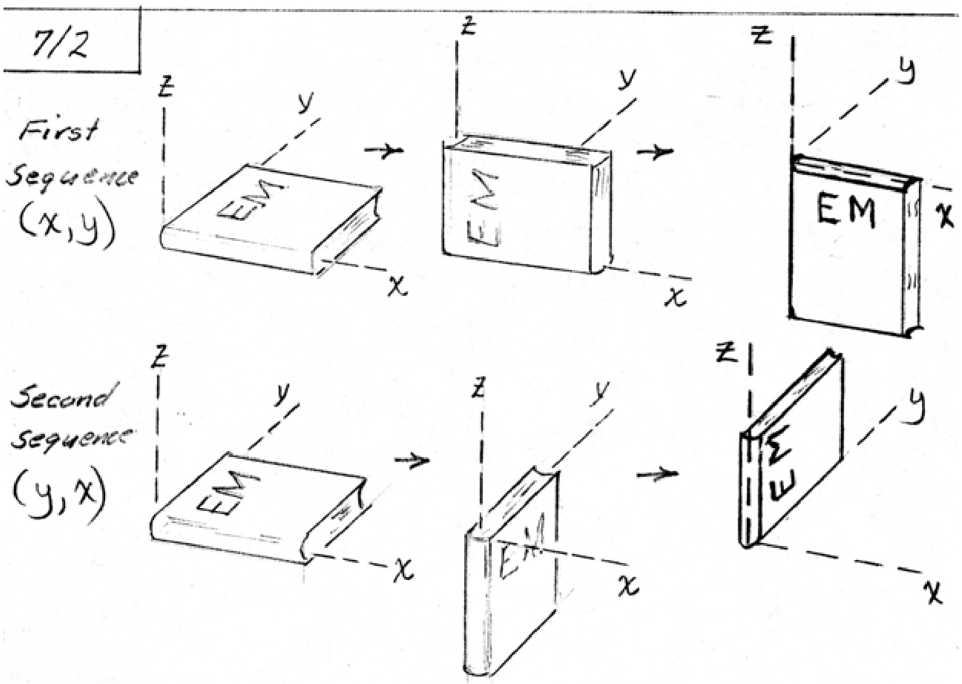
7/1 Line O-1 can rotate through π radians (180°) about any axis through O which lies in the x-z plane.
Line O-2 can rotate through π radians about any axis through O which lies in a plane perpendicular to the line from 2 to 2'.
The intersection of these planes is the unique axis along the 45° line in the x-z plane. Thus $\underline{\theta} = \pi\left(\frac{\underline{i}}{\sqrt{2}} + \frac{\underline{k}}{\sqrt{2}}\right)$

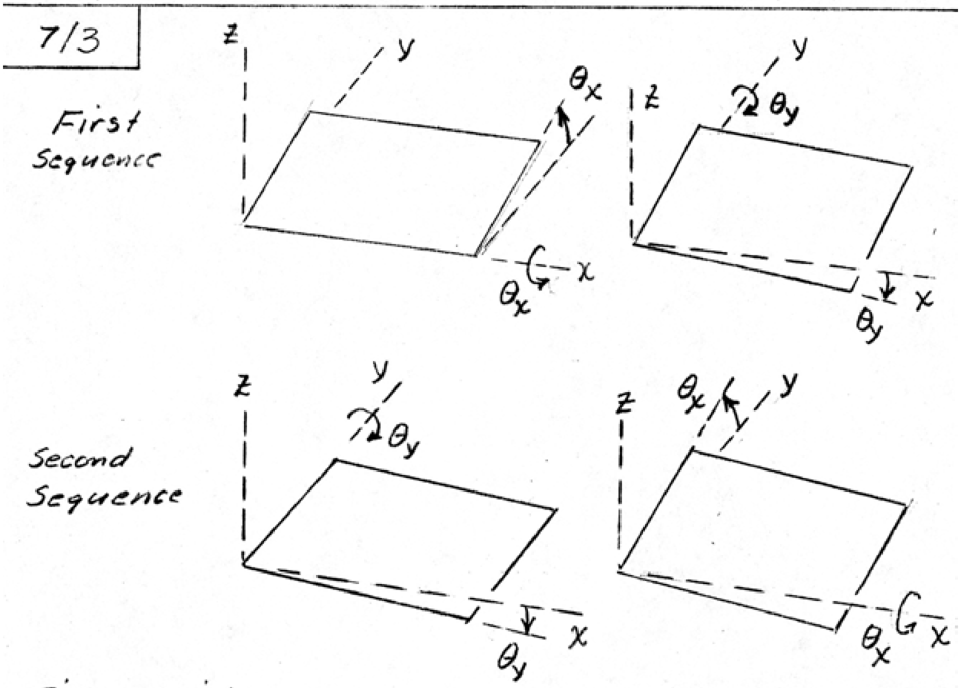
$$\underline{\theta} = \frac{\pi}{\sqrt{2}}(\underline{i} + \underline{k})$$

7/2



Final positions are different so finite rotations cannot be added as proper vectors

7/3



Final positions essentially the same - the more so the smaller the angle. Infinitesimal angles add as proper vectors.

7/4

$$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}), \quad \underline{r} = \vec{OC}, \quad \dot{\underline{\omega}} = \underline{0}$$

$$\underline{r} = 10(2\underline{i} + 0\underline{j} + 8\underline{k}) \text{ mm}, \quad \underline{\omega} = 30(3\underline{i} + 2\underline{j} + 6\underline{k}) \text{ rad/s}$$

$$\underline{v} = \underline{\omega} \times \underline{r} = 300 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 2 & 6 \\ 2 & 0 & 8 \end{vmatrix} = 300(16\underline{i} - 12\underline{j} - 4\underline{k}) \ \frac{\text{mm}}{\text{s}}$$

$$\underline{a} = \underline{\omega} \times \underline{v} = 30(300) \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 2 & 6 \\ 16 & -12 & -4 \end{vmatrix} = 9000(64\underline{i} + 108\underline{j} - 68\underline{k}) \ \text{mm/s}^2$$

$$a = 9\sqrt{64^2 + 108^2 + (-68)^2} = 9\sqrt{20384} = \underline{1285 \text{ m/s}^2}$$

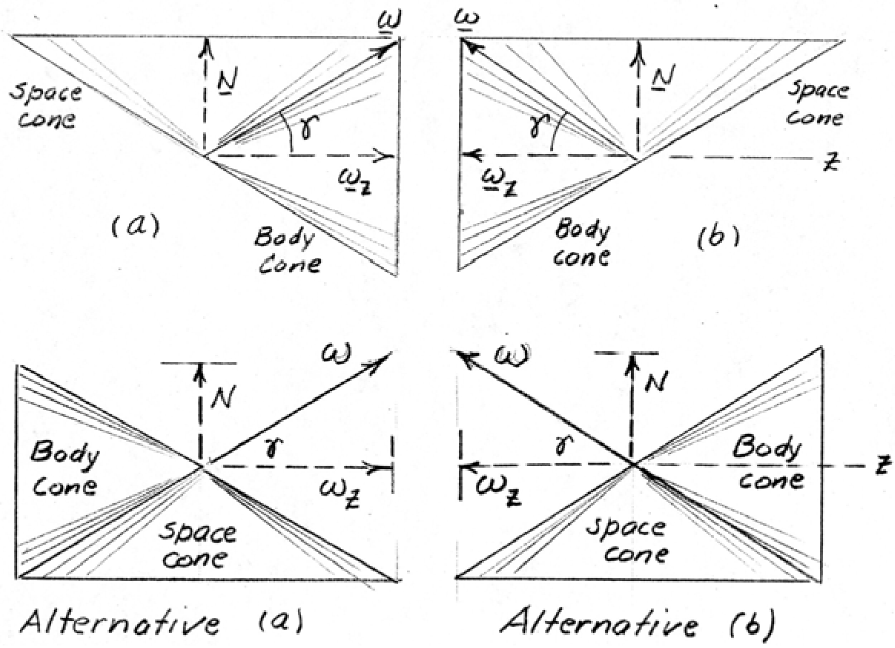
7/5 $\underline{v}_A = \underline{\omega} \times \underline{r} = (-4\underline{j} - 3\underline{k}) \times (0.5\underline{i} + 1.2\underline{j} + 1.1\underline{k})$

$$= \underline{-0.8\underline{i} - 1.5\underline{j} + 2\underline{k} \text{ m/s}}$$

The rim speed of any point B is

$$v_B = \sqrt{0.8^2 + 1.5^2 + 2^2} = \underline{2.62 \text{ m/s}}$$

7/6 $\tan\gamma = \dfrac{N}{\omega_z} = \dfrac{10}{15} = 0.667, \quad \gamma = 33.7°$



7/7 $\underline{r}_{OA} = \underline{r} = 0.260\underline{i} + 0.240\underline{j} + 0.473\underline{k}$ m

Unit vector along OB is

$$\underline{n} = (0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k})/\sqrt{0.2^2 + 0.4^2 + 0.3^2}$$

$$\underline{\omega} = \omega\underline{n} = \frac{1200(2\pi)}{60}\ \frac{0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}}{0.539}$$

$$= 233(0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}) \text{ rad/s}$$

$$\underline{v} = \underline{\omega} \times \underline{r} = 233(0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}) \times (0.260\underline{i} + 0.240\underline{j} + 0.473\underline{k})$$

$$= 233(0.1172\underline{i} - 0.0166\underline{j} - 0.056\underline{k}) \text{ m/s}$$

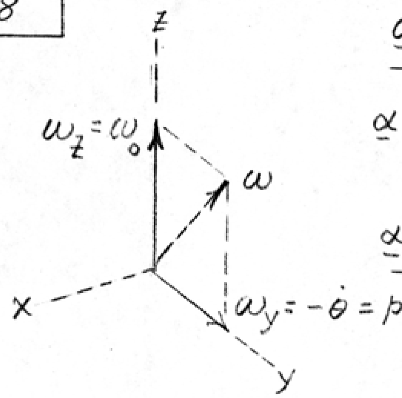
$$= \underline{27.3\underline{i} - 3.87\underline{j} - 13.07\underline{k} \text{ m/s}}$$

$$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = 0 + \underline{\omega} \times \underline{v}$$

$$= 233(0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}) \times (27.3\underline{i} - 3.87\underline{j} - 13.07\underline{k})$$

$$= \underline{-949\underline{i} + 2520\underline{j} - 2730\underline{k} \text{ m/s}^2}$$

7/8



$$\underline{\omega} = p\underline{j} + \omega_0\underline{k}$$

$$\underline{\alpha} = \underline{\omega}_z \times \underline{\omega} = \underline{\omega}_z \times \underline{\omega}_y$$

$$= \omega_0 \underline{k} \times p\underline{j}$$

$$\underline{\alpha} = -p\omega_0\underline{i}$$

7/9

$\underline{\omega} \cdot \underline{v} = 0, \quad 10(\underline{i} + 2\underline{j} + 2\underline{k}) \cdot (120\underline{i} - 80\underline{j} + v_z\underline{k}) = 0$

$120 - 160 + 2v_z = 0, \quad v_z = 20$ in./sec

$v = \sqrt{120^2 + 80^2 + 20^2} = \underline{145.6 \text{ in./sec}}$

$v = R\omega, \quad R = \dfrac{145.6}{30} = \underline{4.85 \text{ in.}}$

where $\omega = 10\sqrt{1^2 + 2^2 + 2^2} = 10(3) = 30$ rad/sec

$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{0} + \underline{\omega} \times \underline{v}$

$= 10(\underline{i} + 2\underline{j} + 2\underline{k}) \times (120\underline{i} - 80\underline{j} + 20\underline{k})$

$= 10(200\underline{i} + 220\underline{j} - 320\underline{k})$

$a = 10\sqrt{200^2 + 220^2 + 320^2} = 10\sqrt{190800} = \underline{4370 \text{ in./sec}^2}$

(or simply $a = a_n = r\omega^2 = 4.85(30^2) = 4370$ in./sec^2)

7/10 $\underline{\alpha} = \underline{\Omega} \times \underline{\omega} = 0.6\,\underline{k} \times 2\,\underline{j} = \underline{-1.2\,\underline{i}}$ rad/sec^2

$\underline{a}_p = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$, $\underline{\omega} = \underline{\Omega} + \underline{\omega}_0$

$\dot{\underline{\omega}} = \underline{\alpha} = -1.2\,\underline{i}$ rad/sec^2

$\underline{r} = 34\underline{j} + 20\,\underline{k}$ in. (for $\beta = 90°$)

Carry out algebra to obtain

$\underline{a_p = 35.8\underline{j} - 80\,\underline{k}}$ in./sec^2

7/11 $\underline{v} = \underline{\omega} \times \underline{r} \qquad \underline{a} = \underline{\omega} \times (\underline{\omega} \times \underline{r})$ where $\dot{\underline{\omega}} = \underline{0}$

$$\underline{\omega} = 20\left(\frac{4}{5}\underline{j} + \frac{3}{5}\underline{k}\right) = 4(4\underline{j} + 3\underline{k}) \text{ rad/sec}$$

$$\underline{r}_A = 1.5\underline{i} + 4.75\underline{j} + 2\underline{k} \text{ in.}$$

Thus $\underline{v} = 4(4\underline{j} + 3\underline{k}) \times (1.5\underline{i} + 4.75\underline{j} + 2\underline{k})$

$$= 4(-6.25\underline{i} + 4.5\underline{j} - 6\underline{k}) \text{ in./sec}$$

$$v = 4\sqrt{6.25^2 + 4.5^2 + 6^2} = 4(9.76) = \underline{39.1 \text{ in./sec}}$$

$$\underline{a} = \underline{\omega} \times \underline{v} = 4(4\underline{j} + 3\underline{k}) \times 4(-6.25\underline{i} + 4.5\underline{j} - 6\underline{k})$$

$$= 16(-37.5\underline{i} - 18.75\underline{j} + 25\underline{k})$$

$$a = 16\sqrt{37.5^2 + 18.75^2 + 25^2}$$

$$= 16(48.8) = \underline{781 \text{ in./sec}^2}$$

$v = R\omega, \quad R = 39.1/20 = \underline{1.953 \text{ in.}}$

7/12 $\underline{\alpha} = \underline{\omega}_x \times \underline{\omega}_z = -\dot{\gamma}\underline{i} \times \omega_0 \underline{k} = -3\pi\underline{i} \times 4\pi\underline{k}$

$$= 12\pi^2 \underline{j} \text{ rad/sec}^2$$

$\underline{r} = 5\underline{j} + 10\underline{k}$ in.

$$\underline{v} = \underline{\omega} \times \underline{r} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -3\pi & 0 & 4\pi \\ 0 & 5 & 10 \end{vmatrix} = 5\pi(-4\underline{i} + 6\underline{j} - 3\underline{k}) \text{ in./sec}$$

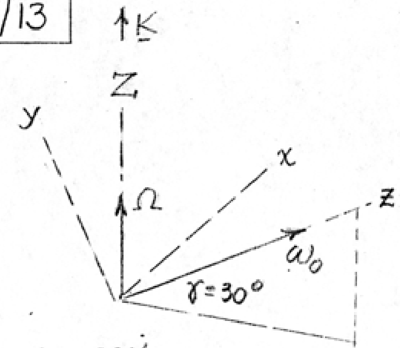
$$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{\alpha} \times \underline{r} + \underline{\omega} \times \underline{v}$$

$$= 12\pi^2 \underline{j} \times (5\underline{j} + 10\underline{k}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -3\pi & 0 & 4\pi \\ -4 & 6 & -3 \end{vmatrix} 5\pi$$

$$= 120\pi^2 \underline{i} - 120\pi^2 \underline{i} - 125\pi^2 \underline{j} - 90\pi^2 \underline{k}$$

$$= -5\pi^2(25\underline{j} + 18\underline{k}) \text{ in./sec}^2$$

7/13



$\omega_0 = \alpha_0 t$

when $t = 2$ sec, $\omega_0 = \frac{3000(2\pi)}{60}$

$= 100\pi$ rad/sec

so $\alpha_0 = 100\pi/2 = 50\pi$ rad/sec^2

$\omega_0 = 50\pi t$

$\underline{\omega}_0 = 50\pi t \underline{k}$ rad/sec

$\underline{\alpha} = \dot{\underline{\omega}}_0 = 50\pi \underline{k} + 50\pi t \dot{\underline{k}}$

But $\dot{\underline{k}} = \underline{\Omega} \times \underline{k}$

$= \pi \underline{K} \times \underline{k}$

$= (\sqrt{3}/2)\pi \underline{i}$

$\Omega = 30 \times 2\pi/60$

$= \pi$ rad/sec

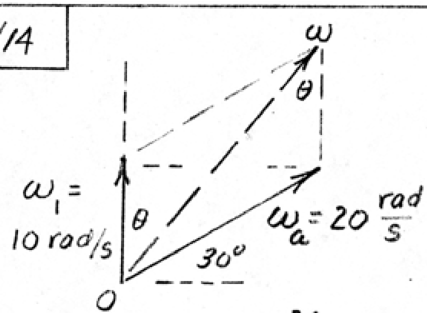
Thus for $t = 1/3$ s,

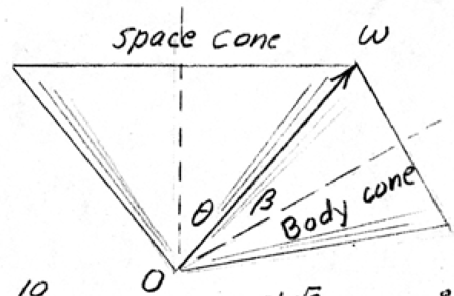
$\underline{\alpha} = 50\pi \underline{k} + 50\pi(\frac{1}{3})(\sqrt{3}/2)\pi \underline{i}$

$= 50\pi(\frac{\pi}{2\sqrt{3}} \underline{i} + \underline{k})$ rad/sec^2

(Note: Total angular velocity is $\underline{\omega} = \underline{\Omega} + \underline{\omega}_0$
& $\underline{\alpha} = \dot{\underline{\omega}} = \dot{\underline{\Omega}} + \dot{\underline{\omega}}_0 = \underline{0} + \dot{\underline{\omega}}_0$)

7/14

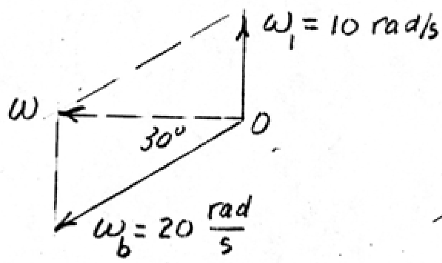


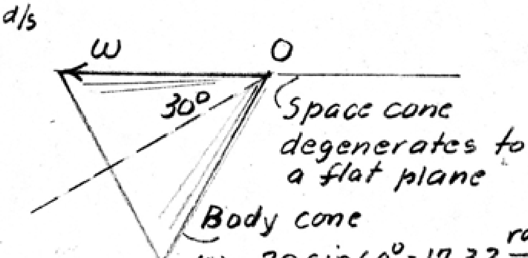


Law of sines $\frac{20}{\sin\theta} = \frac{10}{\sin(60-\theta)}$, $\theta = \tan^{-1}\frac{\sqrt{3}}{2} = 40.9°$

$\omega = \sqrt{(20\cos 30°)^2 + (20\sin 30° + 10)^2}$, $\beta = 60° - \theta = 19.1°$

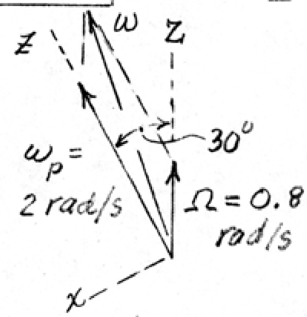
$\underline{\omega = 26.5 \text{ rad/s}}$





$\omega = 20\sin 60° = \underline{17.32 \frac{\text{rad}}{\text{s}}}$

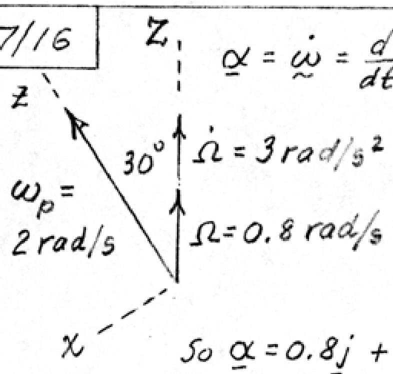
7/15



$$\underline{\omega} = \underline{\omega}_p + \underline{\Omega}$$
$$= 2\underline{k} + 0.8\cos 30^\circ \underline{k} - 0.8\sin 30^\circ \underline{i}$$
$$= \underline{-0.4\underline{i} + 2.69\underline{k} \text{ rad/s}}$$

$$\underline{\alpha} = \underline{\Omega} \times \underline{\omega}_p$$
$$= 0.8(-0.5\underline{i} + 0.866\underline{k}) \times 2\underline{k}$$
$$= 1.6(0.5\underline{j} + 0)$$
$$\underline{\underline{\alpha} = 0.8\underline{j} \text{ rad/s}^2}$$

7/16



$\underline{\alpha} = \dot{\underline{\omega}} = \frac{d}{dt}(\underline{\omega}_p + \underline{\Omega}) = \underline{\Omega} \times \underline{\omega}_p + \dot{\underline{\Omega}}$

$\underline{\Omega} \times \underline{\omega}_p =$

$0.8(\cos 30° \underline{k} - \sin 30° \underline{i}) \times 2\underline{k}$

$= 0.8\underline{j}$ rad/s²

So $\underline{\alpha} = 0.8\underline{j} + 3(\cos 30° \underline{k} - \sin 30° \underline{i})$

$= \underline{-1.5\underline{i} + 0.8\underline{j} + 2.60\underline{k}}$ rad/s²

7/17

$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2 = 2\underline{k} + 1.5\underline{i}$

$\omega = \sqrt{2^2 + 1.5^2} = \underline{2.5 \text{ rad/s}}$

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega}_2 = 2\underline{k} \times 1.5\underline{i} = \underline{3\underline{j} \text{ rad/s}^2}$

7/18

$$\underline{\omega} = \underline{\omega}_1 + \omega_s$$

$$= 2\underline{k} + 0.8(\underline{j}\cos 30° + \underline{k}\sin 30°)$$

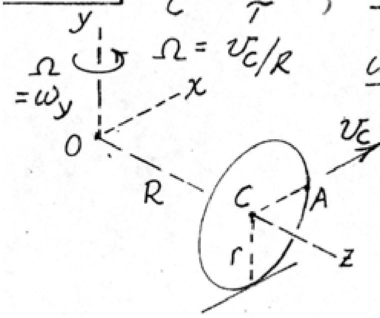
$$\underline{\underline{\omega} = 0.693\underline{j} + 2.40\underline{k} \text{ rad/s}}$$

$$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega}_s = 2\underline{k} \times 0.8(\underline{j}\cos 30° + \underline{k}\sin 30°)$$

$$\underline{\underline{\alpha} = -1.386\underline{i} \text{ rad/s}^2}$$

7/19

$v_C = \frac{2\pi R}{\tau}$; $\underline{\omega}_z = -\frac{v_c}{r}\underline{k} = -\frac{2\pi R}{\tau r}\underline{k}$



$\Omega = v_c/R$

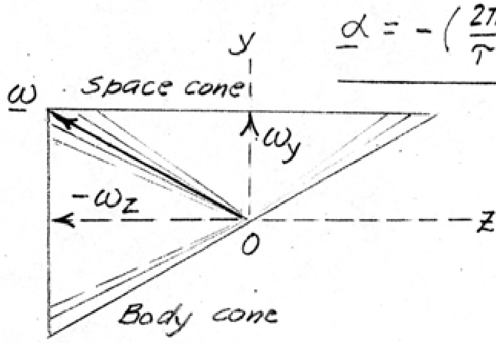
$\underline{\omega} = \underline{\omega}_y + \underline{\omega}_z = \frac{v_c}{R}\underline{j} - \frac{v_c}{r}\underline{k}$

$|\underline{\omega}| =$ const. so $\underline{\alpha} = \underline{\Omega}\times\underline{\omega}$

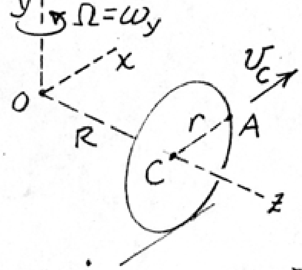
$$\underline{\alpha} = \frac{v_c}{R}\underline{j}\times\left(\frac{v_c}{R}\underline{j} - \frac{v_c}{r}\underline{k}\right)$$

$$= -\frac{v_c^2}{rR}\underline{i} = -\left(\frac{2\pi R}{\tau}\right)^2\frac{1}{rR}\underline{i}$$

$$\underline{\alpha} = -\left(\frac{2\pi}{\tau}\right)^2\frac{R}{r}\underline{i}$$



7/20 $v_C = \frac{2\pi R}{\tau}$; $\omega_y = \Omega = \frac{v_C}{R}$, $\omega_z = -\frac{v_C}{R}$



$$\underline{\omega} = \underline{\omega}_y + \underline{\omega}_z = v_C\left(\frac{1}{R}\underline{j} - \frac{1}{r}\underline{k}\right)$$

$$\underline{v}_A = \underline{\omega}\times\underline{r}_A = v_C\left(\frac{1}{R}\underline{j} - \frac{1}{r}\underline{k}\right)\times\left(r\underline{i} + R\underline{k}\right)$$

$$= \frac{2\pi R}{\tau}\left(\underline{i} - \underline{j} - \frac{r}{R}\underline{k}\right)$$

$$\underline{a} = \dot{\underline{\omega}}\times\underline{r}_A + \underline{\omega}\times\underline{v}_A$$

$$\dot{\underline{\omega}} = v_C\left(\frac{1}{R}\dot{\underline{j}} - \frac{1}{r}\dot{\underline{k}}\right) = v_C\left(\underline{0} - \frac{1}{r}[\Omega\underline{j}\times\underline{k}]\right) = -\left(\frac{2\pi}{\tau}\right)^2\frac{R}{r}\underline{i}$$

$$\dot{\underline{\omega}}\times\underline{r}_A = -\left(\frac{2\pi}{\tau}\right)^2\frac{R}{r}\underline{i}\times\left(r\underline{i} + R\underline{k}\right) = \left(\frac{2\pi}{\tau}\right)^2\frac{R^2}{r}\underline{j}$$

$$\underline{\omega}\times\underline{v}_A = \frac{2\pi R}{\tau}\left(\frac{1}{R}\underline{j} - \frac{1}{r}\underline{k}\right)\times\frac{2\pi R}{\tau}\left(\underline{i} - \underline{j} - \frac{r}{R}\underline{k}\right)$$

$$= \left(\frac{2\pi}{\tau}\right)^2 R^2\left(-\left[\frac{1}{r} + \frac{r}{R^2}\right]\underline{i} - \frac{1}{r}\underline{j} - \frac{1}{R}\underline{k}\right)$$

