Longitudinal stability and control

In this chapter, we will start to investigate the stability of the entire aircraft. This can be split up into two parts: longitudinal and lateral stability. In this chapter, we will only look at longitudinal stability.

1 Stick fixed longitudinal stability

1.1 Effects of the wing and the tail on stability

To start our investigation in the stability of an aircraft, we reexamine the moment equation. In an earlier chapter, we found that

$$C_m = C_{m_{ac}} + C_{N_{w_\alpha}}(\alpha - \alpha_0)\frac{x_{cg} - x_w}{\bar{c}} - C_{N_h}\left(\frac{V_h}{V}\right)^2 \frac{S_h l_h}{S\bar{c}} = 0, \tag{1.1}$$

where C_{N_h} is given by

$$C_{N_h} = C_{N_{h_\alpha}}\left((\alpha - \alpha_0)\left(1 - \frac{d\varepsilon}{d\alpha}\right) + (\alpha_0 + i_h)\right) + C_{N_{h_{\delta_e}}}\delta_e. \tag{1.2}$$

We can also rewrite the moment equation to $C_m = C_{m_w} + C_{m_h}$. In this equation, C_{m_w} is the contribution due to the wings. Similarly, C_{m_h} is the contribution from the horizontal tailplane. They are both given by

$$C_{m_w} = C_{m_{ac}} + C_{N_{w_\alpha}}(\alpha - \alpha_0)\frac{x_{cg} - x_w}{\bar{c}} \qquad \text{and} \qquad C_{m_h} = -C_{N_h}\left(\frac{V_h}{V}\right)^2 \frac{S_h l_h}{S\bar{c}}. \tag{1.3}$$

Taking a derivative of the moment equation will give us $C_{m_\alpha} = C_{m_{\alpha_w}} + C_{m_{\alpha_h}}$, where

$$C_{m_{\alpha_w}} = C_{N_{w_\alpha}}\frac{x_{cg} - x_w}{\bar{c}} \qquad \text{and} \qquad C_{m_{\alpha_h}} = -C_{N_{h_\alpha}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(\frac{V_h}{V}\right)^2 \frac{S_h l_h}{S\bar{c}}. \tag{1.4}$$

To achieve stability for our aircraft, we should have $C_{m_\alpha} < 0$. Usually, the wing is in front of the CG. We thus have $x_{cg} - x_w > 0$ and also $C_{m_{\alpha_w}} > 0$. The wing thus destabilizes the aircraft. Luckily, the horizontal tailplane has a stabilizing effect. This is because $C_{m_{\alpha_h}} < 0$. To achieve stability, the stabilizing effect of the tailplane should be bigger than the destabilizing effect of the wings. We should thus have

$$|C_{m_{\alpha_w}}| < |C_{m_{\alpha_h}}|. \tag{1.5}$$

1.2 Effects of the center of gravity on stability

We will now examine the effects of the CG on the stability. To do this, we suppose x_{cg} increases (the CG moves to the rear). However, the other parameters (including δ_e) stay constant. The movement of the CG causes C_{m_α} to increase. At a certain point, we will reach $C_{m_\alpha} = 0$. When the CG moves beyond this position, the aircraft becomes unstable.

Let's examine the point at which $C_{m_\alpha} = 0$. We remember, from a previous chapter, that this point is called the neutral point. And, because the stick deflection is constant (δ_e is constant), we call this point the **stick fixed neutral point**. Its x coordinate is denoted by $x_{n_{fix}}$. To find it, we can use

$$C_{m_\alpha} = C_{N_{w_\alpha}}\frac{x_{n_{fix}} - x_w}{\bar{c}} + C_{N_{h_\alpha}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(\frac{V_h}{V}\right)^2 \frac{S_h}{S}\frac{x_{n_{fix}} - x_h}{\bar{c}}. \tag{1.6}$$

