8/1

$$k = \frac{W}{\delta_{st}} = \frac{3(9.81)}{0.042} = \underline{701\ \text{N/m}}$$

$$k = 701\ \frac{\text{N}}{\text{m}}\left(\frac{1\ \text{lb/in.}}{175.13\ \text{N/m}}\right) = \underline{4.00\ \text{lb/in.}}$$

$$k = 4.00\ \frac{\text{lb}}{\text{in.}}\left(\frac{12\ \text{in.}}{\text{ft}}\right) = \underline{48.0\ \text{lb/ft}}$$

8/2

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{k}{W/g}} = \sqrt{\frac{g}{W/k}} = \sqrt{\frac{g}{\delta_{st}}}$$

8/3

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{54(12)}{2}} = \underline{18 \text{ rad/sec}}$$

$$f_n = \left(18 \frac{\text{rad}}{\text{sec}}\right)\left(\frac{1 \text{ cycle}}{2\pi \text{ rad}}\right) = \underline{2.86 \text{ Hz}}$$

8/4 $x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$

$x_0 = -2$ in., $\dot{x}_0 = 0$, $\omega_n = 18$ rad/sec, so

$\underline{x = -2 \cos 18t \text{ in.}}$

$\dot{x} = +36 \sin 18t$ in./sec $\Rightarrow \underline{v_{max} = 36 \text{ in./sec}}$

$\ddot{x} = 36(18) \cos 18t$ in./sec^2 $\Rightarrow \underline{a_{max} = 648 \text{ in./sec}^2}$

(or $\underline{v_{max} = 3 \text{ ft/sec}}$, $\underline{a_{max} = 54 \text{ ft/sec}^2}$)

8/5

$$x = C \sin(\omega_n t + \psi)$$

$$C = \left[x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2\right]^{1/2} = \left[2^2 + \left(\frac{-9}{18}\right)^2\right]^{1/2} = \underline{2.06 \text{ in.}}$$

$$\psi = \tan^{-1}\left(\frac{x_0 \omega_n}{\dot{x}_0}\right) = \tan^{-1}\left(\frac{2(18)}{-9}\right) = 1.816 \text{ rad}$$

So $\underline{x = 2.06 \sin(18t + 1.816) \text{ in.}}$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{18} = \underline{0.349 \text{ sec}}$$

(ω_n from solution to Prob. 8/3)

8/6

$$\delta_{st} = \frac{W}{k} = \frac{2(9.81)}{98} = \underline{0.200 \text{ m}}$$

$$\omega_n = \sqrt{k/m} = \sqrt{98/2} = 7 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{7} = \underline{0.898 \text{ s}}$$

$$v_{max} = C\omega_n = 0.1(7) = \underline{0.7 \text{ m/s}}$$

8/7 $\omega_n = 7$ rad/s (from Prob. 8/6)

$$y = y_0 \cos \omega_n t + \frac{\dot{y}_0}{\omega_n} \sin \omega_n t$$

$$= 0.1 \cos 7t \text{ m}$$

$$v = \dot{y} = -0.7 \sin 7t \text{ m/s}$$

$$a = \ddot{y} = -4.9 \cos 7t \text{ m/s}^2$$

When $t = 3$ s:

$$y = 0.1 \cos(7 \cdot 3) = \underline{-0.0548 \text{ m}}$$

$$v = -0.7 \sin(7 \cdot 3) = \underline{-0.586 \text{ m/s}}$$

$$a = -4.9 \cos(7 \cdot 3) = \underline{+2.68 \text{ m/s}^2}$$

$$\underline{a_{max} = 4.9 \text{ m/s}^2}$$

8/8

$F = kx: \quad 30 \times 9.81 = k(0.050), \; k = 5890 \text{ N/m}$

$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{5890}{30}} = \underline{2.23 \text{ Hz}}$

8/9 $\ddot{x} + \omega_n^2 x = 0$ where $\omega_n = \sqrt{k/m} = 2\pi(2.23) = 14.01$ rad/s (Prob. 8/8)

$x = A\cos\omega_n t + B\sin\omega_n t$, $\dot{x} = -A\omega_n \sin\omega_n t + B\omega_n \cos\omega_n t$

When $t = 0$, $\dot{x} = 0$, $0 = 0 + B\omega_n$, $B = 0$

" $t = 0$, $x = 0.025$ m, $0.025 = A \times 1$, $A = 0.025$ m

$x = 0.025 \cos 14.01t$ meters

or $\underline{x = 25 \cos 14.01t \text{ mm}}$ (t in seconds)

8/10

$$\Sigma F_y = m\ddot{y} : \quad C_2 - k_2 y + mg - C_1 - k_1 y = m\ddot{y}$$

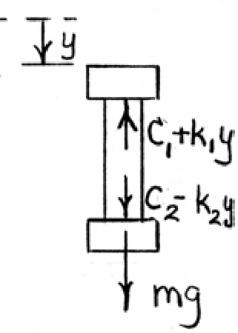
At equilibrium, $C_2 + mg - C_1 = 0$

So $\quad -(k_1 + k_2)y = m\ddot{y}$

$$\ddot{y} + \frac{k_1 + k_2}{m} y = 0$$

$$\omega_n = \sqrt{\frac{k_1 + k_2}{m}} = \sqrt{\frac{3600 + 1800}{2.5}} = 46.5 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{46.5}{2\pi} = \underline{7.40 \text{ Hz}}$$



8/11 $\omega_n = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2(180{,}000)}{100}} = 60 \text{ rad/s}$

$x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$

$= 0 + \frac{0.5}{60} \sin 60t = 8.33(10^{-3}) \sin 60t$

$\dot{x} = 60(8.33)(10^{-3}) \cos 60t = 0.5 \cos 60t$

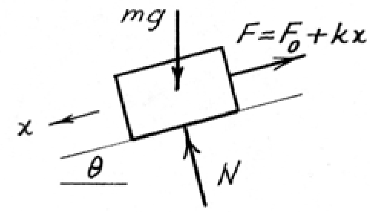
$\ddot{x} = -60(0.5) \sin 60t = -30 \sin 60t$

$a_{max} = 30 \text{ m/s}^2$

8/12

For equilibrium position

$F_0 = mg\sin\theta$



$\Sigma F_x = m\ddot{x}: \quad mg\sin\theta - (mg\sin\theta + kx) = m\ddot{x}$

$\ddot{x} + \frac{k}{m}x = 0$ so $\underline{f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$ independent of θ

8/13

$$\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4(17{,}500)}{1000}} = 8.37 \ \frac{\text{rad}}{\text{sec}}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{8.37}{2\pi} = \underline{1.332 \text{ Hz}}$$

We have assumed the unsprung mass (wheels, axles, etc.) to be a small fraction of the total car mass.

8/14

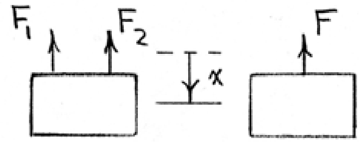
(a) From Eq. 8/3 frequency $f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

$$3 = \frac{1}{2\pi}\sqrt{\frac{3k}{4000}}, \quad k = 474\times10^3 \text{ N/m} \quad \text{or} \quad \underline{k = 474 \text{ kN/m}}$$

(b) For $m = 4000 + 40\,000 = 44\,000$ kg,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3(474\times10^3)}{44\times10^3}} = \underline{0.905 \text{ Hz}}$$

8/15



(a) $F = F_1 + F_2$

$kx = k_1 x + k_2 x$

$\underline{k = k_1 + k_2}$

(b) $F = F_1 = F_2$

$x_1 = \dfrac{F_1}{k_1}$, $x_2 = \dfrac{F_2}{k_2}$, $x = \dfrac{F}{k}$

From $x = x_1 + x_2$, we have $\dfrac{F}{k} = \dfrac{F_1}{k_1} + \dfrac{F_2}{k_2}$

or $\dfrac{F}{k} = \dfrac{F}{k_1} + \dfrac{F}{k_2}$. Thus $\underline{\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2}}$

8/16 Deflect the system as shown and experimentally observe its natural frequency ω_1. Then add m_2 and observe ω_2.

$$\omega_1 = \sqrt{\frac{k}{m_1}}, \qquad \omega_2 = \sqrt{\frac{k}{m_1+m_2}}$$

Solve simultaneously to obtain

$$\begin{cases} m_1 = \dfrac{m_2\omega_2^2}{\omega_1^2 - \omega_2^2} \\[2ex] k = \dfrac{m_2\,\omega_1^2\omega_2^2}{\omega_1^2 - \omega_2^2} \end{cases}$$

We could have used the natural frequencies in Hz – f_1 and f_2 – rather than ω_1 and ω_2 in rad/sec. Alternative solutions:

(a) Place m_1, then m_2 (but not both) on spring.

(b) Place m_2, then m_1+m_2 on spring.

$\boxed{8/17}$ $f_n = \frac{1}{2\pi}\sqrt{k/m}$

$\frac{1}{0.6} = \frac{1}{2\pi}\sqrt{\frac{k}{90}}$, $k = 9870$ N/m

$\delta_{st} = \frac{W}{k} = \frac{90(9.81)}{9870} = 0.0895$ m or $\underline{89.5 \text{ mm}}$

$\boxed{8/18}$ $f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{k/m_{Tot}}$

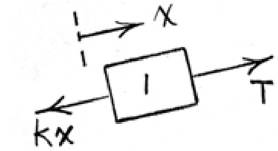
$\frac{1}{0.75} = \frac{1}{2\pi}\sqrt{600/(m+6)}$, $\underline{m = 2.55 \text{ kg}}$

$\omega_n = \sqrt{k/m_{Tot}} = \sqrt{600/(6+2.55)} = 8.38 \text{ rad/s}$

$a_{max} = \omega_n^2 C = 8.38^2(0.050) = 3.51 \text{ m/s}^2$

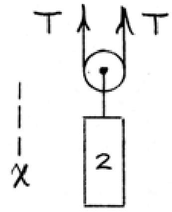
$a_{max} = \mu_s g: \quad 3.51 = \mu_s(9.81)$, $\underline{\mu_s = 0.358}$

8/19



Free body diagrams show dynamic forces only. x is the displacement from equilibrium.

From constraint, $a_2 = \frac{1}{2} a_1$



$\Sigma F_x = m\ddot{x}$:

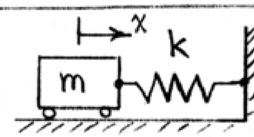
① $-kx + T = m\ddot{x}$

② $-2T = m\left(\frac{1}{2}\ddot{x}\right)$

Eliminating T : $\ddot{x} + \frac{4k}{5m} x = 0$

$$\omega_n = \sqrt{\frac{4k}{5m}}$$

8/20 Equivalent system :



$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{(3000)(12)}{2500/32.2}} = 21.5 \text{ rad/sec}$$

$$x = \cancel{x_0}^{\,0}\cos\omega_n t + \frac{\dot{x}_0}{\omega_n}\sin\omega_n t$$

$$= \frac{5(5280/3600)}{21.5}\sin 21.5t = 0.341\sin 21.5t$$

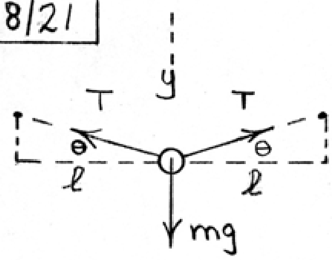
$$x_{max} = 0.341 \text{ ft or } \underline{4.09 \text{ in.}}$$

$$v = (0.341)(21.5)\cos 21.5t = \underline{7.33\cos 21.5t}$$
(in ft/sec)

or $\underline{v = 88.0\cos 21.5t \text{ in./sec}}$

8/21

$$\sin\theta \cong \theta \cong \frac{y+\delta_{st}}{\ell}$$



$y+\delta_{st}$ $\begin{cases} \delta_{st} = \text{static deflection} \\ y = \text{dynamic deflection} \end{cases}$

$$\Sigma F_y = m\ddot{y} : \quad -2T\sin\theta + mg = m\ddot{y}$$

$$-2T\left(\frac{y+\delta_{st}}{\ell}\right) + mg = m\ddot{y}$$

$$-2T\frac{y}{\ell} \cancel{- 2T\frac{\delta_{st}}{\ell}} \cancel{+ mg} = m\ddot{y}$$

$$\ddot{y} + \left(\frac{2T}{m\ell}\right)y = 0 \quad , \quad \underline{\omega_n = \sqrt{\frac{2T}{m\ell}}}$$

Although done above, the inclusion of the forces $+mg$ and $-2T\frac{\delta_{st}}{\ell}$, which sum to zero, is not necessary.

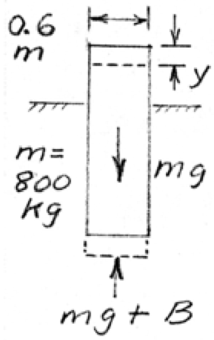
8/22

$$f_n = \frac{1}{2\pi}\sqrt{k/m}$$

$$\frac{1}{0.5} = \frac{1}{2\pi}\sqrt{k/4000}\,, \qquad k = 632 \text{ kN/m}$$

$$\delta_{st} = \frac{W}{k} = \frac{4000(9.81)}{632(10^3)} = 0.0621 \text{ m or } \underline{62.1 \text{ mm}}$$

8/23



B = added buoyancy due to deflection y below equil. position

$$B = \text{Vol.} \times \text{density} \times g$$

$$= \frac{\pi d^2}{4} y \rho g = \frac{\pi (0.6^2)}{4} y\,(1.03 \times 10^3)(9.81)$$

$$= 2857y \quad \text{N}$$

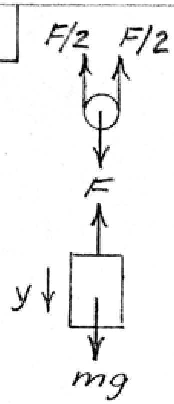
$$\Sigma F_y = ma_y; \quad mg - (mg + B) = m\ddot{y}$$

$$\ddot{y} + \frac{2857}{800} y = 0$$

$$\omega_n = \sqrt{\frac{2857}{800}} = 1.890 \frac{\text{rad}}{\text{sec}}, \quad f_n = \frac{\omega_n}{2\pi} = \frac{1.890}{2\pi}$$

$$= \underline{0.301 \text{ Hz}}$$

8/24



When $y = 0$, $F_0 = mg$

For $y = y$, spring stretched $2y$

So $F/2 = \frac{mg}{2} + k(2y)$

Hence $F = mg + 4ky$

$\Sigma F_y = m\ddot{y}$; $mg - (mg + 4ky) = m\ddot{y}$

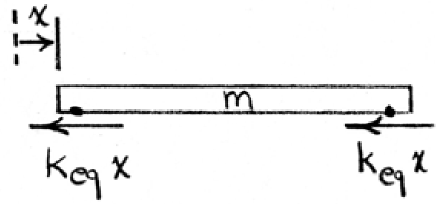
$$\underline{\ddot{y} + 4\frac{k}{m}y = 0}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{4k}{m}}} = \underline{\pi\sqrt{\frac{m}{k}}}$$

8/25 For one upright $P = \left(\frac{12EI}{L^3}\right)\delta = k_{eq}\delta$

So $k_{eq} = \frac{12EI}{L^3}$.

FBD of top mass:
(dynamic forces only)

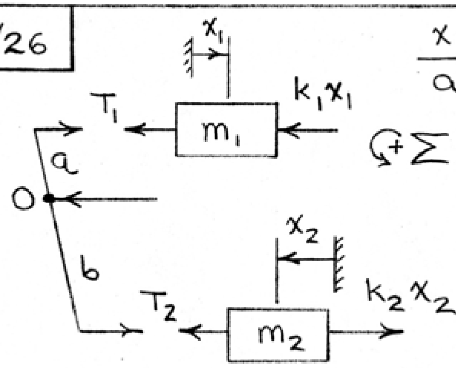


$$\Sigma F_x = m\ddot{x} : \quad -2k_{eq}x = m\ddot{x}$$

$$\ddot{x} + \frac{2k_{eq}}{m}x = 0 \quad \text{or} \quad \ddot{x} + \frac{24EI}{mL^3}x = 0$$

$$\omega_n = \sqrt{\frac{24EI}{mL^3}} = \underline{2\sqrt{\frac{6EI}{mL^3}}}$$

8/26



$$\frac{x_1}{a} = \frac{x_2}{b}, \quad x_2 = \frac{b}{a}x_1$$

$$\circlearrowleft + \Sigma M_0 = 0: \quad -T_1 a + T_2 b = 0$$

$$T_2 = \frac{a}{b}T_1$$

$$\Sigma F_x = m\ddot{x}: \quad -T_1 - k_1 x_1 = m_1 \ddot{x}_1 \qquad (a)$$

$$T_2 - k_2 x_2 = m_2 \ddot{x}_2$$

Second eq. : $\frac{a}{b}T_1 - k_2\left(\frac{b}{a}x_1\right) = m_2 \frac{b}{a}\ddot{x}_1$ (b)

Solve (b) for T_1 and substitute into (a):

$$\underline{\left[m_1 + \frac{b^2}{a^2}m_2\right]\ddot{x}_1 + \left[k_1 + \frac{b^2}{a^2}k_2\right]x_1 = 0}$$

For $k_1 = k_2 = k$, $m_1 = m_2 = m$: $\underline{m\ddot{x}_1 + kx_1 = 0, \quad \omega_n' = \sqrt{k/m}}$

8/27 The velocity of the putty after dropping 2 m is

$$v = \sqrt{2gh} = \sqrt{2(9.81)(2)} = 6.26 \text{ m/s}$$

The additional static deflection due to the 3-kg mass is

$$\delta_{st} = \frac{W}{4k} = \frac{3(9.81)}{4(800)} = 9.20(10^{-3}) \text{ m}$$

Velocity after impact : $\dot{x}_0 = \dfrac{3(6.26)}{31} = 0.606$ m/s

Natural frequency of motion : $\omega_n = \sqrt{\dfrac{4k}{m}} = 10.16 \dfrac{\text{rad}}{\text{s}}$

$$x = \delta_{st} + x_0 \cos\omega_n t + \frac{\dot{x}_0}{\omega_n} \sin\omega_n t$$

$$= 9.20(10^{-3}) - 9.20(10^{-3}) \cos 10.16t + 59.7(10^{-3}) \sin 10.16t$$

$$= \underline{9.20(10^{-3})(1 - \cos 10.16t) + 59.7(10^{-3}) \sin 10.16t \text{ m}}$$

8/28

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{3(12)}{8/32.2}} = 12.04 \ \frac{\text{rad}}{\text{sec}}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{2.5}{2\left(\frac{8}{32.2}\right)(12.04)} = \underline{0.418}$$

8/29 $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{30\,000}{35}} = 29.3$ rad/s

