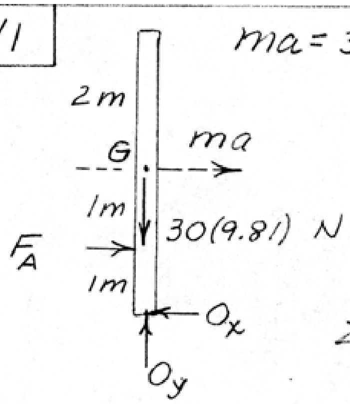
6/1



$ma = 30(20) = 600\ N$

$\Sigma M_A = m\bar{a}d_A$

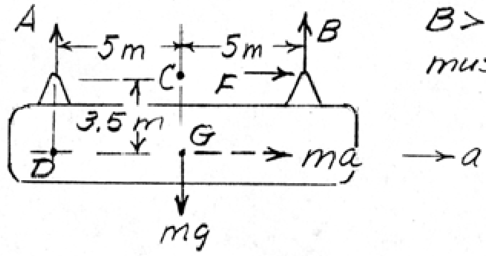
$F_A(1) = 600(2)$

$\underline{F_A = 1200\ N}$

$\Sigma F_x = m\bar{a}_x;\quad 1200 - O_x = 600$

$\underline{O_x = 600\ N}$

6/2



$B > A$ since ΣM_C must be CCW.

$F_{max} = \mu B$
$= 0.25B$

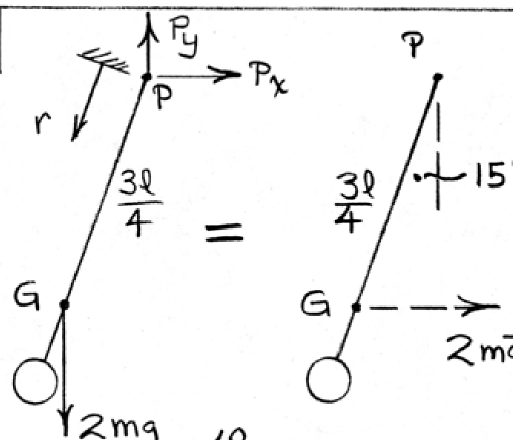
$\Sigma M_D = 0$; $5mg + 0.25B(3.5) - 10B = 0$

$B = 5mg/9.125 = 0.548\,mg$

$\Sigma F = ma$; $0.25B = ma$, $a = \dfrac{0.25(0.548)mg}{m}$

$a = 0.25(0.548)(9.81) = \underline{1.344\ m/s^2}$

6/3



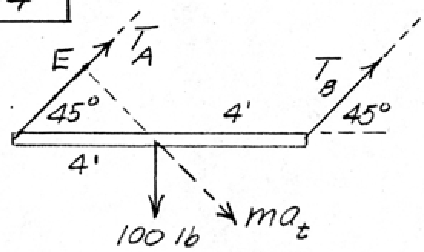
$2m\bar{a} = 2ma$

$$\bar{r} = \frac{\Sigma m_i r_i}{\Sigma m_i} = \frac{m(l/2) + m(l)}{m+m} = \frac{3}{4}l$$

$$\curvearrowleft + \ \Sigma M_P = \cancel{\bar{I}\alpha}^{\,0} + m\bar{a}d \ : \ 2mg\left(\frac{3l}{4}\sin 15°\right) = 2ma\left(\frac{3l}{4}\cos 15°\right)$$

$$\Rightarrow a = g\tan 15° = \underline{0.268g}$$

6/4

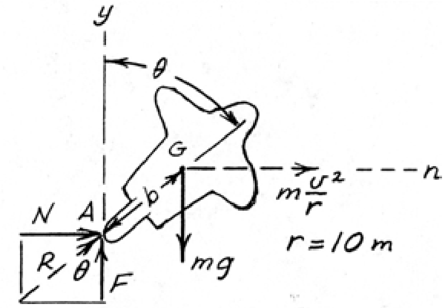


$\Sigma M_E = 0;$

$100(2) - T_B \frac{8}{\sqrt{2}} = 0$

$T_B = 25\sqrt{2} = \underline{35.4 \text{ lb}}$

6/5



$\overset{\curvearrowleft}{\Sigma M_A} = m\bar{a}d$: $mgb\sin\theta = m\frac{v^2}{r}b\cos\theta$, $v^2 = gr\tan\theta$

But $\tan\theta = N/F = 1/\mu$ so $v^2 = \frac{gr}{\mu}$, $v = \sqrt{\frac{9.81\times10}{0.70}}$

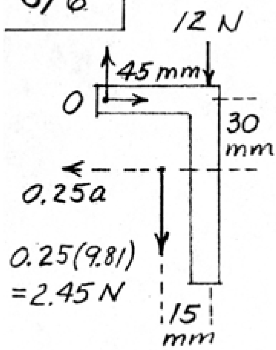
$= 11.84$ m/s

or $\underline{v = 42.6 \text{ km/h}}$

$\theta = \tan^{-1}\frac{v^2}{gr} = \tan^{-1}\frac{11.84^2}{9.81\times10} = \underline{55.0°}$

Note: The fact that in reality this is a rigid body rotating about the central axis does not invalidate the plane-motion analysis as a translating body so long as $\dot{\theta} = 0$.

6/6

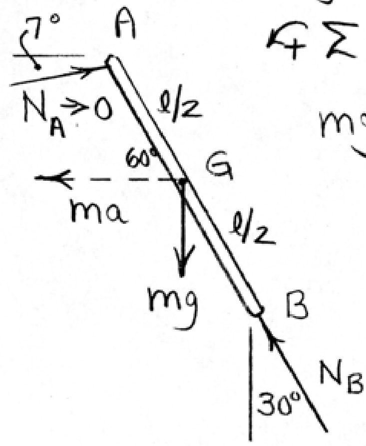


$$\Sigma M_O = m a d_O$$

$$12(45) + 0.25(9.81)(30) = 0.25a(30)$$

$$a = 81.8 \text{ m/s}^2 = \underline{8.34g}$$

6/7



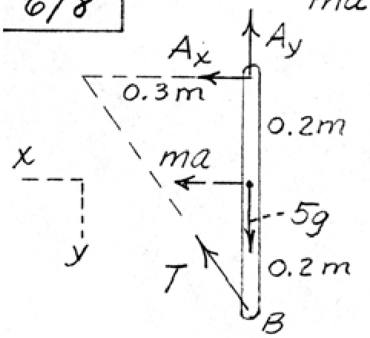
Tipping impends when $N_A \to 0$.

$\circlearrowleft + \Sigma M_B = m a d$:

$$mg \frac{l}{2} \sin 30° = ma \frac{l}{2} \cos 30°$$

$$\underline{a = g \tan 30° = 5.66 \text{ m/s}^2}$$

6/8



$ma = 5(0.6g) = 3g$

$\Sigma M_B = mad_B$

$0.4A_x = 3g(0.2)$

$A_x = \frac{0.6}{0.4}(9.81) = 14.72$ N

$\Sigma M_A = mad_A$

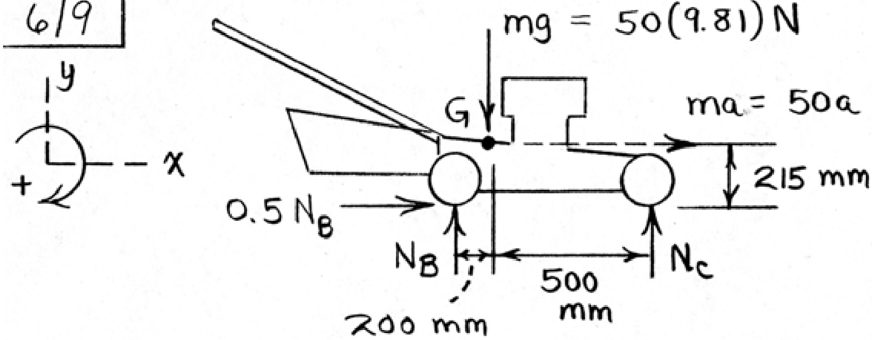
$(\frac{3}{5}T)0.4 = 3g(0.2)$

$\underline{T = 24.5 \text{ N}}$

$\Sigma F_y = 0; \quad \frac{4}{5}(24.5) + A_y - 5(9.81) = 0, \quad A_y = 29.4$ N

$A = \sqrt{29.4^2 + 14.72^2} = \underline{32.9 \text{ N}}$

6/19



$\Sigma F_x = ma: \quad 0.5 N_B = 50a$

$\Sigma F_y = 0: \quad N_B + N_C - 50(9.81) = 0$

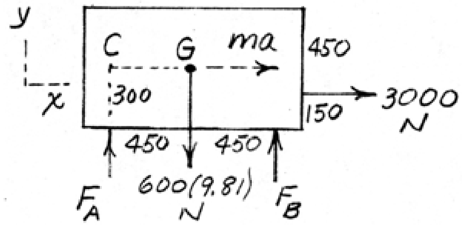
$\Sigma M_B = mad: \quad 50(9.81)(0.2) - N_C(0.7) = 50a(0.215)$

Simultaneous solution : $\begin{cases} N_B = 414 \text{ N} \\ N_C = 76.6 \text{ N} \\ \underline{a = 4.14 \text{ m/s}^2} \end{cases}$

6/10 $\Sigma M_C = 0;\ 600(9.81)450 - 900F_B - 3000(150) = 0$

$F_B = 2440$ N

Dimensions in mm



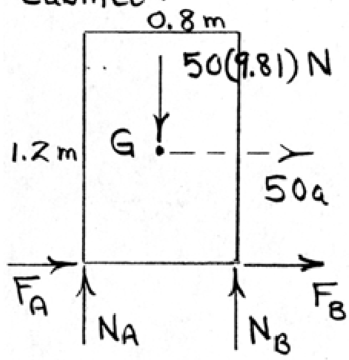
$\Sigma F_y = 0;$

$F_A + 2440 - 600(9.81) = 0$

$F_A = 3440$ N

6/11

Cabinet :



N_B & $F_B \rightarrow 0$ when tipping impends

$\overset{+}{\curvearrowright} \Sigma M_A = mad: \ mg(0.4) = ma(0.6)$

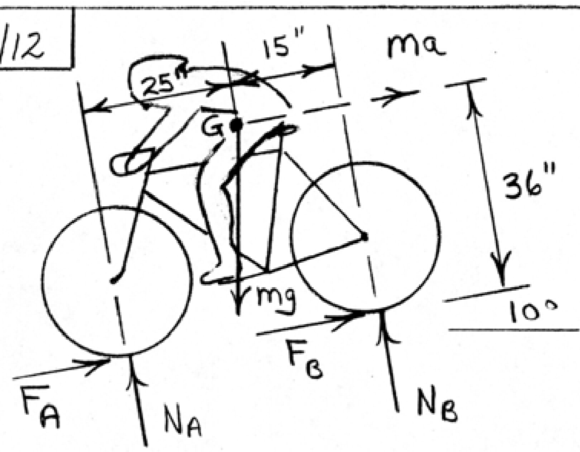
$a = \frac{2}{3} g$ or $6.54 \ m/s^2$

As a whole : $\overset{+}{\rightarrow} \Sigma F = ma$

$P = 60(6.54) = \underline{392 \ N}$

$\underline{\mu_s > \frac{a}{g} = \frac{2}{3}}$

6/12

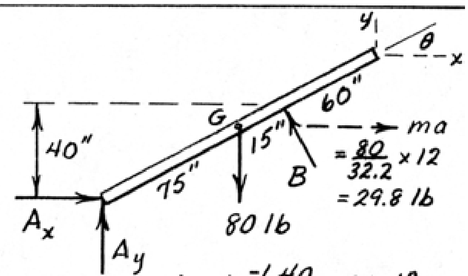


Tipping at front wheel : $N_B, F_B \to 0$

$$+\circlearrowleft \Sigma M_A = mad : \quad mg\,(25 \cos 10° - 36 \sin 10°) = ma\,(36)$$

Solve to obtain $a = \underline{0.510g}$ ($\underline{16.43 \text{ ft/sec}^2}$)

6/13

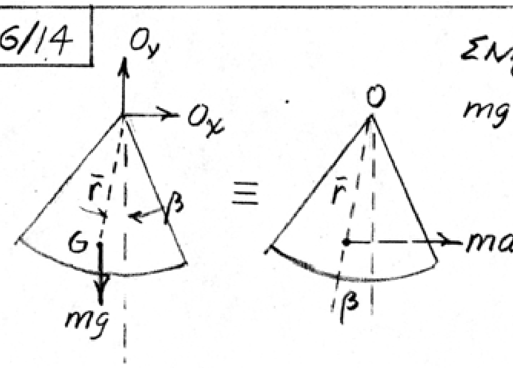


$\theta = \sin^{-1}\frac{40}{90} = 26.4°$

$\Sigma M_A = mad$: $80 \times \frac{75}{12} \times \cos 26.4° - \frac{90}{12} B$

$= 29.8 \times \frac{75}{12} \sin 26.4°$, $\underline{B = 48.7 \text{ lb}}$

6/14



$\Sigma M_O = mad$

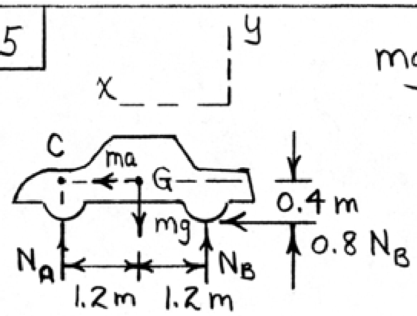
$mg\bar{r}\sin\beta = ma\bar{r}\cos\beta$

$a = g\tan\beta$

But $\theta = \frac{120}{20}\beta = 6\beta$

so $\underline{a = g\tan\theta/6}$

6/15



$$mg = 1650(9.81) = 16.19(10^3)\text{ N}$$

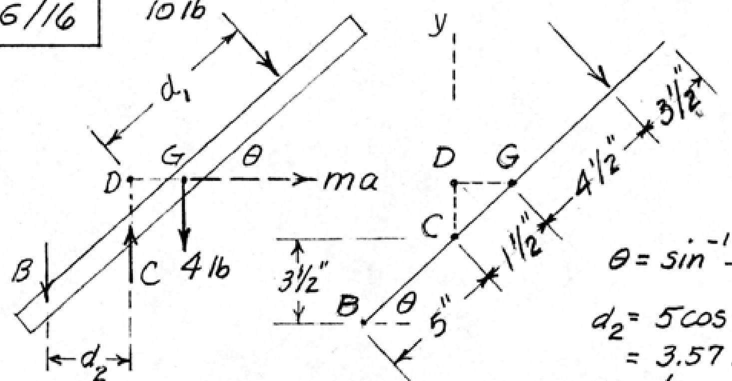
$$\circlearrowleft + \sum M_C = m\bar{a}d = 0: \quad N_B(2.4) - 0.8N_B(0.4) - 16.19(10^3)1.2 = 0$$

$$N_B = 9.34(10^3)\text{ N or } \underline{N_B = 9.34\text{ kN}}$$

$$\sum F_y = 0: \quad N_A + 9.34(10^3) - 16.19(10^3) = 0$$

$$N_A = 6.85(10^3)\text{ N} \quad \text{or} \quad \underline{N_A = 6.85\text{ kN}}$$

6/16



$\theta = \sin^{-1}\frac{3.5}{5} = 44.4°$

$d_2 = 5\cos 44.4°$
$= 3.57$ in.

$d_1 = 4.5 + 1.5(\cos 44.4°)^2$
$= 5.27$ in.

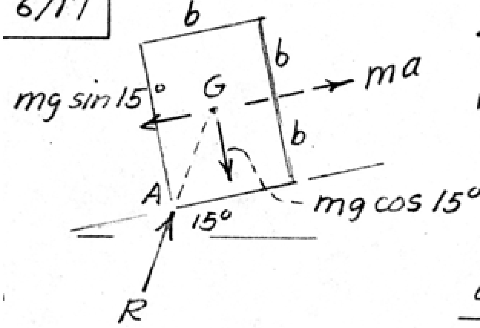
$\Sigma M_D = 0;\quad 10(5.27) + 4(1.5\cos 44.4°) - 3.57B = 0$

$\underline{B = 15.94 \text{ lb}}$

$\Sigma F_y = 0;\quad C - 4 - 15.94 - 10\cos 44.4° = 0$

$\underline{C = 27.1 \text{ lb}}$

6/17



$\Sigma M_A = ma d$

$(mg \cos 15°)\frac{b}{2} - (mg \sin 15°) b = mab$

$g\left(\frac{0.966}{2} - 0.259\right) = a$

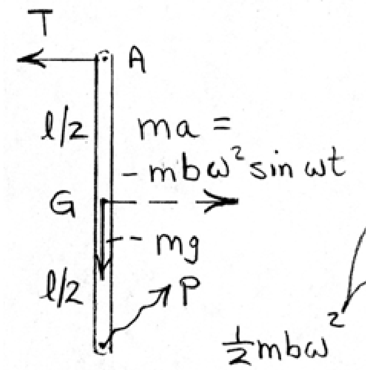
$a = 0.224g$

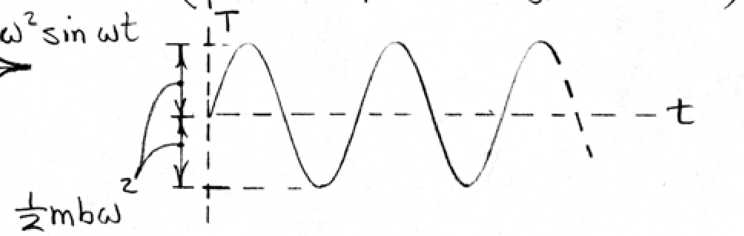
6/18

$\overset{+}{\curvearrowleft}\ \Sigma M_P = m a d: \quad -T\ell = -mb\omega^2 \sin\omega t \left(\frac{\ell}{2}\right)$

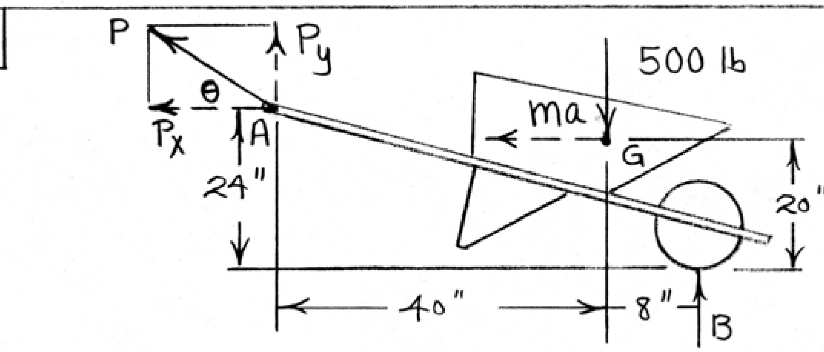
$\Rightarrow T = \frac{1}{2} mb\omega^2 \sin\omega t$

(positive T is tension in link)





6/19



Static equilibrium : $P_x = ma = 0$

$\circlearrowleft\!+\ \sum M_A = 0 : \ 500(40) - B(48) = 0, \quad \underline{B_{st} = 417 \text{ lb}}$

Dynamic : $\circlearrowleft\!+\ \sum M_A = mad$:

$500(40) - B(48) = \frac{500}{32.2}(5)(4), \quad \underline{B = 410 \text{ lb}}$

$\overset{+}{\leftarrow} \sum F_x = ma : P_x = \frac{500}{32.2}(5) = 77.6 \text{ lb}$

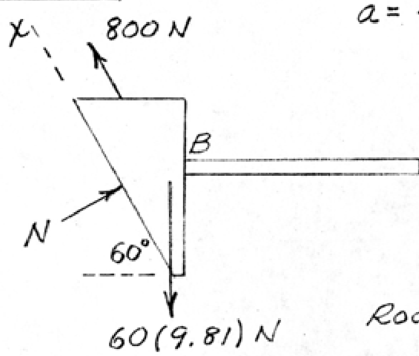
$+\uparrow \sum F_y = 0 : \ B - 500 + P_y = 0, \ P_y = 89.8 \text{ lb}$

$\therefore \underline{P = 118.7 \text{ lb}}$

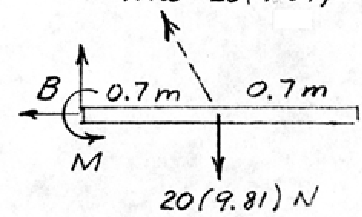
$\underline{\theta = 49.2^\circ}$

6/20 $\Sigma F_x = ma_x$; $800 - 60(9.81)\sin 60^\circ = 60a$

$a = 4.84\ \text{m/s}^2$



$ma = 20(4.84) = 96.8\ \text{N}$



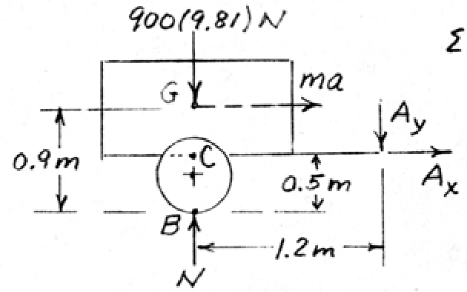
Rod: $\Sigma M_B = ma\,d$

$M - 20(9.81)0.7 = 96.8(0.7\sin 60^\circ)$

$\underline{M = 196.0\ \text{N}\cdot\text{m}}$

6/21

$v^2 = 2as, \quad a = \frac{v^2}{2s} = \frac{(60/3.6)^2}{2(30)} = 4.63 \text{ m/s}^2$

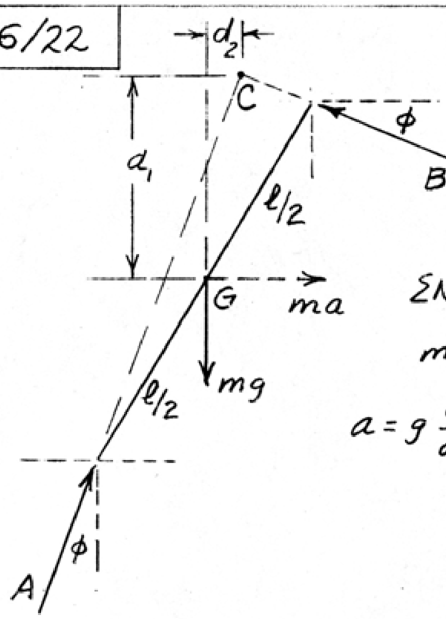


$\Sigma M_C = mad;$

$1.2 A_y = 900(4.63)(0.9 - 0.5)$

$A_y = 1389 \text{ N}$

6/22



$\phi = \tan^{-1}\mu = \tan^{-1} 0.4 = 21.8°$

Moment arms measure

$d_1 = 0.486\ell$

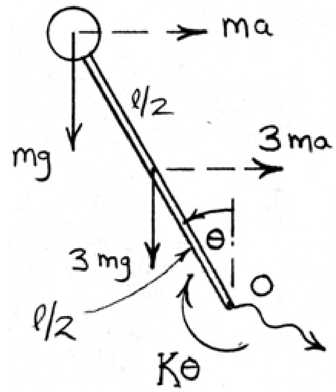
$d_2 = 0.118\ell$

$\Sigma M_C = mad$

$mg\, d_2 = ma\, d_1$

$a = g\,\dfrac{d_2}{d_1} = 9.81\,\dfrac{0.118\ell}{0.486\ell} = \underline{2.38 \text{ m/s}^2}$

6/23



$\curvearrowright\!+\ \Sigma M_O = \Sigma m a d: \ K\theta - 3mg\left(\frac{\ell}{2}\sin\theta\right)$

$- mg\,(\ell \sin\theta) = 3ma\left(\frac{\ell}{2}\cos\theta\right)$

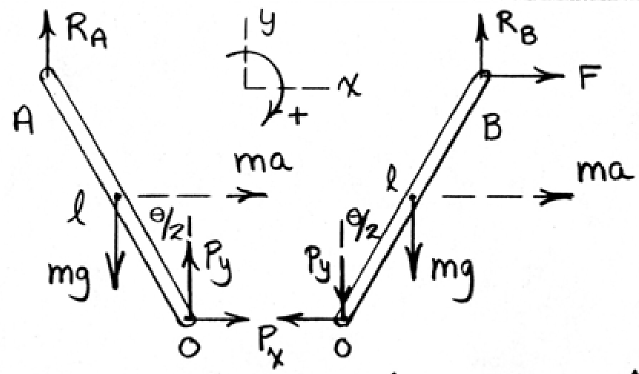
$+ ma\,(\ell\cos\theta)$

Simplify to

$K\theta - \frac{5}{2}mg\ell\sin\theta = \frac{5}{2}ma\ell\cos\theta$

With $m = 0.5$ kg, $\ell = 0.6$ m, $a = 2g$, and $\theta = 20°$, K is found to be $\underline{K = 46.8\ \frac{\text{N}\cdot\text{m}}{\text{rad}}}$

6/24



Ⓐ $\Sigma M_O = m\bar{a}d$: $R_A l \sin\frac{\theta}{2} - mg\frac{l}{2}\sin\frac{\theta}{2} = ma\frac{l}{2}\cos\frac{\theta}{2}$

Ⓑ $\Sigma M_O = m\bar{a}d$: $Fl\cos\frac{\theta}{2} + mg\frac{l}{2}\sin\frac{\theta}{2} - R_B l\sin\frac{\theta}{2} = ma\frac{l}{2}\cos\frac{\theta}{2}$

Two bars together :

$\Sigma F_y = 0$: $R_A + R_B - 2mg = 0$

Subtract Eq. Ⓐ from Ⓑ, combine with y-eq. to obtain $\underline{\theta = 2\tan^{-1}\frac{F}{mg}}$

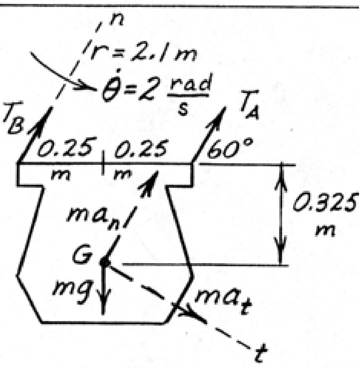
Both bars together: $\Sigma F_x = ma_x$: $F = 2ma$, $a = \frac{g}{2}\tan\frac{\theta}{2}$

From Ⓑ: $mg\tan\frac{\theta}{2}\, l\cos\frac{\theta}{2} + mg\frac{l}{2}\sin\frac{\theta}{2} - R_B l\sin\frac{\theta}{2} = m\left(\frac{g}{2}\tan\frac{\theta}{2}\right)\frac{l}{2}\cos\frac{\theta}{2}$

$\Rightarrow \underline{R_B = \frac{5}{4}mg}$

Finally, from y-eq., $\underline{R_A = \frac{3}{4}mg}$

6/25



$\circlearrowleft \Sigma M_G = 0: (T_A \sin 60^\circ)(0.25) - (T_B \sin 60^\circ)(0.25)$
$- (T_A \cos 60^\circ)(0.325) - (T_B \cos 60^\circ)(0.325) = 0$
$0.0540\, T_A = 0.379\, T_B \quad ---(1)$

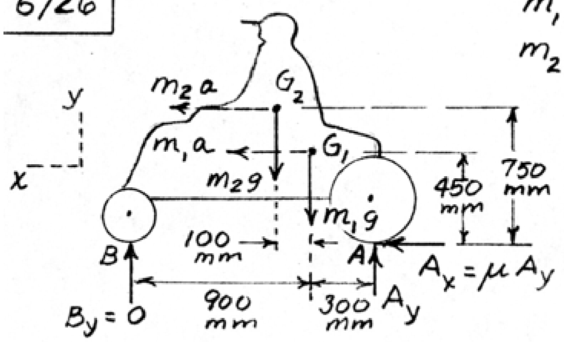
$\Sigma F_n = ma_n: \ T_A + T_B - 10(9.81)\sin 60^\circ$
$= 10(2.1)(2^2) \ ---(2)$

Solve (1) & (2) & get $\underline{T_A = 147.9\text{ N}}$, $\underline{T_B = 21.1\text{ N}}$

6/26

$m_1 g = 140(9.81) = 1373$ N

$m_2 g = 90(9.81) = 883$ N



$\Sigma F_y = 0$

$A_y - 1373 - 883 = 0$

$A_y = 2256$ N

$\Sigma M_A = \Sigma mad$; $883(0.4) + 1373(0.3) = 90a(0.75) + 140a(0.45)$

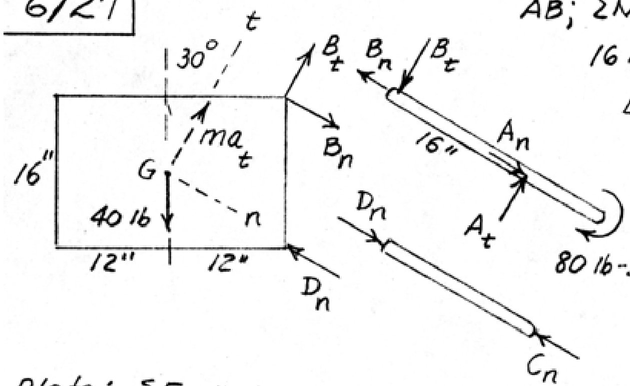
$765.2 = 130.5a$

$a = 5.86\ m/s^2$

$\Sigma F_x = \Sigma ma_x$; $\mu(2256) = (140 + 90)5.86$

$\underline{\mu = 0.598}$

6/27



AB; $\Sigma M_A = 0$;

$16 B_t - 80(12) = 0$

$B_t = 60$ lb

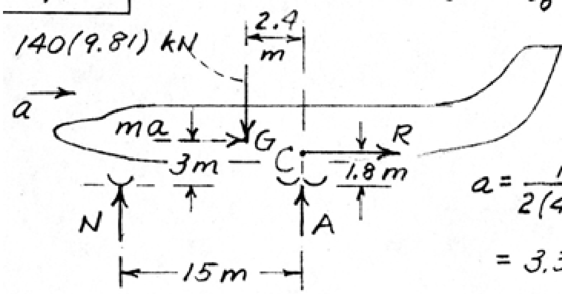
Plate; $\Sigma F_t = m a_t$; $60 - 40\cos 30° = \frac{40}{32.2} a_t$, $a_t = 20.4$ ft/sec^2

$\Sigma M_B = m a d$; $D_n(16\cos 30°) - 40(12)$

$= \frac{40}{32.2}(20.4)\left[12\cos 30° - 8\sin 30°\right]$

$D = D_n = \underline{46.3 \text{ lb}}$

6/28



$$v^2 = v_0^2 + 2as$$

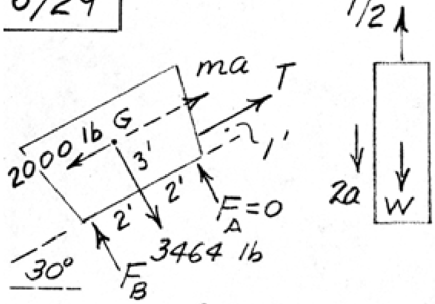
$$a = \frac{1}{2(425)}\left[(200)^2 - (60)^2\right]\frac{1}{(3.6)^2}$$

$$= 3.30 \text{ m/s}^2$$

$\Sigma M_C = mad;$ $15N - 140(9.81)(2.4) = 140(3.30)(3 - 1.8)$

$\underline{N = 257 \text{ kN}}$

6/29



$4000 \sin 30° = 2000$ lb

$4000 \cos 30° = 3464$ lb

Car: $\Sigma M_B = mad_B$

$T(1) + 3464(2) - 2000(3) = \frac{4000}{32.2} a(3)$

$\Sigma F = ma$; $T - 2000 = \frac{4000}{32.2} a$

Solve & get $T = 3464$ lb

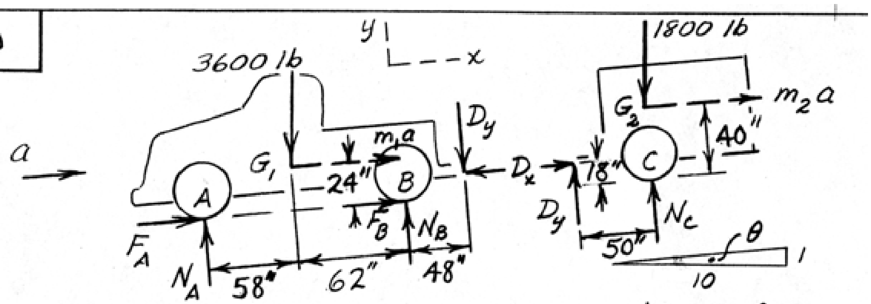
$a = 11.79$ ft/sec^2

Counterweight:

$\Sigma F = ma$; $W - 3464/2 = \frac{W}{32.2}(2 \times 11.79)$

$\underline{W = 6460 \text{ lb}}$

6/30



For const. accel., $\theta = \tan^{-1}\frac{1}{10} = 5.71°$

$v^2 = v_0^2 + 2as$: $44^2 = 88^2 - 2a(360)$, $a = 8.07$ ft/sec^2 decel.

$m_1 a = \frac{3600}{32.2} \times 8.07 = 902$ lb, $m_2 a = \frac{1800}{32.2} \times 8.07 = 451$ lb

Trailer: $\Sigma F_x = ma_x$: $D_x - 1800 \sin 5.71° = 451$, $\underline{D_x = 630 \text{ lb}}$

$\Sigma M_C = mad$: $50 D_y + 630(18) - 1800 \sin 5.71°(40) = 451(40)$, $\underline{D_y = 277 \text{ lb}}$

$\Sigma F_y = 0$: $N_C - 1800 \cos 5.71° + 277 = 0$, $\underline{N_C = 1514 \text{ lb}}$

Truck:

$\Sigma M_A = mad$: $3600 \cos 5.71° \times 58 - 3600 \sin 5.71° \times 24 - 120 N_B$
$+ 277(168) - 630(18) = 902(24)$

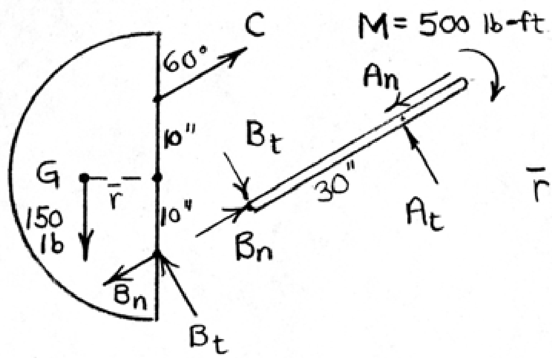
$\underline{N_B = 1773 \text{ lb}}$

6/31

$AB: \begin{cases} \sum M_A = 0: \quad 30\, B_t = 500(12), \quad B_t = 200 \text{ lb} \\ \sum F_t = 0 \Rightarrow \underline{A_t = 200 \text{ lb}} \end{cases}$

Plate: $\sum F_t = ma_t: \quad 200 - 150\frac{\sqrt{3}}{2} = \frac{150}{32.2} a_t$

$a_t = 15.05 \text{ ft/sec}^2$



$\bar{r} = \frac{4r}{3\pi} = \frac{4(20)}{3\pi} = 8.49 \text{ in.}$

$\sum M_C = m\bar{a}d: \quad 200(20)\left(\frac{1}{2}\right) + B_n\left(20\frac{\sqrt{3}}{2}\right)$

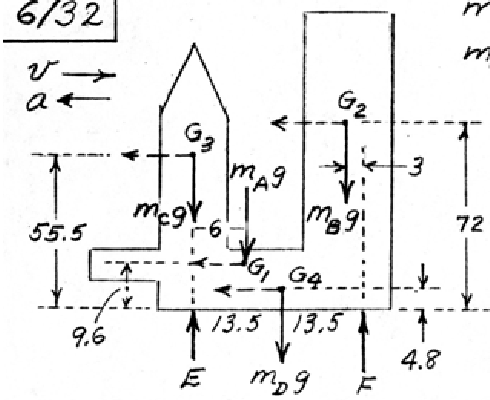
$- 150(8.49) = \frac{150}{32.2} 15.05\left(\frac{\sqrt{3}}{2} 8.49 + \frac{1}{2} 10\right)$

$A_n = B_n = \underline{8.03 \text{ lb}}$

6/32

$m_A = 3$ Gg, $m_B = 3.3$ Gg

$m_C = 0.23$ Gg, $m_D = 3$ Gg



$v^2 = 2as$

$$a = \frac{v^2}{2s} = \frac{(1.5/3.6)^2}{2(0.1)} = 0.868\ \frac{m}{s^2}$$

$$\Sigma M_E = \Sigma mad$$

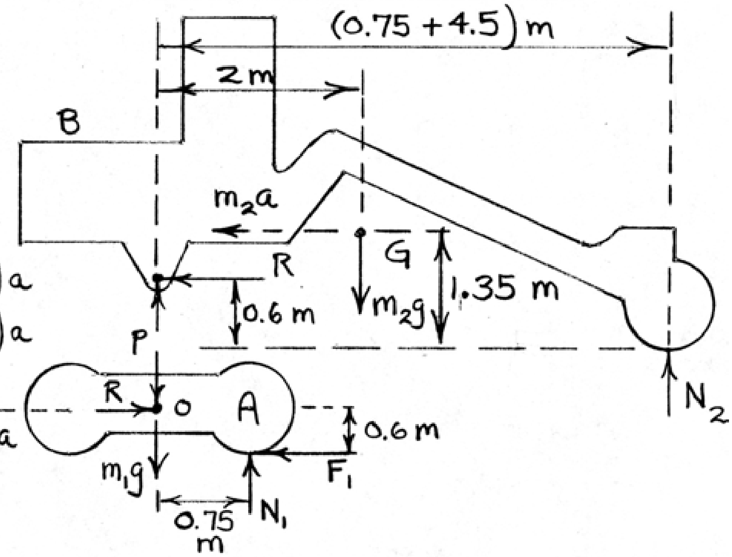
Dimensions in meters

$$27F - [3(6) + 3.3(27-3) + 3(13.5)]9.81$$

$$= [3(9.6) + 3.3(72) + 0.23(55.5) + 3(4.8)]0.868$$

$$27F = 1350.8 + 254.8, \quad \underline{F = 59.5\ \text{MN}}$$

6/33



For rear wheels of unit A to lift off ground:

(A) $\Sigma M_{N_1} = m_1 a d_1$: $[P + 3(10^3)(9.81)](0.750) - 0.6R = 3(10^3)a(0.6)$

(B) $\Sigma M_{N_2} = m_2 a d_2$: $10(10^3)(9.81)(4.5 + 0.75 - 2)$
$- P(4.5 + 0.75) + 0.6R = 10(10^3)a(1.35)$

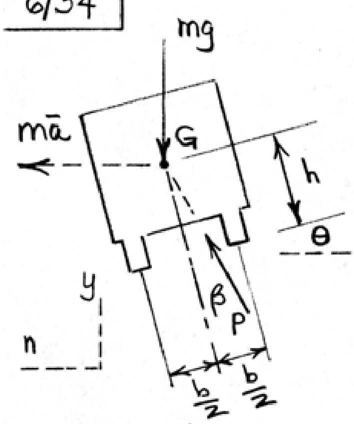
$\Sigma F_x = ma_x$: $R = 10(10^3)a$

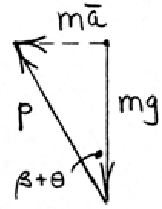
Solve the above three equations to obtain

$R = 76.2$ kN, $P = 49.8$ kN, $a = 7.62$ m/s^2

For constant acceleration, $s = \dfrac{v^2}{2a} = \dfrac{(40/3.6)^2}{2(7.62)} = \underline{8.10 \text{ m}}$

6/34





(a) For no tendency to slip, $\beta = 0$.