Combine & get $\underline{a} = -\left(\frac{2\pi}{\tau}\right)^2 R\left[\left(\frac{R}{r} + \frac{r}{R}\right)\underline{i} + \underline{k}\right]$

7/21

$\underline{r} = \overrightarrow{OB} = -120 \sin 30^\circ \underline{i} + 120 \cos 30^\circ \underline{j} + 200\underline{k}$ mm

$= -60\underline{i} + 103.9\underline{j} + 200\underline{k}$ mm

$\underline{\omega} = \underline{\omega}_x + \underline{\omega}_z = 10\underline{i} + 20\underline{k}$ rad/s

$\underline{v} = \underline{\omega} \times \underline{r} = 10(\underline{i} + 2\underline{k}) \times (-60\underline{i} + 103.9\underline{j} + 200\underline{k})$

$= 10(-208\underline{i} - 320\underline{j} + 103.9\underline{k})$

$v = 10\sqrt{208^2 + 320^2 + 103.9^2} = 3950$ mm/s

or $\underline{v = 3.95 \text{ m/s}}$

$a = \dot{\omega} \times r + \omega \times (\omega \times r)$

where $\underline{\dot{\omega}} = \underline{\alpha} = \underline{\omega}_x \times \underline{\omega} = \underline{\omega}_x \times \underline{\omega}_z = 10\underline{i} \times 20\underline{k} = -200\underline{j}$ $\frac{\text{rad}}{\text{s}^2}$

$\underline{\dot{\omega}} \times \underline{r} = -200\underline{j} \times (-60\underline{i} + 103.9\underline{j} + 200\underline{k})$

$= -4000(10\underline{i} + 3\underline{k})$ mm/s²

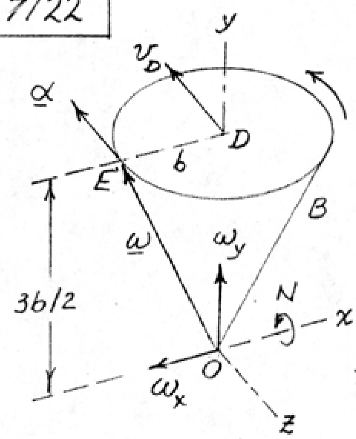
$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{\omega} \times \underline{v} = 10(\underline{i} + 2\underline{k}) \times 10(-208\underline{i} - 320\underline{j} + 103.9\underline{k})$

$= 100(640\underline{i} - 520\underline{j} - 320\underline{k})$

$\underline{a} = 24.0\underline{i} - 52.0\underline{j} - 44.0\underline{k}$ m/s²

$a = \sqrt{24.0^2 + 52.0^2 + 44.0^2} = \underline{72.2 \text{ m/s}^2}$

7/22



$v_D = \dfrac{3b}{\ell}\,\dfrac{60(2\pi)}{60} = 3\pi b$; $v_E = 0$ (Gear C fixed)

$\omega_y = \dfrac{v_D}{b} = 3\pi$ rad/s

$\omega_x = -\dfrac{60(2\pi)}{60} = -2\pi$ rad/s

$\underline{\omega} = -2\pi\underline{i} + 3\pi\underline{j} = \underline{\pi(-2\underline{i} + 3\underline{j})}$ rad/s

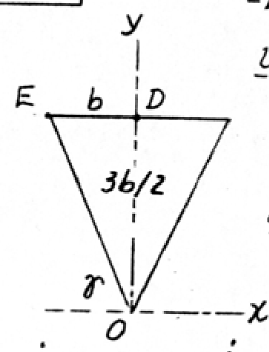
$\underline{\alpha} = \dot{\underline{\omega}}$; $\dot{\underline{i}} = 0$, $\dot{\underline{j}} = \underline{\omega} \times \underline{j} = -2\pi\underline{k}$

so $\underline{\alpha} = 0 + 3\pi(-2\pi\underline{k}) = \underline{-6\pi^2\underline{k}}$ rad/s²

Body cone is the pitch cone of gear B

Space " " " " " " " C

7/23



$$\underline{v}_E = \frac{3b}{2}\,\frac{20(2\pi)}{60}(-\underline{k}) = -b\pi\underline{k}$$

$$\underline{v}_D = \frac{3b}{2}\,\frac{60(2\pi)}{60}(-\underline{k}) = -3b\pi\underline{k}$$

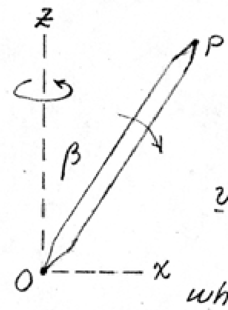
$$\underline{\omega}_y = \frac{v_D - v_E}{b}\underline{j} = \frac{3\pi - \pi}{b}b\underline{j} = 2\pi\underline{j} \quad \text{rad/s}$$

$$\underline{\omega}_x = -\frac{60(2\pi)}{60}\underline{i} = -2\pi\underline{i} \quad \text{rad/s}$$

$$\underline{\omega} = -2\pi\underline{i} + 2\pi\underline{j} = \underline{2\pi(-\underline{i}+\underline{j}) \text{ rad/s}}$$

$$\underline{\alpha} = \dot{\underline{\omega}} = \underline{0} + 2\pi\dot{\underline{j}} = 2\pi(-2\pi\underline{k}) = \underline{-4\pi^2\underline{k} \quad \text{rad/s}^2}$$

7/24 $\overline{OP} = 24\ m$, $\dot{\beta} = 0.10\ rad/s$ const., $\beta = 30°$



$\underline{r} = \overrightarrow{OP} = (24 \sin 30°)\underline{i} + (24 \cos 30°)\underline{k}$

$= 12\underline{i} + 20.78\underline{k}\ m$

$\underline{\omega} = \frac{2(2\pi)}{60}\underline{k} + 0.10\underline{j} = 0.209\underline{k} + 0.10\underline{j}\ \frac{rad}{s}$

$\underline{v} = \underline{\omega} \times \underline{r} = (0.209\underline{k} + 0.10\underline{j}) \times (12\underline{i} + 20.78\underline{k})$

$= 2.078\underline{i} + 2.513\underline{j} - 1.2\underline{k}\ m/s$

where $v = |\underline{v}| = \sqrt{(2.078)^2 + (2.513)^2 + (-1.2)^2} = \underline{3.48\ \frac{m}{s}}$

$\underline{a} = \underline{\dot{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{\alpha} \times \underline{r} + \underline{\omega} \times \underline{v}$

$\underline{\alpha} = \underline{\dot{\omega}} = \underline{\omega}_z \times \underline{\omega}_y = 0.209\underline{k} \times 0.10\underline{j} = -0.0209\underline{i}\ rad/s^2$

$\underline{\dot{\omega}} \times \underline{r} = \underline{\alpha} \times \underline{r} = -0.0209\underline{i} \times (12\underline{i} + 20.78\underline{k}) = 0.435\underline{j}\ m/s^2$

$\underline{\omega} \times \underline{v} = (0.209\underline{k} \times 0.10\underline{j}) \times (2.078\underline{i} + 2.513\underline{j} - 1.2\underline{k})$

$= -0.646\underline{i} + 0.435\underline{j} - 0.208\underline{k}\ m/s^2$

$\underline{a} = -0.646\underline{i} + 0.870\underline{j} - 0.208\underline{k}\ m/s^2$

$a = |\underline{a}| = \sqrt{(-0.646)^2 + (0.870)^2 + (-0.208)^2} = \underline{1.104\ m/s^2}$

7/25 $\underline{\omega} = \Omega \underline{k} + \dot{\gamma}\underline{i} - \omega_0 \cos\gamma \underline{j} - \omega_0 \sin\gamma \underline{k}$

$\underline{\alpha} = \dot{\underline{\omega}} = \Omega \dot{\underline{k}} + \dot{\gamma}\dot{\underline{i}} + \omega_0 \dot{\gamma} \sin\gamma \underline{j} - \omega_0 \cos\gamma \dot{\underline{j}}$

$\quad - \omega_0 \dot{\gamma} \cos\gamma \underline{k} - \omega_0 \sin\gamma \dot{\underline{k}}$

where $\Omega = 4$ rad/s const.

$\omega_0 = 3$ rad/s " $\qquad \gamma = 30°$

$\dot{\gamma} = -\pi/4$ rad/s "

& $\dot{\underline{i}} = \underline{\Omega} \times \underline{i} = \Omega \underline{k} \times \underline{i} = \Omega \underline{j}$; $\dot{\underline{j}} = \underline{\Omega} \times \underline{j} = \Omega \underline{k} \times \underline{j} = -\Omega \underline{i}$; $\dot{\underline{k}} = \Omega \underline{k} \times \underline{k} = \underline{0}$

so $\underline{\alpha} = \underline{0} + \dot{\gamma}\Omega \underline{j} + \omega_0 \dot{\gamma} \sin\gamma \underline{j} + \omega_0 \Omega \cos\gamma \underline{i} - \omega_0 \dot{\gamma} \cos\gamma \underline{k} + 0$

$= \omega_0 \Omega \cos\gamma \underline{i} + \dot{\gamma}(\Omega + \omega_0 \sin\gamma) \underline{j} - \omega_0 \dot{\gamma} \cos\gamma \underline{k}$

$= 3(4)(0.866)\underline{i} - \frac{\pi}{4}(4 + 3 \times 0.5)\underline{j} + 3(\pi/4)(0.866)\underline{k}$

$= 10.392\underline{i} - 4.320\underline{j} + 2.040\underline{k}$ rad/s²

$\alpha = |\underline{\alpha}| = \sqrt{(10.392)^2 + (4.320)^2 + (2.040)^2} = \underline{11.44 \text{ rad/s}^2}$

$\underline{\omega} = -\frac{\pi}{4}\underline{i} - 3(0.866)\underline{j} + (4 - 3 \times 0.5)\underline{k}$

$= \underline{-0.785\underline{i} - 2.60\underline{j} + 2.5\underline{k} \text{ rad/s}}$

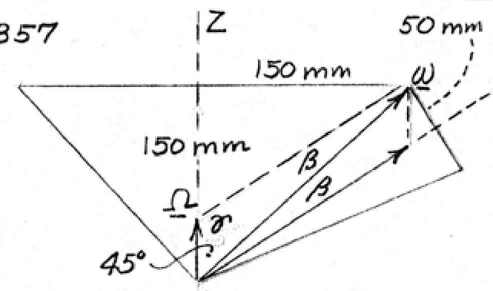
▶**7/26** $\sin\beta = \frac{50}{150\sqrt{2}} = 0.2357$

$\beta = 13.63°$

$\Omega = \frac{2\pi}{4} = \pi/2 \text{ rad/s}$

Law of sines

$\frac{\omega}{\sin\gamma} = \frac{\Omega}{\sin\beta}, \quad \omega = \Omega\frac{\sin\gamma}{\sin\beta}$



$|\underline{\alpha}| = |\underline{\Omega} \times \underline{\omega}| = \Omega\omega\sin 45° = \Omega^2\frac{\sin\gamma}{\sin\beta}\sin 45°$

$\sin\gamma = \sin(180 - 45 - 13.63) = 0.8539$

So $\alpha = (\frac{\pi}{2})^2 \frac{0.8539}{0.2357} 0.7071 = \underline{6.32 \text{ rad/s}^2}$

▶ 7/27 For $t=0$ $\theta=0$ and position vector of B is $\underline{r} = 4\underline{i} - 8\underline{k}$ in.

$$\omega_x = -\dot{\theta} = -\frac{\pi}{6}3\pi\cos 3\pi t = -\frac{\pi}{2}^2 \text{ rad/sec for } t=0$$

$$\omega_z = 2\pi \text{ rad/sec}$$

$$\underline{\omega} = \omega_x\underline{i} + \omega_z\underline{k} = -\frac{\pi^2}{2}\underline{i} + 2\pi\underline{k} \text{ rad/sec for } t=0$$

$$\underline{v} = \underline{\omega}\times\underline{r} = \left(-\frac{\pi^2}{2}\underline{i} + 2\pi\underline{k}\right)\times(4\underline{i} - 8\underline{k}) = -4\pi^2\underline{j} + 8\pi\underline{j} = 4\pi(2-\pi)\underline{j} \text{ in./sec}$$

or $\underline{v} = -14.35\underline{j}$ in./sec

$$\underline{a} = \dot{\underline{\omega}}\times r + \underline{\omega}\times(\underline{\omega}\times\underline{r})$$

$$\dot{\underline{\omega}} = \dot{\omega}_x\underline{i} + \omega_x\dot{\underline{i}} + \dot{\omega}_z\underline{k} + \omega_z\dot{\underline{k}} = +\frac{\pi^2}{2}(3\pi)\sin 3\pi t\,\underline{i} - \frac{\pi^2}{2}\cos 3\pi t(\omega_z\underline{j}) + \underline{0} + \underline{0}$$

$$\dot{\underline{\omega}}_{t=0} = \underline{0} - \frac{\pi^2}{2}2\pi\underline{j} = -\pi^3\underline{j}, \quad \underline{\alpha} = \dot{\underline{\omega}} = -\pi^3\underline{j} = -31.0\underline{j} \text{ rad/sec}^2$$

so $\underline{a} = -\pi^3\underline{j}\times(4\underline{i} - 8\underline{k}) + \left(-\frac{\pi^2}{2}\underline{i} + 2\pi\underline{k}\right)\times 4\pi(2-\pi)\underline{j}$

$$= 16\pi^2(\pi-1)\underline{i} + 2\pi^4\underline{k} \text{ in./sec}^2$$

$$\underline{a} = 338\underline{i} + 194.8\underline{k} \text{ in./sec}^2$$

▶ 7/28 Angular velocity $\underline{\omega}$ of link A cannot have a component along the y-axis so $\underline{\omega} \cdot \underline{j} = 0$. A vector in the $\underline{j}$-direction is $\underline{h} \times \underline{n}$ as is $\underline{h} \times (\underline{r} \times \underline{h})$. The magnitude is immaterial. Thus

$$\underline{\omega} \cdot (\underline{h} \times \underline{n}) = 0 \quad \text{or} \quad \underline{\omega} \cdot (\underline{h} \times [\underline{r} \times \underline{h}]) = 0$$

$$\text{or} \quad \underline{\omega \cdot \underline{h} \times (\underline{r} \times \underline{h}) = 0}$$

7/29 $\underset{-}{p} = \omega \cos 20° \underset{-}{k} = 30\,(0.9397)\,\underset{-}{k}$ rad/s

$p = \underline{28.2 \text{ rad/s}}$

$\underset{-}{v}_{B/A} = \underset{-}{\omega} \times \underset{-}{r}_{B/A} = \underset{-}{\omega}_y \times \underset{-}{r}_{B/A} = 30 \sin 20° \underset{-}{j} \times 0.4 \underset{-}{k}$

$= \underline{4.10 \underset{-}{i} \text{ m/s}}$

7/30 Angular velocity of rotor is

$\underline{\omega} = p\underline{k} - q\underline{i}$, $\underline{\alpha} = \dot{\underline{\omega}} = p\dot{\underline{k}} - q\dot{\underline{i}} = \underline{\Omega} \times (p\underline{k} - q\underline{i})$

where $\underline{\Omega}$ = angular velocity of axes $= -q\underline{i}$

Thus $\underline{\alpha} = -q\underline{i} \times (p\underline{k} - q\underline{i}) = \underline{pq\underline{j}}$

or from Eq. 7/7, $\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = \underline{0} + \underline{\Omega} \times \underline{\omega}$

$= -q\underline{i} \times (p\underline{k} - q\underline{i}) = \underline{pq\,j}$

7/31 $\underline{\omega} = \underline{\Omega} + \underline{p} = 4\underline{i} + 10\underline{k}$, $\omega = \sqrt{4^2 + 10^2} = 10.77 \frac{rad}{s}$

$\underline{\alpha} = \underline{\Omega} \times \underline{p} = 4\underline{i} \times 10\underline{k} = -40\underline{j}$ rad/s^2

7/32 $\underline{\alpha} = \frac{d}{dt}\underline{\omega} = \frac{d}{dt}(\underline{\Omega} + \underline{p}) = 0 + \frac{d}{dt}(p\underline{k})$

$$= \dot{p}\underline{k} + p\dot{\underline{k}} = \dot{p}\underline{k} + p(\underline{\Omega}\times\underline{k}) = \dot{p}\underline{k} + p\Omega(-\underline{j})$$

$$\underline{\alpha} = 6\underline{k} - 10(4)\underline{j} = \underline{-40\underline{j} + 6\underline{k}} \text{ rad/s}^2$$

7/33 | Angular velocity of x-y-z axes is $\underline{\Omega} = 4\underline{i}$ rad/s

$\underline{v}_A = \underline{v}_C + \underline{\Omega} \times \underline{r}_{A/C} + \underline{v}_{rel}$

$\underline{v}_C = 0.4(4)(-\underline{j}) = -1.6\underline{j}$ m/s

$\underline{\Omega} \times \underline{r}_{A/C} = 4\underline{i} \times 0.3\underline{j} = 1.2\underline{k}$ m/s

$\underline{v}_{rel} = 0.3(10)(-\underline{i}) = -3\underline{i}$ m/s

So $\underline{v}_A = -1.6\underline{j} + 1.2\underline{k} - 3\underline{i}$, $\underline{\underline{v}_A = -3\underline{i} - 1.6\underline{j} + 1.2\underline{k} \text{ m/s}}$

$\underline{a}_A = \underline{a}_C + \dot{\underline{\Omega}} \times \underline{r}_{A/C} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/C}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_C = 0.4(4^2)(-\underline{k}) = -6.4\underline{k}$ m/s^2 , $\dot{\underline{\Omega}} = \underline{0}$

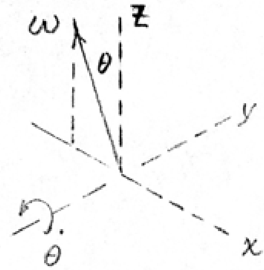
$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/C}) = 4\underline{i} \times 1.2\underline{k} = -4.8\underline{j}$ m/s^2

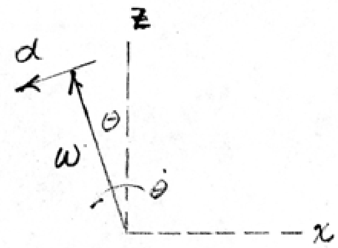
$2\underline{\Omega} \times \underline{v}_{rel} = 2(4\underline{i}) \times (-3\underline{i}) = \underline{0}$

$\underline{a}_{rel} = 0.3(10^2)(-\underline{j}) = -30\underline{j}$ m/s^2

So $\underline{a}_A = -6.4\underline{k} - 4.8\underline{j} - 30\underline{j}$, $\underline{\underline{a}_A = -34.8\underline{j} - 6.4\underline{k} \text{ m/s}^2}$

7/34





$$\omega = \frac{2\pi N}{60} = \frac{2\pi(360)}{60} = 12\pi \text{ rad/s}$$

$$\underset{\sim}{\alpha} = -\dot{\theta}\underline{j} \times \underline{\omega} = -0.2\underline{j} \times 12\pi(-\sin\theta\,\underline{i} + \cos\theta\,\underline{k})$$

$$= 2.4\pi(-0.5k - 0.866\underline{i})$$

$$= -1.2\pi(\sqrt{3}\,\underline{i} + \underline{k}) \text{ rad/s}^2$$

7/35 $\overline{OB} = \sqrt{7^2 - 2^2 - 3^2} = 6$ ft ; $\underline{v}_A = -3\underline{i}$ ft/sec

$\underline{v}_B = \underline{v}_A + \underline{\omega}_n \times \underline{r}_{B/A}$, $\underline{r}_{B/A} = -2\underline{i} - 3\underline{j} + 6\underline{k}$ ft

so $v_B\underline{k} = -3\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -2 & -3 & 6 \end{vmatrix}$; Equate coefficients of $\underline{i}, \underline{j}, \underline{k}$ terms & get

$1 = 2\omega_y + \omega_z$, $\omega_z = -3\omega_x$, $v_B = -3\omega_x + 2\omega_y$

Eliminate ω's & get $\underline{v}_B = 1.0\underline{k}$ ft/sec

Now ω_n is $\perp$ to AB, so $\underline{\omega}_n \cdot \underline{r}_{B/A} = 0$ which gives $-2\omega_x - 3\omega_y + 6\omega_z = 0$. Combine with above & get

$\omega_x = -3/49$ rad/sec, $\omega_y = 20/49$ rad/sec, $\omega_z = 9/49$ rad/sec

so $\underline{\omega}_n = \frac{1}{49}(-3\underline{i} + 20\underline{j} + 9\underline{k})$ rad/sec

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(Alternative solution for v_B

$x^2 + y^2 + z^2 = l^2$, $x\dot{x} + y\dot{y} + z\dot{z} = 0$; $\dot{y} = 0$, $\dot{x} = -3$ ft/sec

so $\dot{z} = -\frac{x\dot{x}}{z} = -\frac{2(-3)}{6} = 1.0$ ft/sec)

7/36 Angular velocity of OA is $\underline{\omega} = -\dot{\beta}\underline{i} + p\sin\beta\underline{j} + (p\cos\beta + \Omega)\underline{k}$

Eq. 7/7a, $[\,\underline{\ }\,] = \underline{\omega}$, $\left(\frac{d[\,\underline{\ }\,]}{dt}\right)_{XYZ} = \left(\frac{d[\,\underline{\ }\,]}{dt}\right)_{xyz} + \underline{\Omega} \times [\,\underline{\ }\,]$

$\left(\frac{d\underline{\omega}}{dt}\right)_{xyz} = \underline{0} + p\dot{\beta}\cos\beta\underline{j} + (-p\dot{\beta}\sin\beta + 0)\underline{k}$

$\underline{\Omega} \times \underline{\omega} = \Omega\underline{k} \times (-\dot{\beta}\underline{i} + p\sin\beta\underline{j} + [p\cos\beta + \Omega]\underline{k})$

$= -\Omega\dot{\beta}\underline{j} - \Omega p\sin\beta\underline{i}$

so $\underline{\alpha} = (p\dot{\beta}\cos\beta - \Omega\dot{\beta})\underline{j} - \Omega p\sin\beta\underline{i} - p\dot{\beta}\sin\beta\underline{k}$

$\underline{\alpha} = -\Omega p\sin\beta\underline{i} + \dot{\beta}(p\cos\beta - \Omega)\underline{j} - p\dot{\beta}\sin\beta\underline{k}$

7/37

$$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$$

$$\underline{a}_A = \underline{a}_B + \underline{\dot{\omega}} \times \underline{r}_{A/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{A/B})$$

$\underline{\omega} = 1.4\underline{i} + 1.2\underline{j}$ rad/sec ; $\underline{\dot{\omega}} = 2\underline{i} + 3\underline{j}$ rad/sec^2

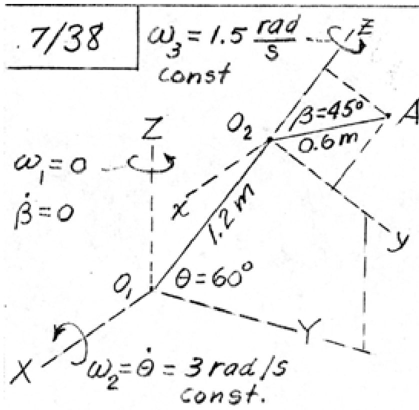
$\underline{r}_{A/B} = 5\underline{i}$ ft , $\underline{v}_B = 3.2\underline{j}$ ft/sec , $\underline{a}_B = 4\underline{j}$ ft/sec^2

Substitution and simplification yield

$\underline{v}_A = 3.2\underline{j} - 6\underline{k}$ ft/sec $\Rightarrow$ $\underline{v_A = 6.8 \text{ ft/sec}}$

$\underline{a}_A = -7.2\underline{i} + 12.4\underline{j} - 15\underline{k}$ ft/sec^2 $\Rightarrow$ $\underline{a_A = 20.8 \text{ ft/sec}^2}$

7/38



Attach axes x-y-z with origin at O_2 and x parallel to X. So x-y-z axes have angular velocity

$\underline{\Omega} = \dot{\theta}\underline{i} = 3\underline{i}$ rad/s

$\underline{v}_A = \underline{v}_{O_2} + \underline{\Omega} \times \underline{r}_{A/O_2} + \underline{v}_{rel}$

$\underline{v}_{O_2} = \underline{\omega}_2 \times \underline{r}_{O_2} = 3\underline{i} \times 1.2\underline{k} = -3.6\underline{j}$ m/s

$\underline{v}_{rel} = \underline{\omega}_3 \times \underline{r}_{A/O_2} = 1.5\underline{k} \times \frac{0.6}{\sqrt{2}}(\underline{j} + \underline{k})$

$= -0.636\underline{i}$ m/s

$\underline{\Omega} \times \underline{r}_{A/O_2} = 3\underline{i} \times \frac{0.6}{\sqrt{2}}(\underline{j}+\underline{k}) = 1.273(\underline{k} - \underline{j})$ $\frac{m}{s}$

So $\underline{v}_A = -3.6\underline{j} + 1.273(\underline{k} - \underline{j}) - 0.636\underline{i} = -0.636\underline{i} - 4.873\underline{j} + 1.273\underline{k}$ m/s

7/39 Sol. I $X^2+Y^2+Z^2=L^2$

$X\dot{X}+Y\dot{Y}+0=0$, $Z=\text{const.}$, $L=\text{const.}$

$\dot{Y}=v_A=-\frac{X}{Y}\dot{X}=-\frac{0.3}{0.2}4=-6 \text{ m/s}$ (−Y-dir.)

Sol. II $\underline{v}_A=\underline{v}_B+\underline{\omega}\times\underline{r}_{A/B}$, $\underline{\omega}\cdot\underline{r}_{A/B}=0$ taking $\omega \perp AB$

$$v_A\underline{j}=4\underline{i}+\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ \omega_x & \omega_y & \omega_z\\ -0.3 & 0.2 & 0.6\end{vmatrix}$$

$$(\underline{i}\omega_x+\underline{j}\omega_y+\underline{k}\omega_z)\cdot(-0.3\underline{i}+0.2\underline{j}+0.6\underline{k})=0$$

Expand, equate coefficients & get

$$0.6\omega_y-0.2\omega_z=-4 \quad (1)$$

$$-0.6\omega_x-0.3\omega_z=v_A \quad (2)$$

$$0.2\omega_x+0.3\omega_y=0 \quad (3)$$

$$-0.3\omega_x+0.2\omega_y+0.6\omega_z=0 \quad (4)$$

Solve simultaneously & get

$\omega_x=7.35$ rad/s, $\omega_y=-4.90$ rad/s, $\omega_z=5.31$ rad/s

$\underline{v}_A=-6\underline{j}$ m/s

7/40 Angular velocity of axes is $\underline{\Omega} = p\underline{k}$

" " " A " $\underline{\omega} = \Omega - \dot{\beta}\underline{i}$

$$\underline{\alpha} = \dot{\underline{\omega}} = \dot{\underline{\Omega}} - \ddot{\beta}\underline{i} - \dot{\beta}\dot{\underline{i}} = \dot{\underline{\Omega}} - \ddot{\beta}\underline{i} - \dot{\beta}\,\underline{\Omega}\times\underline{i}$$

$$= 0 - \ddot{\beta}\underline{i} - \dot{\beta}p\underline{j}$$

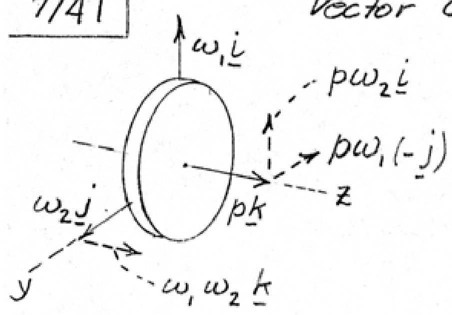
(a) before; $\dot{\beta}\,d\dot{\beta} = \ddot{\beta}\,d\beta$, $\ddot{\beta} = \dot{\beta}\dfrac{d\dot{\beta}}{d\beta} = \left(2\dfrac{2\pi}{360}\right)\dfrac{2}{18}$

$$= 0.00388 \text{ rad/s}^2$$

$$\underline{\alpha} = -0.00388\underline{i} - \frac{2\pi}{180}\frac{1}{10}\underline{j} = \underline{-(3.88\underline{i} + 3.49\underline{j})10^{-3}\ \frac{\text{rad}}{\text{s}^2}}$$

(b) after; $\ddot{\beta} = 0$, $\underline{\alpha} = \underline{-3.49(10^{-3})\underline{j}\ \text{rad/s}^2}$

7/41



Vector $\omega_1 \underline{i}$ does not change orientation so $\frac{d}{dt}(\underline{i}\omega_1) = 0$

Accel. components give

$$\underline{\alpha} = p\omega_2 \underline{i} - p\omega_1 \underline{j} + \omega_1 \omega_2 \underline{k}$$

$\boxed{7/42}$ Let γ = angle between AB & y-axis

Angular velocity of AB is $\underline{\omega} = -\dot{\gamma}\underline{i} + \Omega\underline{k}$

So $\underline{\alpha} = \dot{\underline{\omega}} = -\ddot{\gamma}\underline{i} - \dot{\gamma}\dot{\underline{i}} + \underline{0}$

But $z = \ell \sin\gamma$, $v_A = \dot{z} = \ell\dot{\gamma}\cos\gamma$

& $\dot{v}_A = 0 = -\ell\dot{\gamma}^2\sin\gamma + \ell\ddot{\gamma}\cos\gamma$

So $\dot{\gamma} = \dfrac{v_A}{\ell\cos\gamma} = \dfrac{8}{5(4/5)} = 2$ rad/sec

$\ddot{\gamma} = \dot{\gamma}^2\tan\gamma = 2^2(3/4) = 3$ rad/sec^2

Also $\dot{\underline{i}} = \Omega\underline{k}\times\underline{i} = \Omega\underline{j} = 2\underline{j}$ rad/sec

Thus $\underline{\alpha} = -3\underline{i} - 2(2\underline{j}) = \underline{-3\underline{i} - 4\underline{j}}$ rad/sec^2

7/43 $\underline{\Omega}$ = angular velocity of axes x-y-z = $\frac{2\pi N}{60}\underline{j} = \pi\underline{j}$ $\frac{rad}{s}$

$\underline{v} = \underline{v}_A = \underline{v}_B + \underline{\Omega} \times \underline{r}_{A/B} + \underline{v}_{rel}$

where $\underline{v}_B = \pi\underline{j} \times \underline{r}_{OB} = \pi\underline{j} \times (-0.18\underline{i} + 0.1\underline{k}) = \pi(0.1\underline{i} + 0.18\underline{k})$ m/s

$\underline{\Omega} \times \underline{r}_{A/B} = \pi\underline{j} \times 0.1\underline{i} = -0.1\pi\underline{k}$ m/s

$\underline{v}_{rel} = p\underline{k} \times \underline{r}_{A/B} = \frac{240(2\pi)}{60}\underline{k} \times 0.1\underline{i} = 0.8\pi\underline{j}$ m/s

Collect terms & get $\underline{v} = \pi(0.1\underline{i} + 0.8\underline{j} + 0.08\underline{k})$ m/s

$\underline{a} = \underline{a}_A = \underline{a}_B + \dot{\underline{\Omega}} \times \underline{r}_{A/B} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$, $\dot{\underline{\Omega}} = \underline{0}$

where $\underline{a}_B = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{B/O}) = \pi\underline{j} \times (\pi\underline{j} \times [-0.18\underline{i} + 0.1\underline{k}])$

$= \pi^2(0.18\underline{i} - 0.1\underline{k})$ m/s²

$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) = \pi\underline{j} \times (-0.1\pi\underline{k}) = -0.1\pi^2\underline{i}$ m/s²

$2\underline{\Omega} \times \underline{v}_{rel} = 2\pi\underline{j} \times 0.8\pi\underline{j} = \underline{0}$

$\underline{a}_{rel} = \dot{p}\underline{k} \times \underline{r}_{A/B} + \underline{r}_{A/B}\, p^2(-\underline{i}) = \underline{0} - (8\pi)^2 0.1\underline{i} = -6.4\pi^2\underline{i}$ $\frac{m}{s^2}$

Collect terms & get

$\underline{a} = -0.1\pi^2\underline{i} - 6.4\pi^2\underline{i} + 0.18\pi^2\underline{i} - 0.1\pi^2\underline{k}$

$\underline{a} = -\pi^2(6.32\underline{i} + 0.1\underline{k})$ m/s²

7/44 $\underline{\Omega}$ = angular velocity of disk & axes x-y-z

$$= \frac{2\pi}{60}(N\underline{j} + p\underline{k}) = \frac{\pi}{30}(30\underline{j} + 240\underline{k}) = \pi(\underline{j} + 8\underline{k}) \text{ rad/s}$$

$$\underline{v} = \underline{v}_A = \underline{v}_B + \underline{\Omega} \times \underline{r}_{A/B} + \underline{v}_{rel}$$

Where $\underline{v}_B = \pi(0.1\underline{i} + 0.18\underline{k})$ m/s from Prob. 7/43

$$\underline{\Omega} \times \underline{r}_{A/B} = \pi(\underline{j} + 8\underline{k}) \times 0.1\underline{i} = \pi(0.8\underline{j} - 0.1\underline{k}) \text{ m/s}$$

$$\underline{v}_{rel} = \underline{0}$$

Thus $\underline{v} = \pi(0.1\underline{i} + 0.18\underline{k}) + \pi(0.8\underline{j} - 0.1\underline{k})$

$$= \underline{\pi(0.1\underline{i} + 0.8\underline{j} + 0.08\underline{k}) \text{ m/s}} \quad \text{(agrees with 7/43)}$$

$$\underline{a} = \underline{a}_A = \underline{a}_B + \dot{\underline{\Omega}} \times \underline{r}_{A/B} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$$

where $\dot{\underline{\Omega}} = \pi(\dot{\underline{j}} + 8\dot{\underline{k}})$ with $\dot{\underline{j}} = \underline{\Omega} \times \underline{j} = \pi(\underline{j} + 8\underline{k}) \times \underline{j} = -8\pi\underline{i}$

$$\dot{\underline{k}} = \underline{\Omega} \times \underline{k} = \pi(\underline{j} + 8\underline{k}) \times \underline{k} = \pi\underline{i}$$

so $\dot{\underline{\Omega}} = \pi(-8\pi\underline{i} + 8\pi\underline{i}) = \underline{0}$

$$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) = \pi(\underline{j} + 8\underline{k}) \times \pi(0.8\underline{j} - 0.1\underline{k}) = -6.5\pi^2\underline{i} \text{ m/s}^2$$

$$2\underline{\Omega} \times \underline{v}_{rel} = 2\pi(\underline{j} + 8\underline{k}) \times \underline{0} = \underline{0}$$

$$\underline{a}_{rel} = \underline{0}$$

Collect terms & get

$$\underline{\underline{a} = -\pi^2(6.32\underline{i} + 0.1\underline{k}) \text{ m/s}^2} \quad \text{(agrees with 7/43)}$$

7/45 From Eqs. 7/6

$\underline{v}_A = \underline{v}_O + \underline{\Omega} \times \underline{r}_{A/O} + \underline{v}_{rel}$

$\underline{v}_O = -R\Omega \underline{i}$, $\underline{\Omega} = \Omega \underline{k}$, $\underline{r}_{A/O} = b\sin\beta \underline{j} + b\cos\beta \underline{k}$, $\underline{v}_{rel} = b\dot{\beta}(\cos\beta \underline{j} - \sin\beta \underline{k})$

$\underline{v}_A = -R\Omega \underline{i} + \Omega \underline{k} \times b(\sin\beta \underline{j} + \cos\beta \underline{k}) + b\dot{\beta}(\cos\beta \underline{j} - \sin\beta \underline{k})$

$\underline{v}_A = -\Omega(R + b\sin\beta)\underline{i} + b\dot{\beta}\cos\beta \underline{j} - b\dot{\beta}\sin\beta \underline{k}$

$\underline{a}_A = \underline{a}_O + \dot{\underline{\Omega}} \times \underline{r}_{A/O} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_O = -R\Omega^2 \underline{j}$, $\dot{\underline{\Omega}} = \underline{0}$, $\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) = \Omega \underline{k} \times (\Omega \underline{k} \times b[\sin\beta \underline{j} + \cos\beta \underline{k}])$

$2\underline{\Omega} \times \underline{v}_{rel} = 2\Omega \underline{k} \times b\dot{\beta}(\cos\beta \underline{j} - \sin\beta \underline{k})$, $\underline{a}_{rel} = b\dot{\beta}^2(\sin\beta \underline{j} + \cos\beta \underline{k})$

Combine, collect terms, & get

$\underline{a}_A = -2b\Omega\dot{\beta}\cos\beta \underline{i} - (\Omega^2[R + b\sin\beta] + b\dot{\beta}^2\sin\beta)\underline{j} - b\dot{\beta}^2\cos\beta \underline{k}$

7/46 Precession is steady so $\underline{\alpha} = \underline{\Omega} \times \underline{p}$

$\underline{\alpha} = 4\pi\underline{k} \times 10\pi\underline{j} = \underline{-40\pi^2\underline{i} \text{ rad/s}^2}$

$\underline{a}_A = \underline{a}_O + \dot{\underline{\Omega}} \times \underline{r}_{A/O} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_O = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_O) = -r_O\Omega^2\underline{i} = -0.3(4\pi)^2\underline{i} = -4.8\pi^2\underline{i} \text{ m/s}^2$

$\dot{\underline{\Omega}} = 0$; $\underline{\Omega} \times \underline{r}_{A/O} = 4\pi\underline{k} \times 0.1\underline{k} = \underline{0}$

$\underline{v}_{rel} = \underline{p} \times \underline{r}_{A/O} = 10\pi\underline{j} \times 0.1\underline{k} = \pi\underline{i} \text{ m/s}$

$2\underline{\Omega} \times \underline{v}_{rel} = 2(4\pi\underline{k}) \times \pi\underline{i} = 8\pi^2\underline{j} \text{ m/s}^2$

$\underline{a}_{rel} = \underline{p} \times (\underline{p} \times \underline{r}_{A/O}) = -0.1(10\pi)^2\underline{k} = -10\pi^2\underline{k} \text{ m/s}^2$

$\underline{a}_A = -4.8\pi^2\underline{i} + 8\pi^2\underline{j} - 10\pi^2\underline{k}$

$= \underline{2\pi^2(-2.4\underline{i} + 4\underline{j} - 5\underline{k}) \text{ m/s}^2}$

7/47 Angular velocity of axes $\underline{\Omega} = \Omega \underline{k}$

" " " panels $\underline{\omega} = -\dot{\theta}\underline{j} + \Omega \underline{k}$

$\dot{\underline{\omega}} = -\dot{\theta}\dot{\underline{j}} + \Omega\dot{\underline{k}} = -\dot{\theta}(\underline{\Omega}\times\underline{j}) + \Omega(\underline{\Omega}\times\underline{k}) = \underline{\Omega}\times\underline{\omega} = \Omega\dot{\theta}\underline{i}$

$= \frac{1}{2}\frac{1}{4}\underline{i} = \underline{\frac{1}{8}\underline{i} \text{ rad/sec}^2}$

$\underline{a}_A = \underline{a}_o + \dot{\underline{\Omega}}\times\underline{r}_{A/o} + \underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/o}) + 2\underline{\Omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_o = \underline{0}$; $\quad \underline{\Omega}\times\underline{r}_{A/o} = \frac{1}{2}\underline{k}\times(-\underline{i} + 8\underline{j} + \sqrt{3}\underline{k}) = -\frac{1}{2}\underline{j} - 4\underline{i}$ $\frac{ft}{sec}$

$\underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/o}) = \frac{1}{2}\underline{k}\times(-\frac{1}{2}\underline{j} - 4\underline{i}) = \frac{1}{4}\underline{i} - 2\underline{j}$ ft/sec^2

$2\underline{\Omega}\times\underline{v}_{rel} = 2(\frac{1}{2}\underline{k})\times(-\frac{\sqrt{3}}{4}\underline{i} - \frac{1}{4}\underline{k}) = -\frac{\sqrt{3}}{4}\underline{j}$ ft/sec^2

$\underline{a}_{rel} = 2(\frac{1}{4})^2(\frac{1}{2}\underline{i} - \frac{\sqrt{3}}{2}\underline{k}) = \frac{1}{16}\underline{i} - \frac{\sqrt{3}}{16}\underline{k}$ ft/sec^2

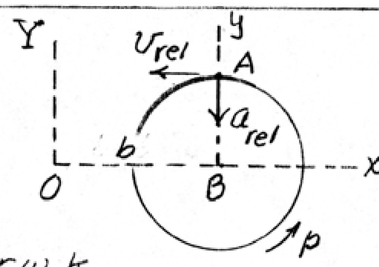
$\underline{a}_A = (\frac{1}{4} + \frac{1}{16})\underline{i} + (-2 - \frac{\sqrt{3}}{4})\underline{j} - \frac{\sqrt{3}}{16}\underline{k}$

$= \underline{0.313\underline{i} - 2.43\underline{j} - 0.1083\underline{k} \text{ ft/sec}^2}$

with $a_A = 2.45$ ft/sec^2

7/48 Angular velocity of x-y-z axes is

$\underline{\Omega} = -\omega_1 \underline{i} + \omega_2 \underline{J}$



$\underline{v} = \underline{v}_A = \underline{v}_B + \underline{\Omega} \times \underline{r}_{A/B} + \underline{v}_{rel}$

where $\underline{v}_B = b\omega_2(-\underline{k}) = -b\omega_2 \underline{k}$

$\underline{\Omega} \times \underline{r}_{A/B} = (-\omega_1 \underline{i} + \omega_2 \underline{J}) \times r\underline{j} = -r\omega_1 \underline{k}$

$\underline{v}_{rel} = -rp\underline{i}$

Thus $\underline{v} = -b\omega_2 \underline{k} - r\omega_1 \underline{k} - rp\underline{i} = \underline{-rp\underline{i} - (r\omega_1 + b\omega_2)\underline{k}}$

$\underline{a} = \underline{a}_A = \underline{a}_B + \dot{\underline{\Omega}} \times \underline{r}_{A/B} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

where $\underline{a}_B = -b\omega_2^2 \underline{i}$

$\dot{\underline{\Omega}} = -\omega_1 \dot{\underline{i}} + \omega_2 \dot{\underline{J}} = -\omega_1 \underline{\Omega} \times \underline{i} = \omega_1 \omega_2 \underline{k}$

$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) = (-\omega_1 \underline{i} + \omega_2 \underline{J}) \times (-r\omega_1 \underline{k}) = -r\omega_1(\omega_1 \underline{j} + \omega_2 \underline{i})$

$2\underline{\Omega} \times \underline{v}_{rel} = 2(-\omega_1 \underline{i} + \omega_2 \underline{J}) \times (-rp\underline{i}) = 2rp\omega_2 \underline{k}$

$\underline{a}_{rel} = -rp^2 \underline{j}$, $\dot{\underline{\Omega}} \times \underline{r}_{A/B} = \omega_1 \omega_2 \underline{k} \times r\underline{j} = -r\omega_1 \omega_2 \underline{i}$

Substitute, combine & get

$$\underline{a} = -\omega_2(b\omega_2 + 2r\omega_1)\underline{i} - r(\omega_1^2 + p^2)\underline{j} + 2rp\omega_2 \underline{k}$$

▸ 7/49 From Sample Problem 7/2

$\Omega = 2\pi$ rad/sec, $\omega_y = \sqrt{3}\pi$ rad/sec, $\omega_z = 5\pi$ rad/sec, $\omega_0 = 4\pi \frac{rad}{sec}$

Also $\omega_x = -\dot{\gamma} = -3\pi$ rad/sec

In general $\underline{\omega} = (-\dot{\gamma}\underline{i} + \Omega\cos\gamma\,\underline{j} + [\omega_0 + \Omega\sin\gamma]\underline{k})$

For $\gamma = 30°$, $\underline{\omega} = \pi(-3\underline{i} + \sqrt{3}\underline{j} + 5\underline{k})$ rad/sec

From Eq. 7/7 $\underline{\alpha} = [d\underline{\omega}/dt]_{XYZ} = [d\underline{\omega}/dt]_{xyz} + \underline{\omega}_{axes} \times \underline{\omega}$

But $[d\underline{\omega}/dt]_{xyz} = (\underline{0} - \Omega\dot{\gamma}\sin\gamma\,\underline{j} + \Omega\dot{\gamma}\cos\gamma\,\underline{k})$

$= 6\pi^2(-\frac{1}{2}\underline{j} + \frac{\sqrt{3}}{2}\underline{k}) = 3\pi^2(-\underline{j} + \sqrt{3}\underline{k})$ rad/sec^2

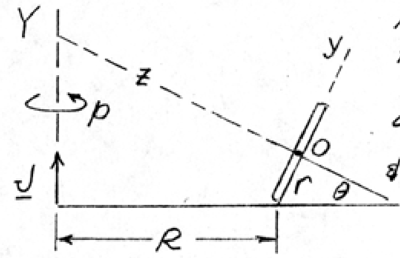
$\underline{\omega}_{axes} = \underline{\omega} - \omega_0\underline{k}$ & $\underline{\omega}_{axes} \times \underline{\omega} = (\underline{\omega} - \omega_0\underline{k}) \times \underline{\omega} = -\omega_0\underline{k} \times \underline{\omega}$

so $\underline{\omega}_{axes} \times \underline{\omega} = -4\pi\underline{k} \times \pi(-3\underline{i} + \sqrt{3}\underline{j} + 5\underline{k}) = 4\pi^2(\sqrt{3}\underline{i} + 3\underline{j})\ \frac{rad}{sec^2}$

Thus $\underline{\alpha} = 3\pi^2(-\underline{j} + \sqrt{3}\underline{k}) + 4\pi^2(\sqrt{3}\underline{i} + 3\underline{j})$

$= \pi^2(4\sqrt{3}\underline{i} + 9\underline{j} + 3\sqrt{3}\underline{k})$ rad/sec^2

▸ 7/50



$\underline{\omega} = \underline{J}p + \frac{Rp}{r}\underline{k} = (p\cos\theta)\underline{j} + (p\sin\theta + \frac{Rp}{r})\underline{k}$

Angle $d\phi$ measured in x-y-z turned by wheel in time dt is

$d\phi = \frac{R(p\,dt)}{r}$ so $\dot{\phi} = \frac{Rp}{r}$

& $\underline{\omega} = p\left[\underline{j}\cos\theta + \underline{k}\left(\sin\theta + \frac{R}{r}\right)\right]$

Angular velocity of axes is $\Omega = \underline{J}p$ so

$\underline{\omega} = \underline{\Omega} + (\frac{Rp}{r})\underline{k}$; Now use $\left(\frac{d[\;]}{dt}\right)_{XYZ} = \left(\frac{d[\;]}{dt}\right)_{xyz} + \underline{\Omega}\times[\;]$

Noting $\underline{\Omega}$ is constant in XYZ & xyz.

Thus $\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = 0 + \underline{\Omega}\times\left[\underline{\Omega}\times\frac{Rp}{r}\underline{k}\right] = \underline{\Omega}\times\frac{Rp}{r}\underline{k}$

$\alpha = \left[(p\cos\theta)\underline{j} + (p\sin\theta)\underline{k}\right]\times\frac{Rp}{r}\underline{k}$, $\underline{\alpha} = (\frac{Rp^2}{r}\cos\theta)\underline{i}$

or merely $\underline{\alpha} = \dot{\underline{\omega}} = 0 + \frac{Rp}{r}\dot{\underline{k}} = \frac{Rp}{r}(\underline{\Omega}\times\underline{k})$, etc.

▶ 7/51 Angular velocity of axes $= \underline{\Omega}$

" " " rotor $= \underline{\omega} = \underline{\Omega} + p\underline{k}$

where $p = 100(2\pi)/60 = 10\pi/3$ rad/s

$\underline{\Omega} = -\dot{\gamma}\underline{i} + \underline{j}\,\omega_1 \cos\gamma + \underline{k}\,\omega_1 \sin\gamma$, $\omega_1 = \frac{2\pi}{60}20 = \frac{2\pi}{3}\frac{rad}{s}$

$\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = \left(\frac{d\underline{\omega}}{dt}\right)_{xyz} + \underline{\Omega}\times\underline{\omega}$ (Eq. 8/7)

$\left(\frac{d\underline{\omega}}{dt}\right)_{xyz} = \left(\frac{d\underline{\Omega}}{dt}\right)_{xyz} + 0 = 0 - j\,\dot{\gamma}\omega_1 \sin\gamma + \underline{k}\,\dot{\gamma}\omega_1 \cos\gamma$

$\underline{\Omega}\times\underline{\omega} = \underline{\Omega}\times(\underline{\Omega} + p\underline{k}) = \underline{\Omega}\times p\underline{k} = \dot{\gamma}p\underline{j} + p\omega_1\cos\gamma\,\underline{i}$

$\underline{\alpha} = (\dot{\gamma}p - \dot{\gamma}\omega_1 \sin\gamma)\,\underline{j} + p\omega_1\cos\gamma\,\underline{i} + \dot{\gamma}\omega_1\cos\gamma\,\underline{k}$

substitute $\dot{\gamma} = 4$ rad/s, $p = 10\pi/3$ rad/s, $\omega_1 = 2\pi/3$ rad/s

& get

$\underline{\alpha} = \left(4\frac{10\pi}{3} - 4\frac{2\pi}{3}\frac{1}{2}\right)\underline{j} + \frac{10\pi}{3}\frac{2\pi}{3}\frac{\sqrt{3}}{2}\underline{i} + 4\frac{2\pi}{3}\frac{\sqrt{3}}{2}\underline{k}$

$= 12\pi\underline{j} + \frac{10\pi^2}{3\sqrt{3}}\underline{i} + \frac{4\pi}{\sqrt{3}}\underline{k} = 18.99\underline{i} + 37.70\underline{j} + 7.25\underline{k}\ \frac{rad}{s^2}$

$\alpha = \sqrt{\overline{18.99}^2 + \overline{37.70}^2 + \overline{7.25}^2} = \underline{42.8\ rad/s^2}$

▶7/52 $\underline{v}_A = \underline{v}_B + \underline{\omega}_n \times \underline{r}_{A/B}$ where $\underline{\omega}_n \cdot \underline{r}_{A/B} = 0$

$200^2 + 300^2 + z^2 = 700^2$, $z = 600$ mm

$\underline{r}_{A/B} = 100(3\underline{i} + 2\underline{j} - 6\underline{k})$ mm

$$2\underline{j} = v_B\underline{k} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_{n_x} & \omega_{n_y} & \omega_{n_z} \\ 3 & 2 & -6 \end{vmatrix} (0.1) \quad \text{m/s}$$

Equate coefficients of like terms and get

$\omega_{n_z} + 3\omega_{n_y} = 0$, $2\omega_{n_x} + \omega_{n_z} = 20/3$, $v_B = -0.2\omega_{n_x} + 0.3\omega_{n_y}$

Eliminate ω_n's & get $v_B = -\frac{2}{3}$ m/s, $\underline{\underline{v}_B = -\frac{2}{3}\underline{k}\text{ m/s}}$

$(\omega_{n_x}\underline{i} + \omega_{n_y}\underline{j} + \omega_{n_z}\underline{k}) \cdot (3\underline{i} + 2\underline{j} - 6\underline{k}) = 0$

$3\omega_{n_x} + 2\omega_{n_y} - 6\omega_{n_z} = 0$. Combine with above & get

$\omega_{n_x} = \frac{1}{3}\frac{400}{49}$, $\omega_{n_y} = -\frac{20}{49}$, $\omega_{n_z} = \frac{60}{49}$ rad/s

$\underline{\underline{\omega}_n = \frac{10}{49}\left(\frac{40}{3}\underline{i} - 2\underline{j} + 6\underline{k}\right)\text{ rad/s}}$

Second solution for v_B

$0.3^2 + y^2 + z^2 = 0.7^2$

$0 + 2y\dot{y} + 2z\dot{z} = 0$, $\dot{z} = v_B = -\frac{y}{z}\dot{y} = -\frac{200}{600}2 = \underline{-\frac{2}{3}\text{ m/s}}$

7/53 $\omega_x = \omega_y = 0$, $\omega_z = \omega$

For these conditions, Eq. 7/11 is

$$\underline{H} = \omega\left[-I_{xz}\,\underline{i} - I_{yz}\,\underline{j} + I_{zz}\,\underline{k}\right]$$

$$\begin{cases} I_{xz} = 0 \\ I_{yz} = mR\left(\frac{L}{3}\right) - mR\left(\frac{2L}{3}\right) = -mRL/3 \\ I_{zz} = 2mR^2 \end{cases}$$

So $\underline{\underline{H} = mR\omega\left[\frac{L}{3}\underline{j} + 2R\,\underline{k}\right]}$

$$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_o = \frac{1}{2}\omega\underline{k}\cdot mR\omega\left[\frac{L}{3}\underline{i} + 2R\,\underline{k}\right] = \underline{mR^2\omega^2}$$

(By inspection, $T = \frac{1}{2}I_{zz}\,\omega_z^2 = \frac{1}{2}\left(2mR^2\right)\omega^2 = \underline{mR^2\omega^2}$)

7/54 | x-y-z are principal axes so

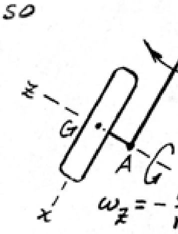
$\underline{H} = I_{xx}\omega_x \underline{i} + I_{yy}\omega_y \underline{j} + I_{zz}\omega_z \underline{k}$

