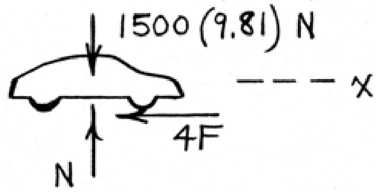
3/1

$$v_2^2 - v_1^2 = 2a(x_2 - x_1)$$

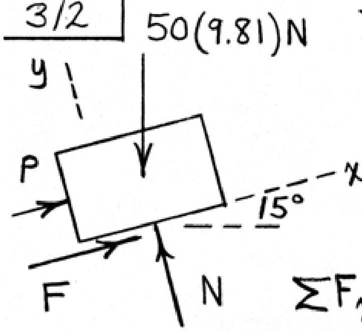
$$0^2 - \left(\frac{100}{3.6}\right)^2 = 2a_x(50), \quad a_x = -7.72 \text{ m/s}^2$$



$$\Sigma F_x = ma_x: \quad -4F = 1500(-7.72)$$

$$\underline{F = 2890 \text{ N}}$$

3/2



$\Sigma F_y = 0: \; N - 50(9.81)\cos 15° = 0$

$N = 474$ N throughout

(a) $P = 0$

Equilibrium check:

$\Sigma F_x = 0: \; F - 50(9.81)\sin 15° = 0$

$F = 127.0$ N

$F_{max} = \mu_s N = 0.2(474) = 94.8 \text{ N} < F$: motion ↙

$\Sigma F_x = ma_x: \; 0.15(474) - 50(9.81)\sin 15° = 50a_x$

$\underline{a_x = -1.118 \text{ m/s}^2}$

(b) $P = 150$ N; Equilibrium check:

$\Sigma F_x = 0: \; 150 + F - 50(9.81)\sin 15° = 0$

$F = -23.0$ N, $|F| < F_{max}$ so no motion: $\underline{a = 0}$

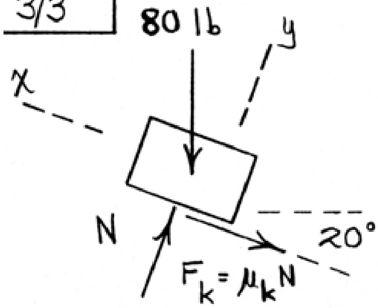
(c) $P = 300$ N; Equilibrium check yields $F = -173.0$ N

$|F| > F_{max}$, so motion ↗, $F = F_k$ ↙.

$\Sigma F_x = ma_x: \; 300 - 0.15(474) - 50(9.81)\sin 15° = 50a_x$

$\underline{a_x = 2.04 \text{ m/s}^2}$

3/3



$\Sigma F_y = 0: \; N - 80 \cos 20° = 0$

$N = 75.2 \text{ lb}$

$\Sigma F_x = ma_x:$

$-0.25(75.2) - 80 \sin 20° = \frac{80}{32.2} a$

$a = -18.58 \text{ ft/sec}^2$

$v = v_0 + at: \quad 0 = +30 - 18.58t, \quad \underline{t = 1.615 \text{ sec}}$

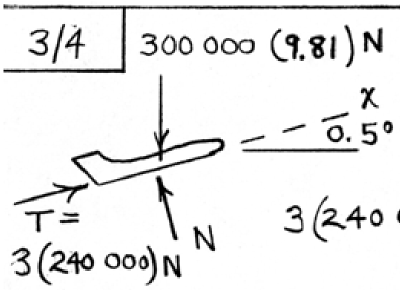
$v^2 = v_0^2 + 2a(s - s_0): \quad 0^2 = 30^2 + 2(-18.58)\,d$

$\underline{d = 24.2 \text{ ft}}$

$v^2 = v_0^2 + 2a(s - s_0): \quad 15^2 = 30^2 + 2(-18.58)\,d'$

$\underline{d' = 18.17 \text{ ft}}$

3/4

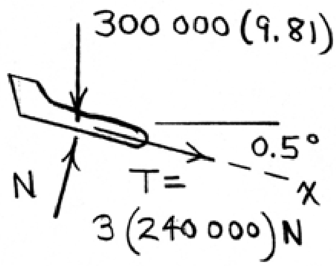


$\sum F_x = ma_x$:

$3(240\,000) - 300\,000(9.81)\sin\frac{1}{2}^\circ = 300\,000\, a_x$

$a_x = 2.31\ \text{m/s}^2$

$v^2 = 2a_x s$: $(220/3.6)^2 = 2(2.31)s$, $\underline{s_u = 807\ \text{m}}$



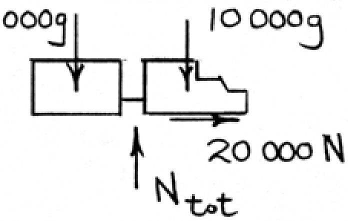
$\sum F_x = ma_x$:

$3(240\,000) + 300\,000(9.81)\sin\frac{1}{2}^\circ = 300\,000\, a_x$

$a_x = 2.49\ \text{m/s}^2$

$v^2 = 2a_x s$: $(220/3.6)^2 = 2(2.49)s$, $\underline{s_d = 751\ \text{m}}$

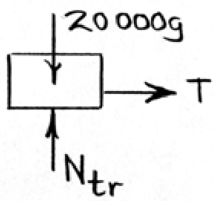
3/5 For entire unit:



$\Sigma F = ma$: $20\,000 = 30\,000a$

$\underline{a = 0.667 \text{ m/s}^2}$

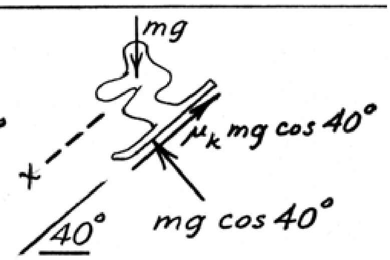
For trailer alone, $T = 20\,000(0.667) = 13\,330$ N



or $\underline{T = 13.33 \text{ kN}}$

3/6

$\Sigma F_x = ma_x: \; mg \sin 40^\circ - \mu_k mg \cos 40^\circ = ma$



$a = 9.81(\sin 40^\circ - \mu_k \cos 40^\circ)$
$= 6.31 - 7.51\mu_k$

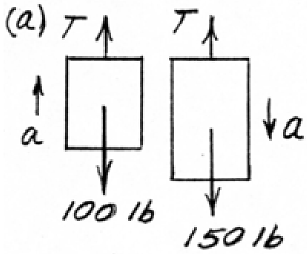
For constant accel. $s = v_0 t + \frac{1}{2} a t^2$:

$20 = 0 + \frac{1}{2}(6.31 - 7.51\mu_k)2.58^2$

$\underline{\mu_k = 0.0395}$

3/7 $\Sigma F = ma$; $T - 100 = \frac{100}{32.2}a$

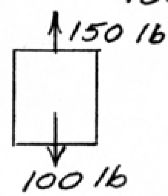
(a)



$150 - T = \frac{150}{32.2}a$

$50 = \frac{250}{32.2}a$, $a = \frac{32.2}{5} = 6.44 \frac{ft}{sec^2}$

(b)

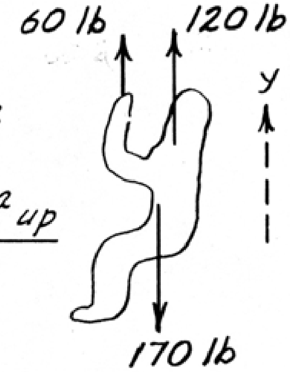


$150 - 100 = \frac{100}{32.2}a$, $a = \frac{32.2}{2} = 16.10 \frac{ft}{sec^2}$

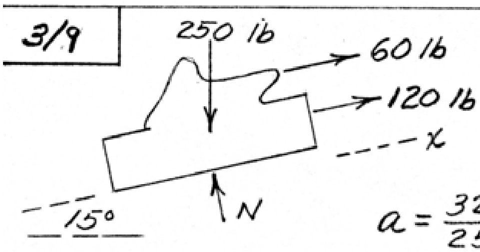
3/8

$$\Sigma F_y = ma_y: \; 180 - 170 = \frac{170}{32.2} a$$

$$\underline{a = 1.894 \text{ ft/sec}^2 \text{ up}}$$



3/9



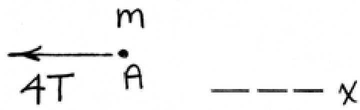
$$\Sigma F_x = m a_x$$

$$60 + 120 - 250 \sin 15^\circ = \frac{250}{32.2} a$$

$$a = \frac{32.2}{250}(180 - 64.7) = \underline{14.85 \text{ ft/sec}^2}$$

3/10

$\overset{+}{\leftarrow} \Sigma F = ma: \quad 4(40{,}000) = \dfrac{750{,}000}{32.2}\, a$



$a = 6.87 \text{ ft/sec}^2$

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = -6.87\underline{i} - \underline{0} = \underline{-6.87\underline{i} \text{ ft/sec}^2}$

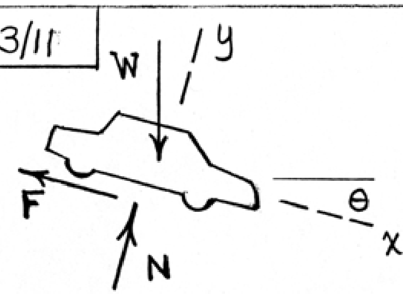
$v_A = (v_A)_0 + at = 0 + 6.87(10) = 68.7 \text{ ft/sec}$

$v_B = 15\left(\dfrac{88}{60}\right) = 22 \text{ ft/sec}$

$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = -68.7\underline{i} - 22(\cos 30°\underline{i} + \sin 30°\underline{j})$

$= \underline{-87.7\underline{i} - 11\underline{j} \text{ ft/sec}}$

3/11



When $\theta = \theta_1$, $a = 0$:

$$\Sigma F_x = 0 = -F + W\sin\theta_1$$

$$F = W\sin\theta_1$$

When $\theta = \theta_2$,

$$\Sigma F_x = ma: \quad W\sin\theta_2 - W\sin\theta_1 = \frac{W}{g}a$$

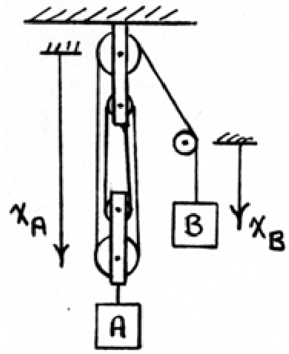
$$\underline{a = g(\sin\theta_2 - \sin\theta_1)}$$

$$\left.\begin{aligned}\theta_1 &= 6^\circ\\ \theta_2 &= 2^\circ\end{aligned}\right\} \quad a = g(\sin 2^\circ - \sin 6^\circ) = \underline{-0.0696\,g}$$

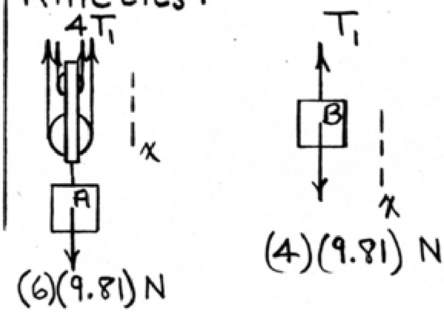
$$\underline{(-2.24\ \text{ft/sec}^2 \text{ or } -0.683\ \text{m/s}^2)}$$

3/12 Kinematics: $4x_A + x_B = L_{rope} + \text{constant}$

$$\therefore \quad 4a_A + a_B = 0 \qquad (1)$$



Kinetics:



$A: \Sigma F_x = ma_x: \quad 6(9.81) - 4T_1 = 6a_A \qquad (2)$

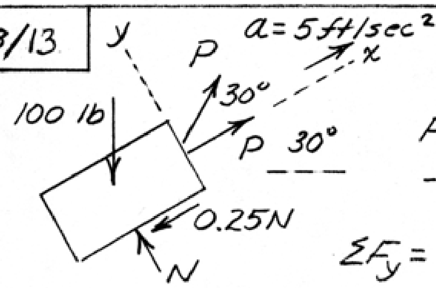
$B: \Sigma F_x = ma_x: \quad 4(9.81) - T_1 = 4a_B \qquad (3)$

Solution of Eqs. (1)–(3):
$$\begin{cases} a_A = -1.401 \text{ m/s}^2 \\ a_B = 5.61 \text{ m/s}^2 \\ \underline{T_1 = 16.82 \text{ N}} \end{cases}$$

Tension in cable above A

$T_2 = 4T_1 = \underline{67.3 \text{ N}}$

3/13



$\Sigma F_x = ma_x$;

$P(1 + \cos 30°) - 0.25N - 100 \sin 30° = \frac{100}{32.2}(5)$

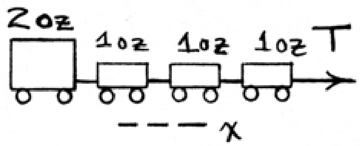
$\Sigma F_y = 0$; $N + P \sin 30° - 100 \cos 30° = 0$

$$\left.\begin{aligned} 1.866P - 0.25N &= 65.53 \\ 0.5P + N &= 86.6 \end{aligned}\right\}$$ solve simultaneously & get $N = 64.7$ lb

$\underline{P = 43.8 \text{ lb}}$

3/14 Coupler 1 will fail first, because it must accelerate more mass than any other coupler.

Rear part of train:

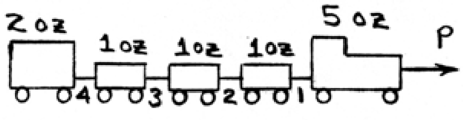


$$\Sigma F_x = ma_x$$

$$T = 0.2 = \left(\frac{5/16}{32.2}\right) a$$

$$a = 20.6 \text{ ft/sec}^2$$

Whole train:

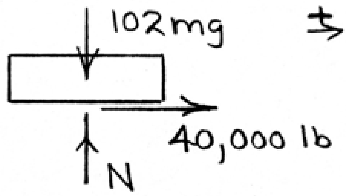


$$\Sigma F_x = ma_x$$

$$P = \left(\frac{10/16}{32.2}\right)(20.6)$$

$$\underline{P = 0.4 \text{ lb}}$$

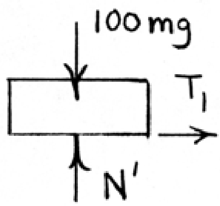
3/15 Let m be the mass of each car and $2m$ that of the locomotive.



$\overset{+}{\rightarrow} \Sigma F = ma$:

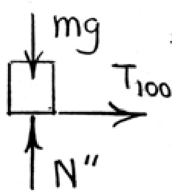
$$40{,}000 = \frac{102(200{,}000)}{32.2} a$$

$$a = 0.0631 \text{ ft/sec}^2$$



$\overset{+}{\rightarrow} \Sigma F = ma$: $T_1 = \dfrac{100(200{,}000)}{32.2} 0.0631$

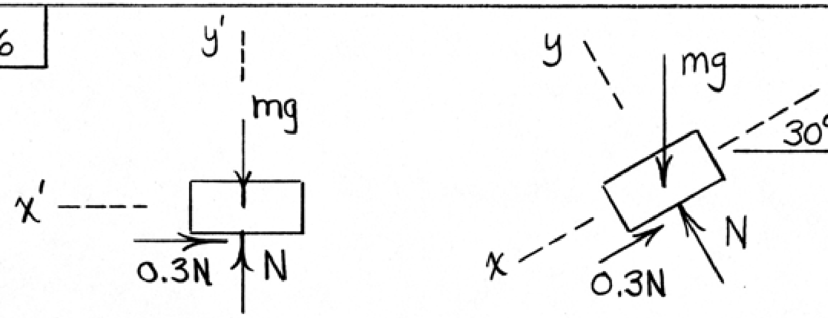
$$\underline{T_1 = 39{,}200 \text{ lb}}$$



$\overset{+}{\rightarrow} \Sigma F = ma$: $T_{100} = \dfrac{1(200{,}000)}{32.2} 0.0631$

$$\underline{T_{100} = 392 \text{ lb}}$$

3/16



A to B:

$\Sigma F_y = 0 \Rightarrow N = 0.866mg$

$\Sigma F_x = ma_x$: $mg\sin 30° - 0.3(0.866mg) = ma$

$a_x = 2.36 \text{ m/s}^2$

$v_B^2 = v_A^2 + 2a_x d$: $v_B^2 = 0.8^2 + 2(2.36)(2)$

$v_B = 3.17 \text{ m/s}$

B to C:

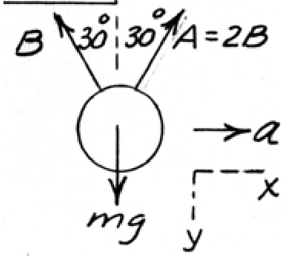
$\Sigma F_{y'} = 0 \Rightarrow N = mg$

$\Sigma F_{x'} = ma_{x'}$: $-0.3(mg) = ma_{x'}$, $a_{x'} = -2.94 \text{ m/s}^2$

$v_C^2 = v_B^2 + 2a_{x'}s$: $0 = 3.17^2 - 2(2.94)s$

$\underline{s = 1.710 \text{ m}}$

3/17



$\Sigma F_x = ma_x$; $2B \sin 30° - B \sin 30°$

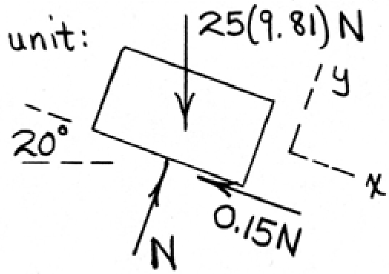
$= ma$

$\Sigma F_y = 0$; $2B \cos 30° + B \cos 30°$

$- mg = 0$

Eliminate B & set $\underline{a = g/3\sqrt{3}}$

3/18 Frame & sphere as a unit:



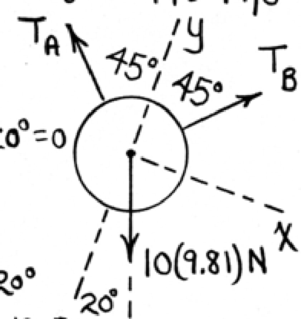
$\Sigma F_y = 0: \; N - 25(9.81)\cos 20° = 0$

$N = 230 \text{ N}$

$\Sigma F_y = ma_x:$

$25(9.81)\sin 20° - 0.15(230) = 25a, \quad a = 1.973 \text{ m/s}^2$

Sphere alone:



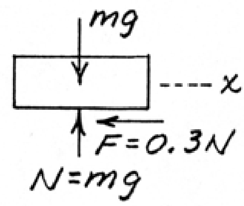
$\Sigma F_y = 0: \; (T_A + T_B)\cos 45° - 10(9.81)\cos 20° = 0$

$T_A + T_B = 130.4 \text{ N}$

$\Sigma F_x = ma_x: \; (T_B - T_A)\sin 45° + 98.1\sin 20°$
$= 10(1.973), \text{ or } T_B - T_A = -19.56 \text{ N}$

Solution: $\underline{T_A = 75.0 \text{ N}}, \quad \underline{T_B = 55.4 \text{ N}}$

3/19 Let m = mass of crate



$\Sigma F_x = ma_x$; $-0.3mg = ma_x$

$a_x = -0.3g = -0.3(9.81) = -2.94 \text{ m/s}^2$

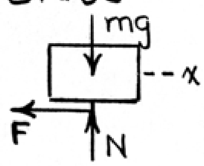
$\int_v^0 v\,dv = \int_0^s a_x\,dx$; $-\frac{v^2}{2} = a_x s$

$s = \frac{-(70/3.6)^2/2}{-2.94} = \underline{64.3 \text{ m}}$

3/20 Truck :

$$\begin{cases} v^2 - v_0^2 = 2a_T(x - x_0) \\ 0^2 - (19.44)^2 = 2a_T(50-0) \\ a_T = -3.78 \text{ m/s}^2 \end{cases}$$

Crate :



$\Sigma F_x = ma_x : \; -F = m(-3.78)$

$F = 3.78\, m$

$F_{max} = \mu_s N = 0.3(m\, 9.81) = 2.94\, m$

$F > F_{max}$, crate slips , $F = \mu_k N$

$\therefore \; \Sigma F_x = ma_x : \; -0.25\, mg = ma_c \, , \; a_c = -2.45 \text{ m/s}^2$

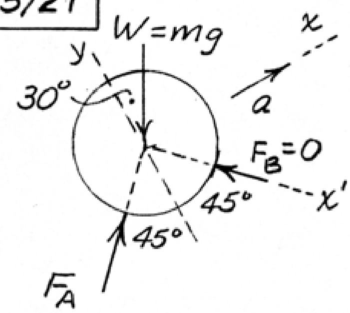
$a_{c/T} = a_c - a_T = -2.45 - (-3.78) = 1.328 \text{ m/s}^2$

$v_{c/T}^2 - v_{c/T_0}^2 = 2a_{c/T}(x_{c/T} - x_{c/T_0})$

$v_{c/T}^2 - 0^2 = 2(1.328)(3-0)$, $\underline{v_{c/T} = 2.82 \text{ m/s}}$

(Truck stopping time = 5.14 s, crate impacts at 2.13 s)

3/21



$\Sigma F_{x'} = m a_{x'}$;

$mg \cos(45° + 30°) = m a \cos 45°$

$a = g \dfrac{\cos 75°}{\cos 45°} = 9.81 \dfrac{0.2588}{0.7071}$

$= \underline{0.366g}$

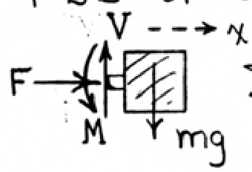
3/22

$$x = X \sin \omega t$$

$$\dot{x} = X \omega \cos \omega t$$

$$\ddot{x} = -X \omega^2 \sin \omega t, \quad \ddot{x}_{max} = X \omega^2$$

FBD of circuit board:

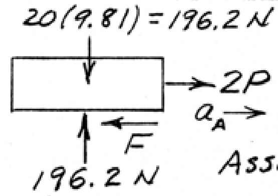


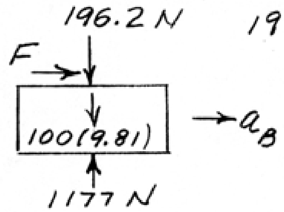
$$\Sigma F_x = m a_x : \quad F = m\left(-X \omega^2 \sin \omega t\right)$$

$$\underline{F_{max} = m X \omega^2}$$

3/23

$\mu = 0.5$





(a) $2P = 120$ N

$F_{max} = 0.5(196.2) = 98.1 \text{ N} < 2P$

Assume slipping occurs & $F = 98.1$ N

A; $\Sigma F = ma$; $120 - 98.1 = 20\,a_A$

$\underline{a_A = 1.095 \text{ m/s}^2}$

B; $\Sigma F = ma$; $98.1 = 100\,a_B$

$a_A > a_B$ so assumption OK.

$\underline{a_B = 0.981 \text{ m/s}^2}$

(b) $2P = 80 \text{ N} < F_{max}$ so no slipping occurs & for block & cart combined,

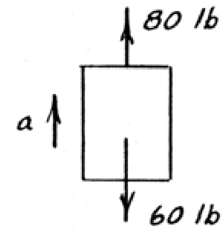
$\Sigma F = ma$; $80 = 120a$, $\underline{a_A = a_B = a = 0.667 \text{ m/s}^2}$

3/24

(a) $\Sigma F = ma$:

$$80 - 60 = \frac{60}{32.2} a,$$

$$\underline{a = 10.73 \text{ ft/sec}^2}$$



(a)

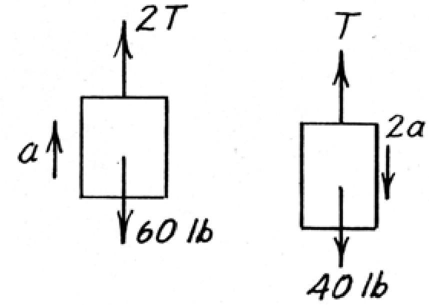
(b) $\Sigma F = ma$:

$$[60\text{-lb}] \quad 2T - 60 = \frac{60}{32.2} a$$

$$[40\text{-lb}] \quad 40 - T = \frac{40}{32.2}(2a)$$

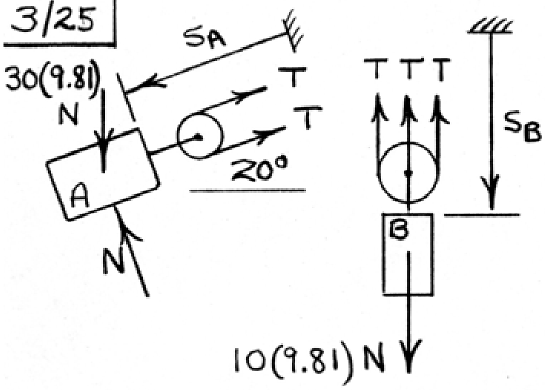
Solve & get $T = 32.7$ lb

$$\underline{a = 2.93 \text{ ft/sec}^2}$$



(b)

3/25



Kinematic constraint: $L = 2s_A + 3s_B$

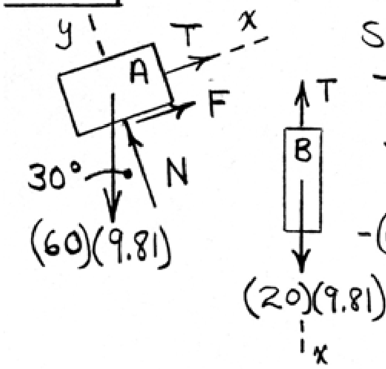
$$\Rightarrow \quad 0 = 2a_A + 3a_B \qquad (1)$$

$\overset{+}{\swarrow}$ $\Sigma F = m_A a_A$: $30(9.81)\sin 20° - 2T = 30a_A \qquad (2)$

$+\downarrow$ $\Sigma F = m_B a_B$: $10(9.81) - 3T = 10a_B \qquad (3)$

Solution of Eqs. (1)-(3): $\begin{cases} \underline{a_A = 1.024 \text{ m/s}^2} \\ \underline{a_B = -0.682 \text{ m/s}^2} \\ T = 35.0 \text{ N} \end{cases}$

3/26



Check for motion. Assume static equilibrium. From B, $T = 196.2$ N. Mass A:

$\Sigma F_x = 0$: $196.2 + F - (60)(9.81)\sin 30° = 0$, $F = 98.1$ N

$F_{max} = \mu_s N = (0.25)(60)(9.81) \times \cos 30° = 127.4$ N (a)

No motion for (a), so $\underline{a = 0}$, $\underline{T = 196.2\text{ N}}$

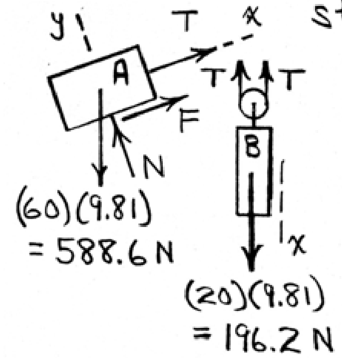
$F_{max} = (0.15)(60)(9.81)\cos 30° = 76.5$ N, motion for (b) ($\swarrow$)

A: $\Sigma F_x = ma_x$: $T - (60)(9.81)\sin 30° + (0.1)(60)(9.81)\cos 30° = 60a$

B: $\Sigma F_y = ma_x$: $(20)(9.81) - T = 20a$

Solution : $\underline{a = -0.589 \text{ m/s}^2}$, $\underline{T = 208 \text{ N}}$

3/27



Check for motion by assuming static equilibrium.

$B:\quad 2T = 196.2\,,\quad T = 98.1\text{ N}$

$A:\quad \Sigma F_x = 0:\quad 98.1 - 588.6\sin 30^\circ + F = 0\,,\quad F = 196.2\text{ N}$

$F_{max} = \mu_s N = (0.25)(588.6)\cos 30^\circ = 127.4\text{ N}$

$F > F_{max} \Rightarrow$ motion $(\swarrow)$

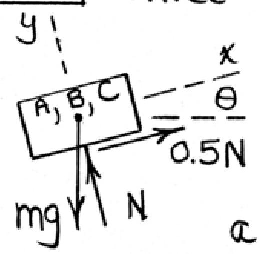
From kinematics, $a_A = 2a_B = 2a$

$A:\ \Sigma F_x = ma_x:\ T + 0.2\,(588.6\cos 30^\circ) - 588.6\sin 30^\circ = 60\,(2a)$

$B:\ \Sigma F_x = ma_x:\ -2T + 196.2 = 20\,a$

Solution: $\underline{a = -0.725\text{ m/s}^2}$, $\underline{T = 105.4\text{ N}}$

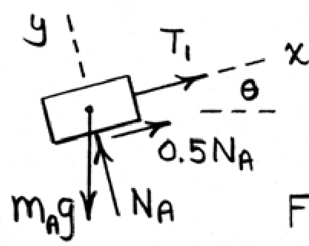
3/28 Three-car unit: $\left(\theta = \tan^{-1}\left(\frac{5}{100}\right) = 2.86°\right)$



$$\begin{cases} \Sigma F_y = 0 \Rightarrow N = mg\cos\theta \\ \Sigma F_x = ma_x : 0.5mg\cos\theta - mg\sin\theta = ma \end{cases}$$

$$a = g(0.5\cos\theta - \sin\theta) = \underline{4.41 \text{ m/s}^2}$$

Car A:



$\Sigma F_y = 0$: $N_A = m_A g\cos\theta$

$\Sigma F_x = ma_x$: $T_1 + 0.5m_A g\cos\theta - m_A g\sin\theta = m_A(4.41)$, $\underline{T_1 = 0}$

From FBD of Car C, $\underline{T_2 = 0}$

By similar analyses:

	(b)	(c)	(d)
a	2.78 m/s^2	2.78 m/s^2	2.78 m/s^2
T_1	32 700 N (T)	16 330 N (C)	16 330 N (C)
T_2	16 330 N (T)	16 330 N (T)	32 700 N (C)

3/29 $\overset{x}{\rightarrow}$ $\underset{D}{\leftarrow}$ (Neglect weight for now)

$$\Sigma F_x = ma_x : \; -D = -C_D \tfrac{1}{2} \rho v^2 S = mv \frac{dv}{dx}$$

$$\int_0^x \left(-C_D \tfrac{1}{2} \rho S\right) dx = m \int_{v_0}^{v} \frac{dv}{v}$$

$$\Rightarrow \quad v = v_0 \, e^{\left(-\frac{1}{2} C_D \rho S x / m\right)}$$

$$= v_0 \, e^{\left(-\frac{1}{2}(0.3)\left(\frac{0.07530}{32.2}\right)(\pi)\left(\frac{9.125/2\pi}{12}\right)^2 x \Big/ \frac{5.125}{16 \cdot 32.2}\right)}$$

$$\underline{v = v_0 \, e^{-1.623(10^{-3})x}}$$

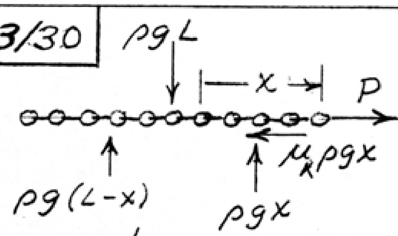
For $v_0 = 90$ mi/hr and $x = 60$ ft: $\underline{v = 81.7 \text{ mi/hr}}$

Comment on y-motion. Assume $v = 90$ mi/hr = constant. Time t to plate is

$$t = \frac{60}{90\left(5280/3600\right)} = 0.455 \text{ sec}$$

$v_y = v_{y_0} - gt = -32.2(0.455) = -14.64$ ft/sec, which would not appreciably change $v = \sqrt{v_x^2 + v_y^2}$.