$\zeta \overset{1}{=} \frac{c}{2m\omega_n}$, $c = 2m\omega_n = 2(35)(29.3)$

$= 2050 \ \frac{\text{N}\cdot\text{s}}{\text{m}}$

8/30

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3(12)}{8/32.2}} = 12.04 \text{ rad/sec}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{2.5}{2\left(\frac{8}{32.2}\right)(12.04)} = 0.418$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 12.04\sqrt{1-0.418^2} = 10.94 \frac{\text{rad}}{\text{sec}}$$

$$x = (A_3 \cos\omega_d t + A_4 \sin\omega_d t)e^{-\zeta\omega_n t}$$

$$= (A_3 \cos 10.94t + A_4 \sin 10.94t)e^{-5.03t}$$

Initial conditions: $x_0 = A_3$

$$\dot{x} = -5.03(A_3 \cos 10.94t + A_4 \sin 10.94t)e^{-5.03t}$$

$$+ 10.94(-A_3 \sin 10.94t + A_4 \cos 10.94t)e^{-5.03t}$$

$$0 = -5.03A_3 + 10.94A_4$$

$$A_4 = 0.460A_3 = 0.460x_0$$

So $\underline{x = x_0(\cos 10.94t + 0.460 \sin 10.94t)e^{-5.03t}}$

8/31

$$\tau_d = 1.25\,\tau$$

$$\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.25\frac{2\pi}{\omega_n}\,, \quad \underline{\zeta = 0.6}$$

8/32 Log decrement $\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln\left(\frac{4.65}{4.3}\right)$

$= 0.0783$

Then $\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} = 0.01245$

From $\zeta = \frac{c}{2m\omega_n}$, $c = 2m\omega_n \zeta$

$c = 2(1.1)(10 \cdot 2\pi)(0.01245) = \underline{1.721 \ \frac{N \cdot s}{m}}$

$\boxed{8/33}$ $\omega_n = \sqrt{k/m} = \sqrt{\frac{200(12)}{80/32.2}} = 31.1$ rad/sec

$\zeta \nearrow 1$

$\zeta = \frac{c}{2m\omega_n}$, $c = 2m\omega_n = 2\left(\frac{80}{32.2}\right)(31.1)$

$= \underline{154.4 \text{ lb-sec/ft}}$

8/34 $\frac{x_1}{x_2} = 4$, so log decrement δ is

$$\delta = \ln \frac{x_1}{x_2} = \ln 4 = 1.386$$

Viscous damping factor $\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

$$= \frac{1.386}{\sqrt{4\pi^2 + 1.386^2}} = \underline{0.215}$$

Natural frequency $\omega_n = \frac{2\pi}{\tau_n} = \frac{2\pi}{0.75} = 8.38 \frac{\text{rad}}{\text{s}}$

From $\omega_n = \sqrt{\frac{k}{m}}$, $k = m\omega_n^2 = 2.5(8.38)^2 = \underline{175.5 \frac{\text{N}}{\text{m}}}$

Damping ratio $\zeta = \frac{c}{2m\omega_n}$

So $c = 2m\omega_n\zeta = 2(2.5)(8.38)(0.215) = \underline{9.02 \frac{\text{N}\cdot\text{s}}{\text{m}}}$

8/35

$$\frac{x_0}{x_N} = \frac{Ce^{-\zeta\omega_n t_0}}{Ce^{-\zeta\omega_n(t_0+N\tau_d)}} = e^{\zeta\omega_n N\tau_d}$$

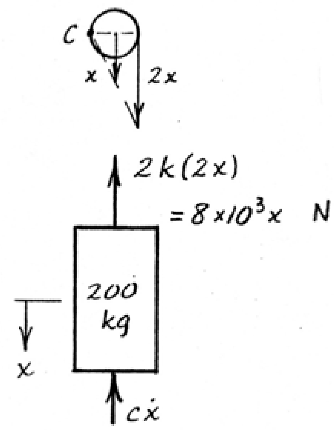
Define $\underline{\delta_N = \ln\frac{x_0}{x_N}} = \zeta\omega_n N\tau_d$, $\tau_d = \dfrac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$

so $\delta_N = \zeta\cancel{\omega_n} N\dfrac{2\pi}{\cancel{\omega_n}\sqrt{1-\zeta^2}} = \dfrac{2\pi N\zeta}{\sqrt{1-\zeta^2}}$

Solve for ζ and get $\underline{\zeta = \dfrac{\delta_N}{\sqrt{(2\pi N)^2+\delta_N^2}}}$

8/36 Dynamic forces shown only.

x measured from equilibrium position



$$\Sigma F_x = m\ddot{x}: \; -c\dot{x} - 8\times10^3 x = 200\ddot{x}$$

$$200\ddot{x} + c\dot{x} + 8\times10^3 x = 0$$

$$\omega_n = \sqrt{\frac{8\times10^3}{200}} = 2\sqrt{10} \text{ rad/s}$$

Let $\zeta = \dfrac{c}{2m\omega_n} = \dfrac{c}{400\times2\sqrt{10}}$,

$\zeta = 1$ for critical damping

so $c = 800\sqrt{10} = \underline{2530 \text{ N}\cdot\text{s/m}}$

8/37 Eq. 8/11 : $x = (A_3 \cos\omega_d t + A_4 \sin\omega_d t)e^{-\zeta\omega_n t}$

At time $t = 0$: $x_0 = A_3$

$\dot{x} = -\zeta\omega_n (A_3 \cos\omega_d t + A_4 \sin\omega_d t)e^{-\zeta\omega_n t}$

$\quad + \omega_d(-A_3 \sin\omega_d t + A_4 \cos\omega_d t)e^{-\zeta\omega_n t}$

At time $t = 0$: $\dot{x}_0 = -\zeta\omega_n A_3 + \omega_d A_4 = 0$

$A_4 = x_0\omega_n\zeta / \omega_d$

Thus $x = x_0\left[\cos\omega_d t + \dfrac{\zeta\omega_n}{\omega_d}\sin\omega_d t\right]e^{-\zeta\omega_n t}$

At time $t = \tau_d = 2\pi/(\omega_n\sqrt{1-\zeta^2})$

$x_{\tau_d} = x_0[1+0]e^{-\zeta\omega_n\left(\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}\right)}$

$\dfrac{x_0}{2} = x_0 e^{-2\pi\zeta/\sqrt{1-\zeta^2}}$

$\ln\left(\frac{1}{2}\right) = -\dfrac{2\pi\zeta}{\sqrt{1-\zeta^2}}$, $\underline{\zeta = 0.1097}$

8/38 $\frac{x_8}{x_{20}} = \frac{e^{-\zeta\omega_n t_8}}{e^{-\zeta\omega_n(t_8+12\tau_d)}} = e^{\zeta\omega_n(12\tau_d)}$

Then $\ln\frac{x_8}{x_{20}} = \zeta\omega_n(12\tau_d)$

But $\omega_n\tau_d = \frac{2\pi}{\sqrt{1-\zeta^2}}$

So $\ln(16) = 12\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$, $\underline{\zeta = 0.0367}$

8/39 Combined $c = 2m\omega_n \zeta$ where $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3 \times 474 \times 10^3}{4000}}$

$= 18.85$ rad/s

and $\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$

But the logarithmic decrement $\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln 4 = 1.386$

so the viscous damping factor $\zeta = 1.386/\sqrt{(2\pi)^2 + 1.386^2}$

$= 0.215$

Thus combined $c = 2(4000)(18.85)(0.215) = 32.5 \times 10^3$

& for each damper $c = 32.5 \times 10^3/2 = \underline{16.24 \times 10^3 \text{ N·s/m}}$

8/40

$$\omega_n = \sqrt{k/m} = \sqrt{98/2} = 7 \text{ rad/s}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{42}{2(2)(7)} = 1.5$$

$$s_{1,2} = \omega_n\left[-\zeta \pm \sqrt{\zeta^2-1}\right]$$

$$= 7\left[-1.5 \pm \sqrt{1.5^2-1}\right] = \begin{cases} -2.674 \\ -18.326 \end{cases}$$

So $x = A_1 e^{-2.674t} + A_2 e^{-18.326t}$

At $t=0$: $x_0 = A_1 + A_2$ (a)

$\dot{x} = -2.674 A_1 e^{-2.674t} - 18.326 A_2 e^{-18.326t}$

At $t=0$: $0 = -2.674 A_1 - 18.326 A_2$ (b)

Solve (a) and (b) for A_1 and A_2.

Then $x = x_0\left[1.171 e^{-2.67t} - 0.1708 e^{-18.33t}\right]$

$\boxed{8/41}$ $\omega_n = \sqrt{k/m} = \sqrt{108/3} = 6 \text{ rad/s}$

$$\zeta = \frac{c}{2m\omega_n} = \frac{18}{2(3)(6)} = 0.5$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6\sqrt{1-0.5^2} = 5.196 \text{ rad/s}$$

$$x = (A_1 \cos\omega_d t + A_2 \sin\omega_d t)\, e^{-\zeta\omega_n t}$$

$$x(t=0) = A_1 = x_0$$

$$\dot{x} = -\zeta\omega_n (A_1 \cos\omega_d t + A_2 \sin\omega_d t)\, e^{-\zeta\omega_n t}$$
$$+ \omega_d(-A_1 \sin\omega_d t + A_2 \cos\omega_d t)\, e^{-\zeta\omega_n t}$$

$$\dot{x}(t=0) = -\zeta\omega_n A_1 + A_2\omega_d = 0$$

$$A_2 = \zeta\omega_n A_1/\omega_d = 0.5(6)x_0/5.196 = 0.577\,x_0$$

So $x = x_0[\cos 5.196t + 0.577 \sin 5.196t]\,e^{-3t}$

and $x(t = \frac{\tau_d}{2}) = x(t = 0.605) = \underline{-0.1630\,x_0}$

8/42 $x = (A_1 + A_2 t) e^{-\omega_n t}$

$x(t=0) = A_1 = x_0$

$\dot{x} = A_2 e^{-\omega_n t} - \omega_n (A_1 + A_2 t) e^{-\omega_n t}$

$\dot{x}(t=0) = A_2 - \omega_n A_1 = \dot{x}_0$

$A_2 = \dot{x}_0 + \omega_n x_0$

So $x = [x_0 + (\dot{x}_0 + \omega_n x_0) t] e^{-\omega_n t}$

For x to become negative with $x_0 > 0$,

$\dot{x}_0 + \omega_n x_0 < 0$, $\dot{x}_0 < -\omega_n x_0$ or $\underline{(\dot{x}_0)_c = -\omega_n x_0}$

8/43 $\omega_n = \sqrt{k/m} = \sqrt{\dfrac{12}{96.6/32.2}} = 2$ rad/sec

(a) $\zeta = \dfrac{c}{2m\omega_n} = \dfrac{12}{2(3)(2)} = \underline{1}$ (Critical damping)

$x = (A_1 + A_2 t)e^{-\omega_n t}$

Consideration of initial conditions yields

$x = (0.5 + t)\,e^{-2t}$, $x(t = 0.5) = 0.368$ ft $\underline{(4.42 \text{ in.})}$

(b) $\zeta = \dfrac{c}{2m\omega_n} = \dfrac{18}{2(3)(2)} = 1.5$ (Overdamped)

$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

Determine A_1 and A_2 in usual fashion:

$x = \left(\dfrac{\lambda_2 x_0}{\lambda_2 - \lambda_1}\right)e^{\lambda_1 t} + \left(\dfrac{\lambda_1 x_0}{\lambda_1 - \lambda_2}\right)e^{\lambda_2 t}$, where

$\lambda_{1,2} = \omega_n\left[-\zeta \pm \sqrt{\zeta^2 - 1}\right] = -0.7639, -5.236$

$x = 0.585\,e^{-0.764t} - 0.085\,e^{-5.24t}$

$x(t = 0.5) = \underline{0.393 \text{ ft } (4.72 \text{ in.})}$

8/44

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} \text{ where } \delta = \ln\left(\frac{x_1}{x_2}\right) = \ln\frac{3}{1/2} = 1.792$$

$$\zeta = \frac{1.792}{\sqrt{(2\pi)^2 + 1.792^2}} = \underline{0.274}$$

$c = 2m\omega_n\zeta$ where equivalent m for each absorber is

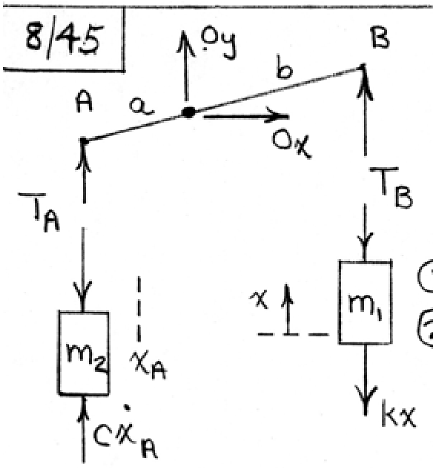
$$\frac{1}{2}\left(\frac{1}{2}\,\frac{3400}{32.2}\right) = 26.4 \text{ lb-sec}^2/\text{ft}$$

$$k = F/\delta_{st} = 100/\tfrac{3}{12} = 400 \text{ lb/ft (for both)}$$

$$\omega_n = \sqrt{k/m} = \sqrt{400/52.8} = 2.75 \text{ rad/sec}$$

so for each shock, $c = 2(26.4)(2.75)(0.274) = \underline{39.9 \text{ lb-sec/ft}}$

8/45



$\frac{x}{b} = \frac{x_A}{a}$, $\dot{x}_A = \left(\frac{a}{b}\right)\dot{x}$

$\ddot{x}_A = \left(\frac{a}{b}\right)\ddot{x}$

$\Sigma F_x = m\ddot{x}$:

(1) $-kx - T_B = m_1\ddot{x}$

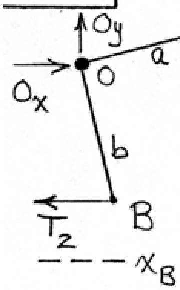
(2) $T_A - c\dot{x}_A = m_2\ddot{x}_A$

$\Sigma M_O = 0$: $T_B b - T_A a = 0$

Elimination of T_B from Eq. (1) yields

$$\left[m_1 + \frac{a^2}{b^2}m_2\right]\ddot{x} + \frac{a^2}{b^2}c\dot{x} + kx = 0$$

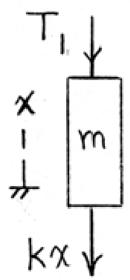
8/46



$$\frac{x}{a} = \frac{x_B}{b}, \quad T_2 = c\dot{x}_B = c\,\frac{b}{a}\dot{x}$$

$$\circlearrowleft \Sigma M_O = 0: \quad T_1 a - T_2 b = 0$$

$$T_1 = \frac{b}{a} T_2 = \frac{b}{a}\left(c\,\frac{b}{a}\dot{x}\right) = c\,\frac{b^2}{a^2}\dot{x}$$



$$\Sigma F_x = m\ddot{x}: \; -T_1 - kx = m\ddot{x}$$

$$-c\,\frac{b^2}{a^2}\dot{x} - kx = m\ddot{x}$$

$$\underline{\ddot{x} + \frac{b^2}{a^2}\frac{c}{m}\dot{x} + \frac{k}{m}x = 0}$$

$$2\zeta\omega_n = \frac{b^2}{a^2}\frac{c}{m}$$

$$\zeta = \frac{b^2}{a^2}\frac{c}{m}\frac{1}{2\sqrt{\frac{k}{m}}} = \underline{\frac{1}{2}\frac{b^2}{a^2}\frac{c}{\sqrt{km}}}$$

8/47 With negligible damping,

$$\frac{X}{F_o/k} = \frac{1}{\left|1-(\omega/\omega_n)^2\right|} \quad \begin{cases} \omega_n^2 = \dfrac{k}{m} = \dfrac{k}{24} \\ X = 0.30 \text{ mm} \\ F_o/k = \delta_{st} = 0.60 \text{ mm} \\ \omega = 2\pi(4) = 8\pi \text{ rad/s} \end{cases}$$

For $\dfrac{X}{F_o/k} = M = \dfrac{1}{2} < 1$, $1-\left(\dfrac{\omega}{\omega_n}\right)^2$ is negative.

Thus $-\dfrac{0.30}{0.60} = \dfrac{1}{1-(8\pi)^2/(k/24)}$, $\underline{k = 5050 \text{ N/m}}$

8/48 $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100,000}{10}} = 100$ rad/s

$\zeta = \frac{c}{2m\omega_n} = \frac{500}{2(10)(100)} = 0.25$ for (a)

$$X = \frac{F_0/k}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}} = \frac{1000/100,000}{\left\{[1-1.2^2]^2+[2(0.25)(1.2)]^2\right\}}$$

$$= \frac{1000/100\,000}{\left\{[1-1.2^2]^2+[2(0.25)(1.2)]^2\right\}^{1/2}} = \underline{1.344(10^{-2})\text{ m}}$$

(b) With $\zeta = 0$, $\underline{X = 2.27(10^{-2})\text{ m}}$

8/49

$$(M)_{\frac{\omega}{\omega_n}=1} = 8(M)_{\frac{\omega}{\omega_n}=2}$$

$$\frac{1}{\{[1-1^2]^2+[2\zeta(1)]^2\}^{1/2}} = \frac{8}{\{[1-2^2]^2+[2\zeta(2)]^2\}^{1/2}}$$

Square both sides and solve for ζ to obtain $\underline{\zeta = 0.1936}$

8/50 $\omega_n = \sqrt{k/m} = \sqrt{6(12)\Big/\frac{64.4}{32.2}} = 6$ rad/sec

$$X = \frac{F_o/k}{1-(\omega/\omega_n)^2} = \frac{5/6(12)}{1-\frac{\omega^2}{6^2}}$$

Because $|X| < \frac{3}{12}$, we set $X < \frac{3}{12}$ & $X > -\frac{3}{12}$

and obtain $\underline{\omega > 6.78 \text{ rad/sec}}$ & $\underline{\omega < 5.10 \text{ rad/sec}}$

8/51 $\omega_n = 6$ rad/sec (from Prob. 8/50)

$$X = \frac{F_0/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$\boxed{\zeta = \frac{c}{2m\omega_n} = \frac{2.4}{2(2)6} = 0.1}$$

$$= \frac{5/6(12)}{\sqrt{\left[1-\left(\frac{\omega}{6}\right)^2\right]^2 + \left[2(0.1)\frac{\omega}{6}\right]^2}} < \frac{3}{12}$$

Square both sides to obtain a quadratic in ω^2. Solution: $\omega < 5.32$ rad/sec

$\underline{\omega > 6.50 \text{ rad/sec}}$

As expected, damping allows a wider range of ω than when $\zeta = 0$ (Prob. 8/50).

