5/1

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{900 - 300}{6/60} = 6000 \text{ rev/min}^2$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta, \quad \theta = N = \frac{\omega_2^2 - \omega_1^2}{2\alpha}$$

$$= \frac{(900)^2 - (300)^2}{2(6000} = \underline{60 \text{ rev}}$$

5/2 (a) $\underline{v}_A = \underline{\omega} \times \underline{r}_{A/O} = -6\underline{k} \times 45\underline{j}$

$= \underline{270\underline{i} \text{ mm/s}}$

$\underline{a}_A = \underline{\alpha} \times \underline{r}_{A/O} - \omega^2 \underline{r}_{A/O} = 4\underline{k} \times 45\underline{j} - 6^2(45\underline{j})$

$= \underline{-180\underline{i} - 1620\underline{j} \text{ mm/s}^2}$

(b) $\underline{v}_B = \underline{\omega} \times \underline{r}_{B/O} = -6\underline{k} \times (-30\underline{i} + 45\underline{j})$

$= \underline{270\underline{i} + 180\underline{j} \text{ mm/s}}$

$\underline{a}_B = \underline{\alpha} \times \underline{r}_{B/O} - \omega^2 \underline{r}_{B/O}$

$= 4\underline{k} \times (-30\underline{i} + 45\underline{j}) - 6^2(-30\underline{i} + 45\underline{j})$

$= \underline{900\underline{i} - 1740\underline{j} \text{ mm/s}^2}$

5/3 $\omega = 12 - 3t^2$; when $\omega = 0$, $t^2 = 4$, $t = 2$ s

$\int_0^{\Delta\theta} d\theta = \int_0^t \omega\, dt$; $\Delta\theta = \int_0^3 (12 - 3t^2)\, dt = \left[12t - t^3\right]_0^3$

$= \underline{9 \text{ rad}}$

$\theta_1 = \int_0^2 (12 - 3t^2)\, dt = \left[12t - t^3\right]_0^2 = 16 \text{ rad (CW)}$

$\theta_2 = \int_2^3 (12 - 3t^2)\, dt = \left[12t - t^3\right]_2^3 = -7 \text{ rad} \quad \text{(CCW)}$

The total number of turns is

$N = (16 + 7)/2\pi = \underline{3.66} \text{ rev}$

$\boxed{5/4}$ Let $\underline{k}$ be a unit vector out of paper.

(a) $\underline{v}_A = \underline{\omega} \times \underline{r}_{A/o} = 3\underline{k} \times (-0.4\underline{e}_n) = \underline{1.2\underline{e}_t \text{ m/s}}$

$\underline{a}_A = \underline{\alpha} \times \underline{r}_{A/o} - \omega^2 \underline{r}_{A/o} = -14\underline{k} \times (-0.4\underline{e}_n) - 3^2(-0.4\underline{e}_n)$

$= \underline{-5.6\underline{e}_t + 3.6\underline{e}_n \text{ m/s}^2}$

(b) $\underline{v}_B = \underline{\omega} \times \underline{r}_{B/o} = 3\underline{k} \times (-0.4\underline{e}_n + 0.1\underline{e}_t)$

$= \underline{1.2\underline{e}_t + 0.3\underline{e}_n \text{ m/s}}$

$\underline{a}_B = \underline{\alpha} \times \underline{r}_{B/o} - \omega^2 \underline{r}_{B/o}$

$= -14\underline{k} \times (-0.4\underline{e}_n + 0.1\underline{e}_t) - 3^2(-0.4\underline{e}_n + 0.1\underline{e}_t)$

$= \underline{-6.5\underline{e}_t + 2.2\underline{e}_n \text{ m/s}^2}$

5/5 For v constant $a_t = 0$ & $a = a_n = v^2/r$

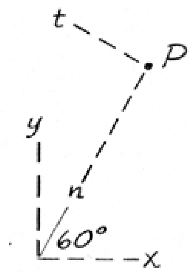
$\left(\frac{v^2}{r}\right)_A = \frac{2}{3}\left(\frac{v^2}{3}\right)_B$, $\underline{r = 4.5 \text{ in.}}$

5/6 For $\theta = 90°$, $\underline{a} = -a_t \underline{i} - a_n \underset{\sim}{j}$ so $a_t = r\alpha = 1.8\ m/s^2$,

$$\alpha = \frac{1.8}{0.3} = \underline{6\ rad/s^2}$$

$$\&\ a_n = r\omega^2 = 4.8\ m/s^2,\ \omega = \sqrt{4.8/0.3} = \underline{4\ rad/s}$$

5/7



$a_x = -3.02 \text{ m/s}^2$

$a_y = -1.624 \text{ m/s}^2$

$a_t = 3.02 \sin 60° - 1.624 \cos 60° = 1.803 \ \frac{m}{s^2}$

$a_n = 3.02 \cos 60° + 1.624 \sin 60° = 2.92 \ \frac{m}{s^2}$

$a_t = r\alpha: \ \alpha = 1.803/0.3 = \underline{6.01 \text{ rad/s}^2}$

$a_n = r\omega^2: \ \omega^2 = 2.92/0.3 = 9.72 \text{ (rad/s)}^2; \ \underline{\omega = 3.12 \text{ rad/s}}$

5/8

$$\alpha = -k\omega^2 = \omega \frac{d\omega}{d\theta}$$

$$-k\int_{\theta_0}^{\theta} d\theta = \int_{\omega_0}^{\omega} \frac{d\omega}{\omega}$$

$$-k(\theta-\theta_0) = \ln\left(\frac{\omega}{\omega_0}\right) \Rightarrow \omega = \omega_0{}^{-k(\theta-\theta_0)}$$

When $\omega = \frac{\omega_0}{3}$: $\frac{\omega_0}{3} = \omega_0 e^{-k(\theta-\theta_0)}$

With $k = 0.1$, $\underline{(\theta-\theta_0) = 10.99 \text{ rad}}$

Now set $\alpha = -k\omega^2 = \frac{d\omega}{dt}$

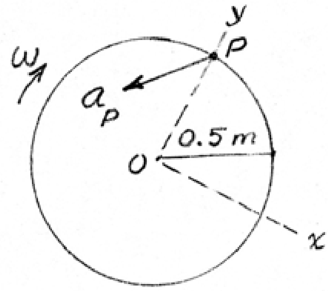
$$-k\int_0^t dt = \int_{\omega_0}^{\omega} \frac{d\omega}{\omega^2}$$

$$-kt = -\left(\frac{1}{\omega} - \frac{1}{\omega_0}\right)$$

When $\omega = \omega_0/3$, with $k = 0.1$: $t = \frac{20}{\omega_0}$

With $\omega_0 = 12$ rad/s, $\underline{t = 1.667 \text{ s}}$

5/9 For point P, $\underset{\sim}{a}_P = -3\underset{\sim}{i} - 4\underset{\sim}{j}$ m/s²



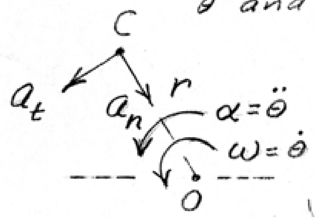
$a_n = r\omega^2$, $4 = 0.5\,\omega^2$, $\omega = \sqrt{8}$ $\frac{rad}{s}$

$\underline{\underset{\sim}{\omega} = -\sqrt{8}\ \underset{\sim}{k}\ rad/s}$

$a_t = r\alpha$, $3 = 0.5\alpha$, $\alpha = 6$ rad/s²

$\underline{\underset{\sim}{\alpha} = 6\underset{\sim}{k}\ rad/s^2}$

5/10 All lines including OC have the same $\dot{\theta}$ and $\ddot{\theta}$; $r = \frac{2}{3}(0.150)\frac{\sqrt{3}}{2} = 0.0866$ m



$a_n = r\omega^2$, $\dot{\theta} = \omega = \sqrt{a_n / r}$

$= \sqrt{80/0.0866}$

$= \underline{30.4 \text{ rad/s}}$

$a_t = r\alpha$, $\ddot{\theta} = \alpha = a_t / r$

$= 30/0.0866$

$= \underline{346 \text{ rad/s}^2}$

5/11

$\omega_{av} = \dfrac{\Delta\theta}{\Delta t}$ where $\Delta\theta = \pi/2$ rad

$\Delta t = \dfrac{\Delta s}{v} = \dfrac{40}{10} = 4$ sec

So $\omega_{av} = \dfrac{\pi/2}{4} = \underline{0.393 \text{ rad/sec}}$

$\boxed{5/12}$ $\underline{v}_P = \underline{\omega} \times \underline{r} = 2\underline{k} \times [0.5\underline{i} + 0.2\underline{j} + 0.050\underline{k}]$

$$= \underline{-0.4\underline{i} + \underline{j} \quad m/s}$$

$$\underline{a}_P = \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$$

$$= -3\underline{k} \times [0.5\underline{i} + 0.2\underline{j} + 0.050\underline{k}]$$

$$+ 2\underline{k} \times [2\underline{k} \times (0.5\underline{i} + 0.2\underline{j} + 0.050\underline{k})]$$

$$= \underline{-1.4\underline{i} - 2.3\underline{j} \quad m/s^2}$$

Note that $\underline{r}$ could have been taken as $0.5\underline{i} + 0.2\underline{j}$ m

The magnitudes of the above results are

$$v_P = 1.077 \text{ m/s} \quad \text{and} \quad a_P = 2.69 \text{ m/s}^2.$$

These magnitudes check with

$$v_P = r_{xy}\,\omega = \sqrt{0.5^2 + 0.2^2}\,(2) = 1.077 \text{ m/s}^2 \checkmark$$

and $a_P = \sqrt{a_t^2 + a_n^2} = \sqrt{(r_{xy}\alpha)^2 + (r_{xy}\omega^2)^2}$

$$= \sqrt{0.5^2 + 0.2^2}\,\sqrt{3^2 + 2^4} = 2.69 \text{ m/s}^2 \checkmark$$

5/13

$\underline{\omega} = 40\left(\frac{3}{5}\underline{j} + \frac{4}{5}\underline{k}\right) = 8(3\underline{j} + 4\underline{k})$ rad/sec

$\underline{r} = 15\underline{i} + 16\underline{j} - 12\underline{k}$ in.

$$\underline{v} = \underline{\omega}\times\underline{r} = 8\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 3 & 4 \\ 15 & 16 & -12 \end{vmatrix} = -800i + 480j - 360\underline{k} \text{ in./sec}$$

$$= \underline{40(-20\underline{i} + 12\underline{j} - 9\underline{k}) \text{ in./sec}}$$

$$\underline{a} = \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times\underline{v}$$

$$= 0 + 8\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 3 & 4 \\ -800 & 480 & -360 \end{vmatrix} = 800(-30\underline{i} - 32\underline{j} + 24\underline{k}) \frac{\text{in.}}{\text{sec}^2}$$

$$= \underline{1600(-15\underline{i} - 16\underline{j} + 12\underline{k}) \text{ in./sec}^2}$$

$r = \sqrt{\overline{15}^2 + \overline{16}^2 + \overline{12}^2} = 25$ in.

$v = r\omega = 25(40) = 1000$ in./sec ; $|\underline{v}| = 40\sqrt{\overline{20}^2 + \overline{12}^2 + \overline{9}^2} = 40(25) = 1000$ in./sec

$a_n = r\omega^2 = 25(40)^2 = 40(10^3)$ in./sec^2 ; $|\underline{a}| = 1600\sqrt{\overline{15}^2 + \overline{16}^2 + \overline{12}^2}$

$= 1600(25) = 40(10^3)$ in./sec^2

(Checks)

5/14 $\underline{\omega}_{OA} = \underline{\omega}_{BC} = -6\underline{k}$ rad/s

$\underline{r}_A = 0.3\underline{i} + 0.28\underline{j}$ m

$\underline{v}_A = \underline{\omega} \times \underline{r}_A = -6\underline{k} \times (0.3\underline{i} + 0.28\underline{j}) = -1.8\underline{j} + 1.68\underline{i}$ m/s

$\underline{v}_A = 1.68\underline{i} - 1.8\underline{j}$ m/s

$\underline{a}_A = \dot{\underline{\omega}} \times \underline{r}_A + \underline{\omega} \times \underline{v}_A = 0 + (-6\underline{k}) \times (1.68\underline{i} - 1.8\underline{j})$

$= -10.08\underline{j} - 10.8\underline{i}$

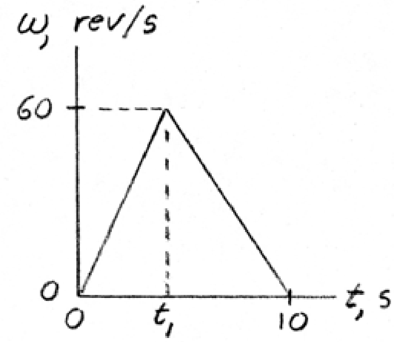
$\underline{a}_A = -10.8\underline{i} - 10.08\underline{j}$ m/s^2

5/15

$N = \Delta\theta = \int_0^{10} \omega \, dt = \text{area}$

$= \frac{1}{2}(10)(60) = \underline{300 \text{ rev}}$

(independent of t_1)

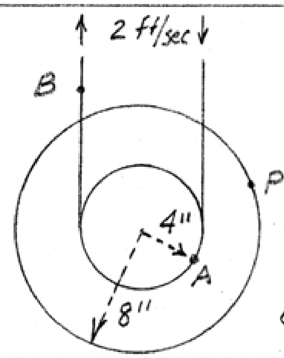


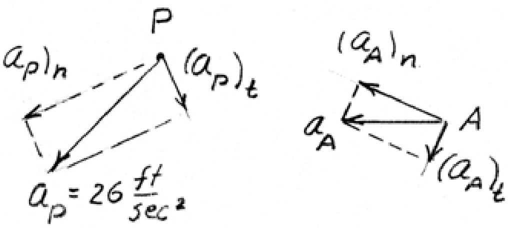
5/16 At B, $v = \frac{50}{30} 44 = 73.3$ ft/sec, $r = 180 - \frac{18}{12} = 178.5$ ft

$\omega = v/r = 73.3/178.5 = \underline{0.411 \text{ rad/sec}}$

Between A & B $\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{30}{180}\pi/1.52 = \underline{0.344 \text{ rad/sec}}$

5/117





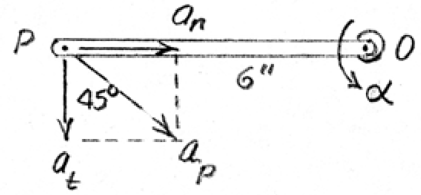
$\omega = v/r = \frac{2}{4/12} = 6 \text{ rad/sec}$

$(a_P)_n = r\omega^2 = \frac{8}{12} 6^2 = 24 \text{ ft/sec}^2$

$(a_P)_t = \sqrt{26^2 - 24^2} = 10 \text{ ft/sec}^2, \quad \alpha = \frac{a_t}{r} = \frac{10}{8/12} = \underline{15 \text{ rad/sec}^2}$

$a_B = (a_A)_t = \frac{4}{8}(a_P)_t = \frac{4}{8} 10 = \underline{5 \text{ ft/sec}^2}$

5/18



$\alpha = \frac{600(2\pi)}{60}\frac{1}{2} = 10\pi \text{ rad/sec}^2$

$a_t = r\alpha = 6(10\pi) = 60\pi \text{ in./sec}^2$

$a_n = r\omega^2 = 60\pi \text{ in./sec}^2$ for 45°

so $\omega^2 = 60\pi/6 = 10\pi$, $\omega = 5.60$ rad/s

$\omega = \omega_0 + \alpha t$: $5.60 = 0 + 10\pi t$, $\underline{t = 0.1784 \text{ sec}}$

5/19 $\Delta\theta = (30-0)2\pi = 60\pi$ rad

$\alpha = 10 + k\theta$, $20 = 10 + 60\pi k$, $k = \frac{1}{6\pi}$

so $\alpha = 10 + \frac{\theta}{6\pi}$

$\int_{\omega_0}^{90} \omega \, d\omega = \int_0^{60\pi} \left(10 + \frac{\theta}{6\pi}\right) d\theta$, $(90)^2 - \omega_0^2 = 2\left[10\theta + \frac{\theta^2}{12\pi}\right]_0^{60\pi}$

$\omega_0^2 = 8100 - 2\left[600\pi + 300\pi\right] = 2445$, , $\underline{\omega_0 = 49.4 \text{ rad/s}}$

5/20 (a) $\alpha = -0.05\omega = \frac{d\omega}{dt}$

$$-0.05\,dt = \frac{d\omega}{\omega}$$

$$-0.05\int_0^t dt = \int_{\omega_0}^{\omega} \frac{d\omega}{\omega}$$

$$-0.05t = \ln\left(\frac{\omega}{\omega_0}\right)$$

$$\Rightarrow \omega = \omega_0 e^{-0.05t}, \quad \omega = 100e^{-0.05(10)} = \underline{60.7 \frac{rad}{s}}$$

(b) $\alpha = -0.05\omega = \omega \frac{d\omega}{d\theta}$

$$-0.05\,d\theta = d\omega$$

$$-0.05\int_0^\theta d\theta = \int_{\omega_0}^{\omega} d\omega$$

$$-0.05\theta = \omega - \omega_0$$

$$\omega = \omega_0 - 0.05\theta, \quad \omega = 100 - 0.05(10\cdot 2\pi)$$

$$= \underline{96.9 \text{ rad/s}}$$

5/21 $\omega\,d\omega = \alpha\,d\theta$, $\dfrac{d\omega}{d\theta} = k$, so $\dfrac{\alpha}{\omega} = k$,

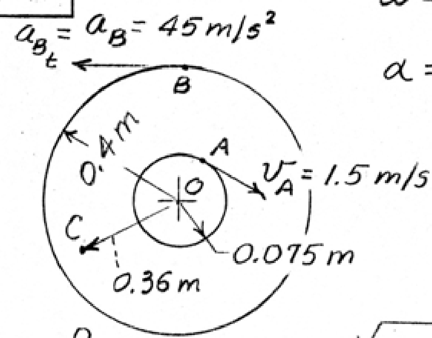
$\dfrac{d\omega}{dt}\dfrac{1}{\omega} = k$, $\displaystyle\int_{\omega_0}^{\omega} \frac{d\omega}{\omega} = \int_0^t k\,dt \Rightarrow \underline{\omega = \omega_0 e^{kt}}$

From $\dfrac{d\theta}{dt} = \omega_0 e^{kt}$, $\displaystyle\int_0^\theta d\theta = \int_0^t \omega_0 e^{kt}\,dt$

$$\Rightarrow \underline{\theta = \frac{\omega_0}{k}\left(e^{kt} - 1\right)}$$

$\alpha = \dot{\omega} = \underline{\omega_0 k e^{kt}}$

5/22



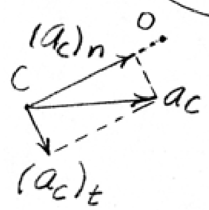
$\omega = v/r = \frac{1.5}{0.075} = 20\ rad/s$

$\alpha = a_t/r = \frac{45}{0.4} = 112.5\ rad/s^2$

$(a_C)_n = r\omega^2 = 0.36(20)^2 = 144\ m/s^2$

$(a_C)_t = r\alpha = 0.36(112.5) = 40.5\ m/s^2$

$a_C = \sqrt{(144)^2 + (40.5)^2} = \underline{149.6\ m/s^2}$



5/23 $\omega = v_A/r_A = \frac{10}{8/12} = 15$ rad/sec, $\underline{\omega} = 15\underline{k}$ rad/sec

$\alpha = (a_A)_t/r_A = \frac{24}{8/12} = 36$ rad/sec^2, $\underline{\alpha} = -36\underline{k}$ rad/sec^2

$$\underline{a}_B = \underline{\alpha} \times \underline{r}_B + \underline{\omega} \times (\underline{\omega} \times \underline{r}_B)$$

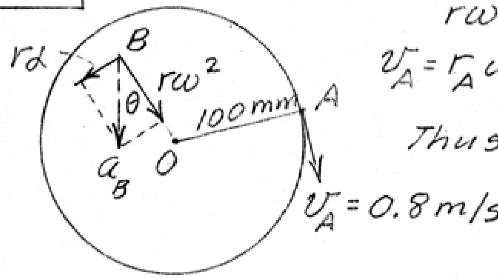
$$= -36\underline{k} \times \frac{6}{12}\underline{j} + 15\underline{k} \times (15\underline{k} \times \frac{6}{12}\underline{j}) = 18\underline{i} - 112.5\underline{j} \text{ ft/sec}^2$$

5/24 For gear A, $\Delta\omega = \int_2^6 \alpha_A \, dt$, $N_A = 2N_B$

$$(N_A - 600)\frac{2\pi}{60} = \frac{4+8}{2}(6-2), \; N_A = 600 + 229 = 829 \text{ rev/min}$$

so at $t = 6$ s, $N_B = \frac{829}{2} = \underline{415 \text{ rev/min}}$

5/25

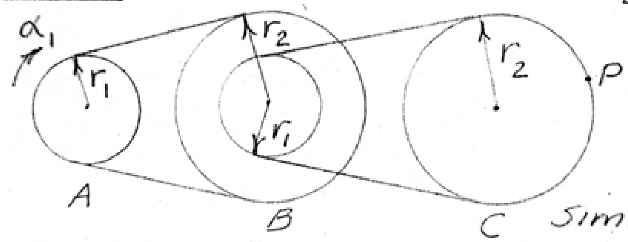


$$\tan\theta = \frac{r\alpha}{r\omega^2} = \frac{\alpha}{\omega^2} = 0.6$$

$$v_A = r_A\omega, \quad \omega = \frac{800}{100} = 8 \text{ rad/s}$$

Thus $\alpha = 0.6(8^2) = \underline{38.4 \ \frac{\text{rad}}{\text{s}^2}}$

▸5/26



$r_2\omega_C = r_1\omega_B$

$r_2\omega_B = r_1\omega_A$

so that

$\omega_C = \left(\frac{r_1}{r_2}\right)^2\omega_A$

Similarly $\alpha_C = \left(\frac{r_1}{r_2}\right)^2\alpha_1$

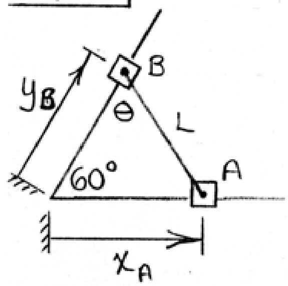
$\omega_A = \alpha_1 t$ so that $\underline{\omega_C = \left(\frac{r_1}{r_2}\right)^2\alpha_1 t}$

For P, $a_n = r_2\omega_C^2 = r_2\left[\left(\frac{r_1}{r_2}\right)^2\alpha_1 t\right]^2$

$a_t = r_2\alpha_C = r_2\left(\frac{r_1}{r_2}\right)^2\alpha_1$

$a_P = \sqrt{a_n^2 + a_t^2} = \underline{\frac{r_1^2}{r_2}\alpha_1\sqrt{1 + \left(\frac{r_1}{r_2}\right)^4\alpha_1^2 t^4}}$

5/27



$$\frac{x_A}{\sin\theta} = \frac{L}{\sin 60^\circ} \qquad (1)$$

$$x_A = \frac{2}{\sqrt{3}} L \sin\theta$$

$$\dot{x}_A = v = \frac{2}{\sqrt{3}} L \cos\theta\, \dot{\theta} \qquad (2)$$

We need $\cos\theta$ in terms of x_A.

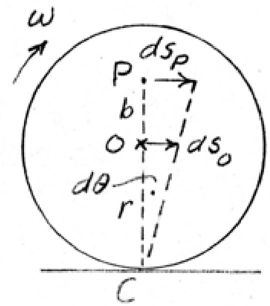
From (1): $\sin\theta = \frac{\sqrt{3}}{2}\frac{x_A}{L}$

Then $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{3}{4}\frac{x_A^2}{L^2}}$

(2): $\dot{\theta} = \omega = \frac{\sqrt{3}\,v}{2L\cos\theta} = \frac{\sqrt{3}\,v}{2L\sqrt{1-\frac{3}{4}\frac{x_A^2}{L^2}}}$

$(0 \leq x_A \leq L)$

5/28



$ds_P = \overline{PC}\, d\theta$

$v_P = \dfrac{ds_P}{dt} = \overline{PC}\,\dot\theta = \overline{PC}\,\omega$

$= \underline{(b+r)\omega}$

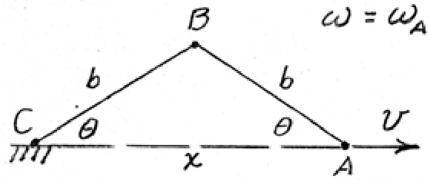
$\dfrac{ds_P}{b+r} = \dfrac{ds_O}{r}$ so $\dfrac{v_P}{b+r} = \dfrac{v_O}{r}$

$\underline{v_P = \dfrac{b+r}{r} v_O}$

5/29

$x = 2b\cos\theta, \quad \dot{x} = -2b\dot{\theta}\sin\theta, \quad v = \dot{x}$

$\omega = \omega_{AB} = \dot{\theta}$ so $\omega = \dfrac{-v}{2b\sin\theta}$ CW

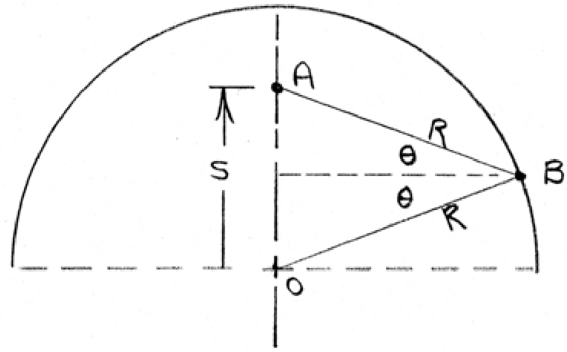


For $a = \ddot{x}$ const., $\dot{x}^2 = 2ax$

$v = \sqrt{2ax}$

so $\omega = \dfrac{\sqrt{2ax}}{2b\sqrt{1-\cos^2\theta}} = \dfrac{\sqrt{2ax}}{\sqrt{4b^2 - x^2}}$

5/30



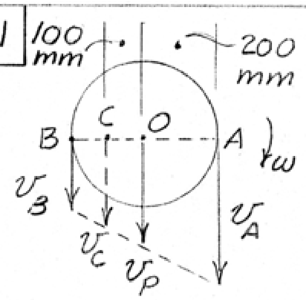
$s = 2(R\sin\theta)$, $\dot{s} = v = 2R\cos\theta\,\dot{\theta}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{s^2}{4R^2}}$

So

$$\dot{\theta} = \omega = \frac{v}{2R\sqrt{1-\frac{s^2}{4R^2}}}$$

5/31



$v_A = 0.4$ m/s, $v_B = 0.2$ m/s

$$\omega = \frac{v_A - v_B}{\overline{AB}} = \frac{0.4 - 0.2}{0.400} = \underline{0.5 \frac{\text{rad}}{\text{s}}}$$

CW

$v_P = v_B + \overline{BO}\omega$

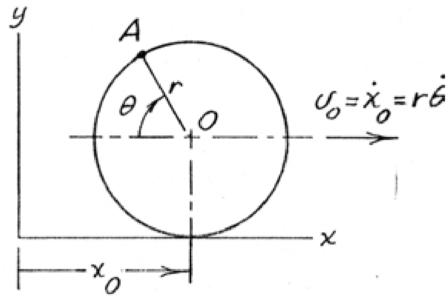
$= 0.2 + 0.200(0.5) = \underline{0.3 \text{ m/s}}$

$v_C = v_B + \overline{BC}\omega$

$= 0.2 + 0.100(0.5) = \underline{0.25 \text{ m/s}}$

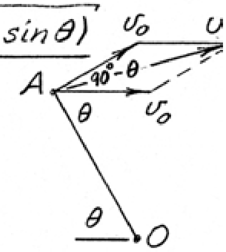
5/32 Coordinates of A are

$$x = x_0 - r\cos\theta$$
$$y = r + r\sin\theta$$



$$\dot{x} = \dot{x}_0 + r\dot{\theta}\sin\theta = v_0(1+\sin\theta)$$
$$\dot{y} = r\dot{\theta}\cos\theta = v_0\cos\theta$$

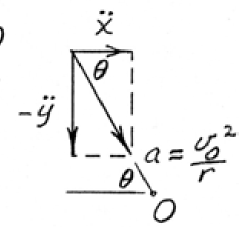
$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = v_0\sqrt{(1+\sin\theta)^2 + \cos^2\theta} = \underline{v_0\sqrt{2(1+\sin\theta)}}$$



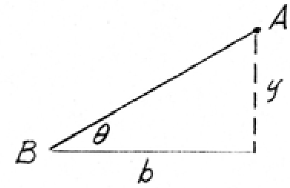
$$\ddot{x} = v_0\dot{\theta}\cos\theta = v_0\left(\frac{v_0}{r}\right)\cos\theta = \frac{v_0^2}{r}\cos\theta$$

$$\ddot{y} = -v_0\dot{\theta}\sin\theta = -v_0\left(\frac{v_0}{r}\right)\sin\theta = -\frac{v_0^2}{r}\sin\theta$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \frac{v_0^2}{r}\sqrt{\cos^2\theta + \sin^2\theta} = \underline{\frac{v_0^2}{r} \text{ toward } O}$$



5/33



$y = b \tan\theta$

$\dot{y} = -v = b\dot{\theta}\sec^2\theta$

v = constant so that

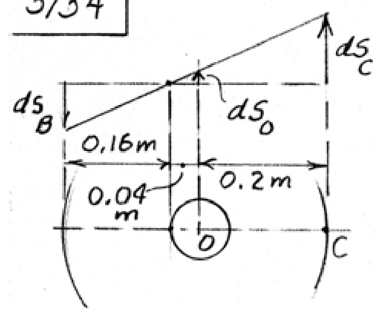
$0 = b\ddot{\theta}\sec^2\theta + 2b\dot{\theta}(\sec\theta \cdot \sec\theta\tan\theta)\dot{\theta}$

$0 = b\sec^2\theta(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)$

$\alpha = \ddot{\theta} = -2\dot{\theta}^2\tan\theta = -2\left(\frac{-v\cos^2\theta}{b}\right)^2\tan\theta = -\frac{2v^2}{b^2}\sin\theta\cos^3\theta$

$$\text{or } \underline{\alpha = -\frac{v^2}{b^2}\sin 2\theta\cos^2\theta}$$

5/34



$v_B = \sqrt{2a_B s_B}$ for constant accel.

$v_B = \sqrt{2(0.2)(1.6)} = 0.8$ m/s

$\frac{ds_B}{0.16} = \frac{ds_0}{0.04} = \frac{ds_C}{0.24}$

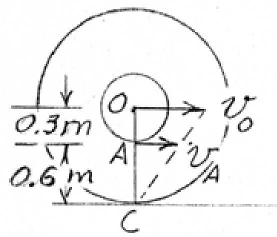
$ds_0 = \frac{0.04}{0.16} ds_B$, $a_0 = \frac{0.04}{0.16} a_B$

$a_0 = \frac{0.2}{4} = \underline{0.05 \text{ m/s}^2}$ up

$v_C = \frac{0.24}{0.16} v_B = \frac{3}{2}(0.8) = \underline{1.2 \text{ m/s}}$ up

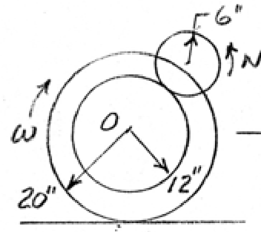
5/35

$v_O = \overline{OC}\,\omega = \frac{\overline{OC}}{\overline{AC}} v_A = \frac{0.9}{0.6} 0.8 = \underline{1.2 \quad m/s}$



$\omega = \frac{v_A}{\overline{AC}} = \frac{v_O}{\overline{OC}} = \frac{1.2}{0.9} = \underline{1.333 \text{ rad/s}}$ CW

5/36



$\rightarrow v = 60\ mi/hr$

$= 88\ ft/sec$

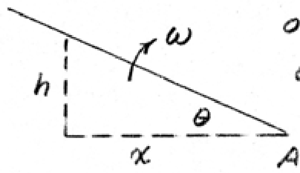
$\omega = v/r = \dfrac{88}{20/12}\ \dfrac{60}{2\pi} = 504$ rev/min

$N = \dfrac{12}{6}(504)$

$= 1008\ \dfrac{rev}{min}$

5/37

$v_A = r\omega_0 = -\dot{x}, \quad h = x \tan\theta$



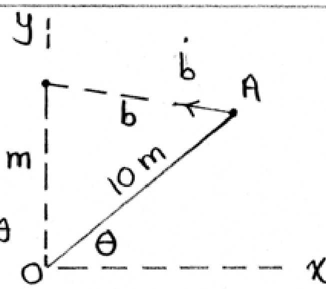
$0 = \dot{x} \tan\theta + x\dot{\theta} \sec^2\theta$

$\omega = \dot{\theta} = -\frac{\dot{x}}{x} \sin\theta \cos\theta$

$= -\frac{\dot{x}}{x} \frac{hx}{x^2+h^2}$

$$\omega = \frac{rh\omega_0}{x^2+h^2}$$

5/38 | $\dot{b} = \frac{1}{4} v_B = \frac{1}{4}(3.2) = 0.8 \ \frac{ft}{sec}$



$b^2 = 6^2 + 10^2 - 2(6)(10)\sin\theta$

$2b\dot{b} = -120\cos\theta\,\dot{\theta}$

$2\dot{b}^2 + 2b\ddot{b} = 120\,\dot{\theta}^2\sin\theta - 120\,\ddot{\theta}\cos\theta$

For $\theta = 30°$, $\dot{b} = -0.8$ m/s, $\ddot{b} = 0$,

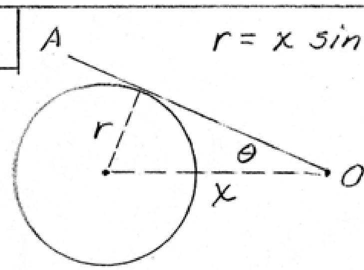
$b^2 = 36 + 100 - 120\left(\frac{1}{2}\right)$, $b = 8.72$ m

$2(8.72)(-0.8) = -120\,\frac{\sqrt{3}}{2}\,\dot{\theta}$, $\underline{\dot{\theta} = \omega = 0.1342 \text{ rad/s}}$

$2(-0.8)^2 + 2(8.72)(0) = 120\,(0.1342)^2\left(\frac{1}{2}\right) - 120\,\ddot{\theta}\,\frac{\sqrt{3}}{2}$

$\underline{\ddot{\theta} = -0.001916 \text{ rad/s}^2}$

5/39



$r = x \sin\theta, \quad 0 = \dot{x}\sin\theta + x\dot{\theta}\cos\theta$

$$\omega = \dot{\theta} = -\frac{\dot{x}}{x}\tan\theta$$

But $v = -\dot{x}$

& $\tan\theta = \dfrac{r}{\sqrt{x^2 - r^2}}$

So $\omega = \dfrac{v}{x}\dfrac{r}{\sqrt{x^2 - r^2}} = \underline{\dfrac{v}{x\sqrt{(x/r)^2 - 1}}}$

5/40

$y = 2L\sin\frac{\theta}{2}$

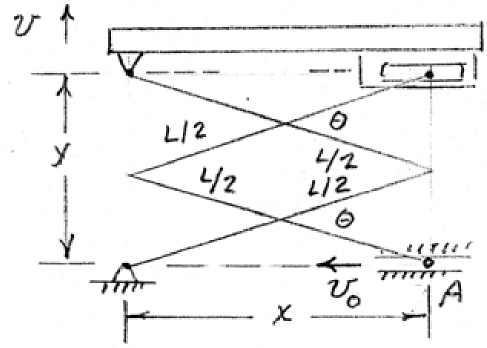
$v = \dot{y} = L\dot{\theta}\cos\frac{\theta}{2}$

$x = L\cos\frac{\theta}{2}$

$\dot{x} = -v_0 = -\frac{L}{2}\dot{\theta}\sin\frac{\theta}{2}$

so $L\dot{\theta} = 2v_0/\sin\frac{\theta}{2}$

& $v = \frac{2v_0}{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}$, $\underline{v = 2v_0 \operatorname{ctn}\frac{\theta}{2}}$



5/41

$$y = \frac{h}{2}\left(1 + \cos\frac{\pi x}{b}\right)$$

$$\dot{y} = -\frac{\pi h}{2b}\dot{x}\sin\frac{\pi x}{b} = -\frac{\pi h}{2b}v\sin\frac{\pi x}{b}$$

$$\ddot{y} = -\left(\frac{\pi v}{b}\right)^2\frac{h}{2}\cos\frac{\pi x}{b}, \quad \ddot{y}_{max} = 2g = \left(\frac{\pi v}{b}\right)^2\frac{h}{2}$$

where $a_G = g = \frac{1}{2}\ddot{y}_{max}$

So $\underline{h = 4g\left(\frac{b}{\pi v}\right)^2}$

For $b = 1$ m, $v = \frac{20}{3.6} = 5.56$ m/s :

$$h = 4(9.81)\left(\frac{1}{\pi\, 5.56}\right)^2 = 0.1288 \text{ m}$$

or $\underline{h = 128.8 \text{ mm}}$

5/42

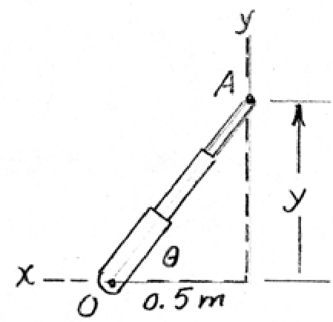
$$y = 0.5 \tan\theta$$

$$\dot{y} = 0.5 \sec^2\theta\, \dot{\theta}$$

$$\ddot{y} = 0 = \sec\theta(\tan\theta \sec\theta)\dot{\theta}^2 + 0.5 \sec^2\theta\, \ddot{\theta}$$

$$\dot{\theta} = 2\dot{y}/\sec^2\theta$$

$$\ddot{\theta} = -2\tan\theta\, \dot{\theta}^2$$



For $y = 0.6$ m, $\tan\theta = \frac{0.6}{0.5} = 1.2$, $\theta = 50.2°$

$\sec\theta = 1.562$

So for $\dot{y} = 0.2$ m/s, $\dot{\theta} = \frac{2(0.2)}{(1.562)^2} = \underline{0.1639 \text{ rad/s}}$

$\ddot{\theta} = -2(1.2)(0.1639)^2 = \underline{-0.0645 \text{ rad/s}^2}$

5/43

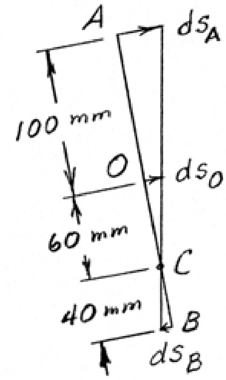
$v_B = 30(4) = 120$ mm/s

Also $v_B = \frac{ds_B}{dt} = \frac{40\,d\theta}{dt} = 40\omega$,

$\underline{\omega = \frac{120}{40} = 3 \text{ rad/s CW}}$

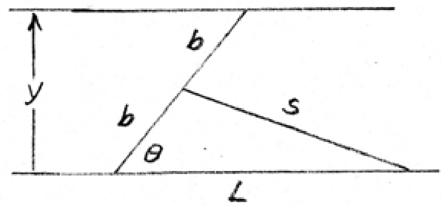
$v_A = \frac{ds_A}{dt} = 160\frac{d\theta}{dt} = 160\omega = 160(3) = \underline{480 \text{ mm/s}}$

$v_O = \frac{ds_O}{dt} = 60\frac{d\theta}{dt} = 60\omega = 60(3) = \underline{180 \text{ mm/s}}$



From Sample Problem 5/4 $a_C = r\omega^2 = 60(3^2) = \underline{540 \text{ mm/s}^2}$ toward O

5/44



$y = 2b \sin\theta$

$v = \dot{y} = 2b\dot{\theta}\cos\theta$

$s^2 = b^2 + L^2 - 2bL\cos\theta$

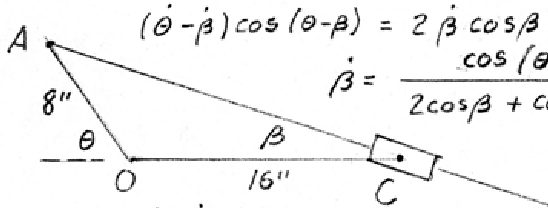
$2s\dot{s} = 0 + 0 + 2bL\dot{\theta}\sin\theta$

$\dot{\theta} = \dfrac{s\dot{s}}{bL\sin\theta}$

so $v = 2b\,\dfrac{s\dot{s}}{bL\sin\theta}\cos\theta = 2\,\dfrac{\sqrt{b^2+L^2-2bL\cos\theta}}{L\tan\theta}\,\dot{s}$

5/45 Law of sines $\frac{8}{\sin\beta} = \frac{16}{\sin(\theta-\beta)}$

$$(\dot{\theta}-\dot{\beta})\cos(\theta-\beta) = 2\dot{\beta}\cos\beta$$



$$\dot{\beta} = \frac{\cos(\theta-\beta)}{2\cos\beta + \cos(\theta-\beta)}\,\dot{\theta}$$

$\tan\beta = \frac{8\sin\theta}{16+8\cos\theta}$, For $\theta = 60^\circ$, $\tan\beta = \frac{8\sin 60^\circ}{16+8\cos 60^\circ}$

$= 0.346$, $\beta = 19.11^\circ$

$\theta - \beta = 60 - 19.11 = 40.9^\circ$

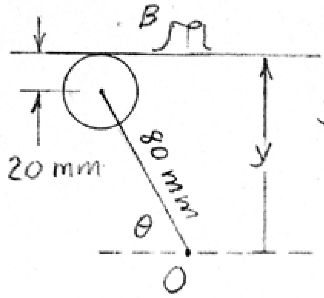
$\cos(\theta-\beta) = 0.756$, $\cos\beta = 0.945$

$$\omega = \dot{\beta} = \frac{0.756}{2(0.945)+0.756}\;\frac{600(2\pi)}{60} = \underline{17.95 \text{ rad/sec CW}}$$

5/46

$y = 20 + 80 \sin\theta$, $\dot{y} = 80\dot{\theta}\cos\theta$

$\ddot{y} = 80\ddot{\theta}\cos\theta - 80\dot{\theta}^2\sin\theta$



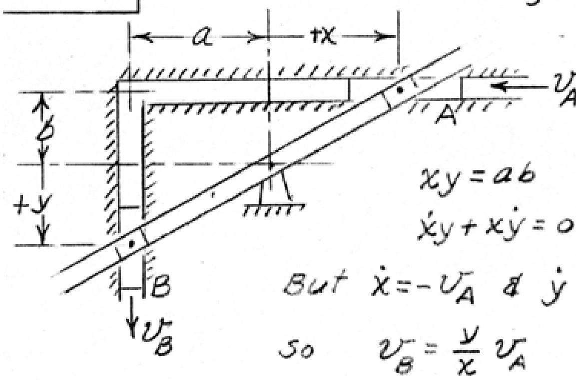
For $\theta = 60°$, $\dot{\theta} = 4\ \frac{rad}{s}$, $\ddot{\theta} = 8\ \frac{rad}{s^2}$,

$\ddot{y} = 80(8)(\frac{1}{2}) - 80(4)^2\frac{\sqrt{3}}{2}$

$= 320 - 1109 = -789\ mm/s^2$

Thus $a_B = \underline{789\ mm/s^2\ down}$

5/47.