After some mathematical trickery, we can find the position of the stick fixed neutral point, with respect to the wing. It is given by

$$\frac{x_{n_{fix}} - x_w}{\bar{c}} = \frac{C_{N_{h_\alpha}}}{C_{N_\alpha}} \left(1 - \frac{d\varepsilon}{d\alpha}\right) \left(\frac{V_h}{V}\right)^2 \frac{S_h l_h}{S\bar{c}}. \tag{1.7}$$

From this, we can also derive the position of the stick fixed neutral point, with respect to the aircraft CG. This is given by

$$C_{m_\alpha} = C_{N_\alpha} \frac{x_{cg} - x_{n_{fix}}}{\bar{c}}. \tag{1.8}$$

The quantity $\frac{x_{cg} - x_{n_{fix}}}{\bar{c}}$ is known as the **(stick fixed) stability margin**. It is an indication of how much the CG can move, before the aircraft becomes unstable.

1.3 The elevator trim curve

Now let's examine the effects of the elvator deflection δ_e. We know from a previous chapter that the elevator deflection necessary to keep the aircraft in equilibrium is

$$\delta_e = -\frac{1}{C_{m_{\delta_e}}} \left(C_{m_0} + C_{m_\alpha} \left(\alpha - \alpha_0\right)\right). \tag{1.9}$$

δ_e depends on α. To see how, we plot δ_e versus α. We usually do this, such that the y axis is reversed. (Positive δ_e appear below the horizontal axis.) Now we examine the slope of this graph. It is given by

$$\frac{d\delta_e}{d\alpha} = -\frac{C_{m_\alpha}}{C_{m_{\delta_e}}}. \tag{1.10}$$

We always have $C_{m_{\delta_e}} < 0$. To make sure we have $C_{m_\alpha} < 0$ as well, we should have $d\delta_e/d\alpha < 0$. The line in the δ_e, α graph should thus go upward as α increases. (Remember that we have reversed the y axis of the graph!)

δ_e also depends on the aircraft velocity V. To see how, we will rewrite equation (1.9). By using $C_N \approx C_{N_\alpha} \left(\alpha - \alpha_0\right) \approx \frac{W}{\frac{1}{2}\rho V^2 S}$, we find that

$$\delta_e = -\frac{1}{C_{m_{\delta_e}}} \left(C_{m_0} + \frac{C_{m_\alpha}}{C_{N_\alpha}} \frac{W}{\frac{1}{2}\rho V^2 S}\right). \tag{1.11}$$

We can now plot δ_e against V. (Again, we reverse the δ_e axis.) We have then created the so-called **elevator trim curve**. Its slope is given by

$$\frac{d\delta_e}{dV} = \frac{4W}{\rho V^3 S} \frac{1}{C_{m_{\delta_e}}} \frac{C_{m_\alpha}}{C_{N_\alpha}}. \tag{1.12}$$

To have $C_{m_\alpha} < 0$, we should have $d\delta_e/dV > 0$. The line in the graph should thus go downward. Also, if you want to fly faster in a stable aircraft, you should push your stick forward.

2 Stick free longitudinal stability

2.1 The stick free elevator deflection

Previously, we have assumed that δ_e is constant. The pilot has his stick fixed. But what will happen if the pilot releases his stick? It would be nice if the aircraft remains stable as well.

Let's suppose the pilot releases the stick. In that case, aerodynamic force will give the elevator a certain **stick free elevator deflection** $\delta_{e_{free}}$. To find $\delta_{e_{free}}$, we examine the moments H_e about the elevator hinge point. (Or, to be more precise, we look at the non-dimensional version C_{h_e}.) Contributing to this hinge moment are the horizontal tailplane, the elevator and the trim tab. By using a linearization, we find that

$$C_{h_{e_{free}}} = C_{h_\alpha}\alpha_h + C_{h_\delta}\delta_{e_{free}} + C_{h_{\delta_t}}\delta_{t_e} = 0. \tag{2.1}$$

It follows that the stick free elevator deflection is

$$\delta_{e_{free}} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\alpha_h - \frac{C_{h_{\delta_t}}}{C_{h_\delta}}\delta_{t_e}. \tag{2.2}$$

From this, we can also derive that

$$\left(\frac{d\delta_e}{d\alpha}\right)_{free} = -\frac{C_{h_\alpha}}{C_{h_\delta}}\left(1 - \frac{d\varepsilon}{d\alpha}\right). \tag{2.3}$$

The elevator deflection thus changes as the angle of attack is changed.