8/52 $\omega_n = 6$ rad/sec (from Prob. 8/50)

$$X = \frac{F_o/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{5/6(12)}{\sqrt{\left[1-\left(\frac{6}{6}\right)^2\right]^2 + \left[2\zeta\frac{6}{6}\right]^2}} = \frac{3}{12}$$

$\zeta = 0.1389$

$\zeta = \frac{c}{2m\omega_n}$, $c = 2\zeta m\omega_n = 2(0.1389)(2)(6)$

$= \underline{3.33 \text{ lb-sec/ft}}$

8/53 $\omega_n = \sqrt{2k/m} = \sqrt{\dfrac{2(6)(12)}{4/32.2}} = 34.0$ rad/sec

Steady-state amplitude is

$$X = \frac{b}{1-\left(\frac{\omega}{\omega_n}\right)^2}, \text{ where } X = \tfrac{1}{2}(10-8) = 1 \text{ in.}$$

So $1 = \dfrac{0.5}{1-(\omega/34.0)^2}$, $\omega = 24.1$ rad/sec

Shaker frequency $f = \dfrac{\omega}{2\pi} = \dfrac{24.1}{2\pi} = \underline{3.83 \text{ Hz}}$

8/54 $\omega_n = \sqrt{2k/m} = \sqrt{\frac{2(200)}{100/32.2}} = 11.35$ rad/sec

(a) $\zeta = 0$: $X = \left| \frac{F_o/k_{eff}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right|$

$$= \left| \frac{75/(2 \cdot 200)}{1 - \left(\frac{15}{11.35}\right)^2} \right| = \underline{0.251 \text{ ft}}$$

(b) $\zeta \neq 0$: $\zeta = \frac{c}{2m\omega_n} = \frac{60}{2\left(\frac{100}{32.2}\right)(11.35)}$

$$\zeta = 0.851$$

$$X = \frac{F_o/k_{eff}}{\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

$$= \frac{75/(2 \cdot 200)}{\left\{\left[1 - \left(\frac{15}{11.35}\right)^2\right]^2 + \left[2(0.851)\left(\frac{15}{11.35}\right)\right]^2\right\}^{1/2}} = \underline{0.0791 \text{ ft}}$$

$$\delta_{st} = \frac{W}{k_{eff}} = \frac{100}{2(200)} = \underline{0.25 \text{ ft}}$$

8/55

$$M = \frac{1}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

$$M_1 = \frac{1}{\{[1-1^2]^2 + [2(0.1)(1)]^2\}^{1/2}} = 5$$

$$M_1' = \frac{1}{\{[1-1^2]^2 + [2(0.2)(1)]^2\}^{1/2}} = 2.5$$

$$R_1 = \frac{M_1 - M_1'}{M_1}(100) = \underline{50\%}$$

$$M_2 = \frac{1}{\{[1-2^2]^2 + [2(0.1)(2)]^2\}^{1/2}} = 0.3304$$

$$M_2' = \frac{1}{\{[1-2^2]^2 + [2(0.2)(2)]^2\}^{1/2}} = 0.3221$$

$$R_2 = \frac{M_2 - M_2'}{M_2}(100) = \underline{2.52\%}$$

8/56 $\omega_n = \sqrt{\frac{k}{m}}$, $\zeta = 1 = \frac{c}{2m\omega_n}$

$$c = 2m\omega_n = 2m\sqrt{\frac{k}{m}} = 2 \times 90\sqrt{\frac{4 \times 30 \times 10^6}{90}} = 180 \times 1155 = \underline{208 \times 10^3\ \text{N}\cdot\text{s/m}}$$

$$\omega = \frac{1}{2}\left(3600 \times \frac{2\pi}{60}\right) = 188.5\ \text{rad/s}, \quad \omega/\omega_n = \frac{188.5}{1155} = 0.1632$$

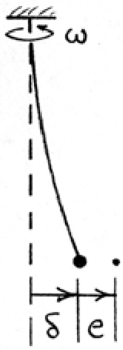
Eq. 8/23 with $\zeta = 1$ becomes $M = \frac{1}{1 + (\omega/\omega_n)^2} = \frac{1}{1 + 0.1632^2} = \underline{0.974}$

8/57 The condition for the maxima is

$$\frac{dM}{d\left(\frac{\omega}{\omega_n}\right)} = \frac{d}{d\left(\frac{\omega}{\omega_n}\right)}\left[\frac{1}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}\right] = 0$$

Differentiate to obtain $\frac{\omega}{\omega_n} = \underline{\sqrt{1-2\zeta^2}}$

8/58



$F = m\bar{a}_n: \quad k\delta = m(\delta + e)\omega^2$

$$\delta = \frac{\frac{m}{k} e\omega^2}{1 - \frac{m}{k}\omega^2}$$

$$= \frac{e(\omega/\omega_n)^2}{1 - (\omega/\omega_n)^2}$$

$\omega_c = \sqrt{k/m}$

8/59 Equivalent spring constant

$k = P/\delta_{st} = \frac{2}{4\times10^{-3}} = 500\ N/m$

$y_0 = \frac{b}{1-(\omega/\omega_n)^2}$ where $\omega_n^2 = \frac{k}{m} = \frac{500}{0.5} = 1000\ (\frac{rad}{s})^2$

& $\omega^2 = (4\times 2\pi)^2 = 632\ (\frac{rad}{s})^2$

Thus $y_0 = \frac{3}{1-\frac{632}{1000}} = \underline{8.15\ mm}$

8/60 $x_i = x_B + x$, $x = x_i - x_B$

From text: $-c\dot{x} - kx = m\frac{d^2}{dt^2}(x + x_B)$

$-c(\dot{x}_i - \dot{x}_B) - k(x_i - x_B) = m\ddot{x}_i$

$\ddot{x} + \frac{c}{m}\dot{x}_i + \frac{k}{m}x_i = \frac{k}{m}x_B + \frac{c}{m}\dot{x}_B$

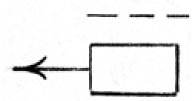
$x_B = b\sin\omega t$, $\dot{x}_B = b\omega\cos\omega t$

So $\underline{\ddot{x}_i + 2\zeta\omega_n\dot{x}_i + \omega_n^2 x_i = \frac{k}{m}b\sin\omega t + \frac{c}{m}b\omega\cos\omega t}$

With 2 forcing terms, we must find the particular solution corresponding to each term and then add (legal for a linear system). Alternatively, we could first combine the two forcing terms into one.

8/61 Let x_m be the absolute cart displacement.

Then $x_m = x_B + x$, x = mass relative displacement $= (x_m - x_B)$



$k(x_m - x_B) = kx$

$\Sigma F_x = ma_x: \; -k(x_m - x_B) = m(\ddot{x}_B + \ddot{x})$

$\ddot{x} + \frac{k}{m} x = b\omega^2 \sin \omega t$

Assume $x = X \sin \omega t$ & obtain $X = \dfrac{b(\omega/\omega_n)^2}{1 - (\omega/\omega_n)^2}$

Requirement : $|X| < 2b$

So $\dfrac{b(\omega/\omega_n)^2}{1 - (\omega/\omega_n)^2} < 2b$ and $\dfrac{b(\omega/\omega_n)^2}{1 - (\omega/\omega_n)^2} > -2b$

$\Downarrow$ $\Downarrow$

$\underline{\dfrac{\omega}{\omega_n} < \sqrt{2/3}}$ and $\underline{\dfrac{\omega}{\omega_n} > \sqrt{2}}$

See Fig. 8/14.

$\boxed{8/62}$ $\omega_n = \sqrt{k/m} = \sqrt{2(2.1)(10^3)/20} = 14.49$ rad/s

$$\zeta = \frac{c}{2m\omega_n} = \frac{2(58)}{2(20)(14.49)} = 0.200$$

$$M^2 = \frac{1}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2} \; ; \text{ let } r = \frac{\omega}{\omega_n} = \frac{\omega}{14.49}$$

$$2^2 = \frac{1}{[1-r^2]^2 + 4(0.200)^2 r^2}$$

$r^4 - 1.84r^2 + 0.75 = 0$, $r^2 = 0.610$ or $r^2 = 1.230$

Thus $\frac{\omega}{14.49} = r = \sqrt{0.610}$, $\omega = 11.32$ rad/s

or 108.1 rev/min

or $\frac{\omega}{14.49} = r = \sqrt{1.230}$, $\omega = 16.07$ rad/s

or 153.5 rev/min

Summary : $\underline{N \leq 108.1 \; \frac{\text{rev}}{\text{min}} \text{ or } N \geq 153.5 \; \frac{\text{rev}}{\text{min}}}$

8/63 $W = k_{eq}\delta_{st}$; $k_{eq} = \frac{W}{\delta_{st}} = \frac{mg}{\delta_{st}}$

$\omega_n = \sqrt{k_{eq}/m} = \sqrt{\frac{mg/\delta_{st}}{m}} = \sqrt{g/\delta_{st}}$

For maximum response, $\omega = \omega_n = \sqrt{g/\delta_{st}}$,

and $f = \frac{\omega}{2\pi} = \underline{\frac{1}{2\pi}\sqrt{\frac{g}{\delta_{st}}}}$

8/64 $\frac{X}{b} = \left| \frac{1}{1-(\omega/\omega_n)^2} \right| = \frac{0.15}{0.10} = 1.5$

$$\left.\begin{array}{ll} \text{For } \omega < \omega_n, & \frac{\omega}{\omega_n} = 0.577 \\ \text{For } \omega > \omega_n, & \frac{\omega}{\omega_n} = 1.291 \end{array}\right\}$$

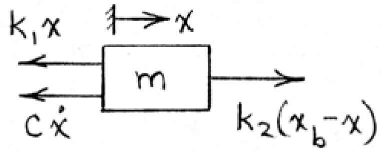
$$\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4(7200)}{43}} = 25.9 \text{ rad/s}$$

For $\omega < \omega_n$, $\omega = 0.577(25.9) = 14.94$ rad/s

For $\omega > \omega_n$, $\omega = 1.291(25.9) = 33.4$ rad/s

Thus prohibited range is $\underline{2.38 < f_n < 5.32 \text{ Hz}}$

8/65



$\Sigma F_x = m\ddot{x}$:

$$-k_1 x - c\dot{x} + k_2(x_b - x) = m\ddot{x}$$

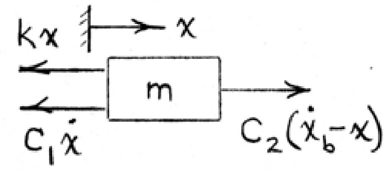
$$m\ddot{x} + c\dot{x} + (k_1 + k_2)x = k_2 x_b$$

$$= k_2 b \cos \omega t$$

$$\omega_c = \sqrt{\frac{k_1 + k_2}{m}}$$

(It is assumed that the damping is light so that the forced response is a maximum at $(\omega/\omega_n) \cong 1$.)

8/66



$$\Sigma F_x = m\ddot{x}:$$

$$-kx - c_1\dot{x} + c_2(\dot{x}_b - x) = m\ddot{x}$$

$$\underline{m\ddot{x} + (c_1 + c_2)\dot{x} + kx =} \; c_2\dot{x}_b$$

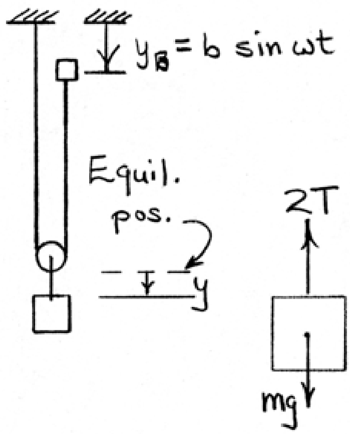
$$= \underline{-c_2 b\,\omega \sin \omega t}$$

$$\underline{\omega_c = \sqrt{\frac{k}{m}}}$$

$$2\zeta\omega_n = \frac{c_1 + c_2}{m}, \quad \underline{\zeta = \frac{c_1 + c_2}{2\sqrt{km}}}$$

(See assumption in solution of Prob. 8/65)

8/67



In equilibrium position, the spring tension is $T_0 = \frac{1}{2}mg$

In displaced position, spring is stretched $2y - y_B$, so spring force is

$$T = \tfrac{1}{2}mg + k(2y - y_B)$$

For $\Sigma F_y = m\ddot{y}$ on m:

$$mg - 2\left[\tfrac{1}{2}mg + k(2y - y_B)\right] = m\ddot{y}$$

or $$\underline{\ddot{y} + \frac{4k}{m}y = \frac{2k}{m}b\sin\omega t}$$

For particular solution, assume $y = Y\sin\omega t$ and obtain $Y = \dfrac{b/2}{1-(\omega/\omega_n)^2}$ where $\omega_n = 2\sqrt{k/m}$.

Thus $\underline{\omega_c = 2\sqrt{k/m}}$

8/68 The maximum value of the force transmitted to the base, from Sample Problem 8/6, is $(F_{tr})_{max} = X\sqrt{k^2 + c^2\omega^2}$

$$= (F_0/k)M\sqrt{k^2 + (4\zeta^2 m^2 \omega_n^2)\omega^2}$$

$$= (F_0/k)M\sqrt{k^2 + \frac{4\zeta^2 m^2 \omega_n^2 \omega^2}{1} \cdot \frac{k^2/m^2}{\omega_n^4}}$$

$$= (F_0/k)Mk\sqrt{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}$$

$$= MF_0\sqrt{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}$$

Then transmission ratio T is

$$T = \frac{(F_{tr})_{max}}{F_0} = \underline{M\sqrt{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}$$

(M = magnification factor)

8/69

$$F_o = 2m_o e\omega^2 = 2(1)(0.012)\left(1800\,\frac{2\pi}{60}\right)^2$$
$$= 853 \text{ N}$$

Force transmitted $kX = 1500$ N

But $kX = \left|\dfrac{F_o}{1-(\omega/\omega_n)^2}\right|$ or $1500 = \left|\dfrac{853}{1-(\omega/\omega_n)^2}\right|$

Solving, $\left(\dfrac{\omega}{\omega_n}\right)^2 = 1.568$ or 0.432

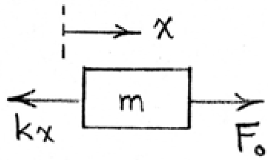
With $\omega_n^2 = k/m$, we obtain

$$k = \frac{m\omega^2}{1.568} \quad \text{or} \quad k = \frac{m\omega^2}{0.432}$$

With $m = 10$ kg and $\omega = 1800\left(\frac{2\pi}{60}\right)$, we obtain

$k = \underline{227 \text{ kN/m}}$ or $\underline{823 \text{ kN/m}}$

8/70



$$\Sigma F = m\ddot{x} : \quad -kx + F_o = m\ddot{x}$$

$$\ddot{x} + \frac{k}{m}x = \frac{F_o}{m}$$

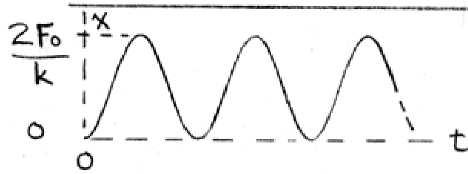
$$x = x_c + x_p$$

or $$x = \left(A_1 \cos\omega_n t + A_2 \sin\omega_n t\right) + \frac{F_o}{k}$$

$$x(t=0) = A_1 + \frac{F_o}{k} = 0, \quad A_1 = -\frac{F_o}{k}$$

$$\dot{x}(t=0) = A_2\omega_n = 0, \quad A_2 = 0$$

So $$x = \frac{F_o}{k}\left(1 - \cos\omega_n t\right)$$



8/71 For steady-state motion,

$$\frac{X}{b} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$\frac{X}{b} = \frac{24}{18} = \frac{4}{3}\ ; \quad \omega_n = \sqrt{k/m} = \sqrt{\frac{1500}{2}} = 27.4\ \frac{\text{rad}}{\text{s}}$$

$$\omega = 5(2\pi) = 31.4\ \text{rad/s} \Rightarrow \frac{\omega}{\omega_n} = 1.147,\ \left(\frac{\omega}{\omega_n}\right)^2 = 1.316$$

So $$\left(\frac{4}{3}\right)^2 = \frac{1.316^2}{[1-1.316]^2+[4(1.316)\zeta^2]}\ , \quad \zeta = 0.408$$

From $\zeta = \frac{c}{2m\omega_n}$, $c = 2\zeta m\omega_n = 2(0.408)(2)(27.4)$

$$= \underline{44.6\ \text{N}\cdot\text{s/m}}$$

▸8/72 Take road contour to be $x = b \sin \omega t$

Wavelength $L = v\tau = v \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi v}{L}$

Thus $x = b \sin \frac{2\pi v}{L} t$.