By similar triangles

$$\frac{b}{x} = \frac{y}{a}$$

so $\underline{y = \frac{ab}{x}}$

$$xy = ab$$

$$\dot{x}y + x\dot{y} = 0 \quad , \quad \dot{y} = -\frac{y}{x}\dot{x}$$

But $\dot{x} = -v_A$ & $\dot{y} = +v_B$

so $\underline{v_B = \frac{y}{x} v_A}$

$$v_B x = v_A y \,, \quad a_B x + v_B \dot{x} = \dot{v}_A y + v_A \dot{y} \,, \quad \dot{v}_A = 0$$

$$a_B x = v_A v_B - v_B(-v_A) = 2 v_A v_B$$

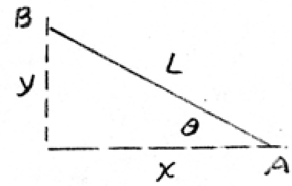
$$a_B = \frac{2 v_A v_B}{x} = \underline{\frac{2 v_A^2 y}{x^2}}$$

5/48

$x = L\cos\theta$

$\dot{x} = -v_0 = -L\dot{\theta}\sin\theta$

$\omega = \dot{\theta} = \dfrac{v_0}{L\sin\theta}$



where $L\sin\theta = y = \sqrt{L^2 - x^2}$

so $\omega = \dfrac{v_0}{\sqrt{L^2-x^2}}$

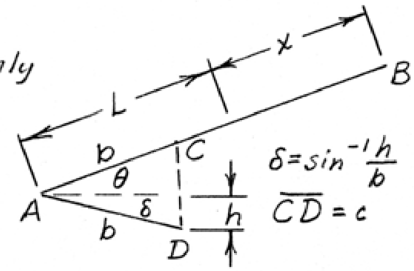
$\alpha = \ddot{\theta} = \dfrac{v_0}{L}\dfrac{d}{dt}\csc\theta = \dfrac{v_0}{L}(-\text{ctn}\,\theta\,\csc\theta)\dot{\theta}$

$= -\dfrac{v_0}{L}\dfrac{x}{y}\dfrac{L}{y}\dot{\theta} = \dfrac{-xv_0^2}{y^2\sqrt{L^2-x^2}}$

$= \dfrac{-xv_0^2}{(L^2-x^2)^{3/2}}$

5/49

For vertical motion only of B, its horizontal coordinate remains constant so



$\delta = \sin^{-1}\frac{h}{b}$

$\overline{CD} = c$

$$\frac{d}{dt}\left\{(L+x)\cos\theta\right\} = 0$$

or $-(L-x)\dot{\theta}\sin\theta + \dot{x}\cos\theta = 0$,

$$\dot{x} = (L+x)\dot{\theta}\tan\theta$$

$$\overline{CD}^2 = c^2 = b^2 + b^2 - 2b^2\cos(\theta+\delta) = 2b^2(1-\cos(\theta+\delta))$$

$$2c\dot{c} = 2b^2\dot{\theta}\sin(\theta+\delta),\quad \dot{\theta} = \frac{c\dot{c}}{b^2\sin(\theta+\delta)} = \frac{\sqrt{2}\sqrt{1-\cos(\theta+\delta)}}{b\sin(\theta+\delta)}\dot{c}$$

Thus $\dot{x} = (L+x)\tan\theta\,\dfrac{\sqrt{2}\sqrt{1-\cos(\theta+\delta)}}{b\sqrt{1-\cos^2(\theta+\delta)}}\dot{c}$

$$= \frac{L+x}{b}\tan\theta\,\frac{\sqrt{2}}{\sqrt{1+\cos(\theta+\delta)}}\dot{c} = \frac{L+x}{b}\,\frac{\tan\theta}{\cos\frac{1}{2}(\theta+\delta)}\dot{c}$$

where $\delta = \sin^{-1}\frac{h}{b}$

5/50 Belt velocity is the same for both pulleys

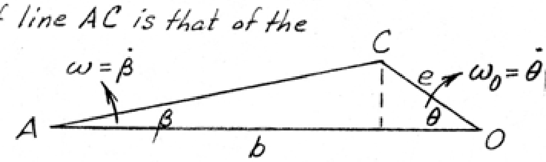
so $r_1\omega_1 = r_2\omega_2$

Thus $\dot{r}_1\omega_1 + r_1\dot{\omega}_1 = \dot{r}_2\omega_2 + r_2\dot{\omega}_2$

For $\dot{\omega}_1 = 0$ & $\alpha_2 = \dot{\omega}_2$, we have

$$\alpha_2 = \dot{\omega}_2 = \frac{\dot{r}_1\omega_1 - \dot{r}_2\omega_2}{r_2} = \underline{\frac{\dot{r}_1 r_2 - r_1\dot{r}_2}{r_2^2}\,\omega_1}$$

5/51 Angular velocity of line AC is that of the fork, whose sides are parallel to AC.



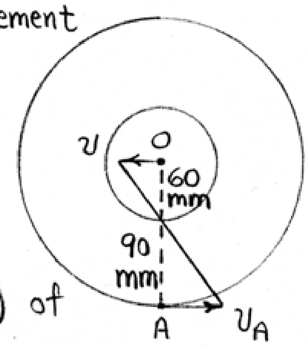
$$\tan\beta = \frac{e\sin\theta}{b - e\cos\theta}$$

$$\sec^2\beta\,\dot{\beta} = \frac{(b-e\cos\theta)e\dot{\theta}\cos\theta - e\sin\theta(e\dot{\theta}\sin\theta)}{(b-e\cos\theta)^2} = \frac{b\cos\theta - e}{(b-e\cos\theta)^2}e\dot{\theta}$$

Substitute $\sec^2\beta = 1 + \tan^2\beta = \dfrac{b^2 - 2be\cos\theta + e^2}{(b-e\cos\theta)^2}$ & get

$$\underline{\omega = \dot{\beta} = \frac{b\cos\theta - e}{b^2 - 2be\cos\theta + e^2}e\omega_0} \quad \text{where } \omega_0 = \dot{\theta}$$

5/52 Let ds = differential movement



$$\frac{ds_O}{60} = \frac{ds_A}{90}$$

So $\frac{v_O}{60} = \frac{v_A}{90}$, $v_O = \frac{2}{3} v_A$

Pitch (distance between teeth) of large gear is $\pi \frac{300}{48} = 19.63$ mm

19.63 mm is the advancement per revolution of worm.

Thus $v_A = 19.63 \left(\frac{120}{60}\right) = 39.3$ mm/s

So $v_O = \frac{2}{3}(39.3) = \underline{26.2 \text{ mm/s}}$

5/53 Given $\dot{s} = 0.260$ m/s

$s = 2(0.2)\sin\frac{\theta}{2}$

$\dot{s} = 0.2\dot{\theta}\cos\frac{\theta}{2}$

For $\theta = 60°$

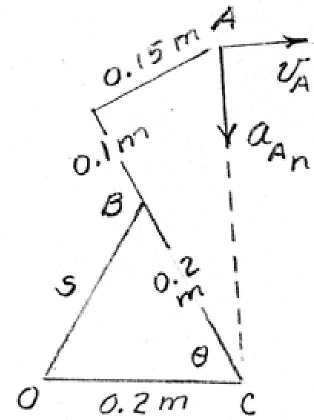
$\dot{s} = 0.260 = 0.2\dot{\theta}\cos\frac{60°}{2}$

$\dot{\theta} = \omega_{AC} = \frac{0.260}{0.2\cos 30°} = 1.501$ rad/s

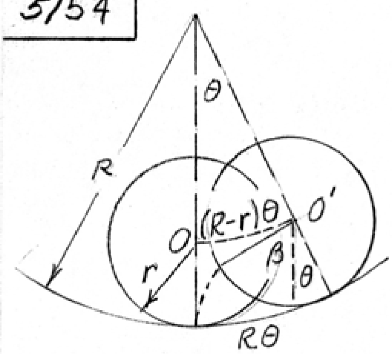
$\overline{AC} = \sqrt{0.3^2 + 0.15^2} = 0.335$ m

$a_{A_n} = \overline{AC}\,\omega_{AC}^2$

$= 0.335(1.501)^2 = \underline{0.756 \text{ m/s}^2}$



5/54



$v_O = v = (R-r)\dot{\theta}$

$R\theta = r(\theta+\beta)$

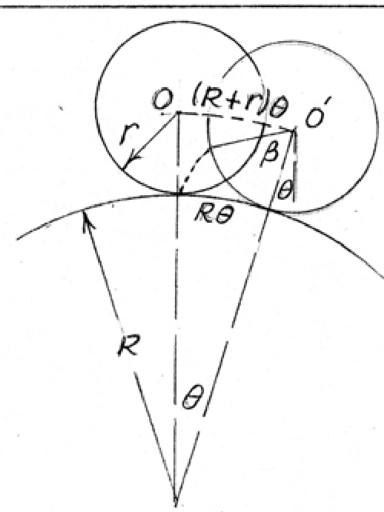
so $\theta(R-r) = r\beta$

$\dot{\theta}(R-r) = r\dot{\beta}$

so $v = r\dot{\beta}$ & $\omega = \dot{\beta}$

$\underline{v = r\omega}$ so $\underline{a_t = r\alpha}$

(β = absolute angle)



$v_O = v = (R+r)\dot{\theta}$

$R\theta = r\beta$ so $\theta+\beta = \left(\frac{r+R}{r}\right)\theta$

so $r(\dot{\theta}+\dot{\beta}) = (R+r)\dot{\theta}$

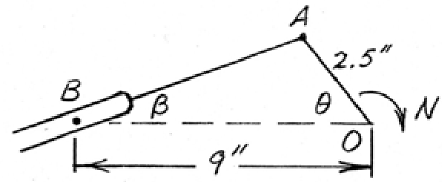
where $\omega = (\dot{\beta}+\dot{\theta})$

so $\underline{v = r\omega}$ so $\underline{a_t = r\alpha}$

($\beta+\theta$ = absolute angle)

5/55

$\tan\beta = \dfrac{2.5\sin\theta}{9-2.5\cos\theta}$



$\dot{\theta} = \dfrac{2\pi N}{60} = \dfrac{120}{30}\pi = 12.57 \text{ rad/s}$

$$\sec^2\beta\,\dot{\beta} = \frac{(9-2.5\cos\theta)2.5\dot{\theta}\cos\theta - 2.5\sin\theta(2.5\dot{\theta}\sin\theta)}{(9-2.5\cos\theta)^2}$$
$$= \frac{22.5\cos\theta - 6.25}{(9-2.5\cos\theta)^2}\dot{\theta}$$

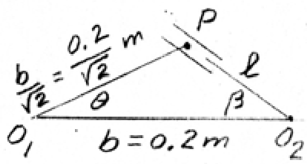
$$\dot{\beta} = \frac{22.5\cos\theta - 6.25}{(9-2.5\cos\theta)^2}\dot{\theta}\cos^2\beta$$

But $\cos^2\beta = \dfrac{(9-2.5\cos\theta)^2}{9^2+2.5^2-2(9)(2.5)\cos\theta}$

so $\dot{\beta} = \dfrac{22.5\cos\theta - 6.25}{87.2-45\cos\theta}12.57$ or $\underline{\dot{\beta} = \dfrac{12.57}{2}\dfrac{\cos\theta - 0.278}{1.939-\cos\theta}\dfrac{\text{rad}}{\text{sec}}}$

▸5/56

$$\frac{b/\sqrt{2}}{\sin\beta} = \frac{b}{\sin(\pi-\theta-\beta)} = \frac{b}{\sin(\theta+\beta)}$$



so $\sqrt{2}\sin\beta = \sin(\theta+\beta)$ - - - - - (a)

$\sqrt{2}\,\dot\beta\cos\beta = (\dot\theta+\dot\beta)\cos(\theta+\beta)$

$$\omega_2 = -\dot\beta = \dot\theta\,\frac{\cos(\theta+\beta)}{\cos(\theta+\beta)-\sqrt{2}\cos\beta} \quad \text{- - - (b)}$$

From (a) $\sin\beta(\sqrt{2}-\cos\theta) = \sin\theta\cos\beta$, $\tan\beta = \dfrac{\sin\theta}{\sqrt{2}-\cos\theta}$

For $\theta = 20°$, $\beta = \tan^{-1}\dfrac{0.3420}{\sqrt{2}-0.9397} = \tan^{-1}0.7208 = 35.8°$

& for $\dot\theta = -2$ rad/s, Eq. (b) gives

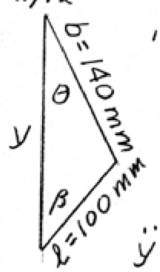
$$\omega_2 = -2\,\frac{\cos(20°+35.8°)}{\cos(20°+35.8°)-\sqrt{2}\cos 35.8°} = -2\,\frac{0.5623}{-0.5849}$$

$\underline{\omega_2 = 1.923 \text{ rad/s}}$

▶5/57 $\theta = \theta_o \sin 2\pi t$, $\dot\theta = 2\pi\theta_o \cos 2\pi t$, $\ddot\theta = -4\pi^2\theta_o \sin 2\pi t$

$\theta_o = \pi/12$ when $\theta = 0$, $t = 1/2\ s$ & $\dot\theta = 2\pi\theta_o = \pi^2/6$ rad/s, $\ddot\theta = 0$

" $\theta = \frac{\pi}{12}$, $t = 1/4\ s$ & $\dot\theta = 0$, $\ddot\theta = -4\pi^2\theta_o = -\pi^3/3$ rad/s²



$\ell^2 = y^2 + b^2 - 2yb\cos\theta$, $0 = y\dot y + yb\dot\theta\sin\theta - \dot y b\cos\theta$

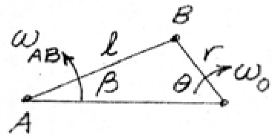
$0 = y\ddot y + \dot y^2 + \dot y b\dot\theta\sin\theta + yb\ddot\theta\sin\theta + yb\dot\theta^2\cos\theta$
$\quad - \ddot y b\cos\theta + \dot y b\dot\theta\sin\theta$

$\ddot y(b\cos\theta - y) = \dot y^2 + 2\dot y b\dot\theta\sin\theta + yb\ddot\theta\sin\theta + yb\dot\theta^2\cos\theta$

(a) $\theta = 0$, $\ddot y(b - [b+\ell]) = 0 + 0 + 0 + (b+\ell)b(\pi^2/6)^2$
$\dot y = 0$
$\ddot\theta = 0$ $\quad \ddot y = \frac{\pi^4}{36}\frac{b(b+\ell)}{-\ell} = \frac{-\pi^4}{36}\frac{0.14(0.24)}{0.1} = -0.909\ \frac{m}{s^2}$
(up)

(b) $\theta = \pi/12$, $b/\sin\beta = \ell/\sin\frac{\pi}{12}$, $\beta = \sin^{-1}\left(\frac{0.14}{0.1}\sin\frac{\pi}{12}\right) = 21.24°$
$\dot y = 0$ $\quad y = b\cos\theta + \ell\cos\beta = 0.14\cos\frac{\pi}{12} + 0.1\cos 21.24° = 0.2284$ m
$\dot\theta = 0$ $\quad \ddot y\left(0.14\cos\frac{\pi}{12} - 0.2284\right) = 0 + 0 + 0.2284(0.14)\left(-\frac{\pi^3}{3}\right)\sin\frac{\pi}{12} + 0$
$\ddot y(-0.09320) = -0.08555$, $\underline{\ddot y = 0.918\ m/s^2}$ (down)

▶5/58 $\ell \sin\beta = r\sin\theta, \quad \ell\dot\beta\cos\beta = r\dot\theta\cos\theta$

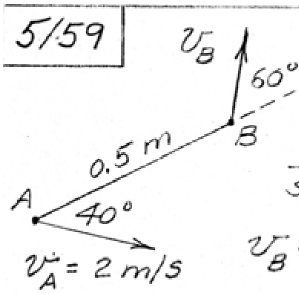


so $\omega_{AB} = \dot\beta = \dfrac{r}{\ell}\dot\theta\dfrac{\cos\theta}{\cos\beta} = \dfrac{r}{\ell}\omega_0\dfrac{\cos\theta}{\sqrt{1-\left(\frac{r}{\ell}\sin\theta\right)^2}}$

$\ell\ddot\beta\cos\beta - \ell\dot\beta^2\sin\beta = -r\dot\theta^2\sin\theta, \quad \ddot\theta = \dot\omega_0 = 0$

$\alpha_{AB} = \ddot\beta = \dfrac{\ell\dot\beta^2\sin\beta - r\dot\theta^2\sin\theta}{\ell\cos\beta} = \dfrac{r\omega_0^2}{\ell}\sin\theta\dfrac{\frac{r^2}{\ell^2}-1}{\left(1-\frac{r^2}{\ell^2}\sin^2\theta\right)^{3/2}}$

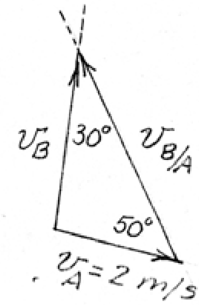
5/59



$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$$

$$\frac{v_B}{\sin 50^\circ} = \frac{2}{\sin 30^\circ}$$

$$v_B = \frac{2 \sin 50^\circ}{\sin 30^\circ} = \underline{3.06 \text{ m/s}}$$



$$v_{B/A} = v_B \cos 30^\circ + v_A \cos 50^\circ$$
$$= 3.06 \cos 30^\circ + 2 \cos 50^\circ = 3.94 \text{ m/s}$$

$$\omega_{AB} = \frac{v_{B/A}}{\overline{AB}} = \frac{3.94}{0.5} = \underline{7.88 \text{ rad/s ccw}}$$

5/60 $\underline{v}_A = \underline{v}_O + \underline{v}_{A/O}$ where $v_{A/O} = \overline{AO}\,\omega = \frac{10}{12}\omega \ \frac{ft}{sec}$

$v_O = 4$ ft/sec

(a) $v_A = 4$ $v_O = 4$

$v_{A/O} = 8$ ft/sec $\quad \omega = \frac{8}{10/12} = 9.6 \frac{rad}{sec}$, $N = 9.6 \frac{60}{2\pi} = 91.7 \frac{rev}{min}$ CCW

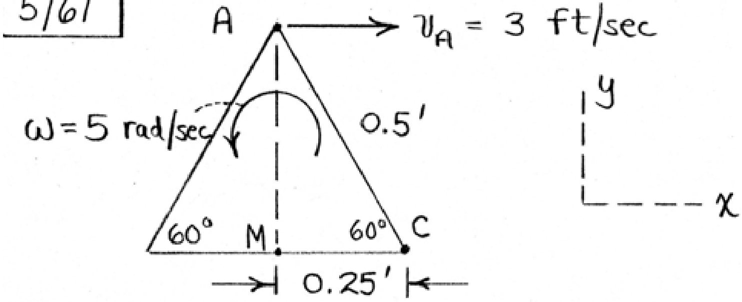
(b) $v_O = 4$

$v_{A/O} = 4$ ft/sec $\quad v_A = 0$, $\omega = \frac{4}{10/12} = 4.8 \frac{rad}{sec}$, $N = 45.8 \frac{rev}{min}$ CCW

(c) $v_O = 4$ $v_{A/O} = 4$

$v_A = 8$ ft/sec $\quad \omega = \frac{4}{10/12} = 4.8 \frac{rad}{sec}$, $N = 45.8 \frac{rev}{min}$ CW

5/61



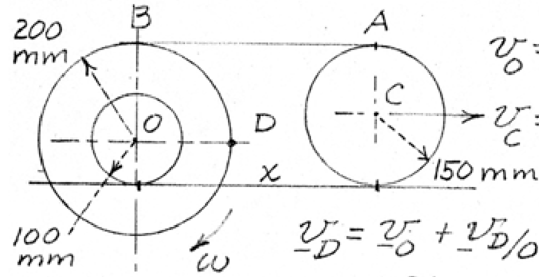
$$\overline{AM} = 0.25\sqrt{3} = 0.433 \text{ ft}$$

$$\underline{v}_C = \underline{v}_A + \underline{v}_{C/A} = \underline{v}_A + \underline{\omega} \times \underline{r}_{C/A}$$

$$= 3\underline{i} + 5\underline{k} \times [0.25\underline{i} - 0.433\underline{j}]$$

$$= \underline{5.17\underline{i} + 1.25\underline{j} \text{ ft/sec}}$$

6/62 $v_B = v_A = 2v_C = 0.8\ m/s$, $\omega = \frac{v_B}{r} = \frac{0.8}{0.3} = \frac{8}{3}\ \frac{rad}{s}$

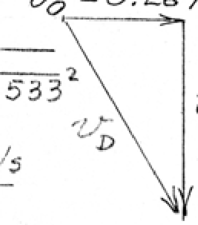


$v_O = \frac{0.1}{0.3} v_B = \frac{0.8}{3} = 0.267\ \frac{m}{s}$

$v_C = 0.4\ m/s$

$\mathbf{v}_D = \mathbf{v}_O + \mathbf{v}_{D/O}$, $v_{D/O} = \overline{OD}\,\omega = 0.2\left(\frac{8}{3}\right) = 0.533\ \frac{m}{s}$

$v_O = 0.267\ m/s$



$v_{D/O} = 0.533\ m/s$

$v_D = \sqrt{0.267^2 + 0.533^2}$

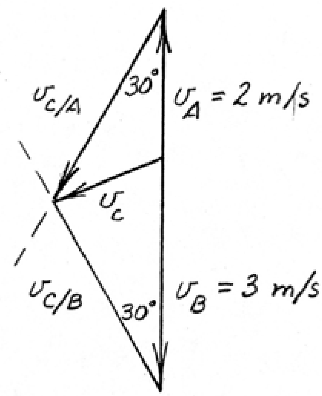
$= \underline{0.596\ m/s}$

$\dot{x} = v_{C/O} = (v_C - v_O) = (0.4 - 0.267) = \underline{0.133\ m/s}$

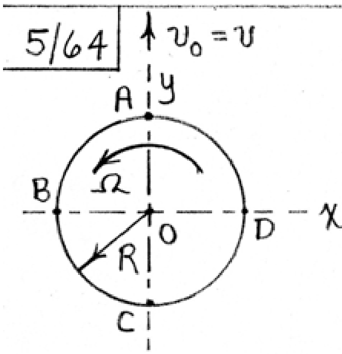
5/63 $\underline{v}_C = \underline{v}_A + \underline{v}_{C/A} = \underline{v}_B + \underline{v}_{C/B}$

From geometry of isosceles triangle

$v_C = \sqrt{(2.5 \tan 30^\circ)^2 + 0.5^2} = \underline{1.528 \text{ m/s}}$



5/64



$\underline{v}_0 = 107\,257\underline{j}$ km/h

$R\Omega = 6371(10^3)[7.292(10^{-5})]$

$= 465 \frac{m}{s} \left(3.6 \frac{km/h}{m/s}\right)$

$= 1672$ km/h

$\underline{v}_A = \underline{v}_0 + \underline{v}_{A/0} = \underline{-1672\underline{i} + 107\,257\underline{j} \text{ km/h}}$

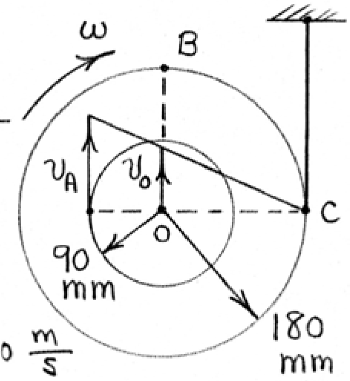
$\underline{v}_B = \underline{v}_0 + \underline{v}_{B/0} = 107\,257\underline{j} - 1672\underline{j} = \underline{105\,585\underline{j} \text{ km/h}}$

$\underline{v}_C = \underline{v}_0 + \underline{v}_{C/0} = \underline{1672\underline{i} + 107\,257\underline{j} \text{ km/h}}$

$\underline{v}_D = \underline{v}_0 + \underline{v}_{D/0} = (107\,257 + 1672)\underline{j} = \underline{108\,929\underline{j} \text{ km/h}}$

5/65

$$v_O = \frac{180}{180+90} v_A = \frac{2}{3}(0.9) = \underline{0.6 \frac{m}{s}}$$



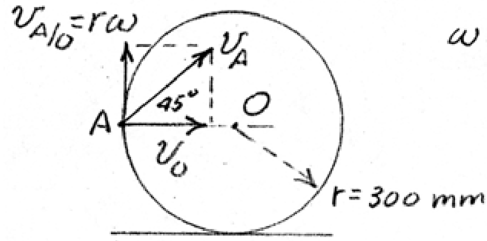
$$\underline{v}_B = \underline{v}_O + \underline{v}_{B/O}, \quad \omega = \omega_{BO} = \frac{0.6}{0.180}$$

$$= 3.33 \text{ rad/s}$$

$$\underline{v}_{B/O} = \overline{BO}\, \omega_{OB} = 0.180\,(3.33) = 0.60 \frac{m}{s}$$

$$v_B = 0.6\sqrt{2} = \underline{0.849 \text{ m/s}}$$

5/66 $|\underline{v}_O| = |v_{A/O}| = r\omega = 12 \cos 45° = \underline{8.49 \text{ m/s}}$

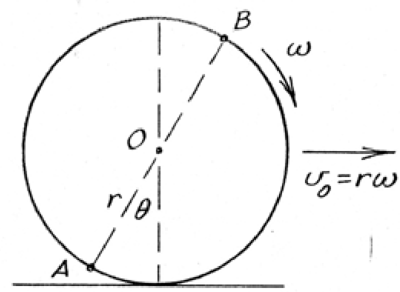


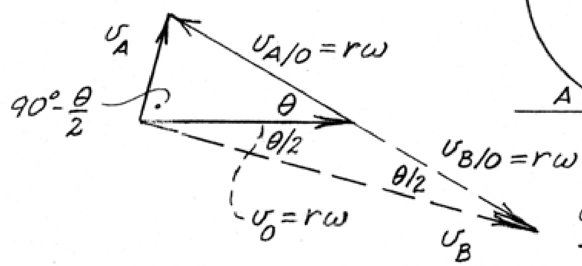
$\omega = \dfrac{8.49}{0.325} = \underline{26.1 \text{ rad/s}}$

5/67

$\underline{v}_A = \underline{v}_O + \underline{v}_{A/O}$

$\underline{v}_B = \underline{v}_O + \underline{v}_{B/O}$

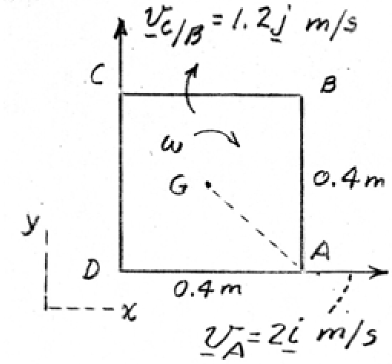




$$\underline{v_A = 2r\omega \sin\frac{\theta}{2}}$$

Angle between $\underline{v}_A$ & $\underline{v}_B$ is 90°

5/68



$v_{C/B} = \overline{CB}\,\omega\ ,\quad \omega = \frac{1.2}{0.4} = 3$ rad/s CW

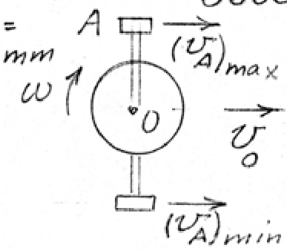
$\underline{\omega} = -3\underline{k}$ rad/s

$$\underline{v}_G = \underline{v}_A + \underline{\omega} \times \underline{r}_{AG}$$
$$= 2\underline{i} - 3\underline{k} \times (-0.2\underline{i} + 0.2\underline{j})$$
$$= 2\underline{i} + 0.6\underline{j} + 0.6\underline{i}$$

$$\underline{v}_G = 2.6\underline{i} + 0.6\underline{j} \quad \text{m/s}$$

5/69 $v_O = \frac{16000}{3600} = 4.444$ m/s, $2r = 660$ mm

$\overline{AO} = 160$ mm



$$\omega = \frac{70}{170}\frac{v_O}{r} = \frac{70}{170}\frac{4.44}{0.33} = 5.55 \text{ rad/s}$$

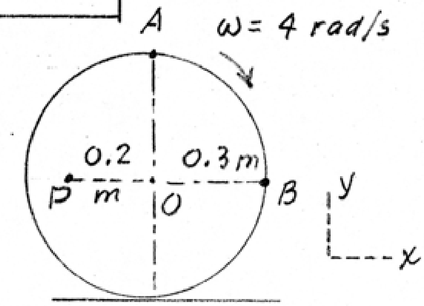
$$\underline{v}_A = \underline{v}_O + \underline{v}_{A/O}$$

$$|\underline{v}_{A/O}| = \overline{AO}\,\omega_{AO} = 0.160(5.55) = 0.887 \text{ m/s}$$

$$(v_A)_{max} = 4.444 + 0.887 = \underline{5.33 \text{ m/s}}$$

$$(v_A)_{min} = 4.444 - 0.887 = \underline{3.56 \text{ m/s}}$$

5/70



$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B}$

$\underline{\omega} = -4\underline{k}$ rad/s

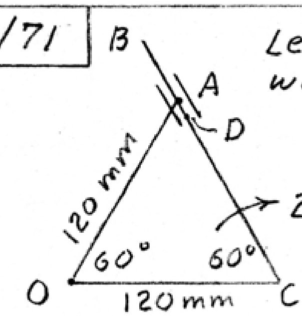
$\underline{r}_{A/B} = -0.3\underline{i} + 0.3\underline{j}$ m

$\underline{v}_{A/B} = -4\underline{k} \times 0.3(-\underline{i} + \underline{j})$

$= \underline{1.2(\underline{i} + \underline{j})}$ m/s

$\underline{v}_P = \underline{v}_O + \underline{v}_{P/O} = r\omega\underline{i} + \overline{PO}\,\omega\underline{j} = \underline{4(0.3\underline{i} + 0.2\underline{j})}$ m/s

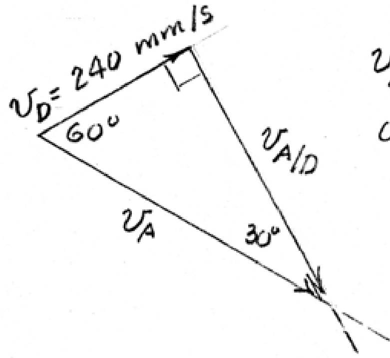
5/71



Let D be point on BC coincident with A for $\theta = 60°$

$$\underline{v}_A = \underline{v}_D + \underline{v}_{A/D}$$

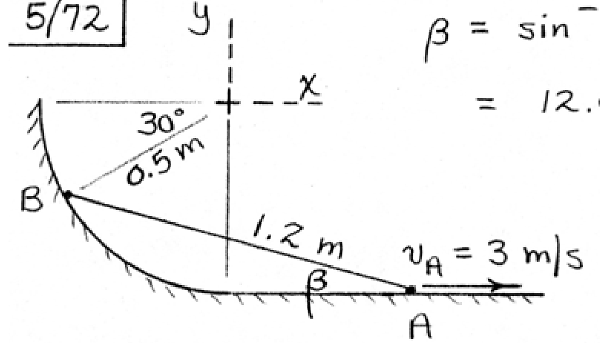
where $v_D = \overline{DC}\,\omega_{CB}$
$= 120(2)$
$= 240 \text{ mm/s}$



$$v_A = \frac{240}{\cos 60°} = 480 \text{ mm/s}$$

$$\omega_{OA} = \frac{v_A}{\overline{OA}} = \frac{480}{120} = \underline{4 \text{ rad/s CW}}$$

5/72



$$\beta = \sin^{-1}\frac{0.5 - 0.5\sin 30°}{1.2}$$
$$= 12.02°$$

$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A} = \underline{v}_A + \underline{\omega} \times \underline{r}_{B/A}$$

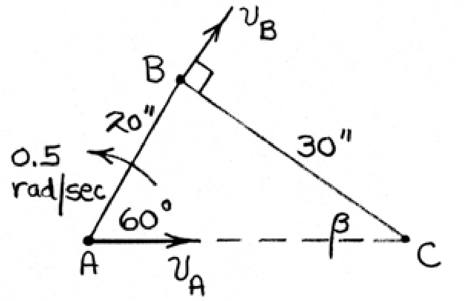
$$v_B(\sin 30°\,\underline{i} - \cos 30°\,\underline{j}) = 3\underline{i} + \omega\underline{k} \times 1.2(-\cos\beta\,\underline{i} + \sin\beta\,\underline{j})$$
$$= 3\underline{i} + \omega\underline{k} \times 1.2(-\cos 12.02°\,\underline{i} + \sin 12.02°\,\underline{j})$$
$$= 3\underline{i} - 1.174\omega\underline{j} - 0.250\omega\underline{i}$$

$$\left.\begin{aligned} \underline{i}: \ & \tfrac{1}{2}v_B = 3 - 0.250\omega \\ \underline{j}: \ & -\tfrac{\sqrt{3}}{2}v_B = -1.174\omega \end{aligned}\right\} \quad \begin{aligned} v_B &= 4.38 \text{ m/s} \\ \omega &= 3.23 \text{ rad/s} \end{aligned}$$

5/73

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

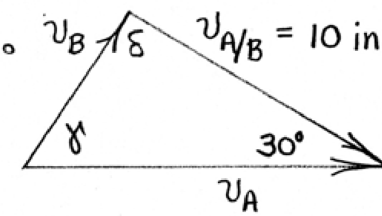
$$\frac{20}{\sin\beta} = \frac{30}{\sin 60^\circ}, \quad \beta = 35.3^\circ$$



$$v_{A/B} = \overline{AB}\,\omega_{AB} = 20(0.5)$$
$$= 10 \text{ in./sec}$$

$$\gamma = 90^\circ - 35.3^\circ = 54.7^\circ$$

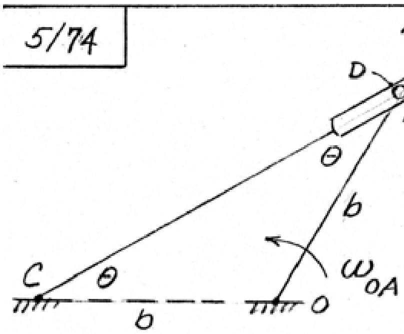
$$\delta = 180^\circ - 54.7^\circ - 30^\circ = 95.3^\circ$$



$$\frac{v_A}{\sin 95.3^\circ} = \frac{10}{\sin 54.7^\circ}$$

$$\underline{v_A = 12.20 \text{ in./sec}}$$

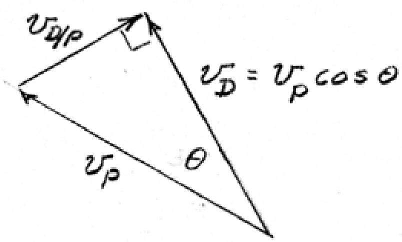
5/74



Let P = pin
D = point on CB coincident with P

$v_P = r\omega = b\omega_{OA}$

$\underline{v}_D = \underline{v}_P + \underline{v}_{D/P}$

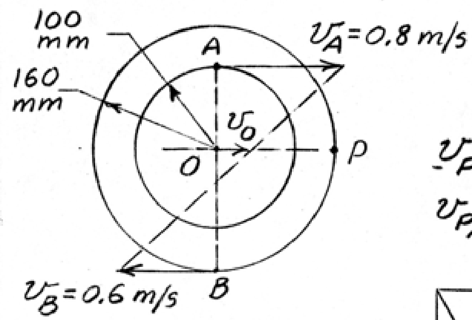


$v_D = \overline{CD}\,\omega_{CB} = 2b\cos\theta\;\omega_{CB}$

So $2b\cos\theta\;\omega_{CB} = b\omega_{OA}\cos\theta$

Thus $\underline{\omega_{CB} = \frac{1}{2}\omega_{OA}}$

5/75

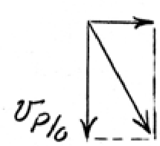


$$\omega = \frac{v_A + v_B}{\overline{AB}} = \frac{0.8 + 0.6}{0.26} = 5.38\ \frac{\text{rad}}{\text{s}}\ \text{CW}$$

$$v_O = v_A - \overline{AO}\,\omega$$
$$= 0.8 - 0.1(5.38) = 0.262\ \frac{\text{m}}{\text{s}}$$

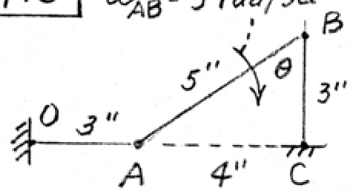
$$\underline{v}_P = \underline{v}_O + \underline{v}_{P/O}$$

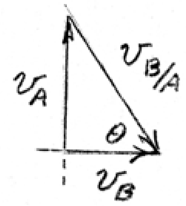
$$v_{P/O} = \overline{PO}\,\omega = 0.16(5.38) = 0.862\ \frac{\text{m}}{\text{s}}$$



$$v_P = \sqrt{\overline{0.262}^2 + \overline{0.862}^2}$$
$$= \underline{0.900\ \text{m/s}}$$

5/76 $\omega_{AB} = 3$ rad/sec





$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$, $\omega_{BC} = \dfrac{v_B}{\overline{BC}}$

$v_{B/A} = \overline{AB}\,\omega_{AB}$

$= 5(3) = 15$ in./sec

$\theta = \cos^{-1}\dfrac{3}{5}$

$v_B = v_{B/A}\cos\theta$

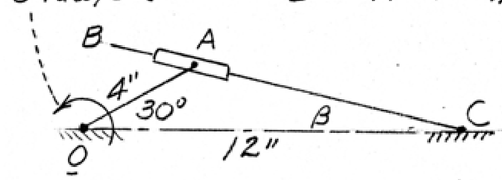
$= 15(3/5) = 9$ in./sec

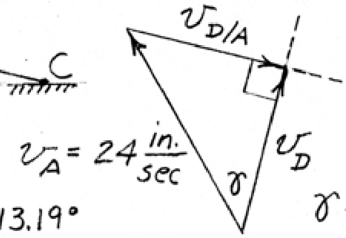
$\omega_{BC} = 9/3 = \underline{3 \text{ rad/sec CW}}$

5/77 Let D be point on BC coincident with A

$\underline{v}_D = \underline{v}_A + \underline{v}_{D/A}$, $v_A = r\omega = 4(6)$

$= 24$ in./sec





$\beta = \tan^{-1} \dfrac{4 \sin 30°}{12 - 4 \cos 30°} = 13.19°$

$\gamma = 30 + \beta$
$= 43.19°$

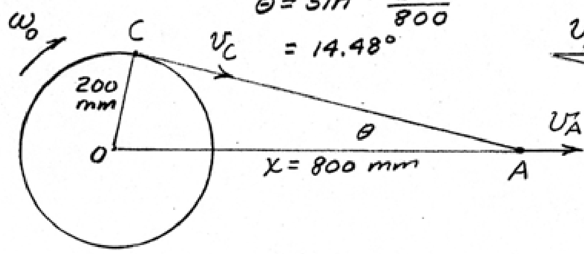
$v_D = v_A \cos \gamma = 24 \cos 43.19° = 17.50$ in./sec

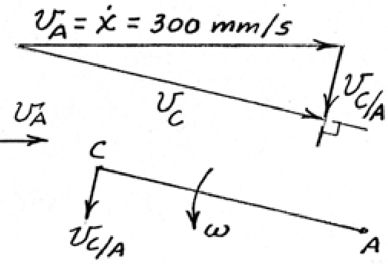
$\omega_{CB} = v_D / \overline{DC}$ where $\overline{DC} = \sqrt{4^2 + \overline{12}^2 - 2(4)(12)\cos 30°} = 8.77$ in.

