc_u, \tag{1.2}$$

where c_u is the tangential velocity of the air. Let's integrate over the entire rotor. We then find that

$$M = \int_1^2 dM = \int_1^2 r\,dF_u = \dot{m}\int_1^2 r\,dc_u = \dot{m}\left(c_{u,2}r_2 - c_{u,1}r_1\right). \tag{1.3}$$

The power is now given by

$$P = M\omega = \dot{m}\left(c_{u,2}r_2 - c_{u,1}r_1\right)\omega = \dot{m}\left(c_{u,2}u_2 - c_{u,1}u_1\right). \tag{1.4}$$

In this equation, u denotes the speed of the rotor at a certain radius r. We have also used the fact that $\omega = u_1/r_1 = u_2/r_2$. The above equation is known as **Euler's equation for turbomachinery**.

1.2 The axial compressor power

Let's examine an axial compressor. This compressor has **rotors** and **stators**. The rotors are moving. They increase the kinetic energy of the gas. The stators are not moving. They are used to turn the kinetic energy into an increase in pressure.

Let's examine the velocities of the gas, as it passes through a rotor and a stator. The situation is shown in figure 1. At the point we're examining, the rotor is moving with a velocity u. The velocity of the gas, relative to the rotor, is denoted by v. The angle between the flow velocity c and the shaft axis is denoted by α. The angle between the rotor blade angle and the shaft axis is denoted by β.

The component of the velocity c in axial direction is denoted by c_a. It is assumed to be constant along the compressor. There is a relation between c_a and u. It is given by

$$u = c_a\left(\tan\alpha_1 + \tan\beta_1\right) = c_a\left(\tan\alpha_2 + \tan\beta_2\right). \tag{1.5}$$

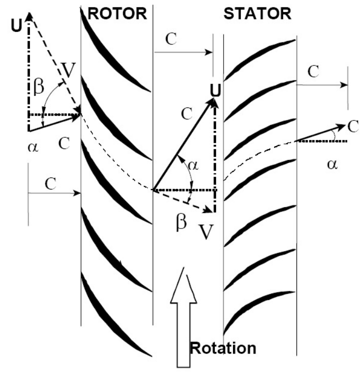

Figure 1: The velocities of the gas in the axial compressor.

It follows that

$$(\tan \alpha_1 + \tan \beta_1) = (\tan \alpha_2 + \tan \beta_2)\,. \tag{1.6}$$

Let's try to use Euler's equation for turbomachinery. We know that $u_1 = u_2 = u$. We can also use the relations

$$c_{u,1} = c_a \tan \alpha_1 \qquad \text{and} \qquad c_{u,2} = c_a \tan \alpha_2. \tag{1.7}$$

Finally, we divide Euler's equation by the mass flow. This gives us the **specific power** W_s. We thus find that

$$W_s = u\,(c_{u,2} - c_{u,2})\,. \tag{1.8}$$

In reality, this isn't the compressor power. This is mainly because the axial flow velocity c_a varies for different radial distances r. To take this effect into account, we use the **work-done factor** λ. The power of the compressor is now given by

$$P = \lambda \dot{m} W_s. \tag{1.9}$$

Generally, λ becomes small for compressors with multiple stages.

1.3 Axial compressor properties

An important effect of the compressor is a rise in the total (stagnation) temperature. In fact, this rise is given by

$$\Delta T_{0s} = \frac{W_s}{c_p} = \frac{\lambda u c_a}{c_p}\,(\tan \alpha_2 - \tan \alpha_1)\,. \tag{1.10}$$

If we take into account the isentropic efficiency η_{is} of the compression stage, then we can also find the pressure ratio. It is given by

$$\varepsilon = \left(1 + \eta_{is} \frac{\Delta T_{0s}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\eta_{is} \lambda u c_a (\tan \alpha_2 - \tan \alpha_1)}{c_p T_{01}}\right)^{\frac{\gamma}{\gamma-1}}. \tag{1.11}$$

From this, we can also derive the degree of reaction. The **degree of reaction** Λ states which part of the total temperature rise is caused by the rotor. We thus have

$$\Lambda = \frac{\Delta T_{0s,rotor}}{\Delta T_{0s,rotor} + \Delta T_{0s,stator}}. \tag{1.12}$$

It can be shown that, for an axial compressor, we have

$$\Lambda = \frac{c_a}{2u}\,(\tan \beta_2 + \tan \beta_1) = 1 - \frac{c_a}{2u}\,(\tan \alpha_1 + \tan \alpha_2)\,. \tag{1.13}$$

1.4 Three-dimensional axial compressors

Previously, we have considered axial compressors as two-dimensional. In reality, they are three-dimensional. The most important effect is that the flow is pushed to the outside of the compressor. This causes a pressure gradient, according to

$$\frac{dp}{dr} = \rho \frac{c_u^2}{r}. \tag{1.14}$$

We can combine this equation with the thermodynamic relation

$$dh = T\,ds + \frac{1}{\rho} dp = T\,ds + \frac{c_u^2}{r} dr. \tag{1.15}$$

We should also examine the definition of the total enthalpy. It is given by

$$h_0 = h + \frac{1}{2} c^2 = h + \frac{1}{2}\left(c_u^2 + c_a^2\right). \tag{1.16}$$

Let's take the derivative of the last two equations, with respect to r. The entropy usually doesn't vary much in radial direction, so we may neglect ds/dr. Combining all data will then give

$$\frac{dh_0}{dr} = \frac{c_u^2}{r} + c_u \frac{dc_u}{dr} + c_a \frac{dc_a}{dr}. \tag{1.17}$$

This equation is called the **vortex energy equation**. Usually, we also assume that the total enthalpy h_0 and the axial velocity c_a don't vary in radial direction. (So $dc_a/dr = dh_0/dr = 0$.) In this case, we can solve for c_u. It is given by

$$c_u r = \text{constant}. \tag{1.18}$$

This condition is known as the **free vortex condition**. It's often used, when designing compressors. The necessary twist of the compressor blades can be derived from it.