2.2 Differences in the moment due to the stick free evelator

The free elevator deflection effects the contribution C_{m_h} of the horizontal tailplane to the moment C_m. Let's investigate this. We can remember that

$$C_{m_h} = -\left(C_{N_{h_\alpha}}\alpha_h + C_{N_{h_\delta}}\delta_e\right)\left(\frac{V_h}{V}\right)^2\frac{S_h l_h}{S\bar{c}}. \tag{2.4}$$

We now substitute δ_e by $\delta_{e_{free}}$. If we also differentiate with respect to α, and work things out, we will get

$$C_{m_{\alpha_{h_{free}}}} = -\left(C_{N_{h_\alpha}} - C_{N_{h_\delta}}\frac{C_{h_\alpha}}{C_{h_\delta}}\right)\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(\frac{V_h}{V}\right)^2\frac{S_h l_h}{S\bar{c}}. \tag{2.5}$$

If we compare this equation to the right side of equation (1.4), we see that only $C_{N_{h_\alpha}}$ has changed. In fact, we can define

$$C_{N_{h_{\alpha_{free}}}} = C_{N_{h_\alpha}} - C_{N_{h_\delta}}\frac{C_{h_\alpha}}{C_{h_\delta}}. \tag{2.6}$$

If we use $C_{N_{h_{\alpha_{free}}}}$, instead of $C_{N_{h_\alpha}}$, then our stability analysis is still entirely valid.

Let's take a closer look at the differences between $C_{N_{h_{\alpha_{free}}}}$ and $C_{N_{h_\alpha}}$. This difference is the term $C_{N_{h_\delta}}\frac{C_{h_\alpha}}{C_{h_\delta}}$. We know that $C_{N_{h_\delta}} > 0$. The term C_{h_δ} is interesting. If it would be positive, then it can be shown that the elevator position is unstable. So, we have to have $C_{h_\delta} < 0$. Finally there is C_{h_α}. This term can be either positive or negative. If it is positive ($C_{h_\alpha} > 0$), then the stick free aircraft will be more stable than the stick fixed aircraft. If, however, it is negative ($C_{h_\alpha} < 0$), then it will be less stable, or possibly even unstable.

2.3 The stick free neutral point

Let's find the **stick free neutral point** $x_{n_{free}}$. Finding $x_{n_{free}}$ goes similar to finding $x_{n_{fix}}$. In fact, we can adjust equations (1.7) and (1.8) to

$$\frac{x_{n_{free}} - x_w}{\bar{c}} = \frac{C_{N_{h_{\alpha_{free}}}}}{C_{N_\alpha}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(\frac{V_h}{V}\right)^2\frac{S_h l_h}{S\bar{c}}, \tag{2.7}$$

$$C_{m_{\alpha_{free}}} = C_{N_{\alpha_{free}}} \frac{x_{cg} - x_{n_{free}}}{\bar{c}}. \tag{2.8}$$

In this equation, we have $C_{N_{\alpha_{free}}} \approx C_{N_\alpha}$. This is because the elevator has a negligible influence on C_{N_α}, compared to the influence of the wing.

We can also find the position of the stick free neutral point, with respect to the stick fixed neutral point. Subtracting equation (1.7) from equation (2.7) gives

$$\frac{x_{n_{free}} - x_{n_{fix}}}{\bar{c}} = -\frac{C_{N_{h_\delta}}}{C_{N_\alpha}} \frac{C_{h_\alpha}}{C_{h_\delta}} \left(1 - \frac{d\varepsilon}{d\alpha}\right) \left(\frac{V_h}{V}\right)^2 \frac{S_h l_h}{S\bar{c}} = \frac{C_{m_\delta}}{C_{N_\alpha}} \frac{C_{h_\alpha}}{C_{h_\delta}} \left(1 - \frac{d\varepsilon}{d\alpha}\right). \tag{2.9}$$

