Flight and Orbital Mechanics

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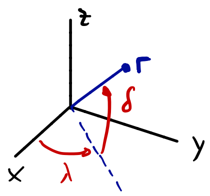
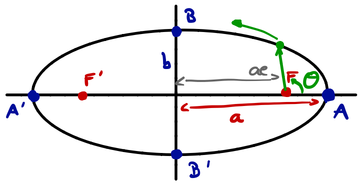
Lecture 1 Kepler Orbits

Kepler orbits: describe where your spacecraft is at in an orbit.

coordinate system



CLOSED ELLIPSE



$$\boxed{r = \frac{a(1-e^2)}{1 + e\cos\theta}}$$

Pericenter : $\theta = 0°$ $\quad r_p = a(1-e)$

Apocenter : $\theta = 180°$ $\quad r_a = a(1+e)$

$r_a + r_p = 2a \qquad a = \frac{r_a + r_p}{2}$

$$\boxed{e = \frac{(r_a - r_p)}{(r_a + r_p)}}$$

2-D

a : semi major axis

e : eccentricity

t_p, τ : time of pericenter passage

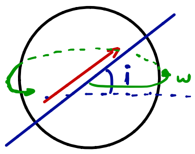
3-D

i : inclination

Ω : ascending nodes

ω : argument of pericenter

determines the "starting point at equador"



ascending node

GRAVITATIONAL INTERACTION:

Newton $\quad F = G \cdot \frac{mass_1 \cdot mass_2}{distance^2}$

Elementary force: $d\vec{F} = -\frac{G\, m_{sat}}{(\Delta r)^2} \cdot \frac{\Delta\vec{r}}{\Delta r} \cdot \rho\, dV$

Total force: $\vec{F} = -\int_F \frac{G\, m_{sat}}{(\Delta r)^2} \cdot \frac{\Delta\vec{r}}{\Delta r} \rho \cdot dv$

Total acceleration due to symmetrical Earth:

$$= -\frac{G \cdot M}{r^3} \cdot \vec{r}$$

Radial acceleration

$$= \frac{-G \cdot M}{r^2} = -\frac{\mu}{r^2}$$

Potential

$$= -\frac{\partial U}{\partial r} = -\frac{\partial}{\partial r}\left[-\frac{\mu}{r}\right]$$

because of earth irregularities.

nonsymmetric earth.

GRAVITY FIELD EARTH

Gravity Potential:

$$U = -\frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n \sin\delta + \sum_{n=2}^{\infty}\sum_{m=1}^{n} J_{n,m}\left(\frac{R_e}{r}\right)^n P_{n,m}(\sin\delta)\cdot\cos\left(m(\lambda - \lambda_{n,m})\right)\right]$$

Legendre function:

$$P_{n,m}(x) = (1-x^2)^{m/2} \cdot \frac{d^m P_n(x)}{dx^m}$$

Legendre Polynomial:

$$P_n(x) = \frac{1}{(-2)^n n!} \cdot \frac{d^n}{dx^n}(1-x^2)^n$$

EXAMPLE:

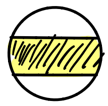
n	$J_n\ [10^{-6}]$
2	1082.63
3	−2.5327
4	−1.6196
5	−0.2273

$$-\frac{\mu}{r}\left[1 - J_2 - J_3 - J_4 \ldots + J_{2,1} + J_{2,2} + J_{3,1} + J_{3,2}\ldots\right]$$

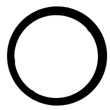
GRAVITY FIELD EARTH

Main term: $U_0 = -\dfrac{\mu}{r}$

SIDE VIEW



TOP VIEW



Most prominent irregularity: $J_2 \longrightarrow U_2 = \dfrac{\mu}{r} J_2 \left(\dfrac{R_e}{r}\right)^2 P_2 \sin\delta$

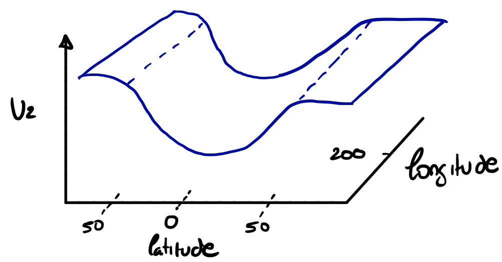
compare the perturbation by J_2: first work out with a parameter x, not delta.

Step 1 $\quad P_2(x) = \dfrac{1}{(-2)^2\, 2!}\dfrac{\partial^2}{\partial x^2}\underbrace{(1-x^2)^2}_{\text{no chain rule needed.}} = \dfrac{1}{8}\dfrac{\partial^2}{\partial x^2}(1-2x^2+x^4) = \dfrac{1}{8}\dfrac{d}{dx}(-4x+4x^3)$

$P_2(x) = -\dfrac{1}{2} + \dfrac{3}{2}x^2$

Step 2 $\quad \boxed{U_2 = \mu J_2 R_e^2 r^{-3}\left(-\dfrac{1}{2} + \dfrac{3}{2}\sin^2\delta\right)}$

PLOT POTENTIAL

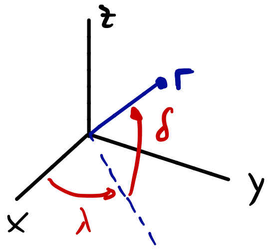


Accelerations:

$a_x = -\dfrac{dU}{dx}$ $\qquad a_r = -\dfrac{dU}{dr}$

$a_y = -\dfrac{dU}{dy}$ $\qquad a_\delta = -\dfrac{1}{r}\dfrac{dU}{d\delta}$

$a_z = -\dfrac{dU}{dz}$ $\qquad a_\lambda = -\dfrac{1}{r\cos\delta}\cdot\dfrac{dU}{d\lambda}$



$a_{r,2} = -\dfrac{\partial U_2}{\partial r}$

EXAMPLE:

West acceleration due to $J_{3,2}$

Step 1

$$U_{3,2} = -\frac{\mu}{r}\left[+ J_{3,2} \left(\frac{Re}{r}\right)^3 P_{3,2} \cdot \sin(\delta) \cdot \cos(2(\lambda - \lambda_{3,2}))\right]$$

Step 2

$$P_3(x) = \frac{1}{(-2)^3 3!} \frac{d^3}{dx^3}(1-x^2)^3 = \frac{5}{2}x^3 - \frac{3}{2}x$$

Step 3:

$$P_{3,2}(x) = (1-x^2)^{2/2} \frac{d^2 P_3(x)}{dx^2} = 15(1-x^2)x$$

Step 4:

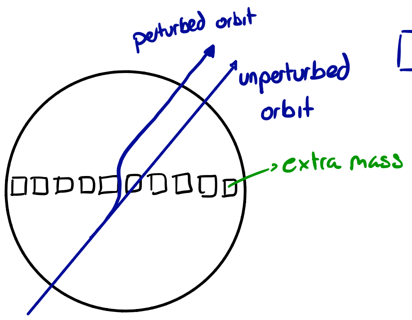
$$U_{3,2} = -\mu J_{3,2} Re^3 r^{-4} \cdot 15 \cdot \cos^2\delta \cdot \sin\delta \cdot \cos(2(\lambda - \lambda_{3,2}))$$

Step 5:

$$a_{ew\,3,2} = -\frac{1}{r\cos\delta} \cdot \frac{\partial U_{3,2}}{\partial \lambda} = -30\mu J_{3,2} \cdot Re^3 \cdot r^{-5} \cos\delta \cdot \sin\delta \cdot \sin(2(\lambda - \lambda_{3,2}))$$

Linear Perturbations on orbital elements due to J_2

- $\Delta a_{2\pi} = 0$
- $\Delta e_{2\pi} = 0$
- $\Delta i_{2\pi} = 0$
- $\Delta \Omega_{2\pi} = -3\pi J_2 \left(\frac{Re}{p}\right)^2 \cos(i)$
- $\Delta \omega_{2\pi} = 1{,}5\pi J_2 \left(\frac{Re}{p}\right)^2 (5\cos^2(i) - 1)$



$\boxed{i < 90°}$

This is dangerous because it can decrease the accuracy of measurements done by the satellites.

Sun-synchronous orbit: always with same degrees corresponding to the sun.

Earth-repeat orbit:

- Req 1: Ground track repeats after j orbital revolutions and k "days"
- Req 2: Effects are measured w.r.t. Earth surface.

$$\Delta L_1 = -2\pi \frac{T}{T_E} \quad \text{(contribution earth rotation)}$$

T → satellite period

T_E → earth period.

$$\Delta L_2 = -\frac{3\pi J_2 Re^2 \cos i}{a^2(1-e^2)^2} \quad \text{(contribution of } J_2\text{)}$$

L = Longitude positive East

Eccentricity = 0 for 95%

$$j\,|\Delta L_1 + \Delta L_2| = k\,2\pi$$

$\boxed{\text{Other terms may affect the repetition!}}$

└→ atmosferic drag f.e.

Developing the previous equation

$$j\left|-2\pi\,\frac{2\pi\sqrt{a^3/\mu}}{T_E}-\frac{3\pi J_2 R_e^2\cos(i)}{a^2(1-e^2)^2}\right| = k2\pi \qquad e=0 \text{ assumption.}$$

assume $a(i)$ or $i(a)$ direct solution
iteration.

a higher altitude corresponds with longer orbital period.

Circular orbits: a = semimajor axis minus Earth radius.

$(j,k) = (14,1)$

Days | every day

$a = 7200$ km $\rightarrow$ $i = 47.2°$

$a = 7300$ km $\rightarrow$ $i = 119.5°$

$a = 7500$ km $\rightarrow$ no solution.

Equatorial spacing: $\Delta = 2\pi\frac{R_e}{j}$: driven by the total number of revolutions, before the repeat patern repeats itself again. $(42,3) = (14,1)$ for example.

Sun-Synchronous Orbit:

Always: $$\boxed{\dot\Omega = \frac{\Delta\Omega_{2\pi}}{T} = -3\pi J_2\left(\frac{R_e}{P}\right)^2\cos i\,\frac{1}{T}}$$

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

T: satellite
T_E: sideral day
T_{ES}: orbital period of Earth around the sun.

Requirement $\dot\Omega = \frac{2\pi}{T_{ES}}$ $\quad\boxed{e=0}$

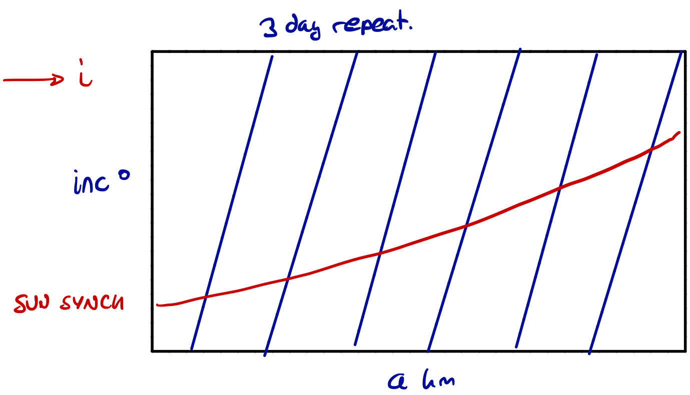
inclination is alway greater than 90°

Combined orbits:

$$\underbrace{j\left|\Delta L_1+\Delta L_2\right| = k2\pi \quad\text{and}\quad \frac{d\Omega}{dt} = \frac{\Delta L_2}{T} = \frac{2\pi}{T_{ES}}}$$

$$\boxed{j\left|-2\pi\frac{T}{T_E}+2\pi\frac{T}{T_{ES}}\right| = k2\pi} \longrightarrow jT\left(\frac{1}{T_E}-\frac{1}{T_{ES}}\right) = k \qquad a(j,k,T_E,T_{ES},\mu)$$

Finally: $$-3\pi J_2\left(\frac{R_e}{a(1-e^2)}\right)^2\cos(i)\,\frac{1}{T} = \frac{2\pi}{T_{ES}} \longrightarrow i$$



GEOSTATIONARY ORBIT

Characteristics:

- $T = 23h\ 56m\ 4s \longrightarrow a = 42164$ km
- $e = 0$
- $i = 0°$

Effects of gravity field:

J_2 and $J_{2,2}$

↳ constant East-West acc on GEO satellites.

$a_\lambda = -5.6 \cdot 10^{-8} \sin(2(\lambda + 14.9))$

ΔV budget:

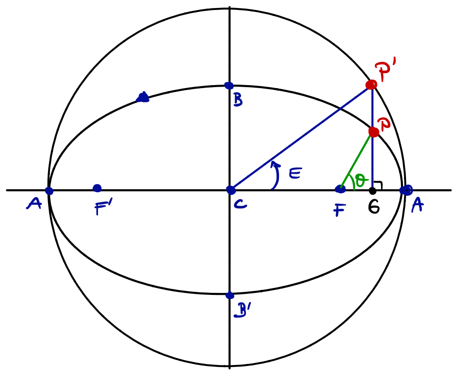
$J_{2,2}$: $1.7 \sin(2(\lambda - 75))$

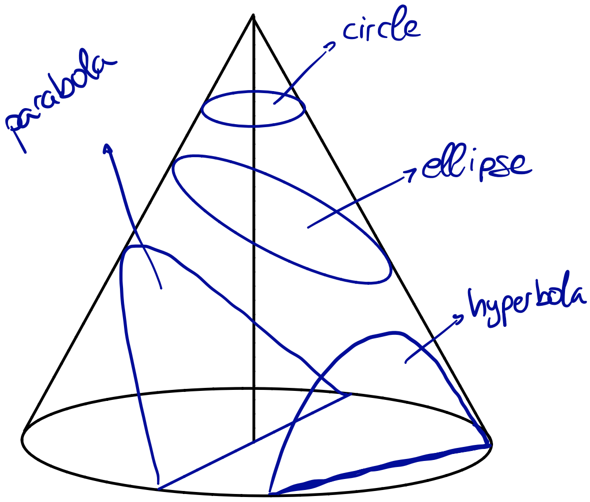
Sun + Moon = 51.4

2nd LECTURE

Kepler orbits:

two dimensional:





EQUATIONS:

$$r = \frac{a(1-e^2)}{1+e\cos\theta} = \frac{P}{1+e\cos\theta} \qquad r_p = a(1-e) \qquad r_a = a(1+e)$$

$$E_{TOT} = E_{kin} + E_{pot} = \frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \qquad T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$\boxed{V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)} \qquad \boxed{V_{circ} = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{\mu}{a}}} \qquad \boxed{V_{esc} = \sqrt{\frac{2\mu}{r}}}$$

more equations: **ELLIPSE**

Ellips: $0 \le e < 1$ $\quad a > 0 \quad E_{TOT} < 0$

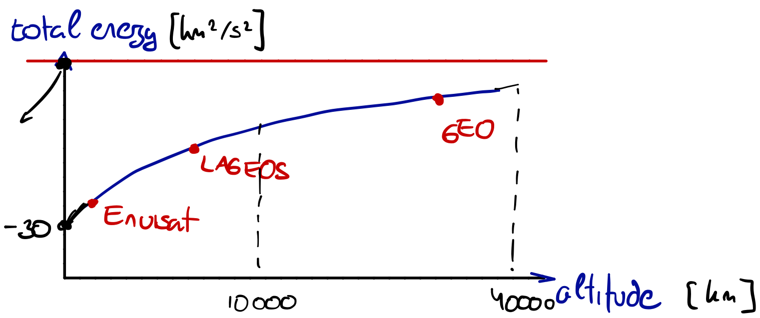
keplers equation, link between position and time.

$$\boxed{n = \sqrt{\frac{\mu}{a^3}}} \qquad \boxed{\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}} \cdot \tan\frac{E}{2}} \qquad \boxed{M = E - e\sin E} \quad \boxed{M = n(t - t_0)}$$

t_0 → pericenter passage

$$\boxed{E_{i+1} = E_i + \frac{M - E_i + e\sin(E_i)}{1 - e\cos E_i}} \qquad \boxed{r = a(1 - e\cos E)}$$

Specific energy: $E = \frac{1}{2}V^2 - \mu/r$



PARABOLA

$e = 1 \qquad a = \infty \qquad E_{tot} = 0$

$$r = \frac{p}{1 + \cos\theta}$$

$$M = \frac{1}{2}\tan\left(\frac{\theta}{2}\right) + \frac{1}{6}\tan^3\frac{\theta}{2}$$

$$M = n(t - t_0)$$

$$n = \sqrt{\frac{\mu}{p^3}}$$

$$V^2 = V_{esc}^2 = \frac{2\mu}{r}$$

HYPERBOLA

$e > 1 \qquad a < 0 \qquad E_{tot} > 0$

$$\tan\frac{\theta}{2} = \sqrt{\frac{e+1}{e-1}} \cdot \tanh\frac{F}{2}$$

$$M = e \sinh F - F$$

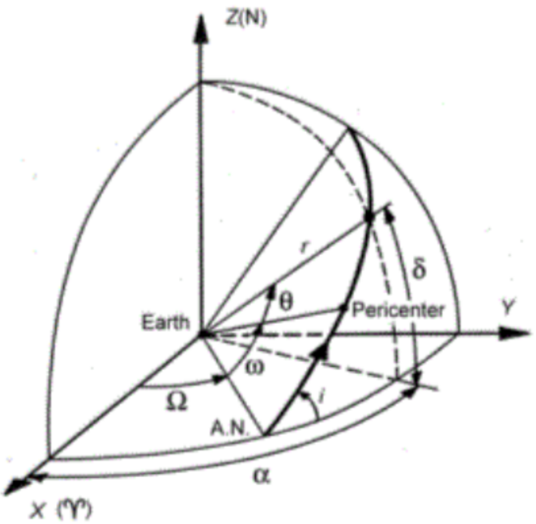
$$n = \sqrt{\frac{\mu}{(-a)^3}}$$

$$M = n(t - t_0)$$

$$r = a(1 - e\cosh F)$$

$$V^2 = V_{esc}^2 + V_\infty^2 = \frac{2\mu}{r} + V_\infty^2$$

Three Dimensional Kepler Orbits:



i: inclination [deg] no more than 180°

Ω: right ascension of the ascending node, or longitude of the ascending node [deg]

ω: argument of pericenter [deg]

$u = \omega + \theta$: argument of latitude [deg]

Coordinate Transforms

Spherical to cartesian:

$$x = r\cos\delta\cos\lambda$$

$$y = r\cos\delta\sin\lambda$$

$$z = r\sin\delta$$

Cartesian to spherical:

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r_{xy} = \sqrt{x^2 + y^2}$$

$$\lambda = \mathrm{atan2}\left(\frac{y}{r_{xy}}, \frac{x}{r_{xy}}\right)$$

$$\delta = \mathrm{asin}(z/r)$$

atan → ignores outside −90° and +90°

atan2 → one response $\boxed{360°}$

Perturbations in Orbit Modelling

Option 1: include directly in equation of motion.