$= \dfrac{17.50}{8.77} = \underline{2.00 \text{ rad/sec CW}}$

5/78 $\underline{v}_C = \underline{v}_A + \underline{v}_{C/A}$, $v_A = 300$ mm/s

$\theta = \sin^{-1}\frac{200}{800} = 14.48°$





$v_C = 300 \cos 14.48°$
$= 300(0.9682) = 290$ mm/s

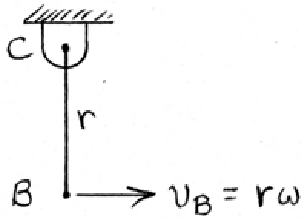
$v_{C/A} = 300 \sin 14.48° = 300/4 = 75$ mm/s

$\overline{CA} = 800 \cos 14.48° = 775$ mm

$\omega_{AB} = v_{C/A}/\overline{CA} = 75/775 = \underline{0.0968 \text{ rad/s CCW}}$

$\omega_o = v_C/\overline{CO} = 290/200 = \underline{1.452 \text{ rad/s CW}}$

5/79 (a) $v_A = v_B = r\omega$ (right)



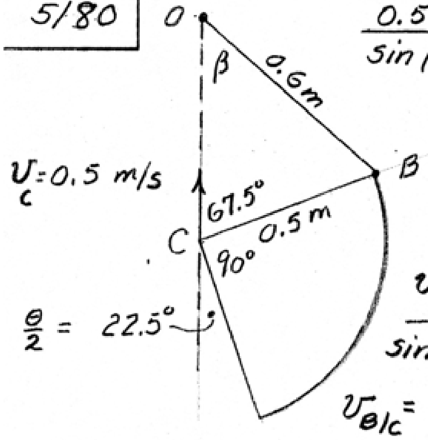
$$\omega_{BC} = \frac{v_B}{\overline{BC}} = \frac{r\omega}{r}$$

$$= \underline{\omega \quad CCW}$$

(b) $v_A = v_B = 2r\omega$ (right)

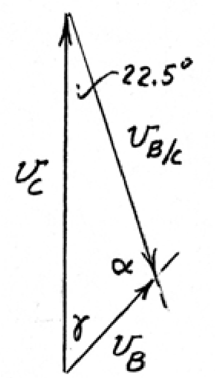
$$\omega_{BC} = \frac{v_B}{\overline{BC}} = \frac{2r\omega}{r} = \underline{2\omega \quad CCW}$$

5/80



$$\frac{0.5}{\sin\beta} = \frac{0.6}{\sin 67.5°}, \quad \beta = \sin^{-1} 0.7699 = 50.3°$$

$$\underline{v}_B = \underline{v}_C + \underline{v}_{B/C}$$



$$\frac{v_{B/C}}{\sin 39.7°} = \frac{v_C}{\sin 117.8°}$$

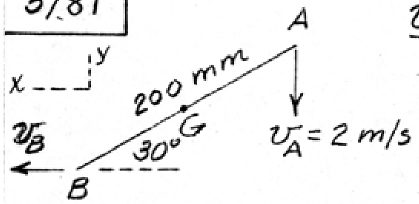
$$v_{B/C} = 0.361 \text{ m/s}$$

$$\omega = \omega_{BC} = \frac{0.361}{0.5} = \underline{0.722 \text{ rad/s}}$$

$$\gamma = 90 - \beta = 39.7°$$

$$\alpha = 180 - 39.7 - 22.5 = 117.8°$$

5/81

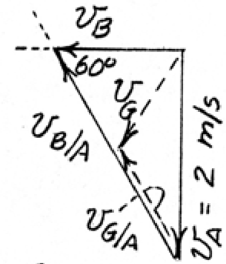


$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$\omega = \dfrac{v_{B/A}}{\overline{BA}}$

$= \dfrac{2/\cos 30°}{0.200}$

$= \underline{11.55 \text{ rad/s CW}}$



$\underline{v}_G = \underline{v}_A + \underline{v}_{G/A}$

$v_{G/A} = \overline{GA}\,\omega = \frac{1}{2} v_{B/A}$

From diagram $\quad v_G = 2/\sqrt{3} = \underline{1.155 \text{ m/s}}$

$\underline{v}_A = -2\underline{j}$ m/s , $\underline{v}_B = v_B \underline{i}$, $\underline{\omega}_{AB} = \omega_{AB}\underline{k}$

$v_B \underline{i} = -2\underline{j} + \omega_{AB}\underline{k} \times (0.2 \cos 30° \underline{i} - 0.2 \sin 30° \underline{j})$

$= (-2 + 0.1732\,\omega_{AB})\underline{j} + 0.1\,\omega_{AB}\underline{i}$

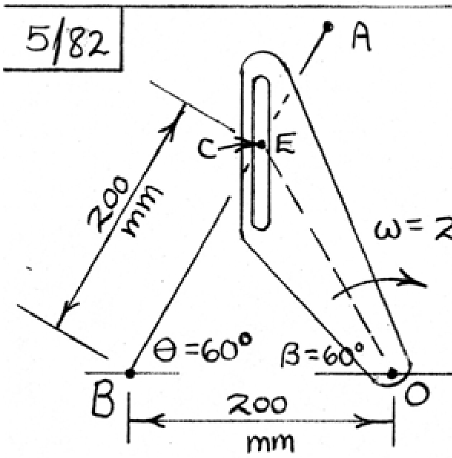
$\omega_{AB} = \dfrac{2}{0.1732} = \underline{11.55 \text{ rad/s}}$ CW ,

$\underline{v}_G = -2\underline{j} + 11.55\underline{k} \times (0.1 \cos 30° \underline{i} - 0.1 \sin 30° \underline{j})$

$= (-2 + 1.00)\underline{j} + 0.577\underline{i} = -\underline{j} + 0.577\underline{i}$ m/s

$v_G = \sqrt{1^2 + 0.577^2} = \underline{1.155 \text{ m/s}}$

5/82

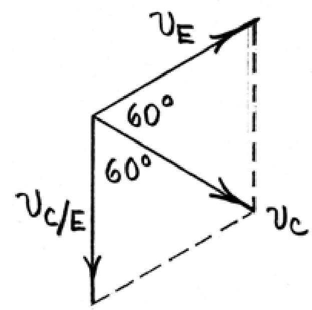


Let E be a point on D coincident with pin C.

$$\underline{v}_C = \underline{v}_E + \underline{v}_{C/E}$$

where $v_E = \overline{EO}\omega = 200\,(2)$

$= 400$ mm/s



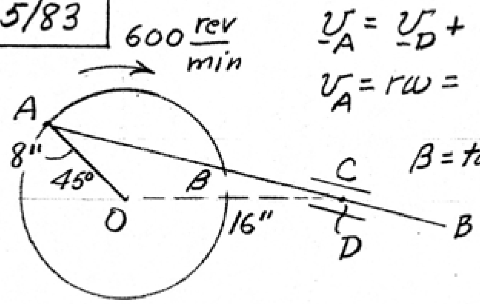
From vector triangles,

$v_C = 400$ mm/s

$$v_A = \frac{200+130}{200}(400)$$

$= \underline{660 \text{ mm/s}}$

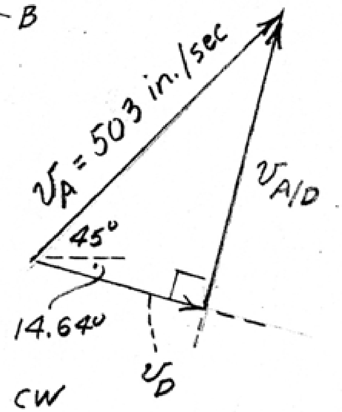
5/83



$\underline{v}_A = \underline{v}_D + \underline{v}_{A/D}$

$v_A = r\omega = 8\,\dfrac{600(2\pi)}{60} = 503$ in./sec

$\beta = \tan^{-1}\dfrac{8\sin 45°}{16 + 8\cos 45°} = 14.64°$



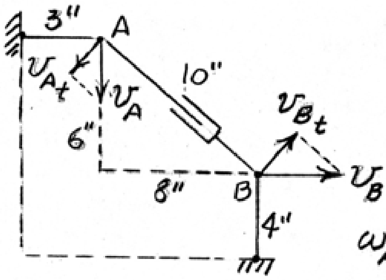
$v_{A/D} = 503\sin(45° + 14.64°)$
$= 434$ in./sec

$\omega_{AB} = \omega_{AD} = \dfrac{v_{A/D}}{\overline{AD}}$

$\overline{AD} = \dfrac{8\cos 45°}{\sin 14.64°} = 22.4$ in.

$\omega_{AB} = \dfrac{434}{22.4} = \underline{19.38 \text{ rad/sec CW}}$

5/84



$v_A = 3(0.5) = 1.5$ in./sec

$v_{A_t} = 1.5\left(\frac{4}{5}\right) = 1.2$ in./sec

$v_B = 4(0.5) = 2.0$ in./sec

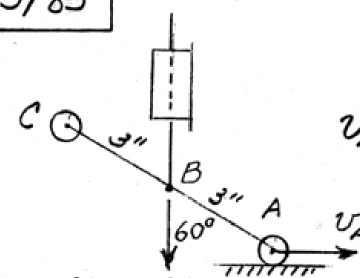
$v_{B_t} = 2.0\left(\frac{3}{5}\right) = 1.2$ in./sec

$$\omega_{AB} = \omega = \frac{v_{A_t} + v_{B_t}}{\overline{AB}}$$

$$\omega = \frac{1.2+1.2}{10} = \underline{0.24 \text{ rad/sec}}$$

CCW

5/85

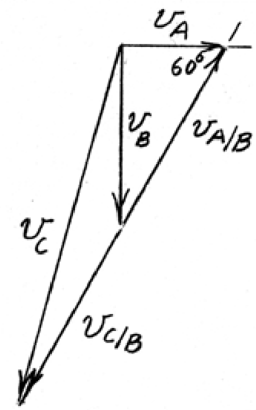


$v_B = 3$ ft/sec

$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$$

$$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B}$$

$$v_{A/B} = \overline{AB}\omega$$
$$= \overline{CB}\omega$$
$$= v_{C/B}$$



From geometry

$v_{A/B} = 3/\sin 60° = 3.46$ ft/sec, $v_A = 3/\tan 60° = 1.732 \frac{ft}{sec}$

$v_{C/B} = 3.46$ ft/sec

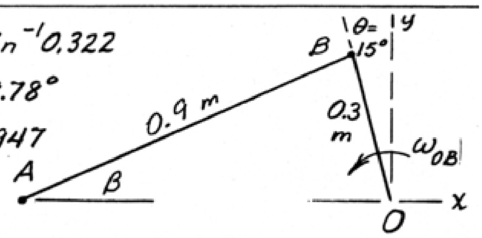
$$v_C = \sqrt{(3+3)^2 + (1.732)^2} = \sqrt{39} = \underline{6.24 \text{ ft/sec}}$$

5/86

$0.9 \sin\beta = 0.3 \cos 15°, \ \beta = \sin^{-1} 0.322$

$= 18.78°$

$\cos\beta = 0.947$



$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

I (Vector algebra) $\underline{v}_A = v_A \underline{i}, \ \underline{v}_B = \underline{\omega}_{OB} \times \underline{r}_{OB}$

$= \omega_{OB} \underline{k} \times (-0.3 \times 0.259 \underline{i} + 0.3 \times 0.966 \underline{j}) = \omega_{OB} (-0.0776 \underline{j} - 0.290 \underline{i})$

$\underline{v}_{A/B} = \underline{\omega}_{AB} \times \underline{r}_{AB} = -0.086 \underline{k} \times 0.9(-0.947 \underline{i} - 0.322 \underline{j}) = 0.0733 \underline{j} - 0.0249 \underline{i}$ m/s

So $v_A \underline{i} = -0.0776 \omega_{OB} \underline{j} - 0.290 \omega_{OB} \underline{i} + 0.0733 \underline{j} - 0.0249 \underline{i}$

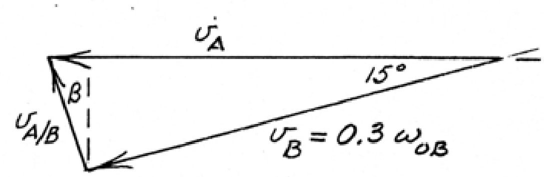
$\underline{j}$-terms: $\omega_{OB} = \frac{0.0733}{0.0776} = \underline{0.944 \text{ rad/s CCW}}$

$\underline{i}$-terms: $v_A = -0.290(0.944) - 0.0249 = \underline{-0.298 \text{ m/s}}$ (neg. x-dir)

II (Vector geometry)

$v_{A/B} = 0.9(0.086)$

$= 0.0774$ m/s

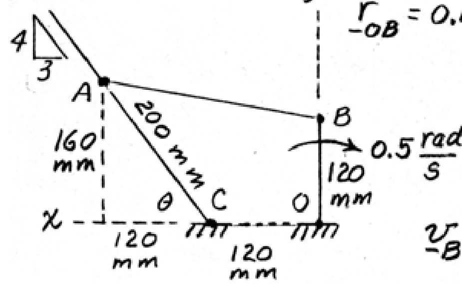


Law of sines: $\frac{0.0774}{\sin 15°} = \frac{0.3\,\omega_{OB}}{\sin(90° - 18.78°)}, \ \underline{\omega_{OB} = 0.944 \text{ rad/s CCW}}$

$v_A = 0.3(0.944) \cos 15° + 0.0774 \sin 18.78°,$

$\underline{v_A = 0.298 \text{ m/s to the left}}$

5/87



$\underline{r}_{CA} = 0.12\underline{i} + 0.16\underline{j}$ m

$\underline{r}_{OB} = 0.12\underline{j}$ m, $\underline{r}_{BA} = 0.24\underline{i} + 0.04\underline{j}$ m

$$\underline{v}_A = \underline{\omega}_{AC} \times \underline{r}_{CA}$$
$$= \omega_{AC}\underline{k} \times (0.12\underline{i} + 0.16\underline{j})$$
$$= 0.12\omega_{AC}\underline{j} - 0.16\omega_{AC}\underline{i}$$

$$\underline{v}_B = \underline{\omega}_{OB} \times \underline{r}_{OB} = 0.5\underline{k} \times 0.12\underline{j}$$
$$= -0.06\underline{i} \text{ m/s}$$

$$\underline{v}_{A/B} = \underline{\omega}_{AB} \times \underline{r}_{BA} = \omega_{AB}\underline{k} \times (0.24\underline{i} + 0.04\underline{j})$$
$$= 0.24\omega_{AB}\underline{j} - 0.04\omega_{AB}\underline{i}$$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$, so $0.12\omega_{AC}\underline{j} - 0.16\omega_{AC}\underline{i} = -0.06\underline{i}$
$+ 0.24\omega_{AB}\underline{j} - 0.04\omega_{AB}\underline{i}$

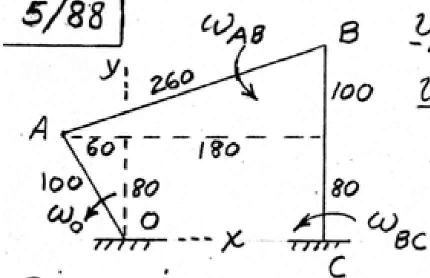
Equate coefficients & get

$$0.16\omega_{AC} - 0.04\omega_{AB} = 0.06$$
$$0.12\omega_{AC} - 0.24\omega_{AB} = 0$$

solve & get

$$\underline{\omega}_{AB} = 0.214\underline{k} \text{ rad/s}, \quad \underline{\omega}_{CA} = 0.429\underline{k} \text{ rad/s}$$

5/88



Dimen. in mm

$\omega_{AO} = \omega_o = 10$ rad/s

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$\underline{v}_A = \underline{\omega}_{AO} \times \underline{r}_{AO}$

$= 10\underline{k} \times (-0.06\underline{i} + 0.08\underline{j})$

$= -0.6\underline{j} - 0.8\underline{i}$ m/s

$\underline{v}_B = \underline{\omega}_{BC} \times \underline{r}_{BC}$

$= \omega_{BC}\underline{k} \times 0.18\underline{j} = -0.18\,\omega_{BC}\,\underline{i}$ m/s

$\underline{v}_{A/B} = \underline{\omega}_{AB} \times \underline{r}_{A/B}$

$= \omega_{AB}\underline{k} \times (-0.24\underline{i} - 0.1\underline{j})$

Thus, $\quad = -0.24\,\omega_{AB}\underline{j} + 0.1\,\omega_{AB}\underline{i}$ m/s

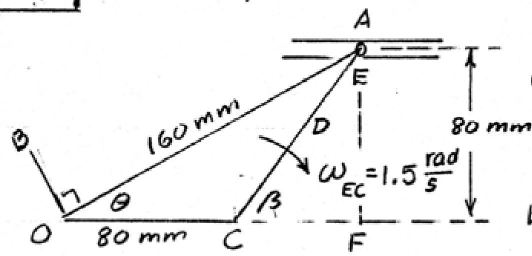
$-0.6\underline{j} - 0.8\underline{i} = -0.18\,\omega_{BC}\underline{i} - 0.24\,\omega_{AB}\underline{j} + 0.1\,\omega_{AB}\underline{i}$

Equate $\underline{j}$ terms & set $\omega_{AB} = \dfrac{0.6}{0.24} = 2.5$ rad/s

$\underline{\omega}_{BC} = 5.83\underline{k}$ rad/s

$\underline{\omega}_{AB} = 2.5\underline{k}$ rad/s

5/89 Let E be point on member D coincident with A



$\theta = \sin^{-1} \frac{80}{160} = 30°$

$\overline{CF} = 160 \cos 30° - 80$
$= 58.6 \text{ mm}$

$\beta = \tan^{-1} \frac{80}{58.6} = 53.8°$

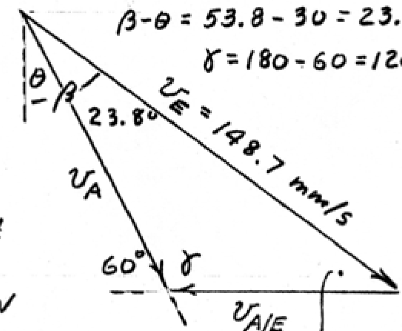
$\overline{EC} = \frac{80}{\sin 53.8°} = 99.1 \text{ mm}$

$\underline{v}_A = \underline{v}_E + \underline{v}_{A/E}$

$v_E = 99.1(1.5) = 148.7 \frac{mm}{s}$

$\beta - \theta = 53.8 - 30 = 23.8°$

$\gamma = 180 - 60 = 120°$



$90 - \beta = 36.2°$

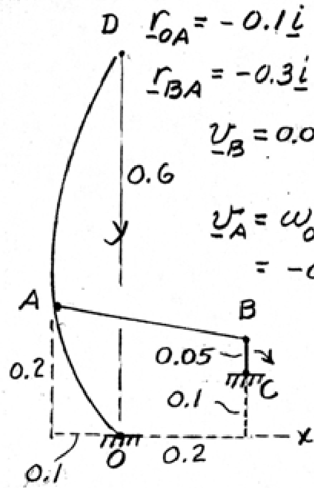
$$\frac{v_A}{\sin 36.2°} = \frac{148.7}{\sin 120°}$$

$$v_A = 148.7 \frac{0.591}{0.866} = 101.4 \frac{mm}{s}$$

$$\omega_{AOB} = \frac{101.4}{160} = \underline{0.634 \text{ rad/s CW}}$$

Alternatively, draw vector triangle to scale & measure $v_A \approx 101$ mm/s. Etc.

5/90 $\omega_{CB} = -\frac{2\pi}{2}\frac{rad}{s}$ or $\underline{\omega}_{CB} = -\pi\underline{k}$ rad/s



$\underline{r}_{OA} = -0.1\underline{i} + 0.2\underline{j}$ m , $\underline{r}_{CB} = 0.05\underline{j}$ m

$\underline{r}_{BA} = -0.3\underline{i} + 0.05\underline{j}$ m, $\underline{r}_{OD} = 0.6\underline{j}$,

$\underline{v}_B = 0.05\pi\underline{i}$ m/s, $\underline{v}_{A/B} = \omega_{AB}\underline{k} \times (-0.3\underline{i} + 0.05\underline{j})$

$= -0.3\omega_{AB}\underline{j} - 0.05\omega_{AB}\underline{i}$

$\underline{v}_A = \omega_{OA}\underline{k} \times (-0.1\underline{i} + 0.2\underline{j})$

$= -0.1\omega_{OA}\underline{j} - 0.2\omega_{OA}\underline{i}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$ so

$-0.1\omega_{OA}\underline{j} - 0.2\omega_{OA}\underline{i} = 0.05\pi\underline{i}$

$-0.3\omega_{AB}\underline{j} - 0.05\omega_{AB}\underline{i}$

Dimensions in meters

Thus $-0.2\omega_{OA} + 0.05\omega_{AB} = 0.05\pi$

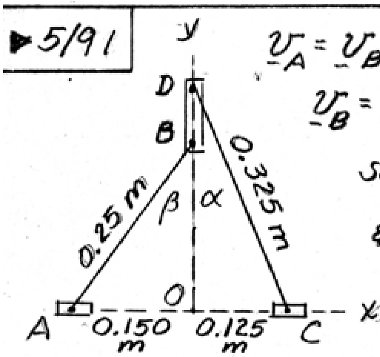
$-0.1\omega_{OA} + 0.3\omega_{AB} = 0$

Solve & get $\underline{\omega}_{AB} = -0.0909\pi\underline{k} = -0.286\underline{k}$ rad/s (CW)

$\underline{\omega}_{OA} = -0.273\pi\underline{k} = -0.857\underline{k}$ rad/s (CW)

$v_E = v_D = 0.6\,\omega_{OD} = 0.6\,\omega_{OA} = 0.6(0.857) = \underline{0.514 \text{ m/s}}$

▶5/91



$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$, $\underline{v}_C = \underline{v}_D + \underline{v}_{C/D}$

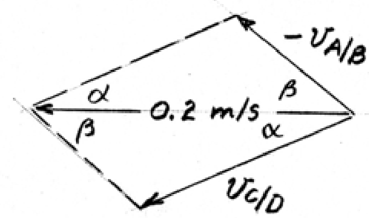
$\underline{v}_B = \underline{v}_D$, $\underline{v}_C - \underline{v}_A = -0.2\underline{i}$ m/s

So $\underline{v}_A = (\underline{v}_C - \underline{v}_{C/D}) + \underline{v}_{A/B}$

& $\underline{v}_{C/D} - \underline{v}_{A/B} = \underline{v}_C - \underline{v}_A = -0.2\underline{i}$ m/s

$\overline{OD} = \sqrt{(0.325)^2 - (0.125)^2}$
$= 0.3$ m

$\overline{OB} = \sqrt{(0.25)^2 - (0.15)^2}$
$= 0.2$ m



$\sin\alpha = 0.125/0.325 = 5/13$

$\cos\alpha = 0.3/0.325 = 12/13$

$\sin\beta = 0.150/0.250 = 3/5$

$\cos\beta = 0.2/0.25 = 4/5$

$v_{C/D}\cos\alpha + v_{A/B}\cos\beta = 0.2$

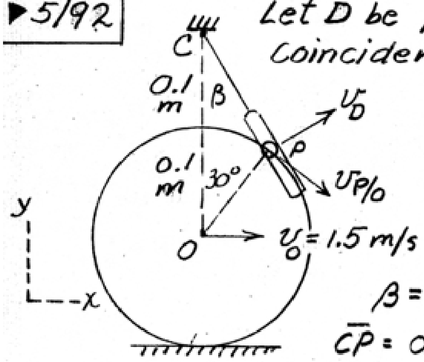
$v_{C/D}\sin\alpha - v_{A/B}\sin\beta = 0$

solve & get $v_{C/D} = \frac{39}{280} = 0.1393$ m/s

$\underline{v} = \underline{v}_D = \underline{v}_C - \underline{v}_{C/D}$; $\underline{v}_J = v_C\underline{i} - \left[\frac{39}{280}\right](-\underline{i}\cos\alpha - \underline{j}\sin\alpha)$

so $v = \frac{39}{280}\frac{5}{13} = 3/56 = \underline{0.0536 \text{ m/s}}$

▶5/92



Let D be point on slotted arm coincident with P

$$\underline{v}_D = \underline{v}_P + \underline{v}_{D/P} \qquad \text{---- (a)}$$

$$\underline{v}_P = \underline{v}_O + \underline{v}_{P/O}$$

$$v_{P/O} = \overline{PO}\,\omega_{PO} = \overline{PO}\,\frac{v_O}{\overline{PO}} = v_O$$

$$\beta = \tan^{-1}\frac{0.1\sin 30^\circ}{0.2 - 0.1\cos 30^\circ} = 23.8^\circ$$

$$\overline{CP} = 0.1\,\frac{\sin 30^\circ}{\sin 23.8^\circ} = 0.1239 \text{ m}$$

$$\underline{v}_D = v_D(\underline{i}\cos\beta + \underline{j}\sin\beta) = v_D(0.915\underline{i} + 0.403\underline{j})$$

$$\underline{v}_P = 1.5\underline{i} + (1.5\cos 30^\circ)\underline{i} - (1.5\sin 30^\circ)\underline{j} = 2.799\underline{i} - 0.75\underline{j} \text{ m/s}$$

$$\underline{v}_{D/P} = v_{D/P}(-\underline{i}\sin\beta + \underline{j}\cos\beta) = v_{D/P}(-0.403\underline{i} + 0.915\underline{j})$$

Substitute in Eq. (a) & separate $\underline{i}$ & $\underline{j}$ terms to get

$$\left.\begin{array}{l} 0.915 v_D - 2.799 + 0.403 v_{D/P} = 0 \\ 0.403 v_D + 0.75 - 0.915 v_{D/P} = 0 \end{array}\right\} \text{solve \& get}$$

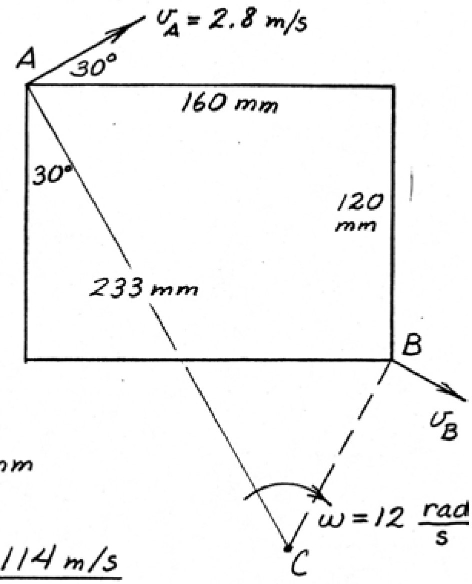
$v_D = 2.26$ m/s, $v_{D/P} = 1.816$ $\frac{m}{s}$

Thus $\omega = \omega_{CD} = \dfrac{2.26}{0.1239} = \underline{18.22 \text{ rad/s CCW}}$

5/93 Instantaneous center C of zero velocity must lie on the perpendicular to v_A at a distance from A of

$v = r\omega$, $\overline{AC} = r = \dfrac{v}{\omega} = \dfrac{2.8}{12}$

$= 0.233$ m or 233 mm

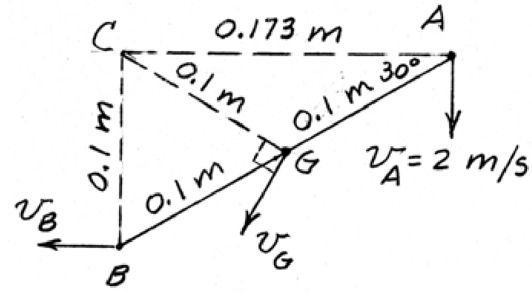


$\overline{CB}^2 = (160 - 233 \sin 30°)^2 + (233 \cos 30° - 120)^2$

$= 8614 \text{ mm}^2$, $\overline{CB} = 92.8$ mm

$v_B = \overline{CB}\,\omega = 0.0928(12) = \underline{1.114 \text{ m/s}}$

5/94



$\omega = v_A / \overline{AC}$

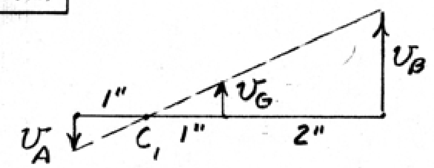
$= 2/0.1732 = \underline{11.55 \text{ rad/s}}$ $\underline{\text{CW}}$

$v_G = \overline{CG}\,\omega$

$= 0.1(11.55)$

$= \underline{1.155 \text{ m/s}}$

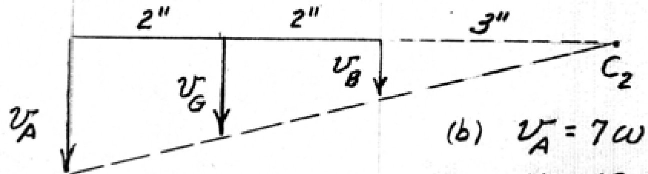
5/95



(a) $v_A = (1)\omega = 1(6)$

$\underline{v_A = 6 \text{ in./sec}}$

$\underline{v_G = 6 \text{ in./sec}}$

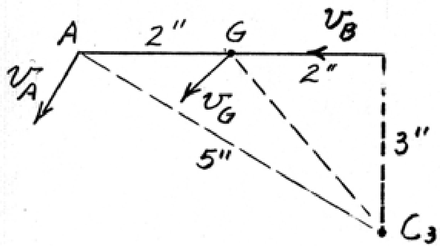


(b) $v_A = 7\omega = 7(6)$

$\underline{v_A = 42 \text{ in./sec}}$

$v_G = 5\omega = 5(6)$

$\underline{v_G = 30 \text{ in./sec}}$

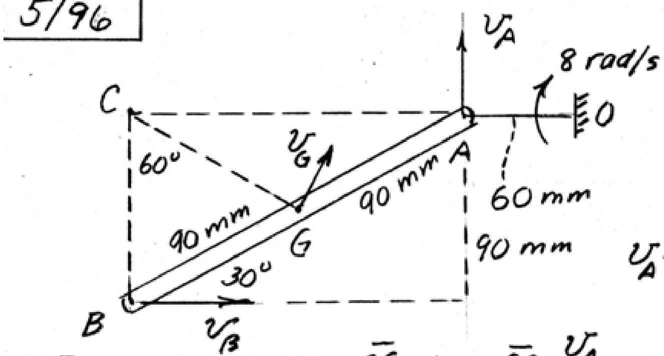


(c) $v_A = 5\omega = 5(6) = \underline{30 \text{ in./sec}}$

$v_G = \overline{C_3G}\,\omega = \sqrt{3^2+2^2}\,(6)$

$= \underline{21.6 \text{ in./sec}}$

5/96



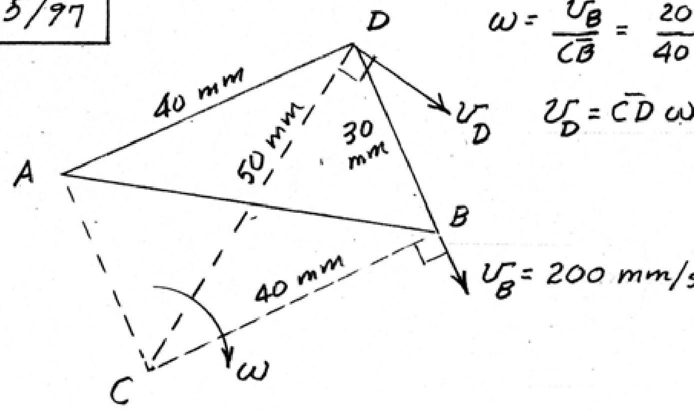
$v_A = \overline{OA}\,\omega_{OA}$

$= 60(8) = 480$ mm/s

$\overline{CG} = 90$ mm, $\quad v_G = \overline{CG}\,\omega_{CG} = \overline{CG}\,\dfrac{v_A}{\overline{AC}} = \dfrac{90}{180\cos 30^\circ}\,480$

$\underline{v_G = 277 \text{ mm/s}}$

5/97

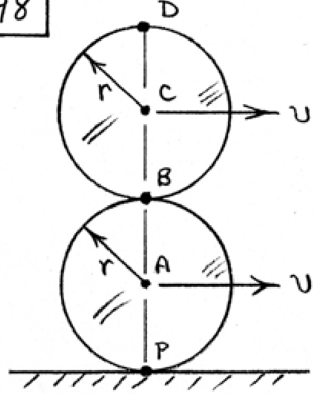


$\omega = \frac{v_B}{\overline{CB}} = \frac{200}{40} = 5$ rad/s

$v_D = \overline{CD}\,\omega = 50(5)$

$= 250$ mm/s

5/98



(a) $\omega_l = \frac{v}{r}$ CW

(b) $\omega_u = \frac{v}{r}$ CCW

(c) $v_A = v$ (right)

$v_B = 2v$ (right)

$v_C = v$ (right)

$v_D = v_P = 0$

The mechanic's hands have no absolute velocity!

