AEROSPACE FLIGHT DYNAMICS AND SIMULATION

AE3202

EXAMINATION

April 11, 2012

Delft University of Technology
Faculty of Aerospace Engineering

This exam contains 5 questions.

You may use the formulas on the given formula sheets.

PLEASE NOTE

Always write down the correct units for each computed parameter value. Be mindful for any required conversion before making any computations. **Always** write down the derivations of your answers.

Question 1 (10 points)

Which of the following statements are true and which are false (answer ALL questions):

(a) The sign of C_{Y_β} for a conventional aircraft is negative and the dominant contribution to this stability derivative is made by the main wing.

(b) The natural frequency is at most equal or less than the undamped frequency for any eigenmotion.

(c) C_{Z_α} and C_{X_α} have a significant effect on the short period eigenmotion as well as the phugoid motion.

(d) By increasing dihedral enough one can make the spiral motion (more) stable while making the Dutch Roll motion unstable.

(e) Shifting the position of the center of gravity forward will decrease the magnitude of the required control forces exerted by the pilot during normal operations.

(f) In the aerodynamic center the change in aerodynamic moment due to a change in angle of attack is zero. Moreover, the second order derivative of C_m with respect to α is zero.

Question 2 (20 points): static and dynamic transformations

In general Newton's Laws hold for a non-rotating inertial frame. If one wants to express the equations of motion in a rotating frame, one has to compensate for this rotation. This process involves both static and dynamic transformations. In the following, these transformations are addressed.

(a) **(3 points)** State the three (static) unit-axis transformation matrices for a rotation α around the X-, Y- and Z-axis.

(b) **(7 points)** In a dynamic rotation we assume that the angular displacement $d\alpha$ (the rotation) takes place in a certain time dt. Given the three frames A, B and C with corresponding rotations in Fig. 1 you are asked to:

- i) Set up the static transformation from frame A to C, $\mathbb{T}_{CA}$, in terms of the product of individual $\mathbb{T}$.
- ii) From the sequence of transformations, derive the angular rate of frame C with respect to frame A, $\mathbf{\Omega}_{CA}$. Make sure to express all components of $\mathbf{\Omega}_{CA}$ in components of frame C.

(c) **(10 points)** The position vector $\mathbf{R}$ is defined in the E-frame by $\mathbf{R} = -R\mathbf{z_E}$ (Fig. 2). The time derivative of $\mathbf{R}$ can be derived from this definition, and would include a component *relative* to the E-frame and one due to the *rotation* of the E-frame. You are asked to derive an expression for $\frac{d\mathbf{R}}{dt}$ expressed in components of the E-frame. Use can be made of the following:

$$\frac{d\mathbf{x_E}}{dt} = \mathbf{\Omega}^E_{EC} \times \mathbf{x_E} \qquad \frac{d\mathbf{y_E}}{dt} = \mathbf{\Omega}^E_{EC} \times \mathbf{y_E} \qquad \frac{d\mathbf{z_E}}{dt} = \mathbf{\Omega}^E_{EC} \times \mathbf{z_E}$$

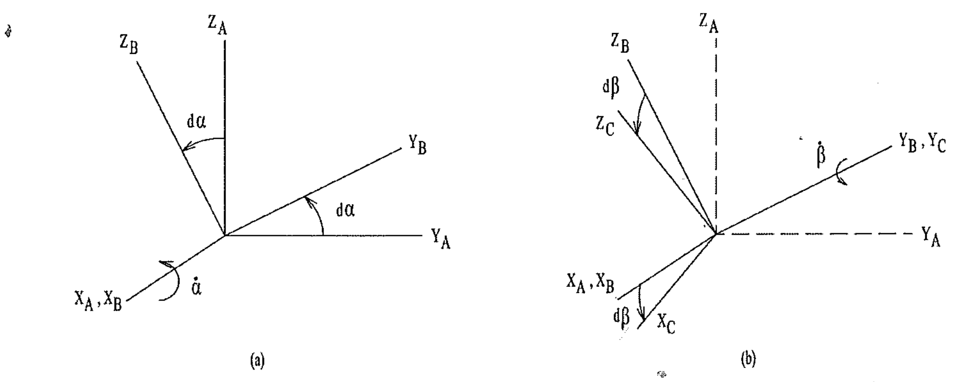

Figure 1: Transformation from A to C frame

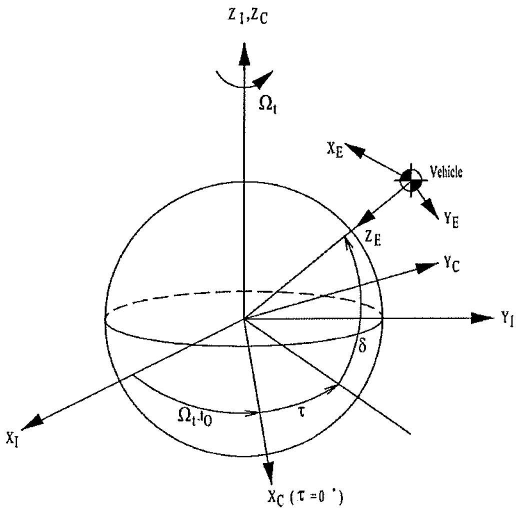

Figure 2: Relation between C and E frame

Question 3 (25 points): linearization

The open-loop flight behavior of an entry capsule is characterized by very strong oscillations around all three axes. Individual components of motion are hard to distinguish, because of a strong dynamic coupling, partially the result of an offset in the location of the center of mass in Z direction. This offset gives a product of inertia I_{xz} that is too large to be ignored.