$X = \left| \frac{b}{1-(\omega/\omega_n)^2} \right|$. Set $b = 25$ mm

$\omega = \frac{2\pi\left(\frac{25}{3.6}\right)}{1.2} = 36.4$ rad/s, and $\omega_n = \sqrt{k/m}$

$= \left(\frac{75(9.81)/0.003}{500}\right)^{1/2} = 22.1$ rad/s to

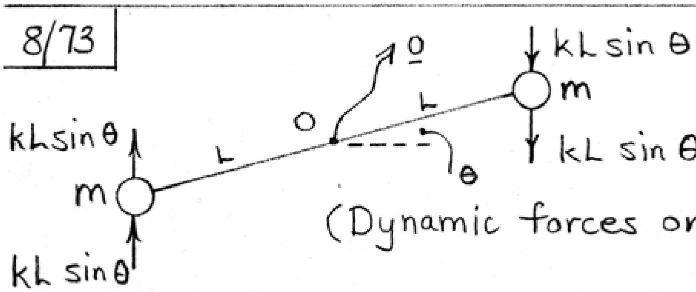
obtain $\underline{X = 14.75 \text{ mm}}$

Critical speed: $\omega_c = \omega_n$

$\frac{2\pi v_c}{L} = \sqrt{k/m} = 22.1$

$v_c = 4.23$ m/s or $\underline{15.23 \frac{km}{h}}$

8/73



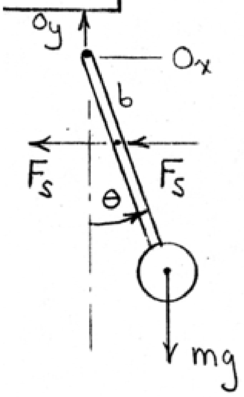
$$\curvearrowleft + \Sigma M_o = I_o \ddot{\theta}: \quad -4kL\sin\theta(L\cos\theta) = 2mL^2\ddot{\theta}$$

For small θ: $\ddot{\theta} + \frac{2k}{m}\theta = 0$

$$\underline{\omega_n = \sqrt{\frac{2k}{m}}}, \quad \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{2k}}$$

$$= \underline{\pi\sqrt{\frac{2m}{k}}}$$

8/74



Each spring force is $F_s = kb\sin\theta$

(Springs are assumed to remain horizontal)

$$\Sigma M_O = I_O\ddot{\theta}: -mgl\sin\theta - 2F_s\, b\cos\theta = ml^2\ddot{\theta}$$

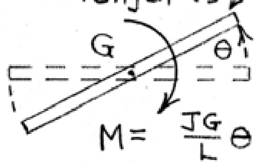
Simplify to obtain (for small θ)

$$\ddot{\theta} + \left(\frac{g}{l} + \frac{2kb^2}{ml^2}\right)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\Big/\sqrt{\frac{g}{l} + \frac{2kb^2}{ml^2}}$$

8/75

mass m, length ℓ



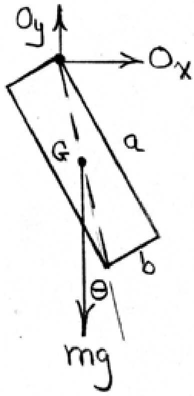
$\curvearrowleft + \Sigma M_G = \bar{I}\ddot{\theta} : -\frac{JG}{L}\theta = \frac{1}{12}m\ell^2\ddot{\theta}$

$$\ddot{\theta} + \left(\frac{12JG}{m\ell^2 L}\right)\theta = 0$$

$$\omega_n = \left(\frac{12JG}{m\ell^2 L}\right)^{1/2}$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\left(\frac{m\ell^2 L}{12JG}\right)^{1/2}}$$

8/76



$$\sum M_O = I_O \ddot{\theta} : -mg\,\frac{\sqrt{a^2+b^2}}{2}\sin\theta$$

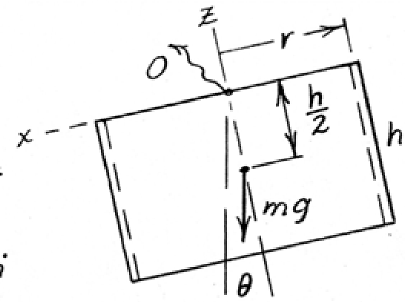
$$= \left[\frac{1}{12} m\left(a^2+b^2\right) + m\left(\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2\right)\right]\ddot{\theta}$$

For small θ:

$$\ddot{\theta} + \frac{\frac{3}{2}g}{\sqrt{a^2+b^2}}\,\theta = 0$$

$$\omega_n = \frac{\sqrt{\frac{3g}{2}}}{4\sqrt{a^2+b^2}}$$

8/77



$\Sigma M_O = I_O \alpha$: $I_O = I_{yy}$

$= \frac{1}{2}mr^2 + \frac{1}{3}mh^2$

from Table D/4

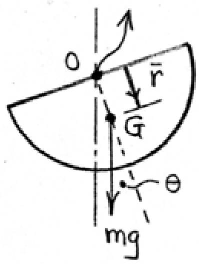
$-mg\frac{h}{2}\sin\theta = \left(\frac{1}{2}mr^2 + \frac{1}{3}mh^2\right)\ddot{\theta}$

For small angles $\sin\theta \approx \theta$

$$\ddot{\theta} + \frac{gh}{2}\frac{1}{\frac{r^2}{2}+\frac{h^2}{3}}\theta = 0, \quad \underline{\omega_n = \sqrt{\frac{gh}{2}}\Big/\sqrt{\frac{r^2}{2}+\frac{h^2}{3}}}$$

8/78

$\bar{r} = \frac{4r}{3\pi}, \quad I_O = \frac{1}{2}mr^2$



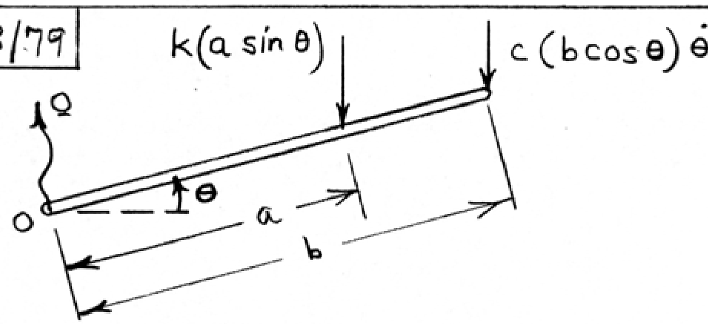
$\circlearrowleft + \sum M = I_O\ddot{\theta}: \ -mg\frac{4r}{3\pi}\sin\theta = \frac{1}{2}mr^2\ddot{\theta}$

For small θ, $\sin\theta \cong \theta$, so

$$\ddot{\theta} + \frac{8g}{3\pi r}\theta = 0$$

$\omega_n = \sqrt{\frac{8g}{3\pi r}}, \quad f_n = \frac{\omega_n}{2\pi} = \underline{\frac{1}{\pi}\sqrt{\frac{2g}{3\pi r}}}$

8/79



$$\stackrel{+}{\curvearrowleft} \Sigma M_O = I_O\ddot{\theta} : \quad -(ka\sin\theta)a\cos\theta - cb\cos\theta\,\dot{\theta}\,(b\cos\theta) = \frac{1}{3}mb^2\ddot{\theta}$$

Small θ: $\quad \ddot{\theta} + \frac{3c}{m}\dot{\theta} + \frac{3a^2}{b^2}\frac{k}{m}\theta = 0$

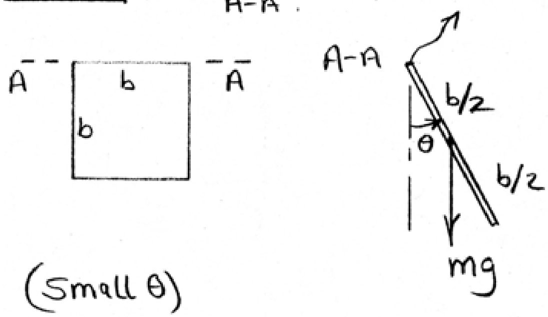
$$\omega_n = \sqrt{\frac{3a^2k}{b^2m}} \; ; \quad 2\zeta\omega_n = \frac{3c}{m} \, , \quad \zeta = \frac{3c}{2m\omega_n}$$

or $\quad \zeta = \frac{3c}{2m}\sqrt{\frac{b^2m}{3a^2k}} = \underline{\frac{cb}{2a}\sqrt{\frac{3}{km}}}$

For $\zeta = 1$, $\quad \underline{c_{cr} = \frac{2a}{b}\sqrt{\frac{km}{3}}}$

8/80

$$I_{A-A} = \frac{1}{12} m b^2 + m\left(\frac{b}{2}\right)^2 = \frac{1}{3} m b^2$$



(Small θ)

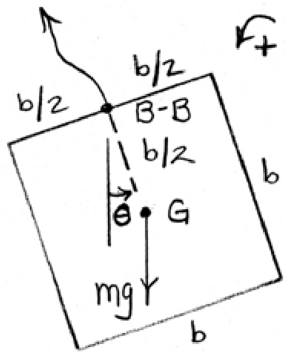
$$\sum M_{A-A} = I_{A-A}\ddot{\theta}: \quad -mg\frac{b}{2}\theta = \frac{1}{3} m b^2 \ddot{\theta}$$

$$\ddot{\theta} + \frac{3g}{2b}\theta = 0, \quad \omega_n = \sqrt{\frac{3g}{2b}}$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{2b}{3g}}}$$

8/81

$$I_{B-B} = \frac{1}{6}mb^2 + m\left(\frac{b}{2}\right)^2 = \frac{5}{12}mb^2$$



$$\circlearrowleft + \Sigma M_{B-B} = I_{B-B}\ddot{\theta} : \quad (\text{small } \theta)$$

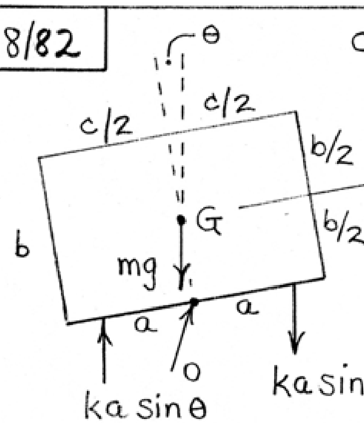
$$-mg\frac{b}{2}\theta = \frac{5}{12}mb^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{6}{5}\frac{g}{b}\theta = 0$$

$$\omega_n = \sqrt{\frac{6g}{5b}}, \quad \tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{5b}{6g}}}$$

(11.8 % higher than result of Prob. 8/80)

8/82



$c = 0.9\text{ m},\quad b = 0.6\text{ m},\quad a = 0.375\text{ m}$

$m = 250\text{ kg}$

$\circlearrowleft + \Sigma M_O = I_O\ddot{\theta}:\quad mg\frac{b}{2}\sin\theta$

$- 2ka\sin\theta\,(a\cos\theta) = I_O\ddot{\theta}$

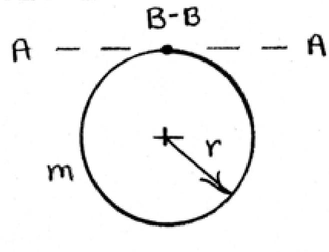
For small θ:

$$\ddot{\theta} + \frac{2ka^2 - mg\frac{b}{2}}{I_O}\,\theta = 0$$

For harmonic oscillation, coefficient of θ must be positive.

Thus $k_{min} = \dfrac{mgb}{4a^2} = \dfrac{250(9.81)(0.6)}{4(0.375)^2} = \underline{2620\text{ N/m}}$

8/83



$$I_{A-A} = \tfrac{1}{2}mr^2 + mr^2 = \tfrac{3}{2}mr^2$$

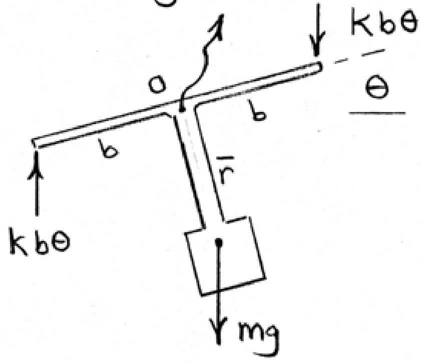
$$I_{B-B} = mr^2 + mr^2 = 2mr^2$$

Desired ratio $R = \dfrac{\tau_{B-B}}{\tau_{A-A}}$

or $R = \dfrac{\omega_{A-A}}{\omega_{B-B}}$

Each natural frequency is proportional to $\dfrac{1}{\sqrt{I}}$, so $R = \dfrac{1/\sqrt{I_{A-A}}}{1/\sqrt{I_{B-B}}} = \dfrac{\sqrt{2mr^2}}{\sqrt{\frac{3}{2}mr^2}} = \underline{\dfrac{2}{\sqrt{3}}}$

8/84 Dynamic forces, small θ:



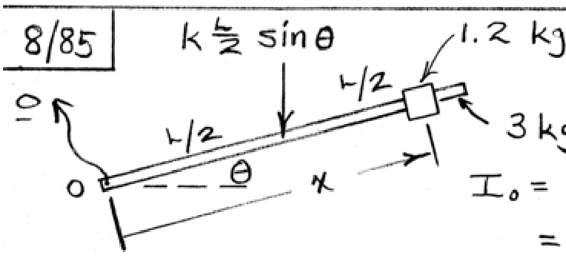
$$\circlearrowleft + \ \Sigma M_O = I_O \ddot{\theta} \ : \ -2kb\theta(b) - mg\bar{r}\theta = mk_o^2\ddot{\theta}$$

$$\ddot{\theta} + \left[\frac{2kb^2 + mg\bar{r}}{mk_o^2}\right]\theta = 0$$

$$\omega_n = \frac{1}{k_o}\sqrt{\frac{2kb^2}{m} + g\bar{r}}$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi k_o \Big/ \sqrt{\frac{2kb^2}{m} + g\bar{r}}}$$

8/85



Let L be the 0.8-m rod length

$I_o = \frac{1}{3}(3)L^2 + 1.2x^2$
$= L^2 + 1.2x^2$

$\curvearrowleft \Sigma M_O = I_O\ddot{\theta}: -k\frac{L}{2}\sin\theta\left(\frac{L}{2}\cos\theta\right) = (L^2 + 1.2x^2)\ddot{\theta}$

$\Rightarrow \quad \ddot{\theta} + \dfrac{kL^2}{4(L^2+1.2x^2)}\theta = 0$

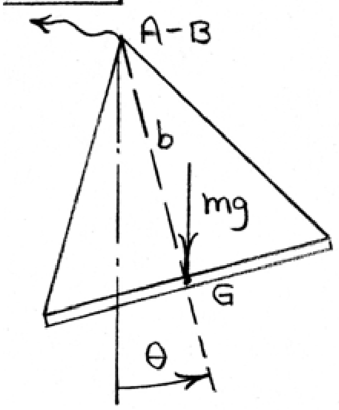
$\tau = \dfrac{2\pi}{\omega_n} = 2\pi\sqrt{\dfrac{4(L^2+1.2x^2)}{kL^2}} = 4\pi\sqrt{\dfrac{L^2+1.2x^2}{kL^2}}$

So $\quad 4\pi\sqrt{\dfrac{0.8^2+1.2x^2}{250\,(0.8)^2}} = 1, \quad \underline{x = 0.558\text{ m}}$

8/86

$I_{A-B} = I_G + mb^2$

$= \frac{1}{12}mb^2 + mb^2 = \frac{13}{12}mb^2$



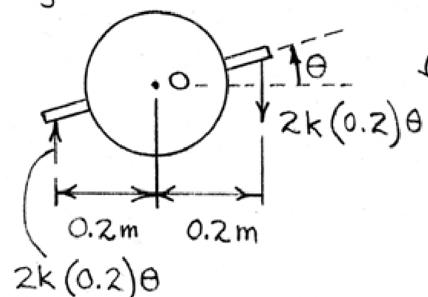
$\sum M_{A-B} = I_{A-B}\ddot{\theta}: \; -mgb\sin\theta = \frac{13}{12}mb^2\ddot{\theta}$

Small θ: $\ddot{\theta} + \frac{12g}{13b}\theta = 0$

$\omega_n = \sqrt{\frac{12g}{13b}}$, $\tau = \frac{2\pi}{\omega_n} = \frac{1}{\pi}\sqrt{\frac{3g}{13b}}$

8/87

Dynamic forces:



$$I_O = mk_O^2 = 43(0.1)^2$$

$$= 0.43 \ \text{kg}\cdot\text{m}^2$$

$$\circlearrowleft\!+ \ \Sigma M_O = I_O\ddot{\theta} : \ -4k(0.2)^2\theta$$

$$= 0.43\,\ddot{\theta}$$

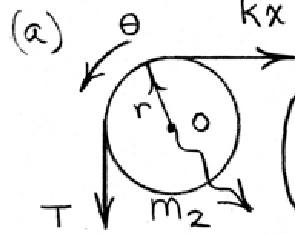
$$\ddot{\theta} + \frac{0.16\,k}{0.43}\,\theta = 0$$

$$\omega_n = \left(\frac{0.16k}{0.43}\right)^{1/2} = 360\left(\frac{2\pi}{60}\right)$$

$$\underline{k = 3820 \ \text{N/m}}$$

8/88

(a)



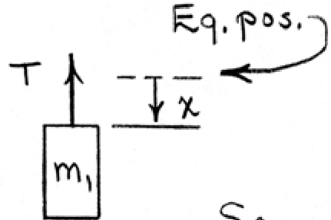
(Dynamic forces only)

$\circlearrowleft + \sum M_O = I_O\ddot{\theta} : \; r(T - kx) = \frac{1}{2}m_2 r^2 \ddot{\theta}$

$+\downarrow \sum F = ma : \; -T = m_1 \ddot{x}$

With $x = r\theta$:

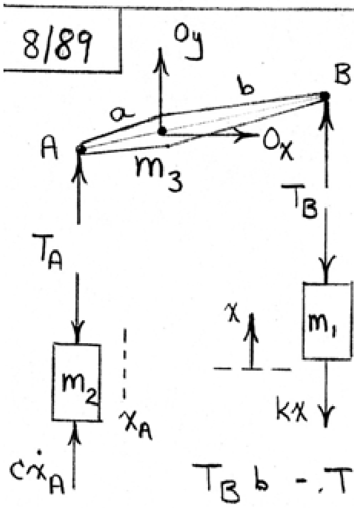
$\ddot{x}\left[m_1 + \frac{1}{2}m_2\right] + kx = 0$



Equation of motion for (b) is $m_{eff}\ddot{x} + kx = 0$

So $\underline{m_{eff} = m_1 + \frac{1}{2}m_2}$

8/89



$$\frac{x}{b} = \frac{x_A}{a}, \quad x_A = \frac{a}{b}x, \quad \ddot{x}_A = \frac{a}{b}\ddot{x}$$

$$x = b\theta, \quad \ddot{x} = b\ddot{\theta}$$

$$\Sigma F_x = m\ddot{x}:$$

$$(1) \quad -kx - T_B = m_1\ddot{x}$$

$$(2) \quad T_A - c\dot{x}_A = m_2\ddot{x}_A$$

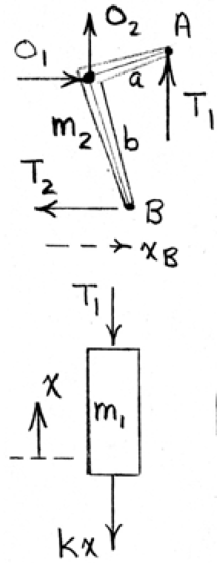
$$\circlearrowleft \Sigma M_O = I_O\ddot{\theta}:$$

$$T_B b - T_A a = m_3 k_O^2 \ddot{\theta}$$

Elimination of T_B from Eq. (1) yields

$$\left[m_1 + \frac{a^2}{b^2}m_2 + \frac{k_O^2}{b^2}m_3\right]\ddot{x} + \left[\frac{a^2}{b^2}c\right]\dot{x} + kx = 0$$

8/90



$\frac{x}{a} = \frac{x_B}{b}$, $x_B = \frac{b}{a}x$, $\dot{x}_B = \frac{b}{a}\dot{x}$

$\frac{x}{a} = \theta$, $\frac{\ddot{x}}{a} = \ddot{\theta}$, $T_2 = c\dot{x}_B$

$$\begin{cases} \Sigma F_x = m\ddot{x} : \; -T_1 - kx = m\ddot{x} \\ \Sigma M_O = I_O\ddot{\theta} : \; aT_1 - b_2T_2 = m_2 k_o^2 \ddot{\theta} \end{cases}$$

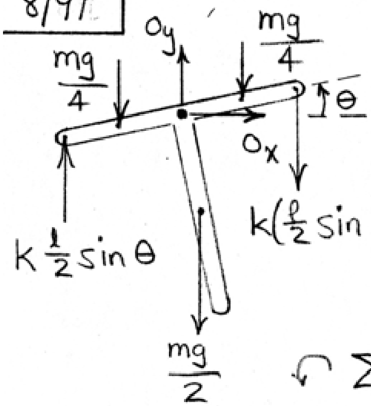
Elimination of T_1 from the x-equation yields

$$\left[m_1 + m_2\frac{k_o^2}{a^2}\right]\ddot{x} + \frac{b^2}{a^2}c\dot{x} + kx = 0$$

$2\zeta\omega_n = \dfrac{cb^2/a^2}{m_1 + k_o^2 m_2/a^2}$, $\omega_n = \sqrt{\dfrac{k}{m_1 + m_2(k_o/a)^2}}$

$$\zeta = \frac{cb^2/a^2}{2\sqrt{k\left(m_1 + \left(\frac{k_o}{a}\right)^2 m_2\right)}}$$