2.4 Elevator stick forces

Now we will examine the stick forces which the pilot should exert. We denote the **stick deflection** by s_e. By considering the work done by the pilot, we find that $F_e ds_e + H_e d\delta_e = 0$. From this follows that the **stick force** F_e is given by

$$F_e = -\frac{d\delta_e}{ds_e} H_e = -\frac{d\delta_e}{ds_e} C_{h_e} \frac{1}{2} \rho V_h^2 S_e \bar{c}_e. \tag{2.10}$$

By the way S_e is the **elevator surface** and $\bar{c}_e$ is the **mean elevator chord**. If we massively rewrite the above equation, we can eventually find that

$$F_e = -\frac{d\delta_e}{ds_e} S_e \bar{c}_e \left(\frac{V_h}{V}\right)^2 \left(C'_{h_0} \frac{1}{2} \rho V^2 + C'_{h_\alpha} \frac{W}{S} \frac{1}{C_{N_\alpha}}\right). \tag{2.11}$$

We see that F_e consists of two parts. One part varies with the airspeed, while the other part does not. By the way, the coefficients C'_{h_0} and C'_{h_α} are given by

$$C'_{h_0} = -\frac{C_{h_\delta}}{C_{m_\delta}} C_{m_{ac}} - \frac{C_{h_\delta}}{C_{N_{h_\delta}}} C_{N_{h_{\alpha_{free}}}} (\alpha_0 + i_h) + C_{h_{\delta_t}} \delta_{t_e}, \tag{2.12}$$

$$C'_{h_\alpha} = -\frac{C_{h_\delta}}{C_{m_\delta}} C_{m_{\alpha_{free}}} = -\frac{C_{h_\delta}}{C_{m_\delta}} C_{N_\alpha} \frac{x_{cg} - x_{n_{free}}}{\bar{c}}. \tag{2.13}$$

We see that C'_{h_0} depends on δ_{t_e}. To simplify our equation, we can apply a small trick. We define $\delta_{t_{e_0}}$ to be the value of δ_{t_e} for which $C'_{h_0} = 0$. It follows that

$$\delta_{t_{e_0}} = \frac{1}{C_{h_{\delta_t}}} \left(\frac{C_{h_\delta}}{C_{m_\delta}} C_{m_{ac}} + \frac{C_{h_\delta}}{C_{N_{h_\delta}}} C_{N_{h_{\alpha_{free}}}} (\alpha_0 + i_h)\right). \tag{2.14}$$

We can now rewrite the stick deflection force as

$$F_e = \frac{d\delta_e}{ds_e} S_e \bar{c}_e \left(\frac{V_h}{V}\right)^2 \left(\frac{W}{S} \frac{C_{h_\delta}}{C_{m_{\delta_e}}} \frac{x_{cg} - x_{n_{free}}}{\bar{c}} - \frac{1}{2} \rho V^2 C_{h_{\delta_t}} \left(\delta_{t_e} - \delta_{t_{e_0}}\right)\right). \tag{2.15}$$

The control forces, which the pilots need to exert, greatly determine how easy and comfortable it is to fly an airplane. The above equation is therefore rather important.

We can also derive something else from the above equation. Let's define the **trim speed** V_{tr} to be the speed at which $F_e = 0$. We now examine the derivative dF_e/dV at this trim speed. (So at $F_e = 0$.) If it is positive ($dF_e/dV > 0$), then the aircraft is said to have **elevator control force stability** in the current flight condition. It can be shown that this derivative is given by

$$\left(\frac{dF_e}{dV}\right)_{F_e=0} = -2 \frac{d\delta_e}{ds_e} S_e \bar{c}_e \left(\frac{V_h}{V_{tr}}\right)^2 \frac{W}{S} \frac{C_{h_\delta}}{C_{m_{\delta_e}}} \frac{x_{cg} - x_{n_{free}}}{\bar{c}} \frac{1}{V_{tr}}. \tag{2.16}$$

It's the job of the designer to keep this derivative positive.