(a) **(12 points)** Linearize the following equations for pitch motion:

$$\dot{q} = \frac{M_y}{I_{yy}} + \frac{I_{xz}}{I_{yy}}\left(r^2 - p^2\right) + \frac{I_{zz} - I_{xx}}{I_{yy}} pr$$

$$\dot{\alpha} \approx q - \frac{L}{mV}$$

The external moment M_y contains an aerodynamic moment $\mathcal{M}$ and a reaction-control moment $M_{T,y}$.
Assume the following (READ THIS CAREFULLY):

- Both pitch moment $\mathcal{M}$ and lift L are a function of the angle of attack, α, and Mach number, M.
- For the rotational motion considered, the atmospheric properties are constant.
- Due to the oscillatory nature of the rotational motion, the nominal angular rates p_0, q_0 and r_0 cannot be considered small
- The Mach number is *not* a state and has to be properly linearized.
- Translational and rotational motion are *not* decoupled.
- Consider all state and control variables (even though remaining state equations are not shown here)

Write your answer in the form $\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta\mathbf{u}$ as two scalar equations. Carefully indicate the state variables $\Delta\mathbf{x}$ and control variables $\Delta\mathbf{u}$.

(b) **(6 points)** To analytically study the open-loop pitch motion, we make some simplifications. In that case, the state-space model is given by:

$$\Delta\dot{q} = \frac{1}{I_{yy}} C_{m_\alpha} \bar{q}_0 S_{ref} c_{ref} \Delta\alpha + \frac{\Delta M_{T,y}}{I_{yy}}$$

$$\Delta\dot{\alpha} = \Delta q - \frac{1}{mV_0} C_{L_\alpha} \bar{q}_0 S_{ref} \Delta\alpha$$

You are asked to derive an analytical expression for the eigenvalues related to the longitudinal oscillation.

(e) **(5 points)** A numerical representation of the eigenvalues found under (b) is $\lambda_{1,2}$ = -1.9744$\cdot 10^{-2} \pm 1.5534 \cdot 10^{1} j$. You are asked to calculate the *dimensional* values of the damping factor, ζ, period, P, and amplitude half (or double) time, $T_{\frac{1}{2}}$ or T_2, depending on the nature of the eigenmotion. First state the used equation, then your numerical value.

(d) **(2 points)** Is the mode under (c) stable or unstable? Explain why.

Question 4 (25 points): Static stability of an aircraft with a canard

The force and moment equilibrium for an aircraft with a canard configuration differs significantly from that of an aircraft with a conventional horizontal tail plane.

(a) In Figure 3 of the Results Sheet, draw the non-dimensional forces and moments necessary for longitudinal equilibrium for an aircraft with a canard configuration and the given center of gravity. Make sure that the forces and moments have the correct direction! Assume that for the canard, C_{T_h} and $C_{m_{ac_h}}$ are negligible. For the main wing, assume that the contribution of C_{T_w} to the aerodynamic moment is negligible.

(b) Formulate the equilibrium equation for the pitching moment coefficient C_m. Note that for a canard configuration it is safe to assume that $\left(\frac{V_h}{V}\right)^2 = 1$, while the factor $\frac{S_h}{S}$ must not be neglected.

(c) Derive from the moment equation the expression for the static stability C_{m_α}.

(d) Derive an expression for the x-coordinate of the neutral point stick-fixed $x_{n_{fix}}$.

(e) The situation in Figure 3 is not very efficient nor statically stable. Suggest a configuration change in Figure 3 resulting in a more efficient, and stable design. Mark the changed forces, moments, lengths and points with an asterisk (*).

Question 5 (20 %)

Consider a conventional statically and dynamically stable aircraft (low wing configuration) at cruising flight, i.e. α is small. Please provide your answers in the following table form:

Coefficient	Sign	Case 1	Case 2	Case 3

(a) What is the conventional sign of the following longitudinal stability and control derivatives: C_{m_α}, C_{m_q}, C_{X_u}, $C_{Z_{\delta_e}}$? Give your answer using the following notation: $> 0, < 0, = 0$.

(b) What happens to these stability and control derivatives in the following cases:

- 1) A canard is fitted to the aircraft. The canard has a fixed orientation with respect to the aircraft. All aircraft parameters remain the same.
- 2) The horizontal tail length l_h is doubled. Note that all other parameters may be assumed to remain unchanged.
- 3) The center of gravity is moved forward. All other aircraft parameters remain the same.

Give your answer in terms of the following statements: more/less negative/positive or unchanged.

(c) What is the conventional sign of the following lateral stability and control derivatives: C_{n_β}, C_{l_r}, C_{n_p}, $C_{l_{\delta_a}}$? Give you answer using the following notation: $> 0, < 0, = 0$.

(d) What happens to these stability and control derivatives in the following cases:

- 1) Increase of dihedral Γ. All other aircraft parameters remain unchanged.
- 2) Increase of wing sweep angle Λ. All other aircraft parameters remain unchanged.
- 3) The vertical tail length l_v is doubled. All other aircraft parameters remain unchanged.

Give your answer in terms of the following statements: more/less negative/positive or unchanged.