8/91



$$I_O = \frac{1}{3}\frac{m}{2}l^2 + 2\left(\frac{1}{3}\frac{m}{4}\left(\frac{l}{2}\right)^2\right)$$
$$= \frac{5}{24}ml^2$$

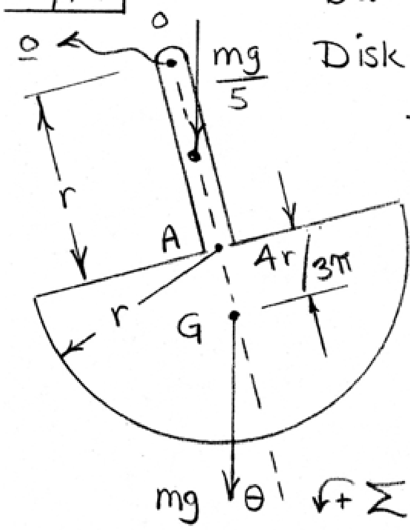
$$\curvearrowleft \Sigma M_O = I_O\ddot{\theta}:$$

$$-\left(k\frac{l}{2}\sin\theta\right)\frac{l}{2}\cos\theta - k\left(\frac{l}{2}\sin\theta - y_B\right)\frac{l}{2}\cos\theta$$
$$-\frac{mg}{2}\left(\frac{l}{2}\sin\theta\right) = \frac{5}{24}ml^2\ddot{\theta}$$

Small θ: $\ddot{\theta} + \left[\frac{12}{5}\frac{k}{m} + \frac{6}{5}\frac{g}{l}\right]\theta = \frac{12}{5}\frac{kb}{ml}\sin\omega t$

$$\omega_c = \sqrt{\frac{6}{5}\left(\frac{2k}{m} + \frac{g}{l}\right)}$$

8/92



Bar: $I_O = \frac{1}{3}\left(\frac{m}{5}\right)r^2 = \frac{1}{15}mr^2$

Disk: $I_A = \frac{1}{2}mr^2$

$$I_G = \frac{1}{2}mr^2 - \left(\frac{4r}{3\pi}\right)^2 m$$

$$I_O = I_G + md^2$$

$$= \frac{1}{2}mr^2 - \left(\frac{4r}{3\pi}\right)^2 m + m\left(r + \frac{4r}{3\pi}\right)^2$$

$$= mr^2\left[\frac{3}{2} + \frac{8}{3\pi}\right]$$

$\circlearrowleft + \Sigma M_O = I_O\ddot{\theta}$:

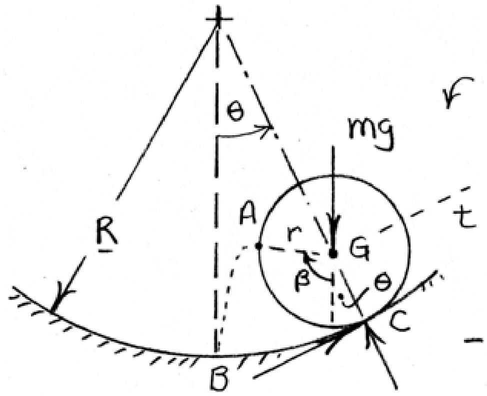
$$-\frac{mg}{5}\left(\frac{r}{2}\sin\theta\right) - mg\left(r + \frac{4r}{3\pi}\right)\sin\theta = mr^2\left(\frac{1}{15} + \frac{3}{2} + \frac{8}{3\pi}\right)\ddot{\theta}$$

Small θ: $\ddot{\theta} + \left(\frac{33\pi + 40}{47\pi + 80}\right)\frac{g}{r}\theta = 0$

$$f_n = \frac{\omega_n}{2\pi} = \underline{\sqrt{\left(\frac{33\pi + 40}{47\pi + 80}\right)\frac{g}{r}}\Big/2\pi}$$

8/93 Arc AC = Arc BC : $r(\beta+\theta) = R\theta$

$$\beta = \left(\frac{R}{r}-1\right)\theta$$



$\curvearrowleft \Sigma M_G = \bar{I}\ddot{\beta}$:

$$Fr = \frac{1}{2}mr^2(-\ddot{\beta})$$

$\Sigma F_t = ma_t$:

$$-mg\sin\theta + F = mr\ddot{\beta}$$

Eliminate F, substitute $\ddot{\beta} = \left(\frac{R}{r}-1\right)\ddot{\theta}$, & assume small θ: $\ddot{\theta} + \frac{2g}{3(R-r)}\theta = 0$

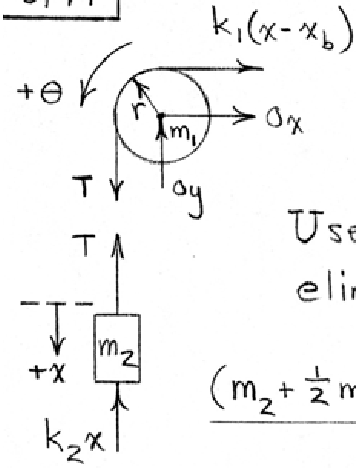
$\omega_n = \sqrt{\frac{2g}{3(R-r)}}$, $\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{3(R-r)}{2g}}}$

Solution of differential eq.: $\theta = \theta_0 \sin\omega_n t$

$\dot{\theta} = \theta_0\omega_n\cos\omega_n t$, $\dot{\theta}_{max} = \theta_0\omega_n$.

$\omega = \dot{\beta}_{max} = \left(\frac{R}{r}-1\right)\theta_0\omega_n = \underline{\frac{\theta_0}{r}\sqrt{2g(R-r)/3}}$

8/94



$$\Sigma M_O = I_O\ddot{\theta}:$$

$$r\left(T - k_1(x - x_b)\right) = \frac{1}{2} m_1 r^2 \ddot{\theta}$$

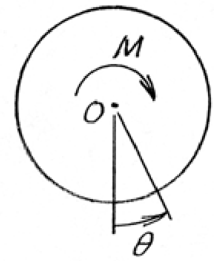
$$\Sigma F_x = m\ddot{x}: \ -T - k_2 x = m_2 \ddot{x}$$

Use the constraint $x = r\ddot{\theta}$ and eliminate T to obtain

$$\underline{(m_2 + \tfrac{1}{2} m_1)\ddot{x} + (k_1 + k_2)x = k_1 b \cos \omega t}$$

8/95

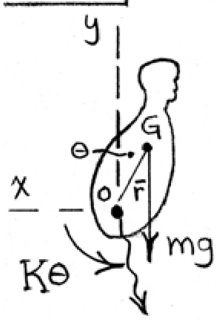
$\circlearrowright \Sigma M_O = I_O \alpha: \quad -\frac{JG}{L}\theta = I\ddot{\theta}$



$$\ddot{\theta} + \frac{JG}{IL}\theta = 0, \quad \omega_n = \sqrt{\frac{JG}{IL}}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{JG}{IL}}$$

8/96



(z in)

$$\Sigma \underline{M}_O = \dot{\underline{H}}_O + \bar{\underline{\rho}} \times m\ddot{\underline{r}}_O$$

$$\Sigma \underline{M}_O = (mg\bar{r}\sin\theta - K\theta)\,\underline{k}$$

$$\dot{\underline{H}}_O = I_O\ddot{\theta}\,\underline{k} = mk_O^2\ddot{\theta}\,\underline{k}$$

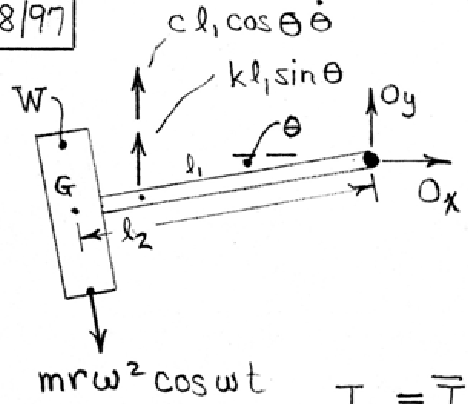
$$\bar{\underline{\rho}} = \bar{r}(-\sin\theta\,\underline{i} + \cos\theta\,\underline{j})$$

$$\ddot{\underline{r}}_O = a\underline{i}$$

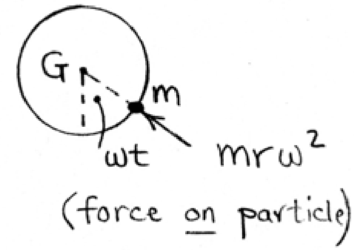
$$\therefore\ mg\bar{r}\sin\theta - K\theta = mk_O^2\ddot{\theta} - ma\bar{r}\cos\theta$$

Rearrange : $mk_O^2\ddot{\theta} + K\theta - m\bar{r}(g\sin\theta + a\cos\theta) = 0$

▶ 8/97



View of wheel from left side :



$$I_O = \bar{I} + \frac{W}{g} l_2^2 = 1 + \frac{100}{32.2}(3)^2$$
$$= 28.95 \text{ lb-sec}^2\text{-ft}$$

$\Sigma M_O = I_O \ddot\theta$:

$$-(k l_1 \sin\theta)(l_1 \cos\theta) - (c l_1 \dot\theta \cos\theta)(l_1 \cos\theta)$$
$$+ (mr\omega^2 \cos\omega t)\cos\theta\, l_2 = I_O \ddot\theta$$

For small θ :

$$\ddot\theta + \frac{c l_1^2}{I_O}\dot\theta + \frac{k l_1^2}{I_O}\theta = \frac{mr\omega^2 l_2 \cos\omega t}{I_O}$$

$$\omega_n = \sqrt{\frac{k l_1^2}{I_O}} = \sqrt{\frac{(50)(12)(\frac{27}{12})^2}{28.95}} = \underline{10.24 \ \frac{\text{rad}}{\text{sec}}}$$

$$v = r\omega_n = \frac{14}{12}(10.24) = \underline{11.95 \ \text{ft/sec}}$$

$$2\zeta\omega_n = \frac{c l_1^2}{I_O}, \qquad \zeta = \frac{c l_1^2}{2 I_O \omega_n}$$

$$\zeta = \frac{(200)(\frac{27}{12})^2}{2(10.24)(28.95)} = \underline{1.707}$$

▶ 8/98 The particular solution to the differential equation of the previous solution is

$$\theta = \Theta \cos(\omega t - \phi), \quad \text{where}$$

$$\Theta = \left(\frac{M_0}{k_t}\right) M = \left(\frac{mr\omega^2 l_2}{k l_1^2}\right) M$$

$$= \frac{mr\omega^2 l_2 / k l_1^2}{\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

(a) $\frac{\omega}{\omega_n} = \frac{10.24}{10.24} = 1$, $\Theta = 5.50(10^{-4})$ rad

Vertical motion $X = l_2 \Theta = 3(5.50)(10^{-4})$

$= 1.650(10^{-3})$ ft

$= \underline{1.980(10^{-2}) \text{ in.}}$

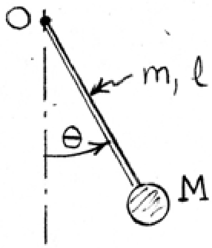
(b) $v = 55 \frac{mi}{hr} = 80.67$ ft/sec, $\omega = \frac{v}{r} = 69.14 \frac{rad}{sec}$

$\frac{\omega}{\omega_n} = \frac{69.14}{10.24} = 6.75$

$\Theta = 1.705(10^{-3})$ rad, $X = 5.12(10^{-3})$ ft

$= \underline{6.14(10^{-2}) \text{ in.}}$

8/99



$$I_O = \frac{1}{3}ml^2 + Ml^2 = \left(M + \frac{m}{3}\right)l^2$$

$$E = T + V = \frac{1}{2}I_O\dot{\theta}^2$$

$$+ mg\frac{l}{2}(1-\cos\theta) + Mgl(1-\cos\theta)$$

Differentiate with respect to time and let θ be small to obtain

$$\ddot{\theta} + \frac{3}{2}\frac{g}{l}\left(\frac{m+2M}{m+3M}\right)\theta = 0$$

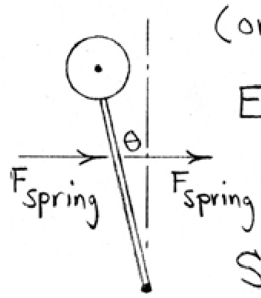
$\boxed{8/100}$ Energy $E = T + V = 8\dot{x}^2 + 64x^2 = \text{constant}$

so $dE/dt = 16\dot{x}\ddot{x} + 128x\dot{x} = 0$, $\underline{\ddot{x} + 8x = 0}$

$\omega_n = \sqrt{8} = 2\sqrt{2}$ rad/sec, $\tau = \dfrac{2\pi}{\omega_n} = \dfrac{2\pi}{2\sqrt{2}} = \underline{2.22 \text{ sec}}$

8/101

Let the initial spring stretch (or compression) be δ.



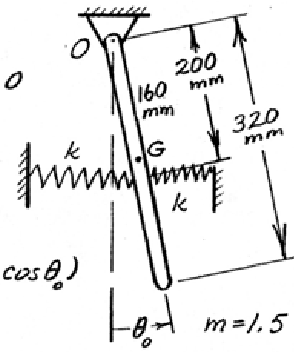
$$E = T + V = \frac{1}{2} m(l\dot{\theta})^2 - mgl(1-\cos\theta) + \frac{1}{2}k(\delta + b\sin\theta)^2 + \frac{1}{2}k(\delta - b\sin\theta)^2$$

Set $\frac{dE}{dt} = 0$ and assume small θ to obtain $\ddot{\theta} + \left[\frac{2kb^2 - mgl}{ml^2}\right]\theta = 0$

$$f_n = \frac{\omega_n}{2\pi} = \underline{\frac{1}{2\pi}\sqrt{\frac{2kb^2}{ml^2} - \frac{g}{l}}}$$

$$\frac{2kb^2}{ml^2} > \frac{g}{l}, \quad \underline{k > \frac{mgl}{2b^2}}$$

8/102 $V_{max} = T_{max}$



$m = 1.5$ kg

$k = 120$ N/m

Take $V = V_e + V_g = 0$ at equil. position $\theta = 0$

For small θ_o spring deflection δ is

$$\delta \approx 0.200\,\theta_o$$

so $V_{max} = \frac{1}{2}k\delta^2 - \frac{1}{2}k(-\delta)^2 + mgh$

$$= 120(0.200\,\theta_o)^2 + 1.5(9.81)(0.160)(1 - \cos\theta_o)$$

& with $\cos\theta_o = 1 - \frac{\theta_o^2}{2!} + \cdots,$

$$V_{max} = 4.80\,\theta_o^2 + 1.177\,\theta_o^2 = 5.98\,\theta_o^2 \text{ J}$$

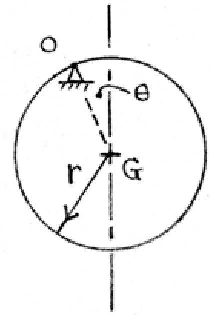
$T_{max} = \frac{1}{2} I_o \dot{\theta}_{max}^2$ where $\dot{\theta}_{max} = \theta_o\,\omega_n$

$$= \frac{1}{2}\left(\frac{1}{3} \times 1.5 \times 0.320^2\right)\theta_o^2\omega_n^2 = 0.0256\,\theta_o^2\omega_n^2 \text{ J}$$

Thus $5.98\,\theta_o^2 = 0.0256\,\theta_o^2\omega_n^2$, $\omega_n = 15.28$ rad/s

$$f_n = \frac{\omega_n}{2\pi} = \frac{15.28}{2\pi} = \underline{2.43 \text{ Hz}}$$

8/103



$$I_O = \bar{I} + md^2 = mr^2 + mr^2$$
$$= 2mr^2$$

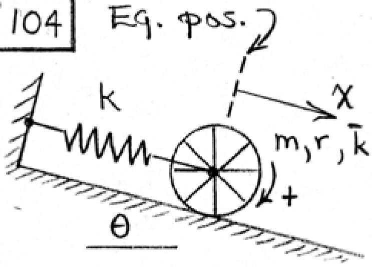
$$E = T + V = \frac{1}{2} I_O \dot{\theta}^2 + mgr(1 - \cos\theta)$$

Set $\frac{dE}{dt} = 0$ and assume small θ to obtain

$$\ddot{\theta} + \frac{g}{2r}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{2r}{g}}}$$

8/104



Choose $V = 0$ @ $x = 0$

Then $V = \frac{1}{2}kx^2$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}(m\bar{k}^2)\left(\frac{\dot{x}}{r}\right)^2$$

$$= \frac{1}{2}m\left(1 + \frac{\bar{k}^2}{r^2}\right)\dot{x}^2$$

$$E = T + V = \frac{1}{2}m\left(1 + \frac{\bar{k}^2}{r^2}\right)\dot{x}^2 + \frac{1}{2}kx^2$$

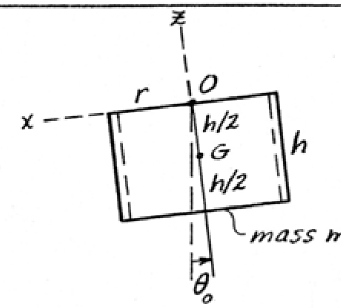
$$\frac{dE}{dt} = m\left(1 + \frac{\bar{k}^2}{r^2}\right)\dot{x}\ddot{x} + kx\dot{x} = 0$$

$$\ddot{x} + \frac{k}{m\left(1 + \frac{\bar{k}^2}{r^2}\right)}x = 0$$

$$\underline{\omega_n = \sqrt{\frac{k}{m(1 + \bar{k}^2/r^2)}}}$$

$$\begin{cases} \bar{k} = 0: & \underline{\omega_n = \sqrt{k/m}} \\ \bar{k} = r: & \underline{\omega_n = \sqrt{k/2m}} \end{cases}$$

8/105 $V_{max} = T_{max}$



$V_{max} = mg\frac{h}{2}(1-\cos\theta_o)$

where for small θ_o, $\cos\theta_o \approx 1 - \frac{\theta_o^2}{2!}$

so $V_{max} = mg\frac{h}{2}\frac{\theta_o^2}{2} = \frac{1}{4}mgh\theta_o^2$

$T_{max} = \frac{1}{2}I_o\dot{\theta}_{max}^2$

with $\theta = \theta_o \sin\omega_n t$, $\dot{\theta}_{max} = \theta_o\omega_n$

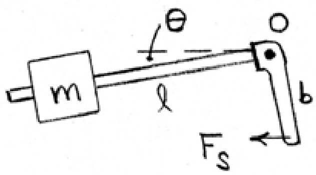
From Table D/4, $I_o = \frac{1}{2}mr^2 + \frac{1}{3}mh^2$

so $T_{max} = \frac{1}{2}m\left(\frac{r^2}{2}+\frac{h^2}{3}\right)\theta_o^2\omega_n^2$

Thus $\frac{1}{4}mgh\theta_o^2 = \frac{1}{2}m\left(\frac{r^2}{2}+\frac{h^2}{3}\right)\theta_o^2\omega_n^2$, $\omega_n^2 = \dfrac{gh/2}{\frac{r^2}{2}+\frac{h^2}{3}}$

