Lateral stability and control

In this chapter, we will examine lateral stability and control. How should we control an aircraft in a non-symmetrical steady flight?

1 The equations of motion

1.1 Derivation of the equations of motion for asymmetric flight

Let's examine an aircraft in a steady asymmetric flight. It has a roll angle φ and a sideslip angle β. By examining equilibrium, we can find that

$$W \sin\varphi + Y = mVr, \qquad L = 0 \quad \text{and} \quad N = 0. \tag{1.1}$$

Non-dimensionalizing these equations gives

$$C_L \varphi - 4\mu_b \frac{rb}{2V} + C_Y = 0, \qquad C_l = 0 \quad \text{and} \quad C_n = 0, \tag{1.2}$$

where we have $\mu_b = \frac{m}{\rho S b}$. We can also apply linearization to the above equations. This will then give us

$$C_L \varphi + C_{Y_\beta} \beta + (C_{Y_r} - 4\mu_b) \frac{rb}{2V} + C_{Y_{\delta_a}} \delta_a + C_{Y_{\delta_r}} \delta_r = 0, \tag{1.3}$$

$$C_{l_\beta} \beta + C_{l_r} \frac{rb}{2V} + C_{l_{\delta_a}} \delta_a + C_{l_{\delta_r}} \delta_r = 0, \tag{1.4}$$

$$C_{n_\beta} \beta + C_{n_r} \frac{rb}{2V} + C_{n_{\delta_a}} \delta_a + C_{n_{\delta_r}} \delta_r = 0. \tag{1.5}$$

1.2 Simplifying the equations of motion

Let's examine the equations of the previous paragraph. There are quite some terms in these equations that are negligible. They are $C_{Y_{\delta_a}}$, $C_{l_{\delta_r}}$, C_{Y_r}, $C_{n_{\delta_a}}$ and $C_{Y_{\delta_r}}$. By using these neglections, and by putting the above equations into matrix form, we will get

$$\begin{bmatrix} C_L & C_{Y_\beta} & -4\mu_b & 0 & 0 \\ 0 & C_{l_\beta} & C_{l_r} & C_{l_{\delta_a}} & 0 \\ 0 & C_{n_\beta} & C_{n_r} & 0 & C_{n_{\delta_r}} \end{bmatrix} \begin{bmatrix} \varphi \\ \beta \\ \frac{rb}{2V} \\ \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{1.6}$$

Let's assume that the velocity V is already set. We then still have five unknowns and three equations. That means that there are infinitely many solutions. This makes sense: You can make a turn in infinitely many ways. How do we deal with this? We simply set one of the parameters. We then express three of the remaining parameters as a function of the last parameter. (This fifth parameter is usually $\frac{rb}{2V}$.) So let's do that.

2 Steady horizontal turns

2.1 Turns using ailerons only

Let's try to turn the aircraft, by only using ailerons. We do not use the rudder and thus have $\delta_r = 0$. We can insert this into the equations of motion. We then solve for the parameters β, φ and δ_a. This will give us

$$\frac{d\beta}{d\frac{rb}{2V}} = -\frac{C_{n_r}}{C_{n_\beta}} > 0 \qquad \text{(since } C_{n_r} < 0 \text{ and } C_{n_\beta} > 0\text{)}, \tag{2.1}$$

$$\frac{d\varphi}{d\frac{rb}{2V}} = \frac{4\mu_b + C_{Y_\beta}\frac{C_{n_r}}{C_{n_\beta}}}{C_L} > 0, \tag{2.2}$$

$$\frac{d\delta_a}{d\frac{rb}{2V}} = \frac{1}{C_{l_{\delta_a}}}\frac{C_{l_\beta}C_{n_r} - C_{l_r}C_{n_\beta}}{C_{n_\beta}}. \tag{2.3}$$

The sign of the last equation is still a point of discussion. We would like to have $d\delta_a/d\frac{rb}{2V}$. If this is the case, then we have so-called **spiral stability**. We know that $C_{l_{\delta_a}} < 0$ and $C_{n_\beta} > 0$. So spiral stability is achieved if

$$C_{l_\beta}C_{n_r} - C_{l_r}C_{n_\beta} > 0. \tag{2.4}$$

We will find out in the next chapter why they call this the spiral stability condition.