$I_{zz} = mk^2$

$= 45(0.370)^2 = 6.16 \text{ kg}\cdot\text{m}^2$

$I_{xx} + I_{yy} = I_{zz}$ & $I_{xx} = I_{yy}$

so $I_{yy} = \frac{1}{2} I_{zz} = 3.08 \text{ kg}\cdot\text{m}^2$



$\dot{\theta} = \frac{30}{180}\pi = 0.524$ rad/s $= -\omega_y$

$\omega_z = -\frac{v}{r} = -\frac{200(10^3)/3600}{0.920/2}$

$= -120.8$ rad/s

$\omega_x = 0$

About G, $\underline{H}_G = \underline{0} + 3.08(-0.524)\underline{j} + 6.16(-120.8)\underline{k}$

$$\underline{H}_G = -1.613\,\underline{j} - 744\,\underline{k} \text{ kg}\cdot\text{m}^2/\text{s}$$

About A, $I_{yy} = \bar{I}_{yy} + md^2 = 3.08 + 45(0.215)^2 = 5.16 \text{ kg}\cdot\text{m}^2$

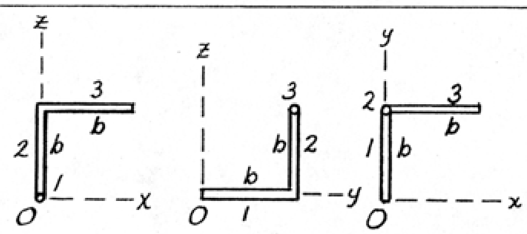
$$\underline{H}_A = \underline{0} + 5.16(-0.524)\underline{j} + 6.16(-120.8)\underline{k}$$

$$\underline{H}_A = -2.70\,\underline{j} - 744\,\underline{k} \text{ kg}\cdot\text{m}^2/\text{s}$$

7/55 With $\omega_x = \omega_y = 0$,

Eq. 7/11 gives

$\underline{H}_O = (-I_{xz}\underline{i} - I_{yz}\underline{j} + I_{zz}\underline{k})\omega$



Part	I_{xz}	I_{yz}	I_{zz}
1	0	0	$\frac{1}{3}\rho b^3$
2	0	$\frac{1}{2}\rho b^3$	ρb^3
3	$\frac{1}{2}\rho b^3$	ρb^3	$\frac{4}{3}\rho b^3$
Totals	$\frac{1}{2}\rho b^3$	$\frac{3}{2}\rho b^3$	$\frac{8}{3}\rho b^3$

$$\left\{ (I_{zz})_3 = \frac{1}{12}\rho b\, b^2 + \rho b\left(b^2 + \left[\frac{b}{2}\right]^2\right) = \frac{4}{3}\rho b^3 \right\}$$

so $\underline{H}_O = \rho b^3\left(-\frac{1}{2}\underline{i} - \frac{3}{2}\underline{j} + \frac{8}{3}\underline{k}\right)\omega$

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O = \frac{1}{2}\omega \cdot \frac{8}{3}\rho b^3\omega$, $\quad T = \frac{4}{3}\rho b^3\omega^2$

7/56 From Eq. 7/14 using O for A,

$$\underline{H}_O = \underline{H}_G + \bar{\underline{r}} \times m\bar{\underline{v}} \quad \text{where } \bar{r} = \Sigma m\underline{r}/\Sigma m$$

$\bar{r}_x = \rho b(0+0+\frac{b}{2})/3\rho b = b/6$, $\bar{r}_y = \rho b(\frac{b}{2}+b+b)/3\rho b = \frac{5}{6}b$,

$\bar{r}_z = \rho b(0+\frac{b}{2}+b)/3\rho b = \frac{1}{2}b$

$\bar{\underline{v}} = \underline{\omega} \times \bar{\underline{r}} = \omega\underline{k} \times b(\frac{1}{6}\underline{i} + \frac{5}{6}\underline{j} + \frac{1}{2}\underline{k}) = \frac{\omega b}{6}(-5\underline{i} + \underline{j})$

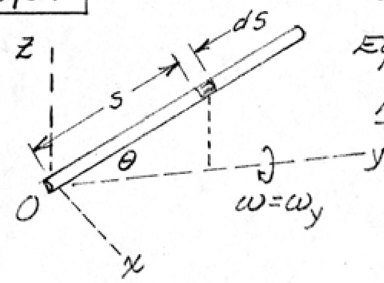
$\bar{\underline{r}} \times m\bar{\underline{v}} = b(\frac{1}{6}\underline{i} + \frac{5}{6}\underline{j} + \frac{1}{2}\underline{k}) \times 3\rho b\left(\frac{\omega b}{6}\right)(-5\underline{i} + \underline{j}) = \frac{\rho b^3\omega}{4}(-\underline{i} - 5\underline{j} + \frac{26}{3}\underline{k})$

From Prob. 7/55 $\underline{H}_O = \rho b^3(-\frac{1}{2}\underline{i} - \frac{3}{2}\underline{j} + \frac{8}{3}\underline{k})\omega$

Thus $\underline{H}_G = \underline{H}_O - \bar{\underline{r}} \times m\bar{\underline{v}} = \rho b^3\omega(-\frac{1}{2}\underline{i} - \frac{3}{2}\underline{j} + \frac{8}{3}\underline{k} + \frac{1}{4}\underline{i} + \frac{5}{4}\underline{j} - \frac{13}{6}\underline{k})$

$$\underline{H}_G = \rho b^3\omega(-\tfrac{1}{4}\underline{i} - \tfrac{1}{4}\underline{j} + \tfrac{1}{2}\underline{k}), \quad \underline{\underline{H}_G = \tfrac{1}{4}\rho b^3\omega(-\underline{i} - \underline{j} + 2\underline{k})}$$

7/57



$\omega_x = \omega_z = 0, \quad \omega_y = \omega,$ so Eq. 7/11 gives

$$\underline{H} = (-\underline{i}\, I_{xy} + \underline{j}\, I_{yy} - \underline{k}\, I_{yz})\omega$$

But $I_{xy} = 0$

$$I_{yy} = \frac{1}{3} m (\ell \sin\theta)^2$$

& $I_{yz} = \int yz\, dm = \int_0^{\ell} (s\cos\theta)(s\sin\theta)\, \rho\, ds$

where ρ = mass per unit length

so $I_{yz} = \rho \sin\theta\cos\theta \dfrac{\ell^3}{3} = \dfrac{1}{3} m\ell^2 \sin\theta\cos\theta$

& $\underline{H} = \left[\underline{i}(0) + \underline{j}\frac{1}{3} m\ell^2 \sin^2\theta - \underline{k}\frac{1}{3} m\ell^2 \sin\theta\cos\theta\right]\omega$

$$= \frac{1}{3} m\ell^2 \omega \sin\theta\, (\underline{j}\sin\theta - \underline{k}\cos\theta)$$

7/58

$\omega_x = \omega_y = 0\ ,\ \omega_z = \omega$

$I_{xz} = 0,\ I_{yz} = 0 + m\left(\frac{4r}{3\pi}\right)\left(c + \frac{b}{2}\right),\ I_{zz} = \frac{1}{2}mr^2$

So $\underline{H} = -I_{yz}\,\omega_z\,\underline{j} + I_{zz}\,\omega_z\,\underline{k}$

$$\underline{H} = mr\omega\left[-\frac{2(2c+b)}{3\pi}\,\underline{j} + \frac{r}{2}\,\underline{k}\right]$$

$\boxed{7/59}$ Eq. 7/14: $\underline{H}_O = \underline{H}_G + \underline{\bar{r}} \times m\underline{\bar{v}}$

$\underline{H}_G = \bar{I}_{xx}\omega_x \underline{i} + \bar{I}_{yy}\omega_y \underline{j} + \bar{I}_{zz}\omega_z \underline{k}$

$\omega_x = \omega, \quad \omega_y = p, \quad \omega_z = 0$

$\bar{I}_{xx} = \frac{3}{20}mr^2 + \frac{3}{80}mb^2 = \bar{I}_{zz}$ (from Table D/4)

$\bar{I}_{yy} = \frac{3}{10}mr^2$

$\underline{\bar{r}} = h\underline{k} - \frac{b}{4}\underline{j}, \quad \underline{\bar{v}} = -h\omega\underline{j} - \frac{b}{4}\omega\underline{k}$

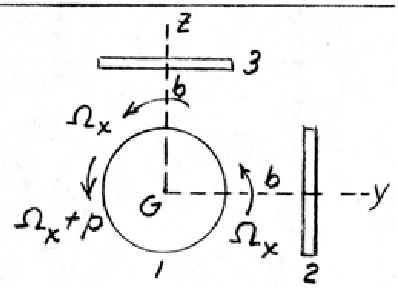
Substitution and simplification yield

$\underline{H}_O = \underline{\left[m\omega\left(\frac{3}{20}r^2 + \frac{1}{10}b^2 + h^2\right)\underline{i} + \frac{3}{10}mr^2p\underline{j}\right]}$

From $T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O$,

$T = \underline{\frac{1}{2}m\omega^2\left(\frac{3}{20}r^2 + \frac{1}{10}b^2 + h^2\right) + \frac{3}{20}mr^2p^2}$

7/60 About G,



$H_{x_1} = I(\Omega_x + p)$

$H_{x_2} = (\frac{I}{2} + mb^2)\Omega_x$

$H_{x_3} = (\frac{I}{2} + mb^2)\Omega_x$

So $H_x = I(\Omega_x + p) + (I + 2mb^2)\Omega_x$

$= Ip + 2(I + mb^2)\Omega_x$

similarly

$H_y = Ip + 2(I + mb^2)\Omega_y$

$H_z = Ip + 2(I + mb^2)\Omega_z$

Thus $\underline{H}_G = \underline{Ip(\underline{i} + \underline{j} + \underline{k}) + 2(I + mb^2)\underline{\Omega}}$

where $\underline{\Omega} = \Omega_x \underline{i} + \Omega_y \underline{j} + \Omega_z \underline{k}$

7/61 $\underline{\omega} = p\underline{k} - \dot{\gamma}\underline{i} + N(\cos\gamma\,\underline{j} + \sin\gamma\,\underline{k})$

$p = 100\left(\frac{2\pi}{60}\right) = \frac{10\pi}{3}$ rad/sec , $\gamma = 30°$

$\dot{\gamma} = 4$ rad/sec , $N = 20\left(\frac{2\pi}{60}\right) = \frac{2\pi}{3}$ rad/sec

So $\underline{\omega} = -4\underline{i} + 1.814\underline{j} + 11.52\underline{k}$ rad/sec

Eq. 7/11 yields $\underline{H} = I_{xx}\omega_x\underline{i} + I_{yy}\omega_y\underline{j} + I_{zz}\omega_z\underline{k}$

With numbers: $\underline{H}_o = -0.01\underline{i} + 0.0045\underline{j} + 0.0576\underline{k}$ lb-ft-sec

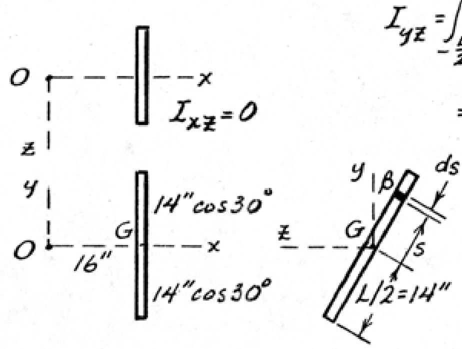
7/62 From Eq. 7/11 with $\omega_x = \omega_y = 0$,

$$\underline{H}_O = -I_{xz}\,\omega_z\,\underline{i} - I_{yz}\,\omega_z\,\underline{j} + I_{zz}\,\omega_z\,\underline{k}$$

$$I_{yz} = \int_{-\frac{L}{2}}^{\frac{L}{2}} (s\cos\beta)(-s\sin\beta)\rho\,ds$$

where ρ = mass/unit length

$$= -\rho\sin\beta\cos\beta\,\frac{s^3}{3}\Big|_{-L/2}^{L/2} = -\rho\frac{L^3}{24}\sin 2\beta$$



$$= -\frac{6.20/32.2}{28/12}\,\frac{(28/12)^3}{24}\sin 60^\circ$$

$$= -0.0378 \text{ lb-ft-sec}^2$$

$$I_{zz} = \bar{I}_O = \frac{1}{12}mL^2 + md^2$$

$$= \frac{6.20}{32.2}\left[\left(\frac{28\cos 30^\circ}{12}\right)^2\frac{1}{12} + \left(\frac{16}{12}\right)^2\right]$$

$$= 0.408 \text{ lb-ft-sec}^2$$

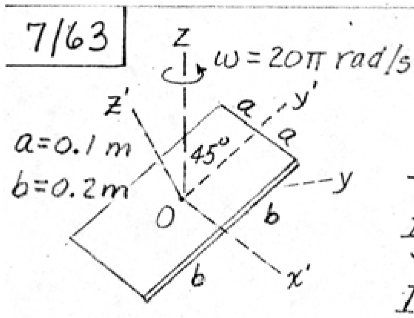
$$\underline{H}_O = (-I_{xz}\underline{i} - I_{yz}\underline{j} + I_{zz}\underline{k})\omega_z = (\underline{0} - [-0.0378]\underline{j} + 0.408\underline{k})\frac{600\times 2\pi}{60}$$

$$\underline{\underline{H}_O = 2.38\,\underline{j} + 25.6\,\underline{k} \text{ lb-ft-sec}}$$

From Eq. 7/18 $\quad T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O = \frac{1}{2}\omega_z\underline{k}\cdot\underline{H}_O$

$$= \frac{1}{2}\,\frac{600\times 2\pi}{60}\times 25.6 = \underline{805 \text{ ft-lb}}$$

7/63



Introduce axes x'-y'-z'

$\omega_{x'} = 0, \ \omega_{y'} = \frac{\omega}{\sqrt{2}}, \ \omega_{z'} = \frac{\omega}{\sqrt{2}}$

$I_{x'y'} = 0, \ I_{y'y'} = \frac{1}{12}m(2a)^2 = \frac{1}{3}ma^2$

$I_{y'z'} = 0, \ I_{x'z'} = 0$

$I_{z'z'} = \frac{1}{12}m\left([2a]^2 + [2b]^2\right) = \frac{1}{3}m(a^2 + b^2)$

Eq. 7/11 applied to x'-y'-z' gives $\underline{H} = \underline{j}' I_{y'y'} \omega_{y'} + \underline{k}' I_{z'z'} \omega_{z'}$

$$= \underline{j}'\left(\frac{1}{3}ma^2\right)\frac{\omega}{\sqrt{2}} + \underline{k}'\left(\frac{1}{3}m[a^2+b^2]\right)\frac{\omega}{\sqrt{2}}$$

But $\underline{j}' = \underline{j}\cos 45° + \underline{k}\sin 45° = \frac{1}{\sqrt{2}}(\underline{j} + \underline{k})$

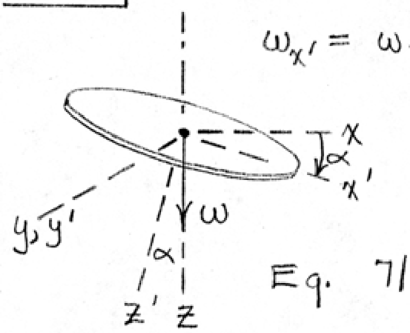
$\underline{k}' = -\underline{j}\sin 45° + \underline{k}\cos 45° = \frac{1}{\sqrt{2}}(-\underline{j} + \underline{k})$

So $\underline{H} = \frac{1}{6}m\omega\left(-b^2\underline{j} + [2a^2+b^2]\underline{k}\right) = \frac{3}{6}20\pi\left(-0.04\underline{j} + 0.06\underline{k}\right)$

$= \underline{\pi(-0.4\underline{j} + 0.6\underline{k}) \ \text{N·m·s}}$

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H} = \frac{1}{2}(20\pi\underline{k})\cdot\pi(-0.4\underline{j} + 0.6\underline{k}) = 6.0\pi^2 = \underline{59.2 \ \text{J}}$

7/64 Introduce axes x'-y'-z' as shown.



$$\omega_{x'} = \omega \sin\alpha, \quad \omega_{y'} = 0, \quad \omega_{z'} = \omega\cos\alpha$$

$$I_{x'x'} = I_{y'y'} = \frac{1}{4} mr^2$$

$$I_{z'z'} = \frac{1}{2} mr^2$$

Eq. 7/11 yields

$$\underline{H} = \left(\frac{1}{4} mr^2\right)\omega\sin\alpha\, \underline{i}' + \left(\frac{1}{2} mr^2\right)\omega\cos\alpha\, \underline{k}'$$

But $\begin{cases} \underline{i}' = \underline{i}\cos\alpha + \underline{k}\sin\alpha \\ \underline{k}' = -\underline{i}\sin\alpha + \underline{k}\cos\alpha \end{cases}$

Thus $\underline{H} = \frac{1}{4} mr^2\omega \Big[(-\sin\alpha\cos\alpha)\underline{i}$

$$+ (\sin^2\alpha + 2\cos^2\alpha)\underline{k} \Big]$$

$$\beta = \cos^{-1}\left(\frac{\underline{H}\cdot\underline{k}}{H}\right) = \underline{4.96°} \quad \text{for } \alpha = 10°$$

$\boxed{7/65}$ $\omega_x = \Omega$, $\omega_y = 0$, $\omega_z = p$

$$I_{xx} = I_{yy} = \frac{3}{20} mr^2 + \frac{3}{5} mh^2$$

$$I_{zz} = \frac{3}{10} mr^2, \quad I_{xy} = I_{xz} = I_{yz} = 0$$

Eq. 7/11 yields $\underline{H} = I_{xx}\omega_x \underline{i} + I_{yy}\omega_y \underline{j} + I_{zz}\omega_z \underline{k}$

$$\underline{H}_O = \left(\frac{3}{20} r^2 + \frac{3}{5} h^2\right) m\Omega \underline{i} + \frac{3}{10} mr^2 p \underline{k}$$

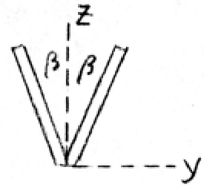
or $\underline{\underline{H}_O = \frac{3}{10} mr^2 \left[\left(\frac{1}{2} + 6\frac{h^2}{r^2}\right)\Omega \underline{i} + p\underline{k}\right]}$

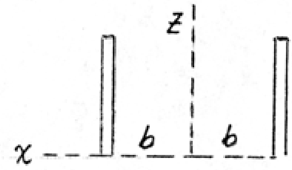
$T = \frac{1}{2} \underline{\omega} \cdot \underline{H}_O = \underline{\frac{3}{10} mr^2 \left[\left(\frac{1}{4} + \frac{h^2}{r^2}\right)\Omega^2 + \frac{1}{2} p^2\right]}$

7/66 With $\omega_x = \omega_y = 0$, $\omega_z = \omega$, the components of $\underline{H}_O$ are $H_{O_x} = -I_{xz}\,\omega_z$, $H_{O_y} = -I_{yz}\,\omega_z$, $H_{O_z} = I_{zz}\,\omega_z$

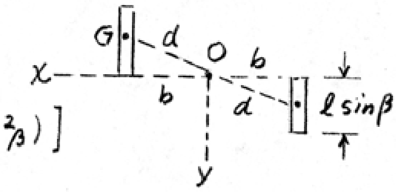
By inspection

$I_{yz} = 0,\ I_{xz} = 0$





$$I_{zz} = 2(I_G + md^2)$$
$$= 2\left[\tfrac{1}{12} m (\ell \sin\beta)^2 + m\left(b^2 + \tfrac{\ell^2}{4}\sin^2\beta\right)\right]$$
$$= 2m\left[\tfrac{1}{3}\ell^2 \sin^2\beta + b^2\right]$$



Thus $\underline{H}_O = 2m\left[\tfrac{1}{3}\ell^2 \sin^2\beta + b^2\right]\omega\,\underline{k}$

7/67 Let $\underline{\Omega}$ = angular velocity of x-y-z about z_o

For axes: $\Omega_x = -\Omega \sin\theta$, $\Omega_y = \dot{\theta} = 0$, $\Omega_z = \Omega \cos\theta$; $\Omega = 2\pi f$

Capsule: $\omega_x = -\Omega \sin\theta$, $\omega_y = 0$, $\omega_z = \Omega \cos\theta + p$

$H_{G_x} = I_{xx}\omega_x = mk'^2(-2\pi f \sin\theta)$, $H_{G_y} = I_{yy}\omega_y = 0$

$H_{G_z} = I_{zz}\omega_z = mk^2(2\pi f \cos\theta + p)$

$\underline{H}_G = 2\pi m f(-k'^2 \sin\theta\, \underline{i} + k^2 \cos\theta\, \underline{k}) + mk^2 p\, \underline{k}$

7/68 $\omega_x = -\omega_1,\ \omega_y = \omega_2,\ \omega_z = p$

Eq. 7/14, $\underline{H}_O = \underline{H}_B + \overrightarrow{OB} \times \underline{G}$, $\overrightarrow{OB} = b\underline{i}$, $\underline{G} = m\underline{v}_B = -mb\omega_2\underline{k}$

$$\overrightarrow{OB} \times \underline{G} = b\underline{i} \times (-mb\omega_2\underline{k}) = mb^2\omega_2\underline{j}$$

$I_{xx} = \frac{1}{4}mr^2,\ I_{yy} = \frac{1}{4}mr^2,\ I_{zz} = \frac{1}{2}mr^2,\ I_{xy} = I_{xz} = I_{yz} = 0$

Eq. 7/11, $H_B = \frac{1}{4}mr^2(-\omega_1)\underline{i} + \frac{1}{4}mr^2\omega_2\underline{j} + \frac{1}{2}mr^2p\underline{k}$

so $\underline{H}_O = -\frac{1}{4}mr^2\omega_1\underline{i} + m\omega_2\left(b^2 + \frac{r^2}{4}\right)\underline{j} + \frac{1}{2}mr^2p\underline{k}$

$$= \frac{1}{4}mr^2\left\{-\omega_1\underline{i} + \left(1 + \frac{4b^2}{r^2}\right)\omega_2\underline{j} + 2p\underline{k}\right]$$

From Eq. 7/15 $T = \frac{1}{2}\bar{v}\cdot m\bar{v} + \frac{1}{2}\omega\cdot\underline{H}_B$

So $T = \frac{1}{2}mb^2\omega_2^2 + \frac{1}{2}(-\omega_1\underline{i} + \omega_2\underline{j} + p\underline{k})\cdot\left(-\frac{1}{4}mr^2\omega_1\underline{i} + \frac{1}{4}mr^2\omega_2\underline{j} + \frac{1}{2}mr^2p\underline{k}\right)$

$$= \frac{1}{2}mb^2\omega_2^2 + \frac{1}{8}mr^2(\omega_1^2 + \omega_2^2 + 2p^2)$$

$$= \frac{mr^2}{8}\left\{\omega_1^2 + \left(1 + \frac{4b^2}{r^2}\right)\omega_2^2 + 2p^2\right\}$$

7/69 x'-y'-z' are principal axes of inertia

so $\underline{H}_{O'} = \underline{i}\, I_{x'x'}\omega_x + \underline{j}\, I_{y'y'}\omega_y + \underline{k}\, I_{z'z'}\omega_z$

where $I_{x'x'} = I_{z'z'} = \frac{1}{4}mr^2$, $I_{y'y'} = \frac{1}{2}mr^2$

$\omega_x = \omega$, $\omega_y = p$, $\omega_z = 0$

so $\underline{H}_{O'} = \frac{1}{4}mr^2\omega\,\underline{i} + \frac{1}{2}mr^2 p\,\underline{j} = \frac{1}{2}mr^2\left(\frac{\omega}{2}\underline{i} + p\underline{j}\right)$

$$= \frac{1}{2}\frac{6}{32.2}\left(\frac{4}{12}\right)^2\left(\frac{10\pi}{2}\underline{i} + 40\pi\underline{j}\right) = \underline{0.1626(\underline{i} + 8\underline{j})}$$

lb-ft-sec

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_{O'} + \frac{1}{2}\bar{\underline{v}}\cdot\underline{G} = \frac{1}{2}(\omega\underline{i} + p\underline{j})\cdot\frac{1}{2}mr^2\left(\frac{\omega}{2}\underline{i} + p\underline{j}\right)$

$+ \frac{1}{2}(-\bar{r}\omega\underline{j})\cdot(-m\bar{r}\omega\underline{j})$ where $\bar{r} = 10\underline{k}$ in.

$= \frac{1}{4}mr^2\left(\frac{1}{2}\omega^2 + p^2\right) + \frac{1}{2}m\bar{r}^2\omega^2$

$= \frac{1}{4}\frac{6}{32.2}\left(\frac{4}{12}\right)^2\left(\frac{1}{2}\overline{10\pi}^2 + \overline{40\pi}^2\right) + \frac{1}{2}\frac{6}{32.2}\left(\frac{10}{12}10\pi\right)^2$

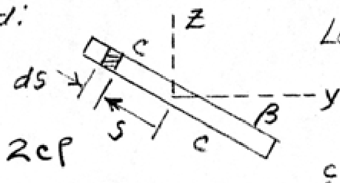
$= 84.29 + 63.85$

$= \underline{148.1 \text{ ft-lb}}$

7/70 With $\omega_x = \omega_y = 0$ & $\omega_z = \omega$, the components of angular momentum become

$H_{O_x} = -I_{xz}\,\omega_z$, $H_{O_y} = -I_{yz}\,\omega_z$, $H_{O_z} = I_{zz}\,\omega_z$

Rod:



Let ρ = mass per unit length

$dI_{yz} = yz\,dm = yz\,\rho\,ds$

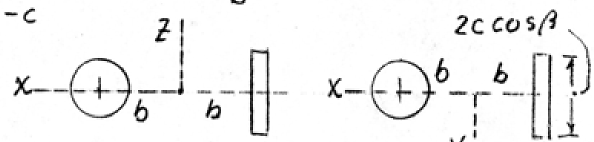
$m = 2c\rho$ $\qquad = (-s\cos\beta)(s\sin\beta)\,\rho\,ds$

$$I_{yz} = -\rho\sin\beta\cos\beta\int_{-c}^{c} s^2 ds = -\tfrac{1}{6}mc^2\sin 2\beta$$

Sphere: $I_{yz} = 0$

By inspection

$I_{xz} = I_{xy} = 0$



Sphere $I_{zz} = \frac{2}{5}mr^2 + mb^2$, Rod $I_{zz} = \frac{1}{12}m(2c\cos\beta)^2 + mb^2$

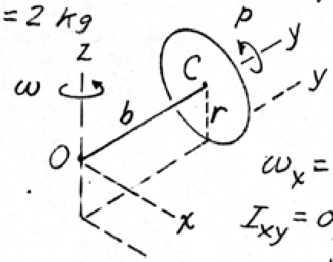
Thus $\underline{H}_O = 0\underline{i} - (-\frac{1}{6}mc^2\sin 2\beta)\omega\underline{j} + (\frac{2}{5}mr^2 + \frac{1}{3}mc^2\cos^2\beta + 2mb^2)\omega\underline{k}$

$$\underline{H}_O = m\omega\left[\tfrac{1}{6}c^2\sin 2\beta\,\underline{j} + \left(\tfrac{2}{5}r^2 + \tfrac{1}{3}c^2\cos^2\beta + 2b^2\right)\underline{k}\right]$$

7/71 $r = 100\,mm \quad \omega = 4\pi$ rad/s

$b = 200\,mm \quad p = \frac{v_C}{r} = \frac{b}{r}\omega = 8\pi$ rad/s

$m = 2\,kg$



Eq. 7/11 holds for point O as a fixed point on axis of disk

$\omega_x = 0,\ \omega_y = -p = -8\pi$ rad/s, $\omega_z = \omega = 4\pi\ \frac{rad}{s}$

$I_{xy} = 0;\ I_{yy} = \frac{1}{2}mr^2 = \frac{1}{2}(2)(0.1)^2 = 0.01\ kg \cdot m^2$

$I_{yz} = 0,\ I_{xz} = 0,\ I_{zz} = \frac{1}{4}mr^2 + mb^2 = 2\left(\frac{1}{4}\overline{0.1}^2 + \overline{0.2}^2\right)$

$= 0.085\ kg \cdot m^2$

So $\underline{H}_O = \underline{j} I_{yy}\omega_y + \underline{k} I_{zz}\omega_z = \underline{j}\left(-\frac{1}{2}mr^2 p\right) + \underline{k}\left(\frac{1}{4}mr^2 + mb^2\right)\omega$

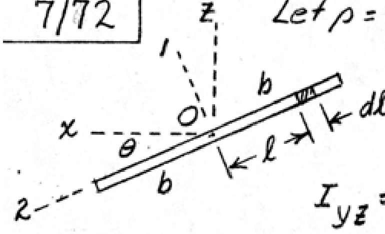
$= mr^2\omega\left(-\frac{1}{2}\frac{b}{r}\underline{j} + \left[\frac{1}{4} + \frac{b^2}{r^2}\right]\underline{k}\right)$

$= 2(0.1)^2 4\pi\left(-\frac{1}{2}2\underline{j} + \left[\frac{1}{4} + 4\right]\underline{k}\right) = \underline{0.251(-\underline{j} + 4.25\underline{k})\ N \cdot m \cdot s}$

$T = \frac{1}{2}\underline{\omega} \cdot \underline{H}_O = \frac{1}{2}(-8\pi\underline{j} + 4\pi\underline{k}) \cdot 0.251(-\underline{j} + 4.25\underline{k})$

$= 3.15 + 6.71 = \underline{9.87\ J}$

7/72



Let ρ = mass per unit of panel area

$$I_{xz} = \int_{-b}^{b} (-l\cos\theta)(l\sin\theta)2c\rho\, dl$$

$$= -\frac{2}{3}\rho c b^3 \sin 2\theta \quad \text{for 2 panels}$$

$I_{yz} = I_{xy} = 0$ by symmetry

$I_{zz} = \bar{I}_{zz} + md^2$ for each panel

For total, $$I_{zz} = 2\left\{\frac{2bc\rho}{12}\left[c^2 + (2b\cos\theta)^2\right] + 2bc\rho\left[a + \frac{c}{2}\right]^2\right\}$$

$$= 4bc\rho\left\{\frac{c^2}{3} + \frac{b^2}{3}\cos^2\theta + a^2 + ac\right\}$$

$\underline{H}_O = -I_{xz}\omega_z \underline{i} + I_{zz}\omega_z \underline{k}$, $m = 4bc\rho$ (total)

$$\underline{H}_O = \frac{m}{6} b^2 \omega \sin 2\theta\, \underline{i} + m\omega\left\{\frac{c^2}{3} + \frac{b^2}{3}\cos^2\theta + a^2 + ac\right\}\underline{k}$$

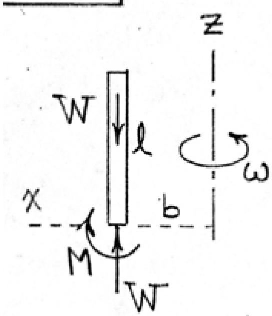
By symmetry, principal axes are O-1, O-2, O-y

$I_1 = m\left\{\frac{c^2 + b^2}{3} + a^2 + ac\right\}$ (max)

$I_2 = m\left\{\frac{1}{3}c^2 + a^2 + ac\right\}$ (intermediate)

$I_3 = \frac{1}{3}mb^2$ (minimum)

7/73

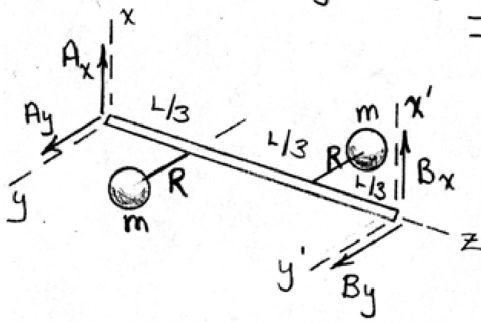


$$\Sigma M_y = -I_{xz}\,\omega_z^2 :$$

$$-M = -m\,\frac{b\ell}{2}\,\omega^2$$

$$M = \frac{mb\ell}{2}\,\omega^2$$

7/74 $\omega_x = \omega_y = 0, \quad \omega_z = \omega, \quad \dot{\omega}_z = 0$



$I_{yz} = m\frac{L}{3}R - m\frac{2L}{3}R$

$= -mLR/3$

$I_{xz} = 0$

$I_{y'z} = -mLR/3$

$I_{x'z} = 0$

x-y axes: $\Sigma M_x = I_{yz}\,\omega_z^2$ (from Eq. 7/23)

$-B_y L = -\frac{mLR}{3}\omega^2, \quad \underline{B_y = \frac{mR\omega^2}{3}}$

$\Sigma M_y = 0, \quad \underline{B_x = 0}$

x'-y' axes: $\Sigma M_{x'} = I_{y'z}\,\omega_z^2$

$A_y L = -\frac{mLR}{3}\omega^2, \quad A_y = \underline{-\frac{mR\omega^2}{3}}$

$\Sigma M_{y'} = 0, \quad \underline{A_x = 0}$

7/75

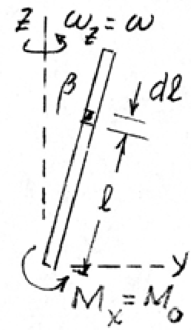
$$dI_{yz} = yz\,dm = (l\sin\beta)(l\cos\beta)\,dm$$

$$I_{yz} = \frac{1}{2}\sin 2\beta \int l^2 dm = \frac{1}{2}\sin 2\beta (I_{xx})$$

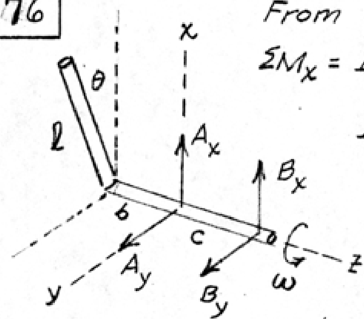
$$= \frac{1}{6} m L^2 \sin 2\beta$$

Eq. 7/23 $\Sigma M_x = I_{yz}\,\omega_z^2$

$$M_0 = \frac{1}{6} m L^2 \omega^2 \sin 2\beta$$



7/76



From Eqs. 7/23, with $\dot{\omega}_z = \dot{\omega} = 0$,

$\Sigma M_x = I_{yz}\omega_z^2$, $\Sigma M_y = -I_{xz}\omega_z^2$, $\Sigma M_z = 0$

$I_{yz} = -m\frac{bl}{2}\sin\theta$, $I_{xz} = -m\frac{bl}{2}\cos\theta$

So $-B_y c = -m\frac{bl}{2}\sin\theta\ (\omega^2)$

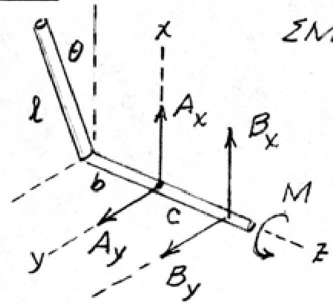
$$B_y = \frac{mbl\omega^2}{2c}\sin\theta$$

& $+B_x c = m\frac{bl}{2}\cos\theta\,(\omega^2)$

$$B_x = \frac{mbl\omega^2}{2c}\sin\theta$$

& $\underline{B} = \frac{mbl\omega^2}{2c}(\underline{i}\sin\theta + \underline{j}\cos\theta)$, $B = |\underline{B}| = \frac{mbl\omega^2}{2c}$

7/77



From Eq. 7/23, with $\omega_z = 0$

$\Sigma M_z = I_{zz}\dot{\omega}_z$; $M = \frac{1}{3}ml^2\dot{\omega}$

$\dot{\omega} = \frac{3M}{ml^2}$

$\Sigma M_x = -I_{xz}\dot{\omega}_z$, $\Sigma M_y = -I_{yz}\dot{\omega}_z$

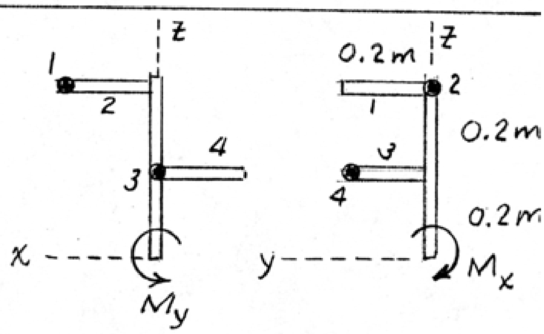
so $-B_y c = -\left(-m\frac{bl}{2}\cos\theta\right)\frac{3M}{ml^2}$

$\underline{B_y = -\frac{3Mb}{2lc}\cos\theta}$

& $B_x c = -\left(-\frac{mbl}{2}\sin\theta\right)\frac{3M}{ml^2}$, $\underline{B_x = \frac{3Mb}{2lc}\sin\theta}$

7/78

$\omega_z = \frac{1200(2\pi)}{60} = 125.7 \ \frac{rad}{s}$



Eq. 7/23, $\Sigma M_x = I_{yz}\,\omega_z^2$

$\Sigma M_y = -I_{xz}\,\omega_z^2$

$I_{yz} = m_1(0.1)(0.4) + m_2(0) + m_3(0.1)(0.2) + m_4(0.2)(0.2)$

$= 0.12(0.04 + 0.02 + 0.04) = 0.012 \ kg\cdot m^2$

$I_{xz} = m_1(0.2)(0.4) + m_2(0.1)(0.4) + m_3(0) + m_4(-0.1)(0.2)$

$= 0.12(0.08 + 0.04 - 0.02) = 0.012 \ kg\cdot m^2$

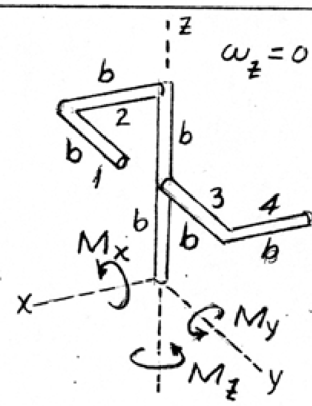
Thus $M_x = 0.012(125.7)^2 = 189.5 \ N\cdot m$

$M_y = -0.012\,(125.7)^2 = -189.5 \ N\cdot m$

$M = \sqrt{M_x^2 + M_y^2} = 189.5\sqrt{2} = \underline{268 \ N\cdot m}$

7/79 $m_1 = m_2 = m_3 = m_4 = 0.12$ kg

$b = 0.2$ m, $M_z = 64$ N·m



Eq. 7/23 $\Sigma M_x = -I_{xz}\dot{\omega}_z$

$\Sigma M_y = -I_{yz}\dot{\omega}_z$

$\Sigma M_z = I_{zz}\dot{\omega}_z$

For ① $I_{zz} = \frac{1}{12}mb^2 + m\left(b^2 + \frac{b^2}{4}\right) = \frac{4}{3}mb^2$

② & ③ $I_{zz} = \frac{1}{3}mb^2$

④ $I_{zz} = \frac{4}{3}mb^2$

Total $I_{zz} = \frac{10}{3}mb^2 = \frac{10}{3}(0.12)(0.2)^2 = 0.016$ kg·m^2

From Sol. to Prob. 7/78, $I_{yz} = I_{xz} = 0.012$ kg·m^2

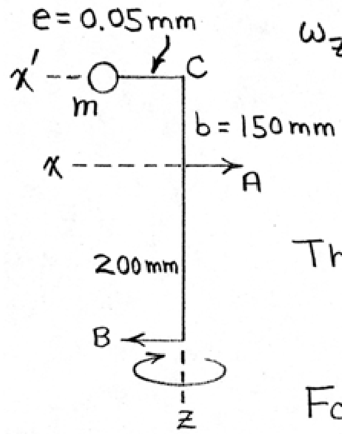
So $64 = 0.016\dot{\omega}_z$, $\dot{\omega}_z = 4000$ rad/s^2

$M_x = -0.012(4000) = -48$ N·m

$M_y = -0.012(4000) = -48$ N·m

$M = \sqrt{M_x^2 + M_y^2} = \underline{48\sqrt{2} \text{ N·m}}$

7/80



$\Sigma M_y = -I_{yz}\dot{\omega}_z - I_{xz}\omega_z^2$, $\dot{\omega}_z = 0$

$\omega_z = \omega = 10{,}000\left(\frac{2\pi}{60}\right) = 1047\ \frac{\text{rad}}{\text{sec}}$

$I_{xz} = -mbe = -6(0.15)(50)(10^{-6})$

$= -45(10^{-6})\ \text{kg}\cdot\text{m}^2$

Thus $B(0.20) = 45(10^{-6})(1047)$

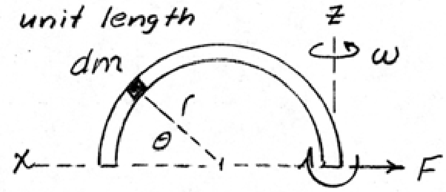
$= \underline{247\ \text{N}}$

For origin of coordinates x'-y'-z at C, $\Sigma M_{y'} = 0$, since $I_{x'z} = 0$.

Thus $0.35B - 0.15A = 0$, $A = \frac{0.35}{0.15}(247) = \underline{576\ \text{N}}$

7/81 Let ρ = mass per unit length

$\Sigma M_y = -I_{xz}\,\omega_z^2$

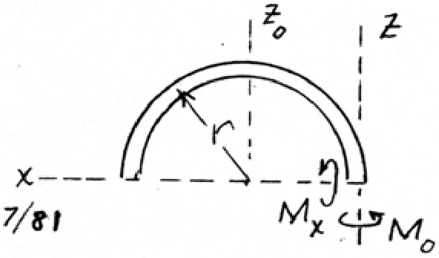


$I_{xz} = \int xz\,dm = \int_0^{\pi} (r + r\cos\theta)(r\sin\theta)\,\rho r\,d\theta$

$= \rho r^3 \left[-\cos\theta - \frac{1}{4}\cos 2\theta\right]_0^{\pi} = 2\rho r^3 = \frac{2}{\pi} m r^2$

So $-M = -\frac{2}{\pi} m r^2 \omega^2$, $\underline{M = \frac{2}{\pi} m r^2 \omega^2}$

7/82 $m = \rho \pi r$



$I_{zz} = I_{z_0 z_0} + mr^2$

$= \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$

$I_{xz} = \frac{2}{\pi}mr^2$ from Prob. 7/81

Eq. 7/23 with $\omega = \omega_z = 0$, $\dot{\omega} = \dot{\omega}_z$

$\Sigma M_z = I_{zz}\dot{\omega}_z$: $M_0 = \frac{3}{2}mr^2\dot{\omega}_z$, $\dot{\omega}_z = \frac{2M_0}{3mr^2}$

$\Sigma M_x = -I_{xz}\dot{\omega}_z$: $M = M_x = -\frac{2}{\pi}mr^2\left(\frac{2M_0}{3mr^2}\right) = \underline{-\frac{4M_0}{3\pi}}$

7/83 $\Sigma M_{z_1} = I_{z_1} \alpha$ where I_{z_1} is given by Eq. B/10 with $l = \cos\theta$, $m = 0$, $n = \sin\theta$

$I_{xy} = I_{xz} = I_{yz} = 0$

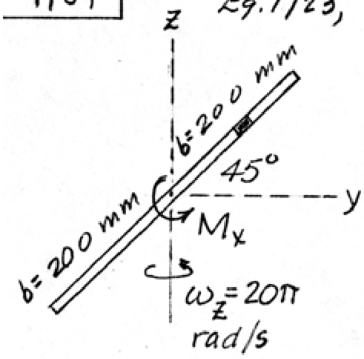
Thus $I_{z_1} = I_{xx} l^2 + I_{yy} m^2 + I_{zz} n^2 + 0$

$= I_0 \cos^2\theta + 0 + I \sin^2\theta$

so $M = (I_0 \cos^2\theta + I \sin^2\theta)\alpha$

$$\alpha = \frac{M}{I_0 \cos^2\theta + I \sin^2\theta}$$

7/84



Eq. 7/23, $\Sigma M_x = I_{yz}\,\omega_z^2$

$$I_{yz} = \int yz\,dm = \int y^2 \rho\, d\ell = \int_{-b/\sqrt{2}}^{b/\sqrt{2}} y^2 \rho \sqrt{2}\, dy$$

$$= \frac{\rho\sqrt{2}}{3}\left(\frac{b^3}{2\sqrt{2}} - \frac{-b^3}{2\sqrt{2}}\right) = \frac{mb^2}{6} = \frac{3(0.2)^2}{6} = 0.02 \text{ kg}\cdot\text{m}^2$$

$$M_x = 0.02\,(20\pi)^2$$

$$= 79.0 \text{ N}\cdot\text{m}$$

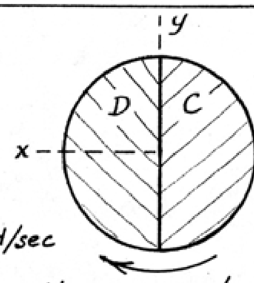
On plate, $\underline{M}_x = 79.0\,\underline{i}$ N·m

but acting on shaft, $\underline{M} = -79.0\,\underline{i}$ N·m

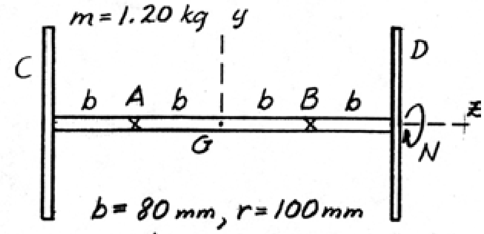
7/85

$\omega_x = \omega_y = 0,$

$\omega_z = \dfrac{1200 \times 2\pi}{60}$
$= 125.7$ rad/sec

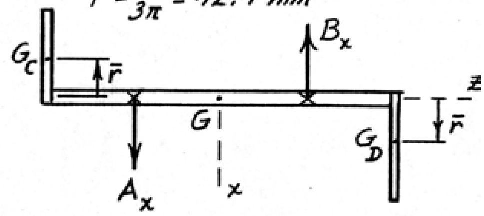


$N = 1200$ rev/min



$b = 80$ mm, $r = 100$ mm

$\bar{r} = \dfrac{4r}{3\pi} = 42.4$ mm



From Eqs. 7/23, about G,

$\Sigma M_x = I_{yz}\omega_z^2,\ \Sigma M_y = -I_{xz}\omega_z^2,$

$\Sigma M_z = 0$

$I_{yz} = 0,\ I_{xz} = \{0 + m(2b)(\bar{r})\} + \{0 + m(-2b)(-\bar{r})\} = 4mb\bar{r}$

$= 4(1.20)(0.080)(0.0424) = 0.01630\ \text{kg}\cdot\text{m}^2$

$\Sigma F_x = 0$ so $A_x = B_x$

$\Sigma M_y = -A_x b - B_x b = -4mb\bar{r}\omega_z^2,\ A_x = B_x = 2mb\bar{r}\omega_z^2/b$

$A_x = B_x = \frac{1}{2}(0.01630)(125.7)^2/0.080$

$= 1608$ N

$\Sigma M_x = 0,\ A_y = B_y = 0$

$\underline{\mathbf{F}_A = 1608\,\mathbf{i}\ \text{N}},\ \underline{\mathbf{F}_B = -1608\,\mathbf{i}\ \text{N}}$

7/86 With $\omega_x = \omega_y = \omega_z = \dot{\omega}_x = \dot{\omega}_y = 0$, $\dot{\omega}_z = 900 \text{ rad/s}^2$,

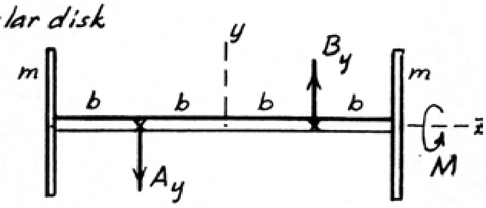
Eqs. 7/23 become

$\Sigma M_x = -I_{xz}\alpha$, $\Sigma M_y = -I_{yz}\alpha$, $\Sigma M_z = I_{zz}\alpha$

From the solution to Prob. 7/85, $I_{yz} = 0$, $I_{xz} = 0.01630 \text{ kg}\cdot\text{m}^2$

Also $I_{zz} = \frac{1}{2}(2m)r^2 = 1.20(0.100)^2 = 0.012 \text{ kg}\cdot\text{m}^2$

where m = mass of semicircular disk



$\Sigma F_y = 0$ so $A_y = B_y$

$\Sigma M_x = -0.080 A_y - 0.080 B_y$

$= -0.01630(900)$

$A_y = B_y = 91.7 \text{ N}$

so $\underline{\mathbf{F}_A = -91.7\mathbf{j} \text{ N}, \mathbf{F}_B = 91.7\mathbf{j} \text{ N}}$

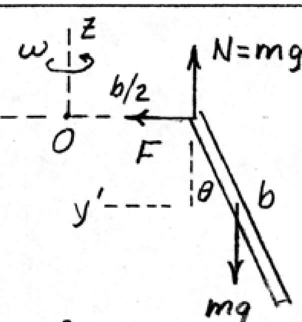
$b = 80 \text{ mm}$

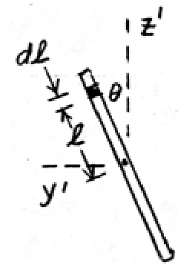
$m = 1.20 \text{ kg}$

$\alpha = \dot{\omega}_z = 900 \text{ rad/s}^2$

$M = \Sigma M_z = 0.012(900) = \underline{10.8 \text{ N}\cdot\text{m}}$

7/87





$I_{yz} = I_{y'z'} + m d_y d_z$

$I_{y'z'} = \int \ell \sin\theta \, \ell \cos\theta \, dm$

$= \sin\theta \cos\theta \int \ell^2 dm$

$= \sin\theta \cos\theta \, I_{x'x'}$

$= \frac{1}{2} \sin 2\theta \, \frac{1}{12} m b^2 = \frac{1}{24} m b^2 \sin 2\theta$

$I_{yz} = \frac{1}{24} m b^2 \sin 2\theta + m\left(-\frac{b}{2} - \frac{b}{2}\sin\theta\right)\left(-\frac{b}{2}\cos\theta\right)$

$= \frac{mb^2}{4}\left(\frac{2}{3}\sin 2\theta + \cos\theta\right)$

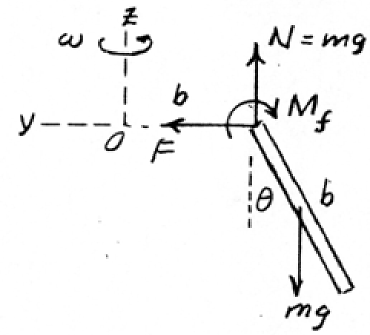
Eq. 7/23 $\Sigma M_x = 0 + I_{yz}\omega_z^2$

$mg\left(\frac{b}{2} + \frac{b}{2}\sin\theta\right) - mg\frac{b}{2} = \frac{mb^2}{4}\left(\frac{2}{3}\sin 2\theta + \cos\theta\right)$

$g \tan\theta = b\left(\frac{2}{3}\sin\theta + \frac{1}{2}\right)\omega^2$

$$\omega = \sqrt{\frac{1}{b} \cdot \frac{6g\tan\theta}{4\sin\theta + 3}}$$

7/88



Eq. 7/23 $\Sigma M_x = 0 + I_{yz}\,\omega_z^2$

From sol. to Prob. 7/87

$$I_{yz} = \frac{mb^2}{4}\left(\frac{2}{3}\sin 2\theta + \cos\theta\right)$$

$$M_f + mg\left(\frac{b}{2} + \frac{b}{2}\sin\theta\right) - mg\frac{b}{2}$$
$$= \frac{mb^2}{4}\left(\frac{2}{3}\sin 2\theta + \cos\theta\right)\omega^2$$

$$M_f = \underline{\frac{mb}{2}\left\{\cos\theta\left[\frac{2}{3}\sin\theta + \frac{1}{2}\right]b\omega^2 - g\sin\theta\right\}}$$

where $\omega^2 > \dfrac{6g\tan\theta}{b(4\sin\theta + 3)}$

7/89 From Eq. 7/23 with $\omega_z = \omega$, $\dot{\omega}_z = 0$,

$\Sigma M_x = I_{yz}\,\omega^2$

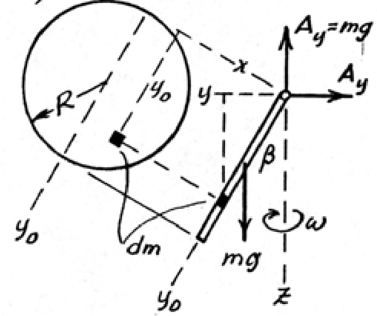
$I_{yz} = \int yz\,dm = \int (y_0 \sin\beta)(y_0 \cos\beta)\,dm$

$= \sin\beta\cos\beta \int y_0^2\,dm$

$= \frac{1}{2}\sin 2\beta\, I_{xx}$

$= \frac{1}{2}\sin 2\beta\left(\frac{1}{4}mR^2 + mR^2\right)$

$= \frac{5}{8}mR^2 \sin 2\beta$

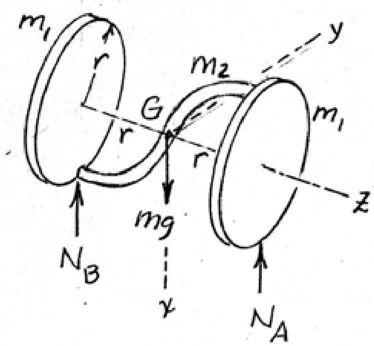


So $mgR\sin\beta = \left(\frac{5}{8}mR^2\sin 2\beta\right)\omega^2$,

$\sin\beta\left(g - \frac{5}{8}R\omega^2 \times 2\cos\beta\right) = 0$, $\underline{\beta = \cos^{-1}\frac{4g}{5R\omega^2} \text{ if } \omega^2 \geq \frac{4g}{5R}}$;

$\underline{\text{otherwise } \beta = 0}$

7/90 From Sample Problem 7/7:



$$I_{xz} = -\rho r^3 = -\frac{m_2 r^2}{\pi}$$

$$\Sigma M_y = -I_{xz}\,\omega_z^2 \quad , \quad \omega_z = v/r$$

$$N_B r - N_A r = \frac{m_2 r^2}{\pi}\frac{v^2}{r^2} = \frac{m_2 v^2}{\pi}$$

Also $\Sigma F_x = 0$; $N_A + N_B - mg = 0$

Combine & get

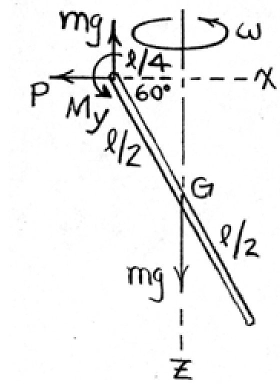
$$\underline{N_A = \frac{mg}{2} - \frac{m_2 v^2}{2\pi r}}$$

$m = 2m_1 + m_2$

$$\underline{N_B = \frac{mg}{2} + \frac{m_2 v^2}{2\pi r}}$$

7/91

$\Sigma F_x = m\bar{a}_x : \quad P = 0$



$\Sigma M_y = -I_{xz}\,\omega_z^2$

$I_{xz} = \int xz\,dm = \int_{-\ell/4}^{\ell/4} x\sqrt{3}\left(\frac{\ell}{4}+x\right)\rho\,dx$

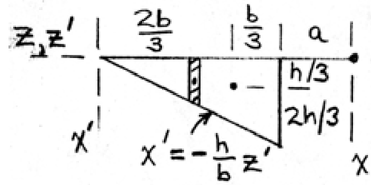
where ρ = mass/(x-comp. of length)

$I_{xz} = \frac{\sqrt{3}}{48}\,m\ell^2$, where $m = \frac{\rho\ell}{2}$

So $\quad M_y - mg\frac{\ell}{4} = -\frac{\sqrt{3}}{48}\,m\ell^2\omega^2$

& for $M_y = 0$, $\quad \omega = 2\sqrt{\frac{\sqrt{3}\,g}{\ell}}$

▸7/92



$$I_{x'z'} = \int x_c' z_c' \, dm$$

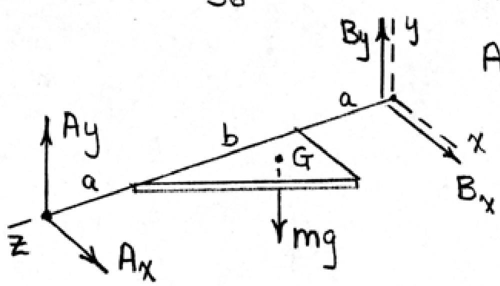
$$= \int \left(-\frac{h}{2b} z'\right)(z') \, \rho \, (x' dz')$$

$$= \int \left(-\frac{h}{2b} z'\right)(z') \, \rho \left(+\frac{h}{b} z' dz'\right)$$

$$I_{x'z'} = \frac{-h^2 \rho}{2b^2} \int_0^{-b} z'^3 \, dz' = -\frac{1}{4} mhb, \text{ since } m = \frac{\rho hb}{2}$$

$$\bar{I}_{x'z'} = I_{x'z'} - m d_{x'} d_{y'} = -\frac{1}{4} mhb - m\left(+\frac{h}{3}\right)\left(-\frac{2b}{3}\right)$$

$$= -\frac{1}{36} mhb. \quad \text{Similarly, } I_{xz} = \frac{1}{12} mhb + \frac{1}{3} mha$$



Also, $I_{zz} = \frac{1}{6} mh^2$, $I_{yz} = 0$

Eqs. 7/23: $\Sigma M_z = I_{zz} \dot{\omega}_z$

$$-mg \frac{h}{3} = \frac{1}{6} mh^2 \dot{\omega}_z$$

$$\dot{\omega}_z = -2g/h$$

$$\Sigma M_y = 0 \Rightarrow A_x = 0$$

$$\Sigma M_x = -I_{xz} \dot{\omega}_z : \; -A_y(2a+b) + mg\left(a + \frac{b}{3}\right) = -I_{xz} \dot{\omega}_z$$

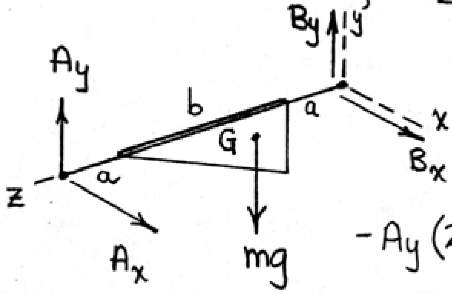
Simplifying, $A_y = A = \underline{mg/6}$

▸7/93

$$U = \Delta T + \Delta V_e + \Delta V_g$$

$$0 = \frac{1}{2} I_{zz}\, \omega_z^2 - mg(h/3)$$

From Prob. 7/92, $I_{zz} = \frac{1}{6} m h^2$, so $\omega_z = 2\sqrt{\frac{g}{h}}$



$$\Sigma M_z = 0, \quad \dot{\omega}_z = 0$$

$$\Sigma M_x = I_{yz}\, \omega_z^2,$$

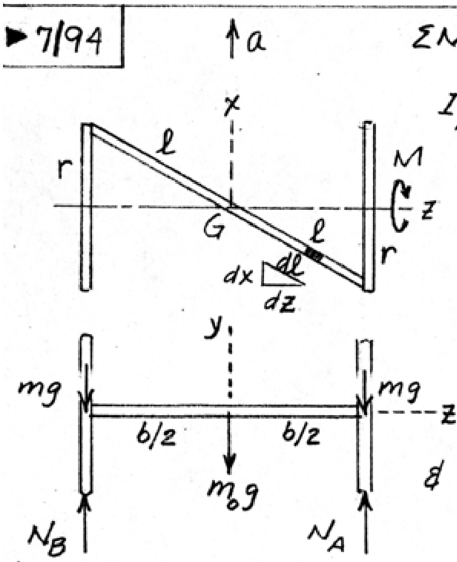
$$-A_y(2a+b) + mg\left(a + \frac{b}{3}\right) = - m h\left(\frac{b}{12} + \frac{a}{3}\right) 4 \frac{g}{h}$$

$$A_y = \frac{mg}{3}\left[\frac{7a+2b}{2a+b}\right]$$

$$\Sigma M_y = 0 : \quad A_x(2a+b) = 0, \quad A_x = 0$$

$$A = \sqrt{A_x^2 + A_y^2} = \underline{\frac{mg}{3}\left[\frac{7a+2b}{2a+b}\right]}$$

▶ 7/94



$\Sigma M_x = -I_{xz}\dot{\omega}_z$ from Eq. 7/23

$I_{xz} = \int xz\,dm = \int (-x)\left(\frac{b}{2r}x\right)\rho\,dl$

$= -\frac{b}{2r}\rho\int_{-r}^{r} x^2\frac{l}{r}\,dx = -\frac{1}{6}brm_0$

where $m_0 = 2\rho l$

Thus $N_B\frac{b}{2} - N_A\frac{b}{2} = +\frac{1}{6}brm_0\left(-\frac{a}{r}\right)$

$N_B - N_A = -\frac{1}{3}m_0 a$

& $\Sigma F_y = 0$; $N_A + N_B = (m_0 + 2m)g$

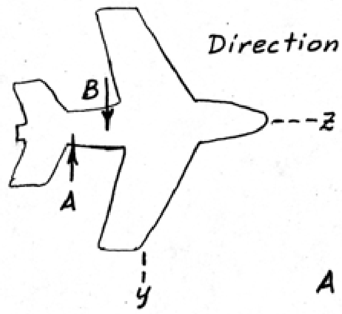
Combine & get

$\underline{N_A = mg + \frac{m_0 g}{2}\left(1 + \frac{a}{3g}\right)}$

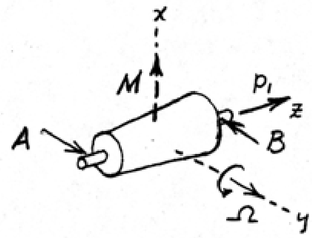
$\underline{N_B = mg + \frac{m_0 g}{2}\left(1 - \frac{a}{3g}\right)}$

$\dot{\omega}_z = -\frac{a}{r}\underline{k}$, $\omega_z = \omega_x = \omega_y = 0$

7/95



Direction of bearing forces acting on airframe



To satisfy $\underline{M} = I\underline{\Omega} \times \underline{p}$

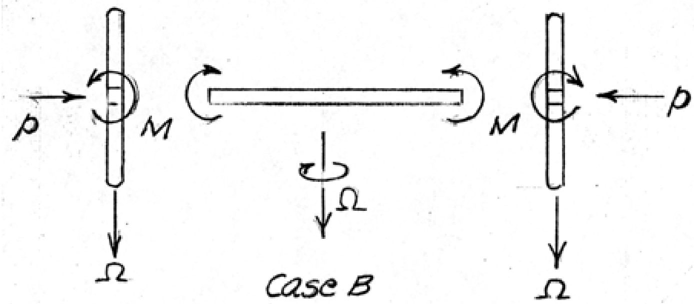
$\underline{p \text{ must be } p_1}$

7/96 $\underline{M} = I\underline{\Omega} \times \underline{p}$: $-M\underline{i} = I\underline{\Omega} \times p\underline{j}$

$\underline{\Omega}$ is in $+\underline{k}$ direction

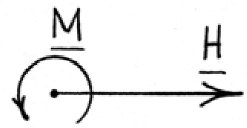
So precession is CCW when viewed from above.

7/97

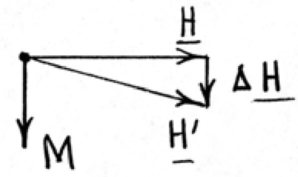


Case B

7/98



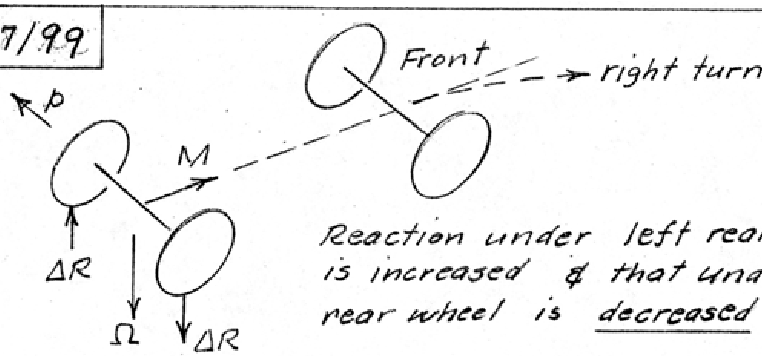
(Side view)



(Overhead view)

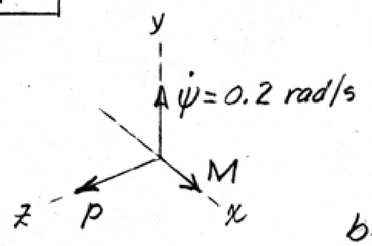
$\underline{M}$ is the moment exerted on the handle by the student; $\underline{H}$ is the wheel angular momentum. From $\underline{M} = \dot{\underline{H}} \cong \frac{\Delta \underline{H}}{\Delta t}$, we see that $\Delta \underline{H}$ is in the same direction as $\underline{M}$. $\underline{H}'$ is the new angular momentum. The student will sense a tendency of the wheel to rotate to her right.

7/99



Reaction under left rear wheel is increased & that under right rear wheel is decreased

7/100



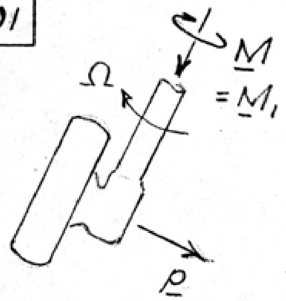
$$M = I\dot{\psi}p$$

$$0.8(9.81)(b-0.180) = 2.2(0.06)^2(0.2)\,\frac{1725(2\pi)}{60}$$

$$b - 0.180 = 0.0364$$

$$b = 0.216 \text{ m} \quad \text{or} \quad \underline{b = 216 \text{ mm}}$$

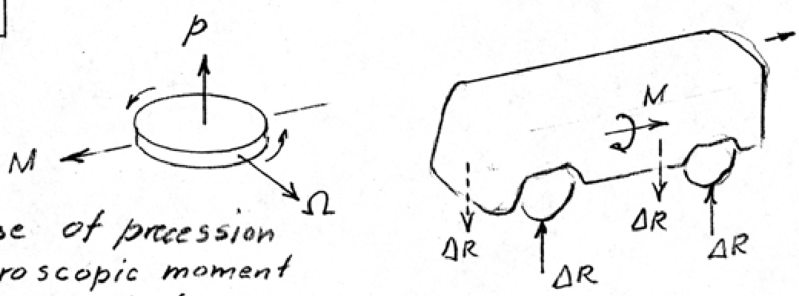
7/101



$M = I\Omega p$

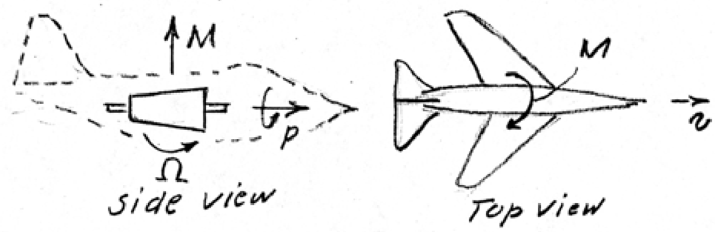
$M = M_1 = mk^2\Omega \frac{v}{r}$

7/102



Because of precession Ω, gyroscopic moment on rotor points to the rear and reacting moment on bus is forward. Result is that the force under the right-hand tires is increased.

7/103



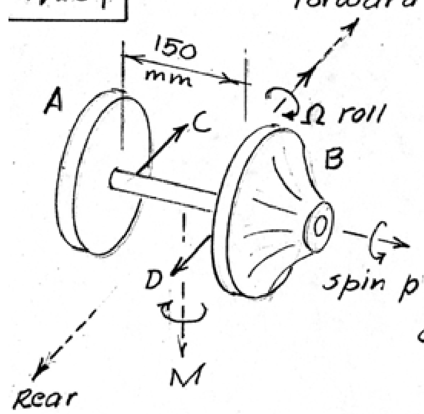
Pilot would apply ***left rudder*** *to counter the clockwise (viewed from above) reaction to the gyroscopic moment*

$$M = I\Omega p = 210(0.220)^2\left[\frac{1200(1000)}{3600}\Big/3800\right]\frac{18000\times 2\pi}{60}$$

$$= (10.16)(0.0877)(1885)$$

$$= \underline{1681\ \text{N}\cdot\text{m}}$$

7/104



$p = 20\,000 \frac{2\pi}{60} = 2094 \text{ rad/s}$

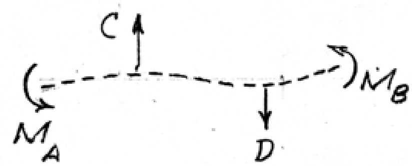
$\Omega = 2 \text{ rad/s}$

$I = 3.5(0.079)^2 + 2.4(0.071)^2$
$= 0.0339 \text{ kg}\cdot\text{m}^2$

$M = I\Omega p \quad (= M_A + M_B)$

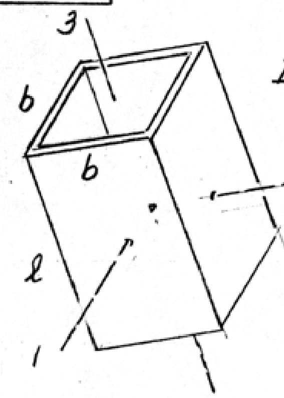
$0.15C = 0.0339(2)(2094)$

$\underline{C = D = 948 \text{ N}}$



7/105

Let m = mass of each of the four sides



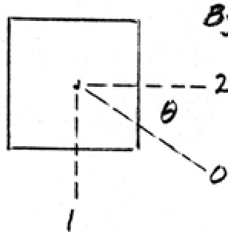
$$I_1 = I_2 = 2\left\{\frac{m}{12}(b^2+\ell^2) + \frac{1}{12}m\ell^2 + m\left(\frac{b}{2}\right)^2\right\}$$

$$= \frac{m}{3}(2b^2+\ell^2)$$

$$I_3 = 4\left\{\frac{m}{12}b^2 + m\left(\frac{b}{2}\right)^2\right\} = \frac{4}{3}mb^2$$

$I_1 = I_3$ if $\frac{m}{3}b^2(2+[\ell/b]^2) = \frac{4mb^2}{3}$

or $\ell/b = \sqrt{2}$



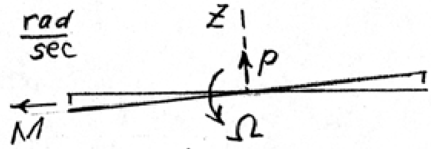
By Eq. B/10, $I_0 = I_1 = I_2 = \frac{m}{3}(2b^2+\ell^2)$

If $\underline{\ell > b\sqrt{2}}$, $I_0 > I_3$ <u>direct precession</u>

If $\underline{\ell < b\sqrt{2}}$, $I_0 < I_3$ <u>retrograde precession</u>

7/106

$\Omega = \frac{10}{180}\pi = 0.1745 \ \frac{rad}{sec}$



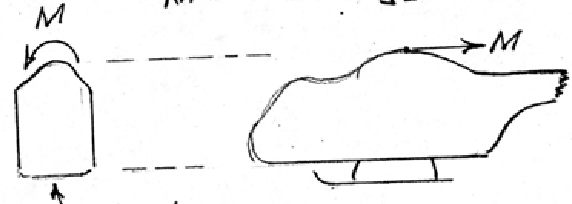
$p = \frac{500}{60} 2\pi = 52.4 \ rad/sec$

$I = \frac{140}{32.2} 10^2 = 435$ lb-ft-sec^2

$M = I\Omega p$

$= 435(0.1745)52.4$

$= \underline{3970 \text{ lb-ft}}$



As viewed by passenger looking forward

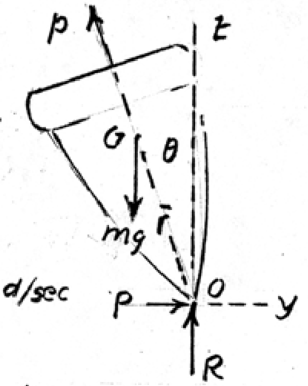
Conclusion: <u>CCW deflection</u>

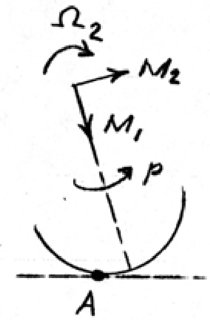
7/107

Neglect momentum about z-axis compared with that about spin axis.

$\bar{r} = 2.5$ in., $\bar{k} = 0.62$ in.

$p = 3600(2\pi)/60 = 377$ rad/sec





Eq. 7/24a

$$\underline{M}_O = I\underline{\Omega}\times\underline{p}: \quad mg\bar{r}\sin\theta\,\underline{i} = I\Omega\underline{k}\times p(\cos\theta\,\underline{k} - \sin\theta\,\underline{j})$$

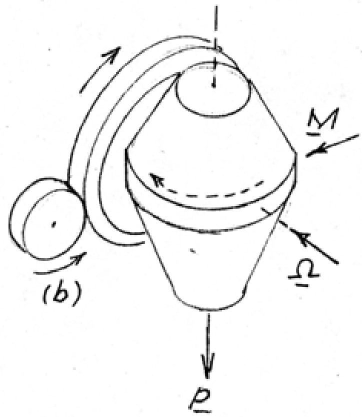
$$mg\bar{r}\sin\theta = I\Omega p\sin\theta$$

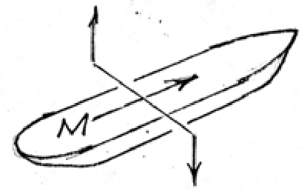
$$\text{or } g\bar{r} = \bar{k}^2\Omega p, \quad \Omega = \frac{g\bar{r}}{k^2 p} \quad (\text{Eq. } 7/25)$$

$$\text{So } \Omega = \frac{32.2(2.5/12)}{(0.62/12)^2\,377} = 6.67 \text{ rad/sec or } \underline{\underline{\Omega} = 6.67\underline{k} \text{ rad/sec}}$$

Friction force at A is into the paper ($-x$-dir) which produces a moment M_1 to slow the spin and a moment M_2 which causes a precession Ω_2 that decreases θ.

7/108

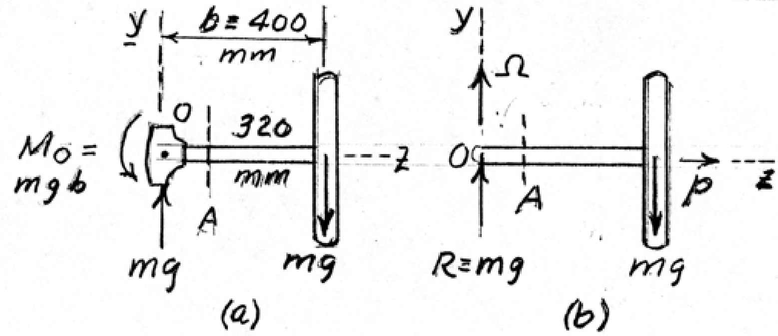




M needed on structure of ship to counteract roll to port (left).
Reaction on gyro is opposite to M on ship.
Proper directions of $\underline{p}$, $\underline{\Omega}$, $\underline{M}$ shown - requiring rotation (b) of motor.

$$M = I\Omega p = 80(1.45)^2\, 960\, \frac{2\pi}{60}\, 0.320 = \underline{5410 \text{ kN}\cdot\text{m}}$$

7/109



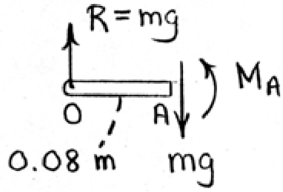
Case (a) $\Sigma M_x = 0$: so no precession

$M_A = 4(9.81)0.320 = \underline{12.56 \text{ N}\cdot\text{m}}$

Case (b) $\Sigma M_x = mgb = 4(9.81)\,0.4 = 15.70 \text{ N}\cdot\text{m}$

$\Sigma M_x = I_{zz}\Omega p$: $15.70 = 4(0.12)^2 \Omega \dfrac{3600(2\pi)}{60}$

$\underline{\Omega = 0.723 \text{ rad/s}}$



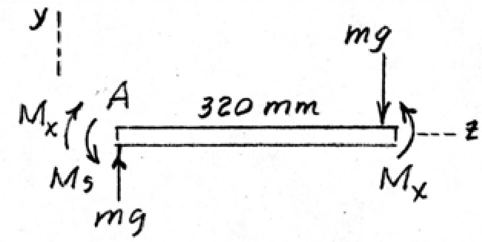
$\Sigma M_{A_x} = 0$: $M_A = mg(0.08)$

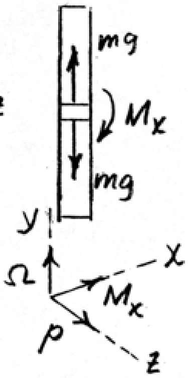
$M_A = 4(9.81)(0.08) = \underline{3.14 \text{ N}\cdot\text{m}}$

7/110

$mg = 4(9.81)$
$= 39.2 \text{ N}$

$\Omega = 2$ rad/s const





For rotor

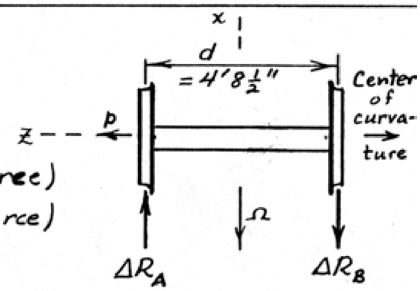
$M_x = I_{zz}\Omega p = 4(0.12)^2 2 \dfrac{3600(2\pi)}{60} = 43.4 \text{ N·m}$

So $M_A = M_x - M_s = 43.4 - 39.2(0.320)$
$= \underline{30.9 \text{ N·m}}$

$\Sigma M_y = I_{yy}\dot{\Omega}$ but $\Omega = $ const. so $\dot{\Omega} = 0$ & $\underline{M_y = M_O = 0}$

7/111 $\underline{M} = I\underline{\Omega} \times \underline{p} = I(-\Omega \underline{i} \times p\underline{k})$

$= I\Omega p \underline{j}$



(view from rear)

so $\underline{M}$ is into the paper &

ΔR_A is up (increase of normal force)

& ΔR_B is down (decrease of normal force)

For wheel & axle unit,

$I_{zz} = \Sigma \frac{1}{2} mr^2 = 2\left(\frac{1}{2}\frac{560}{32.2}\left[\frac{33/2}{12}\right]^2\right) + \frac{1}{2}\frac{300}{32.2}\left(\frac{5/2}{12}\right)^2$

$= 32.9 + 0.202 = 33.1$ lb-ft-sec^2

$p = v/r = \left(\frac{80}{30}44\right) / \frac{33/2}{12} = 85.3$ rad/sec

$\Omega = v/\rho = \left(\frac{80}{30}44\right)/717 = 0.1636$ rad/sec

$d = 4'8\frac{1}{2}'' = 4.71$ ft

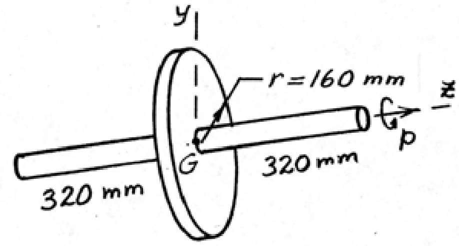
So $M = \Delta R\,(4.71) = 33.1 \times 85.3 \times 0.1636$, $\underline{\Delta R = 98.1 \text{ lb}}$

7/112 From Eq. 7/30 with θ small so that $\cos\theta \approx 1$, the precessional rate is

$$\dot{\psi} = \frac{I p}{I_0 - I} = \frac{p}{(I_0/I) - 1} = \frac{3}{\frac{1}{2} - 1} = \underline{-6 \text{ rev/min}}$$

Where the minus sign indicates <u>retrograde precession</u>

7/113



$I_{zz_{disk}} = \frac{1}{2}mr^2 = \frac{1}{2}8(0.160)^2$
$= 0.1024 \text{ kg}\cdot\text{m}^2$

$I_{yy_{disk}} = \frac{1}{4}mr^2 = 0.0512 \text{ kg}\cdot\text{m}^2$

$I_{zz_{rod}} \approx 0, \; I_{yy_{rod}} = \frac{1}{12}3(0.640)^2 = 0.1024 \text{ kg}\cdot\text{m}^2$

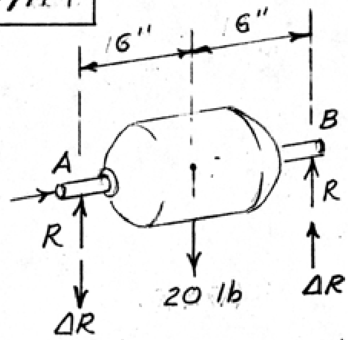
From Eq. 7/30 $\dot{\psi} = \dfrac{Ip}{(I_0 - I)\cos\theta}$

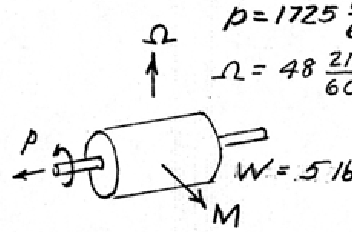
where $I = I_{zz} = 0.1024 \text{ kg}\cdot\text{m}^2$
$I_0 = I_{yy} = 0.0512 + 0.1024 = 0.1536 \text{ kg}\cdot\text{m}^2$
$\theta = 15°, \; p = 60 \text{ rad/s}$

so $\dot{\psi} = \dfrac{0.1024(60)}{(0.1536 - 0.1024)\cos 15°} = \underline{124.2 \text{ rad/s}}$

$I_0 - I$ is plus, so precession is direct & $\underline{\dot{\psi} \text{ is } \dot{\psi}_1}$

7/114





$p = 1725 \frac{2\pi}{60} = 180.6 \frac{\text{rad}}{\text{sec}}$

$\Omega = 48 \frac{2\pi}{60} = 5.03 \frac{\text{rad}}{\text{sec}}$

static reactions

$R = \frac{1}{2}\, 20 = 10 \text{ lb}$

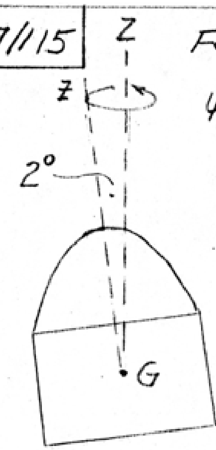
$M = I\Omega p$; $2(\Delta R)(6/12) = \frac{5}{32.2}\left(\frac{1.5}{12}\right)^2(5.03)(180.6)$

$\Delta R = 2.20 \text{ lb}$

$R_A = 10 - 2.20 = \underline{7.80 \text{ lb}}$

$R_B = 10 + 2.20 = \underline{12.20 \text{ lb}}$

7/115



For zero moment Eq. 7/30 is

$$\dot{\psi} = \frac{Ip}{(I_0 - I)\cos\theta} = \frac{p}{\left(\frac{k_0^2}{\bar{k}^2} - 1\right)\cos\theta}$$

where $\bar{k} = 0.72$ m

$k_0 = 0.54$ m

$p = 1.5$ rad/s

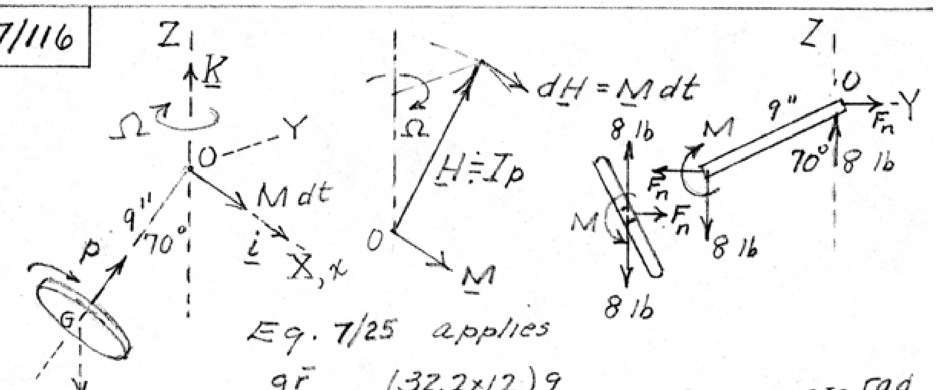
$\theta = 2°$

$(I = \bar{k}^2 m) > (I_0 = k_0^2 m)$ so retrograde precession with p in negative z-dir.

Period $\tau = \left|\frac{2\pi}{\dot{\psi}}\right| = 2\pi \left|\frac{(k_0^2/\bar{k}^2 - 1)}{p}\cos\theta\right|$

$= 2\pi \left|\frac{(0.54/0.72)^2 - 1}{1.5}\cos 2°\right| = \underline{1.831 \text{ s}}$

7/116



Eq. 7/25 applies

$$\Omega = \frac{g\bar{r}}{k^2 p} = \frac{(32.2 \times 12)9}{3^2(3000 \times 2\pi/60)}, \quad \underline{\boldsymbol{\Omega} = -1.23\mathbf{K}\ \frac{\text{rad}}{\text{sec}}}$$

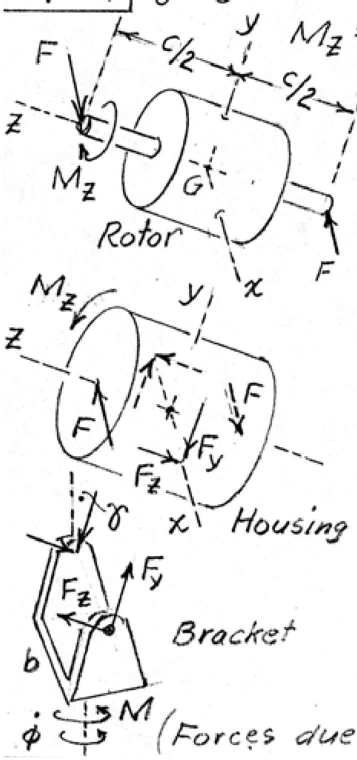
Results are independent of 70°-angle. $\left(\text{or } \frac{1.23 \times 60}{2\pi} = 11.75 \text{ rev/min}\right)$

$$M = I\Omega p \sin 70° = mk^2\left(\frac{g\bar{r}}{k^2 p}\right)p \sin 70° = mg\bar{r}\sin 70°$$

which agrees with static analysis of shaft where $\Sigma M_O = 0$ gives $M = 8 \times 9 \sin 70°$

$$\underline{\mathbf{M} = 67.7\mathbf{i} \text{ lb-in.}}$$

7/117 $\dot\theta = 0$ where $\theta = \pi/2 - \gamma$; $\dot\psi = 0$, $\dot\phi = p$ const.



M_z = torque of electric field to maintain $\dot\phi$ constant (ω_z is not constant)

Apply Eq. 7/26 with

$I = mk_z^2$, $I_0 = mk_x^2$, $\gamma = \pi/2 - \theta$

$\Sigma M_x = 0 = I_0(\ddot\theta - 0) + 0$, $\ddot\theta = -\ddot\gamma = 0$

$\Sigma M_y = Fc = F_z b = mk_x^2(\ddot\psi \cos\gamma + 0) + 0$

$\Sigma M_z = -M_z = -F_y b = mk_z^2 \frac{d}{dt}(\dot\psi \sin\gamma + p)$

$-F_y b = mk_z^2(\ddot\psi \sin\gamma + 0 + 0)$

From bracket $\Sigma M_z = 0$ so

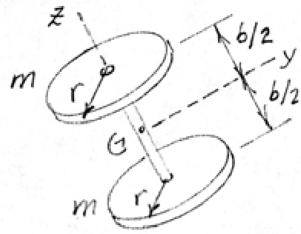
$M = F_z b \cos\gamma - F_y b \sin\gamma$

substitute $F_z b$ & $F_y b$ & get

$$\ddot\psi = \frac{M/m}{k_x^2 \cos^2\gamma + k_z^2 \sin^2\gamma}$$

(Forces due to gyroscopic action only are shown)

7/118



$I = I_{zz} = 2\left(\frac{1}{2}mr^2\right) = mr^2$

$I_0 = I_{yy} = 2\left(\frac{1}{4}mr^2 + m\left[\frac{b}{2}\right]^2\right)$

$= \frac{1}{2}mr^2 + \frac{1}{2}mb^2$

Precession is not possible when $I = I_0$ $(\theta = \beta = 0)$

So $\frac{1}{2}mr^2 + \frac{1}{2}mb^2 = mr^2$, $\underline{b = r}$

7/119 From Eq. 7/30,

$$\dot{\psi} = \frac{Ip}{(I_o - I)\cos\theta} = \frac{p}{[(I_o/I) - 1]\cos\theta}$$

where $I_o/I = \dfrac{\frac{1}{4}mr^2}{\frac{1}{2}mr^2} = \frac{1}{2}$, $p = \dfrac{300(2\pi)}{60} = 10\pi$ rad/s

$\tau = 2\pi/|\dot{\psi}|$ $\qquad\cos\theta = \cos 5^\circ = 0.9962$

$$\tau = 2\pi\,\frac{|(1/2 - 1|\,0.9962}{10\pi} = \underline{0.0996\ s}$$

Precession is <u>retrograde</u> since $I > I_o$

7/120 Case (a) $p = \frac{120 \times 2\pi}{60} = \underline{4\pi \text{ rad/s}}$

$\underline{\theta = \beta = 0 \quad , \dot{\psi} = 0}$

Case (b) $p = 4\pi \quad , \theta = 10^\circ, \quad I_0/I = 1/0.3$

From Eq. 7/30, the precessional rate is

$$\dot{\psi} = \frac{p}{\left(\frac{I_0}{I} - 1\right)\cos\theta} = \frac{4\pi}{\left(\frac{1}{0.3} - 1\right)\cos 10^\circ}$$

$$= \underline{5.47 \text{ rad/s}}$$

From Eq. 7/29,

$$\tan\beta = \frac{I}{I_0}\tan\theta = 0.3 \tan 10^\circ, \quad \underline{\beta = 3.03^\circ}$$

Case (c) $\theta = \beta = 90^\circ, \ \underline{p = 0}$

$\underline{\dot{\psi} = 4\pi \text{ rad/s}}$

7/121 I = moment of inertia about its longitudinal axis $= \frac{1}{12} m (a^2 + a^2)$, $a = 4''$

I_0 = moment of inertia about transverse axis through O $= \frac{1}{12} m (a^2 + \ell^2)$, $\ell = 8'' = 2a$

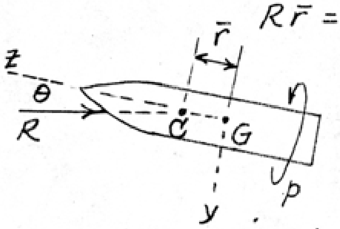
$I_0 / I = \frac{1}{12} m (a^2 + 4a^2) \Big/ \frac{1}{6} m a^2 = 5/2$

Eq. 7/30 $\dot{\psi} = \dfrac{p}{\left(\frac{I_0}{I} - 1\right)\cos\theta} = \dfrac{200}{\left(\frac{5}{2} - 1\right)\cos 10°} = 135.4$ rev/min

period of wobble $\tau = \dfrac{60}{135.4} = \underline{0.443 \text{ sec}}$

▶ 7/122 $\Sigma M_x = R\bar{r}\sin\theta$ & from Eq. 7/27 we have

$$R\bar{r} = \dot{\psi}\left[I(\dot{\psi}\cos\theta + p) - I_0\dot{\psi}\cos\theta\right]$$



or

$$\dot{\psi}^2 + \frac{Ip}{(I-I_0)\cos\theta}\dot{\psi} - \frac{R\bar{r}}{(I-I_0)\cos\theta} = 0$$

Solve for $\dot{\psi}$ & rearrange to give

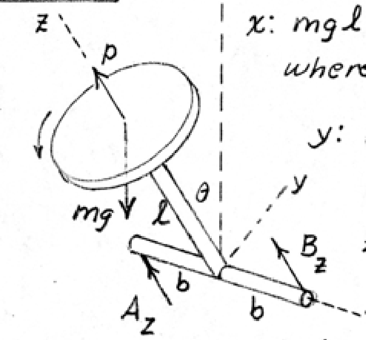
$$\dot{\psi} = \frac{Ip}{2(I_0-I)\cos\theta}\left[1 \pm \sqrt{1 - \frac{4R\bar{r}(I_0-I)\cos\theta}{I^2p^2}}\right]$$

Expression under radical is (+) if

$$p > \frac{2}{I}\sqrt{R\bar{r}(I_0-I)\cos\theta}$$ = min. value of

p for which precession at constant θ can occur.

▸ 7/123



$\dot\psi = \ddot\psi = 0$; From moment Eqs. 7/26

x: $mg\ell \sin\theta = m\left(\frac{r^2}{4}+\ell^2\right)\ddot\theta$ ------- (a)

where $I_0 = I_{xx} = \frac{1}{4}mr^2 + m\ell^2$

y: $(A_z - B_z)b = -\frac{1}{2}mr^2\dot\theta p$ ------- (b)

where $I = \frac{1}{2}mr^2$

z: $0 = I\dot p$ where $\omega_z = 0 + p$ ---- (c)

From (c), $p = \text{const}$

From (a) with $\dot\theta d\dot\theta = \ddot\theta d\theta$, $\int_0^{\pi/2} g\ell \sin\theta\, d\theta = \left(\frac{r^2}{4}+\ell^2\right)\int_0^{\dot\theta} \dot\theta\, d\dot\theta$

which gives $\dot\theta^2 = 8g\ell/(r^2+4\ell^2)$

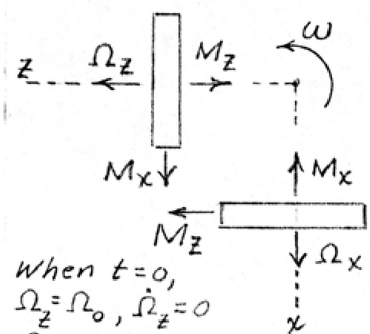
From (b) $-A_z + B_z = \frac{1}{2}m\frac{r^2}{b}\dot\theta p$ for $\theta = \pi/2$ ------ (d)

Also for $\theta = \pi/2$, $\Sigma F_z = m\bar a_z$; $-A_z - B_z = m\ell\dot\theta^2$ ------ (e)

Solve (d) & (e) & get

$$\left.\begin{aligned} A_z &= -\frac{m\dot\theta}{2}\left(\frac{r^2p}{2b} + \ell\dot\theta\right) \\ B_z &= \frac{m\dot\theta}{2}\left(\frac{r^2p}{2b} - \ell\dot\theta\right)\end{aligned}\right\} \text{where } \dot\theta = 2\sqrt{\frac{2g\ell}{r^2+4\ell^2}}$$

▶7/124 $\omega = \frac{2\pi}{\tau}$ = constant precessional rate about y-axis



When $t=0$,
$\Omega_z = \Omega_o$, $\dot{\Omega}_z = 0$
$\Omega_x = 0$

M_x = gyroscopic moment on z-wheel $= I\Omega_z\omega$

$-M_x$ = moment to accelerate x-wheel $= I\dot{\Omega}_x$

so $I\Omega_z\omega = -I\dot{\Omega}_x$, $\dot{\Omega}_x + \omega\Omega_z = 0$ (a)

M_z = gyroscopic moment on x-wheel $= -I\Omega_x\omega$

$-M_z$ = moment to accelerate z-wheel $= +I\dot{\Omega}_z$

so $I\Omega_x\omega = +I\dot{\Omega}_z$, $\dot{\Omega}_z - \omega\Omega_x = 0$ (b)

Combine (a) & (b) & get $\ddot{\Omega}_z + \omega^2\Omega_z = 0$ & $\ddot{\Omega}_x + \omega^2\Omega_x = 0$

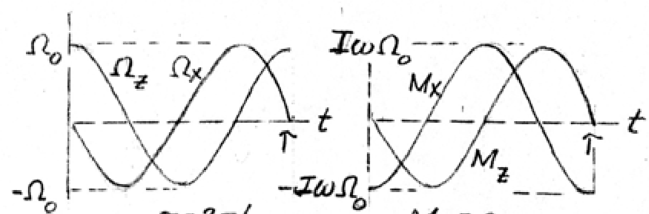
For given conditions at $t=0$, $\begin{cases} \Omega_x = -\Omega_o \sin \omega t \\ \Omega_z = \Omega_o \cos \omega t \end{cases}$

Thus motor torques on shafts are

$\underline{M_x = -I\omega\Omega_o \cos \omega t}$

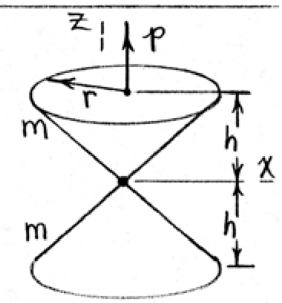
$\underline{M_z = -I\omega\Omega_o \sin \omega t}$

$M = \sqrt{M_x^2 + M_z^2}$
$= I\omega\Omega_o$
constant



$\underline{M_y = 0}$

▶7/125 $\dot{\psi} = \dfrac{I p}{(I_0 - I)\cos\theta}$



(a) No precession if $I_0 = I$

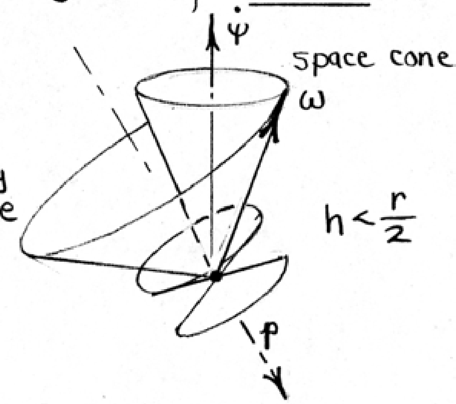
From Table D4,

$I = I_{zz} = 2\left(\frac{3}{10} m r^2\right) = \frac{3}{5} m r^2$

$I_0 = I_{xx} = 2\left(\frac{3}{20} m r^2 + \frac{3}{5} m h^2\right) = \frac{3}{10} m r^2 + \frac{6}{5} m h^2$

$I = I_0: \ \frac{3}{5} m r^2 = \frac{3}{10} m r^2 + \frac{6}{5} m h^2, \quad \underline{h = \frac{r}{2}}$

(b) For $h < \frac{r}{2}$, $I_0 < I$; retrograde precession



(c) $h = r$, $I_0 = \frac{3}{10} m r^2 + \frac{6}{5} m r^2 = \frac{3}{2} m r^2$

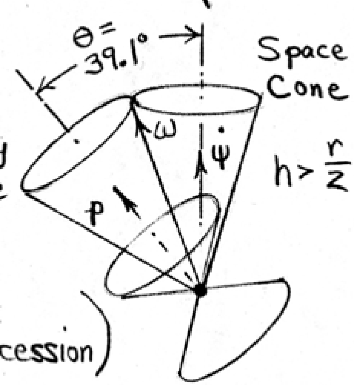
$\dfrac{I}{I_0 - I} = \dfrac{3/5}{3/2 - 3/5} = \dfrac{2}{3}$

$p = 200\left(\frac{2\pi}{60}\right) = 20.9 \ \frac{\text{rad}}{\text{s}}$

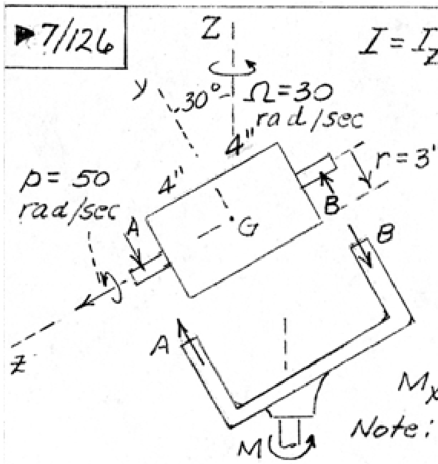
$\theta = \cos^{-1}\left[\dfrac{I}{I_0 - I}\dfrac{p}{\dot{\psi}}\right]$

$= \cos^{-1}\left[\dfrac{2}{3}\,\dfrac{20.9}{18}\right]$

$= 39.1°$



▶7/126



$$I = I_{zz} = \frac{1}{2}mr^2 = \frac{1}{2}\,\frac{64.4}{32.2}\left(\frac{3}{12}\right)^2 = 0.0625 \text{ lb-ft-sec}^2$$

$$\underline{I}_0 = I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$$

$$= \frac{64.4}{32.2}\left(\frac{1}{4}\left[\frac{3}{12}\right]^2 + \frac{1}{12}\left[\frac{8}{12}\right]^2\right)$$

$$= 0.1053 \text{ lb-ft-sec}^2$$

From Eq. 7/27 with $\dot{\psi} = \Omega$

$$M_x = \Omega \sin\theta\left[I(\Omega\cos\theta + p) - I_0\Omega\cos\theta\right]$$

Note: $\theta = \pi/2 + 30°$; $\sin\theta = \sqrt{3}/2$, $\cos\theta = -\frac{1}{2}$

$$M_x = 30\frac{\sqrt{3}}{2}\left[0.0625(30[-\tfrac{1}{2}] + 50) - 0.1053(30)(-\tfrac{1}{2})\right]$$

$= 25.98\,[2.188 + 1.580] = 97.9$ lb-ft, so $\underline{M} = 97.9\underline{i}$ lb-ft

$\omega_x = 0$, $\omega_y = 30\frac{\sqrt{3}}{2} = 25.98\ \frac{\text{rad}}{\text{sec}}$, $\omega_z = 50 - 30(\frac{1}{2}) = 35$ rad/sec

$\bar{H}_x = 0$, $\bar{H}_y = I_0\omega_y = 0.1053(25.98) = 2.736$ lb-ft-sec

$\bar{H}_z = I\omega_z = 0.0625(35) = 2.188$ lb-ft-sec

$T = \frac{1}{2}\underline{\omega}\cdot\underline{\bar{H}} = \frac{1}{2}(25.98\underline{j} + 35\underline{k})\cdot(2.736\underline{j} + 2.188\underline{k}) = \underline{73.8 \text{ ft-lb}}$

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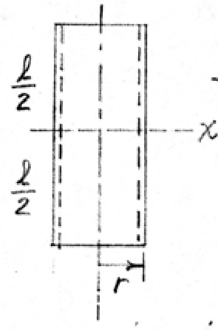


$I = I_{zz} = mr^2$

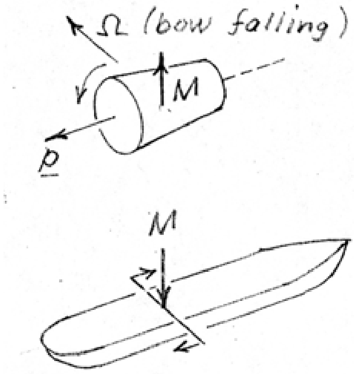
$I_0 = I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$

$\frac{I_0}{I} = \frac{1}{2} + \frac{1}{12}\left(\frac{l}{r}\right)^2$

Direct precession if $I_0/I > 1$; $\frac{1}{2} + \frac{1}{12}\left(\frac{l}{r}\right)^2 > 1$, $\frac{l}{r} > \sqrt{6}$

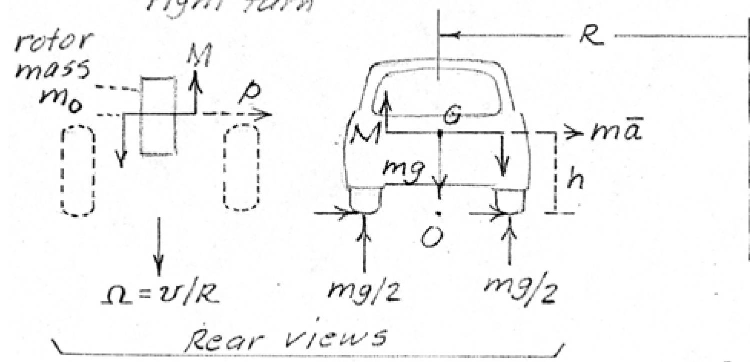
Retrograde " if $I_0/I < 1$; $\frac{l}{r} < \sqrt{6}$

7/128



Reaction of M on hull tends to swing bow to starboard (right)

7/129 Assume right turn



$m\bar{a} = mv^2/R$; $\Sigma M_O = m\bar{a}h$ so $M = mv^2h/R$

$M = I\Omega p$; $\frac{mv^2h}{R} = m_0 k^2 \frac{v}{R} p$

$$p = \frac{m}{m_0}\frac{vh}{k^2}$$ opposite direction to rotation of wheels

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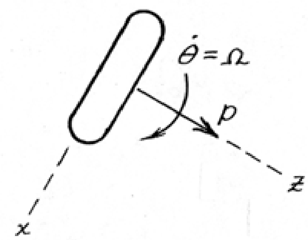
$$p = \frac{v}{r} = \frac{150(10^3)}{60^2 \times 0.560/2} = 148.8 \text{ rad/s}$$



$\underline{p} = 148.8\ \underline{k}$ rad/s

$$\Omega = \frac{30\pi}{180} = 0.524 \text{ rad/s}$$

$\underline{\Omega} = 0.524\ \underline{j}$ rad/s

$\underline{\alpha} = \underline{\Omega} \times \underline{p} = 0.524\ \underline{j} \times 148.8\ \underline{k} = 77.9\ \underline{i}$ rad/s^2; $\underline{\alpha = 77.9\ \underline{i} \text{ rad/s}^2}$

7/131 Angular velocity $\underline{\omega}$ and velocity $\underline{v}$ of point A are perpendicular.

Thus $\underline{\omega} \cdot \underline{v} = 0$

$$\underline{\omega} = \omega(300\,\underline{i} + 150\,\underline{j} + 300\,\underline{k})/\sqrt{300^2+150^2+300^2} = \frac{\omega}{3}(2\underline{i} + \underline{j} + 2\underline{k})$$

$$\underline{v} = 15\underline{i} - 20\underline{j} + v_z\underline{k} \text{ m/s}$$

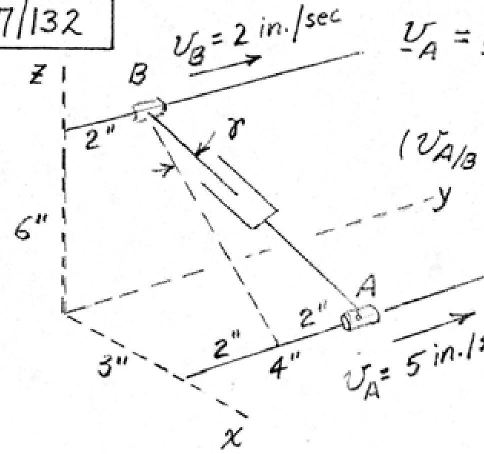
Thus $\frac{\omega}{3}(2\underline{i} + \underline{j} + 2\underline{k}) \cdot (15\underline{i} - 20\underline{j} + v_z\underline{k}) = 0$

$$30 - 20 + 2v_z = 0, \quad v_z = -5 \text{ m/s}$$

$$v = \sqrt{15^2 + 20^2 + 5^2} = \underline{25.5 \text{ m/s}}$$

$$v = \frac{d}{2}\omega, \quad d = \frac{2v}{\omega} = \frac{2(25.5)}{1720 \times 2\pi/60} = 0.283 \text{ m or } \underline{d = 283 \text{ mm}}$$

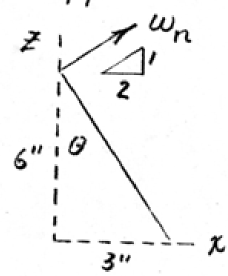
7/132



$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}, \quad \underline{v}_{A/B} = (5-2)\underline{j}$$
$$= 3\underline{j} \text{ in./sec}$$

$$(v_{A/B})_{\substack{normal \\ to\ AB}} = 3\cos\gamma$$
$$= 3\,\frac{\sqrt{6^2+3^2}}{\sqrt{6^2+3^2+2^2}} = \frac{9\sqrt{5}}{7} \text{ in./sec}$$

$$\omega_n = \frac{9\sqrt{5}/7}{7} = \frac{9\sqrt{5}}{49} \text{ rad/sec}$$

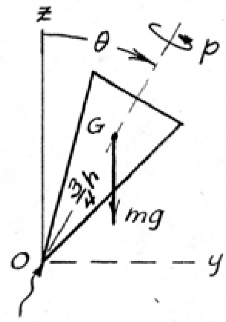


$$\underline{\omega}_n = \frac{9\sqrt{5}}{49}(\underline{i}\cos\theta + \underline{k}\sin\theta)$$
$$= \frac{9\sqrt{5}}{49}\left(\frac{2}{\sqrt{5}}\underline{i} + \frac{1}{\sqrt{5}}\underline{k}\right)$$
$$\underline{\omega}_n = \frac{9}{49}(2\underline{i} + \underline{k}) \text{ rad/sec}$$

7/133

$\underline{M}_O = mg\frac{3}{4}h\sin\theta\,(-\underline{i})$

so change in angular-momentum vector is in $-x$ direction and precession is designated by $\Omega\underline{k}$. Eq. 7/25 gives the precession, so the period is



$\tau = 2\pi/\Omega$

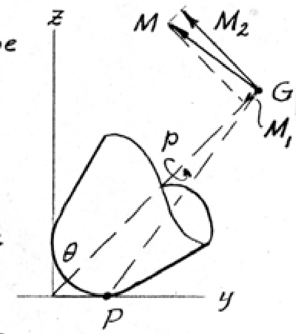
$\tau = 2\pi\Big/\left(\frac{g\bar{r}}{k^2 p}\right)$. For the solid cone, $\bar{r} = \frac{3}{4}h$

& from Table D/4, $I = \frac{3}{10}mr^2$ so $k^2 = \frac{3}{10}r^2$

Thus

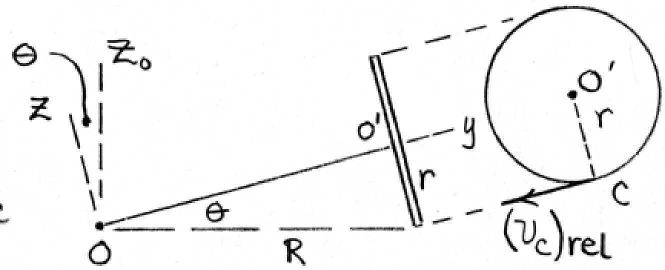
$$\tau = \frac{2\pi}{\dfrac{3gh/4}{\frac{3}{10}r^2 p}} = \underline{\frac{4\pi r^2 p}{5gh}}$$ independent of θ for large p.

7/134 For the given direction of spin p, the friction force acting on the cone at P will be in the $+x$-direction. This force produces a moment M about G, a small component of which, M_1, is along the spin axis and tends to reduce the spin. The other component M_2 causes a change in the principal angular momentum Ip in the direction of M_2, thus causing θ to decrease.



7/135

Let $\underline{\Omega}$ be the angular velocity of the axes xyz.



$$\underline{\Omega} = \frac{2\pi}{\tau}\left(\underline{j}\sin\theta + \underline{k}\cos\theta\right)$$

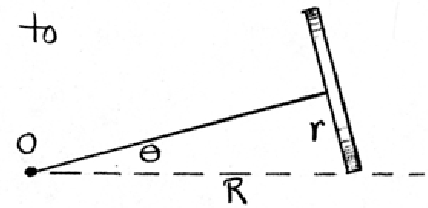
Relative to the xyz axes, O' is fixed and C moves with speed $(v_c)_{rel} = R\,\frac{2\pi}{\tau}$

So $\underline{\omega}_{rel} = \frac{(v_c)_{rel}}{r}(-\underline{j}) = -\frac{2\pi}{\tau}\frac{R}{r}\underline{j}$

Thus $\underline{\omega} = \frac{2\pi}{\tau}\left[-\frac{R}{r}\underline{j} + \underline{j}\sin\theta + \underline{k}\cos\theta\right]$

$$= \frac{2\pi}{\tau}\left[\left(-\frac{R}{r} + \frac{r}{R}\right)\underline{j} + \frac{\sqrt{R^2 - r^2}}{R}\,\underline{k}\right]$$

7/136 From the solution to Prob. 7/135, the absolute angular velocity of the disk is



$$\underline{\omega} = \frac{2\pi}{\tau}\left[\left(-\frac{R}{r}+\frac{r}{R}\right)\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right]$$

$\underline{\alpha} = \dot{\underline{\omega}}$; Need $\dot{\underline{j}} = \underline{\Omega} \times \underline{j} = \frac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta)\times\underline{j}$

$$= -\frac{2\pi}{\tau}\cos\theta\,\underline{i} = \frac{2\pi}{\tau}\left(-\frac{\sqrt{R^2-r^2}}{R}\,\underline{i}\right)$$

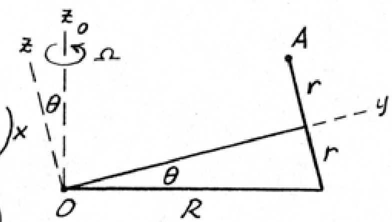
and $\dot{\underline{k}} = \underline{\Omega}\times\underline{k} = \frac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta)\times\underline{k}$

$$= \frac{2\pi}{\tau}\sin\theta\,\underline{i} = \frac{2\pi}{\tau}\frac{r}{R}\underline{i}$$

So $\underline{\alpha} = \left(\frac{2\pi}{\tau}\right)^2\left\{\left[\frac{r}{R}-\frac{R}{r}\right]\left(-\frac{\sqrt{R^2-r^2}}{R}\right)\underline{i} + \frac{\sqrt{R^2-r^2}}{R}\frac{r}{R}\underline{i}\right\}$

$$= \left(\frac{2\pi}{\tau}\right)^2\frac{\sqrt{R^2-r^2}}{r}\,\underline{i}$$

7/137 From Eq. 7/6



$\underline{v}_A = \underline{v}_O + \underline{\Omega} \times \underline{r}_{A/O} + \underline{v}_{rel}$

$\underline{v}_O = \underline{0}$, $\underline{\Omega} \times \underline{r}_{A/O} = \frac{2\pi}{\tau}\left(\frac{r}{R}\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right) \times (\sqrt{R^2-r^2}\,\underline{j} + r\underline{k})$

$= \frac{2\pi}{\tau}\left(\frac{2r^2}{R} - R\right)\underline{i}$

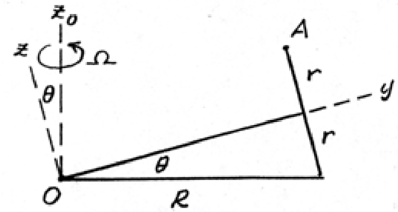
$\underline{\Omega} = \frac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta)$

$= \frac{2\pi}{\tau}\left(\frac{r}{R}\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right)$

$\underline{v}_{rel} = -r\omega_{rel}\underline{i} = -r\left(\frac{R}{r}\frac{2\pi}{\tau}\right)\underline{i} = \frac{2\pi}{\tau}R\underline{i}$

$\underline{v}_A = \frac{2\pi}{\tau}\left[\frac{2r^2}{R} - R - R\right]\underline{i}$, $\underline{v}_A = -\frac{4\pi}{\tau}\left(R - \frac{r^2}{R}\right)\underline{i}$

7/138 Using Eqs. 7/6



$$\underline{a}_A = \underline{a}_O + \dot{\underline{\Omega}} \times \underline{r}_{A/O} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$$

$$\underline{a}_O = \underline{0}, \ \dot{\underline{\Omega}} = \underline{0}$$

$$\underline{\Omega} = \frac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta) = \frac{2\pi}{\tau}\left(\frac{r}{R}\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right)$$

$$\underline{\Omega} \times \underline{r}_{A/O} = \frac{2\pi}{\tau}\left(\frac{r}{R}\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right) \times (\sqrt{R^2-r^2}\,\underline{j} + r\underline{k}) = \frac{2\pi}{\tau}\left(\frac{2r^2}{R} - R\right)\underline{i}$$

$$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/O}) = \left(\frac{2\pi}{\tau}\right)^2\left(\frac{r}{R}\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right) \times \left(\frac{2r^2}{R} - R\right)\underline{i}$$

$$= \left(\frac{2\pi}{\tau}\right)^2\left(\frac{2r^2}{R^2} - 1\right)\left(\sqrt{R^2-r^2}\,\underline{j} - r\underline{k}\right)$$

$$\underline{v}_{rel} = -r\omega_{rel}\underline{i} = -r\left(\frac{R}{r}\frac{2\pi}{\tau}\right)\underline{i} = -\frac{2\pi}{\tau}R\underline{i}$$

$$2\underline{\Omega} \times \underline{v}_{rel} = \frac{4\pi}{\tau}\left(\frac{r}{R}\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right) \times \left(-\frac{2\pi}{\tau}R\underline{i}\right) = -2\left(\frac{2\pi}{\tau}\right)^2\left(\sqrt{R^2-r^2}\,\underline{j} - r\underline{k}\right)$$

$$\underline{a}_{rel} = -r\omega_{rel}^2\underline{k} = -r\left(\frac{R}{r}\frac{2\pi}{\tau}\right)^2\underline{k} = -\left(\frac{2\pi}{\tau}\right)^2\frac{R^2}{r}\underline{k}$$

Substitute, simplify, & get

$$\underline{\underline{a}_A = \left(\frac{2\pi}{\tau}\right)^2\left[\sqrt{R^2-r^2}\left(\frac{2r^2}{R^2} - 3\right)\underline{j} + \left(3r - \frac{R^2}{r} - \frac{2r^3}{R^2}\right)\underline{k}\right]}$$

7/139

$I_{zz} = mr^2, \quad k = r = 0.060 \text{ m}$

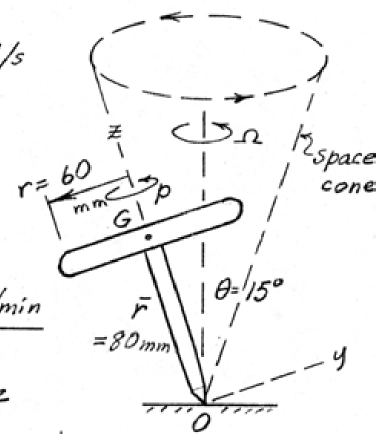
$p = 10\,000\,(2\pi/60) = 1047 \text{ rad/s}$

From Eq. 7/25,

$$\Omega \approx \frac{g\bar{r}}{k^2 p} = \frac{9.81(0.080)}{(0.060)^2(1047)}$$

$$= 0.208 \text{ rad/s}$$

$$N = \frac{\Omega}{2\pi}60 = \frac{0.208}{2\pi}\times 60 = \underline{1.988 \text{ cycles/min}}$$



With $\Omega = \dot{\psi}$ very small, the body cone is too small to observe, so space cone is the only relatively apparent cone.

(Note direction of precession on diagram.)

7/140 Eq. 7/14 becomes $\underline{H}_O = \underline{H}_C + \underline{\bar{r}} \times m\underline{\bar{v}}$, $\underline{\bar{r}} = \overrightarrow{OC}$, $\underline{\bar{v}} = \underline{v}_C$

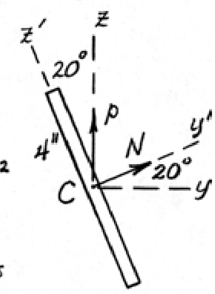
For disk, $\omega_{y'} = \frac{300 \times 2\pi}{60} + \frac{60 \times 2\pi}{60} \sin 20^\circ = 33.6$ rad/sec

$\omega_{z'} = \frac{60 \times 2\pi}{60} \cos 20^\circ = 5.90$ rad/sec

$\omega_{x'} = 0$

$I_{y'y'} = \frac{1}{2} mr^2 = \frac{1}{2} \frac{8}{32.2} \left(\frac{4}{12}\right)^2 = 0.01380$ lb-ft-sec^2

$I_{z'z'} = \frac{1}{4} mr^2 = 0.00690$ lb-ft-sec^2



With $\omega_x = 0$ & principal axes $x-y'-z'$, Eq. 7/13 gives

$\underline{H}_c = I_{y'y'} \omega_{y'} \underline{j}' + I_{z'z'} \omega_{z'} \underline{k}' = 0.01380(33.6)\underline{j}' + 0.00609(5.90)\underline{k}'$

$= 0.463\underline{j}' + 0.0407\underline{k}' = 0.421\underline{j} + 0.1967\underline{k}$

$\underline{\bar{r}} = \frac{10}{12}\underline{i} = 0.833\underline{i}$ ft

$\underline{\bar{v}} = p\underline{k} \times \underline{\bar{r}} = \frac{60 \times 2\pi}{60} \underline{k} \times 0.833\underline{i} = 5.24\underline{j}$ ft/sec

$\underline{\bar{r}} \times m\underline{\bar{v}} = 0.833\underline{i} \times \frac{8}{32.2}(5.24\underline{j}) = 1.084\,\underline{k}$ lb-ft-sec

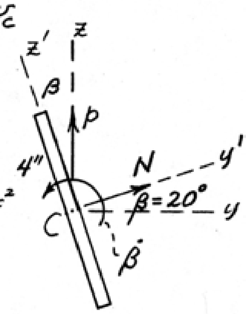
$\underline{H}_o = 0.421\underline{j} + 0.1967\underline{k} + 1.084\underline{k} = \underline{0.421\underline{j} + 1.281\underline{k} \text{ lb-ft-sec}}$

$T = \frac{1}{2}\underline{\bar{v}} \cdot \underline{G} + \frac{1}{2}\underline{\omega} \cdot \underline{H}_G \quad (G = C \text{ here})$

$= \frac{1}{2}\, 5.24\underline{j} \cdot \frac{8}{32.2}(5.24\underline{j}) + \frac{1}{2}(29.5\underline{j} + 17.03\underline{k}) \cdot (0.421\underline{j} + 0.1967\underline{k})$

$= \underline{11.30 \text{ ft-lb}}$

7/141 Eq. 7/14 becomes $\underline{H}_O = \underline{H}_C + \underline{\bar{r}} \times m\underline{\bar{v}}$, $\underline{\bar{r}} = \overline{OC}$, $\underline{\bar{v}} = \underline{v}_C$



For disk, $\omega_x = \dot{\beta} = \frac{120 \times 2\pi}{60} = 12.57$ rad/sec

$$\omega_{y'} = \frac{300 \times 2\pi}{60} + \frac{60 \times 2\pi}{60} \sin 20° = 33.6 \text{ rad/sec}$$

$$\omega_{z'} = \frac{60 \times 2\pi}{60} \cos 20° = 5.90 \text{ rad/sec}$$

$$I_{xx} = I_{z'z'} = \frac{1}{4} mr^2 = \frac{1}{4} \frac{8}{32.2} \left(\frac{4}{12}\right)^2 = 0.00690 \text{ lb-ft-sec}^2$$

$$I_{y'y'} = \frac{1}{2} mr^2 = 0.01380 \text{ lb-ft-sec}^2$$

For principal axes x-y'-z' Eq. 7/13 gives

$$\underline{H}_C = I_{xx}\omega_x \underline{i} + I_{y'y'}\omega_{y'}\underline{j}' + I_{z'z'}\omega_{z'}\underline{k}'$$

$$= 0.00690(12.57)\underline{i} + 0.01380(33.8)\underline{j}' + 0.00690(5.90)\underline{k}'$$

$$\underline{H}_C = 0.0867\underline{i} + 0.463\underline{j}' + 0.0407\underline{k}'$$

$$= 0.0867\underline{i} + 0.421\underline{j} + 0.1967\underline{k} \text{ lb-ft-sec}$$

$$\underline{\bar{r}} = \frac{10}{12}\underline{i} = 0.833\underline{i} \text{ ft}$$

$$\underline{\bar{v}} = p\underline{k} \times \underline{\bar{r}} = \frac{60 \times 2\pi}{60}\underline{k} \times 0.833\underline{i} = 5.24\underline{j} \text{ ft/sec}$$

$$\underline{\bar{r}} \times m\underline{\bar{v}} = 0.833\underline{i} \times \frac{8}{32.2}(5.24\underline{j}) = 1.084\underline{k} \text{ lb-ft-sec}$$

$$\underline{H}_O = 0.0867\underline{i} + 0.421\underline{j} + 0.1967\underline{k} + 1.084\underline{k} = \underline{0.0867\underline{i} + 0.421\underline{j} + 1.281\underline{k}}$$

$\underline{\text{lb-ft-sec}}$

$$T = \tfrac{1}{2}\underline{\bar{v}} \cdot \underline{G} + \tfrac{1}{2}\underline{\omega} \cdot \underline{H}_G \quad (G = C \text{ here})$$

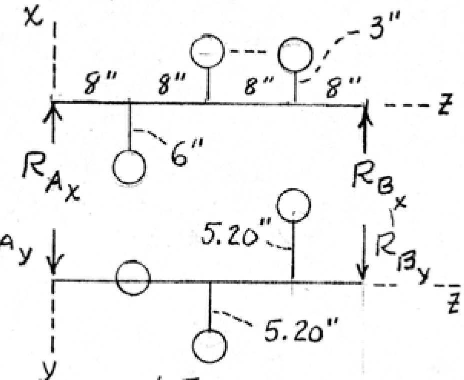
$$= \tfrac{1}{2} 5.24\underline{j} \cdot \frac{8}{32.2}(5.24\underline{j}) + \tfrac{1}{2}(12.57\underline{i} + 29.5\underline{j} + 17.03\underline{k}) \cdot (0.0867\underline{i} + 0.421\underline{j} + 0.1967\underline{k})$$

$$= \underline{11.85 \text{ ft-lb}}$$

7/142

$\omega_z = \dfrac{1200(2\pi)}{60}$

$= 40\pi$ rad/sec



Eq. 7/23

$\Sigma M_x = I_{yz}\omega_z^2$

$\Sigma M_y = -I_{xz}\omega_z^2$

Where

$I_{yz} = m(5.20 \times 16 - 5.20 \times 24)$
$= -161.4(10^{-3})$ in.-lb-sec^2

$m = \dfrac{1.5}{32.2}\dfrac{1}{12} = 3.88(10^{-3})$ lb-sec^2/in.

$I_{xz} = m(-6\times 8 + 3\times 16 + 3\times 24)$
$= 280(10^{-3})$ in.-lb-sec^2

$\Sigma M_x = -32R_{B_y} = -0.1614(40\pi)^2$, $R_{B_y} = 79.6$ lb

$\Sigma M_y = +32R_{B_x} = -0.280(40\pi)^2$, $R_{B_x} = -137.9$ lb

Because mass center has no acceleration

$R_{A_y} = -R_{B_y}$, $R_{A_x} = R_{B_x}$

$|R_A| = |R_B| = \sqrt{79.6^2 + 137.9^2} = \underline{159.3 \text{ lb}}$

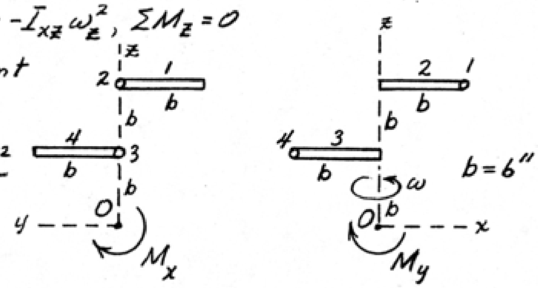
7/143 With $\omega_x = \omega_y = 0$, $\omega_z = \frac{1200 \times 2\pi}{60} = 125.7$ rad/sec,

$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$, Eqs. 7/23 about O become

$\Sigma M_x = I_{yz}\omega_z^2$, $\Sigma M_y = -I_{xz}\omega_z^2$, $\Sigma M_z = 0$

Let m = mass of each segment of length b

$= \frac{1.4}{32.2} = 0.0435 \ \frac{\text{lb-sec}^2}{\text{ft}}$



Static forces produce no moment so are not shown.

$$I_{xz} = \overset{①}{m(b)(2b)} + \overset{②}{m\left(\tfrac{b}{2}\right)(2b)} + \overset{③}{m\left(-\tfrac{b}{2}\right)(b)} + \overset{④}{m(-b)(b)} = \frac{3}{2}mb^2$$

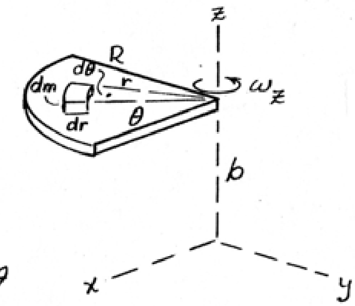
$$M_y = -\frac{3}{2}mb^2\omega_z^2 = -\frac{3}{2}(0.0435)\left(\frac{6}{12}\right)^2(125.7)^2 = -257 \text{ lb-ft}$$

$$I_{yz} = \overset{①}{m\left(-\tfrac{b}{2}\right)(2b)} + \overset{②}{m(0)} + \overset{③}{m(0)} + \overset{④}{m\left(\tfrac{b}{2}\right)(b)} = -\frac{1}{2}mb^2$$

$$M_x = -\frac{1}{2}mb^2\omega_z^2 = -\frac{1}{2}(0.0435)\left(\frac{6}{12}\right)^2(125.7)^2 = -85.8 \text{ lb-ft}$$

$$M = \sqrt{M_x^2 + M_y^2} = \sqrt{85.8^2 + 257^2} = \underline{271 \text{ lb-ft}}$$

7/144 Let m = mass of each plate



mass per unit area $= m/(\pi R^2/4)$

$= 4m/\pi R^2$

$$dm = \frac{4m}{\pi r^2} r\,dr\,d\theta$$

$$I_{xz} = \int xz\,dm = \frac{4m}{\pi R^2}\int_0^{\frac{\pi}{2}}\int_0^R (r\cos\theta)\,b\,r\,dr\,d\theta$$

$$= \frac{4mbR}{3\pi}$$

$$I_{yz} = \int yz\,dm = \frac{4m}{\pi R^2}\int_0^{\frac{\pi}{2}}\int_0^R (-r\sin\theta)\,b\,r\,dr\,d\theta = -\frac{4mbr}{3\pi}$$

Top plate $I_{xz} = -I_{yz} = \dfrac{4(2)(0.150)(0.150)}{3\pi} = 0.01910\ \text{kg}\cdot\text{m}^2$

Lower plate $I_{xz} = -\dfrac{4mbR}{3\pi}$, $I_{yz} = \dfrac{4mbR}{3\pi}$ where $b = 0.075$ m ($\frac{1}{2}$ of 0.150)

$I_{xz} = -I_{yz} = -0.01910/2 = -0.00955\ \text{kg}\cdot\text{m}^2$

From Eq. 7/23 with $\omega_x = \omega_y = 0$, $\omega_z = \dfrac{2\pi(300)}{60} = 10\pi$ rad/s, $\dot{\omega}_z = 0$

$$\Sigma M_x = I_{yz}\,\omega_z^2 = (-0.01910 + 0.00955)(10\pi)^2 = -9.42\ \text{N}\cdot\text{m}$$

$$\Sigma M_y = -I_{xz}\,\omega_z^2 = -(0.01910 - 0.00955)(10\pi)^2 = -9.42\ \text{N}\cdot\text{m}$$

$$M = \sqrt{9.42^2 + 9.42^2} = \underline{13.33\ \text{N}\cdot\text{m}}$$

7/145 With $\omega_x = \omega_y = \omega_z = 0$ & $\dot{\omega}_z = 200\ rad/s^2$, Eq. 7/23 gives

$$\Sigma M_x = -I_{xz}\dot{\omega}_z\ ,\ \Sigma M_y = -I_{yz}\dot{\omega}_z$$

From solution to Prob. 7/144,

$$I_{xz} = 0.01910 - 0.00955 = 0.00955\ kg\cdot m^2$$

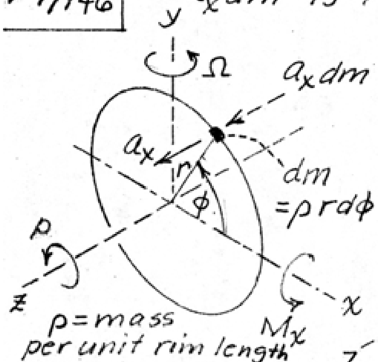
$$I_{yz} = -0.01910 + 0.00955 = -0.00955\ kg\cdot m^2$$

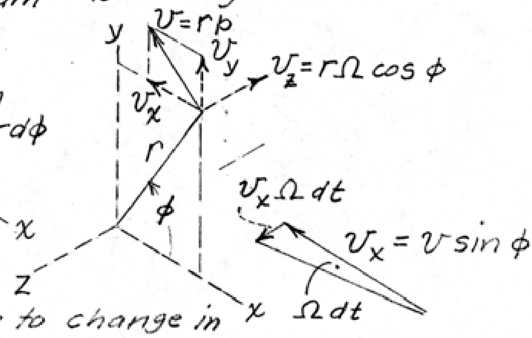
So $\Sigma M_x = -0.00955(200) = -1.910\ N\cdot m$

$\Sigma M_y = 0.00955(200) = 1.910\ N\cdot m$

$M = \sqrt{1.910^2 + 1.910^2} = \underline{2.70\ N\cdot m}$

▶7/146 $a_x dm$ is the only force on dm which exerts a moment about x- and y-axes





Accel. in z-dir. due to change in dir. of v_x is $v_x\Omega = rp\Omega\sin\phi$

Accel. in z-dir. due to change in mag. of v_z is

$-\frac{d}{dt}(r\Omega\cos\phi) = r\Omega\dot{\phi}\sin\phi = r\Omega p\sin\phi$

Thus $a_z = 2rp\,\Omega\sin\phi$

$$M_x = \int r\sin\phi\,(a_x dm) = 2\rho r^3 p\,\Omega\int_0^{2\pi}\sin^2\phi\,d\phi = 2\rho r^3 p\,\Omega\pi$$

$$= mr^2\Omega p = \underline{I\Omega p}$$

$$M_y = -\int r\cos\phi\,(a_x dm) = 2\rho r^3 p\,\Omega\int_0^{2\pi}\sin\phi\cos\phi\,d\phi = 0$$