$\tau = 2\pi/\omega_n = \underline{2\pi\frac{\sqrt{2}}{\sqrt{gh}}\sqrt{\frac{r^2}{2}+\frac{h^2}{3}}}$

8/106 At equilibrium, $\Sigma M_O = 0$ to obtain



$$\delta_{st} = \frac{mgl}{kb}$$

$$E = T + V$$

$$= \frac{1}{2} m (l\dot{\theta})^2 + \frac{1}{2} k \left(\frac{mgl}{kb} + b \sin\theta \right)^2$$

$$- mgl \sin\theta$$

Set $\frac{dE}{dt} = 0$ and assume θ small

to obtain $\ddot{\theta} + \frac{kb^2}{ml^2}\theta = 0$

$$f_n = \frac{\omega_n}{2\pi} = \underline{\frac{1}{2\pi}\frac{b}{l}\sqrt{\frac{k}{m}}}$$

8/107 Let y be the downward displacement from the equilibrium position where $V = V_e + V_g$ is taken to be zero.

$$(T_{max})_{y=0} = (V_{max})_{y=y_{max}}$$

$$T_{max} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(70)\dot{y}_{max}^2 + \frac{1}{2}(40)(0.2)^2\left(\frac{\dot{y}_{max}}{0.3}\right)^2$$

$$= 43.9\,\dot{y}_{max}^2$$

But $y = y_{max}\sin\omega_n t$, $\quad \dot{y}_{max} = y_{max}\,\omega_n$

So $T_{max} = 43.9\,y_{max}^2\,\omega_n^2$

$$V_{max} = \frac{1}{2}(2000)(2y_{max})^2 = 4000\,y_{max}^2$$

Thus $43.9\,y_{max}^2\,\omega_n^2 = 4000\,y_{max}^2$

$\omega_n = 9.55$ rad/s, $f_n = \dfrac{\omega_n}{2\pi} = \underline{1.519\text{ Hz}}$

8/108 Let $\theta = 0$ be the angular position of static equilibrium & choose $V = 0$ there.

$$T = \frac{1}{2} I_o \dot{\theta}^2 + \frac{1}{2}(2mr^2)\dot{\theta}^2 = \frac{1}{2}\left[I_o + 2mx^2\right]\dot{\theta}^2$$

$$V = \frac{1}{2} K \theta^2$$

Set $\frac{dE}{dt} = 0$: $(I_o + 2mx^2)\ddot{\theta} + K\theta = 0$

$$\omega_n = \sqrt{\frac{K}{(I_o + 2mx^2)}} \; ; \; \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_o + 2mx^2}{K}}$$

Solve for x as $x = \sqrt{\frac{\tau^2 K / 4\pi^2 - I_o}{2m}}$

8/109 Let x be the displacement (downward) from the equilibrium position, where V is taken to be zero.

$$T = \frac{1}{2} m \dot{x}^2 , \quad V = 2\left[\frac{1}{2} k (2x)^2\right]$$

$$E = T + V = \frac{1}{2} m \dot{x}^2 + 4 k x^2$$

$$\frac{dE}{dt} = m \dot{x} \ddot{x} + 8 k x \dot{x} = 0 , \quad \ddot{x} + \frac{8k}{m} x = 0$$

$$\omega_n = \sqrt{\frac{8k}{m}} , \quad \tau = \frac{2\pi}{\omega_n} = \pi \sqrt{\frac{m}{2k}}$$

Numbers: $\tau = \pi \sqrt{\frac{50/32.2}{2(6)(12)}} = \underline{0.326 \text{ sec}}$

$\boxed{8/110}$ Take $\theta = 0$ to be the position where $V = 0$.

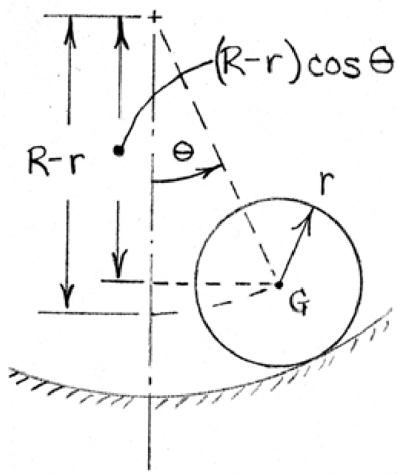
$$E = T + V = \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m (r\dot{\theta})^2 + mgr(1 - \cos\theta)\sin\alpha$$

Set $\frac{dE}{dt} = 0$ to obtain, for small angles,

$$\ddot{\theta} + \left[\frac{mgr\sin\alpha}{I + mr^2} \right] \theta = 0$$

So $\underline{\omega_n = \sqrt{\frac{mgr\sin\alpha}{I + mr^2}}}$

8/111



$$E = T + V = \frac{1}{2} m v_G^2 + \frac{1}{2}\bar{I}\omega^2 + mg(R-r)(1-\cos\theta)$$

Now, $v_G = (R-r)\dot{\theta}$

$$\omega = \frac{v_G}{r} = \frac{(R-r)\dot{\theta}}{r}$$

Thus $E = \frac{3}{4} m (R-r)^2 \dot{\theta}^2 + mg(R-r)(1-\cos\theta)$

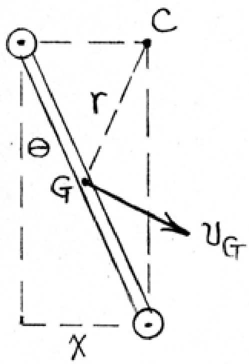
Set $\frac{dE}{dt} = 0$: $\ddot{\theta} + \frac{2}{3}\frac{g}{R-r}\sin\theta = 0$

For small θ, $\tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{6(R-r)}{g}}$

8/112

$v_G = r\dot{\theta} = \left[\left(\frac{l}{2}\cos\theta\right)^2 + \left(\frac{l}{2}\sin\theta\right)^2\right]^{1/2}\dot{\theta}$

$= \frac{l}{2}\dot{\theta}$



$E = T + V = \frac{1}{2}mv_G^2 + \frac{1}{2}\bar{I}\dot{\theta}^2$

$+ \frac{1}{2}k(\delta_{st} - l\sin\theta)^2 + \frac{1}{2}k(\delta_{st} + l\sin\theta)^2$

$- mg\frac{l}{2}(1-\cos\theta)$, where

δ_{st} is the spring deflection at $\theta = 0$.

Substitute $v_G = \frac{l}{2}\dot{\theta}$, $\bar{I} = \frac{1}{12}ml^2$ into expression for E, set $\frac{dE}{dt} = 0$ and assume θ small to obtain

$$\ddot{\theta} + \left[\frac{6k}{m} - \frac{3g}{2l}\right]\theta = 0$$

$$\underline{\omega_n = \sqrt{\frac{6k}{m} - \frac{3}{2}\frac{g}{l}}}, \quad \underline{k > \frac{mg}{4l}}$$

$\boxed{8/113}$ For the bar, $I_O = \frac{1}{12} m_2 l^2 + m_2 \left(\frac{3}{10} l\right)^2$

$$= \frac{13}{75} m_2 l^2$$

Combined: $I_O = \frac{1}{2} m_1 \left(\frac{l}{5}\right)^2 + \frac{13}{75} m_2 l^2$

$$= \frac{1}{50} m_1 l^2 + \frac{13}{75} m_2 l^2$$

Let $\theta = 0$ be the equilibrium position shown &
choose $V = 0$ @ $\theta = 0$; $V = \frac{1}{2} k \left(\frac{3l}{5}\theta\right)^2 = \frac{9}{50} k l^2 \theta^2$

$E = T + V = \frac{1}{2} I_O \dot{\theta}^2 + \frac{9}{50} k l^2 \theta^2$

$$= l^2 \left[\left(\frac{1}{100} m_1 + \frac{13}{150} m_2\right) \dot{\theta}^2 + \frac{9}{50} k \theta^2\right]$$

Set $\frac{dE}{dt} = 0$ to obtain

$$\ddot{\theta} + \frac{54k}{3m_1 + 26m_2} \theta = 0, \quad \underline{\omega_n = 3\sqrt{\frac{6k}{3m_1 + 26m_2}}}$$

$\theta = \theta_0 \sin \omega_n t$, $\dot{\theta} = \theta_0 \omega_n \cos \omega_n t$

$$\dot{\theta}_{max} = \omega = \underline{3\theta_0 \sqrt{\frac{6k}{3m_1 + 26m_2}}}$$

8/114 $E = T + V = 2\left(\frac{1}{2} M v_0^2\right) + 2\left(\frac{1}{2} \bar{I} \dot{\theta}^2\right)$

$$+ \frac{1}{2} m v_A^2 + mg r_0 (1 - \cos\theta)$$

But $v_0 = r\dot{\theta}$ and $\bar{I} = \frac{1}{2} M r^2$. Also,

from kinematics, $v_A^2 = \dot{\theta}^2 (r^2 + r_0^2) - 2 r r_0 \dot{\theta}^2 \cos\theta$

Thus $E = \left[\frac{3}{2} M r^2 + \frac{1}{2} m \left(r^2 + r_0^2 - 2 r r_0 \cos\theta\right)\right] \dot{\theta}^2$

$$+ mg r_0 [1 - \cos\theta] = \text{constant}$$

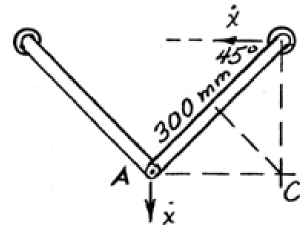
Set $\frac{dE}{dt} = 0$: $2\left[\frac{3}{2} M r^2 + \frac{1}{2} m (r^2 + r_0^2 - 2 r r_0 \cos\theta\right] \ddot{\theta}$

$$+ \dot{\theta}^2 [m r r_0 \sin\theta] + mg r_0 \sin\theta = 0$$

Small $\theta, \dot{\theta}$: $\ddot{\theta} + \left[\frac{mg r_0}{3 M r^2 + m(r^2 + r_0^2 - 2 r r_0)}\right] \theta = 0$

$$f_n = \frac{1}{2\pi} \left[\frac{mg r_0}{3 M r^2 + m (r - r_0)^2}\right]^{1/2}$$

8/115 Take $V = V_g + V_e = 0$ at equilibrium position, where spring tension is $2(W/2) = W$



so for downward displacement x_0 from equilibrium position,

$$V_{max} = \Delta V_e + \Delta V_g = \left(Wx_0 + \frac{1}{2}kx_0^2\right) + \left(-2W\frac{x_0}{2}\right)$$

$$= \frac{1}{2}kx_0^2 = \frac{1}{2}1050\,x_0^2$$

$$T_{max} = 2\left(\frac{1}{2}I_c\,\omega^2\right) = I_c\left(\frac{\dot{x}_{max}}{0.300/\sqrt{2}}\right)^2$$

where $I_c = I_A = \frac{1}{3}m\ell^2 = \frac{1}{3}1.5 \times 0.300^2 = 0.045\ \text{kg}\cdot\text{m}^2$

$$T_{max} = 0.045\,\frac{2\dot{x}_{max}^2}{0.09} = \dot{x}_{max}^2, \text{ at } x = 0$$

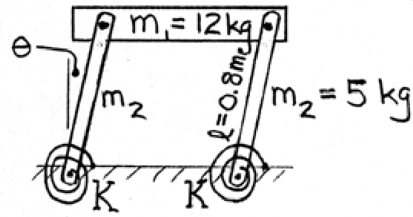
But $\dot{x}_{max} = x_0\,\omega_n$ for harmonic motion

so $T_{max} = V_{max}$ gives $x_0^2\omega_n^2 = 525\,x_0^2$

$$\omega_n = \sqrt{525} = 22.9\ \text{rad/s},$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{22.9}{2\pi} = \underline{3.65\ \text{Hz}}$$

8/116



(θ small)

$$V_g = V_{g_1} + V_{g_2}$$
$$= -m_1 g l (1 - \cos\theta) - 2 m_2 g \frac{l}{2}(1 - \cos\theta)$$
$$= -(m_1 + m_2) g l (1 - \cos\theta)$$
$$\cong -(m_1 + m_2) g l \frac{\theta^2}{2}$$

$$V_e = 2\left(\tfrac{1}{2} K\theta^2\right) = K\theta^2$$

$$V_{max} = -(m_1 + m_2) g l \frac{\theta_{max}^2}{2} + K\theta_{max}^2$$
$$= \left[K - \frac{m_1 + m_2}{2} g l\right]\theta_{max}^2$$
$$= \left[500 - \frac{12+5}{2}(9.81)(0.8)\right]\theta_{max}^2 = 433.3\ \theta_{max}^2$$

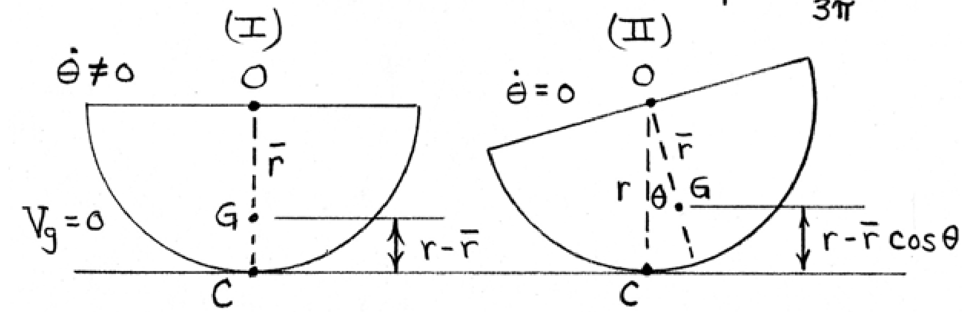
$$T = \tfrac{1}{2} m_1 v_1^2 + 2\left(\tfrac{1}{2} I \dot\theta^2\right) = \tfrac{1}{2} m_1 (l\dot\theta)^2 + 2\left(\tfrac{1}{2}\cdot\tfrac{1}{3} m_2 l^2 \dot\theta^2\right)$$
$$= \left(\tfrac{1}{2} m_1 + \tfrac{1}{3} m_2\right) l^2 \dot\theta^2$$

$$T_{max} = \left(\frac{12}{2} + \frac{5}{3}\right)(0.8)^2 \dot\theta_{max}^2 = 4.907\,\dot\theta_{max}^2$$
$$= 4.907(\theta_{max}\,\omega_n)^2 = 4.907\,\omega_n^2\,\theta_{max}^2$$

Set $T_{max} = V_{max}$ & obtain $\omega_n = 9.40$ rad/s, $\underline{f_n = 1.496 \text{ Hz}}$

8/117

$\bar{r} = \frac{4r}{3\pi}$



In position II, $(V_g)_{max} = mg\left[(r - \bar{r}\cos\theta) - (r - \bar{r})\right]$

$$= mg\left(\frac{4r}{3\pi}\right)(1 - \cos\theta)$$

In position I, $T_{max} = \frac{1}{2} I_C \dot{\theta}^2$

$$I_C = \bar{I} + m(r - \bar{r})^2 = I_O - m\bar{r}^2 + m(r - \bar{r})^2$$

$$= \frac{1}{2}mr^2 + mr^2 - 2mr\bar{r} = \left(\frac{3}{2} - \frac{8}{3\pi}\right)mr^2$$

$T_{max} = (V_g)_{max}$: $\quad mg\frac{4r}{3\pi}(1 - \cos\theta) = \frac{1}{2}\left(\frac{3}{2} - \frac{8}{3\pi}\right)mr^2\dot{\theta}^2$

For small θ, replace $\cos\theta$ by $1 - \frac{\theta^2}{2}$

For harmonic oscillation, $\dot{\theta}_{max} = \theta\omega_n$

So $\frac{4g}{3\pi}\left[1 - \left(1 - \frac{\theta^2}{2}\right)\right] = \left(\frac{3}{4} - \frac{4}{3\pi}\right)r\theta^2\omega_n^2$

$$\omega_n = 0.807\sqrt{g/r}, \quad \tau = \frac{2\pi}{\omega_n} = \underline{7.78\sqrt{r/g}}$$

8/118 $E = T + V \neq$ constant. Because $\omega =$ constant, the system will have more kinetic energy of rotation when the blocks are in outer positions than when the blocks are in inner positions of the same potential energy.