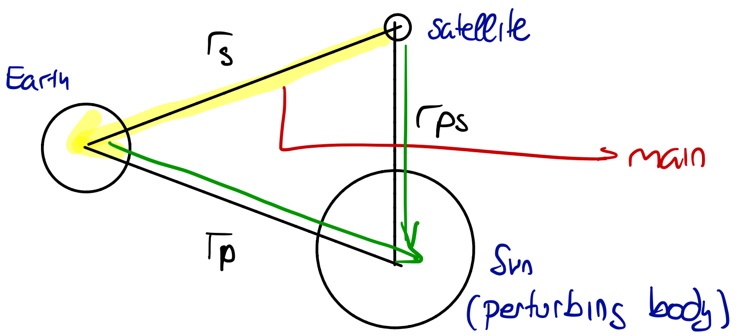
$$\frac{dx^2}{dt^2} = a_{main} + a_{\Delta grav} + a_{drag} + a_{solrad} + a_{3rd body} + \text{etcetera.}$$

Option 2: express as variation of orbital elements.

e.g. $$\frac{da}{dt} = \frac{2a^2}{\sqrt{\mu p}}\left[\underbrace{S e \cdot \sin(\theta)}_{\text{radial.}} + \underbrace{N \cdot \frac{p}{r}}_{\text{transverse.}}\right]$$

Irregularities in Gravity Field — Treated before

Third Body Perturbations



Difference between the acceleration of the satellite and the acceleration on the origin of the reference frame.

May have contributions to total ΔV budget.

Atractional forces:

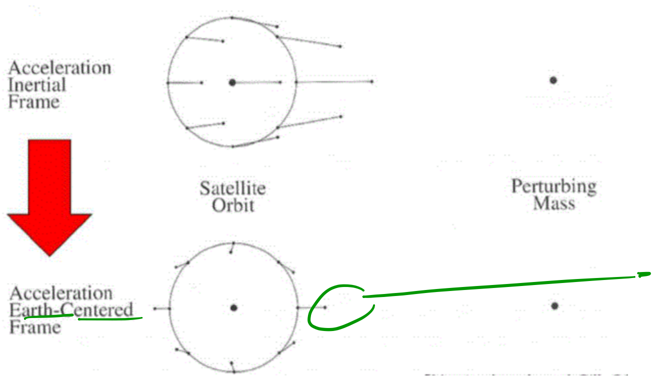
- Earth atracts satellites
- Perturbing body attracts satellite
- Perturbing body attracts Earth.
- Net effect counts

$$\ddot{r}_s = -G\frac{M_{main}}{r_s^3} r_s + G M_p\left(\frac{r_{ps}}{r_{ps}^3} - \frac{r_p}{r_p^3}\right)$$

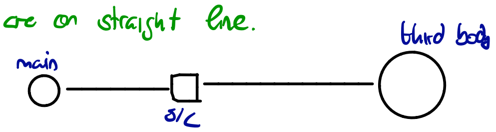
The satellite does not attract the other bodies.

Acceleration: $$a_p = \mu_p\left\{\frac{r_{ps}}{\|r_{ps}\|^3} - \frac{r_p}{\|r_p\|^3}\right\}$$

$$\left(\frac{a_p}{a_{main}}\right)_{max} = 2 \cdot \frac{m_p}{m_{main}}\left(\frac{r_s}{r_p}\right)^3$$



maximum when they are on straight line.



Two situations:

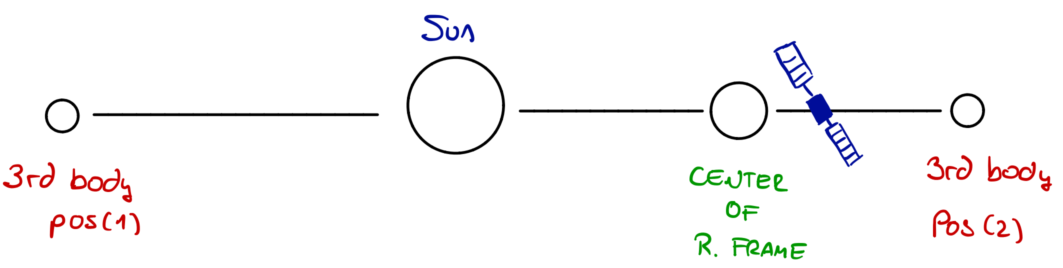
HELIOCENTRIC:

- The influence of the sun decreases with distance to Sun while the influence of the planets increases.
- Acceleration from sun $O(10^{-2})\,m/s^2$; dominant.
- Near planet, the third body becomes dominant.
- Solar system: Sun dominant
- Near earth: Earth dominant.
- SPHERE OF INFLUENCE
  - → Area around planet where gravity from the planet is dominant
  - → Approximation: sphere with constant radius and acceleration μ/r^2
  - → Boundary: Not the equilibrium of the two bodies. what is it then?

$$\frac{acc_{sun,3rd}}{acc_{Earth,main}} = \frac{acc_{Earth,3rd}}{acc_{sun,main}}$$

$$r_{SoI} = r_{3d}\left(\frac{\mu_{main}}{\mu_{3rd}}\right)^{0.4}$$

PLANETOCENTRIC



- Influence of sun as third body increases with distance from Earth.
- Effective third-body acceleration by Sun $O(10^{-6})\,m/s^2$
- Influence of Moon as third body increases with distance from Earth.
- Effective third-body acceleration by Moon $O(10^{-5})\,m/s^2$ at GEO
- Next to earth: Earth dominant, Moon most important third body. Other planets 4 order smaller.

Atmospheric Drag:

$$a_{drag} = \ominus \frac{C_D}{m} S \cdot \frac{1}{2} \rho V^2 \cdot \frac{\vec{V}}{V}$$

negative.

velocity must be taken using rotating atmosphere.

also changes depending on day, night. { • max day • min night

$$\rho = \rho_0 \exp\left(-\frac{\Delta h}{H}\right)$$

specific, reference altitude.

distance w.r.t

density scale height constant

The drag causes a loss of energy, lowers the apocenter and makes the orbit circular.

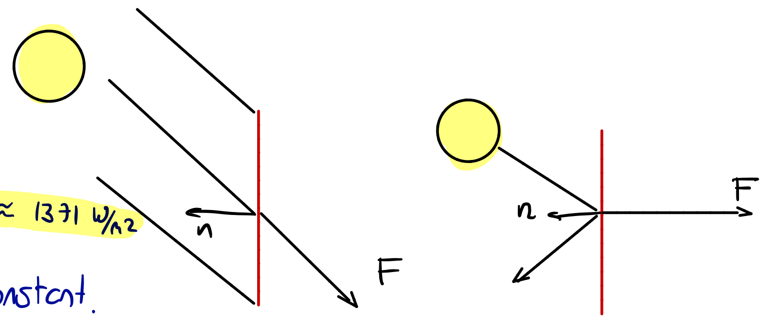
Circular Orbits: EFFECTS AFTER ONE COMPLETE REVOLUTION:

$$\Delta a_{2\pi} = -2\pi \left(\frac{C_D \cdot A}{m}\right) \cdot \rho \cdot a^2 \qquad \Delta V_{2\pi} = \pi \left(\frac{C_D \cdot A}{m}\right) \rho a V \qquad L = -\frac{H}{\Delta a_{2\pi}}$$

Lifetime.

$$\Delta T_{2\pi} = -6\pi^2 \left(\frac{C_D \cdot A}{m}\right) \rho \cdot a^2 / V \qquad \Delta e_{2\pi} = 0$$

Solar Radiation Pressure



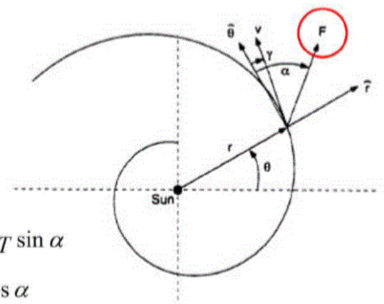
- amount of energy emitted by sun at 1 AU ≈ 1371 W/m^2
- depends on solar activity: SC: solar constant.
- solar radiation pressure: $\frac{SC}{c} = N/m^2 \longrightarrow \text{Energy}(r) = SC/(r_{AU})^2$

$$a_{rad} = \underbrace{(1+\rho)}_{\text{"}C_R\text{"}} \frac{1}{(\|r_{SUN-SAT}/AU\|)^2} \cdot \frac{SC}{c} \cdot \frac{A}{m} \cdot \frac{r_{SUN-SAT}}{\|r_{SUN-SAT}\|}$$

→ distance in AU

Thrust

Acceleration $a_T = \frac{F}{M_{SAT}}$ I.C $a_T = 0$ → unperturbed orbit.



[Petropoulos, 2001]

$$\ddot{r} - r\dot{\theta}^2 + \mu/r^2 = a_T \sin\alpha$$
$$\frac{1}{r}\frac{d}{dt}\left(r^2 \dot{\theta}\right) = a_T \cos\alpha$$
$$(or: \; \ddot{\theta} r + 2\dot{\theta}\dot{r} = a_T \cos\alpha)$$

- High Thrust:
  can compete against central gravity.
  instantaneous velocity changes
- Low Thrust:
  attractive since high I_{sp}
  primary propulsion interplanetary
  station keeping. → transfer GEO.

Low Earth Orbit

- J_2 is dominant perturbation for all LEO.
- Low thrust already important.
- Atmospheric drag dominant perturbation at very low altitudes.
- Solar, lunar and $J_{2,2}$ accelerations very small but build up for LEO.
- Kepler orbit very good first-order approximation.

Interplanetary Orbit

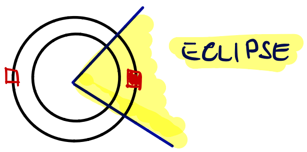
- Low thrust important
- Solar radiation can be important
- Kepler orbit very good first-order approximation

Chapter 3 Eclipse, Maneuvers

Eclipse: Obstruction of sunlight

- consequences: power, thermal control, attitude control.

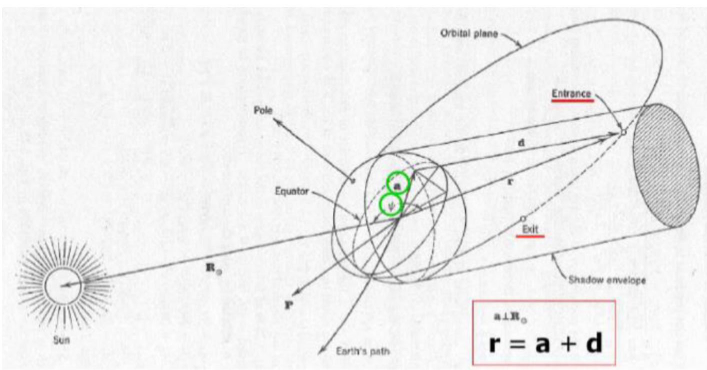
- Planar



- two-dimensional
- sun at infinite distance
- satellite in circular orbit
- length of eclipse: $\sin(\lambda) = R_e/a \to \lambda \to T_{eclipse} = \left(\frac{2\lambda}{360}\right) \cdot T_{orbit}$

42 % time at low orbits

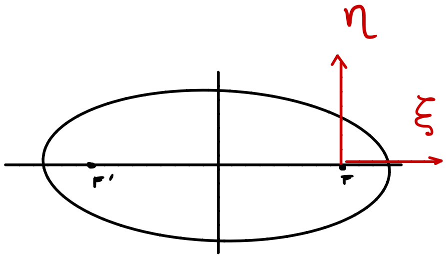
- Three - Dimensional



$r = a + d$

Eclipse conditions

1. Satellite on night-side of earth $\psi > 90°$
2. Satellite "hides" behind earth. $a < R_e$



in-plane position satellite: (r_{sat}, θ)

where: $$r_{sat} = \frac{a(1-e^2)}{1+e\cos\theta} = \frac{p}{1+e\cos\theta}$$

3D-Position: transformation: $\boxed{r_{3d} : R_3(\Omega) R_1(i) R_3(\omega)}$ → elementary rotation about z axis

$$R_3 = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} X_{sat} \\ Y_{sat} \\ Z_{sat} \end{pmatrix} = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \\ n_1 & n_2 \end{pmatrix} \begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \\ n_1 & n_2 \end{pmatrix} = \begin{pmatrix} r_{sat} \cdot \cos\theta \\ r_{sat} \cdot \sin\theta \end{pmatrix}$$

Angle ψ:

$$R_{sun} \cdot r_{sat} = R_{sun} \cdot r_{sat} \cdot \cos\psi \quad \text{or} \quad \cos\psi = \bar{\alpha}\cos\theta + \bar{\beta}\sin\theta$$

Where:

$$\bar{\alpha} = (l_1 \cdot X_{sun} + m_1 \cdot Y_{sun} + n_1 \cdot Z_{sun}) / R_{sun}$$
$$\bar{\beta} = (l_2 \, X_{sun} + m_2 \, Y_{sun} + n_2 \, Z_{sun}) / R_{sun}$$

ENTERING:

$a = R_e$

$\sin^2\psi + \cos^2\psi = 1$

$r_{sat} = p/(1+e\cos\theta)$

Conditions:

1. $\cos\psi = \bar{a}\cos\theta + \bar{\beta}\sin\theta < 0$
2. $a = r_{sat} \cdot \sin\psi < R_e$

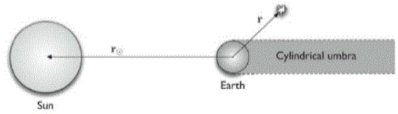
SHADOW FUNCTION:

$$\boxed{S(\theta) = R_e^2(1+e\cos\theta]^2 + p^2(\bar{\alpha}\cos\theta + \bar{\beta}\sin\theta)^2 - p^2}$$

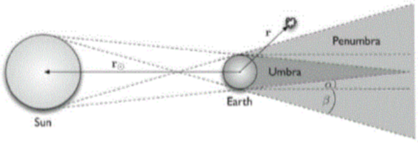
	S < 0	S = 0	S > 0
Ψ < 90°	in sunlight	in sunlight	in sunlight
Ψ > 90°	in sunlight	entering/leaving shadow cone	in shadow cone

Complication 1: Umbra, Penumbra.

- Idealized: idealized:



- Reality: reality:



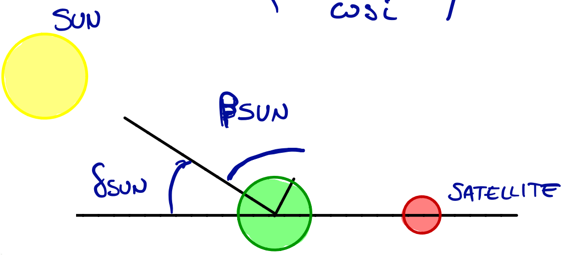
Long-Term Eclipse Behaviour

- direction of the sun
- orbit normal n

Eclipse if:

$$a\bar{n} \cdot \hat{R}_{SUN} \leq Re$$

$$\bar{n} = \begin{pmatrix} \sin i \cdot \sin\Omega \\ -\sin i \cdot \cos\Omega \\ \cos i \end{pmatrix}$$

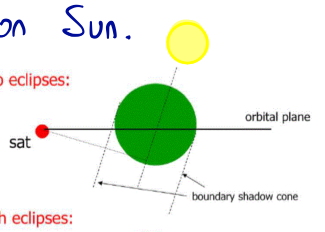


Complication 2: flattening Earth.

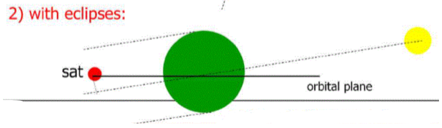
Complication 3: atmosphere Earth.

Complication 4: motion Sun.

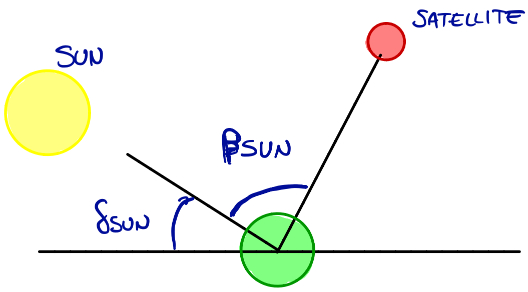
1. w/o eclipse:



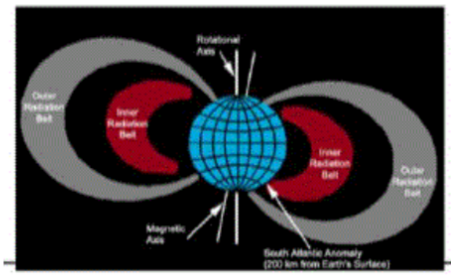
2. with eclipse:

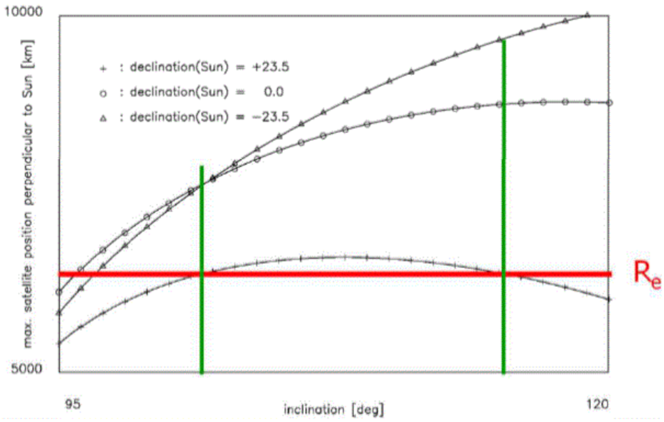


Permanent Sunlight ?



- Optimal $h > 1400$ m not good because Van Allen Belt





3.

- Sun-Synchronous orbit $\rightarrow a(i)$
- $\delta_{SUN} + \beta_{SUN} + i = 180°$ $\rightarrow \beta_{SUN}(i)$
- $a \cdot \sin(\beta_{SUN}) = a(i) \cdot \sin\beta_{SUN}(i) = f(i) \geq Re$

2.

- Line of nodes perpendicular to direction to Sun.
- Extreme situations: $\delta_{SUN} = -23.5°, 0°, 23.5°$

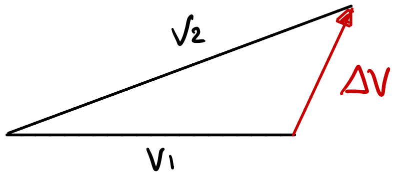
Maneuvers

- Change in velocity, to obtain change in orbit.

$\Delta \vec{V} = \vec{V}_2 - \vec{V}_1$

$\Delta V = ||\Delta \vec{V}||$

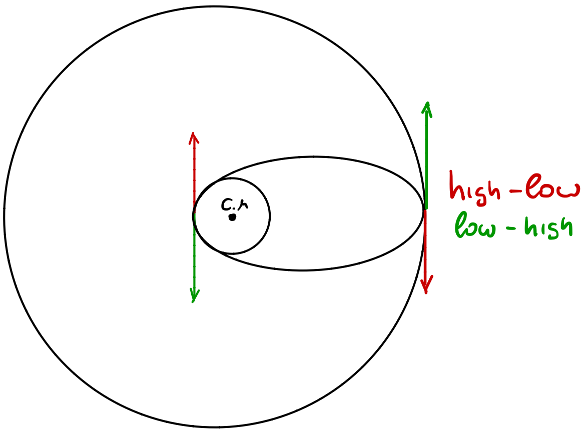
$\Delta E = \frac{1}{2}(V_2^2 - V_1^2) = \frac{1}{2}(\Delta V)^2 + \vec{V}_1 \cdot \Delta \vec{V}$



Dog-leg maneuver: most efficient when velocity is smallest. Highest point of orbit. Inclination at lowest.

