Examining an entire aircraft

In the previous chapter, we've only considered the wing of an aircraft. Now we're going to add the rest of the aircraft too. How do the various components of the aircraft influence each other?

1 Adding a fuselage

1.1 Changes in moment coefficient

Previously, we have only considered a wing. This wing had a moment coefficient C_{m_w}. Now let's add a fuselage. The combination of wing and fuselage has a moment coefficient $C_{m_{wf}}$. The change in moment coefficient ΔC_m is now defined such that

$$C_{m_{wf}} = C_{m_w} + \Delta C_m. \tag{1.1}$$

Let's take a closer look at this change ΔC_m. What does it consist of? We know that a fuselage in a flow usually has a moment coefficient C_{m_f}. However, the wing causes the flow around the fuselage to change. This also causes a moment coefficient induced on the fuselage, denoted by $\Delta C_{m_{fi}}$. Finally, the fuselage effects the flow around the wing. There is thus also a factor $\Delta C_{m_{wi}}$. We thus have

$$\Delta C_m = C_{m_f} + \Delta C_{m_{fi}} + \Delta C_{m_{wi}}. \tag{1.2}$$

In this equation, the coefficients C_{m_f} and $\Delta C_{m_{fi}}$ are usually considered together as $C_{m_{fi}}$.

1.2 Effects of the fuselage

We can use inviscid incompressible flow theory to examine the fuselage. We then find that the moment coefficient of the fuselage, in the induced velocity field, is

$$C_{m_{fi}} = C_{m_f} + \Delta C_{m_{fi}} = \frac{\pi}{2S\bar{c}} \int_0^{l_f} b_f(x)^2 \, \alpha_f(x) \, dx. \tag{1.3}$$

Here, $b_f(x)$ is the **fuselage width** and $\alpha_f(x)$ is the (effective) **fuselage angle of attack**. We also integrate over the entire length l_f of the fuselage.

If there was only a fuselage (and no wing), then the fuselage would have a constant angle of attack. However, the wing causes the angle of attack to vary. In front of the wing, the flow goes up a bit. Behind the wing, there is a downwash. To deal with these complicated effects, we apply linearization. We thus approximate $\alpha_f(x)$ as

$$\alpha_f(x) = \alpha_{f_0} + \frac{d\alpha_f(x)}{d\alpha}(\alpha - \alpha_0). \tag{1.4}$$

Here, α_0 is the **zero normal force angle of attack** $\alpha_{C_N=0}$. α_{f_0} is the corresponding fuselage angle of attack. By using the above equation, we can find a relation for $C_{m_{fi}}$. We get

$$C_{m_{fi}} = \frac{\pi \alpha_{f_0}}{2S\bar{c}} \int_0^{l_f} b_f(x)^2 \, dx + \frac{\pi(\alpha - \alpha_0)}{2S\bar{c}} \int_0^{l_f} b_f(x)^2 \, \frac{d\alpha_f(x)}{d\alpha} \, dx. \tag{1.5}$$

1.3 The shift of the aerodynamic center

Adding the fuselage causes the aerodynamic center to shift. We know that

$$C_{m_w} = C_{m_{ac_w}} + C_{N_w} \frac{x - x_{ac_w}}{\bar{c}} \qquad \text{and} \qquad C_{m_{wf}} = C_{m_{ac_{wf}}} + C_{N_{wf}} \frac{x - x_{ac_{wf}}}{\bar{c}}. \tag{1.6}$$

Let's assume that adding the fuselage doesn't effect the normal force. Thus $C_{N_w} = C_{N_{wf}} = C_N$. In this case, we have

$$\Delta C_m = C_{m_{wf}} - C_{m_w} = \Delta C_{m_{ac}} - C_N \frac{x_{ac_{wf}} - x_{ac_w}}{\bar{c}}. \tag{1.7}$$

We can differentiate this equation with respect to α. From the definition of the AC follows that $d(\Delta C_{m_{ac}})/d\alpha = 0$. If we then also use the fact that $dC_N/d\alpha = C_{N_\alpha}$, we find that

$$\frac{x_{ac_{wf}} - x_{ac_w}}{\bar{c}} = \frac{\Delta x_{ac}}{\bar{c}} = -\frac{1}{C_{N_\alpha}} \frac{d(\Delta C_m)}{d\alpha}. \tag{1.8}$$

Part of this shift is caused by the fuselage, while the other part is caused by the new flow on the wing. The shift in angle of attack, due to the fuselage, is

$$\left(\frac{\Delta x_{ac}}{\bar{c}}\right)_{fi} = -\frac{1}{C_{N_\alpha}} \frac{dC_{m_{fi}}}{d\alpha} = -\frac{1}{C_{N_\alpha}} \frac{\pi}{2S\bar{c}} \int_0^{l_f} b_f(x)^2 \, \frac{d\alpha_f(x)}{d\alpha} \, dx. \tag{1.9}$$

The shift due to the flow induced on the wing is denoted by $\left(\frac{\Delta x_{ac}}{\bar{c}}\right)_{wi}$. We don't have a clear equation for this part of the shift. However, it is important to remember that this shift is only significant for swept wings. If there is a positive sweep angle, then the AC moves backward.

2 Adding the rest of the aircraft

2.1 The three parts

It is now time to examine an entire aircraft. The CG of this aircraft is positioned at (x_{cg}, z_{cg}). We split this aircraft up into three parts.

- First, there is the wing, with attached fuselage and nacelles. The position of the AC of this part is (x_w, z_w). Two forces and one moment are acting in this AC. There are a normal force N_w (directed upward), a tangential force T_w (directed to the rear) and a moment M_{ac_w}.
- Second, we have a horizontal tailplane. The AC of this part is at (x_h, z_h). In it are acting a normal force N_h, a tangential force T_h and a moment M_{ac_h}.
- Third, there is the propulsion unit. Contrary to the other two parts, this part has no moment. It does have a normal force N_p and a tangential force T_p. However, these forces are all tilted upward by the **thrust inclination** i_p. (So they have different directions then the force T_w, T_h, N_w and N_h.) Also, the tangential force T_p is defined to be positive when directed forward. (This is contrary to the forces T_h and T_w, which are positive when directed backward.)

2.2 The equations of motion

We will now derive the equations of motion for this simplified aircraft. We assume that the aircraft is in a fully symmetric flight. We then only need to consider three equations of motion. Taking the sum of forces in X direction gives

$$T = T_w + T_h - T_p \cos i_p + N_p \sin i_p = -W \sin\theta. \tag{2.1}$$

Similarly, we can take the sum of forces in Z direction, and the sum of moments about the CG. (This is done about the Y axis.) We then get

$$N = N_w + N_h + N_p \cos i_p + T_p \sin i_p = W \cos\theta, \tag{2.2}$$

$$\begin{aligned} M = & M_{ac_w} + N_w(x_{cg} - x_w) - T_w(z_{cg} - z_w) + M_{ac_h} + N_h(x_{cg} - x_h) - T_h(z_{cg} - z_h) + \dots \\ & \dots + (N_p \cos i_p + T_p \sin i_p)(x_{cg} - x_p) + (T_p \cos i_p - N_p \sin i_p)(z_{cg} - z_p) = 0. \end{aligned} \tag{2.3}$$

We can simplify these equations, by making a couple of assumptions. We want to examine the stability of the aircraft. The propulsion doesn't influence the stability of the aircraft much. So we neglect propulsion effects. We also neglect T_h, since it is very small compared to T_w. We assume that $(z_{cg} - z_w) \approx 0$. And finally, we neglect M_{ac_h}. This gives us

$$T = T_w = -W \sin\theta, \tag{2.4}$$

$$N = N_w + N_h = W \cos\theta, \tag{2.5}$$

$$M = M_{ac_w} + N_w(x_{cg} - x_w) + N_h(x_{cg} - x_h) = 0. \tag{2.6}$$

That simplifies matters greatly.

2.3 Non-dimensionalizing the equations of motion

Let's non-dimensionalize the equations of motion of the previous paragraph. For that, we divide the force equations by $\frac{1}{2}\rho V^2 S$ and the moment equation by $\frac{1}{2}\rho V^2 S\bar{c}$. This then gives us

$$C_T = C_{T_w} = -\frac{W}{\frac{1}{2}\rho V^2 S} \sin\theta, \tag{2.7}$$

$$C_N = C_{N_w} + C_{N_h} \left(\frac{V_h}{V}\right)^2 \frac{S_h}{S} = \frac{W}{\frac{1}{2}\rho V^2 S} \cos\theta, \tag{2.8}$$

$$C_m = C_{m_{ac_w}} + C_{N_w} \frac{x_{cg} - x_w}{\bar{c}} - C_{N_h} \left(\frac{V_h}{V}\right)^2 \frac{S_h l_h}{S\bar{c}} = 0. \tag{2.9}$$

A lot of new coefficients have suddenly disappeared. These coefficients are defined, such that

$$N = C_N \frac{1}{2}\rho V^2 S, \qquad T = C_T \frac{1}{2}\rho V^2 S, \qquad M = C_m \frac{1}{2}\rho V^2 S\bar{c}, \tag{2.10}$$

$$N_w = C_{N_w} \frac{1}{2}\rho V^2 S, \qquad T_w = C_{T_w} \frac{1}{2}\rho V^2 S, \qquad M_{ac_w} = C_{m_{ac_w}} \frac{1}{2}\rho V^2 S\bar{c}, \tag{2.11}$$

$$N_h = C_{N_h} \frac{1}{2}\rho V_h^2 S_h, \qquad T_h = C_{T_h} \frac{1}{2}\rho V_h^2 S_h, \qquad M_{ac_h} = C_{m_{ac_h}} \frac{1}{2}\rho V_h^2 S_h \bar{c}_h. \tag{2.12}$$

Here, $\frac{1}{2}\rho V_h^2$ is the average local dynamic pressure on the horizontal tail plain. Also, S_h is the tailplane surface area and $\bar{c}_h$ is the MAC of the tailplane. The quantity $\frac{S_h l_h}{S\bar{c}}$ is known as the **tailplane volume**. And finally, we have defined the **tail length** $l_h = x_h - x_w \approx x_h - x_{cg}$.

3 The horizontal tailplane

3.1 Important angles

We will now take a closer look at the horizontal tailplane. There are three parameters that describe the configuration of the horizontal tailplane. These parameters are the **effective horizontal tailplane angle of attack** α_h, the **elevator deflection** δ_e and the **elevator trim tab deflection** δ_{t_e}. The three angles are visualized in figure 1.

There is one angle which we will examine more closely now. And that is the effective angle of attack α_h. It is different from the angle of attack of the aircraft α. There are two important causes for this. First,

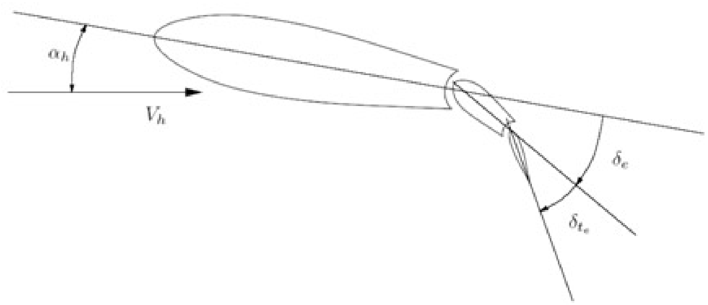

Figure 1: The angles of the horizontal tailplane.

the horizontal tail plane has an **incidence angle** i_h, relative to the MAC of the wing. And second, the tailplane experiences downwash, caused by the wing of the aircraft. The **average downwash angle** is denoted by ε. By putting this all together, we find that

$$\alpha_h = \alpha + i_h - \varepsilon. \tag{3.1}$$

We can elaborate a bit further on this. The downwash ε mainly depends on α. Linearization thus gives $\varepsilon \approx \frac{d\varepsilon}{d\alpha}(\alpha - \alpha_0)$. It follows that

$$\alpha_h = \left(1 - \frac{d\varepsilon}{d\alpha}\right)(\alpha - \alpha_0) + (\alpha_0 + i_h). \tag{3.2}$$

From this follows that the derivate $d\alpha_h/d\alpha$ is given by

$$\frac{d\alpha_h}{d\alpha} = 1 - \frac{d\varepsilon}{d\alpha}. \tag{3.3}$$

This derivative is thus generally smaller than 1.

3.2 The horizontal tailplane normal force

Let's examine the normal force C_{N_h} of the horizontal tailplane. This is a function of the three angles α_h, δ_e and δ_{t_e}. Applying linearization gives

$$C_{N_h} = C_{N_{h_0}} + \frac{\partial C_{N_h}}{\partial \alpha_h}\alpha_h + \frac{\partial C_{N_h}}{\partial \delta_e}\delta_e + \frac{\partial C_{N_h}}{\partial \delta_{e_t}}\delta_{e_t}. \tag{3.4}$$

The effect of the trim tab to the normal force is usually negligible. So, $\partial C_{N_h}/\partial \delta_{e_t} \approx 0$. Also, since most horizontal tailplanes are (nearly) symmetric, we have $C_{N_{h_0}} \approx 0$. This simplifies the above equation to

$$C_{N_h} = \frac{\partial C_{N_h}}{\partial \alpha_h}\alpha_h + \frac{\partial C_{N_h}}{\partial \delta_e}\delta_e = C_{N_{h_\alpha}}\alpha_h + C_{N_{h_\delta}}\delta_e. \tag{3.5}$$

Note that we have used a shorter notation in the right part of the above equation. The variables $C_{N_{h_\alpha}}$ and $C_{N_{h_\delta}}$ are quite important for the balance of the control surface. If they are both negative, then the control surface is called **aerodynamically underbalanced**. If, however, they are both positive, then the control surface is called **aerodynamically overbalanced**.

3.3 The elevator deflection necessary for equilibrium

We can ask ourselves, what elevator deflection δ_e should we have, to make sure our aircraft is in equilibrium? For that, we examine the moment equation (2.9). In this equation are the coefficients C_{N_w} and

C_{N_h}. We can replace these by the linearizations

$$C_{N_w} = C_{N_{w_\alpha}}(\alpha - \alpha_0) \qquad \text{and} \qquad C_{N_h} = C_{N_{h_\alpha}}\alpha_h + C_{N_{h_\delta}}\delta_e. \tag{3.6}$$

If we do this, we find that

$$C_m = C_{m_{ac_w}} + C_{N_{w_\alpha}}(\alpha - \alpha_0)\frac{x_{cg} - x_w}{\bar{c}} - \left(C_{N_{h_\alpha}}\alpha_h + C_{N_{h_\delta}}\delta_e\right)\left(\frac{V_h}{V}\right)^2 \frac{S_h l_h}{S\bar{c}} = 0. \tag{3.7}$$

We can now also substitute the relation (3.2) for α_h. Doing this, and working the whole equation out, gives

$$C_m = C_{m_0} + C_{m_\alpha}(\alpha - \alpha_0) + C_{m_{\delta_e}}\delta_e = 0, \tag{3.8}$$

where C_{m_α} is known as the **static longitudinal stability** and $C_{m_{\delta_e}}$ is the **elevator effectivity**. Together with the constant C_{m_0}, they are defined as

$$C_{m_0} = C_{m_{ac_w}} - C_{N_{h_\alpha}}(\alpha_0 + i_h)\left(\frac{V_h}{V}\right)^2 \frac{S_h l_h}{S\bar{c}}, \tag{3.9}$$

$$C_{m_\alpha} = C_{N_{w_\alpha}}\frac{x_{cg} - x_w}{\bar{c}} - C_{N_{h_\alpha}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(\frac{V_h}{V}\right)^2 \frac{S_h l_h}{S\bar{c}}, \tag{3.10}$$

$$C_{m_{\delta_e}} = -C_{N_{h_\delta}}\left(\frac{V_h}{V}\right)^2 \frac{S_h l_h}{S\bar{c}}. \tag{3.11}$$

We can now solve for δ_e. It is simply given by

$$\delta_e = -\frac{C_{m_0} + C_{m_\alpha}(\alpha - \alpha_0)}{C_{m_{\delta_e}}}. \tag{3.12}$$

This is a nice expression. But do remember that we have made several linearizations to derive this equation. The above equation is thus only valid, when all the linearizations are allowed.