8/119 Let y_o = amplitude of vertical deflection of frame & body

$\frac{12}{18}y_o = \frac{2}{3}y_o$ = corresponding spring deflection

ΔV = change in $V_e + V_g$ due only to y_o so

$\Delta V = 2\left(\frac{1}{2}\right)(270)\left(\frac{2}{3}y_o\right)^2 = 120\,y_o^2$ in.-lb

$\Delta T = T_{max} = \frac{1}{2}\,\frac{1800}{32.2 \times 12}\,\dot{y}_{max}^2$ but $\dot{y}_{max} = y_o\omega_n$

so $T_{max} = 2.33\,y_o^2\omega_n^2$

Thus with $T_{max} = \Delta V$, $2.33\,y_o^2\omega_n^2 = 120\,y_o^2$

& $\omega_n^2 = \frac{120}{2.33}$, $\omega_n = 7.18\ \frac{rad}{sec}$, $f_n = \frac{\omega_n}{2\pi} = \frac{7.18}{2\pi}$

$= \underline{1.142\ \text{Hz}}$

8/120 $V_{max} = T_{max}$

$V_{max} = mgh = mg\ell(1-\cos\beta_o) = mg\ell(1-[1-\frac{\beta_o^2}{2!}+\dots]) = \frac{1}{2}mg\ell\beta_o^2$

But $\ell\beta_o \approx b\theta_o$ so $V_{max} = \frac{1}{2}mg\frac{b^2\theta_o^2}{\ell}$ where

θ_o = max. angular twist

$T_{max} = \frac{1}{2}I_o\dot{\theta}^2_{max}$ where $\dot{\theta} = \theta_o\omega_n\cos\omega_n t$ & $\dot{\theta}^2_{max} = \theta_o^2\omega_n^2$

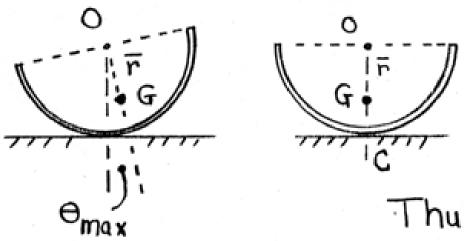
& $T_{max} = \frac{1}{2}(\frac{1}{12}m[2b]^2)\theta_o^2\omega_n^2 = \frac{1}{6}mb^2\theta_o^2\omega_n^2$

Thus $\frac{1}{2}mg\frac{b^2\theta_o^2}{\ell} = \frac{1}{6}mb^2\theta_o^2\omega_n^2$, $\omega_n = \sqrt{3g/\ell}$

so $\underline{\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{\ell}{3g}}}$

8/121

$T = 0,\ V = V_{max}$ $\qquad$ $V = 0,\ T = T_{max}$



$$V_{max} = mg\bar{r}(1 - \cos\theta_{max})$$

$$= mg\frac{2r}{\pi}(1 - \cos\theta_{max})$$

For θ small, $\cos\theta \cong 1 - \theta^2/2$

Thus $V_{max} = mgr\,\theta_{max}^2/\pi$

$$T = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\left[\bar{I} + m(r-\bar{r})^2\right]\omega^2$$

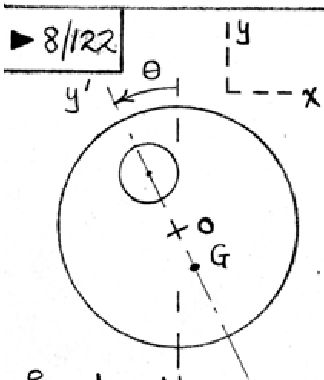
$$= \frac{1}{2}\left[I_O - m\bar{r}^2 + m(r-\bar{r})^2\right]\omega^2 = mr^2\left(1 - \frac{2}{\pi}\right)\omega^2$$

$$T_{max} = mr^2\left(1 - \frac{2}{\pi}\right)(\omega_n\theta_{max})^2$$

$$T_{max} = V_{max}:\ mr^2\left(1 - \frac{2}{\pi}\right)\omega_n^2\theta_{max}^2 = mgr\,\theta_{max}^2/\pi$$

$$\omega_n^2 = \frac{g}{\pi r(1 - 2/\pi)},\quad \tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{(\pi-2)r}{g}}}$$

► 8/122



$$\bar{Y}' = \frac{\Sigma \bar{y}' A}{\Sigma A} = \frac{-\frac{R}{2}(\pi)\left(\frac{R}{4}\right)^2}{\pi R^2 - \pi\left(\frac{R}{4}\right)^2}$$

$$= -\frac{R}{30}, \quad \text{So } \overline{OG} = R/30$$

$$I_o = I_{whole} - I_{hole}$$

$$= \frac{1}{2} m_1 R^2 - \left[\frac{1}{2} m_2 \left(\frac{R}{4}\right)^2 + m_2 \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{247}{512} \pi t \rho R^4$$

ρ = density

t = length

$$\text{But } m = \pi \rho t\left[\pi R^2 - \pi\left(\frac{R}{4}\right)^2\right] = \frac{15}{16}\pi \rho t R^2$$

$$\therefore I_o = \frac{247}{512} \pi t \rho R^4 \left(\frac{m}{\frac{15}{16}\pi \rho t R^2}\right) = \frac{247}{480} m R^2$$

$$\bar{I} = I_o - m\left(\overline{OG}^2\right) = \frac{247}{480} m R^2 - m\left(\frac{R}{30}\right)^2$$

$$= 0.5135\, m R^2$$

$$\underline{v}_G = \underline{v}_o + \underline{\omega} \times \underline{r}_{G/O} = -R\omega \underline{i} + \omega \underline{k} \times \frac{R}{30}\left[\sin\theta \underline{i} - \cos\theta \underline{j}\right]$$

$$= \omega\left[\left(\frac{R}{30}\cos\theta - R\right)\underline{i} + \left(\frac{R}{30}\sin\theta\right)\underline{j}\right]$$

$$v_G^2 = \omega^2\left[1.001 R^2 - \frac{1}{15} R^2 \cos\theta\right]$$

$$\text{Now}, \quad E = T + V$$

$$E = \frac{1}{2}\bar{I}\dot{\theta}^2 + \frac{1}{2} m v_G^2 + mg\frac{R}{30}(1 - \cos\theta)$$

$$E = \frac{1}{2}\left(0.5135\, mR^2\right)\dot{\theta}^2 + \frac{1}{2}m\left[\dot{\theta}^2\left(1.001\,R^2 - \frac{1}{15}R^2\cos\theta\right)\right]$$

$$+\ mg\,\frac{R}{30}\left(1-\cos\theta\right)$$

$$\frac{dE}{dt} = \left(0.5135\,\cancel{m}R^2\right)\cancel{\dot{\theta}}\,\ddot{\theta} + \cancel{m}\,\cancel{\dot{\theta}}\,\ddot{\theta}\left(1.001\,R^2 - \frac{1}{15}R^2\cos\theta\right)$$

$$+\ \frac{1}{2}\cancel{m}\,\dot{\theta}^2\left(\frac{1}{15}R^2\sin\theta\,\cancel{\dot{\theta}}\right) + \cancel{m}g\,\frac{R}{30}\sin\theta\,\cancel{\dot{\theta}} = 0$$

We now assume small θ:

$$1.448\,R^2\ddot{\theta} + 0.0333\,gR\theta = 0$$

$$\ddot{\theta} + 0.0230\,\frac{g}{R}\,\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{41.4\sqrt{\frac{R}{g}}}$$

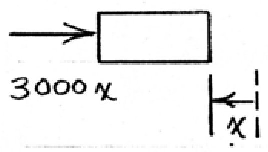
8/123 Linear momentum is conserved during impact

$G_1 = G_2$: $0.1(500) = 10.1\,v$, $v = 4.95$ m/s

After impact, energy is conserved

$T_1 = V_e$: $\frac{1}{2}(10.1)(4.95)^2 = \frac{1}{2}(3000)X^2$

$$\underline{X = 0.287 \text{ m}}$$



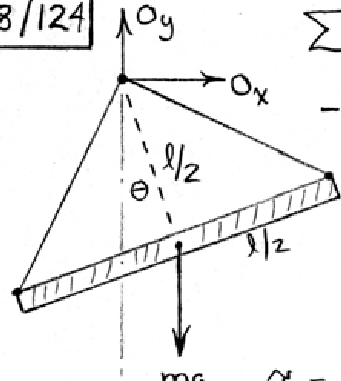
$\Sigma F_x = m\ddot{x}$: $-3000x = 10.1\,\ddot{x}$

$$\ddot{x} + 297x = 0$$

$\omega_n = \sqrt{297} = 17.23$ rad/s

$\tau = \frac{2\pi}{\omega_n} = \underline{0.365 \text{ s}}$

8/124



$\sum M_O = I_O \ddot{\theta}$:

$$-mg\frac{l}{2}\sin\theta = \left[\frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2\right]\ddot{\theta}$$

For small θ,

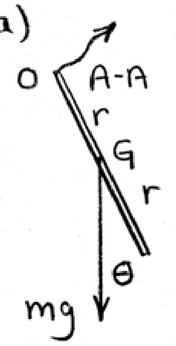
$$\ddot{\theta} + \frac{3g}{2l}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2l}{3g}} = \underline{7.33\ s}$$

8/125 $I_{A\text{-}A} = \frac{1}{4}mr^2 + mr^2 = \frac{5}{4}mr^2$

$I_{B\text{-}B} = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$

(a)

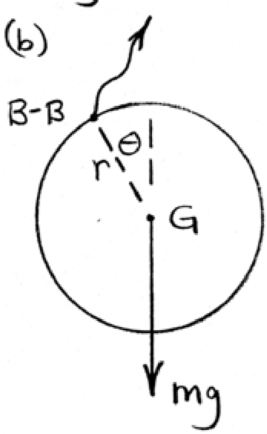


$\circlearrowleft + \; \Sigma M_0 = I_0\ddot{\theta}: \; -mgr\sin\theta = \frac{5}{4}mr^2\ddot{\theta}$

Small θ: $\ddot{\theta} + \frac{4g}{5r}\theta = 0$

$\underline{\omega_n = 2\sqrt{\frac{g}{5r}}}$

(b)

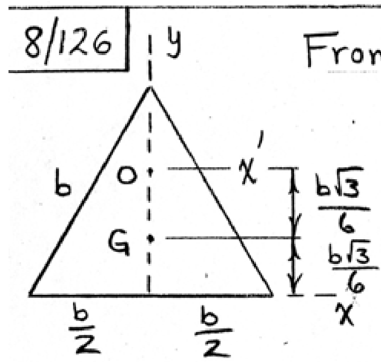


$\circlearrowleft \; \Sigma M_0 = I_0\ddot{\theta}: \; -mgr\sin\theta = \frac{3}{2}mr^2\ddot{\theta}$

Small θ: $\ddot{\theta} + \frac{2}{3}\frac{g}{r}\theta = 0$

$\underline{\omega_n = \sqrt{\frac{2g}{3r}}}$

8/126



From Table D/3, $I_x = \frac{1}{12} b\left(\frac{b\sqrt{3}}{2}\right)^3$

$= \frac{\sqrt{3}}{32} b^4$

$I_{xx} = I_x \rho t = \frac{\sqrt{3}}{32} b^4 \rho t$

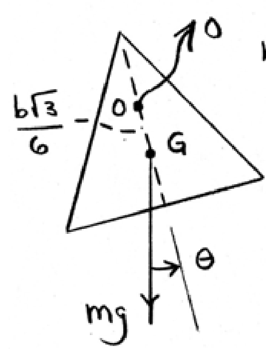
$= \frac{\sqrt{3}}{32} b^4 \rho t \left(\frac{m}{\rho\left(\frac{1}{2} b \frac{\sqrt{3}}{2} b\right) t}\right)$

$= \frac{1}{8} m b^2 = I_{x'x'}$

Also, $I_y = 2 \frac{1}{12}\left(\frac{\sqrt{3}}{2} b\right)\left(\frac{b}{2}\right)^3 = \frac{\sqrt{3}}{96} b^4$

$I_{yy} = I_y \rho t = \frac{\sqrt{3}}{96} b^4 \rho t \left(\frac{m}{\rho\left(\frac{1}{2} b \frac{\sqrt{3}}{2} b\right) t}\right) = \frac{1}{24} m b^2$

$I_{zz} = I_O = I_{x'x'} + I_{yy} = \frac{1}{6} m b^2$

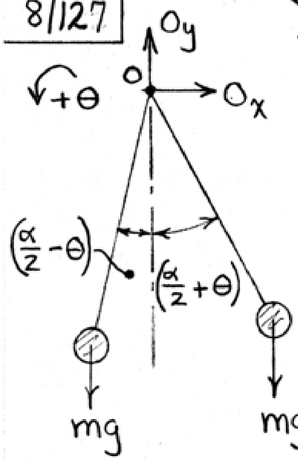


$\circlearrowleft + \Sigma M_O = I_O \ddot{\theta} : mg \frac{b\sqrt{3}}{6} \theta = \frac{1}{6} m b^2 \ddot{\theta}$

$\ddot{\theta} + \frac{g\sqrt{3}}{b} \theta = 0$

$\omega_n = \sqrt{\frac{g\sqrt{3}}{b}}$

8/127



$\sum M_O = I_O \ddot{\theta}$:

$$-mgl \sin\left(\frac{\alpha}{2}+\theta\right) + mgl \sin\left(\frac{\alpha}{2}-\theta\right) = 2ml^2\ddot{\theta} \qquad (a)$$

$$\sin\left(\frac{\alpha}{2}+\theta\right) = \sin\frac{\alpha}{2}\cos\theta + \cos\frac{\alpha}{2}\sin\theta$$

$$(\text{for } \theta \text{ small}) = \sin\frac{\alpha}{2} + \theta\cos\frac{\alpha}{2}$$

Similarly,

$$\sin\left(\frac{\alpha}{2}-\theta\right) = \sin\frac{\alpha}{2} - \theta\cos\frac{\alpha}{2}$$

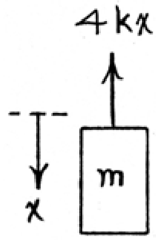
The equation of motion (a) becomes

$$\ddot{\theta} + \left(\frac{g}{l}\cos\frac{\alpha}{2}\right)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{l}{g\cos(\alpha/2)}}}$$

For $\alpha \to 0$, $\underline{\tau \to 2\pi\sqrt{l/g}}$; For $\alpha \to 180°$, $\underline{\tau \to \infty}$

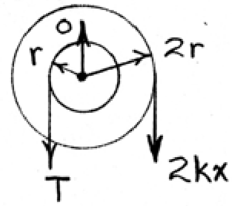
8/128



$$\sum F_x = m\ddot{x}: \quad -4kx = m\ddot{x}$$

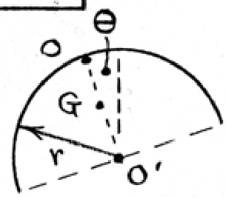
$$\ddot{x} + \frac{4k}{m}x = 0$$

$$f_n = \frac{\omega_n}{2\pi} = \underline{\frac{1}{\pi}\sqrt{\frac{k}{m}}}$$



Note that $\Sigma M_O = 0$ yields $T = 4kx$.

8/129 From Appendix D, $\overline{O'G} = 2r/\pi$



$$\bar{I} = I_{O'} - m(\overline{O'G})^2$$
$$= mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1 - \frac{4}{\pi^2}\right)$$
$$I_O = \bar{I} + m(\overline{OG}^2)$$
$$= mr^2\left(2 - \frac{4}{\pi}\right)$$

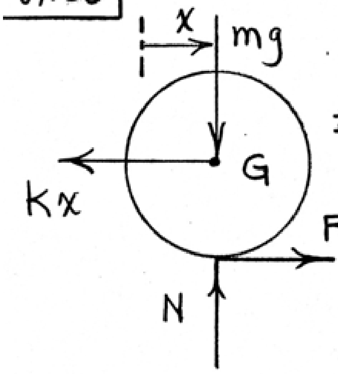
$$E = T + V = \frac{1}{2} I_O \dot{\theta}^2 + mg\,\overline{OG}\,(1 - \cos\theta)$$
$$= \frac{1}{2} mr^2\left(2 - \frac{4}{\pi}\right)\dot{\theta}^2 + mgr\,\frac{\pi - 2}{\pi}\,(1 - \cos\theta)$$

Set $\dfrac{dE}{dt} = 0$ and assume small θ:

$$\ddot{\theta} + \frac{g}{2r}\theta = 0, \quad \underline{f_n = \frac{1}{2\pi}\sqrt{\frac{g}{2r}}}$$

(Same result as for full circular shape!)

8/130



$\Sigma F_y = 0 \Rightarrow N = mg$

$\Sigma F_x = m\ddot{x} : F - kx = m\ddot{x}$ (1)

$\circlearrowleft + \Sigma M_G = \bar{I}\ddot{\theta} : -Fr = \frac{1}{2}mr^2\ddot{\theta}$ (2)

Constraint : $\ddot{x} = r\ddot{\theta}$ (3)

Combine (1), (2), & (3):

$$\ddot{x} + \frac{2k}{3m}x = 0$$

$x = x_0 \sin \omega_n t$, $\ddot{x} = -x_0 \omega_n^2 \sin \omega_n t$, $\ddot{x}_{max} = x_0 \omega_n^2$

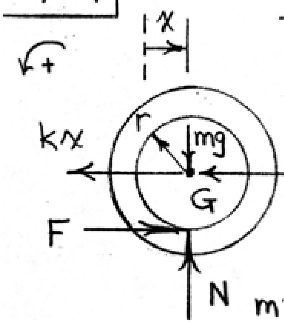
$$\ddot{\theta}_{max} = \frac{\ddot{x}_{max}}{r} = \frac{x_0 \omega_n^2}{r}$$

Eq. (2) : $|F_{max}| r = \frac{1}{2} mr^2 \ddot{\theta}_{max}$

$$\mu_s mg\, r = \frac{1}{2} mr^2 \frac{x_0 \omega_n^2}{r}$$

$$x_0 = \frac{2\mu_s g}{\omega_n^2} = \frac{2\mu_s g}{2k/3m} = \underline{\frac{3\mu_s mg}{k}}$$

8/131



$$\begin{cases} \Sigma F_x = m\ddot{x}: \; -kx - c\dot{x} + F = m\ddot{x} \\ \Sigma M_G = \bar{I}\alpha: \; Fr = mk_G^2\alpha \end{cases}$$

Roll with no slip: $\ddot{x} = -r\alpha$

The x-equation reduces to

$$m\left[1 + \frac{k_G^2}{r^2}\right]\ddot{x} + c\dot{x} + kx = 0$$

$$\omega_n = \sqrt{\frac{k}{m(1 + k_G^2/r^2)}} = \sqrt{\frac{15(12)}{\frac{20}{32.2}\left(1 + \frac{5.5^2}{6^2}\right)}} = 12.55 \frac{\text{rad}}{\text{sec}}$$

$$\zeta = \frac{c}{2\omega_n m\left(1 + \frac{k_G^2}{r^2}\right)} = \frac{2}{2(12.55)\left(\frac{20}{32.2}\right)\left(1 + \frac{5.5^2}{6^2}\right)}$$

$\underline{\zeta = 0.0697}$

8/132 $Q = (T+V)_1 - (T+V)_2$

But at x_1 & x_2, $\dot{x} = 0$ so $T_1 = T_2 = 0$ & $Q = V_1 - V_2 = \frac{1}{2}k(x_1^2 - x_2^2)$

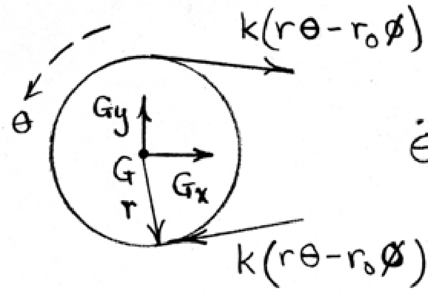
For the damped linear oscillator (case III, underdamped, of Art. 8/2b)

$$\frac{x_1}{x_2} = e^{\delta}$$

So $Q = \frac{1}{2}kx_1^2\left(1 - \left[\frac{x_2}{x_1}\right]^2\right) = \underline{\frac{1}{2}kx_1^2(1 - e^{-2\delta})}$

where $\underline{\delta = \frac{c}{2m}\tau_d = \pi \Big/ \sqrt{km/c^2 - \frac{1}{4}}}$

8/133



$$\Sigma M_G = \bar{I}\ddot{\theta}:$$

$$-2k(r\theta - r_0\phi)r = m\bar{k}^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{2kr^2}{m\bar{k}^2}\theta = \frac{2krr_0\phi_0}{m\bar{k}^2}\cos\omega t$$

Assume $\theta = \theta_{max}\cos\omega t$, substitute, and solve for $\theta_{MAX} = \phi_0 \dfrac{r_0/r}{1-\left(\frac{\omega}{\omega_n}\right)^2}$,

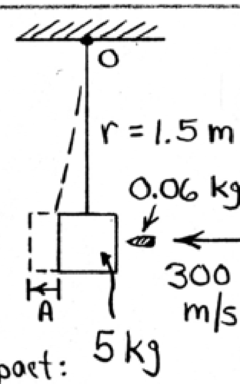
where $\omega_n = \dfrac{r}{\bar{k}}\sqrt{\dfrac{2k}{m}}$

8/134 For seismic instruments,

$$\frac{X}{b} = \frac{(\omega/\omega_n)^2}{\left\{\left[1-(\omega/\omega_n)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

For $X = 0.75$ mm, $\zeta = 0.5$, $\frac{\omega}{\omega_n} = \frac{\frac{180}{60}(3)}{3} = 3$,

solve for $b = \delta_0 = \underline{0.712 \text{ mm}}$



8/135 Angular momentum about O is conserved during impact:

$H_{O_2} = H_{O_1}$: $(5+0.06)vr = 0.06(300)r$

$v = 3.56$ m/s

With neglect of energy loss after impact:

$T_{max} = V_{max}$: $\frac{1}{2}(5.06)(3.56)^2 = \frac{1}{2}kA^2$

But $f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}$, so $4 = \frac{1}{2\pi}\sqrt{\frac{k}{5.06}}$

$k = 3200$ N/m

Then $A = 3.56\sqrt{\frac{5.06}{3200}} = \underline{0.1415 \text{ m}}$

For damped vibration, $\ln\left(\frac{x_0}{x_n}\right) = n\zeta\omega_n\tau_d$

$\ln\left(\frac{x_0}{x_n}\right) = n\zeta\omega_n\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$, where $n = 10$

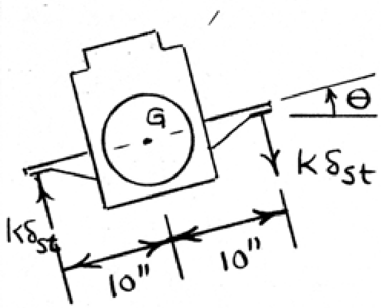
$\ln\left(\frac{1}{0.6}\right) = 10\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$, $\zeta = 0.00813$

$c = 2m\omega_n\zeta = 2(5.06)\left(\sqrt{\frac{3200}{5.06}}\right)0.00813 = \underline{2.07 \frac{\text{N}\cdot\text{s}}{\text{m}}}$

8/136 Vertical vibration:

$$(f_n)_y = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{2\pi}\sqrt{\frac{2(600)(12)}{480/32.2}} = \underline{4.95\text{ Hz}}$$

Rotation about G:



Force changes for position of static equilibrium.