- Combined maneuvers are more efficient than separated ones.

Hohmann Orbit



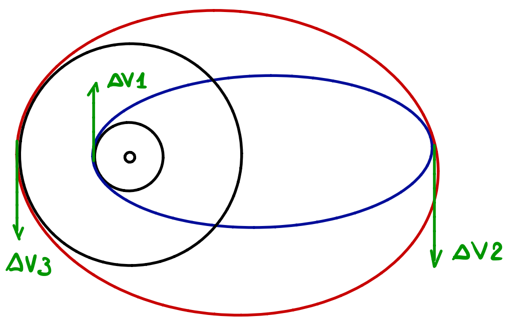
- $a_t = \frac{1}{2}(r_1 + r_2)$
- $\Delta V_1 = V_{per} - V_{c,1} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_t}\right)} - \sqrt{\frac{\mu}{r_1}}$
- $\Delta V_2 = V_{c,2} - V_{apo} = \sqrt{\frac{\mu}{r_2}} - \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_t}\right)}$
- $T_{transfer} = \frac{1}{2} T_t = \pi \sqrt{\frac{a_t^3}{\mu}}$

Optimal change LEO to GEO at (250 km, $i = 28.5°$)

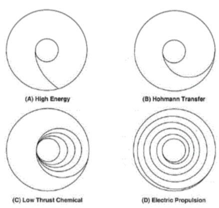
Combined maneuver, 4° plane change at perigee, 24.5° plane change at apogee.

Can we do better?

1. Use super-synchronous apocenter.



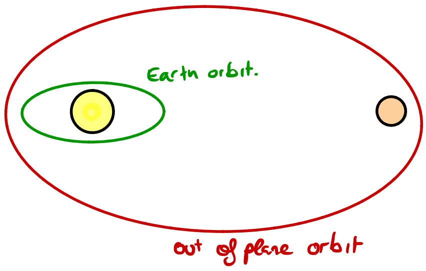
2. Use alternative techniques for in-plane transfer.



transfer type	orbit type	typical acceleration	ΔV	transfer time
high-energy	elliptic or hyperbolic	10 g	> Hohmann	< Hohmann
Hohmann	elliptic	1-5 g	Hohmann	Hohmann
low-thrust chemical prop.	Hohmann segments	0.02-0.5 g	< Hohmann	6-8 * Hohmann
electric propulsion	spiral transfer	0.0001-0.001 g	diff. between V_{circ}	60-120 * Hohmann

3. Use alternative techniques for out-of-plane transfer.

method	mechanism
ΔV at lowest velocity	small velocity is easier to change
combine ΔV with orbit raising	vector sum is smaller than sum of components
three-burn transfer	use intermediate high altitude for ΔV and then lower
differential node rotation	natural mechanism (J_2) for plane change: f(Δa,i)
aero-assist	aerodynamic forces cause plane change
planetary fly-by	gravity pull of planet causes plane change



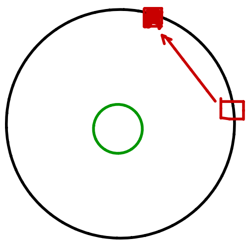
DISPOSAL (1): LEO

- Lower orbit and burn up in atmosphere
- One tangential maneuver

DISPOSAL (2):

- Raise circular GEO orbit to circular graveyard orbit (GEO + 300km)
- Two tangential maneuvers Hohmann
- $\Delta V = 10\, m/s$
- 6.8 kg propellant. ($m_{sat\,dry} = 2000$ kg, $I_{sp} = 300s$)

IN PLANE PHASE SHIFT

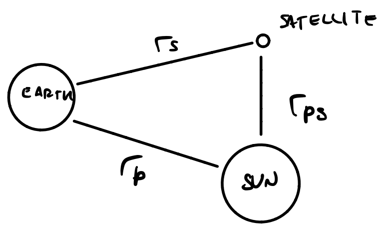


- Orbital plane remains the same
- Reposition satellite at new slot
- GEO, GPS, Iridium examples

LECTURE 4+5 INTERPLANETARY FLIGHT

- INTRODUCTION : planets disposition:
  - much greater than earth missions
  - Orbits of planets are more or less circular (except Mercury and pluto)
  - Orbits of planets more or less coplanar (except Mercury and Pluto)
  - 2-D situation with circular orbits is a good first model.

BASICS : Interaction between three bodies :



Sphere of influence:

$$\frac{acc\ Sun,3rd}{acc\ Earth,main} = \frac{acc\ Earth\ 3rd}{acc\ Sun,main} \qquad r_{SOI} = r_{3rd}\left(\frac{M_{main}}{M_{3rd}}\right)^{0.4}$$

- $acc_{3rd}/acc_{main} = O(10^{-6})$
- SoI Earth: ~ 930 000 km (0.006 AU)

Kepler orbits

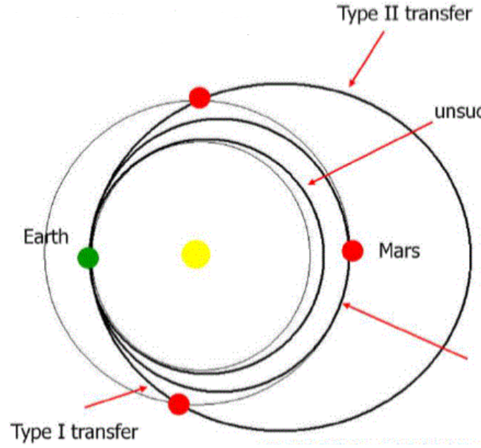
symbol	meaning	ellipse	hyperbola
a	semi-major axis	> 0	< 0
e	eccentricity	< 1	> 1
E	(specific) energy	< 0	> 0
r(θ)	radial distance	$a(1-e^2)/(1+e\cos(\theta))$	
r_{min}	minimum distance (pericenter)	a(1-e)	
r_{max}	maximum distance (apocenter)	a(1+e)	∞
V	velocity	$\sqrt{[\mu(2/r - 1/a)]}$	
V	velocity		$\sqrt{[V_{esc}^2 + V_\infty^2]}$

Parabola (e=1) boundary between ellipse and hyperbola.

General equation for velocity is the vis-viva.

Excess velocity V_∞ is obtained at an infinite distance from the central body and dependent on the value of the semi-major axis: $V_\infty = \sqrt{-\mu/a}$

TRANSFERS BETWEEN PLANETS



Type II transfer : longer transfer, it arrives at the planet at a later orbit than Type I.

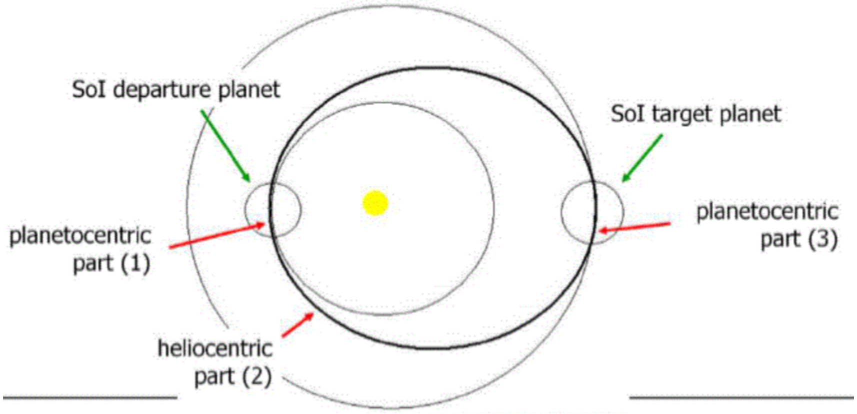
unsuccessful transfer : Not enough energy to reach Mars.

Hohmann transfer : touches both orbits

Hohman Transfer: around Sun:

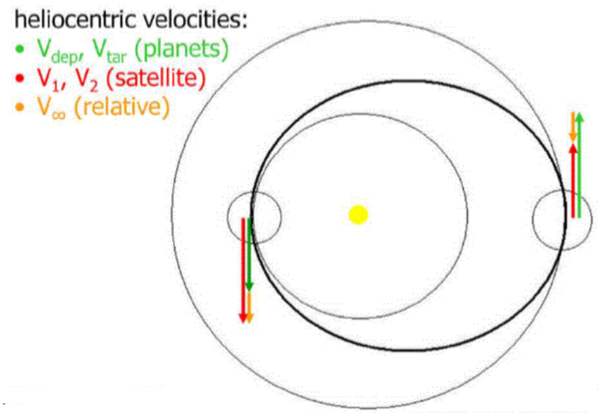
- Coplanar orbits
- Circular orbits departure and target planet.
- Impulsive shots
- Transfer orbit touches tangentially
- Minimum energy.

Interplanetary trajectory: succesion of three influence areas.



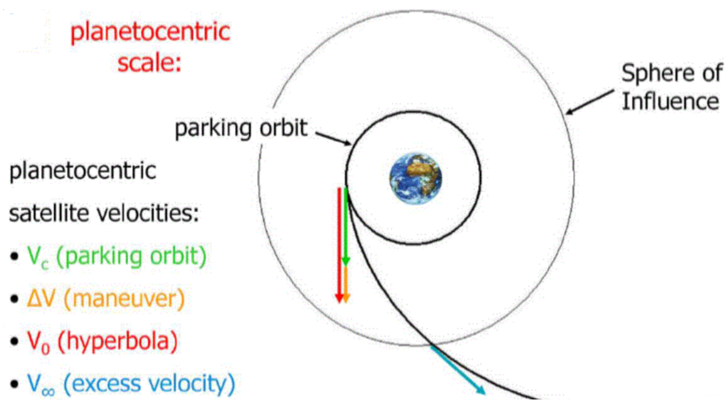
We end up with a series of three successive two-body problems, each one described to first order by a kepler orbit.

Heliocentric velocities



The velocities are parallel. The satellite needs to increase its heliocentric velocity in order to reach a target planet which is located further away from the sun.

Planetocentric Scale.



ΔV maneuvre transfer, from circular to hyperbolic.

Results in the excess velocity V_∞

$V_\infty < V_0$

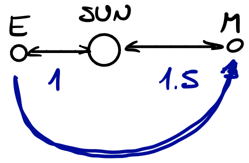
Lecture 28-02-2018

Hohmann Transfer (continued)

- Main elements of computation interplanetary Hohmann transfer:
  - → Semi-major axis heliocentric transfer orbit
  - → V_0 at departure and target planet.
  - → pericenter velocity of planetocentric hyperbolae
  - → ΔV's

step	parameter	expression	example
1	V_{dep} (heliocentric velocity of departure planet)	$V_{dep} = \sqrt{(\mu_{Sun}/r_{dep})}$	29.785 km/s
2	V_{tar} (heliocentric velocity of target planet)	$V_{tar} = \sqrt{(\mu_{Sun}/r_{tar})}$	24.130 km/s
3	V_{c0} (circular velocity around departure planet)	$V_{c0} = \sqrt{(\mu_{dep}/r_0)}$	7.793 km/s
4	V_{c3} (circular velocity around target planet)	$V_{c3} = \sqrt{(\mu_{tar}/r_3)}$	3.315 km/s
5	a_{tr} (semi-major axis of transfer orbit)	$a_{tr} = (r_{dep} + r_{tar}) / 2$	188.77×10^6 km
6	e_{tr} (eccentricity of transfer orbit)	$e_{tr} = \lvert r_{tar} - r_{dep} \rvert / (r_{tar} + r_{dep})$	0.208
7	V_1 (heliocentric velocity at departure position)	$V_1 = \sqrt{[\mu_{Sun}(2/r_{dep} - 1/a_{tr})]}$	32.729 km/s
8	V_2 (heliocentric velocity at target position)	$V_2 = \sqrt{[\mu_{Sun}(2/r_{tar} - 1/a_{tr})]}$	21.481 km/s
9	$V_{\infty,1}$ (excess velocity at departure planet)	$V_{\infty,1} = \lvert V_1 - V_{dep} \rvert$	2.945 km/s
10	$V_{\infty,2}$ (excess velocity at target planet)	$V_{\infty,2} = \lvert V_2 - V_{tar} \rvert$	2.649 km/s
11	V_0 (velocity in pericenter of hyperbola around departure planet)	$V_0 = \sqrt{(2\mu_{dep}/r_0 + V_{\infty,1}^2)}$	11.408 km/s
12	V_3 (velocity in pericenter of hyperbola around target planet)	$V_3 = \sqrt{(2\mu_{tar}/r_3 + V_{\infty,2}^2)}$	5.385 km/s
13	ΔV_0 (maneuver in pericenter around departure planet)	$\Delta V_0 = \lvert V_0 - V_{c0} \rvert$	3.615 km/s
14	ΔV_3 (maneuver in pericenter around target planet)	$\Delta V_3 = \lvert V_3 - V_{c3} \rvert$	2.070 km/s
15	ΔV_{tot} (total velocity change)	$\Delta V_{tot} = \Delta V_0 + \Delta V_3$	5.684 km/s
16	T_{tr} (transfer time)	$T_{tr} = \pi \sqrt{(a_{tr}^3/\mu_{Sun})}$	0.709 yr

Semi major axis of transfer orbit



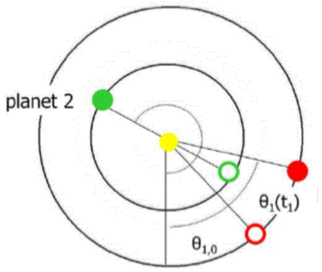
Timing Period

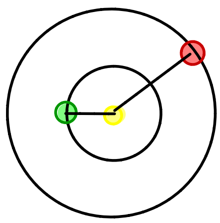
SYNODIC PERIOD: time interval after which relative geometry repeats.

$$\Delta\theta(t_1) = \theta_2(t_1) - \theta_1(t_1)$$

$$t_2 = t_1 + T_{syn}$$

$$\Delta\theta(t_2) = \theta_2(t_2) - \theta_1(t_2) = \Delta\theta(t_1) + 2\pi$$





Position of planet 1 and 2

$$\theta_1(t) = \theta_{1,0} + n_1(t - t_0)$$

$$\theta_2(t) = \theta_{2,0} + n_2(t - t_0)$$

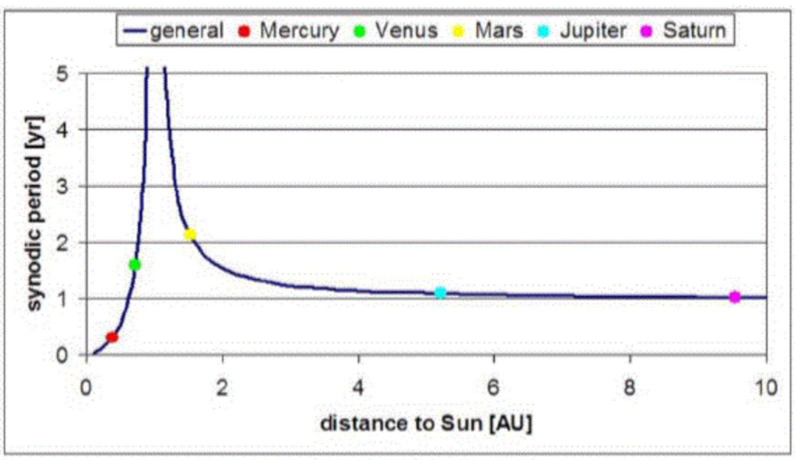
↳ mean motion of an object.

Difference

$$\Delta\theta(t) = \theta_2(t) - \theta_1(t) = (\theta_{2,0} - \theta_{1,0}) + (n_2 - n_1)(t - t_0)$$

Geometry repeats after T_{syn}: $t_2 = t_1 + T_{syn}$

$$\Delta\theta(t_2) - \Delta\theta(t_1) = 2\pi = (n_2 - n_1)(t_2 - t_1) = (n_2 - n_1)T_{syn}$$



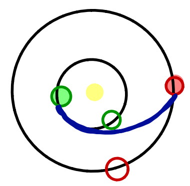
$$\frac{1}{T_{syn}} = \left| \frac{1}{T_2} - \frac{1}{T_1} \right|$$

TRANSFER TIME:

$$T_H = \frac{1}{2} T_{orbit} = \pi \sqrt{\frac{a^3}{\mu}}$$

n [°/day]

WHEN TO DEPART SO



- Planet 1 at t_1
- Planet 1 at t_2
- Planet 2 at t_2
- Planet 2 at t_1

POSITIONS AT EPOCH 1:

$\theta_1(t_1) = \theta_1(t_0) + n_1(t_1 - t_0)$

$\theta_2(t_1) = \theta_2(t_0) + n_2(t_1 - t_0)$

$\theta_{SAT}(t_1) = \theta_1(t_1) = \theta_1(t_0) + n_1(t_1 - t_0)$

POSITIONS AT EPOCH 2:

$\theta_2(t_2) = \theta_1(t_1) + n_1 T_H$

$\theta_2(t_2) = \theta_2(t_1) + n_2 T_H$

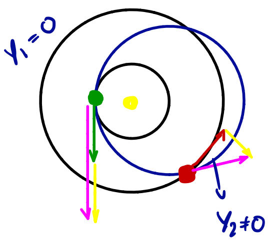
$\theta_{sat}(t_2) = \theta_{sat}(t_1) + \pi = \theta_1(t_1) + \pi = \theta_2(t_2)$

$\theta_1(t_0) + n_1(t_1 - t_0) + \pi = \theta_2(t_0) + n_2(t_1 - t_0) + n_2 T_H$

$$t_1 = t_0 + \frac{\theta_2(t_0) - \theta_1(t_0) + n_2 T_H - \pi}{n_1 - n_2}$$

$$t_2 = t_1 + T_H$$

FAST TRAJECTORIES



- Faster than HOHMANN TRANSFER
  - Earth
  - Mars
  - Satellite in transfer orbit
  - V_∞ at Earth
  - V_∞ at Mars

↳ flight path angle.