3/30



$\Sigma F_x = m a_x$;

$P - \mu_k \rho g x = \rho L a_x$

$\int_0^v v\,dv = \int_0^L a_x\,dx$

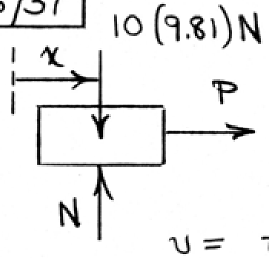
$\frac{v^2}{2} = \int_0^L \left(\frac{P}{\rho L} - \frac{\mu_k g x}{L}\right) dx = \frac{P}{\rho} - \frac{\mu_k g L}{2}$, $\underline{v = \sqrt{\frac{2P}{\rho} - \mu_k g L}}$

From $\frac{v^2}{2} = \int_0^x \left(\frac{P}{\rho L} - \frac{\mu_k g x}{L}\right) dx$, we obtain

$v(x) = \sqrt{2 \frac{x}{L}\left(\frac{P}{\rho} - \mu_k g \frac{x}{2}\right)}$

Note that $v(L) \geq 0$ if $\underline{P \geq \mu_k \rho g \frac{L}{2} = P_{min}}$

3/31



$\Sigma F_x = ma_x : P = 10a_x$

$\frac{P}{10} = \frac{dv}{dt}$, $v = \int_0^t \frac{P}{10}\, dt$

For $P_1 = 10t$:

$v = t^2/2$, $s = t^3/6$

At $t = 5$ s, $\underline{v = 12.5 \text{ m/s}}$, $\underline{s = 20.8 \text{ m}}$

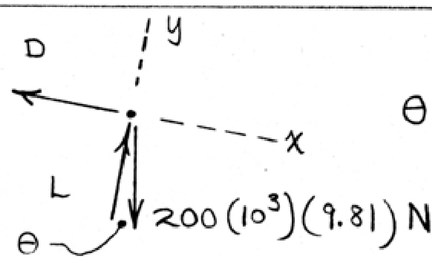
For $P_2 = kt^2$: $50 = k(5)^2$, $k = 2 \text{ N/s}^2$

So $P_2 = 2t^2$

$v = \int_0^t \frac{2t^2}{10}\, dt = \frac{t^3}{15}$, $s = t^4/60$

At $t = 5$s, $\underline{v = 8.33 \text{ m/s}}$, $\underline{s = 10.42 \text{ m}}$

3/32



$$\theta = \tan^{-1}\frac{250}{2000} = 7.13°$$

$$v_B^2 - v_A^2 = 2a_x(s_B - s_A):$$

$$\left(\frac{200}{3.6}\right)^2 - \left(\frac{300}{3.6}\right)^2 = 2a_x\left(\frac{2000}{\cos 7.13°}\right),\quad a_x = -0.957\ \text{m/s}^2$$

$$\Sigma F_x = ma_x:\ -D + 200(10^3)(9.81)\sin 7.13° = 200(10^3)(-0.957),\quad \underline{D = 435\ \text{kN}}$$

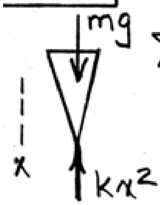
$$\Sigma F_y = 0:\ L - 200(10^3)(9.81)\cos 7.13° = 0$$

$$\underline{L = 1.947\ \text{MN}}$$

The net aerodynamic force is then

$$R = \sqrt{L^2 + D^2} = \sqrt{1.947^2 + 0.435^2} = \underline{1.995\ \text{MN}}$$

3/33 FBD of cone during penetration:



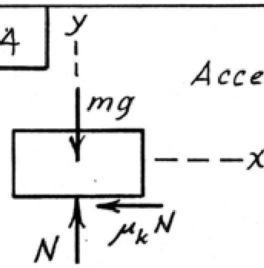
$$\Sigma F_x = ma_x: \quad mg - kx^2 = mv\,\frac{dv}{dx}$$

$$\int_0^d \left(g - \frac{k}{m}x^2\right)dx = \int_{v_0}^{0} v\,dv$$

$$gd - \frac{k}{3m}d^3 = -\frac{v_0^{\,2}}{2}, \quad \text{where } v_0 = \sqrt{2gh}$$

$$\therefore \quad \underline{k = \frac{3mg}{d^3}(h+d)}$$

3/34



Accel. down: $\Sigma F_y = ma_y$: $-mg + N = -ma$,

$N = m(g-a)$

$\Sigma F_x = ma_x$: $-\mu_k m(g-a) = ma_x$,

$a_x = -\mu_k (g-a)$

Accel. up: $\Sigma F_y = ma_y$: $N - mg = ma$,

$N = m(g+a)$

$\Sigma F_x = ma_x$: $-\mu_k m(g+a) = ma_x$,

$a_x = -\mu_k (g+a)$

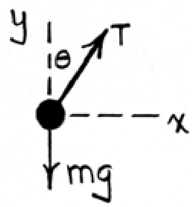
$v^2 = v_0^2 + 2as$: Down: $0 = v^2 - 2\mu_k (g-a) s_1$

Up: $0 = v^2 - 2\mu_k (g+a) s_2$

Eliminate v^2 & get $(g-a)s_1 = (g+a) s_2$,

$$\underline{a = g\frac{s_1 - s_2}{s_1 + s_2}}$$

3/35 Mass m :

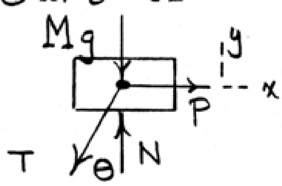


$$\Sigma F_y = 0: \quad T\cos\theta - mg = 0$$
$$T = mg/\cos\theta$$

$$\Sigma F_x = ma_x: \quad T\sin\theta = ma$$

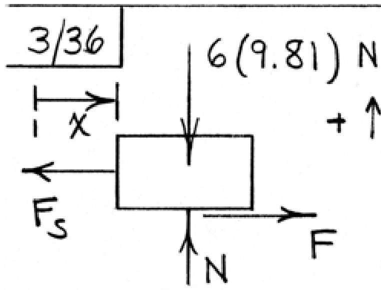
$$\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \cancel{m}a, \quad \underline{\theta = \tan^{-1}\left(\frac{a}{g}\right)}$$

Cart M :



$$\Sigma F_x = ma_x: \quad P - T\sin\theta = Ma$$
$$P = ma + Ma = \underline{(m+M)a}$$

3/36



$F_s = 150x + 400x^2$ (N)

$+\uparrow \Sigma F = 0 \Rightarrow \quad N = 58.9$ N

$F_{max} = \mu_s N = 0.30(58.9)$
$= 17.66$ N

(a) $x = 50$ mm : $\quad F_s = 150(0.050) + 400(0.050)^2$
$= 8.5 \text{ N} < F_{max}$

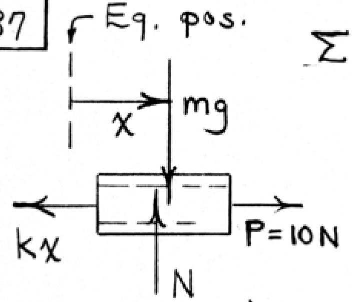
So $\underline{a = 0}$

(b) $F_s = 150(0.1) + 400(0.1)^2 = 19 \text{ N} > F_{max}$

$\Sigma F_x = ma_x : \; -19 + 0.25(58.9) = 6a$

$\underline{a = -0.714 \text{ m/s}^2}$

3/37



$$\Sigma F_x = m\ddot{x}: \; P - kx = m\ddot{x}$$

$$10 - 200x = 2\ddot{x}$$

$$\ddot{x} = 5 - 100x$$

$$5 - 100x = v\frac{dv}{dx}$$

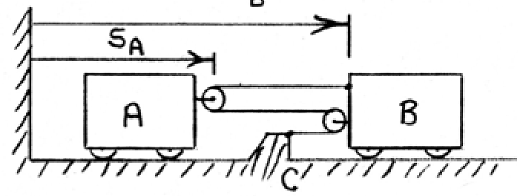
$$\int_0^{0.040} (5 - 100x)\,dx = \int_0^v v\,dv, \quad 5x - 50x^2\Big|_0^{0.040} = \frac{v^2}{2}\Big|_0^v$$

$$\underline{v = 0.490 \text{ m/s}}$$

$$5x - 50x^2 = 0$$

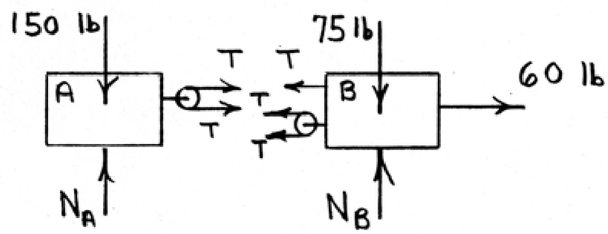
$x = 0$ (initial condition) or $x = 0.10$ m or $\underline{100 \text{ mm}}$

3/38



$$L = 2(s_B - s_A) + (s_B - s_C) + \text{constants}$$

$$\Rightarrow \quad 0 = 3a_B - 2a_A \qquad (1)$$

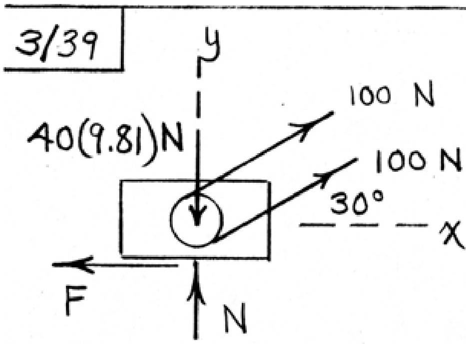


$\xrightarrow{+} \Sigma F = ma$: (A) $\quad 2T = \frac{150}{32.2} a_A \qquad (2)$

(B) $\quad 60 - 3T = \frac{75}{32.2} a_B \qquad (3)$

Solve Eqs. (1) – (3) : $\begin{cases} a_A = 7.03 \text{ ft/sec}^2 \\ a_B = 4.68 \text{ ft/sec}^2 \\ T = 16.36 \text{ lb} \end{cases}$

3/39



$\mu_s = 0.5$
$\mu_k = 0.4$

$\Sigma F_y = 0: \quad N + 200 \sin 30° - 40(9.81) = 0$

$N = 292 \text{ N}$

Assume static equilibrium:

$\Sigma F_x = 0: \quad -F + 200 \cos 30° = 0, \quad F = 173.2 \text{ N}$

$F_{max} = \mu_s N = 0.5(292) = 146.2 \text{ N} < F$

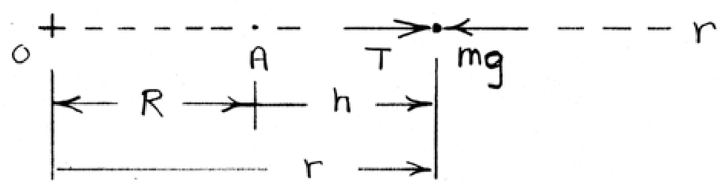
Assumption wrong, motion exists $\rightarrow$.

$F = \mu_k N = 0.4(292) = 117.0 \text{ N}$

$\Sigma F_x = ma_x: \quad -117 + 200 \cos 30° = 40 a_x$

$a_x = a = 1.406 \text{ m/s}^2$

3/40



$g = g_0 \frac{R^2}{r^2}$ (all pertaining to moon)

$\Sigma F_r = ma_r : \quad T - mg_0 \frac{R^2}{r^2} = mv \frac{dv}{dr}$

$$\int_R^{2R} \left(\frac{T}{m} - g_0 \frac{R^2}{r^2} \right) dr = \int_0^v v\, dv$$

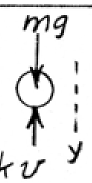
$$\Rightarrow \quad v = \sqrt{\frac{2TR}{m} - g_0 R} = \sqrt{R\left(\frac{2T}{m} - g_0\right)}$$

Numbers :

$$v = \sqrt{\frac{3476(1000)}{2}\left(\frac{2(2500)}{1200} - 1.62\right)}$$

$$= \underline{2100 \text{ m/s}}$$

3/41



$R = kv$

$\Sigma F_y = ma_y;\quad mg - kv = ma$

$$a = g - \frac{k}{m}v$$

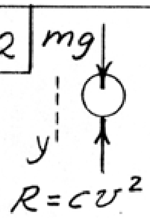
$$v\,dv = a\,dy, \quad \int_0^v \frac{v\,dv}{g - \frac{k}{m}v} = \int_0^h dy$$

$$\frac{m^2}{k^2}\left[\left(g - \frac{k}{m}v\right) - g\ln\left(g - \frac{k}{m}v\right)\right]_0^v = h$$

$$h = \frac{m^2}{k^2}\left[-\frac{k}{m}v - g\ln\left(1 - \frac{kv}{mg}\right)\right]$$

$$h = \frac{m^2}{k^2}\, g\ln\left(\frac{1}{1 - \frac{kv}{mg}}\right) - \frac{mv}{k}$$

3/42



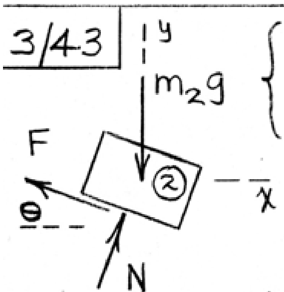
$$\Sigma F_y = ma_y; \quad mg - cv^2 = ma$$

$$a = g - \frac{c}{m}v^2$$

$$v\,dv = a\,dy, \quad \int_0^v \frac{v\,dv}{g - \frac{c}{m}v^2} = \int_0^h dy$$

$$-\frac{m}{2c}\ln\left(g - \frac{c}{m}v^2\right)\Big]_0^v = h, \quad h = \frac{m}{2c}\ln\left(\frac{mg}{mg - cv^2}\right)$$

3/43



$$\begin{cases} \Sigma F_x = ma_x : -F\cos\theta + N\sin\theta = m_2 a \\ \Sigma F_y = 0 : \; F\sin\theta + N\cos\theta - m_2 g = 0 \end{cases}$$

Solve to obtain

(slipping impends ↘)
$$\begin{cases} F = m_2(g\sin\theta - a\cos\theta) \\ N = m_2(a\sin\theta + g\cos\theta) \end{cases}$$

For impending slip, $F = \mu_s N$, or

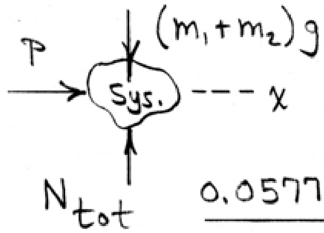
$$m_2(g\sin\theta - a\cos\theta) = \mu_s m_2(a\sin\theta + g\cos\theta)$$

Solving for a: $\quad a = g\,\dfrac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}$

With numbers, $a = 0.0577g$ (Note: $\tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70°$)

Let slipping impend up the inclined block (reverse F on above FBD) & obtain

$$a = g\,\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = 0.745g$$

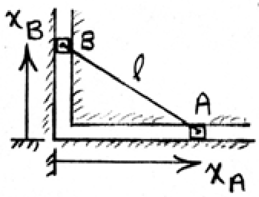


$\Sigma F_x = ma_x : P = (m_1+m_2)a$

So

$$\underline{0.0577(m_1+m_2)g \le P \le 0.745(m_1+m_2)g}$$

3/44



$$x_A^2 + x_B^2 = l^2$$

$$\cancel{2} x_A \dot{x}_A + \cancel{2} x_B \dot{x}_B = 0$$

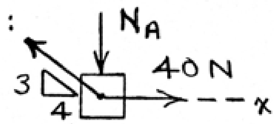
$$x_A \ddot{x}_A + \dot{x}_A^2 + x_B \ddot{x}_B + \dot{x}_B^2 = 0$$

So $\dot{x}_B = -\dfrac{x_A \dot{x}_A}{x_B} = \dfrac{-(0.4)(0.9)}{0.3} = -1.2$ m/s

$$\ddot{x}_B = \frac{-\dot{x}_B^2 - \dot{x}_A^2 - x_A \ddot{x}_A}{x_B} = \frac{-1.2^2 - 0.9^2 - 0.4\ddot{x}_A}{0.3}$$

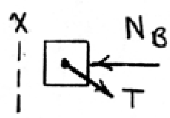
$$= -7.5 - \frac{4}{3}\ddot{x}_A \quad \text{or} \quad a_B = -7.5 - \frac{4}{3}a_A \quad (1)$$

A:



$$\Sigma F_x = ma_x: \; 40 - \frac{4}{5}T = 2a_A \quad (2)$$

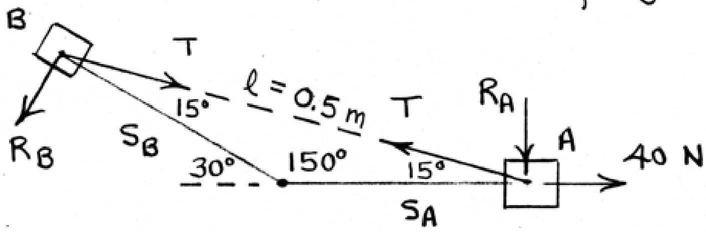
B:



$$\Sigma F_x = ma_x: \; -\frac{3}{5}T = 3a_B \quad (3)$$

Solution of Eqs. (1)-(3): $\begin{cases} a_A = 1.364 \text{ m/s}^2 \\ a_B = -9.32 \text{ m/s}^2 \\ T = 46.6 \text{ N} \end{cases}$

3/45 $\sin 150^\circ / l = \sin 15^\circ / s_B$, $s_B = s_A = 0.259$ m



Law of cosines: $l^2 = s_A^2 + s_B^2 - 2 s_A s_B \cos 150^\circ$

$$2l\dot{l} = 0 = 2 s_A v_A + 2 s_B v_B - 2\left(-\frac{\sqrt{3}}{2}\right)(s_A v_B + s_B v_A)$$

$$s_A v_A + s_B v_B + \frac{\sqrt{3}}{2}(s_A v_B + v_A s_B) = 0^*$$

With $s_A = s_B = 0.259$ m, $v_A = 0.4$ m/s : $v_B = -0.4$ m/s

Differentiate *: $v_A^2 + s_A a_A + v_B^2 + s_B a_B + \frac{\sqrt{3}}{2}(s_A a_B + v_A v_B + a_A s_B + v_A v_B) = 0$

Numbers: $0.483 a_A + 0.483 a_B + 0.0429 = 0$ (1)

Kinetics:

$\overset{+}{\nwarrow}$ $\Sigma F = m a_B$: $-T \cos 15^\circ = 3 a_B$ (2)

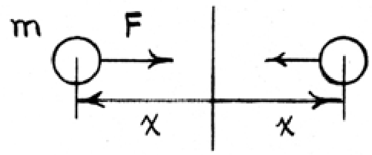
$\xrightarrow{+}$ $\Sigma F = m a_A$: $40 - T\cos 15^\circ = 2 a_A$ (3)

Solution of Eqs. (1)-(3): $T = 25.0$ N

$a_A = 7.95$ m/s^2

$a_B = -8.04$ m/s^2

▶3/46



$$F = \frac{Gm^2}{x^2}$$

$$m = \rho V = 7210\left(\tfrac{4}{3}\pi\, 0.05^3\right)$$
$$= 3.775 \text{ kg}$$

$$\Sigma F_x = ma_x: \; -\frac{Gm^2}{(2x)^2} = mv\frac{dv}{dx}$$

$$-\frac{Gm}{4}\int_{x_0=0.5}^{x}\frac{dx}{x^2} = \int_{v_0=0}^{v} v\,dv$$

$$v = \sqrt{Gm}\sqrt{\frac{1}{2x}-1} = \sqrt{(6.673\times10^{-11})(3.775)}\sqrt{\frac{1}{2(0.05)}-1}$$
$$= \underline{4.76\times10^{-5} \text{ m/s}}$$

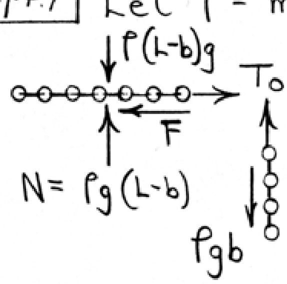
Now, $\dfrac{dx}{dt} = -\sqrt{Gm}\sqrt{\dfrac{\frac{1}{2}-x}{x}}$

$$\int_{x_0=0.5}^{x=0.05}\frac{\sqrt{x}\,dx}{\sqrt{\frac{1}{2}-x}} = -\sqrt{Gm}\int_0^t dt$$

$$\left[-\sqrt{x}\sqrt{\tfrac{1}{2}-x}+\tfrac{1}{2}\sin^{-1}\sqrt{2x}\right]_{x_0=0.5}^{x=0.05} = -\sqrt{Gm}\,t$$

Solving, $\underline{t = 48{,}800 \text{ s}}$ or $\underline{t = 13 \text{ hr } 33 \text{ min}}$

▸3/47 Let ρ = mass/length. Length b to get started:

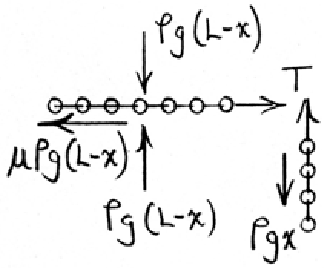


$$F = \mu N = \mu g \rho (L-b)$$

$$\Sigma F = 0: \ T_0 - \mu g \rho (L-b) = 0$$

and $T_0 = \rho g b$

Solve to obtain $b = \dfrac{\mu L}{1+\mu}$



$$\Sigma F = ma: \ T - \mu \rho g (L-x) = \rho (L-x) a$$

and $\rho g x - T = \rho x a$

Eliminate T to obtain

$$a = \ddot{x} = \frac{g}{L}\left[x(1+\mu) - \mu L\right]$$

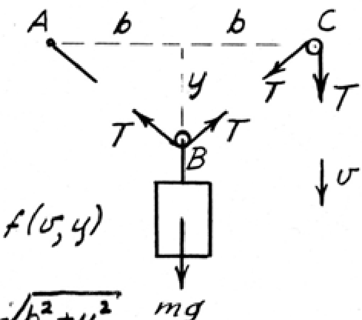
$$v\,dv = \ddot{x}\,dx: \quad \int_0^v v\,dv = \int_b^L \frac{g}{L}\left[x(1+\mu) - \mu L\right] dx$$

$$\frac{1}{2}v^2 = \frac{g}{L}\left[\frac{x^2}{2}(1+\mu) - \mu L x\right]_b^L$$

Substitute $b = \dfrac{\mu L}{1+\mu}$, simplify, and obtain

$$\underline{v = \sqrt{\frac{gL}{1+\mu}}}$$

▸3/48



$$+\uparrow \Sigma F = ma: \; 2T\frac{y}{\sqrt{b^2+y^2}} - mg = ma, \quad a = -\ddot{y}$$

$$T = \frac{m(a+g)\sqrt{b^2+y^2}}{2y} \text{ where } a = f(v, y)$$

Let L = length of cable $ABC = 2\sqrt{b^2+y^2}$

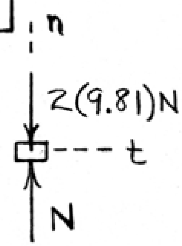
$$-v = \dot{L} = 2\frac{y\dot{y}}{\sqrt{b^2+y^2}}, \quad \ddot{L} = 2\frac{\sqrt{b^2+y^2}\,(\dot{y}^2+y\ddot{y})}{b^2+y^2} - 2\frac{y\dot{y}(y\dot{y})}{(b^2+y^2)\sqrt{b^2+y^2}} = 0$$

$$\text{so } \sqrt{b^2+y^2}\left(\frac{v^2(b^2+y^2)}{4y^2} + y\ddot{y}\right) = \frac{y^2}{\sqrt{b^2+y^2}}\,\frac{v^2(b^2+y^2)}{4y^2}$$

Simplify and get $\ddot{y} = -\dfrac{b^2v^2}{4y^3} = -a$

$$\text{Thus } T = \frac{m\left(g + \frac{b^2v^2}{4y^3}\right)\sqrt{b^2+y^2}}{2y}, \quad \underline{T = \frac{m}{2y}\sqrt{b^2+y^2}\left(g + \frac{b^2v^2}{4y^3}\right)}$$

3/49



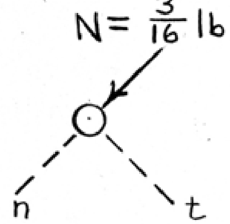
$$\sum F_n = ma_n = m\frac{v^2}{\rho}:$$

$$N - 2(9.81) = 2\,\frac{4^2}{1.5}$$

$$\underline{N = 41.0 \text{ N}} \quad \text{up}$$

Any friction present would not enter the normal equation.

3/50



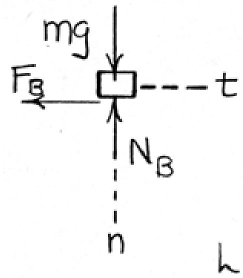
$$\Sigma F_n = ma_n = m\frac{v^2}{\rho}:$$

$$\frac{3}{16} = \frac{2/16}{32.2}\left(\frac{5^2}{\rho}\right)$$

$$\underline{\rho = 0.518 \text{ ft}}$$

3/51

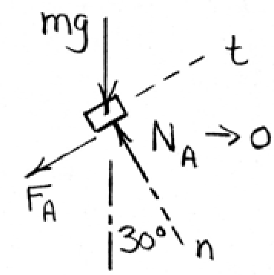
$$\Sigma F_n = ma_n = m\frac{v^2}{\rho}:$$



$$2(9.81) - N = 2\,\frac{3.5^2}{2.4}$$

$$\underline{N_B = 9.41 \text{ N}}$$

Loss of contact at A: $N_A \to 0$

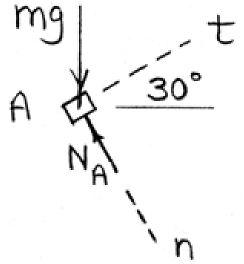


$$\Sigma F_n = ma_n = m\frac{v^2}{\rho}:$$

$$\cancel{m}g \cos 30^\circ = \cancel{m}\frac{v^2}{2.4}$$

$$\underline{v = 4.52 \text{ m/s}}$$

3/52



$\Sigma F_n = ma_n$: $-N_A + mg\cos 30° = m\dfrac{v_A^2}{\rho}$

$N_A = m\left(g\cos 30° - \dfrac{v_A^2}{\rho}\right)$

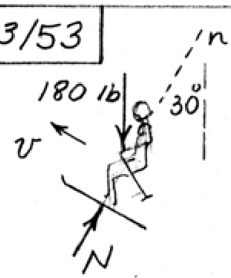
$= 2\left(9.81\cos 30° - \dfrac{4.5^2}{2.4}\right)$

$= \underline{0.1164\text{ N}}$

$\Sigma F_t = ma_t$: $-mg\sin 30° = ma_t$

$a_t = -\dfrac{g}{2} = \underline{-4.90\text{ m/s}^2}$

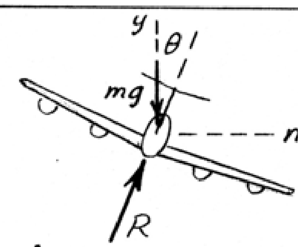
3/53



$\Sigma F_n = ma_n$; $N - 180\cos 30° = \frac{180}{32.2}\frac{(80)^2}{150}$

$N = 180(0.866 + 1.33) = \underline{394 \text{ lb}}$

3/54



$\Sigma F_y = 0$: $R\cos\theta - mg = 0$, $R\cos\theta = mg$

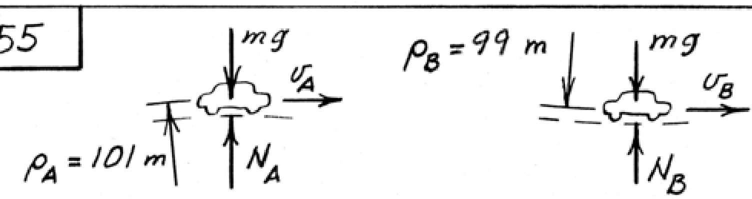
$\Sigma F_n = ma_n$: $R\sin\theta = mv^2/\rho$

Combine & get $\tan\theta = \dfrac{v^2}{\rho g}$, $\theta = \tan^{-1}\dfrac{v^2}{\rho g}$

where $v = \dfrac{400 \times 5280}{3600} = 587$ ft/sec, $\rho = 2 \times 5280 = 10{,}560$ ft

so $\theta = \tan^{-1}\dfrac{587^2}{10{,}560 \times 32.2} = \tan^{-1} 1.012$, $\underline{\theta = 45.3°}$

3/55



$\Sigma F_n = ma_n$:

$$A: \; mg - N_A = m\frac{v_A^2}{\rho_A} \qquad B: \; N_B - mg = m\frac{v_B^2}{\rho_B}$$

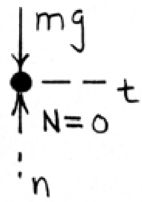
For $N_B = 2N_A$, $\quad m\left(\frac{v_B^2}{\rho_B} + g\right) = 2m\left(g - \frac{v_A^2}{\rho_A}\right)$

$$v_B^2 = \rho_B g - 2v_A^2\frac{\rho_B}{\rho_A} = 99(9.81) - 2\left(\frac{60\times1000}{3600}\right)^2\frac{99}{101}$$

$$= 427 \text{ m}^2/\text{s}^2$$

$v_B = 20.7$ m/s $\quad$ or $\quad \underline{v_B = 74.4 \text{ km/h}}$

3/56 FBD of object inside airplane:



$$\Sigma F_n = ma_n: \quad \cancel{m}g = \cancel{m}\frac{v^2}{\rho}$$

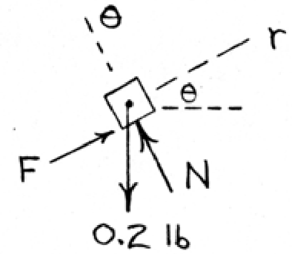
$$\rho = \frac{v^2}{g} = \frac{\left[(600)\left(\frac{5280}{3600}\right)\right]^2}{32.2}$$

$$\underline{\rho = 24{,}050 \text{ ft}}$$

$\boxed{3/57}$ $\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$: $N - 0.2\cos 30°$

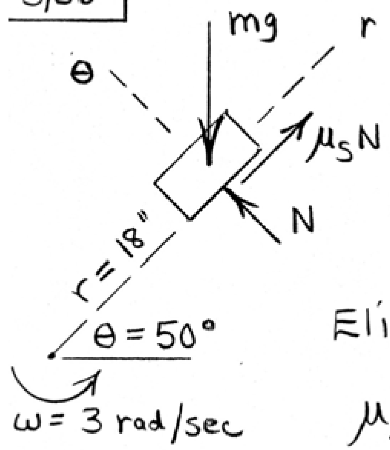
Slider:

$$= \frac{0.2}{32.2}\left(r\cancelto{0}{\ddot{\theta}} + 2(-4)(3)\right)$$



$\underline{N = 0.024 \text{ lb}}$

3/58



$\Sigma F_\theta = ma_\theta : \; N - mg\cos\theta = 0$

$N = mg\cos\theta$

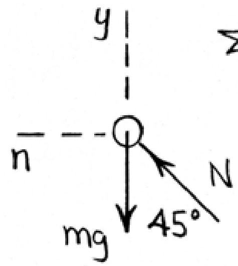
$\Sigma F_r = ma_r : \; \mu_s N - mg\sin\theta = m(0 - r\omega^2)$

Eliminate N:

$$\mu_s = \tan\theta - \frac{r\omega^2}{g\cos\theta}$$

Numbers: $\mu_s = \tan 50° - \dfrac{(18/12)(3)^2}{32.2\cos 50°} = \underline{0.540}$

3/59



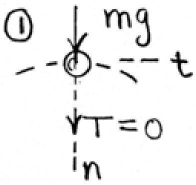
$$\Sigma F_y = 0: \; N\frac{\sqrt{2}}{2} - mg = 0, \quad N = \frac{2}{\sqrt{2}}mg$$

$$\Sigma F_n = ma_n: \; N\frac{\sqrt{2}}{2} = m\left(3R + R\frac{\sqrt{2}}{2}\right)\Omega^2$$

$$\frac{2}{\sqrt{2}}mg\left(\frac{\sqrt{2}}{2}\right) = mR\left(3 + \frac{\sqrt{2}}{2}\right)\Omega^2$$

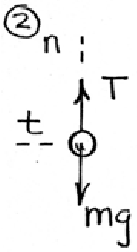
With $R = 0.200$ m, $\underline{\Omega = 3.64 \frac{\text{rad}}{\text{s}}}$

3/60

① 

$$\Sigma F_n = m\frac{v^2}{\rho}: \quad mg = m\frac{v^2}{1}$$

$$\underline{v = \sqrt{g} = 3.13 \text{ m/s}}$$

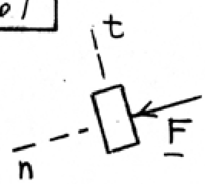
② 

$$\Sigma F_n = m\frac{v^2}{\rho}: \quad T - mg = m\frac{g}{1}$$

$$T = 2mg = 2(0.050)(9.81)$$

$$\underline{= 0.981 \text{ N}}$$

3/61



$$a_n = \frac{v^2}{\rho} = \frac{\left[(35)\left(\frac{5280}{3600}\right)\right]^2}{100}$$

$$= 26.4 \frac{ft}{sec^2}\left(\frac{1g}{32.2\ ft/sec^2}\right)$$

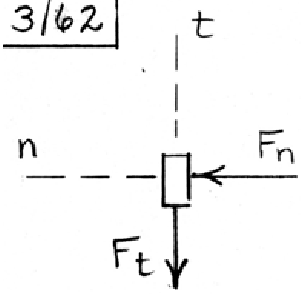
$$= \underline{0.818\ g}$$

$$\Sigma F_n = ma_n : \quad F = \frac{3000}{32.2}(26.4)$$

$$= \underline{2460\ lb}$$

(An average of 614 lb per tire!)