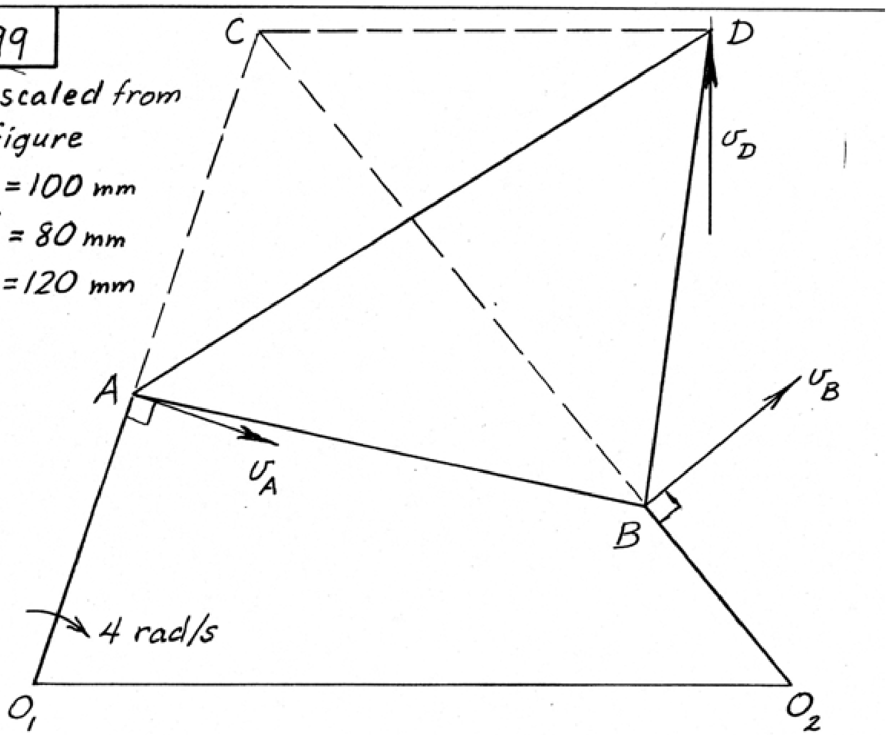
5/99

As scaled from figure

$\overline{AC} = 100$ mm

$\overline{O_1A} = 80$ mm

$\overline{CD} = 120$ mm



$$v_A = \overline{O_1A}\,\omega = 0.80(4) = 0.32 \text{ m/s}$$

$$v_D/\overline{CD} = v_A/\overline{AC}, \quad v_D = \frac{0.120}{0.100}\,0.32 = \underline{0.38 \text{ m/s}}$$

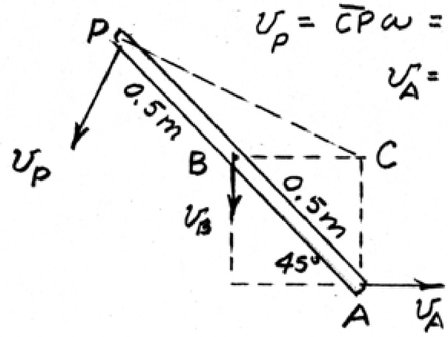
5/100

$\overline{CP} = \sqrt{(1 \times \cos 45°)^2 + (0.5 \sin 45°)^2} = 0.791$ m

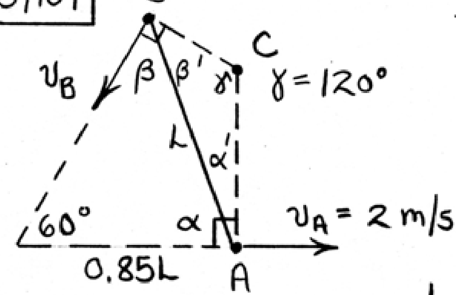
$v_P = \overline{CP}\omega = 0.791(2) = \underline{1.581 \text{ m/s}}$

$v_A = \overline{CA}\omega = 0.5 \sin 45°(2)$

$\underline{v_A = 0.707 \text{ m/s}}$



5/101



$$\frac{\sin 60°}{L} = \frac{\sin\beta}{0.85L}$$

$$\beta = 47.4°$$

$$\alpha = 180° - 60° - 47.4° = 72.6°$$

$$\beta' = 42.6°, \quad \alpha' = 17.40°$$

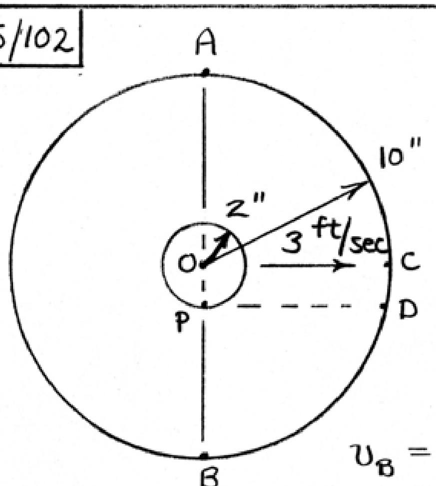
$$\frac{\sin\gamma}{L} = \frac{\sin\beta'}{\overline{AC}}, \quad \overline{AC} = 0.782L$$

$$\frac{\sin\gamma}{L} = \frac{\sin\alpha'}{\overline{BC}}, \quad \overline{BC} = 0.345L$$

$$v_A = \overline{AC}\,\omega, \quad \omega = \frac{2}{0.782L} = \frac{2}{0.782(0.8)} = \underline{3.20 \ \frac{\text{rad}}{\text{s}}}$$

$$v_B = \overline{BC}\,\omega = 0.345(0.8)(3.20) = \underline{0.884 \text{ m/s}}$$

5/102



Point P is the instantaneous center

$v_O = \overline{OP}\,\omega$, $\omega = \frac{3}{2/12}$

$\omega = 18$ rad/sec CW

$v_A = \overline{AP}\,\omega = \frac{12}{12}(18)$

$= \underline{18 \text{ ft/sec} \rightarrow}$

$v_B = \overline{BP}\,\omega = \frac{8}{12}(18) = \underline{12\,\frac{\text{ft}}{\text{sec}} \leftarrow}$

$v_C = \overline{CP}\,\omega = \sqrt{\left(\frac{2}{12}\right)^2 + \left(\frac{10}{12}\right)^2}\,(18) = \underline{15.30 \text{ ft/sec}}$

$\alpha = \tan^{-1}\frac{2}{12} = 9.46°$

$v_D = \overline{DP}\,\omega = \frac{10}{12}(18) = \underline{15 \text{ ft/sec} \downarrow}$

5/103

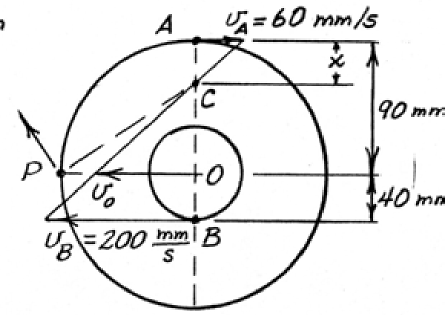
$$\frac{60}{x} = \frac{60+200}{90+40}, \quad x = 30 \text{ mm}$$

$$\frac{v_O}{\overline{CO}} = \frac{v_A}{\overline{AC}}, \quad v_O = \frac{60}{30}60 = \underline{120 \ \frac{\text{mm}}{\text{s}}}$$

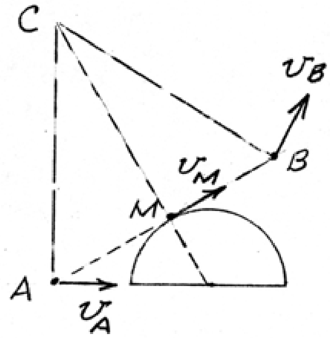
$$\overline{CP} = \sqrt{60^2 + 90^2} = 108.2 \text{ mm}$$

$$v_P = \overline{CP}\,\omega = \overline{CP}\,\frac{v_A}{\overline{AC}} = 108.2\,\frac{60}{30}$$

$$= \underline{216 \text{ mm/s}}$$



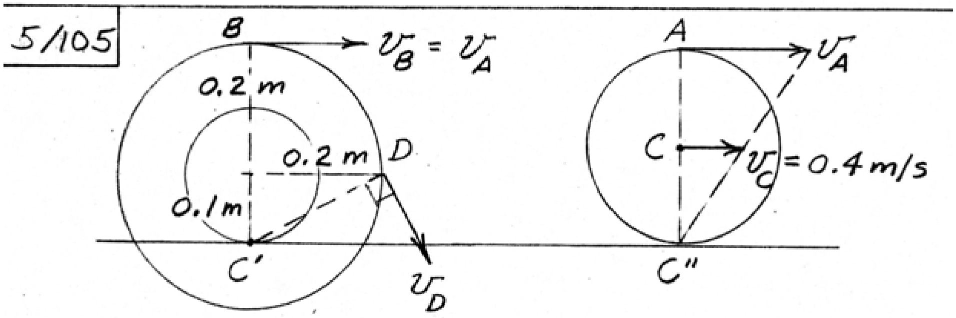
5/104



For given position
$\overline{CB} = \overline{CA}$ so

$v_B = \overline{CB}\,\omega = \overline{CB}\,\dfrac{v_A}{\overline{AC}} = v_A$

5/105



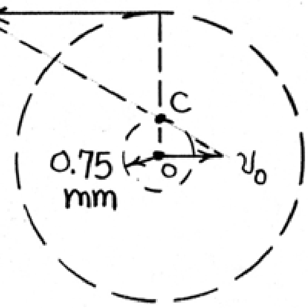
$$v_B = v_A = \frac{\overline{AC''}}{\overline{CC''}} v_C = 2(0.4) = 0.8 \text{ m/s}$$

$$v_D = \overline{C'D}\,\omega = \overline{C'D}\,\frac{v_B}{\overline{BC'}} = \frac{\sqrt{\overline{0.1}^2 + \overline{0.2}^2}}{0.3}\,0.8 = \underline{0.596 \text{ m/s}}$$

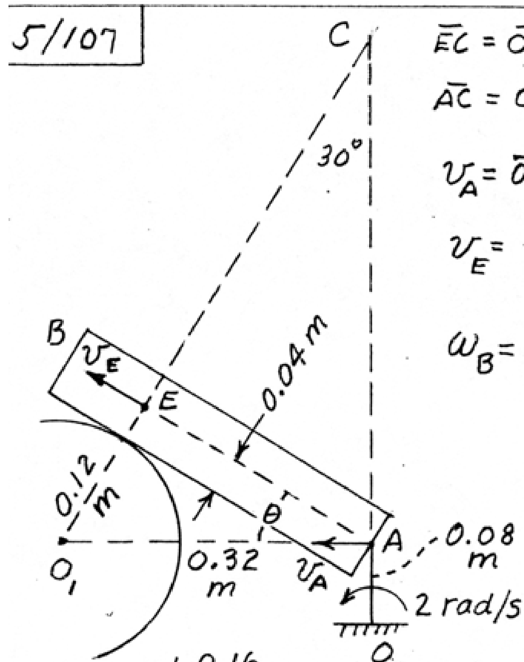
5/106 C = instantaneous center

$v_o = \overline{OC}\,\omega = 0.75(10^{-3})\left(\frac{1800 \cdot 2\pi}{60}\right)$

$= \underline{0.1414 \text{ m/s}}$



5/107



$\theta = \sin^{-1}\dfrac{0.16}{0.32} = 30°$

$\overline{EC} = \overline{O_1C} - 0.16 = 0.64 - 0.16 = 0.48$ m

$\overline{AC} = \overline{O_1A}\ \text{ctn}\ 30° = 0.32\sqrt{3} = 0.554$ m

$v_A = \overline{OA}\,\omega_{OA} = 0.08(2) = 0.16$ m/s

$v_E = \dfrac{\overline{EC}}{\overline{AC}}\,v_A = \dfrac{0.48}{0.554}\,0.16 = 0.1386$ m/s

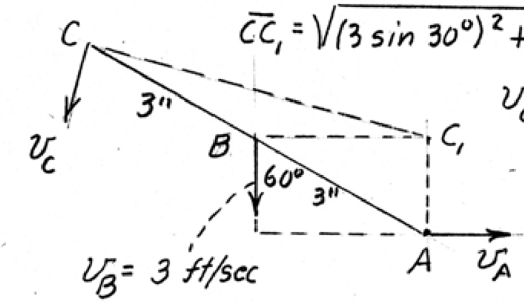
$\omega_B = \dfrac{v_A}{\overline{AC}} = \dfrac{0.16}{0.554} = 0.289$ rad/s CW

5/108

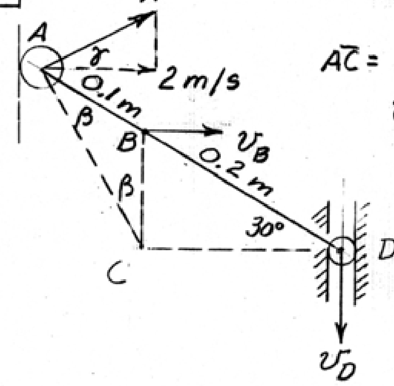
$v_C = \overline{CC_1}\,\omega_{AC}$, $\omega_{AC} = \omega_{AB} = v_B/\overline{BC_1} = \dfrac{3}{\frac{3}{12}\sin 60^\circ} = 13.86$ rad/s

$\overline{CC_1} = \sqrt{(3\sin 30^\circ)^2 + (6\cos 30^\circ)^2} = 5.41$ in.

$v_C = \dfrac{5.41}{12}(13.86) = \underline{6.24 \ \dfrac{ft}{sec}}$



5/109



$\overline{CB} = 0.1\text{ m} = \overline{AB}$, $2\beta + (180-60) = 180$

$\beta = 30°$, $\gamma = 30°$

$\overline{AC} = 2(0.1)\cos 30° = 0.1732\text{ m}$

$v_A = \dfrac{2}{\cos 30°} = 2.31\text{ m/s}$

$\omega = \dfrac{v_A}{AC} = \dfrac{2.31}{0.1732} = \underline{13.33\ \dfrac{\text{rad}}{\text{s}}}$

$v_D = \overline{CD}\,\omega = 0.2\cos 30°(13.33)$

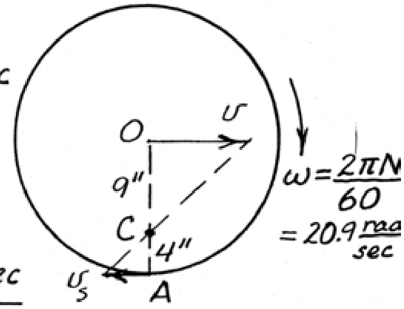
$= \underline{2.31\text{ m/s}}$

5/110

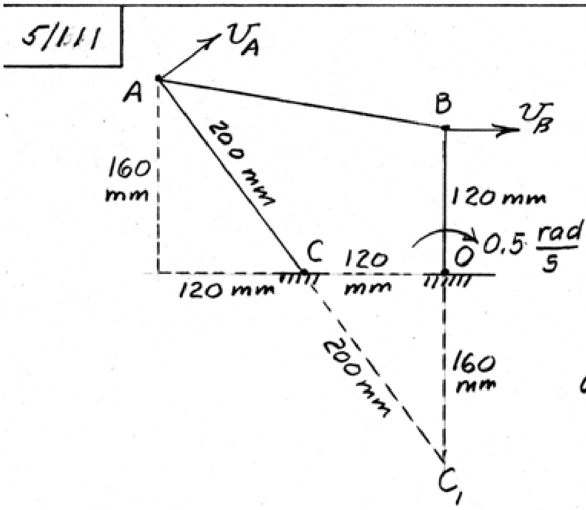
$\omega = v/\overline{OC}$, $v = \frac{9}{12} \cdot 20.9 = 15.71$ ft/sec

or $\underline{v = 10.71 \text{ mi/hr}}$

$v_s = \frac{4}{9} v = \frac{4}{9}(15.71)$, $\underline{v_s = 6.98 \text{ ft/sec}}$



5/111



$$v_B = 0.120(0.5) = 0.060 \text{ m/s}$$

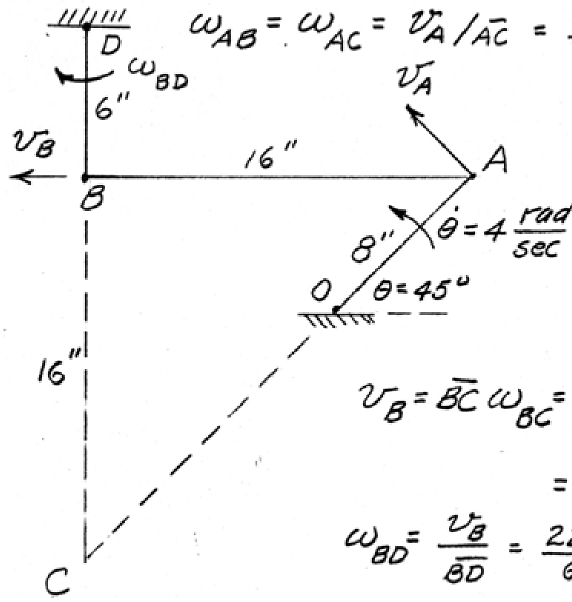
$$\omega_{CA} = v_A / \overline{CA}$$

$$v_A = \frac{\overline{AC_1}}{\overline{BC_1}} v_B = \frac{0.400}{0.280}(0.060) = 0.0857 \text{ m/s}$$

$$\omega_{CA} = \frac{0.0857}{0.200} = \underline{0.429 \ \frac{\text{rad}}{\text{s}}}$$

5/112 $v_A = \overline{AO}\,\dot{\theta} = 8(4) = 32$ in./sec

$\omega_{AB} = \omega_{AC} = v_A/\overline{AC} = \dfrac{32}{16\sqrt{2}} = \sqrt{2} = 1.414\ \dfrac{\text{rad}}{\text{sec}}$ <u>CCW</u>

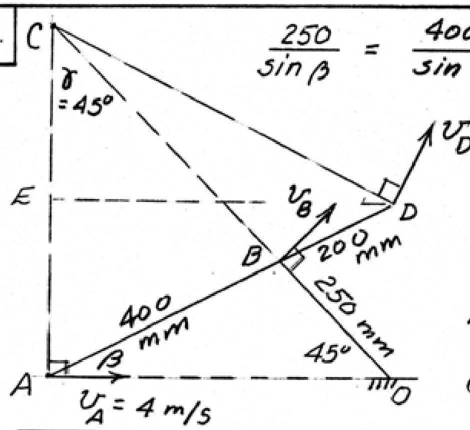


$v_B = \overline{BC}\,\omega_{BC} = \overline{BC}\,\dfrac{v_A}{\overline{AC}} = \dfrac{16}{16\sqrt{2}}(32)$

$= 22.6$ in./sec

$\omega_{BD} = \dfrac{v_B}{\overline{BD}} = \dfrac{22.6}{6} = $ <u>3.77 rad/sec CW</u>

5/113.



$$\frac{250}{\sin\beta} = \frac{400}{\sin 45°}, \quad \beta = 26.2°$$

$\overline{AO} = 400\cos 26.2° + 250\cos 45° = 535.6$ mm

$\overline{AC} = \overline{AO}\tan 45° = 535.6$ mm

$\overline{ED} = 600\cos 26.2° = 538.2$ mm

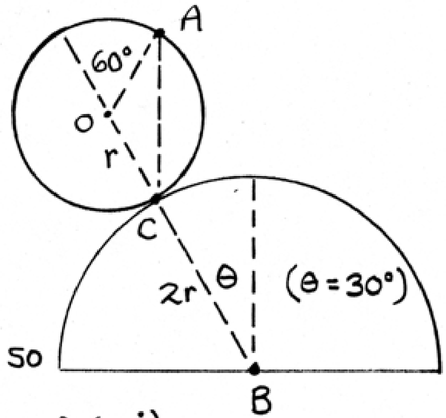
$\overline{CE} = 535.6 - 600\sin 26.2° = 270.4$ mm

$$\overline{CD} = \sqrt{(270.4)^2 + (538.2)^2} = 602.4 \text{ mm}$$

$$v_D = v_A \frac{\overline{CD}}{\overline{CA}} = 4\,\frac{602.4}{535.6} = \underline{4.50 \text{ m/s}}$$

$$\omega_{ABD} = \omega = v_A/\overline{CA} = \frac{4000}{535.6} = \underline{7.47 \text{ rad/s}}$$

5/114



$v_O = \overline{OB}\,\dot{\theta} = 3r\dot{\theta}$

$\omega_{OC} = \omega_{AC} = \dfrac{v_O}{\overline{OC}} = \dfrac{3r\dot{\theta}}{r}$

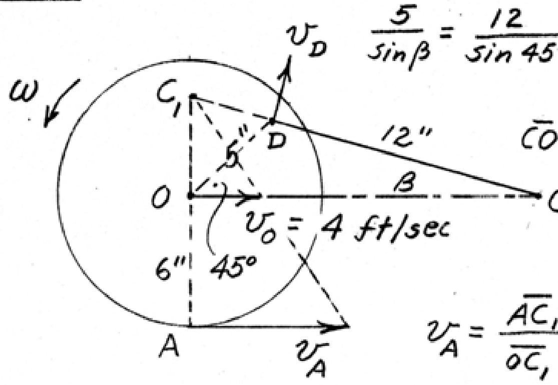
$= 3\dot{\theta}$

C is the instantaneous center of zero velocity, so

$v_A = \overline{AC}\,\omega_{AC} = 2r\cos 30^\circ\,(3\dot{\theta})$

$= 2r\,\dfrac{\sqrt{3}}{2}\,(3\dot{\theta}) = \underline{3\sqrt{3}\,r\dot{\theta}}$

5/115 C_1 = instantaneous center



$\frac{5}{\sin\beta} = \frac{12}{\sin 45°}$, $\beta = \sin^{-1}\left(\frac{5}{12}\frac{1}{\sqrt{2}}\right)$

$= 17.14°$

$\overline{CO} = 5\cos 45° + 12\cos 17.14°$

$= 15.00$ in.

$\overline{C_1O} = 15.00 \tan 17.14°$

$= 4.63$ in.

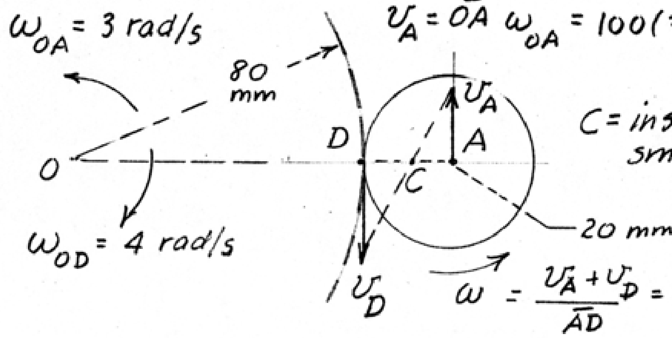
$v_A = \frac{\overline{AC_1}}{\overline{OC_1}} v_O = \frac{6 + 4.63}{4.63} 4$

$= \underline{9.19 \text{ ft/sec}}$

5/116

$v_D = \overline{OD}\,\omega_{OD} = 80(4) = 320 \text{ mm/s}$

$v_A = \overline{OA}\,\omega_{OA} = 100(3) = 300 \text{ mm/s}$



C = instant. center of small gear

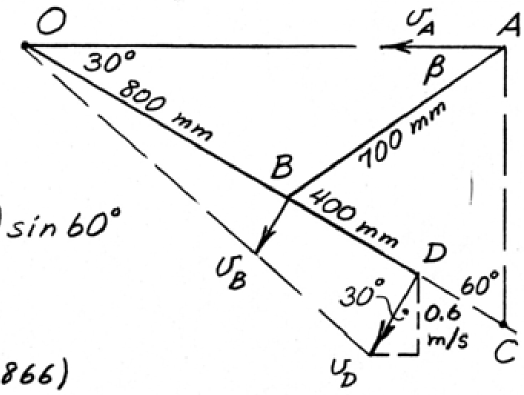
$$\omega = \frac{v_A + v_D}{\overline{AD}} = \frac{320 + 300}{20}$$

$\omega = 31$ rad/s CCW

5/117

$800 \sin 30° = 700 \sin\beta$

$\beta = \sin^{-1}\frac{4}{7} = 34.8°$



$700 \cos 34.8° = (400 + \overline{DC}) \sin 60°$

$\overline{DC} = 263$ mm

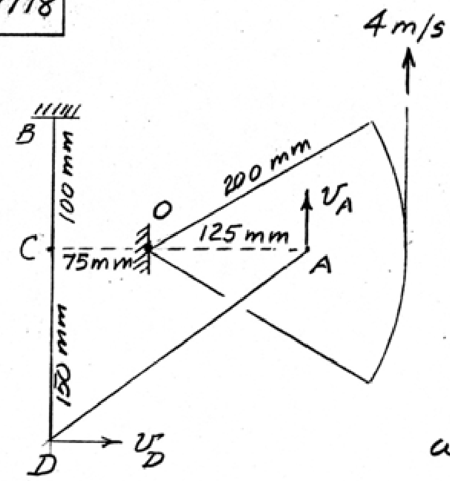
$v_B = \frac{800}{1200} v_D = \frac{8}{12}(0.6/0.866)$
$= 0.462$ m/s

C = inst. center for AB

$\overline{AC} = 700 \sin 34.8° + (400+263)\cos 60° = 732$ mm

$v_A/\overline{AC} = v_B/\overline{BC}$, $v_A = \frac{732}{400+263} 0.462 = \underline{0.509 \text{ m/s}}$

5/118



C = instantaneous center of zero velocity for AD.

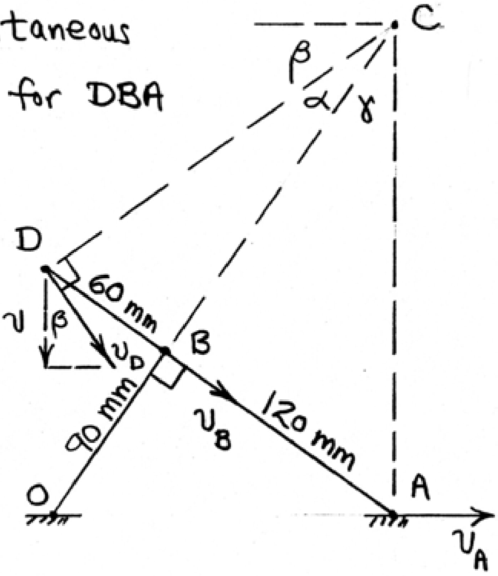
$v_A = \frac{125}{200}(4) = 2.5 \text{ m/s}$

$\omega_{AD} = \frac{v_A}{\overline{AC}} = \frac{2.5}{0.200} = 12.5 \frac{\text{rad}}{\text{s}}$

$v_D = \overline{CD}\,\omega_{CD} = \overline{CD}\,\omega_{AD}$
$= 0.150\ (12.5)$
$= 1.875 \text{ m/s}$

$\omega_{BD} = \frac{v_D}{BD} = \frac{1.875}{0.25} = 7.5 \text{ rad/s}$

5/119 C is the instantaneous center of zero velocity for DBA



From geometry,

$\overline{AC} = \frac{5}{3}(120) = 200$ mm

$\overline{BC} = 160$ mm

$\overline{DC} = \sqrt{60^2 + 160^2}$

$= 170.9$ mm

$\gamma = \sin^{-1}\frac{120}{200} = 36.9°$

$\alpha = \tan^{-1}\frac{60}{160} = 20.6°$

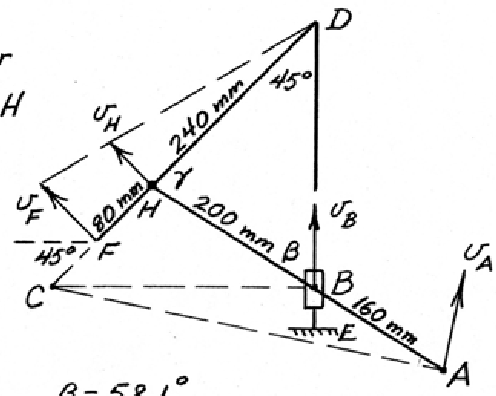
$\beta = 90 - 36.9 - 20.6 = 32.6°$

$v_D = \frac{v}{\cos\beta} = \frac{0.2}{\cos 32.6°} = 0.237$ m/s

$\frac{v_D}{\overline{DC}} = \frac{v_A}{\overline{AC}}$; $v_A = \frac{200}{170.9}(0.237) = \underline{0.278 \text{ m/s}}$

5/120

C = instantaneous center of zero velocity of ABH



$v_F \cos 45° = v_G = 2$ m/s

so $v_F = 2.83$ m/s

& $v_H = \dfrac{240}{80+240} 2.83 = 2.12$ m/s

Law of sines, $\dfrac{240}{\sin\beta} = \dfrac{200}{\sin 45°}$, $\beta = 58.1°$

$\gamma = 180° - (58.1° + 45°) = 76.9°$

$\dfrac{\overline{BD}}{\sin 76.9°} = \dfrac{200}{\sin 45°}$, $\overline{BD} = 276$ mm & $\overline{DC} = \dfrac{276}{\cos 45°} = 390$ mm

$\overline{CA}^2 = 276^2 + 160^2 - 2(276)(160)\cos(90° + 58.1°)$, $\overline{CA} = 420$ mm

$\overline{CH} = \overline{CD} - 240 = 390 - 240 = 149.7$ mm

$v_A / \overline{AC} = v_H / \overline{CH}$, $v_A = 2.12 \dfrac{420}{149.7} = \underline{5.95 \text{ m/s}}$

►5/121 $\overline{OB} = 4$ in., $\overline{BD} = 30$ in.

$\overline{AE} = \overline{ED} = \overline{DF} = 15$ in.

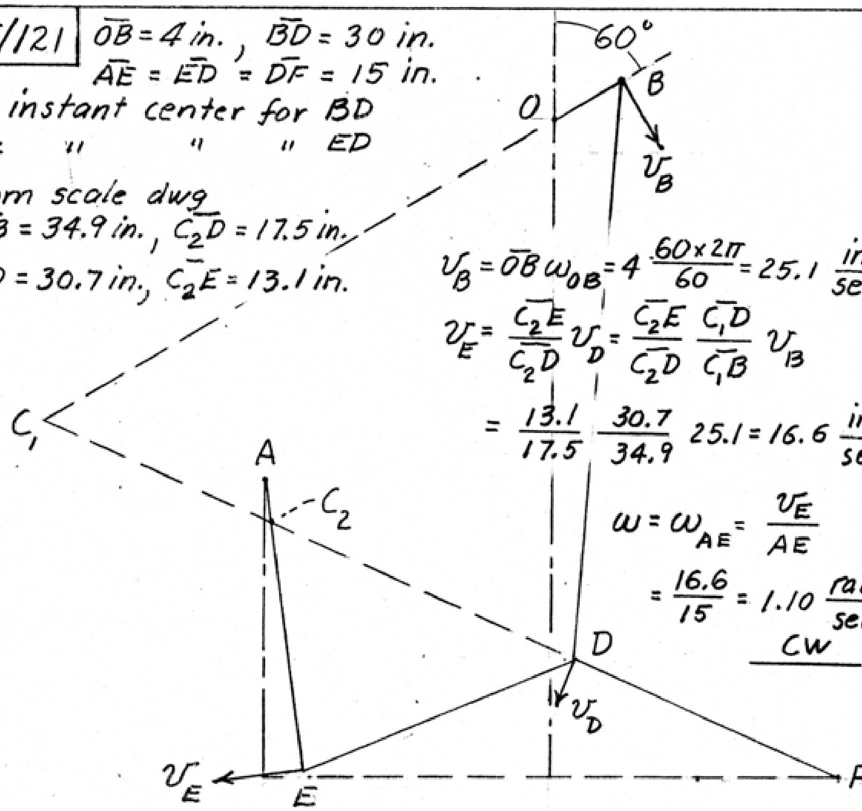
C_1 = instant center for BD

C_2 = " " " ED

From scale dwg

$\overline{C_1B} = 34.9$ in., $\overline{C_2D} = 17.5$ in.,

$\overline{C_1D} = 30.7$ in., $\overline{C_2E} = 13.1$ in.



$$v_B = \overline{OB}\,\omega_{OB} = 4\,\frac{60 \times 2\pi}{60} = 25.1\ \frac{\text{in.}}{\text{sec}}$$

$$v_E = \frac{\overline{C_2E}}{\overline{C_2D}}\,v_D = \frac{\overline{C_2E}}{\overline{C_2D}}\,\frac{\overline{C_1D}}{\overline{C_1B}}\,v_B$$

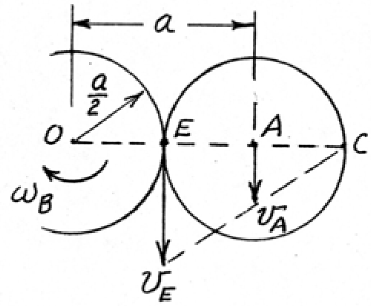
$$= \frac{13.1}{17.5}\,\frac{30.7}{34.9}\,25.1 = 16.6\ \frac{\text{in.}}{\text{sec}}$$

$$\omega = \omega_{AE} = \frac{v_E}{AE}$$

$$= \frac{16.6}{15} = 1.10\ \frac{\text{rad}}{\text{sec}}$$

CW

▶5/122

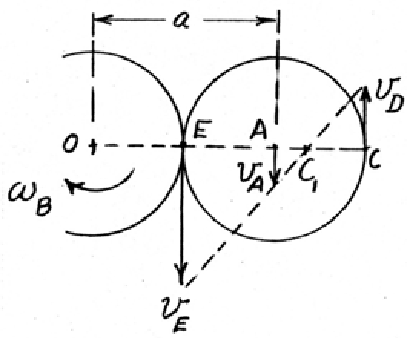


(a) $v_A = \omega_{OA}\, a$

$v_E = 2v_A = 2a\,\omega_{OA}$

$\omega_B = \dfrac{v_E}{a/2} = \dfrac{2a\,\omega_{OA}}{a/2} = 4(90)$

$= \underline{360 \text{ rev/min}}$



(b) $v_D = \overline{OC}\,\omega_D = \dfrac{3a}{2}\,80 = 120a$

$v_A = \overline{OA}\,\omega_{OA} = 90a$

$\dfrac{v_A + v_D}{a} = \dfrac{v_E - v_A}{a}$, $v_E = v_D + 2v_A$

$= 300a$

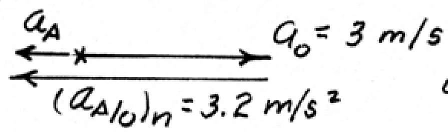
$\omega_{OE} = \omega_B = \dfrac{v_E}{a/2} = \dfrac{300a}{a/2} = \underline{600 \dfrac{\text{rev}}{\text{min}}}$

5/123 $\underset{\sim}{a}_A = \underset{\sim}{a}_O + (\underset{\sim}{a}_{A/O})_n + (\underset{\sim}{a}_{A/O})_t$ not function of v_O or sense of ω

$(a_{A/O})_n = \overline{OA}\,\omega^2 = 0.8(2^2) = 3.2\ \text{m/s}^2$

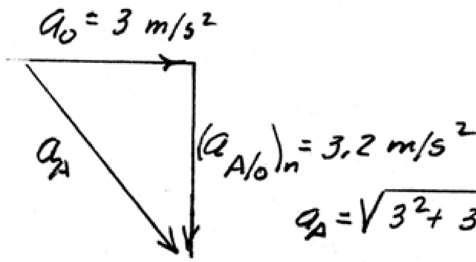
$(a_{A/O})_t = \overline{OA}\,\alpha = 0$

(a) $\theta = 0$



$a_A = 0.2\ \text{m/s}^2$

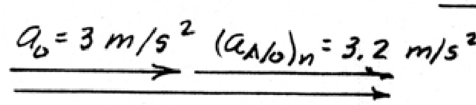
(b) $\theta = 90^\circ$



$a_A = \sqrt{3^2 + 3.2^2}$

$= 4.39\ \text{m/s}^2$

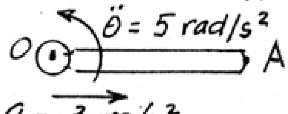
(c) $\theta = 180^\circ$



$a_A = 3 + 3.2 = 6.2\ \text{m/s}^2$

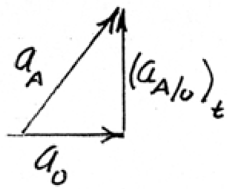
5/124 $\underline{a}_A = \underline{a}_O + (\underline{a}_{A/O})_n + (\underline{a}_{A/O})_t$

$\overline{OA} = 0.8$ m



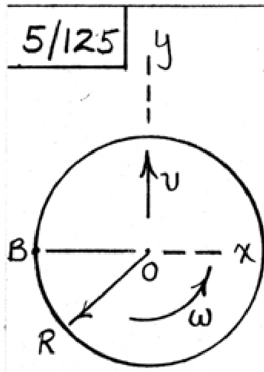
$(a_{A/O})_n = \overline{AO}\,\dot{\theta}^2 = 0$

$(a_{A/O})_t = \overline{AO}\,\ddot{\theta} = 0.8(5) = 4$ m/s^2



$a_A = \sqrt{3^2 + 4^2} = \underline{5 \text{ m/s}^2}$

5/125



$$a_o = \frac{G m_s}{r^2}$$

$$= \frac{6.673(10^{-11})[5.976 \cdot 10^{24} \cdot 333\,000]}{[149.6(10^9)]^2}$$

$$= 0.00593 \text{ m/s}^2 \ (\leftarrow)$$

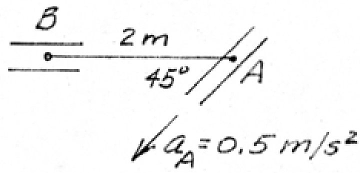
$$R\omega^2 = 6371(10^3)[7.292(10^{-5})]^2$$

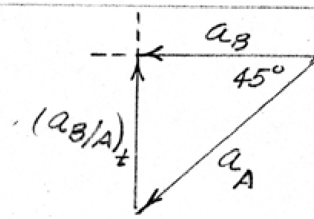
$$= 0.0339 \text{ m/s}^2 \ (\rightarrow)$$

$$\underline{a}_B = \underline{a}_o + \underline{a}_{B/o} = -0.00593\,\underline{i} + 0.0339\,\underline{i}$$

$$= \underline{0.0279\,\underline{i} \text{ m/s}^2}$$

5/126 $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$





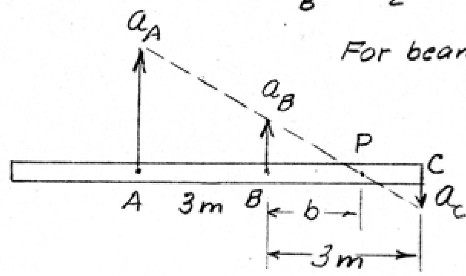
$(a_{B/A})_t = 0.5 \sin 45° = 0.354\ m/s^2$

$\alpha_{AB} = \dfrac{0.354}{2} = \underline{0.1768\ rad/s^2}$
CW

5/127

$a_A = r\alpha_1 = 0.5(0.5) = 0.25\ m/s^2$

$a_B = r\alpha_2 = 0.5(0.2) = 0.1\ m/s^2$



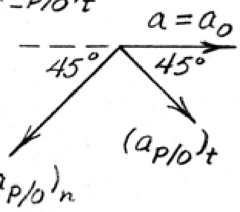
For beam $\alpha = \dfrac{a_{A/B}}{\overline{AB}} = \dfrac{0.25 - 0.1}{3}$

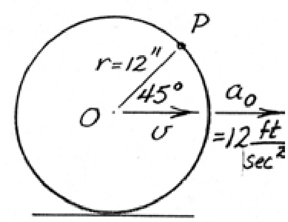
$= \underline{0.05\ rad/s^2\ CW}$

$\overset{+}{\downarrow}\ \underline{a}_C = \underline{a}_B + \underline{a}_{C/B} = -0.1 + 3(0.05) = \underline{0.05\ m/s^2\ down}$

$\overset{+}{\downarrow}\ \underline{a}_P = \underline{0} = \underline{a}_B + \underline{a}_{P/B} = -0.1 + b(0.05),\ \underline{b = 2\ m}$

5/128 $\underline{a}_P = \underline{a}_O + (\underline{a}_{P/O})_n + (\underline{a}_{P/O})_t$





$$(a_{P/O})_n = r\omega^2 = r\left(\frac{v}{r}\right)^2 = \frac{v^2}{r}$$

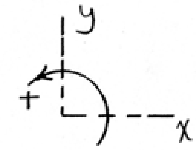
$$(a_{P/O})_t = r\alpha = r\left(\frac{a_O}{r}\right) = a_O$$

For $(a_P)_{horiz} = 0$, $\frac{v^2}{1}\cos 45^\circ = 12 + 12\cos 45^\circ$

$$v^2 = 29.0 \text{ ft}^2/\text{sec}^2$$

$v = 5.38$ ft/sec or $\underline{v = 3.67 \text{ mi/hr}}$

5/129 In the coordinates shown, the no-slip kinematic constraints are $v_0 = -r\omega$, $a_0 = -r\alpha$.