3 Longitudinal control

3.1 Special manoeuvres

Previously, we have only considered steady flight. Now we suppose that we are performing some special manoeuvre. We will consider both a steady pull-up manoeuvre and a horizontal steady turn.

During these manoeuvres, we will have a certain **load factor** $n = N/W$. There are two parameters that are important for the manoeuvres. They are the **elevator deflection per** g, denoted by $d\delta_e/dn$, and the **stick force per** g, denoted by dF_e/dn. Both these parameters should be negative. And they may not be too high or too low either.

3.2 The elevator deflection per g

We will now find an expression for $d\delta_e/dn$. Let's suppose we're initially in a horizontal steady flight. But after a brief moment, we'll be in one of the special manoeuvres. In this brief moment, several aircraft parameters have changed.

Let's examine the change in normal force ΔC_N and the change in moment ΔC_m. The change in normal force is effected by the angle of attack α and the pitch rate q. This gives us

$$\Delta C_N = \frac{\Delta N}{\frac{1}{2}\rho V^2 S} = \frac{W}{\frac{1}{2}\rho V^2 S}\Delta n = C_{N_\alpha}\Delta\alpha - C_{Z_q}\Delta\frac{q\bar{c}}{V}. \tag{3.1}$$

Similarly, the change in moment is effected by the angle of attack α, the pitch rate q and the elevator deflection δ_e. This gives us

$$\Delta C_m = 0 = C_{m_\alpha}\Delta\alpha + C_{m_q}\Delta\frac{q\bar{c}}{V} + C_{m_{\delta_e}}\Delta\delta_e. \tag{3.2}$$

You may wonder, why is $\Delta C_m = 0$? This is because both in the initial situation and the final situation, we have a steady manoeuvre. There is thus no angular acceleration present. The moment must thus stay constant.

From the first of the above two equations, we can find the derivative of α with respect to n. It is given by

$$\frac{d\alpha}{dn} = \frac{1}{C_{N_\alpha}}\frac{W}{\frac{1}{2}\rho V^2 S} + \frac{C_{Z_q}}{C_{N_\alpha}}\frac{d\frac{q\bar{c}}{V}}{dn}. \tag{3.3}$$

From the second of these equations, we can find that

$$\frac{d\delta_e}{dn} = -\frac{1}{C_{m_{\delta_e}}}\left(C_{m_\alpha}\frac{d\alpha}{dn} + C_{m_q}\frac{d\frac{q\bar{c}}{V}}{dn}\right). \tag{3.4}$$

Inserting the value of $d\alpha/dn$ will eventually give us

$$\frac{d\delta_e}{dn} = -\frac{1}{C_{m_{\delta_e}}}\left(\frac{C_{m_\alpha}}{C_{N_\alpha}}\frac{W}{\frac{1}{2}\rho V^2 S} + \left(\frac{C_{m_\alpha}C_{Z_q}}{C_{N_\alpha}} + C_{m_q}\right)\frac{d\frac{q\bar{c}}{V}}{dn}\right). \tag{3.5}$$

We will determine the term $d\frac{q\bar{c}}{V}/dn$ later, since it depends on the type of manoeuvre that is being performed.

3.3 The stick force per g

It's time to find an expression for dF_e/dn. From equation (2.10), we can derive that

$$\frac{dF_e}{dn} = -\frac{d\delta_e}{ds_e}\frac{1}{2}\rho V_h^2 S_e\bar{c}_e\left(C_{h_\alpha}\frac{d\alpha_h}{dn} + C_{h_\delta}\frac{d\delta_e}{dn}\right). \tag{3.6}$$

We already have an expression for $d\delta_e/dn$. The expression for α_h is a bit tricky. This is because we also have a rotation q. If we take this into account, we will have

$$\alpha_h = (\alpha - \alpha_0)\left(1 - \frac{d\varepsilon}{d\alpha}\right) + (\alpha_0 + i_h) + \frac{l_h}{\bar{c}}\frac{q\bar{c}}{V}. \tag{3.7}$$

The derivative of α_h, with respect to n, will then be

$$\frac{d\alpha_h}{dn} = \left(1 - \frac{d\varepsilon}{d\alpha}\right)\frac{d\alpha}{dn} + \frac{l_h}{\bar{c}}\frac{d\frac{q\bar{c}}{V}}{dn}. \tag{3.8}$$

Luckily, we still remember $d\alpha/dn$ from equation (3.3). From this, we can derive an equation that's way too long to write down here. However, once we examine specific manoeuvres, we will mention the final equation.