3/62



$\Sigma F_n = ma_n$: $F_n = \dfrac{3000}{32.2} \dfrac{\left(25 \cdot \frac{5280}{3600}\right)^2}{100}$

$F_n = 1253$ lb

$\sqrt{F_n^2 + F_t^2} = F_{tot}$

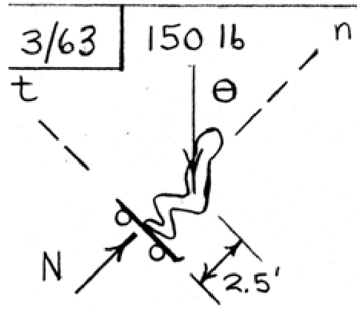
$1253^2 + F_t^2 = 2400^2$

$F_t = 2047$ lb

$\Sigma F_t = ma_t$: $-2047 = \dfrac{3000}{32.2} a_t$

$\underline{a_t = -22.0 \text{ ft/sec}^2}$

3/63



$\rho = 15 - 2.5 = 12.5'$

$$\Sigma F_n = ma_n: \quad N - 150 \cos\theta = \frac{150}{32.2}\frac{v^2}{12.5}$$

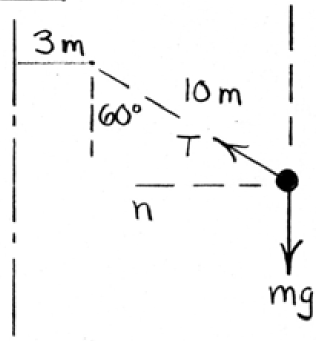
$$N = 150\left(\cos\theta + \frac{v^2}{402.5}\right)$$

$\theta = 0(+)$: $\quad N_0 = 150\left(1 + \frac{28^2}{402.5}\right) = \underline{442 \text{ lb}}$

$\theta = 45°$: $\quad N_{45°} = 150\left(\frac{\sqrt{2}}{2} + \frac{20^2}{402.5}\right) = \underline{255 \text{ lb}}$

$\theta = 90°$: $\quad \underline{N_{90°} = 0}$

3/64



$\Sigma F_n = ma_n$:

$T\sin 60° = m[3 + 10\sin 60°]\omega^2$

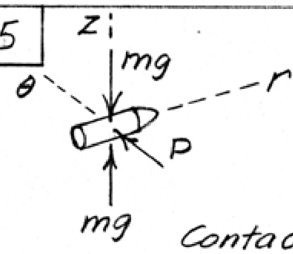
$\Sigma F_y = 0$: $T\cos 60° - mg = 0$

$\Rightarrow \tan 60° = \dfrac{3 + 10\sin 60°}{9.81}\omega^2$

$\omega = 1.207$ rad/s

$N = 1.207\left(\dfrac{60}{2\pi}\right) = \underline{11.53 \text{ rev/min}}$

3/65



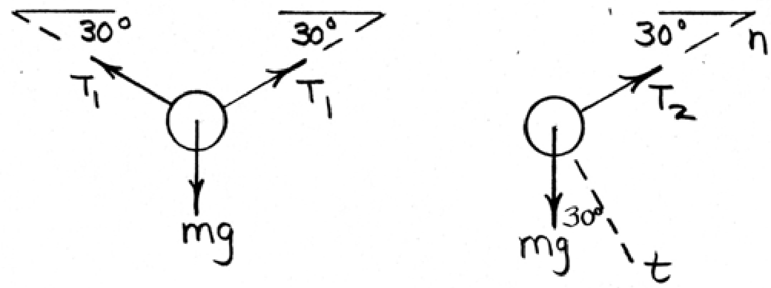
$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$P = 0.06\,(0 + 2[600][0.5])$$

$$= \underline{36\text{ N}}$$

Contact is against <u>right-hand</u> side of barrel.

3/66



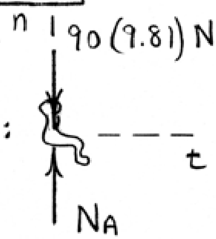
Equilibrium : $\Sigma \underline{F} = \underline{0} \Rightarrow T_1 = mg$

Motion : $\Sigma F_n = ma_n = 0$: $T_2 - mg \sin 30° = 0$

$$k = \frac{T_2}{T_1} = \frac{mg \sin 30°}{mg} = \underline{0.5}$$

3/67

A:

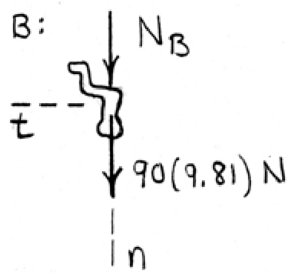


$\Sigma F_n = ma_n$:

$$N_A - 90(9.81) = 90\,\frac{(600/3.6)^2}{1000}$$

$$\underline{N_A = 3380\ \text{N}}$$

B:



$\Sigma F_n = ma_n$:

$$N_B + 90(9.81) = 90\,\frac{(600/3.6)^2}{1000}$$

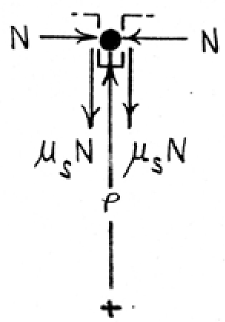
$$\underline{N_B = 1617\ \text{N}}$$

(Note static normal $mg = 90(9.81) = 883$ N)

$\underline{3/68}$ $\omega = \left(4000 \ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)\left(\frac{2\pi \text{ rad}}{\text{sec}}\right)$

$= 418.9 \text{ rad/sec}$

FBD of pebble :



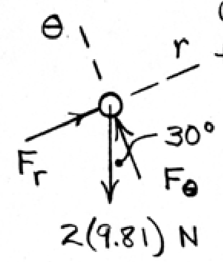
Tire Center

$\Sigma F_n = ma_n : \ 2\mu_s N = m r \omega^2$

$$N = \frac{m r \omega^2}{2\mu_s} = \frac{(0.010)(0.350)(418.9)^2}{2(0.95)}$$

$$\underline{N = 323 \text{ N}}$$

3/69



F_r and F_θ are the r- and θ-components of the total friction force F.

$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2):$$

$$F_r - 19.62 \sin 30° = 2[0 - 1(-0.873)^2]$$

$$F_r = 8.29 \text{ N}$$

$$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$F_\theta - 19.62 \cos 30° = 2[(1)(3.49) + 2(-0.5)(-0.873)]$$

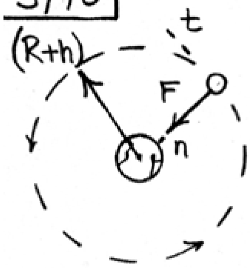
$$F_\theta = 25.7 \text{ N}$$

$$F = \sqrt{F_r^2 + F_\theta^2} = 27.0 \text{ N}$$

$$P = \frac{F/2}{\mu_s} = \frac{27.0/2}{0.5} = \underline{27.0 \text{ N}}$$

(Static gripping force = $\underline{19.62 \text{ N}}$)

3/70



$\Sigma F_n = ma_n$: $F = \dfrac{Gm_e m}{(R+h)^2} = m\dfrac{v^2}{(R+h)}$

But $v = \dfrac{s}{t} = \dfrac{2\pi(R+h)}{(23.944)(3600)}$

Combining the two equations :

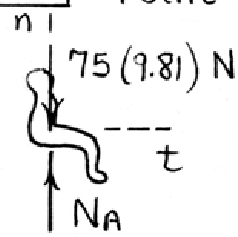
$$v = \frac{2\pi(R+h)}{(23.944)(3600)} = \sqrt{\frac{Gm_e}{(R+h)}}$$

Solve for h to obtain $\underline{h = 3.580 \times 10^7 \text{ m}}$

$(35{,}800 \text{ km})$

3/71

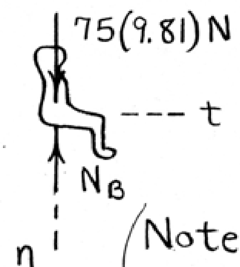
Point A: $\Sigma F_n = ma_n$:



$$N_A - 75(9.81) = 75\,\frac{22^2}{40}$$

$$\underline{N_A = 1643\text{ N}}$$

Point B: $\Sigma F_n = ma_n$:

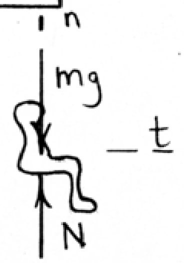


$$75(9.81) - N_B = 75\,\frac{12^2}{20}$$

$$\underline{N_B = 195.8\text{ N}}$$

(Note static normal of magnitude
$N = mg = 75(9.81) = 736\text{ N}$)

3/72



$$\theta = \frac{\pi}{3}\sin 0.950t$$

$$\dot\theta = \frac{\pi}{3}(0.950)\cos 0.950t$$

$$\dot\theta_{max} = \frac{\pi}{3}(0.950) = 0.995 \text{ rad/s}$$

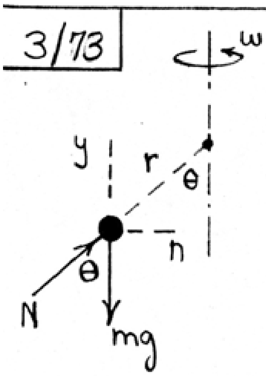
when $\theta = 0$.

$$\Sigma F_n = ma_n: \quad N - mg = mr\dot\theta^2$$

$$N = mg + m(11)(0.995)^2 = \underline{20.7m} \quad \left\{ \begin{array}{l} N \text{ in newtons} \\ \text{when } m \text{ in} \\ \text{kg} \end{array} \right.$$

Riders near center experience the greatest normal force; those at ends of unit experience the smallest normal force when θ is at a maximum (or minimum).

3/73



$\Sigma F_y = 0: \quad N\cos\theta - mg = 0$

$N = mg/\cos\theta$

$\Sigma F_n = ma_n: \quad N\sin\theta = m(r\sin\theta)\omega^2$

$\left(\frac{\cancel{m}g}{\cos\theta}\right)\cancel{\sin}\theta = \cancel{m}r\cancel{\sin}\theta\,\omega^2$

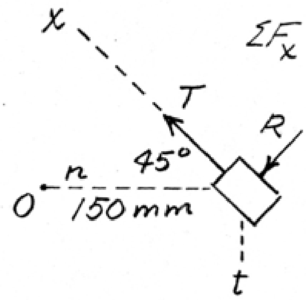
$$\underline{\omega = \sqrt{\frac{g}{r\cos\theta}}}$$

Note that $\cos\theta = \frac{g}{r\omega^2} \leq 1$

$\therefore \quad \omega^2 \geq \frac{g}{r}$ is a restriction.

3/74

$a_n = r\dot{\theta}^2 = 0.15\left(300\,\frac{2\pi}{60}\right)^2 = 148.0\ \text{m/s}^2$

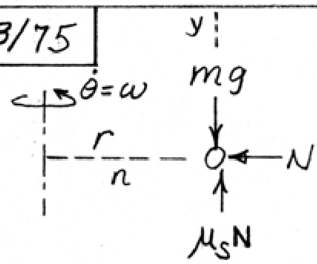


$\Sigma F_x = ma_x;\ T = 3(148.0 \cos 45°)$

$= \underline{314\ \text{N}}$

Direction of rotation does not change accel., hence has no influence on T or R.

3/75

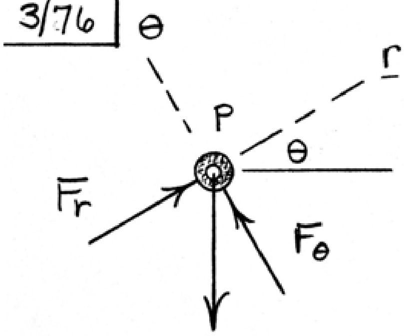


$\Sigma F_n = ma_n$; $N = mr\omega^2$

$\Sigma F_y = 0$; $\mu_s(mr\omega^2) = mg$

$$\omega^2 = \frac{g}{\mu_s r}, \quad \underline{\omega = \sqrt{\frac{g}{\mu_s r}}}$$

3/76



$\theta = 30°$

$\dot{\theta} = 40\left(\frac{\pi}{180}\right) = 0.698 \text{ rad/s}$

$\ddot{\theta} = 120\left(\frac{\pi}{180}\right) = 2.09 \text{ rad/s}^2$

$r = 1.25 \text{ m}$

$\dot{r} = 0.4 \text{ m/s}$

$\ddot{r} = -0.3 \text{ m/s}^2$

$1.2(9.81) = 11.77 \text{ N}$

$\Sigma F_r = ma_r$: $F_r - 11.77 \sin 30° = 1.2\left[-0.3 - 1.25(0.698)^2\right]$

$\underline{F_r = 4.79 \text{ N}}$

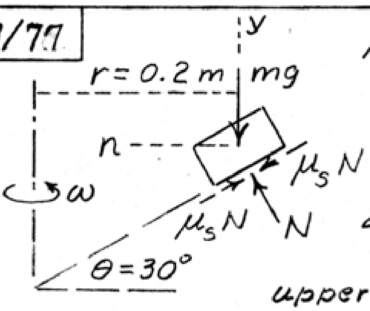
$\Sigma F_\theta = ma_\theta$: $F_\theta - 11.77 \cos 30° = 1.2\left[1.25(2.09) + 2(0.4)(0.698)\right]$

$\underline{F_\theta = 14.00 \text{ N}}$

For static case, set $a_\theta = a_r = 0$ & obtain

$\underline{(F_r)_{st} = 5.89 \text{ N}}$, $\underline{(F_\theta)_{st} = 10.19 \text{ N}}$

3/77



$\mu_s N$ is down for ω_{max}

" " up " ω_{min}

$\Sigma F_y = 0;\ N\cos\theta \mp \mu_s N \sin\theta = mg$

$\Sigma F_n = ma_n;\ N\sin\theta \pm \mu_s N\cos\theta = mr\omega^2$

upper sign for ω_{max}

lower sign for ω_{min}

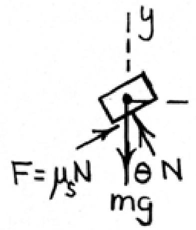
Combine & get $\dfrac{\sin\theta \pm \mu_s \cos\theta}{\cos\theta \mp \mu_s \sin\theta} = \dfrac{r\omega^2}{g}$

$$\omega = \sqrt{\frac{g}{r}}\sqrt{\frac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s \sin\theta}} = \sqrt{\frac{9.81}{0.2}}\sqrt{\frac{0.5 \pm 0.3(0.866)}{0.866 \mp 0.3(0.5)}}$$

Upper sign $\omega_{max} = \underline{7.21\ \text{rad/s}}$

Lower sign $\omega_{min} = \underline{3.41\ \text{rad/s}}$

3/78



$$\begin{cases} \Sigma F_y = 0: & N\cos\theta - mg + \mu_s N \sin\theta = 0 \\ \Sigma F_n = ma_n: & -N\sin\theta + \mu_s N\cos\theta = mr\omega^2 \end{cases}$$

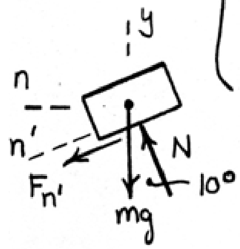
Solving for ω:

$$\omega = \sqrt{\frac{g}{r}\,\frac{(\mu_s\cos\theta - \sin\theta)}{(\cos\theta + \mu_s\sin\theta)}} = \underline{2.73 \text{ rad/s}}$$

3/79

$$\begin{cases} \Sigma F_y = 0: & N\cos 10° - F_{n'}\sin 10° - mg = 0 \\ \Sigma F_n = ma_n: & F_{n'}\cos 10° + N\sin 10° = m\dfrac{(2t)^2}{30} \\ \Sigma F_t = ma_t: & F_t = m(2) \end{cases}$$

Crate:



(t, F_t into paper)

Solve first two equations for N and $F_{n'}$ to obtain

$$F_{n'} = m\left[\frac{4t^2\cos 10°}{30} - g\sin 10°\right]$$

$$N = m\left[\frac{4t^2\sin 10°}{30} + g\cos 10°\right]$$

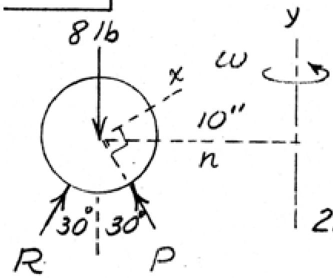
Condition for slipping: $\sqrt{F_t^2 + F_{n'}^2} = \mu_s N$

$$\sqrt{2^2 + \left(\frac{4t^2\cos 10°}{30} - g\sin 10°\right)^2} = 0.3\left[\frac{4t^2\sin 10°}{30} + g\cos 10°\right]$$

Square both sides and solve for t:

$\underline{t = 5.58\text{ s}}$

3/80



$\omega = 30 \times 2\pi/60 = \pi$ rad/sec

Sol. I

$\Sigma F_y = 0;\ (R+P)\cos 30° = 8$

$\Sigma F_n = ma_n;\ (R-P)\sin 30° = \frac{8}{32.2}\,\frac{10}{12}\,(\pi)^2$

$2R = 8\left[\frac{1}{0.866} + \frac{10\pi^2}{32.2(12)}\,\frac{1}{0.5}\right]$

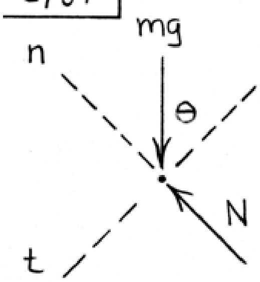
$R = 4(1.155 + 0.511) = \underline{6.66\text{ lb}}$

Sol. II (one force equation)

$\Sigma F_x = ma_x;\ R\cos 30° - 8\sin 30° = \frac{8}{32.2}\,\frac{10}{12}\,(\pi)^2\cos 30°$

$R = 8\left(\tan 30° + \frac{10\pi^2}{32.2\times 12}\right) = \underline{6.66\text{ lb}}$

3/81



Treat the child as a particle.

$$\begin{cases} \Sigma F_t = ma_t : \ mg\cos\theta = ma_t & (1) \\ \Sigma F_n = ma_n : \ N - mg\sin\theta = m\frac{v^2}{R} & (2) \end{cases}$$

From (1) : $g\cos\theta = v\frac{dv}{ds} = v\frac{dv}{R\,d\theta}$

$$\int_{\theta_0 = 20^\circ}^{\theta} Rg\cos\theta\, d\theta = \int_{v_0 = 0}^{v} v\, dv$$

$$v = \left[2Rg\left(\sin\theta - \sin 20^\circ\right)\right]^{1/2}$$

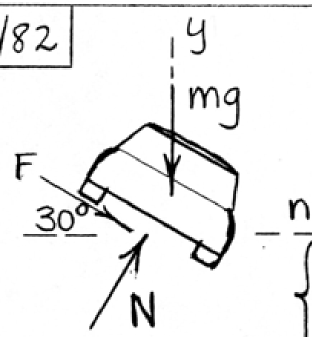
(2) : $N = m\left(g\sin\theta + \frac{v^2}{R}\right)$

Numbers $(R = 2.5\text{ m},\ g = 9.81\text{ m/s}^2)$

$\theta = 30^\circ$: $\begin{cases} \underline{a_t = 8.50\text{ m/s}^2} \\ \underline{v = 2.78\text{ m/s}} \\ \underline{N = 280\text{ N}} \end{cases}$

$\theta = 90^\circ$: $\begin{cases} \underline{a_t = 0} \\ \underline{v = 5.68\text{ m/s}} \\ \underline{N = 795\text{ N}} \end{cases}$

3/82



For no slipping tendency, set F to zero on FBD.

$$\begin{cases} \Sigma F_y = 0: & N\cos 30° - mg = 0 \\ \Sigma F_n = m\frac{v^2}{\rho}: & N\sin 30° = m\frac{v^2}{1200} \end{cases}$$

Solve: $N = 1.155 mg$, $v = 149.4$ ft/sec

or $\underline{v = 101.8 \text{ mi/hr}}$

$\underline{v_{min} = 0}$, as $\theta_{max} = \tan^{-1}\mu_s = \tan^{-1}(0.9)$
$= 42.0° > 30°$

For v_{max}, set $F = F_{max} = \mu_s N$:

$$\begin{cases} \Sigma F_y = 0: & N\cos 30° - mg - \mu_s N\sin 30° = 0 \\ \Sigma F_n = m\frac{v^2}{\rho}: & \mu_s N\cos 30° + N\sin 30° = m\frac{v_{max}^2}{\rho} \end{cases}$$

With $\mu_s = 0.9$: $N = 2.40 mg$

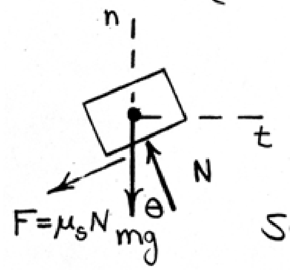
$v_{max} = 345$ ft/sec $(\underline{235 \text{ mi/hr}})$

3/83

Package:

$$\begin{cases} \sum F_t = ma_t: & -\mu_s N \cos\theta - N\sin\theta = -m\,\dfrac{g}{2} \\ \sum F_n = ma_n: & N\cos\theta - \mu_s N \sin\theta - mg \end{cases}$$

$$= m\left(\frac{19.44^2}{80}\right)$$



First eq.:

$$N = \frac{mg/2}{\sin\theta + \mu_s \cos\theta}$$

Second eq.:

$$\left(\frac{\cancel{m}g/2}{\sin\theta + \mu_s\cos\theta}\right)\left(\cos\theta - \mu_s\sin\theta\right) - \cancel{m}g = \cancel{m}\,(4.726)$$

$$\tan\theta = \left(\frac{1 - 2.9635\mu_s}{\mu_s + 2.9635}\right)$$

For $\mu_s = 0.2$, $\underline{\theta = 7.34^\circ}$

For $\mu_s = 0.4$, $\underline{\theta = -3.16^\circ}$!!

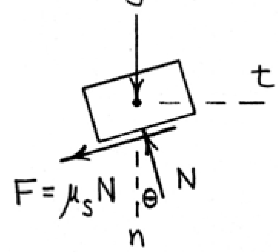
(Note: $N > 0$ for $\theta = -3.16^\circ$)

3/84

$$\Sigma F_t = ma_t: \; -\mu_s N \cos\theta - N\sin\theta = -mg/2$$

Package:

$$N = \frac{mg/2}{\sin\theta + \mu_s \cos\theta}$$



$$\Sigma F_n = ma_n: \; -N\cos\theta + \mu_s N \sin\theta + mg = m\left(\frac{19.44^2}{80}\right)$$

$$\left(\frac{mg/2}{\sin\theta + \mu_s\cos\theta}\right)(\mu_s \sin\theta - \cos\theta) + mg = m\,(4.726)$$

$$\tan\theta = \left[\frac{1 - 1.036\,\mu_s}{\mu_s + 1.036}\right]$$

For $\mu_s = 0.2$, $\theta = 32.7°$ and $N > 0$.

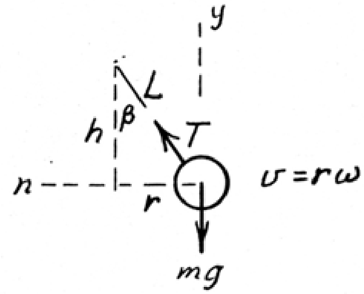
For $\mu_s = 0.4$, $\underline{\theta = 22.2° \text{ and } N > 0.}$

3/85

$\Sigma F_y = 0: \; T\cos\beta - mg = 0, \; T\cos\beta = mg$

$\Sigma F_n = ma_n: \; T\sin\beta = mv^2/r$

Divide & get $\tan\beta = \dfrac{v^2}{gr} = \dfrac{r\omega^2}{g}$



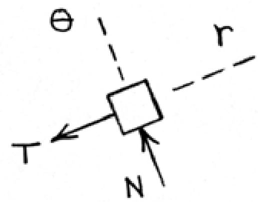
But $r = L\sin\beta$ so $\tan\beta = L\omega^2\sin\beta/g$

or $L\cos\beta = g/\omega^2$

And $h = L\cos\beta$ so $\underline{h = g/\omega^2}$ (depends only on ω & g)

Then $\underline{T} = \dfrac{mg}{\cos\beta} = \dfrac{mg}{h/L} = \dfrac{mgL}{g/\omega^2} \underline{= mL\omega^2}$

3/86



$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2):$$

$$-T = \frac{3}{32.2}\left(0 - \frac{9}{12}6^2\right)$$

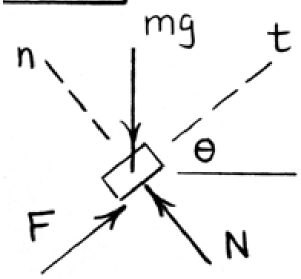
$$\underline{T = 2.52 \text{ lb}}$$

$$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}):$$

$$N = \frac{3}{32.2}\left[\frac{9}{12}(-2) + 2\left(-\frac{2}{12}\right)(6)\right]$$

$\underline{N = -0.326 \text{ lb}}$ (Contact on side B)

3/87



$a_n = r\Omega^2 = \frac{13}{12}(7.5)^2 = 60.9 \text{ ft/sec}^2$

$\Sigma F_n = ma_n$:

$N - mg\cos\theta = 60.9\,m$ (1)

$\Sigma F_t = ma_t$:

$F - mg\sin\theta = 0$ (2)

Slip impends when $F = F_{max} = \mu_s N$. From (1) & (2): $\mu_s = \dfrac{32.2\sin\theta}{60.9 + 32.2\cos\theta}$

(a) $\theta = 50°$: $\underline{\mu_s = 0.302}$

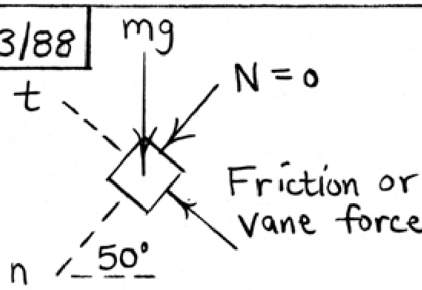
(b) $\theta = 100°$: $\underline{\mu_s = 0.573}$

From (1) $N = m(60.9 + g\cos\theta) > 0$ for all θ

So contact is maintained.

(Look ahead to solution of Prob.*3/365)

3/88

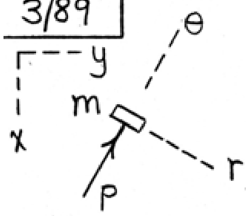


$$\Sigma F_n = ma_n: \quad mg\sin 50° = mr\Omega^2$$

$$\Omega = \sqrt{\frac{g\sin 50°}{r}} = \sqrt{\frac{9.81\sin 50°}{0.330}} = \underline{4.77 \text{ rad/s}}$$

$$(45.6 \text{ rev/min})$$

3/89



(Note: mg & static normal $\perp$ to paper)

$\sum F_r = ma_r: \quad 0 = m(\ddot{r} - r\Omega^2) \quad (1)$

$\sum F_\theta = ma_\theta: \quad P = m(r\underbrace{\ddot{\theta}}_{0} + 2\dot{r}\Omega) \quad (2)$

$(1): \quad \ddot{r} = \dot{r}\dfrac{d\dot{r}}{dr} = r\Omega^2$

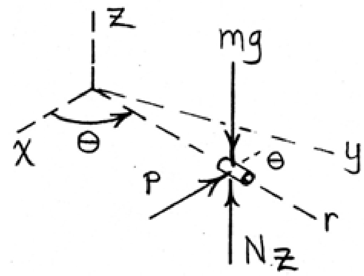
$$\int_{\dot{r}_0}^{\dot{r}} \dot{r}\,d\dot{r} = \int_{r_0}^{r} \Omega^2 r\,dr \Rightarrow \dot{r}^2 = \dot{r}_0^2 + \Omega^2(r^2 - r_0^2)$$

Numbers: (at end of tube) $\dot{r} = \left[60^2 + 7^2\left(3^2 - \left(\frac{6}{12}\right)^2\right)\right]^{1/2} = 63.5\ \frac{\text{ft}}{\text{sec}}$

$(2): \quad P = m(2\dot{r}\Omega) = \dfrac{5/16}{32.2}(2)(63.5)(7)$

$= \underline{8.62\ \text{lb}}$

3/90



$\Sigma F_z = 0 \Rightarrow N_z = mg$

From the solution to Prob. 3/89,

$$\dot{r} = \left\{ \dot{r}_0^{\,2} + \Omega^2 (r^2 - r_0^2) \right\}^{1/2}$$

So $$\int_{r_0}^{r} \frac{dr}{\sqrt{\dot{r}_0^{\,2} + \Omega^2(r^2 - r_0^2)}} = \int_0^t dt$$

$$\frac{1}{\Omega} \ln \left[r + \sqrt{r^2 + \frac{\dot{r}_0^{\,2}}{\Omega^2} - r_0^2} \right]_{r_0}^{r} = t$$

$$\frac{1}{\Omega} \ln \left[\frac{r + \sqrt{r^2 + \frac{\dot{r}_0^{\,2}}{\Omega^2} - r_0^2}}{r_0 + \frac{\dot{r}_0}{\Omega}} \right] = t$$

With numbers ($r = 3$ ft, $r_0 = 0.5$ ft, $\Omega = 7 \frac{\text{rad}}{\text{sec}}$, $\dot{r}_0 = 60$ ft/sec),

$t = 0.0408$ sec; Then $\theta = \Omega t = 0.285$ rad

From solution to Prob. 3/89, $P = 8.62$ lb

$P_x = -P \sin\theta = -8.62 \sin 0.285^R = \underline{-2.43 \text{ lb}}$

$P_y = P \cos\theta = 8.62 \cos 0.285^R = \underline{8.28 \text{ lb}}$

3/91 The distance traveled from A to C is

$(s_C - s_A) = 100 + 250\left(30\frac{\pi}{180}\right) = 231$ ft

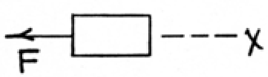
Uniform tangential acceleration : $v_C^2 = v_A^2 + 2a_t(s_C - s_A)$

$0^2 = \left[60\,\frac{5280}{3600}\right]^2 + 2a_t(231)\,,\quad a_t = -16.77$ ft/sec^2

Speed at B : $v_B^2 = v_A^2 + 2a_t(s_B - s_A)$

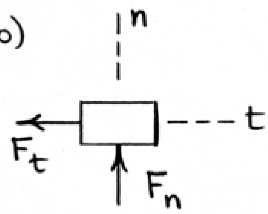
$v_B^2 = \left[60\,\frac{5280}{3600}\right]^2 + 2(-16.77)(100)\,,\quad v_B = 66.3$ ft/sec

(a) $\Sigma F_x = ma_x : \; -F = \frac{3000}{32.2}(-16.77)$



$\underline{F = 1562 \text{ lb}}$

(b) $\Sigma F_t = ma_t : \; -F_t = \frac{3000}{32.2}(-16.77)$



$F_t = 1562$ lb

$\Sigma F_n = m\frac{v^2}{\rho} : \quad F_n = \frac{3000}{32.2}\,\frac{66.3^2}{250}$

$F_n = 1636$ lb

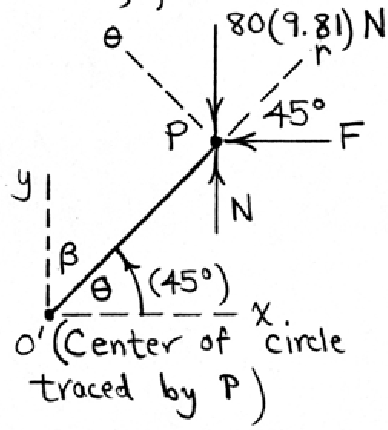
$F = \sqrt{F_t^2 + F_n^2} = \underline{2260 \text{ lb}}$

(c) v and therefore F_n go to zero;

$\underline{F = F_t = 1562 \text{ lb}}$

(In all FBDs, there is a weight into the paper and a static normal force out of the paper.)

3/92 FBD of rider at P (could be any rider!), treated as a particle:



$O'P = 6$ m

$\theta = 45°$

$\dot{\theta} = -0.8$ rad/s

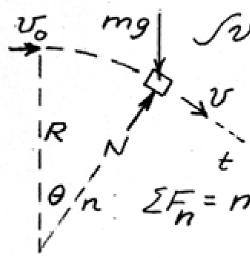
$\ddot{\theta} = -0.4$ rad/s^2

$$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2): -80(9.81)\frac{\sqrt{2}}{2} + N\frac{\sqrt{2}}{2} - F\frac{\sqrt{2}}{2} = 80\left[0 - 6(-0.8)^2\right] \quad (1)$$

$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}): -80(9.81)\frac{\sqrt{2}}{2} + N\frac{\sqrt{2}}{2} + F\frac{\sqrt{2}}{2} = 80\left[6(-0.4) + 0\right] \quad (2)$$

Solve (1) & (2): $\begin{cases} N = 432 \text{ N} \\ F = 81.5 \text{ N} \end{cases}$ || Static: $\begin{cases} N_S = 785 \text{ N} \\ F_S = 0 \end{cases}$

3/93 $\Sigma F_t = ma_t$; $mg\sin\theta = ma_t$, $a_t = g\sin\theta$



$\int v\,dv = \int a_t\,ds$; $\int_{v_0}^{v} v\,dv = \int_0^{\theta} g\sin\theta\,(R\,d\theta)$

$v^2 = v_0^2 + 2gR(1-\cos\theta)$

$\Sigma F_n = ma_n$; $mg\cos\theta - N = m\dfrac{v^2}{R}$

$N = mg\cos\theta - \dfrac{m}{R}v_0^2 - 2mg(1-\cos\theta)$

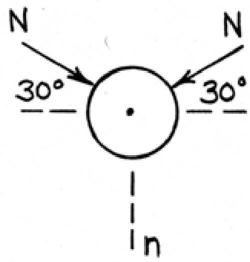
$= mg\left(3\cos\theta - 2 - \dfrac{v_0^2}{gR}\right)$

When $N = 0$, $\theta = \beta$ so $3\cos\beta = 2 + \dfrac{v_0^2}{gR}$

$\beta = \cos^{-1}\left(\dfrac{2}{3} + \dfrac{v_0^2}{3gR}\right)$

For $v_0 = 0$, $\beta = \cos^{-1}\left(\dfrac{2}{3}\right) = \underline{48.2°}$

3/94 $\Sigma F_n = ma_n = mr\omega^2$:

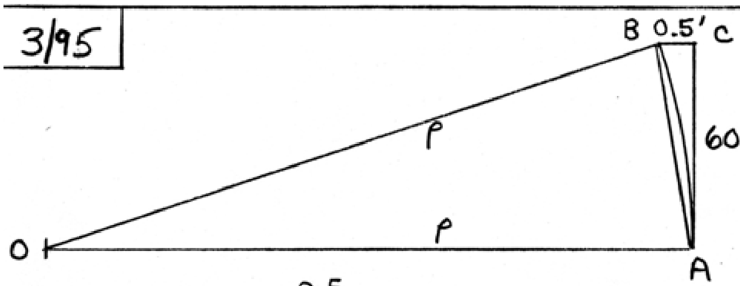


$$2N \sin 30° = 2.5(0.150)\left[\frac{600(2\pi)}{60}\right]^2$$

$$N = 1480 \text{ N}$$

$$F = 4N \cos 30° = \underline{5130 \text{ N}}$$

3/95



$\measuredangle BAC = \tan^{-1}\frac{0.5}{60} = 0.477°$

$\measuredangle OBA = \measuredangle OAB = (90 - 0.477) = 89.5°$

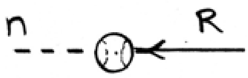
$\measuredangle BOA = 180 - 2(89.5) = 0.955° = 2\measuredangle BAC$

$\overline{AB} = \sqrt{60^2 + 0.5^2} = 60.002'$

$\frac{\sin 0.955°}{60.002'} = \frac{\sin 89.5°}{\rho}$, $\underline{\rho = 3600 \text{ ft}}$

FBD: (horizontal forces)

$\Sigma F_n = ma_n : R = \frac{5.125/16}{32.2}\ \frac{120^2}{3600}$



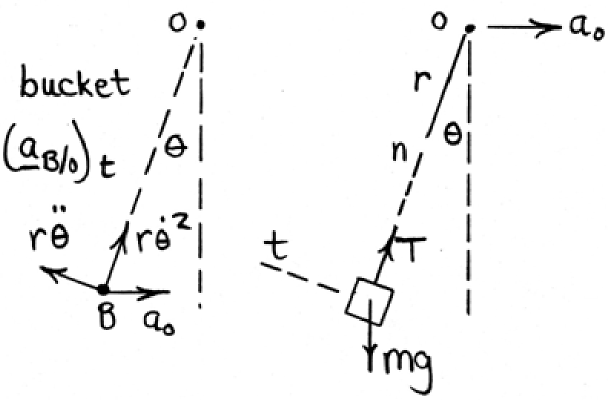
$\underline{R = 0.0398 \text{ lb}}$

(Note: R = 0.637 oz represents 12.4% of the weight of the baseball)

3/96

Acceleration of bucket

$\underline{a}_B = \underline{a}_0 + (\underline{a}_{B/0})_n + (\underline{a}_{B/0})_t$



$\Sigma F_t = ma_t: \; -mg\sin\theta = m(r\ddot{\theta} - a_0\cos\theta)$

$\ddot{\theta} = +\frac{1}{r}(a_0\cos\theta - g\sin\theta)$

$\dot{\theta}$ is a maximum when $\ddot{\theta} = 0: \; a_0\cos\theta = g\sin\theta$

$\underline{\theta = \tan^{-1}\left(\frac{a_0}{g}\right)}$

With $\ddot{\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta}$:

$$\int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \int_0^{\theta} \frac{1}{r}(a_0\cos\theta - g\sin\theta)\, d\theta$$

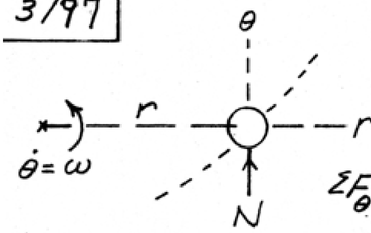
$$\dot{\theta}^2 = \frac{2}{r}(a_0\sin\theta + g\cos\theta - g)$$

$\Sigma F_n = ma_n: \; T - mg\cos\theta = m(r\dot{\theta}^2 + a_0\sin\theta)$

Substitute expression for $\dot{\theta}^2$:

$\underline{T = m(3a_0\sin\theta + 3g\cos\theta - 2g)}$

3/97



$\Sigma F_r = ma_r;\quad 0 = m(\ddot{r} - r\omega^2)$

sol. is $r = r_0 \cosh \omega t$

$\dot{r} = r_0 \omega \sinh \omega t$

$\Sigma F_\theta = ma_\theta;\quad N = m(0 + 2\dot{r}\omega)$

$= 2mr_0\omega^2 \sinh \omega t$

But $\cosh^2 \omega t - \sinh^2 \omega t = 1$, $\sinh^2 \omega t = \cosh^2 \omega t - 1$

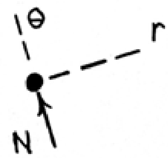
$\sinh \omega t = \sqrt{\left(\frac{r}{r_0}\right)^2 - 1}$

So $N = 2mr_0\omega^2\sqrt{\left(\frac{r}{r_0}\right)^2 - 1} = \underline{2m\omega^2\sqrt{r^2 - r_0^2}}$

3/98

$\Sigma F_r = ma_r: \quad 0 = m(\ddot{r} - r\dot{\theta}^2)$

Particle:



$$\ddot{r} = r\dot{\theta}^2 = r\omega_0^2$$

$$\dot{r}\frac{d\dot{r}}{dr} = r\omega_0^2$$

$$\int_0^{\dot{r}} \dot{r}\,d\dot{r} = \omega_0^2 \int_{r_0}^{r} r\,dr$$

$$\Rightarrow \quad \underline{\dot{r} = \omega_0\sqrt{r^2 - r_0^2} = v_r}$$

$$\frac{dr}{dt} = \omega_0\sqrt{r^2 - r_0^2}$$

$$\int_{r_0}^{r} \frac{dr}{\sqrt{r^2 - r_0^2}} = \omega_0 \int_0^t dt$$

$$\ln\left[r + \sqrt{r^2 - r_0^2}\right]\Big|_{r_0}^{r} = \omega_0 t \Rightarrow \underline{r = \frac{r_0}{2}\left[e^{-\omega_0 t} + e^{\omega_0 t}\right]}$$

$$v_\theta = r\dot{\theta} = r\omega_0 = \underline{\frac{r_0\omega_0}{2}\left[e^{-\omega_0 t} + e^{\omega_0 t}\right]}$$

As a function of t, $\underline{v_r = \frac{r_0\omega_0}{2}\left(e^{\omega_0 t} - e^{-\omega_0 t}\right)}$.

In terms of hyperbolic functions,

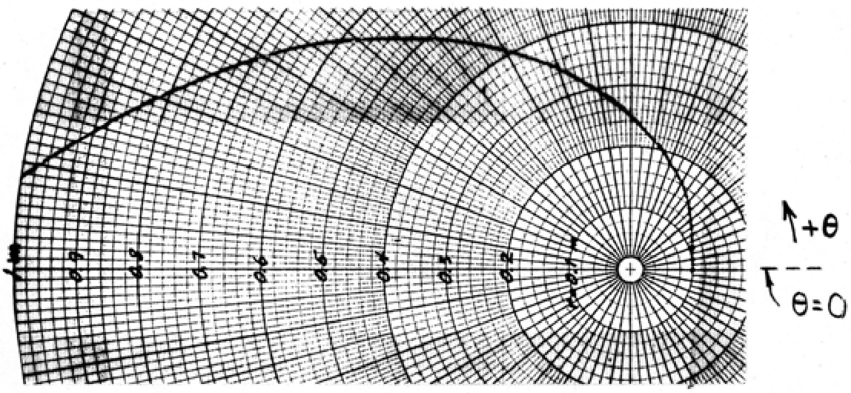
$$v_r = r_0 \omega_0 \sinh \omega_0 t$$

$$r = r_0 \cosh \omega_0 t$$

$$v_\theta = r_0 \omega_0 \cosh \omega_0 t$$

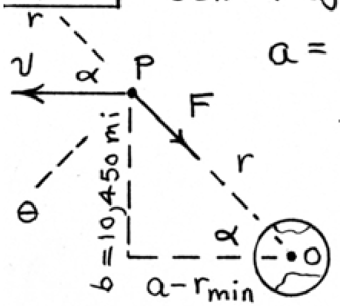
With numbers,

$$\begin{cases} v_r = 0.1 \sinh t \\ r = 0.1 \cosh t \\ v_\theta = 0.1 \cosh t \end{cases}$$



($r = 1.0$ m @ $\theta = 171.5°$)

3/99 Semi-major axis of ellipse is



$$a = \frac{r_{max} + r_{min}}{2} = \frac{26,259 + 4159}{2}$$

$$= 15,209 \text{ mi}$$

$$\alpha = \tan^{-1} \frac{b}{a - r_{min}}$$

$$= \tan^{-1} \frac{10,450}{15,209 - 4159} = 43.4°$$

$$\dot{r} = v_r = v \cos\alpha = 13,244 \cos 43.4° = \underline{9620 \text{ ft/sec}}$$

$$r\dot{\theta} = v_\theta = v \sin\alpha, \quad \dot{\theta} = \frac{v \sin\alpha}{r}, \quad \text{where}$$

$$r = \sqrt{b^2 + (a - r_{min})^2} = \sqrt{10,450^2 + (15,209 - 4159)^2}$$

$$= 15,208 \text{ mi}; \text{ so } \dot{\theta} = \frac{13,244 \sin 43.4°}{15,208(5280)} = \underline{1.133(10^{-4}) \frac{\text{rad}}{\text{sec}}}$$

$$\Sigma F_r = ma_r: \; -\cancel{m}\frac{gR^2}{r^2} = \cancel{m}(\ddot{r} - r\dot{\theta}^2)$$

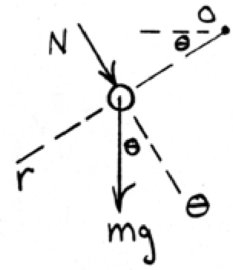
$$\ddot{r} = r\dot{\theta}^2 - \frac{gR^2}{r^2} = 15,208(5280)\left[(1.133)(10^{-4})\right]^2 - \frac{32.23(3959)^2}{15,208^2}$$

$$= \underline{-1.153 \text{ ft/sec}^2}$$

$$\Sigma F_\theta = ma_\theta: \quad 0 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} = -\frac{2(9620)(1.133)(10^{-4})}{15,208(5280)} = \underline{-2.72(10^{-8}) \frac{\text{rad}}{\text{sec}^2}}$$

▸3/100



$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$:

$\cancel{m}g \sin\theta = \cancel{m}(\ddot{r} - r\omega_0^2)$

$\ddot{r} - \omega_0^2 r = g \sin\omega_0 t$

Assume $r_h = Ce^{st}$ and substitute into equation to obtain $s_1 = -\omega_0$, $s_2 = \omega_0$. Also, assume a particular solution of form $r_p = D\sin\omega_0 t$, substitute, and obtain $D = -g/2\omega_0^2$.

So $r = r_h + r_p = C_1 e^{-\omega_0 t} + C_2 e^{\omega_0 t} - \frac{g}{2\omega_0^2}\sin\omega_0 t$

Initial conditions :

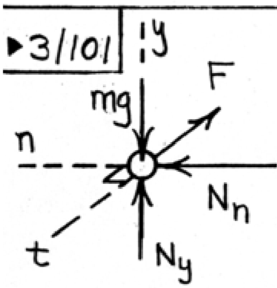
$$\begin{cases} r(0) = C_1 + C_2 = 0 \\ \dot{r}(0) = -\omega_0 C_1 + \omega_0 C_2 - \frac{g}{2\omega_0} = 0 \end{cases}$$

Solve for C_1 and C_2 to obtain

$r = -\frac{g}{4\omega_0^2}e^{-\theta} + \frac{g}{4\omega_0^2}e^{\theta} - \frac{g}{2\omega_0^2}\sin\theta$

or $\underline{r = \frac{g}{2\omega_0^2}\left[\sinh\theta - \sin\theta\right]}$

▸3/101



$$\Sigma F_y = 0: \quad N_y = mg$$

$$\Sigma F_n = ma_n: \quad N_n = m\frac{v^2}{r}$$

$$F = \mu_k N_{tot} = \mu_k \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$$

$$= \frac{\mu_k m}{r}\sqrt{r^2g^2 + v^4}$$

$$\Sigma F_t = ma_t: \quad -\frac{\mu_k \cancel{m}}{r}\sqrt{r^2g^2+v^4} = \cancel{m}v\frac{dv}{ds}$$

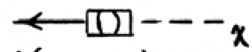
$$-\frac{\mu_k}{r}\int_0^s ds = \int_{v_0}^0 \frac{v\,dv}{\sqrt{v^4+r^2g^2}} = \int_{v_0^2}^0 \frac{\frac{1}{2}dx}{\sqrt{x^2+r^2g^2}}$$

where $x = v^2$, $dx = 2v\,dv$

Integrating,

$$-\frac{\mu_k}{r}s = \frac{1}{2}\ln\left[x + \sqrt{x^2+r^2g^2}\right]_{v_0^2}^0$$

$$\text{or} \quad \underline{s = \frac{r}{2\mu_k}\ln\left[\frac{v_0^2 + \sqrt{v_0^4 + r^2g^2}}{rg}\right]}$$

▶3/102 Motion from A to B : 

$$\Sigma F_x = ma_x : \; -4(2500) = 1350\,a$$

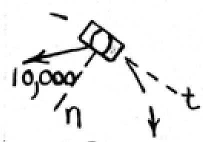
$$a = -7.407 \text{ m/s}^2 , \qquad v_B^{\,2} - v_A^{\,2} = 2a(x_B - x_A)$$

$$v_B^{\,2} - 25^2 = 2(-7.407)(10)$$

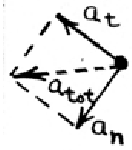
$$v_B = 21.84 \text{ m/s}$$

Beyond B : $\quad F = ma_{tot}\,,\; a_{tot} = \dfrac{10{,}000}{1350}$

$$= 7.407 \text{ m/s}^2$$



$$a_{tot} = \sqrt{a_n^2 + a_t^2} = \sqrt{\frac{v^4}{\rho^2} + a_t^2}$$



$$a_t = -\sqrt{a_{tot}^2 - \frac{v^4}{\rho^2}} = v\,\frac{dv}{ds}$$

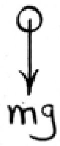
$$\int_{10}^{s} ds = -\rho \int_{v_B}^{0} \frac{v\,dv}{\sqrt{\rho^2 a_{tot}^2 - v^4}}$$

Let $x = v^2$: $\quad s - 10 = -\rho \displaystyle\int_{v_B^2}^{0} \frac{dx/2}{\sqrt{\rho^2 a_{tot}^2 - x^2}}$

$$s = 10 + \frac{\rho}{2}\sin^{-1}\left(\frac{v_B^{\,2}}{\rho a_{tot}}\right) = \underline{47.4 \text{ m}}$$

3/103 State 1 : launch; State 2 : apex

$T_1 + U_{1-2} = T_2 : \frac{1}{2} m v_0^2 - mgh = 0$



$\Rightarrow h = \underline{\frac{v_0^2}{2g}}$

For $v_0 = 50$ m/s : $h = \frac{50^2}{2(9.81)} = \underline{127.4 \text{ m}}$

3/104

(a) $U_{1-2} = \frac{1}{2} k (x_1^{\,2} - x_2^{\,2})$

$= \frac{1}{2}(3)(12)\left[\left(\frac{6}{12}\right)^2 - \left(\frac{3}{12}\right)^2\right] = \underline{3.38 \text{ ft-lb}}$

(b) $U_{1-2} = -mgh = -14\left(\frac{9}{12}\right)\sin 15°$

$= \underline{-2.72 \text{ ft-lb}}$

3/105

$$T_A + V_{A-B} = T_B$$

$$\frac{1}{2}\cancel{m}v_A^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$$

$$v_B^2 = v_A^2 + 2gh = 4^2 + 2(9.81)(1.8)$$

$$\underline{v_B = 7.16 \text{ m/s}}$$

Knowledge of the shape of the track is unnecessary, as long as it is known that the cart passes the highest point.

$\boxed{3/106}$ $T_A + U_{A-B} = T_B$

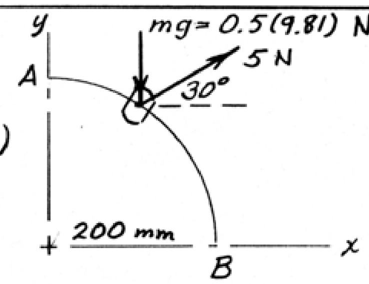
$$\frac{1}{2} m v_A^2 + U_f + mgh = \frac{1}{2} m v_B^2$$

$$U_f = m\left(\frac{v_B^2}{2} - \frac{v_A^2}{2} - gh\right)$$

$$= 3\left(\frac{6^2}{2} - \frac{4^2}{2} - 9.81(1.8)\right) = \underline{-23.0 \text{ J}}$$

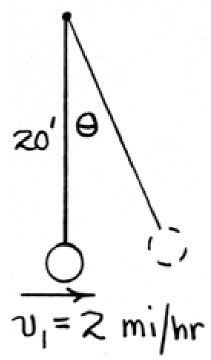
3/107

$U = \Delta T$: $5\cos 30°(0.2) - 5\sin 30°(0.2)$
$+ 0.5(9.81)(0.2)$
$= \frac{1}{2}0.5(v^2 - 0)$



$v^2 = 5.39\ (m/s)^2$, $\underline{v = 2.32\ m/s}$

3/108

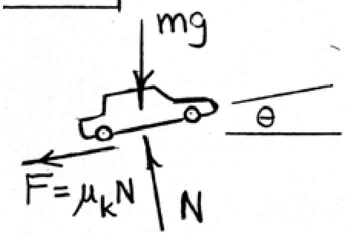


$$T_1 + U_{1-2} = T_2: \quad \frac{1}{2}\cancel{m}v_1^2 - \cancel{m}gh = 0$$

$$\frac{1}{2}\left[2\,\frac{5280}{3600}\right]^2 - 32.2\left[20\left(1-\cos\theta\right)\right] = 0$$

$$\underline{\theta = 6.63^\circ}$$

3/109



$$\theta = \tan^{-1}\frac{6}{100} = 3.43°$$

$$T_A + U_{A-B} = T_B$$

$$\frac{1}{2}\cancel{m}v_0^2 - \mu_k \cancel{m}g\cos\theta\, s - \cancel{m}g\, s\sin\theta = 0$$

$$\frac{1}{2}\left(65\,\frac{5280}{3600}\right)^2 - 32.2s\left[+0.6\cos 3.43° + \sin 3.43°\right] = 0$$

$$\underline{s = 214\text{ ft}}$$

Going downhill $(B \rightarrow A)$: $T_B + U_{B-A} = T_A$

$$\frac{1}{2}\cancel{m}v_0^2 - \mu_k\cancel{m}g\cos\theta\, s + \cancel{m}g\, s\sin\theta = 0$$

$$\frac{1}{2}\left(65\,\frac{5280}{3600}\right)^2 + 32.2s\left[-0.6\cos 3.43° + \sin 3.43°\right] = 0$$

$$\underline{s = 262\text{ ft}}$$

$\boxed{3/110}$ For collar, $U_{1-2} = \Delta T = 0$

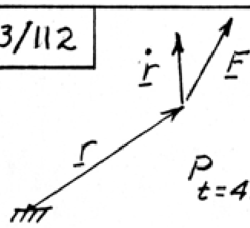
$$U_{1-2} = 50\left(\frac{50-30}{12}\right) - 30\,\frac{40}{12}\sin 30° - \frac{1}{2}k\left(\frac{6}{12}\right)^2 = 0$$

$$\underline{k = 267 \text{ lb/ft}}$$

3/111 $U_{1-2} = \Delta T$; $2\left(\frac{1}{2} k x^2\right) = \frac{1}{2} m v^2 - 0$

$$k = \frac{1}{2} \frac{m v^2}{x^2} = \frac{1}{2} \frac{3500}{32.2} \left(\frac{5}{30} 44\right)^2 \frac{1}{(6/12)^2} \frac{1}{12} = \underline{974 \text{ lb/in.}}$$

3/112



Power $P = \underline{F} \cdot \dot{\underline{r}}$

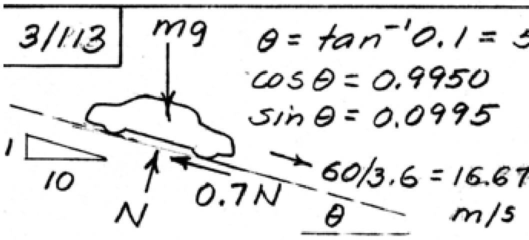
$$P = (40\underline{i} - 20\underline{j} - 36\underline{k}) \cdot (8\underline{i} + 2.4t\underline{j} - 1.5t^2\underline{k})$$

$$P_{t=4s} = (40\underline{i} - 20\underline{j} - 36\underline{k}) \cdot (8\underline{i} + 9.6\underline{j} - 24\underline{k})$$

$$= 320 - 192 + 864 = 992 \text{ W}$$

or $\underline{P = 0.992 \text{ kW}}$

3/113



$\theta = \tan^{-1} 0.1 = 5.71°$

$\cos\theta = 0.9950$

$\sin\theta = 0.0995$

$N = mg\cos\theta$

$= 0.9950(9.81)m$

$= 9.76\,m$

$U = \Delta T$; $-0.7(9.76m)s + 9.81m(0.0995)s = -\frac{m}{2}(16.67)^2$

$5.86\,s = 138.9$, $\underline{s = 23.7\ \text{m}}$

3/114 $P = W\dot{y}$ where $\dot{y} = v \sin\theta$

$\theta = \tan^{-1} 0.05 = 2.86°$, $\sin\theta = 0.0499$

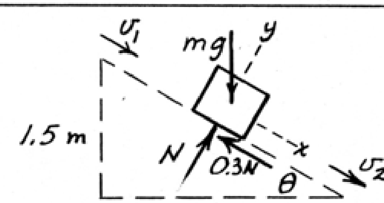
$P = 200\left(\frac{15}{30}44\right)0.0499 = 219.7$ ft-lb/sec

or $P = \frac{219.7}{550} = \underline{0.400 \text{ hp}}$

3/115

$\Sigma F_y = 0: \; N - mg\cos\theta = 0$

$U = \Delta T: \; (mg\sin\theta - 0.3\,mg\cos\theta)\dfrac{1.5}{\sin\theta} = \dfrac{1}{2}m(0.14^2 - 0.40^2)$



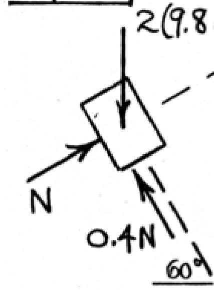
$v_1 = 0.40$ m/s

$v_2 = 0.14$ m/s

$1.5(9.81)(1 - \dfrac{0.3}{\tan\theta}) = -0.0702$

$\tan\theta = 0.299$, $\underline{\theta = 16.62^\circ}$

3/116



$\Sigma F_y = 0: \quad N - 2(9.81)\cos 60° = 0$

$N = 9.81$ N

(a) $U_{1-2} = \Delta T: \quad 2(9.81)(0.5 \sin 60°)$

$- 0.4(9.81)(0.5) = \frac{1}{2} 2 v^2$

$\underline{v = 2.56 \text{ m/s}}$

(b) $U_{1-3} = \Delta T: \quad 2(9.81)(0.5 + x)\sin 60° - 0.4(9.81)(0.5 + x)$

$- \frac{1}{2}(1600)x^2 = 0$

$800x^2 - 13.07x - 6.53 = 0$

$x = 0.0989$ m or $\underline{x = 98.9 \text{ mm}}$

3/117 $P = Wh/\Delta t$

or $P = \dfrac{120(9)}{5} / 550 = \underline{0.393 \text{ hp}}$

Conversions : $h = 9 \text{ ft} \left(\dfrac{0.3048 \text{ m}}{\text{ft}} \right) = 2.74 \text{ m}$

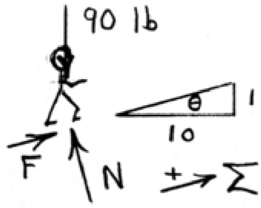
$W = 120 \text{ lb} \left(\dfrac{4.4482 \text{ N}}{\text{lb}} \right) = 534 \text{ N}$

$P = \dfrac{Wh}{\Delta t} = \dfrac{534(2.74)}{5} = \underline{293 \text{ watts}}$

Check : $0.393 \text{hp} \left(\dfrac{745.7 \text{ watts}}{\text{hp}} \right) = 293 \text{ watts}$ ✓

3/118

$$v_B = 5\,\frac{5280}{3600} = 7.33 \text{ ft/sec}$$



$$v_B^{\,2} = 2as\,, \quad a = \frac{7.33^2}{2(50)} = 0.538 \frac{\text{ft}}{\text{sec}^2}$$

$$\theta = \tan^{-1}(0.1) = 5.71°$$

$$\xrightarrow{+} \Sigma F = ma: \quad F - 90 \sin 5.71° = \frac{90}{32.2}(0.538)$$

$$F = 10.46 \text{ lb}$$

$$P = Fv = 10.46(7.33) = 76.7 \frac{\text{ft-lb}}{\text{sec}}$$

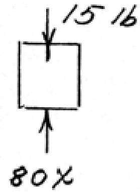
$$\text{or } P = 76.7/550 = \underline{0.1394 \text{ hp}}$$