So $\omega = -\dfrac{v_0}{r} = -\dfrac{3}{0.4} = -7.5$ rad/s

$\alpha = -\dfrac{a_0}{r} = -\dfrac{-5}{0.4} = 12.5$ rad/s^2

$$\underline{v}_A = \underline{v}_0 + \underline{v}_{A/0} = \underline{v}_0 + \underline{\omega} \times \underline{r}_{A/0}$$

$$= 3\underline{i} + (-7.5\underline{k}) \times 0.4[-\cos 45°\underline{i} + \sin 45°\underline{j}]$$

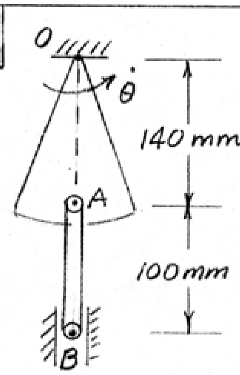
$$= \underline{5.12\underline{i} + 2.12\underline{j} \text{ m/s}}$$

$$\underline{a}_B = \underline{a}_0 + \underline{a}_{B/0} = \underline{a}_0 + \underline{\alpha} \times \underline{r}_{B/0} - \omega^2 \underline{r}_{B/0}$$

$$= -5\underline{i} + 12.5\underline{k} \times 0.2\underline{i} - (-7.5)^2(0.2\underline{i})$$

$$= \underline{-16.25\underline{i} + 2.5\underline{j} \text{ m/s}^2}$$

5/130



$\theta = \frac{\pi}{12}\sin 2\pi t$, $\dot\theta = \frac{\pi^2}{6}\cos 2\pi t$

$\ddot\theta = -\frac{\pi^3}{3}\sin 2\pi t$

$\theta = 0$, $\dot\theta = \pi^2/6$ rad/s², $\ddot\theta = 0$

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$, $a_A = 0.140\left(\frac{\pi^2}{6}\right)^2$

$= 0.379$ m/s² ↑

$v_A = 0.140\left(\frac{\pi^2}{6}\right) = 0.230$ m/s

$\omega_{AB} = \frac{0.230}{0.100} = 2.30$ rad/s

$a_{B/A} = (a_{B/A})_n = 0.100(2.30)^2 = 0.530$ m/s² ↑

$a_B = 0.379 + 0.530 = \underline{0.909 \text{ m/s}^2 \text{ (up)}}$

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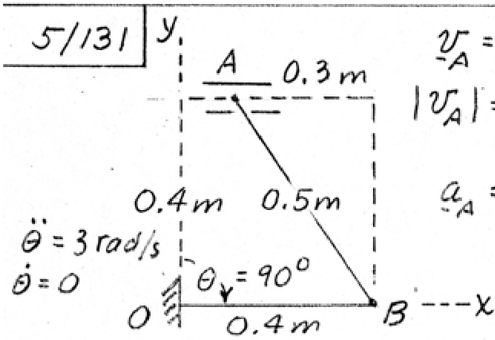

$$\underset{\sim}{v}_A = \underset{\sim}{v}_B + \underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{A/B}$$

$$|v_A| = |v_B| = |\omega_{AB}| = 0 \quad \text{for } \dot\theta = 0$$

$$\underset{\sim}{a}_A = \underset{\sim}{a}_B + (\underset{\sim}{a}_{A/B})_t \,, \quad (\underset{\sim}{a}_{A/B})_n = \underset{\sim}{0}$$

$$a_A \underset{\sim}{i} = 0.4(3)(-\underset{\sim}{j}) + \alpha_{AB}\underset{\sim}{k} \times (-0.3\underset{\sim}{i} + 0.4\underset{\sim}{j})$$

$$= -1.2\underset{\sim}{j} - 0.3\alpha_{AB}\underset{\sim}{j} - 0.4\alpha_{AB}\underset{\sim}{i}$$

$$a_A = -0.4\alpha_{AB} \quad \& \quad 0 = -1.2 - 0.3\alpha_{AB}$$

$$\alpha_{AB} = -4 \text{ rad/s}^2, \quad \underset{\sim}{\alpha}_{AB} = \underline{-4\underset{\sim}{k} \text{ rad/s}^2}$$

$$a_A = -0.4(-4) = +1.6 \text{ m/s}^2, \quad \underset{\sim}{a}_A = \underline{1.6\underset{\sim}{i} \text{ m/s}^2}$$

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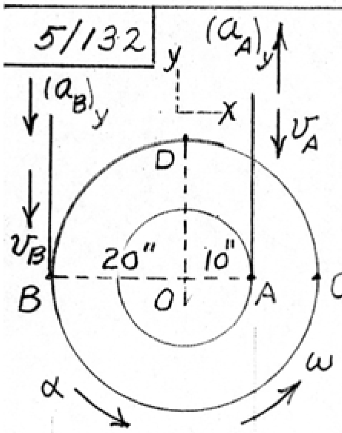


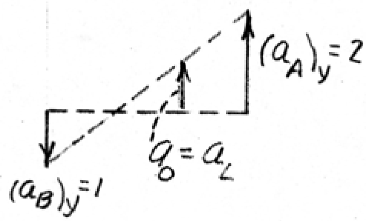
$v_A = \frac{6}{12}4 = 2$ ft/sec, $(a_A)_y = \frac{6}{12}4 = 2$ ft/sec^2

$v_B = \frac{6}{12}6 = 3$ ft/sec, $(a_{B_y}) = \frac{6}{12}2 = 1$ ft/sec^2

$\omega = \frac{3-2}{30/12} = 0.4$ rad/sec CCW

$\alpha = \frac{2+1}{30/12} = 1.2$ rad/sec^2 CCW

From diagram, $\underline{a}_L = \underline{a}_O = 1\underline{j}$ ft/sec^2



$$\underline{a}_C = \underline{a}_O + (\underline{a}_{C/O})_n + (\underline{a}_{C/O})_t$$
$$= 1\underline{j} - \frac{20}{12}(0.4)^2\underline{i} + \frac{20}{12}(1.2)\underline{j}$$
$$= 3\underline{j} - 0.267\underline{i} \text{ ft/sec}^2$$

$$\underline{a}_D = \underline{a}_O + (\underline{a}_{D/O})_n + (\underline{a}_{D/O})_t$$
$$= 1\underline{j} - \frac{20}{12}(0.4)^2\underline{j} - \frac{20}{12}(1.2)\underline{i}$$
$$= 0.733\underline{j} - 2\underline{i} \text{ ft/sec}^2$$

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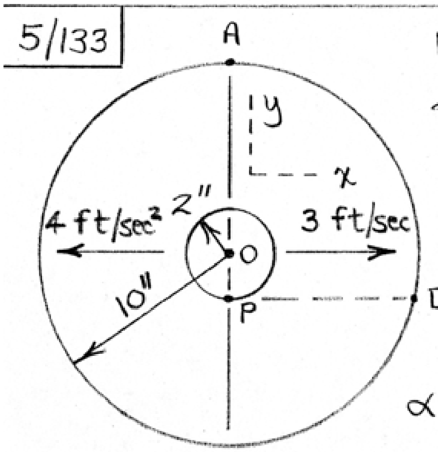


From the solution to Prob. 5/102 or from $v_0 = r\omega$, $\omega = \frac{v_0}{r} = \frac{3}{2/12} = 18$ rad/sec CW.

From $a_0 = r\alpha$, $\alpha = \frac{a_0}{r} = \frac{4}{2/12} = 24 \frac{\text{rad}}{\text{sec}^2}$ CCW

$$\underline{a}_A = \underline{a}_0 + \underline{a}_{A/0} = \underline{a}_0 + \underline{\alpha} \times \underline{r}_{A/0} - \omega^2 \underline{r}_{A/0}$$

$$= -4\underline{i} + 24\underline{k} \times \frac{10}{12}\underline{j} - 18^2\left(\frac{10}{12}\underline{j}\right)$$

$$= \underline{-24\underline{i} - 270\underline{j} \ \text{ft/sec}^2}$$

$$\underline{a}_D = \underline{a}_0 + \underline{a}_{D/0} = \underline{a}_0 + \underline{\alpha} \times \underline{r}_{D/0} - \omega^2 \underline{r}_{D/0}$$

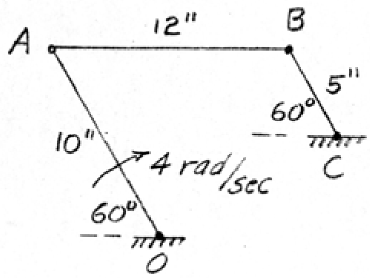
$$= -4\underline{i} + 24\underline{k} \times \left(\frac{10}{12}\cos\sin^{-1}\frac{2}{10}\underline{i} - \frac{2}{12}\underline{j}\right)$$

$$- 18^2\left(\frac{10}{12}\cos\sin^{-1}\frac{2}{10}\underline{i} - \frac{2}{12}\underline{j}\right)$$

$$= \underline{-265\underline{i} + 73.6\underline{j} \ \text{ft/sec}^2}$$

(Could use P as a base point for $\underline{a}_D$.)

5/134 $v_A = r\omega = 10(4) = 40 \text{ in./sec} = v_B$

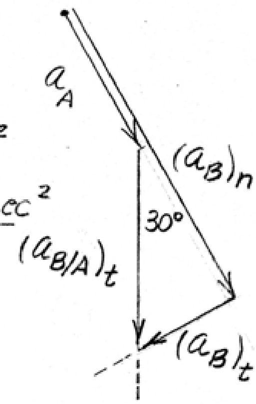


$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$\omega_{AB} = 0$ so $(a_{B/A})_n = 0$

$a_A = 10(4^2) = 160 \text{ in/sec}^2$

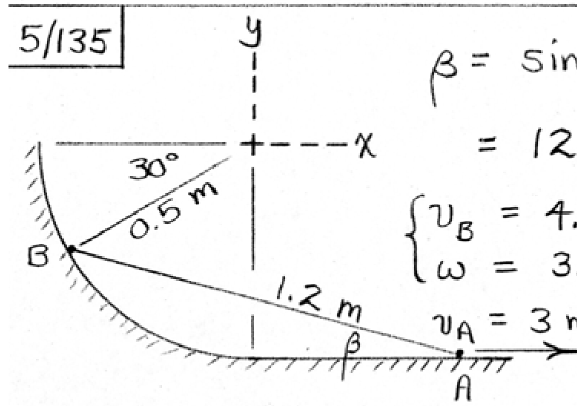
$(a_B)_n = v_B^2/\overline{BC} = \overline{40}^2/5 = 320 \frac{\text{in.}}{\text{sec}^2}$



$(a_{B/A})_t = \frac{160}{\cos 30°} = 185 \frac{\text{in.}}{\text{sec}^2}$

$\alpha_{AB} = \frac{185}{12} = \underline{15.40 \text{ rad/sec}^2}$
CW

$\boxed{5/135}$



$$\beta = \sin^{-1} \frac{0.5 - 0.5 \sin 30°}{1.2} = 12.02°$$

$$\begin{cases} v_B = 4.38 \text{ m/s} & (\text{Prob. } 5/68) \\ \omega = 3.23 \text{ rad/s} \end{cases}$$

$v_A = 3$ m/s = constant

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = \underline{a}_A + \underline{\alpha} \times \underline{r}_{B/A} - \omega^2 \underline{r}_{B/A}$$

$$a_{B_t}(\sin 30° \underline{i} - \cos 30° \underline{j}) + \frac{4.38^2}{0.5}(\cos 30° \underline{i} + \sin 30° \underline{j})$$

$$= \underline{0} + \alpha \underline{k} \times 1.2(-\cos 12.02° \underline{i} + \sin 12.02° \underline{j})$$

$$- 3.23^2 (1.2)(-\cos 12.02° \underline{i} + \sin 12.02° \underline{j})$$

Carry out vector algebra & equate coefficients:

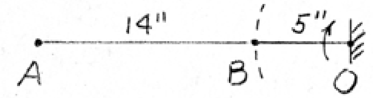
$\underline{i}$: $\frac{1}{2} a_{B_t} + 33.3 = -0.250\alpha + 12.28$

$\underline{j}$: $-\frac{\sqrt{3}}{2} a_{B_t} + 19.21 = -1.174\alpha - 2.61$

Solution: $\underline{a_{B_t} = -23.9 \text{ m/s}^2}$, $\underline{\alpha = -36.2 \text{ rad/s}^2}$

5/136 $\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t \quad \dashrightarrow +x$

(a)

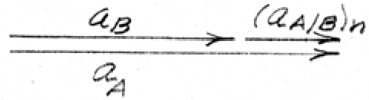


From Sample Prob. 5/15

$a_B = (a_B)_n = 10{,}280$ ft/sec^2

$v_{A/B} = v_B = 65.4$ ft/sec $\qquad v_B = 65.45$ ft/sec

$(a_{A/B})_n = (65.45)^2 / \frac{14}{12} = 3670$ ft/sec^2

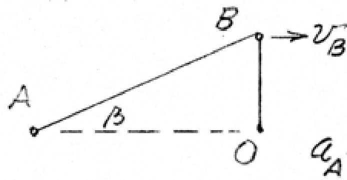


$\underline{\underline{a}_A = 13{,}950\,\underline{i}\ \text{ft/sec}^2}$

(b)

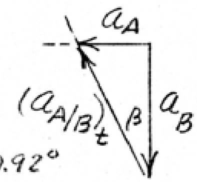
$\beta = \sin^{-1}\frac{5}{14}$

$= 20.92°$



$\omega_{AB} = 0$

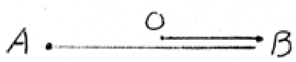
$(a_{A/B})_n = 0$

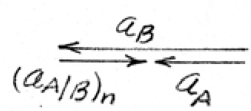


$a_A = 10{,}280 \tan 20.92°$

$\underline{a}_A = \underline{-3930\,\underline{i}\ \text{ft/sec}^2}$

(c)

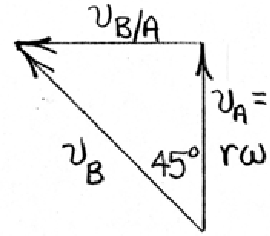




$a_A = 10{,}280 - 3670$ ft/sec^2

$\underline{a}_A = \underline{-6610\,\underline{i}\ \text{ft/sec}^2}$

5/137 $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$



$v_{B/A} = r\omega$, $\omega_{AB} = \dfrac{r\omega}{r} = \omega$

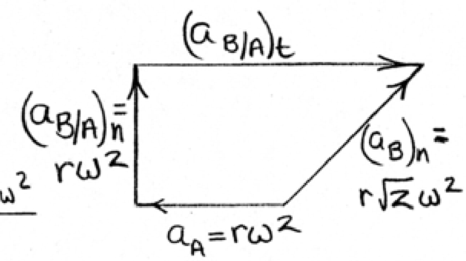
$v_B = r\omega\sqrt{2}$, $\omega_{BC} = \dfrac{r\omega\sqrt{2}}{r\sqrt{2}} = \omega$

$(\underline{a}_B)_n + (\underline{a}_B)_t = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$a_A = r\omega^2 \leftarrow$; $(a_{B/A})_n = r\omega^2 \uparrow$

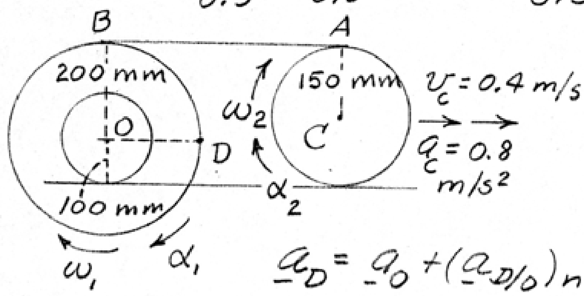
$(a_B)_n = r\sqrt{2}\,\omega^2 \; \angle 45°$

$(a_B)_t = 0$ so $\underline{\alpha_{BC} = 0}$



$(a_{B/A})_t = 2r\omega^2$, so $\alpha_{AB} = \dfrac{2r\omega^2}{r} = \underline{2\omega^2}$

5/138 $\omega_1 = \frac{v_B}{0.3} = \frac{v_A}{0.3} = \omega_2 = \frac{2(0.4)}{0.3} = \frac{8}{3}$ rad/s



$\alpha_1 = \frac{a_{B_x}}{0.3} = \frac{a_{A_x}}{0.3} = \frac{2(0.8)}{0.3} = \frac{16}{3} \frac{\text{rad}}{\text{s}^2}$

$\underline{a}_D = \underline{a}_O + (\underline{a}_{D/O})_n + (\underline{a}_{D/O})_t$

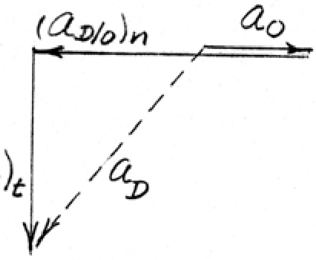
$a_O = \frac{100}{300} a_{B_x} = \frac{1}{3} a_{A_x} = \frac{1}{3}(2)(0.8) = \frac{1.6}{3} = 0.533$ m/s²

$(a_{D/O})_n = 0.2(8/3)^2 = 1.422$ m/s²

$(a_{D/O})_t = 0.2(16/3) = 1.067$ m/s²

$a_D = \sqrt{(1.422 - 0.533)^2 + (1.067)^2}$

$= \sqrt{1.928} \quad = \underline{1.388 \text{ m/s}^2}$



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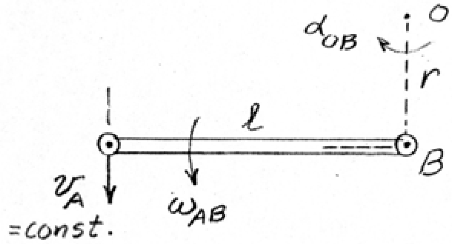

$\omega_{AB} = v_A/\ell$

$v_B = 0$ so $(a_B)_n = \dfrac{v_B^2}{r} = 0$

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$\underline{a}_{B_t} = 0 + (\underline{a}_{B/A})_n + 0$

$(a_B)_t$

$(a_{B/A})_n = \ell\,\omega_{AB}^2 = v_A^2/\ell$

Thus $\alpha_{OB} = \dfrac{(a_B)_t}{r} = \underline{\dfrac{v_A^2}{r\ell}}$

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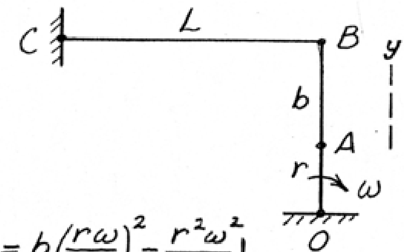


For this position $(v_A)_y = 0$

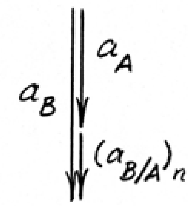
so $v_B = 0, \ \omega_{BC} = 0$

$\omega_{AB} = v_A / b = r\omega / b$ CCW

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}, \ a_A = r\omega^2 \downarrow, \ (a_{B/A})_n = b\left(\frac{r\omega}{b}\right)^2 = \frac{r^2\omega^2}{b}\downarrow$

$(a_B)_n = L\,\omega_{BC}^2 = 0$

$a_B = (a_B)_t = r\omega^2 + \frac{r}{b} r\omega^2 = r\omega^2\left(1 + \frac{r}{b}\right)$



$\alpha_{BC} = \frac{(a_B)_t}{L} = \underline{\frac{r\omega^2}{L}\left(1 + \frac{r}{b}\right) \text{ CW}}$

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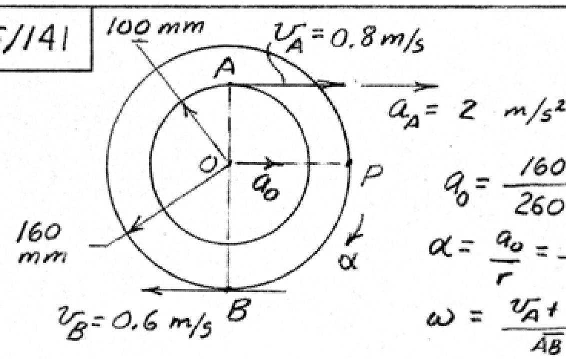


$$a_O = \frac{160}{260}(2) = 1.231 \text{ m/s}$$

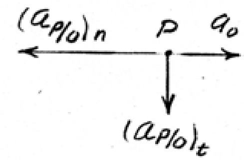
$$\alpha = \frac{a_O}{r} = \frac{1.231}{0.160} = 7.69 \text{ rad/s}^2$$

$$\omega = \frac{v_A + v_B}{\overline{AB}} = \frac{0.8+0.6}{0.260} = 5.38 \frac{\text{rad}}{\text{s}}$$

$$\underline{a}_P = \underline{a}_O + (\underline{a}_{P/O})_n + (\underline{a}_{P/O})_t$$

$$(a_{P/O})_n = \overline{PO}\,\omega^2 = 0.160(5.38)^2 = 4.64 \text{ m/s}^2$$

$$(a_{P/O})_t = \overline{PO}\,\alpha = 0.16(7.69) = 1.231 \text{ m/s}^2$$



$$a_P = \sqrt{(4.64 - 1.231)^2 + (1.231)^2} = \underline{3.62 \text{ m/s}^2}$$

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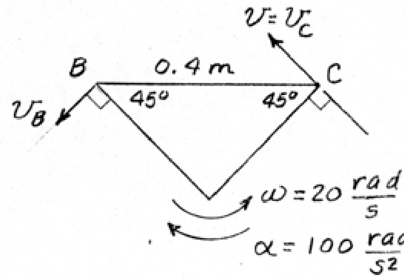


$\underline{v} = \underline{v}_C = \underline{v}_B + \underline{v}_{C/B}$

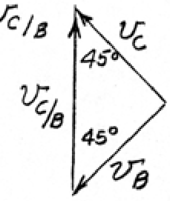
$v_{C/B} = \overline{CB}\,\omega$

$= 0.4(20)$

$= 8$ m/s

$v = v_C = 8/\sqrt{2}$

$= \underline{5.66 \text{ m/s}}$



$\underline{a} = \underline{a}_C = \underline{a}_B + (\underline{a}_{C/B})_n + (\underline{a}_{C/B})_t$

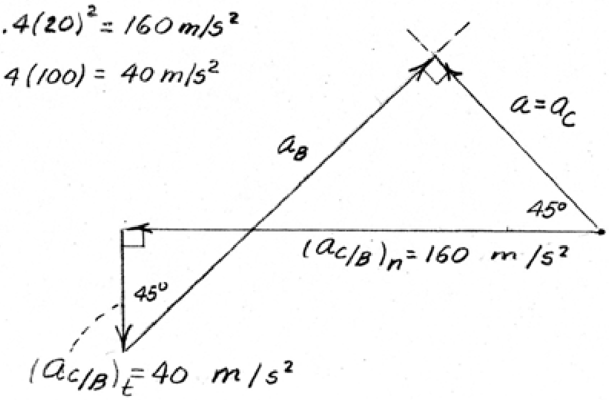
$(a_{C/B})_n = \overline{CB}\,\omega^2 = 0.4(20)^2 = 160 \text{ m/s}^2$

$(a_{C/B})_t = \overline{CB}\,\alpha = 0.4(100) = 40 \text{ m/s}^2$

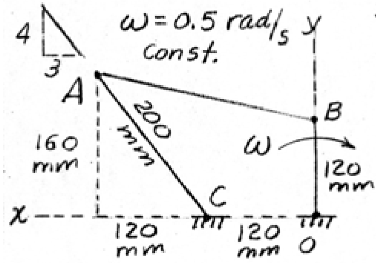
From diagram

$a = 160/\sqrt{2} - 40/\sqrt{2}$

$= \underline{84.9 \text{ m/s}^2}$



5/143 From Prob. 5/87, $\underline{\omega}_{AB} = 0.214\underline{k}$, $\underline{\omega}_{CA} = 0.429\underline{k}$ $\frac{rad}{s}$



$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$ ---- (a)

$\underline{a}_A = \underline{\omega}_{CA} \times (\underline{\omega}_{CA} \times \underline{r}_{CA}) + \underline{\alpha}_{CA} \times \underline{r}_{CA}$

$= (0.429)^2 (\underline{k} \times [\underline{k} \times \{0.12\underline{i} + 0.16\underline{j}\}])$

$+ \alpha_{CA}\underline{k} \times (0.12\underline{i} + 0.16\underline{j})$

$= 0.1837(-0.12\underline{i} - 0.16\underline{j})$

$+ 0.12\alpha_{CA}\underline{j} - 0.16\alpha_{CA}\underline{i}$

$\underline{a}_B = \underline{\omega} \times (\underline{\omega} \times \underline{r}_{OB}) = (0.5)^2(0.12)(-\underline{j}) = -0.03\underline{j}$ m/s²

$(\underline{a}_{A/B})_n = \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_{BA}) = (0.214)^2 (\underline{k} \times [\underline{k} \times \{0.24\underline{i} + 0.04\underline{j}\}])$

$= 0.0459(-0.24\underline{i} - 0.04\underline{j})$ m/s²

$(\underline{a}_{A/B})_t = \alpha_{AB}\underline{k} \times \underline{r}_{BA} = \alpha_{AB}\underline{k} \times (0.24\underline{i} + 0.04\underline{j}) = \alpha_{AB}(0.24\underline{j} - 0.04\underline{i})$

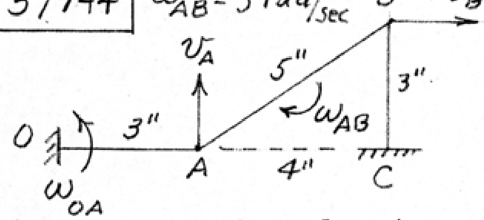
Substitute terms into Eq. (a) & equate separately $\underline{i}$ & $\underline{j}$ coefficients & get $\alpha_{AB} - 4\alpha_{CA} = 0.2755$

$2\alpha_{AB} - \alpha_{CA} = 0.02041$

Solve & get $\alpha_{CA} = -0.0758$ rad/s², $\alpha_{AB} = -0.0277$ rad/s²

$\underline{\alpha}_{CA} = \underline{-0.0758\,\underline{k}\ \text{rad/s}^2}$

5/144 $\omega_{AB} = 3\ rad/sec$



C = instant. center of AB

$\omega_{AB} = \dfrac{v_A}{\overline{AC}}$, $v_A = \dfrac{4}{12} 3 = 1\ ft/sec$

$v_B = \overline{BC}\, \omega_{AB} = \dfrac{3}{12} 3 = 0.75\ ft/sec$

$(\alpha_{OA} = 0)$ $\quad \underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$(a_B)_n = v_B^2/\overline{BC} = (0.75)^2/\frac{3}{12} = 2.25\ ft/sec^2$

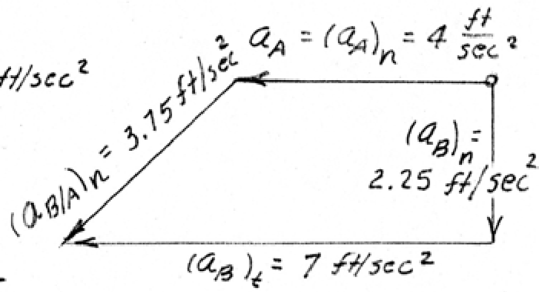
$(a_B)_t = \overline{BC}\, \alpha_{BC}$

$(a_{B/A})_n = \overline{AB}\, \omega_{AB}^2 = \frac{5}{12} 3^2 = 3.75\ ft/sec^2$

$(a_{B/A})_t = \overline{AB}\, \alpha_{AB}$

$(a_A)_n = v_A^2/\overline{AO} = 1^2/\frac{3}{12} = 4\ ft/sec^2$

$(a_A)_t = \overline{AO}\, \alpha_{AO} = 0$



From diag,

$(a_{B/A})_t = 0$ so $\underline{\alpha_{AB} = \alpha_{ABD} = 0}$

$\alpha_{BC} = 7/\frac{3}{12} = \underline{28\ rad/sec^2\ CCW}$

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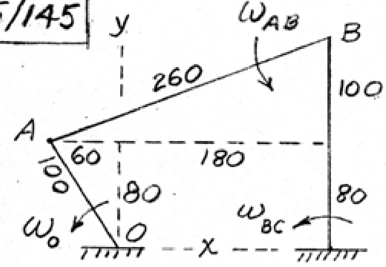


Dim. in mm

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

$$\underline{a}_B = \underline{\omega}_{BC} \times (\underline{\omega}_{BC} \times \underline{r}_{B/C}) + \underline{\alpha}_{BC} \times \underline{r}_{B/C}$$

$$= 5.83\underline{k} \times (5.83\underline{k} \times 0.18\underline{j}) + \alpha_{BC}\underline{k} \times 0.18\underline{j} \quad m/s^2$$

$$= -6.125\underline{j} - 0.18\alpha_{BC}\underline{i} \quad m/s^2$$

$$\underline{a}_A = \underline{\omega}_o \times (\underline{\omega}_o \times \underline{r}_{A/O}) = 10\underline{k} \times (10\underline{k} \times [-0.06\underline{i} + 0.08\underline{j}])$$

$$= 6\underline{i} - 8\underline{j} \ m/s^2 \qquad (\underline{\alpha}_{OA} = 0)$$

$$(\underline{a}_{B/A})_n = \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_{B/A}) = 2.5\underline{k} \times (2.5\underline{k} \times [0.24\underline{i} + 0.1\underline{j}])$$

$$= -1.5\underline{i} - 0.625\underline{j} \ m/s^2$$

$$(\underline{a}_{B/A})_t = \alpha_{AB}\underline{k} \times (0.24\underline{i} + 0.1\underline{j}) = -0.1\alpha_{AB}\underline{i} + 0.24\alpha_{AB}\underline{j}$$

Substitute in accel. equation & equate coefficients & get

$$\left.\begin{aligned} -0.18\alpha_{BC} &= 6 - 1.5 - 0.1\alpha_{AB} \\ -6.125 &= -8 - 0.625 + 0.24\alpha_{AB} \end{aligned}\right\}$$

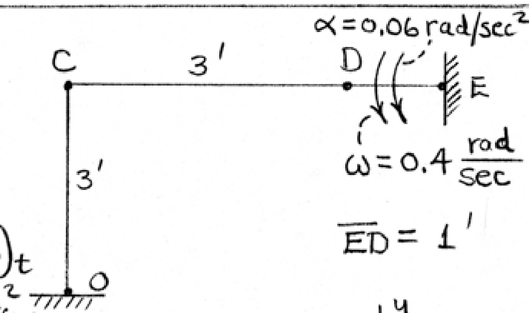
Sol. is

$\underline{\alpha}_{AB} = 10.42\underline{k}$ rad/s^2

$(\underline{\alpha}_{BC} = -19.21\underline{k}$ rad/s$^2)$

5/146

$v_C = 0$



$\omega_{CD} = \frac{v_D}{\overline{CD}} = \frac{1(0.4)}{3}$

$= 0.1333$ rad/sec

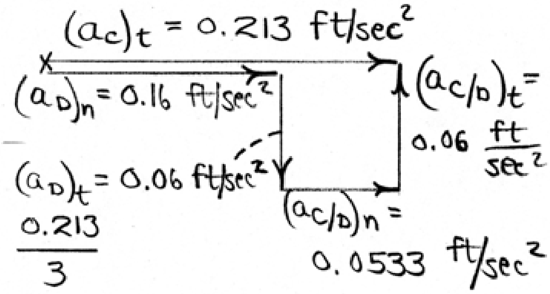
$\underline{a}_C = \underline{a}_D + (\underline{a}_{C/D})_n + (\underline{a}_{C/D})_t$

$(a_D)_n = 1(0.4)^2 = 0.16$ ft/sec^2 $\rightarrow$

$(a_D)_t = 1(0.06) = 0.06$ ft/sec^2 $\downarrow$

$(a_{C/D})_n = 3(0.1333)^2 = 0.0533$ ft/sec^2 $\rightarrow$

$(a_C)_n = \frac{v_C^2}{\overline{CO}} = 0$



$\alpha_{AB} = \alpha_{CO} = \frac{(a_C)_t}{\overline{CO}} = \frac{0.213}{3}$

$= 0.0711$ rad/sec^2 CW

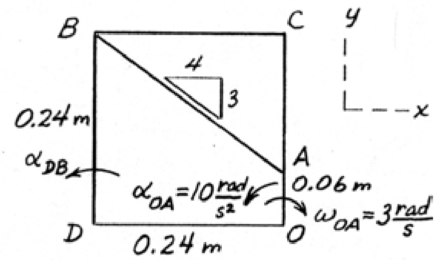
$\omega_{AB} = \omega_{CO} = \frac{v_C}{\overline{CO}} = 0$

So $(\underline{a}_{A/B}) = (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t = \underline{0} + \overline{AB}\,\alpha_{AB}\,\underline{j}$

$= (4+6)(0.0711)\,\underline{j} = \underline{0.711\underline{j} \text{ ft/sec}^2}$

5/147 $v_B = v_A = \overline{OA}\,\omega_{OA} = 0.06(3) = 0.18\ m/s^2$

$\omega_{DB} = v_B/\overline{DB} = 0.18/0.24 = 0.75\ rad/s$



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$\underline{a}_{B_n} + \underline{a}_{B_t} = \underline{a}_{A_n} + \underline{a}_{A_t} + \underline{a}_{B/A_n} + \underline{a}_{B/A_t}$

$\underline{a}_{B_n} = \overline{DB}\,\omega_{DB}^2(-\underline{j}) = 0.24(0.75)^2(-\underline{j})$
$= -0.135\underline{j}\ m/s^2$

$\underline{a}_{B_t} = \overline{DB}\,\alpha_{DB}(-\underline{i}) = -0.24\alpha_{DB}\underline{i}$

$\underline{a}_{A_n} = \overline{OA}\,\omega_{OA}^2(-\underline{j}) = 0.06(3^2)(-\underline{j}) = -0.54\underline{j}\ m/s^2$

$\underline{a}_{A_t} = \overline{OA}\,\alpha_{OA}(-\underline{i}) = 0.06(10)(-\underline{i}) = -0.6\underline{i}\ m/s^2$

$(\underline{a}_{B/A})_n = \overrightarrow{BA}\,\omega_{AB}^2 = \underline{0}$ since $\omega_{AB} = 0$

$(\underline{a}_{B/A})_t = \alpha_{AB}\underline{k} \times \overrightarrow{AB} = \alpha_{AB}\underline{k} \times (-0.24\underline{i} + 0.18\underline{j}) = -0.24\alpha_{AB}\underline{j} - 0.18\alpha_{AB}\underline{i}$

$-0.135\underline{j} - 0.24\alpha_{DB}\underline{i} = -0.54\underline{j} - 0.6\underline{i} + \underline{0} - 0.24\alpha_{AB}\underline{j} - 0.18\alpha_{AB}\underline{i}$

$\underline{j}$-terms: $-0.135 = -0.54 - 0.24\alpha_{AB}$, $\alpha_{AB} = -1.688\ rad/s^2$ (CW)

$\underline{i}$-terms: $-0.24\alpha_{DB} = -0.6 - 0.18(-1.688)$,

$\underline{\alpha_{DB} = 1.234\ rad/s^2\ CCW}$

5/148 Using C as the instant. center for AB gives

$v_B = 0.150(40) = 6\ m/s$, $\omega_{BC} = \frac{6}{0.15} = 40\ rad/s$

$v_A = 0.2(40) = 8\ m/s$, $\omega_{AO} = \frac{8}{0.1} = 80\ rad/s$

$(\underline{a}_A)_n + (\underline{a}_A)_t = (\underline{a}_B)_n + (\underline{a}_B)_t + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$

$(\underline{a}_A)_n = 0.1(80)^2(-\underline{i}) = -640\underline{i}\ m/s^2$

$(\underline{a}_B)_n = 0.150(40)^2(-\underline{j}) = -240\underline{j}\ m/s^2$

$(\underline{a}_{A/B})_n = \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_{A/B}) = 40\underline{k} \times (40\underline{k} \times [0.2\underline{i} - 0.15\underline{j}])$

$= -320\underline{i} + 240\underline{j}\ m/s^2$

$(\underline{a}_{A/B})_t = \underline{\alpha}_{AB} \times \underline{r}_{A/B} = 0$ for ω_{AB} const.

Substitute & equate $\underline{i}$ & $\underline{j}$ coefficients & get

$(\underline{a}_A)_t = -240\underline{j} + 240\underline{j} = 0$ so $\underline{\alpha_{OA} = 0}$

$(\underline{a}_B)_t = -320\underline{i}\ m/s^2$, $(\underline{a}_D)_n = \frac{225}{150}(240)(-\underline{j}) = -360\underline{j}\ m/s^2$

$(\underline{a}_D)_t = \frac{225}{150}(320)(-\underline{i}) = -480\underline{i}\ m/s^2$, $\underline{a}_D = \underline{-120(4\underline{i} + 3\underline{j})\ m/s^2}$

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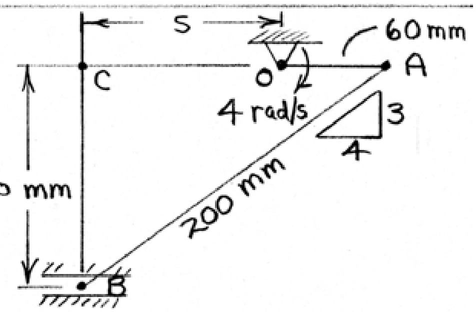


$(60+s)^2 + 120^2 = 200^2$

$s = 100 \text{ mm}$

$v_A = 0.06(4) = 0.24 \text{ m/s}$

$\omega_{AB} = \dfrac{v_A}{\overline{AC}} = \dfrac{0.24}{0.160}$

$= 1.5 \text{ rad/s}$

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$; $a_A = (a_A)_n = 0.06(4)^2$

$= 0.96 \ \frac{\text{m}}{\text{s}^2} \leftarrow$

$(a_{B/A})_n = 0.2(1.5)^2$

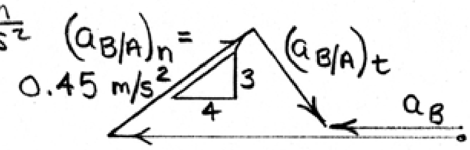
$= 0.45 \text{ m/s}^2$ ↗ 45°

From the diagram,

$(a_{B/A})_t = \frac{3}{4}(0.45) = 0.338 \ \frac{\text{m}}{\text{s}^2}$

$\alpha_{AB} = (a_{B/A})_t / \overline{AB}$

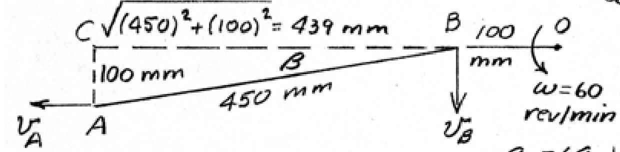
$= \dfrac{0.338}{0.2} = \underline{1.688 \text{ rad/s}^2 \text{ CCW}}$



5/150 C = instantaneous center of zero velocity for AB

$\omega_{AB} = \dfrac{v_B}{\overline{CB}} = \dfrac{\overline{OB}\,\omega}{\overline{CB}}$

$= \dfrac{100}{439}\ \dfrac{60(2\pi)}{60} = 1.432\ \dfrac{rad}{s}$

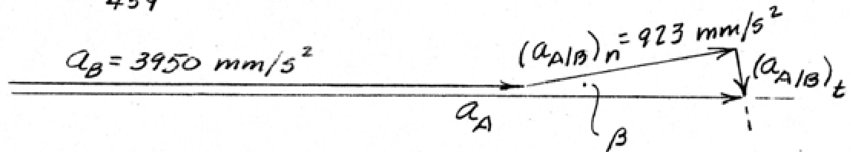


$a_B = (a_B)_n = 100(2\pi)^2 = 3950\ mm/s^2$

$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t\ ;$

$(a_{A/B})_n = 450(1.432)^2 = 923\ mm/s^2$

$\beta = \tan^{-1}\dfrac{100}{439} = 12.82°$



$(a_{A/B})_t = 923 \tan 12.82 = 923\ \dfrac{100}{439} = 210\ mm/s^2$

$\alpha_{AB} = \dfrac{(a_{A/B})_t}{\overline{AB}} = \dfrac{210}{450} = \underline{0.467\ rad/s^2\ CCW}$

$a_A = 3950 + 923\cos 12.82° + 210 \sin 12.82° = 4890\ mm/s^2$

or $\underline{a_A = 4.89\ m/s^2}$

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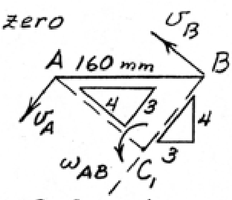
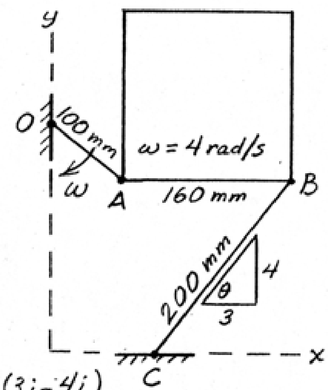
C_1 = inst. center of zero vel. for AB





$\overline{AC_1} = \frac{4}{5}160 = 128$ mm

$\overline{BC_1} = \frac{3}{5}160 = 96$ mm

$\omega_{AB} = v_A/\overline{AC_1} = 0.1(4)/0.128 = 3.12$ rad/s

$v_B = \overline{C_1B}\,\omega_{AB} = 0.096(3.12) = 0.3$ m/s

$(\underline{a}_B)_n + (\underline{a}_B)_t = (\underline{a}_A)_n + (\underline{a}_A)_t + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$(\underline{a}_B)_n = \frac{0.3^2}{0.2}\left(-\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}\right) = -0.09(3\underline{i} + 4\underline{j})$ m/s²

$(\underline{a}_B)_t = \underline{\alpha}_{CB} \times \underline{r}_{CB} = \alpha_{CB}\underline{k} \times 0.2\left(\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}\right) = 0.04\alpha_{CB}(3\underline{j} - 4\underline{i})$

$(\underline{a}_A)_n = 0.1(4^2)\left(-\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}\right) = 0.32(-4\underline{i} + 3\underline{j})$ m/s²

$(\underline{a}_A)_t = \underline{0}$

$(\underline{a}_{B/A})_n = 0.160(3.12^2)(-\underline{i}) = -1.562\,\underline{i}$ m/s²

$(\underline{a}_{B/A})_t = \underline{\alpha}_{AB} \times \underline{r}_{AB} = \alpha_{AB}\underline{k} \times 0.16\underline{i} = 0.16\alpha_{AB}\underline{j}$

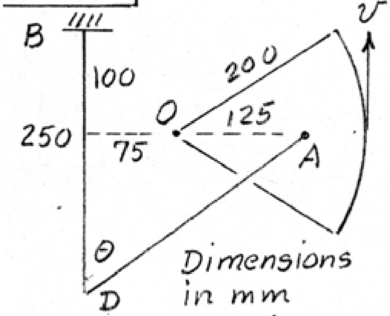
Thus $-0.09(3\underline{i} + 4\underline{j}) + 0.04\alpha_{CB}(3\underline{j} - 4\underline{i}) = 0.32(-4\underline{i} + 3\underline{j}) - 1.562\underline{i} + 0.16\alpha_{AB}\underline{j}$

Equate $\underline{i}$-terms: $-0.27 - 0.16\alpha_{CB} = -1.28 - 1.562$, $\underline{\alpha_{CB} = 16.08 \text{ rad/s}^2 \text{ CCW}}$

" $\underline{j}$-terms: $-0.36 + 0.12(16.08) = 0.96 + 0.16\alpha_{AB}$,

$\underline{\alpha_{AB} = 3.81 \text{ rad/s}^2 \text{ CCW}}$

5/152 $v = 4$ m/s const.