2.2 Turns using the rudder only

We can also make a turn using only the rudder. So we have $\delta_a = 0$. This again gives us three equations, being

$$\frac{d\beta}{d\frac{rb}{2V}} = -\frac{C_{l_r}}{C_{l_\beta}} > 0 \qquad \text{(since } C_{l_r} > 0 \text{ and } C_{l_\beta} < 0\text{)}, \tag{2.5}$$

$$\frac{d\varphi}{d\frac{rb}{2V}} = \frac{4\mu_b + C_{Y_\beta}\frac{C_{l_r}}{C_{l_\beta}}}{C_L} > 0, \tag{2.6}$$

$$\frac{d\delta_r}{d\frac{rb}{2V}} = -\frac{1}{C_{n_{\delta_r}}}\frac{C_{l_\beta}C_{n_r} - C_{l_r}C_{n_\beta}}{C_{l_\beta}}. \tag{2.7}$$

In the last equation, we have $C_{n_{\delta_r}} < 0$ and $C_{l_\beta} < 0$. If there is also spiral stability, then we have $d\delta_r/d\frac{rb}{2V} < 0$.

2.3 Coordinated turns

In a **coordinated turn**, we have $\beta = 0$. This means that there is no sideward component of the force acting on the aircraft. This is an important factor for passenger comfort. For the coordinated turn, we again have three equations. They are

$$\frac{d\varphi}{d\frac{rb}{2V}} = \frac{4\mu_b}{C_L} > 0, \tag{2.8}$$

$$\frac{d\delta_a}{d\frac{rb}{2V}} = -\frac{C_{l_r}}{C_{l_{\delta_a}}} > 0 \qquad \text{(since } C_{l_r} > 0 \text{ and } C_{l_{\delta_a}} < 0\text{)}, \tag{2.9}$$

$$\frac{d\delta_r}{d\frac{rb}{2V}} = -\frac{C_{n_r}}{C_{n_{\delta_r}}} < 0 \qquad \text{(since } C_{n_r} < 0 \text{ and } C_{n_{\delta_r}} < 0\text{)}. \tag{2.10}$$

2.4 Flat turns

If we want the aircraft to stay flat during the turns, then we have $\varphi = 0$. It then follows that

$$\frac{d\beta}{d\frac{rb}{2V}} = \frac{4\mu_b}{C_{Y_\beta}} < 0. \tag{2.11}$$

From this, we can also derive that

$$\frac{d\delta_a}{d\frac{rb}{2V}} > 0 \qquad \text{and} \qquad \frac{d\delta_r}{d\frac{rb}{2V}} < 0. \tag{2.12}$$

3 Other flight types

3.1 Steady straight sideslipping flight

Let's examine a steady straight sideslipping flight. This type of flight is usually only used during landings with strong sidewinds. However, sometimes the aircraft is brought into a steady straight sideslipping flight involuntarily. It is therefore important to know how the aircraft behaves.

In a straight flight, we have $\frac{rb}{2V} = 0$. We can now derive that

$$\frac{d\varphi}{d\beta} = -\frac{C_{Y_\beta}}{C_L} > 0, \qquad \frac{d\delta_a}{d\beta} = -\frac{C_{l_\beta}}{C_{l_{\delta_a}}} \qquad \text{and} \qquad \frac{d\delta_r}{d\beta} = -\frac{C_{n_\beta}}{C_{n_{\delta_r}}}. \tag{3.1}$$

We generally want to have $d\delta_a/d\beta < 0$ and $d\delta_r/d\beta > 0$. We also always have $C_{l_{\delta_a}} < 0$ and $C_{n_{\delta_r}} < 0$. This implies that we should have $C_{l_\beta} < 0$ and $C_{n_\beta} > 0$.

3.2 Stationary flight with asymmetric power

Let's suppose one of the engines of the aircraft doesn't work anymore. In this case, a yawing moment will be present. This moment has magnitude

$$C_{n_e} = k\frac{\Delta T_p y_e}{\frac{1}{2}\rho V^2 S b}. \tag{3.2}$$

The variable ΔT_p consists of two parts. First there is the reduction in thrust. Then there is also the increase in drag of the malfunctioning engine. y_e is the Y coordinate of the malfunctioning engine. Finally, k is an additional parameter, taking into account other effects. Its value is usually between 1.5 and 2.

Now let's try to find a way in which we can still perform a steady straight flight. (We should thus have $r = 0$.) We now have four unknowns and three equations. So we can still set one parameter. Usually, we would like to have $\varphi = 0$ as well. In this case, a sideslip angle β is unavoidable. If the right engine is inoperable, then a positive rudder deflection and sideslip angle will be present.

We could also choose to have $\beta = 0$. In this case, we will constantly fly with a roll angle φ. The wing with the inoperable engine then has to be lower than the other wing. So if the right wing malfunctions, then we have a positive roll angle.