3/119 Net power required $= 30(140)(24)/33{,}000$

$= 3.05$ hp

$$\text{Mechanical efficiency} = \frac{\text{Power required}}{\text{Power supplied}} = \frac{3.05}{4.00} = \underline{0.764}$$

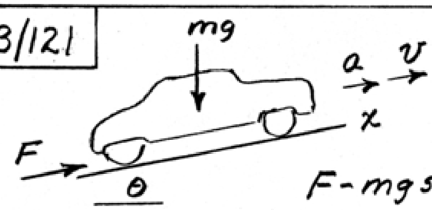
3/120 $U_{1-2} = \Delta T$; $15(18+2) - \frac{1}{2}80(2^2) = \frac{1}{2}\frac{15v^2}{32.2}(12)$



$300 - 160 = 2.795v^2$, v in ft/sec

$v^2 = 50.09$, $\underline{v = 7.08 \text{ ft/sec}}$

3/121



$\theta = \tan^{-1} 0.1, \ \sin\theta = 0.0995$

$\Sigma F_x = ma_x$ where $v^2 = 2ax$

$F - mg\sin\theta = m\dfrac{v^2}{2x}$

$$P = Fv = mgv\sin\theta + \frac{mv^3}{2x}$$

$$= 1500(9.81)\frac{50000}{3600}0.0995 + \frac{1500\,(50\,000/3600)^3}{2(100)}$$

$$= 20\,336 + 20\,094 = 40\,430 \text{ W}$$

or $\underline{P = 40.4 \text{ kW}}$

3/122 For $x = 75$ mm, $U = \Delta T$ &

$\frac{1}{2}(0.075)R_{max} = \frac{1}{2}(0.25)(600)^2$, $R_{max} = 1.2$ MN

For $x = 25$ mm, $R = \frac{25}{75}(1.2) = 0.4$ MN or $0.4(10^6)$ N

$U = \Delta T$; $\frac{1}{2}(0.025)(0.4)10^6 = \frac{1}{2}(0.25)(\overline{600}^2 - v^2)$

$v^2 = 320(10^3)\ (m/s)^2$, $\underline{v = 566\ m/s}$

3/123 Power output = rate of doing work

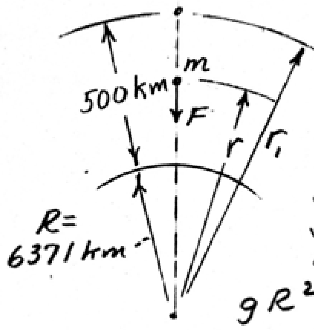
$$= 300(9.81)(2) - 100(9.81)(4)$$

$$= 1962 \text{ J/s (W)}$$

$$= 1.962 \text{ kW}$$

Efficiency $e = \dfrac{\text{Power output}}{\text{Power input}} = \dfrac{1.962}{2.20} = \underline{0.892}$

3/124 $F =$ gravitational force $= Gmm_e/r^2 = gR^2m/r^2$



$U = \Delta T$

$$\int_{r_1}^{r} F(-dr) = \frac{1}{2}mv^2 - 0$$

$$-\int_{r_1}^{r} \frac{gR^2m}{r^2}dr = \frac{1}{2}mv^2, \quad -gR^2\left(-\frac{1}{r}\right)_{r_1}^{r} = \frac{v^2}{2}$$

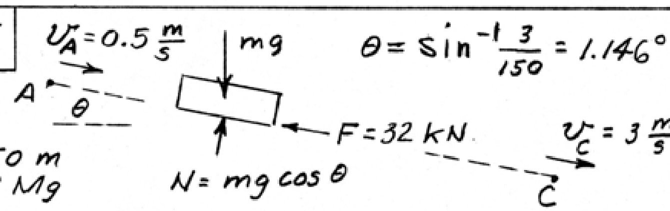
$$gR^2\left(\frac{1}{r} - \frac{1}{r_1}\right) = \frac{v^2}{2}, \quad v = R\sqrt{2g\left(\frac{1}{r} - \frac{1}{r_1}\right)}$$

$$v = 6371\sqrt{\frac{2(9.825)}{1000}\left(\frac{1}{6371+500-100} - \frac{1}{6371+500}\right)} \quad \frac{\text{km}}{\text{s}}$$

$$= 6371\sqrt{4.2237(10^{-8})} = \underline{1.309 \text{ km/s}}$$

3/125

$\theta = \sin^{-1}\frac{3}{150} = 1.146°$



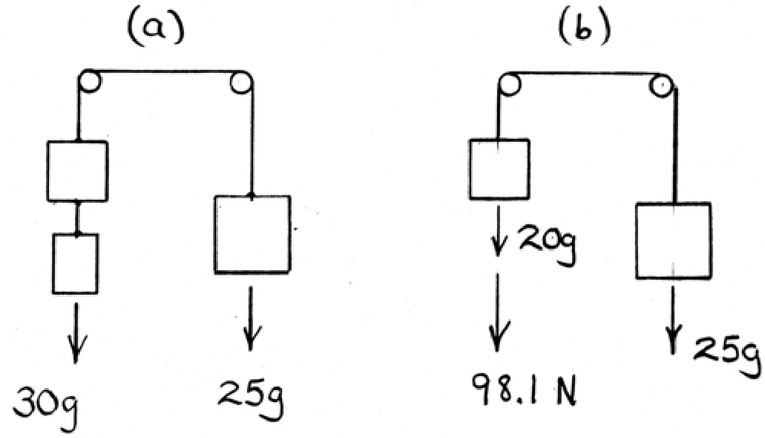
$\overline{AC} = 150$ m

$m = 68$ Mg

$U_{1\text{-}2} = \Delta T$; $\quad 68(10^3)(9.81)(3) - 32(10^3)x = \frac{1}{2}68(10^3)(3^2 - 0.5^2)$

$2001 - 32x = 297.5, \quad \underline{x = 53.2 \text{ m}}$

3/126 Active-force diagrams for system:



$U = \Delta T$ for system:

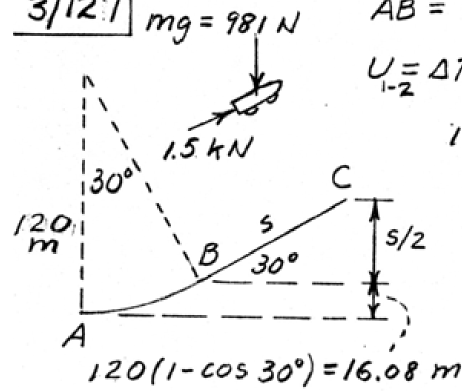
(a) $(30-25)(9.81)2 = \frac{1}{2}(30+25)v^2$

$\underline{v = 1.889 \text{ m/s}}$

(b) $\left[(20-25)(9.81) + 98.1\right]2 = \frac{1}{2}(20+25)v^2$

$\underline{v = 2.09 \text{ m/s}}$

3/127



$\overline{AB} = r\theta = 120\frac{\pi}{6} = 62.8$ m

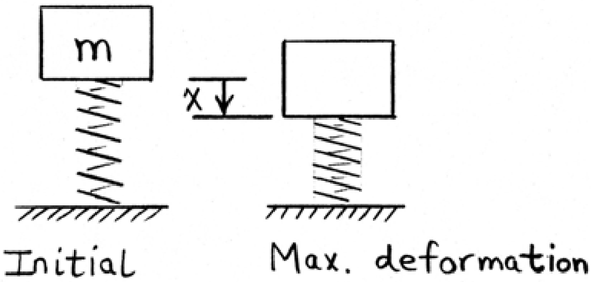
$U_{1-2} = \Delta T = 0$ since $T_C = T_A = 0$

$$1500(62.8) - 981\left(16.08 + \frac{s}{2}\right) = 0$$

$$s = 2(94248 - 15771)/981$$

$$\underline{= 160.0 \text{ m}}$$

3/128



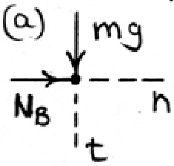
The maximum force $F = kx$ occurs when x is a maximum with $\dot{x} = 0$.

$U_{1-2} = \Delta T$: $mgx - \frac{1}{2}kx^2 = 0$, $x = \frac{2mg}{k}$

So $F = kx = \underline{2mg}$

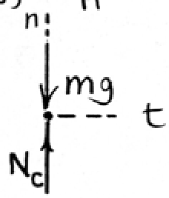
3/129 $T_A + U_{A-B} = T_B : \quad 0 + 2mgR = \frac{1}{2}mv_B^2, \quad v_B^2 = 4gR$

(a)



$\Sigma F_n = ma_n : \quad N_B = m\,\frac{4gR}{R} = \underline{4mg}$

(b) $T_A + U_{A-C} = T_C : \quad 0 + 3mgR = \frac{1}{2}mv_C^2, \quad v_C^2 = 6gR$



$\Sigma F_n = ma_n : \quad N_C - mg = m\,\frac{6gR}{R}$

$\underline{N_C = 7mg}$

(c) Call stopping point E :

$T_A + U_{A-E} = T_E$

$0 + 2mgR - mg\left(\frac{1}{2}s\right) - \mu_k \frac{\sqrt{3}}{2}mgs = 0$

$\underline{s = \frac{4R}{1+\mu_k\sqrt{3}}}$

(Note: Normal force on incline is
$N = mg\cos 30^\circ = \frac{\sqrt{3}}{2}mg$)

3/130 Let s = distance down incline before reversal of direction.

$$U_{1-2} = 110(2)(10+s-s) - 300\,(10+s-s)\frac{5}{13} = 1046 \text{ ft-lb}$$

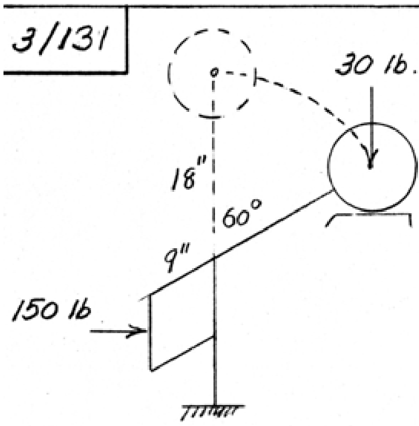
$$\Delta T = \frac{1}{2}\,\frac{300}{32.2}\left[v^2 - (\pm 9)^2\right] = 4.66v^2 - 377 \text{ ft-lb}$$

$$U_{1-2} = \Delta T : \quad 1046 = 4.66v^2 - 377$$

$$\underline{v = 17.48 \text{ ft/sec}}$$

The initial kinetic energy is positive regardless of the velocity direction.

3/131



$$U_{1-2} = \Delta T$$

$$150\left(\frac{9}{12}\sin 60^\circ\right) - 30\,\frac{18}{12}\left(1-\cos 60^\circ\right)$$

$$= \frac{1}{2}\,\frac{30}{32.2}\left(v^2 - 0^2\right)$$

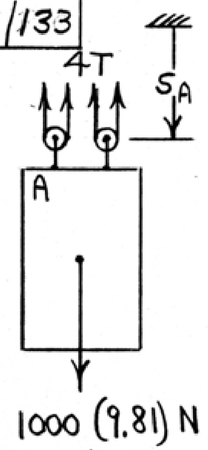
$v^2 = 160.8$, $\underline{v = 12.68 \text{ ft/sec}}$

3/132 $U_{1-2} = \Delta T$; $mg(0.8 - 1.2\cos 60°)$

$$= \frac{1}{2}m(v_C^2 - 3^2)$$

$9.81(0.20) = \frac{1}{2}(v_C^2 - 9)$, $v_C^2 = 12.92$, $\underline{v_C = 3.59 \text{ m/s}}$

3/133



$+\downarrow \Sigma F = 0: \ 9810 - 4T = 0, \ T = 2450 \text{ N}$

Length of cable $L = 4s_A + \text{constants}$

$\dot{L} = 4v_A = 4(-3) = -12 \text{ m/s}$

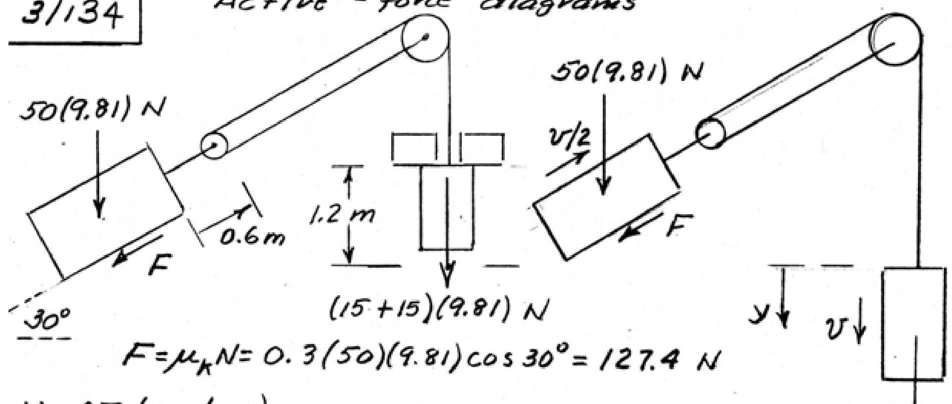
$P_{out} = -T\dot{L} = -2450(-12) = 29\,400 \text{ watts}$

or $P_{out} = 29.4 \text{ kW}$

$e = \dfrac{P_{out}}{P_{in}}, \quad P_{in} = \dfrac{P_{out}}{e} = \dfrac{29.4}{0.8}$

$\underline{P_{in} = 36.8 \text{ kW}}$

3/134 Active - force diagrams



$F = \mu_k N = 0.3(50)(9.81)\cos 30° = 127.4$ N

$U = \Delta T$ (system)

First interval;

$$30(9.81)1.2 - [50(9.81)0.5 + 127.4]\frac{1.2}{2} = \frac{1}{2}30v^2 + \frac{1}{2}50(v/2)^2$$

$v^2 = 6.096\ (m/s)^2, \quad v = 2.469$ m/s

Second interval

$$15(9.81)(y) - [50(9.81)0.5 + 127.4]\frac{y}{2} = 0 - \frac{1}{2}15(6.097) - \frac{1}{2}50\left(\frac{6.097}{4}\right)$$

$y = 2.14$ m, $\quad s = \frac{1}{2}(1.2 + 2.14) = \underline{1.67 \text{ m}}$

$\boxed{3/135}$ $U = \Delta T$; $-\int_0^4 (3x^2 + 60x)\,dx = \frac{1}{2}\frac{48}{32.2}(0 - v^2)12$

$x^3 + 30x^2\Big]_0^4 = \frac{288}{32.2}v^2$, v in ft/sec.

$v^2 = \frac{32.2}{288}(64 + 480) = 60.82\ (\text{ft/sec})^2$, $\underline{v = 7.80\ \text{ft/sec}}$

$\boxed{3/136}$ $\theta = \tan^{-1} \frac{6}{100} = 3.43°$

$U_{1-2} = \Delta T : \quad U_f + mgh = \frac{1}{2} m(v_2^2 - v_1^2)$

$$U_f = -1400(9.81)(200 \sin 3.43°) + \frac{1}{2}\, 1400 \left[\left(\frac{20}{3.6}\right)^2 - \left(\frac{100}{3.6}\right)^2\right]$$

$$= -683\,000 \text{ J} \quad \text{or} \quad -683 \text{ kJ}$$

Energy lost $\underline{Q = 683 \text{ kJ}}$

$\boxed{3/137}$ The power output of the drivetrain is

$$P_{out} = Fv = 560\left(\frac{90}{3.6}\right) = 14\,000 \text{ W}$$

The power input to the drivetrain:

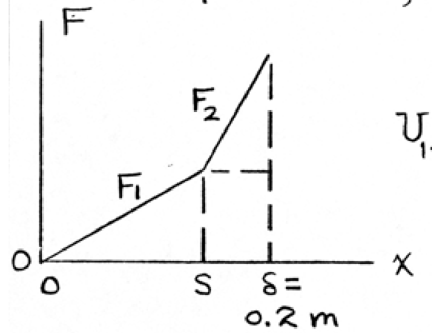
$$P_{in} = \frac{P_{out}}{e} = \frac{14\,000}{0.70} = 20\,000 \text{ W}$$

So the motor output $\underline{P = 20 \text{ kW}}$

3/138

$F_1 = 300x\,,\quad F_2 = 300x + 150(x - s)$

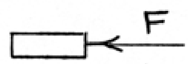
$= 450x - 150s$



$$U_{1-2} = -\frac{1}{2}300s^2 - \int_s^{0.2\text{ m}} (450x - 150s)\,dx$$

$$= -75s^2 + 30s - 9 \quad \text{J}$$

$U_{1-2} = \Delta T$:



$$-75s^2 + 30s - 9 = \frac{1}{2}(0.5)(0 - 5^2)$$

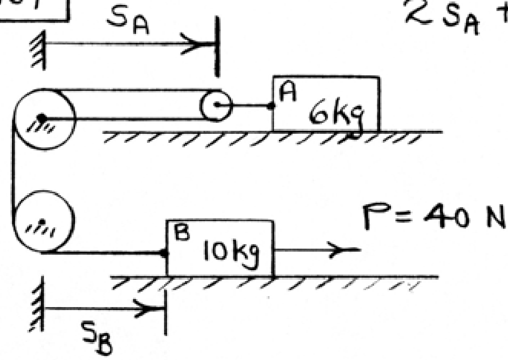
$$75s^2 - 30s + 2.75 = 0$$

$s = 0.1423$ m or 0.257 m

0.257 m > 200 mm , impossible

So $\underline{s = 142.3 \text{ mm}}$

3/139



$2s_A + s_B = \text{constant}$

$2v_A + v_B = 0$

(velocities)

$T_1 + U_{1\text{-}2} = T_2$

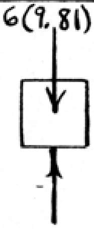
$0 + 40(0.8) = \frac{1}{2} 6 v_A^{\,2} + \frac{1}{2} 10 (2v_A)^2$

$\underline{v_A = 1.180 \text{ m/s}}$

$\underline{v_B = 2v_A = 2.36 \text{ m/s}}$

Speeds

3/140.



$U_{1-2} = \Delta T = 0:$

$$6(9.81)(0.1+\delta) - \int_{0.05}^{0.05+\delta} 4000x\,dx = 0$$

$$2000\delta^2 + 141.1\delta - 5.89 = 0$$

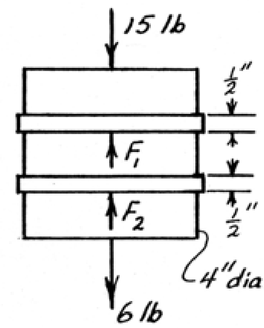
$\delta = 0.0294$ m or $\underline{\delta = 29.4 \text{ mm}}$

(Positive result taken from quadratic formula)

3/141 $F_1 + F_2 = \mu_k (\text{Area}) p$

$$= 0.15 \times 2 \times 4\pi \times \frac{1}{2} p$$

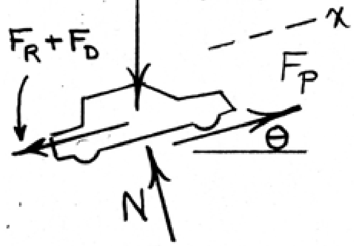
$$= 1.885 p \text{ lb}$$



$U = \Delta T$: $(15 + 6 - 1.885p)\frac{10}{12} = \frac{1}{2}\frac{6}{32.2}(8^2 - 0^2)$

Solve & get $p = \underline{7.34 \text{ lb/in.}^2}$

3/142 $mg = 2000\text{ lb}$



$\theta = 0$ or $\theta = \tan^{-1}\frac{6}{100} = 3.43°$

$F_D = kv^2$: $50 = k(60)^2$

$\Rightarrow k = 0.01389 \ \frac{\text{lb-hr}^2}{\text{mi}^2}$

& $F_D = 0.01389 v^2$

$\Sigma F_x = 0$: $F_P - F_R - F_D - mg\sin\theta = 0$

$F_P = F_R + F_D + mg\sin\theta$

(a) $\underline{\theta = 0}$ $v = 30$ mi/hr : $F_D = 0.01389(30^2) = 12.50$ lb

$F_P = F_R + F_D = 50 + 12.50 = 62.5$ lb

$P_{30} = Fv = 62.5\left(30\ \frac{5280}{3600}\right)/550 = \underline{5\text{ hp}}$

$v = 60$ mi/hr : $F_D = 50$ lb, $F_P = F_R + F_D = 100$ lb

$P_{60} = Fv = 100\left(60\ \frac{5280}{3600}\right)/550 = \underline{16\text{ hp}}$

(b) $\underline{\theta = 3.43°}$ $F_P = 50 + 50 + 2000\sin 3.43° = 220$ lb

$P_{up} = 220\left(60\ \frac{5280}{3600}\right)/550 = \underline{35.2\text{ hp}}$

Down: $F_P = 50 + 50 - 2000\sin 3.43° = -19.78$ lb

$P_{down} = -19.78\left(60\ \frac{5280}{3600}\right)/550 = \underline{-3.17\text{ hp}}$ (brakes!)

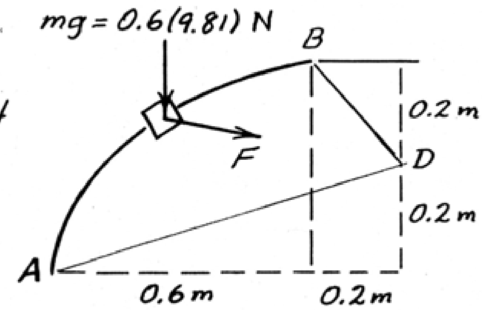
(c) $\Sigma F_x = 0$: $50 + kv^2 - 2000\sin 3.43° = 0$, $\underline{v = 70.9\ \frac{\text{mi}}{\text{hr}}}$

3/143 $U = \Delta T$

$U = F(\overline{AD} - \overline{BD}) - 0.6(9.81)0.4$

$= F(0.825 - 0.283) - 2.35$

$= 0.542F - 2.35$ J



$\overline{AD} = \sqrt{0.8^2 + 0.2^2} = 0.825$ m

$\overline{BD} = \sqrt{0.2^2 + 0.2^2} = 0.283$ m

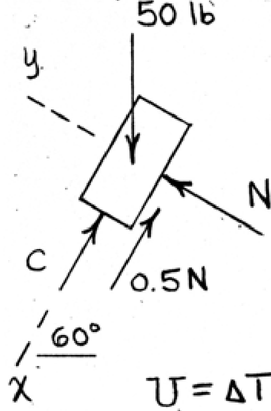
$\Delta T = \frac{1}{2}m(v_B^2 - v_A^2)$

$= \frac{1}{2}0.6(4^2 - 0) = 4.8$ J

Thus $0.542F - 2.35 = 4.8$,

$\underline{F = 13.21 \text{ N}}$

3/144



$\Sigma F_y = 0: \; N - 50\cos 60^\circ = 0, \; N = 25 \text{ lb}$

Displacement is $3 + \frac{4}{12} = 3.33$ ft

$$U_{1-2} = (50 \sin 60^\circ - 0.5 \cdot 25)\, 3.33 - \frac{1}{12}\int_0^{4''} (100x + 9x^2)\,dx$$

$$= 20.0 \text{ ft-lb}$$

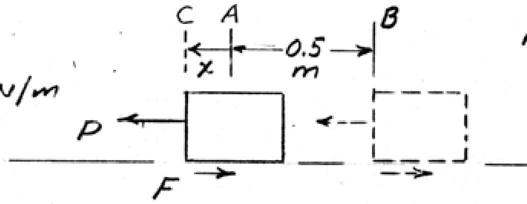
$U_{1-2} = \Delta T: \quad 20.0 = \frac{1}{2}\frac{50}{32.2}(v^2 - 2^2)$

$\underline{v = 5.46 \text{ ft/sec}}$

3/145

$k = 300$ N/m

$F = \mu_k N$
$= 0.30(10)(9.81)$
$= 29.43$ N



(a)

From B to A: $U_{1-2} = \Delta T$

$$\frac{1}{2}(300)(0.5)^2 - 29.43(0.5) = \frac{1}{2}(10)v^2$$

$$v^2 = 4.557\ (\text{m/s})^2, \quad \underline{v = 2.13\ \text{m/s}}$$

(b) From A to C; $U_{1-2} = \Delta T$

$$-\frac{1}{2}(300)x^2 - 29.43x = 0 - \frac{1}{2}(10)(4.557)$$

$$x^2 + 0.1962x - 0.1519 = 0$$

$$x = -\frac{0.1962}{2} \pm \frac{1}{2}\sqrt{(0.1962)^2 + 4(0.1519)}$$

$$= -0.0981 \pm 0.4019, \quad \underline{x = 0.304\ \text{m}} \quad (x = -0.48)$$

3/146 $P = Fv$; $F = ma$, so $P = mav$

& $a = \frac{P}{mv}$.

But $v\,dv = a\,ds$, so $mv^2 dv = P\,ds$

$\int_{v_1}^{v_2} mv^2 dv = \int_0^s P\,ds$; $\frac{m}{3}(v_2^3 - v_1^3) = Ps$

$$\underline{v_2 = \left(\frac{3Ps}{m} + v_1^3\right)^{1/3}}$$

3/147 $U'_{1-2} = 0 = \Delta T + \Delta V_g + \Delta V_e$

$\Delta T = \frac{1}{2} 3 (v^2 - 0) = \frac{3}{2} v^2$

$\Delta V_g = -3(9.81)(0.8) = -23.5 \text{ J}$

$\Delta V_e = \frac{1}{2} 200 \left[\left(\sqrt{0.8^2 + 0.6^2} - 0.4 \right)^2 - (0.8 - 0.4)^2 \right]$

$= 20 \text{ J}$

So $0 = \frac{3}{2} v^2 - 23.5 + 20$, $\underline{v = 1.537 \text{ m/s}}$

3/148 For the system, $U'_{1-2} = 0$, so $\Delta V_g + \Delta T = 0$

$$-mg\left(\frac{18}{12}\right) + \frac{1}{2}(2m)(v^2 - 0) = 0$$

$\underline{v = 6.95 \text{ ft/sec}}$

3/149 Establish datum @ A.

(a) $T_A + V_A = T_B + V_B$

$0 + 0 = \frac{1}{2} m v_B^2 - m g h_B$

$v_B = \sqrt{2 g h_B} = \sqrt{2(9.81)(4.5)} = \underline{9.40 \text{ m/s}}$

(b) State F: spring fully compressed

$T_A + V_A = T_F + V_F$

$0 + 0 = 0 - m g h_f + \frac{1}{2} k \delta^2$

$\delta = \sqrt{\frac{2 m g h_f}{k}} = \sqrt{\frac{2(1.2)(9.81)(3)}{24\,000}} = 0.0542 \text{ m}$

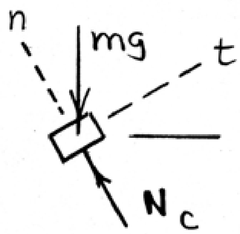
or $\underline{\delta = 54.2 \text{ mm}}$

3/150 Establish datum @ A.

$$T_A + V_A = T_C + V_C : 0 + 0 = \frac{1}{2}mv_C^2 - mgh_C$$

$$v_C = \sqrt{2gh_C} = \sqrt{2(9.81)(3 + 1.5\cos 30°)}$$

$$= 9.18 \text{ m/s}$$



(a) $\Sigma F_n = m\frac{v^2}{\rho} : N_C - 1.2(9.81)\cos 30° = 1.2\frac{9.18^2}{1.5}$

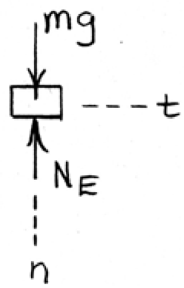
$$\underline{N_C = 77.7 \text{ N}}$$

(b) $\Sigma F_n = 0 : N_C - 1.2(9.81)\cos 30° = 0$

$$\underline{N_C = 10.19 \text{ N}}$$

$$T_A + V_A = T_E + V_E : 0 + 0 = \frac{1}{2}mv_E^2 - mgh_E$$

$$v_E = \sqrt{2gh_E} = \sqrt{2(9.81)(3)} = 7.67 \text{ m/s}$$



$\Sigma F_n = m\frac{v^2}{\rho} : -N_E + 1.2(9.81) = 1.2\frac{7.67^2}{1.5}$

$$\underline{N_E = -35.3 \text{ N}} \text{ (down)}$$

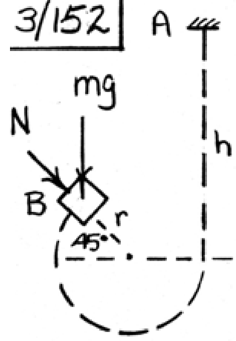
3/151 $\Delta T + \Delta V_e + \Delta V_g = 0, \quad \Delta T = 0$

$\Delta V_e = \frac{1}{2} k (x_2^2 - x_1^2) = \frac{1}{2} 500 (0.050^2 - 0.100^2) = -1.875 \text{ J}$

$\Delta V_g = mg\Delta h = 2(9.81)h = 19.62 h$

Thus $0 - 1.875 + 19.62 h = 0, \quad h = 0.0956 \text{ m}$ or $\underline{h = 95.6 \text{ mm}}$

3/152



$$U'_{1-2} = \Delta T + \Delta V_g = 0$$

$$\frac{1}{2} m v^2 - mg\left(h - \frac{r}{\sqrt{2}}\right) = 0$$

$$v^2 = 2g\left(h - \frac{r}{\sqrt{2}}\right)$$

$$\Sigma F_n = ma_n: \quad N + \frac{mg}{\sqrt{2}} = m\frac{v^2}{r}$$

$$\Rightarrow N = mg\left[\left(\frac{h}{r} - \frac{1}{\sqrt{2}}\right)2 - \frac{1}{\sqrt{2}}\right]$$

$$= mg\left[2\frac{h}{r} - \frac{3}{\sqrt{2}}\right]$$

With $m = 0.25$ kg, $r = 0.15$ m, & $h = 0.6$m,

$N = 14.42$ N

3/153 $T_A + V_A = T_B + V_B$, datum @ B

$$0 + mgR + \frac{1}{2}k\left[R\sqrt{2} - R\right]^2 = \frac{1}{2}mv_B^2 + 0$$

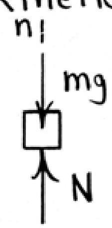
$$\underline{v_B = \sqrt{2gR + \frac{kR^2}{m}\left(3 - 2\sqrt{2}\right)}}$$

$T_A + V_A = T_C + V_C$, datum @ C

$$0 + 2mgR + \frac{1}{2}k\left[R\sqrt{2} - R\right]^2 = \frac{1}{2}mv_C^2 + 0$$

$$\underline{v_C = \sqrt{4gR + \frac{kR^2}{m}\left(3 - 2\sqrt{2}\right)}}$$

Kinetics at C:



$\Sigma F_n = ma_n$: $N - mg = m\frac{v_C^2}{R}$

$$\Rightarrow \underline{N = m\left[5g + \frac{kR}{m}\left(3 - 2\sqrt{2}\right)\right]}$$