$k\delta_{st} = 600(10\theta)$
$= 6000\theta$ lb
(θ small)

$$\Sigma M_G = I_G\ddot{\theta} : -2(6000\theta)\left(\frac{10}{12}\right) = \frac{48.0}{32.2}\left(\frac{4.60}{12}\right)^2\ddot{\theta}$$

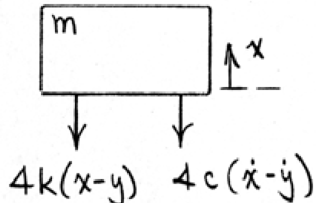
$$\ddot{\theta} + 4570\theta = 0, \quad \omega_n = 67.6 \text{ rad/sec}$$

$$(f_n)_\theta = \omega_n/2\pi = \underline{10.75\text{ Hz}}$$

Critical speed: $N = \omega_n = 10.75(60) = \underline{645\ \frac{\text{rev}}{\text{min}}}$

▶ 8/137 Let $y = y_0 \sin \omega t$ be the floor motion.

$\Sigma F_x = m\ddot{x}$ yields



$$m\ddot{x} + 4c\dot{x} + 4kx = 4ky + 4c\dot{y}$$

$$= 4ky_0 \sin \omega t + 4c\omega y_0 \cos \omega t$$

$$\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4(250{,}000)}{200}} = 70.71 \text{ rad/s}$$

$$\zeta = \frac{4c}{2m\omega_n} = \frac{4(1000)}{2(200)(70.71)} = 0.1414$$

Assume $x_p = X \sin(\omega t - \alpha)$ to obtain

$$X = \frac{\left[1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2\right]^{1/2} y_0}{\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

Originally, $X = 0.325\, y_0$

With damping doubled, $X' = 0.418\, y_0$

So amplitude increases by $\underline{28.9\ \%}$!

***8/138**

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{100(12)}{50/32.2}} = 27.8 \text{ rad/sec}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{18}{2\left(\frac{50}{32.2}\right)(27.8)} = 0.208$$

$$\frac{\omega}{\omega_n} = \frac{60}{27.8} = 2.158, \quad \omega_d = 27.8\sqrt{1-0.208^2} = 27.2 \text{ rad/sec}$$

$$X = \frac{160/1200}{\left\{\left[1-2.158^2\right]^2 + \left[2(0.208)(2.158)\right]^2\right\}^{1/2}} = 0.03539 \text{ ft}$$

$$\phi = \tan^{-1}\left[\frac{2(.208)(2.158)}{1-12.158^2}\right] = 2.90 \text{ rad}$$

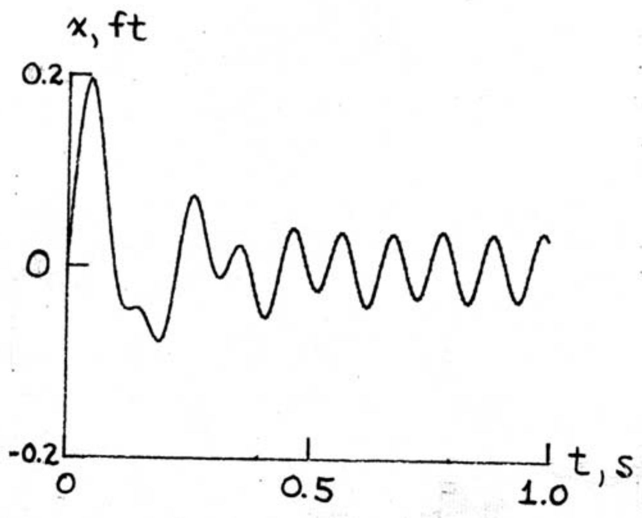
So $x = Ce^{-\zeta\omega_n t}\cos(\omega_d t - \psi) + X\cos(\omega t - \phi)$

$$= Ce^{-5.796t}\cos(27.2t - \psi) + 0.0354\cos(60t - 2.9)$$

Determine C and ψ : $\begin{cases} C = 0.212 \text{ ft} \\ \psi = 1.408 \text{ rad} \end{cases}$

$x_{max} = 0.1955$ ft @ $t = 0.046$ sec

$x_{min} = -0.079$ ft @ $t = 0.192$ sec



$*$8/139 For $\zeta = 1$, $x = (A_1 + A_2 t)e^{-\omega_n t}$

$x_0 = A_1$

$\dot{x} = -\omega_n (A_1 + A_2 t) e^{-\omega_n t} + A_2 e^{-\omega_n t}$

$\dot{x}_0 = -\omega_n A_1 + A_2 = -\omega_n x_0 + A_2 = 0$, $A_2 = \omega_n x_0$

So $x = x_0 (1 + \omega_n t) e^{-\omega_n t} = x_0 (1 + 4t) e^{-4t}$

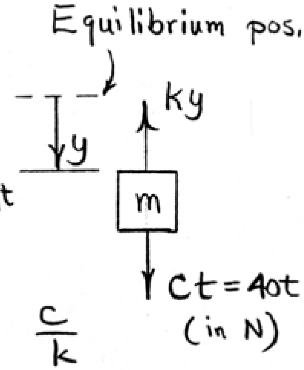
When $x = 0.1 x_0$, $0.1 x_0 = x_0 (1 + 4t) e^{-4t}$

or $f(t) = 4te^{-4t} + e^{-4t} - 0.1 = 0$

$f'(t) = -16 t e^{-4t}$

By Newton's method, $\underline{t = 0.972 \text{ s}}$

***8/140** $\Sigma F_y = m\ddot{y}$: $Ct - ky = m\ddot{y}$



$$\ddot{y} + \omega_n^2 y = \frac{Ct}{m}, \quad \omega_n^2 = \frac{k}{m}$$

$y = y_H + y_P$, $y_H = A\cos\omega_n t + B\sin\omega_n t$

Try $y_P = B_2 t$: $\omega_n^2 B_2 t = \frac{Ct}{m}$

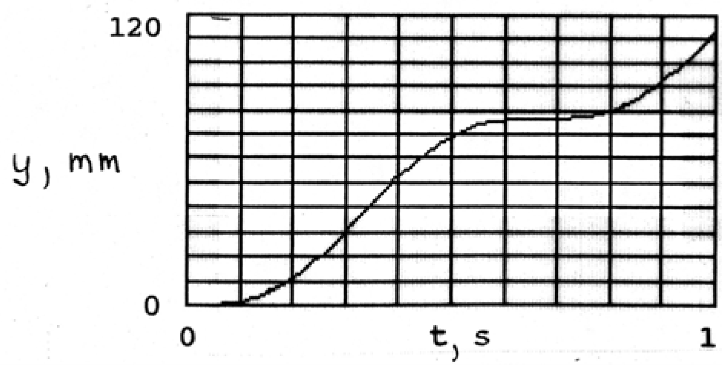
$$B_2 = \frac{C}{m\omega_n^2} = \frac{C}{k}$$

So $y = A\cos\omega_n t + B\sin\omega_n t + \frac{C}{k}t$

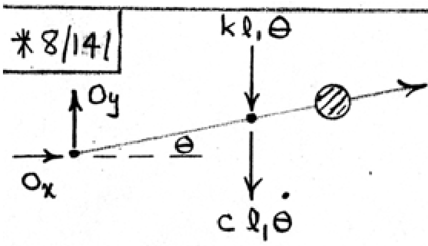
Quiet initial conditions yield $y = \frac{C}{k}\left[t - \frac{1}{\omega_n}\sin\omega_n t\right]$

With $C = 40\ \frac{N}{s}$, $k = 350$ N/m, $\omega_n = \sqrt{k/m} = 9.35$ rad/s:

$y = 114.3\ [t - 0.1069 \sin 9.35t]$ (in mm)



*8/141



Eq. 4/13 : (with $P=0$)

$$\Sigma \underline{M}_O = \dot{\underline{H}}_{O_r} + \bar{\underline{\rho}} \times m\underline{a}_O$$

For small θ,

$$-k l_1^2 \theta - c l_1^2 \dot\theta = m l_2^2 \ddot\theta + m l_2 \ddot y_B$$

or $$\ddot\theta + \frac{c l_1^2}{m l_2^2}\dot\theta + \frac{k l_1^2}{m l_2^2}\theta = \frac{b}{l_2}\omega^2 \sin \omega t$$

Steady-state amplitude :

$\Theta = M b \left(\frac{\omega}{\omega_n}\right)^2 \frac{1}{l_2}$, where M = magnification factor

Pen amplitude $= l_3 \Theta = M b \left(\frac{\omega}{\omega_n}\right)^2 \frac{l_3}{l_2} = A$

Set up computer program to determine range of k for which $A \leq 1.5b$. Note that $\omega_n = \frac{l_1}{l_2}\sqrt{\frac{k}{m}}$, $2\zeta\omega_n = \frac{c l_1^2}{m l_2^2}$ or

$\zeta = \frac{c l_1}{2 l_2}\sqrt{\frac{1}{km}}$. Answer : $\underline{0 < k < 1.895 \ \frac{\text{lb}}{\text{ft}}}$

$\boxed{8/142}$ Eq. 8/9 : $\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 0$

Solution : $y = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

where $\lambda_{1,2} = \omega_n\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$

$\omega_n = \sqrt{k/m} = \sqrt{800/4} = 14.14$ rad/s ; $\zeta = \dfrac{c}{2m\omega_n}$

(a) $\zeta = \dfrac{124}{2(4)(14.14)} = 1.096$; (b) $\zeta = \dfrac{80}{2(4)(14.14)} = 0.707$

(a) $\zeta > 1$ (overdamped)

$\lambda_1 = 14.14\left(-1.096 + \sqrt{1.096^2 - 1}\right) = -9.16 \text{ s}^{-1}$

$\lambda_2 = 14.14\left(-1.096 - \sqrt{1.096^2 - 1}\right) = -21.8 \text{ s}^{-1}$

Initial condition considerations :

$$\left.\begin{aligned} y_0 &= 0.1 = A_1 + A_2 \\ \dot{y}_0 &= 0 = A_1\lambda_1 + A_2\lambda_2 \end{aligned}\right\} \Rightarrow \begin{aligned} A_1 &= 0.1722 \text{ m} \\ A_2 &= -0.0722 \text{ m} \end{aligned}$$

Solution : $\underline{y = 0.1722e^{-9.16t} - 0.0722e^{-21.8t} \text{ m}}$

(b) $\zeta < 1$ (underdamped)

Eq. (8/12): $y = Ce^{-\zeta\omega_n t}\sin[\omega_d t + \psi]$

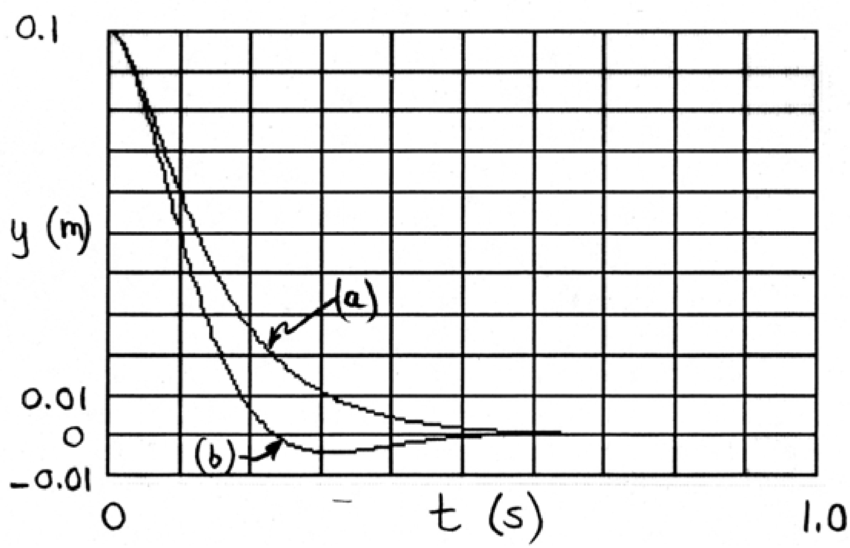
$\omega_d = \omega_n\sqrt{1-\zeta^2} = 14.14\sqrt{1-0.707^2} = 10 \text{ rad/s}$

Initial condition considerations:

$$\left.\begin{aligned} y_0 &= 0.1 = C\sin\psi \\ \dot{y}_0 &= 0 = -\zeta\omega_n C\sin\psi + C\omega_d\cos\psi \end{aligned}\right\} \begin{aligned} C &= 0.1414 \text{ m} \\ \psi &= 0.785 \text{ rad} \end{aligned}$$

Solution: $y = 0.1414e^{-0.707(14.14)t}\sin[10t + 0.785]$

$= \underline{0.1414e^{-10t}\sin[10t + 0.785] \text{ m}}$



***8/143**

$\Sigma F_x = m\ddot{x}: \ bt - kx = m\ddot{x},$

$\ddot{x} + \frac{k}{m}x = \frac{bt}{m}$

Sol. is $x = x_c + x_p$ where

$x_c = C_1 \sin\omega_n t + C_2 \cos\omega_n t, \ x_p = C_3 t$ with $C_3 = \frac{b}{k}$

so $x = C_1 \sin\omega_n t + C_2 \cos\omega_n t + \frac{b}{k}t$

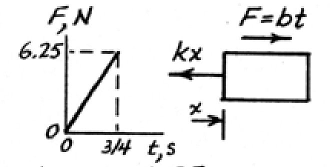
When $t = 0$, $\dot{x} = 0$ & $x = 0$ giving $C_1 = -\frac{b}{\omega_n k}$,

$C_2 = 0$

& $x = -\frac{b}{\omega_n k}\sin\omega_n t + \frac{b}{k}t = \frac{b}{k}\left(t - \frac{1}{\omega_n}\sin\omega_n t\right)$

where $\omega_n = \sqrt{k/m} = \sqrt{90/0.75} = 10.95$ rad/s, $\frac{1}{\omega_n} = 0.0913$ s

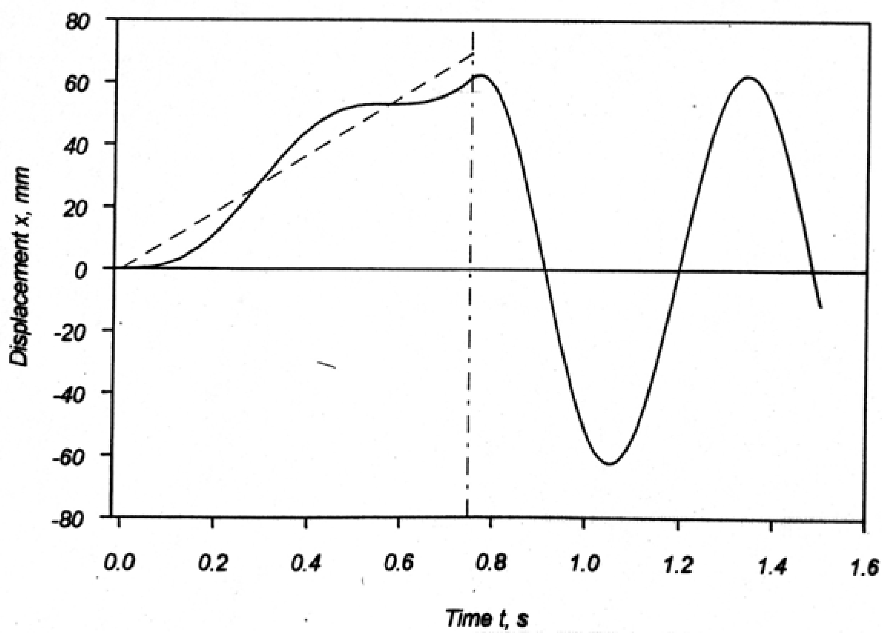
Thus $\underline{x = 0.0926(t - 0.0913 \sin 10.95t) \text{ m}}$ for first 3/4 s



$b = \frac{6.25}{3/4} = 8.33$ N/s

$k = 90$ N/m

$b/k = \frac{8.33}{90} = 0.0926$ m/s



*8/144 $I = \int F dt = m\dot{x}$, $8 = 4\dot{x}$, $\dot{x} = 2$ m/s at $t \approx 0$

After impulse, oscillator obeys Eq. 8/9 with $\zeta = 0.1 < 1$ so underdamped with solution given by Eq. 8/12

$$x = Ce^{-\zeta\omega_n t}\sin(\omega_d t + \psi), \quad C, \psi \text{ constants}$$

$$\dot{x} = -C\zeta\omega_n e^{-\zeta\omega_n t}\sin(\omega_d t + \psi) + Ce^{-\zeta\omega_n t}\omega_d\cos(\omega_d t + \psi)$$

where $\omega_n = \sqrt{k/m} = \sqrt{200/4} = 7.07$ rad/s,

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 7.07\sqrt{1-0.1^2} = 7.04 \text{ rad/s}$$

When $t = 0$, $x = 0$, so $0 = C\sin\psi$, $\psi = 0$

" " , $\dot{x} = 2$ m/s, so $2 = -C(0.1)(7.07)(0) + C \times 7.04$, $C = 0.284$ m

Thus $\underline{x = 0.284\, e^{-0.707t}\sin 7.04t}$