2 Compressor and turbine behaviour

Compressors and turbines often show strange behaviour. Let's take a look at the kind of phenomena that may occur, and how we can analyze them.

2.1 The characteristic

The compressor has several important parameters. There are the mass flow $\dot{m}$, the initial and final temperatures T_{02} and T_{03}, the initial and final pressures p_{02} and p_{03}, the shaft speed ω (also denoted as N or Ω), the rotor diameter D, and so on.

Let's suppose we'll be working with different kinds of compressors. In this case, it would be nice if we could compare these parameters in some way. To do that, dimensionless parameters are used. By using dimensional analysis, we can find that there are four dimensionless parameter groups. They are

$$\frac{\dot{m}\sqrt{RT_{02}}}{p_{02}D^2}, \qquad \frac{p_{03}}{p_{02}}, \qquad \frac{\omega D}{\sqrt{RT_{02}}} \qquad \text{and} \qquad \eta. \tag{2.1}$$

These parameter groups are known as the **mass flow parameter group**, the **pressure ratio**, the **shaft speed parameter group** and the **efficiency**. The efficiency can be either polytropic or isentropic. (These two efficiencies depend on each other anyway.)

The relation between the four dimensionless parameters can be captured in a graph, known as a **characteristic**. An example of a characteristic is shown in figure 2.

When applying dimensional analysis to a turbine, the same results will be found. However, this time the initial and final pressures are p_{04} and p_{05}. The initial and final temperatures are T_{04} and T_{05}.

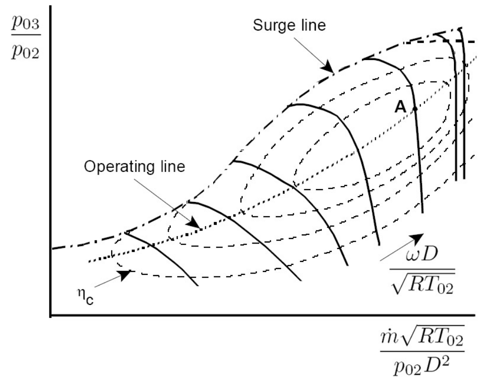

Figure 2: The characteristic graph of a compressor.

2.2 Stall

Let's examine the air entering the rotor. Previously, we have assumed that this air has exactly the right **angle of incidence** i to follow the curvature of the rotor blade. In reality, this is of course not the case. In fact, if the angle of incidence is too far off, then the flow can't follow the curvature of the rotor blades. This is called **stall**.

Stall usually starts at one rotor blade. However, this stall alters the flow properties of the air around it. Because of this, stall spreads around the rotor. And it does this opposite to the direction of rotation of the rotor. This phenomenon is called **rotating stall**.

Often, only the tips of the rotor blades are subject to stall. This is because the velocity is highest there. This is called **part span stall**. If, however, the stall spreads to the root of the blade, then we have **full span stall**.

For high compressor speeds ω, stall usually occurs at the last stages. On the other hand, for low compressor speeds, stall occurs at the first stages. Generally, the possibility of stalling increases if we get further to the left of the characteristic. (See also figure 2.)

2.3 Surge

Let's suppose we control the mass flow $\dot{m}$ in a compressor, runnning at a constant speed ω. The mass flow $\dot{m}$ effects the pressure ratio p_{03}/p_{02}. There can either be a positive or a negative relation between these two.

Let's examine the case where there is a negative relation between these two parameters. Now let's suppose we increase the mass flow $\dot{m}$. The pressure at the start of the compressor will thus decrease. However, the pressure upstream in the compressor hasn't noticed the change yet. There is thus a higher pressure upstream than downstream. This can cause **flow reversal** in the compressor.

Flow reversal itself is already bad. However, it doesn't stop here. The flow reversal causes the pressure upstream in the compressor to drop. This causes the compressor to start running again. The pressure upstream again increases. Also, the mass flow increases. But this again causes the pressure downstream to increase. Flow reversal thus again occurs. A rather unwanted cycle has thus been initiated.

This cyclic phenomenon is called **surge**. It causes the whole compressor to start vibrating at a high frequency. Surge is different from stall, in that it effects the entire compressor. However, the occurance of stall can often lead to surge.

There are several ways to prevent surge. We can **blow-off** bleed air. This happens halfway through the compressor. This provides an escape for the air. Another option is to use **variable stator vanes**

(VSVs). By adjusting the stator vanes, we try to make sure that we always have the correct angle of incidence i. Finally, the compressor can also be split up into parts. Every part will then have a different speed ω.

Contrary to compressors, turbines aren't subject to surge. Flow simply never tends to move upstream in a turbine.

2.4 Choked flow

Let's examine the pressure ratio p_{04}/p_{05} in a turbine. Increasing this pressure ratio usually leads to an increase in mass flow $\dot{m}$. However, after a certain point, the mass flow will not increase further. This is called **choked flow**. It occurs, when the flow reaches supersonic velocities.

Choked flow can also occur at the compressor. If we look at the right side of figure 2, we see vertical lines. So, when we change the pressure ratio p_{03}/p_{02} at constant compressor speed ω, then the mass flow remains constant.