3/154 For the system, $T_1 + V_1 + U'_{1\text{-}2} = T_2 + V_2$

$$\tfrac{1}{2} m \cancel{v_1^2}^{\,0} + \tfrac{1}{2} k x_1^2 + 0 = \tfrac{1}{2} m v^2 + \tfrac{1}{2} k x_2^2 - mgh,$$

where the datum is the initial position and h is the drop distance. Note that the spring deflection runs at twice that of the cylinder. Numbers:

$$\tfrac{1}{2}\, 6(12)\left[\frac{3}{12}\right]^2 = \frac{1}{2}\,\frac{100}{32.2} v^2 + \frac{1}{2}\, 6(12)\left[\frac{3+2\left(\frac{1}{2}\right)}{12}\right]^2 - 100\left(\frac{1/2}{12}\right)$$

$$\underline{v = 1.248 \text{ ft/sec}}$$

3/155 (a) $\Delta T + \Delta V_g = 0$

$$\frac{1}{2}\frac{5}{32.2}v^2 + \frac{1}{2}\frac{10}{32.2}\left(\frac{12}{18}v\right)^2 + 5\,\frac{18}{12}\sin 60^\circ - 10\,\frac{12}{12}\sin 60^\circ = 0$$

$$0.1467\,v^2 = 2.165, \quad v^2 = 14.76\ (\text{ft/sec})^2$$

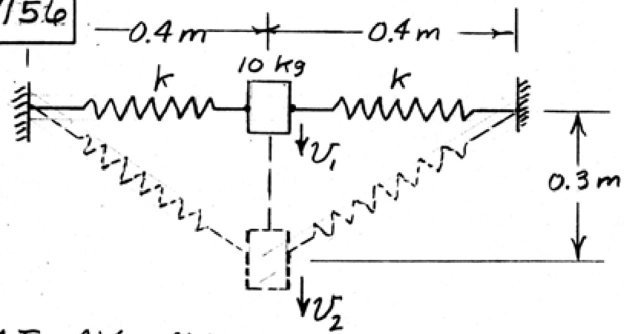
$$\underline{v = 3.84\ \text{ft/sec}}$$

(b) For entire interval $\Delta T = 0$, $\Delta V_g + \Delta V_e = 0$

$$-2.165(12) + \frac{1}{2}(200)x^2 = 0, \quad x^2 = 0.2598\ (\text{in})^2$$

$$\underline{x = 0.510\ \text{in.}}$$

3/156



$k = 800$ N/m

$v_1 = 2$ m/s

Initial stretch

$\delta_1 = 0.1$ m

$\delta_2 = 0.5 - 0.3$
$= 0.2$ m

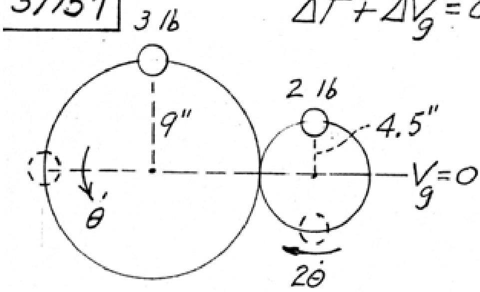
$\Delta T + \Delta V_g + \Delta V_e = 0$

$\frac{1}{2} 10(v_2^2 - 2^2) - 10(9.81)(0.3) + \frac{2}{2} 800(0.2^2 - 0.1^2) = 0$

$5v_2^2 = 20 + 29.43 - 24, \quad v_2^2 = 5.086 \ (m/s)^2$

$\underline{v_2 = 2.26 \text{ m/s}}$

3/157



$$\Delta T + \Delta V_g = 0; \quad \Delta T = \frac{1}{2}\frac{3}{32.2}\left(\frac{9}{12}\dot{\theta}\right)^2$$

$$+ \frac{1}{2}\frac{2}{32.2}\left(\frac{4.5}{12}[2\dot{\theta}]\right)^2$$

$$= 0.04367\,\dot{\theta}^2 \text{ ft-lb}$$

$$\Delta V_g = -3\left(\frac{9}{12}\right) - 2\left(\frac{4.5+4.5}{12}\right)$$

$$= -\frac{15}{4} = -3.75 \text{ ft-lb}$$

Thus $0.04367\dot{\theta}^2 - 3.75 = 0$, $\dot{\theta}^2 = 85.87\ (\text{rad/sec})^2$

$$\underline{\dot{\theta} = 9.27 \text{ rad/sec}}$$

$\boxed{3/158}$ Let m be the mass of the car

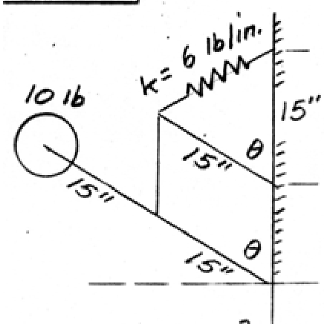
$U'_{1-2} = \Delta T + \Delta V_g: \quad 0 = \frac{1}{2}m(v^2 - v_0^2) + mgy$

$$v^2 = v_0^2 - 2gy$$

$a_n = \frac{v^2}{\rho}: \quad \frac{v_0^2}{\rho_0} = \frac{v_0^2 - 2gy}{\rho}, \quad \underline{\rho = \rho_0\left(1 - \frac{2gy}{v_0^2}\right)}$

For car to remain in contact with the track at the top, $a_n > g$, so for constant a_n, $v_0^2/\rho_0 > g$ so $\underline{v_{0_{min}} = \sqrt{\rho_0 g}}$

3/159 For the interval from $\theta = 60°$ to $\theta = 90°$,



spring stretch is

$\delta = 15\sqrt{2} - 15 = 6.21$ in.

$\Delta V_e = \frac{1}{2} 6 (6.21)^2 = 115.8$ in.-lb

$\Delta V_g = -mg\Delta h = -10(30)\cos 60°$

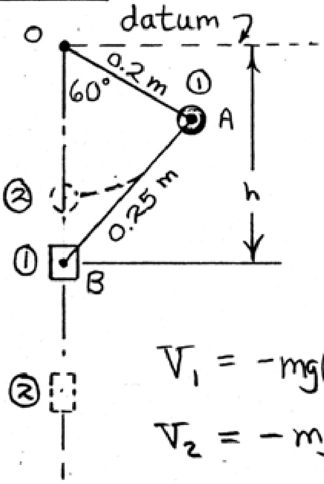
$= -150$ in.-lb.

$\Delta T + \Delta V_g + \Delta V_e = 0$

$\frac{1}{2}\frac{10}{32.2(12)} v^2 - 150 + 115.8 = 0, \quad v^2 = 2642, \quad v = 51.4 \frac{\text{in.}}{\text{sec}}$

or $v = 4.28$ ft/sec

3/160



$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2} m v_A^2$$

Note that

$$h = 0.2 \cos 60° + \sqrt{0.25^2 - (0.2 \sin 60°)^2}$$

$$= 0.280 \text{ m}$$

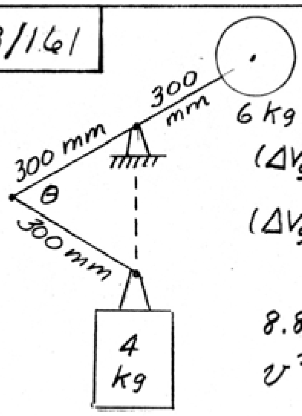
$$V_1 = -mg(0.2 \cos 60) - mg(0.280) = -0.380 mg$$

$$V_2 = -mg(0.2) - mg(0.45) = -0.650 mg$$

So $\quad -0.380 mg = \frac{1}{2} m v_A^2 - 0.650 mg$

$$\underline{v_A = 2.30 \text{ m/s}}$$

3/161



For motion from $\theta = 60°$ to $\theta = 180°$

$\Delta V_g + \Delta T = 0$

$(\Delta V_g)_{6\,kg} = 6(9.81)(0.3)(1 - \sin 30°) = 8.829\ J$

$(\Delta V_g)_{4\,kg} = -4(9.81)(2)(0.3)(1 - \sin 30°)$

$= -11.772\ J$

$8.829 - 11.772 + \frac{1}{2}6v^2 + 0 = 0$

$v^2 = 0.981\ (m/s)^2$, $\underline{v = 0.990\ m/s}$

3/162 Establish datum at release point.

$$T_A + V_A = T_B + V_B$$

$$0 + \frac{1}{2}k_A x_A^2 = 0 + mg(x_A + d + x_B) + \frac{1}{2}k_B x_B^2$$

$$\frac{1}{2}(48)(12)\left(\frac{5}{12}\right)^2 = 14\left(\frac{5 + 14 + x_B}{12}\right) + \frac{1}{2}(10)(12)\left(\frac{x_B}{12}\right)^2$$

$$\underline{x_B = 6.89 \text{ in.}}$$

The fact that $x_B > x_A$ is due to the difference in spring stiffnesses (along with the particular value $d = 20 - 6 = 14''$). Note that $d = 14''$ is the distance which the collar moves when out of contact with the springs.

3/163 A force analysis reveals that A will move down & B will move up.

Kinematics : $3v_A = 2v_B$ (speeds)

$T_1 + V_1 = T_2 + V_2$, datum @ initial position

$$0 + 0 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B \left(\frac{3}{2} v_A\right)^2 + m_B g h_B - m_A g h_A$$

$$0 = \frac{1}{2}(40) v_A^2 + \frac{1}{2} 8 \frac{9}{4} v_A^2 + 8(9.81)(1) - 40(9.81)\left(\frac{2}{3}(1)\sin 20°\right)$$

$\underline{v_A = 0.616 \text{ m/s}}$, $\underline{v_B = \frac{3}{2} v_A = 0.924 \text{ m/s}}$

$\boxed{3/164}$ 1st interval of motion (0.4 m) $\Delta T + \Delta V_g = 0$ for system

$\frac{1}{2}(4+6+8)v^2 + 9.81 \times 0.4(8-4-6) = 0, \; v^2 = 0.872 \; (m/s)^2$

$v = 0.934$ m/s

2nd interval for 6- & 8-kg cylinders $\Delta T + \Delta V_g = 0$

$0 - \frac{1}{2}(6+8)(0.872) + 9.81(h-0.4)(8-6) = 0, \; h = 0.711$ m

or $\underline{h = 711 \text{ mm}}$

Kinetic energy of collar is dissipated into heat & sound during impact with bracket.

3/165 Constant total energy is $E = T_A + V_{g_A} = T_P + V_{g_A}$

Thus $\frac{1}{2} m v_A^2 - \frac{mgR^2}{r_A} = \frac{1}{2} m v_P^2 - \frac{mgR^2}{r_P}$

$v_A^2 = v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right)$, $\underline{v_A = \sqrt{v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right)}}$

3/166 $U'_{1-2} = \Delta T + \Delta V_e + \Delta V_g$ for system

$U'_{1-2} = 50(1.5)\cos 30° = 64.95$ J

$\Delta T = \frac{1}{2} 2 v^2 = v^2$

$\Delta V_e = \frac{1}{2} 30 \left[\left(\sqrt{2^2 + 1.5^2} - 1.5 \right)^2 - (2 - 1.5)^2 \right] = 11.25$ J

$\Delta V_g = 2(9.81)1.5 = 29.43$ J

So $64.95 = v^2 + 11.25 + 29.43$, $v^2 = 24.27$, $\underline{v = 4.93 \frac{m}{s}}$

3/167 $\Delta T + \Delta V_g = 0$, $V_g = -\dfrac{mgR^2}{r}$

Mean radius of earth is $R = 6371$ km

$g = 9.825(3600)^2/1000 = 127.3(10^3)$ km/h^2

Thus

$$\frac{1}{2}m\left(v_B^2 - [24\,000]^2\right) + 127.3(10^3)(6371)^2 m\left(-\frac{1}{6500} + \frac{1}{7000}\right) = 0$$

$$\frac{1}{2}v_B^2 - 288(10^6) + 5167(10^9)(-0.01099)(10^{-3}) = 0$$

$$v_B^2 = 2\left[288 + 56.8\right]10^6 = 690\,(10^6),\quad \underline{v_B = 26\,300 \text{ km/h}}$$

3/168

$$\Delta T + \Delta V_g + \Delta V_e + U_f = 0$$

Spring elongation at A is $\delta_A = \sqrt{3^2+4^2} - 2 = 3$ ft

" " " B " $\delta_B = \sqrt{3^2+3^2} - 2 = 2.24$ ft

$$\Delta V_e = \frac{1}{2}k(\delta_B^2 - \delta_A^2) = \frac{1}{2}2(2.24^2 - 3^2) = -3.97 \text{ ft-lb}$$

$$\Delta T = \frac{1}{2}m(v_B^2 - v_A^2) = \frac{1}{2}\frac{5}{32.2}(10^2 - 6^2) = 4.97 \text{ ft-lb}$$

$$\Delta V_g = W\Delta z = 5(0-4) = -20 \text{ ft-lb}$$

Thus $4.97 - 20 - 3.97 + U_f = 0$, $\underline{U_f = 19.00 \text{ ft-lb (loss)}}$

$U_f = F_{av}s$, $F = \dfrac{19.00}{5} = \underline{3.80 \text{ lb}}$

3/169 Ellipse eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$

$= \sqrt{1 - \frac{0.6^2}{0.8^2}} = 0.661$

$r_{min} = a(1-e) = 0.8(1-0.661) = 0.271$ m

$r_{max} = a(1+e) = 0.8(1+0.661) = 1.329$ m

$T_A + V_A = T_C + V_C$

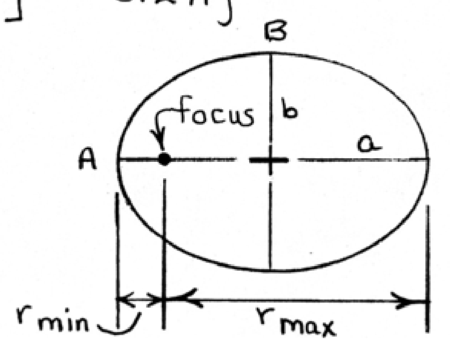
$\frac{1}{2} m v_A^2 + 0 = 0 + \frac{1}{2} k x_C^2$

$\frac{1}{2}(0.4) v_A^2 = \frac{1}{2}(3)\left[1.329 - 0.271\right]^2$, $v_A = 2.90$ m/s

Then $T_A + V_A = T_B + V_B$

$\frac{1}{2}(0.4)(2.90)^2 + 0 = \frac{1}{2}(0.4) v_B^2 + \frac{3}{2}\left\{\left[(0.8 - 0.271)^2 + (0.6)^2\right]^{1/2} - 0.271\right\}^2$

$\underline{v_B = 2.51 \text{ m/s}}$



$\boxed{3/170}$ $U'_{1-2} = \Delta T + \Delta V_g = 0$

$$\Delta T = \frac{1}{2} m \left[v^2 - \left(2000 \frac{44}{30}\right)^2 \right]$$

$$\Delta V_g = -mgR^2 \left(\frac{1}{R} - \frac{1}{2R}\right) = -\frac{mgR}{2}$$

$$= -\frac{1}{2} m \; 5.32 \, (1080)(5280)$$

So $v^2 - \left(2000 \frac{44}{30}\right)^2 = 5.32 \, (1080)(5280)$

$$\underline{v = 6240 \text{ ft/sec}} \quad \text{or} \quad \underline{4250 \text{ mi/hr}}$$

$\boxed{3/171}$ $U'_{1-2} = 0$ so $T_1 + V_{g_1} = T_2 + V_{g_2}$

Take datum $V_g = 0$ at ground level.

$T_1 = \frac{1}{2} \frac{175 + 10}{32.2} v^2 = 2.87 v^2$, $T_2 = 0$

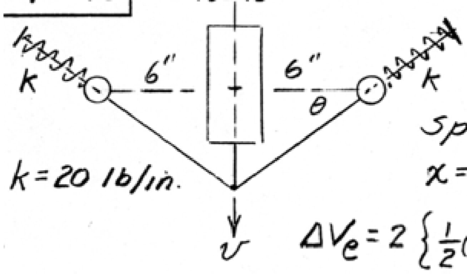
$V_{g_1} = (175 + 10) \frac{42}{12} = 648$ ft-lb

$V_{g_2} = 175(18) + 10(8) = 3230$ ft-lb

So $2.87 v^2 + 648 = 0 + 3230$

$\underline{v = 30.0 \text{ ft/sec}}$ or $\underline{20.4 \text{ mi/hr}}$

3/172



For interval $\theta = 0$ to $\theta = 30^\circ$

$\Delta V_g = -10(6 \tan 30^\circ)$
$= -34.64$ in.-lb

Spring compression is

$x = \frac{6}{\cos 30^\circ} - 6 = 0.928$ in.

$\Delta V_e = 2\left\{\frac{1}{2}(20)(0.928)^2\right\} = 17.23$ in.-lb

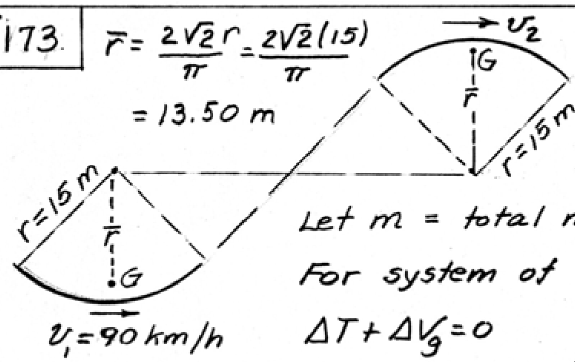
$\Delta T = \frac{1}{2}\,\frac{10}{32.2}\,\frac{1}{12}\,v^2$, ($v$ in in./sec)
$= 0.01294\,v^2$

$\Delta T + \Delta V_g + \Delta V_e = 0$; $0.01294\,v^2 - 34.64 + 17.24 = 0$
$v^2 = 1345$ (in./sec)^2, $v = 36.7$ in/sec
or $\underline{v = 3.06 \text{ ft/sec}}$

3/173

$\bar{r} = \frac{2\sqrt{2}\,r}{\pi} = \frac{2\sqrt{2}(15)}{\pi}$

$= 13.50$ m



Let m = total mass of train

For system of cars

$\Delta T + \Delta V_g = 0$

$\frac{1}{2}m(v_2^2 - v_1^2) + mg(2\bar{r}) = 0, \quad v_2^2 = v_1^2 - 4g\bar{r}$

$v_2^2 = \left[\frac{90(1000)}{3600}\right]^2 - 4(9.81)(13.50) = 625 - 529.9 = 95.07\ (\text{m/s})^2$

$v_2 = 9.75$ m/s or $\underline{v_2 = 35.1 \text{ km/h}}$

3/174 $U' = \Delta T + \Delta V_g = 0$ where $V_g = -\frac{mgR^2}{r}$

$$\Delta V_g = -9.825\,m\left[6371(10^3)\right]^2\left(\frac{1}{(2500+6371)10^3} - \frac{1}{(2200+6371)10^3}\right)$$

$$= 1.573(10^6)m$$

$$\Delta T = \frac{1}{2}m\left(v_B{}^2 - \left[\frac{25\,000\times10^3}{3600}\right]^2\right)$$

Thus $\frac{1}{2}v_B{}^2 - \frac{1}{2}\left[\frac{25\,000}{3.6}\right]^2 + 1.573(10^6) = 0$

$v_B{}^2 = 45.08\,(10^6)\,\left(\frac{m}{s}\right)^2$, $v_B = 6714$ m/s

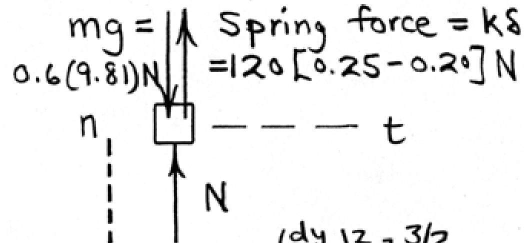
or $\underline{v_B = 24\,170 \text{ km/h}}$

3/175 $T_A + V_A = T_B + V_B$, datum @ B.

$$0 + 0.6(9.81)(0.5) + \tfrac{1}{2}120\left[\sqrt{0.25^2+0.5^2} - 0.2\right]^2$$
$$= \tfrac{1}{2}(0.6)v_B^2 + \tfrac{1}{2}120\left[0.25-0.20\right]^2$$

$\underline{v_B = 5.92 \text{ m/s}}$

Kinetics at B:



Radius of curvature $\rho = \dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$y = kx^2$: $0.5 = k(0.5)^2 \Rightarrow k = 2$

$y = 2x^2$, $\dfrac{dy}{dx} = 4x$, $\dfrac{d^2y}{dx^2} = 4$

When $x = 0$, $\rho = \dfrac{[1+0^2]^{3/2}}{4} = 0.25 \text{ m}$

$$\Sigma F_n = ma_n : N + 120(0.05) - 0.6(9.81) = 0.6\,\frac{5.92^2}{0.25}$$

$\underline{N = 84.1 \text{ N}}$

$\boxed{3/176}$ $\Delta T + \Delta V_g + \Delta V_e = 0$

$\Delta T = \frac{1}{2} m \dot{y}^2$; $\Delta V_g = -mgy$

$\Delta V_e = 2\left\{\frac{1}{2} k x^2\right\} = k(y \sin\theta)^2 = ky^2(1-\cos^2\theta)$

$= ky^2(1 - c^2/b^2)$

$\frac{1}{2} m \dot{y}^2 - mgy + ky^2(1 - c^2/b^2) = 0$

$$\underline{\dot{y} = \sqrt{2y\left(g - \frac{k}{m} y \frac{b^2 - c^2}{b^2}\right)}}$$

$y_{max} = y$ for $\dot{y} = 0$, so $2gy - \frac{2k}{m} y^2 (1 - c^2/b^2) = 0$

Hence ($y_{min} = 0$), $\underline{y_{max} = \frac{mg}{k} \frac{b^2}{b^2 - c^2}}$

$\boxed{3/177}$ $x^2 + y^2 = 0.9^2$, $x\dot{x} + y\dot{y} = 0$, $v_A = -\dot{y} = \frac{x}{y}\dot{x} = \frac{x}{y} v_B$

$\Delta T + \Delta V_g = 0$; $\frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + mg\left(y - \frac{0.9}{\sqrt{2}}\right) = 0$

$\dot{x}^2\left(1 + \frac{x^2}{y^2}\right) = 2(9.81)\left(\frac{0.9}{\sqrt{2}} - y\right)$, $\dot{x}^2 \frac{x^2+y^2}{y^2} = 19.62\left(\frac{0.9}{\sqrt{2}} - y\right)$

$0.9^2 \dot{x}^2 = 19.62\left(\frac{0.9}{\sqrt{2}} y^2 - y^3\right)$

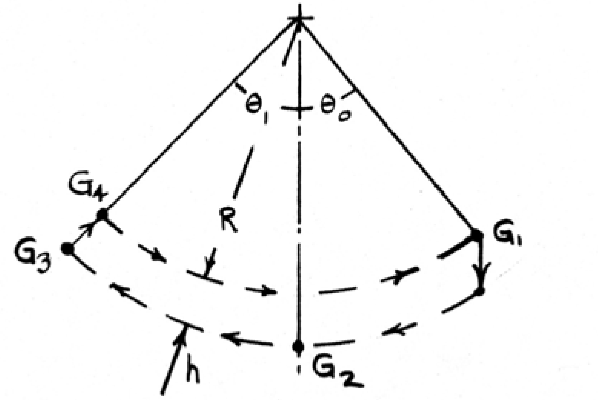
For max. $\dot{x}$, $\frac{d(\dot{x}^2)}{dy} = \frac{19.62}{0.81}\left(\frac{1.8}{\sqrt{2}} y - 3y^2\right) = 0$

So $y\left(\frac{1.8}{\sqrt{2}} - 3y\right) = 0$, $y = 0.6/\sqrt{2}$ m

& $\dot{x}^2 = \frac{19.62}{0.81}\left(\frac{0.9}{\sqrt{2}} \frac{0.36}{2} - \frac{0.108}{\sqrt{2}}\right) = \frac{19.62\sqrt{2}}{30}$

$v_{B_{max}} = \dot{x} = \sqrt{\frac{19.62\sqrt{2}}{30}} = \underline{0.962 \text{ m/s}}$

3/178



$$U'_{1-3} = \Delta T + \Delta V_g$$

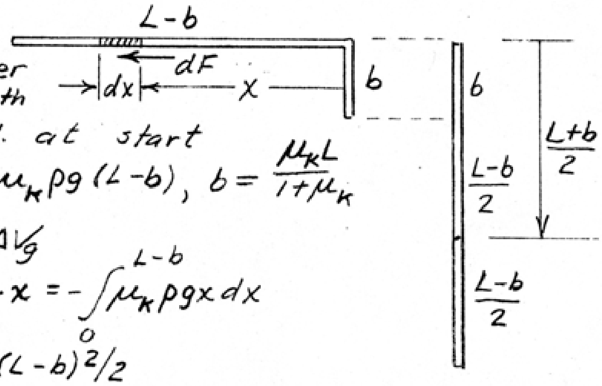
$$U'_{1-2} = U'_{2-3} = 0, \quad \Delta T = 0, \quad \text{so} \quad V_{g_1} = V_{g_3}$$

Thus $$R\cos\theta_0 = (R+h)\cos\theta_1$$

$$\underline{\underline{\theta_1 = \cos^{-1}\left(\frac{R}{R+h}\cos\theta_0\right)}}$$

▶3/179

ρ = mass per unit length



For equil. at start

$$\rho g b = \mu_k \rho g (L-b), \quad b = \frac{\mu_k L}{1+\mu_k}$$

$$U = \Delta T + \Delta V_g$$

$$U = -\int dF \cdot x = -\int_0^{L-b} \mu_k \rho g x \, dx$$

$$= -\mu_k \rho g (L-b)^2/2$$

$$\Delta T = \frac{1}{2} \rho L v^2$$

$$\Delta V_g = -\rho g (L-b)\left(\frac{L+b}{2}\right)$$

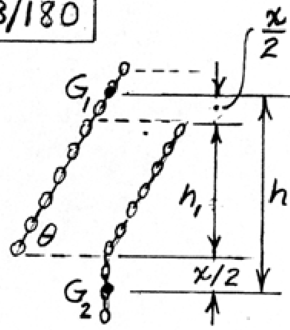
Thus $-\mu_k \rho g (L-b)^2/2 = \frac{1}{2}\rho L v^2 - \rho g \frac{L^2-b^2}{2}$

$v^2 = g(1-b/L)(L+b-\mu_k[L-b])$; Now substitute b

so $v^2 = g\left(1-\frac{\mu_k}{1+\mu_k}\right)\left(L\left[1+\frac{\mu_k}{1+\mu_k}\right]-\mu_k\left[L-\frac{\mu_k L}{1+\mu_k}\right]\right)$

$$= \frac{gL}{1+\mu_k}, \qquad \underline{v = \sqrt{\frac{gL}{1+\mu_k}}}$$

3/180



$h_1 = (L-x)\sin\theta$

$h = (L-x)\sin\theta + \frac{x}{2}\sin\theta + \frac{x}{2}$

$= L\sin\theta + \frac{x}{2}(1-\sin\theta)$

Let ρ = mass per unit length

$\Delta V_g + \Delta T = 0$

ΔV_g is that of the length x dropping a distance h

$\Delta V_g = -\rho g x h = -\rho g\left[Lx\sin\theta + \frac{x^2}{2}(1-\sin\theta)\right]$

$\Delta T = \frac{1}{2}\rho L v^2$

Thus $-\rho g\left[Lx\sin\theta + \frac{x^2}{2}(1-\sin\theta)\right] + \frac{1}{2}\rho L v^2 = 0$

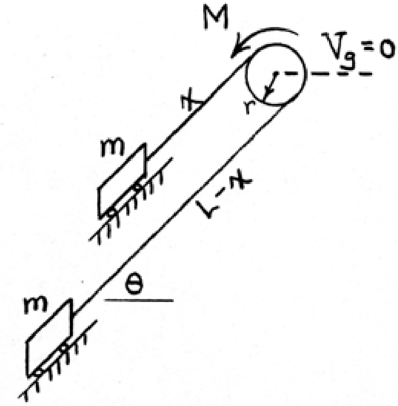
$$v = \sqrt{2gx\left[\sin\theta + \frac{x}{2L}(1-\sin\theta)\right]}$$

▶3/181

$U'_{1-2} = \Delta T + \Delta V$

$U'_{1-2} = M \frac{x}{r}$

$\Delta V_e = 0$



$$V_{g_2} = -g\left[m(L-x) + mx + \rho(L-x)\frac{L-x}{2} + \rho x \frac{x}{2}\right]\sin\theta$$

$$= -g\sin\theta\left\{mL + \frac{\rho}{2}\left[(L-x)^2 + x^2\right]\right\}$$

$$V_{g_1} = -g\sin\theta\left\{mL + \rho L \frac{L}{2}\right\}$$

$$\Delta V_g = -g\sin\theta\left\{mL + \frac{\rho}{2}\left[(L-x)^2 + x^2\right] - mL - \frac{\rho L^2}{2}\right\}$$

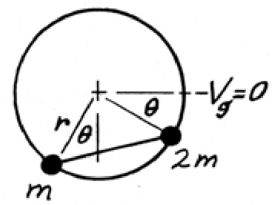
$$= -g\sin\theta\left\{\frac{\rho}{2}\left[2x^2 - 2Lx\right]\right\}$$

$$\Delta T = \frac{1}{2}(2m + \rho L)v^2$$

$$\therefore\ M\frac{x}{r} = \frac{1}{2}(2m+\rho L)v^2 - g\sin\theta\left\{\frac{\rho}{2}\left[2x^2 - 2Lx\right]\right\}$$

Solving, $v = \sqrt{\dfrac{2}{2m+\rho L}}\sqrt{\dfrac{Mx}{r} - \rho g x(L-x)\sin\theta}$

▸3/182 For the unit $U' = \Delta T + \Delta V_g = 0$



$\Delta V_g = (-2mgr\sin\theta - mgr\cos\theta) - (-mgr + 0)$

$= mgr(-2\sin\theta - \cos\theta + 1)$

so $\frac{1}{2}3mv^2 - 0 + mgr(-2\sin\theta - \cos\theta + 1) = 0$

or $v^2/gr = \frac{2}{3}(2\sin\theta + \cos\theta - 1)$

(a) Rod is horiz. when $\theta = 45°$

$v^2/gr = \frac{2}{3}(2\sin 45° + \cos 45° - 1) = 0.748$, $\underline{v_{45°} = 0.865\sqrt{gr}}$

(b) $\frac{d}{d\theta}\left(\frac{v^2}{gr}\right) = \frac{2}{3}(2\cos\theta - \sin\theta) = 0$ for max v^2 & hence max v

$\tan\theta = 2$, $\theta = \tan^{-1}2 = 63.4°$

so $v^2_{max}/gr = \frac{2}{3}(2\sin 63.4° + \cos 63.4° - 1) = 0.824$

$\underline{v_{max} = 0.908\sqrt{gr}}$

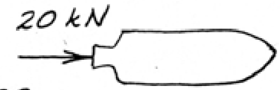
(c) $\theta = \theta_{max}$ when $T = \Delta T = 0$ so $2\sin\theta + \cos\theta - 1 = 0$

$2\sqrt{1-\cos^2\theta} = 1 - \cos\theta$, $5\cos^2\theta - 2\cos\theta - 3 = 0$

$\cos\theta = 0.2 \pm 0.8 = 1$ or -0.6, $\theta = 0$ or $\underline{\theta_{max} = 126.9°}$