- Put more energy in departure
- Semi-major axis "a" is larger than Hohmann value
- Transfer orbit touches orbit of first planet, arbitrary V_∞ parallel to V_{dep}
- Transfer orbit intersect orbit of target planet at arbitrary angle and with arbitrary V_∞
- Unperturbed Kepler orbit
- Forward propagation: assume V_∞

step	parameter	expression	example
1	V_{dep} (heliocentric velocity of departure planet)	$V_{dep} = \sqrt{(\mu_{Sun}/r_{dep})}$	29.784 km/s
2	V_{tar} (heliocentric velocity of target planet)	$V_{tar} = \sqrt{(\mu_{Sun}/r_{tar})}$	24.129 km/s
3	V_{c0} (circular velocity around departure planet)	$V_{c0} = \sqrt{(\mu_{dep}/r_0)}$	7.793 km/s
4	V_{c3} (circular velocity around target planet)	$V_{c3} = \sqrt{(\mu_{tar}/r_3)}$	3.315 km/s

step	parameter	expression		example
		to more outer planet	to more inner planet	
5	V_1 (heliocentric velocity at departure position)	$V_1 = V_{dep} + V_{\infty,1}$	$V_1 = V_{dep} - V_{\infty,1}$	39.784 km/s
6	H (angular impulse momentum)	$H = r_{dep} V_1$		5.95×10^9 km²/s
7	a_{tr} (semi-major axis of transfer orbit)	$a_{tr} = ½\, \mu_{Sun} / (\mu_{Sun}/r_{dep} - ½\, V_1^2)$		6.93×10^8 km
8	e_{tr} (eccentricity of transfer orbit)	$e_{tr} = 1 - r_{dep}/a_{tr}$	$e_{tr} = r_{dep}/a_{tr} - 1$	0.784
9	r_p (pericenter distance)	$r_p = r_{dep}$	$r_p = a_{tr}(1-e_{tr})$	1.49×10^8 km
10	r_a (apocenter distance)	$r_a = a_{tr}(1+e_{tr})$	$r_a = r_{dep}$	1.24×10^9 km

step	parameter	expression	example
11	V_2 (heliocentric velocity at target position)	$\sqrt{[\mu_{Sun}(2/r_{tar} - 1/a_{tr})]}$	31.193 km/s
12	$V_{2,trans}$ (transverse component of heliocentric velocity at target position)	$V_{2,trans} = H/r_{tar}$	26.111 km/s
13	$V_{2,rad}$ (radial component of heliocentric velocity at target position) (**)	$V_{2,rad} = \sqrt{(V_2^2 - V_{2,trans}^2)}$	17.066 km/s
14	$V_{\infty,2}$ (excess velocity at target position)	$V_{\infty,2} = \sqrt{[V_{2,rad}^2 + (V_{2,trans} - V_{tar})^2]}$	17.180 km/s
15	γ_2 (flight path angle at target position) (*)	$\gamma_2 = \mathrm{atan}(V_{2,rad} / V_{2,trans})$	33.17°
16	V_0 (velocity in pericenter of hyperbola around departure planet)	$V_0 = \sqrt{(2\mu_{dep}/r_0 + V_{\infty,1}^2)}$	14.882 km/s
17	V_3 (velocity in pericenter of hyperbola around target planet)	$V_3 = \sqrt{(2\mu_{tar}/r_3 + V_{\infty,2}^2)}$	17.809 km/s
18	ΔV_0 (maneuver in pericenter around departure planet)	$\Delta V_0 = \lvert V_0 - V_{c0} \rvert$	7.089 km/s
19	ΔV_3 (maneuver in pericenter around target planet)	$\Delta V_3 = \lvert V_3 - V_{c3} \rvert$	14.493 km/s
20	ΔV_{tot} (total velocity increase)	$\Delta V_{tot} = \Delta V_0 + \Delta V_3$	21.582 km/s

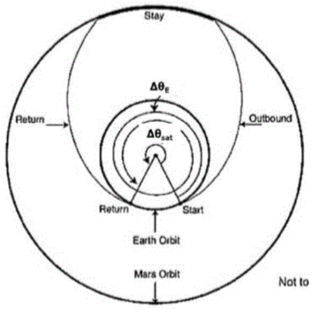
step	parameter	expression		example
		to more outer planet	to more inner planet	
21	θ_1 (true anomaly at departure position) (*)	$\theta_1 = 0°$	$\theta_1 = 180°$	0.0°
22	θ_2 (true anomaly at target position) (*)	$\theta_2 = \mathrm{acos}[\{a_{tr}(1-e_{tr}^2)/r_{tar} - 1\} / e_{tr}]$		77.40°
23	E_1 (eccentric anomaly at departure position)	$E_1 = 0$	$E_1 = \pi$	0.0 rad
24	E_2 (eccentric anomaly at target position)	$E_2 = 2\, \mathrm{atan}[\sqrt{((1-e_{tr})/(1+e_{tr}))}\, \tan(\theta_2/2)]$		0.54 rad
25	M_1 (mean anomaly at departure position)	$M_1 = 0$	$M_1 = \pi$	0.0 rad
26	M_2 (mean anomaly at departure position)	$M_2 = E_2 - e \sin(E_2)$		0.14 rad
27	T_{tr} (transfer time)	$T_{tr} = \lvert M_2 - M_1 \rvert / \sqrt{(\mu_{Sun}/a^3)}$		6.9×10^6 s = 0.219 yrs

Not the best w.r.t fuel and ΔV. Not used!

Hohmann is ideal energy transfer.

FASTER TRAJECTORIES

TIMING ROUND-TRIP MISSIONS



ANGLE COVERED BY EARTH:

$\Delta\theta_E = \Delta\theta_{sat} + 2\pi N$ (N=1) fastest for Mars   (N=−1) fastest for venus.

TOTAL TRIP TIME

$T = 2T_H + t_{stay}$ $\Delta\theta_E = \omega_E T = \omega_E \cdot (2T_H + t_{stay}) = \Delta\theta_{sat} + 2\pi N$

$$= \underbrace{\pi + \omega_M \cdot t_{stay} + \pi}_{\text{HOHMANN}} + 2\pi N$$

$$t_{stay} = \frac{2\pi(N+1) - 2\omega_E T_H}{\omega_E - \omega_M}$$

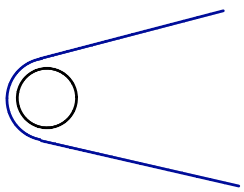
$$T = \frac{2\pi(N+1) - 2\omega_M T_H}{\omega_E - \omega_M}$$

N: are integers, varies depending on the planet.

GRAVITY ASSIST

Hohmann is not useful for far away planets.

HYPERBOLA



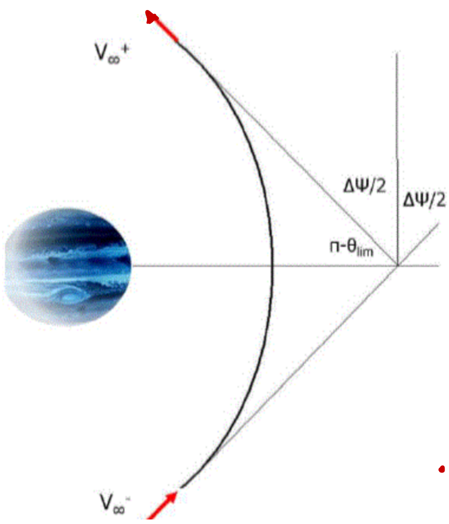
$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

$a < 0$ $e > 1$

$\cos(\theta_{lim}) = -\frac{1}{e}$

$|V_\infty^-| = |V_\infty^+|$

$V_\infty^{(-)} \neq V_\infty^{(+)}$ refer to situation before and after the passing of the central body.



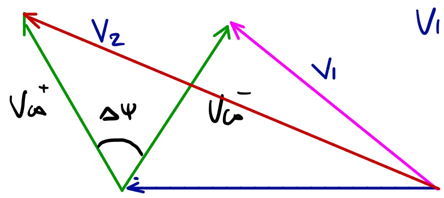
$$\Delta\Psi = 2\left[\frac{\pi}{2} - (\pi - \theta_{lim})\right] = 2\theta_{lim} - \pi$$

$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \qquad \frac{V_\infty^2}{2} = -\frac{\mu}{2a} \qquad a = -\frac{\mu}{V_\infty^2}$$

$$r_p = a(1-e) \longrightarrow e = 1 - \frac{r_p}{a}$$

$$a(V_\infty),\ e(V_\infty, r_p),\ \theta_{lim}(V_\infty, r_p),\ \Delta\Psi(V_\infty, r_p)$$

- The bending angle increases for heavier planets and for smaller pericenter distances
- The bending angle decreases with increasing excess velocity.



$V_1 \neq V_2$

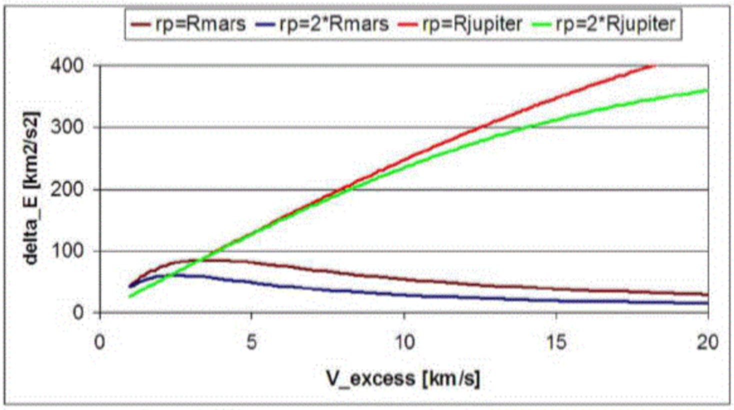
$$V_1^2 = V_{planet}^2 + V_\infty^2 - 2V_{planet} V_\infty \cos\left(\frac{\pi}{2} - \frac{\Delta\Psi}{2}\right)$$

$$V_2^2 = V_{planet}^2 + V_\infty^2 - 2V_{planet} \cdot V_\infty \cos\left(\frac{\pi}{2} + \frac{\Delta\Psi}{2}\right)$$

$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$ and $\cos\left(\frac{\pi}{2} - \alpha\right) = -\sin(\alpha)$

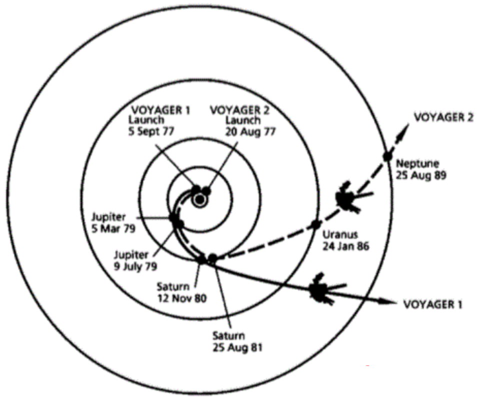
ENERGY GAIN OF SATELLITE

$$\Delta E = \frac{V_2^2}{2} - \frac{V_1^2}{2} = 2 V_{planet} V_\infty \cdot \sin\left(\frac{\Delta\psi}{2}\right)$$



- Related to mass of planets
- Increases with decreasing pericenter distances
- Strong dependence on excess velocity.

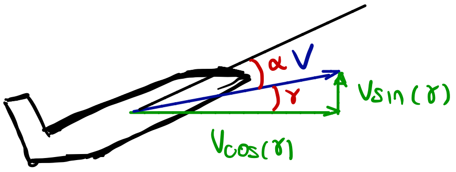
THE GRAND TOUR:



FLIGHT AND ORBITAL MECHANICS — Introduction

UNSTEADY FLIGHT

RATE OF CLIMB AND CLIMB ANGLE



γ: flight path angle [rad]

$V\sin(\gamma) = RC$ rate of climb. [m/s]

easier for pilot. in the cockpit.

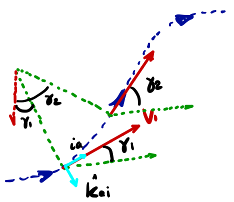
V_x – IAS for maximum climb range

V_y – IAS for maximum rate of climb

- Optimum flight condition for rate of climb is not the same as the optimum flight condition for flight path angle.

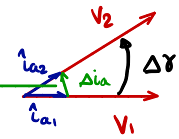
GENERAL EQUATIONS OF MOTION

Assume symmetrical conditions, symmetric flight. Assume constant mass, realistic because climb takes only 10 min.



$$\boxed{\vec{F} = m\vec{a}}$$

$$\vec{a} = \frac{\partial(V\hat{i}_a)}{dt} = \frac{dV}{dt}\cdot\hat{i}_a + V\cdot\boxed{\frac{d\hat{i}_a}{dt}}$$



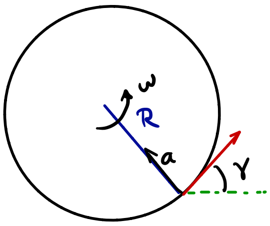
$di = d\gamma$

$$\boxed{\vec{a} = \underbrace{\frac{dV}{dt}\cdot\hat{i}_a}_{\text{along trajectory}} + V\cdot\frac{d\gamma}{dt}\cdot(-\hat{k}_a)}$$

⟶ one side, now look at forces.

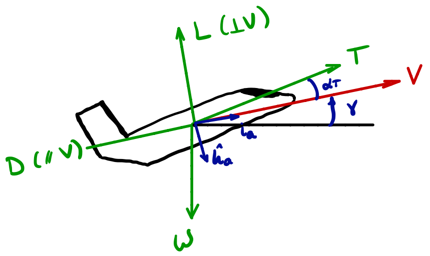
CENTRIPETAL ACCELERATION:

use this because R → does not have a meaning for pilots.



$$\boxed{a = V\cdot\frac{d\gamma}{dt}} \quad \left.\omega = \frac{d\gamma}{dt}\right\} \longrightarrow a = V\cdot\omega = V\cdot\frac{V}{R} = \boxed{\frac{V^2}{R}}$$

$R\omega = V \qquad \omega = \frac{V}{R}$



$$\boxed{\begin{aligned} T\cos\alpha_T - D - W\sin\gamma &= \frac{W}{g}\cdot\frac{dV}{dt} \\ L + T\sin\alpha_T - W\cos\gamma &= \frac{W}{g}V\cdot\frac{d\gamma}{dt} \end{aligned}}$$

$\cos(\gamma) = 1$
$\sin(\gamma) = \sin(\gamma)$

Small angle approx. to simplify.

$$\boxed{\begin{aligned} T - D - W\sin(\gamma) &= \frac{W}{g}\cdot\frac{dV}{dt} \\ L + \underbrace{\underbrace{T}_{\text{small}}\cdot\underbrace{\sin(\gamma)}_{\text{small}}}_{\approx 0} - W &= \frac{W}{g}V\cdot\frac{d\gamma}{dt} \end{aligned}}$$

CLIMB PERFORMANCE

Given: W, $H(\rho)$

Pilot sets: T, V

$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$

$T - D - W\sin\gamma = 0 \quad (1)$

$L = W \quad (2)$

$C_L\frac{1}{2}\rho V^2 S = W \longrightarrow \begin{matrix} C_L = \dots \\ C_D = \dots \end{matrix}$

$T - C_D\frac{1}{2}\rho V^2 S - W\sin\gamma = 0 \qquad \gamma = \dots$

$RC = V\sin(\gamma)$

Climb Procedures

Steady climb performance

$$T - D - W\sin\gamma = \cancel{\frac{W}{g}\cdot\frac{dV}{dt}} \to 0$$

$$L - W = \cancel{\frac{W}{g}\cdot V\cdot\frac{d\gamma}{dt}} \to 0$$

$$\underbrace{T\cdot V}_{P_a} - \underbrace{D\cdot V}_{P_r} - W\boxed{V\sin\gamma} = 0$$

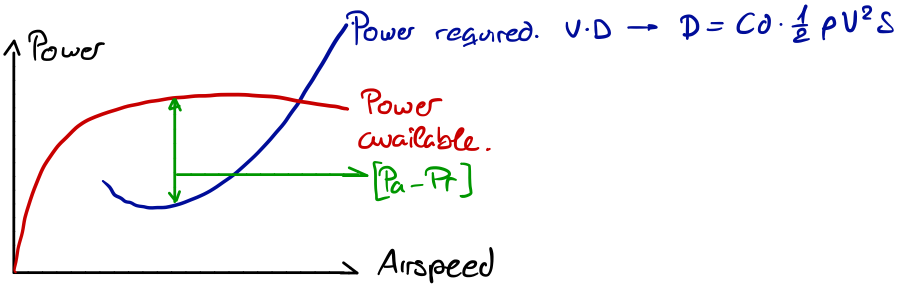
$$P_a - P_r - W\cdot RC = 0$$

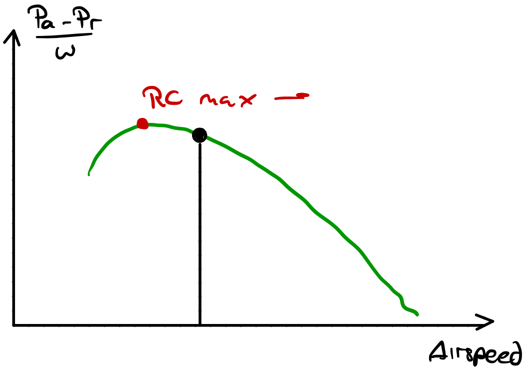
$$\boxed{RC = \frac{\overbrace{P_a - P_r}^{\text{excess power}}}{W}}$$ used to gain altitude.

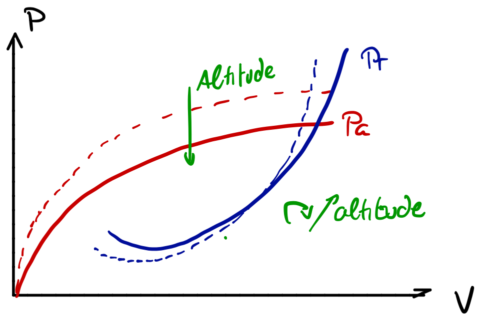
$$\boxed{E_p = m\cdot g\cdot H}$$ potential energy

$$\boxed{\frac{dE_p}{dt} = W\cdot\frac{dH}{dt}}$$

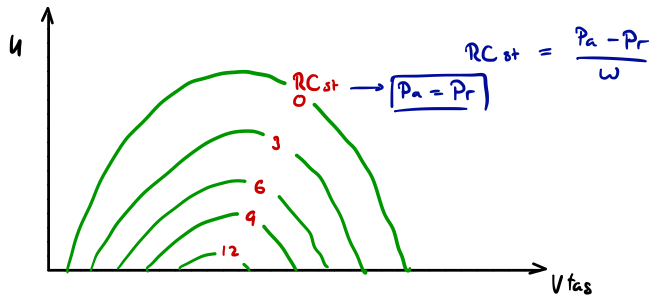
Performance diagram



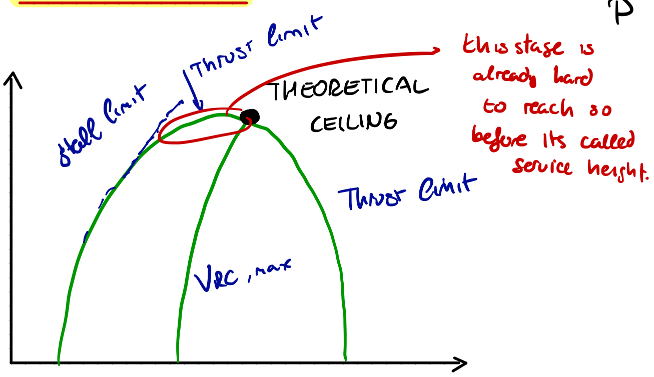


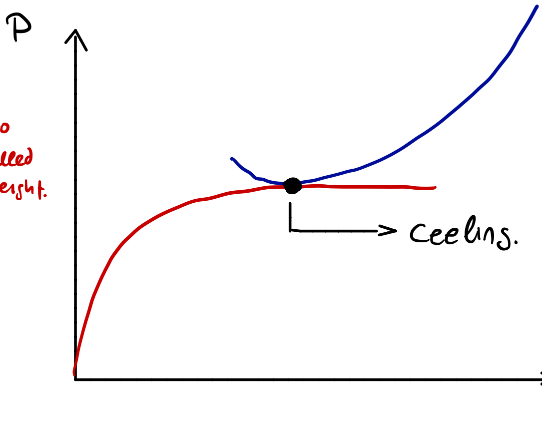


Constant rate of climb

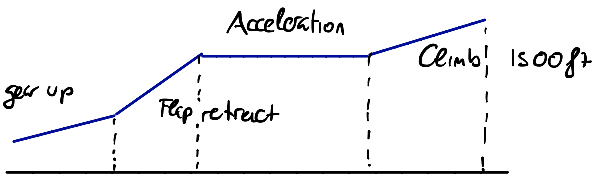


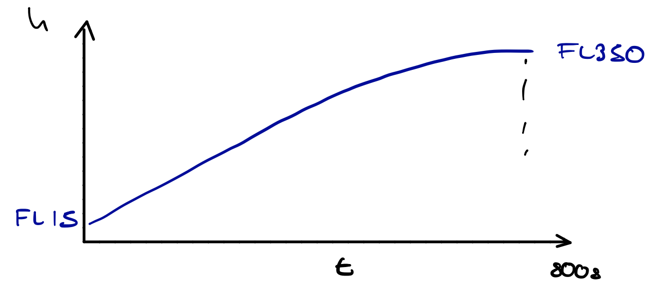
Flight ceiling





REAL EN-ROUTE CLIMB PERFORMANCE





- Constant (IAS)
- Max continuous power.