From Prob. 5/118

$v_A = 2.5\ \frac{m}{s}$, $v_D = 1.875\ \frac{m}{s}$

$\omega_{AD} = 12.5$ rad/s

$\theta = \tan^{-1}\frac{200}{150} = 53.1°$

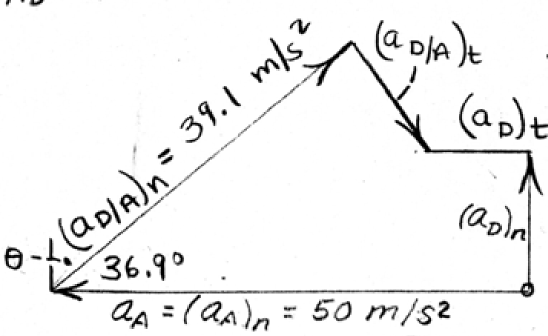
$\bar{AD} = 250$ mm

$\underline{a}_D = \underline{a}_A + \underline{a}_{D/A}$

$(a_D)_n = v_D^2/\bar{BD} = \frac{1.875^2}{0.250} = 14.06\ m/s^2$

$a_A = (a_A)_n = v_A^2/\bar{OA} = \frac{2.5^2}{0.125} = 50\ m/s^2$

$(a_{D/A})_n = \bar{AD}\,\omega_{AD}^2 = 0.250(12.5)^2 = 39.1\ m/s^2$



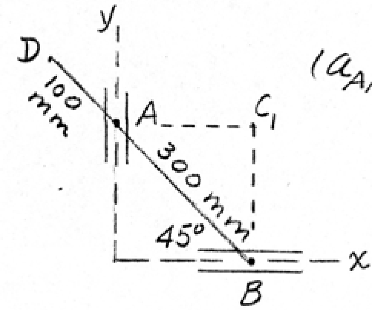
Solution of polygon gives $(a_{D/A})_t = 11.72\ m/s^2$

$(a_D)_t = 11.72\ m/s^2$

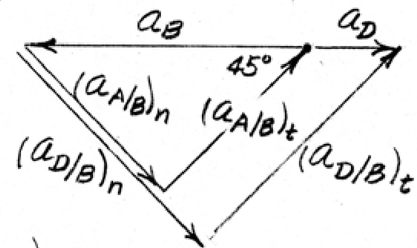
$\alpha_{BD} = (a_D)_t/\bar{BD}$

$= \frac{11.72}{0.25} = 46.9\ \frac{rad}{s^2}$ CW

5/153 $\underline{a}_A = \underline{0} = \underline{a}_B + \underline{a}_{A/B}$; $\omega_{AB} = \dfrac{v_A}{\overline{AC_1}} = \dfrac{0.5}{0.3/\sqrt{2}} = 2.36 \dfrac{\text{rad}}{\text{sec}}$



$(a_{A/B})_n = 0.3(2.36)^2 = 1.667\ m/s^2$



$\underline{a}_D = \underline{a}_B + (\underline{a}_{D/B})_n + (\underline{a}_{D/B})_t$

where $(a_{D/B})_n = \dfrac{400}{300}(a_{A/B})_n = \dfrac{4}{3}(1.667) = 2.22\ m/s^2$

$(a_{D/B})_t = \dfrac{400}{300}(a_{A/B})_t = 2.22\ m/s^2$

$a_D = (2.22 - 1.667)\sqrt{2} = 0.786\ m/s^2$

or $\underline{\underline{a}_D = 0.786\,\underline{i}\ m/s^2}$

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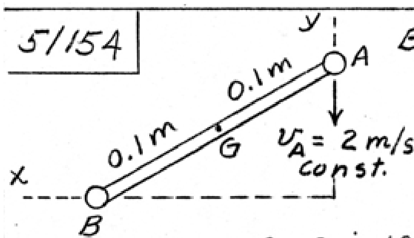


By inspection using instant center,

$\underline{\omega}_{AB} = \frac{2}{0.2}\,\frac{2}{\sqrt{3}}\,\underline{k} = \frac{20}{\sqrt{3}}\underline{k}$ rad/s

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t\,,\ \underline{a}_A = \underline{0}$

Vector algebra: $\underline{a}_B = a_B\underline{i}\,,\ (\underline{a}_{B/A})_n = \underline{\omega}_{AB}\times(\underline{\omega}_{AB}\times\underline{r}_{AB})$

$= \left(\frac{20}{\sqrt{3}}\right)^2\underline{k}\times\left[\underline{k}\times 0.2\left(\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}\right)\right]$

$= \frac{40}{3}(-\sqrt{3}\,\underline{i} + \underline{j})$ m/s^2

$(a_{B/A})_t = \underline{\alpha}_{AB}\times\underline{r}_{AB} = \alpha_{AB}\underline{k}\times 0.2\left(\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}\right)$

$= \frac{\alpha_{AB}}{10}(\sqrt{3}\,\underline{j} + \underline{i})$

Thus $a_B\underline{i} = \underline{0} + \frac{40}{3}(-\sqrt{3}\,\underline{i} + \underline{j}) + \frac{\alpha_{AB}}{10}(\sqrt{3}\,\underline{j} + \underline{i})$

so $a_B = -\frac{40}{\sqrt{3}} + \frac{\alpha_{AB}}{10}$ & $0 = \frac{40}{3} + \frac{\alpha_{AB}}{10}\sqrt{3}$

giving $\alpha_{AB} = -\frac{400}{3\sqrt{3}}$ rad/s^2 & $a_B = -\frac{160}{3\sqrt{3}}$ m/s^2

$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A} = 0 + \frac{1}{2}(\underline{a}_{B/A})_n + \frac{1}{2}(\underline{a}_{B/A})_t = \frac{20}{3}(-\sqrt{3}\,\underline{i} + \underline{j}) - \frac{20}{3\sqrt{3}}(\sqrt{3}\,\underline{j} + \underline{i})$

$= -\frac{80}{3\sqrt{3}}\underline{i} = \underline{-15.40\,\underline{i}\ \text{m/s}^2}$

Vector geometry:

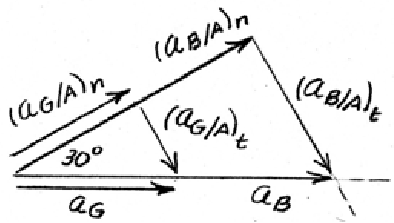
$(a_{B/A})_n = 0.2(20/\sqrt{3})^2 = 80/3$ m/s^2

$(a_{B/A})_t = 0.2\,\alpha_{AB}$

$(a_{G/A})_n = \frac{1}{2}(a_{B/A})_n = 40/3$ m/s^2

$(a_{G/A})_t = \frac{1}{2}(a_{B/A})_t = 0.1\,\alpha_{AB}$

$a_G = \frac{40}{3}\,\frac{2}{\sqrt{3}} = \underline{15.40\ \text{m/s}^2}$



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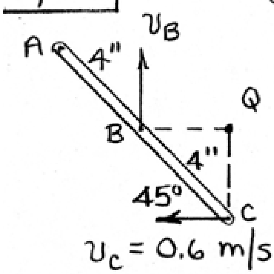


Q is instantaneous center of zero velocity for bar AC.

$$\omega_{AC} = \frac{v_C}{\overline{QC}} = \frac{0.6}{\frac{4}{12}\cos 45°}$$

$$= 2.55 \text{ rad/sec CW}$$

$$\underline{a}_B = \underline{a}_C + (\underline{a}_{B/C})_n + (\underline{a}_{B/C})_t$$

$$(a_{B/C})_n = \overline{BC}\,\omega_{AC}^2 = \frac{4}{12}(2.55)^2$$

$$= 2.16 \text{ ft/sec}^2$$

$$\alpha_{BC} = \frac{(a_{B/C})_t}{\overline{BC}} = \frac{2.16}{4/12}$$

$$= 6.48 \text{ rad/sec}^2 \text{ CCW}$$

$$\underline{a}_A = \underline{a}_C + (\underline{a}_{A/C})_n + (\underline{a}_{A/C})_t$$

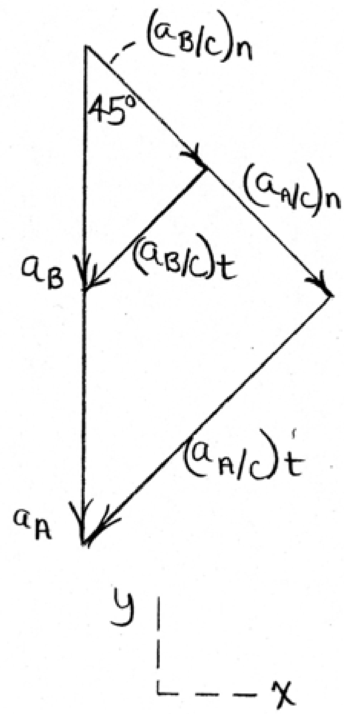
$$(a_{A/C})_n = \overline{AC}\,\omega_{AC}^2 = \frac{8}{12}(2.55)^2$$

$$= 4.32 \text{ ft/sec}^2$$

$$(a_{A/C})_t = 4.32 \text{ ft/sec}^2$$

$$a_A = 4.32\sqrt{2} = 6.11 \text{ ft/sec}^2$$

$$\therefore \underline{a}_A = -6.11\underline{j} \text{ ft/sec}^2$$



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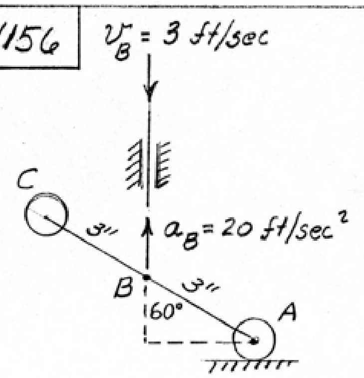


From solution to Prob. 5/85

$v_{A/B} = 3.46$ ft/sec

$\omega_{AB} = 3.46/\frac{3}{12} = 13.86$ rad/sec

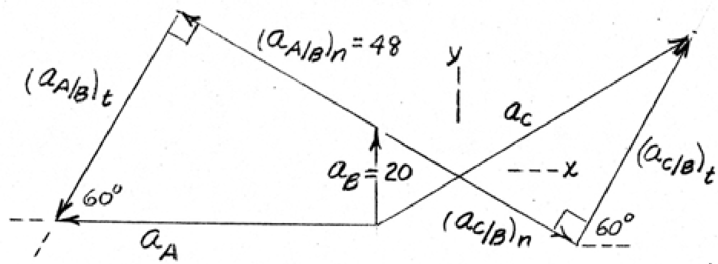
$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$

$(a_{A/B})_n = \frac{3}{12}(13.86)^2 = 48$ ft/sec²

From diagram $(a_{A/B})_t = \dfrac{20 + 24}{\sin 60°}$

$= 50.8 \ \frac{ft}{sec^2}$

$= (a_{C/B})_t$



Also $(a_{C/B})_n$
$= (a_{A/B})_n = 48 \ \frac{ft}{sec^2}$

Components of a_C are

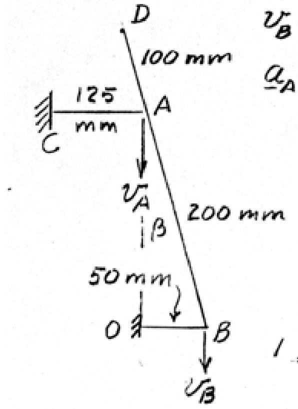
$(a_C)_x = 48\cos 30° + 50.8\cos 60° = 66.97 \ \frac{ft}{sec^2}$

$(a_C)_y = 20 - 48\sin 30° + 50.8\sin 60° = 40 \ \frac{ft}{sec^2}$

$a_C = \sqrt{(66.97)^2 + (40)^2}$

$= \underline{78.0 \text{ ft/sec}^2}$

▶5/157 $v_B = r\omega = 50\,\frac{(120)2\pi}{60} = 200\pi = 628$ mm/s

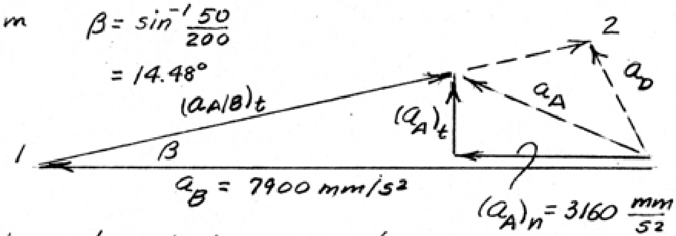


v_B parallel to v_A so $v_A = v_B$ & $\omega_{AB} = 0$

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$; $a_B = (a_B)_n = \frac{v_B^2}{r} = \frac{(200\pi)^2}{50} = 7900$ mm/s²

$(a_A)_n = \frac{(200\pi)^2}{125} = 3160$ mm/s²

$\beta = \sin^{-1}\frac{50}{200}$
$= 14.48°$



From solution by vector algebra or vector geometry, $(a_{A/B})_t = 4890$ mm/s²

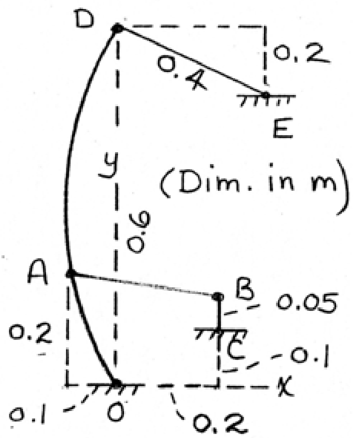
$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B}$; $a_{D/B} = (a_{D/B})_t = \frac{\overline{BD}}{\overline{BA}}(a_{A/B})_t = \frac{300}{200}(4890) = 7340$ $\frac{mm}{s^2}$

$(= 1-2)$

$a_D = \sqrt{(7340 \sin 14.48°)^2 + (7900 - 7340 \cos 14.48°)^2}$

$= \underline{1997 \text{ mm/s}^2}$

▶ 5/158 $\underline{\omega}_{CB} = -\pi \underline{k}$ rad/s , $\underline{r}_{OA} = -0.1\underline{i} + 0.2\underline{j}$ m



$\underline{r}_{CB} = 0.05\underline{j}$ m, $\underline{r}_{BA} = -0.3\underline{i} + 0.05\underline{j}$ m

$\underline{r}_{OD} = 0.6\underline{j}$ m , $\underline{v}_B = 0.05\pi\,\underline{i}$ m/s

$\underline{v}_{A/B} = \omega_{AB}\underline{k} \times (-0.3\underline{i} + 0.05\underline{j})$

$= -0.3\,\omega_{AB}\,\underline{j} - 0.05\,\omega_{AB}\,\underline{i}$

$\underline{v}_A = \omega_{OA}\underline{k} \times (-0.1\underline{i} + 0.2\underline{j})$

$= -0.1\omega_{OA}\,\underline{j} - 0.2\omega_{OA}\,\underline{i}$

From $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$:

$-0.1\,\omega_{OA}\,\underline{j} - 0.2\,\omega_{OA}\,\underline{i} = 0.05\pi\,\underline{i} - 0.3\,\omega_{AB}\,\underline{j} - 0.05\,\omega_{AB}\,\underline{i}$

Equate like coefficients: $\begin{cases} \underline{\omega}_{AB} = -0.286\underline{k} \text{ rad/s} \\ \underline{\omega}_{OA} = -0.857\underline{k} \text{ rad/s} \end{cases}$

Now, $\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$ $*$

$\underline{a}_A = -\omega_{OA}^2\,\underline{r}_{OA} + \underline{\alpha}_{OA} \times \underline{r}_{OA}$

$= 0.734(0.1\underline{i} - 0.2\underline{j}) + \alpha_{OA}(-0.1\underline{j} - 0.2\,\underline{i})$

$(\underline{a}_{A/B})_n = -\omega_{AB}^2\,\underline{r}_{BA} = 0.0816(0.3\underline{i} - 0.05\underline{j})$ m/s^2

$(\underline{a}_{A/B})_t = \underline{\alpha}_{AB} \times \underline{r}_{BA} = \alpha_{AB}(-0.3\underline{j} - 0.05\underline{i})$ m/s^2

Substitute into $*$, equate like coefficients, & obtain

$\alpha_{OA} = -0.0519$ rad/s^2 , $\alpha_{AB} = -1.186$ rad/s^2

$\underline{a}_E = (\underline{a}_D)_n + (\underline{a}_D)_t + (\underline{a}_{E/D})_n + (\underline{a}_{E/D})_t$, $(\underline{a}_{E/D})_n = \underline{0}$ since $\omega_{DE} = 0$

$a_E\,\underline{i} = -0.6(0.857)^2\underline{j} + 0.6(0.0519)\underline{i} + \alpha_{ED}\underline{k} \times (\sqrt{0.12}\,\underline{i} - 0.2\underline{j})$

Solve to obtain $\alpha_{ED} = 1.272$ rad/s^2, $\underline{a_E = 0.285 \text{ m/s}^2}$

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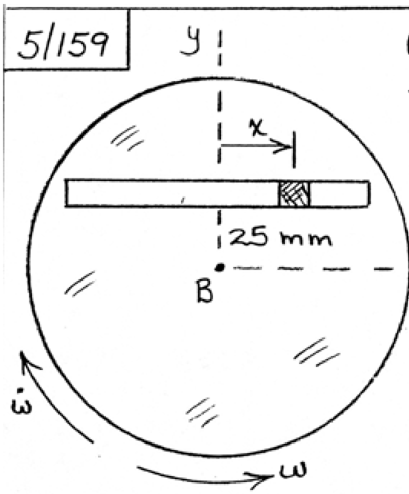


Attach Bxy to disk as shown.
In Eqs. 5/12 & 5/14:

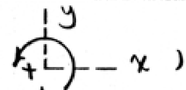
$$\bar{x}\begin{cases} \underline{v}_B = \underline{a}_B = \underline{0} \\ \underline{\omega} = 5\underline{k}\ \frac{\text{rad}}{\text{s}},\quad \dot{\underline{\omega}} = -3\underline{k}\ \frac{\text{rad}}{\text{s}^2} \\ \underline{r} = 36\underline{i} + 25\underline{j}\ \text{mm} \\ \underline{v}_{rel} = -100\underline{i}\ \text{mm/s} \\ \underline{a}_{rel} = 150\underline{i}\ \text{mm/s}^2 \end{cases}$$

(5/12): $\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$ gives

$$\underline{v}_A = -225\underline{i} + 180\underline{j}\ \text{mm/s}$$

(5/14): $\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

gives $\underline{a}_A = -675\underline{i} - 1733\underline{j}\ \text{mm/s}^2$

5/160 For the coordinates 

, the no-slip constraints are $v_0 = -r\omega$ & $a_0 = -r\alpha$. So

$$\omega = -\frac{v_0}{r} = -\frac{-3}{0.30} = 10 \text{ rad/s}$$

$$\alpha = -\frac{a_0}{r} = -\frac{5}{0.30} = -16.67 \text{ rad/s}^2$$

Use the frame Oxy as disk-fixed.

(5/12): $\underline{v}_A = \underline{v}_0 + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

(5/14): $\underline{a}_A = \underline{a}_0 + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

Ingredients:

$$\begin{cases} \underline{v}_0 = -3\underline{i} \text{ m/s} \\ \underline{a}_0 = 5\underline{i} \text{ m/s}^2 \\ \underline{\omega} = 10\underline{k} \text{ rad/s} \\ \underline{\alpha} = -16.67\underline{k} \text{ rad/s}^2 \end{cases}$$

$\underline{r} = 0.24\underline{j}$ m

$\underline{v}_{rel} = 2\underline{i}$ m/s

$\underline{a}_{rel} = -7\underline{i} - \frac{2^2}{0.24}\underline{j}$

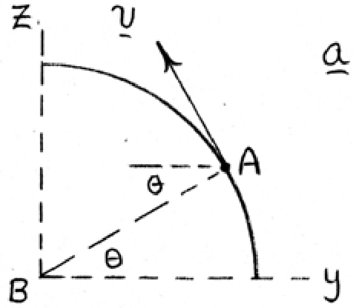
$= -7\underline{i} - 16.67\underline{j}$ m/s^2

Substitute into (5/12) & (5/14) & simplify:

$$\underline{v}_A = -3.4\underline{i} \text{ m/s}$$

$$\underline{a}_A = 2\underline{i} - 0.667\underline{j} \text{ m/s}^2$$

5/161



$\underline{v} = v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$

$\underline{a}_{cor} = 2\underline{\omega} \times \underline{v}$

$= 2\Omega\underline{k} \times v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$

$= \underline{2\Omega v \sin\theta\,\underline{i}}$ (west)

For $v = 500$ km/h,

(a) Equator, $\theta = 0$: $\underline{a_{cor} = 0}$

(b) North pole, $\theta = 90°$: $a_{cor} = 2(7.292\cdot10^{-5})\dfrac{500}{3.6}$

$= \underline{0.0203 \text{ m/s}^2}$

The track provides the necessary westward acceleration so that the velocity vector is properly rotated and reduced in magnitude.

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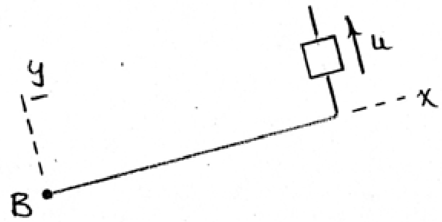
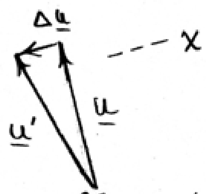
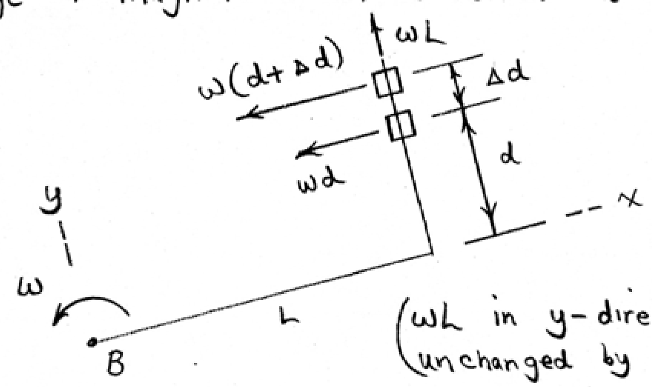
$$\underline{a}_{cor} = 2\underline{\omega} \times \underline{v}_{rel}$$
$$= 2\omega\underline{k} \times u\underline{j} = \underline{-2\omega u\underline{i}}$$



Change-of-direction effect is in $-x$ direction:



Change-of-magnitude effect is in $-x$ direction:



(ωL in y-direction unchanged by Δd.)

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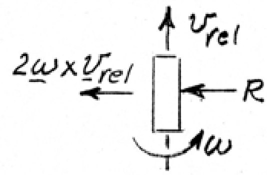
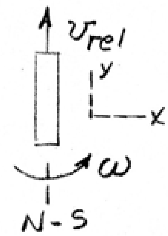
(a) North pole



Only horizontal component of acceleration is $|2\underline{\omega}\times\underline{v}_{rel}| = 2(0.7292)(10^{-4})(15) = 0.00219\ m/s^2$

$\Sigma F = ma_j$ $\quad R = 50\,000\,(0.00219) = \underline{109.4\ N}$

(b) Equator



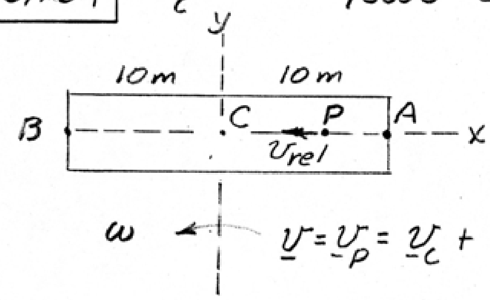
$\omega_{xy} = 0$ where x-y is horizontal plane

so $2\underline{\omega}\times\underline{v}_{rel} = 0$ & there is no other horizontal acceleration so $\underline{R = 0}$

5/164 $v_C = 25000/3600 = 6.94$ m/s

$\omega = v_C/\rho = 6.94/60$

$= 0.1157$ rad/s



$\underline{v}_{rel} = -1.5\underline{i}$ m/s

$\underline{\omega} = 0.1157\underline{k}$ rad/s

$\underline{v} = \underline{v}_P = \underline{v}_C + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

For A; $\underline{r} = 10\underline{i}$ m; $\underline{v}_A = -6.94\underline{i} + 0.1157\underline{k} \times 10\underline{i} - 1.5\underline{i}$

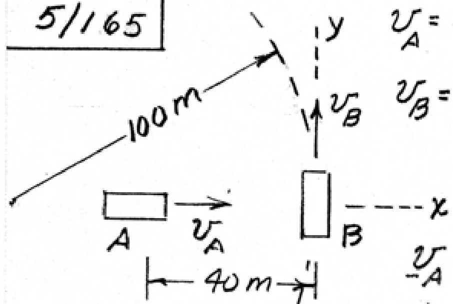
$= \underline{-8.44\underline{i} + 1.157\underline{j} \text{ m/s}}$

For C; $r = 0$, $\underline{v}_C = -6.94\underline{i} - 1.5\underline{i} = \underline{-8.44\underline{i}}$ m/s

For B; $\underline{r} = -10\underline{i}$ m; $\underline{v}_B = -6.94\underline{i} - 1.157\underline{j} - 1.5\underline{i}$

$= \underline{-8.44\underline{i} - 1.157\underline{j} \text{ m/s}}$

5/165



$$v_A = \frac{72 \times 1000}{3600} = 20\ \text{m/s}$$

$$v_B = \frac{54 \times 1000}{3600} = 15\ \text{m/s}$$

$$\omega = \omega_B = \frac{15}{100} = 0.15\ \text{rad/s}$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{rel}$$

$$20\mathbf{i} = 15\mathbf{j} + 0.15\mathbf{k} \times (-40\mathbf{i}) + \mathbf{v}_{rel}$$

$$\mathbf{v}_{rel} = 20\mathbf{i} - 9\mathbf{j}\ \text{m/s}$$

$(v_{rel})_{\text{rotating axes}}$ differs from $(v_{rel})_{\text{translating axes}}$ by $\boldsymbol{\omega} \times \mathbf{r}$

5/166 From Prob. 5/165 $\underline{v}_{rel} = 20\underline{i} - 9\underline{j}$ m/s

$\underline{\omega} = 0.15\underline{k}$ rad/s

$\underline{a}_A = \underline{a}_B + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \dot{\underline{\omega}} \times \underline{r} + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_A = \underline{0}$, $\underline{a}_B = \frac{v_B^2}{R}(-\underline{i}) = -\frac{15^2}{100}\underline{i} = -2.25\underline{i}$ m/s^2

$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = 0.15\underline{k} \times (0.15\underline{k} \times [-40\underline{i}] = 0.90\underline{i}$ m/s^2

$\dot{\underline{\omega}} \times \underline{r} = \underline{0}$

$2\underline{\omega} \times \underline{v}_{rel} = 2(0.15\underline{k}) \times (20\underline{i} - 9\underline{j}) = 2.7\underline{i} + 6\underline{j}$ m/s^2

Thus $\underline{0} = -2.25\underline{i} + 0.90\underline{i} + \underline{0} + 2.7\underline{i} + 6\underline{j} + \underline{a}_{rel}$

$\underline{a}_{rel} = \underline{-1.35\underline{i} - 6\underline{j}}$ m/s^2

5/167 $x = 2\sin 4\pi t$, $\dot{x} = 8\pi\cos 4\pi t$, $\ddot{x} = -32\pi^2 \sin 4\pi t$

$\theta = 0.2\sin 8\pi t$, $\dot{\theta} = 1.6\pi\cos 8\pi t$, $\ddot{\theta} = -12.8\pi^2\sin 8\pi t$

(a) For $x = 0$ & $\dot{x}\ (+)$; $t = 0$, $v_{rel} = \dot{x} = 8\pi$ in./sec

$\underline{a}_A = 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$ $\quad a_{rel} = \ddot{x} = 0$

$= 2(1.6\pi)(8\pi)\underline{j} + 0$ $\quad \omega = \dot{\theta} = 1.6\pi$ rad/sec

$= 253\underline{j}$ in./sec^2 $\quad \dot{\omega} = \ddot{\theta} = 0$

(b) For $x = +2$ in., $\sin 4\pi t = 1$, $\cos 4\pi t = 0$, $t = 1/8$ sec

$\theta = 0$

$v_{rel} = \dot{x} = 0$, $\ddot{x} = -32\pi^2$ in./sec^2

$\omega = \dot{\theta} = -1.6\pi$ rad/sec

$\dot{\omega} = \ddot{\theta} = 0$

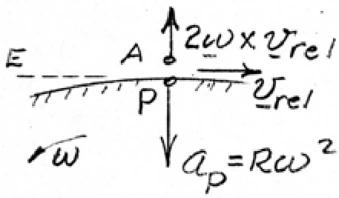
$\underline{a}_A = \underline{\dot{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{\dot{\omega}}\times\underline{r} = 0$, $\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -2(1.6\pi)^2\underline{i} = -5.12\pi^2\underline{i}$ in./sec^2

$2\underline{\omega}\times\underline{v}_{rel} = 0$, $\underline{a}_{rel} = \ddot{x}\underline{i} = -32\pi^2\underline{i}$ in./sec^2

$\underline{a}_A = -5.12\pi^2\underline{i} - 32\pi^2\underline{i} = -366\underline{i}$ in./sec^2

5/168 Let P be a point on the road coincident with A. $\underline{a}_A = \underline{a}_P + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



For zero vertical acceleration,

$|2\underline{\omega} \times \underline{v}_{rel}| = R\omega^2$

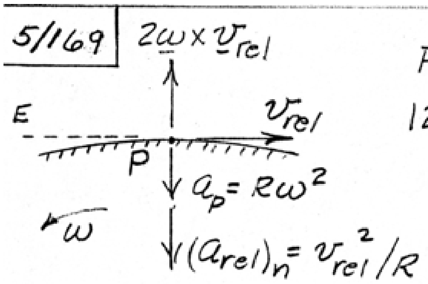
$v_{rel} = \frac{1}{2} R\omega$

For $R = 6378$ km, $\omega = 0.7292(10^{-4})$ rad/s,

$v_{rel} = \frac{1}{2}(6378)(10^3)(0.7292 \times 10^{-4}) = 233$ m/s

or $v_{rel} = 233(3.6) = \underline{837 \text{ km/h}}$

5/169



For zero vertical accel,

$$|2\omega \times v_{rel}| = R\omega^2 + v_{rel}^2/R$$

$$v_{rel}^2 - 2\omega R v_{rel} + R^2\omega^2 = 0$$

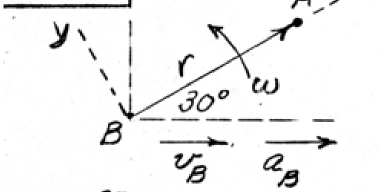
$$(v_{rel} - R\omega)^2 = 0, \quad \underline{v_{rel} = R\omega}$$

(zero absolute velocity)

[$v_{rel} = 6378(0.7292)(10^{-4}) = 0.4651$ km/s

or $0.4651(3600) = \underline{1674 \text{ km/h}}$]

5/170



$\underline{r} = (20+b)\underline{i} = 25\underline{i}$ ft

$\underline{v}_{rel} = \dot{r}\underline{i} = 2\underline{i}$ ft/sec

$\underline{a}_{rel} = \ddot{r}\underline{i} = -1\underline{i}$ ft/sec^2

$\underline{\omega} = \frac{10}{180}\pi\underline{k} \doteq 0.1745\underline{k}$ rad/sec

$\dot{\underline{\omega}} = \underline{0}$

$v_B = \frac{35}{30}44 = 51.3$ ft/sec , $\underline{v}_B = 51.3(\underline{i}\cos 30° - \underline{j}\sin 30°)$

$a_B = -10$ ft/sec^2 $\qquad \underline{a}_B = -10(\underline{i}\cos 30° - \underline{j}\sin 30°)$

Eq. 5/14, $\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

so $\underline{a}_A = -10(0.866\underline{i} - 0.5\underline{j}) + \underline{0} + (0.1745)^2\underline{k}\times(\underline{k}\times 25\underline{i})$

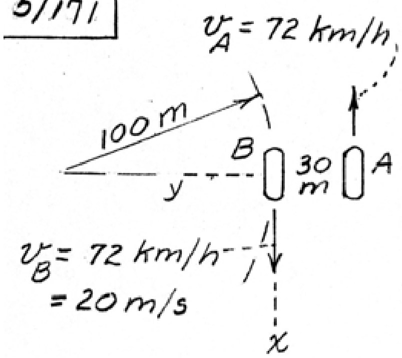
$+ 2(0.1745\underline{k})\times 2\underline{i} - 1\underline{i}$

(b) $\underline{a}_A = -10.42\underline{i} + 5.70\underline{j}$ ft/sec^2 with respect to ground

(a) $\underline{a}_A - \underline{a}_B = -10.42\underline{i} + 5.70\underline{j} - (-10)(0.866\underline{i} - 0.5\underline{j})$

$= -1.76\underline{i} + 0.70\underline{j}$ ft/sec^2 with respect to truck

5/171



$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

Angular velocity of axes is $\omega = \dfrac{72/3.6}{100} = 0.2$ rad/s

so $\underline{\omega} = 0.2\underline{k}$ rad/s

& $\underline{r} = \underline{r}_{A/B} = -30\underline{j}$ m

Thus $-20\underline{i} = 20\underline{i} + 0.2\underline{k} \times (-30\underline{j}) + \underline{v}_{rel}$

$\underline{v}_{rel} = -40\underline{i} - 6\underline{i} = \underline{-46\underline{i}\ \text{m/s}}$

Curvature of road for A has no effect on $\underline{v}_{rel}$ & hence $\underline{v}_A$.