3/183

$$\int \Sigma F \, dt = \Delta G$$

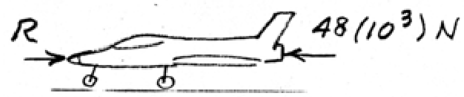


$$(20\,000)(3 \times 60) = 30\,000\,(v - 24\,000)\frac{1000}{3600}$$

$$\underline{v = 24\,400 \text{ km/h}}$$

3/184

$$\int \Sigma F\,dt = m\Delta v$$



$$[48(10^3) - R]10 = 6450\left(\frac{250 \times 1000}{3600} - 0\right)$$

$$R = 3208 \text{ N} \quad \text{or} \quad \underline{R = 3.21 \text{ kN}}$$

$\boxed{3/185}$ $\int F\,dt = m\Delta v$

$$2(26)10^3\, t = 90(10^3)\left[28\,100 - 28\,000\right]/3.6$$

$$\underline{t = 48.1 \text{ s}}$$

3/186

$$\begin{cases} \underline{v} = 1.5t^3\underline{i} + (2.4 - 3t^2)\underline{j} + 5\underline{k} & (\text{m/s}) \\ \dot{\underline{v}} = 4.5t^2\underline{i} - 6t\underline{j} & (\text{m/s}^2) \end{cases}$$

$$\text{At } t = 2\text{s}: \begin{cases} \underline{v} = 12\underline{i} - 9.6\underline{j} + 5\underline{k} \ \text{m/s} \\ \dot{v} = 18\underline{i} - 12\underline{j} \ \text{m/s}^2 \end{cases}$$

$$\text{Then } \underline{G} = m\underline{v} = 1.2(12\underline{i} - 9.6\underline{j} + 5\underline{k})$$

$$= \underline{14.40\underline{i} - 11.52\underline{j} + 6\underline{k} \ \text{kg}\cdot\text{m/s}}$$

$$G = \sqrt{14.40^2 + 11.52^2 + 6^2} = \underline{19.39 \ \text{kg}\cdot\text{m/s}}$$

$$\Sigma\underline{F} = \dot{\underline{G}} : \ \underline{R} = m\dot{\underline{v}} = 1.2(18\underline{i} - 12\underline{j})$$

$$= \underline{21.6\underline{i} - 14.4\underline{j} \ \text{N}}$$

3/187 Conservation of system linear momentum:

$\xrightarrow{+}$ $0.075(600) = 50.075 v_f$, $v_f = 0.899$ m/s

Initial energy $T_1 = \frac{1}{2}(0.075)(600)^2 = 13\ 500$ J

Final energy $T_2 = \frac{1}{2}(50.075)(0.899)^2 = 20.2$ J

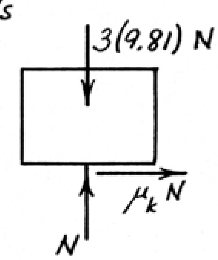
Absolute energy loss $|\Delta E| = T_1 - T_2 = \underline{13\ 480 \text{ J}}$

Percent lost: $n = \frac{|\Delta E|}{T_1}(100\%) = \underline{99.9\ \%}$

3/188. For system of bullet and block $\Delta G = 0$, $G_1 = G_2$:

$$(0.060)(600) = (0.060)(400) + 3v$$

Initial velocity of block is $v = 4$ m/s



For block $U = \Delta T$: $-\mu_k(3 \times 9.81)(2.70)$

$$= \frac{1}{2}3(0 - 4^2)$$

$$\underline{\mu_k = 0.302}$$

3/189 $\Delta G = 0$; $150{,}000 \times 2 + 120{,}000 \times 3$

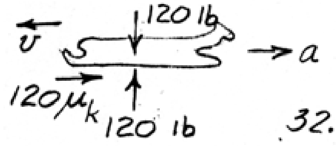
$$= (150{,}000 + 120{,}000)\, v\ , \quad \underline{v = 2.44\ \text{mi/hr}}$$

$$|\Delta E| = \frac{1}{2} m_A v_A^{\,2} + \frac{1}{2} m_B v_B^{\,2} - \frac{1}{2}(m_A + m_B) v^2$$

$$= \frac{1}{2(32.2)} \left(\frac{44}{30}\right)^2 \left[150{,}000 \times 2^2 + 120{,}000 \times 3^2 - 270{,}000 \times \overline{2.44}^{\,2}\right]$$

$$\underline{= 2230 \text{ ft-lb loss}}$$

3/190 $\Delta G = 0; \quad 100(15) = 120v, \quad v = 12.5 \text{ ft/sec}$



$v^2 = 2as; \quad a = F/m = \dfrac{120\mu_k}{120/g} = \mu_k g$

$32.2\mu_k = \dfrac{\overline{12.5}^2}{2(80)}, \quad \underline{\mu_k = 0.030}$

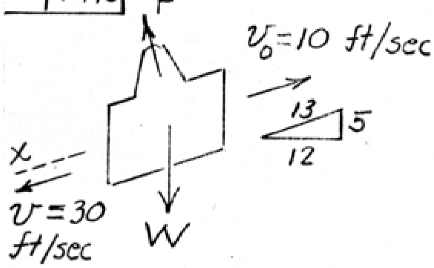
3/191 No difference between cases (a) & (b).

$G_1 = G_2: \quad mv = (3m)v', \quad \underline{v' = \frac{v}{3}}$

$T = \frac{1}{2}mv^2, \quad T' = \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \frac{1}{6}mv^2$

$n = \frac{T - T'}{T} = \frac{\frac{1}{2}mv^2 - \frac{1}{6}mv^2}{\frac{1}{2}mv^2} = \underline{\frac{2}{3}}$

3/192

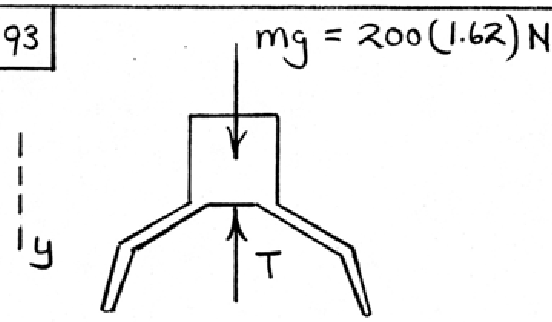


$$\int \Sigma F_x \, dt = m \Delta v_x$$

$$W\left(\frac{5}{13}\right)t = \frac{W}{32.2}(30 - [-10])$$

$$\underline{t = 3.23 \text{ sec}}$$

3/193



$\int \Sigma F_y \, dt = m \Delta v_y$:

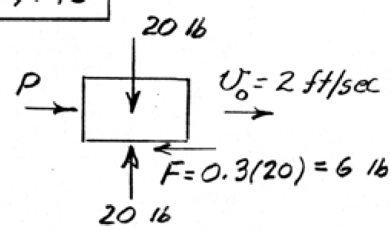
$200(1.62)(5) - \left[\frac{1}{2} 2(800) + 2(800)\right] = 200(v-6)$

$\underline{v = 2.10 \text{ m/s}}$

$\boxed{3/194}$ $\Delta G = 0;\quad 600(18\,000) - \{400 v_3 + 200(18060)\} = 0$

$\underline{v_3 = 17\,970 \text{ km/h}}$

3/195

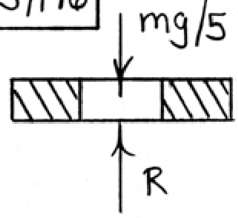


$$\int_0^t \Sigma F\,dt = m\Delta v$$

$$16(0.2) + 8(0.2) - 6(0.4) = \frac{20}{32.2}(v - 2)$$

$$\underline{v = 5.86 \text{ ft/sec}}$$

3/196



Washer :

$$+\downarrow \quad \frac{m}{5}v + \left[\frac{mg}{5} - R\right]\Delta t = 0$$

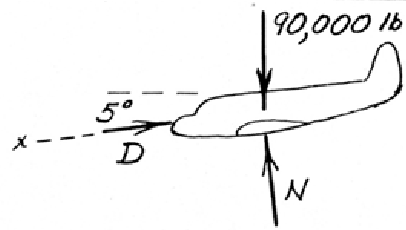
$$\underline{R = \frac{m}{5}\left(\frac{v}{\Delta t} + g\right)}$$

Initial energy : $\frac{1}{2}\frac{6m}{5}v^2 = \frac{3}{5}mv^2$

Final energy : $\frac{1}{2}mv^2$

$$n = \frac{\frac{3}{5} - \frac{1}{2}}{3/5}(100\%) = \underline{16.67\ \%}$$

3/197



$\int \Sigma F_x \, dt = \Delta G_x$:

$$(90{,}000 \sin 5° - D)\,120 = \frac{90{,}000}{32.2}(360 - 400)\frac{5280}{3600}$$

$\underline{D = 9210 \text{ lb}}$

$\boxed{3/198}$ $\Delta G = 0$; $(0.140)(600) - [0.140 + 3 \times 0.100]v = 0$

$$\underline{v = 190.9 \text{ m/s}}$$

$|\Delta E| = \frac{1}{2}(0.140)(600)^2 - \frac{1}{2}(0.140 + 0.300)(190.9)^2$

$= 25.2(10^3) - 8.018(10^3) = \underline{17.18(10^3) \text{ J}}$ loss

3/199 $\int F\,dt = m\Delta v$

$$(50{,}000 \cos 20°)\,t = \frac{150{,}000 \times 2240}{32.2}\;\frac{1 \times 1.151}{1}\;\frac{44}{30}$$

$$46{,}985\,t = 17.62 \times 10^6$$

$t = 375$ sec or $\underline{t = 6.25 \text{ min}}$

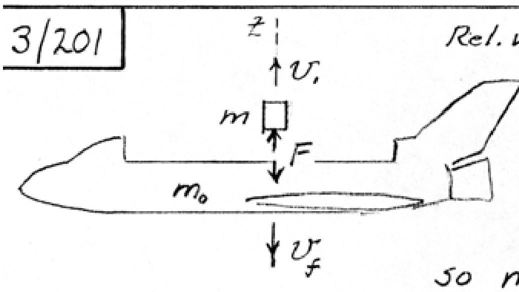
$\boxed{3/200}$ $\Delta G = 0;\quad 320(28) - (320 + 20\times 18)v = 0$

Initial velocity of chain is $\underline{v = 13.18 \text{ m/s}}$

$\int \Sigma F\,dt = m\Delta v;\quad (20\times 18)9.81(0.7)t = (320 + 20\times 18)(13.18)$

$$\underline{t = 3.62 \text{ s}}$$

3/201



Rel. velocity is

$v_1 + v_f = 0.3$ m/s --(1)

$\int F\,dt = m v_1$

$\int -F\,dt = m_f(-v_f)$

so $m v_1 = m_0 v_f$

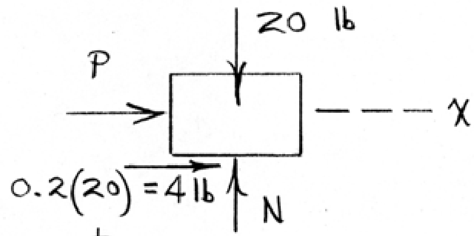
$800\, v_1 = 90\,000\, v_f$ ----- (2)

Solve (1) & (2) & get $v_f = 0.3 - \frac{90\,000}{800} v_f$

$\underline{v_f = 0.00264 \text{ m/s}}$

So $F_{av}\int_0^4 dt = 90\,000\,(0.00264)$, $F_{av} = \frac{90(2.64)}{4} = \underline{59.5 \text{ N}}$

3/202



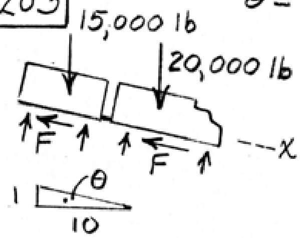
$\xrightarrow{+}$ $mv_1 + \int_0^t \Sigma F\,dt = 0:$

$$-\frac{20}{32.2}(4) + 5(0.2) + 2.5(t - 0.2) + 4t = 0$$

$$\underline{t = 0.305 \text{ sec}}$$

3/203

$\theta = \tan^{-1} 0.1 = 5.71°, \quad \sin\theta = 0.0995$



$$\int \Sigma F_x \, dt = m \Delta v_x$$

$$[35{,}000 \times 0.0995 - 2F]5 = \frac{35{,}000}{32.2}\left(0 - 20\frac{44}{30}\right)$$

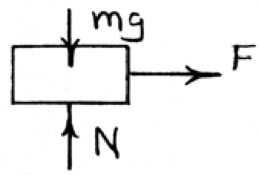
$$F = 4930 \text{ lb}$$

(Trailer)

$$[P - 4930 + 15{,}000 \times 0.0995]5 = \frac{15{,}000}{32.2}\left(0 - 20\frac{44}{30}\right)$$

$$\underline{P = 704 \text{ lb (tension)}}$$

3/204



$\xrightarrow{+}$ $mv_1 + \int \Sigma F\,dt = mv_2$:

$$0 + \int_0^t F_0 e^{-bt}\,dt = mv$$

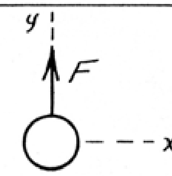
$$\underline{v = \frac{F_0}{mb}\left(1 - e^{-bt}\right)}, \quad \underline{v \rightarrow \frac{F_0}{mb} \text{ as } t \rightarrow \infty}$$

$$\frac{ds}{dt} = \frac{F_0}{mb}\left(1 - e^{-bt}\right)$$

$$\int_{s_0=0}^{s} ds = \int \frac{F_0}{mb}\left(1 - e^{-bt}\right) dt$$

$$\underline{s = \frac{F_0}{mb}\left[t + \frac{1}{b}\left(e^{-bt} - 1\right)\right]}$$

3/205



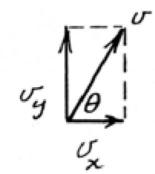
$\int \Sigma F_y \, dt = \Delta G_y$:

$$\int_0^4 \left(2 + \frac{3t^2}{4}\right) dt = 2.4\left(v_y - \left[-\tfrac{3}{5}5\right]\right)$$

$$2t + \frac{t^3}{4}\Big|_0^4 = 2.4\,(v_y + 3), \quad v_y = 7 \text{ m/s}$$

$\int \Sigma F_x \, dt = \Delta G_x$: $0 = 2.4\left(v_x - \frac{4}{5}5\right)$, $v_x = 4$ m/s constant

$$v = \sqrt{4^2 + 7^2} = \underline{8.06 \text{ m/s}}, \quad \theta = \tan^{-1}\frac{7}{4} = \underline{60.3^\circ}$$



3/206 Impact velocity $v_0 = \sqrt{2gh} = \sqrt{2(9.81)(1.4)}$
$= 5.24$ m/s

$\Delta G = 0$; $450(5.24) + 0 = (450 + 240)v$
$\underline{v = 3.42 \text{ m/s}}$

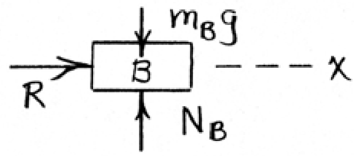
Impulse of weights is negligible compared with impulse of impact forces.

3/207 (a) $m_A v_A = (m_A + m_B) v'$

$$v' = \frac{m_A}{m_A + m_B} v_A = \frac{4000/g}{(4000+2000)/g} 20$$

$$= \underline{13.33 \text{ mi/hr} \quad (19.56 \text{ ft/sec})}$$

(c) Car B :



$$m_B v_B + R\Delta t = m_B v' : 0 + R(0.1) = \frac{2000}{32.2}(19.56)$$

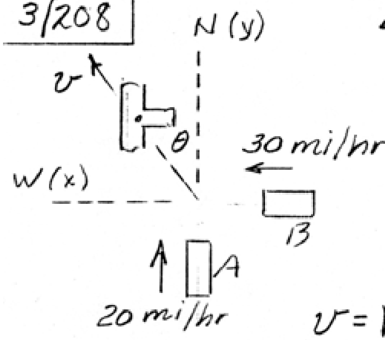
$$\underline{R = 12{,}150 \text{ lb}}$$

(The force which car B exerts on car A is 12,150 lb to the left, by Newton's Third Law.)

(b) $a_A = \dfrac{\Delta v}{\Delta t} = \dfrac{19.56 - 20\left(\frac{5280}{3600}\right)}{0.1} = \underline{-97.8 \dfrac{\text{ft}}{\text{sec}^2}}$

$$a_B = \frac{\Delta v}{\Delta t} = \frac{19.56 - 0}{0.1} = \underline{195.6 \frac{\text{ft}}{\text{sec}^2}}$$

3/208



$\Delta G_x = 0;\ \dfrac{3200}{g}(30) = \dfrac{(3200+3400)}{g} v_x$

$v_x = 14.55$ mi/hr

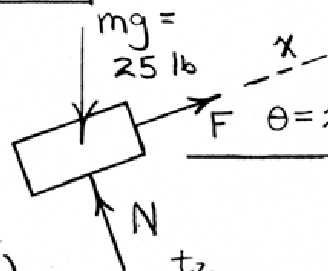
$\Delta G_y = 0;\ \dfrac{3400(20)}{g} = \dfrac{(3200+3400)}{g} v_y$

$v_y = 10.30$ mi/hr.

$v = \sqrt{(14.55)^2 + (10.30)^2} = \underline{17.82 \text{ mi/hr}}$

$\theta = \tan^{-1} \dfrac{v_x}{v_y} = \tan^{-1} \dfrac{14.55}{10.30} = \underline{54.7°}$

3/209



$\theta = 20°$

$F = b + 10 \sin 6t$

(a) $mv_{x_1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = mv_{x_2}$:

$0 - (mg \sin\theta)\Delta t + \int_0^{\Delta t} F \, dt = mv$

$-25 \sin 20°(1.5) + \left[5t - \frac{10}{6} \cos 6t\right]_0^{1.5} = \frac{25}{32.2} v$

$\underline{v = -2.76 \text{ ft/sec}}$

(b) b must equal $mg \sin\theta$

or $b = 25 \sin 20° = \underline{8.55 \text{ lb}}$

$\boxed{3/210}$ $\underline{G}_1 = \underline{G}_2$: $m_s \underline{v}_s + m_m \underline{v}_m = (m_s + m_m)\underline{v}$

$$1000\,(2000)\underline{j} + 10\,(5000)\left[\frac{+5\underline{i} - 4\underline{j} - 2\underline{k}}{\sqrt{5^2 + 4^2 + 2^2}}\right] = (1000+10)\,\underline{v}$$

$$\underline{v} = 36.9\underline{i} + 1951\underline{j} - 14.76\underline{k} \quad \text{m/s}$$

The angle between $\underline{v}_s$ and $\underline{v}$ is

$$\beta = \cos^{-1}\frac{\underline{v}\cdot\underline{v}_s}{v v_s}$$

$$= \cos^{-1}\left[\frac{(36.9\underline{i} + 1951\underline{j} - 14.76\underline{k})\cdot 2000\underline{j}}{\sqrt{36.9^2 + 1951^2 + 14.76^2}\ 2000}\right]$$

$$= 1.167°$$

$\boxed{3/211}$ $\int \underline{F} dt = \underline{F} t = m \Delta \underline{v}$

$\underline{F} = \frac{0.20}{0.04}\left([18 \cos 20°]\underline{i} + [18 \sin 20°]\underline{j} - [-12\underline{i}] \right)$

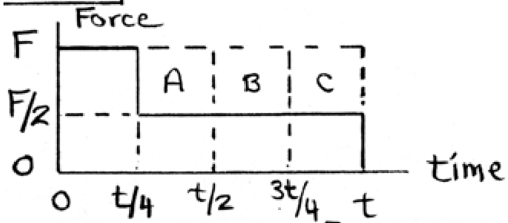
$= 5\left(18 \times 0.9397\underline{i} + 18 \times 0.3420\underline{j} + 12\underline{i} \right)$

$= 30(4.819\underline{i} + 1.026\underline{j})$ N

$F = 30\sqrt{4.819^2 + 1.026^2} = \underline{147.8 \text{ N}}$

$\beta = \tan^{-1} v_y / v_x = \tan^{-1} \frac{1.026}{4.819} = \underline{12.02°}$

3/212



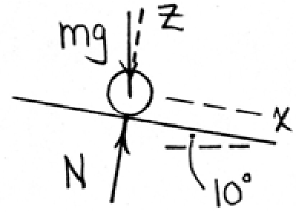
Solid area is $5/8$ of nominal area, so $\underline{n = 62.5\%}$

In order to compensate, areas A, B, & C must be added after time t, so the extra time $\underline{t' = \frac{3}{4}t}$.

3/213 $\int F_x\, dt = m\Delta v_x$:

$(mg \sin 10°)2 = m\left[v_x - (-3 \sin 15°)\right]$

$v_x = 2.63$ m/s

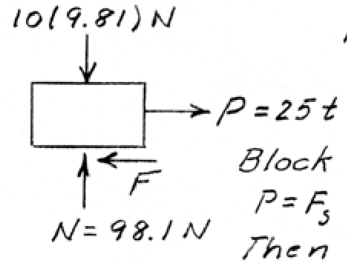


$\int F_y\, dt = m\Delta v_y$: $0 = m\left[v_y - 3 \cos 15°\right]$

$v_y = 2.90$ m/s

$v = \sqrt{v_x^2 + v_y^2} = \underline{3.91 \text{ m/s}}$

3/214



$F_s = \mu_s N = 0.6(98.1) = 58.9\ N$

$F_k = \mu_k N = 0.4(98.1) = 39.2\ N$

Block does not move until

$P = F_s$ or $25t = 58.9$, $t = 2.35\ s$

Then F becomes $39.2\ N$

$$\int \Sigma F\,dt = m\Delta v;\quad \int_{2.35}^{4} (25t - 39.2)\,dt = 10(v - 0)$$

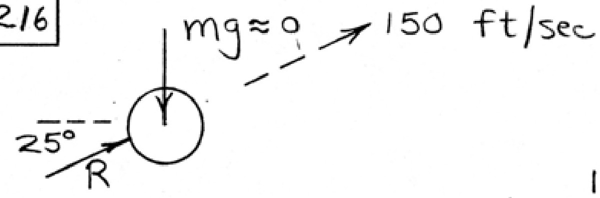
$$\left.\frac{25t^2}{2} - 39.2t\right]_{2.35}^{4} = 10v,\quad 10v = 66.1,\quad \underline{v = 6.61\ m/s}$$

3/215 $\int F_x\,dt = m\Delta v_x:\ 0.2t = \frac{1.2}{32.2}\left[v_x - (-10\sin 30°)\right]$

$$v_x = \frac{dx}{dt} = 5.37t - 5$$

$$\int_0^0 dx = \int_0^t (5.37t - 5)\,dt\ , \qquad \underline{t = 1.863 \text{ sec}}$$

3/216



$$+\nearrow \; R\Delta t = mv \;:\; R(0.001) = \frac{1.62/16}{32.2}(150)$$

$$\underline{R = 472 \text{ lb}}$$

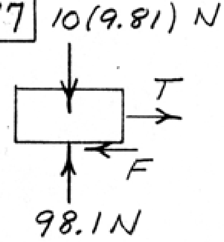
$$+\nearrow \; R = ma \;:\; 472 = \frac{1.62/16}{32.2}\,a$$

$$\underline{a = 150{,}000 \text{ ft/sec}^2 \; (4660g)}$$

$$v^2 - v_0^2 = 2ad \;:\; 150^2 - 0^2 = 2(150{,}000)\,d$$

$$\underline{d = 0.075 \text{ ft or } 0.900 \text{ in.}}$$

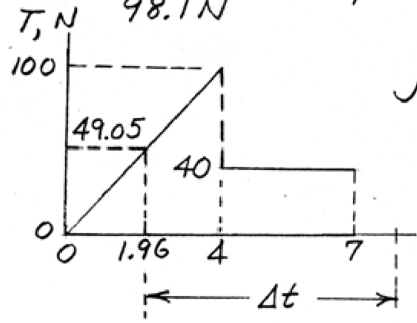
3/217



Block begins to move when $T = F = \mu W = 0.5(98.1)$
$= 49.05\text{ N}$

which occurs at

$$t_1 = \frac{49.05}{100}4 = 1.96\text{ s}$$



$$\int \Sigma F\,dt = m\Delta v$$

Max. velocity reached by block occurs at $t = 4$ s

$$\frac{(100 - 49.05)}{2}(4 - 1.96) = 10(v - 0)$$

$$\underline{v_{max} = 5.19\text{ m/s}}$$

For total motion $\Delta v = 0$, so

$$\frac{100 + 49.05}{2}(4 - 1.96) + 40(7 - 4) - 49.05\,\Delta t = 0$$

$$\underline{\Delta t = 5.54\text{ s}}$$

3/218 $\Delta G = 0, \ G_1 = G_2$

$$\left(\frac{2/16}{32.2} + 0\right)2000 = \frac{2/16 + 50}{32.2} v_2, \quad v_2 = 4.99 \text{ ft/sec}$$

$U = \Delta T$: $v_2 = \sqrt{2gh}$, $4.99^2 = 2(32.2)(6)(1-\cos\theta)$ where $h = 6(1-\cos\theta)$

$$\cos\theta = 0.936, \quad \underline{\theta = 20.7°}$$

$$\% \text{ energy loss} = \frac{\frac{1}{2}m_1 v_1^2 - (m_1 + m_2)gh}{\frac{1}{2}m_1 v_1^2} \times 100\% = \left(1 - \frac{m_1 + m_2}{m_1}\frac{2gh}{v_1^2}\right)100\%$$

$$= \left[1 - \frac{2/16 + 50}{2/16}\frac{2(32.2)6(1-0.936)}{2000^2}\right]100\% = \underline{99.8\%}$$

$\boxed{3/219}$

$$\Sigma F\,dt = m\,dv, \quad (F - R_0 - Kv)\,dt = m\,dv$$

$$\int_0^t dt = \int_0^v \frac{m\,dv}{F - R_0 - Kv}; \quad t = -\frac{m}{K}\ln(F - R_0 - Kv)\Big]_0^v$$

$$t = -\frac{m}{K}\ln\frac{F - R_0 - Kv}{F - R_0}$$

$$t = \frac{m}{K}\ln\frac{F - R_0}{F - R_0 - Kv}$$

$\boxed{3/220}$ For plug: $\Delta T + \Delta V_g = 0$; $\frac{1}{2} m_A v^2 - m_A g r = 0$

$$v = \sqrt{2gr}$$

Plug & block: $\Delta G = 0$; $m_A\sqrt{2gr} = (m_A + m_C)\, v'$

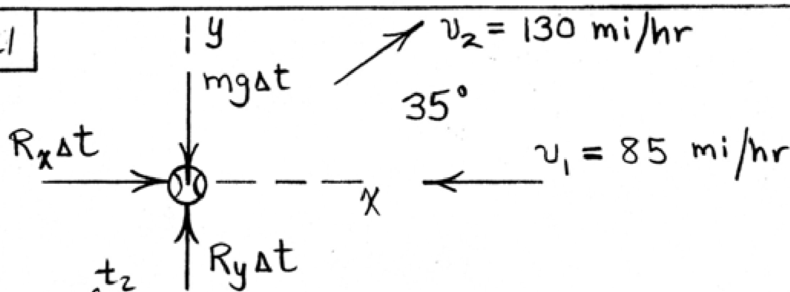
where v' = velocity of block & plug after impact

Friction force $F = \mu_k (m_A + m_C) g$

Deceleration $a = F/(m_A + m_C) = \mu_k g$

$v'^2 = 2as$, $\quad s = \left(\frac{m_A}{m_A + m_C}\right)^2 2gr \, \frac{1}{2\mu_k g} = \underline{\frac{r}{\mu_k}\left(\frac{m_A}{m_A + m_C}\right)^2}$

3/221



$$m v_{x_1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = m v_{x_2}:$$

$$-\frac{5.125/16}{32.2}\left(85 \, \frac{5280}{3600}\right) + R_x(0.005) = \frac{5.125/16}{32.2}\left(130 \, \frac{5280}{3600} \cos 35°\right)$$

$\underline{R_x = 559 \text{ lb}}$

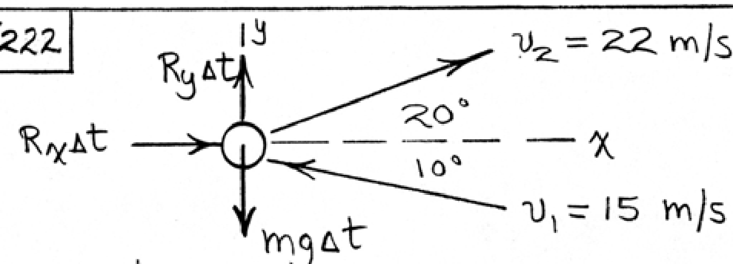
$$m v_{y_1} + \int_{t_1}^{t_2} \Sigma F_y \, dt = m v_{y_2}:$$

$$0 + R_y(0.005) - \frac{5.125}{16}(0.005) = \frac{5.125/16}{32.2}\left(130 \, \frac{5280}{3600} \sin 35°\right)$$

$\underline{R_y = 218 \text{ lb}}$

If mg is neglected during impact, $R_y = 218$ lb – a good assumption! The quantity mg may <u>not</u> be neglected thereafter – otherwise we obtain a record home-run distance!