From diagram,

$$\tan\theta = \frac{m\bar{a}}{mg} = \frac{v^2/r}{g}$$

$$\underline{\theta = \tan^{-1}\frac{v^2}{gr}}$$

(b) $\tan(\beta+\theta) = \dfrac{m\bar{a}}{mg} = \dfrac{v^2/r}{g}$

$$v^2 = gr\tan(\beta+\theta) = gr\,\frac{\tan\beta + \tan\theta}{1-\tan\beta\tan\theta}$$

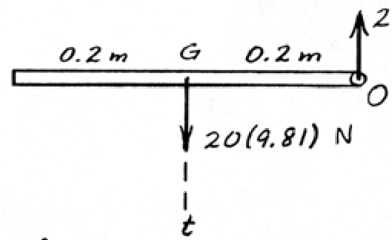
Slips first if $\mu < \dfrac{b/2}{h}$ & $\mu = \tan\beta$

So $\underline{v^2 = gr\,\dfrac{\mu + \tan\theta}{1-\mu\tan\theta}}$

Tips first if $\mu > \dfrac{b/2}{h}$ & $\tan\beta = \dfrac{b}{2h}$:

$$\underline{v^2 = gr\,\frac{\frac{b}{2h} + \tan\theta}{1-\frac{b}{2h}\tan\theta}}$$

6/35



$\Sigma M_O = I_O \alpha$, $20(9.81)(0.2) = \frac{1}{3} 20(0.4)^2 \alpha$

$\alpha = 36.8 \text{ rad/s}^2$

$\bar{a} = \bar{r}\alpha$, $\bar{a} = 0.2 \times 36.8 = 7.36 \text{ m/s}^2$

$\Sigma F_t = m\bar{a}_t$, $20(9.81) - 2F = 20 \times 7.36$

$2F = 49.0$, $\underline{F_A = F_B = F = 24.5 \text{ N}}$

6/36 Accelerating force on rear wheels is

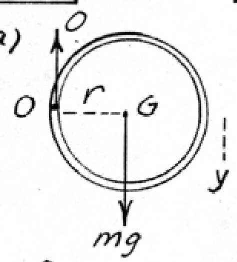
$$F = ma = \frac{5200}{g} 0.5g = 2600 \text{ lb}$$

$$\alpha_{drum} = \frac{a_t}{r} = \frac{0.5(32.2)}{3} = 5.37 \text{ rad/sec}^2$$

$$\Sigma M_O = I_O \alpha; \quad I_O = \frac{2600(3)}{5.37} = \underline{1453 \text{ lb-ft-sec}^2}$$

6/37

(a)



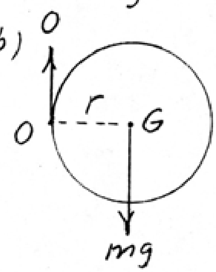
$$\Sigma M_O = I_O\alpha;\quad mgr = 2mr^2\alpha$$

$$\underline{\alpha = g/2r}$$

$$\Sigma F_y = m\bar{a}_y;\quad mg - O = mr\left(\frac{g}{2r}\right)$$

$$\underline{O = mg/2}$$

(b)



$$\Sigma M_O = I_O\alpha;\quad mgr = \left(\tfrac{1}{2}mr^2 + mr^2\right)\alpha$$

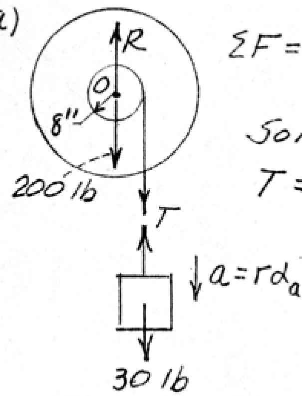
$$\underline{\alpha = 2g/3r}$$

$$\Sigma F_y = m\bar{a}_y;\quad mg - O = mr\left(\frac{2g}{3r}\right)$$

$$\underline{O = mg/3}$$

6/38

(a)



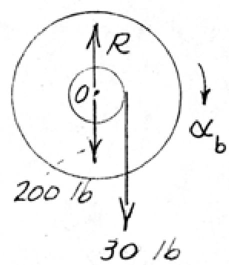
$\Sigma M_O = I_O\alpha;\quad T\frac{8}{12} = \frac{200}{32.2}\left(\frac{15}{12}\right)^2\alpha_a$

$\Sigma F = ma;\quad 30 - T = \frac{30}{32.2}\left(\frac{8}{12}\alpha_a\right)$

Solve simultaneously & get

$T = 28.77$ lb $\quad \underline{\alpha_a = 1.976 \text{ rad/sec}^2}$

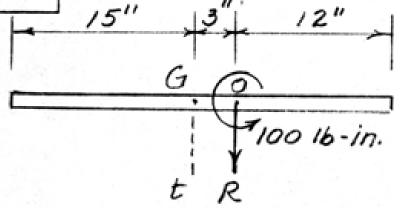
(b)



$\Sigma M_O = I_O\alpha;\quad 30\frac{8}{12} = \frac{200}{32.2}\left(\frac{15}{12}\right)^2\alpha_b$

$\underline{\alpha_b = 2.06 \text{ rad/sec}^2}$

6/39



$$I_O = I_G + md^2$$

$$I_G = \frac{1}{12} m l^2 = \frac{1}{12} \frac{20}{32.2} \left(\frac{30}{12}\right)^2$$

$$= 0.323 \text{ lb-ft-sec}^2$$

$$I_O = 0.323 + \frac{20}{32.2} \left(\frac{3}{12}\right)^2$$

$$= 0.3623 \text{ lb-ft-sec}^2$$

$\Sigma M_O = I_O \alpha$; $\frac{100}{12} = 0.3623 \alpha$, $\alpha = 23.0 \text{ rad/sec}^2$

$\bar{a}_t = \bar{r}\alpha$; $\bar{a}_t = \frac{3}{12}(23.0) = 5.75 \text{ ft/sec}^2$

$\Sigma F_t = m\bar{a}_t$; $R = \frac{20}{32.2}(5.75) = \underline{3.57 \text{ lb}}$

6/40

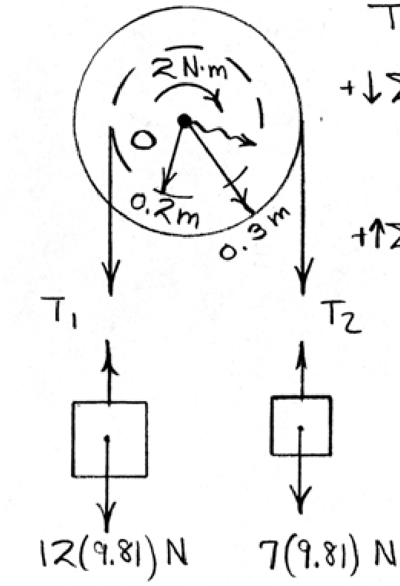
$\circlearrowright + \sum M_O = I_O \alpha$ for drum:

$$T_1(0.2) - T_2(0.3) - 2 = 8(0.225)^2 \alpha \quad (1)$$

$+\downarrow \sum F = ma$ for 12-kg cylinder:

$$12(9.81) - T_1 = 12(0.2\alpha) \quad (2)$$

$+\uparrow \sum F = ma$ for 7-kg cylinder:

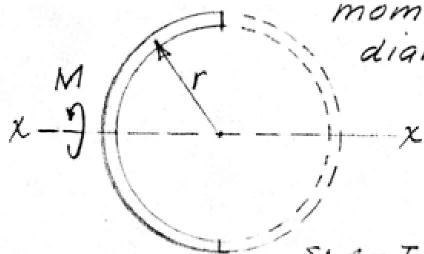


$$T_2 - 7(9.81) = 7(0.3\alpha) \quad (3)$$

Solution of Eqs. (1)-(3):

$$\begin{cases} T_1 = 116.2 \text{ N} \\ T_2 = 70.0 \text{ N} \\ \underline{\alpha = 0.622 \text{ rad/s}^2} \end{cases}$$

6/41



For complete ring of mass 2m moment of inertia about diameter x-x $= \frac{1}{2}(2mr^2)$

So moment of inertia of half ring about x-x is $\frac{1}{2}mr^2$

$\Sigma M_x = I_x\alpha$; $M = \frac{1}{2}mr^2\alpha$, $\underline{\alpha = \dfrac{2M}{mr^2}}$

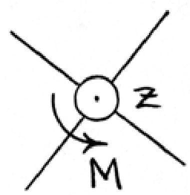
6/42 $\quad \alpha = \frac{\omega}{\tau} \qquad (H = \text{hub} ; \; B = \text{blades})$

$$I_{zz} = \frac{1}{2} m_H r^2 + 4\left[\frac{1}{12} m_B l^2 + m_B \left(r + \frac{l}{2}\right)^2\right]$$

$$= \frac{1}{2}(\rho \pi r^2 d) r^2 + 4(\rho l d t)\left[\frac{1}{12} l^2 + r^2 + rl + \frac{l^2}{4}\right]$$

$$= \frac{1}{2} \rho \pi d r^4 + 4 \rho l d t \left[\frac{1}{3} l^2 + rl + r^2\right]$$

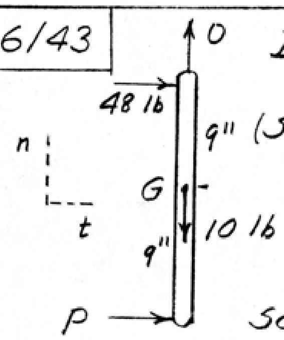
$$= \rho d \left[\frac{1}{2} \pi r^4 + 4 l t \left(\frac{1}{3} l^2 + rl + r^2\right)\right]$$



$\Sigma M_z = I_{zz} \alpha$:

$$M = \frac{\omega \rho d}{\tau}\left[\frac{1}{2} \pi r^4 + 4 l t \left(\frac{1}{3} l^2 + rl + r^2\right)\right]$$

6/43



$\bar{I} = \frac{1}{12} m l^2 = \frac{1}{12} \frac{10}{32.2} \left(\frac{18}{12}\right)^2 = 0.0582 \text{ ft-lb-sec}^2$

(Sol. I)

$\Sigma M_G = \bar{I}\alpha; \quad \frac{9}{12}P - 48\frac{9}{12} = 0.0582\,\alpha$

$\Sigma F_t = m\bar{a}_t; \quad P + 48 = \frac{10}{32.2}\,\frac{9}{12}\alpha$

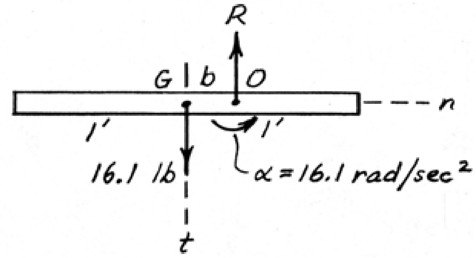
Solve simultaneously & get

$\underline{P = 96.0 \text{ lb}}, \quad \alpha = 618.2 \text{ rad/sec}^2$

(Sol II) $q = k_O^2/\bar{r} = \frac{l^2}{3}\Big/\frac{l}{2} = \frac{2}{3}l = \frac{2}{3}\,\frac{18}{12} = 1 \text{ ft}$

$\Sigma M_Q = 0; \quad P\left(\frac{18}{12} - 1\right) - 48(1) = 0, \quad P = 48/0.5 = \underline{96 \text{ lb}}$

6/44



$$I_O = \frac{1}{12}mL^2 + mb^2$$

$$= \frac{16.1}{32.2}\left(\frac{2^2}{12} + b^2\right)$$

$$= \frac{1}{6} + \frac{b^2}{2} \text{ lb-ft-sec}^2$$

$\Sigma M_O = I_O\alpha$: $16.1\,b = \left(\frac{1}{6} + \frac{b^2}{2}\right)16.1$, $3b^2 - 6b + 1 = 0$

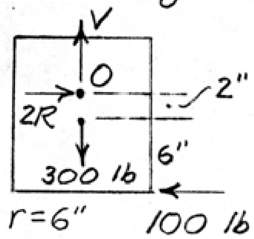
$b = 1 \pm \sqrt{24}/6$, $b = 0.1835$ ft $(1.817$ ft$)$,

$\underline{b = 2.20 \text{ in.}}$

$\Sigma F_t = m\bar{r}\alpha$: $16.1 - R = \frac{16.1}{32.2}0.1835(16.1)$, $\underline{R = 14.62 \text{ lb}}$

6/45

$$I_O = \bar{I} + m\bar{r}^2 = \left(\frac{1}{4}mr^2 + \frac{1}{12}ml^2\right) + m\bar{r}^2$$



$$= \frac{300}{32.2}\left[\frac{1}{4}\left(\frac{6}{12}\right)^2 + \frac{1}{12}\left(\frac{12}{12}\right)^2 + \left(\frac{2}{12}\right)^2\right]$$

$$= 1.617 \text{ lb-ft-sec}^2$$

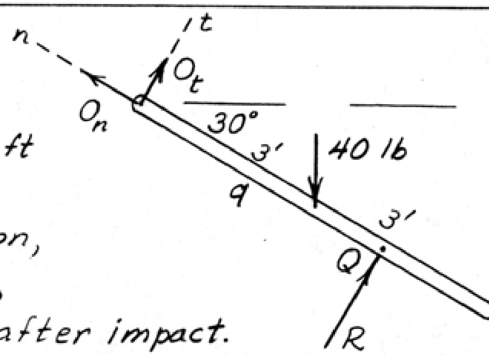
$$\Sigma M_O = I_O\alpha; \quad 100\left(\frac{8}{12}\right) = 1.617\alpha$$

$$\alpha = 41.2 \text{ rad/sec}^2$$

$$\Sigma F_t = m\bar{r}\alpha; \quad 100 - 2R = \frac{300}{32.2}\,\frac{2}{12}(41.2)$$

$$\underline{R = 18 \text{ lb}}$$

6/46



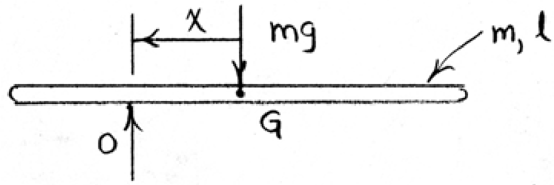
For slender rod,

$$q = \frac{k_O^2}{\bar{r}} = \frac{\frac{1}{3}L^2}{\frac{L}{2}} = \frac{2}{3}(6) = 4 \text{ ft}$$

For fixed-axis rotation,
$\Sigma M_Q = 0$ at all times,
before, during, and after impact.
Thus

$$40(1)\cos 30° - 4\,O_t = 0, \quad \underline{O_t = 8.66 \text{ lb}} \text{ at all times}$$

6/47



$$I_O = I_G + mx^2 = \frac{1}{12}ml^2 + mx^2 = m\left(\frac{l^2}{12} + x^2\right)$$

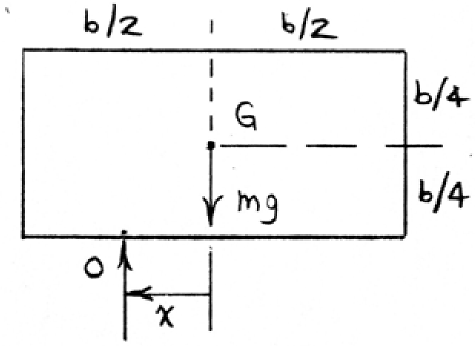
$$\circlearrowleft \Sigma M_O = I_O\alpha : \quad mgx = m\left(\frac{l^2}{12} + x^2\right)\alpha$$

$$\alpha = \frac{gx}{\frac{1}{12}l^2 + x^2}$$

$$\frac{d\alpha}{dx} = \frac{\left(\frac{1}{12}l^2 + x^2\right)g - gx(2x)}{\left(\frac{1}{12}l^2 + x^2\right)^2} = 0 \Rightarrow \underline{x = \frac{l}{2\sqrt{3}}}$$

$$\alpha = \frac{g\,\frac{l}{\sqrt{12}}}{\frac{1}{12}l^2 + \frac{1}{12}l^2} = \underline{\sqrt{3}\,\frac{g}{l}}$$

6/48



$$I_G = \frac{1}{12}m\left[b^2 + \left(\frac{b}{2}\right)^2\right] = \frac{5}{48}mb^2$$

$$I_O = I_G + m\left[\left(\frac{b}{4}\right)^2 + x^2\right] = \frac{1}{6}mb^2 + mx^2$$

$$\Sigma M_O = I_O\alpha: \quad mgx = \left(\frac{1}{6}mb^2 + mx^2\right)\alpha$$

$$\alpha = \frac{gx}{\frac{1}{6}b^2 + x^2}$$

$$\frac{d\alpha}{dx} = \frac{\left(\frac{1}{6}b^2 + x^2\right)g - gx(2x)}{\left(\frac{1}{6}b^2 + x^2\right)} = 0 \Rightarrow \underline{x = \frac{b}{\sqrt{6}}}$$

$$\alpha = \frac{g\frac{b}{\sqrt{6}}}{\frac{1}{6}b^2 + \frac{1}{6}b^2} = \underline{\sqrt{\frac{3}{2}}\,\frac{g}{b}}$$

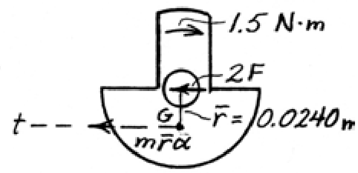
6/49 $\omega^2 = \omega_0^2 + 2\bar{\alpha}\theta$, $\left(\frac{1200 \times 2\pi}{60}\right)^2 = 0 + 2\bar{\alpha}\,(18 \times 2\pi)$, $\bar{\alpha} = 69.8 \text{ rad/s}^2$

Static test $\Sigma M = 0$: $0.660 - 2.8(9.81)\bar{r}$, $\bar{r} = 0.0240$ m

(a) $\Sigma M = I\alpha$: $1.5 = 2.8\,k^2 \times 69.8$, $k = 0.0876$ m or $\underline{k = 87.6 \text{ mm}}$

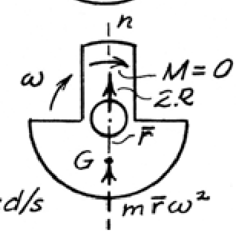
(b) $\Sigma F_t = m\bar{r}\alpha$: $2F = 2.8\,(0.0240)69.8$

$\underline{F = 2.35 \text{ N}}$

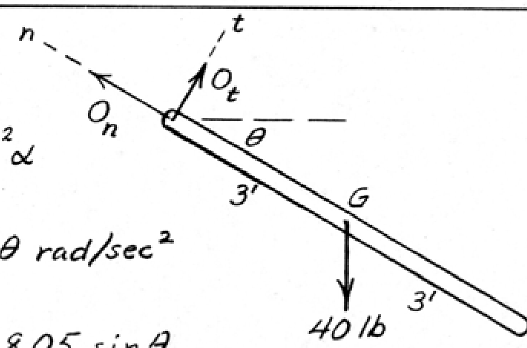


(c) $\Sigma F_n = m\bar{r}\omega^2$: $2R = 2.8(0.0240)(125.7^2)$

$\underline{R = 531 \text{ N}}$



6/50



$\Sigma M_O = I_O \alpha$:

$$40(3)\cos\theta = \frac{1}{3}\frac{40}{32.2}6^2\alpha$$

$$\alpha = 8.05\cos\theta \text{ rad/sec}^2$$

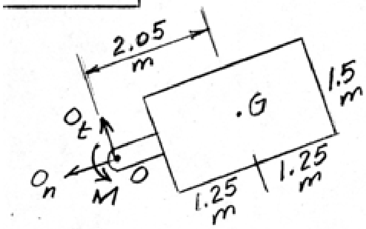
$$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta: \ \frac{\omega^2}{2} = 8.05\sin\theta,$$

$$\omega^2_{\theta=30°} = 2(8.05)0.5 = 8.05 \text{ (rad/sec)}^2$$

$$\Sigma F_n = m\bar{a}_n: \ O_n - 40\sin 30° = \frac{40}{32.2}(3)(8.05), \ \underline{O_n = 50 \text{ lb}}$$

6/51

$m = 6000\text{ kg}$; From Table D/4



$I_G = \frac{1}{12}(6000)\left[(1.5)^2 + (2.5)^2\right]$
$= 4250\text{ kg}\cdot\text{m}^2$

$I_O = I_G + md^2$
$= 4250 + 6000(2.05)^2$
$= 29\,465\text{ kg}\cdot\text{m}^2$

$M = 30\text{ N}\cdot\text{m}$

$\Sigma M_O = I_O\alpha$; $30 = 29\,465\,\alpha$, $\alpha = 1.018(10^{-3})\text{ rad/s}^2$

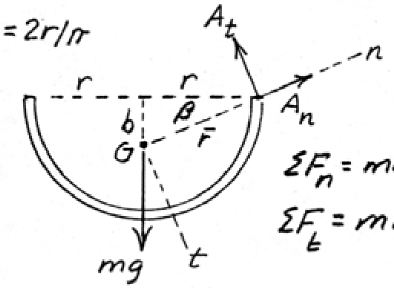
$\theta = \frac{1}{2}\alpha t^2$, $\frac{\pi}{4} = \frac{1}{2}\,1.018(10^{-3})\,t_1^2$, $t_1 = 39.28\text{ s}$

Total time $t = 2t_1 = \underline{78.6\text{ s}}$

6/52

$I_A = \frac{1}{2}(2mr^2 + 2mr^2) = 2mr^2$

$b = 2r/\pi$



$\Sigma M_A = I_A\alpha$; $mgr = 2mr^2\alpha$

$\underline{\alpha = g/2r}$

$\Sigma F_n = m\bar{a}_n = 0$; $A_n = mg \sin\beta$

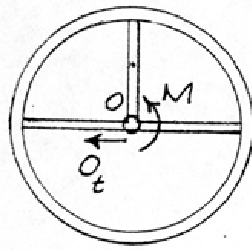
$\Sigma F_t = m\bar{a}_t$; $mg\cos\beta - A_t = m\bar{r}\alpha$

or $A_n = mg\frac{b}{\bar{r}}$, $A_t = mg\left(\frac{r}{\bar{r}} - \frac{\bar{r}}{2r}\right)$

so $A = mg\sqrt{\frac{b^2}{\bar{r}^2} + \frac{r^2}{\bar{r}^2} - 1 + \frac{\bar{r}^2}{4r^2}} = mg\frac{\bar{r}}{2r} = \frac{mg}{2}\sqrt{1 + 4/\pi^2}$

or $\underline{A = 0.593\, mg}$

6/53 Rim: $I_O = mr^2 = \frac{100}{32.2}\left(\frac{18}{12}\right)^2 = 6.99$ lb-ft-sec^2



Each spoke: $I_O = \frac{1}{3}ml^2 = \frac{1}{3}\frac{15}{32.2}\left(\frac{18}{12}\right)^2$

$= 0.349$ lb-ft-sec^2

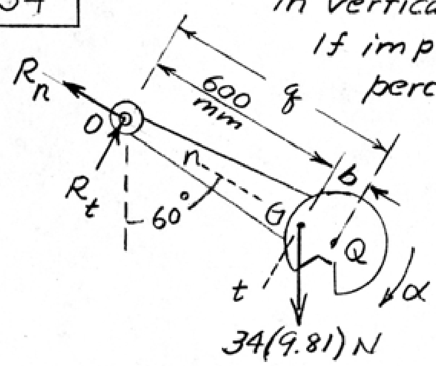
$\Sigma M_O = I_O\alpha$; $\frac{400}{12} = [6.99 + 3(0.349)]\alpha$

$\alpha = 4.15$ rad/sec^2

$\Sigma F_t = \Sigma m\bar{r}\alpha$; $O_t = \frac{15}{32.2}\left(\frac{9}{12}\right)(4.15) + 0$ (middle spoke only)

$O_t = 1.449$ lb

6/54



In vertical position $R_t = 0$

If impact occurs at center of percussion so

$q = k_O^2/\bar{r}$,

$$0.600 + b = \frac{(0.620)^2}{0.600}$$

$b = 0.0407$ m

or $\underline{b = 40.7 \text{ mm}}$

$\Sigma M_Q = 0;\ 34(9.81)(0.0407 \sin 60°) - (0.6407)R_t = 0$

$R_t = 18.35$ N

$\Sigma F_n = m\bar{r}\omega^2 = 0;\ R_n - 34(9.81)\cos 60° = 0$

$R_n = 166.8$ N

$R = \sqrt{(166.8)^2 + (18.35)^2} = \underline{167.8 \text{ N}}$

6/55 For entire assembly,

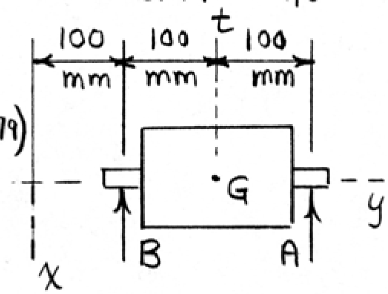
$I_{zz} = 0.60 + (0.080 + 12(0.2)^2) = 1.160 \text{ kg}\cdot\text{m}^2$

$\Sigma M_z = I_{zz}\alpha$: $16 = 1.160\,\alpha$, $\alpha = 13.79 \text{ rad/s}^2$

For cylinder :

$\Sigma F_t = ma_t$: $A + B = 12(0.2)(13.79)$

$= 33.1 \text{ N}$

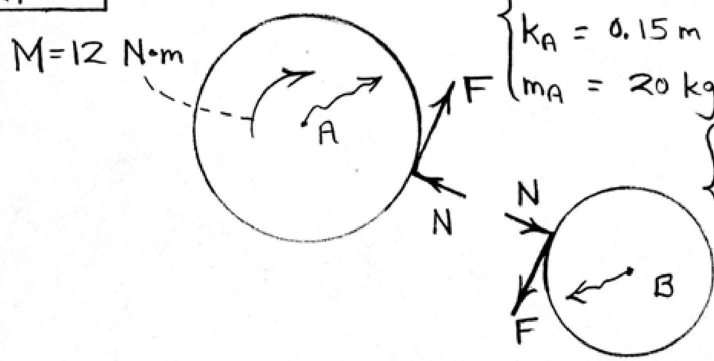


$\Sigma M_O = I_{zz}\alpha$:

$0.3A + 0.1B = [0.080 + 12(0.2)^2]\,13.79$

Simultaneous solution : $\underline{A = 22.1 \text{ N}}$, $\underline{B = 11.03 \text{ N}}$

6/56



$$\begin{cases} r_A = 0.24\ \text{m} \\ k_A = 0.15\ \text{m} \\ m_A = 20\ \text{kg} \end{cases}$$

$$\begin{cases} r_B = 0.18\ \text{m} \\ k_B = 0.1\ \text{m} \\ m_B = 10\ \text{kg} \end{cases}$$

$$\stackrel{+}{\curvearrowright} \Sigma M_A = I_A \alpha_A: \quad 12 - F(0.24) = 20(0.15)^2 \alpha_A \quad (1)$$

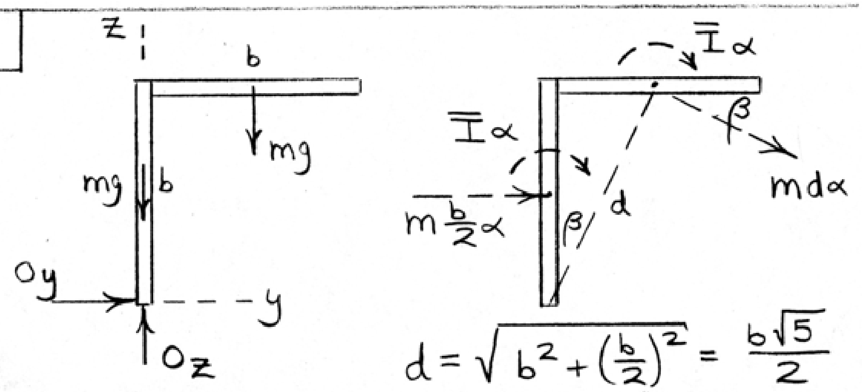
$$\stackrel{+}{\curvearrowleft} \Sigma M_B = I_B \alpha_B: \quad F(0.18) = 10(0.1)^2 \alpha_B \quad (2)$$

Tangential accelerations match: $r_A \alpha_A = r_B \alpha_B$

$$0.24\,\alpha_A = 0.18\,\alpha_B \quad (3)$$

Solution of Eqs. (1)-(3):
$$\begin{cases} F = 14.16\ \text{N} \\ \alpha_A = 19.12\ \text{rad/s}^2\ (\text{CW}) \\ \alpha_B = 25.5\ \text{rad/s}^2\ (\text{CCW}) \end{cases}$$

6/57



$$d = \sqrt{b^2 + \left(\frac{b}{2}\right)^2} = \frac{b\sqrt{5}}{2}$$

$$m = \rho bc\,; \quad I_O = \frac{1}{3}mb^2 + \frac{1}{12}mb^2 + m\left[\left(\frac{b}{2}\right)^2 + b^2\right]$$

$$= \frac{5}{3}mb^2 = \frac{5}{3}\rho b^3 c$$

$$\Sigma M_O = I_O\alpha: \quad g\rho bc\left(\frac{b}{2}\right) = \frac{5}{3}\rho b^3 c\,\alpha, \quad \underline{\alpha = \frac{3g}{10b}}$$

For each plate, $\bar{I} = \frac{1}{12}mb^2 = \frac{1}{12}\rho b^3 c$

$$\cos\beta = \frac{b}{\frac{b}{2}\sqrt{5}} = \frac{2}{\sqrt{5}}, \quad \sin\beta = \frac{b/2}{\frac{b}{2}\sqrt{5}} = \frac{1}{\sqrt{5}}$$

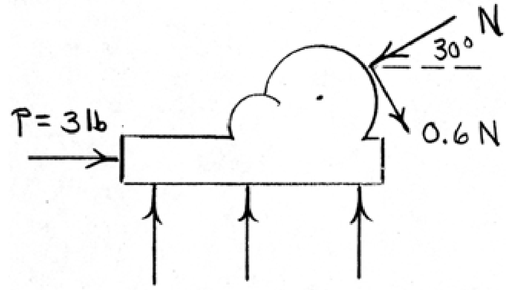
$$\Sigma F_y = m\bar{a}_y: \quad O_y = m\frac{b}{2}\alpha + m\frac{b}{2}\sqrt{5}\,\alpha\frac{2}{\sqrt{5}}$$

$$= \frac{3}{2}mb\alpha = \underline{\frac{9}{20}\rho bcg}$$

$$\Sigma F_z = \Sigma m\bar{a}_z: \quad O_z - 2mg = -md\alpha\sin\beta$$

$$O_z = 2\rho gbc - \rho bc\left(\frac{b}{2}\sqrt{5}\right)\frac{3g}{10b}\frac{1}{\sqrt{5}} = \underline{\frac{37}{20}\rho bcg}$$

6/58 Power unit C:

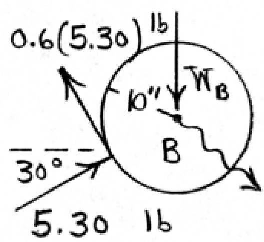


$\xrightarrow{+} \Sigma F = 0$:

$$3 - N\cos 30^\circ + 0.6N\sin 30^\circ = 0$$

$$N = 5.30 \text{ lb}$$

Wheel B : $\overset{+}{\circlearrowleft} \Sigma M_B = I_B\alpha$: $0.6(5.30)\left(\frac{10}{12}\right) = \frac{50}{32.2}\left(\frac{8}{12}\right)^2\alpha$



$$\underline{\alpha = 3.84 \text{ rad/sec}^2}$$

Steady-state speed:

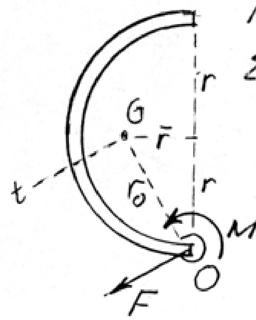
$$r_A\omega_A = r_B\omega_B$$

$$\omega_B = \frac{r_A\omega_A}{r_B} = \frac{8\left[1600\ \frac{2\pi}{60}\right]}{10}$$

$$= 134.0 \text{ rad/sec}$$

$$\omega_B = \cancelto{0}{\omega_{B0}} + \alpha t \ : \quad t = \frac{\omega_B}{\alpha} = \frac{134.0}{3.84} = \underline{\mathbf{34.9} \text{ sec}}$$

6/59



For complete ring $I_O = 2(2m)r^2$

For the half ring $I_O = 2mr^2$

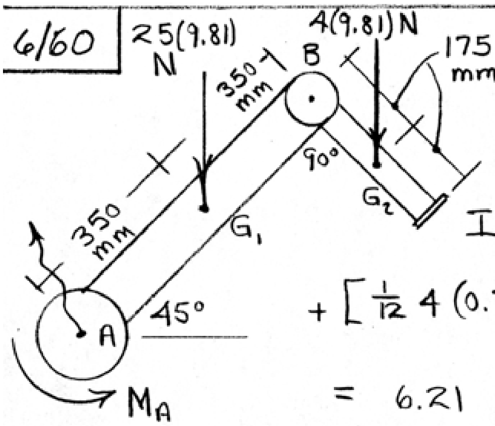
$\Sigma M_O = I_O\alpha$; $M = 2mr^2\alpha$, $\underline{\alpha = \dfrac{M}{2mr^2}}$

$\Sigma F_t = m\bar{a}_t$; $F = m r_0 \alpha$

$= m\sqrt{r^2 + \bar{r}^2}\,\dfrac{M}{2mr^2}$

$F = \dfrac{M}{r}\sqrt{\dfrac{1}{4} + \dfrac{1}{\pi^2}} = \underline{0.593\,M/r}$

6/60

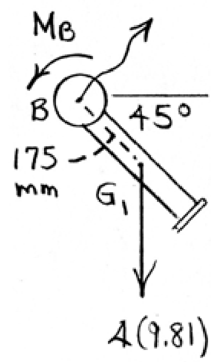


$$I_B = \frac{1}{3}(4)(0.350)^2 = 0.1633\ \text{kg}\cdot\text{m}^2$$

$$I_A = \frac{1}{3}(25)(0.7)^2 + \left[\frac{1}{12}4(0.350)^2 + 4(0.7^2 + 0.175^2)\right] = 6.21\ \text{kg}\cdot\text{m}^2$$

$$\circlearrowleft{+}\ \Sigma M_A = I_A\alpha:\quad M_A - 25(9.81)(0.350\cos 45°) - 4(9.81)(0.700 + 0.175)\cos 45° = 6.21(4)$$

$$\underline{M_A = 109.8\ \text{N}\cdot\text{m}}$$

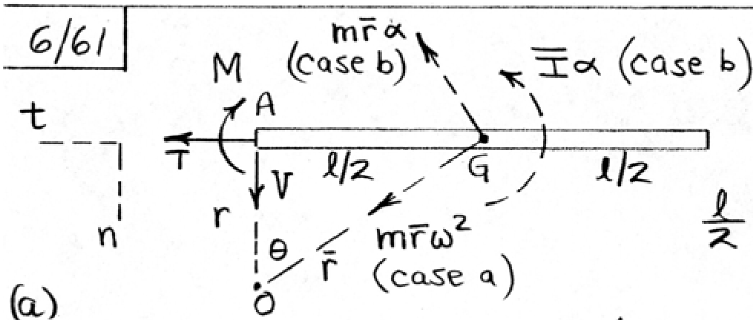


$$\circlearrowleft{+}\ \Sigma M_B = I_B\alpha:$$

$$M_B - 4(9.81)(0.175\cos 45°) = 0.1633(4)$$

$$\underline{M_B = 5.51\ \text{N}\cdot\text{m}}$$

6/61



$\frac{l}{2} = \bar{r}\sin\theta$

(a)

$$\overset{+}{\curvearrowleft}\ \Sigma M_A = m\bar{a}d: \quad M = m\bar{r}\omega^2 \frac{l}{2}\cos\theta$$

$$= m\frac{r}{\cos\theta}\omega^2\frac{l}{2}\cos\theta = \underline{mrl\omega^2/2}$$

$$\Sigma F_n = m\bar{a}_n: \quad V = m\bar{r}\omega^2\cos\theta = m\frac{r}{\cos\theta}\omega^2\cos\theta$$

$$= \underline{mr\omega^2}$$

$$\Sigma F_t = m\bar{a}_t: \quad T = m\bar{r}\omega^2\sin\theta = \underline{ml\omega^2/2}$$

(b)

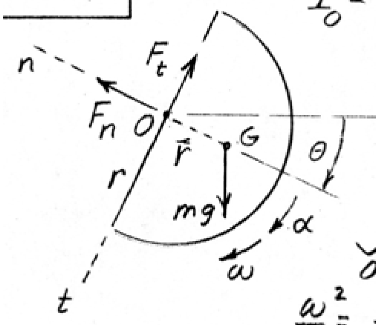
$$\Sigma M_A = m\bar{a}d: \quad M = -m\bar{r}\alpha\frac{l}{2}\sin\theta - \bar{I}\alpha$$

$$M = -m\alpha\frac{l^2}{4} - \frac{1}{12}ml^2\alpha = \underline{-ml^2\alpha/3}$$

$$\Sigma F_n = m\bar{a}_n: \quad V = -m\bar{r}\alpha\sin\theta = \underline{-ml\alpha/2}$$

$$\Sigma F_t = m\bar{a}_t: \quad T = m\bar{r}\alpha\cos\theta = \underline{mr\alpha}$$

6/62



$I_O = \frac{1}{2} m r^2 \, ; \quad \bar{r} = \frac{4r}{3\pi}$

$\Sigma M_O = I_O \alpha \, ; \quad mg \bar{r} \cos\theta = I_O \alpha$

$\alpha = mg \cos\theta \, \frac{\bar{r}}{I_O} = \frac{8}{3\pi} \frac{g}{r} \cos\theta$

$\int_0^\omega \omega \, d\omega = \int_0^\theta \alpha \, d\theta$

$\frac{\omega^2}{2} = \frac{8}{3\pi} \frac{g}{r} \sin\theta \Big]_0^\theta , \quad \omega^2 = \frac{16}{3\pi} \frac{g}{r} \sin\theta$

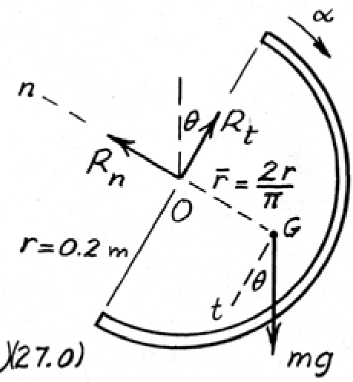
$\Sigma F_n = m \bar{r} \omega^2 \, ; \quad F_n - mg \sin\theta = m \frac{4r}{3\pi} \frac{16}{3\pi} \frac{g}{r} \sin\theta$

$F_n = \left(\frac{64}{9\pi^2} + 1 \right) mg \sin\theta \quad = \underline{1.721 \, mg \sin\theta}$

$\Sigma F_t = m \bar{r} \alpha \, ; \quad mg \cos\theta - F_t = m \frac{4r}{3\pi} \frac{8}{3\pi} \frac{g}{r} \cos\theta$

$F_t = \left(1 - \frac{32}{9\pi^2} \right) mg \cos\theta \quad = \underline{0.640 \, mg \cos\theta}$

6/63



$\circlearrowleft \Sigma M_O = I_O\alpha: \; mg\bar{r}\cos\theta = mr^2\alpha,$

$24.5(0.1273)\cos 30° = 0.1\alpha,$

$\alpha = 2.70/0.1 = 27.0 \text{ rad/s}$

$\Sigma F_t = m\bar{r}\alpha: \; mg\cos\theta - R_t = m\bar{r}\alpha,$

$R_t = 24.5\cos 30° - 2.5(0.1273)(27.0)$

$= 12.63 \text{ N}$

$mg = 2.5(9.81)$

$= 24.5 \text{ N}$

$\Sigma F_n = m\bar{r}\omega^2: \; R_n - mg\sin\theta = 0,$

$R_n = 24.5\sin 30° = 12.26 \text{ N}$

$\bar{r} = \frac{2(0.2)}{\pi} = 0.1273 \text{ m}$

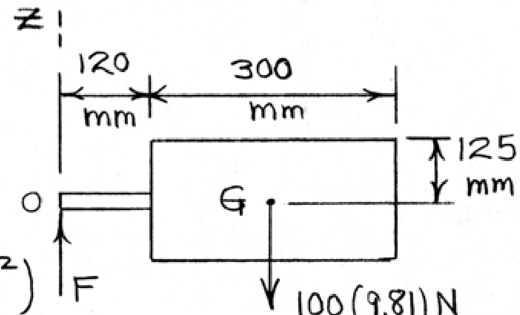
$I_O = mr^2 = 2.5(0.2)^2$

$= 0.1 \text{ kg}\cdot\text{m}^2$

$R = \sqrt{R_n^2 + R_t^2}: \; R = \sqrt{12.26^2 + 12.63^2} = \underline{17.60 \text{ N}}$

6/64

From Table D/4,



$I_G = \frac{1}{4} m r^2 + \frac{1}{12} m l^2$

$= \frac{1}{4} 100 \left(0.125^2 + \frac{1}{3} 0.3^2\right)$

$= 1.141 \text{ kg}\cdot\text{m}^2$

$I_O = I_G + md^2 = 1.141 + 100(0.120 + 0.150)^2 = 8.43 \text{ kg}\cdot\text{m}^2$

$\Sigma M_O = I_O \alpha: \quad 981(0.120 + 0.150) = 8.43\alpha$

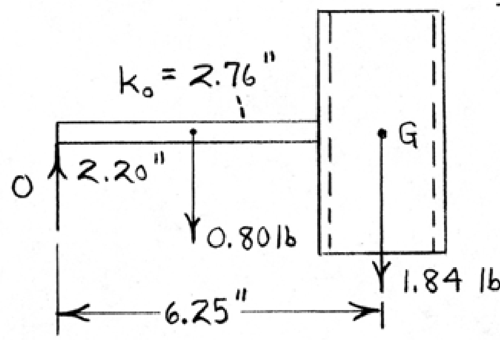
$\underline{\alpha = 31.4 \text{ rad/s}^2}$

$\Sigma F_z = m\bar{a}_z: \quad F - 981 = 100(-0.27)(31.4)$

$\underline{F = 132.7 \text{ N}}$

6/65 From Appendix D, for tube:

$$I_G = \frac{1}{2} m \left(r^2 + \frac{l^2}{6}\right)$$



$$= \frac{1}{2}\,\frac{1.84}{32.2}\left[\left(\frac{1.25}{12}\right)^2 + \frac{1}{6}\left(\frac{4}{12}\right)^2\right]$$

$$= 0.000839 \text{ lb-ft-sec}^2$$

$$I_O = I_G + m\bar{r}^2$$

$$= 0.000839 + \frac{1.84}{32.2}\left(\frac{6.25}{12}\right)^2$$

$$= 0.01634 \text{ lb-ft-sec}^2$$

Link: $I_O = \frac{0.80}{32.2}\left(\frac{2.76}{12}\right)^2 = 0.001314 \text{ lb-ft-sec}^2$

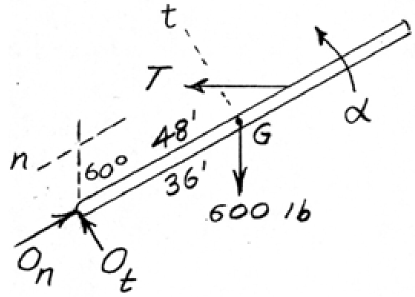
$$\Sigma M_O = I_O \alpha: \; 1.84\left(\frac{6.25}{12}\right) + 0.08\left(\frac{2.20}{12}\right)$$

$$= (0.001314 + 0.01634)\alpha, \quad \underline{\alpha = 62.6 \text{ rad/sec}^2}$$

$$\Sigma F_t = m\bar{a}_t: \; 1.84 + 0.80 - O = \left[\frac{1.84}{32.2}\,\frac{6.25}{12} + \frac{0.80}{32.2}\,\frac{2.20}{12}\right] 62.6$$

$$\underline{O = 0.492 \text{ lb}}$$

6/66 $M = \frac{T}{2} r$, $T = \frac{2(900)}{2} = 900$ lb



$\Sigma M_O = I_O \alpha$;

$900(48 \cos 60°) - 600(36 \sin 60°)$

$= \frac{1}{3} \frac{600}{32.2} \overline{72}^2 \alpha$

$\underline{\alpha = 0.0899 \text{ rad/sec}^2}$

$\Sigma F_t = m\bar{a}_t$; $O_t + 900 \cos 60° - 600 \sin 60° = \frac{600}{32.2}(36)(0.0899)$

$O_t = 129.9$ lb

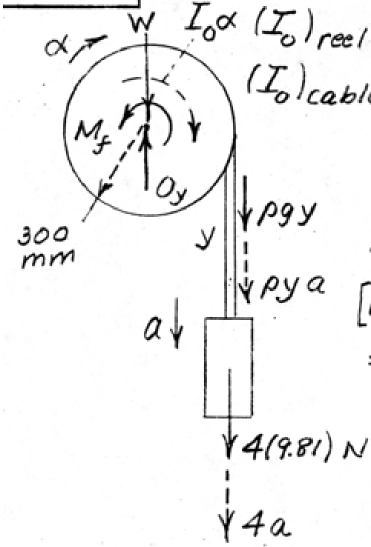
$\Sigma F_n = m\bar{a}_n = 0$; $900 \sin 60° + 600 \cos 60° - O_n = 0$

$O_n = 1079.4$ lb

$O = \sqrt{\overline{129.9}^2 + \overline{1079.4}^2} = \underline{1087 \text{ lb}}$

6/67

$\rho = 0.16\ \text{kg/m}\ ;\ M_f = 4(9.81)(0.3) = 11.772\ \text{N·m}$



$I_O\alpha$ $(I_O)_{reel} = mk_O^2 = 16(0.200)^2 = 0.64\ \text{kg·m}^2$

$(I_O)_{cable} = mr^2 = (60-y)(0.16)(0.300)^2$

$= 0.0144(60-y)\ \text{kg·m}^2$

$\rho g y = 0.16(9.81)y = 1.5696\,y$

$\Sigma M_O = I_O\alpha + \Sigma mar$ for system

$[1.5696y + 4(9.81)]0.300 - 11.772$

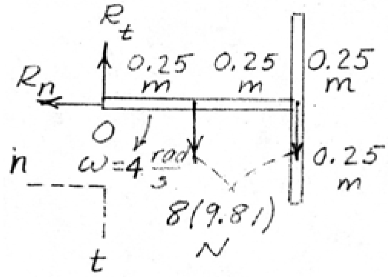
$= [0.64 + 0.0144(60-y)]\dfrac{a}{0.300}$

$+ [0.16y + 4]a(0.300)$

solve for a & get

$\underline{a = 0.0758y}$, a in m/s^2
y in m

6/68 $I_O = \frac{1}{3}ml^2 + \left(\frac{1}{12}ml^2 + ml^2\right) = \frac{17}{12}(8)(0.5)^2$ $kg \cdot m^2$



$\Sigma M_O = I_O \alpha$

$8(9.81)(0.50 + 0.25) = \frac{17}{12}(8)(0.5)^2\alpha$

$\alpha = 20.8\ rad/s^2$

$\Sigma F_t = \Sigma m\bar{a}_t$

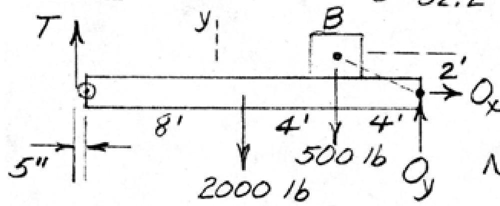
$2(8)(9.81) - R_t = 8(0.25)(20.8) + 8(0.50)(20.8)$

$R_t = 32.3\ N$

$\Sigma F_n = \Sigma m\bar{a}_n$; $R_n = 8(0.25)4^2 + 8(0.50)4^2 = 96.0\ N$

$R = \sqrt{32.3^2 + 96.0^2} = \underline{101.3\ N}$

6/69 Beam, $I_O \approx \frac{1}{3}\frac{2000}{32.2}16^2 + \frac{500}{32.2}(2^2+4^2)$



$= 5300 + 310.6$

$= 5611$ lb-ft-sec^2

$M = Tr;\ T = \frac{600}{6/12} = 1200$ lb

$\Sigma M_O = I_O\alpha;\quad 1200\left(16 + \frac{5}{12}\right) - 2000(8) - 500(4) = 5611\,\alpha$

$\alpha = 0.303$ rad/sec^2

$\Sigma F_y = m\bar{r}\alpha;\ O_y + 1200 - 2000 - 500 = \frac{2000}{32.2}(8)(0.303) + \frac{500}{32.2}(4)(0.303)$

$O_y = 1469$ lb

The $m\bar{r}\alpha$ resultant for the winch has an x-component, so that $O_x \neq 0$.

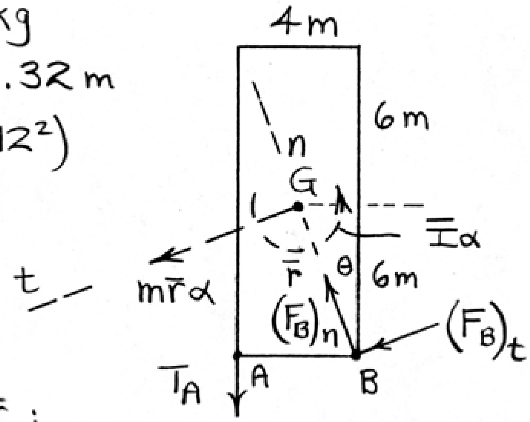
6/70 $m = 3000\ \text{kg}$

$\bar{r} = \sqrt{6^2+2^2} = 6.32\ \text{m}$

$\bar{I} = \frac{1}{12}(3000)(4^2+12^2)$

$= 40\ 000\ \text{kg}\cdot\text{m}^2$

$\theta = \tan^{-1}\frac{2}{6} = 18.43°$



$\circlearrowleft + \Sigma M_B = \bar{I}\alpha + m\bar{a}\bar{r}$:

$4T_A = 40\ 000\alpha + 3000(6.32\alpha)(6.32)$

$\underline{\alpha = 0.05\ \text{rad/s}^2}$

$\Sigma F_n = m\bar{a}_n = 0 : (F_B)_n - 2000\cos 18.43° = 0$

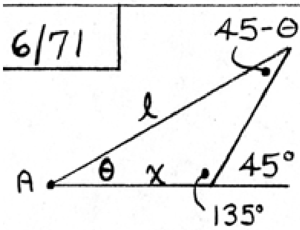
$(F_B)_n = 1897\ \text{N}$

$\Sigma F_t = m\bar{a}_t :\ 2000\sin 18.43° + (F_B)_t = 3000(6.32)(0.05)$

$(F_B)_t = 316\ \text{N}$

$F_B = \sqrt{1897^2 + 316^2} = \underline{1924\ \text{N}}$

6/71



$$\frac{\sin(45^\circ-\theta)}{x} = \frac{\sin 135^\circ}{l}$$

Differentiate WRT time t:

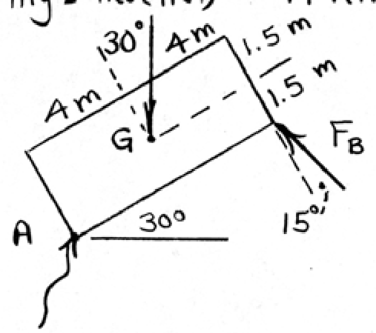
$$-\dot{\theta}\cos(45^\circ-\theta) = \dot{x}\,\frac{\sin 135^\circ}{l}$$

$$-\ddot{\theta}\cos(45^\circ-\theta) + \dot{\theta}^{2}\sin(45^\circ-\theta) = \ddot{x}\,\frac{\sin 135^\circ}{l}$$

So $\ddot{\theta} = -\dfrac{\ddot{x}}{l}\,\dfrac{\sin 135^\circ}{\cos(45^\circ-\theta)}$; For $l = 8$ m & $\theta = 35^\circ$,

and $\ddot{x} = 3\text{ m/s}^2$, $\ddot{\theta} = -0.275\text{ rad/s}^2$ (CW)

$mg = 120(9.81) = 1177$ kN



From Table D/4,

$$I_A = \frac{1}{3}m(b^2+l^2)$$

$$= \frac{1}{3}\,120(10^3)\left[3^2+8^2\right]$$

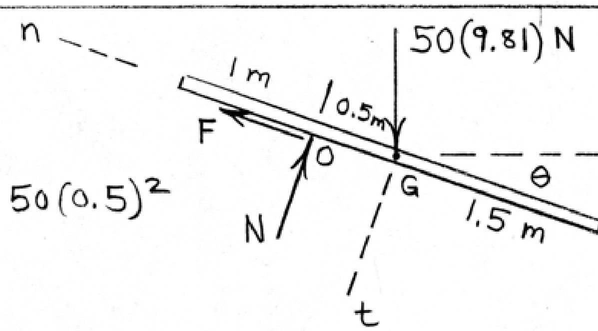
$$= 2920(10^3)\ \text{kg}\cdot\text{m}^2$$

$$\circlearrowright\!+\ \Sigma M_A = I_A\alpha:\ 1177(10^3)\cos 30^\circ(4) - 1177(10^3)\sin 30^\circ(1.5)$$

$$-8F_B\cos 15^\circ = 2920(10^3)(0.275)$$

$$F_B = 310{,}000\text{ N} \quad \text{or} \quad \underline{310\text{ kN}}$$

6/72



$$I_0 = \bar{I} + mr^2$$
$$= \frac{1}{12} 50(3)^2 + 50(0.5)^2$$
$$= 50 \text{ kg}\cdot\text{m}^2$$

$$\Sigma M_0 = I_0 \alpha : \quad 50(9.81)(0.5 \cos\theta) = 50\alpha$$

$$\alpha = 4.905 \cos\theta = \omega \frac{d\omega}{d\theta}, \quad \int_0^\omega \omega\, d\omega = \int_0^\theta 4.905 \cos\theta\, d\theta$$

$$\omega^2 = 9.81 \sin\theta$$

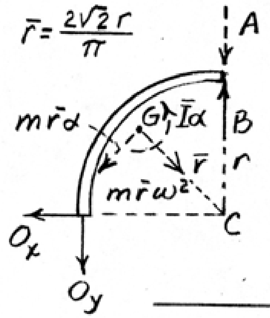
$$\begin{cases} \Sigma F_t = m\bar{a}_t : \quad 50(9.81)\cos\theta - N = 50(0.5)(4.905\cos\theta) \\ \Sigma F_n = ma_n : \quad F - 50(9.81)\sin\theta = 50(0.5)(9.81\sin\theta) \end{cases}$$

Slipping occurs when $F = 0.30N$

$$\left.\begin{array}{ll} 2^{nd} \text{ eq}: & 0.3N = 75(9.81)\sin\theta \\ 1^{st} \text{ eq}: & N = \frac{75}{2}(9.81)\cos\theta \end{array}\right\} \text{ Divide to obtain } \underline{\theta = 8.53°}$$

6/73

$\bar{r} = \frac{2\sqrt{2}\,r}{\pi}$



(a) $\alpha = 0$; $\Sigma M_C = 0$; $O_y = 0$

$\Sigma M_O = m\bar{a}d$; $Ar = m\frac{2\sqrt{2}\,r}{\pi}\omega^2\frac{r}{\sqrt{2}}$

$$\underline{A = \frac{2}{\pi}mr\omega^2}$$

By symmetry, $\Sigma M_A = m\bar{a}d$ or $\Sigma F_n = m\bar{a}_n$

$$O = -O_x = A = \underline{\frac{2}{\pi}mr\omega^2}$$

(b) $\omega = 0$; $\Sigma M_C = I_C\alpha$; $O_y r = mr^2\alpha$, $O_y = mr\alpha$

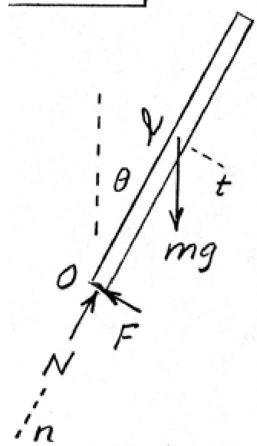
$\Sigma F_x = m\bar{a}_x$; $O_x = m\frac{2\sqrt{2}\,r}{\pi}\alpha\frac{1}{\sqrt{2}} = \frac{2}{\pi}mr\alpha$

$\Sigma F_y = m\bar{a}_y$; $O_y - B = m\frac{2\sqrt{2}\,r}{\pi}\alpha\frac{1}{\sqrt{2}}$, $B = mr\alpha - \frac{2mr\alpha}{\pi}$

$$= \underline{mr\alpha\left(1 - \frac{2}{\pi}\right)}$$

Thus $O = \sqrt{O_x^2 + O_y^2} = \underline{mr\alpha\sqrt{1 + \frac{4}{\pi^2}}}$

6/74



$\Sigma M_O = I_O\alpha$; $mg\frac{l}{2}\sin\theta = \frac{1}{3}ml^2\alpha$

$$\alpha = \frac{3g}{2l}\sin\theta$$

$\int_0^\omega \omega\,d\omega = \int_0^\theta \alpha\,d\theta$, $\omega^2 = \frac{3g}{l}(-\cos\theta)\Big|_0^\theta$

$$\omega^2 = \frac{3g}{l}(1-\cos\theta)$$

$\Sigma F_n = m\bar{a}_n$; $mg\cos\theta - N = m\frac{l}{2}\omega^2$

$N = mg\cos\theta - m\frac{l}{2}\frac{3g}{l}(1-\cos\theta)$

$= mg\left[\cos\theta - \frac{3}{2}(1-\cos\theta)\right] = \frac{mg}{2}(5\cos\theta - 3)$

$\Sigma F_t = m\bar{a}_t$; $mg\sin\theta - F = m\frac{l}{2}\frac{3g}{2l}\sin\theta$, $F = \frac{mg}{4}\sin\theta$

(a) Slips at $\theta = 30°$, $\mu_s = F/N = \dfrac{mg\sin 30°/4}{\frac{mg}{2}(5\cos 30° - 3)} = \underline{0.188}$

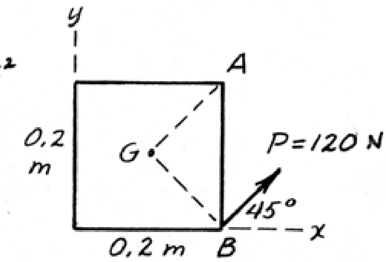
(b) No slip: $N = 0$ when $\cos\theta = 3/5$, $\underline{\theta = 53.1°}$

6/75

$\Sigma F = m\bar{a}: \; 120 = 6\bar{a}, \; \bar{a} = a_G = 20 \text{ m/s}^2$

$\Sigma M_G = \bar{I}\alpha: \; 120\frac{0.2}{\sqrt{2}} = \frac{1}{6}(6)(0.2^2)\alpha,$

$\alpha = 424 \text{ rad/s}^2$



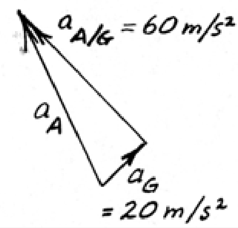
$\bar{I} = \frac{1}{6}ml^2$

$\underset{\sim}{a}_A = \underset{\sim}{a}_G + \underset{\sim}{a}_{A/G}$ where $a_{A/G} = (a_{A/G})_t$

$= \overline{AG}\alpha$

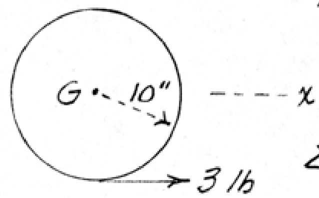
$= \frac{0.2}{\sqrt{2}} \times 424$

$= 60 \text{ m/s}^2$



$a_A = \sqrt{60^2 + 20^2} = \underline{63.2 \text{ m/s}^2}$

6/76



$\Sigma F_x = m\bar{a}_x; \quad 3 = \frac{64.4}{32.2} a, \quad a = 1.5 \text{ ft/sec}^2$

$v^2 = 2ax, \quad v^2 = 2(1.5)(3) = 9$

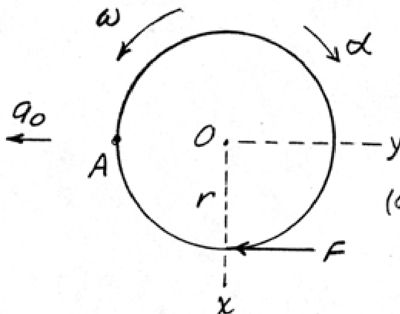
$\underline{v = 3 \text{ ft/sec}}$

$\Sigma M_G = \bar{I}\alpha; \quad 3\frac{10}{12} = \frac{1}{2}\frac{64.4}{32.2}\left(\frac{10}{12}\right)^2 \alpha$

$\alpha = 3.6 \text{ rad/sec}^2$

$\omega = \alpha t; \quad \underline{\omega = 3.6(2) = 7.2 \text{ rad/sec}}$

6/77 $\Sigma M_O = I_O\alpha$; $Fr = mk^2\alpha$, $\alpha = \dfrac{Fr}{mk^2}$



$\Sigma F_y = ma_y$; $-F = m(-a_o)$

$a_o = F/m$

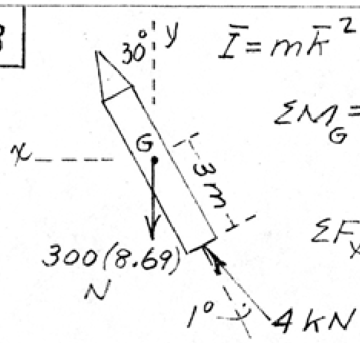
$\underline{a}_A = \underline{a}_o + \underline{a}_{A/O}$

$(a_{A/O})_n = r\omega^2 \underline{j}$, $(a_{A/O})_t = -r\alpha\, \underline{i} = -\dfrac{Fr^2}{mk^2}\underline{i}$

$a_o = -\dfrac{F}{m}\underline{j}$

so $\underline{a}_A = -\dfrac{Fr^2}{mk^2}\underline{i} - \left(\dfrac{F}{m} - r\omega^2\right)\underline{j}$

6/78



$\bar{I} = m\bar{k}^2 = 300(1.5)^2 = 675\ \text{kg}\cdot\text{m}^2$

$\Sigma M_G = \bar{I}\alpha$; $4000 \sin 1° (3) = 675\alpha$

$\underline{\alpha = 0.310\ \text{rad/s}^2}$

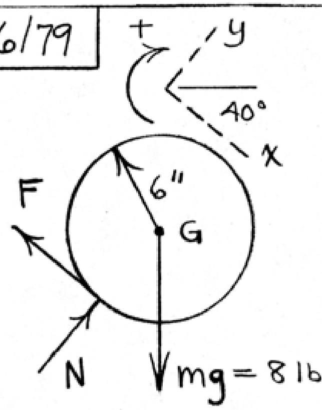
$\Sigma F_x = m\bar{a}_x$; $4000 \sin 31° = 300\bar{a}_x$

$\underline{\bar{a}_x = 6.87\ \text{m/s}^2}$

$\Sigma F_y = m\bar{a}_y$; $4000 \cos 31° - 300(8.69) = 300\bar{a}_y$

$\underline{\bar{a}_y = 2.74\ \text{m/s}^2}$

6/79



$$\begin{cases} mg = 8 \text{ lb}, & \bar{I} = \frac{1}{2}mr^2 \\ \mu_s = 0.3, & \mu_k = 0.20 \\ \theta = 40° \end{cases}$$

$\Sigma F_x = m\bar{a}_x$: $-F + 8\sin 40° = \frac{8}{32.2}a$ (1)

$\Sigma F_y = 0$: $N - 8\cos 40° = 0$ (2)

$\Sigma M_G = \bar{I}\alpha$: $F\left(\frac{6}{12}\right) = \frac{1}{2}\frac{8}{32.2}\left(\frac{6}{12}\right)^2\alpha$ (3)

Assume rolling with no slip : $a = \frac{6}{12}\alpha$ (4)

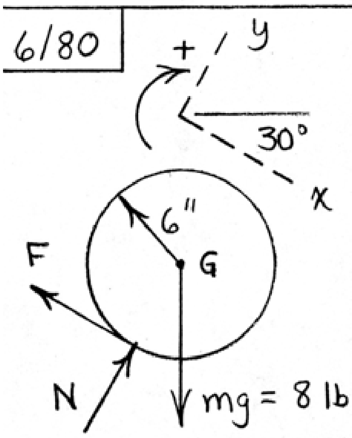
Solution of (1) - (4) : $\underline{F = 1.714 \text{ lb}}$ $\underline{a = 13.80 \frac{\text{ft}}{\text{sec}^2}}$

$N = 6.13$ lb $\alpha = 27.6 \frac{\text{rad}}{\text{sec}^2}$

$F_{max} = \mu_s N = 0.3\ (6.13) = 1.839 \text{ lb} > F$

Assumption valid.

6/80



$$\begin{cases} mg = 8 \text{ lb}, \quad \bar{I} = \frac{1}{2} m r^2 \\ \mu_s = 0.15, \quad \mu_k = 0.10 \\ \theta = 30° \end{cases}$$

$$\Sigma F_x = m\bar{a}_x : \quad -F + 8 \sin 30° = \frac{8}{32.2} a \qquad (1)$$

$$\Sigma F_y = 0 : \quad N - 8 \cos 30° = 0 \qquad (2)$$

$$\Sigma M_G = \bar{I}\alpha : \quad F\left(\frac{6}{12}\right) = \frac{1}{2} \frac{8}{32.2} \left(\frac{6}{12}\right)^2 \alpha \qquad (3)$$

Assume rolling with no slip: $a = \frac{6}{12}\alpha$ (4)

Solution of (1) - (4): $F = 1.333$ lb $\quad a = 10.73 \frac{\text{ft}}{\text{sec}^2}$

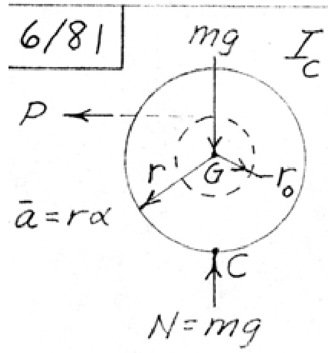
$N = 6.93$ lb $\quad \alpha = 21.5 \frac{\text{rad}}{\text{sec}^2}$

$F_{max} = \mu_s N = 0.15(6.93) = 1.039 \text{ lb} < F \Rightarrow$ Slips

$F = \mu_k N = 0.10(6.93) = \underline{0.693 \text{ lb}}$

From Eqs. (1) & (3): $\underline{a = 13.31 \text{ ft/sec}^2}$, $\alpha = 11.15 \frac{\text{rad}}{\text{sec}^2}$

6/81



$I_C = \bar{I} + mr^2 = m(\bar{k}^2 + r^2)$

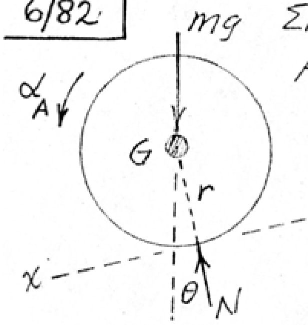
$\Sigma M_C = I_C\alpha;\ P(r + r_o) = m(\bar{k}^2 + r^2)\alpha$

$\Sigma F = m\bar{a};\ P = m\bar{a} = mr\alpha$

Thus $mr\alpha(r + r_o) = m(\bar{k}^2 + r^2)\alpha$

$r^2 + rr_o = \bar{k}^2 + r^2,\ \underline{r_o = \dfrac{\bar{k}^2}{r}}$

6/82

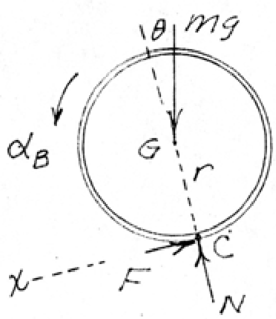


$\Sigma M_G = \bar{I}\alpha$, but $\bar{I} = 0$ so $\Sigma \bar{M} = 0$

Hence no friction force & $\mu_s = 0$

$\Sigma F_x = ma_x$; $mg \sin\theta = mr\alpha_A$

$$\underline{\alpha_A = \frac{g}{r}\sin\theta}$$



$\Sigma M_C = I_C\alpha$; $mgr\sin\theta = 2mr^2\alpha_B$

$$\underline{\alpha_B = \frac{g}{2r}\sin\theta}$$

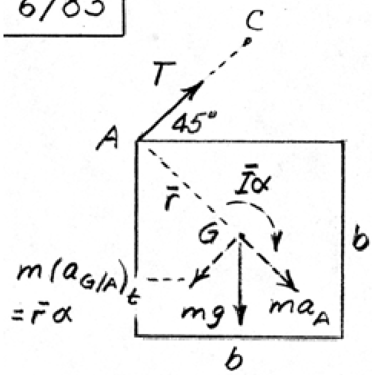
$\Sigma \bar{M} = \bar{I}\alpha$; $Fr = mr^2 \frac{g}{2r}\sin\theta$

$$F = \frac{1}{2} mg \sin\theta$$

$$\mu_s = \frac{F}{N} = \frac{1}{2} mg \sin\theta / mg\cos\theta$$

$$\underline{\mu_s = \frac{1}{2}\tan\theta}$$

6/83



$$\Sigma M_A = \bar{I}\alpha + m\bar{a}d$$

$$\frac{mgb}{2} = \frac{1}{6}mb^2\alpha + m\frac{b}{\sqrt{2}}\alpha\frac{b}{\sqrt{2}}$$

$$\alpha = \frac{3g}{4b}$$

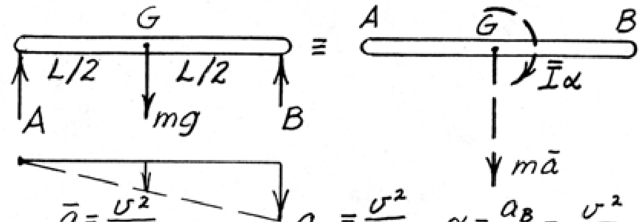
$$\Sigma M_G = \bar{I}\alpha$$

$$T\frac{b}{\sqrt{2}} = \frac{1}{6}mb^2\left(\frac{3g}{4b}\right)$$

$$T = \frac{\sqrt{2}}{8}mg = \frac{\sqrt{2}}{8}(12)(9.81)$$

$$= \underline{20.8\ N}$$

6/84



$a_A = 0$

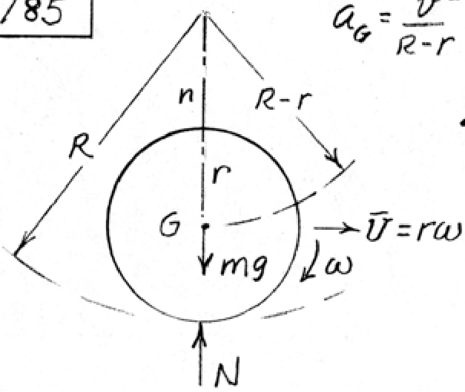
$\omega_{AB} = 0$ $\quad \bar{a} = \dfrac{v^2}{2r}$ $\quad a_B = \dfrac{v^2}{r}$ $\quad \alpha = \dfrac{a_B}{L} = \dfrac{v^2}{Lr}$

$$\Sigma M_A = m\bar{a}\frac{L}{2} + \bar{I}\alpha : \; mg\frac{L}{2} - BL = m\frac{v^2}{2r}\frac{L}{2} + \frac{1}{12}mL^2\frac{v^2}{Lr}$$

$$\underline{B = m\left(\frac{g}{2} - \frac{v^2}{3r}\right)}$$

$B = 0$ if $\dfrac{g}{2} - \dfrac{v^2}{3r} = 0$, $\quad \underline{v = \sqrt{3gr/2}}$

6/85



$$a_G = \frac{\bar{v}^2}{R-r} = \frac{r^2\omega^2}{R-r} = \bar{a}_n$$

$$\Sigma F_n = m\bar{a}_n$$

$$N - mg = m\frac{r^2\omega^2}{R-r}$$

$$N = m\left(g + \frac{r^2\omega^2}{R-r}\right)$$

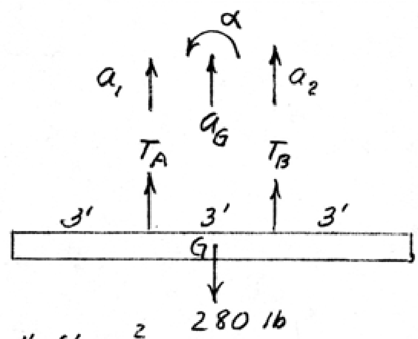
6/86

$a_1 = r\alpha_1 = \frac{15/2}{12} 4 = 2.5 \text{ ft/sec}^2$

$a_2 = r\alpha_2 = \frac{15/2}{12} 6 = 3.75 \text{ ft/sec}^2$

$a_G = (2.5 + 3.75)/2 = 3.13 \text{ ft/sec}^2$

$\alpha = \frac{a_{B/A}}{\overline{AB}} = \frac{3.75 - 2.5}{3} = 0.417 \frac{\text{rad}}{\text{sec}^2}$



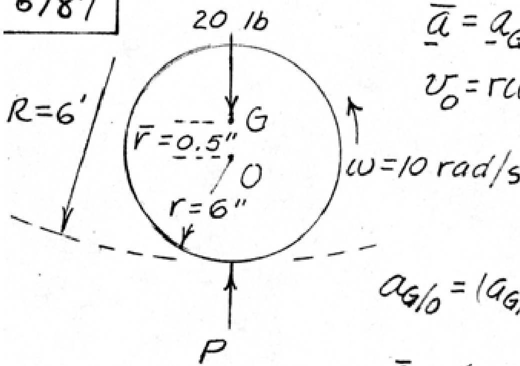
$I_G = \frac{1}{12} mL^2 = \frac{1}{12} \frac{280}{32.2} 9^2 = 58.7 \text{ lb-ft-sec}^2$

$\Sigma F = ma_G$: $T_A + T_B - 280 = \frac{280}{32.2}(3.13)$, $T_A + T_B = 307$

$\Sigma M_G = I_G\alpha$: $(T_B - T_A)\frac{3}{2} = 58.7(0.417)$, $T_B - T_A = 16.30$

Solve & get $\underline{T_A = 145.4 \text{ lb}}$, $\underline{T_B = 161.7 \text{ lb}}$

6/87



$\underline{\bar{a}} = \underline{a}_G = \underline{a}_O + \underline{a}_{G/O}$

$v_O = r\omega = (6/12)(10) = 5$ ft/sec

$a_O = v_O^2/(R-r)$

$= \dfrac{5^2}{6-6/12} = 4.55\ \dfrac{ft}{sec^2} \uparrow$

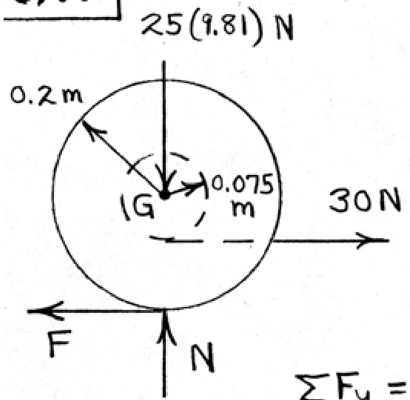
$a_{G/O} = (a_{G/O})_n = \bar{r}\omega^2 = (0.5/12)\overline{10}^2$

$= 4.17$ ft/sec$^2 \downarrow$

$\bar{a} = 4.55 - 4.17 = 0.38$ ft/sec$^2 \uparrow$

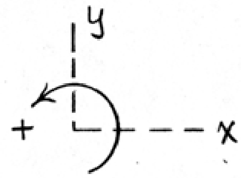
$\Sigma F = m\bar{a}$; $P - 20 = \dfrac{20}{32.2}\,0.38$, $\underline{P = 20.2 \text{ lb}}$

6/88



$\bar{k} = 0.175$ m

$\mu_s = 0.1$, $\mu_k = 0.08$



$\Sigma F_y = 0 \Rightarrow N = 25(9.81) = 245$ N

$\Sigma F_x = m\bar{a}_x : \quad 30 - F = 25a \qquad (1)$

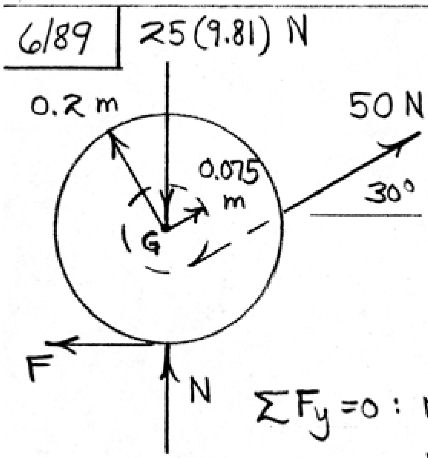
$\Sigma M_G = \bar{I}\alpha : \quad 30(0.075) - F(0.2) = 25(0.175)^2\alpha \quad (2)$

Assume rolling with no slip : $a = -r\alpha \qquad (3)$

Solution of Eqs. (1)-(3) : $\begin{cases} a = 0.425 \text{ m/s}^2 \\ \alpha = -2.12 \text{ rad/s}^2 \\ F = 19.38 \text{ N} \end{cases}$

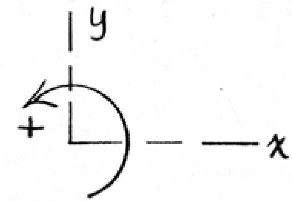
$F_{max} = \mu_s N = 0.1(245) = 24.5 \text{ N} > F$ (assumption OK)

6/89



$\bar{k} = 0.175$ m

$\mu_s = 0.1, \ \mu_k = 0.08$



$\Sigma F_y = 0: \ N - 25(9.81) + 50 \sin 30° = 0$

$N = 220 \ N$

$$\Sigma F_x = m\bar{a}_x: \quad 50 \cos 30° - F = 25a \qquad (1)$$

$$\Sigma M_G = \bar{I}\alpha: \quad 50(0.075) - F(0.2) = 25(0.175)^2\alpha \qquad (2)$$

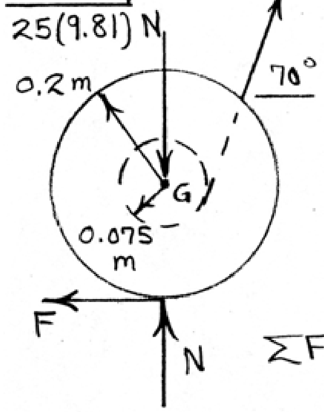
Assume rolling with no slip : $a = -r\alpha$ (3)

Solution of (1)-(3) : $\begin{cases} a = 0.556 \text{ m/s}^2, \ \alpha = -2.78 \text{ rad/s}^2, \\ F = 29.4 \text{ N} \end{cases}$

$F_{max} = \mu_s N = 0.1(220) = 22.0 \text{ N} < F$: slips, $\underline{F = \mu_k N = 17.62 \text{ N}}$

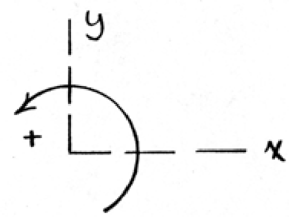
From Eqs. (1) & (2): $\underline{a = 1.027 \text{ m/s}^2, \ \alpha = 0.295 \text{ rad/s}^2}$

6/90



$\bar{k} = 0.0175$ m

$\mu_s = 0.1$, $\mu_k = 0.08$



$\Sigma F_y = 0: \quad N - 25(9.81) + 30 \sin 70° = 0$

$N = 217$ N

$$\Sigma F_x = m\bar{a}_x : \quad 30 \cos 70° - F = 25a \qquad (1)$$

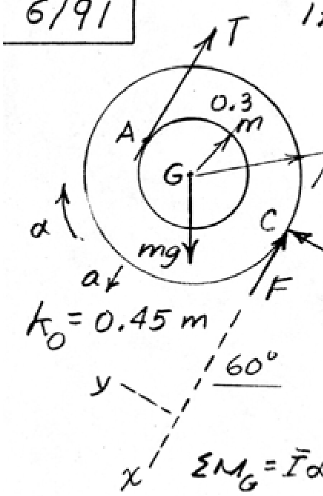
$$\Sigma M_G = \bar{I}\alpha : \quad 30(0.075) - F(0.2) = 25(0.175)^2 \alpha \qquad (2)$$

Assume rolling with no slip : $a = -r\alpha$ (3)

Solution of Eqs. (1)-(3) : $\begin{cases} a = -0.0224 \text{ m/s}^2 \\ \alpha = 0.1121 \text{ rad/s}^2 \\ F = 10.82 \text{ N} \end{cases}$

$F_{max} = \mu_s N = 0.1(217) = 21.7 > F$ (assumption OK)

6/91



If wheel were in equilibrium,

$\Sigma M_A = 0$; $0.9F - 0.3mg \sin 60°$

$\Sigma F_y = 0$; $N - mg\cos 60° = 0$

$\min. \mu_s = \frac{F}{N} = \frac{(mg/3)\sin 60°}{mg\cos 60°} = \frac{\tan 60°}{3}$

$= 0.577 > (\mu_s = 0.40)$

so wheel slips & $\mu = \mu_k = 0.30$

$\Sigma F_y = 0$; $N = mg\cos 60° = 30(9.81)(0.5)$
$= 147.2$ N

$F = \mu_k N = 0.3(147.2) = 44.1$ N

$\Sigma M_G = \bar{I}\alpha$ where $\alpha = a/0.3$

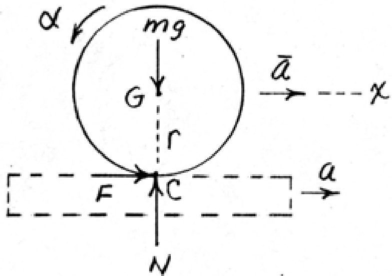
$0.3T - 44.1(0.6) = 30(0.450)^2 \frac{a}{0.3}$ ----(1)

$\Sigma F_x = m a_x$

$30(9.81)\sin 60° - T - 44.1 = 30a$ ----(2)

Solve (1) & (2) & get $\underline{a = 1.256 \text{ m/s}^2}$

6/92



$\alpha = \frac{a_{C/G}}{r} = \frac{a - \bar{a}}{r}$

$\Sigma F_x = m a_x$; $F = m\bar{a}$

$\Sigma M_G = \bar{I}\alpha$; $Fr = \frac{1}{2} m r^2 \frac{a - \bar{a}}{r}$

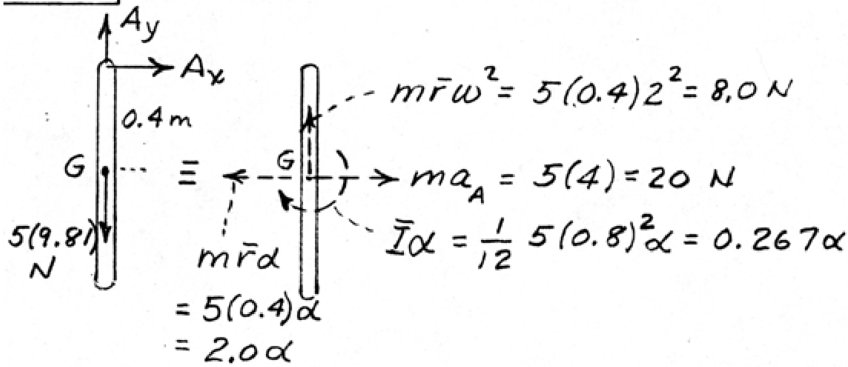
solve & get $\bar{a} = \frac{1}{3} a$

$a_{G/C} = a - \bar{a} = \frac{2}{3} a$ to the left

Rel. to truck, $s = \frac{1}{2} a_{rel} t^2$, $d = \frac{1}{2}\left(\frac{2}{3}a\right)t^2$, $t^2 = \frac{3d}{a}$

Truck $s = \frac{1}{2} a t^2$, $s = \frac{1}{2} a \frac{3d}{a} = \underline{\frac{3d}{2}}$

6/93



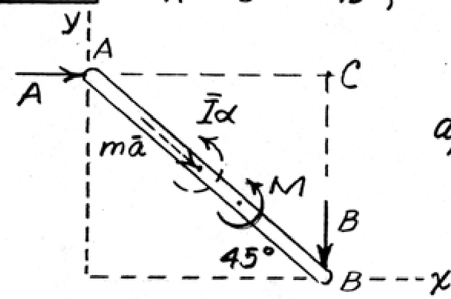
$\Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d$; $\quad 0 = 0.267\alpha + 2.0\alpha(0.4) - 20(0.4)$

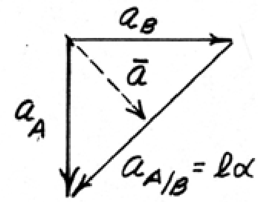
$\alpha = 7.50\text{ rad/s}^2$

$\Sigma F_x = m\bar{a}_x$; $\quad A_x = 20 - 2.0(7.50) = \underline{5\text{ N}}$

$\Sigma F_y = m\bar{a}_y$; $\quad A_y - 5(9.81) = 8, \quad A_y = \underline{57.1\text{ N}}$

6/94 $\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$, $a_{A/B} = (a_{A/B})_t = l\alpha$





$\bar{a} = l\alpha/2$

$\Sigma M_C = \bar{I}\alpha + m\bar{a}d$; $M = \frac{1}{12}ml^2\alpha + m\frac{l\alpha}{2}\frac{l}{2} = \frac{1}{3}ml^2\alpha$

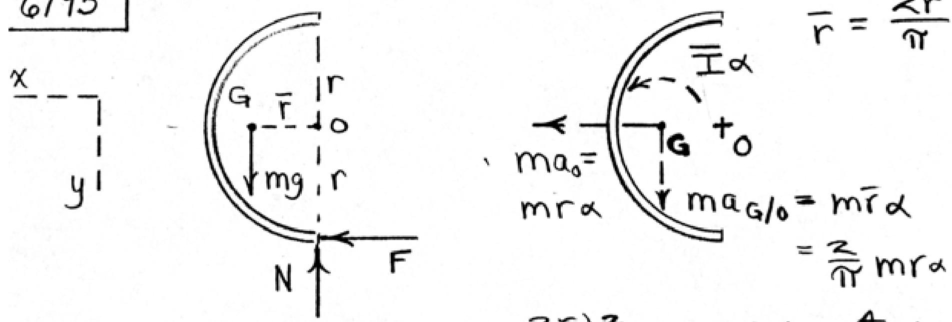
$\alpha = \frac{3M}{ml^2}$

$\Sigma F_x = m\bar{a}_x$; $A = m\frac{l\alpha}{2}\frac{1}{\sqrt{2}} = \frac{ml}{2\sqrt{2}}\frac{3M}{ml^2}$, $\underline{A} = \frac{3M}{2\sqrt{2}l}\underline{i}$

$\Sigma F_y = m\bar{a}_y$; $-B = m\left(-\frac{l\alpha}{2}\frac{1}{\sqrt{2}}\right)$ $\underline{B} = -\frac{3M}{2\sqrt{2}l}\underline{j}$

6/95

$\bar{r} = \frac{2r}{\pi}$



$\bar{I} = I_o - m\bar{r}^2 = mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1 - \frac{4}{\pi^2}\right)$

$\Sigma F_x = m\bar{a}_x$: $F = mr\alpha$

$\Sigma F_y = m\bar{a}_y$: $mg - N = \frac{2}{\pi} mr\alpha$, $N = m\left(g - \frac{2r\alpha}{\pi}\right)$

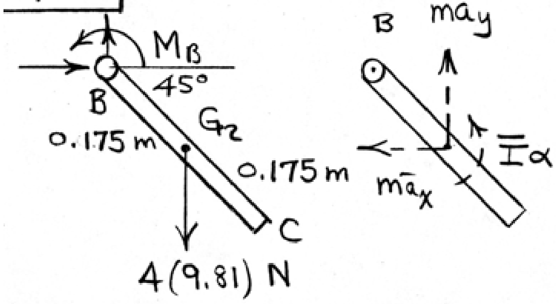
$\Sigma M_G = \bar{I}\alpha$: $N\left(\frac{2r}{\pi}\right) - Fr = mr^2\left(1 - \frac{4}{\pi^2}\right)\alpha$

Solve simultaneously : $\alpha = \frac{g}{\pi r}$

$\therefore$ $F = mr\alpha = mr\frac{g}{\pi r} = m\frac{g}{\pi}$ & $N = mg\left(1 - \frac{2}{\pi^2}\right)$

Thus $\mu_s = \frac{F}{N} = \frac{mg/\pi}{mg\left(1 - \frac{2}{\pi^2}\right)} = \frac{\pi}{\pi^2 - 2} = \underline{0.399}$

6/96



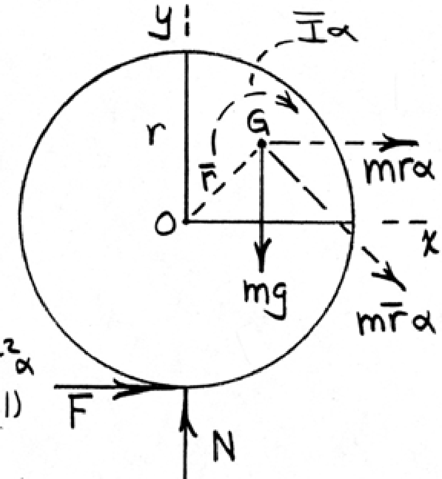
$\underline{\omega}_{BC} = \underline{\omega}_{AB} = 2\underline{k}$ rad/s

$\underline{\alpha}_{BC} = \underline{\alpha}_{AB} = 4\underline{k}$ rad/s^2

$\underline{a}_{G_2} = \underline{\alpha} \times \underline{r}_{AG_2} - \omega^2 \underline{r}_{AG_2} = 4\underline{k} \times [(0.7 + 0.175)\cos 45°\underline{i}$
$+ (0.7 - 0.175)\sin 45°\underline{j}] - 2^2[(0.7 + 0.175)\cos 45°\underline{i}$
$+ (0.7 - 0.175)\sin 45°\underline{j}] = -3.96\underline{i} + 0.990\underline{j}$ m/s^2

$\Sigma M_B = \bar{I}\alpha + \Sigma m\bar{a}d$: $M_B - 4(9.81)(0.175 \sin 45°) =$
$\frac{1}{12}(4)(0.35)^2(4) + 4(0.990)(0.175 \cos 45°)$
$- 4(3.96)(0.175 \sin 45°)$, $\underline{M_B = 3.55 \text{ N}\cdot\text{m}}$ (CCW)

6/97 $\bar{r} = \frac{4r}{3\pi}\sqrt{2}$



$$\bar{I} = I_O - m\bar{r}^2$$
$$= \frac{1}{2}mr^2 - m\left[\frac{4r}{3\pi}\sqrt{2}\right]^2$$
$$= 0.1397\,mr^2$$

$\circlearrowright +$ $\Sigma M_G = \bar{I}\alpha$:

$$N\left(\frac{4r}{3\pi}\right) - F\left(r + \frac{4r}{3\pi}\right) = 0.1397\,mr^2\alpha \qquad (1)$$

$\Sigma F_x = m\bar{a}_x$:

$$F = mr\alpha + m\left(\frac{4r}{3\pi}\sqrt{2}\right)\frac{1}{\sqrt{2}}\alpha \qquad (2)$$

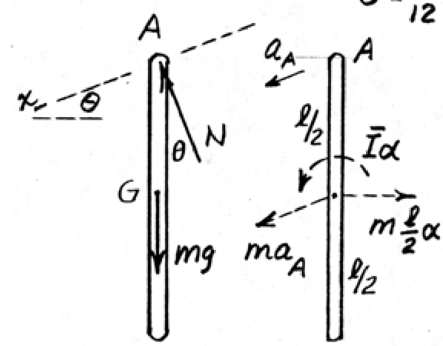
$\Sigma F_y = m\bar{a}_y$:

$$N - mg = -m\left(\frac{4r}{3\pi}\sqrt{2}\right)\frac{1}{\sqrt{2}}\alpha \qquad (3)$$

Solve Eqs. (1)-(3) to obtain

$$\begin{cases} \underline{F = 0.257\,mg} \\ N = 0.923\,mg \\ \alpha = 0.1807\,\frac{g}{r} \end{cases}$$

6/98



$$\Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d$$

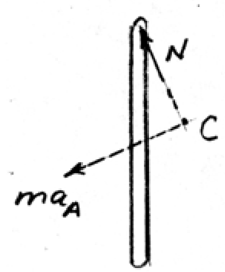
$$0 = \frac{1}{12}m\ell^2\alpha + m\frac{\ell}{2}\alpha\frac{\ell}{2} - ma_A\frac{\ell}{2}\cos\theta$$

$$\Sigma F_x = m\bar{a}_x$$

$$mg\sin\theta = m\left(a_A - \frac{\ell}{2}\alpha\cos\theta\right)$$

Eliminate α & get

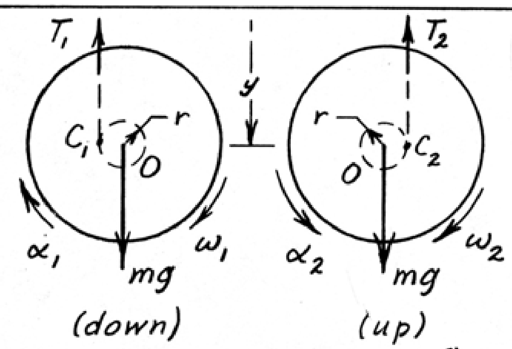
$$a_A = \frac{g\sin\theta}{1-\frac{3}{4}\cos^2\theta}$$



Alternative solution:

Pt. C may be used as a moment center thus eliminating reference to a_A & N giving one equation in α.