5/172 Refer to figure and solution for Prob. 5/171 where $\underline{v}_{rel} = -46\underline{i}$ m/s

$\underline{\omega} = 0.2\underline{k}$ rad/s const.

$$\underline{a}_A = \underline{a}_B + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + \dot{\underline{\omega}}\times\underline{r} + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$$

$$\underline{a}_A = (v^2/\rho)(-\underline{j}) = -\frac{\overline{20}^2}{100}\underline{j} = -4\underline{j}\ \text{m/s}^2, \quad (a_A)_t = 0$$

$$\underline{a}_B = (v^2/\rho)(+\underline{j}) = +4\underline{j}\ \text{m/s}^2 \quad (a_B)_t = 0$$

$$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = 0.2\underline{k}\times(0.2\underline{k}\times[-30\underline{j}]) = 1.2\underline{j}\ \text{m/s}^2$$

$$\dot{\underline{\omega}}\times\underline{r} = 0$$

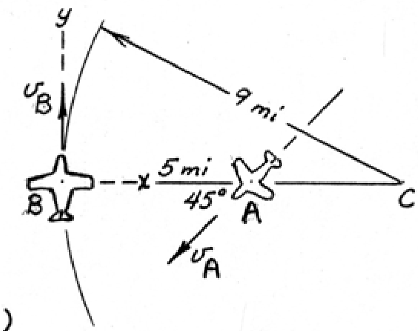
$$2\underline{\omega}\times\underline{v}_{rel} = 2(0.2\underline{k})\times(-46\underline{i}) = -18.4\underline{j}\ \text{m/s}^2$$

So $\underline{a}_{rel} = -4\underline{j} - 4\underline{j} - 1.2\underline{j} + 18.4\underline{j} = \underline{9.2\underline{j}\ \text{m/s}^2}$

5/173

$v_B = 480 \frac{44}{30} = 704$ ft/sec

$v_A = 360 \frac{44}{30} = 528$ ft/sec



$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

Angular vel. of axes $= \underline{\omega} = \frac{v_B}{\rho}(-\underline{k})$

$= \frac{-704}{9 \times 5280} \underline{k} = -0.01481 \underline{k}$ rad/sec

v_{rel} = vel. of A rel. to B

$\underline{r} = 5(5280)\underline{i} = 26{,}400\,\underline{i}$ ft

Thus $528(-0.707\underline{i} - 0.707\underline{j}) = 704\underline{j} - 0.01481\,\underline{k} \times 26{,}400\,\underline{i} + \underline{v}_{rel}$

$\underline{v}_{rel} = -373\underline{i} - 686\underline{j}$ ft/sec with $v_{rel} = 781$ ft/sec or 533 mi/hr

5/174 Refer to solution for Prob. 5/173 .

$$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$$

$$\underline{a}_A = \underline{0},\ \underline{a}_B = \frac{v_B^2}{\rho}\underline{i} = \frac{704^2}{9\times5280}\underline{i} = 10.43\,\underline{i}\ ft/sec^2$$

$$\dot{\underline{\omega}}\times\underline{r} = \underline{0}$$

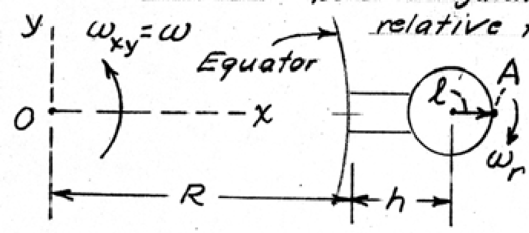
$$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -0.01481\,\underline{k}\times(-0.01481\,\underline{k}\times26{,}400\,\underline{i}) = -5.79\,\underline{i}\ ft/sec^2$$

$$2\underline{\omega}\times\underline{v}_{rel} = 2(-0.1481\,\underline{k})\times(-373\,\underline{i} - 686\,\underline{j}) = 11.05\underline{j} - 23.0\underline{i}\ ft/sec^2$$

$$\underline{a}_{rel} = \underline{0} - 10.43\underline{i} - \underline{0} + 5.79\underline{i} - 11.05\underline{j} + 20.3\underline{i} = \underline{15.69\,\underline{i} - 11.05\,\underline{j}\ ft/sec^2}$$

where $\underline{a_{rel} = 19.19\ ft/sec^2}$

5/175 Attach x-y axes to earth with z-axis pointing toward N-pole. Angular velocity of hour hand relative to clock is $\omega_r = 2\pi/12$ CW



But $\omega = 2\pi/24$ CCW

so $\omega_r = 2\omega$

$\underline{v}_{rel} = \ell\omega_r(-\underline{j}) = 2\ell\omega(-\underline{j})$

$$\underline{v}_A = \underline{v}_O + \underline{\omega}\times\underline{r} + \underline{v}_{rel} = \underline{0} + \omega\underline{k}\times(R+h+\ell)\underline{i} + 2\ell\omega(-\underline{j})$$

$$\underline{v}_A = \underline{(R+h-\ell)\omega\underline{j}}$$

$$\underline{a}_A = \underline{a}_O + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$$

$$\underline{a}_O = \underline{0},\ \dot{\underline{\omega}} = \underline{0},\ \underline{\omega}\times(\underline{\omega}\times\underline{r}) = -(R+h+\ell)\omega^2\underline{i}$$

$$2\underline{\omega}\times\underline{v}_{rel} = 2\omega\underline{k}\times(-2\ell\omega\underline{j}) = 4\ell\omega^2\underline{i}$$

$$\underline{a}_{rel} = -\ell\omega_r^2\underline{i} = -4\ell\omega^2\underline{i}$$

so $\underline{a}_A = -(R+h+\ell)\omega^2\underline{i} + 4\ell\omega^2\underline{i} - 4\ell\omega^2\underline{i}$

$$\underline{a}_A = \underline{-(R+h+\ell)\omega^2\underline{i}}$$

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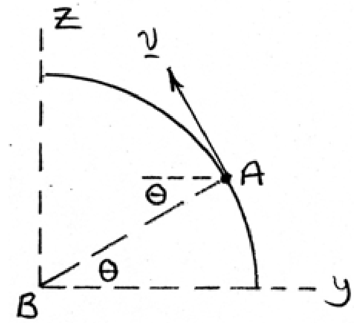


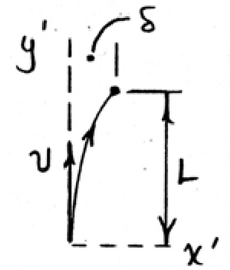
$$\underline{v} = v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$$

$$\underline{a}_{cor} = 2\underline{\omega} \times \underline{v}$$

$$= 2\Omega\underline{k} \times v(-\sin\theta\underline{j} + \cos\theta\underline{k})$$

$$= 2\Omega v \sin\theta\,\underline{i} \quad \text{(west)}$$

With no westward force mechanism available, the ball will drift to the east (relative to the ground) with an acceleration of magnitude a_{cor}.



$$a_{x'} = 2\Omega v \sin\theta$$

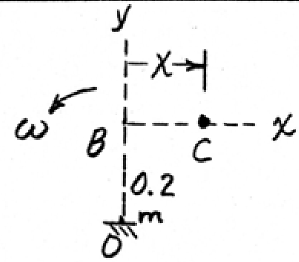
$$\delta = \frac{1}{2}a_{x'}t^2 = \frac{1}{2}(2\Omega v \sin\theta)\left(\frac{L}{v}\right)^2$$

$$= \underline{\frac{\Omega L^2}{v}\sin\theta} \quad (\text{assumes } \delta \ll L)$$

With $\Omega = 7.292(10^{-5})$ rad/sec, $v = 15$ ft/sec, $L = 60$ ft, & $\theta = 40°$: $\delta = 0.01125$ ft $\underline{(0.1350 \text{ in.})}$

5/177

	(a) $t = 3$ s	(b) $t = 0.5$ s
$x = 0.04 \sin \pi t$ =	0	0.04
$\dot{x} = 0.04\pi \cos \pi t$ =	-0.04π	0
$\ddot{x} = -0.04\pi^2 \sin \pi t$ =	0	$-0.04\pi^2$
$\omega = 2 \sin \frac{\pi}{2} t$ =	-2	$\sqrt{2}$
$\dot{\omega} = \pi \cos \frac{\pi}{2} t$ =	0	$\pi/\sqrt{2}$



$\underline{a}_C = \underline{a}_B + \underline{\dot{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

(a) $\underline{a}_B = 0.2(-2)^2(-\underline{j}) + 0\underline{i} = -0.8\underline{j}$ m/s^2

$\underline{\dot{\omega}} \times \underline{r} = \underline{0}$, $\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -2\underline{k} \times (-2\underline{k} \times \underline{0}) = \underline{0}$

$2\underline{\omega} \times \underline{v}_{rel} = 2(-2\underline{k}) \times (-0.04\pi \underline{i}) = 0.503\underline{j}$ m/s^2, $\underline{a}_{rel} = \underline{0}$

Substitute & get $\underline{\underline{a}_C = -0.297\underline{j} \text{ m/s}^2}$

(b) $\underline{a}_B = -0.2\sqrt{2}^2\underline{j} - 0.2\frac{\pi}{2}\underline{i} = -0.444\underline{i} - 0.4\underline{j}$ m/s^2

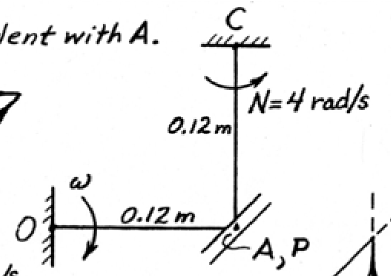
$\underline{\dot{\omega}} \times \underline{r} = \pi/\sqrt{2}\, \underline{k} \times 0.04\underline{i} = 0.0889\underline{j}$ m/s^2

$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = \sqrt{2}\underline{k} \times (\sqrt{2}\underline{k} \times 0.04\underline{i}) = -0.08\underline{i}$ m/s^2

$2\underline{\omega} \times \underline{v}_{rel} = 2\sqrt{2}\underline{k} \times \underline{0} = \underline{0}$, $\underline{a}_{rel} = -0.04\pi^2\underline{i} = -0.395\underline{i}$ $\frac{m}{s^2}$

Substitute & get $\underline{a}_C = \underline{-0.919\underline{i} - 0.311\underline{j}}$ m/s^2

5/178 | Let P = point on ODE coincident with A.

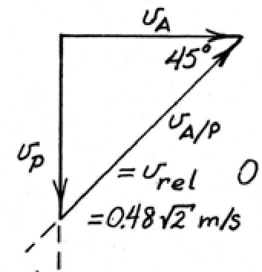


$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$

$v_A = 0.12(4) = 0.48 \text{ m/s}$

$v_P = 0.48 \text{ m/s},$

$\omega_{OP} = \omega = \dfrac{0.48}{0.12} = \underline{4 \text{ rad/s CW}}$



$\underline{a}_A = \underline{a}_P + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$a_A = (a_A)_n = 0.12(4^2) = 1.92 \text{ m/s}^2 \uparrow$

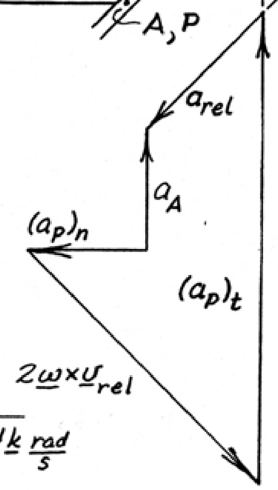
$|2\underline{\omega} \times \underline{v}_{rel}| = 2(4)(0.48\sqrt{2}) = 5.43 \text{ m/s}^2 \searrow$

$(a_P)_n = 0.12(4^2) = 1.92 \text{ m/s}^2 \leftarrow$

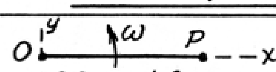
From diagram, $a_{rel} = 2.72 \text{ m/s}^2$,

$(a_P)_t = 7.68 \text{ m/s}^2$

$\alpha_{ODE} = \alpha = 7.68/0.12 = \underline{64.0 \text{ rad/s}^2 \text{ CCW}}$



Alternatively, with



$\underline{\omega} = -4\underline{k} \ \frac{\text{rad}}{\text{s}}$

$\underline{a}_A = (\underline{a}_A)_n = 0.12(4^2)\underline{j} = 1.92\underline{j} \text{ m/s}^2$

$(\underline{a}_P)_n = 0.12(4^2)(-\underline{i}) = -1.92\underline{i} \text{ m/s}^2$

$(\underline{a}_P)_t = \alpha\underline{k} \times 0.12\underline{i} = 0.12\alpha\underline{j}$

$2\underline{\omega} \times \underline{v}_{rel} = 2(-4\underline{k}) \times 0.48(\underline{i} + \underline{j}) = 3.84(\underline{i} - \underline{j}) \text{ m/s}^2$

$\underline{a}_{rel} = a_{rel}\dfrac{1}{\sqrt{2}}(\underline{i} + \underline{j})$

Substitute: $\underline{i}$-terms give $a_{rel}/\sqrt{2} = -1.92$, $a_{rel} = -2.72 \text{ m/s}^2$ so $a_{rel} = \swarrow$

$\underline{j}$-terms: $0.12\alpha = 1.92 + 3.84 + 1.92$, $\underline{\alpha = 64 \text{ rad/s}^2 \text{ CCW}}$

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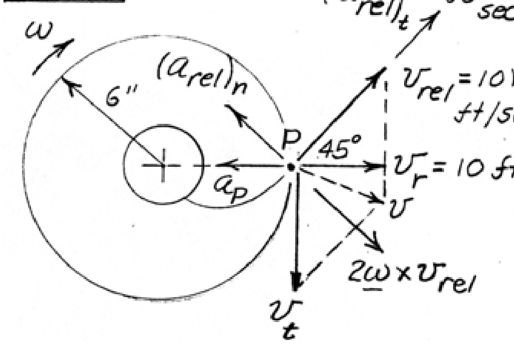


$\underline{a} = \underline{a}_P + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$a_P = r\omega^2 = \frac{6}{12}\left[\frac{200 \times 2\pi}{60}\right]^2$

$= 219$ ft/sec^2

$|2\underline{\omega} \times \underline{v}_{rel}| = 2\left[\frac{200 \times 2\pi}{60}\right] 10\sqrt{2}$

$= 592$ ft/sec^2

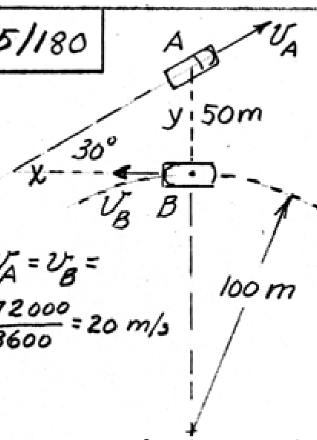
$(a_{rel})_n = \frac{v_{rel}^2}{\rho} = \frac{(10\sqrt{2})^2}{8/12} = 300 \frac{ft}{sec^2}$

Let $\underline{n}_1$ = unit vector in n-dir. & $\underline{t}_1$ = unit vector in t-dir.

$\underline{a} = \frac{219}{\sqrt{2}}\underline{n}_1 - \frac{219}{\sqrt{2}}\underline{t}_1 - 592\underline{n}_1 + 80\underline{t}_1 + 300\underline{n}_1$

$= -137.3\underline{n}_1 - 75.1\underline{t}_1$ ft/sec^2, $a = \sqrt{137.3^2 + 75.1^2} = \underline{156.5 \text{ ft/sec}^2}$

5/180



$v_A = v_B = \frac{72000}{3600} = 20$ m/s

$$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$$

$$\underline{v}_A = 20(-0.866\underline{i} + 0.5\underline{j})\ m/s$$

$$\underline{v}_B = 20\underline{i}\ m/s$$

$$\underline{\omega} = \frac{20}{100}(-\underline{k}) = -0.2\underline{k}\ rad/s$$

$$\underline{\omega} \times \underline{r} = -0.2\underline{k} \times 50\underline{j} = 10\underline{i}\ m/s$$

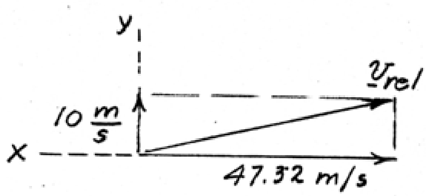
$$\underline{v}_{rel} = 20(-0.866\underline{i} + 0.5\underline{j}) - 20\underline{i} - 10\underline{i}$$

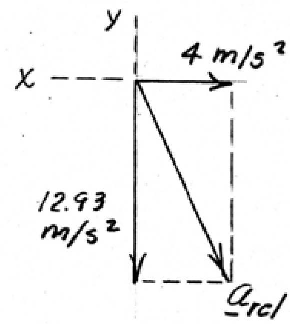
$$= \underline{-47.3\underline{i} + 10\underline{j}\ m/s}$$

$$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$$

$$\underline{0} = \frac{(20)^2}{100}(-\underline{j}) + \underline{0} - 0.2\underline{k} \times 10\underline{i} - 2(0.2\underline{k}) \times (-47.32\underline{i} + 10\underline{j}) + \underline{a}_{rel}$$

$$\underline{a}_{rel} = \underline{-4\underline{i} - 12.93\underline{j}}\ \ m/s^2$$





5/181 $\underline{a}_c = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

where $\underline{\omega} = \dot{\theta}\underline{k} = 2\pi f_1 \theta_0 \cos 2\pi f_1 t\, \underline{k}$

$\dot{\underline{\omega}} = \ddot{\theta}\underline{k} = -4\pi^2 f_1^2 \theta_0 \sin 2\pi f_1 t\, \underline{k}$

$\underline{r} = (8 + y)\underline{j}$ ft , $\underline{v}_{rel} = \dot{\underline{r}} = \dot{y}\underline{j} = 2\pi f_2 y_0 \cos 2\pi f_2 t\, \underline{j}$

$\underline{a}_{rel} = \ddot{\underline{r}} = \ddot{y}\underline{j} = -4\pi^2 f_2^2 y_0 \sin 2\pi f_2 t\, \underline{j}$

For $t = 2$ sec, $\theta_0 = \pi/4$ rad, $f_1 = \frac{1}{4}$ cycle/sec, $f_2 = \frac{1}{2}$ cycle/sec, $y_0 = 6$ in.

$\underline{\omega} = -\pi^2/8\, \underline{k}$ rad/sec, $\dot{\underline{\omega}} = \underline{0}$, $\underline{r} = (8 + y)\underline{j} = 8\underline{j}$ ft

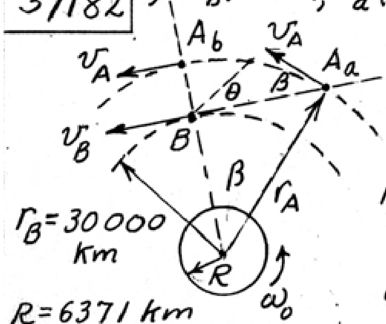
$\underline{v}_{rel} = \frac{\pi}{2}\underline{j}$ ft/sec, $\underline{a}_{rel} = \underline{0}$

so $\underline{a}_c = \underline{0} + \left(-\frac{\pi^2}{8}\underline{k}\right) \times \left(-\frac{\pi^2}{8}\underline{k} \times 8\underline{j}\right) + 2\left(-\frac{\pi^2}{8}\underline{k}\right) \times \frac{\pi}{2}\underline{j}$

$= -\frac{\pi^4}{8}\underline{j} + \frac{\pi^3}{8}\underline{i}$

$\underline{a}_c = \frac{\pi^3}{8}(\underline{i} - \pi\underline{j})$ ft/sec^2

5/182 $A_b(\theta = 90°);\ A_a(\theta = 0)$



$R = 6371$ km

$g = 9.825\ m/s^2$

$\omega_0 = 0.7292(10^{-4})$ rad/s

$\omega = \omega_{xy} = v_B / r_B$

$= \dfrac{13125}{30000} = 0.4375\ \dfrac{rad}{h}$

For circular orbit $v = R\sqrt{g/r}$

For geosyn. orbit $v = r_A \omega_0$

so $r_A \omega_0 = R\sqrt{g/r_A}$, $r_A = \left(\dfrac{R^2 g}{\omega_0^2}\right)^{1/3}$

$$r_A = \left\{\frac{(6371)^2(9.825/10^3)}{[0.7292(10^{-4})]^2}\right\}^{1/3} = 42\,171 \text{ km}$$

$$\beta = \cos^{-1}\frac{30000}{42171} = 44.65°$$

$$\overline{BA} = \sqrt{(42171)^2 - (30000)^2} = 29\,638 \text{ km}$$

$$v_A = 6371\sqrt{\frac{9.825/10^3}{42171}}(3600) = 11070 \text{ km/h}$$

$$v_B = 6371\sqrt{\frac{9.825/10^3}{30000}}(3600) = 13\,125 \text{ km/h}$$

$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$, $\quad \underline{v}_{rel} = \underline{v}_A - \underline{v}_B - \underline{\omega}\times\underline{r}$

(a) $\theta = 0$; $\underline{r} = 29638\underline{i}$ km

$\underline{v}_{rel} = 11070(-\underline{i}\cos\beta + \underline{j}\sin\beta) - (-13125\underline{i}) - 0.4375\underline{k}\times 29638\underline{i}$

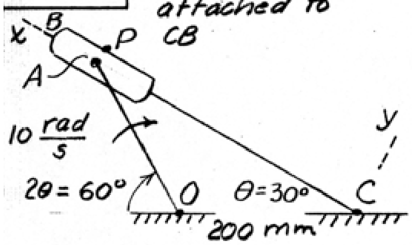
$= \underline{5250\underline{i} - 5190\underline{j}\ \text{km/h}}$

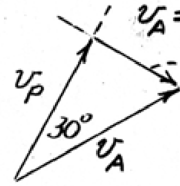
(b) $\theta = 90°$, $\underline{r} = (42171 - 30000)\underline{j} = 12171\underline{j}$ km

$\underline{v}_{rel} = -11070\underline{i} - (-13125\underline{i}) - 0.4375\underline{k}\times 12171\underline{j}$

$= \underline{7380\underline{i}\ \text{km/h}}$

5/183 x-y axes are attached to CB





$v_A = 200(10) = 2000$ mm/s

$v_{A/P} = v_{rel} = 2000\left(\frac{1}{2}\right) = 1000$ mm/s

$v_P = 2000\frac{\sqrt{3}}{2} = 1732\ \frac{mm}{s}$

$\omega_{xy} = \omega = \frac{v_P}{\overline{PC}} = \frac{1732}{2(200)\sqrt{3}/2} = 5$ rad/s CW

$\overline{OA} = 200$ mm

$\underline{a}_A = \underline{a}_C + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\dot{\omega} = \ddot{\theta} = 0$ since $\frac{d^2(2\theta)}{dt^2} = 0$; thus $(\underline{a}_P)_t = \dot{\underline{\omega}} \times \underline{r} = 0$

$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = 5^2 \underline{k} \times (\underline{k} \times 200\sqrt{3}\,\underline{i}) = -8660\,\underline{i}$ mm/s²

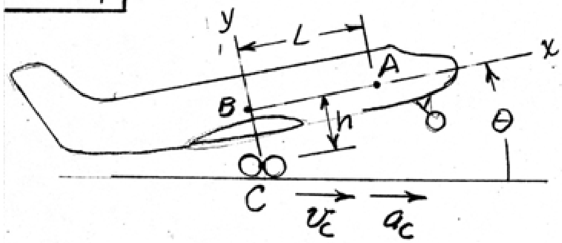
$2\underline{\omega} \times \underline{v}_{rel} = 2(5\underline{k}) \times 1000(-\underline{i}) = -10\,000\,\underline{j}$ mm/s²

$\underline{a}_{rel} = \ddot{x}\,\underline{i}$; $\underline{a}_A = 200(10)^2(-0.866\,\underline{i} - 0.5\,\underline{j})$ mm/s²

Thus $20\,000(-0.866\,\underline{i} - 0.5\,\underline{j}) = \underline{0} + \underline{0} - 8660\,\underline{i} - 10000\,\underline{j} + \ddot{x}\,\underline{i}$

$\ddot{x} = -8660$ mm/s², $\underline{a}_{rel} = -8660\,\underline{i}$ mm/s²

▸5/184



$\underline{\omega} = \omega\underline{k} = \dot{\theta}\underline{k}$, $\underline{\alpha} = \dot{\omega}\underline{k}$

$\underline{r} = L\underline{i}$, $\underline{v}_{rel} = \dot{L}\underline{i}$

$\underline{a}_{rel} = \ddot{L}\underline{i}$

$\underline{v}_C = v_C(\underline{i}\cos\theta - \underline{j}\sin\theta)$

$\underline{a}_C = a_C(\underline{i}\cos\theta - \underline{j}\sin\theta)$

$$\underline{v}_B = \underline{v}_C + \underline{\omega}\times\underline{r}_{CB} = \underline{v}_C + \omega\underline{k}\times h\underline{j} = (v_C\cos\theta - \omega h)\underline{i} - (v_C\sin\theta)\underline{j}$$

$$\underline{a}_B = \underline{a}_C + \dot{\underline{\omega}}\times\underline{r}_{CB} + \underline{\omega}\times(\underline{\omega}\times\underline{r}_{CB}) = \underline{a}_C + \alpha\underline{k}\times h\underline{j} + \omega\underline{k}\times(\omega\underline{k}\times h\underline{j})$$
$$= (a_C\cos\theta - \alpha h)\underline{i} - (a_C\sin\theta + h\omega^2)\underline{j}$$

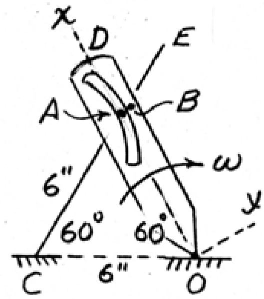
$$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel} = \underline{v}_B + \omega\underline{k}\times L\underline{i} + \dot{L}\underline{i}$$

$$\underline{\underline{v}_A = (v_C\cos\theta - \omega h + \dot{L})\underline{i} + (\omega L - v_C\sin\theta)\underline{j}}$$

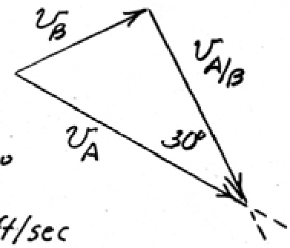
$$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$$
$$= \underline{a}_B + \dot{\omega}\underline{k}\times L\underline{i} + \omega\underline{k}\times(\omega\underline{k}\times L\underline{i}) + 2\omega\underline{k}\times\dot{L}\underline{i} + \ddot{L}\underline{i}$$

$$\underline{a}_A = \underline{(a_C\cos\theta - \alpha h - L\omega^2 + \ddot{L})\underline{i} + (-a_C\sin\theta - h\omega^2 + L\alpha + 2\omega\dot{L})\underline{j}}$$

▶5/185 Let B = point on DO coincident with A



$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$



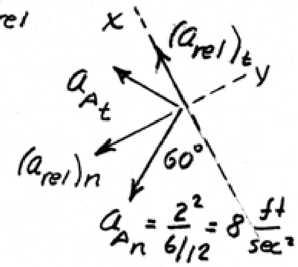
$v_B = \frac{6}{12} 2 = 1$ ft/sec

$v_{A/B} = v_{rel} = v_B / \tan 30°$

$v_A = 2$ ft/sec, $v_{rel} = \sqrt{3}$ ft/sec

$\omega = 2$ rad/sec
$\dot{\omega} = 6$ rad/sec^2

$\underline{a}_A = \underline{a}_o + \underline{\dot{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



$\underline{a}_o = 0, \quad \underline{\dot{\omega}} \times \underline{r} = 6\underline{k} \times \frac{6}{12}\underline{i} = 3\underline{j}$ ft/sec^2

$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = 2\underline{k} \times (2\underline{k} \times \frac{6}{12}\underline{i}) = -2\underline{i}$ ft/sec^2

$2\underline{\omega} \times \underline{v}_{rel} = 4\underline{k} \times (-\sqrt{3}\underline{i}) = -4\sqrt{3}\underline{j}$ ft/sec^2

$(\underline{a}_{rel})_n = \frac{v_{rel}^2}{\rho}(-\underline{j}) = -\frac{3}{6/12}\underline{j} = -6\underline{j}$ ft/sec^2

$\underline{a}_{A_n} = -8\cos 60°\underline{i} - 8\sin 60°\underline{j}$ ft/sec^2

$\underline{a}_{A_t} = a_{A_t}\cos 30°\underline{i} - a_{A_t}\sin 30°\underline{j}$

substitute & get

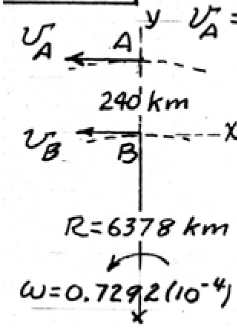
$a_{A_t}\frac{\sqrt{3}}{2}\underline{i} - a_{A_t}\frac{1}{2}\underline{j} - 4\underline{i} - 8\frac{\sqrt{3}}{2}\underline{j} = 3\underline{j} - 2\underline{i} - 4\sqrt{3}\underline{j} - 6\underline{j} + (a_{rel})_t\underline{i}$

Equate $\underline{i}$ & $\underline{j}$ terms & get $(a_{rel})_t = 3.20$ ft/sec^2

$(a_A)_t = 6$ ft/sec^2 so $\alpha_{EC} = 6/\frac{6}{12} = \underline{12 \text{ rad/sec}^2 \text{ CCW}}$

▶5/186 For circular orbit ; At equator $g = 9.814\ m/s^2$

$$v_A = R\sqrt{g/(R+h)} = 6378\sqrt{\frac{9.814/1000}{6378+240}}(3600) = 27\,960\ \frac{km}{h}$$



$$a_A = \frac{v_A^2}{R+h} = g\left(\frac{R}{R+h}\right)^2 = 9.814\left(\frac{6378}{6378+240}\right)^2 = 9.115\ m/s^2$$

$$v_B = R\omega = 6378(0.7292)(10^{-4})(3600) = 1674\ km/h$$

$$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$$

$$\underline{\omega}\times\underline{r} = 0.7292(10^{-4})(3600)(240)(-\underline{i})$$
$$= -63.00\underline{i}\ km/h$$

$$\underline{v}_{rel} = -27\,960\underline{i} - (-1674\underline{i}) - (-63.00\underline{i})$$
$$= -26\,220\underline{i}\ km/h$$

$$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$$

$$\underline{a}_B = -R\omega^2\underline{j} = -6378(10^3)(0.7292)^2(10^{-4})^2\underline{j} = -0.03391\underline{j}\ m/s^2$$

$$\dot{\underline{\omega}} = \underline{0};\ \underline{\omega}\times(\underline{\omega}\times\underline{r}) = (0.7292)(10^{-4})\underline{k}\times(-63.00\underline{i})\frac{1000}{3600} = -0.001276\underline{j}\ m/s^2$$

$$2\underline{\omega}\times\underline{v}_{rel} = 2(0.7292)(10^{-4})\underline{k}\times(-26\,220\underline{i})\frac{1000}{3600} = -1.0623\underline{j}\ m/s^2$$

So

$$-9.115\underline{j} = -0.03391\underline{j} - 0.001276\underline{j} - 1.0623\underline{j} + \underline{a}_{rel}$$

$$\underline{a}_{rel} = \underline{-8.018\underline{j}\ m/s^2}$$

$\boxed{5/187}$ $\underline{\omega} \times \underline{r} = \underline{v}$; $2\underline{k} \times (x\underline{i} + y\underline{j}) = -0.8\underline{i} - 0.6\underline{j}$

$$2x\underline{j} - 2y\underline{i} = -0.8\underline{i} - 0.6\underline{j}$$

So $2x = -0.6$, $\underline{x = -0.3 \text{ m}}$

$-2y = -0.8$, $\underline{y = 0.4 \text{m}}$ $r = \sqrt{0.3^2 + 0.4^2} = \underline{0.5 \text{m}}$

5/188 $\underline{\omega} = 3\underline{k}$ rad/s $\quad \underline{\alpha} = -6\underline{k}$ rad/s

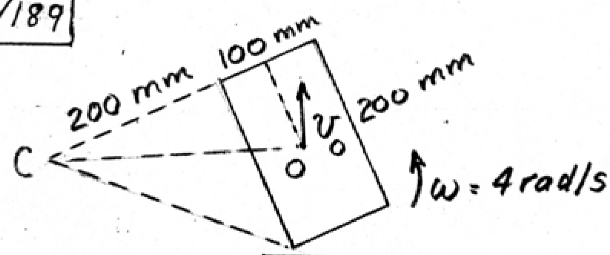
$\underline{r}_P = \underline{r} = -0.1\underline{i} + 0.15\underline{j}$ m

$\underline{v} = \underline{\omega} \times \underline{r} = 3\underline{k} \times (-0.1\underline{i} + 0.15\underline{j}) = -0.45\underline{i} - 0.3\underline{j}$ m/s

$\underline{a}_t = \underline{\alpha} \times \underline{r} = -6\underline{k} \times (-0.1\underline{i} + 0.15\underline{j}) = 0.9\underline{i} + 0.6\underline{j}$ m/s^2

$\underline{a}_n = \underline{\omega} \times \underline{v} = 3\underline{k} \times (-0.45\underline{i} - 0.3\underline{j}) = \underline{0.9\underline{i} - 1.35\underline{j}\ \text{m/s}^2}$

5/189



$v_O = \overline{OC}\,\omega$, $\overline{OC} = \sqrt{100^2 + 250^2} = 269$ mm

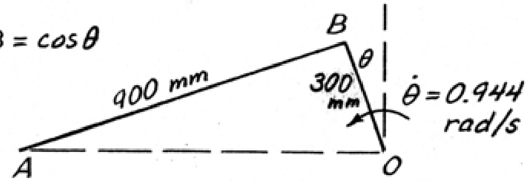
$v_O = 269(4) = 1077$ mm/s or $\underline{1.077\ m/s}$

5/190

$900 \sin\beta = 300 \cos\theta, \quad 3\sin\beta = \cos\theta$

$3\dot{\beta}\cos\beta = -\dot{\theta}\sin\theta,$

$\dot{\beta} = -\frac{0.944}{3}\frac{\sin\theta}{\cos\beta}$



$9\sin^2\beta = \cos^2\theta, \quad 9(1-\cos^2\beta) = \cos^2\theta, \quad \cos\beta = \sqrt{1-\frac{1}{9}\cos^2\theta}$

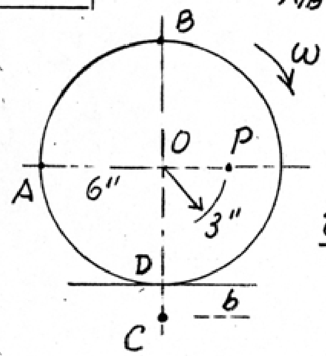
So $\dot{\beta} = -\frac{0.944}{3}\frac{\sin\theta}{\sqrt{1-\frac{1}{9}\cos^2\theta}} = -\frac{0.944}{3}\frac{\sin 20°}{\sqrt{1-\frac{1}{9}\cos^2 20°}}$

$= -0.1133$ rad/s

$\underline{\omega_{AB} = 0.1133 \text{ rad/s CW}}$

5/191

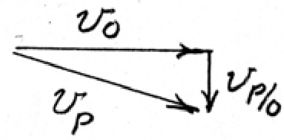
$v_{A/B} = \overline{AB}\,\omega$, $3\sqrt{2} = \frac{1}{2}\sqrt{2}\,\omega$, $\omega = 6$ rad/s



$v_O / \overline{OC} = \omega$, $\frac{4(12)}{6+b} = 6$

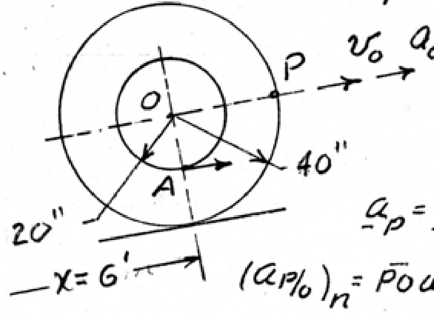
$v_O = 4$ ft/sec $\underline{b = 2 \text{ in}}$

$\mathbf{v}_P = \mathbf{v}_O + \mathbf{v}_{P/O}$, $v_{P/O} = \overline{PO}\,\omega = \frac{3}{12}\,6 = \frac{3}{2}$ ft/sec



$v_P = \sqrt{4^2 + (3/2)^2} = \underline{4.27 \text{ ft/sec}}$

5/192 Displacement, velocity, & acceleration of truck are $20/40 = 0.5$ of x, v_0, a_0



Thus $a_0 = 2(a_{truck})$

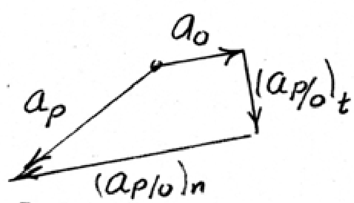
$= 2(2) = 4 \ ft/sec^2$

$$v_0^2 = 2a_0 x = 2(4)(6) = 48\left(\frac{ft}{sec}\right)^2$$

$$\underline{a}_P = \underline{a}_O + (\underline{a}_{P/O})_n + (\underline{a}_{P/O})_t$$

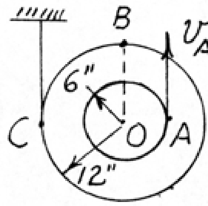
$$(a_{P/O})_n = \bar{PO}\,\omega^2 = \bar{PO}\left(\frac{v_0}{\bar{PO}}\right)^2 = \frac{48}{40/12} = 14.4 \ ft/sec^2$$

$$(a_{P/O})_t = \bar{PO}\,\alpha = a_0 = 4 \ ft/sec^2$$



$$a_P = \sqrt{(14.4-4)^2 + 4^2} = \underline{11.14 \ \frac{ft}{sec^2}}$$

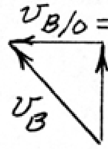
5/193 $v_O = \frac{12}{18} v_A = \frac{2}{3}(3) = \underline{2 \text{ ft/sec}}$



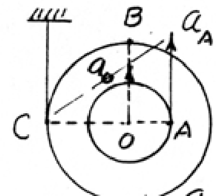
$v_A = 3$ ft/sec

$\underline{v}_B = \underline{v}_O + \underline{v}_{B/O}$

$v_{B/O} = \overline{BO}\,\omega_{BO} = \overline{BO}\,\frac{v_O}{\overline{CO}} = 2$ ft/sec



$v_B = 2\sqrt{2} = \underline{2.83 \text{ ft/sec}}$



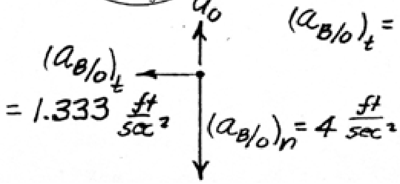
$a_A = 2$ ft/sec^2

$a_O = \frac{12}{18}(2) = \underline{1.333 \text{ ft/sec}^2}$

$\underline{a}_B = \underline{a}_O + (\underline{a}_{B/O})_n + (\underline{a}_{B/O})_t$

$(a_{B/O})_n = \overline{BO}\,\omega_{BO}^2 = \frac{12}{12}\left(\frac{2}{1}\right)^2 = 4$ ft/sec^2

$(a_{B/O})_t = \overline{BO}\,\alpha_{BO} = \frac{12}{12}\,\frac{1.333}{1} = 1.333$ ft/sec^2



$a_B = \sqrt{(4 - 1.333)^2 + (1.333)^2}$

$= \underline{2.98 \text{ ft/sec}^2}$

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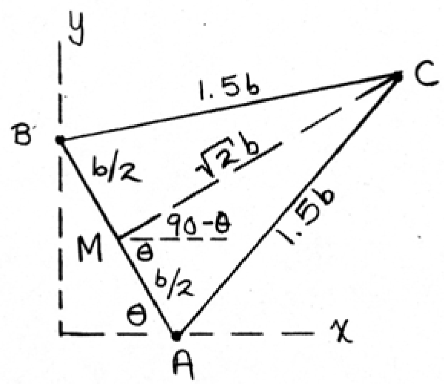

Note:

$\overline{CM}^2 = (1.5b)^2 - \left(\frac{b}{2}\right)^2$

$\overline{CM} = \sqrt{2}\,b$

$$x_C = \frac{b}{2}\cos\theta + \sqrt{2}\,b\cos(90^\circ - \theta)$$
$$= \frac{b}{2}\cos\theta + \sqrt{2}\,b\sin\theta$$
$$\dot{x}_C = -\frac{b}{2}\dot{\theta}\sin\theta + \sqrt{2}\,b\dot{\theta}\cos\theta$$
$$\dot{x}_C = 0 \text{ when } \quad \frac{1}{2}\sin\theta = \sqrt{2}\cos\theta$$
$$\tan\theta = 2\sqrt{2}, \quad \underline{\theta = 70.5^\circ}$$

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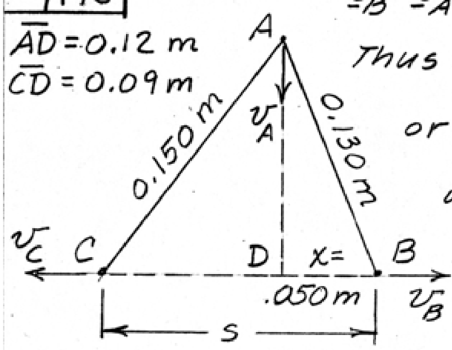
$\overline{AD} = 0.12\ m$

$\overline{CD} = 0.09\ m$

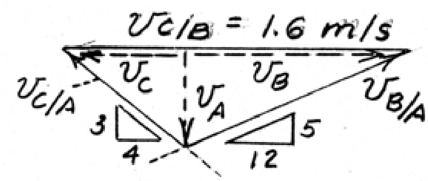


$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}\ ,\quad \underline{v}_C = \underline{v}_A + \underline{v}_{C/A}$

Thus $\underline{v}_B = \underline{v}_C - \underline{v}_{C/A} + \underline{v}_{B/A}$

or $\underline{v}_{C/A} = \underline{v}_{C/B} + \underline{v}_{B/A}$

where $\underline{v}_{C/B} = \underline{v}_C - \underline{v}_B$

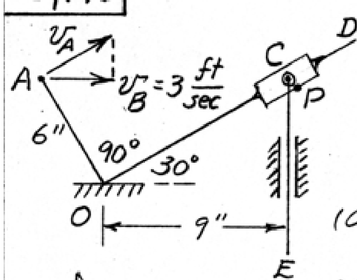


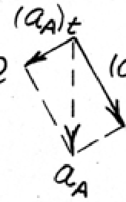
$v_{C/B} = \dot{s} = 1.6\ m/s$

From diagram $\frac{5}{12} v_B = \frac{3}{4} v_C\ ;\quad v_B + v_C = 1.6$

$\frac{5}{12} v_B = \frac{3}{4}(1.6 - v_B),\quad \underline{v_B = 1.029\ m/s}$

5/196



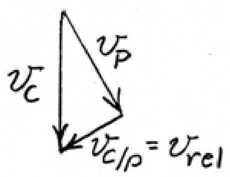


$v_A = v_B/\cos 30° = 3/0.866$
$= 3.46$ ft/sec

$(a_A)_{horiz.} = 0$ since $(v_A)_{horiz}$ is constant

Thus a_A is vertical

$(a_A)_n = v_A^2/\overline{AO} = (2\sqrt{3})^2/\frac{6}{12} = 24$ ft/sec^2



$(a_A)_t = 24/\sqrt{3} = 13.86$ ft/sec^2

$\alpha_{OD} = \alpha_{OA} = \frac{(a_A)_t}{\overline{OA}} = \frac{13.86}{6/12} = 27.7$ rad/sec^2 ccw

$v_P = \frac{\overline{OP}}{\overline{OA}} v_A = \frac{9(2/\sqrt{3})}{6}(3.46) = 6$ ft/sec

$v_{rel} = 6/\sqrt{3}$ ft/sec

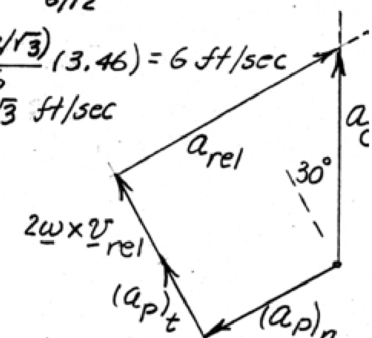
$\underline{a}_C = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$(a_P)_n = v_P^2/\overline{OP} = 6^2/\frac{9}{12}\frac{2}{\sqrt{3}} = 24\sqrt{3}$ ft/sec^2

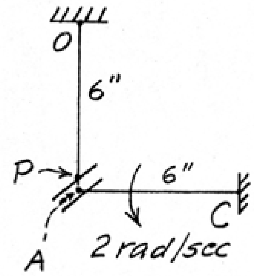
$(a_P)_t = \overline{OP}\alpha_{OD} = \frac{9}{12}\frac{2}{\sqrt{3}}(27.7) = 24$ ft/sec^2

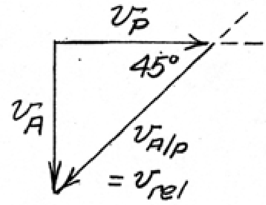
$|2\underline{\omega}\times\underline{v}_{rel}| = 2\frac{2\sqrt{3}}{6/12}\frac{6}{\sqrt{3}} = 48$ ft/sec^2

From diag. $a_C = (24+48)\frac{2}{\sqrt{3}} = \underline{83.1 \frac{ft}{sec^2} \text{ up}}$



5/197 Let P be a point on EBO coincident with A





$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$

$v_A = 6(2) = 12$ in./sec

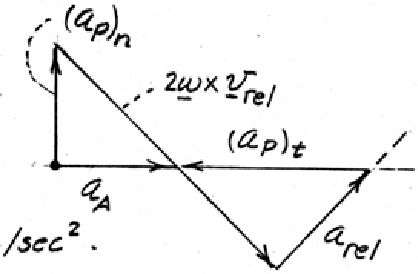
$\omega_{PO} = \omega = \dfrac{v_P}{\overline{PO}} = \dfrac{12}{6} = 2 \dfrac{\text{rad}}{\text{sec}}$ CCW

$\underline{a}_A = \underline{a}_P + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$a_A = (a_A)_n = 6(2^2) = 24$ in./sec^2

$|2\underline{\omega} \times \underline{v}_{rel}| = 2(2)(12\sqrt{2}) = 48\sqrt{2} \dfrac{\text{in.}}{\text{sec}^2}$

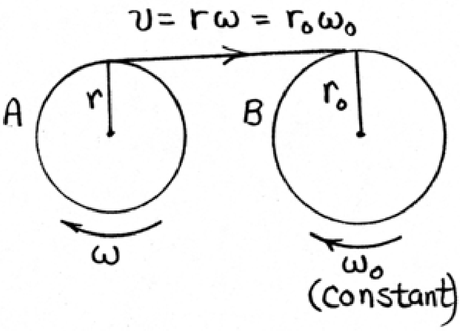
$(a_P)_n = 6(2^2) = 24$ in./sec^2



From diagram $(a_P)_t = 48$ in./sec^2.