} what the pilot has to do.

$$V_{TAS} = V_{EAS} \cdot \sqrt{\frac{\rho_0}{\rho}}$$

AIRSPEED INDICATOR:

- Total pressure
- Static pressure

$$P_t - P_s = \frac{1}{2}\rho V_{TAS}^2 \quad (1) \text{ Ber. for incom. flow}$$

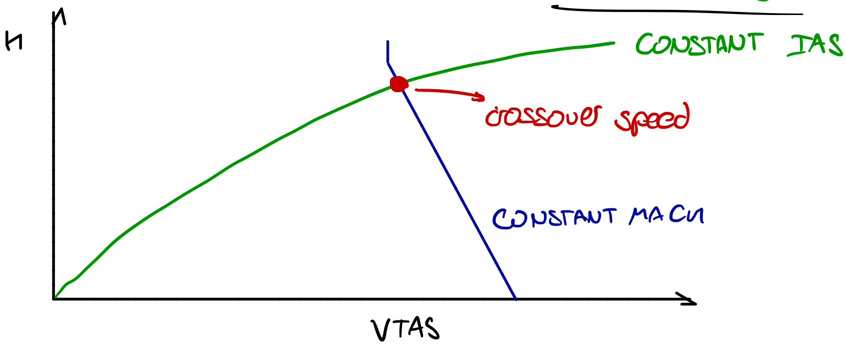
$$P_t - P_s = \frac{1}{2}\underbrace{\rho_0}_{\text{sea level}} \cdot V_{EAS}^2 \quad (2)$$

$$\boxed{V_{EAS} = \sqrt{\frac{2(P_t - P_s)}{\rho_0}}}$$

what the pilot sees.

EAS ≃ IAS incompressible

CAS ≃ IAS compressible.



- Unsteady clim:

$$\frac{dV}{dt} \neq 0$$

- Quasi rectilinear

$$\frac{d\gamma}{dt} \approx 0$$

E.O.M:

$$\frac{W}{g} \cdot \frac{dV}{dt} = T\cancel{\cos\alpha_T}^{1} - D - W\sin\gamma$$

$$\cancel{\frac{W}{g} \cdot V \cdot \frac{d\gamma}{dt}}^{0} = L + T\sin\alpha_T - W\cancel{\cos\gamma}^{1}$$

$$\boxed{T - D - W\sin\gamma = \frac{W}{g} \cdot \frac{dV}{dt}}$$

$$\boxed{L = W}$$

Introduce rate of climb

$$\frac{W}{g} \cdot V \cdot \frac{dV}{dt} = T \cdot V - D \cdot V - W \cdot V \cdot \sin\gamma$$

$$\frac{W}{g} \cdot V \cdot \frac{dV}{dt} = P_a - P_r - W \cdot RC$$

$$\boxed{\underbrace{\frac{P_a - P_r}{W}} = RC + \frac{V}{g} \cdot \frac{dV}{dt}} \quad (1)$$

The excess power is used to ascend

CLIMB PERFORMANCE:

$$\frac{dV}{dt} = 0 \qquad \frac{P_a - P_r}{W} = RC_{steady} + 0 \quad (2)$$

$$RC_{steady} = RC + \frac{V}{g} \cdot \frac{dV}{dt} \longrightarrow \text{substitude time with height.} \quad (3)$$

$$\frac{dV}{dt} = \frac{dV}{dH} \cdot \frac{dH}{dt} \qquad \left(\frac{dH}{dH} = 1\right)$$

$$RC_{ST} = RC + \frac{V}{g} \cdot \frac{dV}{dH} \cdot RC$$

$$\frac{dV}{dt} = RC \cdot \frac{dV}{dH} \quad (4)$$

$$RC_{ST} = \left(1 + \frac{V}{g} \cdot \frac{dV}{dH}\right) \cdot RC$$

constant IAS (≈ EAS)

$$\boxed{\frac{RC}{RC_{ST}} = \frac{1}{1 + \frac{dV}{dH} \cdot \frac{V}{g}}}$$

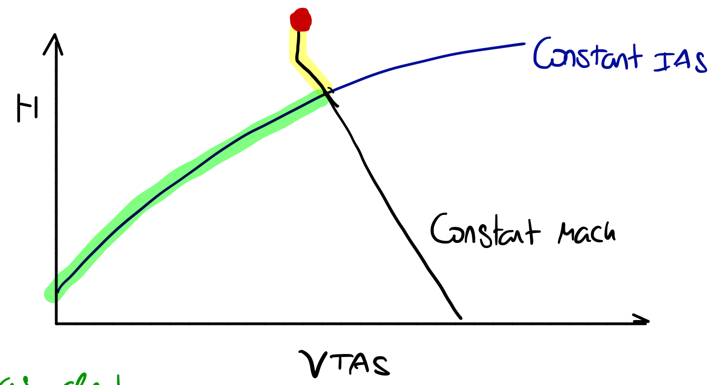
$$V_{TAS} = V_{EAS} \cdot \sqrt{\frac{\rho_0}{\rho}}$$

- En route climb is unsteady
- Actual rate of climb ≠ steady rate of climb
- Atmospheric data required.

$$\boxed{\frac{dV}{dH} = \frac{d\left(V_{EAS}\sqrt{\frac{\rho_0}{\rho}}\right)}{dH} = V_{EAS}\frac{d\left(\sqrt{\frac{\rho_0}{\rho}}\right)}{dH}}$$

DESCENT PROCEDURE IN POWER FLIGHT CONDITIONS

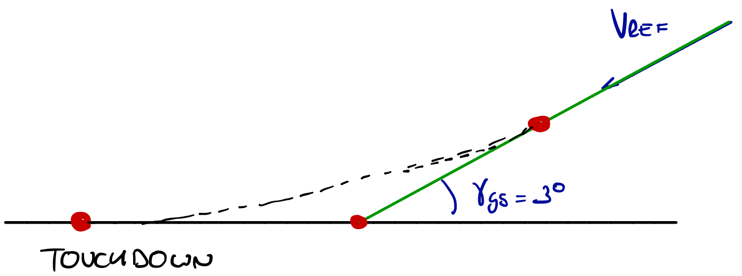
- Accelerated flight while descending at constat Mach.
- Secondly goes at constant IAS.



RESULTS IN: Unsteady flight: $\frac{dV}{dt} \neq 0$

Quasi rectilinear: $\frac{d\gamma}{dt} \approx 0$ } Same as climb.

- Final approach is done at a constant glide slope. 3° at V_{REF}



unsteady speed.

DESCENT PERFORMANCE

FLIGHT CONDITIONS:

$\gamma = 3°$ $\quad V = \dots$

$\rho = \dots$ $\quad W = \dots$

1. Simplify E.O.M
2. C_L from vertical equ
3. C_D from lift-drag polar
4. Drag
5. Thrust

EOM:

$$\frac{W}{g}\underbrace{\cancel{\frac{dV}{dt}}}_{0} = T - D - W\sin\gamma$$

$$\frac{W}{g}\cdot V\cdot \underbrace{\cancel{\frac{d\gamma}{dt}}}_{0} = L - W\cdot \underbrace{\cancel{\cos\gamma}}_{=1}$$

LIFT DRAG POLAR

$$C_D = C_{D_0} + k C_L^2$$

$$C_D = \dots$$

$$T - D - W\sin\gamma = 0$$

$$L = W$$

$$C_L = \frac{1}{2}\rho V^2 \cdot S = W$$

$$C_L = \dots$$

$$D = C_D \cdot \frac{1}{2}\rho V^2 \cdot S = \dots$$

$T = \dots$

Procedure identical to climb calculation.
Low thrust instead of high thrust.
Performance along glide slope is different.

GLIDING FLIGHT

Gliders use thermals to ascend. Failure of all engines can happen.

How much can we glide.

EOM.

- $\frac{W}{g}\cdot\cancel{\frac{dV}{dt}}^{0} = \cancel{T}^{0}\cos\alpha_T - D - W\sin(\gamma)$ $\qquad \frac{dV}{dt}\approx 0 \qquad 0 = -D - W\sin\gamma \qquad \bar{\gamma} = \arcsin\left(\frac{D}{L}\right)$
- $\frac{W}{g}V\cdot\cancel{\frac{d\gamma}{dt}}^{0} = L - W\cancel{\cos}^{1}\gamma + \cancel{T}^{0}\sin\alpha_T$ $\qquad 0 = -D + L\sin(\gamma) \qquad \bar{\gamma} = \arcsin\left(\frac{C_D}{C_L}\right)$

$L = W = C_L\frac{1}{2}\rho V^2\cdot S \qquad V = \sqrt{\frac{W}{S}\cdot\frac{2}{\rho}\cdot\frac{1}{C_L}}$

Minimum descent angle
Airspeed at minimum glide angle.

TOTAL TIME, DISTANCE AND FUEL CONSUMPTION

Integration of points during flight

Start of flight: altitude, weight, airspeed $\longrightarrow$ FULL POWER

Calculate: RC, Fuel flow, ground speed, time $=0$.

500 m height: airspeed altitude.

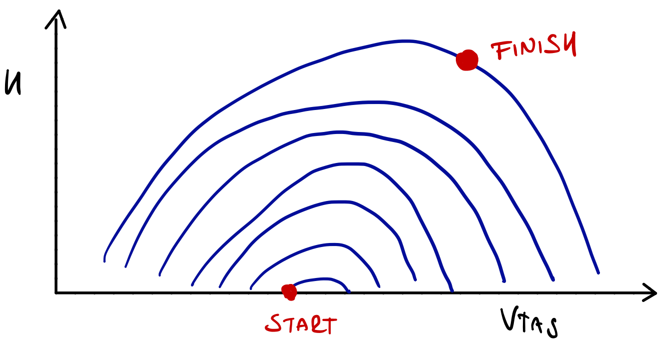
Calculate: time $= \frac{h}{RC}$, weight $=$ initial weight $-$ FF $\cdot t$, RC, FF, ground speed

Continuously until final altitude.

Given flight strategy
Possible to calculate Performance.
} most optimal strategy?

ENERGY HEIGHT

Minimum time to climb



POTENTIAL ENERGY AND KINETIC ENERGY IS USED

$E_P = mgH \qquad E_K = \frac{1}{2}mV^2$

Exchange kinetic energy for potential energy.

Energy height

Total energy:

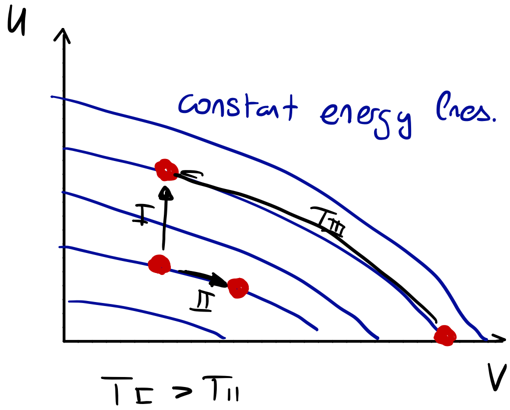
$$E_{TOT} = mgH + \frac{1}{2}mV^2$$

Weight

$$W = mg$$

Energy height:

$$E_H = H + \frac{V^2}{2g}$$

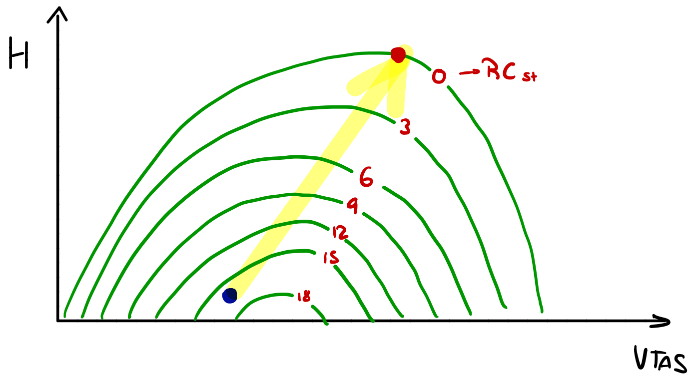


$T_I > T_{II}$

$T_I > T_{III}$

Optimal subsonic climb

minimum time to climb?



Unsteady climb:

$$\frac{RC}{RC_{steady}} = \frac{1}{1 + \frac{V}{g}\cdot\frac{dV}{dH}}$$

Time to climb:

$$t = \int_{t_0}^{t_1} dt$$

$$RC = \frac{dH}{dt} \longleftrightarrow dt = \frac{dH}{RC}$$

$$t = \int_{H_0}^{H_1} \frac{dH}{RC} \qquad t = \int_{H_0}^{H_1} \frac{\left[1 + \frac{V}{g}\cdot\frac{dV}{dH}\right]}{RC_{steady}}\, dH$$

EOM: 2D symmetric flight.

$$\frac{W}{g}\cdot\frac{dV}{dt} = T - D - W\cdot\sin\gamma$$

$$\frac{W}{g}\cdot V\frac{dV}{dt} + W\cdot \underbrace{V\cdot\sin\gamma}_{RC} = \underbrace{T\cdot V}_{P_a} - \underbrace{D\cdot V}_{P_r}$$

$$\frac{V}{g}\cdot\frac{dV}{dt} + RC = \frac{P_a - P_r}{W} \qquad \boxed{VdV = \frac{1}{2}dV^2}$$

$$\frac{1}{2g}\frac{dV^2}{dt} + RC = \frac{P_a - P_r}{W}$$

ENERGY:

$$E = mg\cdot H + \frac{1}{2}mV^2$$

$$\frac{E}{W} = H + \frac{1}{2g}V^2 = H_E$$

$$\frac{dH_E}{dt} = \frac{dH}{dt} + \frac{dV^2}{dt}\cdot\frac{1}{2g}$$

=

$$\frac{dH_e}{dt} = \frac{P_a - P_r}{W} = RC_{ST} \qquad dt = \frac{dH_e}{RC_{st}}$$

$$t = \int \frac{dH_e}{RC_{st}} \longrightarrow \text{fly at condition of maximum RC.}$$

The optimal trajectory is when the constant energy lines are tangential to rate of climb lines.

For pilot it is easy, constant IAS → for propeller.

LECTURE 2: CLIMB AND DESCEND

SUPERSONIC:

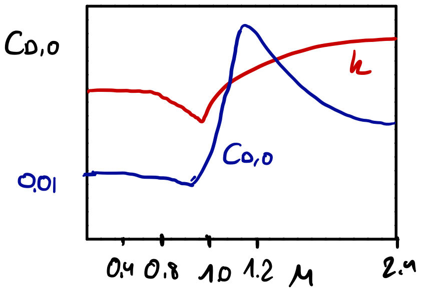
AERODYNAMIC DRAG:

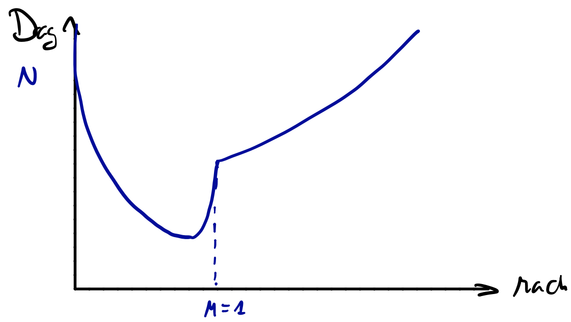
$D = C_D \cdot \frac{1}{2} \rho V^2 \cdot S$

$C_D = C_{D0} + k C_L^2$

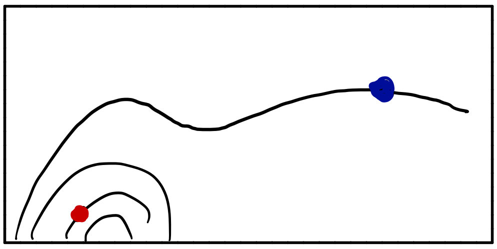
- $k = k(M)$
- $C_{D0} = C_{D0}(M)$

this is because of supersonic regions over the airfoil, increasing drag.





OPTIMAL SUPERSONIC CLIMB ⟶ ALWAYS MAXIMUM THRUST



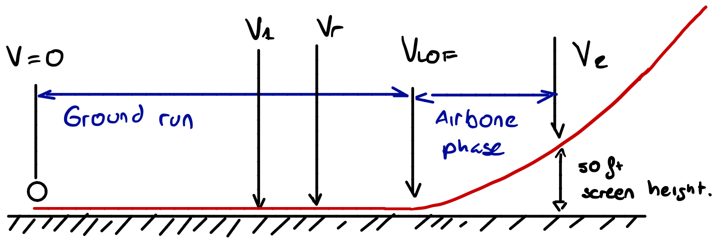
- From low altitude low speed to high altitude high speed. • (0)
- Start at green line, check the best rate of climb. 160. • (1)
- Next energy level: check the highest rate of climb. • (2) • (3) • (4) • (5) • (6) • (7)
- When $M=1$: climb rate very low so drop with the constant energy line to • (8) us. ⟶ not really constant because max thrust.
- Move at best rate of climbs to the right until you reach constant energy line connecting to the red point. • (9) • (10) • (11) • (12) • (13)

LECTURE 3: AIRFIELD SPECIFICATIONS

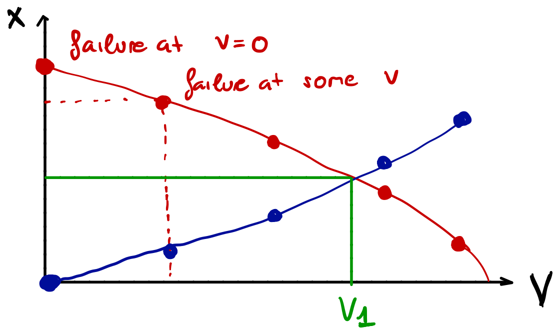
TAKE-OFF MANEUVER

Take off distances:

NORMAL TAKE-OFF



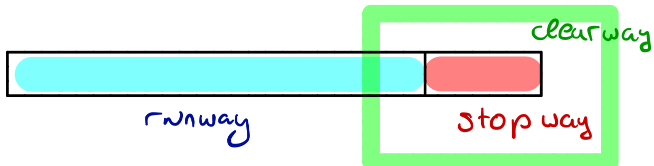
BALANCED FIELD LENGTH



scenario of take off with failure

scenario of stop with failure.