6/99



Down: $\Sigma M_{C_1} = I_{C_1}\alpha_1:\ mgr = m(k^2+r^2)\alpha_1,\ (a_O)_1 = r\alpha_1 = \dfrac{g}{k^2/r^2+1}$ const.

$$\Sigma F_y = m\bar{a}_y:\ mg - T_1 = \frac{mg}{k^2/r^2+1},\ T_1 = \frac{mg}{1+r^2/k^2}$$

$$v^2 = v_0^2 + 2as:\ \underline{v = \sqrt{v_0^2 + \frac{2gL}{k^2/r^2+1}}}$$

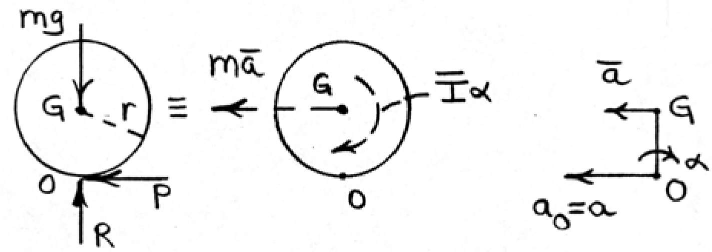
Up: $\Sigma M_{C_2} = I_{C_2}\alpha_2:\ mgr = m(k^2+r^2)\alpha_2,\ (a_O)_2 = r\alpha_2 = \dfrac{g}{k^2/r^2+1}$ const.

$$\Sigma F_y = m\bar{a}_y:\ mg - T_2 = \frac{mg}{k^2/r^2+1},\ T_2 = \frac{mg}{1+r^2/k^2}$$

Thus $\underline{T = \dfrac{mg}{1+r^2/k^2}\ \&\ a = \dfrac{g}{k^2/r^2+1}\ \text{for both motions}}$

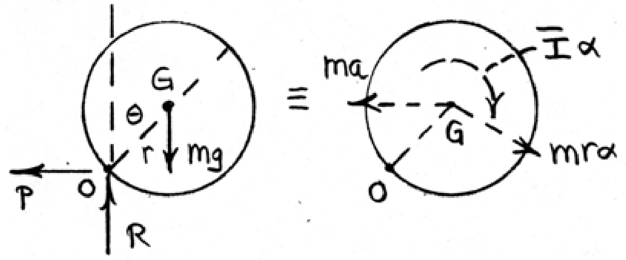
6/100

$\bar{I} = \frac{2}{5} m r^2$



$$\Sigma M_O = \bar{I}\alpha - m\bar{a}r : \quad 0 = \frac{2}{5} m r^2 \alpha - m\bar{a}r, \quad \bar{a} = \frac{2}{5} r\alpha$$

$$\underline{a}_G = \underline{a}_O + \underline{a}_{G/O} : \quad \bar{a} = a - r\alpha = \frac{2}{5} r\alpha \Rightarrow \underline{\alpha = \frac{5}{7}\frac{a}{r}}$$



$a_0 = a$

$$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d : \quad mgr\sin\theta = \frac{2}{5} m r^2 \alpha + m r^2 \alpha - mar\cos\theta$$

$$\alpha = \frac{5}{7r}\left(g\sin\theta + a\cos\theta\right)$$

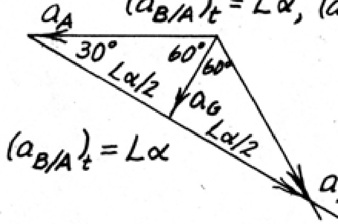
$$\omega\, d\omega = \alpha\, d\theta : \quad \int_0^\omega \omega\, d\omega = \frac{5}{7r}\int_0^\theta \left(g\sin\theta + a\cos\theta\right) d\theta$$

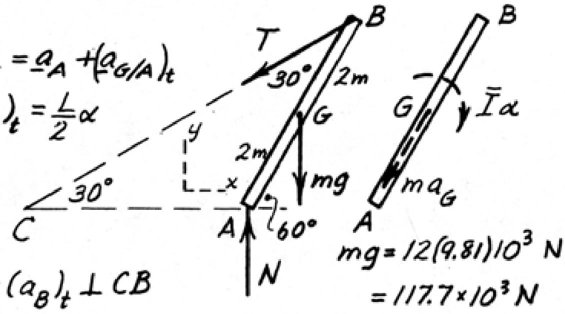
$$\underline{\omega = \sqrt{\frac{10}{7r}}\sqrt{g(1-\cos\theta) + a\sin\theta}}$$

6/101

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_t, \quad \underline{a}_G = \underline{a}_A + (\underline{a}_{G/A})_t$

$(a_{B/A})_t = L\alpha, \quad (a_{G/A})_t = \frac{L}{2}\alpha$





From accel. diag. $a_B = \frac{L}{2}\alpha \sec 30° = \frac{4}{2}\sec 30°\alpha = 2.30\,\alpha$ m/s²

Since $\underline{a}_G$ passes through A,

$\Sigma M_A = \bar{I}\alpha: \; 117.7(10^3)2\cos 60° - T \times 4 \sin 30° = \frac{1}{12}12(10^3)4^2\alpha$

$117.7(10^3) - 2T = 16(10^3)\alpha \qquad --(a)$

$\Sigma F_x = m\bar{a}_x: \; T\cos 30° = 12(10^3)(a_G)_x$ where $a_G = \frac{L}{2}\alpha\tan 30°$,

$(a_G)_x = \frac{L}{2}\tan 30°\alpha\cos 60° = 0.577\alpha \frac{m}{s^2}$

so $T = 8(10^3)\alpha \qquad ---(b)$

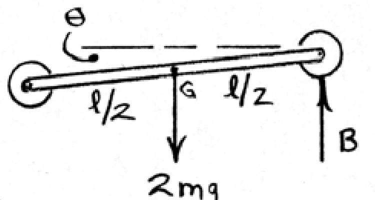
Solve (a) & (b) & get $\alpha = 3.68$ rad/s², $\underline{T = 29.4 \text{ kN}}$

$a_A = \frac{L}{2}\alpha/\cos 30° = \frac{4}{2}(3.68)/\cos 30°$, $\underline{a_A = 8.50 \text{ m/s}^2}$

6/102 Assume that the angle θ present as B clears the surface is very small and that the speed of B is constant (while on surface).

Time t between A & B leaving surface : $t = \frac{l}{v}$.

$$\bar{I} = 2m\left(\frac{l}{2}\right)^2 = ml^2/2$$



$$I_B = \frac{ml^2}{2} + 2m\left(\frac{l}{2}\right)^2 = ml^2$$

$$\Sigma M_B = I_B\ddot{\theta} : 2mg\frac{l}{2} = ml^2\ddot{\theta}, \quad \ddot{\theta} = \frac{g}{l} \text{ (CCW)}$$

$$\omega = \omega_0^{\nearrow 0} + \ddot{\theta}t = \frac{g}{l}\frac{l}{v} = \underline{\frac{g}{v}}$$

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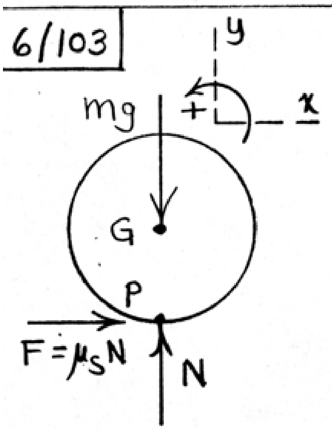


The no-slip constraint is found by equating the horizontal acceleration of point P to the acceleration a_C of the cart:

$$(a_P)_{hor} = a_G + r\alpha = a_C \qquad (1)$$

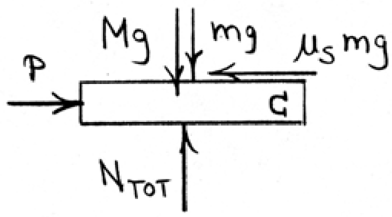
$$\Sigma F_y = 0 \Rightarrow N = mg \qquad (2)$$

$\Sigma F_x = m a_G$: $\quad \mu_s \cancel{m} g = \cancel{m} a_G \qquad (3)$

$\Sigma M_G = \bar{I}\alpha$: $\quad +\mu_s \cancel{m} g r = \cancel{m} \bar{k}^2 \alpha \qquad (4)$

Solution of (1), (3), & (4) :
$$\begin{cases} a_G = \mu_s g \\ a_C = \mu_s g \left[1 + \dfrac{r^2}{\bar{k}^2}\right] \\ \alpha = +\mu_s g r/\bar{k}^2 \end{cases}$$

Cart :



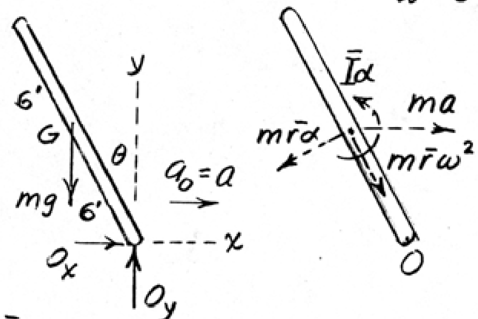
$\Sigma F_x = M a_C$:

$$P - \mu_s mg = M \mu_s g \left[1 + \frac{r^2}{\bar{k}^2}\right]$$

$$\underline{P = \mu_s g \left[m + M\left(1 + \frac{r^2}{\bar{k}^2}\right)\right]}$$

6/104 $m\underline{\bar{a}} = m\underline{a}_O + m\underline{a}_{G/O} = m\underline{a} + m\bar{r}\omega^2 + m\bar{r}\alpha$

$a = 3\ ft/sec^2,\ \bar{r} = 6\ ft$



$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d;$

$$mg(6\sin\theta) = \frac{1}{12}m(12^2)\alpha + m(6\alpha)(6) - m(3)6\cos\theta$$

$$\alpha = \frac{1}{8}(g\sin\theta + 3\cos\theta)$$

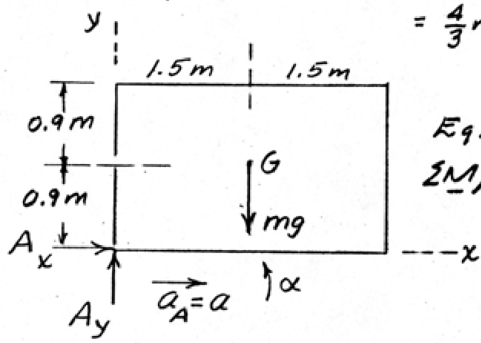
$\int\omega\, d\omega = \int\alpha\, d\theta;\quad \int_0^{\omega}\omega\, d\omega = \frac{1}{8}\int_0^{\pi/2}(g\sin\theta + 3\cos\theta)d\theta$

$$\omega^2 = \frac{1}{4}\left[-32.2\cos\theta + 3\sin\theta\right]_0^{\pi/2} = \frac{1}{4}[32.2 + 3] = \frac{35.2}{4}$$

$$\omega = \frac{1}{2}\sqrt{35.2} = \underline{2.97\ rad/sec}$$

6/105 $I_A = \frac{1}{12}m([1.8]^2 + [3.0]^2) + m([0.9]^2 + [1.5]^2)$

$= \frac{4}{3}m([0.9]^2 + [1.5]^2) = 4.08\ m$ kg·m²



Eq. 6/3

$\Sigma \underline{M}_A = I_A \underline{\alpha} + \bar{\underline{\rho}} \times m\underline{a}_A$

$-m(9.81)(1.5)\underline{k} = 4.08 m\alpha \underline{k} + (1.5\underline{i} + 0.9\underline{j}) \times ma\underline{i}$

$-14.72 = 4.08\alpha - 0.9a$,

when $\alpha = 0$, $a = a_{min} = 14.72/0.9 = \underline{16.35\ m/s^2}$

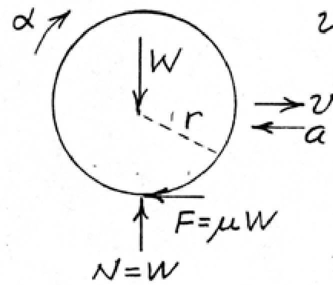
(Max. feasible accel. for $\mu_s \cong 1$ is approx. $g = 9.81\ m/s^2$)

so $a_{min.}$ is not feasible)

If accel is $1.2a = 1.2(16.35)\ m/s^2$,

$\alpha = \frac{1}{4.08}(0.9[1.2]16.35 - 14.72) = \underline{0.721\ rad/s^2}$

6/106 $\Sigma F = ma$; $\mu W = \frac{W}{g} a$, $a = \mu g$



$v = v_0 - \mu g t$, $t = \frac{v_0 - v}{\mu g}$ = time to slow down to vel. v

$\Sigma \bar{M} = \bar{I}\alpha$; $\mu W r = \frac{W}{g} k^2 \alpha$

$\alpha = \frac{\mu g r}{k^2}$

$v = r\omega = r(0 + \alpha t)$

so $t = \frac{v}{r\alpha} = \frac{v k^2}{\mu g r^2}$ = time to acquire $v = r\omega$.

Equate t's & set $\frac{v_0 - v}{\mu g} = \frac{v k^2}{\mu g r^2}$, $v = \frac{v_0 r^2}{k^2 + r^2}$

& $v^2 = v_0^2 - 2as$,

so $s = \frac{-v^2 + v_0^2}{2a} = \frac{v_0^2}{2\mu g} \frac{k^2(k^2 + 2r^2)}{(k^2 + r^2)^2}$; Substitute $v_0 = 20$ ft/sec

$k = 3.28/12$ ft

$r = \frac{1}{12}\left(\frac{27}{2\pi}\right)$ ft

$\mu = 0.20$

& get $\underline{s = 18.66 \text{ ft}}$

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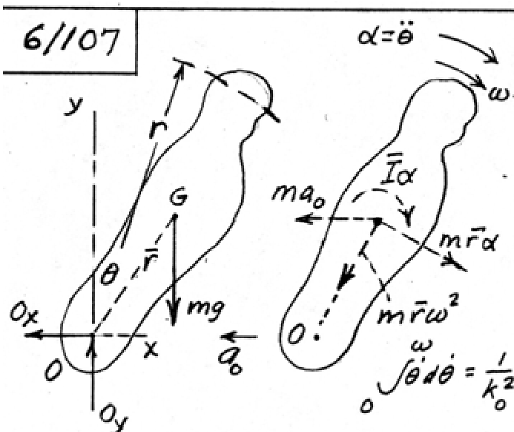

$$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$$

$$mg\bar{r}\sin\theta = m(k_O^2 - \bar{r}^2)\alpha + m\bar{r}\alpha(\bar{r}) - ma_O\bar{r}\cos\theta$$

$$\alpha = \frac{1}{k_O^2}(g\bar{r}\sin\theta + a_O\bar{r}\cos\theta)$$

$$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta$$

$$\int_0^\omega \dot{\theta}\,d\dot{\theta} = \frac{1}{k_O^2}\int_0^\theta (g\bar{r}\sin\theta + a_O\bar{r}\cos\theta)\,d\theta$$

$$\frac{\omega^2}{2} = \frac{1}{k_O^2}\left[g\bar{r}(1-\cos\theta) + a_O\bar{r}\sin\theta\right] \quad \text{where } a_O = a$$

$$v = r\omega = \frac{r\sqrt{2}}{k_O}\sqrt{g\bar{r}(1-\cos\theta) + a\bar{r}\sin\theta}$$

For $\bar{r} = 0.45$ m, $r = 0.8$ m, $k_O = 0.55$ m, $\theta = 45°$, $a = 10g$,

$$v = \frac{0.80\sqrt{2}}{0.55}\sqrt{9.81(0.45)(1 - 1/\sqrt{2}) + 10(9.81)(0.45)\,1/\sqrt{2}} = \underline{11.73 \text{ m/s}}$$

(Alternatively apply Eq. 6/3 with moment center at O)

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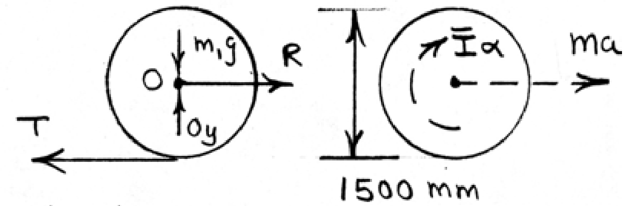


$a = r\alpha: \quad \alpha = \dfrac{0.2(9.81)}{1.500/2} = 2.62 \text{ rad/s}^2$

Spool & cable: $\bar{I} = 140(0.530)^2 + 150\pi(1.5)(0.75)\left(\frac{1.5}{2}\right)^2$

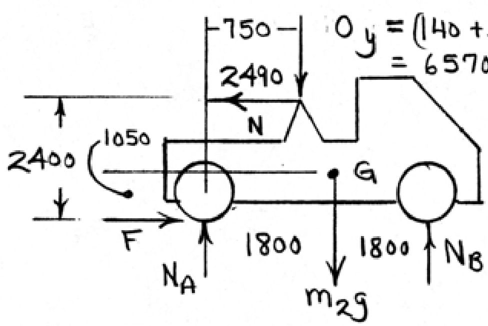
$= 338 \text{ kg}\cdot\text{m}^2$

$\Sigma M_O = I_O\alpha: \quad T\left(\frac{1.500}{2}\right) = 338(2.62), \quad \underline{T = 1177 \text{ N}}$

$\Sigma F = m\bar{a}: \quad R - 1177 = (140 + 150\pi(1.5)(0.75))(0.2)(9.81)$

$R = 2490 \text{ N}$

$O_y = (140 + 530)9.81 = 6570 \text{ N}$



$m_2 g = 2030(9.81) = 19\,910 \text{ N}$

$a = 0.2(9.81) = 1.962 \ \frac{\text{m}}{\text{s}^2}$

(Dim. in mm)

$\Sigma M_A = \Sigma m\bar{a}d: \ 6570(0.750) + 19\,910(1.8) - 2490(2.4) - 3.6 N_B$
$= 2030(1.962)(1.05)$

$\Sigma F_y = 0: \quad N_A + N_B - 6570 - 19\,910 = 0 \Rightarrow \underline{N_A = 17\,980 \text{ N}, \ N_B = 8500 \text{ N}}$

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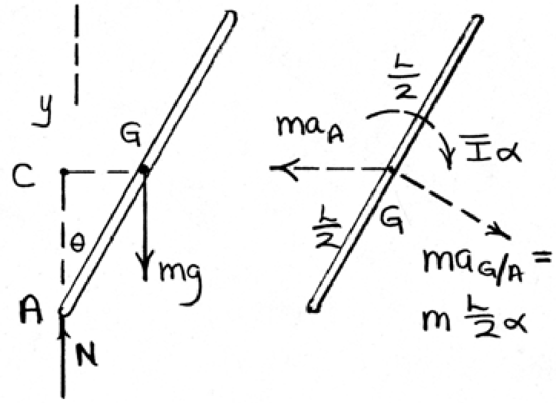


$\overset{+}{\circlearrowright}$ $\Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d$:

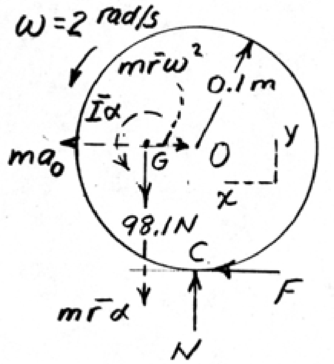
$$mg\frac{L}{2}\sin\theta = \frac{1}{12}mL^2\alpha + \frac{L}{2}\sin^2\theta\left(m\frac{L}{2}\alpha\right)$$

$$\underline{\alpha = \frac{2g}{L}\frac{\sin\theta}{\frac{1}{3}+\sin^2\theta}}$$

$$\Sigma F_y = m\bar{a}_y: \quad mg - N = m\frac{L}{2}\left(\frac{2g}{L}\frac{\sin\theta}{\frac{1}{3}+\sin^2\theta}\right)\sin\theta$$

$$\underline{N = \frac{mg}{1+3\sin^2\theta}}$$

6/110 $\bar{r} = \overline{OG} = 0.040$ m ; $m\bar{r}\omega^2 = 10(0.040)(2^2) = 1.6$ N



$\bar{I} = m\bar{k}^2 = 10(0.064)^2 = 0.0410$ kg-m^2

$m\bar{r}\alpha = 10(0.040)\dfrac{a_0}{0.1} = 4a_0$ N

$\Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d$

$98.1(0.040) = 0.0410\dfrac{a_0}{0.1} + 4a_0(0.040)$

$+ (10a_0 - 1.6)(0.1)$

$a_0 = 2.60$ m/s^2

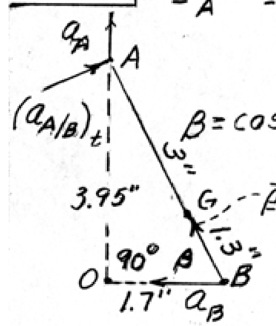
$\Sigma F_x = m\bar{a}_x$; $F = 10(2.60) - 1.6 = \underline{24.4\ N}$

$\Sigma F_y = m\bar{a}_y$; $N - 98.1 = -4(2.60)$, $\underline{N = 87.7\ N}$

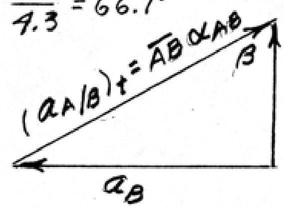
6/111

$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_t\,;\quad (a_{A/B})_n = \overline{AB}\,\omega_{AB}^2 = 0$ since $v_A = v_B$

$a_B = r\omega^2 = \frac{1.7}{12}(100\pi)^2 = 13.98(10^3)$ ft/sec²



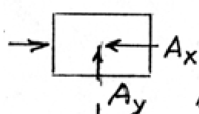
$\beta = \cos^{-1}\frac{1.7}{4.3} = 66.7°$



$(a_{A/B})_t = \frac{13.98(10^3)}{\sin 66.7°} = 15.22(10^3)\ \frac{ft}{sec^2}$

$\alpha_{AB} = \frac{15.22(10^3)}{4.3/12} = 42.5(10^3)$ rad/sec² CW

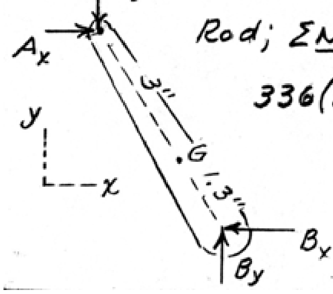
$a_A = 13.98(10^3)/\tan\beta = 6.02(10^3)$ ft/sec²



Piston; $\Sigma F_y = ma_y$; $A_y = \frac{1.80}{32.2}\,6.02(10^3) = 336$ lb

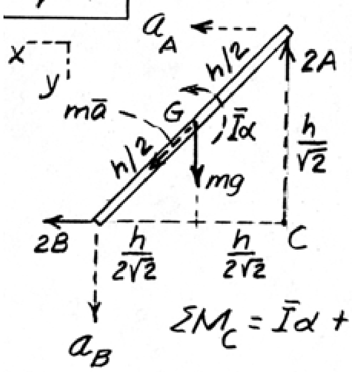
Rod; $\Sigma \underline{M}_B = I_B\underline{\alpha} + \bar{\underline{\rho}}\times m\underline{a}_B$ (Eq. 6/3)

$336(1.7) - A_x(3.95) = \frac{1.20}{32.2(12)}[(1.12)^2 + (1.3)^2](-42.5)10^3$
$+ 1.3\,\frac{1.20}{32.2(12)}(13.98)(10^3)12\sin\beta$



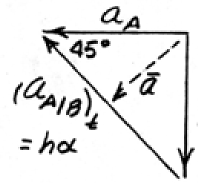
$A_x = 85.6$ lb, $A = \sqrt{(85.6)^2 + (336)^2} = \underline{347\text{ lb}}$

6/112



$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$

with no velocity, $a_{A/B} = (a_{A/B})_t = h\alpha$



$a_A = h\alpha/\sqrt{2}$

$a_B = h\alpha/\sqrt{2}$

$\bar{a} = h\alpha/2$

$\Sigma M_C = \bar{I}\alpha + m\bar{a}d; \quad mg\frac{h}{2\sqrt{2}} = \frac{1}{12}mh^2\alpha + m\frac{h\alpha}{2}\frac{h}{2}$

$$\alpha = \frac{3g}{2\sqrt{2}h}$$

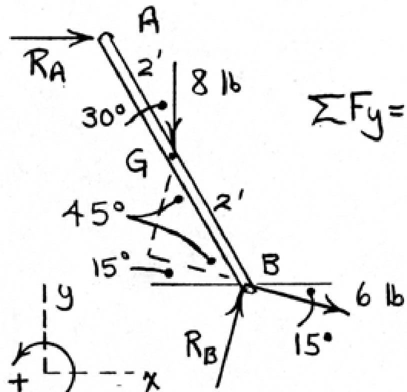
$\Sigma F_x = m\bar{a}_x; \quad 2B = m\frac{h\alpha}{2}\frac{1}{\sqrt{2}}, \quad B = \frac{mh}{4\sqrt{2}}\frac{3g}{2\sqrt{2}h} = \underline{\frac{3}{16}mg}$

$\Sigma F_y = m\bar{a}_y; \quad mg - 2A = m\frac{h\alpha}{2}\frac{1}{\sqrt{2}}$

$2A = mg - \frac{mh}{2\sqrt{2}}\frac{3g}{2\sqrt{2}h}, \quad \underline{A = \frac{5}{16}mg}$

▶6/113

$\Sigma F_x = m\bar{a}_x$: $R_A + 6\cos 15° + R_B \sin 15° = \frac{8}{32.2}\bar{a}_x$ (1)



$\Sigma F_y = m\bar{a}_y$: $R_B\cos 15° - 6\sin 15° - 8 = \frac{8}{32.2}\bar{a}_y$ (2)

$\Sigma M_G = \bar{I}\alpha$:

$$-R_A(2\cos 30°) + R_B(2\cos 45°) + 6(2\sin 45°) = \frac{1}{12}\frac{8}{32.2}(4)^2\alpha \quad (3)$$

Kinematics:

$$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = \bar{a}_x\underline{i} + \bar{a}_y\underline{j} + \underline{\alpha}\times\underline{r}_{A/G} - \omega^2\underline{r}_{A/G}$$

With $\underline{r}_{A/G} = 2[-\sin 30°\underline{i} + \cos 30°\underline{j}]$, we have

$$a_A\underline{j} = [\bar{a}_x - 2\cos 30°\alpha - 2^2\cdot(-2\sin 30°)]\underline{i} + [\bar{a}_y - 2\sin 30°\alpha - 2^2\cdot(2\cos 30°)]\underline{j}$$

$$\Rightarrow \begin{cases} 0 = \bar{a}_x - \sqrt{3}\alpha + 4 & (4) \\ a_A = \bar{a}_y - \alpha - 4\sqrt{3} & (5) \end{cases}$$

$$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G} = \bar{a}_x \underline{i} + \bar{a}_y \underline{j} + \alpha \underline{k} \times \underline{r}_{B/G} - \omega^2 \underline{r}_{B/G}$$

With $\underline{r}_{B/G} = 2[\sin 30° \underline{i} - \cos 30° \underline{j}]$, we have

$$a_B[\cos 15° \underline{i} - \sin 15° \underline{j}] =$$

$$[\bar{a}_x + 2\cos 30° \alpha - 2^2 \cdot (2 \sin 30°)]\ \underline{i}$$

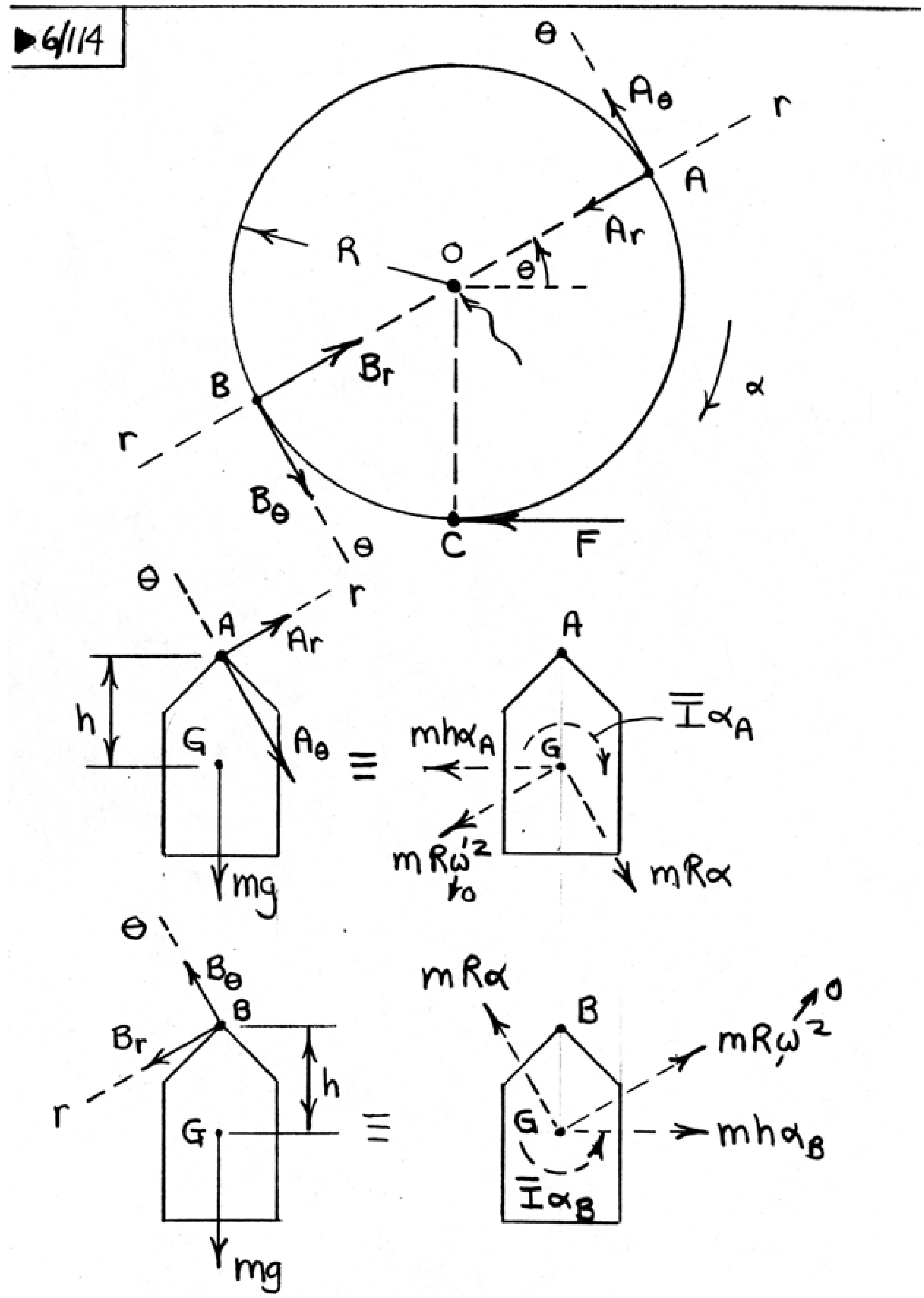
$$+ [\bar{a}_y + 2 \sin 30° \alpha - 2^2 \cdot (-2\cos 30°)]\ \underline{j}$$

$$\Rightarrow \begin{cases} a_B \cos 15° = \bar{a}_x + \sqrt{3}\,\alpha - 4 & (6) \\ -a_B \sin 15° = \bar{a}_y + \alpha + 4\sqrt{3} & (7) \end{cases}$$

Solution of Eqs. (1)–(7):

$$\begin{cases} \underline{R_A = 1.128 \text{ lb}} & \underline{\alpha = 18.18 \text{ rad/sec}^2} \\ \underline{R_B = -0.359 \text{ lb}} & \underline{a_A = -65.0 \text{ ft/sec}^2} \\ \bar{a}_x = 27.5 \text{ ft/sec}^2 & \underline{a_B = 56.9 \text{ ft/sec}^2} \\ \bar{a}_y = -39.8 \text{ ft/sec}^2 & \end{cases}$$

▶6/114



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• Gondola A — $\circlearrowleft \Sigma M_A = \bar{I}\alpha_A + \Sigma m a_A d$:

$$0 = \bar{I}\alpha_A + mh^2\alpha_A - mR\alpha h \sin\theta$$

But $\bar{I} + mh^2 = I_A = mk^2$

So $I_A\alpha_A = mRh\alpha \sin\theta$ or $m\alpha_A = mRh\alpha\sin\theta / k^2$

$\Sigma F_\theta = m\bar{a}_\theta$: $A_\theta + mg\cos\theta = mR\alpha - mh\alpha_A \sin\theta$

$$A_\theta = m(R\alpha - g\cos\theta) - mR\alpha\left(\frac{h\sin\theta}{k}\right)^2 \quad (1)$$

• Gondola B — $\circlearrowleft \Sigma M_B = \bar{I}\alpha_B + \Sigma m a_B d$:

$$0 = \bar{I}\alpha_B + mh^2\alpha_B - mR\alpha h \sin\theta$$

So $I_B\alpha_B = mRh\alpha\sin\theta$ or $m\alpha_B = mRh\alpha\sin\theta / k^2$

(where $I_B = I_A = mk^2$, as above)

$\Sigma F_\theta = m\bar{a}_\theta$: $B_\theta - mg\cos\theta = mR\alpha - mh\alpha_B\sin\theta$

$$B_\theta = m(R\alpha + g\cos\theta) - mR\alpha\left(\frac{h\sin\theta}{k}\right)^2 \quad (2)$$

• Wheel — $\circlearrowleft \Sigma M_O = I_O\alpha$:

$$\left[F - \Sigma(A_\theta + B_\theta)\right]R = I_O\alpha \quad (3)$$

Substitute (1) & (2) into (3)

$$FR - \sum_{1}^{n/2}\left[2mR\alpha - 2mR\alpha\left(\frac{h\sin\theta_n}{k}\right)^2\right]R = I_O\alpha$$

Simplify & solve for F :

$$F = \left\{mR\left[n - 2\frac{h^2}{k^2}\left(\sin^2\theta_1 + \sin^2\theta_2 + \cdots \sin^2\theta_{n/2}\right)\right] + \frac{I_O}{R}\right\}\alpha$$

($n = 0$ corresponds to $\theta = 0$; $n/2$ corresponds to $\theta < \pi$)

Note: The above expression for F simplifies to

$$F = \left\{mRn\left(1 - \frac{h^2}{2k^2}\right) + \frac{I_O}{R}\right\}\alpha$$

6/115 $I_O = \frac{1}{12}ml^2 + m\left(\frac{l}{4}\right)^2 + 2m\left(\frac{3l}{4}\right)^2$

$= \frac{61}{48}ml^2$

$T_1 + U_{1-2} = T_2: \quad 0 + mg\left(\frac{l}{4}\right) + 2mg\left(\frac{3l}{4}\right) = \frac{1}{2}\frac{61}{48}ml^2\omega^2$

$\underline{\omega = 1.660\sqrt{\frac{g}{l}} \quad \text{CW}}$

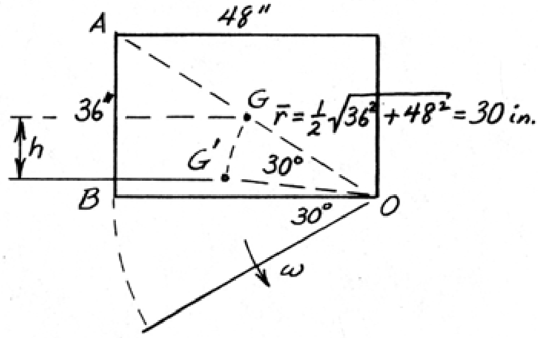
6/116

$\angle AOB = \tan^{-1}\frac{36}{48} = 36.9°$

$\angle G'OB = 36.9° - 30° = 6.87°$

$h = 30\sin 36.9° - 30\sin 6.87°$

$= 14.41$ in.



$U = \Delta T$: $\quad mgh = \frac{1}{2} I_O \omega^2$

$$250\,\frac{14.41}{12} = \frac{1}{2}\,\frac{1}{3}\,\frac{250}{32.2}(3^2 + 4^2)\omega^2$$

$$\omega^2 = 9.28\ (\text{rad/sec})^2,\quad \underline{\omega = 3.05\ \text{rad/sec}}$$

Weight cancels so does not influence the results.