3.4 The pull-up manoeuvre

Let's consider an aircraft in a pull-up manoeuvre. When an aircraft pulls its nose up, the pilot will experience higher g-forces. This will thus cause the load factor n to change.

To be able to study pull-up manoeuvres, we simplify them. We assume that both n and V are constant. If this is the case, the aircraft's path will form a part of a circle. The centripetal accelaration thus is $N - W = mVq$. By using $n = N/W$ and $W = mg$, we can rewrite this as

$$\frac{q\bar{c}}{V} = \frac{g\bar{c}}{V^2}(n-1). \tag{3.9}$$

Differentiating with respect to n gives

$$\frac{d\frac{q\bar{c}}{V}}{dn} = \frac{g\bar{c}}{V^2} = \frac{1}{2\mu_c}\frac{W}{\frac{1}{2}\rho V^2 S}, \qquad \text{where} \quad \mu_c = \frac{m}{\rho S\bar{c}} = \frac{W}{g\rho S\bar{c}}. \tag{3.10}$$

By using this, we can find the elevator deflection per g for a pull-up manoeuvre. It is

$$\frac{d\delta_e}{dn} = -\frac{1}{C_{m_{\delta_e}}}\frac{W}{\frac{1}{2}\rho V^2 S}\left(\frac{C_{m_\alpha}}{C_{N_\alpha}}\left(1 + \frac{C_{Z_q}}{2\mu_c}\right) + \frac{C_{m_q}}{2\mu_c}\right). \tag{3.11}$$

Often the term $C_{Z_q}/2\mu_c$ can be neglected. This simplifies matters a bit. We can also derive a new expression for the stick force per g. We will find that

$$\frac{dF_e}{dn} = \frac{d\delta_e}{ds_e}\frac{W}{S}\left(\frac{V_h}{V}\right)^2 S_e\bar{c}_e\frac{C_{h_\delta}}{C_{m_{\delta_e}}}\left(\frac{C_{m_{\alpha_{free}}}}{C_{N_\alpha}} + \frac{C_{m_{q_{free}}}}{2\mu_c}\right). \tag{3.12}$$

In this equation, we can see the parameters $C_{m_{\alpha_{free}}}$ and $C_{m_{q_{free}}}$. These are the values of C_{m_α} and C_{m_q} when the pilot releases his stick. They are given by

$$C_{m_{\alpha_{free}}} = C_{N_{w_\alpha}}\frac{x_{cg} - x_w}{\bar{c}} + C_{m_{\alpha_{h_{free}}}} \qquad \text{and} \qquad C_{m_{q_{free}}} = C_{m_q} - C_{m_{\delta_e}}\frac{C_{h_\alpha}}{C_{N_\alpha}}\frac{l_h}{\bar{c}}. \tag{3.13}$$

(The relation for $C_{m_{\alpha_{h_{free}}}}$ was already given in equation (2.5).)

3.5 The steady horizontal turn

Now let's consider an aircraft in a steady horizontal turn. It is performing this turn with a constant roll angle φ. From this, we can derive that

$$N\cos\varphi = W \qquad \text{and} \qquad N - W\cos\varphi = mVq. \tag{3.14}$$

If we combine the above relations, and rewrite them, we will get

$$\frac{q\bar{c}}{V} = \frac{g\bar{c}}{V^2}\left(n - \frac{1}{n}\right). \tag{3.15}$$

Differentiating with respect to n will then give us

$$\frac{d\frac{q\bar{c}}{V}}{dn} = \frac{1}{2\mu_c}\frac{W}{\frac{1}{2}\rho V^2 S}\left(1 + \frac{1}{n^2}\right). \tag{3.16}$$