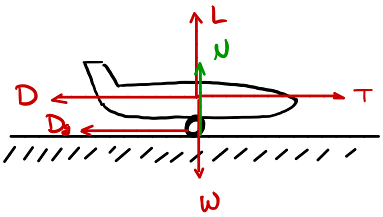
balanced field length. ⟶ also Normal TO distance without engine failure + 15 %



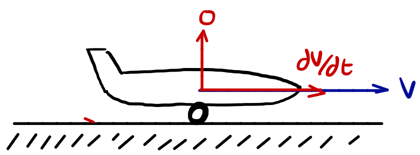
max elevation angle 1.25°

E.O.M (GROUND RUN)

FBD



KD



$$\left.\begin{aligned} \frac{W}{g}\cdot\frac{dV}{dt} &= T - D - D_g \\ 0 &= N + L - W \\ D_g &= \mu N \end{aligned}\right\} \quad \frac{W}{g}\cdot\frac{dV}{dt} = T - D - \mu(W-L)$$

DISTANCE REQUIRED

$$x = \int dx = \int_0^{V_{LOF}} \frac{V}{a}\,dV = \frac{1}{\bar{a}}\int^{V_{LOF}} V\,dV$$

$$\boxed{\bar{a} = \frac{g}{W}\cdot(\bar{T} - \bar{D} - \mu(\bar{W} - \bar{L}))}$$

$$\boxed{\bar{x} = \frac{V_{LOF}^2}{2a}}$$ → lift off ?

• Assumptions:

→ No ground effect.

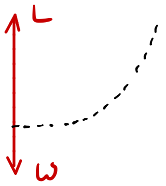
→ No wind.

→ No runaway slope.

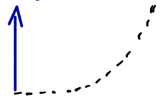
$$\boxed{V = \frac{V_{LOF}}{\sqrt{2}}}$$

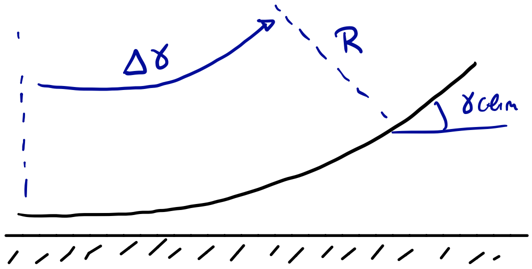
$$\boxed{V_{LOF} = 1.05 \cdot V_{MIN}}$$

AIRBONE DISTANCE TAKE-OFF



$$\frac{mV^2}{R} = mV \cdot \frac{d\gamma}{dt}$$





$$\frac{mV^2}{R} = L - W \qquad n = \frac{L}{W} = \begin{matrix} 1.10 \\ 1.15 \end{matrix}$$

$$\frac{V^2}{g \cdot R} = \frac{L}{W} - 1 = 0.15 \qquad \frac{V_{LOF}^2}{0.15\,g} = R$$

$$x_{total} = x_{transition} + x_{climb}$$

$$x_{tr} = R \sin \gamma_{climb} = \frac{V_{LOF}^2}{0.15 g} \cdot \sin(\gamma_{climb})$$

$$h_{climb} = h_{scr} - h_{transition} \qquad h_{tr} = R - R \cos \gamma_{climb}$$

$$\tan \gamma_{climb} = \frac{h_{climb}}{x_{climb}} \qquad h_{tr} = (1 - \cos \gamma_{climb}) \frac{V_{LOF}^2}{0.15 g}$$

EQUATIONS

$$x_{trans} = \frac{V_{LOF}^2}{0.15\,g} \sin(\gamma_{climb})$$

$$x_{climb} = \frac{h_{scr} - (1 - \cos\gamma) \frac{V_{LOF}^2}{0.15 g}}{\tan \gamma}$$

$$x_{total\text{-}airbone} = x_{trans} + x_{climb}$$

$$x_{total} = x_{ground} + x_{airbone}.$$

BENEFICIAL FOR TAKE OFF:

- High C_L
- Low Weight
- Powerfull engines

FACTORS AFFECTING TAKE OFF PERFORMANCE

ENGINE THRUST

$\frac{T}{W} = 0.25$ → but not really needed for cruise after.

AIRCRAFT TIRES

Friction coefficient, creates friction drag, $F_D = \mu \cdot (W - L)$

LIFT OFF SPEED

$$V_{LOF} = 1.05\, V_{min}$$

$$V_{min} = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_{L_{max}}}}$$

HIGH LIFT DEVICES:

The more complex → the heavier but more C_L

AERODYNAMICS

Less drag coefficient, smaller area.

But lift of speed is better.

WIND:

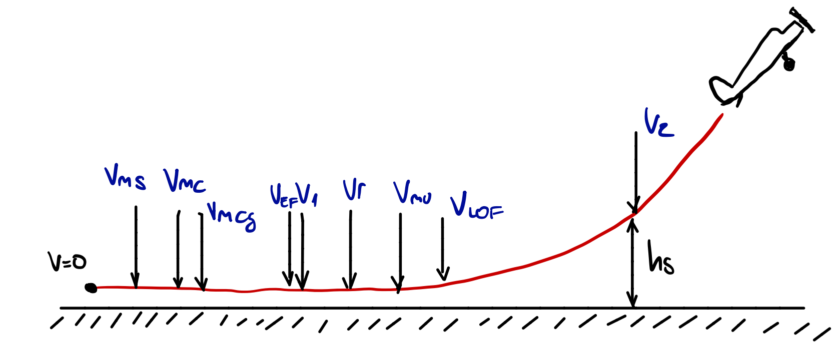
If there is wind, the ground speed and the airspeed are not the same

GROUND EFFECT:

Flow field, C_L increases, C_D decreases.

RUNWAY SLOPE

The Flight Manual

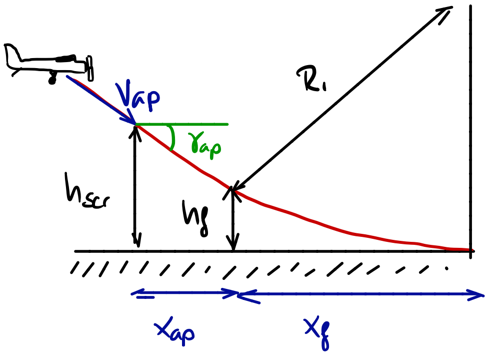


- V_{MS}: Minimum stall speed.
- V_{MC}: Minimum control speed
- V_{MCG}: Minimum control speed ground
- V_{EF}: Engine failure speed: (realize pilot)
- V_1: Decision speed.
- V_r: Rotation speed.
- V_{MU} Minimum unstick speed
- V_{LOF} lift of speed.
- V_2 Free air safety speed.

- Depend on weight, configuration and weather

Airbone Distance Landing

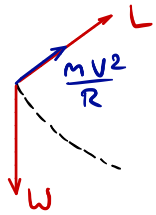
Landing approach and flare.



$V_{ap} = 1.3\, V_{min}$

How much distance x_{ap} and x_F ?

Forces on airplane: EOM



$$m\frac{V_{ap}^2}{R} = L - W\cancel{\cos(\gamma_{ap})}\ 1$$

$$\frac{V_{ap}^2}{gR} = \frac{L}{W} - 1$$

$$\boxed{n = \frac{L}{W}}$$ 0.1 Jet; 0.15 propeller

$$R = \frac{V_{ap}^2}{0.1g}$$

$R =$ known

$V_{ap} = 1.3\, V_{min}$

$V_{ap} = 1.3\sqrt{\frac{W}{S}\cdot\frac{2}{\rho}\cdot\frac{1}{C_{L_{max}}}}$

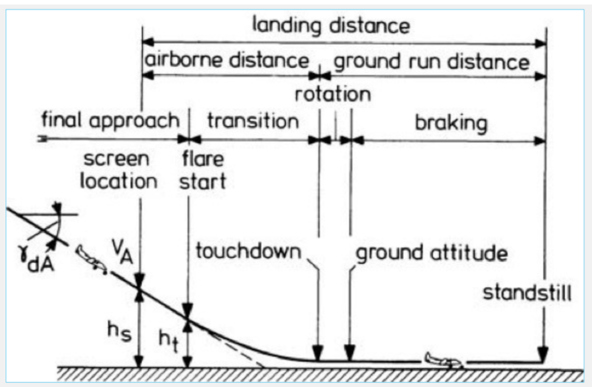
$x_f = R\cdot\sin(\gamma_{ap})$

$h_f = R - R\cdot\cos(\gamma_{ap})$

$h_{ap} = h_{scr} - h_f = h_{scr}\,(1-\cos(\gamma_{ap}))R$

$X_{ap} = \frac{h_{ap}}{\tan(\gamma_{ap})}$

Ground Round Distance Landing



- $V_{ap} = 1.3\, V_{min}$
- $t_{tr} = 2\,s$
- $x_{tr} = t_{tr}\cdot V_{ap}$
- $x_{tr} = 2.6\cdot V_{min}$.

GROUND ROUND DISTANCE LANDING

$$a = \frac{g}{W}\left(T - D - \mu(W - L)\right) \quad \text{E.O.M}$$

T reversed. → average.

μ braking the wheels

$$a = \frac{dV}{dt} = \frac{dV}{dx}\cdot\frac{dx}{dt}$$

$$\boxed{x = \int dx = \int_{V_a}^{0} \frac{V\,dV}{a}} = -\frac{1.3^2 \cdot V_{min}^2}{2\cdot \bar{a}}$$

$\bar{a}$ → average.

when $V = \frac{V_A}{\sqrt{2}}$

$$\boxed{x_{brake} = \frac{1.3^2 \cdot \frac{W}{S}\cdot\frac{2}{\rho}\cdot\frac{1}{C_{Lmax}}}{\frac{g}{W}\left(-T_{rev} - \bar{D} - \mu_{brake}(W - L)\right)}}$$

TOTAL BRAKING DISTANCES

$X_{tr} = 2.6\,V_{min}$ $\qquad X_{total} = X_{tr} + X_{brake}$

FACTORS INFLUENCING LANDING PERFORMANCE

DESIGN FACTORS

- MINIMUM AIRSPEED:
  - Lower, less distance
  - C_{Lmax} as high in landing as possible, but not in take off. (More D)
  - Lower weight better.
- AIRCRAFT WEIGHT.
  - Lower mass easier to move, less thrust.
- AERODYNAMIC DRAG:
  - Desirable in landing. (parachutes, airbrakes)
  - Spoilers: two effects, less lift, more drag.
- BRAKING:
  - Deformation of wheels during landing for higher coefficient.
- THRUST REVERSERS:

Operational Factors

- A steeper descent: shorter landing distance
- Screen height: airworthiness
- Load factor: Higher load factor, reduced distance.

External Factors

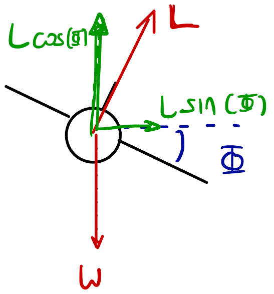
- Head wind, reduces landing distance.
- Braking less effective on a wet runaway.

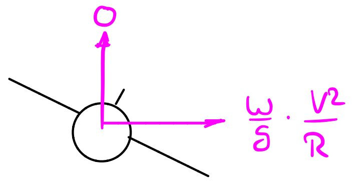
Turning Flight

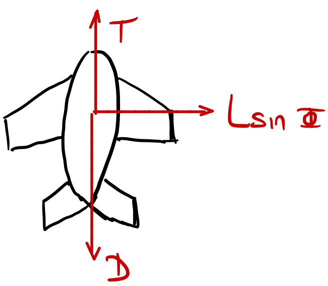
Depends on: Flight conditions and aircraft design.

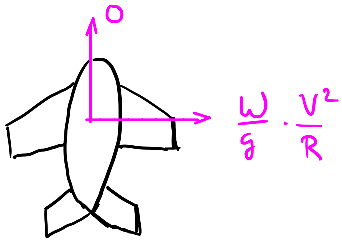
Equations of motion and load factor

- Steady, coordinated, horizontal.







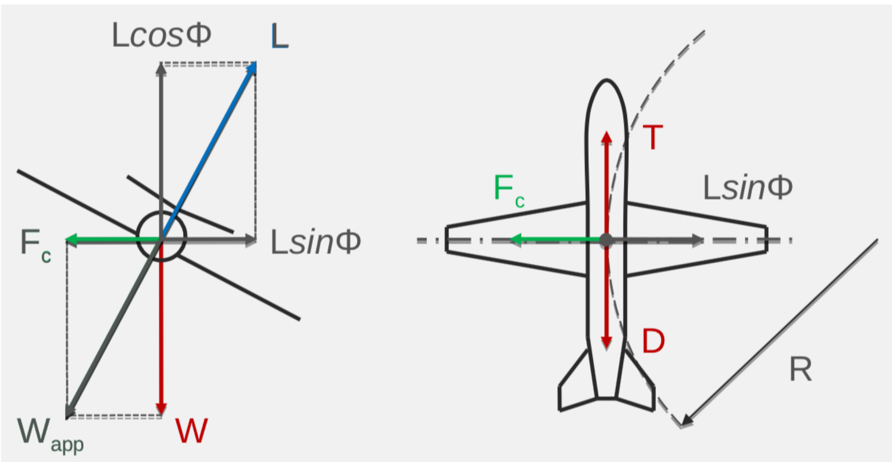


(1) $0 = L\cos(\Phi) - W$

$\boxed{L\cos(\Phi) = W}$ stay altitude

(2) $\boxed{\frac{W}{g}\cdot\frac{V^2}{R} = L\sin(\Phi)}$

(3) $0 = T - D$ $T = D$



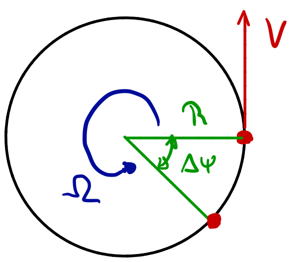
$n = \frac{L}{W} = 2 \rightarrow 2g \rightarrow$ 2 times gravity.

$L\cos(\Phi) = W$

$$n = \frac{1}{\cos(\Phi)}$$

Standard Turning

- Rate 1/2: 1.5 deg/s $\longrightarrow$ bank angle =
- Rate 1: 3 deg/s $\longrightarrow$ bank angle = 15°
- Rate 2: 6 deg/s $\longrightarrow$ bank angle =



$$\Omega = \frac{\Delta\psi}{\Delta t}$$

$$V = \Omega R$$

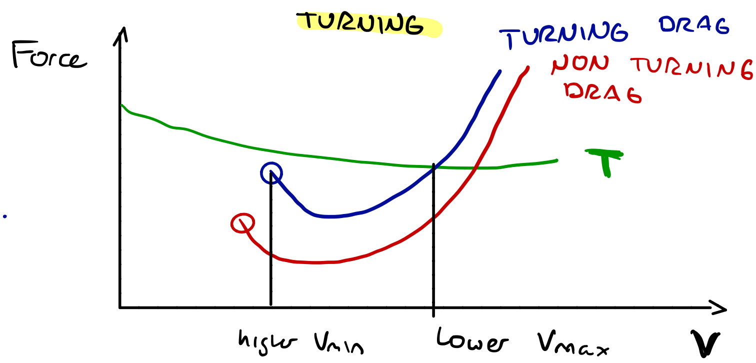
$\left.\begin{matrix}(2)\\(1)\end{matrix}\right\}$ E.O.M

$$\frac{L\sin(\Phi)}{L\cos(\Phi)} = \tan(\Phi) = \frac{1}{g}\frac{V^2}{R}$$

$$\boxed{R = \frac{V^2}{g}\cdot\tan(\Phi)}$$

The pilot uses the giroscope.

Maximum Load Factors



Maximum turning performance

Steepest turn: higest load factor possible.

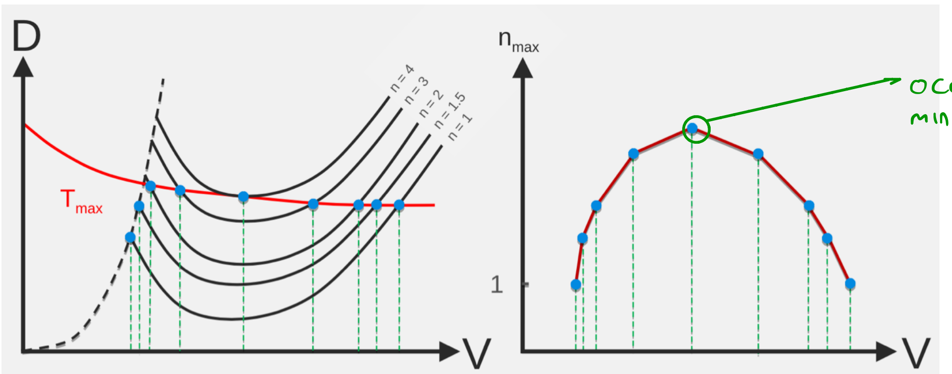
$$n = \frac{L}{W} = C_L \cdot \frac{1}{2}\rho V^2 \cdot S$$

V_{tot}

$$V = \sqrt{n} \cdot \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}}$$

$$D = C_D \cdot \frac{1}{2}\rho V^2 S = C_D \cdot \cancel{\frac{1}{2}} \cancel{\rho} \cdot \frac{nW}{\cancel{S}} \cdot \frac{\cancel{2}}{\cancel{\rho}} \cdot \frac{1}{C_L} \cancel{S}$$

$$\boxed{D = \frac{C_D}{C_L} \cdot n \cdot W} \qquad \boxed{D \propto n}$$



occurs near minimum drag condition.

Conclusion:

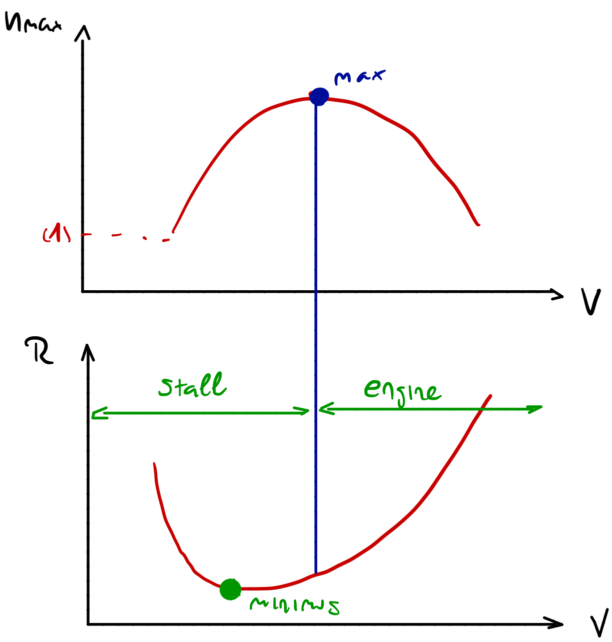
Maximum n:

- Function of airspeed.
- Determine using performance diagram.