6/117 $T_1 + U_{1-2} = T_2$

$$T_1 = \tfrac{1}{2}8(0.3)^2 + \tfrac{1}{2}12(0.210)^2\left(\frac{0.3}{0.2}\right)^2 = 0.955 \text{ J}$$

$$U_{1-2} = 8(9.81)(1.5) - 3\left(\frac{1.5}{0.2}\right) = 95.2 \text{ J}$$

$$T_2 = \tfrac{1}{2}8v^2 + \tfrac{1}{2}12(0.210)^2\left(\frac{v}{0.2}\right)^2 = 10.62v^2$$

So $0.955 + 95.2 = 10.62v^2$, $\underline{v = 3.01 \text{ m/s}}$

6/118

$$U'_{1-2} = \Delta T + \Delta V_g$$

$$0 = \frac{1}{2}m\left(4^2 - 0^2\right) - mg(5)\left(1 - \cos\theta\right)$$

$$\underline{\theta = 33.2°}$$

6/119

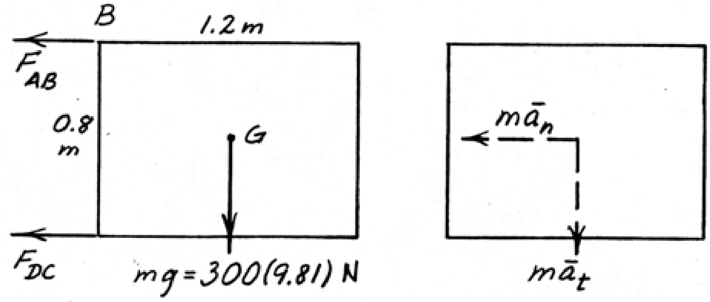


Plate has curvilinear translation so $T = \frac{1}{2}mv^2$

$U = \Delta T$: $300(9.81)(0.8 \cos 60°) = \frac{1}{2}(300)v^2$, $v = 2.80$ m/s

$\omega = v/r$: Angular velocity of links is $\omega = 2.80/0.8 = \underline{3.50 \text{ rad/s}}$

$\Sigma F_t = m\bar{a}_t$: $\bar{a}_t = 9.81$ m/s^2

$\bar{a}_n = v^2/r$: $\bar{a}_n = 2.80^2/0.8 = 9.81$ m/s^2

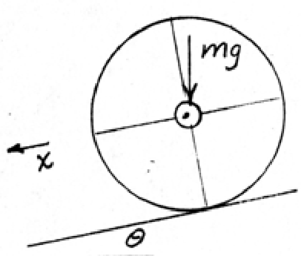
$\Sigma M_B = m\bar{a}d$: $300(9.81)(0.6) + F_{DC}(0.8)$

$= 300(9.81)(0.6) + 300(9.81)(0.4)$

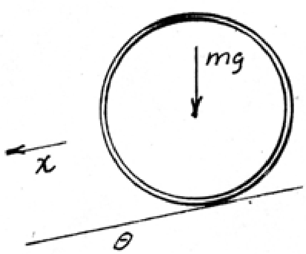
$\underline{F_{DC} = 1472 \text{ N}}$

6/120

A



B



$U = \Delta T$

$U = mgx\sin\theta$

$\Delta T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

Case A: $\Delta T = \frac{1}{2}mv^2 + 0$

case B: $\Delta T = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\left(\frac{v}{r}\right)^2$

$= mv^2$

Case A: $mgx\sin\theta = \frac{1}{2}mv^2$

$\underline{v_A = \sqrt{2gx\sin\theta}}$

Case B: $mgx\sin\theta = mv^2$

$\underline{v_B = \sqrt{gx\sin\theta}}$

6/121 $U = \Delta T$; $U = 40(2 \times 3) = 240$ J

$$\Delta T_{hoop} = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}m\omega^2(r^2 + r^2) = mr^2\omega^2$$

$$= 10(0.3)^2\omega^2 = 0.9\omega^2$$

$$\Delta T_{\substack{\text{each pair}\\ \text{spokes}}} = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}\frac{1}{12}m(2r)^2\omega^2$$

$$= \frac{2}{3}mr^2\omega^2$$

$$\Delta T_{\text{both pair}} = \frac{4}{3}mr^2\omega^2 = \frac{4}{3}4(0.3)^2\omega^2 = 0.48\omega^2$$

Thus $240 = 0.9\omega^2 + 0.48\omega^2$, $\omega^2 = 173.9$

$$\underline{\omega = 13.19 \text{ rad/s}}$$

6/122

For rotation: $T_{rot} = \frac{1}{2} I_C \omega^2 = \frac{1}{2}(4)\left(\frac{1}{12}mb^2 + m\left(\frac{b}{2}\right)^2\right)\omega^2$

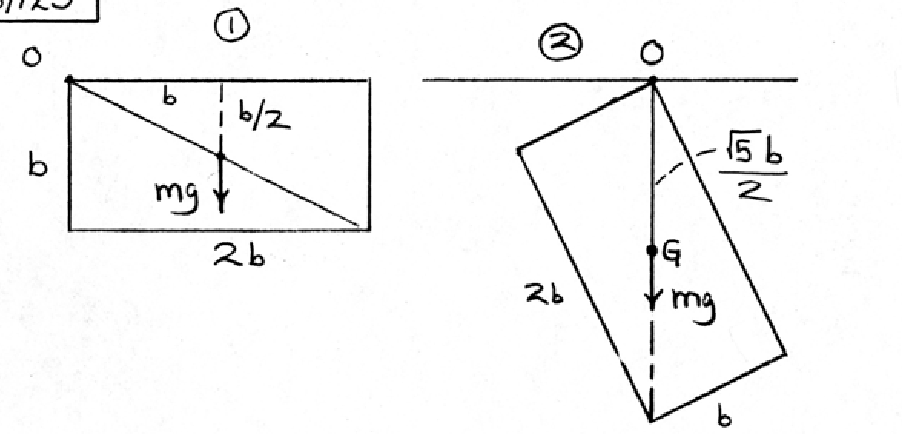
$= \frac{2}{3} m b^2 \omega^2$

For translation: $T_{tran} = \frac{1}{2}(4m)v^2 = 2mv^2$

For $T_{tran} = T_{rot}$: $2mv^2 = \frac{2}{3} m b^2 \omega^2$

$$\underline{v = \frac{b\omega}{\sqrt{3}}}$$

6/123



$$I_O = \bar{I} + md^2 = \frac{1}{12}m\left[b^2 + (2b)^2\right] + m\left[b^2 + \left(\frac{b}{2}\right)^2\right]$$

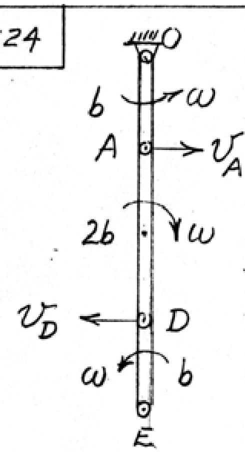
$$= \frac{5}{3}mb^2$$

$T_1 + U_{1\text{-}2} = T_2$:

$$0 + mgb\left[\frac{\sqrt{5}}{2} - \frac{1}{2}\right] = \frac{1}{2}\left[\frac{5}{3}mb^2\right]\omega^2$$

$$\omega^2 = \frac{3g}{5b}\left(\sqrt{5} - 1\right), \quad \underline{\omega = 0.861\sqrt{\frac{g}{b}}}$$

6/124



Final position of OADE (all bars)

$\Delta V_g = 8b\rho g(-2b) = -16\rho g b^2$

$\Delta T = 8(\frac{1}{3} m b^2)\omega^2$

$= \frac{8}{3}\rho b^3 \omega^2$

$U = 0 = \Delta T + \Delta V_g$

$0 = \frac{8}{3}\rho b^3 \omega^2 - 16\rho g b^2$

$\omega^2 = 6g/b$, $\underline{\omega = \sqrt{6g/b}}$

6/125 Note: the wheel has no motion in initial or final positions so $\Delta T_{wheel} = 0$

$U' = \Delta V_g + \Delta T$; $U' = Fb \sin\theta$

$$\Delta V_g = -2m_o g \frac{b}{2} \sin\theta$$

$$\Delta T = 2\left(\frac{1}{2} I_C \omega^2\right) = \frac{1}{3} m_o b^2 \omega^2$$

Thus $Fb \sin\theta = -m_o g b \sin\theta + \frac{1}{3} m_o b^2 \omega^2$

$$\underline{\omega = \sqrt{\frac{3(F + m_o g)\sin\theta}{m_o b}}}$$

6/126 Power $P = \frac{d(\text{Energy})}{dt} = \frac{\Delta E}{t}$

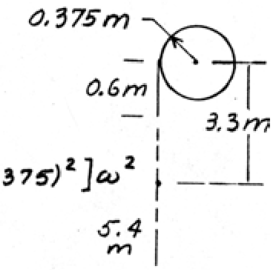
$$\Delta E = \frac{1}{2}\bar{I}(\omega_2^2 - \omega_1^2) = \frac{1}{2}(1200)(0.4)^2([5000]^2 - [3000]^2)\left(\frac{2\pi}{60}\right)^2$$

$$= 16.84(10^6)\ J$$

$$P = \frac{16.84(10^6)}{2(60)} = 140.4(10^3)\ J/s \text{ or } W$$

so $\underline{P = 140.4\ kW}$ or $P = \frac{140.4(10^3)}{7.457(10^2)} = \underline{188\ hp}$

6/127 $\Delta V_g + \Delta T = 0$



$$\Delta V_g = -5.4(3.08)(9.81)(3.3) = -538 \text{ J}$$

$$\Delta T = \frac{1}{2}6.0(3.08)(0.375\omega)^2 + \frac{1}{2}\left[41(0.30)^2 + (3.08)(18-6)(0.375)^2\right]\omega^2$$

$$= 1.299\omega^2 + 4.44\omega^2 = 5.74\omega^2$$

Thus $-538 + 5.74\omega^2 = 0$, $\omega^2 = 93.8$,

$$\underline{\omega = 9.68 \text{ rad/s}}$$

$\boxed{6/128}$ $U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$

$U'_{1-2} = M\theta = \frac{\pi}{2} M = 1.571\, M$ in.-lb

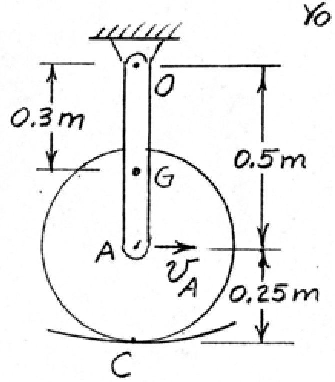
$\Delta T = \frac{1}{2} I_O \omega^2 - 0 = \frac{1}{2}\left(\frac{12}{32.2 \times 12} \times 10^2\right) 4^2 = 24.8$ in.-lb

$\Delta V_g = Wh = 12(-8) = -96$ in.-lb

$\Delta V_e = \frac{1}{2} k (x_2^2 - x_1^2) = \frac{1}{2} 3\left([30 - 15\sqrt{2}\,]^2 - 0\right) = 115.8$ in.-lb

Thus $1.571 M = 24.8 - 96 + 115.8$, $\underline{M = 28.4 \text{ lb-in.}}$

6/129 For system $\Delta T + \Delta V_g = 0$ since $U = 0$



Yoke: $\Delta V_g = 3 \times 9.81(-0.3) = -8.83\ \text{J}$

$$\Delta T = \frac{1}{2} I_O \omega^2 = \frac{1}{2}\left(3 \times \overline{0.35}^2\right)\left(\frac{v_A}{0.5}\right)^2$$

$$= 0.735\, v_A^2$$

Hoop: $\Delta V_g = 4 \times 9.81(-0.5) = -19.62\ \text{J}$

$$\Delta T = \frac{1}{2} I_C \omega^2 = \frac{1}{2}\left(2 \times 4 \times \overline{0.25}^2\right)\left(\frac{v_A}{0.25}\right)^2$$

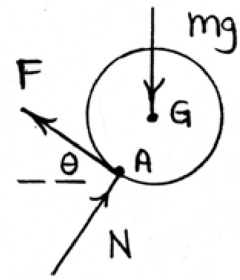
$$= 4 v_A^2$$

Thus $0.735\, v_A^2 + 4 v_A^2 - 8.83 - 19.62 = 0$

$4.735\, v_A^2 = 28.45, \quad v_A^2 = 6.01, \quad \underline{v_A = 2.45\ \text{m/s}}$

6/130 (a) $v = 0$

$v_A = 0$, so F and N are applied at a stationary point and thus do no work.



(b) $v \neq 0$, $v_A \neq 0$: F and N do work.

6/131

For the top position $\omega_B = \dfrac{v}{0.080}$, $\omega_{OA} = \dfrac{v}{0.280}$

For entire system $U'_{1-2} = \Delta T + \Delta V_g$

$U'_{1-2} = M\theta = 4(\pi/2) = 6.28$ J

$$\Delta T_{OA} = \frac{1}{2} I_O \omega_{OA}^2 = \frac{1}{2} 0.8(0.140^2)(v/0.280)^2 = 0.1\, v^2 \text{ J}$$

$$\Delta T_B = \frac{1}{2} m v^2 + \frac{1}{2} \bar{I} \omega^2 = \frac{1}{2} 0.9 v^2 + \frac{1}{2}\left[\frac{1}{2} 0.9 \times 0.080^2\right]\left(\frac{v}{0.080}\right)^2$$
$$= 0.675\, v^2 \text{ J}$$

$(\Delta V_g)_{OA} = mgh = 0.8(9.81)(0.100) = 0.785$ J

$(\Delta V_g)_B = mgh = 0.9(9.81)(0.280) = 2.47$ J

Thus $6.28 = 0.1 v^2 + 0.675 v^2 + 0.785 + 2.47$,

$$v^2 = 3.90 \text{ (m/s)}^2$$

$$\underline{v = 1.976 \text{ m/s}}$$

6/132 $T_1 + U_{1-2} = T_2$

$T_1 = 0$

$U_{1-2} = \int_1^2 M\,d\theta = \int_0^{5(2\pi)} 2(1-e^{-0.1\theta})d\theta$

$= \left(2\theta + 20\,e^{-0.1\theta}\right)\Big|_0^{5(2\pi)}$

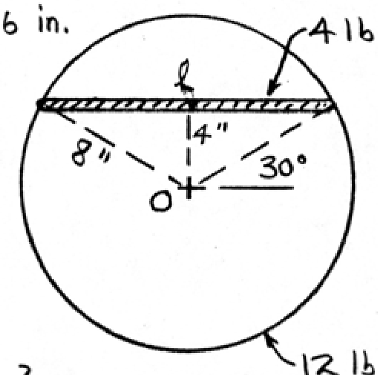
$= 2(5)(2\pi) + 20\,e^{-0.1(5)(2\pi)} - 20$

$= 43.7\ \text{J}$

$T_2 = \frac{1}{2} I \omega^2 = \frac{1}{2}(50)(0.4)^2 \omega^2 = 4\omega^2$

So $0 + 43.7 = 4\omega^2$, $\underline{\omega = 3.31\ \text{rad/s}}$

6/133 $l = 2(8)\cos 30^\circ = 13.86$ in.



$$I_O = \frac{1}{2}\frac{12}{32.2}\left(\frac{8}{12}\right)^2 + \left[\frac{1}{12}\frac{4}{32.2}\left(\frac{13.86}{12}\right)^2 + \frac{4}{32.2}\left(\frac{4}{12}\right)^2\right]$$

$$= 0.1104 \text{ lb-sec}^2\text{-ft}$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 4\left(\frac{8}{12}\right) = \frac{1}{2}(0.1104)\omega^2$$

$$\underline{\omega = 6.95 \text{ rad/sec}}$$

6/134 $I = mk^2 = 10(0.090)^2 = 0.081\ \text{kg}\cdot\text{m}^2$

$M = I\dot{\omega}$, $\dot{\omega} = M/I = -2.10/0.081 = -25.9\ \text{rad/s}^2$

$\omega_0 = 80\,000\left(\frac{2\pi}{60}\right) = 8380\ \text{rad/s}$

$P = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = I\omega\dot{\omega}$

(a) $t = 0$: $P = I\omega\dot{\omega} = (0.081)(8380)(25.9)$

$= 17\,590\ \text{W}$ or $\underline{17.59\ \text{kW}}$

(b) $t = 120\ \text{s}$: $\omega = \omega_0 + \dot{\omega}t = 8380 - 25.9\,(120)$

$= 5270\ \text{rad/s}$

$P = I\omega\dot{\omega} = (0.081)(5270)(25.9) = 11\,060\ \text{W}$

or $\underline{P = 11.06\ \text{kW}}$

6/135

$$T_1 + U_{1-2} = T_2$$

$$mg\left(\frac{l}{2}-x\right) = \frac{1}{2}\left[\frac{1}{12}ml^2 + m\left(\frac{l}{2}-x\right)^2\right]\omega^2$$

$$\omega^2 = \frac{g\left(\frac{l}{2}-x\right)}{\frac{l^2}{6} - \frac{lx}{2} + \frac{x^2}{2}}$$

Set $\frac{d\omega^2}{dx} = 0$ & obtain $\underline{x = 0.789l}$

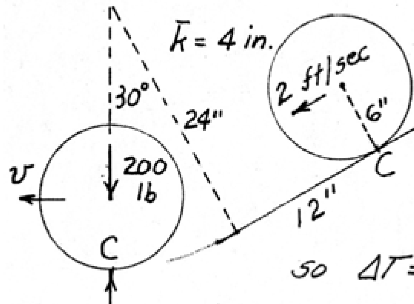
or $\underline{x = 0.211l}$

$$\omega_{max} = \omega_{x=0.211} = \sqrt{\frac{g\left(\frac{l}{2} - 0.211l\right)}{\frac{l^2}{6} - \frac{0.211l^2}{2} + \frac{(0.211l)^2}{2}}}$$

$$= \underline{1.861\sqrt{\frac{g}{l}}}$$

(The solution $x = 0.789l$ would yield the same ω_{max}, only then the motion is CCW.)

6/136 $0 = \Delta V_g + \Delta T; \; \Delta V_g = -200\left[\frac{12}{12}\sin 30° + \frac{18}{12}(1-\cos 30°)\right]$



$= -140.2$ ft-lb

$\Delta T = \frac{1}{2} I_C (\omega_2^2 - \omega_1^2)$

$I_C = \frac{200}{32.2}\left(\left[\frac{4}{12}\right]^2 + \left[\frac{6}{12}\right]^2\right) = 2.24$ lb-ft-sec^2

$\omega_1 = \frac{2}{6/12} = 4$ rad/sec, $\omega_2 = \frac{v}{6/12} = 2v$

so $\Delta T = \frac{1}{2} 2.24 (4v^2 - 4^2) = 4.49v^2 - 17.94$ ft-lb

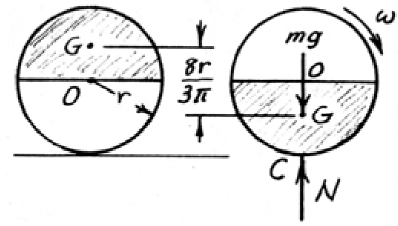
Thus $0 = -140.2 + 4.49v^2 - 17.94$

$v^2 = 35.30, \quad v = 5.94$ ft/sec

$\Sigma F_n = m\bar{a}_n; \quad N - 200 = \frac{200}{32.2}\,\frac{35.30}{18/12}, \quad \underline{N = 346 \text{ lb}}$

6/137

$U = \Delta T: \; mg\left(\frac{8r}{3\pi}\right) = \frac{1}{2} I_C \omega^2$



$I_G = I_O - m\bar{r}^2; \; I_C = I_G + m(r - \bar{r})^2$

so $I_C = I_O - m\bar{r}^2 + m(r - \bar{r})^2$

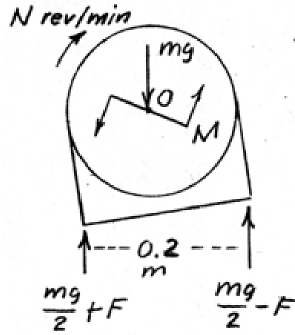
$I_C = m\left(\frac{1}{2}r^2 - \bar{r}^2 + r^2 - 2r\bar{r} + \bar{r}^2\right) = m\left(\frac{3}{2}r^2 - 2r\left[\frac{4r}{3\pi}\right]\right) = mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)$

So $mg\left(\frac{8r}{3\pi}\right) = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\omega^2; \; \omega^2 = \frac{32}{9\pi - 16}\frac{g}{r}, \; \underline{\omega = \sqrt{\frac{g}{r}\frac{32}{9\pi - 16}} \; \frac{\text{rad}}{\text{s}}}$

$\Sigma F_n = m\bar{a}_n: \; N - mg = m\bar{r}\omega^2, \; N = mg + m\frac{4r}{3\pi}\omega^2$

$$\underline{N = mg\left(1 + \frac{128}{3\pi(9\pi - 16)}\right)}$$

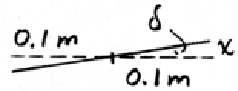
6/138 Power $P = M\omega = M\,\frac{2\pi N}{60}$, $M = \frac{4000(60)}{2\pi(1725)} = 22.14\ \text{N}\cdot\text{m}$



$\Sigma M_O = 0$; $2F(0.2) - 22.14 = 0$ $F = 55.4$ N

$F = kx$, $x = \frac{55.4}{15(10^3)} = 0.00369$ m

or $x = 3.69$ mm



$\delta = \tan^{-1}\frac{x}{0.1} = \tan^{-1}\frac{3.69}{100} = \underline{2.11^\circ}$

<u>Motor shaft turns CW</u>

6/139 $U = \Delta T$

For treads $T = 2(T_{hoop} + T_{top\ section})$, $T_{bottom\ section} = 0$

$$T_{hoop} = \frac{1}{2} I_C \omega^2 = \frac{1}{2}\left[2\pi r\rho(r^2 + r^2)\frac{v^2}{r^2}\right] = 2\pi\rho r v^2$$

$$T_{top\ section} = \frac{1}{2}(\rho b)(2v)^2 = 2\rho b v^2$$

$$U = M\theta = M\frac{s}{r}$$

Thus $M\frac{s}{r} = 2(2\pi\rho r v^2 + 2\rho b v^2)$, $\underline{M = 4\rho\frac{r}{s}v^2(\pi r + b)}$

6/140 $\Delta V_e = \frac{1}{2}(1500)\left[(0.1+2\times0.05)^2 - \overline{0.1}^2\right] = 22.5 \text{ J}$

$\Delta V_g = -(150)(9.81)(0.05) = -73.58 \text{ J}$

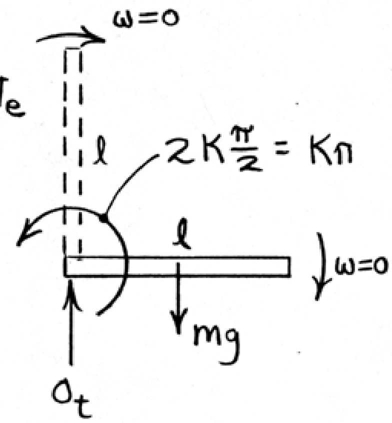
$\Delta T = \sum \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(150)v^2 + \frac{1}{2}(50)(0.3)^2\left(\frac{v}{0.4}\right)^2$

$= 75v^2 + 14.06v^2 = 89.06v^2$

$\Delta T + \Delta V_g + \Delta V_e = 0; \quad 89.06v^2 - 73.58 + 22.5 = 0$

$v^2 = 0.573, \quad \underline{v = 0.757 \text{ m/s}}$

6/141



For dropping, $U' = \Delta T + \Delta V_g + \Delta V_e$

$U' = \Delta T = 0$, $\Delta V_g = -mg\frac{l}{2}$

$\Delta V_e = 2\int_0^{\pi/2} K\theta \, d\theta$

$= K\theta^2 \Big|_0^{\pi/2} = \frac{\pi^2}{4}K$

So $0 = -mg\frac{l}{2} + \frac{\pi^2}{4}K$

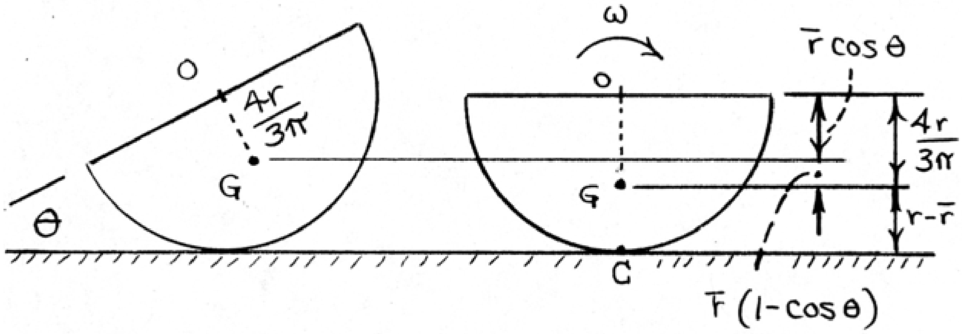
$\underline{K = \frac{2l}{\pi^2}mg}$

Release from rest at $\theta = \pi/2$:

$\Sigma M_0 = I_0 \alpha$: $\frac{2l}{\pi^2}mg\pi - mg\frac{l}{2} = \frac{1}{3}ml^2\alpha$, $\underline{\alpha = 0.410\frac{g}{l}}$

Lid would not stay down; $K = \frac{2l}{\pi^2}mg$ is not practical.

6/142



$U' = \Delta T + \Delta V_g = 0$

$I_C = \bar{I} + m(r-\bar{r})^2 = (I_O - m\bar{r}^2) + m(r-\bar{r})^2$

$= I_O + m(r^2 - 2r\bar{r}) = \frac{1}{2}mr^2 + mr^2\left(1 - \frac{8}{3\pi}\right)$

$= mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)$

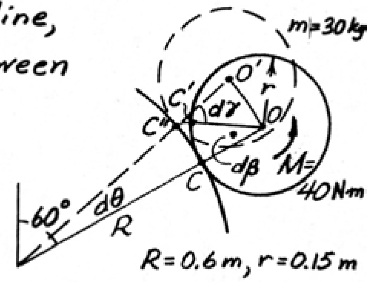
$\Delta T = \frac{1}{2}I_C\omega^2 - 0 = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\omega^2$

$\Delta V_g = -mg\frac{4r}{3\pi}(1-\cos\theta)$

So $0 = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\omega^2 - mg\frac{4r}{3\pi}(1-\cos\theta)$

$\omega = 4\sqrt{\frac{g(1-\cos\theta)}{(9\pi - 16)r}}$

6/143 During rotation $d\theta$ of radial line, disk rotates through angle $d\gamma$ between lines OC' and $O'C''$. $CC' = CC''$ so



$$R\,d\theta = r\,d\beta \ \ \&\ \ d\gamma = d\theta + d\beta$$

$$= \left(1 + \frac{R}{r}\right) d\theta$$

$$\text{or } \ \gamma = \left(1 + \frac{R}{r}\right)\theta$$

For $\theta = \frac{\pi}{3}$, $\gamma = (1 + 0.6/0.15)\frac{\pi}{3} = 5\pi/3$ rad

$U' = \Delta T + \Delta V_g$: $\quad U' = M\gamma = 40\frac{5\pi}{3} = 209$ J

$$\Delta T = \frac{1}{2} I_c \omega^2 = \frac{1}{2}\left(\frac{3}{2} m r^2\right)\left(\frac{v}{r}\right)^2 = \frac{3}{4} m v^2 = \frac{3}{4}(30) v^2$$

$$= 22.5\, v^2$$

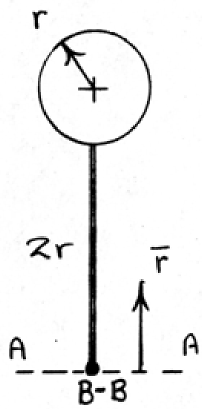
$$\Delta V_g = mgh = mg(R+r)(1 - \cos 60^\circ)$$

$$= 30(9.81)(0.75)\left(\tfrac{1}{2}\right) = 110.4 \text{ J}$$

$209 = 22.5 v^2 + 110.4$, $v^2 = 4.40\ (\text{m/s})^2$, $\underline{v = 2.10 \text{ m/s}}$

6/144 Total mass $m = 2r\rho + 2\pi r\rho = 2r\rho(1+\pi)$

where ρ = mass per unit length.



$$\bar{r} = \frac{\Sigma \bar{r} m}{\Sigma m} = \frac{2r\rho(r) + 2\pi r\rho(3r)}{2r\rho + 2\pi r\rho}$$

$$= r\,\frac{1+3\pi}{1+\pi}$$

$$I_{B-B} = \frac{1}{3}(2r\rho)(2r)^2 + \left[2\pi r\rho r^2 + 2\pi r\rho(3r)^2\right]$$

$$= \frac{4+30\pi}{3(1+\pi)}\,mr^2$$

$$I_{A-A} = \frac{1}{3}(2r\rho)(2r)^2 + \left[\frac{1}{2}\,2\pi r\rho r^2 + 2\pi r\rho(3r)^2\right]$$

$$= \frac{8+57\pi}{6(1+\pi)}\,mr^2$$

$$T_1 + U_{1-2} = T_2$$

(a) $$0 + mgr\,\frac{1+3\pi}{1+\pi} = \frac{1}{2}\,\frac{8+57\pi}{6(1+\pi)}\,mr^2\omega^2$$

$$\underline{\omega = 2\sqrt{\frac{3+9\pi}{8+57\pi}\,\frac{g}{r}}}$$

(b) $$0 + mgr\,\frac{1+3\pi}{1+\pi} = \frac{1}{2}\,\frac{4+30\pi}{3(1+\pi)}\,mr^2\omega^2$$

$$\underline{\omega = \sqrt{\left(\frac{3+9\pi}{2+15\pi}\right)\frac{g}{r}}}$$

6/145 $P = \frac{dU}{dt} = \frac{d}{dt}(T + V_g) + Rv$

$$P = \frac{d}{dt}\left[\Sigma \frac{1}{2} m v^2 + \Sigma \frac{1}{2} \bar{I} \omega^2\right] + \frac{d}{dt}(mgh) + Rv$$

$$= \Sigma m v \frac{dv}{dt} + \Sigma \bar{I} \omega \frac{d\omega}{dt} + mgv \sin\theta + Rv$$

$$= mva + 4\bar{I}\omega\alpha + (mg \sin\theta + R)v$$

$$= (mv + 4\bar{I}\frac{v}{r^2})a + (mg \sin\theta + R)v$$

(a) with $a = 0$, $P = 0 + (500 \times 9.81 \times \frac{1}{\sqrt{101}} + 400)\frac{72}{3.6}$

$= 17\,761$ W or $\underline{P = 17.76 \text{ kW}}$

(b) with $a = 3$ m/s^2

$$P = \left(500 \frac{72}{3.6} + 4(40)(0.4)^2 \frac{72/3.6}{0.6^2}\right)3 + 17\,761$$

$= 30\,000 + 4267 + 17\,761$

$= 52\,028$ W or $\underline{P = 52.0 \text{ kW}}$

6/146 $\Delta T_{translational} = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(10000)\left(\frac{72}{3.6}\right)^2 - 0$

$= 2(10^6)$ J

$\Delta T_{rotation} = \frac{1}{2}I(\omega_2^2 - \omega_1^2)$

$= \frac{1}{2}(1500)(0.5)^2\left(\omega_2^2 - \left[\frac{4000 \times 2\pi}{60}\right]^2\right)$

$= 187.5\omega_2^2 - 32.90 \times 10^6$ J

$\Delta E = 0.1(187.5\omega_2^2 - 32.90 \times 10^6) = 18.75\omega_2^2 - 3.29 \times 10^6$ J

$\Delta V_g = mgh = 10000(9.81)(20) = 1.96 \times 10^6$ J

$\Delta E = \Delta T + \Delta V_g$;

$18.75\omega_2^2 - 3.29 \times 10^6 = 2 \times 10^6 + 187.5\omega_2^2 - 32.90 \times 10^6 + 1.96 \times 10^6$

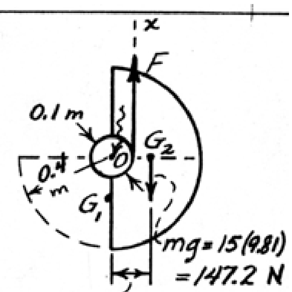
$168.75\omega_2^2 = 25.65 \times 10^6$, $\omega_2^2 = 152\,000$ (rad/s)^2

$\omega_2 = 390$ rad/s or $N = \frac{390 \times 60}{2\pi} = \underline{3720 \text{ rev/min}}$

6/147 For equil. $\Sigma M_O = 0$,

$0.05 F_o - 147.2\,(0.1698) = 0, \quad F_o = 500\ \text{N}$

$F_o = 2k\delta$, where δ = initial spring stretch in equil. position. $\delta = \dfrac{500}{2 \times 2.6 \times 10^3} = 0.0961\ \text{m}$



$U' = \Delta T + \Delta V_e + \Delta V_g$ where $U' = 0$

$\Delta T = \frac{1}{2} I_O \omega^2 - 0 = \frac{1}{2}\left(\frac{1}{2} \times 15 \times 0.4^2\right)\omega^2 = 0.6\,\omega^2$

$\bar{r} = \dfrac{4 \times 0.4}{3\pi} = 0.1698\ \text{m}$

$\Delta V_e = 2\left(\frac{1}{2} k \Delta[x^2]\right) = 2.6 \times 10^3 (0.0961^2 - [0.0961 + 0.05\pi/2]^2)$

$= 2.6 \times 10^3 (0.0961^2 - 0.1746^2) = -55.3\ \text{J}$

$\Delta V_g = mg\Delta h = mg\bar{r} = 15 \times 9.81 \times 0.1698 = 25.0\ \text{J}$

Thus $0 = 0.6\,\omega^2 - 55.3 + 25.0$, $\omega^2 = 50.5\ (\text{rad/s})^2$,

$\underline{\omega = 7.11\ \text{rad/s}}$

6/148 Each spring stretches 4 ft.

so $\Delta V_e = 2\left(\frac{1}{2}kx^2\right) = 2\left(\frac{1}{2}50[4]^2\right) = 800$ ft-lb

$\Delta V_g = -200\,(9-4) = -1000$ ft-lb

$U' = \Delta T + \Delta V_g + \Delta V_e$: $\quad 0 = \frac{1}{2}\frac{200}{32.2}v^2 - 1000 + 800$

$v^2 = 64.4$, $v = \underline{8.02 \text{ ft/sec}}$

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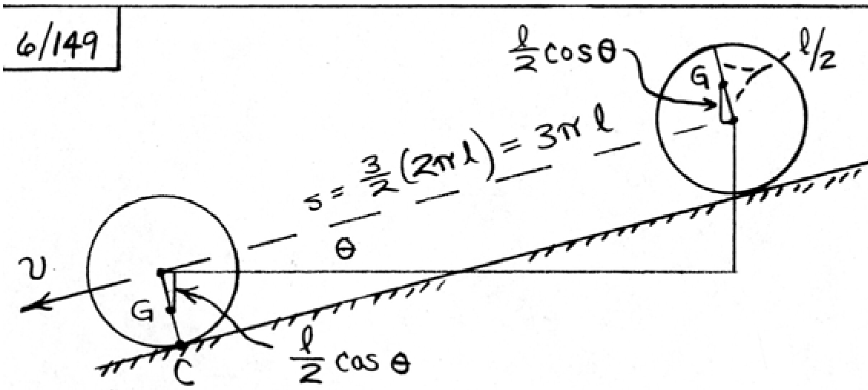


Mass center drops $h = 2\left(\frac{l}{2}\cos\theta\right) + (3\pi l)\sin\theta$

$$= l\left(\cos\theta + 3\pi\sin\theta\right)$$

$U' = \Delta T + \Delta V_g \quad : \quad U' = 0$

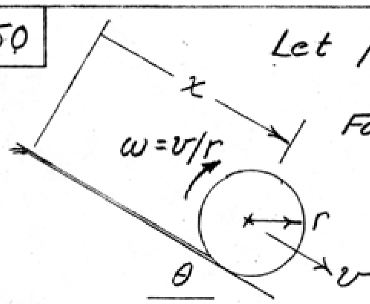
$T = \frac{1}{2} I_C \omega^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\left(\frac{v}{l}\right)^2 = \frac{1}{6}mv^2$

$\Delta V_g = -mgh = -mgl\left(\cos\theta + 3\pi\sin\theta\right)$

So $\quad 0 = \frac{1}{6}mv^2 - mgl\left(\cos\theta + 3\pi\sin\theta\right)$

$$\underline{v = \sqrt{6gl\left(\cos\theta + 3\pi\sin\theta\right)}}$$

6/150



Let ρ = mass of paper per unit length

For general position $m = \rho(L-x)$

$$\Delta T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2$$

$$= \frac{1}{2}\frac{1}{2}mr^2\omega^2 + \frac{1}{2}mv^2$$

$$= \frac{3}{4}mv^2 = \frac{3}{4}\rho(L-x)v^2$$

$$\Delta V_g = -\rho g(L-x)x\sin\theta - \rho g x\frac{x}{2}\sin\theta$$

$$= -\rho g x(L-\frac{x}{2})\sin\theta$$

$$U' = 0 = \Delta T + \Delta V_g; \quad 0 = \frac{3}{4}\rho(L-x)v^2 - \rho g x(L-\frac{x}{2})\sin\theta$$

$$v^2 = \frac{4}{3}\frac{gx(L-x/2)\sin\theta}{L-x}, \quad v = 2\sqrt{\frac{gx}{3}\frac{L-x/2}{L-x}\sin\theta}$$

As $x \to L$, $v \to \infty$ so that the loss of potential energy $-\rho g L\sin\theta/2$ is concentrated in the kinetic energy of the last bit of moving paper. Abrupt termination of motion causes abrupt energy loss at the end.

6/151 Let x = distance moved by center O in m.

$\theta = \tan^{-1}\frac{1}{5} = 11.31^\circ$, $\sin\theta = 0.1961$

$\Delta V_g = mg\,\Delta h = mgx\sin\theta = 10(9.81)\,x\,(0.1961) = 19.24x$

$\Delta V_e = \frac{1}{2}k\left(x_2^2 - x_1^2\right) = \frac{1}{2}(600)\left[\left(0.225 - \frac{275}{200}x\right)^2 - (0.225)^2\right]$

$567.2x^2 - 185.6x$

$\Delta T = \frac{1}{2}mv^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(10)v^2 + \frac{1}{2}(10)(0.125)^2\left(\frac{v}{0.2}\right)^2$

$= 6.95v^2$. For system, $U' = \Delta T + \Delta V_g + \Delta V_e$:

$0 = 6.95v^2 + 19.24x + 567.2x^2 - 185.6x$

$v^2 = 23.93x - 81.57x^2$

Set $\frac{dv^2}{dx} = 0$ to get $x = 0.1467$ m for v_{max}

$v_{max}^2 = 23.93(0.1467) - 81.57(0.1467)^2$, $\underline{v_{max} = 1.325\ \frac{m}{s}}$

6/152 $U' = M\theta$

$\Delta V_g = 2mg\left(\frac{b}{2} - \frac{b}{2}\cos\theta\right) = mgb(1-\cos\theta)$

Bar BO is rotating about O so

$\Delta T_{BO} = \frac{1}{2} I_O \omega^2 - 0 = \frac{1}{2}\frac{1}{3}mb^2\left(\frac{v_B}{b}\right)^2$

But in the limit as $\theta \to 0$, $v_B = \frac{1}{2}v_A$

so $\Delta T_{BO} = \frac{1}{6}m\frac{v_A^2}{4} = \frac{1}{24}mv_A^2$

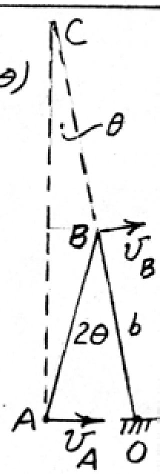
Also AB is rotating about C so

$\Delta T_{AB} = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\left[\frac{1}{12}mb^2 + m\left(\frac{3b}{2}\right)^2\right]\left(\frac{v_A}{2b}\right)^2$

$= \frac{7}{24}mv_A^2$

$U' = \Delta T + \Delta V_g$; $M\theta = \frac{7}{24}mv_A^2 + \frac{1}{24}mv_A^2 + mgb(1-\cos\theta)$

$v_A = \sqrt{3}\sqrt{\frac{M\theta}{m} - gb(1-\cos\theta)}$



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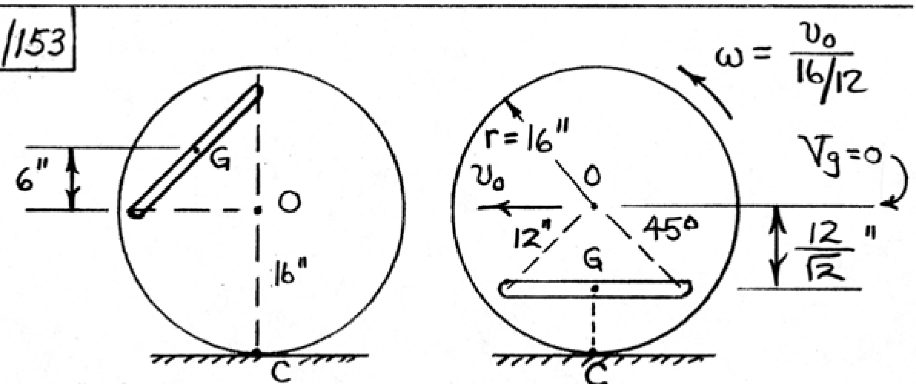

$v_G = \frac{\overline{CG}}{r} v_0, \quad \bar{I}_G = \frac{1}{12}\frac{20}{32.2}\left(\frac{12\sqrt{2}}{12}\right)^2 = 0.1035$ lb-ft-sec^2

$U_{1-2}' = 0 = \Delta T + \Delta V_g \qquad (1)$

$\Delta T_{disk} = \frac{1}{2}\frac{100}{32.2} v_0^2 + \frac{1}{2}\left[\frac{1}{2}\frac{100}{32.2}\left(\frac{16}{12}\right)^2\right]\left(\frac{v_0}{16/12}\right)^2 - 0$

$= 2.33\, v_0^2$

$\Delta T_{bar} = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}\frac{20}{32.2}\left[\frac{16 - \frac{12}{\sqrt{2}}}{16} v_0\right]^2$

$+ \frac{1}{2}(0.1035)\left(\frac{v_0}{16/12}\right)^2 - 0 = 0.0976\, v_0^2$

$\Delta V_g = -mgh = -20\left(\frac{6 + 12/\sqrt{2}}{12}\right) = -24.1$ ft-lb

Eq. (1): $0 = (2.33 + 0.0976)v_0^2 - 24.1$

$v_0 = 3.15$ ft/sec

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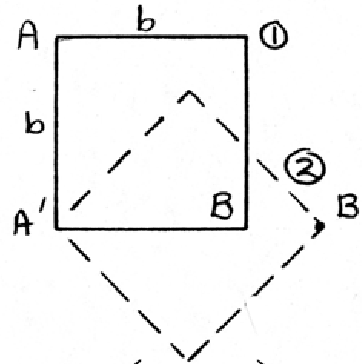
$$\bar{I} = 4\left[\frac{1}{12}\frac{m}{4}b^2 + \frac{m}{4}\left(\frac{b}{2}\right)^2\right]$$

$$= \frac{1}{3}mb^2$$

$$I_B = \frac{1}{3}mb^2 + m\left(\frac{b\sqrt{2}}{2}\right)^2$$

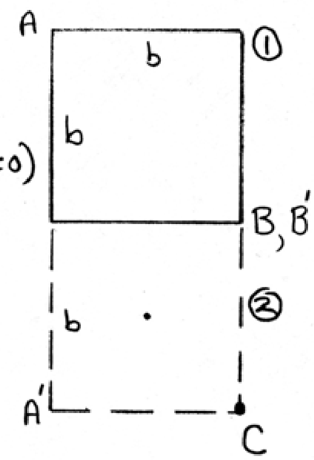
$$= \frac{5}{6}mb^2$$



(a) A has dropped distance b ($v_{B'} = 0$)

$$T_1 + V_{1-2} = T_2: \quad 0 + \frac{mgb}{2} = \frac{1}{2}\left[\frac{5}{6}mb^2\right]\omega^2$$

$$\omega = \sqrt{\frac{6g}{5b}}, \quad v_A = b\omega\sqrt{2} = \underline{\sqrt{\frac{12}{5}gb}}$$



(b) A has dropped distance 2b ($v_c = 0$)

$$T_1 + V_{1-2} = T_2:$$

$$0 + mgb = \frac{1}{2}\left[\frac{5}{6}mb^2\right]\omega^2$$

$$\omega = \sqrt{\frac{12g}{5b}}, \quad v_A = b\omega = \underline{\sqrt{\frac{12}{5}gb}}$$

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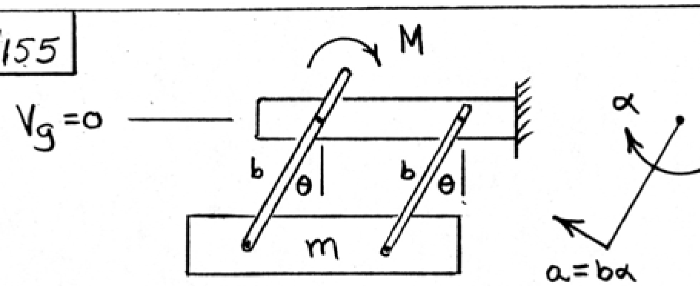


$$dU' = dT + dV_g$$

$$dU' = M\,d\theta$$

$$dT = d\left(\tfrac{1}{2}mv^2\right) = mv\,dv = m\underline{a}\cdot d\underline{s}$$

