Dynamics Formula Overview

1 General Equations:

$$\mathbf{v} = \frac{d\mathbf{s}}{dt} \tag{1}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{s}}{dt^2} \tag{2}$$

$$\mathbf{a}\, d\mathbf{s} = \mathbf{v}\, d\mathbf{v} \tag{3}$$

$$\Sigma\mathbf{F} = m\mathbf{a} \tag{4}$$

2 Polar Coordinate Systems:

$$\mathbf{r} = r\mathbf{e_r} \tag{5}$$

$$\mathbf{v} = \dot{r}\mathbf{e_r} + r\dot{\theta}\mathbf{e}_\theta \tag{6}$$

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e_r} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\mathbf{e}_\theta \tag{7}$$

3 Relative Velocity/Acceleration:

$$\mathbf{v_{A/B}} = \mathbf{v_A} - \mathbf{v_B} = -\mathbf{v_{B/A}} \tag{8}$$

$$\mathbf{a_{A/B}} = \mathbf{a_A} - \mathbf{a_B} = -\mathbf{a_{B/A}} \tag{9}$$

4 Friction:

$$F_w = \mu_k N \tag{10}$$

$$F_w \leq \mu_s N \tag{11}$$

5 Energy:

$$U = \int \mathbf{F} \cdot d\mathbf{s} \tag{12}$$

$$T = \frac{1}{2}mv^2 \tag{13}$$

$$V^g = mgh \tag{14}$$

$$V^e = \frac{1}{2}k(\mathbf{r} - \mathbf{r_0})^2 \tag{15}$$

$$U'_{1,2} = \Delta T + \Delta V^g + \Delta V^e \tag{16}$$

6 Momentum:

$$\mathbf{G} = m\mathbf{v} \tag{17}$$

$$\dot{\mathbf{G}} = m\dot{\mathbf{v}} = m\mathbf{a} = \Sigma\mathbf{f} \tag{18}$$

$$\mathbf{H} = m\mathbf{r} \times \mathbf{v} \tag{19}$$

$$\dot{\mathbf{H}} = m\mathbf{r} \times \dot{\mathbf{v}} = \mathbf{r} \times (m\mathbf{a}) = \mathbf{r} \times \Sigma\mathbf{f} = \Sigma\mathbf{M_0} \tag{20}$$

7 Collisions

$$e = -\frac{v1' - v2'}{v1 - v2} \tag{21}$$

Plastic collision $\Rightarrow e = 0$, elastic collision $\Rightarrow e = 1$.

8 Rotations

$$\omega = \frac{d\theta}{dt} \tag{22}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \tag{23}$$

$$\alpha\, d\theta = \omega\, d\omega \tag{24}$$

$$\mathbf{v} = \omega \times \mathbf{r} \tag{25}$$

$$\mathbf{a} = \alpha \times \mathbf{r} - \omega \times (\omega \times \mathbf{r}) \tag{26}$$

9 Moment of Inertia

$$I_G = \int_\Omega r^2 dA \tag{27}$$

$$I_A = I_G + m\rho_A^2 = mk_A^2 \tag{28}$$

$$I_{G_{bar}} = \frac{1}{12}ml^2 \tag{29}$$

$$I_{G_{disc}} = \frac{1}{2}mr^2 \tag{30}$$

10 Kinetic Energy for Rotations

$$T = \frac{1}{2}I_0\omega^2 \tag{31}$$

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2 \tag{32}$$

11 Angular Momentum Equations

$$\mathbf{H_A} = I_G\omega + \mathbf{r_{AG}} \times m\mathbf{v_G} \tag{33}$$

$$\mathbf{M_A} = \dot{\mathbf{H}}_\mathbf{A} + \mathbf{v_A} \times m\mathbf{v_G} \tag{34}$$

$$\mathbf{M_A} = \dot{\mathbf{H}}_\mathbf{G} + \mathbf{r_{AG}} \times m\mathbf{a_G} \tag{35}$$

12 Table of Useful Equations

$$\begin{array}{ll} \Sigma\mathbf{F} = m\mathbf{a} & \Sigma\mathbf{M} = I\alpha \\ \mathbf{G} = m\mathbf{v} & \mathbf{H} = I\omega \\ \dot{\mathbf{G}} = \Sigma\mathbf{F} & \dot{\mathbf{H}} = \Sigma\mathbf{M} \\ T = \frac{1}{2}mv^2 & T = \frac{1}{2}I\omega^2 \end{array} \tag{36}$$