Depends on:

- Aerodynamic and propulsion characteristics
- Air density & aircraft weight.

Minimum turn radius:



EOM:

(1) $= L\cos(\Phi) = W$

(2) $= L\sin(\Phi) = \frac{W}{S} \cdot \frac{V^2}{R}$

(3) $= T = D$

$$\sin(\Phi) = \sqrt{1 - \frac{1}{n^2}}$$

$$n = \frac{L}{W} = \frac{1}{\cos(\Phi)}$$

(2) $= n\sin(\Phi) = \frac{V^2}{g \cdot R}$

$$R = \frac{V^2}{g \cdot n \cdot \sin(\Phi)}$$

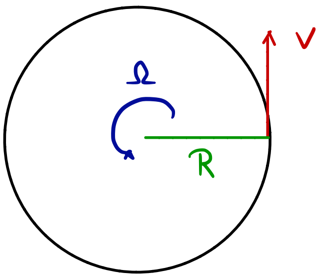
$$R = \frac{V^2}{g \cdot n \cdot \sqrt{1 - \frac{1}{n^2}}}$$

$$\boxed{R = \frac{V^2}{g\sqrt{n^2 - 1}}}$$

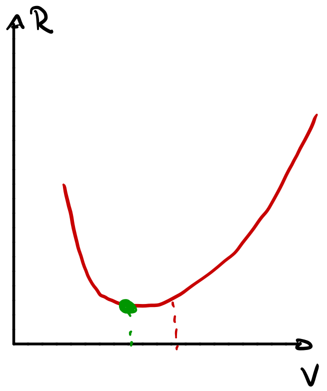
Minimum Time to Turn

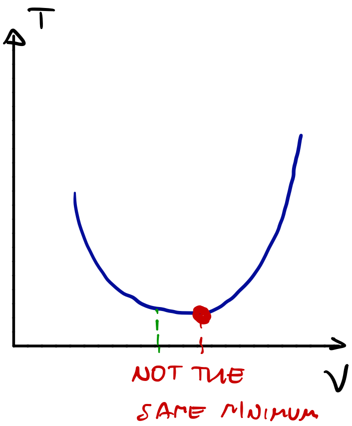
$$R = \frac{V^2}{g\sqrt{n^2-1}}$$

larger distance at a higher speed maybe reduce the turning time



$$T_{2\pi} = \frac{2\pi R}{V}$$



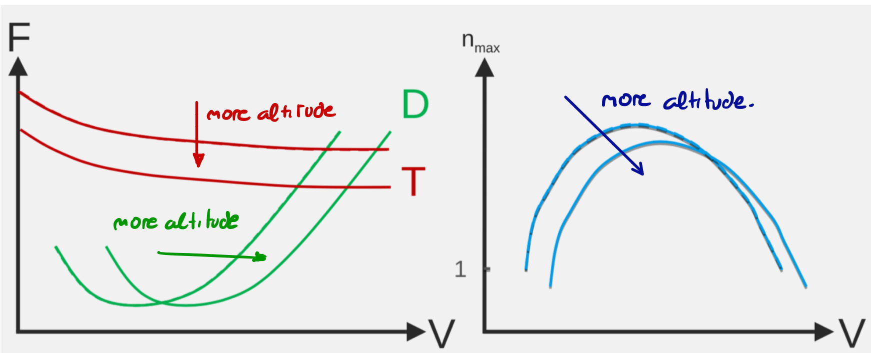


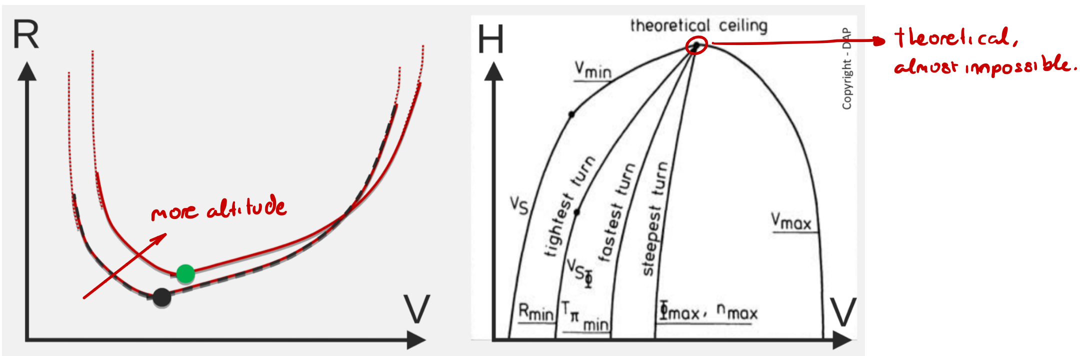
- $V_{R\,min} \leq V_{T\,min} \leq V_{n\,max}$

Altitude Effects

Turning Performance:

Based on: EOM, Propulsion system, Aerodynamics, Aircraft Weight. And on Altitude





Higher altitude, turning performance decreases

- Thrust decreases.
- Higher velocity required.

CRUISE FLIGHT

CRUISE PERFORMANCES

- Best Range $\left(\frac{V}{F}\right)_{max}$ $\left[\frac{m}{kg}\right]$
- Maximum speed $(V)_{max}$
- Best Endurance $(F)_{min}$ $\left[\frac{kg}{s}\right]$

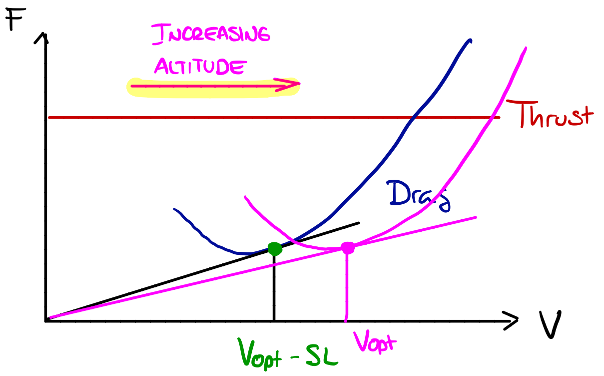
Flying high is always better!

BEST RANGE:

1. Point Performance - simplified jet
2. Path Performance - simplified jet
3. Point Performance - propeller
4. Path Performance - propeller

5. UNIFIED RANGE EQUATION
6. Cruise performance of turbofan aircraft at transonic conditions.

POINT PERFORMANCE IN SIMPLIFIED JET



Specific Range

V/F

- $F = C_T \cdot T$
- $L = W$
- $T = D$

$$\frac{V}{F} = \frac{V}{C_T \cdot T} = \frac{V}{C_T \cdot D}$$

C_T → constant during flight.

$$\boxed{\left(\frac{V}{D}\right)_{max} \quad \left(\frac{D}{V}\right)_{min}}$$

$L = W$

$$V = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}} \qquad D = C_D \cdot \frac{1}{2} \rho V^2 S$$

$$D = \frac{C_D}{C_L} \cdot W$$

$$\left(\frac{V}{D}\right)_{max} = \frac{\sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}}}{\frac{C_D}{C_L} \cdot W} = \left(\sqrt{\frac{1}{WS} \cdot \frac{2}{\rho} \cdot \boxed{\frac{C_L}{C_D^2}}}\right)_{max}$$

maximize.

$$\left(\frac{C_L}{C_D^2}\right)_{max} \Leftrightarrow \left(\frac{X}{y^2}\right)_{max} \qquad \frac{d}{dx}\left(\frac{x}{y^2}\right) = 0 \qquad \frac{y^2 \cdot 1 - x \cdot 2y \cdot \frac{dy}{dx}}{y^4} = 0$$

$$\left(y^2 - x \cdot 2y \frac{dy}{dx}\right) = 0$$

$$\boxed{\frac{1}{2} \cdot \frac{y}{x} = \frac{dy}{dx}}$$

$$\boxed{\begin{array}{l} C_D = C_{D0} + \dfrac{C_L^2}{\pi A e} \\ y = a x^2 + b \end{array} \quad \begin{cases} a = \dfrac{1}{\pi A e} \\ b = C_{D0} \end{cases}}$$

$$\frac{1}{2} \frac{ax^2 + b}{x} = 2ax$$

$$ax^2 + b = 4ax^2 \qquad x = \sqrt{\frac{b}{3a}}$$

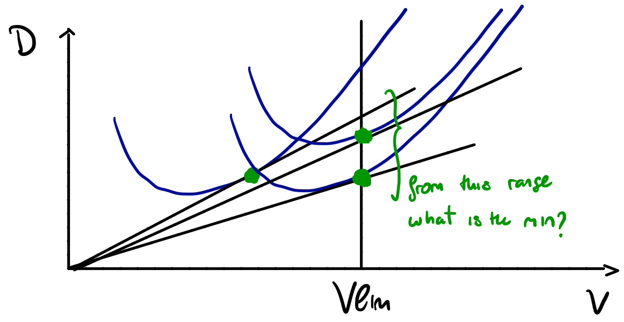
$$\boxed{C_{L_{opt}} = \sqrt{\frac{1}{3} C_{D0} \pi A e}}$$

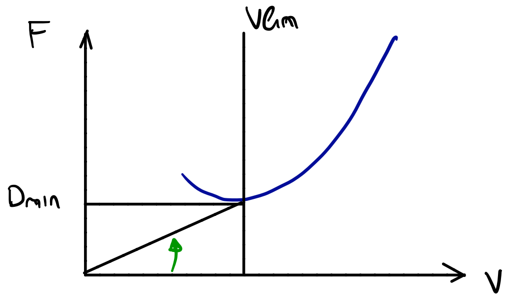
IF ALTITUDE INCREASES
- Faster flight
- Same drag
- Thrust - Fuel flow.

LIMITS:

OPERATIONAL LIMITS:

- Maximum operating speed V_{MO}
- Maximum operating Mach number M_{MO}



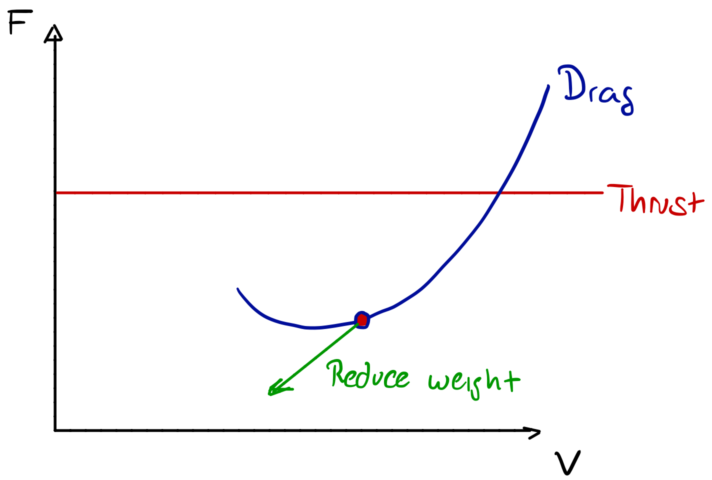


$$D_{min} = \left(\frac{C_D}{C_L}\right)_{min} \cdot W$$

$$\frac{C_D}{C_L} = \frac{C_{D0} + \frac{C_L^2}{\pi A e}}{C_L} = \frac{C_{D0}}{C_L} + \frac{C_L}{\pi A e} = \frac{a}{x} + b x$$

$$d\left(\frac{C_D}{C_L}\right) = -a x^{-2} + b = 0 \qquad x = \sqrt{\frac{a}{b}} \qquad \boxed{C_L = \sqrt{C_{D0} \pi A e}}$$

RANGE OF A SIMPLIFIED JET AIRCRAFT



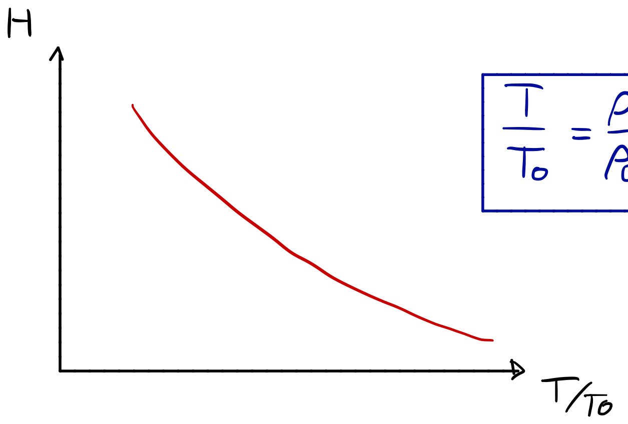
EFFECT OF WEIGHT

$L = W$, $T = D$

$$V = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}} \qquad D = \frac{C_D}{C_L} \cdot W$$

$V \sim \sqrt{W}$ Reduce $\qquad D \sim W$ Reduce

HOW MUCH AN AIRCRAFT CAN CLIMB FOR A GIVEN WEIGHT REDUCTION



$$\boxed{\frac{T}{T_0} = \frac{\rho}{\rho_0}}$$

$T_{max} = k \cdot \rho$ $\quad W\downarrow ?$ $\quad$ best flying cond. C_L ; C_D constant

(1) $W_1 = \rho\,?$ $\qquad V_1 = \sqrt{\frac{W_1}{S} \cdot \frac{2}{\rho_1} \cdot \frac{1}{C_L}}$

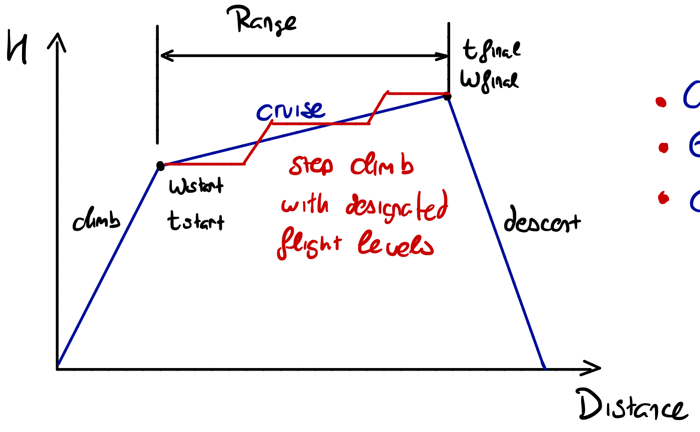
(2) $W_2 = \rho\,?$ $\qquad V_2 = \sqrt{\frac{W_2}{S} \cdot \frac{2}{\rho_2} \cdot \frac{1}{C_L}}$

$$\boxed{\frac{V_1}{V_2} = \sqrt{\frac{W_1}{W_2} \cdot \frac{\rho_2}{\rho_1}}}$$

$$D_1 = \frac{C_D}{C_L} W_1, \quad D_2 = \frac{C_D}{C_L} \cdot W_2 \quad \Big\} \quad \frac{D_1}{D_2} = \boxed{\frac{W_1}{W_2} = \frac{T_1}{T_2} = \frac{h\rho_1}{h\rho_2}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{\rho_1}{\rho_2} \cdot \frac{\rho_2}{\rho_1}} = 1 \qquad \boxed{V_1 = V_2}$$

keep airspeed angle of attack constant.



- Constant throttle
- Gradual climb
- Constant V and α

In case of operational speed limits, at high altitudes $T =$ constant, Maximum Mach number is at a constant speed. No issue.

Maximum Distance Range

$$R = \int_{t_0}^{t_1} V\,dt = \int_{W_0}^{W} -\frac{V}{F} \cdot dW = \boxed{\int_{W_1}^{W_0} \frac{V}{F} \cdot dW} = \int_{W_1}^{W_0} \frac{V}{C_T \cdot D} dW$$

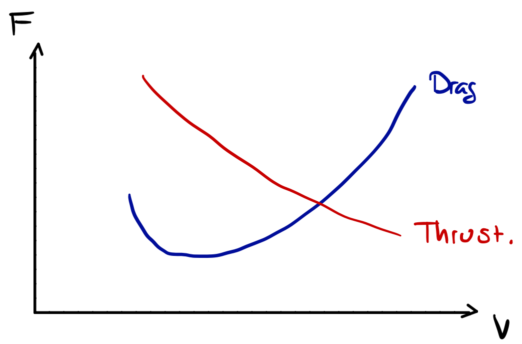
$$\dot{F} = -\frac{dW}{dt} \iff dt = \frac{-dW}{F}$$

$$\boxed{R = \int_{W_1}^{W_0} \frac{V}{C_T} \cdot \frac{C_L}{C_D} \cdot \frac{1}{W} \cdot \frac{L}{L} \, dW}$$

constant

$$\boxed{R = \frac{V}{C_T} \cdot \frac{C_L}{C_D} \cdot \int_{W_1}^{W_0} \frac{dW}{W} = \frac{V}{C_T} \cdot \frac{C_L}{C_D} \cdot \ln\left(\frac{W_0}{W_1}\right)}$$

Cruise Performance of Propeller Aircraft

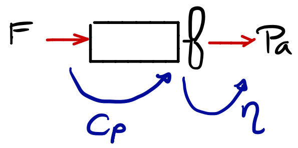


$P_a = \eta P_{br}$ (η → constant)

Best Range $\left(\frac{V}{F}\right)_{max}$

Fuel flow $F = C_p \cdot P_{br}$ (C_p → constant)

$$P_a = T \cdot V = D \cdot V = \frac{C_D}{C_L} \cdot W \cdot V$$



$F = C_p \cdot P_{br}$

$P_a = \eta P_{br}$

$$\boxed{F = \frac{C_p}{\eta} \cdot P_a}$$

Range:

$$R = \int V dt = \int_{W_1}^{W_0} \frac{V}{F} \cdot dW = \int_{W_1}^{W_0} \frac{\eta}{C_p} \cdot \frac{V}{T \cdot V} dW = \int_{W_1}^{W_0} \frac{\eta}{C_p} \cdot \frac{C_L}{C_D} \cdot \frac{dW}{W}$$

constants

$$\boxed{R = \frac{\eta}{C_p} \cdot \frac{C_L}{C_D} \cdot \int_{W_1}^{W_0} \frac{1}{W} \cdot dW = \frac{\eta}{C_p} \cdot \frac{C_L}{C_D} \cdot \ln\left(\frac{W_{start}}{W_{final}}\right)}$$

Altitude does not matter for Range only for time.