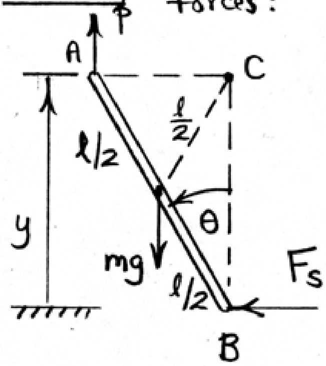
$$= mb\alpha(b\,d\theta) = mb^2\alpha\,d\theta$$

$$dV_g = d(-mgb\cos\theta) = mgb\sin\theta\,d\theta$$

Thus $\quad M\,d\theta = mb^2\alpha\,d\theta + mgb\sin\theta\,d\theta$

$$\underline{\alpha = \frac{M}{mb^2} - \frac{g}{b}\sin\theta}$$

6/156 Active forces:



$$I_C = \frac{1}{12}m\ell^2 + m\left(\frac{\ell}{2}\right)^2 = \frac{1}{3}m\ell^2$$

$$dU' = dT + dV$$

Due to the equilibrium condition, The work due to the weight and the spring add to zero, So that $dU' = P\,dy$

$$= P\,d(\ell\cos\theta) = -P\ell\sin\theta\,d\theta$$

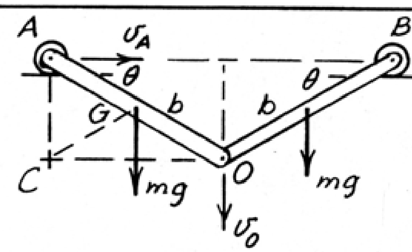
$$dT = d\left(\frac{1}{2}I_C\omega^2\right) = I_C\omega\,d\omega = I_C\alpha\,d\theta$$

So $\quad -P\ell\sin\theta\,d\theta = I_C\alpha\,d\theta = \frac{1}{3}m\ell^2\alpha\,d\theta$

$$\underline{\alpha = -\frac{3P\sin\theta}{m\ell}} \quad \left(\begin{array}{l}\text{minus sign indicates} \\ \alpha \text{ is CW}\end{array}\right)$$

6/157 $dU = dT$

C = instantaneous center of zero velocity for AO

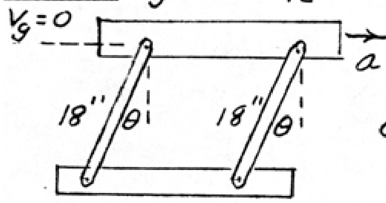


$$dU = 2mg\,d\left(\frac{b}{2}\sin\theta\right)$$
$$= mgb\cos\theta\,d\theta$$
$$dT = 2d\left(\frac{1}{2}I_C\omega^2\right) = 2I_C\,\omega\,d\omega = \frac{2}{3}mb^2\alpha\,d\theta$$

So $mgb\cos\theta\,d\theta = \frac{2}{3}mb^2\ddot{\theta}\,d\theta$ where $\alpha \equiv \frac{d^2\theta}{dt^2}$

$$\underline{\alpha = \ddot{\theta} = \frac{3g\cos\theta}{2b}}$$

6/158 $V_g = -2(8)\frac{9}{12}\cos\theta - 12\frac{18}{12}\cos\theta = -30\cos\theta$ ft-lb



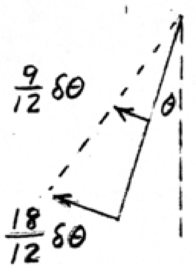
$\delta V_g = 30\sin\theta\,\delta\theta$

$$\delta T = \Sigma m\bar{a}\,\delta s$$

$$= 2\,\frac{8}{32.2}\,4\left(-\frac{9}{12}\delta\theta\cos\theta\right)$$

$$+\frac{12}{32.2}\,4\left(-\frac{18}{12}\delta\theta\cos\theta\right)$$

$$= -3.73\cos\theta\,\delta\theta$$



$\delta T + \delta V_g = 0;\ -3.73\cos\theta\,\delta\theta + 30\sin\theta\,\delta\theta = 0$

$$\tan\theta = \frac{3.73}{30} = 0.1242$$

$$\underline{\theta = 7.08^\circ}$$

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$dU' = 0 = dT + dV_e + dV_g$

$dV_g = 2(6)\,d(8\cos\theta) + 10\,d(18\cos\theta)$

$= 276\,d(\cos\theta) = -276\sin\theta\,d\theta$ in.-lb

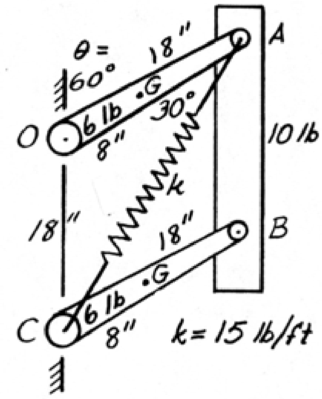
$dT_{bar} = d(\frac{1}{2}mv^2) = mv\,dv = ma_t\,ds$

$= \frac{10}{32.2\times 12}(18\alpha)18\,d\theta = 8.39\,\alpha\,d\theta$

where $a_t = r\alpha$, $ds = r\,d\theta$

$dT_{links} = 2d(\frac{1}{2}I_O\omega^2) = 2I_O\omega\,d\omega = 2I_O\alpha\,d\theta$

$= 2\left(\frac{6}{32.2\times 12}10^2\right)\alpha\,d\theta = 3.11\,\alpha\,d\theta$



$\overline{CA} = 2(18)\cos 30° = 31.2$ in.; stretch $x = 2(18)\cos\frac{\theta}{2} - 18$,

$dx = 36\left(-\sin\frac{\theta}{2}\,\frac{d\theta}{2}\right)$

$dV_e = d(\frac{1}{2}kx^2) = kx\,dx = -\frac{15}{12}18\left(2\cos\frac{\theta}{2} - 1\right)36\sin\frac{\theta}{2}\,\frac{d\theta}{2}$

$= -148.2\,d\theta$ in.-lb

Thus $0 = (8.39 + 3.11)\alpha\,d\theta - 148.2\,d\theta - 276\sin 60°\,d\theta$

$\underline{\alpha = 33.7 \text{ rad/sec}^2}$

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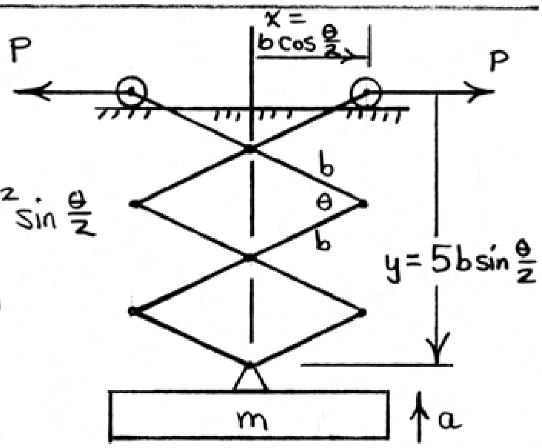
$V_g = 0$



$\dot{y} = \frac{5}{2} b \dot{\theta} \cos\frac{\theta}{2}$

$\ddot{y} = \frac{5}{2} b \ddot{\theta} \cos\frac{\theta}{2} - \frac{5}{4} b \dot{\theta}^2 \sin\frac{\theta}{2}$

For $\dot{\theta} = 0$ & $a = -\ddot{y}$

$a = -\frac{5}{2} b \ddot{\theta} \cos\frac{\theta}{2}$

$dU' = dT + dV_g$

$dU' = +2P d\left(b \cos\frac{\theta}{2}\right) = -Pb \sin\frac{\theta}{2}\, d\theta$

$dT = d\left(\frac{1}{2} m v^2\right) = m v\, dv = ma\,(-dy)$

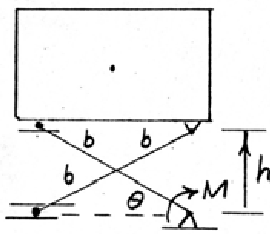
$\quad = -ma\left(\frac{5}{2} b \cos\frac{\theta}{2}\right) d\theta$

$dV_g = d(-mgy) = -mg \frac{5}{2} b \cos\frac{\theta}{2}\, d\theta$

Thus $-Pb \sin\frac{\theta}{2}\, d\theta = -\frac{5}{2} mab \cos\frac{\theta}{2} d\theta - \frac{5}{2} mgb \cos\frac{\theta}{2} d\theta$

$$a = \frac{2P}{5m} \tan\frac{\theta}{2} - g$$

6/161 $dU' = dT + dV_g \, ; \; dU' = M\,d\theta$



$dT = ma\,dh = ma\,d(2b\sin\theta)$

$= 2mba\cos\theta\,d\theta$

$dV_g = mg\,dh = 2mbg\cos\theta\,d\theta$

Thus $M\,d\theta = 2mb\cos\theta\,(a+g)\,d\theta$

$a+g = \dfrac{M}{2mb\cos\theta}$

But $2b\sin\theta = h$

so $\cos\theta = \sqrt{4b^2-h^2}/2b$
$= \sqrt{1-(h/2b)^2}$

so $a = \dfrac{M}{2mb\sqrt{1-(h/2b)^2}} - g$

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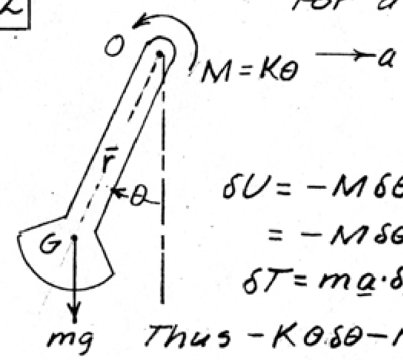

For a virtual displacement $\delta\theta$ from the steady-state configuration,

$$\delta U = \delta T$$

$$\delta U = -M\delta\theta + mg\,\delta(\bar{r}\cos\theta)$$

$$= -M\delta\theta - mg\bar{r}\sin\theta\,\delta\theta$$

$$\delta T = m\underline{a}\cdot\delta\underline{s} = ma(-\bar{r}\delta\theta\cos\theta)$$

Thus $-K\theta\delta\theta - mg\bar{r}\sin\theta\,\delta\theta = -ma\bar{r}\cos\theta\,\delta\theta$

$$K = \frac{m\bar{r}}{\theta}(a\cos\theta - g\sin\theta)$$

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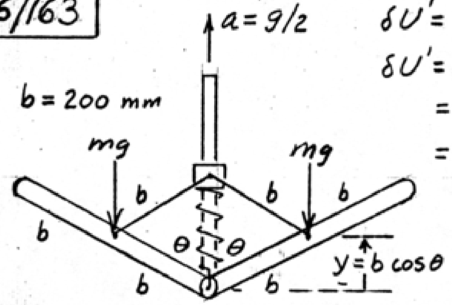


$\delta U' = \delta T + \delta V_e$

$\delta U' = 2mg(-\delta y)$

$= 2(2 \times 9.81)(-\delta[b\cos\theta])$

$= -39.24(0.200)(-\sin\theta)\delta\theta$

$= 7.85 \sin\theta\,\delta\theta$ J

$\delta T = \Sigma m\underline{\bar{a}} \cdot \delta\underline{s} = 2(2)\frac{9.81}{2}\delta y = 19.62\,\delta(b\cos\theta)$

$= -19.62(0.200)\sin\theta\,\delta\theta$

$= -3.92\sin\theta\,\delta\theta$

$\delta V_e = kx\,\delta x = 130(2b - 2b\cos\theta)\delta(2b - 2b\cos\theta)$

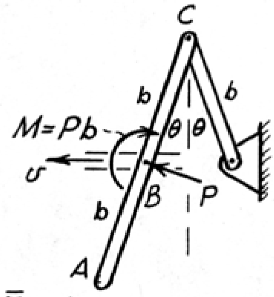
$= 520\,b^2(1-\cos\theta)\sin\theta\,\delta\theta$

Thus $7.85\sin\theta\,\delta\theta = -3.92\sin\theta\,\delta\theta + 520\,b^2(1-\cos\theta)\sin\theta\,\delta\theta$

$\left[(7.85 + 3.92) - 520(0.200)^2(1-\cos\theta)\right]\sin\theta\,\delta\theta = 0$

$1 - \cos\theta = \frac{11.77}{520(0.200)^2}$, $\cos\theta = 1 - 0.5660 = 0.4340$, $\underline{\theta = 64.3°}$

6/164 Replace P by force P at B and couple $M = Pb$



$dU = dT$

$dU = P\cos\theta \, d(2b\sin\theta) + Pb\,d\theta$

$= Pb(2\cos^2\theta + 1)\,d\theta$

$dT_{AC} = d(\frac{1}{2}2mv^2 + \frac{1}{2}\bar{I}\omega^2)$

$= 2mv\,dv + \bar{I}\omega\,d\omega = 2m\,a\,dx + \bar{I}\alpha\,d\theta$

where $x = 2b\sin\theta$, $v = 2b\dot{\theta}\cos\theta$, $a = 2b(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)$

$= 2b\ddot{\theta}\cos\theta$ since $\dot{\theta} = 0$

So $dT_{AC} = 2m(2b\ddot{\theta}\cos\theta)d(2b\sin\theta) + \frac{1}{12}(2m)(2b)^2\ddot{\theta}\,d\theta$

$= 2mb^2(4\cos^2\theta + \frac{1}{3})\ddot{\theta}\,d\theta$

$dT_{OC} = d(\frac{1}{2}I_O\omega^2) = I_O\,\omega\,d\omega = I_O\alpha\,d\theta = \frac{1}{3}mb^2\ddot{\theta}\,d\theta$

So $dT = 2mb^2(4\cos^2\theta + \frac{1}{3})\ddot{\theta}\,d\theta + \frac{1}{3}mb^2\ddot{\theta}\,d\theta$

$= mb^2(8\cos^2\theta + 1)\ddot{\theta}\,d\theta$

$Pb(2\cos^2\theta + 1)d\theta = mb^2(8\cos^2\theta + 1)\ddot{\theta}\,d\theta$,

$$\ddot{\theta} = \alpha = \frac{P(2\cos^2\theta + 1)}{mb(8\cos^2\theta + 1)}$$

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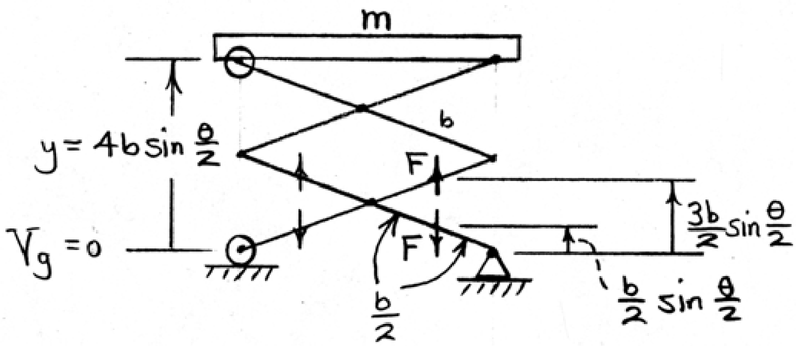

$$dU' = dT + dV_g$$

$$dU' = 2F\,d\left(\frac{3b}{2}\sin\frac{\theta}{2}\right) - 2F\,d\left(\frac{b}{2}\sin\frac{\theta}{2}\right)$$

$$= 2F\,d\left(b\sin\frac{\theta}{2}\right) = Fb\cos\frac{\theta}{2}\,d\theta$$

$$dV_g = d\left(mg\,4b\sin\frac{\theta}{2}\right) = 2mgb\cos\frac{\theta}{2}\,d\theta$$

$$dT = d\left(\frac{1}{2}mv^2\right) = mv\,dv = ma\,dy$$

$$= ma\left(2b\cos\frac{\theta}{2}\,d\theta\right)$$

Thus $Fb\cos\frac{\theta}{2}\,d\theta = 2mgb\cos\frac{\theta}{2}\,d\theta + 2mab\cos\frac{\theta}{2}\,d\theta$

$\underline{a = \dfrac{F}{2m} - g}$ } Both b and θ cancel so a is independent of both b and θ.

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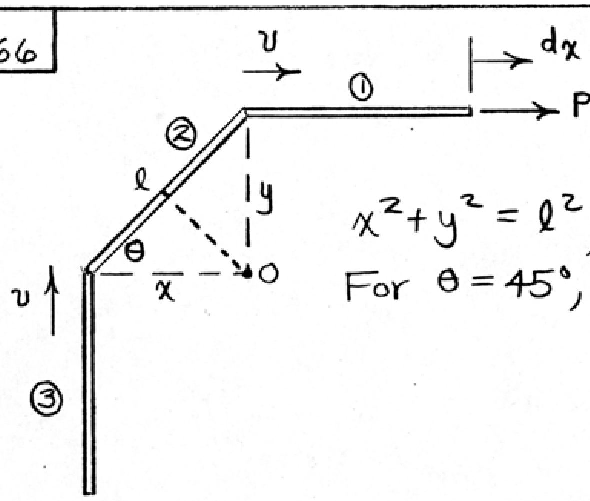

$x^2+y^2=l^2, \quad x\,dx+y\,dy=0$

For $\theta=45°, \quad x=y, \quad dx=-dy$

$dU' = dT + dV_g \quad ; \qquad dU' = P\,dx$

$dT = d(T_1+T_2+T_3) = d\left(\frac{1}{2}mv^2+\frac{1}{2}I_O\omega^2+\frac{1}{2}mv^2\right)$

$\quad = 2mv\,dv + I_O\omega\,d\omega = 2ma\,dx + I_O\alpha\,|d\theta|$

$\quad = 2ma\,dx + \frac{1}{3}ml^2\frac{a}{l/\sqrt{2}}\frac{dx}{l/\sqrt{2}} = \frac{8}{3}ma\,dx$

$dV_g = d(V_{g_1}+V_{g_2}+V_{g_3}) = 0 + mg\frac{dx}{2} + mg\,dx$

$\quad = \frac{3}{2}mg\,dx$

Thus $P\,dx = \frac{8}{3}ma\,dx + \frac{3}{2}mg\,dx$

$$\underline{a = \frac{3}{8}\left(\frac{P}{m} - \frac{3g}{2}\right)}$$

6/167 Radius to each weight is $r = 0.25 + 1.5\sin\theta$ in.

$$\delta T = 2(mr\omega^2)(-\delta r) = 2\frac{12}{16(32.2)}\,\frac{0.25+1.5\sin\theta}{12}\,\omega^2(-\delta r) \quad \text{ft-lb}$$

But $\delta r = 1.5\cos\theta\,\delta\theta$ in.

& $2(1.5) - 2(1.5)\cos\theta = 0.625\sin\beta$, $\beta = 15°$

so $\cos\theta = \dfrac{3 - 0.625\sin 15°}{3} = 0.9461$, $\theta = 18.90°$

$$\text{so } \delta T = \frac{0.25+1.5\sin 18.90°}{8(32.2)}\,\omega^2\left(-\frac{1.5\cos 18.90°}{12}\right)\delta\theta$$

$$= -0.3378(10^{-3})\,\omega^2\delta\theta \quad \text{ft-lb}$$

$$\delta V_e = kx\delta x = 5(12)\,\frac{2(1.5)}{12}(1-\cos\theta)\,\delta\left\{\frac{2(1.5)}{12}(1-\cos\theta)\right\}$$

$$= 3.75(1-\cos\theta)\sin\theta\,\delta\theta =$$

$$= 3.75(1-\cos 18.90°)\sin 18.9°\,\delta\theta = 65.50(10^{-3})\,\delta\theta$$

$$\delta U = \delta T + \delta V_e = 0;\ -0.3378(10^{-3})\,\omega^2\delta\theta + 65.50(10^{-3})\,\delta\theta = 0$$

$$\omega^2 = \frac{65.50}{0.3378} = 193.9 \quad (\text{rad/sec})^2$$

$$\omega = 13.92 \text{ rad/sec},\quad N = \frac{13.92(60)}{2\pi} = \underline{133.0\ \frac{\text{rev}}{\text{min}}}$$

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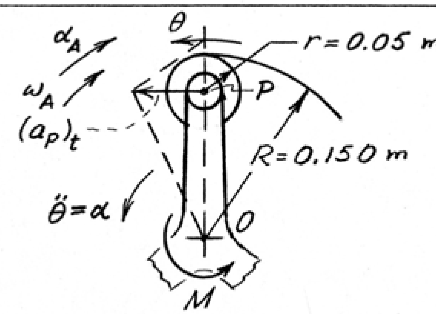


$ds_P = (R-r)d\theta$, $d\theta_A = ds_P/r$
$= \left(\frac{R}{r}-1\right)d\theta$

$v_P = (R-r)\dot{\theta}$, $\omega_A = \frac{v_P}{r} = \left(\frac{R}{r}-1\right)\dot{\theta}$

$(a_P)_t = (R-r)\alpha$, $\alpha_A = \frac{(a_P)_t}{r} = \left(\frac{R}{r}-1\right)\alpha$

$dU = dT_{spider} + dT_{gears}$

$dU = M\,d\theta$

$dT_{spider} = d\left(\frac{1}{2}I_O\omega^2\right) = I_O\,\omega\,d\omega = I_O\,\alpha\,d\theta$

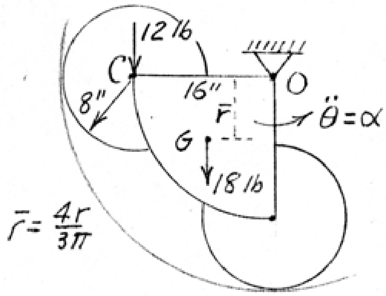
$$dT_{gears} = 3\left\{d\left(\tfrac{1}{2}I_A\omega_A^2\right) + d\left(\tfrac{1}{2}m_A v_P^2\right)\right\} = 3\left\{I_A\alpha_A\,d\theta_A + m_A (a_P)_t\,ds_P\right\}$$
$$= 3\left\{I_A\left(\frac{R}{r}-1\right)^2\alpha\,d\theta + m_A(R-r)^2\alpha\,d\theta\right\}$$
$$= 3(R-r)^2\left(\frac{I_A}{r^2}+m_A\right)\alpha\,d\theta$$

So $M\,d\theta = \left[I_O + 3(R-r)^2\left(\frac{I_A}{r^2}+m_A\right)\right]\alpha\,d\theta$

$$5 = \left[1.2\times0.06^2 + 3(0.150-0.050)^2\left(\frac{0.8\times0.030^2}{0.050^2}+0.8\right)\right]\alpha$$
$$= \left[0.00432 + 0.03\times1.088\right]\alpha,$$

$\underline{\alpha = 135.3\ \text{rad/s}^2}$

6/169 Each wheel: $dT = m_w \bar{a}_w ds_w + \bar{I}_w \alpha_w d\theta_w$



$= \frac{12}{32.2} \frac{16}{12} \alpha \frac{16}{12} d\theta$

$+ \frac{1}{2} \frac{12}{32.2} \left(\frac{8}{12}\right)^2 (2\alpha)(2d\theta)$

$= \frac{32}{32.2} \alpha d\theta$

where $d\theta_w = 2d\theta$

$\alpha_w = 2\alpha$

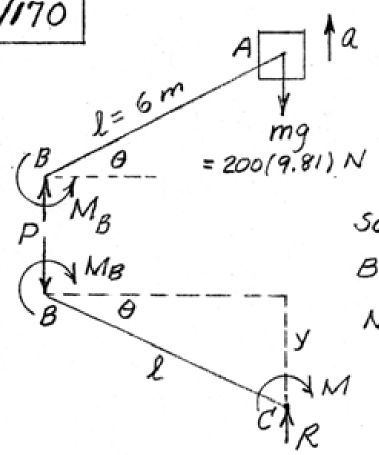
Sector: $dT = I_o \alpha d\theta = \frac{1}{2} \frac{18}{32.2} \left(\frac{16}{12}\right)^2 \alpha d\theta = \frac{16}{32.2} \alpha d\theta$

Combined $dT = 2\left(\frac{32}{32.2} \alpha d\theta\right) + \frac{16}{32.2} \alpha d\theta = \frac{80}{32.2} \alpha d\theta$

$dU = 12 \frac{16}{12} d\theta + 18 \frac{4 \times 16}{3\pi} \frac{1}{12} d\theta = 16\left(1 + \frac{2}{\pi}\right) d\theta = 26.19\, d\theta$

$dU = dT$; $26.19\, d\theta = \frac{80}{32.2} \alpha d\theta$, $\alpha = \frac{26.19(32.2)}{80} = \underline{10.54 \frac{\text{rad}}{\text{sec}^2}}$

6/170



Upper arm ; $dU = dT$

$P\,dy - mg\,2\,dy + M_B\,d\theta = d(\frac{1}{2}mv^2)$

$y = l \sin\theta, \quad dy = l\cos\theta\,d\theta$

$d(\frac{1}{2}m\dot{l}^2) = ma\,d(2y)$

$= 2ma\,l\cos\theta\,d\theta$

So $(P - 2mg)\,l\cos\theta + M_B = 2ma\,l\cos\theta$

But $P - mg = ma$ so

$M_B = mg\,l\cos\theta\,(\frac{a}{g} + 1)$

$= 200(9.81)(6)(0.866)(\frac{1.2}{9.81} + 1)$

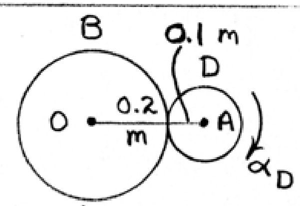
$= 11\,440$ N·m or $\underline{11.44 \text{ kN·m}}$

Lower arm; $\Sigma M = 0$; $M + M_B - Pl\cos\theta = 0$

$M = -mg\,l\cos\theta\,(\frac{a}{g}+1) + mg(\frac{a}{g}+1)\,l\cos\theta, \quad \underline{M = 0}$

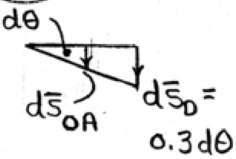
$M = 0$ can be obtained by inspection since m is directly above C. Also, problem can be solved directly by F-m-a equations.

6/171 $dU' = dT + dV_g$



$dU' = \Sigma m_i a_i \cdot ds_i + \Sigma \bar{I}_i \alpha_i \cdot d\theta_i + \Sigma m_i g\, dh_i$

Let $\begin{cases} \alpha = \text{angular acceleration of OA} \\ d\theta = \text{angular displacement of OA} \end{cases}$



Arm OA: $\bar{a} = \frac{0.3}{2}\alpha$, $d\bar{s} = \frac{0.3}{2}d\theta$, $dh = -\frac{0.3}{2}d\theta$

$\bar{I} = \frac{1}{12}(4)(0.3)^2 = 0.03 \text{ kg}\cdot\text{m}^2$

$dU'_{arm} = 4\left(\frac{0.3}{2}\alpha\right)\left(\frac{0.3}{2}d\theta\right) + 0.03\alpha\, d\theta - 4(9.81)\left(\frac{0.3}{2}d\theta\right)$

$= 0.12\alpha\, d\theta - 5.89\, d\theta$

Gear D: $\bar{a} = a_A = 0.3\alpha$, $d\bar{s}_D = 0.3d\theta$, $dh = -0.3d\theta$

$\alpha_D = 3\alpha$, $d\theta_D = 3\,d\theta$

$\bar{I} = m\bar{k}^2 = 5(0.064)^2 = 0.0205 \text{ kg}\cdot\text{m}^2$

$dU'_D = 5(0.3\alpha)(0.3d\theta) + 0.0205(3\alpha)(3d\theta) - 5(9.81)(0.3d\theta) = 0.634\alpha\, d\theta - 14.72\, d\theta$

For system: $dU' = dU'_{arm} + dU'_D = 0$

$0.12\alpha\, d\theta - 5.89\, d\theta + 0.634\alpha\, d\theta - 14.72\, d\theta = 0$

$\underline{\alpha = 27.3 \text{ rad/s}^2}$

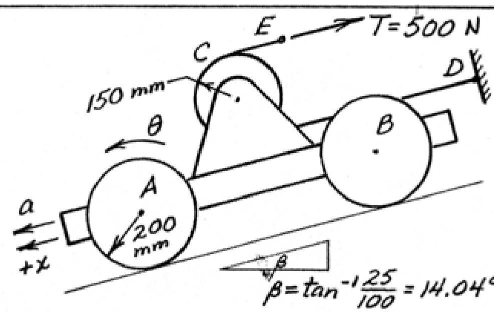
$\boxed{6/172}$ $dU'_{1-2} = dT + dV_g$

Let x = displacement of vehicle down slope

$\bar{I}_A = \bar{I}_B = mk^2 = 140(0.150)^2$

$= 3.15\ kg\cdot m^2$

$\bar{I}_C = 40(0.100)^2 = 0.4\ kg\cdot m^2$



$a = r\alpha$

$dU'_{1-2} = -500\,(2\,dx) = -1000\,dx$

$(dT_{wheels})_{rotation\ only} = 2d(\frac{1}{2}\bar{I}_A\,\omega^2) = 2\bar{I}_A\omega\,d\omega = 2\bar{I}_A\alpha\,d\theta = 2\bar{I}_A\frac{a}{r_A}\frac{dx}{r_A}$

$= 2\times 3.15\,\frac{a\,dx}{0.2^2} = 157.5\,a\,dx$

$(dT_{drum})_{rotation\ only} = d(\frac{1}{2}\bar{I}_C\,\omega_C^2)$

$= \bar{I}_C\,\omega_C\,d\omega_C = \bar{I}_C\,\alpha_C\,d\theta_C = \bar{I}_C\,\frac{a}{r_C}\frac{dx}{r_C} = 0.4\frac{a\,dx}{0.150^2} = 17.78\,a\,dx$

$dT_{vehicle\ translation} = d(\frac{1}{2}mv^2) = mv\,dv = ma\,dx = 520\,a\,dx$

$dV_g = -mg\,dh = -520(9.81)\,dx\,\sin 14.04° = -1237\,dx$

Thus $-1000\,dx = (157.5 + 17.78 + 520)\,a\,dx - 1237\,dx$,

$\underline{a = 0.341\ m/s^2}$

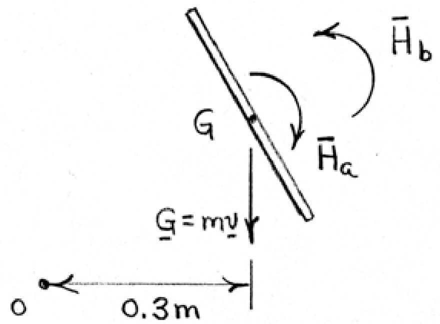
6/173

$$\int_{t_1}^{t_2} M_o \, dt = H_{o_2} - H_{o_1}$$

$$\int_0^3 90 \cos 15° (0.8) \, dt = 4\left(\frac{1}{3}\right)(60)(1.2)^2 \omega$$

$$\underline{\omega = 1.811 \text{ rad/s}}$$

6/174



$$\bar{H} = \bar{I}\omega = \frac{1}{12} m l^2 \omega = \frac{1}{12} 0.8 (0.4)^2 10$$

$$= 0.1067 \ \text{kg}\cdot\text{m}^2/\text{s}$$

$$G = mv = 0.8(2) = 1.6 \ \text{kg}\cdot\text{m/s}$$

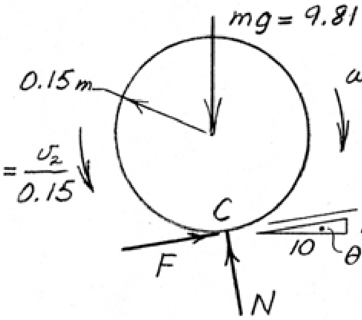
(a) $H_o = \bar{H}_a + Gr = 0.1067 + 1.6(0.3)$

$$= \underline{0.587 \ \text{kg}\cdot\text{m}^2/\text{s}}$$

(b) $H_o = -\bar{H}_b + Gr = -0.1067 + 1.6(0.3)$

$$= \underline{0.373 \ \text{kg}\cdot\text{m}^2/\text{s}}$$

6/175



$\omega_1 = \frac{v_1}{0.15} = \frac{2}{0.15} = 13.33$ rad/s

$\omega_2 = \frac{v_2}{0.15}$

$\theta = \tan^{-1} 0.1 = 5.71°$

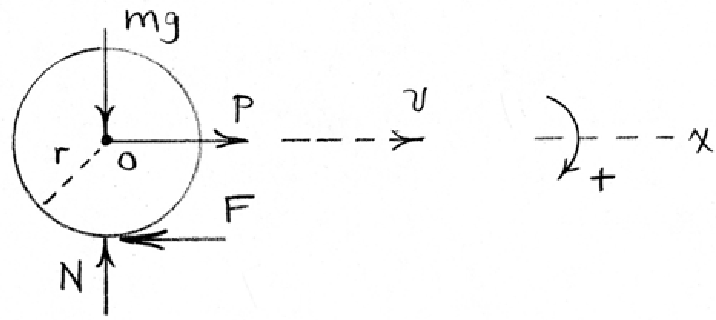
$\Sigma M_C = I_C \alpha$:

$\Sigma M_C t = I_C(\omega_2 - \omega_1)$

$9.81 \cancel{m} \sin 5.71° (0.15)6 = \cancel{m}(0.090^2 + 0.150^2)\left(\frac{v_2}{0.15} - [-13.33]\right)$

$\underline{v_2 = 2.31 \text{ m/s}}$

6/176



$\int \Sigma F_x \, dt = \Delta m v_x : \quad (P - F)t = mv - 0$

$\int \Sigma M_O \, dt = \Delta I_O \omega : \quad Frt = \frac{1}{2} m r^2 \left(\frac{v}{r} - 0 \right)$

Eliminate F & obtain $\underline{v = \dfrac{2Pt}{3m}}$

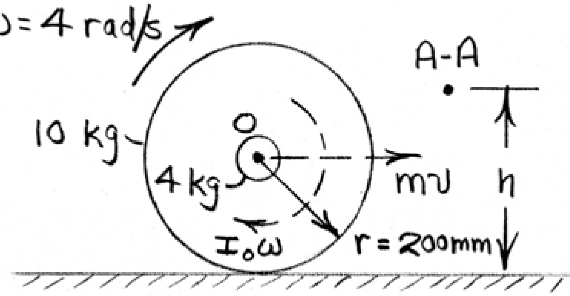
6/177

$$\int_{t_1}^{t_2} \Sigma M_G \, dt = \bar{I}(\omega_2 - \cancel{\omega_1}^{\,0}) = m\bar{k}^2\omega$$

$$\int_0^3 10(1 - e^{-t})\,dt = 75(0.5)^2\,\omega$$

$$10\left[t + e^{-t}\right]\Big|_0^3 = 75(0.5)^2\,\omega, \quad \underline{\omega = 1.093 \text{ rad/s}}$$

6/178



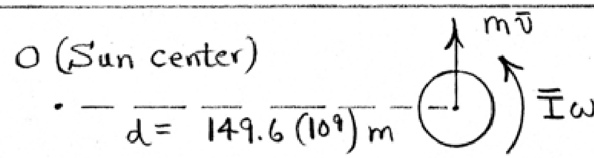
$v = r\omega = 0.2(4) = 0.8$ m/s

$I_O = 10(0.180)^2 + \overset{\sim 0}{I_{Shaft}} = 0.324$ kg·m^2

$\overset{+}{\circlearrowleft} H_{A-A} = I_o\omega - mvd: \; 0.324(4) - (10+4)(0.8)(h-0.2) = 0$

$h = 0.316$ m or $\underline{316 \text{ mm}}$

6/179



$$\bar{H} = \bar{I}\omega = \frac{2}{5}mr^2\left(\frac{2\pi}{\tau}\right)$$

$$= \frac{2}{5}(5.976\cdot10^{24})(6.371\cdot10^6)^2\ \frac{2\pi}{23.9344(3600)}$$

$$= 7.08(10^{33})\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$\bar{v} = \sqrt{\frac{Gm_s}{d}} = \sqrt{\frac{6.673(10^{-11})(333{,}000)(5.976\cdot10^{24})}{149.6(10^9)}}$$

$$= 29\,800\ \text{m/s}$$

$$m\bar{v}d = 5.976(10^{24})(29\,800)(149.6\cdot10^9)$$

$$= 2.66(10^{40})\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$\bar{H} = \bar{I}\omega + m\bar{v}d = \underline{2.66(10^{40})\ \text{kg}\cdot\text{m}^2/\text{s}}$$

(The $\bar{I}\omega$ term is insignificant compared with the $m\bar{v}d$ term.)

6/180

System $\xrightarrow{+}$ $\int_0^{10} \Sigma F\,dt = \Delta G : 400(10) = (1200+800)[v-(-1.5)]$

$$\underline{v = 0.5 \text{ m/s} \quad (\text{right})}$$

Drum $\overset{+}{\circlearrowright}$ $\int \Sigma M_O\,dt = \Delta H_O : 400(0.500)(10) = 800(0.480)^2[\omega-(-3)]$

$$\underline{\omega = 7.85 \text{ rad/s CW}}$$

The rotation of the drum does not affect the linear momentum of the system, so $v = 0.5$ m/s is independent of ω.

6/181 $M\,dt = d(I\omega) = I\,d\omega$

$M = -k\omega^2$ so $-k\omega^2 dt = I\,d\omega$

$$-k\int_0^t dt = I\int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega^2}, \quad -kt = I\left(-\frac{1}{\omega}\right)_{\omega_0}^{\omega_0/2} = I\left(\frac{-1}{\omega_0/2} + \frac{1}{\omega_0}\right)$$

$$= -I/\omega_0$$

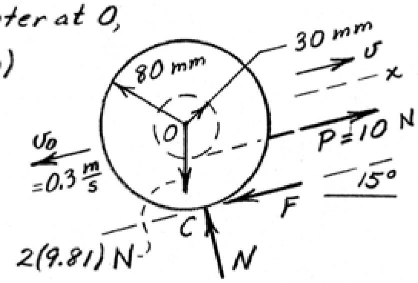
so $t = \dfrac{I}{\omega_0 k}$

6/182 For no slipping & mass center at O,

$\Sigma M_C = I_C \alpha$ so $\int \Sigma M_C \, dt = \Delta H_C = \Delta(I_C \omega)$

$I_C = m(k_O^2 + r^2) = 2(0.060^2 + 0.080^2)$

$= 0.02 \text{ kg}\cdot\text{m}^2$



$\circlearrowleft + \int_0^5 (10[0.080 - 0.030] - 2(9.81)(0.080)\sin 15°)\,dt$

$= 0.02\left(\frac{v}{0.080} - \left[\frac{-0.3}{0.080}\right]\right)$

$0.469 = 0.25(v + 0.3)$, $\underline{v = 1.575 \text{ m/s}}$ up the incline

Alternative sol. if preferred:

$\circlearrowleft + \int \Sigma M_G \, dt = \Delta H_G : \int_0^5 (F \times 0.080 - 10 \times 0.030)\,dt = 2 \times 0.060^2\left(\frac{v}{0.080} - \left[\frac{-0.3}{0.080}\right]\right)$

$0.080F(5) - 0.3(5) = 0.09(v + 0.3) \qquad (1)$

$\int \Sigma F_x \, dt = \Delta G_x : \int_0^5 (10 - F - 2(9.81)\sin 15°)\,dt = 2(v - [-0.3])$

$50 - 25.4 - 5F = 2v + 0.6 \qquad (2)$

Solve & get $\underline{v = 1.575 \text{ m/s}}$ ($F = 4.17$ N,

$N = 2(9.81)\cos 15° = 18.95$ N

$(\mu_s)_{min} = 4.17/18.95 = 0.220$)

6/183 $H_{O_1} = H_{O_2}: \quad mvb = (I_O + mb^2)\,\omega$

$$\frac{2}{16}\,\frac{1}{32.2}\,(1500)\,\frac{15}{12} = \left[\frac{1}{3}\,\frac{20}{32.2}\left(\frac{30}{12}\right)^2 + \frac{2}{16}\,\frac{1}{32.2}\left(\frac{15}{12}\right)^2\right]\omega$$

$$\underline{\omega = 5.60 \text{ rad/sec}}$$

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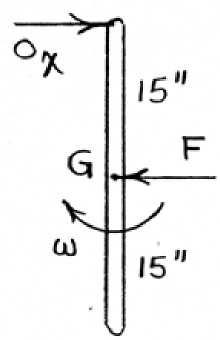

$\int \Sigma M_G \, dt = \bar{H}_2 - \bar{H}_1$:

$$O_x \frac{15}{12}(0.001) = \frac{1}{12}\frac{20}{32.2}\left(\frac{30}{12}\right)^2(\omega - 0),$$

where $\omega = 5.60$ rad/sec from Prob. 6/183.

$\Rightarrow \underline{O_x = 1449 \text{ lb}}$

6/185

$\overset{+}{\curvearrowleft}$ $H_{o_1} = H_{o_2}$ for system

$$m v h = (I_o + m h^2)\omega$$

$$\left(\frac{1/16}{32.2}\right)(1600)\left(\frac{43}{12}\right) = \left[\frac{55}{32.2}\left(\frac{37}{12}\right)^2 + \frac{1/16}{32.2}\left(\frac{43}{12}\right)^2\right]\omega$$

$$\underline{\omega = 0.684 \text{ rad/sec}}$$

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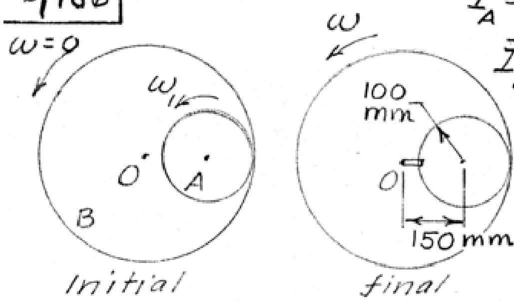

$\bar{I}_A = \frac{1}{2}mr^2 = \frac{1}{2}8(0.1)^2 = 0.04\ kg\cdot m^2$

$\bar{I}_B = mk^2 = 60(0.2)^2 = 2.4\ kg\cdot m^2$

Initial

$\bar{H}_A = (H_A)_O = \bar{I}_A\omega_1$

$= 0.04(80)$

$= 3.2\ kg\cdot m^2/s$

Final $H_O = (I_{A_O} + \bar{I}_B)\omega = (0.04 + 8\times\overline{0.15}^2 + 2.4)\omega$

$= 2.62\omega$

$\Delta H_O = 0;\ 3.2 = 2.62\omega,\ \underline{\omega = 1.221\ rad/s}$

(Note: Overbars refer to center of mass.)

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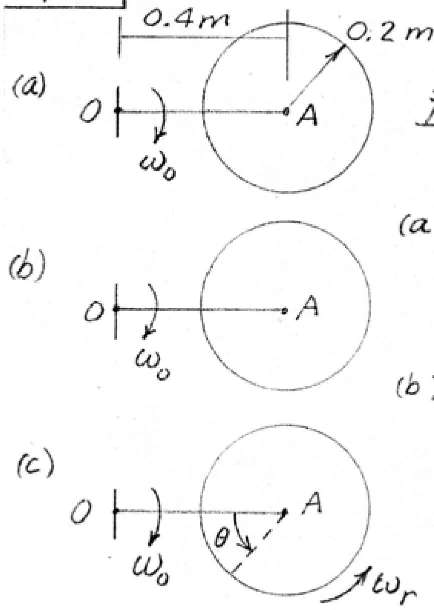

$\omega_0 = 4$ rad/s

ω = angular velocity of disk

$\bar{I} = I_A = \frac{1}{2}mr^2 = \frac{1}{2}25(0.2)^2$

$= \frac{1}{2}$ kg·m^2

(a) $\omega = \omega_0$

$H_O = I_O\omega = \left(\frac{1}{2} + 25[0.4]^2\right)4$

$= \underline{18 \text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $\omega = 0$

$H_O = m\bar{v}d = 25(0.4)(4)(0.4)$

$= \underline{16 \text{ kg}\cdot\text{m}^2/\text{s}}$

(c) $\omega = \omega_0 - \omega_r = 4 - 8 = -4$ rad/s

$H_O = \bar{I}\omega + m\bar{v}d$

$= \frac{1}{2}(-4) + 16$

$= \underline{14 \text{ kg}\cdot\text{m}^2/\text{s}}$

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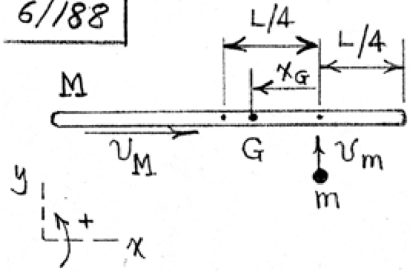


G is <u>system</u> mass center.

$$x_G = \frac{ML/4}{M+m}$$

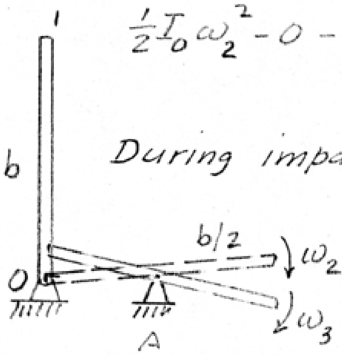
x mom. : $M v_M = (M+m) v_x$, $\underline{v_x = \dfrac{M v_M}{M+m}}$

y mom : $m v_m = (M+m) v_y$, $\underline{v_y = \dfrac{m v_m}{M+m}}$

ang. mom_G: $m v_m \left(\dfrac{ML/4}{M+m}\right) = \left[\dfrac{1}{12} M L^2 + M\left(\dfrac{L}{4} - \dfrac{ML/4}{M+m}\right)^2 + m\left(\dfrac{ML/4}{M+m}\right)^2\right] \omega$

Solving , $\underline{\omega = \dfrac{12 v_m}{L}\left(\dfrac{m}{4M+7m}\right)}$ ↶

6/189 From 1 to 2, $\Delta T + \Delta V_g = 0$



$\frac{1}{2}I_O\omega_2^2 - 0 - mg\frac{b}{2} = 0$, $\frac{1}{3}mb^2\omega_2^2 = mgb$

$\omega_2 = \sqrt{3g/b}$

During impact with A, $\Delta H_A = 0$, $H_{A_2} = H_{A_3}$

$H_{A_2} = \bar{I}\omega_2$

$H_{A_3} = \bar{I}\omega_3$

so $\omega_3 = \omega_2$

$= \sqrt{3g/b}$

6/190 Approximate the diver's body as a uniform slender bar in the first case and as a sphere in the second case. Conservation of angular momentum $H_1 = H_2$:

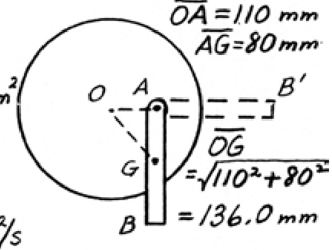
$$\frac{1}{12} \cancel{m} l^2 N_1 = \frac{2}{5} \cancel{m} r^2 N_2$$

$$\frac{1}{12} (2)^2 (0.3) = \frac{2}{5} \left(\frac{0.7}{2}\right)^2 N_2$$

$$\underline{N_2 = 2.04 \text{ rev/s}}$$

6/191 $\Sigma M_O = 0 = \Delta H_O$ so $H_{O_1} = H_{O_2}$

Initial conditions:



$(I_O)_{\text{each rod}} = 0.84(\frac{1}{12} \times 0.160^2 + 0.136^2) = 0.01733$ kg·m^2

$(I_O)_{\text{disk}} = 30(0.090)^2 = 0.243$ kg·m^2

$\omega_1 = 600 \times 2\pi/60 = 62.8$ rad/s

$H_{O_1} = [4(0.01733) + 0.243]62.8 = 19.62$ kg·m^2/s

Final conditions:

$(I_O)_{\text{each rod}} = 0.84(\frac{1}{12} \times 0.160^2 + [0.110 + 0.080]^2) = 0.0321$ kg·m^2

$(I_O)_{\text{disk}} = 0.243$ kg·m^2

$H_{O_2} = [4(0.0321) + 0.243]\,\omega_2 = 0.371\,\omega_2$

Thus $19.62 = 0.371\,\omega_2$, $\omega_2 = 52.8$ rad/s, $\underline{N = 504 \text{ rev/min}}$

Energy loss:

$T_1 = \Sigma \frac{1}{2} I_O \omega^2 = \frac{1}{2}(4 \times 0.01733 + 0.243)(62.8)^2 = 617$ J

$T_2 = \Sigma \frac{1}{2} I_O' \omega'^2 = \frac{1}{2}(4 \times 0.0321 + 0.243)(52.8)^2 = 518$ J

$|\Delta E| = T_1 - T_2 = 617 - 518 = \underline{\mathbf{98.1} \text{ J loss}}$

Direction of rotation & sequence of rod release do not affect the results.

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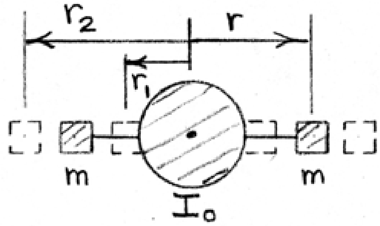

$$H = I_o\omega_o + 2mr^2\omega_o$$

$$\dot{H} = 4mr\dot{r}\omega_o$$

$$r = r_1 + \frac{\Delta r}{\Delta t}t$$

$$= 1.2 + \frac{4.5 - 1.2}{120}t$$

$$= 1.2 + 0.02750t$$

$\dot{r} = 0.02750$ m/s

$M = \dot{H}$, $\quad 2T(1.1) = 4(10)(1.2 + 0.0275t)(0.0275)$

$\times(1.25)$

$$\underline{T = 0.750 + 0.01719t \text{ N}}$$

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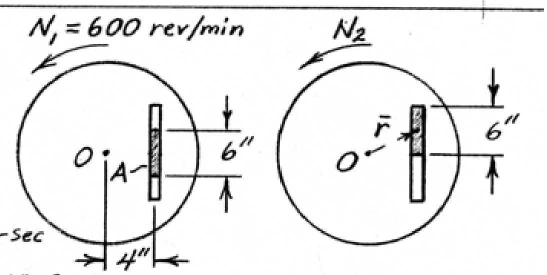


For system, $\Sigma M_O = 0$ so
$\Delta H_O = 0$, $H_1 = H_2$

$$(H_1)_{disk} = \frac{8}{32.2}\left(\frac{6}{12}\right)^2 \frac{600\times 2\pi}{60} \text{ lb-ft-sec}$$

$$(H_1)_{bar} = \left[\frac{1}{12}\frac{2}{32.2}\left(\frac{6}{12}\right)^2 + \frac{2}{32.2}\left(\frac{4}{12}\right)^2\right]\frac{600\times 2\pi}{60} \text{ lb-ft-sec}$$

$\bar{r}^2 = 4^2 + 3^2 = 5^2$ in.^{2}

$$(H_2)_{disk} = \frac{8}{32.2}\left(\frac{6}{12}\right)^2 \frac{2\pi N_2}{60} \text{ lb-ft-sec, } N_2 \text{ in rev/min}$$

$$(H_2)_{bar} = \left[\frac{1}{12}\frac{2}{32.2}\left(\frac{6}{12}\right)^2 + \frac{2}{32.2}\left(\frac{5}{12}\right)^2\right]\frac{2\pi N_2}{60} \text{ lb-ft-sec}$$

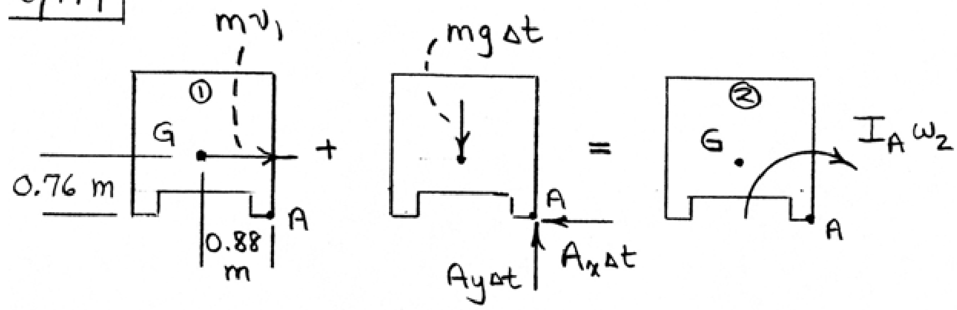
Factor out $\frac{1}{32.2}\times\frac{1}{12^2}\times\frac{2\pi}{60}$ & get

$$\left(8\times 6^2 + \frac{2}{12}\times 6^2 + 2\times 4^2\right)600 = \left(8\times 6^2 + \frac{2}{12}\times 6^2 + 2\times 5^2\right)N_2,$$

$$\underline{N_2 = 569 \text{ rev/min}}$$

Friction forces in the slot are internal so have no effect on ΣM_O. Hence the final value of N_2, as well as the loss of energy, is unaffected.

6/194

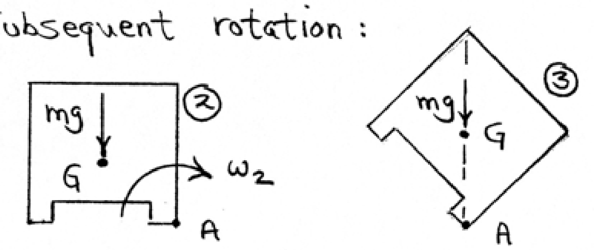


$+\circlearrowright \; H_{A_1} = H_{A_2}$ (ignoring nonimpulsive $mg\Delta t$)

$$2300 v_1 (0.76) = \left[900 + 2300\left(0.76^2 + 0.88^2\right)\right] \omega_2$$

$$\omega_2 = 0.436 v_1$$

Subsequent rotation:



$T_2 + U_{2\text{-}3} = T_3$:

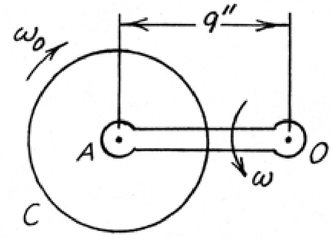
$$\frac{1}{2} I_A \omega_2^2 - mgh = 0$$

$$\frac{1}{2}\left[900 + 2300\left(0.76^2 + 0.88^2\right)\right]\left[0.436 v_1\right]^2 - 2300(9.81)\left[\sqrt{0.76^2 + 0.88^2} - 0.76\right] = 0$$

$\underline{v_1 = 4.88 \text{ m/s}}$ (not very fast!)

$\boxed{6/195}$

Let ω_0 = true angular velocity of disk & armature $= \omega_{rel} - \omega$



$\Sigma M_O = 0$ so $\Delta H_O = 0$;

$H_{O_{initial}} = 0$ so $H_{O_{final}} = 0$

$\omega_{rel} = \frac{300 \times 2\pi}{60} = 31.4$ rad/sec

OA: $H_O = I_O\omega = \frac{10}{32.2}\left(\frac{7}{12}\right)^2 \omega = 0.1057\,\omega$ lb-ft-sec CCW

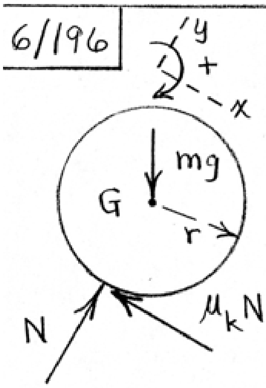
C: $H_O = I_{z'}\omega_0 - mr\omega = \frac{15}{32.2}\left(\frac{4}{12}\right)^2[31.4 - \omega] - \frac{15}{32.2}\left(\frac{9}{12}\right)^2 \omega$

$= 1.626 - 0.314\,\omega$ lb-ft-sec CW

$0.1057\,\omega = 1.626 - 0.314\,\omega$, $\omega = 3.88$ rad/sec

$N = \frac{3.88 \times 60}{2\pi} = \underline{37.0 \text{ rev/min}}$

6/196



$$\int_0^t \Sigma F_y \, dt = m(v_y - v_{y_0}) = 0 \Rightarrow N = mg\cos\theta$$

$$\int_0^t \Sigma F_x \, dt = m(v_x - v_{x_0}) :$$

$$(-\mu_k mg\cos\theta + mg\sin\theta)t = m(v - v_0) \quad (1)$$

$$\int_0^t \Sigma M_G \, dt = \bar{I}(\omega - \omega_0) :$$

$$(\mu_k mg\cos\theta \, r)t = \frac{2}{5} m r^2 \omega \quad (2)$$

We desire the time t when $v = r\omega$ (3)

Solution of Eqs. (1)–(3) :

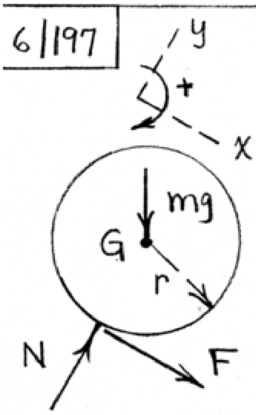
$$\begin{cases} t = \dfrac{2v_0}{g(7\mu_k\cos\theta - 2\sin\theta)} \\ v = \dfrac{5v_0\mu_k}{7\mu_k - 2\tan\theta} \\ \omega = \dfrac{5v_0\mu_k/r}{7\mu_k - 2\tan\theta} \end{cases}$$

For slipping to cease,

$7\mu_k\cos\theta > 2\sin\theta$

or $\mu_k > \frac{2}{7}\tan\theta$

6/197



$$\int_0^t \Sigma F_y\,dt = m(v_y - v_{y_0}) = 0 \Rightarrow N = mg\cos\theta$$

$$\int_0^t \Sigma F_x\,dt = m(v_x - v_{x_0}):$$

$$(+\mu_k mg\cos\theta + mg\sin\theta)t = mv \quad (1)$$

$$\int_0^t \Sigma M_G\,dt = \bar{I}(\omega - \omega_0):$$

$$(-\mu_k mgr\cos\theta)t = \tfrac{2}{5}mr^2(\omega - \omega_0) \quad (2)$$

Slipping ceases when $v = r\omega$ (3)

Solution of Eqs. (1)-(3): $t = \dfrac{2r\omega_0}{g(2\sin\theta + 7\mu_k\cos\theta)}$

Note that the effect of the ramp is to decrease t.

$$v = \frac{2r\omega_0(\sin\theta + \mu_k\cos\theta)}{(2\sin\theta + 7\mu_k\cos\theta)}$$

$$\omega = \frac{2\omega_0(\sin\theta + \mu_k\cos\theta)}{(2\sin\theta + 7\mu_k\cos\theta)}$$

6/198 Conservation of angular momentum about the vertical spin axis of the platform:

$$H_1 = H_2$$

$$\left[10(0.3)^2\right]\left(250\ \frac{2\pi}{60}\right) = \left[I + \frac{1}{2}(10)(0.3)^2 + 10(0.6)^2\right] \times \left(30\ \frac{2\pi}{60}\right)$$

$$\underline{I = 3.45\ \mathrm{kg \cdot m^2}}$$

6/199 Conservation of angular momentum about the vertical spin axis of the platform:

$$H_1 = H_2$$

$$[10(0.3)^2][250] = [3.45 + 10(0.6)^2]N - 10(0.3)^2[250]$$

$$\underline{N = 63.8 \text{ rev/min}}$$

$\boxed{6/200}$ Bar B : $U'_{1-2} = 0 = \Delta T + \Delta V_g$

$\Delta V_g = -mgh = -8(9.81)(0.180) = -14.13 \text{ J}$

$\Delta T = \frac{1}{2} I \omega_B^2 = \frac{1}{2}(8)(0.220)^2 \omega_B^2 = 0.1936\, \omega_B^2$

So $0 = 0.1936\,\omega_B^2 - 14.13$, $\omega_B = 8.54 \text{ rad/s}$

Prior to impact : $H_O = I\omega_B = 8(0.220)^2(8.54) = 3.31 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$

For system after impact :

$H_O = I_{tot}\,\omega = [2 \cdot 20(0.3)^2 + 8(0.220)^2]\,\omega = 3.99\omega$

$\Delta H_O = 0 : \; 3.99\omega - 3.31 = 0$, $\omega = 0.830 \text{ rad/s}$

After impact : $U'_{1-2} = 0 = \Delta T + \Delta V_g$

$\Delta V_g = mgh = 2 \cdot 20(9.81)(0.25)(1-\cos\theta)$
$+ 8(9.81)(0.18)(1-\cos\theta) = 112.2(1-\cos\theta)$

$\Delta T = 0 - \frac{1}{2}I\omega^2 = -\frac{1}{2}[2 \cdot 20(0.3)^2 + 8(0.220)^2](0.830)^2$
$= -1.372 \text{ J}$

So $0 = 112.2(1-\cos\theta) - 1.372$, $\underline{\theta = 8.97°}$

Loss of energy $|\Delta E| = (V_g)_{before} - (V_g)_{after}$
$= 14.13 - 112.2(1 - \cos 8.97°) = \underline{12.75 \text{ J}}$

6/201 $\Delta H = 0;$

Initial: $H_{rods} = 2I\omega = 2(1.5)(0.060)^2 \frac{300\times 2\pi}{60}$ N·m·s

$H_{base} = mk^2\omega = 4(0.040)^2 \frac{300\times 2\pi}{60}$ N·m·s

Final: $H_{rods} = 2\left[\bar{I} + md^2\right]\omega = 2m\left[\frac{\ell^2}{12} + d^2\right]\frac{2\pi N}{60}$

$= 2(1.5)\left[\frac{0.3^2}{12} + (0.150 + 0.060)^2\right]\frac{2\pi N}{60}$

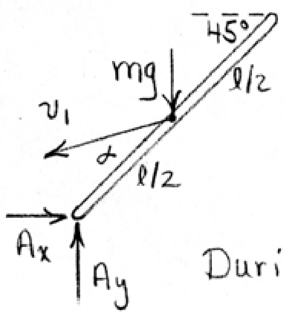
$= 0.1548\left(\frac{2\pi N}{60}\right)$ N·m·s

$H_{base} = 4(0.040)^2 \frac{2\pi N}{60} = 0.0064\left(\frac{2\pi N}{60}\right)$

Thus $\left[3(0.06)^2 + 4(0.04)^2\right]300 = \left[0.1548 + 0.0064\right]N$

$0.0172(300) = 0.1612\,N,$ $\underline{N = 32.0 \text{ rev/min}}$

6/202 Neglecting impulse of weight, $\Delta H_A = 0$ during impact:



$$mv_1 \frac{l}{2} \sin\alpha = \frac{1}{3} ml^2 \omega_2$$

$$\omega_2 = \frac{3v_1}{2l} \sin\alpha$$

During subsequent rotation about A,

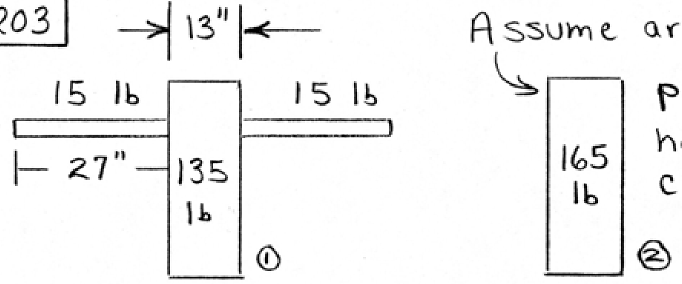
$$V = \Delta T \text{ or } -mg\frac{l}{2}\left(1-\cos 45°\right) = 0 - \frac{1}{2}I_A\omega_2^2$$

$$\omega_2 = \sqrt{\frac{3g}{l}\left(1-\frac{\sqrt{2}}{2}\right)}$$

So $\sqrt{\frac{3g}{l}\left(1-\frac{\sqrt{2}}{2}\right)} = \frac{3v_1}{2l}\sin\alpha$

$$\sin\alpha = \frac{0.625}{v_1}\sqrt{gl} \quad \left(0 \le \alpha \le 45°\right)$$

6/203



Assume arms become part of homogeneous cylinder

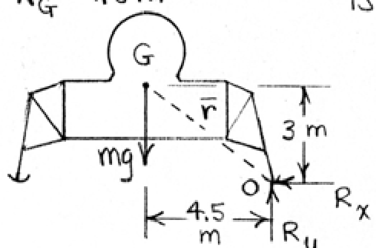
Conservation of angular momentum about a vertical axis : $H_1 = H_2$

$$\left\{\frac{1}{2}\,\frac{135}{32.2}\left(\frac{13}{2\cdot 12}\right)^2 + 2\left[\frac{1}{12}\,\frac{15}{32.2}\left(\frac{27}{12}\right)^2 + \frac{15}{32.2}\left(\frac{13+27}{2\cdot 12}\right)^2\right]\right\} \times 1 = \left\{\frac{1}{2}\,\frac{165}{32.2}\left(\frac{13}{2\cdot 12}\right)^2\right\} N$$

$\underline{N = 4.78 \text{ rev/sec}}$

6/204

$k_G = 1.8\text{ m}$



Impulse of mg during impact interval is small and is neglected.

Before impact, $\bar{v} = \frac{8}{3.6} = 2.22 \frac{m}{s}$

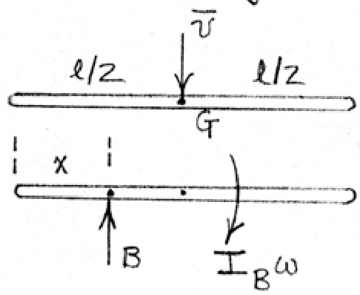
$\Delta H_O = 0$:

$$\cancel{m}\bar{v}d = \cancel{m}(\bar{k}^2 + \bar{r}^2)\omega$$

$$2.22(4.5) = ((1.8)^2 + (4.5)^2 + (3)^2)\omega$$

$$\underline{\omega = 0.308 \text{ rad/s}}$$

6/205 Velocity of bar at impact $= \sqrt{2gh} = \bar{v}$



Neglect small impulse of weight.

$\Delta H_B = 0$

$I_B \omega = m\bar{v}\left(\frac{l}{2} - x\right)$

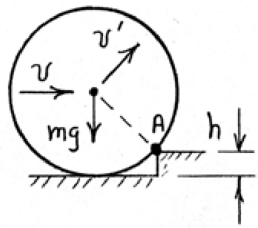
$$I_B = \frac{1}{12} m l^2 + m\left(\frac{l}{2} - x\right)^2 = \frac{1}{3} m l^2 - m l x + m x^2$$

Thus $$\omega = \frac{\left(\frac{l}{2} - x\right)\sqrt{2gh}}{\left(\frac{1}{3} l^2 - l x + x^2\right)}$$

$$\omega_{x=0} = \underline{\frac{3}{2l}\sqrt{2gh}}\ , \quad \omega_{x=l/2} = \underline{0}$$

$$\omega_{x=l} = \underline{-\frac{3}{2l}\sqrt{2gh}}$$

6/206 Angular impulse of mg is negligible.



Before impact: $H_A = \bar{I}\omega + mv(r-h)$

$$= mk^2 \frac{v}{r} + mv(r-h)$$

Just after impact:

$$H_A' = I_A \frac{v'}{r} = m(k^2+r^2)\frac{v'}{r}$$

$\Delta H_A = 0$: $mv\left(\frac{k^2}{r} + r - h\right) = m(k^2+r^2)\frac{v'}{r}$

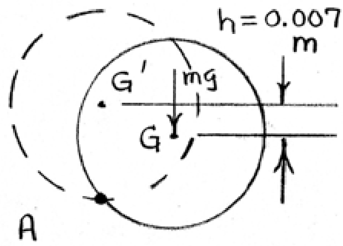
$$v' = v\left(1 - \frac{rh}{k^2+r^2}\right)$$

During roll on curb point, $\Delta T + \Delta V_g = 0$

$$\left[0 - \frac{1}{2}m(k^2+r^2)\frac{v'^2}{r^2}\right] + \left[mgh - 0\right] = 0$$

Solve for v: $\underline{v = \frac{r}{k^2+r^2-rh}\sqrt{2gh(k^2+r^2)}}$

6/207 Process II - roll about fixed point A



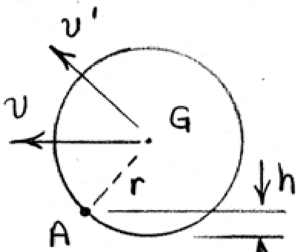
$$U_{1-2} = \Delta T$$

$$-mgh = \frac{1}{2}\left(\frac{3}{2}mr^2\right)\left(\omega'^2 - 0\right)$$

$$\omega' = \sqrt{\frac{4gh}{3r^2}} = \sqrt{\frac{4(9.81)(0.007)}{3(0.035)^2}}$$

$$= 8.65 \text{ rad/s}$$

Process I - impact at A



$$\Delta H_A = 0: \quad mv(r-h) = I_A \omega'$$

$$= \left(\frac{3}{2}mr^2\right)\omega'$$

With $v = 0.5\Omega$:

$$\frac{1}{2}\Omega(r-h) = \frac{3}{2}r^2\omega'$$

$$\Omega = \frac{3r^2\omega'}{r-h} = \frac{3(0.035)^2(8.65)}{0.035-0.007} = \underline{1.135 \frac{\text{rad}}{\text{s}}}$$

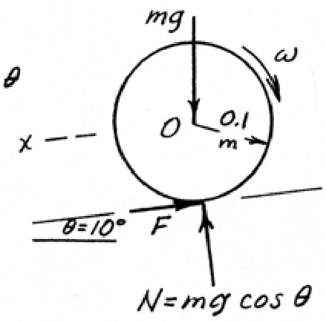
▶ **6/208** During slipping $(a_O)_x = 0$, so

$\Sigma F_x = 0$, $F - mg\sin\theta = 0$; $F = \mu_k mg\cos\theta$

so $mg\sin\theta = \mu_k mg\cos\theta$,

$\mu_k = \tan\theta = \tan 10°$

$\underline{\mu_k = 0.1763}$



$\Sigma M_O \times t = \Delta H_O$:

$0.1763(30)(9.81)\cos 10°(0.1)t$

$= 0 - (-30 \times 0.075^2)\frac{2\pi \times 300}{60}$, $\underline{t = 1.037\text{ s}}$

During rolling (assume no slip)

$\int_0^4 \Sigma F_x\,dt = m\Delta v_x$: $(30 \times 9.81\sin 10° - F)4 = 30(v - 0)$, $204 - 4F = 30v$

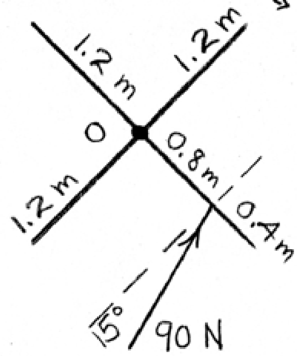
$\int_0^4 \Sigma M_O\,dt = I_O\Delta\omega$: $0.1F \times 4 = 30 \times 0.075^2(v/0.1)$, $4F = 16.88v$

Combine & get $F = 18.40$ N, $\underline{v = 4.36\text{ m/s}}$

Check: $F_{max} = \mu_s N$, & $\mu_k N = 0.1763 \times 30 \times 9.81\cos 10° = 51.1\text{ N} < \mu_s N$

so $18.40 < \mu_k N < \mu_s N$ & assumption of no slip is valid.

6/209 $I_O = 4\left(\frac{1}{3}ml^2\right) = 4\left(\frac{1}{3}60\,(1.2)^2\right) = 115.2 \text{ kg}\cdot\text{m}^2$

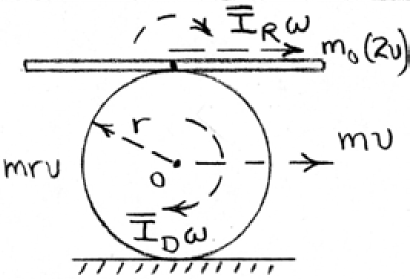


$\circlearrowleft + \Sigma M_O = I_O\alpha$:

$(90\cos 15°)(0.8) = 115.2\alpha$

$\underline{\alpha = 0.604 \text{ rad/s}^2}$

6/21.0



$\omega = v/r$

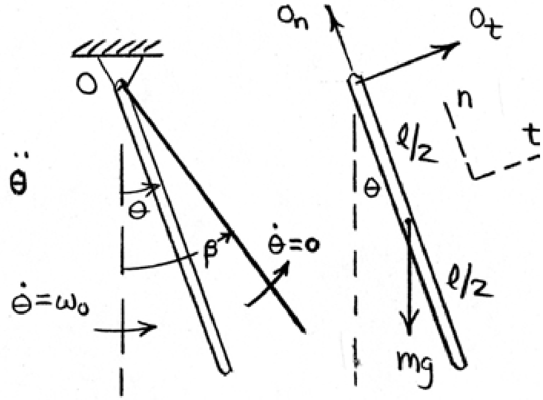
Disk: $\bar{I}_D\,\omega = \frac{1}{2}mr^2\left(\frac{v}{r}\right) = \frac{1}{2}mrv$

Rod : $\bar{I}_R\omega = \frac{1}{12}m_0 l^2 \frac{v}{r}$

Combined : $H_o = \frac{1}{2}mrv + \frac{1}{12}m_0 l^2\frac{v}{r} + m_0(2v)r$

$$= vr\left[\frac{m}{2} + m_0\left(2 + \frac{l^2}{12r^2}\right)\right]$$

6/211



$$\curvearrowright_{+} \Sigma M_O = I_O \ddot\theta :$$

$$-mg\frac{l}{2}\sin\theta = \frac{1}{3}ml^2\ddot\theta$$

$$\ddot\theta = -\frac{3g}{2l}\sin\theta$$

$$\dot\theta\, d\dot\theta = \ddot\theta\, d\theta$$

$$\int_{\omega_0}^{\omega}\dot\theta\, d\dot\theta = \int_0^{\theta} -\frac{3g}{2l}\sin\theta\, d\theta$$

$$\Rightarrow \omega^2 = \omega_0^2 - \frac{3g}{l}(1-\cos\theta)$$

When $\theta = \beta$, $\omega = 0$: $0 = \omega_0^2 - \frac{3g}{l}(1-\cos\beta)$

$$\omega_0^2 = \frac{3g}{l}(1-\cos\beta)$$

So $\omega^2 = \frac{3g}{l}(\cos\theta - \cos\beta)$

$$\Rightarrow \frac{d\theta}{dt} = \sqrt{\frac{3g}{l}}\sqrt{\cos\theta - \cos\beta}$$

So $t = \sqrt{\frac{l}{3g}}\int_0^{\beta}\frac{d\theta}{\sqrt{\cos\theta - \cos\beta}}$

$\boxed{6/212}$ Max. power occurs when dV_g/dt is greatest, which occurs when $\bar{v}_y$ is max. at the start.

$\bar{v}_y = 1.500\,\omega = 1.500\,\frac{4\pi}{180} = 0.1047$ m/s

$P = mg\bar{v}_y = 1600(5)\,9.81(0.1047) = 8218$ W

or $\underline{P = 8.22 \text{ kW}}$

$\boxed{6/213}$ $\Delta V_g + \Delta V_e + \Delta T = 0$

$\Delta V_g = -15(2) = -30$ ft-lb

$\Delta V_e = \frac{1}{2} 3 \left(\sqrt{6^2+4^2} - 2\right)^2 = 40.74$ ft-lb

$\Delta T = 0 - \frac{1}{2} \frac{1}{3} \frac{15}{32.2} 4^2 \omega^2 = -1.242\omega^2$

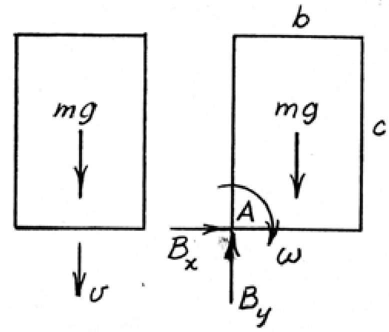
$-30 + 40.74 - 1.242\,\omega^2 = 0$, $\omega^2 = 8.64$, $\underline{\omega = 2.94 \text{ rad/sec}}$

6/214

$v = \sqrt{2gh}$

$\Delta H_B = 0$

$I_A \omega - mv\frac{b}{2} = 0$



$I_A = \frac{1}{12}m(b^2+c^2) + m\left[\left(\frac{b}{2}\right)^2 + \left(\frac{c}{2}\right)^2\right]$

$= \frac{1}{3}m(b^2+c^2)$

$\frac{1}{3}m(b^2+c^2)\omega - m\sqrt{2gh}\,\frac{b}{2} = 0,\quad \underline{\omega = \frac{3b\sqrt{2gh}}{2(b^2+c^2)}}$

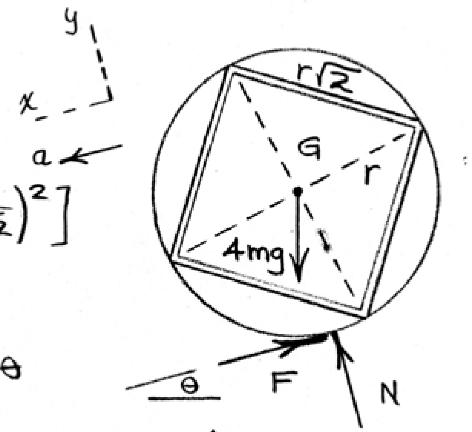
Percentage loss of energy $n = \frac{|\Delta E|}{E} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}I_A\omega^2}{\frac{1}{2}mv^2}$

$= 1 - \frac{I_A\omega^2}{mv^2}$

and for $b = c$, $n = 1 - \frac{2c^2/3}{2gh}\,\frac{9c^2(2gh)}{4(2c^2)}$

$= 1 - \frac{3}{8} = \frac{5}{8}$ or $\underline{n = 62.5\%\ \text{loss}}$

6/215



$$\bar{I} = 4\left[\frac{1}{12}m(2r^2) + m\left(\frac{r}{\sqrt{2}}\right)^2\right]$$
$$= \frac{8}{3}mr^2$$

$\Sigma F_y = 0$: $N = 4mg\cos\theta$

$\Sigma F_x = ma_{G_x}$: $4mg\sin\theta - F = 4ma$ (1)

$\Sigma M_G = \bar{I}\alpha$: $Fr = \frac{8}{3}mr^2\alpha$ (2)

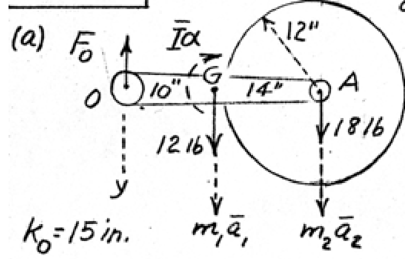
No slipping : $a = r\alpha$ (3)

Solution of (1)-(3) : $\begin{cases} a = \frac{3}{5}g\sin\theta, \ \alpha = \frac{3g}{5r}\sin\theta \\ F = \frac{8}{5}mg\sin\theta \end{cases}$

$$\mu_s = \frac{F}{N} = \frac{\frac{8}{5}mg\sin\theta}{4mg\cos\theta} = \underline{\frac{2}{5}\tan\theta}$$

6/216 Disk has no moment about its center so undergoes curvilinear translation with no $\bar{I}\alpha$.

(a)



$k_O = 15$ in.

For OA;

$m_1\bar{a}_1 = \frac{12}{32.2}\frac{10}{12}\alpha = 0.311\alpha$

$\bar{I}\alpha = \frac{12}{32.2}\left[\left(\frac{15}{12}\right)^2 - \left(\frac{10}{12}\right)^2\right]\alpha = 0.3235\alpha$

For disk;

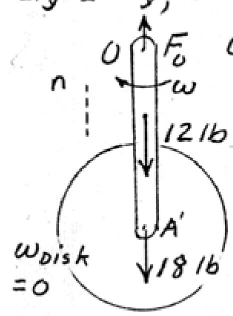
$m_2\bar{a}_2 = \frac{18}{32.2}\,\frac{24}{12}\alpha = 1.1180\,\alpha$

$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d;$

$12\frac{10}{12} + 18\frac{24}{12} = 0.3235\alpha + 0.311\alpha\left(\frac{10}{12}\right) + 1.1180\alpha\left(\frac{24}{12}\right)$

$\alpha = 46/2.818 = 16.32$ rad/sec^2

$\Sigma F_y = \Sigma m\bar{a}_y;\quad 18 + 12 - F_O = (0.311 + 1.1180)16.32,\quad \underline{F_O = 6.68 \text{ lb}}$



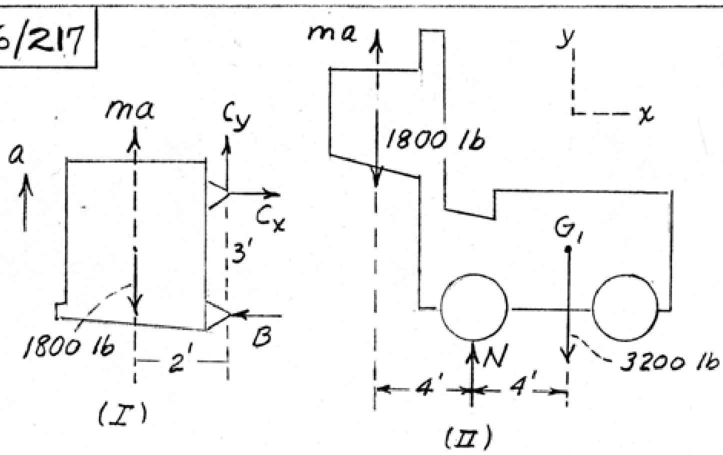
$U = \Delta T;\quad 12\frac{10}{12} + 18\frac{24}{12} = \frac{1}{2}\frac{12}{32.2}\left(\frac{15}{12}\right)^2\omega^2 + \frac{1}{2}\frac{18}{32.2}\left(\frac{24}{12}\omega\right)^2$

$\omega^2 = 32.6\,(\text{rad/s})^2$

$\Sigma F_n = \Sigma m\bar{a}_n;\quad F_O - 12 - 18 = \frac{12}{32.2}\,\frac{10}{12}(32.6) + \frac{18}{32.2}\,\frac{24}{12}(32.6)$

$\underline{F_O = 76.6 \text{ lb}}$

6/217

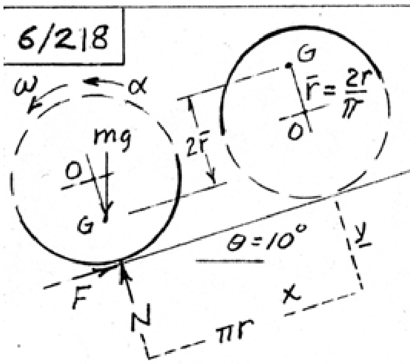


(II) $\Sigma M_N = m\bar{a}d$; $3200(4) - 1800(4) = \frac{1800}{32.2} a(4)$, $a = 25.04$ ft/sec^2

(I) $\Sigma M_C = m\bar{a}d$; $3B - 2(1800) = \frac{1800}{32.2}(25.04)(2)$

$\underline{B = 2130 \text{ lb}}$

6/218



$\bar I = I_O - m\bar r^2 = mr^2 - m\left(\frac{2r}{\pi}\right)^2$

$= mr^2(1 - 4/\pi^2)$

$U = \Delta T;\quad U = mg(2\bar r\cos\theta + \pi r\sin\theta)$

$= mgr\left(\frac{4}{\pi}\cos\theta + \pi\sin\theta\right)$

$\Delta T = \frac{1}{2}m\bar v^2 + \frac{1}{2}\bar I\omega^2$

$= \frac{1}{2}m[(r-\bar r)\omega]^2 + \frac{1}{2}mr^2\left(1 - \frac{4}{\pi^2}\right)\omega^2$

$= mr^2\omega^2(1 - 2/\pi)$

Thus $mgr\left(\frac{4}{\pi}\cos\theta + \pi\sin\theta\right) = mr^2\omega^2(1 - 2/\pi)$

$$\omega^2 = \frac{\left(\frac{4}{\pi}\cos\theta + \pi\sin\theta\right)\frac{g}{r}}{(1-2/\pi)},\quad \underline{\omega = \sqrt{\frac{g}{r}\,\frac{4\cos\theta + \pi^2\sin\theta}{\pi - 2}}}$$

$\Sigma F_y = m\bar a_y;\quad N - mg\cos\theta = m\bar r\omega^2$

$$N = mg\cos\theta + m\frac{2r}{\pi}\,\frac{\frac{4}{\pi}\cos\theta + \pi\sin\theta}{1 - 2/\pi}\,\frac{g}{r}$$

$$N = mg\left[\frac{\pi^2 - 2\pi + 8}{\pi(\pi-2)}\cos\theta + \frac{2\pi}{\pi-2}\sin\theta\right]$$

For $\theta = 10°$, $N = mg\,[3.231\cos 10° + 5.504\sin 10°] = \underline{4.14\,mg}$

6/219 For the entire spacecraft,

$$\Sigma M_x = I_x \alpha: \quad 10^{-6} = 150{,}000\,\alpha$$

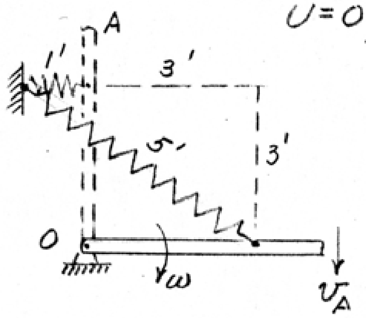
$$\alpha = 6.67 \times 10^{-12} \text{ rad/s}^2$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\frac{1}{3600}\left(\frac{\pi}{180}\right) = 0 + 0 + \frac{1}{2}\left(6.67 \times 10^{-12}\right) t^2$$

$$\underline{t = 1206 \text{ s}}$$

6/220 For system $U = \Delta T + \Delta V_g + \Delta V_e$



$U = 0,\ \Delta T = \frac{1}{2} I_O \omega^2 = \frac{1}{2}\left(\frac{1}{3} m l^2\right)\left(\frac{v_A}{l}\right)^2$