$\alpha_{PO} = \alpha = \dfrac{48}{6} = \underline{8 \text{ rad/sec}^2 \text{ CW}}$

5/198



From $r\omega = r_0\omega_0$,

$$\dot{r}\omega + r\dot{\omega} = \dot{r}_0\omega_0 + r_0\dot{\omega}_0$$

But $\dot{r} = -\dfrac{b}{2\pi/\omega}$

and $\dot{r}_0 = +\dfrac{b}{2\pi/\omega_0}$

So $-\dfrac{b}{2\pi/\omega}\omega + r\dot{\omega} = \dfrac{b}{2\pi/\omega_0}\omega_0$

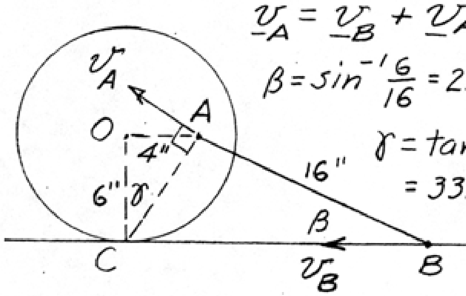
$$\dot{\omega} = \alpha = \frac{b}{2\pi r}\left[\omega_0^2 + \omega^2\right] = \underline{\frac{b\omega_0^2}{2\pi r}\left(1 + \frac{r_0^2}{r^2}\right)}$$

5/199 $v_B = 10$ in./sec, constant

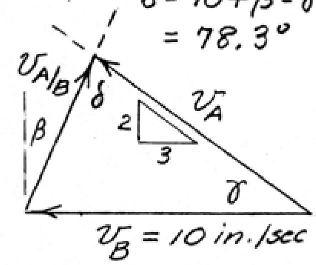
$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$$

$$\beta = \sin^{-1}\frac{6}{16} = 22.0^\circ$$

$$\gamma = \tan^{-1}\frac{2}{3} = 33.7^\circ$$



$$\delta = 90 + \beta - \gamma = 78.3^\circ$$



$$\frac{v_{A/B}}{\sin 33.7^\circ} = \frac{10}{\sin 78.3^\circ}$$

$$v_{A/B} = 10\,\frac{0.555}{0.979} = 5.66 \text{ in./sec.}$$

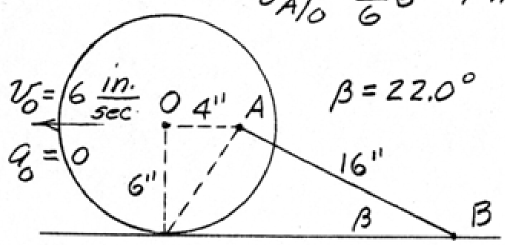
$$\omega_{AB} = v_{A/B}/\overline{AB} = \frac{5.66}{16} = \underline{0.354 \text{ rad/sec CW}}$$

$$\frac{v_A}{\sin(90^\circ - \beta)} = \frac{10}{\sin 78.3^\circ}, \quad v_A = 10\,\frac{0.927}{0.979} = 9.47 \text{ in./sec}$$

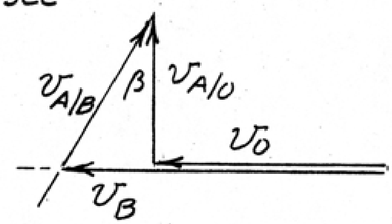
$$v_O = \frac{\overline{OC}}{\overline{AC}}\,v_A = \frac{6}{\sqrt{4^2+6^2}}(9.47) = \underline{7.88 \text{ in./sec}}$$

5/200 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$, $\underline{v}_O + \underline{v}_{A/O} = \underline{v}_B + \underline{v}_{A/B}$

$v_{A/O} = \frac{4}{6} 6 = 4$ in./sec



$\beta = 22.0°$



$v_{A/B} = 4/\cos 22.0°$
$= 4.31$ in./sec

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$\underline{a}_A = \underline{a}_O + \underline{a}_{A/O} = 0 + (\underline{a}_{A/O})_n$

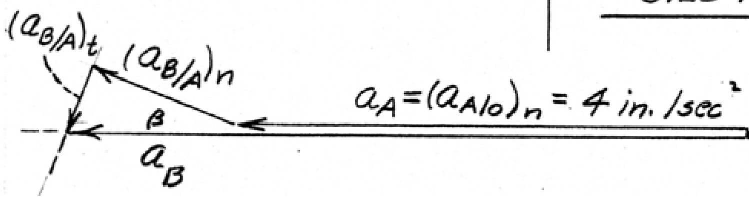
$(a_{A/O})_n = 4^2/4 = 4$ in./sec^2

$(a_{B/A})_n = \overline{4.31}^2/16 = 1.16$ in./sec^2

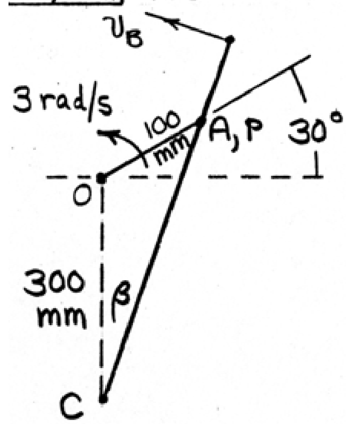
From diagram

$a_B = 4 + 1.16/\cos 22.0°$

$\underline{= 5.25 \text{ in./sec}^2}$



5/201 Let P be a point on BC coincident with A.

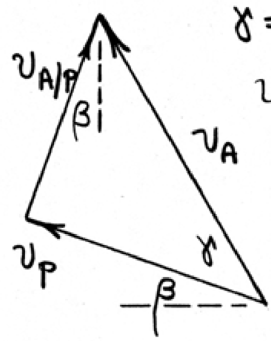


$CA^2 = 300^2 + 100^2 - 2(300)(100)\cos 120°$

$= 361$ mm

$\frac{100}{\sin\beta} = \frac{361}{\sin 120°}$, $\beta = 13.90°$

$\gamma = 60 - \beta = 46.1°$



$v_P = v_A \cos\gamma$

$= 300 \cos 46.1°$

$= 208$ mm/s

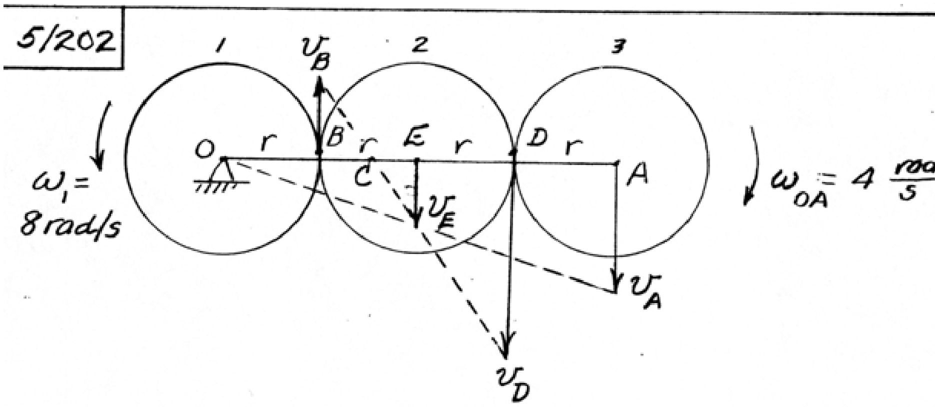
$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$

$v_A = 100(3)$

$= 300$ mm/s

$v_B = \overline{CB}\,\frac{v_P}{\overline{CA}} = 500\,\frac{208}{361} = \underline{288 \text{ mm/s}}$

5/202



Let B be contact point common to gears 1 & 2

" D " " " " " gears 2 & 3

Point C is instantaneous center of zero velocity for gear 2. By similar triangles, $v_D = 3(8r) = 24r$

$v_B = r\omega_1 = 8r$ $\quad v_A = \overline{OA}\,\omega_{OA} = 4r(4) = 16r$

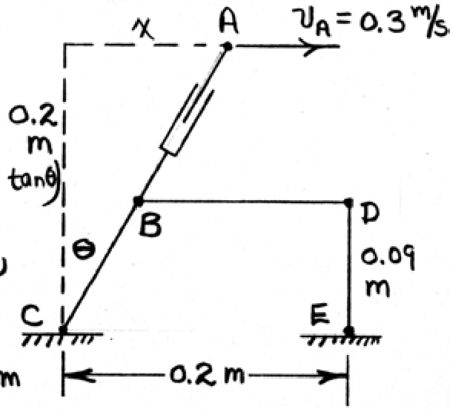
$v_E = 2r\omega_{OA} = 8r$ $\quad \omega_3 = \dfrac{v_{D/A}}{\overline{DA}} = \dfrac{24r - 16r}{r} = \underline{8\text{ rad/s CCW}}$

5/203

$x = 0.2 \tan\theta$

$\dot{x} = 0.2\dot{\theta}\sec^2\theta$

$\ddot{x} = 0.2\ddot{\theta}\sec^2\theta + 0.2(2)\dot{\theta}^2\sec\theta(\sec\theta\tan\theta)$



For $\dot{x} = 0.3\frac{m}{s}$, $\ddot{x} = 0$, $\theta = 30^\circ$,

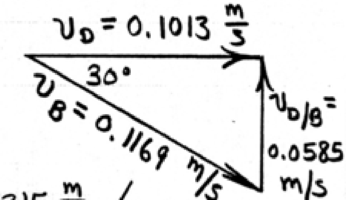
$\dot{\theta} = 1.125 \frac{rad}{s}$, $\ddot{\theta} = -1.461 \frac{rad}{s^2}$

$\overline{BD} = 0.2 - 0.09\tan 30^\circ = 0.1480$ m

$\overline{CB} = \dfrac{0.09}{\cos 30^\circ} = 0.1039$ m

$v_B = \overline{CB}\dot{\theta} = 0.1039(1.125) = 0.1169 \frac{m}{s}$

$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$



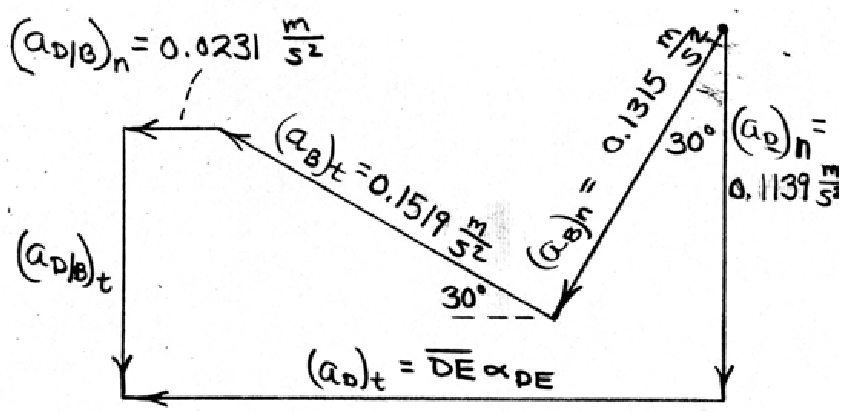
$\underline{a}_D = \underline{a}_B + (\underline{a}_{D/B})_n + (\underline{a}_{D/B})_t$

$(a_B)_n = \overline{CB}\dot{\theta}^2 = 0.1039(1.125)^2 = 0.1315 \frac{m}{s^2}$

$(a_B)_t = \overline{CB}\ddot{\theta} = 0.1039(-1.461) = -0.1519 \frac{m}{s^2}$

$(a_{D/B})_n = \dfrac{v_{D/B}^2}{\overline{DB}} = \dfrac{0.0585^2}{0.1480} = 0.0231 \frac{m}{s^2} \leftarrow$

$(a_D)_n = \dfrac{0.1013^2}{0.09} = 0.1139 \frac{m}{s^2}$

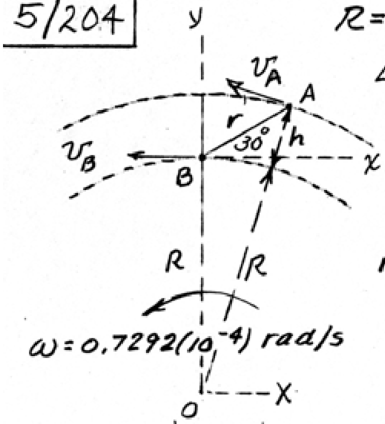


$$(a_D)_t = \overline{DE}\alpha_{DE} = 0.1315 \sin 30^\circ + 0.1519 \cos 30^\circ + 0.0231$$

$$= 0.220 \text{ m/s}^2$$

$$\alpha_{DE} = \frac{0.220}{0.09} = \underline{2.45 \text{ rad/s}^2 \quad \text{CCW}}$$

5/204



$R = 6378$ km, $h = 200$ km

Law of cosines gives

$(R+h)^2 = R^2 + r^2 - 2Rr\cos 120°$

$(6378+200)^2 = (6378)^2 + r^2 - 2(6378)(-0.5)r$

$r^2 + 6378r - 2\,591\,200 = 0$

$r = -\frac{6378}{2} \pm \frac{1}{2}\sqrt{(6378)^2 + 4(2591200)}$

$= 383.2$ km (or -6761 km)

Rel. velocity from nonrotating system X-Y at O is

$\underline{v}_A - \underline{v}_B$. But $\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

so $\underline{\omega}\times\underline{r} = (\underline{v}_A - \underline{v}_B) - (\underline{v}_{rel})$

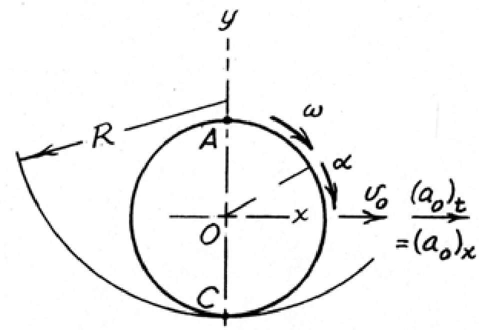
$\underline{\omega}\times\underline{r} = 0.7292(10^{-4})(3600)\underline{k} \times 383.2(0.866\underline{i} + 0.5\underline{j})$

so $\Delta\underline{v}_{rel} = \underline{-50.3\underline{i} + 87.1\underline{j}}$ km/h

5/205

$v_O = r\omega,\ (a_O)_t = r\alpha$

$\underline{a}_C = \underline{a}_O + (\underline{a}_{C/O})_n + (\underline{a}_{C/O})_t$



$\underline{a}_O = r\alpha\underline{i} + \frac{(r\omega)^2}{R-r}\underline{j}$

$(\underline{a}_{C/O})_n = r\omega^2\underline{j},\ (\underline{a}_{C/O})_t = -r\alpha\underline{i}$

$\underline{a}_C = r\alpha\underline{i} + \frac{(r\omega)^2}{R-r}\underline{j} + r\omega^2\underline{j} - r\alpha\underline{i},\ \underline{a}_C = \frac{r\omega^2}{1-r/R}\underline{i}$

$\underline{a}_A = \underline{a}_O + (\underline{a}_{A/O})_n + (\underline{a}_{A/O})_t$

$(\underline{a}_{A/O})_n = -r\omega^2\underline{j},\ (\underline{a}_{A/O})_t = r\alpha\underline{i}$

$\underline{a}_A = r\alpha\underline{i} + \frac{(r\omega)^2}{R-r}\underline{j} - r\omega^2\underline{j} + r\alpha\underline{i},\ \underline{a}_A = 2r\alpha\underline{i} + r\omega^2\frac{2r/R-1}{1-r/R}\underline{j}$

*5/206

$\dot{\theta} = 120 \frac{2\pi}{60} = 4\pi$ rad/sec

$5 \sin\theta = (25 - 5\cos\theta) \tan\beta \quad ---(1)$

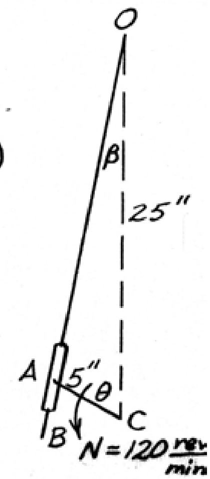
$5\dot{\theta}\cos\theta = 5\dot{\theta}\sin\theta\tan\beta + (25 - 5\cos\theta)\dot{\beta}\sec^2\beta$

$= 5\dot{\theta}\sin\theta\tan\beta + (25 - 5\cos\theta)\dot{\beta}(1 + \tan^2\beta)$

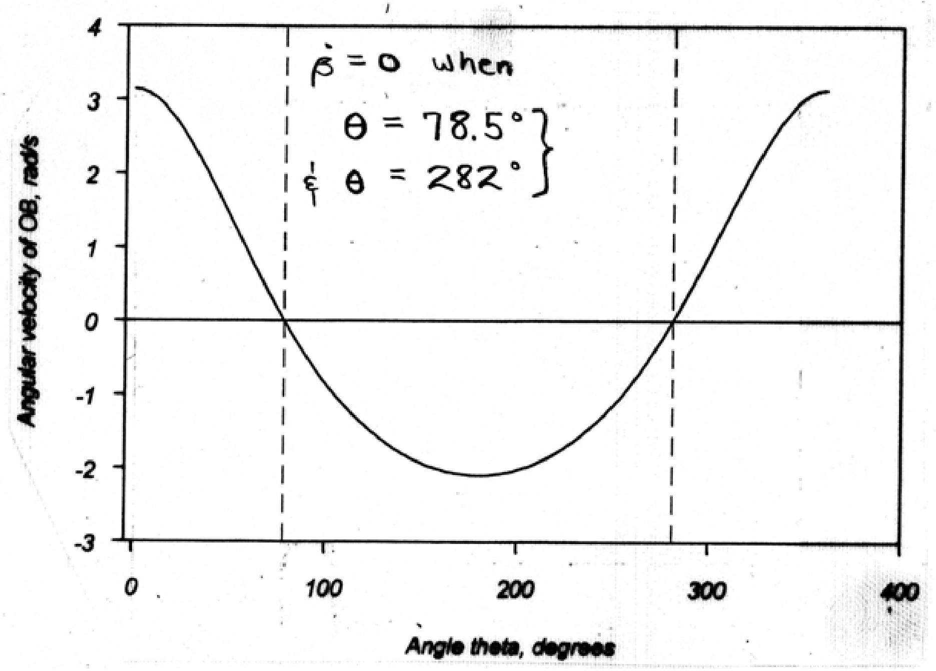
$$\dot{\beta} = \frac{(4\pi)(\cos\theta - \sin\theta\tan\beta)}{(5 - \cos\theta)(1 + \tan^2\beta)}$$

Substitute Eq. (1) & get

$$\dot{\beta} = 4\pi\frac{5\cos\theta - 1}{26 - 10\cos\theta} = 2\pi\frac{5\cos\theta - 1}{13 - 5\cos\theta}$$



Calculate & plot $\dot{\beta}$ vs θ:



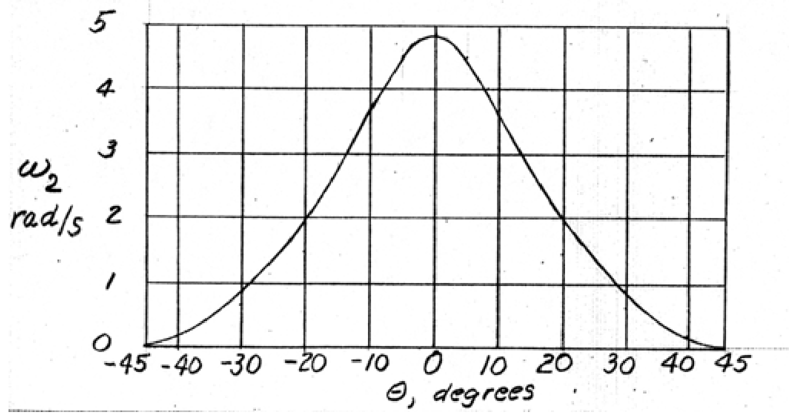
*5/207 From Prob. 5/56 we have

$$\omega_2 = \dot{\theta}\,\frac{\cos(\theta+\beta)}{\cos(\theta+\beta) - \sqrt{2}\cos\beta} \quad \text{where } \beta = \angle O_1O_2P$$

Also $\tan\beta = \dfrac{\sin\theta}{\sqrt{2} - \cos\theta}$

For $\dot{\theta} = -2$ rad/s, $\omega_2 = 2\,\dfrac{\cos(\theta+\beta)}{\sqrt{2}\cos\beta - \cos(\theta+\beta)}$

Set up program to compute β & ω_2 & plot results.



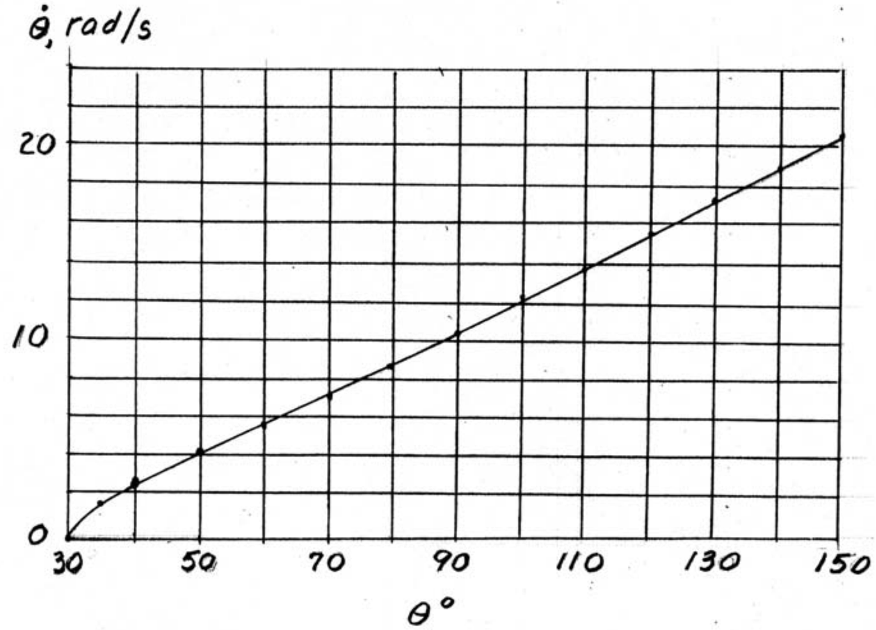
*5/208 $\ddot{\theta} = 100(1-\cos\theta)$ rad/s^2

$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta$ so $\displaystyle\int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = 100\int_{\pi/6}^{\theta}(1-\cos\theta)\,d\theta$

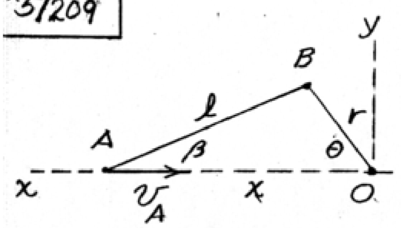
$\dot{\theta}^2 = 200\,(\theta - \sin\theta)\Big|_{\pi/6}^{\theta} = 200\,(\theta - \sin\theta - 0.0236)$

$\dot{\theta} = \dfrac{d\theta}{dt} = 10\sqrt{2}\,\sqrt{\theta - \sin\theta - 0.0236}$ rad/s

$\displaystyle\int_0^t dt = \int_{\pi/2}^{5\pi/6} \frac{d\theta}{10\sqrt{2}\sqrt{\theta - \sin\theta - 0.0236}}$ Numerical integration gives $t = 0.0701$ s



*5/209



$x = \ell\cos\beta + r\cos\theta$

$r\sin\theta = \ell\sin\beta$

$v_A = -\dot{x} = \ell\dot{\beta}\sin\beta + r\dot{\theta}\sin\theta$

$r\dot{\theta}\cos\theta = \ell\dot{\beta}\cos\beta$

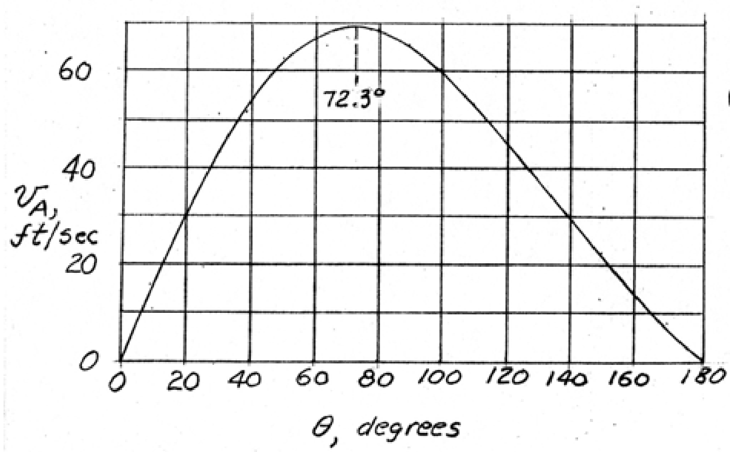
$$\dot{\beta} = \frac{r}{\ell}\frac{\dot{\theta}\cos\theta}{\sqrt{1-\sin^2\beta}} = \frac{\omega\cos\theta}{\sqrt{(\ell/r)^2-\sin^2\theta}}$$

$$v_A = \ell\left[\frac{\omega\cos\theta}{\sqrt{(\ell/r)^2-\sin^2\theta}}\right]\frac{r}{\ell}\sin\theta + r\omega\sin\theta = r\omega\sin\theta\left(1+\frac{\cos\theta}{\sqrt{(\ell/r)^2-\sin^2\theta}}\right)$$

From Sample Problem 5/15 substitute

$\ell = 14/12$ ft, $r = 5/12$ ft, $\omega = 1500(2\pi)/60 = 157.1$ rad/sec & get

$$v_A = 65.45\sin\theta\left(1+\frac{\cos\theta}{\sqrt{7.84-\sin^2\theta}}\right),$$ Set up computer program & solve for $0<\theta<180°$



$(v_A)_{max}$ = 69.6 ft/sec at $\theta = 72.3°$

By symmetry $(v_A)_\theta = -(v_A)_{-\theta}$

***5/210** From the results of Prob. 5/209, we may write

$$a_A = \dot{v}_A = \frac{d}{dt}\left\{ r\omega \sin\theta + r\omega \frac{\sin\theta\cos\theta}{\sqrt{(l/r)^2 - \sin^2\theta}} \right\}$$

$$= r\omega\left\{ \dot{\theta}\cos\theta + \frac{\sqrt{(l/r)^2-\sin^2\theta}\,(\dot{\theta}\cos 2\theta) - \frac{1}{2}\sin 2\theta \frac{(-\frac{1}{2}\sin 2\theta)\dot{\theta}}{\sqrt{(l/r)^2 - \sin^2\theta}}}{(l/r)^2 - \sin^2\theta} \right\}$$

which reduces to

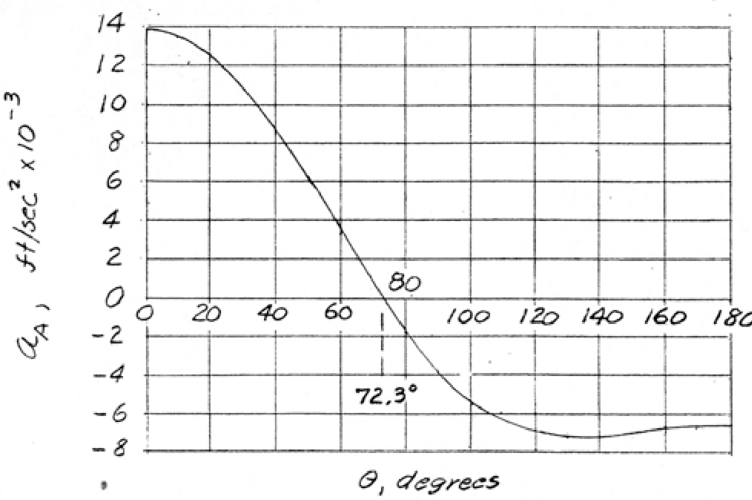
$$a_A = r\omega^2\left[\cos\theta + \frac{r}{l}\,\frac{1 - 2\sin^2\theta + \frac{r^2}{l^2}\sin^4\theta}{(1 - \frac{r^2}{l^2}\sin^2\theta)^{3/2}}\right]$$

From Sample Problem 5/15 substitute

$l = 14/12$ ft, $r = 5/12$ ft, $\omega = 1500(2\pi)/60 = 157.1$ rad/sec & get

$$a_A = 1.028(10^4)\left[\cos\theta + 0.357\,\frac{1 - 2\sin^2\theta + 0.1276\sin^4\theta}{(1 - 0.1276\sin^2\theta)^{3/2}}\right]$$

Set up computer program & solve for $0 < \theta < 180°$



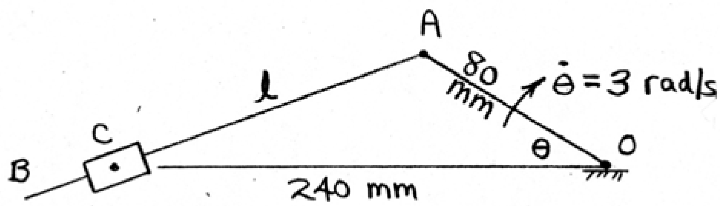
$(a_A)_{max} = 13.95(10^3)$ ft/sec² at $\theta = 0$

$a_A = -6.61(10^3)$ ft/sec² at $\theta = 180°$

$a_A = 0$ at $\theta = 72.3°$

$(a_A)_{+\theta} = +(a_A)_{-\theta}$

*5/211

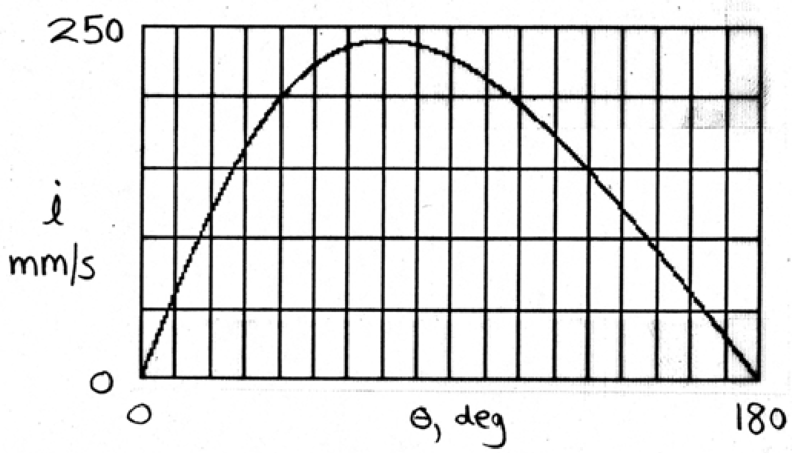


Velocity of AB through collar C is $\dot{l}$.

$l^2 = 240^2 + 80^2 - 2(240)(80)\cos\theta$, $l = 80\sqrt{2}\sqrt{5 - 3\cos\theta}$

$2l\dot{l} = 2(240)(80)\dot{\theta}\sin\theta$

$\dot{l} = \dfrac{720 \sin\theta}{\sqrt{2}\sqrt{5-3\cos\theta}}$ mm/s, $\dot{l}_{max} = 240 \frac{mm}{s}$ @ $\theta = 70.5°$



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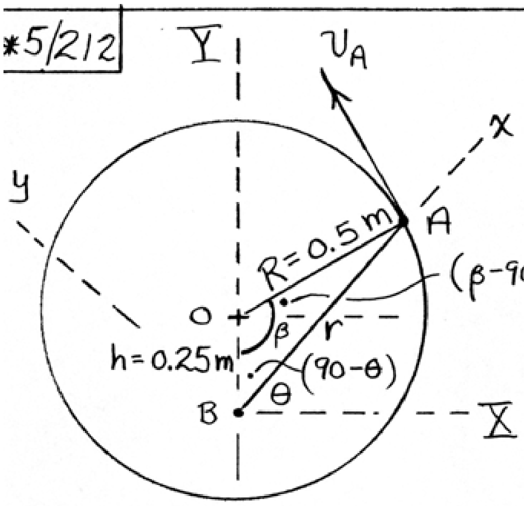

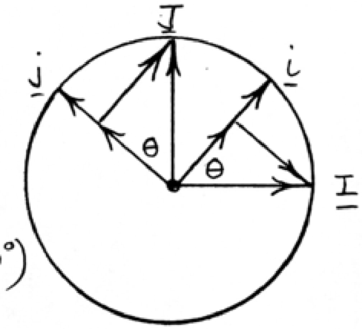

$$\begin{cases} \underline{I} = \underline{i}\cos\theta - \underline{j}\sin\theta \\ \underline{J} = \underline{i}\sin\theta + \underline{j}\cos\theta \end{cases}$$

$$r^2 = h^2 + R^2 - 2hR\cos\beta$$

$$\frac{\sin(90-\theta)}{R} = \frac{\cos\theta}{R} = \frac{\sin\beta}{r}\ , \quad \theta = \cos^{-1}\left[\frac{R}{r}\sin\beta\right]$$

$$\underline{v}_A = R\dot{\beta}\left[-\sin(\beta - 90°)\underline{I} + \cos(\beta - 90°)\underline{J}\right]$$

$$\underline{a}_A = R\dot{\beta}^2\left[-\cos(\beta - 90°)\underline{I} - \sin(\beta - 90°)\underline{J}\right]$$

Substitute the above transformation equations into the expressions for $\underline{v}_A$ & $\underline{a}_A$ and simplify to obtain (with c = cos, s = sin)

$$\underline{v}_A = R\dot{\beta}\left\{\left[-c\theta\, s(\beta-90^\circ)+s\theta\, c(\beta-90^\circ)\right]\underline{i} + \left[s\theta\, s(\beta-90^\circ)+c\theta\, c(\beta-90^\circ)\right]\underline{j}\right\}$$

$$\underline{a}_A = R\dot{\beta}^2\left\{\left[-c\theta\, c(\beta-90^\circ)-s\theta\, s(\beta-90^\circ)\right]\underline{i} + \left[s\theta\, c(\beta-90^\circ)-c\theta\, s(\beta-90^\circ)\right]\underline{j}\right\}$$

Eqs. 5/12 & 5/14, Bxy attached to BD:

$$\begin{cases} \underline{v}_A = \cancel{\underline{v}_B}^{\,0} + \underline{\omega}\times\underline{r} + \underline{v}_{rel} \\ \underline{a}_A = \cancel{\underline{a}_B}^{\,0} + \underline{\alpha}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel} \end{cases}$$

$\left\{\begin{array}{l}\text{Note: } \underline{r} = r\underline{i},\ \underline{v}_{rel} = v_{rel}\underline{i} \\ \underline{a}_{rel} = a_{rel}\underline{i},\ \underline{\omega} = \omega\underline{k},\ \underline{\alpha} = \alpha\underline{k}\end{array}\right.$

$\Rightarrow v_{rel} = v_{A_x}$, $\omega = \dfrac{v_{A_y}}{r}$, $a_{rel} = a_{A_x} + r\omega^2$,

and $\alpha = \dfrac{1}{r}\left(a_{A_y} - 2\omega v_{rel}\right)$

Program above equations & plot: