4/1

$$\bar{\underline{r}} = \frac{\Sigma m_i \underline{r}_i}{\Sigma m_i} = \frac{m(d\underline{i}) + 2m(2d\underline{j}) + 4m(1.5d\underline{k})}{m + 2m + 4m}$$

$$= \frac{d}{7}(\underline{i} + 4\underline{j} + 6\underline{k})$$

$$\dot{\bar{\underline{r}}} = \frac{\Sigma m_i \dot{\underline{r}}_i}{\Sigma m_i} = \frac{m(2v\underline{j}) + 2m(3v\underline{k}) + 4m(v\underline{i})}{7m}$$

$$= \frac{v}{7}(4\underline{i} + 2\underline{j} + 6\underline{k})$$

$$\ddot{\bar{\underline{r}}} = \frac{\Sigma \underline{F}}{\Sigma m_i} = \frac{F\underline{k}}{7m}$$

$$T = \Sigma \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2}\left[m(2v)^2 + 2m(3v)^2 + 4m(v)^2\right]$$

$$= 13mv^2$$

$$\underline{H}_O = \Sigma \underline{r}_i \times m_i \underline{v}_i = 2mvd\underline{k} + 12mvd\underline{i} + 6mvd\underline{j}$$

$$= mvd(12\underline{i} + 6\underline{j} + 2\underline{k})$$

$$\dot{\underline{H}}_O = \Sigma \underline{M}_O = -Fd\underline{j}$$

4/2 From Eq. 4/10 with P replaced by O:

$$\underline{H}_O = \underline{H}_G + \underline{\bar{r}} \times \Sigma m_i \underline{\bar{v}}$$

or

$$\underline{H}_G = \underline{H}_O - \underline{\bar{r}} \times \Sigma m_i \underline{\bar{v}}$$

$$\underline{H}_G = mvd(12\underline{i} + 6\underline{j} + 2\underline{k}) - \frac{d}{7}(\underline{i} + 4\underline{j} + 6\underline{k}) \times 7m \cdot \frac{v}{7}(4\underline{i} + 2\underline{j} + 6\underline{k})$$

$$= \underline{\frac{mvd}{7}(72\underline{i} + 24\underline{j} + 28\underline{k})}$$

($\underline{H}_O$, $\underline{\bar{r}}$, and $\underline{\bar{v}} = \dot{\underline{\bar{r}}}$ from Prob. 4/1)

From Eq. 4/11:

$$\Sigma \underline{M}_O = \dot{\underline{H}}_G + \underline{\bar{r}} \times \Sigma m_i \underline{\bar{a}}$$

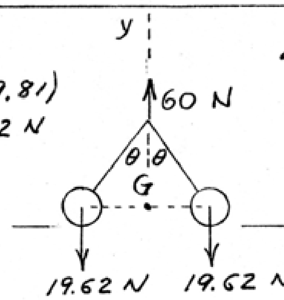
$$\dot{\underline{H}}_G = \Sigma \underline{M}_O - \underline{\bar{r}} \times \Sigma m_i \underline{\bar{a}}$$

$$= -Fd\underline{j} - \frac{d}{7}(\underline{i} + 4\underline{j} + 6\underline{k}) \times 7m\left(\frac{F\underline{k}}{7m}\right)$$

$$= \underline{-\frac{2Fd}{7}(2\underline{i} + 3\underline{j})}$$

**4/3**

$mg = 2(9.81)$
$= 19.62\text{ N}$

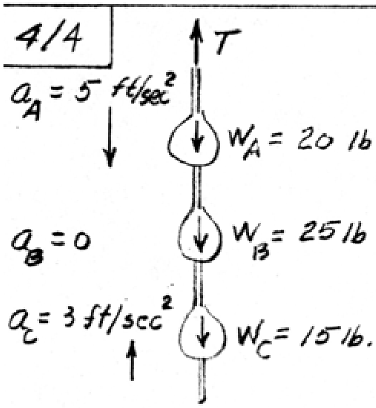



$\Sigma F_y = m\bar{a}_y$; $60 - 2(19.62) = 4\bar{a}_y$

$a_y = \bar{a}_y = \underline{5.19\text{ m/s}^2}$

(independent of $\theta$)

**4/4**




$$\Sigma F = \Sigma m_i a_i$$

$$T - (20 + 25 + 15) = \frac{1}{32.2}\left(20[-5] + 25[0] + 15[3]\right)$$

$$T - 60 = \frac{1}{32.2}(-55)$$

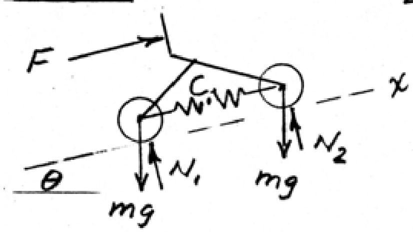
$$T = 60 - 1.708 = \underline{58.3 \text{ lb}}$$

4/5

$\Sigma F = m\bar{a}: \quad 6.4 = (0.8 + 0.5 + 0.3)\bar{a}$

$\underline{\bar{a} = 4 \text{ m/s}^2}$

4/6

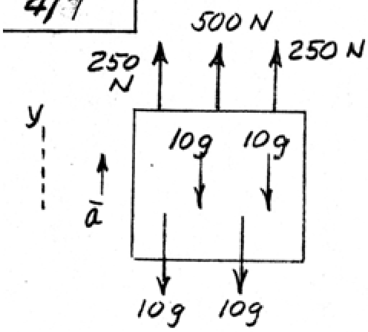



$\Sigma F_x = m\bar{a}_x$ where $a_c = \bar{a}_x$

$F - 2mg\sin\theta = 2ma_c$

$$a_c = \frac{F}{2m} - g\sin\theta$$

**4/7**




$\Sigma F_y = m\bar{a}_y$

$500 + 250 + 250 - 40(9.81) = 40\bar{a}$

$40\bar{a} = 1000 - 392$

$\underline{\bar{a} = 15.19 \text{ m/s}^2}$

4/8 The principle of the motion of the mass center gives $F = m\bar{a}$ for each case, so the mass-center accelerations are identical. In the two cases of hinged members, however, the mass center is not a point attached to a member, and for these two cases the accelerations of the members will differ.

4/9

$$\underline{F}_{av} = \frac{\Delta \underline{G}}{\Delta t} = \left[(3.7-3.4)\underline{i} + (-2.2+2.6)\underline{j} + (4.9-4.6)\underline{k}\right]/0.2$$

$$= 1.5\underline{i} + 2.0\underline{j} + 1.5\underline{k} \text{ N}$$

$$F = |\underline{F}_{av}| = \sqrt{1.5^2 + 2.0^2 + 1.5^2} = \underline{2.92 \text{ N}}$$

4/10 For system, $\Delta T + \Delta V_g = 0$

$\Delta T = 3(\frac{1}{2} m v^2) - 0 = \frac{3}{2} m v^2$

$\Delta V_g = 0 - mg\frac{b}{\sqrt{2}} - mg\frac{2b}{\sqrt{2}} = -\frac{3b}{\sqrt{2}} mg$

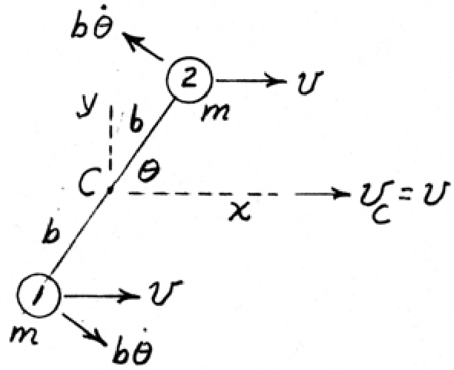
Thus $\frac{3}{2} m v^2 - \frac{3b}{\sqrt{2}} mg = 0$, $v^2 = bg\sqrt{2}$

$\underline{v = \sqrt{bg\sqrt{2}}}$

**4/11**

For sphere 1,

$$\underline{G}_1 = m\left[(v + b\dot{\theta}\sin\theta)\underline{i} - (b\dot{\theta}\cos\theta)\underline{j}\right]$$




For sphere 2

$$\underline{G}_2 = m\left[(v - b\dot{\theta}\sin\theta)\underline{i} + (b\dot{\theta}\cos\theta)\underline{j}\right]$$

$$\underline{G} = \underline{G}_1 + \underline{G}_2 = m\left[v + v\right]\underline{i} = 2mv\,\underline{i}$$

**4/12**

$\underline{H}_O = \underline{H}_G + \underline{\bar{r}} \times \underline{G}$ , $\underline{G} = 3(3\underline{i} + 4\underline{j})$ kg·m/s

$= 1.20\underline{k} + (0.4\underline{i} + 0.3\underline{j}) \times 3(3\underline{i} + 4\underline{j})$

$= 1.20\underline{k} + 3(1.6\underline{k} - 0.9\underline{k})$

$= 1.20\underline{k} + 3(0.7\underline{k}) = \underline{3.3\underline{k}\ \text{kg}\cdot\text{m}^2/\text{s}}$

4/13 Mass center is center of middle bar, so Eq. 4/1 for the entire system gives

$\Sigma F = m\bar{a}$: $\quad 10 = 3\frac{8}{32.2}a, \quad \underline{a = 13.42 \text{ ft/sec}^2}$

4/14

$\Sigma M_0 = \dot{H}_0$ where O-O is the axis of rotation

$$M = \frac{dH_0}{dt}, \quad \int_0^t M\,dt = \int_0^{H_0} dH_0 = H_0$$

$$Mt = 4m(r\omega)r, \quad t = \frac{4mr^2\omega}{M}$$

5/15 $\Sigma M_O = \dot{H}_O = \frac{dH_O}{dt}$ , $\int \Sigma M_O \, dt = \Delta H_O$

$M_O t = \Delta \left| \Sigma m_i r_i (r_i \dot{\theta}) \right| = \Sigma m_i r^2 \Delta \dot{\theta}$

$30 \times 5 = \left[ 3(0.5)^2 + 4(0.4)^2 + 3(0.6)^2 \right] (\dot{\theta}' - 20)$

$150 = 2.47(\dot{\theta}' - 20)$ , $\dot{\theta}' = 60.7 + 20 = \underline{80.7 \ \frac{rad}{s}}$

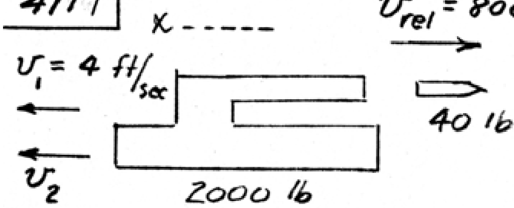
4/16 $\int_0^t M_z\,dt = H_{z_2} - H_{z_1}$ , $H_z = \sum m_i r_i (r_i \dot{\theta})$

$H_z = 2(3)(0.3)^2\dot{\theta} + 2(3)(0.5)^2\dot{\theta} = 2.04\dot{\theta}$

so $30t = 2.04(20-[-20]) = 81.6$

$\underline{t = 2.72 \text{ s}}$

**4/17**




$\Delta G_x = 0, \quad G_1 = G_2$

$$\frac{1}{g}(2000 + 40)4 = \frac{1}{g}\left(2000\, v_2 - 40\left[800 - v_2\right]\right)$$

$$8160 = 2040\, v_2 - 32000$$

$$\underline{v_2 = 19.69 \text{ ft/sec}}$$

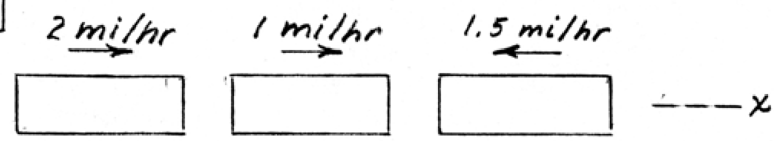
4/18 For entire system $\Delta G_x = 0$, $x$ horiz.

$$(300 + 400 + 100) v - (300 \times 0.6 - 400 \times 0.3 + 100 \times 1.2 \cos 30°) = 0$$

$800 v = 163.9$, $\underline{v = 0.205 \text{ m/s}}$

Momentum is conserved regardless of sequence of events, so final velocity would be the same.

4/19




$W_A =$ 130,000 lb $\quad W_B =$ 100,000 lb $\quad W_C =$ 150,000 lb

$\Sigma F_x = 0$ for system so $\Delta G_x = 0$

$$(130\times 2 + 100\times 1 - 150\times 1.5)\frac{44}{30}\frac{10^3}{32.2}$$

$$-(130+100+150)v\frac{44}{30}\frac{10^3}{32.2} = 0$$

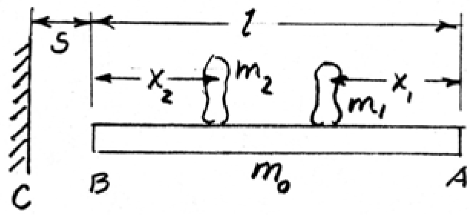
$$v = \frac{260+100-225}{130+100+150} = \underline{0.355 \text{ mi/hr}}$$

$$\%\text{ loss of energy} = \frac{T_i - T_f}{T_i}100 = 100\left(1-\frac{T_f}{T_i}\right) = n$$

$$n = 100\left\{1 - \frac{\frac{1}{2g}(130+100+150)(0.355)^2}{\frac{1}{2g}(130\times 2^2 + 100\times 1^2 + 150\times 1.5^2)}\right\} = 100\left(1-\frac{47.96}{957.5}\right)$$

$$\underline{n = 95.0\%}$$

4/20 With respect to C, $\Sigma m_i x_i$ = constant



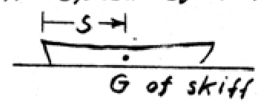

$$m_1 l + m_2(0) + m_0 \frac{l}{2} =$$
$$m_1(s + l - x_1) + m_2(s + x_2)$$
$$+ m_0\left(s + \frac{l}{2}\right)$$

Simplify & get $s = \dfrac{m_1 x_1 - m_2 x_2}{m_0 + m_1 + m_2}$

But they meet when $x_2 + x_1 = l$ so

$$s = \frac{(m_1 + m_2)x_1 - m_2 l}{m_0 + m_1 + m_2}$$

4/21 With neglect of hydraulic forces linear momentum is conserved & velocity $v_2 = v_1 = 1$ knot. Center of mass does not change position with respect to reference axes moving with constant speed of 1 knot.




Thus $(\Sigma m_i x_i)_1 = (\Sigma m_i x_i)_2$

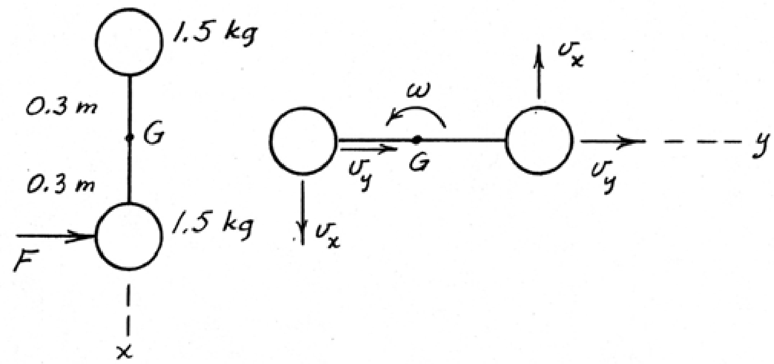
$$\frac{1}{32.2}\left[120(2) + 180(8) + 160(16) + 300(s)\right]$$

$$= \frac{1}{32.2}\left[120(14+x) + 180(4+x) + 160(10+x) + 300(s+x)\right]$$

$$4240 = 4000 + 760x, \quad x = \frac{240}{760} = \underline{0.316 \text{ ft}}$$

Timing & sequence of changed positions does not affect final result because all forces are internal.

4/22



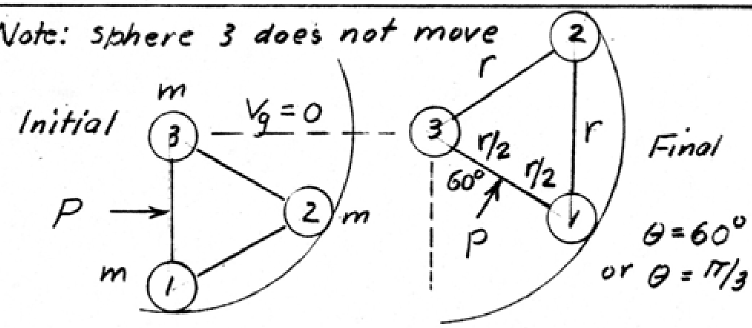

$\int \Sigma F_x \, dt = 0$ so $\Delta G_x = 0$

$\int \Sigma F_y \, dt = \Delta G_y$: $10 = 2(1.5) v_y$, $v_y = 3.33$ m/s

$\int \Sigma M_G \, dt = \Delta H_G$: $10(0.3) = 2(1.5) v_x (0.3)$, $v_x = 3.33$ m/s

$v = 3.33\sqrt{2} = \underline{4.71 \text{ m/s both spheres}}$

**4/23** Note: sphere 3 does not move




(a) $U' = \Delta V_g$ : $U' = P_{min} \frac{r}{2} \frac{\pi}{3}$

$$\Delta V_g = mg\left(r + \frac{r}{2}\right) = \frac{3}{2} mgr$$

Thus $\frac{\pi r}{6} P_{min} = \frac{3}{2} mgr$, $\underline{P_{min} = \frac{9}{\pi} mg}$

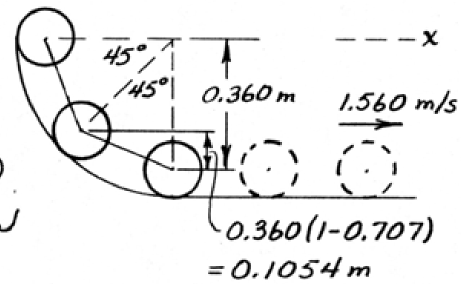
(b) $U' = \Delta T + \Delta V_g$ with $P = 2P_{min} = \frac{18}{\pi} mg$

$$U' = \frac{18}{\pi} mg \frac{r}{2} \frac{\pi}{3} = 3mgr, \quad \Delta V_g = \frac{3}{2} mgr$$

$$\Delta T = 2\left(\frac{1}{2} mv^2\right) = mv^2$$

Thus $3mgr = mv^2 + \frac{3}{2} mgr$, $\underline{v = \sqrt{3gr/2}}$

4/24




$$U'_{1-2} = \Delta T + \Delta V_g$$
$$= 3\left(\tfrac{1}{2} \times 2.75 \times 1.560^2\right) - 0$$
$$- 2.75 \times 9.81(0.360 + 0.1054)$$
$$= 10.04 - 12.56 = -2.52 \text{ J}$$

so loss is $\underline{\Delta Q = 2.52 \text{ J}}$

$$I_x = \int \Sigma F_x \, dt = \Delta G_x = G_2 - G_1, \quad G_2 = 3mv = 3(2.75)(1.560)$$
$$= 12.87 \text{ N·s}, \quad G_1 = 0$$

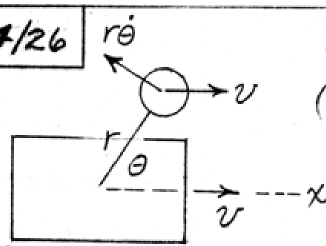
$$\underline{I_x = 12.87 \text{ N·s}}$$

**4/25** (a) $\Sigma F_x = m\bar{a}_x$; $F = 2m\bar{a}$, $\underline{\bar{a} = F/2m}$

(b) $H_G = 2m\left(\frac{L}{2}\right)^2\dot{\theta}$, $\dot{H}_G = mL^2\ddot{\theta}/2$

$\Sigma M_G = \dot{H}_G$; $Fb = mL^2\ddot{\theta}/2$, $\underline{\ddot{\theta} = \dfrac{2Fb}{mL^2}}$

**4/26**




$\Sigma F_x = 0$ for system so $\Delta G_x = 0$

$(G_x)_{\theta=0} = (20+5)(0.6) = 15.0$ N·s

$(G_x)_{\theta=60°} = (20+5)v - 5(1.6)\sin 60°$

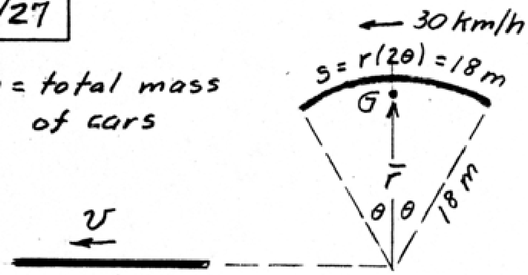
$= 25v - 6.93$ N·s

$r\dot{\theta} = 0.4(4) = 1.6$ m/s

Thus $15.0 = 25v - 6.93$, $v = 21.9/25 = \underline{0.877 \text{ m/s}}$

**4/27**

$m$ = total mass of cars




$\theta = \dfrac{s}{2r} = \dfrac{18}{2(18)} = \dfrac{1}{2}$ rad

$= \dfrac{1}{2}\,\dfrac{180}{\pi} = 28.65°$

$\bar{r} = \dfrac{r\sin\theta}{\theta} = \dfrac{18\sin 28.65°}{1/2}$

$= 17.26$ m

For system $\Delta T + \Delta V_g = 0$

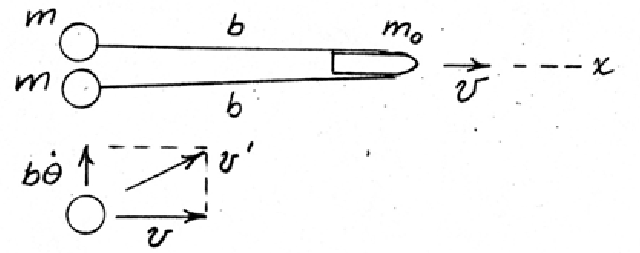
$\frac{1}{2}m\left(v^2 - \left[\frac{30}{3.6}\right]^2\right) - mg(17.26) = 0$

$v^2 = (30/3.6)^2 + 2(9.81)(17.26) = 69.4 + 338.6$

$= 408\ (m/s)^2$

$v = 20.2$ m/s or $v = 20.2(3.6) = \underline{72.7\text{ km/h}}$

4/28




For system $\Delta G_x = 0$: $(m_0 v + 2m v) - m_0 v_0 = 0$

$$\underline{v = \frac{m_0}{m_0 + 2m} v_0}$$

$U = \Delta T$: $0 = \frac{1}{2} m_0 v^2 + 2\left[\frac{1}{2} m \,(v^2 + b^2\dot{\theta}^2)\right] - \frac{1}{2} m_0 v_0^2$

$$(m_0 + 2m) v^2 + 2m b^2 \dot{\theta}^2 = m_0 v_0^2$$

Substitute $v$ & get $\dfrac{m_0^2 v_0^2}{m_0 + 2m} + 2m b^2 \dot{\theta}^2 = m_0 v_0^2$

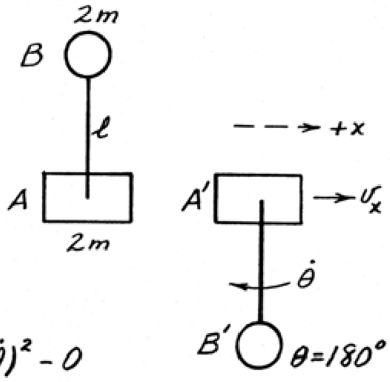
Solve for $\dot{\theta}$ & get $\quad \underline{\dot{\theta} = \dfrac{v_0}{b}\sqrt{\dfrac{m_0}{m_0 + 2m}}}$

**4/29** For entire system,

$\int \Sigma F_x \, dt = \Delta G_x = G_{A'} + G_{B'} - 0$

$0 = 2m v_x + 2m(v_x - l\dot{\theta})$

$2v_x = l\dot{\theta}$ ---- (1)




For entire system,

$U'_{1-2} = \Delta T + \Delta V_g$

$\Delta T = \frac{1}{2}(2m)v_x^2 + \frac{1}{2}(2m)(v_x - l\dot{\theta})^2 - 0$

$= m(2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2)$

$\Delta V_g = -2mg(2l) = -4mgl$

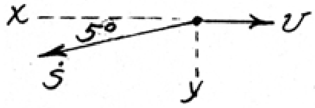
$U'_{1-2} = 0$ so $0 = m(2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2) - 4mgl$

or $2v_x^2 - 2l\dot{\theta}v_x + l^2\dot{\theta}^2 = 4gl$ --- (2)

Combine (1) & (2): $2v_x^2 - 4v_x^2 + 4v_x^2 = 4gl$, $v_x^2 = 2gl$, $\underline{v_x = \sqrt{2gl}}$

$\dot{\theta} = 2v_x/l = 2\sqrt{2gl}/l = \underline{2\sqrt{\frac{2g}{l}}}$

**4/30** System is conservative so $\Delta T + \Delta V_g = 0$




Flatcar; $\Delta T = \frac{1}{2} m v^2 - 0$

$$= \frac{1}{2} \frac{50{,}000}{32.2} v^2$$

$$\Delta V_g = 0$$

Vehicle; $\Delta T = \frac{1}{2} m v^2 - 0 = \frac{1}{2} \frac{15000}{32.2} \left[ (\dot{s}\cos 5° - v)^2 + (\dot{s}\sin 5°)^2 \right] - 0$

$$\Delta V_g = -W\Delta h = -15{,}000\,(40 \sin 5°)$$

Thus $776.4 v^2 + 232.9\left[ (\dot{s}\cos 5° - v)^2 + (\dot{s}\sin 5°)^2 \right] - 52290 = 0$ ----- (1)

Also for system, $\Sigma F_x = 0$ so $\Delta G_x = 0$

$$\frac{15{,}000}{32.2}(\dot{s}\cos 5° - v) - \frac{50{,}000}{32.2} v = 0$$

$\dot{s}\cos 5° - v = 3.33 v$ & $\dot{s}\sin 5° = 4.33 v \tan 5° = 0.379 v$

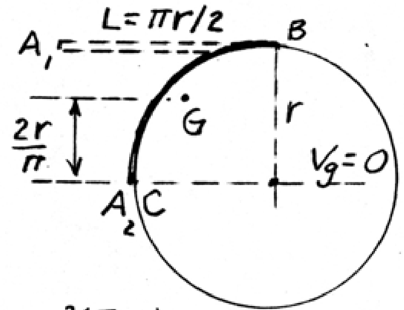
Substitute into (1) & get

$$776.4 v^2 + 232.9\left[ (3.33 v)^2 + (0.379 v)^2 \right] - 52290 = 0$$

$$v^2 (776.4 + 2588 + 33.5) = 52290$$

$v^2 = 15.39\ (\text{ft/sec})^2$, $\underline{v = 3.92\ \text{ft/sec}}$

▶4/31




*Initial energy*

$$V_{g_1} = \rho g \frac{\pi r}{2} r = \rho g \frac{\pi r^2}{2}$$

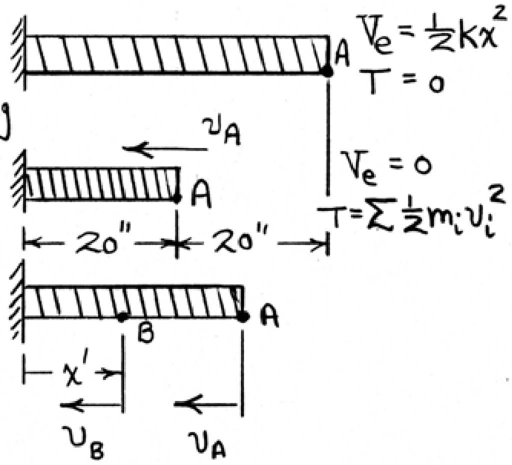
*Final energy*

$$V_{g_2} = \rho g \frac{\pi r}{2} \frac{2r}{\pi} = \rho g r^2$$

Energy loss $\Delta Q = V_{g_1} - V_{g_2} = \rho g r^2\left(\frac{\pi}{2} - 1\right) = \underline{0.571 \rho g r^2}$

With $\Delta T = 0$ $(T_2 = T_1 = 0)$ the loss of potential is dissipated into heat energy due to impact of rope against the drum.

▶ **4/32**




Potential energy of spring

$$V_e = \frac{1}{2}kx^2$$

$$= \frac{1}{2}\frac{200}{20/12}\left(\frac{20}{12}\right)^2$$

$$= 166.7 \text{ ft-lb}$$

During collapse of spring as $x \to 0$:

$$v_B = \frac{x'}{20/12} v_A \quad , \; x' \text{ in feet.}$$

Let $\rho$ = mass per unit length ; $dm = \rho\, dx'$

$$T = \int \frac{1}{2} v_B^2\, dm = \frac{1}{2}\int_0^{20/12} \frac{x'^2}{(20/12)^2} v_A^2 \rho\, dx'$$

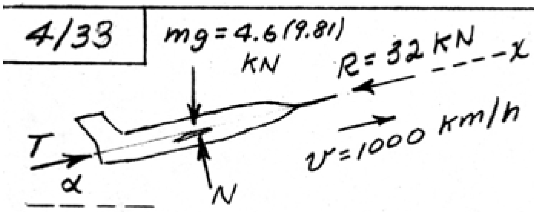
$$= \frac{1}{6}\rho \frac{20}{12} v_A^2, \text{ where } \rho\frac{20}{12} = m$$

$$\text{So } T = \frac{1}{6} m v_A^2 = \frac{1}{6}\frac{3}{32.2} v_A^2 = \frac{v_A^2}{64.4}$$

From $V_e = T$: $166.7 = \frac{v_A^2}{64.4}$ , $\underline{v_A = 103.6 \text{ ft/sec}}$

(Kinetic energy goes into heat, sound, & deformation.)

4/33




$T = m_a'(u - v) + m_f' u$

$= 106(680 - 1000/3.6) + 4(680)$

$= 45400$ N

$\Sigma F_x = ma_x = 0; \quad 45.4 - 32 - 4.6(9.81)\sin\alpha = 0$

$\sin\alpha = 0.2960, \quad \underline{\alpha = 17.22°}$

4/34

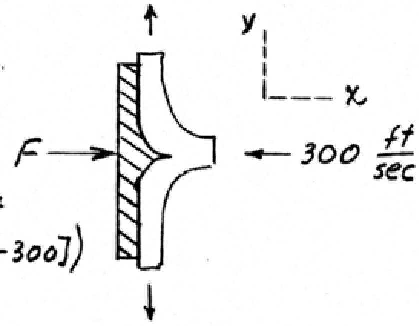
$$m' = \frac{\mu}{g} Q$$

$$= \frac{0.0753}{32.2}(6.50)$$

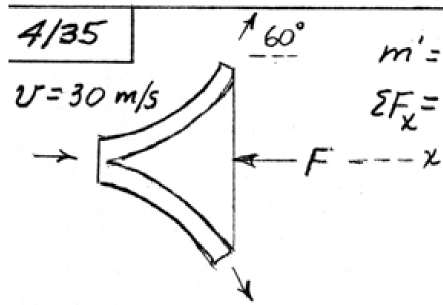
$$= 0.0152 \ \text{lb} \times \text{sec}/\text{ft}$$

$$\Sigma F_x = m' \Delta v_x : \ F = 0.0152(0 - [-300])$$

$$= \underline{4.56 \ \text{lb}}$$



**4/35**




$m' = \rho Q = 1000(0.05) = 50 \text{ kg/s}$

$\Sigma F_x = m' \Delta v_x$; $-F = 50(30 \cos 60° - 30)$

$\underline{F = 750 \text{ N}}$

4/36 | Resistance $R$ equals net thrust $T$

where $T = m'(u - v)$

Nozzle velocity $u = Q/A = \dfrac{0.082}{\frac{\pi(0.050)^2}{4}} = 41.8$ m/s

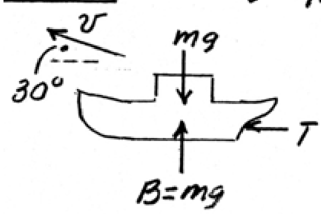
Density of salt water, Table D-1, $\rho = 1030$ kg/m$^3$

$m' = \rho Q = 1030(0.082) = 84.5$ kg/s

$v = 70 \dfrac{1000}{3600} = 19.44$ m/s

$R = T = 84.5(41.8 - 19.44) = \underline{1885\ N}$

4/37




$v = 40$ m/s , $m' = \rho Q = 1030\,(0.080)$

$= 82.4$ kg/s

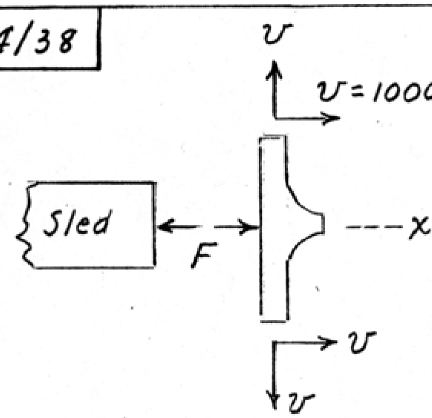
$\Sigma F = m'\Delta v$:

$T = 82.4\,(40 \cos 30° - 0) = 2850$ N

or $\underline{T = 2.85 \text{ kN}}$

**4/38**

$v = 1000$ ft/sec




$$m' = \rho A v = \frac{62.4}{32.2}\,\frac{15}{144}\,1000$$

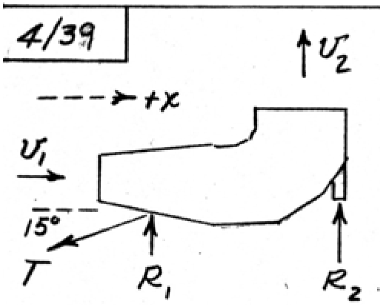
$$= 202 \text{ lb}\times\text{sec/ft}$$

$$\Sigma F_x = m'\Delta v_x$$

$$F = 202(1000 - 0)$$

$$\underline{F = 202{,}000 \text{ lb}}$$

4/39



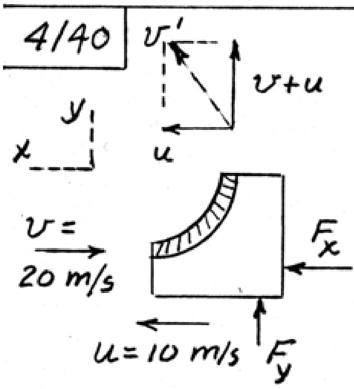

$$\Sigma F_x = m' \Delta v_x$$

$$-T\cos 15° = (43 + 0.8)(0 - 720)$$

$$T = 32\,600 \text{ N}$$

or $\underline{T = 32.6 \text{ kN}}$

4/40




Final velocity of diverted water is $\underline{v}' = \underline{u} + \underline{v}_{rel}$

where $v_{rel} = v + u$

$m' = \rho A v_{rel}$

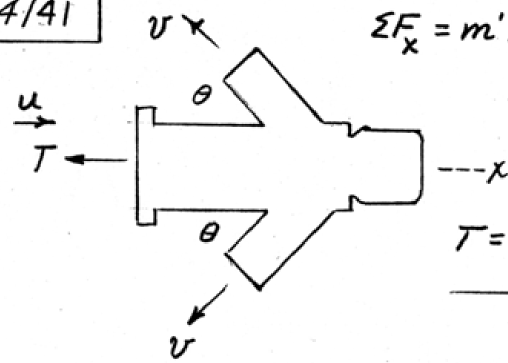
$= 1000 \frac{\pi}{4}(0.025)^2(20+10)$

$= 14.73$ kg/s

$\Sigma F_x = m' \Delta v_x$: $F_x = 14.73(10 - [-20]) = \underline{442 \text{ N}}$

$\Sigma F_y = m' \Delta v_y$: $F_y = 14.73(20 + 10 - 0) = \underline{442 \text{ N}}$

4/41



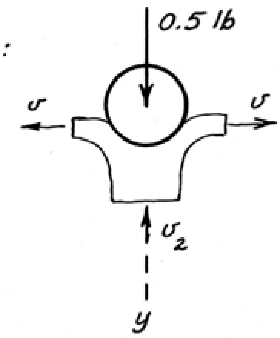

$$\Sigma F_x = m' \Delta v_x \,,$$

$$-T = m'(-v\cos\theta - u)$$

$$T = \frac{\pi d^2}{4} \rho u \left(v\cos\theta + u\right)$$

**4/42**

Ball & stream just under it:




$\Sigma F_y = m' \Delta v_y$:

m' at ball = m' at nozzle

$= \rho A v = \frac{62.4}{32.2} \frac{\pi (0.5)^2}{4} \left(\frac{1}{12}\right)^2 \cdot 35$

$= 0.925$ lb-sec/ft

so $0.5 = 0.925(0 - [-v_2])$

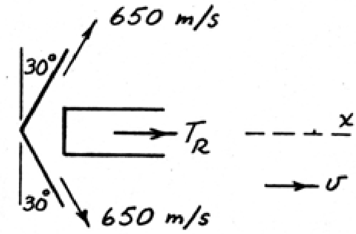
$v_2 = 5.41$ ft/sec

For water stream $\Delta V_g + \Delta T = 0$:

$mgh + \frac{1}{2} m (v_2^{\,2} - v_1^{\,2}) = 0,$

$h = \frac{1}{2 \times 32.2} (35^2 - 5.41^2) = \underline{18.57 \text{ ft}}$

**4/43** $\Sigma F = \Sigma m' u$




$v = 200/3.6 = 55.6$ m/s

With reversers in place,

$T_R = m_g' u \sin 30° + m_a' v$

$T_R = (50 + 0.65)(650)\sin 30°$

$\quad + 50(55.6 - 0)$

$\quad = 16460 + 2780$

$\quad = 19\,240$ N

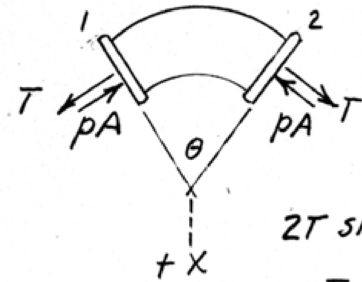
Without reversers $T = m_g' u - m_a' v$

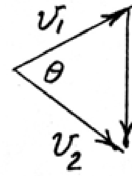
$T = (50 + 0.65)650 - 50(55.6)$

$\quad = 32\,900 - 2780 = 30\,100$ N

so $n = \dfrac{19\,240}{30\,100} = \underline{0.638}$

**4/44**




$\Delta v = 2v \sin\frac{\theta}{2}$

where $v = |\underline{v}_1| = |\underline{v}_2|$

$\Sigma F_x = m' \Delta v_x$ entire system

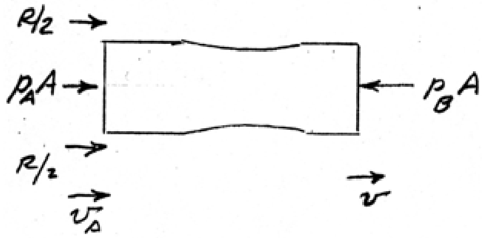
$2T \sin\frac{\theta}{2} - 2pA \sin\frac{\theta}{2} = \rho A v \left(2v \sin\frac{\theta}{2}\right)$

so $T - pA = \rho A v^2$, $\underline{T = A(p + \rho v^2)}$

independent of $\theta$

**4/45** Continuity requires $\rho_A A v_A = \rho_B A v$

so $v_A = v\,\rho_B/\rho_A$




$m' = \rho A v = \rho_B A v$ at $B$

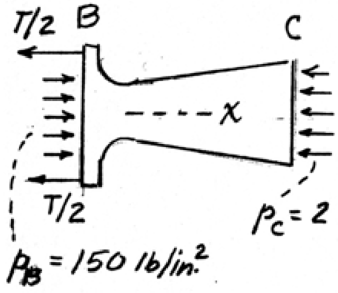
$\Sigma F = m'\Delta v$; $R + p_A A - p_B A = \rho_B A v(v - v_A)$

$= \rho_B A v^2(1 - \rho_B/\rho_A)$

Also, $A = \pi d^2/4$, so $R = \rho_B \frac{\pi d^2}{4} v^2\left(1 - \frac{\rho_B}{\rho_A}\right) + (p_B - p_A)\frac{\pi d^2}{4}$

$$\underline{R = \frac{\pi d^2}{4}\left[\rho_B\left(1 - \frac{\rho_B}{\rho_A}\right)v^2 + (p_B - p_A)\right]}$$

4/46 $A_C = \frac{\pi 4^2}{4(144)} = 0.0873 \text{ ft}^2, \quad A_B = 4A_C = 0.349 \text{ ft}^2$




$m' = \rho A v$

$m'_B = \frac{0.840}{32.2}(0.349)50 = 0.455 \ \frac{\text{lb-sec}}{\text{ft}}$

$m'_C = \frac{0.0760}{32.2}(0.0873)v_C = 2.06(10^{-4})v_C$

$m'_B = m'_C$ so $v_C = \frac{0.455}{2.06(10^{-4})}$

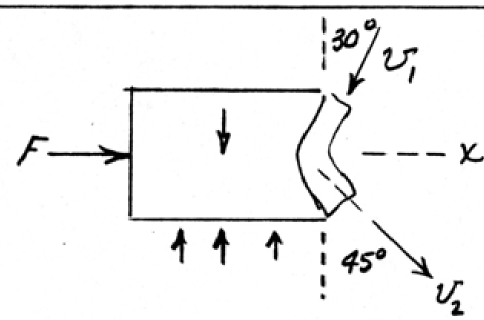
$= 2210 \text{ ft/sec}$

$\Sigma F_x = m' \Delta v_x: \ 150(0.349)144 - 2(0.0873)(144) - T$

$= 0.455(2210 - 50)$

$\underline{T = 6530 \text{ lb}}$

4/47




$Q = Av$:

$\frac{1}{2}(231) = \frac{0.01^2 \pi}{4} v_1$

$v_1 = 1.471(10^6)$ in./min

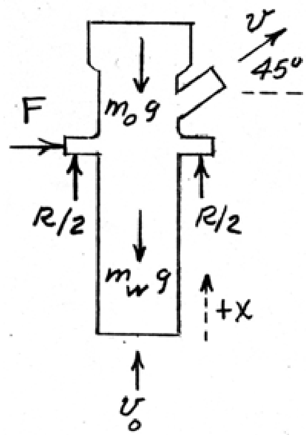
or $v_1 = 2042$ ft/sec & $v_2 = 0.60(2042) = 1225$ ft/sec

$\Sigma F_x = m' \Delta v_x$: $m' = \frac{231}{2} \frac{1}{60} \frac{1}{1728} \frac{68}{32.2} = 2.35(10^{-3})$ lb-sec/ft (slugs/sec)

$F = 2.35(10^{-3})(1225 \sin 45° - [-2042 \sin 30°])$

$F = 4.44$ lb

4/48




$\Sigma F_x = m' \Delta v_x$:

$R - m_o g - m_w g = \rho Q (v \cos 45° - v_o)$

$m_o = 310$ kg

Mass of water $m_w = \rho V$

$= 1000 \frac{\pi}{4} (0.2)^2 (6)$

$= 188.5$ kg

$Q = 0.125 \ m^3/s$

$A = \frac{\pi}{4}(0.1)^2 = 0.00785 \ m^2$

$A_o = \frac{\pi}{4}(0.25)^2 = 0.0491 \ m^2$

$v = Q/A = 0.125/0.00785 = 15.92$ m/s

$v_o = Q/A_o = 0.125/0.0491 = 2.55$ m/s

Thus $R - (310 + 188.5)\, 9.81 = 1000(0.125)(15.92 \cos 45° - 2.55)$

$= 1088$ N

$\underline{R = 5980 \text{ N}}$

**4/49** $kx = m'\Delta v$, $m' = \rho A v = 1000 \frac{\pi}{4}(0.030)^2 v$

$= 0.7069\, v$

$15000(0.150) = 0.7069\, v(v-0)$

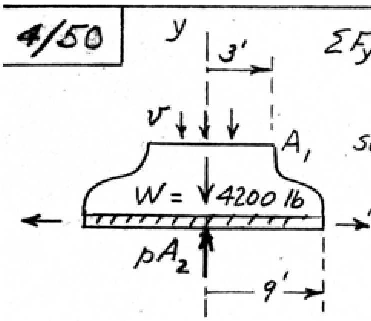
$v^2 = 3183$, $\underline{v = 56.4 \text{ m/s}}$

$\Sigma M_A = m'vd$; $M = 15(150)(15 \sin 75° - 4.8 \cos 75°)$

$= 2250\,(13.25) = 29\,800 \text{ N·m}$

or $\underline{M = 29.8 \text{ kN·m}}$

**4/50**




$\Sigma F_y = m' \Delta v_y$ ; $m' = \rho A_1 v$

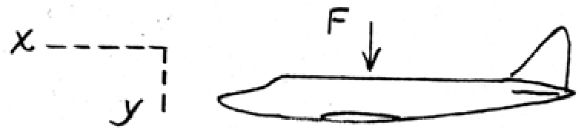
$\Delta v_y = 0 - (-v) = v$

so $pA_2 - W = \rho A_1 v^2$

$p(\pi \times 9^2)(144) - 4200 = \dfrac{0.076}{32.2} \pi \times 3^2 (150^2)$

$p = 0.1556$ lb/in.$^2$

4/51 Consider motion as measured from the inertial reference of the aircraft.




$F = m'\Delta v_y$ where $\Delta v_y = 0 - 20 = -20$ ft/sec

Volume of water striking horizontal surface per second is $Q = Av_y = (2960\ ft^2)\left(\frac{1}{12}\ \frac{ft}{hr}\right)\left(\frac{1}{3600}\ \frac{1}{sec/hr}\right)$

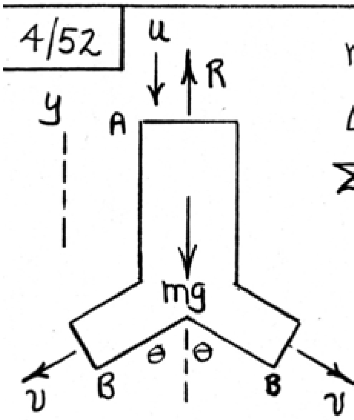
$= 0.0685\ ft^3/sec$

$m' = \rho Q = \frac{62.4}{32.2}(0.0685) = 0.1328$ lb-sec/ft (slugs/sec)

$F = m'|\Delta v_y|$ $F = 0.1328(20)$

$= 2.66$ lb

4/52




$$m' = \rho A v_A = \rho \frac{\pi d^2}{4} u$$

$$\Delta v_y = -v\cos\theta - (-u) = u - v\cos\theta$$

$\Sigma F_y = m'\Delta v_y$:

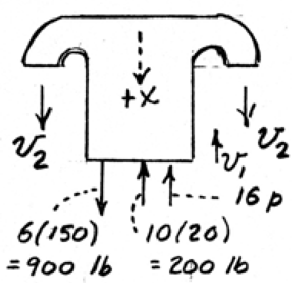
$$R - mg = \rho \frac{\pi d^2}{4} u\,(u - v\cos\theta)$$

$$R = mg + \rho \frac{\pi d^2}{4} u\,(u - v\cos\theta)$$

**4/53**

$Q = 1.6\ ft^3/sec \qquad v_1 = \frac{Q}{A_1} = \frac{1.6}{20/144} = 11.52\ ft/sec$

$v_2 = \frac{Q}{A_2} = \frac{1.6/2}{3.2/144} = 36\ ft/sec$

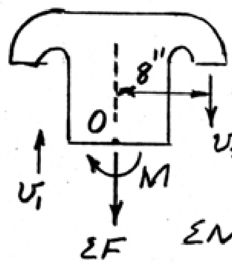



$m' = \rho Q = \frac{64.4}{32.2} 1.6 = 3.2\ lb\text{-}sec/ft$

$\Sigma F_x = m' \Delta v_x$:

$900 - 200 - 16p = 3.2(36 - [-11.52])$

$\underline{p = 34.2\ lb/in.^2}$




For left side blocked off & with

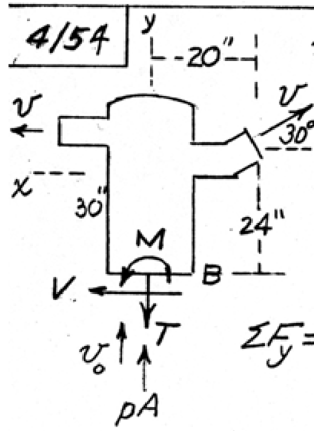
$Q = 1.6/2 = 0.8\ ft^3/sec$,

$v_2 = Q/A_2 = \frac{0.8}{3.2/144} = 36 \frac{ft}{sec}$, $m' = \frac{3.2}{2} = 1.6$ lb-sec/ft

$\Sigma M_O = m'(v_2 d_2 - v_1 d_1)$

$M = 1.6(36 \times 8 - 0) = \underline{461\ lb\text{-}in.}$

**4/54**




$v = \dfrac{\text{Volume rate}}{\text{area}} = \dfrac{10/2}{0.040} = 125 \text{ ft/sec}$

For each outlet $m' = \dfrac{5 \times 62.4}{32.2}$

$= 9.69 \text{ lb-ft}^{-1}\text{-sec}$

$v_o = \dfrac{10}{0.75} = 13.33 \text{ ft/sec}$

$pA = 120(0.75)(144)$
$= 12{,}960 \text{ lb}$

$\Sigma F_y = m' \Delta v_y;\ -T + 12{,}960 = 9.69(0 - 13.33)$
$+ 9.69(+125 \sin 30° - 13.33)$
$= -129.2 + 476.5 - 12{,}960$

$\underline{T = 12{,}610 \text{ lb}}$

$\Sigma F_x = m' \Delta v_x;\ V = 9.69(125 - 0) + 9.69(-125 \cos 30° - 0)$
$= 1211 - 1049 = \underline{162.3 \text{ lb}}$

$\Sigma M_B = \Sigma m' v d;\ M = 9.69(125)\dfrac{30}{12} - 9.69(125 \cos 30°)\dfrac{24}{12}$
$+ 9.69(125 \sin 30°)\dfrac{20}{12}]$

$M = 3028 - 2098 + 1009 = \underline{1939 \text{ lb-ft}}$

4/55 For the truck and plow as a system:

$\Sigma F_x = m' \Delta v_x$ : $P = \frac{60\,000}{60}\left[\frac{20}{3.6} - 0\right] = 5560$ N

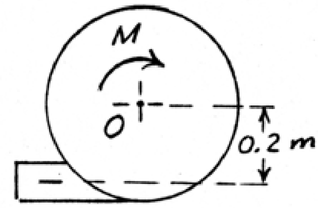
or $\underline{P = 5.56 \text{ kN}}$

$\Sigma F_y = m' \Delta v_y$ : $R = \frac{60\,000}{60}\left[12 \cos 45° - 0\right] = 8490$ N

or $\underline{R = 8.49 \text{ kN}}$

4/56 $M = M_0 = m'(v_2 d_2 - 0)$

$v_2 = \frac{Q}{A} = \frac{16}{\pi(0.150)^2/4}\,\frac{1}{60} = 15.09\ \frac{m}{s}$




From Table D-1, air density is 1.206 $kg/m^3$

so $m' = \rho Q = 1.206(16)/60 = 0.322$ kg/s

$M_0 = 0.322(15.09 \times 0.2 - 0) = 0.971$ N·m

$P = 0.32 + M_0\omega/1000 = 0.32 + \frac{0.971(3450 \times 2\pi/60)}{1000}$

$P = 0.32 + 0.351 = \underline{0.671\ kW}$

4/57 $v = 40$ m/s , $u = 420$ m/s

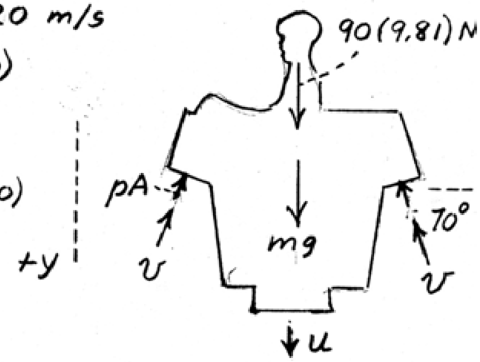
$pA = -1.8(10^3)(0.1320)$

$= -238$ N

$m' = \rho A v$

$= 1.206(0.1320)(40)$

$= 6.37$ kg/s



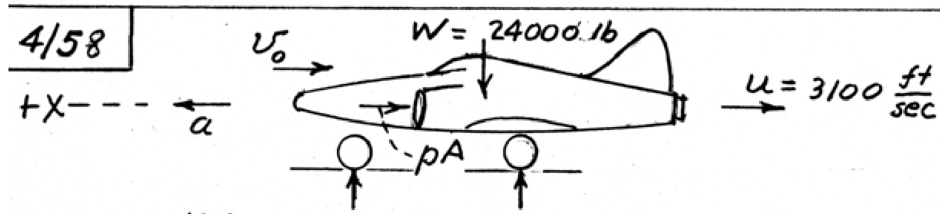

For steady flow

$\Sigma F_y = m' \Delta v_y$

$(90 + m)9.81 - (-238 \sin 70°) = 6.37(420 - [-40 \sin 70°])$

$9.81m + 1106 = 2914$ , $\underline{m = 184.3 \text{ kg}}$

**4/58**




$$m'_{air} = \frac{106}{32.2}\ \frac{\text{lb/sec}}{\text{ft/sec}^2} = 3.29\ \text{lb}\times\text{sec/ft}$$

$$m'_{fuel} = 3.29/18 = 0.1829\ \text{lb}\times\text{sec/ft}$$

air intake velocity $v_o = \dfrac{m'_{air}}{\rho A} = \dfrac{3.29}{(0.0753/32.2)(1800/144)}$

$$= 112.6\ \text{ft/sec}$$

$$m'_{exhaust} = 3.29 + 0.1829$$
$$= 3.47\ \text{lb}\times\text{sec/ft}$$

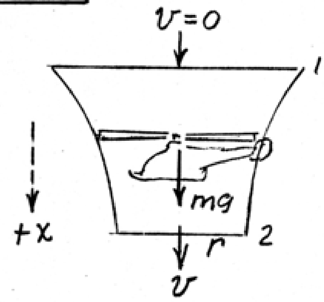
$$pA = -0.30(1800) = -540\ \text{lb}$$

Net thrust $T = m'_{ex}u - m'_{air}v_o - pA$

$$= 3.47(3100) - 3.29(112.6) - (-540)$$
$$= 10{,}940\ \text{lb}$$

$$\Sigma F_x = ma_x:\ 10940 = \frac{24000}{32.2}a,\quad \underline{a = 14.68\ \text{ft/sec}^2}$$

**4/59**




$mg$ = weight of helicopter
= force on system of air stream & helicopter

For system between sections 1 & 2

$$\Sigma F_x = m'\Delta v_x$$

$$mg = \rho \pi r^2 v(v-0)$$

$$v = \frac{1}{r}\sqrt{\frac{mg}{\pi\rho}}$$

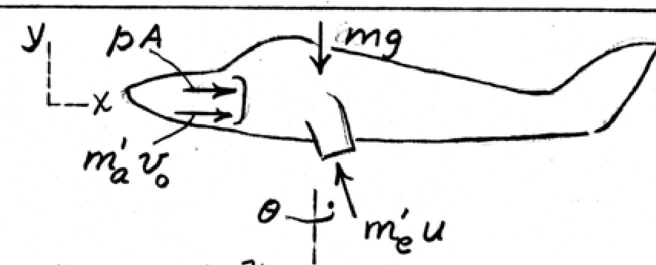
Power = rate of increase of kinetic energy

$$P = \frac{1}{2}m'(v_2^2 - v_1^2) = \frac{1}{2}m'v^2 = m'v\frac{v}{2} = mg\frac{v}{2}$$

$$P = \frac{mg}{2r}\sqrt{\frac{mg}{\pi\rho}}$$

4/60

Simulated FBD




$mg = 8600(9.81) = 84.4(10^3)$ N

mass rate of air $= m_a' = 90$ kg/s

" " " fuel $= 90/18 = 5$ kg/s

" " " exhaust $= m_e' = 95$ kg/s

$pA = -2(10^3)(1.10) = -2200$ N

$v_o = m_a'/\rho A = \frac{90}{1.206(1.10)} = 67.8$ m/s, $m_a' v_o = 90(67.8) = 6110$ N

$m_e' u = 95(1020) = 96\,900$ N

For vertical take off $\Sigma F_x = 0$: $6110 - 2200 - 96\,900 \sin\theta = 0$

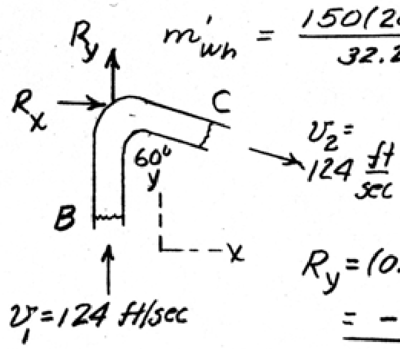
$\sin\theta = 0.0403$, $\underline{\theta = 2.31°}$

$\Sigma F_y = ma_y$: $96\,900 \cos 2.31° - 84\,400 = 8600\, a_y$

$a_y = \underline{1.448 \text{ m/s}^2}$

**4/61** $m'_{air} = \frac{18(2000)}{32.2}\frac{1}{3600} = 0.3106$ slugs/sec

$m'_{wh} = \frac{150(2000)}{32.2}\frac{1}{3600} = 2.588$ slugs/sec




$\Sigma F = m'\Delta v$

$R_x = (0.3106 + 2.588)(124 \sin 60^\circ - 0)$

$= \underline{311 \text{ lb}}$

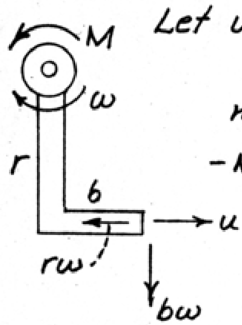
$R_y = (0.3106 + 2.588)(-124 \cos 60^\circ - 124)$

$= \underline{-539 \text{ lb}}$

Forces acting on pipe bend & mass within it

1) tension $pA = 4.42 \frac{\pi(14)^2}{4} = 680$ lb due to vacuum
2) tension in pipe at B
3) " " " " C
4) weight of bend
5) balance of external support forces from crane
6) shear force and bending moment at C

4/62 For entire system $\Sigma M = m'(v_2 d_2 - v_1 d_1)$




components of absolute velocity of water at exit

Let $u$ = velocity of water relative to nozzle $= Q/4A$

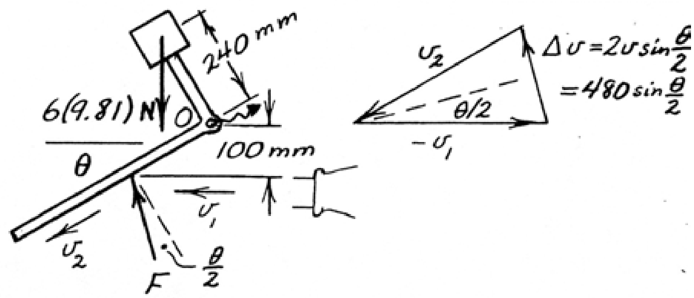
$m' = \rho Q$

$-M = \rho Q\left(r^2\omega + b^2\omega - \frac{Q}{4A}r - 0\right)$

$\underline{M = \rho Q\left(\frac{Qr}{4A} - [r^2+b^2]\omega\right)}$

For $M = 0$, $\omega = \omega_0 = \underline{\dfrac{Qr}{4A(r^2+b^2)}}$

4/63




$F = m'\Delta v$: $m' = \rho A v = 1.206 \frac{\pi \times 0.040^2}{4} 240 = 0.364$ kg/s

$$F = 0.364 \times 480 \sin\frac{\theta}{2} = 174.6 \sin\frac{\theta}{2} \text{ N}$$

For vane:

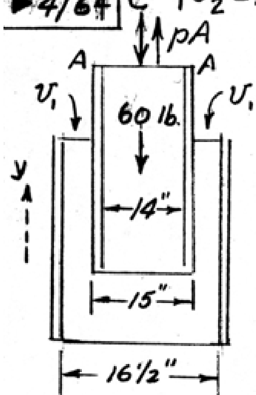
$\Sigma M_O = 0$: $174.6 \sin\frac{\theta}{2} \cos\frac{\theta}{2}\left(\frac{0.100}{\sin\theta}\right) - 6(9.81)(0.240 \sin\theta) = 0$

$$87.3 \times 0.100 = 6(9.81)(0.240 \sin\theta)$$

$\sin\theta = 0.618$, $\underline{\theta = 38.2°}$

Assumption: Entire air stream is diverted downward along the vane, with no flow toward O.

▶4/64 $C \quad \uparrow v_2 = 124$ ft/sec




Air inlet area $= \frac{\pi}{4}([16.5]^2 - [15]^2) = 37.1$ in.$^2$

$= 37.1/144 = 0.258$ ft$^2$

Exit area $= \frac{\pi}{4}(14)^2 = 153.9$ in.$^2$

$= 153.9/144 = 1.069$ ft$^2$

$pA = -(-4.42)(153.9) = 680$ lb

$m'_{air} = \frac{18(2000)}{32.2}\frac{1}{3600} = 0.3106$ slugs/sec

$m'_{wh} = \frac{150(2000)}{32.2}\frac{1}{3600} = 2.588$ slugs/sec

$v_1 = m'_{air}/\rho A = \frac{0.3106}{\left(\frac{0.075}{32.2}\right)(0.258)} = 517$ ft/sec

$\Sigma F_y = m'\Delta v_y;\ -C + 680 - 60 = 0.3106(124 - [-517]) + 2.588(124 - 0)$

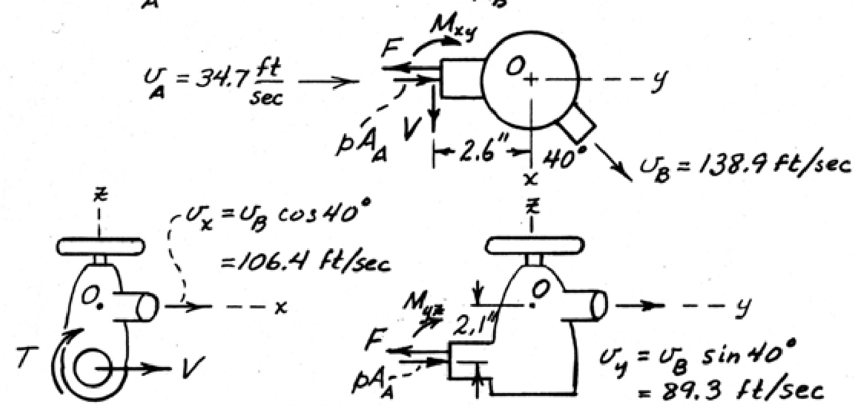
$= 199.2 + 320.9$

$\underline{C = 100.3 \text{ lb}}$

▶4/65 Flow rate $Q = \frac{340 \times 231}{1728 \times 60} = 0.758 \frac{ft^3}{sec}$, $m' = \rho Q = \frac{62.4}{32.2} 0.758 = 1.468 \frac{lb\text{-}sec}{ft}$

Flow area $A_A = \frac{\pi 2^2}{4}/144 = 0.0218\ ft^2$, $A_B = \frac{\pi \times 1^2}{4}/144 = 0.00545\ ft^2$

Velocity $v_A = \frac{Q}{A_A} = \frac{0.758}{0.0218} = 34.7 \frac{ft}{sec}$, $v_B = \frac{Q}{A_B} = \frac{0.758}{0.00545} = 138.9 \frac{ft}{sec}$




$(x\text{-}y)\ \Sigma F_x = m' \Delta v_x$: $150\left(\frac{\pi 2^2}{4}\right) - F = 1.468(89.3 - 34.7)$, $\underline{F = 391\ lb}$

$\Sigma F_y = m' \Delta v_y$: $V = 1.468\ (106.4 - 0)$, $\underline{V = 156.2\ lb}$

$\Sigma M_{A\text{-}A} = m' \Delta(vd)$: $M_{xy} = 1.468\left(106.4 \times \frac{2.6}{12}\right) = 33.8\ lb\text{-}ft$

$(y\text{-}z)\ \Sigma M_{A\text{-}A} = m' \Delta(vd)$: $M_{yz} = 1.468\left(89.3 \times \frac{2.1}{12}\right) = 22.9\ lb\text{-}ft$

$M = \sqrt{M_{xy}^2 + M_{yz}^2} = (33.8^2 + 22.9^2)^{1/2} = \underline{40.9\ lb\text{-}ft}$

$(x\text{-}z)\ \Sigma M_O = 0$: $T - Vd = 0$, $T = 156.2\left(\frac{2.1}{12}\right) = \underline{27.3\ lb\text{-}ft}$

**▸4/66** From Part (b) of Sample Problem $m' = \rho A(v-u)$

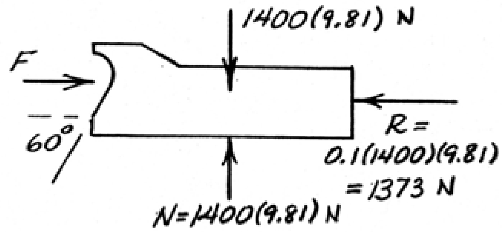
$$= (1000)\frac{\pi \times 0.140^2}{4}(150-u)$$

$$= 15.39(150-u) \text{ kg/s}$$

& $F = \rho A(v-u)^2(1-\cos 120°)$, $\theta = 90° + 30° = 120°$

$$= 15.39(150-u)^2(1-(-0.5))$$

$$= 23.1(150-u)^2$$




$\Sigma F = m\dot{u}$: $23.1(150-u)^2 - 1373 = 1400\,\dot{u}$

$$\int_0^u \frac{du}{0.01649(150-u)^2 - 0.981} = \int_0^3 dt$$

To integrate let $w = 150-u$, $\displaystyle\int_{u=0}^{u=u} \frac{dw}{0.981 - 0.01649 w^2} = 3$

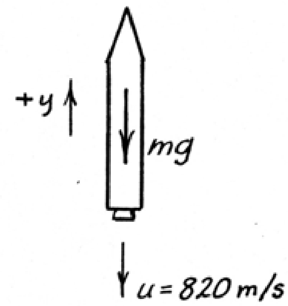
$$\frac{1}{2\sqrt{0.01649}\sqrt{0.981}} \ln\left|\frac{0.990 + 0.1284(150-u)}{0.990 - 0.1284(150-u)}\right|_0^u = 3$$

$$3.93 \ln\frac{1-0.00634u}{1-0.00703u} = 3, \quad \frac{1-0.00634u}{1-0.00703u} = 2.145$$

Solve for $u$ & get $\underline{u = 131.0 \text{ m/s}}$

4/67

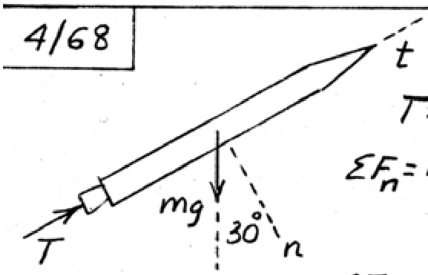
$$\Sigma F_y = ma + \dot{m}u: \quad -9.81m = 6.80m - 220(820)$$




$$m = 10.86\,(10^3)\text{ kg}$$

$$\text{or} \quad \underline{m = 10.86 \text{ Mg}}$$

4/68




$mg = 3(10^3)(9.60) = 28.8(10^3)$ N

$T = m'\Delta v = 130(600) = 78(10^3)$ N

$\Sigma F_n = ma_n$: $28.8(10^3)\cos 30° = 3(10^3)a_n$

$\underline{a_n = 8.31 \text{ m/s}^2}$

$\Sigma F_t = ma_t$: $78(10^3) - 28.8(10^3)\sin 30°$

$= 3(10^3)a_t$

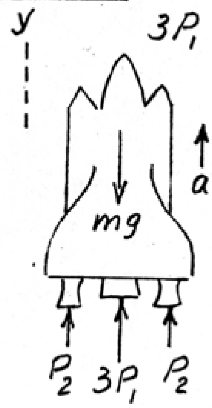
$\underline{a_t = 21.2 \text{ m/s}^2}$

**4/69**

$mg = 2.04(10^6)(9.81) = 20.0(10^6)$ N

$3P_1 = 3(2.00)(10^6) = 6.00(10^6)$ N

$2P_2 = 2(11.80)(10^6) = 23.6(10^6)$ N




Specific impulse $I = \frac{u}{g} = 455$ s

so $u = 455(9.81) = 4460$ m/s

$\Sigma F_y = ma_y: \quad (6.00)10^6 + (23.6)10^6 - 20.0(10^6)$

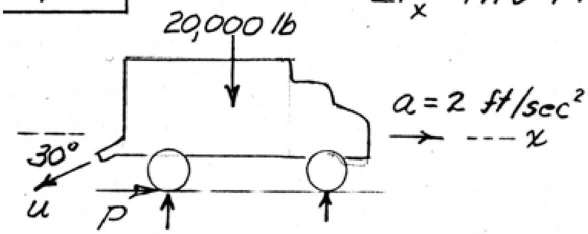
$= 2.04(10^6)\,a$

$\underline{a = 4.70 \text{ m/s}^2}$

$P_1 = m'u, \quad 2.00(10^6) = m'(4460)$

$\underline{m' = 448 \text{ kg/s}}$

4/70




$\Sigma F_x = m\dot{v} + \dot{m}u$

$u = 60$ ft/sec

$\dot{m} = -\frac{80}{32.2} = -2.48 \ \frac{\text{slugs}}{\text{sec}}$

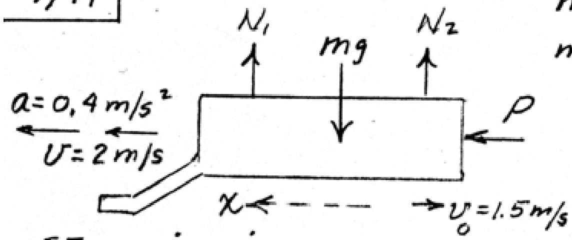
$m = \frac{20{,}000}{32.2} = 621$ slugs

(a) Water on; $P = 621(2) - 2.48(60)\cos 30°$

$= 1242 - 129 = \underline{1113 \text{ lb}}$

(b) water off; $\dot{m} = 0$, $P = \underline{1242 \text{ lb}}$

**4/71**




$m = 50 + 80 = 130$ kg

$\dot{m} = \rho A u = \rho A(v + v_0)$

$= 1000(2000)10^{-6}(2 + 1.5)$

$= 7.0$ kg/s

$u = v + v_0 = 3.5$ m/s

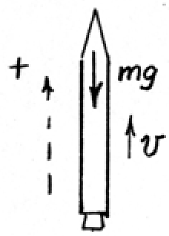
$\Sigma F_x = m\dot{v} + \dot{m}u\ ;\quad P = 130(0.4) + 7.0(3.5)$

$= 52 + 24.5 = \underline{76.5\ N}$

**4/72**

$\Sigma F = m\dot{v} + \dot{m}u \; ; \; -mg = ma + \dot{m}u$

$-m(a+g) = \dot{m}u$

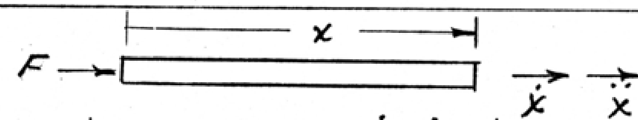



$u\dfrac{dm}{dt} = -m(a+g)$

$\displaystyle\int_{m_0}^{m} \frac{dm}{m} = -\frac{a+g}{u}\int_0^t dt, \quad \ln\frac{m}{m_0} = -\frac{a+g}{u}t$

$\underline{m = m_0 e^{-\frac{a+g}{u}t}}$

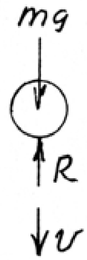
**4/73**




$\Sigma F = m\dot{v} + \dot{m}u$ where $m = \rho x$, $v = \dot{x}$, $\dot{m} = \rho\dot{x}$, $u = \dot{x}$

Thus $F = \rho x\ddot{x} + \rho\dot{x}\dot{x}$ , $\underline{F = \rho(x\ddot{x} + \dot{x}^2)}$

**4/74**



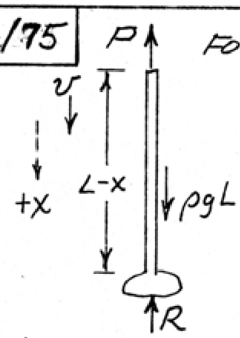

With added moisture particles initially at rest, relative velocity of attachment of mass is $u = v$

Thus with $\Sigma F = m\dot{v} + \dot{m}u$

we have $\Sigma F = m\dot{v} + \dot{m}v = \frac{d}{dt}(mv)$

where $\Sigma F = mg - R$

**4/75**




For $\dot{x} = v = \text{const.}$, $P =$ weight of descending links

$= \rho g (L - x)$

$\Sigma F_x = \dfrac{dG_x}{dt}$; $\rho g L - R - \rho g (L - x) = \dfrac{d}{dt}(\rho [L - x] v)$

$\rho g x - R = -\rho v \dot{x} = -\rho v^2$

So $\underline{R = \rho g x + \rho v^2}$

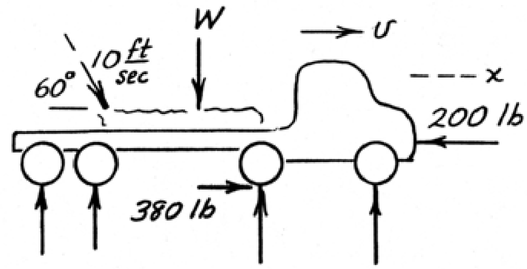
**4/76** $\Sigma F_x = m\dot{v} + \dot{m}u$:

$\Sigma F_x = 380 - 200 = 180$ lb

$m = \dfrac{12000 + 4(220)}{32.2}$

$= 400$ lb-sec²/ft

at $t = 4$ sec.

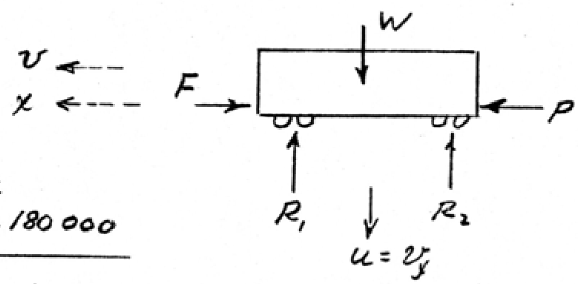



$\dot{m} = 220/32.2 = 6.83$ lb-sec/ft $\quad 1.5$ mi/hr $= 2.20$ ft/sec

$u = 2.20 - 10\cos 60° = -2.80$ ft/sec

So $180 = 400\dot{v} + 6.83(-2.80)$, $a = \dot{v} = \underline{0.498 \text{ ft/sec}^2}$

4/77




$$\Sigma F_x = m\dot{v}_x + \dot{m}u_x$$

$$F = 4\,\frac{54{,}600 + \frac{1}{2}\,180\,000}{2000}$$

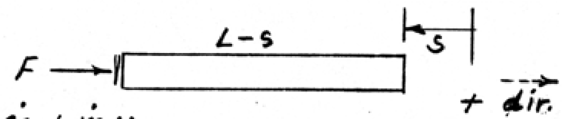
$= 289$ lb frictional resistance

$u_x$ = rel. velocity with respect to the car in x-dir. $= 0$

Thus $P - 289 = \dfrac{54{,}600 + \frac{1}{2}\,180{,}000}{32.2}\,0.15 + 0$

So $P = 674 + 289$, $\underline{P = 963 \text{ lb}}$

**4/78**




Eq. 4/20 $\Sigma F = m\dot{v} + \dot{m}u$

where $\Sigma F = F$, $m = \rho(L-s)$, $\dot{m} = -\rho\dot{s}$, $\dot{v} = -\ddot{s}$, $u = -\dot{s}$

Thus $F = \rho(L-s)(-\ddot{s}) + (-\rho\dot{s})(-\dot{s})$

$\underline{F = \rho\dot{s}^2 - \rho(L-s)\ddot{s}}$

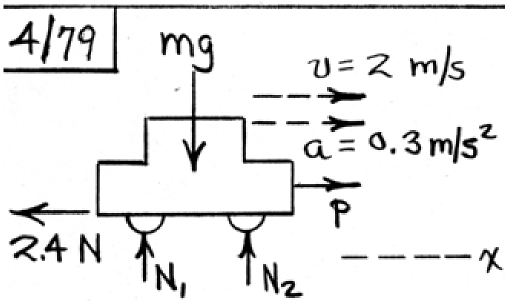
Eq. 4/6 for entire system $\Sigma F = \dot{G}$

$G = \rho(L-s)(-\dot{s}) = -\rho(L-s)\dot{s}$

$\dot{G} = -\rho(L-s)\ddot{s} - \rho(-\dot{s})\dot{s} = \rho\dot{s}^2 - \rho(L-s)\ddot{s}$

Thus $\underline{F = \rho\dot{s}^2 - \rho(L-s)\ddot{s}}$

4/79




$m = 40 + 30(1.2)$
$= 76 \text{ kg}$

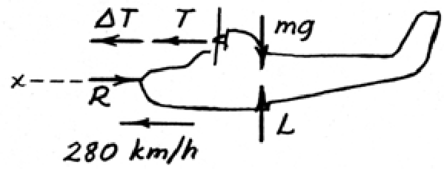
$\dot{m} = -\rho v = -1.2(2)$
$= -2.4 \text{ kg/s}$

$\Sigma F_x = m\dot{v} + \dot{m}u : \quad P - 2.4 = 76(0.3) - 2.4(2)$

$\underline{P = 20.4 \text{ N}}$

4/80

For constant initial speed propeller thrust $T$ = drag $R$.




Added power $= \Delta T \cdot v$,

$$\Delta T \times \frac{280 \times 1000}{3600} = 223.8\,(10^3) \text{ watts (joules/second)}$$

$$\Delta T = 2880 \text{ N}$$

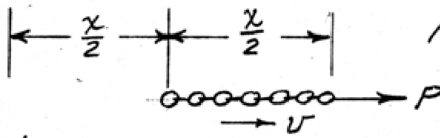
$\Sigma F_x = m\dot{v} + \dot{m}u$ where $\dot{m} = 4.5 \times 1000/12 = 375$ kg/s,

$$u = v = \frac{280 \times 1000}{3600} = 77.8 \text{ m/s}$$

So $2880 = 16.4(10^3)\,\dot{v} + 375(77.8)$, $\dot{v} = \underline{a = -1.603 \text{ m/s}^2}$

(deceleration)

**4/81**




$\rho = 48/8 = 6 \text{ kg/m}$

$\dot{m} = \rho \frac{d}{dt}\left(\frac{x}{2}\right) = \frac{1}{2}\rho \dot{x} = \frac{1}{2}\rho v = \frac{1}{2}(6)(1.5) = 4.5 \text{ kg/s}$

$u = v = 1.5 \text{ m/s}$

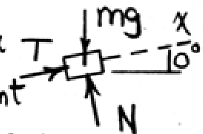
$$\Sigma F = m\dot{v} + \dot{m}u$$

(a) $\dot{v} = 0;\quad P = 0 + 4.5(1.5) = \underline{6.75 \text{ N}}$

(b) $m = \rho \frac{x}{2} = 6\left(\frac{4}{2}\right) = 12 \text{ kg}$

$$20 = 12\dot{v} + 4.5(1.5),\quad a = \dot{v} = \underline{1.104 \text{ m/s}^2}$$

$\boxed{4/82}$ $\Sigma F_x = ma_x$: $T - mg\sin\theta = ma_x$




$T = m'u = \dfrac{2}{32.2}(400) = 24.8$ lb constant

$m = m_0 - m't = \dfrac{1}{32.2}(125 - 2t)$ lb-sec$^2$/ft

Propulsion time $t = \dfrac{20}{2} = 10$ sec

So $m'u - (m_0 - m't)g\sin\theta = (m_0 - m't)\dfrac{dv}{dt}$

$$\int_0^t \left[\frac{m'u}{m_0 - m't} - g\sin\theta\right] dt = \int_0^v dv$$

$$\Rightarrow v = u\ln\left(\frac{m_0}{m_0 - m't}\right) - gt\sin\theta$$

When $t = 10$ sec, $v = 400\ln\left(\dfrac{125}{125-20}\right) - 32.2(10)\sin 10°$

$= \underline{13.83 \text{ ft/sec}}$

**4/83** With $v = \text{const.}$, $\dot{v} = \text{accel.} = 0$

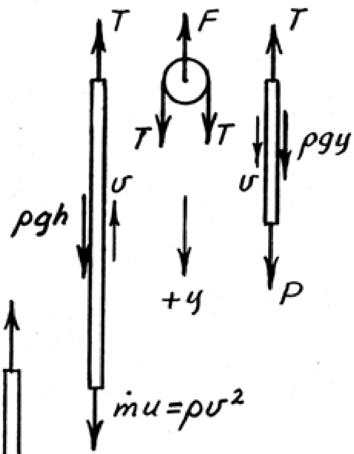
so $\Sigma F_y = 0$ for all bodies:

$$P + \rho g y - T = 0 \qquad \text{---}(1)$$

$$\rho v^2 + \rho g h - T = 0 \qquad \text{---}(2)$$

Eliminate $T$ & get

$$\underline{P = \rho v^2 + \rho g(h-y)}$$




Left-hand portion (constant mass, upper end moving up):

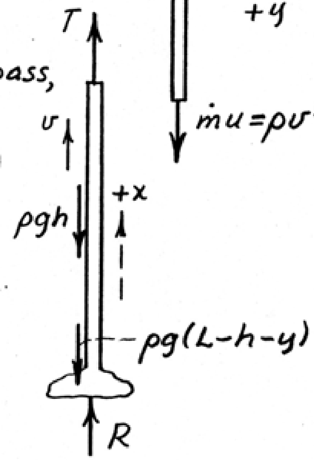
$\Sigma F_x = \dot{G}_x$: $T + R - \rho g h - \rho g(L-h-y) = \dfrac{d}{dt}(\rho h v)$

$\rho v^2 + \rho g h + R - \rho g h - \rho g(L-h-y) = \dfrac{d}{dt}(\rho h v) = \rho v^2$

where $\dot{h} = v$

$\rho v^2 + R - \rho g(L-h-y) = \rho v^2$

$R = \underline{\rho g(L-h-y)}$ = weight of pile of chain



4/84 Let $m_0$ = initial mass of car = $25(10^3)$ kg

$\dot{m} = 4(10^3)$ kg/s

The car acquires mass which has zero initial horizontal velocity, so for horizontal x-dir., $\Sigma F_x = \frac{d}{dt}(mv)$

$0 = \frac{d}{dt}(m_0 + \dot{m}t)v$, $(m_0 + \dot{m}t)a + \dot{m}v = 0$

$$a = \frac{dv}{dt} = -\frac{\dot{m}v}{m_0 + \dot{m}t}$$

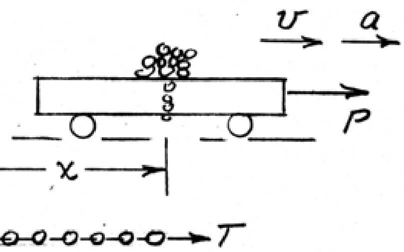
$$\int_{v_0}^{v} \frac{dv}{v} = -\int_0^t \frac{\dot{m}}{m_0 + \dot{m}t}\,dt \Rightarrow v = \frac{dx}{dt} = \frac{m_0 v_0}{m_0 + \dot{m}t}$$

Then $\displaystyle\int_0^x dx = \int_0^t \frac{m_0 v_0}{m_0 + \dot{m}t}\,dt \Rightarrow x = \frac{m_0 v_0}{\dot{m}} \ln\left(\frac{m_0 + \dot{m}t}{m_0}\right)$

With $t = \frac{32}{4} = 8$ s, $x = \dfrac{25(10^3)(1.2)}{4(10^3)} \ln\left(\dfrac{25 + 4(8)}{25}\right)$

$$\underline{x = 6.18 \text{ m}}$$

**4/85** $m = m_0 - \rho x$




$T$ is not transmitted to cart so $\Sigma F = ma$

$$P = (m_0 - \rho x) a$$

$$a = \frac{P}{m_0 - \rho x}$$

$v\,dv = a\,dx$, $\quad \int_{v_0}^{v} v\,dv = \int_0^x \frac{P\,dx}{m_0 - \rho x}$

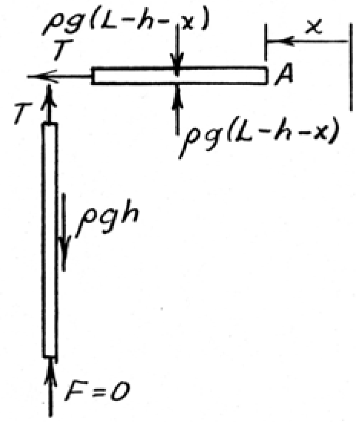
$$\frac{v^2}{2}\Big]_{v_0}^{v} = -\frac{P}{\rho}\ln(m_0 - \rho x)\Big]_0^x = \frac{P}{\rho}\ln\frac{m_0}{m_0 - \rho x}$$

Thus $v = \sqrt{v_0^2 + \frac{2P}{\rho}\ln\frac{m_0}{m_0 - \rho x}}$

$T = m'\Delta v$, $T = \rho v(v)$, $T = \rho v^2$

There is no reaction ($R$ in Fig. 4/6) between departing links & cart, so $\dot{m}u$ is zero & $\Sigma F = ma$

**4/86** Let $x$ be the displacement of the chain & $T$ be the tension in the chain at the corner.




Horiz. part $\Sigma F_x = ma_x$:

$$T = \rho(L-h-x)\ddot{x}$$

Vert. part $\Sigma F_y = ma_y$:

$$\rho gh - T = \rho h\ddot{x}$$

Eliminate $T$ & get

$$\ddot{x} = \frac{gh}{L-x}$$

$$\dot{x}\,d\dot{x} = \ddot{x}\,dx: \int_0^{v_1^2} \frac{1}{2} d(\dot{x}^2) = \int_0^{L-h} \frac{gh}{L-x}\,dx$$

$$\frac{\dot{x}^2}{2}\Bigg|_{\dot{x}=0}^{v_1} = -gh\ln(L-x)\Bigg|_0^{L-h}, \quad \frac{v_1^2}{2} = gh\ln(L-x)\Bigg|_{L-h}^{0} = gh\ln\frac{L}{h}$$

(a) $\underline{v_1 = \sqrt{2gh\ln(L/h)}}$

(b) Free fall of end $A$ gives $v_2^2 = v_1^2 + 2gh = 2gh\ln\frac{L}{h} + 2gh$

$\underline{v_2 = \sqrt{2gh(1+\ln[L/h])}}$

(c) $Q$ = loss of potential energy since $\Delta T = 0$

$Q = \rho gh\frac{h}{2} + \rho g(L-h)h$, $\underline{Q = \rho gh(L - \frac{h}{2}) \text{ loss}}$

**4/87** For airplane plus moving portion of chains

$$\Sigma F = 0 = m\dot{v} + \dot{m}u = (m + 2\rho\tfrac{x}{2})\dot{v} + [2\tfrac{d}{dt}(\rho\tfrac{x}{2})]v$$

$$-(m+\rho x)\frac{dv}{dt} = \rho v\frac{dx}{dt} \qquad \frac{dv}{v} = -\frac{\rho\, dx}{m+\rho x}$$

$$\int_{v_0}^{v}\frac{dv}{v} = -\int_0^x \frac{\rho\, dx}{m+\rho x}\ ; \quad \ln\frac{v}{v_0} = -\ln\frac{m+\rho x}{m}\ , \quad \frac{v}{v_0} = \frac{m}{m+\rho x}$$

or $v = \dfrac{v_0}{1+\rho x/m}$ & for $x = 2L$, $\boxed{v = \dfrac{v_0}{1+\frac{2\rho L}{m}}}$

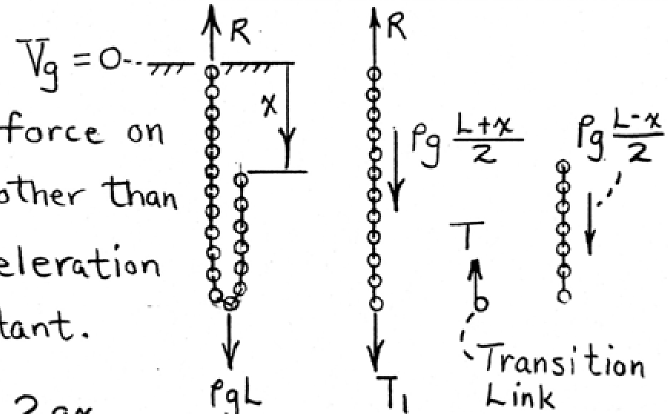
Also, $v = \dfrac{dx}{dt}$ so $\displaystyle\int_0^x (1 + \frac{\rho x}{m})dx = \int_0^t v_0\, dt$

$$x + \frac{\rho x^2}{2m} = v_0 t\ , \quad x^2 + \frac{2m}{\rho}x - \frac{2mv_0 t}{\rho}$$

$$x = -\frac{m}{\rho} \pm \frac{1}{2}\sqrt{\frac{4m^2}{\rho^2} + \frac{8mv_0 t}{\rho}}\ , \quad \underline{x = \frac{m}{\rho}\left[\sqrt{1+\frac{2v_0 t\rho}{m}} - 1\right]}$$

for + root

▶4/88




There is no force on moving part other than weight so acceleration $\ddot{x} = g$ = constant.

Thus, $v^2 = 2gx$

$T_1 = m'\Delta v$: $T_1 = \left[\frac{d}{dt}\left(\rho \frac{L-x}{2}\right)\right]\left[0-v\right]$

$$= \frac{1}{2}\rho v^2 = \frac{1}{2}\rho(2gx) = \underline{\rho g x}$$

Equilibrium of links at rest:

$\Sigma F_x = 0$: $T_1 + \rho g \frac{L+x}{2} - R = 0$

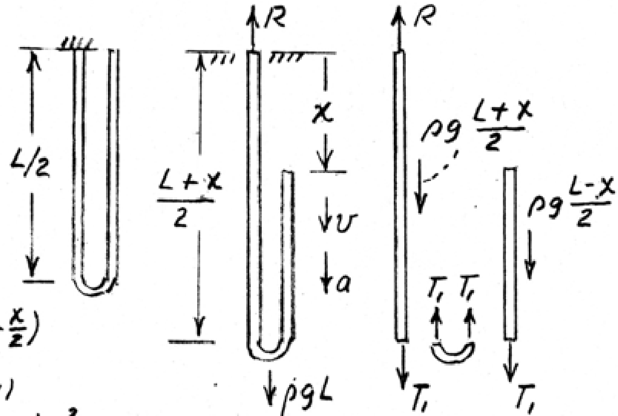
$$\Rightarrow \underline{R = \frac{1}{2}\rho g (L + 3x)}$$

Loss $Q = |V_{g_1} - V_{g_2}| = \left|\rho g L\left(-\frac{L}{4}\right) - \rho g L\left(-\frac{L}{2}\right)\right|$

$$= \underline{\frac{1}{4}\rho g L^2}$$

When $x \to L$, $v \to \infty$. The loss of potential energy equals the gain in kinetic energy, so the gain is concentrated in the last element and is lost during impact when the last element is abruptly brought to rest.

▶ 4/89




$\Delta V_g = \rho g \frac{L-x}{2}(-x)$

$\quad + \rho g \frac{x}{2}(-\frac{x}{2})$

$\quad = -\frac{1}{2}\rho g x (L - \frac{x}{2})$

$\Delta T = \frac{1}{2}\rho \frac{L-x}{2} v^2$

$\Delta V_g + \Delta T = 0$

$\frac{1}{4}\rho(L-x)v^2 = \frac{1}{2}\rho g x(L-\frac{x}{2})$

$v^2 = 2gx\frac{L-x/2}{L-x}$ ---(1)

$v\,dv = a\,dx$ so $a = \frac{1}{2}\frac{dv^2}{dx} = g\frac{(L-x)(L-x) - x(L-x/2)(-1)}{(L-x)^2}$

$$a = g\left(1+\frac{x(L-x/2)}{(L-x)^2}\right) \quad \text{--- (2)}$$

For entire rope

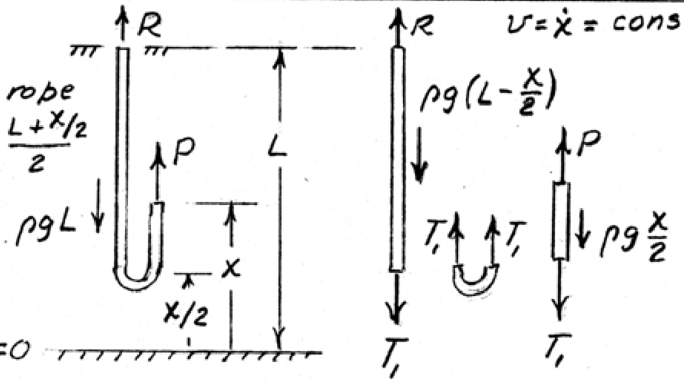
$\Sigma F_x = \dot{G}_x$ : $\rho g L - R = \frac{d}{dt}\left(\rho\frac{L-x}{2}v\right) = \frac{\rho}{2}\left[(L-x)a - v^2\right]$

Sub. (1) & (2) & get $R = \frac{1}{2}\rho g\left[(L+x) + \frac{x(L-x/2)}{L-x}\right]$

Equil of fixed part gives $T_1 = \frac{1}{2}\rho g x(L-x/2)/(L-x)$

**▶4/90**

$v = \dot{x} = \text{const.}$




For entire rope

$$V_g = \rho g\left(L-\frac{x}{2}\right)\frac{L+x/2}{2} + \rho g\frac{x}{2}\,\frac{3x}{4} = \frac{\rho g}{2}\left(L^2+\frac{x^2}{2}\right)$$

Entire rope: $\Sigma F_x = \dot{G}_x$: $R + P - \rho g L = \frac{d}{dt}\left(\rho\frac{x}{2}v\right) = \frac{1}{2}\rho v^2$ ---(1)

Work-energy: $dU' = dT + dV_g$: $P\,dx = d\left(\frac{1}{2}\rho\frac{x}{2}v^2\right) + d\left\{\frac{\rho g}{2}\left(L^2+\frac{x^2}{2}\right)\right\}$

$$= \frac{1}{4}\rho v^2 dx + \frac{1}{2}\rho g x\, dx$$

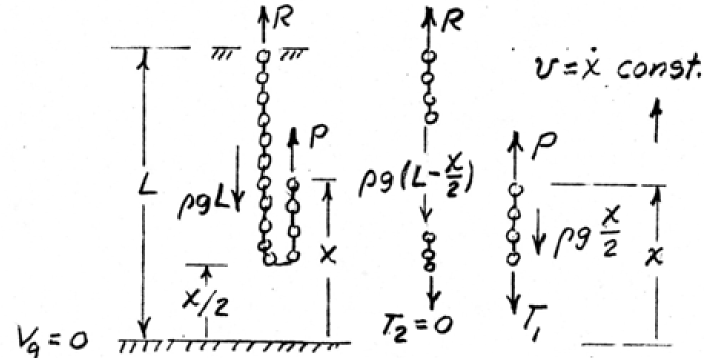
$$P = \frac{1}{4}\rho v^2 + \frac{1}{2}\rho g x \quad \text{---- (2)}$$

Sub. (2) into (1)

$$\underline{R = \frac{1}{4}\rho v^2 + \rho g\left(L - x/2\right)}$$

Equil. of part not moving: $\Sigma F_y = 0$: $R - \rho g\left(L-\frac{x}{2}\right) - T_1 = 0$, $\underline{T_1 = \frac{1}{4}\rho v^2}$

▸4/91




Entire chain: $\Sigma F_x = \dot{G}_x$ $\quad R + P - \rho g L = \frac{d}{dt}(\rho \frac{x}{2} v)$

$R + P = \frac{1}{2}\rho v^2 + \rho g L$ (same as for rope)

$T_1 = m'\Delta v$: $\quad T_1 = \frac{d}{dt}(\rho \frac{x}{2}) v, \quad \underline{T_1 = \frac{1}{2}\rho v^2}$

Moving part: $\Sigma F_x = 0$: $\quad P - \rho g \frac{x}{2} - \frac{1}{2}\rho v^2 = 0, \quad \underline{P = \frac{1}{2}\rho(v^2 + gx)}$

Equil. of part not moving $\quad \Sigma F_x = 0$: $\quad R - \rho g (L - \frac{x}{2}) = 0, \quad \underline{R = \rho g (L - \frac{x}{2})}$

$U_{1-2} = \Delta T + \Delta V_g + Q, \quad \Delta T = \frac{1}{2}\rho \frac{x}{2} v^2, \quad \Delta V_g = \rho g \frac{x}{2} \frac{x}{2}$

$U = Px = \frac{1}{2}\rho(v^2 x + g x^2)$ so $\quad \underline{Q = \frac{1}{4}\rho x (v^2 + gx)}$

▶4/92 For falling part $\Sigma F = m\dot{v} + \dot{m}u$

where $\Sigma F = \rho g x$, $m = \rho x$, $\dot{m} = \rho v$, $u = v = \dot{x}$

Thus $\rho g x = \rho x \dot{v} + \rho v \dot{x}$, $g x\, dt = x\, dv + v\, dx$

or $g x\, dt = d(xv)$; $g x^2 v\, dt = xv\, d(xv)$

so $g x^2 dx = \frac{1}{2} d[(xv)^2]$ & $g\int_0^x x^2 dx = \frac{1}{2}\int_0^{(xv)^2} d[(xv)^2]$

$\frac{g x^3}{3} = \frac{1}{2}(xv)^2$, $\underline{v = \sqrt{\frac{2gx}{3}}}$

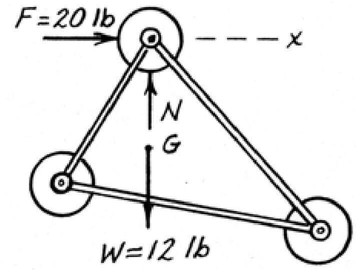
$a = \dot{v} = \sqrt{\frac{2g}{3}}\,\frac{1}{2}x^{-1/2}\dot{x} = \sqrt{\frac{2g}{3}}\,\frac{1}{2\sqrt{x}}\sqrt{\frac{2gx}{3}}$, $\underline{a = \frac{g}{3} \text{ constant}}$

$Q = -\Delta V_g - \Delta T = +\frac{\rho g L^2}{2} - \frac{\rho L}{2} v^2_{x=L} = \frac{\rho g L^2}{2} - \frac{\rho g L^2}{3} = \underline{\frac{\rho g L^2}{6}}$

4/93

$\Sigma F_x = m\bar{a}_x : \ 20 = \frac{12}{32.2}\bar{a}_x$

$\bar{a}_x = \underline{\bar{a} = 53.7 \text{ ft/sec}^2}$



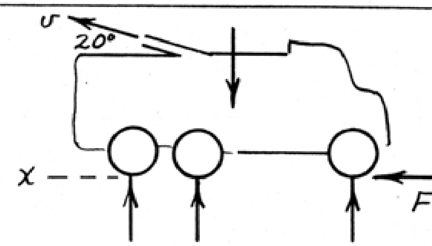
$\boxed{4/94}$ For the system, $\Sigma M_o = \dot{H}_o = 0$, so $H_o$ is conserved:

$$\frac{2}{16}(1000)\frac{10}{12} = \frac{2}{16}\left(\frac{10}{12}\right)^2\omega + 3\left(\frac{20}{12}\right)^2\omega$$

$$\underline{\omega = 12.37 \text{ rad/sec}}$$

A large horizontal force is exerted on the rod by the bearing so that $\Sigma F \neq 0$ in the horizontal direction. Thus $\dot{G}_x \neq 0$ and the linear momentum of the bullet-pendulum system is not conserved.

4/95




$F = m'\Delta v_x: \quad \Delta v_x = v\cos 20°$

$Q = Av: \dfrac{1400 \times 231}{1728}\dfrac{1}{60}\dfrac{ft^3}{sec}$

$= \dfrac{\pi \times 2^2/4}{144}v, \quad v = 143.0 \text{ ft/sec}$

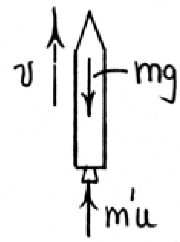
$\Delta v_x = 143.0\cos 20° - 0 = 134.4 \text{ ft/sec}$

$m' = \rho Q = \dfrac{62.4}{32.2}\dfrac{1400 \times 231}{1728 \times 60} = 6.04 \text{ lb-sec/ft}$

$F = 6.04(134.4) = \underline{812 \text{ lb}}$

4/96 $m = m_0 - m't$

$\sum F = ma: \; m'u - (m_0 - m't)g = (m_0 - m't)a$



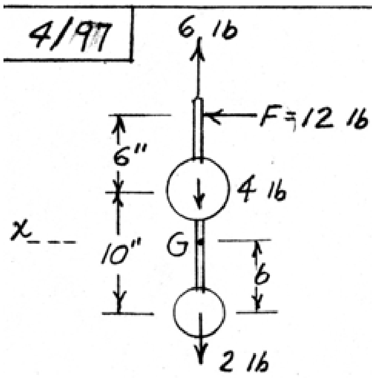

$a = \dfrac{dv}{dt} = \dfrac{m'u}{m_0 - m't} - g$

$\displaystyle\int_0^v dv = \int_0^t \frac{m'u}{m_0 - m't}\,dt - \int_0^t g\,dt$

$v = -u \ln(m_0 - m't)\Big|_0^t - gt\Big|_0^t$

$v = u \ln\left(\dfrac{m_0}{m_0 - m't}\right) - gt$

4/97




$$\Sigma F_x = m\bar{a}_x;\quad 12 = \frac{4+2}{32.2}\bar{a}$$

$$\bar{a} = \underline{64.4\ ft/sec^2}$$

$$4(10-b) = 2b,\quad b = 6.67\ in.$$

$$H_G = \Sigma m r^2 \dot{\theta} = \frac{4(3.33)^2 + 2(6.67)^2}{32.2(12)^2}\dot{\theta}$$

$$= 0.0288\dot{\theta}\ lb\text{-}ft\text{-}sec$$

$$\Sigma M_G = \dot{H}_G;\quad 12\frac{(6+3.33)}{12} = 0.0288\ddot{\theta}$$

$$\ddot{\theta} = \frac{9.33}{0.0288} = \underline{325\ rad/sec^2}$$

4/98

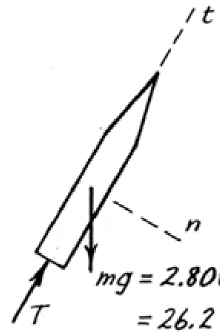
$T = m'u = 120(640) = 76.8(10^3)$ N

$\Sigma F_t = ma_t$ : $76.8(10^3) - 26.2(10^3)\cos 30° = 2.80(10^3)a_t$

$\underline{a_t = 19.34 \text{ m/s}^2}$

$\Sigma F_n = ma_n$ : $26.2(10^3)\sin 30° = 2.80(10^3)a_n$

$\underline{a_n = 4.67 \text{ m/s}^2}$

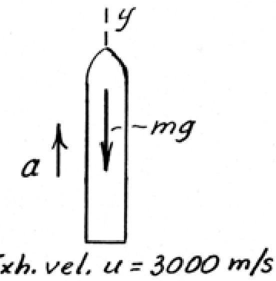



$mg = 2.80(9.34)10^3$ N
$= 26.2(10^3)$ N

4/99

$\dot{m} = -m' = -5.2$ kg/s

$m = 200 + 1200 - 5.2t = 1400 - 5.2t$ kg




$\Sigma F = m\dot{v} + \dot{m}u: \; -mg = ma - 5.2(3000)$

$(1400 - 5.2t)(a + 8.70) = 15\,600$

$a = \dfrac{15\,600}{1400 - 5.2t} - 8.70 \text{ m/s}^2$

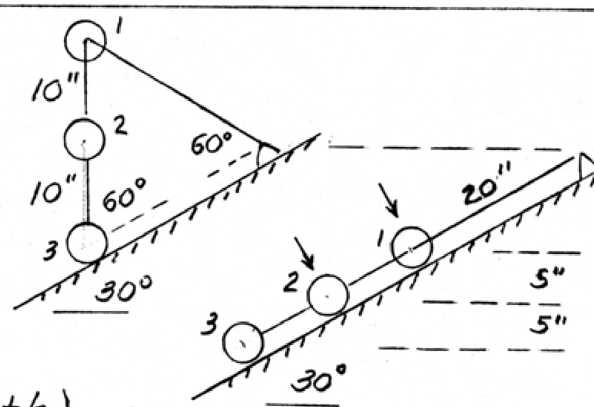
When $t = 60$ s, $a = \dfrac{15\,600}{1400 - 5.2(60)} - 8.70 = 14.34 - 8.70$

$= \underline{5.64 \text{ m/s}^2}$

Max. accel. occurs when $5.2t = 1200$, $\underline{t = 231 \text{ s}}$

$a_{max} = \dfrac{15\,600}{1400 - 5.2(231)} - 8.70 = 78.0 - 8.70 = \underline{69.3 \text{ m/s}^2}$

## 4/100




Vertical drop of spheres is

$h_1 = 20''$

$h_2 = 15''$

$h_3 = 10''$

For system

$\Delta V_g = -mg(h_1 + h_2 + h_3)$

$\Delta T = \frac{1}{2}m(v_1^2 + v_2^2 + 0) = \frac{1}{2}m(v^2 + [\frac{v}{2}]^2) = \frac{5}{8}mv^2$

$U' = \Delta T + \Delta V: \quad 0 = \frac{5}{8}mv^2 - mg(h_1 + h_2 + h_3)$

$v^2 = \frac{8}{5}(32.2)\,\frac{20+15+10}{12} = 193.2 \ (ft/sec)^2$

$\underline{v = 13.90 \ ft/sec}$

Potential energy loss goes into impact energy loss

4/101 $\Sigma F_x = m' \Delta v_x$

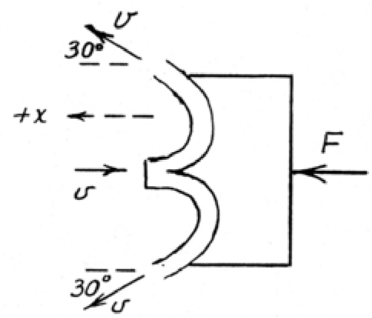
$m' = \rho A v = \frac{62.4}{32.2}\left(\frac{\pi}{4}\frac{(3/4)^2}{144}\right)120$

$= 0.713$ lb-sec/ft

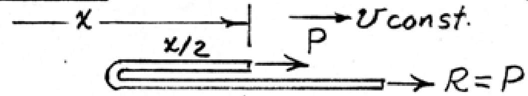
$\Delta v_x = v\cos 30° - (-v) = v(1+\cos 30°)$

$= 120(1+0.866) = 224$ ft/sec

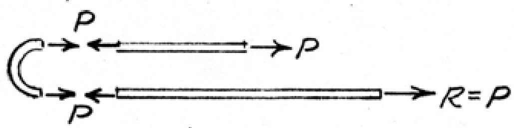
$F = 0.713 \times 224 = \underline{159.8 \text{ lb}}$



**4/102** $dU = dT; \quad P\,dx = d\left(\frac{1}{2}\rho\frac{x}{2}v^2\right) = \frac{1}{4}\rho v^2\,dx$




$$P = \frac{1}{4}\rho v^2$$

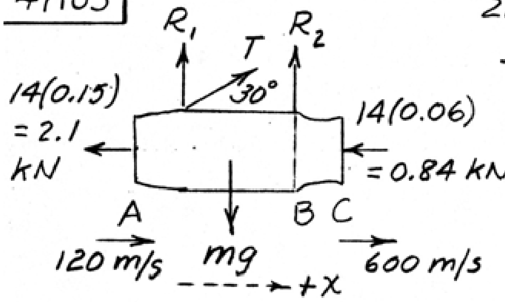



or $\Sigma F_x = \dot{G}_x$

$$P + R = \frac{d}{dt}\left(\rho\frac{x}{2}v\right)$$

$$R = \frac{1}{2}\rho v^2 - \frac{1}{4}\rho v^2 = \frac{1}{4}\rho v^2$$

**4/103**

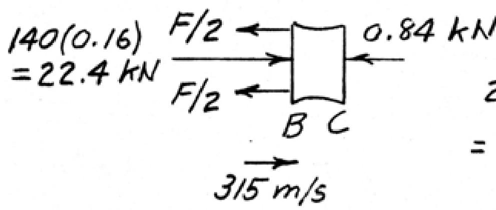



$$\Sigma F_x = m'\Delta v_x$$

$$-2.1 - 0.84 + T\cos 30^\circ$$

$$= [31.6(600) - 30(120)]10^{-3}$$

$$\underline{T = 21.1 \text{ kN}}$$




$$\Sigma F_x = m'\Delta v_x$$

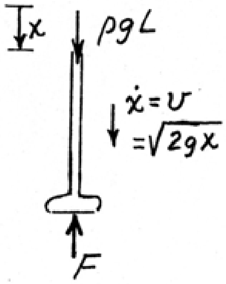
$$22.4 - 0.84 - F$$

$$= 31.6(600 - 315)10^{-3}$$

$$\underline{F = 12.55 \text{ kN}}$$

**4/104**

$$G_x = \rho(L-x)\sqrt{2gx} = \rho\sqrt{2g}\left(Lx^{1/2} - x^{3/2}\right)$$




$\Sigma F_x = \dot{G}_x$ for entire system

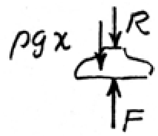
$$\rho g L - F = \rho\sqrt{2g}\left(\tfrac{1}{2}Lx^{-1/2} - \tfrac{3}{2}x^{1/2}\right)\dot{x}$$

$$= \rho\sqrt{2g}\left(\tfrac{1}{2}L\sqrt{2g} - \tfrac{3}{2}\sqrt{2g}\,x\right)$$

$$= \rho g L - 3\rho g x$$

So that $\underline{F = 3\rho g x}$

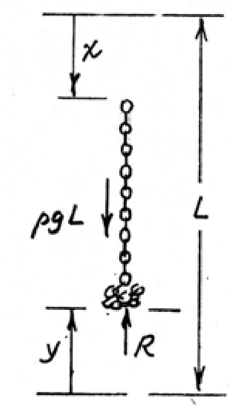
Alternatively




$$R = m'\Delta v = \rho\dot{x}(\dot{x}) = \rho\dot{x}^2 = \rho(2gx) = 2\rho g x$$

$$\Sigma F_x = 0;\quad 2\rho g x + \rho g x - F = 0,\quad \underline{F = 3\rho g x}$$

4/105 Take entire chain as system (constant mass)




$\ddot{x} = g$ , $\ddot{y} = a$ , $G_x = \rho(L-x-y)\dot{x} - \rho(x+y)\dot{y}$

$\dot{x} = gt$ , $\dot{y} = at$

$x = \frac{1}{2}gt^2$, $y = \frac{1}{2}at^2$

$\Sigma F_x = \dot{G}_x$; $\rho g L - R = \rho\left[(-\dot{x}-\dot{y})\dot{x} + (L-x-y)\ddot{x}\right] - \rho\left[(\dot{x}+\dot{y})\dot{y} + (x+y)\ddot{y}\right]$

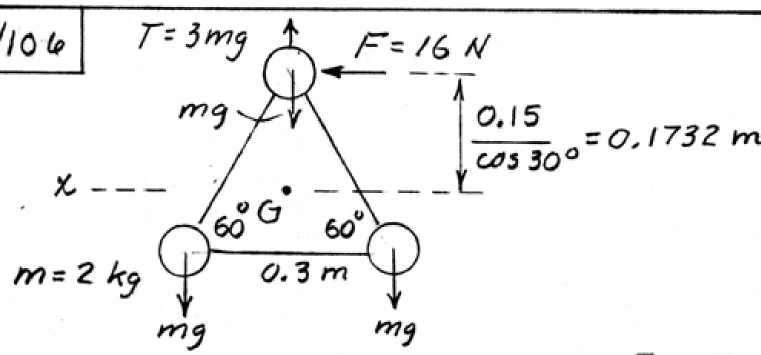
$$\rho g L - R = \rho\left[-(\dot{x}+\dot{y})^2 - (x+y)(\ddot{x}+\ddot{y}) + L\ddot{x}\right]$$

$$= \rho\left[-(a+g)^2t^2 - \frac{1}{2}(a+g)^2t^2 + Lg\right]$$

$$= -\frac{3}{2}\rho(a+g)^2t^2 + \rho g L$$

so $R = \frac{3}{2}\rho(a+g)^2t^2$

4/106




For system: $\Sigma F_x = m\bar{a}_x$: $16 = 3(2)\bar{a}$, $\underline{\bar{a} = 2.67 \frac{m}{s^2}}$

$\Sigma M_G = \dot{H}_G$: $16(0.1732) = \frac{d}{dt}\left(3 \times 2 \times \overline{0.1732}^2 \dot{\theta}\right)$

$\ddot{\theta} = \frac{16(0.1732)}{6(0.1732)^2} = \underline{15.40 \frac{rad}{s^2}}$

For top sphere

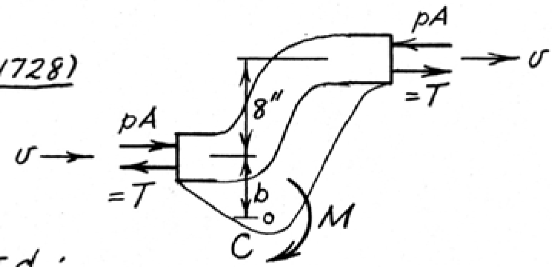
$a = \bar{a} + \bar{r}\ddot{\theta} = 2.67 + 0.1732(15.40) = \underline{5.33 \ m/s^2}$

4/107

$Q = Av$

$$v = Q/A = \frac{5000 \times 231/(60 \times 1728)}{\frac{\pi \times 4^2}{4}\frac{1}{144}}$$

$= 127.7$ ft/sec




$\Sigma M_O = \dot{H}_O = m' v_2 d_2 - m' v_1 d_1$ :

$$m' = \frac{\mu}{g} Q = \frac{62.4}{32.2} \frac{5000 \times 231}{60 \times 1728} = 21.6 \text{ lb-sec/ft}$$

so $M = 21.6\,(127.7)(8/12) = \underline{1837 \text{ lb-ft}}$

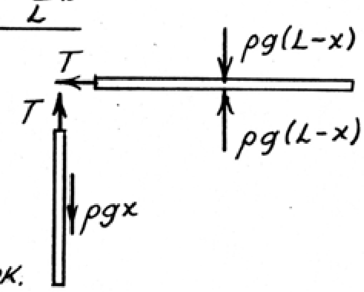
**4/108** System is conservative, so $\Delta V_g + \Delta T = 0$.

$$-\rho g x \frac{x}{2} + \frac{1}{2}\rho L \dot{x}^2 = 0, \quad \frac{g}{L}x^2 = \dot{x}^2, \quad \dot{x} = \sqrt{\frac{g}{L}}\, x$$

(a) accel $a = \ddot{x} = \sqrt{\frac{g}{L}}\,\dot{x} = \sqrt{\frac{g}{L}}\sqrt{\frac{g}{L}}\,x$, so $\underline{a = \frac{g}{L}x}$

(b) $\Sigma F = ma$: $T = \rho(L-x)\frac{g}{L}x$

$$\underline{T = \rho g x\left(1 - \frac{x}{L}\right)}$$

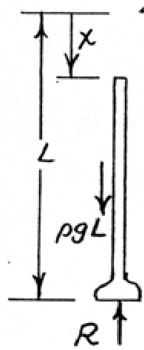



Check from vertical part

$\rho g x - T = \rho x \frac{g}{L}x$, $T = \rho g x\left(1 - \frac{x}{L}\right)$, OK.

(c) $v\,dv = a_x\,dx$: $\int_0^v v\,dv = \frac{g}{L}\int_0^L x\,dx$, $\frac{v^2}{2} = \frac{g}{L}\frac{L^2}{2}$, $\underline{v = \sqrt{gL}}$

▶4/109 For entire rope of constant mass




$\Sigma F_x = \dot{G}_x;\ \rho g L - R = \frac{d}{dt}(\rho[L-x]\dot{x} + 0)$ ----(1)

$\Delta T + \Delta V_g = 0;\ \frac{1}{2}\rho(L-x)\dot{x}^2 = \rho g x\left(L - \frac{x}{2}\right)$

$\dot{x}^2 = \frac{x(2L-x)}{L-x}g$ or $\dot{x}(L-x) = \frac{x}{\dot{x}}(2L-x)g$

Substitute into (1) & get

$$\rho g L - R = \frac{d}{dt}\left(\frac{x(2L-x)}{\dot{x}}\rho g\right) = \rho g\frac{\dot{x}[2L-2x]\dot{x} - x[2L-x]\ddot{x}}{\dot{x}^2}$$

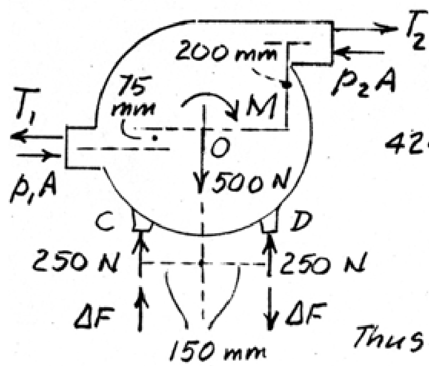
$$= 2\rho g(L-x) - \rho g x(2L-x)\frac{\ddot{x}}{\dot{x}^2}$$

Differentiate $\dot{x}^2$ & get $\ddot{x} = \left[1 + \frac{x(L-\frac{x}{2})}{(L-x)^2}\right]g$. Substitute & get $\rho g L - R = 2\rho g(L-x) - \rho g x(2L-x)\left[1 + \frac{x(L-\frac{x}{2})}{(L-x)^2}\right]g/\dot{x}^2$

Substitute $\dot{x}^2$, simplify, & solve for $R$ & get

$\underline{R = \rho g x\frac{4L-3x}{2(L-x)}}$, (Less than $R_{a=0}$ of Prob. 4/105 with $x < \frac{2L}{3}$)

▶4/110 $T_1 = p_1 A$, $T_2 = p_2 A$




Power $P = M\omega$

$$M = \frac{40(10^3)}{900(2\pi/60)} = 424 \text{ N·m}$$

$$\Sigma M_O = m'(v_2 d_2 - v_1 d_1)$$

$$424 + 0.3\Delta F = \frac{20}{60}(1000)\left[18(0.2) - (18)(-0.075)\right]$$

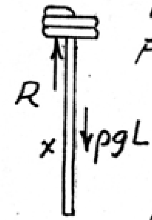
$$\Delta F = \frac{1650 - 424}{0.3} = 4090 \text{ N}$$

Thus $C = 250 + 4090 = \underline{4340 \text{ N}}$ (up)

$D = 4090 - 250 = \underline{3840 \text{ N}}$ (down)

▸4/111 Entire system is conservative so $\Delta V_g + \Delta T = 0$

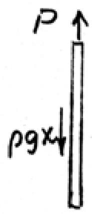
$-\rho g x \frac{x}{2} + \frac{1}{2}\rho x v^2 = 0$, $\underline{v = \sqrt{gx}}$, $a = \dot{v} = \frac{1}{2}\frac{\sqrt{g}}{\sqrt{x}}\sqrt{g}x = g/2$



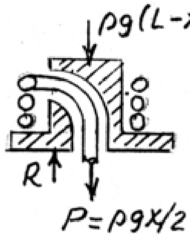

For entire system $\Sigma F_x = \dot{G}_x$

$\rho g L - R = \frac{d}{dt}(\rho x v) = \frac{d}{dt}(\rho \sqrt{g}\, x^{3/2}) = \frac{3}{2}\rho g x$

Thus $R = \rho g L - \frac{3}{2}\rho g x$, $\underline{R = \rho g(L - \frac{3}{2}x)}$




Explanation of $R = 0$ for $x = 2L/3$:
For vertical section $\Sigma F_x = m a_x$; $\rho g x - P = \rho x \frac{g}{2}$
$P = \rho g x/2$. For idealized flow, a frictionless guide must be introduced. Guide & coil of length $L-x$ is isolated. $\Sigma F_x = m' \Delta v_x$ (or $\Sigma F_x = m(0) + \dot{m}u$) gives



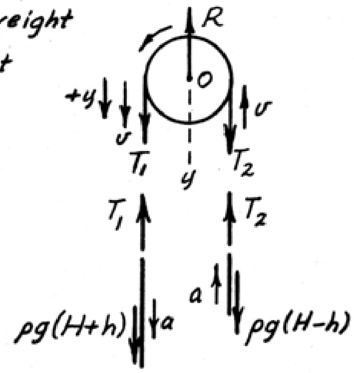

$\rho g x/2 + \rho g(L-x) - R = \rho v(v) = \rho(gx)$

$R = \rho g\left(\frac{x}{2} + L - x - x\right) = \rho g(L - 3x/2)$

& $R = 0$ when $x = 2L/3$ as before.

When $x > 2L/3$, the rate of momentum change requires more force than $P + \rho g(L-x)$ so $R$ reverses.

**▶4/112** With neglect of mass of pulley & weight of small portion of chain in contact with pulley




$\Sigma M_O \approx 0$ so $T_1 = T_2 = T$

$\Sigma F = ma$ for chains

$$\rho g(H+h) - T = \rho(H+h)a \quad \text{---} (1)$$

$$T - \rho g(H-h) = \rho(H-h)a \quad \text{---} (2)$$

Eliminate $T$ from (1) & (2) & get

$$\underline{a = \frac{h}{H}g} \qquad (3)$$

So $T = \rho(H-h)\frac{h}{H}g + \rho g(H-h)$, $T = \rho g\left(H - \frac{h^2}{H}\right)$ (4)

Pulley & chain on it: steady flow gives

$\Sigma F_y = m'\Delta v_y$ : $2T - R = \rho v(v - [-v])$, $R = 2T - 2\rho v^2$ (5)

But $\int_0^v v\,dv = \int_0^h \frac{g}{H}h\,dh$, $v^2 = \frac{g}{H}h^2$, $\underline{v = \sqrt{\frac{g}{H}}\,h}$ (6)

Substitute (4) & (6) into (3) & get

$R = 2\rho g\left(H - \frac{h^2}{H}\right) - 2\rho\frac{g}{H}h^2$, $\underline{R = 2\rho g\left(H - \frac{2h^2}{H}\right)}$