$= \frac{1}{6} m v_A^2 = \frac{1}{6} \frac{60}{32.2} v_A^2 = 0.3106\, v_A^2$

$\Delta V_e = \frac{1}{2} k x^2 - 0$

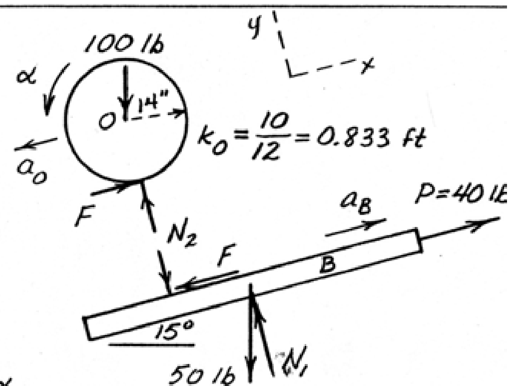
$= \frac{1}{2} 10 (5-1)^2 = 80$ ft-lb

$\Delta V_g = -60(2) = -120$ ft lb

Thus $0 = 0.3106\, v_A^2 - 120 + 80$

$v_A^2 = 128.8,\quad \underline{v_A = 11.35 \text{ ft/sec}}$

6/221



Slab

$\Sigma F_x = ma_x$: $40 - 50\sin 15° - F = \frac{50}{32.2} a_B$ --(1)

Wheel

$\Sigma F_x = m\bar{a}_x$: $F - 100 \sin 15° = \frac{100}{32.2}(-a_o)$ --(2)

$\Sigma M_o = I_o \alpha$: $F\frac{14}{12} = \frac{100}{32.2}(0.833)^2 \alpha$,

$F = \frac{1200}{451}(0.833)^2 \alpha = 1.849\,\alpha$ --(3)

Relative accel.: $(a_B + a_O)/\frac{14}{12} = \alpha$,

$a_o + a_B = 1.167\,\alpha$ --(4)

Solve (1), (2), (3), (4) & get $\underline{a_B = 7.04 \text{ ft/sec}^2 \text{ (+x-dir)}}$

$\underline{a_O = 3.14 \text{ ft/sec}^2 \text{ (-x-dir)}}$

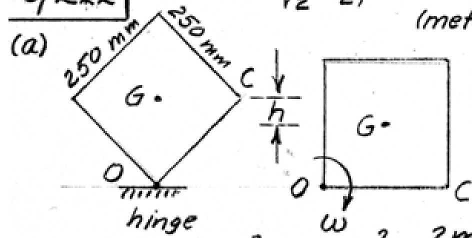
$F = 16.13$ lb

$(\mu_s)_{min} = F/N_2 = \frac{16.13}{100 \cos 15°} = \underline{0.1670}$

6/222 $h = 0.25\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) = 0.05178$ m (meters); $\bar{I} = \frac{1}{6}m(0.25)^2$

$= 0.01042m$, (m = mass)

(a)



$I_O = \bar{I} + m(0.25/\sqrt{2})^2$

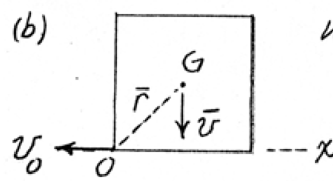
$= 0.04167m$

$\Delta V_g + \Delta T = 0$

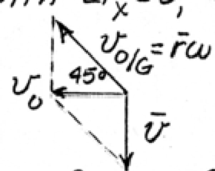
$-mgh + \frac{1}{2}I_O\omega^2 = 0$, $\omega^2 = \frac{2mgh}{I_O} = \frac{2m(9.81)(0.05178)}{0.04167m} = 24.38\ (\text{rad/s})^2$

$\underline{\omega = 4.94 \text{ rad/s}}$

(b)



With $\Sigma F_x = 0$, $\bar{a}$ & hence $\bar{v}$ remain vertical



$\bar{v} = \frac{\bar{r}\omega}{\sqrt{2}} = \frac{0.25}{2}\omega = 0.125\omega$

$\Delta V_g + \Delta T = 0$; $\Delta T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}m(0.125\omega)^2 + \frac{1}{2}(0.01042m)\omega^2$

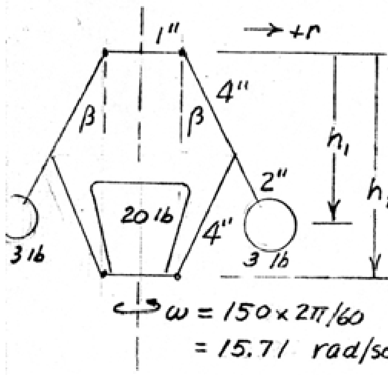
$= 13.02m\omega^2$

So $-mg(0.05178) + 0.01302m\omega^2 = 0$, $\omega^2 = \frac{9.81(0.05178)}{0.01302}$

$= 39.01\ (\text{rad/s})^2$

$\underline{\omega = 6.25 \text{ rad/s}}$

6/223 $\delta T_{\text{both balls}} = 2 m a_r \delta r$, $r = \frac{1}{12}(1 + 6\sin\beta)$ ft



$\omega = 150 \times 2\pi/60$
$= 15.71$ rad/sec

$a_r = -r\omega^2 = -r(15.71)^2$

$= -20.56\,(1 + 6\sin\beta)\ \frac{ft}{sec^2}$

$\delta T = -2\frac{3}{32.2}(20.56)(1 + 6\sin\beta)\frac{\cos\beta}{2}\delta\beta$

$= -1.916\,(1 + 6\sin\beta)\cos\beta\,\delta\beta$

$\delta V_g = -20\,\delta h_2 - 2(3)\delta h_1$

$\delta h_1 = \delta(6\cos\beta) = -6\sin\beta\,\delta\beta$

$\delta h_2 = \delta(2 \times 4\cos\beta) = -8\sin\beta\,\delta\beta$

$\delta V_g = [20(8) + 6(6)]\sin\beta\,\delta\beta/12$

$= 16.33\sin\beta\,\delta\beta$

$\delta U = \delta T + \delta V_e = 0$

$-1.916(1 + 6\sin\beta)\cos\beta\,\delta\beta + 16.33\sin\beta\,\delta\beta = 0$

$1 + 6\sin\beta = 8.526\tan\beta$

Solve by Newton's method of approximations & get $\underline{\beta = 19.26°}$

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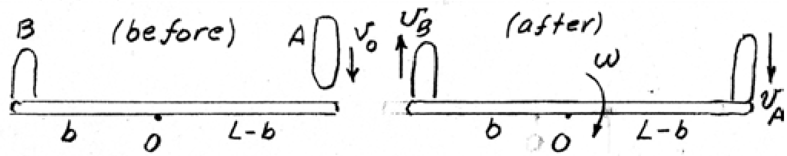


Before: $H_O = m_A v_o (L-b)$

After: $H_O = m_A v_A (L-b) + m_B v_B b$

$\Delta H_O = 0$ along with $\omega = v_B/b = v_A/(L-b)$ give

$$m_A v_o (L-b) = m_A \frac{L-b}{b} v_B (L-b) + m_B v_B b$$

$$v_B = v_o \frac{1}{\frac{L-b}{b} + n\frac{b}{L-b}} \quad \text{where } n = m_B/m_A$$

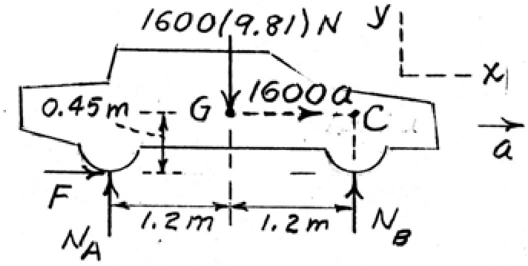
$$\frac{dv_B}{db} = v_o \frac{-\left(\frac{L}{b^2} + n\frac{L-b-b(-1)}{(L-b)^2}\right)}{\left(\frac{L-b}{b} + n\frac{b}{L-b}\right)^2} = v_o \frac{L\left(\frac{1}{b^2} - \frac{n}{(L-b)^2}\right)}{\left(\frac{L-b}{b} + n\frac{b}{L-b}\right)^2} = 0 \text{ for } v_{B_{max}}$$

So $\frac{1}{b^2} = \frac{n}{(L-b)^2}$, $b = \frac{L}{1 \pm \sqrt{n}}$ (+ sign gives positive v_B)

Thus $\underline{b = \frac{L}{1+\sqrt{n}}}$ which gives $v_B = \frac{v_o}{2\sqrt{n}}$

6/225

(a) Max. acceleration occurs when $F = \mu N_A = 0.8 N_A$



$\circlearrowleft + \Sigma M_C = mad = 0$: $1600(9.81)(1.2) - 2.4 N_A + 0.8 N_A (0.45) = 0$

$N_A = 9233$ N, $F = 0.8(9233) = 7386$ N

$\Sigma F_x = m a_x$: $7386 = 1600a$, $\underline{a = 4.62 \text{ m/s}^2}$

(b) Each rear wheel:

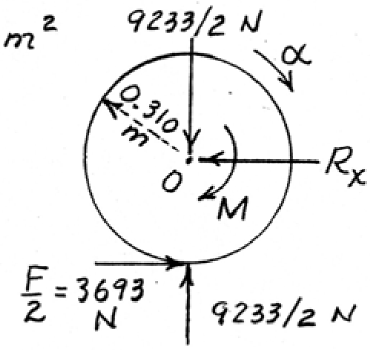
$I_O = mk^2 = 32(0.210)^2 = 1.411 \text{ kg}\cdot\text{m}^2$

$\alpha = \dfrac{a}{r} = \dfrac{4.62}{0.310} = 14.89 \text{ rad/s}^2$

$\Sigma M_O = I_O \alpha$:

$M - 3693(0.310) = 1.411(14.90)$

$\underline{M = 1166 \text{ N}\cdot\text{m}}$



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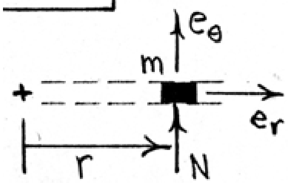


Conservation of angular momentum: $I_0\omega_0 = (I_0 + mr^2)\omega$

$$\dot{\theta} = \omega = \frac{I_0\omega_0}{I_0 + mr^2}$$

$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2): \quad 0 = m(\ddot{r} - r\dot{\theta}^2)$$

$$\ddot{r} = \dot{r}\frac{d\dot{r}}{dr} = r\left(\frac{I_0\omega_0}{I_0 + mr^2}\right)^2$$

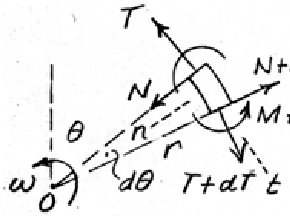
$$\int_0^{\dot{r}} \dot{r}\, d\dot{r} = I_0^2\omega_0^2 \int_0^r \frac{r\,dr}{(I_0 + mr^2)^2}$$

Integrating and solving for $\dot{r}$:

$$\dot{r} = \left(\frac{I_0\omega_0^2 r^2}{I_0 + mr^2}\right)^{1/2} = \underline{\omega_0 r\sqrt{\frac{I_0}{I_0 + mr^2}}}$$

▸6/227 $\Sigma F_n = m\bar{a}_n$; $2T\sin\frac{d\theta}{2} + dT\sin\frac{d\theta}{2} + N\cos\frac{d\theta}{2}$

$- (N+dN)\cos\frac{d\theta}{2} = \rho r d\theta\,(r\omega^2)$



Simplify & get $T - \rho r^2\omega^2 = \frac{dN}{d\theta}$ ----(1)

$\Sigma F_t = m\bar{a}_t = 0$; $-T\cos\frac{d\theta}{2} + (T+dT)\cos\frac{d\theta}{2}$

$+ N\sin\frac{d\theta}{2} + (N+dN)\sin\frac{d\theta}{2} = 0$

Simplify & get $N = -dT/d\theta$ -------- (2)

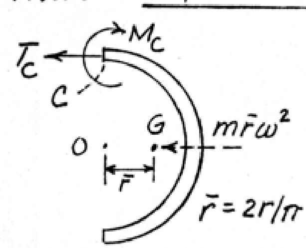
Combine (1) & (2) & get $\frac{d^2N}{d\theta^2} + N = 0$

Sol. $N = A\sin\theta + B\cos\theta$

By symmetry $N = 0$ for $\theta = 0$ so $B = 0$ & $N = A\sin\theta$

From (1) $T = \rho r^2\omega^2 + A\cos\theta$; $T = 0$ when $\theta = \pi$ so $A = \rho r^2\omega^2$

Thus $\underline{N = \rho r^2\omega^2\sin\theta}$ & $\underline{T = \rho r^2\omega^2(1+\cos\theta)}$

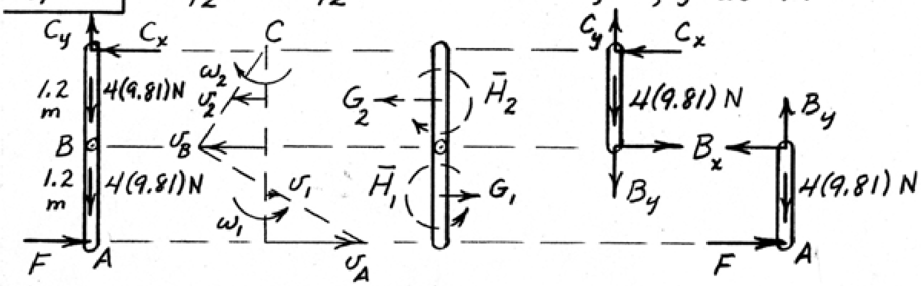


$\Sigma M_C = m\bar{a}d$; $M_C = m\frac{2r}{\pi}\omega^2 r$

$= \rho\pi r\left(\frac{2r^2}{\pi}\omega^2\right)$

$\underline{M_C = 2\rho r^3\omega^2}$

▸6/228 $\bar{I} = \frac{1}{12} m l^2 = \frac{1}{12}(4)(1.2)^2 = 0.48\ \text{kg}\cdot\text{m}^2$, $\int F\,dt = 14\ \text{N}\cdot\text{s}$



$\omega_2 = v_2/0.6$, $\omega_1 = (v_1 + v_B)/0.6 = (v_1 + 2v_2)/0.6$, $m = 4$ kg

System: $\int \Sigma M_C\,dt = \Sigma \Delta H_C$: $14(2.4) = 4v_1(1.8) + 0.48\omega_1$
$$- 4v_2(0.6) - 0.48\omega_2 \qquad (a)$$

AB: $\int \Sigma M_C\,dt = \Delta H_C$: $14(2.4) - \int 1.2 B_x\,dt = 4v_1(1.8) + 0.48\omega_1$ (b)

$\int \Sigma F_x\,dt = \Delta G_x$: $14 - \int B_x\,dt = 4v_1$ (c)

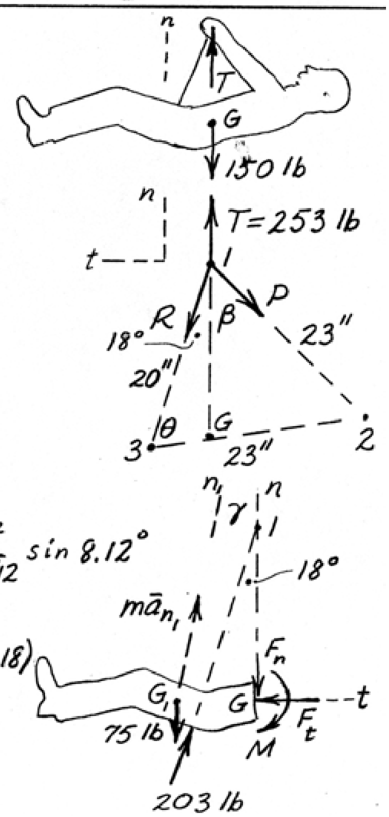
(b) & (c) & ω_1 give $2v_1 + v_2 = 10.5$

(a) & ω_1 & ω_2 give $5v_1 - v_2 = 21$

Combine & get $v_1 = 4.5$ m/s, $v_2 = 1.5$ m/s

& $\underline{\omega_2 = 2.50\ \text{rad/s}}$

▶6/229 *Fixed-axis rotation*



$\Sigma F_n = m\bar{a}_n: \; T - 150 = \dfrac{150}{32.2}\dfrac{13^2}{92/12}$,

$T = 253$ lb

$\theta = \cos^{-1}(10/23) = 64.2°$

$\beta = \theta - 18° = 46.2°$

$\Sigma F_n \doteq 0: \; 253 - R\cos 18° - P\cos 46.2° = 0$

$\Sigma F_t \doteq 0: \; R\sin 18° - P\cos 46.2° = 0$

Solve & get $P = 86.7$ lb, $R = 203$ lb

$\gamma = \sin^{-1}\dfrac{13}{92} = 8.12°$

$\Sigma F_t = m\bar{a}_t: \; 203\sin 18° - F_t = \dfrac{75}{32.2}\dfrac{13^2}{92/12}\sin 8.12°$

$F_t = 55.4$ lb

$\circlearrowleft + \Sigma M_O = I_O\alpha = 0: \; 203\sin 18°(92 - 18.18)$

$-75(13) + 55.4(92) + M = 0$

$\underline{M = 504 \text{ lb-in.}}$

***6/230**

$\Sigma M_O = I_O\alpha$: $78.5(0.220\cos\theta) = 8(0.235^2)\alpha$

$\alpha = 39.1\cos\theta$

$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta$: $\omega^2/2 = 39.1\sin\theta$

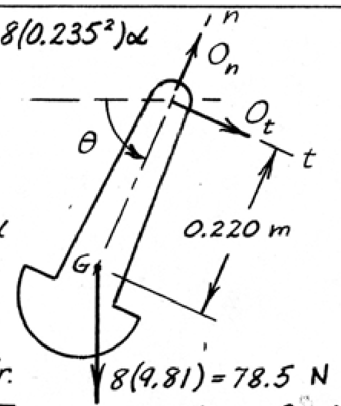
$\omega^2 = 78.2\sin\theta$

$\Sigma F_t = m\bar{a}_t$: $O_t + 78.5\cos\theta = 8\times 0.220\alpha$

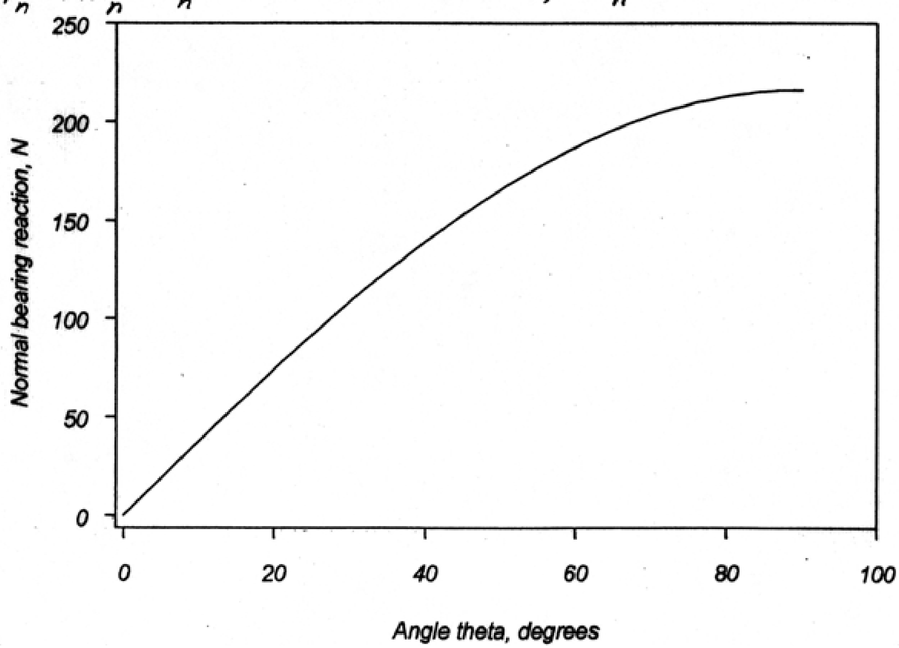
$O_t = 8(0.220)39.1\cos\theta - 78.5\cos\theta$

$O_t = -9.70\cos\theta$ N

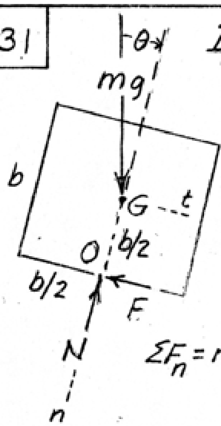
$(O_t)_{max} = 9.70$ N at $\theta = 0$, $-t$-dir.



$\Sigma F_n = m\bar{a}_n$: $O_n - 78.5\sin\theta = 8\times 0.220\omega^2$, $O_n = 216\sin\theta$ N



*6/231



$I_O = \frac{1}{12}m(b^2+b^2) + m(b/2)^2 = \frac{5}{12}mb^2$

$\Sigma M_O = I_O\alpha;\ mg\frac{b}{2}\sin\theta = \frac{5}{12}mb^2\alpha$

$\alpha = \frac{6}{5}\frac{g}{b}\sin\theta$

$\int_0^\omega \omega\,d\omega = \int_0^\theta \alpha\,d\theta;\quad \omega^2 = \frac{12}{5}\frac{g}{b}\int_0^\theta \sin\theta\,d\theta$

$\omega^2 = \frac{12}{5}\frac{g}{b}(1-\cos\theta)$

$\Sigma F_n = m\bar{a}_n;\ mg\cos\theta - N = m\frac{b}{2}\left(\frac{12}{5}\frac{g}{b}\right)(1-\cos\theta)$

$N = \frac{mg}{5}(11\cos\theta - 6)$

$\Sigma F_t = m\bar{a}_t;\ mg\sin\theta - F = m\frac{b}{2}\left(\frac{6}{5}\frac{g}{b}\sin\theta\right)$

$F = \frac{2}{5}mg\sin\theta$

Compute & plot N/mg & F/mg

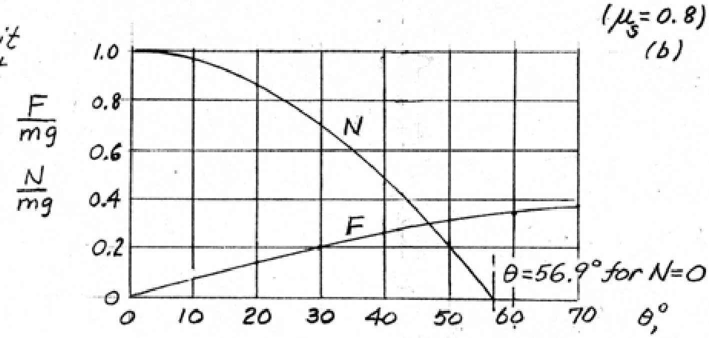
For $F = 0.8N$, $\frac{2}{5}\sin\theta = 0.8\left(\frac{1}{5}\right)(11\cos\theta - 6)$, $5\sin\theta = 22\cos\theta - 12$

Solve by Newton's method & get slip at $\underline{\theta = 45.1°}$

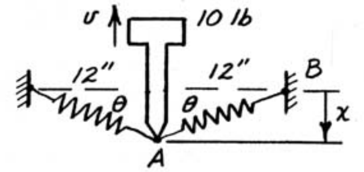
$(\mu_s = 0.8)$

(b)

(a) for no limit on F, contact ceases when $N = 0$ so $\theta = \cos^{-1}6/11$ $\underline{= 56.9°}$



$\boxed{*6/232}$ $U' = \Delta T + \Delta V_e + \Delta V_g$; $U' = 0$



$\Delta T = \frac{1}{2} m v^2 - 0 = \frac{1}{2}\frac{10}{32.2} v^2$ ft-lb

$\Delta V_e = \frac{2}{2} k (x_2^2 - x_1^2) = 6\left[(\sqrt{x^2+12^2}-12)^2 - (15-12)^2\right]\frac{1}{12}$

$= \frac{1}{2}\left[x^2 - 24\sqrt{x^2+144} + 279\right]$ ft-lb $\quad \overline{AB} = 15''$ when $x = 9''$

where x is in inches

$\Delta V_g = 10(9-x)/12 = \frac{5}{6}(9-x)$ ft-lb

$$\frac{5}{32.2} v^2 + \frac{x^2}{2} - 12\sqrt{x^2+144} + \frac{279}{2} + \frac{15}{2} - \frac{5x}{6} = 0$$

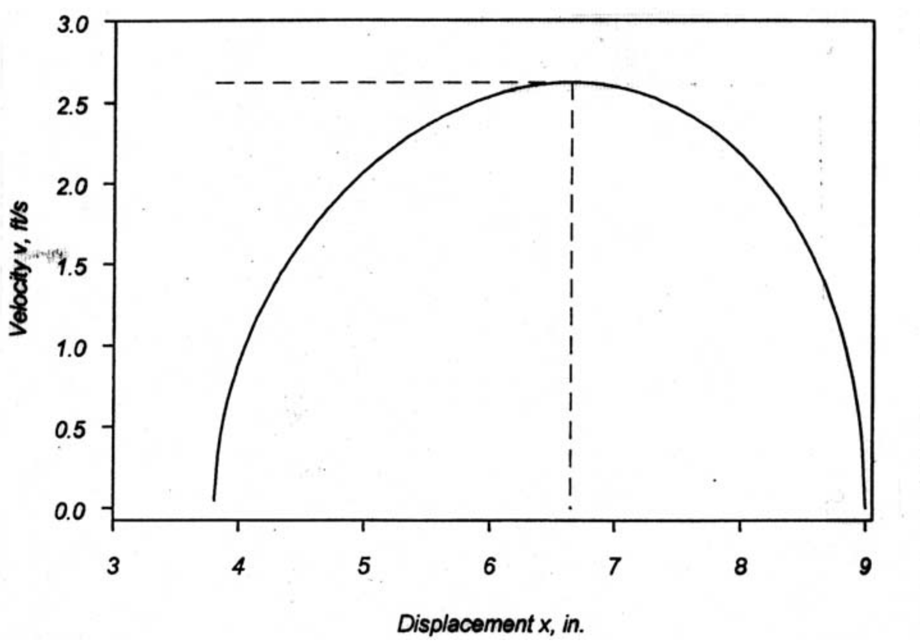
$$v^2 = \frac{32.2}{5}\left\{12\sqrt{x^2+144} - \frac{x^2}{2} + \frac{5x}{6} - 147\right\} \text{ (ft/sec)}^2$$

(where x is in inches)

Plot v vs. x (see continuation)

$v = 0$ at $x = 3.81$ in.

$v_{max} = 2.62$ ft/sec at $x = 6.65$ in.

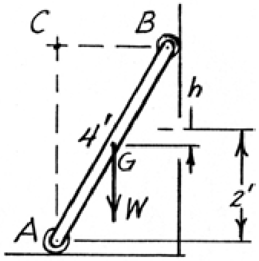


*6/233

$U = \Delta T$: $T = \frac{1}{2} I_C \omega^2 = \frac{1}{2}\frac{1}{3}\frac{W}{g}4^2\omega^2$

$U = Wh = W(2 - 2\cos\theta)$

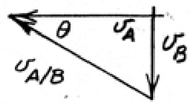
$= 2W(1-\cos\theta)$



Thus $2W(1-\cos\theta) = \frac{8W}{3g}\omega^2$, $\omega^2 = \frac{3g}{4}(1-\cos\theta)$

$\omega = \sqrt{(3\times 32.2/4)(1-\cos\theta)} = 4.91\sqrt{1-\cos\theta}$ rad/sec

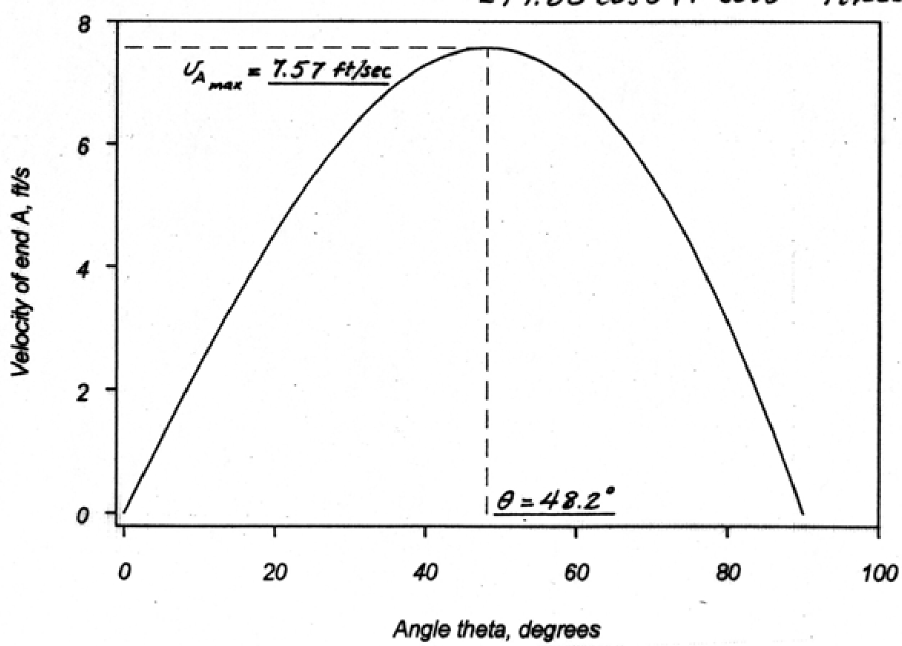
$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$:



$v_A = v_{A/B}\cos\theta = L\omega\cos\theta$

$= 4(4.91)\sqrt{1-\cos\theta}\cos\theta$ ft/sec

$= 19.66\cos\theta\sqrt{1-\cos\theta}$ ft/sec



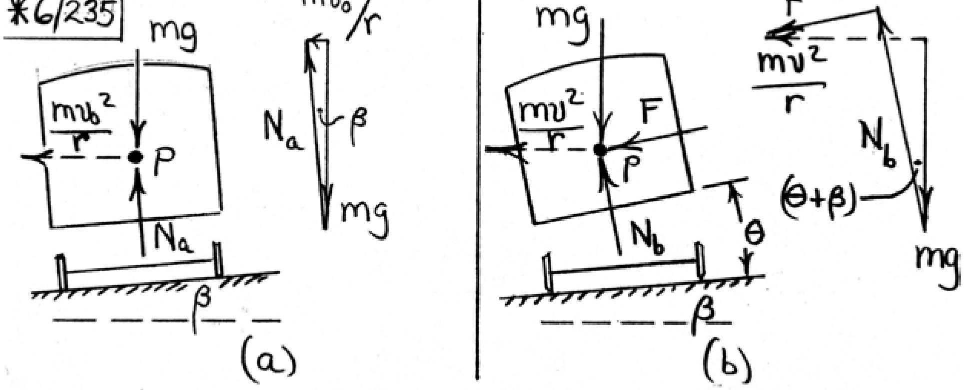
*6/234 From the solution of Prob. 6/23,

$$K\theta - \frac{5}{2} mgl \sin\theta - \frac{5}{2} mal \cos\theta = 0$$

With numbers : $75\theta - 7.36 \sin\theta - 14.72 \cos\theta = 0$

Numerical solution : $\underline{\theta = 12.17^\circ}$

*6/235



(Passenger is shown as particle P above)

Note that $F = 0.3mv^2/r$

(a) $\tan\beta = \dfrac{mv_0^2/r}{mg} = \dfrac{v_0^2}{gr} = \dfrac{(160/3.6)^2}{9.81(1900)}$

$\underline{\beta = 6.05°}$

(b) From the force polygon,

$$mg\sin(\theta+\beta) + \frac{0.3mv^2}{r} = \frac{mv^2}{r}\cos(\theta+\beta)$$

$$9.81\sin(\theta+\beta) + \frac{(260/3.6)^2}{1900}\left(0.3 - \cos(\theta+\beta)\right) = 0$$

$$9.81\sin(\theta+\beta) + 2.75\left[0.3 - \cos(\theta+\beta)\right] = 0$$

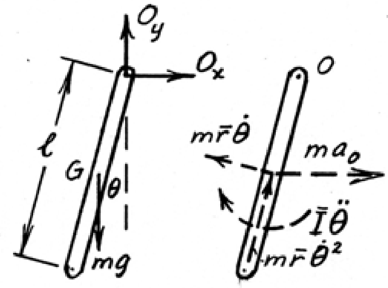
Numerical solution : $\underline{\theta = 4.95°}$

*6/236

$\Sigma M_O = \bar{I}\alpha + m\bar{a}d$:

$\circlearrowleft + \; -mg\frac{l}{2}\sin\theta = \frac{1}{12}ml^2\ddot{\theta} + m\frac{l}{2}\ddot{\theta}\left(\frac{l}{2}\right) - ma_O\frac{l}{2}\cos\theta$

$$\ddot{\theta} = \frac{3}{l}\left(\frac{a_O}{2}\cos\theta - \frac{g}{2}\sin\theta\right)$$



$\int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \int_0^{\theta} \ddot{\theta}\,d\theta$:

$$\frac{\dot{\theta}^2}{2} = \frac{3}{l}\int_0^{\theta}\left(\frac{a_O}{2}\cos\theta - \frac{g}{2}\sin\theta\right)d\theta$$

$$\dot{\theta}^2 = \frac{6}{l}\left(\frac{a_O}{2}\sin\theta - \frac{g}{2}[1-\cos\theta]\right) = 1.5\left(\sin\theta - \frac{9.81}{2}[1-\cos\theta]\right)\frac{\text{rad}^2}{\text{s}^2}$$

$\dot{\theta} = 0$ when $R = \sin\theta - \frac{9.81}{2}(1-\cos\theta) = 0$

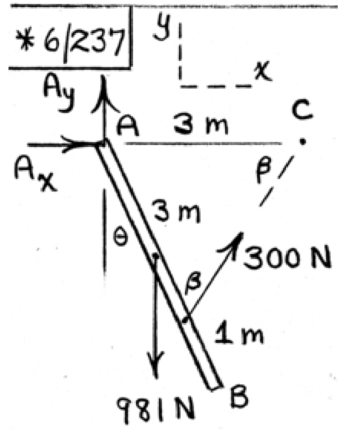
Solve numerically & get

$\underline{\theta_{max} = 23.0^\circ}$

$\dot{\theta}$ is max. when $\ddot{\theta} = 0$: $\frac{a_O}{2}\cos\theta - \frac{g}{2}\sin\theta = 0$ or $\theta = \tan^{-1}\frac{a_O}{g}$

$\underline{\theta = 11.52^\circ}$, $(\dot{\theta}^2)_{max} = 0.1513\ (\text{rad/s})^2$, $\underline{\dot{\theta}_{max} = 0.389\ \text{rad/s}}$

*6/237



$$2\beta + \left(\frac{\pi}{2} - \theta\right) = \pi \quad , \quad \beta = \frac{\pi}{4} + \frac{\theta}{2}$$

$$\sin\beta = \sin\frac{\pi}{4}\cos\frac{\theta}{2} + \cos\frac{\pi}{4}\sin\frac{\theta}{2}$$

$$= \frac{1}{\sqrt{2}}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)$$

$$I_A = \frac{1}{3}ml^2 = \frac{1}{3}(100)(4)^2$$

$$= 533 \text{ kg}\cdot\text{m}^2$$

$$\circlearrowleft + \Sigma M_A = I_A\alpha: \quad 300(3\sin\beta) - 981(2\sin\theta) = 533\alpha$$

$$\frac{900}{\sqrt{2}}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right) - 1962\sin\theta = 533\alpha \quad (1)$$

$$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta: \quad \omega^2 = \frac{2}{533}\int_0^\theta \left[\frac{900}{\sqrt{2}}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right) - 1962\sin\theta\right] d\theta$$

$$\text{or} \quad \omega^2 = \frac{2}{533}\left[\frac{1800}{\sqrt{2}}\left(1 - \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) - 1962(1 - \cos\theta)\right] \quad (2)$$

(a) For max ω, set $\alpha = 0$ in (1) & solve for θ:

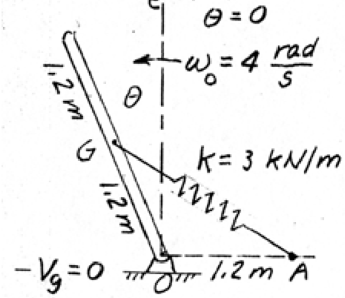
$\underline{\theta = 22.4°}$ From (2): $\underline{\omega_{max} = 0.680 \frac{\text{rad}}{\text{s}}}$

(b) Solve (2) for $\omega = 0$: $\underline{\theta_{max} = 45.9°}$

*6/238

$m = 30$ kg

$V_e = 0$ when $\theta = 0$



$-V_g = 0$

$\Delta V_g = -30(9.81)(1.2)(1-\cos\theta)$
$= -353(1-\cos\theta)$ ✓

$U = 0 = \Delta T + \Delta V_g + \Delta V_e$

$\overline{AG} = 1.2\sqrt{2(1+\sin\theta)}$

$\Delta V_e = \frac{1}{2}k\left[\overline{GA} - 1.2\sqrt{2}\right]^2$

$= \frac{3000}{2}\left[1.2\sqrt{2(1+\sin\theta)} - 1.2\sqrt{2}\right]^2$

$= 4320\left(2 + \sin\theta - 2\sqrt{1+\sin\theta}\right)$ ✓

$\Delta T = \frac{1}{2}I_O(\omega^2 - \omega_0^2)$

$= \frac{1}{2}\left[\frac{1}{3}30(2.4)^2(\omega^2 - 16)\right]$

$= 28.8(\omega^2 - 16)$ ✓

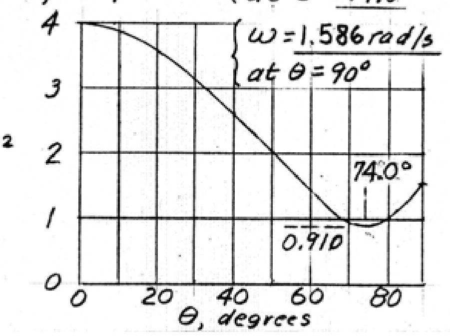
Thus

$0 = 28.8(\omega^2 - 16) - 353(1-\cos\theta)$
$+ 4320\left(2 + \sin\theta - 2\sqrt{1+\sin\theta}\right)$

$\omega^2 = 300\sqrt{1+\sin\theta} - 150\sin\theta$
$- 12.26\cos\theta \quad - 271.7\ (rad/s)^2$

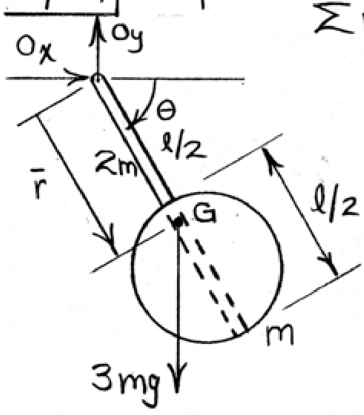
Set up computer program & solve for ω vs θ & get

$\left\{\begin{array}{l}\omega_{min} = \underline{0.910\ \frac{rad}{s}} \\ \text{at } \theta = \underline{74.0°}\end{array}\right.$



*6/239

$$\bar{r} = \frac{\Sigma m\bar{r}}{\Sigma m} = \frac{2m(l/2) + m(3l/4)}{3m} = \frac{7}{12}l$$



$$I_O = \frac{1}{3}(2m)l^2 + \left[\frac{1}{2}m\left(\frac{l}{4}\right)^2 + m\left(\frac{3l}{4}\right)^2\right]$$

$$= \frac{121}{96} m l^2$$

$$\circlearrowright \Sigma M_O = I_O \alpha :$$

$$3mg\left(\frac{7}{12}l\cos\theta\right) = \frac{121}{96} m l^2 \alpha$$

$$\alpha = \frac{168}{121}\frac{g}{l}\cos\theta$$

$$\alpha = \omega\frac{d\omega}{d\theta} = \frac{168}{121}\frac{g}{l}\cos\theta$$

$$\int_{\omega_0}^{\omega} \omega\, d\omega = \frac{168}{121}\frac{g}{l}\int_0^{\theta}\cos\theta\, d\theta$$

$$\Rightarrow \omega = \frac{d\theta}{dt} = \left[\omega_0^2 + \frac{336}{121}\frac{g}{l}\sin\theta\right]^{1/2}$$

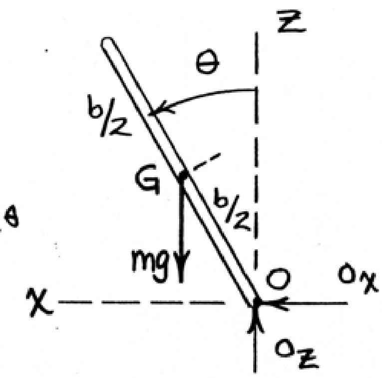
$$\Rightarrow t = \int_0^{\theta}\frac{d\theta}{\left[\omega_0^2 + \frac{336}{121}\frac{g}{l}\sin\theta\right]^{1/2}}$$

Numerical solution with $\begin{cases} \omega_0 = 3 \text{ rad/s} \\ l = 0.8 \text{ m} \\ \theta = \pi/2 \end{cases}$: $\underline{t = 0.302\text{ s}}$

***6/240** $\circlearrowleft + \Sigma M_O = I_O \ddot{\theta}$:

$$mg\frac{b}{2}\sin\theta = \frac{1}{3}mb^2\ddot{\theta}$$

$$\ddot{\theta} = \frac{3}{2}\frac{g}{b}\sin\theta$$



$$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta: \int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \frac{3g}{2b}\int_{\theta_0}^{\theta}\sin\theta\,d\theta$$

$$\frac{\dot{\theta}^2}{2} - \frac{\dot{\theta}_0^2}{2} = \frac{3g}{2b}\left(\cos\theta_0 - \cos\theta\right)$$

$$\frac{d\theta}{dt} = \left[\dot{\theta}_0^2 + \frac{3g}{b}\left(\cos\theta_0 - \cos\theta\right)\right]^{1/2}$$

$$\int_0^t dt = \int_{\theta_0}^{\theta} \frac{d\theta}{\left[\dot{\theta}_0^2 + \frac{3g}{b}\left(\cos\theta_0 - \cos\theta\right)\right]^{1/2}}$$

With $\theta_0 = 10°$ (0.1745 rad), $b = 60'$, $g = 32.2 \frac{ft}{sec^2}$ and $\dot{\theta}_0 = \frac{(v_A)_0}{b} = \frac{4.5}{60} = 0.0750$ rad/sec, a numerical solution yields $\underline{t = 2.85 \text{ sec}}$.

Energy considerations from $\theta_0 = 10°$ to $\theta = 90°$:

$$\Delta T + \Delta V_g = 0$$

$$\Delta T = \frac{1}{2}I_0\left[\frac{v_A}{b}\right]^2 - \frac{1}{2}I_0\left[\frac{(v_A)_0}{b}\right]^2 = \frac{1}{6}m\left[v_A^2 - (v_A)_0^2\right]$$

$$\Delta V_g = -mgh = -mg\frac{b}{2}\cos 10°$$

So $\frac{1}{6}\cancel{m}\left[v_A^2 - 4.5^2\right] - \cancel{m}(32.2)\frac{60}{2}\cos 10° = 0$

$$\underline{v_A = 75.7 \text{ ft/sec}}$$