Unified Equations

Total Efficiency:

$$\eta_{total} = \frac{P_a}{Q} \quad \begin{matrix} \rightarrow T \cdot V \\ \rightarrow \text{energy from fuel} \end{matrix}$$

$$Q = F \cdot \frac{H}{g} \longrightarrow \text{energy per kilo}$$

$$\eta_{total} = \frac{T \cdot V}{F \cdot \frac{H}{g}}$$

Unified Equation

$$R = \underbrace{\frac{H}{g}}_{\text{Fuel quality}} \cdot \overbrace{\eta_{tot}}^{\text{Propulsion efficiency}} \cdot \underbrace{\frac{C_L}{C_D}}_{\text{Aerodynamic Ratio}} \cdot \underbrace{\ln\left(\frac{W_{start}}{W_{final}}\right)}_{\text{Structural efficiency}}$$

Propeller

$$R_{prop} = \frac{\eta}{C_p} \cdot \frac{C_L}{C_D} \cdot \ln\left(\frac{W_s}{W_f}\right)$$

$$T \cdot V = P_a = \eta P_{br} = \frac{\eta}{C_p} \cdot F$$

$$\boxed{\eta_{tot\,p} = \frac{\eta}{C_p} \cdot \frac{g}{H}}$$

$$\boxed{R_{prop} = \frac{H}{g} \cdot \eta_{tot} \cdot \frac{C_L}{C_D} \cdot \ln\left(\frac{W_s}{W_f}\right)}$$

Jet

$$R_{jet} = \frac{V}{C_T} \cdot \frac{C_L}{C_D} \cdot \ln\left(\frac{W_s}{W_f}\right)$$

$$\eta_{tot\,J} = \frac{T \cdot V}{H \cdot \frac{F}{g}} = \frac{\frac{F}{C_T} \cdot V}{H \cdot \frac{F}{g}} = \frac{V \cdot g}{H \cdot C_T}$$

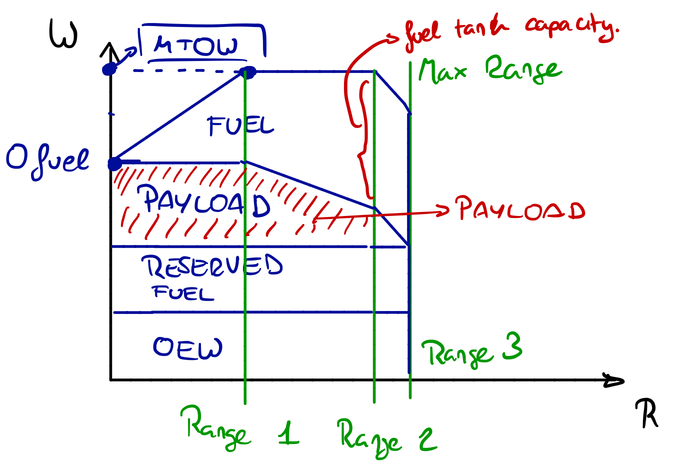
$$\left(\frac{V}{C_T}\right)_{JET} = \eta_{tot} \cdot \frac{g}{H}$$

$$\boxed{R_{JET} = \frac{H}{g} \cdot \eta_{tot} \cdot \frac{C_L}{C_D} \cdot \ln\left(\frac{W_s}{W_f}\right)}$$

=

Economics

Payload-Range Diagram



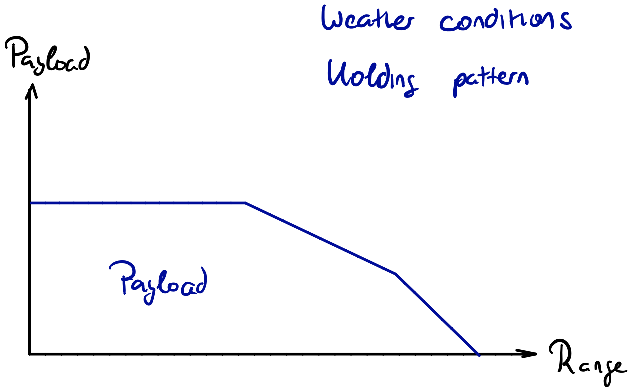
REMOVE SOME PAYLOAD

- Range 1: Design Range
- Range 2: Ultimate Range
- MTOW: maximum take off weight
- MZFW: maximum zero fuel weight
- OEW: operational empty weight

Reserve fuel:

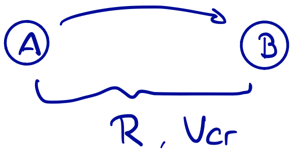
Weather conditions

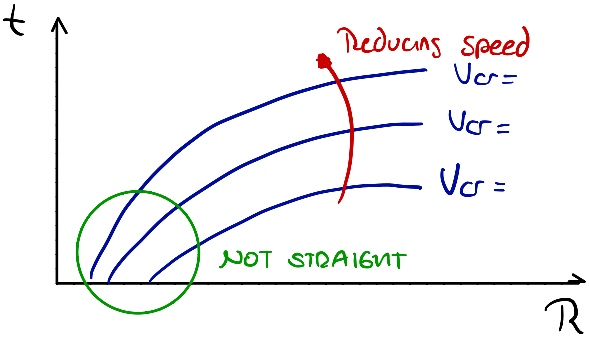
Holding pattern



Speed Range

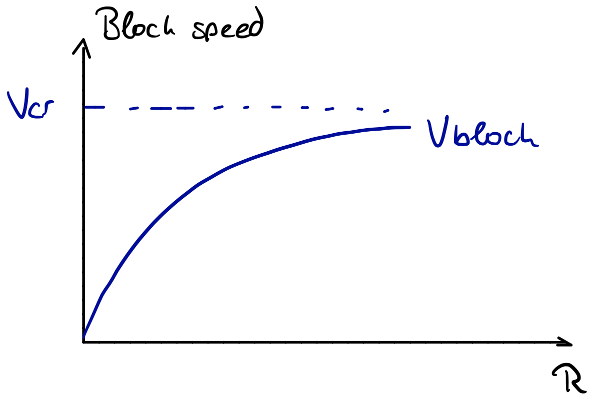
Block time: $t = \frac{R}{V_{cr}} + \Delta t$





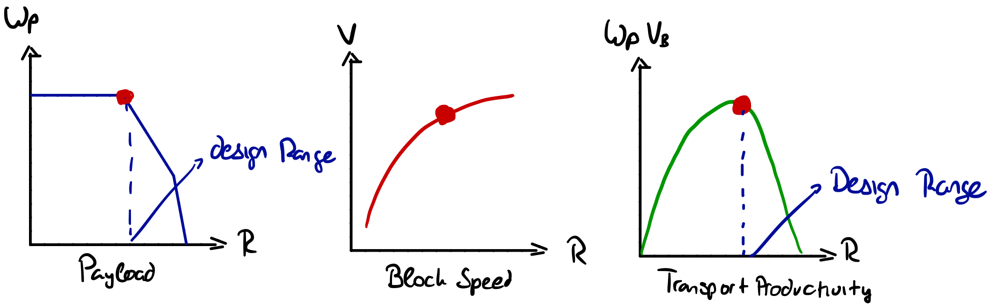
Block Speed

$$\frac{R}{t} = \frac{R}{\frac{R}{V_{cr}} + \Delta t}$$



JOIN PAYLOAD AND SPEED:

$$W_P \cdot V_{block} = \left[pax \frac{km}{hr} \right]$$



COST

Direct:
- Fuel
- Salary
- Maintenance

Indirect:
- Aircraft depreciation

Effects of wind

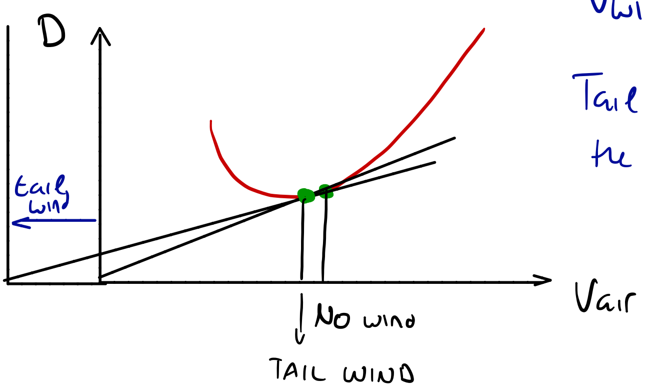
Specific Range parameter

$\frac{V}{F}$ ⟶ Ground speed

$V_{ground} = V_{air} + V_{wind}$

$V_{wind} = 10\ m/s$

Tail wind moves the axis to the left.



Range

$R = \int \frac{V_F}{F} dw$ $\qquad V_g = V + V_w$

$$R = \int \left(\frac{V}{F} + \frac{V_w}{F} \right) dw = \int \frac{V}{F} dw + V_w \cdot \int \frac{dw}{F}$$

$$\boxed{R = R(V_w = 0) + V_w \cdot \text{ENDURANCE}}$$

Graphically solved

CRUISE AT TRANSONIC CONDITIONS

ASSUMPTIONS OF PREVIOUS CRUISE:

- Single, constant, lift drag polar — NOT TRUE
- Maximum power, thrust independent of airspeed — NOT TRUE
- Power / Thrust specific fuel consumption constant — NOT TRUE

} TRANSONIC CONDITIONS.

$$\eta_{total} = \frac{P_a}{Q} = \frac{T \cdot V}{H \cdot \frac{F}{g}} \iff \frac{V}{F} = \eta_{tot} \cdot \frac{H}{g} \cdot \frac{1}{T} \qquad L = W \quad T = D$$

SPECIFIC RANGE

$$\boxed{\frac{V}{F} = \eta_{tot} \cdot \frac{C_L}{C_D} \cdot \frac{H}{g} \cdot \frac{1}{W}}$$

maximize

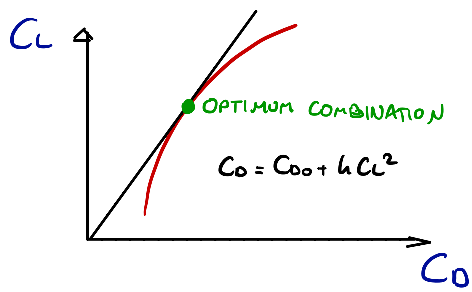
$$\boxed{\eta_{tot}(M, H)\ \frac{C_L}{C_D}(M)}$$

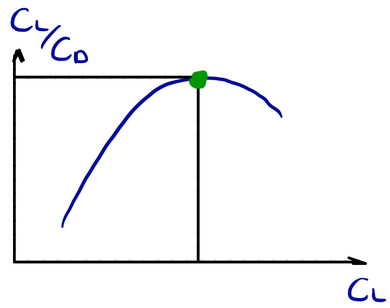
$$\frac{V}{F} = \frac{V}{C_T \cdot T} = \frac{V}{C_T \cdot W} \cdot \frac{C_L}{C_D} = \frac{M}{a} \cdot \frac{C_L}{C_D} \cdot \frac{1}{C_T} \cdot \frac{1}{W}$$

$$\boxed{\frac{V}{F} = \boxed{\frac{M}{a} \cdot \frac{C_L}{C_D}} \cdot \frac{H}{g} \cdot \frac{1}{W}}$$

Maximize

$$D = \frac{C_D}{C_L} \cdot W \qquad M = V \cdot a \qquad C_L, C_D (M)$$





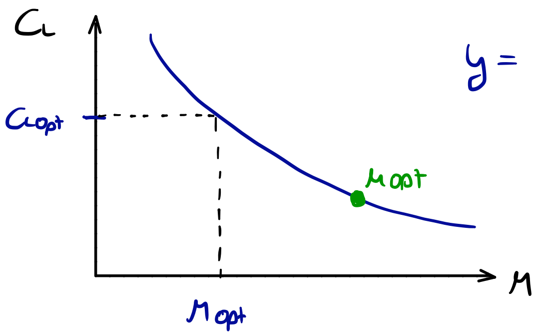
C_{Lopt} ? M ?

$$M \cdot \frac{C_L}{C_D}$$

$$L = W \qquad V = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}} = M \cdot a \qquad \boxed{M^2 = \frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{a^2} \cdot \frac{1}{C_L}}$$

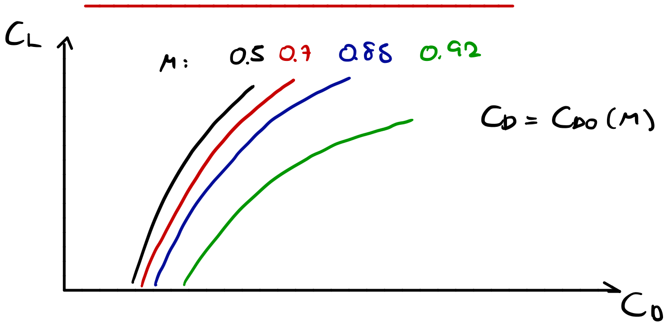
$$\left(\frac{C_L}{C_D}\right)_{max} \Rightarrow \boxed{C_{Lopt} = \sqrt{C_{D0} \cdot \pi A e}}$$

$$y = \frac{(a)}{x^2} \quad (a) \rightarrow \text{constant}$$

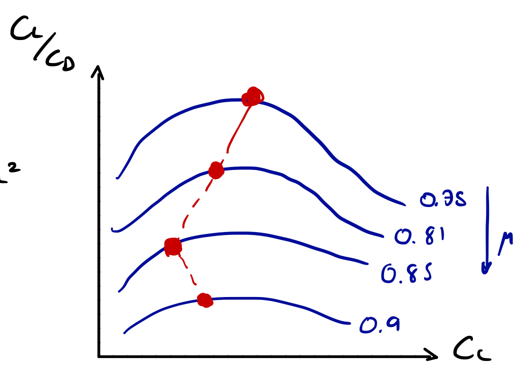


At low Mach numbers we need a high CL to compensate for low dinamic pressure

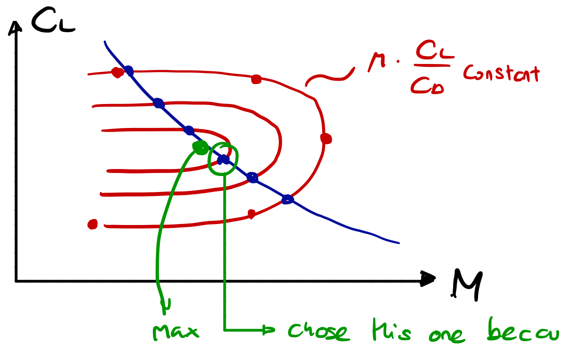
HIGH SPEED DRAG POLARS



$$C_D = C_{D0}(M) + k(M)C_L^2$$



$M \cdot \frac{C_L}{C_D}$ Calculate and Plot in:

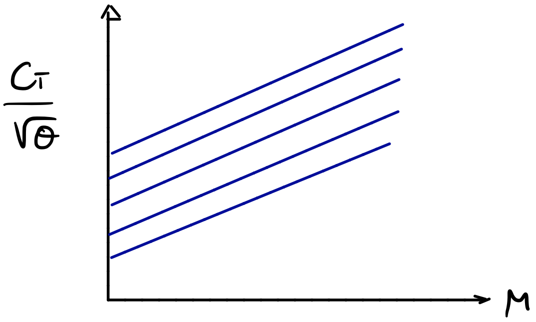


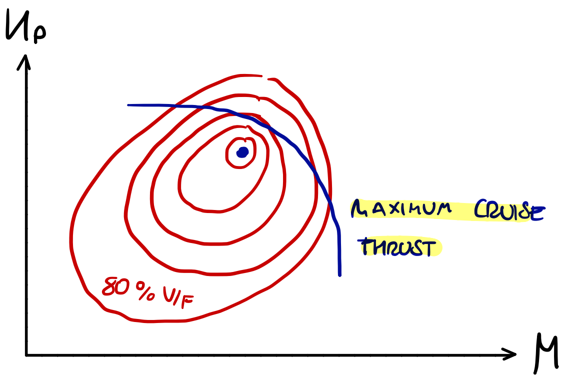
→ chose this one because its the closest option

1. Select M
2. Select C_L to set $\frac{C_L}{C_D}$
3. Draw on graph
4. Draw aircraft C_L/M
5. Find optimum position.

$\eta_{tot}\,(M, V)$

BEHAVIOUR Variation with altitude





APPROACH

1. Single aircraft weight: compute max and min airspeed in steady flight for various altitudes
2. Create a grid on the envelope. compute specific range $\left(\frac{V}{F}\right)$ of each point.

2.b) Compute C_L, C_D, DRAG, THRUST, FUEL FLOW, V/F

3. Create contour lines.

SIMULATION

FLIGHT SIMULATION

Human → Interfaces → Computer → Display → Human

- Aircraft Performance

  Point Mass : 3 EOM   NO ROTATION   PERFORMANCE APP

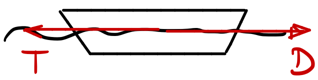
- Used for :
  - Complex situations
  - Accurate results

  } Aircraft operations
  Aircraft design.

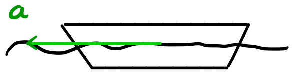
DEVELOP A SIMULATION MODEL

APPROACH:

F.B.D



K.D



$$T - D = m \cdot \frac{dv}{dt}$$

$$\boxed{D = C_D \cdot \frac{1}{2} \rho V^2 S}$$

water

$$F = m \cdot \bar{a}$$

$$D = C_D \cdot \frac{1}{2} \rho V^2 \cdot S \cdot \text{sign}(V)$$

$$\frac{dv}{dt} = \frac{T - C_D \cdot \frac{1}{2} \rho V^2 \cdot S \cdot \text{sign}(V)}{m}$$

constants	control
Independent	state

$$\frac{dx}{dt} = V$$

- Input : T
- Output: X, V

FLOW CHART

INITIAL CONDITION → TIME STEP → SET THRUST → COMPUTE derivatives → COMPUTE NEW X → CONDITION TO STOP → SAVE RESULTS → TIME STEP

INITIAL CONDITION

$i = 1$

$t = 0s$

$V = 0$ m/s

$T = 0N$

$x = 0m$

$D = 0N$

$m = 200$ kg

$C_D = 0.01$

$S = 3m^2$

$\rho = 10^3$

TIME STEP

$t = t + 0.01$

SET THRUST

$T = 5000$ N

COMPUTE derivatives

$dV/dt = (5000 - 0)/200$

COMPUTE NEW X

$x = x + V \cdot 0.01$

CONDITION TO STOP

if $t = 10$

break end.

SAVE RESULTS

time(i) = t

i = i + 1

MOTORBOAT CONTROL:

1. Equation for captain behaviour.

2. Calculate required motorboat thrust.

1. PROPORTIONAL CONTROL:

Position error:

$\epsilon = x_{desired} - x$

Thrust corrective action

$T = k \cdot (x_{desired} - x)$

different captains will be represented.

Thrust control applied is worst than thrust control + speed control.

$T = k_1 (V_{des} - V)$

$V_{des} = k_2 (x_{des} - x)$

• IDEAL CONTROL

• max throttle, and at specific distance full negative throttle.

INITIAL CONDITION → TIME STEP → SET THRUST → COMPUTE derivatives → COMPUTE NEW X → CONDITION TO STOP → SAVE RESULTS → TIME STEP

INITIAL CONDITION

delta = 10

$x_{switch} = 30$

TIME STEP

$x_{switch} = x_{switch} - delta$

SET THRUST

for $x < x_{switch}$

$T = T_{max}$

for $x >= x_{switch}$

$T = T_{min}$

CONDITION TO STOP

If $x = 30$ and $V = 0$

exit.