By using this, we can find the elevator deflection per g for a horizontal steady turn. It is

$$\frac{d\delta_e}{dn} = -\frac{1}{C_{m_{\delta_e}}}\frac{W}{\frac{1}{2}\rho V^2 S}\left(\frac{C_{m_\alpha}}{C_{N_\alpha}} + \left(\frac{C_{m_\alpha}}{C_{N_\alpha}}\frac{C_{Z_q}}{2\mu_c} + \frac{C_{m_q}}{2\mu_c}\right)\left(1 + \frac{1}{n^2}\right)\right). \tag{3.17}$$

Again, we may often assume that $C_{Z_q}/2\mu_c \approx 0$. This again simplifies the equation. We also have the stick force per g. In this case, it is given by

$$\frac{dF_e}{dn} = \frac{d\delta_e}{ds_e}\frac{W}{S}\left(\frac{V_h}{V}\right)^2 S_e\bar{c}_e\frac{C_{h_\delta}}{C_{m_{\delta_e}}}\left(\frac{C_{m_{\alpha_{free}}}}{C_{N_\alpha}} + \frac{C_{m_{q_{free}}}}{2\mu_c}\left(1 + \frac{1}{n^2}\right)\right). \tag{3.18}$$

It is interesting to see the similarities between the pull-up manoeuvre and the steady horizontal turn. In fact, if the load factor n becomes big, the difference between the two manoeuvres disappears.

3.6 The manoeuvre point

An important point on the aircraft, when performing manoeuvres, is the **manoeuvre point**. It is defined as the position of the CG for which $d\delta_e/dn = 0$. First we will examine the **stick fixed manoeuvre point** $x_{m_{fix}}$. To have $d\delta_e/dn = 0$ for a pull-up manoeuvre (neglecting $C_{Z_q}/2\mu_c$), we should have

$$\frac{C_{m_\alpha}}{C_{N_\alpha}} + \frac{C_{m_q}}{2\mu_c} = \frac{x_{cg} - x_{n_{fix}}}{\bar{c}} + \frac{C_{m_q}}{2\mu_c} = 0. \tag{3.19}$$

If the above equation holds, then the CG equals the manoeuvre point. We thus have

$$\frac{x_{m_{fix}} - x_{n_{fix}}}{\bar{c}} = -\frac{C_{m_q}}{2\mu_c} \qquad \text{and also} \qquad \frac{x_{cg} - x_{m_{fix}}}{\bar{c}} = \frac{C_{m_\alpha}}{C_{N_\alpha}} + \frac{C_{m_q}}{2\mu_c}. \tag{3.20}$$

(Remember that the above equations are for the pull-up manoeuvre. For the steady turn, we need to multiply the term with C_{m_q} by an additional factor $\left(1 + 1/n^2\right)$.) By using the above results, we can eventually obtain that

$$\frac{d\delta_e}{dn} = -\frac{1}{C_{m_{\delta_e}}}\frac{W}{\frac{1}{2}\rho V^2 S}\frac{x_{cg} - x_{m_{fix}}}{\bar{c}}. \tag{3.21}$$

By the way, this last equation is valid for both the pull-up manoeuvre and the steady horizontal turn.

We can also find the **stick free manoeuvre point** $x_{m_{free}}$. This goes, in fact, in a rather similar way. We will thus also find, for the pull-up manoeuvre, that

$$\frac{x_{m_{free}} - x_{n_{free}}}{\bar{c}} = -\frac{C_{m_{q_{free}}}}{2\mu_c} \qquad \text{and} \qquad \frac{x_{cg} - x_{m_{free}}}{\bar{c}} = \frac{C_{m_{\alpha_{free}}}}{C_{N_\alpha}} + \frac{C_{m_{q_{free}}}}{2\mu_c}. \tag{3.22}$$

(For the steady turn, we again need to multiply the term with $C_{m_{q_{free}}}$ by $\left(1 + 1/n^2\right)$.)