3/222



$$mv_{x_1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = mv_{x_2}:$$

$$0.060(-15\cos 10°) + R_x(0.05) = 0.060(22\cos 20°)$$

$$R_x = 42.5 \text{ N}$$

$$mv_{y_1} + \int_{t_1}^{t_2} \Sigma F_y \, dt = mv_{y_2}:$$

$$0.060(15\sin 10°) + R_y(0.05) - 0.060(9.81)(0.05) = 0.060(22\sin 20°)$$

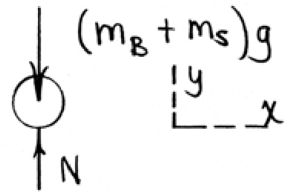
$$R_y = 6.49 \text{ N}$$

Weight $mg = 0.060(9.81) = 0.589$ N is about 9% of R_y – no need to omit mg from analysis!

$$R = \sqrt{R_x^2 + R_y^2} = \underline{43.0 \text{ N}}$$

$$\beta = \tan^{-1}\frac{R_y}{R_x} = \underline{8.68°}$$

3/223 System :



$$m_B v_{Bx} + m_s \cancelto{0}{v_{sx}} = (m_B + m_s) v$$

$$v = \frac{m_B v_{Bx}}{(m_B + m_s)} = \frac{80/32.2}{90\,(32.2)}(16 \cos 30^\circ)$$

$$= \underline{12.32 \text{ ft/sec}}$$

$$m_B v_{By} + m_s \cancelto{0}{v_{sy}} + \int_0^{\Delta t} \left[N - (m_B + m_s) g \right] dt = 0$$

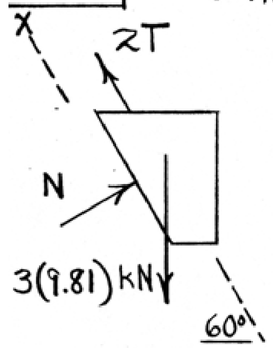
$$-\frac{80}{32.2}(16 \sin 30^\circ) + N(0.05) - 90\,(0.05) = 0$$

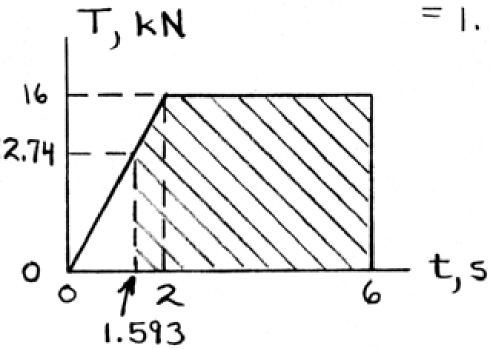
$$\underline{N = 488 \text{ lb}}$$

3/224 Skip begins to move when

$$2T = 3(9.81)\frac{\sqrt{3}}{2}, \quad T = 12.74 \text{ kN}$$

which occurs at $t = \dfrac{12.74}{16} = 1.593$ s





$\int \Sigma F_x \, dt = m \Delta v_x$:

$$2\left[\frac{16 + 12.74}{2}(2 - 1.593) + 16(6-2)\right] - 3(9.81)\frac{\sqrt{3}}{2}(6 - 1.593) = 3(v - 0)$$

$\underline{v = 9.13 \text{ m/s}}$

$\boxed{3/225}$ $T_y = 600 \cos\theta$; $\dot{\theta} = \pi/10$ rad/s, so $dt = \frac{10}{\pi} d\theta$

$\int \Sigma F_y \, dt = m \Delta v_y$; $\int_0^{\pi/2} 600 \cos\theta \left(\frac{10}{\pi} d\theta \right) = 260 (v_y - 0)$

$\frac{6000}{\pi} \sin\theta \Big]_0^{\pi/2} = 260 \, v_y$, $v_y = \frac{6000}{260\pi} = \underline{7.35 \text{ m/s}}$

3/226 (a) $\Delta G = 0$; $m(4) + 0 = mv_A + mv_B$

$$v_A + v_B = 4$$

$\Delta T = 0.4T$; $\frac{1}{2}m(4^2) - \left[\frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2\right] = 0.4\left[\frac{1}{2}m(4^2)\right]$

$$v_A^2 + v_B^2 = 9.6$$

Solve simultaneously & get $(4 - v_B)^2 + v_B^2 = 9.6$

or $v_B^2 - 4v_B + 3.2 = 0$, $v_B = \frac{4}{2} \pm \frac{1}{2}\sqrt{16 - 4(3.2)}$

$$= 2 \pm 0.894$$

(sol. I) $v_B = 2.894$ ft/sec, $v_A = 4 - 2.894 = 1.106$ ft/sec

(sol. II) $v_B = 1.106$ ft/sec, $v_A = 4 - 1.106 = 2.894$ ft/sec

sol. II is ruled out since distance between A & B would be decreasing so that $v_B > v_A$

Thus $\underline{v_B = 2.89 \text{ ft/sec}}$

(b) For initial to final condition

$\Delta G = 0$; $m(4) + 0 = 2m\, v_c$, $\underline{v_c = 2 \text{ ft/sec}}$

3/227 (a) $\underline{H}_O = \underline{r} \times m\underline{v}$

$$\underline{H}_O = (-6\underline{i} + 8\underline{j}) \times 2(7)(-\sin 30°\underline{i} - \cos 30°\underline{j})$$

$$= 128.7\underline{k} \ \ \text{kg}\cdot\text{m}^2/\text{s}$$

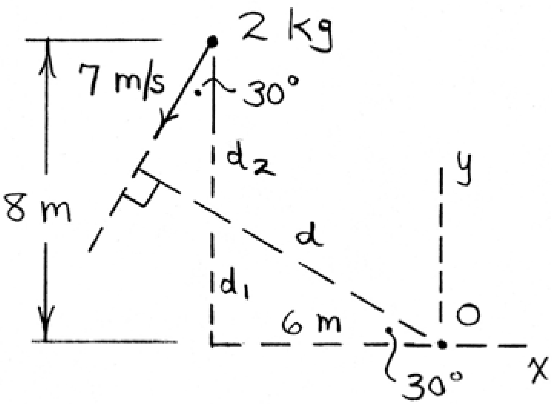
So $\underline{H_o = 128.7 \text{ kg}\cdot\text{m}^2/\text{s}}$

$d_1 = 6 \tan 30°$

$= 3.46$ m

$d_2 = 8 - d_1$

$= 4.54$ m



$$d = \frac{6}{\cos 30°} + 4.54 \sin 30° = 9.20 \text{ m}$$

$$\circlearrowleft + \ H_o = mvd = 2(7)(9.20) = \underline{128.7 \text{ kg}\cdot\text{m}^2/\text{s}}$$

3/228 (a) $\underline{G} = m\underline{v} = 3 \cdot 5 \left(-\cos 30° \underline{i} - \sin 30° \underline{j}\right)$

$$= \underline{-12.99\underline{i} - 7.5\underline{j} \quad \text{kg}\cdot\text{m}/\text{s}}$$

(b) $\underline{H}_o = \underline{r} \times m\underline{v} = \underline{r} \times \underline{G}$

$$= 2\left(\cos 15° \underline{i} - \sin 15° \underline{j}\right) \times \left(-12.99\underline{i} - 7.5\underline{j}\right)$$

$$= \underline{-21.2\underline{k} \quad \text{kg}\cdot\text{m}^2/\text{s}}$$

(c) $T = \frac{1}{2} m v^2 = \frac{1}{2}(3)(5)^2 = \underline{37.5 \text{ J}}$

3/229

$$\underline{H}_o = \underline{r} \times m\underline{v}$$
$$= (a\underline{i} + b\underline{j} + c\underline{k}) \times mv\underline{k}$$
$$= \underline{mv\,(b\underline{i} - a\underline{j})}$$

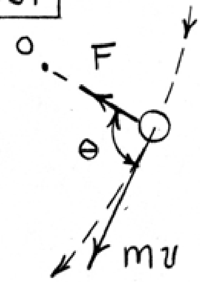
$$\dot{\underline{H}}_o = \underline{M}_o = (a\underline{i} + b\underline{j} + c\underline{k}) \times F\underline{j}$$
$$= \underline{F(-c\underline{i} + a\underline{k})}$$

3/230 Angular momentum about O is conserved:

$H_{O_1} = H_{O_2}$: $3mv(L) + 2mv(L) = 3mL^2\omega$

$$\underline{\omega = \frac{5}{3}\frac{v}{L}}$$

3/231



$\Sigma M_O = \dot{H}_O = 0$, so $H_O = \text{const.}$

$H_{O_A} = H_{O_B}$

$m(4)(0.350 \sin 54°) =$

$m v_B (0.230 \sin 65°)$

$\underline{v_B = 5.43 \text{ m/s}}$

$\boxed{3/232}$ (a) $v_B = \sqrt{2gr}$

$$H_O = mrv_B = \underline{mr\sqrt{2gr}}\ , \quad \underline{\dot{H}_O = mgr}$$

(b) $v_C = \sqrt{2g(2r)} = 2\sqrt{gr}$

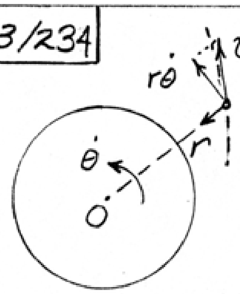
$$H_O = mrv_C = \underline{2mr\sqrt{gr}}\ , \quad \underline{\dot{H}_O = 0}$$

$\boxed{3/233}$ $H_1 + \int_{t_1}^{t_2} M\,dt = H_2$

$$0 + 20(0.1)\,t = 4(3)(0.4)^2\left[150\left(\frac{1}{60}\right)(2\pi)\right]$$

$$\underline{t = 15.08\ \text{s}}$$

3/234



$\Sigma M_O = \dot{H}_O$; $0 = \frac{d}{dt}(m r\dot{\theta} \times r)$

or $\frac{d}{dt}(r^2\dot{\theta}) = 0$

so $\underline{r^2\dot{\theta} = const.}$

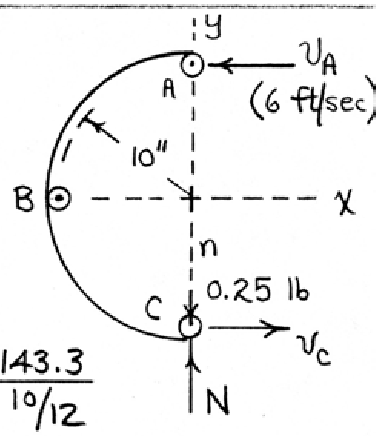
3/235 $T_A + U_{A-C} = T_C$

$\frac{1}{2} m v_A^2 + mgh_{A-C} = \frac{1}{2} m v_C^2$

$v_C^2 = v_A^2 + 2g\,h_{A-C}$

$= 6^2 + 32.2\left(\frac{20}{12}\right)(2)$

$= 143.3 \ \text{ft}^2/\text{sec}^2$



$\Sigma F_y = ma_y : \ N - 0.25 = \frac{0.25}{32.2}\,\frac{143.3}{10/12}$

$N = 1.585 \ \text{lb}$

$\dot{\underline{H}}_B = \underline{M}_B = (1.585 - 0.25)\,\frac{10}{12}\,\underline{k} = \underline{1.113\,\underline{k} \ \ \text{lb-ft}}$

3/236 Velocity of plug upon impact is

$$v = \sqrt{2gh} = \sqrt{2(9.81)(0.6)} = 3.43 \text{ m/s}$$

For system, $\Delta H_O = 0$. Take C.W. positive

Initial $H_O = -4(0.5)^2(2) - 6(0.3)^2(2) + 2(3.43)(0.5)$

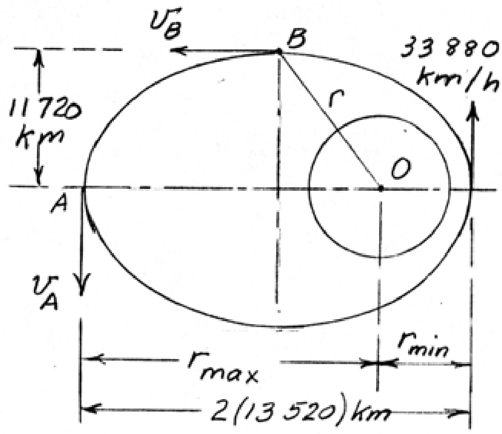
$$= -2 - 1.08 + 3.43 = 0.351 \text{ N·m·s}$$

Final $H_O = [(4+2)(0.5)^2 + 6(0.3)^2]\omega$

$$= 2.04\omega$$

So $0.351 = 2.04\omega$, $\underline{\omega = 0.1721 \text{ rad/s}}$ CW

3/237 $\Sigma M_O = \dot{H}_O = 0$ so $H_O =$ constant



$r_{min} = 6371 + 390 = 6761$ km

$r_{max} = 2(13\,520) - 6761$

$= 20\,279$ km

For H_o constant

$6761(33\,880) = 11720\, v_B$

$= 20\,279\, v_A$

$\underline{v_A = 11\,300 \text{ km/h}}$

$\underline{v_B = 19\,540 \text{ km/h}}$

3/238 For the entire system, $\Sigma M_O = \dot{\underline{H}}_O = 0$, so angular momentum is conserved.

$$H_{O_1} = H_{O_2}: \quad 2mr^2\omega_0 + 0 = 2mr^2\omega + 2m(2r)^2\omega$$

$$\underline{\omega = \omega_0/5}$$

Kinetic energy loss $\Delta Q = T_1 - T_2$

$$\Delta Q = 2\left(\tfrac{1}{2}mr^2\omega_0^{\,2}\right) - \left\{2\left(\tfrac{1}{2}mr^2\omega^2\right) + 2\left(\tfrac{1}{2}m(2r)^2\omega^2\right)\right\}$$

$$= mr^2\omega_0^{\,2} - mr^2\left(5\left(\frac{\omega_0}{5}\right)^2\right) = \frac{4}{5}mr^2\omega_0^{\,2}$$

$$\text{So } n = \frac{\Delta Q}{T_1}(100\%) = \frac{\frac{4}{5}mr^2\omega_0^{\,2}}{2\left(\frac{1}{2}mr^2\omega_0^2\right)}(100\%) = \underline{80\%}$$

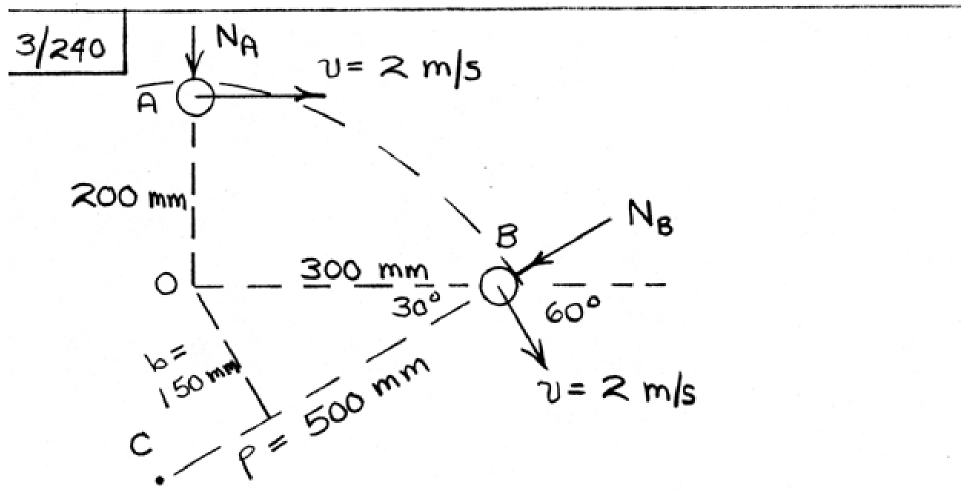
3/239 $\Delta H = 0$; $2m\, r\omega_o (r) - 2m(2r)\omega(2r) = 0$

$$\underline{\omega = \omega_o/4}$$

$$\Delta T = 2\left(\frac{1}{2}m[r\omega_o]^2\right) - 2\left(\frac{1}{2}m\left[2r\frac{\omega_o}{4}\right]^2\right) = mr^2\omega_o^2\,(3/4)$$

$$n = \Delta T/T = \frac{3}{4}mr^2\omega_o^2 \Big/ mr^2\omega_o^2 = \underline{3/4}$$

3/240



$\Sigma M_{O_z} = \dot{H}_{O_z}$:

At A, $\Sigma M_{O_z} = 0$, so $\underline{\dot{H}_{O_z} = 0}$

At B, $\Sigma M_{O_z} = -N_B b$, where $N_B = m\frac{v^2}{\rho} = 0.1\,\frac{2^2}{0.5}$

$= 0.8$ N

So $\dot{H}_{O_z} = -N_B b = -0.8(0.150) = -0.120$ N·m

$\underline{(\text{or } -0.120 \text{ kg}\cdot\text{m}^2/\text{s}^2)}$

3/241 $\sum M_O = \dot{H}_O = 0$, so angular momentum is conserved : $H_{O_1} = H_{O_2}$ (O: any point on axis)

$$0.2\,(0.3\cos 30^\circ)^2 4 = 0.2(0.2\cos 30^\circ)^2\,\omega$$

$$\underline{\omega = 9 \text{ rad/s}}$$

$U'_{1\text{-}2} = \Delta T + \Delta V_g$

$$\Delta T = \tfrac{1}{2}(0.2)\left[(0.2\cos 30^\circ \cdot 9)^2 - (0.3\cos 30^\circ \times 4)^2\right]$$

$$= 0.1350 \text{ J}$$

$$\Delta V_g = 0.2\,(9.81)(0.1\sin 30^\circ) = 0.0981 \text{ J}$$

So $U'_{1\text{-}2} = 0.1350 + 0.0981 = \underline{0.233 \text{ J}}$

3/242 $\int \Sigma M_O \, dt = \Delta H_O = H_{O_B} - H_{O_A}$

$H_{O_A} = 0.02(4)(0.090)\sin 30° = 0.0036 \text{ kg·m}^2/\text{s}$

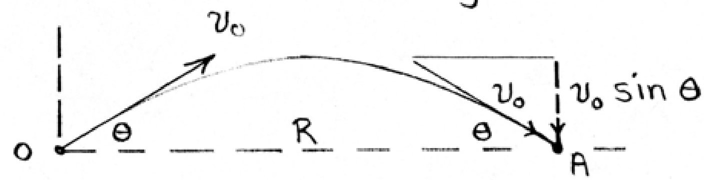
$H_{O_B} = 0.02(6)(0.180)\sin 60° = 0.01871 \text{ kg·m}^2/\text{s}$

$\Delta H_O = 0.01871 - 0.0036 = 0.01511 \text{ kg·m}^2/\text{s}$

$M_{O_{av}} \times 0.5 = 0.01511$, $\underline{M_{O_{av}} = 0.0302 \text{ N·m}}$

3/243 (a) $H_o = 0$ when projectile is at O.

(b) Range $R = \dfrac{2v_0^2 \cos\theta \sin\theta}{g}$

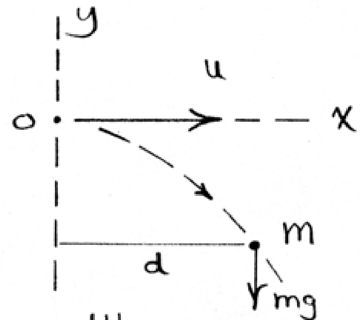


$$H_o = m v_y R = m v_0 \sin\theta \, \frac{2v_0^2 \cos\theta \sin\theta}{g}$$

$$= \frac{2 m v_0^3 \sin^2\theta \cos\theta}{g}$$

The moment of the projectile weight about point O is always increasing the angular momentum about O.

3/244



(Note: $d = ut$)

$$\dot{\underline{H}}_o = \frac{d\underline{H}_o}{dt} = \underline{M}_o = -mgd\,\underline{k}$$

$$\int_0^{H_o} d\underline{H}_o = -\int_0^t mgd\,\underline{k}\,dt = -\int_0^t mgut\,\underline{k}\,dt$$

$$\Rightarrow \underline{H}_o = -\frac{1}{2}mgut^2\,\underline{k}$$

3/245 Conservation of angular momentum:

$$(\cancel{m} r v_\theta)_A = (\cancel{m} r v_\theta)_B$$

$$50(10^6)(188{,}500) = 75(10^6)\, v_\theta$$

$$\underline{v_\theta = 125{,}700 \text{ ft/sec}} \quad @ \ B$$

Energy conservation $T_A + V_A = T_B + V_B$

$$\frac{1}{2}\cancel{m} v_A^2 - \frac{G m_s \cancel{m}}{r_A} = \frac{1}{2}\cancel{m} v_B^2 - \frac{G m_s \cancel{m}}{r_B}$$

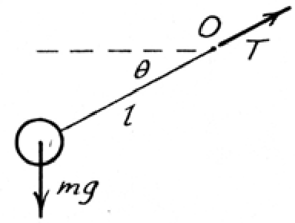
$$\frac{1}{2}(188{,}500)^2 - \frac{1}{2}v_B^2 =$$

$$3.439(10^{-8})(333{,}000)(4.095)10^{23}\left[\frac{1}{50(10^6)} - \frac{1}{75(10^6)}\right]\frac{1}{5280}$$

$$v_B = 153{,}900 \text{ ft/sec}$$

$$v_r = \sqrt{v_B^2 - v_\theta^2} = \sqrt{153{,}900^2 - 125{,}700^2} = \underline{88{,}870 \frac{\text{ft}}{\text{sec}}}$$

3/246



$\Sigma M_O = \dot{H}_O : \; mgl\cos\theta = \frac{d}{dt}(ml^2\dot{\theta})$
$= ml^2\ddot{\theta}$

$\underline{\ddot{\theta} = \frac{g}{l}\cos\theta}$

From $\int \dot{\theta}\,d\dot{\theta} = \int \ddot{\theta}\,d\theta$, $\left.\frac{\dot{\theta}^2}{2}\right|_0^{\dot{\theta}} = \int_0^{\theta} \frac{g}{l}\cos\theta\,d\theta$,

$\dot{\theta}^2 = \frac{2g}{l}\sin\theta$, $\dot{\theta}_{\theta=90°} = \sqrt{\frac{2g}{l}}$

so at $\theta = 90°$, $v = l\dot{\theta} = \underline{\sqrt{2gl}}$

By work-energy $U = \Delta T$, $mgl = \frac{1}{2}mv^2$, $v = \sqrt{2gl}$

3/247 Forces on particle exert no moment about the central axis, so angular momentum is conserved about this axis. Thus $\Delta H_z = 0$ &

$$m v_0 \cos\beta (r) = m v \cos\theta (r), \quad v_0 \cos\beta = v \cos\theta$$

Also energy is conserved so that

$$\Delta T + \Delta V_g = 0; \quad \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 - mgh = 0$$

Eliminate v & get $\cos\theta = \dfrac{v_0 \cos\beta}{\sqrt{v_0^2 + 2gh}}$

or $$\theta = \cos^{-1} \frac{\cos\beta}{\sqrt{1 + \frac{2gh}{v_0^2}}}$$

3/248 System angular momentum conserved during impact: $\curvearrowleft + \; H_{O_1} = H_{O_2}$:

$$0.050(300)(0.4\cos 20°) - 3.2(0.2)^2 6 - 3.2(0.4)^2 6$$
$$= (0.050 + 3.2)(0.4)^2\omega' \quad + 3.2(0.2)^2\omega'$$

$\omega' = 2.77$ rad/s (CCW)

Energy considerations after impact:

$T' + V' = \cancel{T}^{\,0} + V$, choose datum @ O:

$$\tfrac{1}{2}(0.05+3.2)[0.4(2.77)]^2 + \tfrac{1}{2}(3.2)[0.2(2.77)]^2$$
$$+[3.2(0.2) - (3.2+0.05)(0.4)]9.81 \; = \; 0 +$$
$$[3.2(0.2) - (3.2+0.05)(0.4)]9.81\cos\theta$$

$\theta = 52.1°$

3/249 Path form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \left(\begin{array}{l} a = 5 \text{ ft} \\ b = 4 \text{ ft} \end{array} \right.$

Angular momentum about O is conserved:

$$m r_A v_A = m r_B v_B : \quad v_B = \frac{r_A}{r_B} v_A = \frac{a}{b} v_A$$

$$= \frac{5}{4}(8) = 10 \text{ ft/sec}$$

$$y = b\left[1 - \left(\frac{x}{a}\right)^2\right]^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2} b \left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}} \cdot \left(-\frac{2x}{a^2}\right) = -\frac{bx}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{b}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}} - \frac{bx}{a^2}\left(-\frac{1}{2}\right)\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{3}{2}}\left(-\frac{2x}{a^2}\right)$$

$$= -\frac{b}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}} - \frac{bx^2}{a^4}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{3}{2}}$$

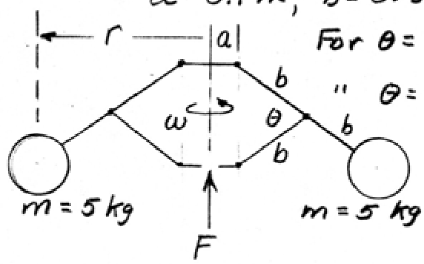
Now, $\left.\frac{dy}{dx}\right|_{x=0} = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{x=0} = -\frac{b}{a^2} = -\frac{4}{5^2} = -\frac{4}{25}$

$$\rho_{xy} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{[1+0]^{3/2}}{-4/25} = -6.25'$$

So $\rho = 6.25'$; $\Sigma F_n = m\frac{v^2}{\rho}$: $T_B = \frac{1.5}{32.2}\,\frac{10^2}{6.25} = \underline{0.745 \text{ lb}}$

▶ 3/250 $\omega_0 = 40(2\pi)/60 = 4.19$ rad/s

$a = 0.1$ m, $b = 0.3$ m



For $\theta = 90°$, $r_0 = 0.1 + 2(0.3)\cos 45° = 0.524$ m

" $\theta = 60°$, $r = 0.1 + 2(0.3)\cos 30° = 0.620$ m

$\Delta H = 0$; $2mr_0^2\omega_0 - 2mr^2\omega = 0$

$\omega = \frac{r_0^2}{r^2}\omega_0 = \left(\frac{0.524}{0.620}\right)^2(4.19)$

$= \underline{3.00 \text{ rad/s}}$

(or $\frac{3.00}{2\pi}60 = 28.6$ rev/min)

$U = \Delta T + \Delta V_g = 2(\frac{1}{2}m)(r^2\omega^2 - r_0^2\omega_0^2) + 2mg\,\Delta h$

where $\Delta h = 2b(\sin 45° - \sin 30°)$

$= 2(0.3)(0.7071 - 0.5) = 0.1243$ m

$U = 5([0.620 \times 3.00]^2 - [0.524 \times 4.19]^2) + 2(5)(9.81)(0.1243)$

$= -6.850 + 12.190 = \underline{5.34 \text{ J}}$

$\boxed{3/251}$ $\quad v = \sqrt{2gh}\ , \quad v' = \sqrt{2gh'}$

$$e = \frac{v'}{v} = \sqrt{\frac{h'}{h}} = \sqrt{\frac{1100}{2100}} = \underline{0.724}$$

$$n = \frac{mgh - mgh'}{mgh}(100\%) = \frac{2100 - 1100}{2100}(100\%)$$

$$= \underline{47.6\%}$$

3/252 System linear momentum :

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{1.5}{32.2}(0.8) + \frac{2}{32.2}(-2.4) = \frac{1.5}{32.2} v_1' + \frac{2}{32.2} v_2'$$

Restitution : $e = \frac{v_2' - v_1'}{v_1 - v_2}$: $0.5 = \frac{v_2' - v_1'}{0.8 - (-2.4)}$

Solve the two equations to obtain

$$v_1' = -1.943 \text{ ft/sec}$$

$$\underline{v_2' = -0.343 \text{ ft/sec}}$$

Original energy : $T_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$$T_1 = \frac{1}{2}\frac{1.5}{32.2}(0.8)^2 + \frac{1}{2}\frac{2}{32.2}(2.4)^2$$

$$= 0.1938 \text{ ft-lb}$$

$$T_2 = \frac{1}{2}\frac{1.5}{32.2}(1.943)^2 + \frac{1}{2}\frac{2}{32.2}(0.343)^2$$

$$= 0.0916 \text{ ft-lb}$$

$$n = \frac{T_1 - T_2}{T_1}(100\%) = \frac{0.1938 - 0.0916}{0.1938}(100\%)$$

$$= \underline{52.7\%}$$

3/253 System momentum:

$$\frac{1.5}{32.2}(0.8) + \frac{2}{32.2}v_2 = \frac{1.5}{32.2}v_1' + 0$$

Restitution: $\frac{-v_1'}{0.8 - v_2} = 0.5$

Solve to obtain $\begin{cases} v_1' = -1.120 \text{ ft/sec} \\ \underline{v_2 = -1.440 \text{ ft/sec}} \end{cases}$

(Note: v_2 assumed totally unknown above - no leftward direction assumed.)

3/254 Consider the case $v_2' = v_1$. Conservation of system linear momentum:

$$m_1 v_1 + m_2 \cancelto{0}{v_2} = m_1 v_1' + m_2 v_2' = m_1 v_1' + m_2 v_1$$

$$v_1' = \frac{(m_1 - m_2)}{m_1} v_1$$

Restitution: $e = \dfrac{v_2' - v_1'}{v_1 - v_2} = \dfrac{v_1 - \left(\dfrac{m_1 - m_2}{m_1}\right) v_1}{v_1}$

$$\frac{m_1}{m_2} = \frac{1}{e}$$

So for $v_2' > v_1$, $\underline{\dfrac{m_1}{m_2} > \dfrac{1}{e}}$

$\boxed{3/255}$ System linear momentum :

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \xrightarrow{+}$$

$$\cancel{m} v + 0 = \cancel{m} v_A' + p\cancel{m} v_B' \qquad (1)$$

Restitution : $e = \dfrac{v_B' - v_A'}{v_A - v_B}$: $\quad 0.1 = \dfrac{v_B' - v_A'}{v - 0} \quad (2)$

Solve (1) & (2) to obtain

$$\underline{v_A' = \left(\frac{1 - 0.1p}{1 + p}\right) v} \, , \qquad \underline{v_B' = \frac{1.1}{1 + p} v}$$

For $p = \frac{1}{2}$: $\underline{v_A' = 0.633 v}$, $\underline{v_B' = 0.733 v}$

$\boxed{3/256}$ Impact velocity $v = \sqrt{2gh} = \sqrt{2(32.2)(4)}$

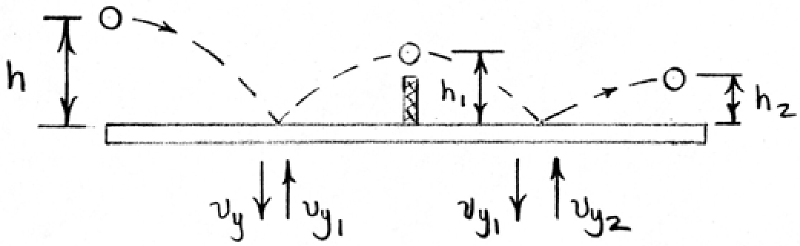
$= 16.05$ ft/sec

$\Delta G = 0;\ 500(16.05) + 0 = 0 + 800 v'$

$\underline{v' = 10.03 \text{ ft/sec}}$

$e = \dfrac{v'}{v} = \dfrac{10.03}{16.05} = \underline{0.625}$

3/257



(Note: Drop distances reduced by $r = 0.75''$)

$v_y = \sqrt{2g(h-r)}$, $v_{y_1} = ev_y = e\sqrt{2g(h-r)}$

$= \sqrt{2g(h_1-r)}$

$\therefore \; 0.90^2(2g)(h-0.75) = (2g)(9-0.75)$, $\underline{h = 10.94 \text{ in.}}$

$v_{y_2} = ev_{y_1}$: $\sqrt{2g(h_2-r)} = 0.9\sqrt{2g(h_1-r)}$

With $r = 0.75$ in. & $h_1 = 9$ in.: $\underline{h_2 = 7.43 \text{ in.}}$

3/258 $\Delta G = 0;\quad m_A v_A + 0 = m_A v_A' + m_B v_B'$

$e = 0;\quad v_A' = v_B'$

Thus $\quad m_A v_A = (m_A + m_B) v_A'$

$$|\Delta T| = -\frac{1}{2} m_A v_A'^2 - \frac{1}{2} m_B v_B'^2 + \frac{1}{2} m_A v_A^2$$

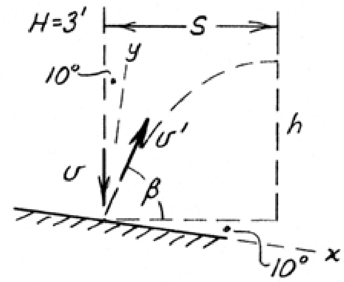
$$= -\frac{1}{2} m_A \left(\frac{m_A}{m_A + m_B} v_A\right)^2 - \frac{1}{2} m_B \left(\frac{m_A}{m_A + m_B} v_A\right)^2 + \frac{1}{2} m_A v_A^2$$

$$= -\frac{1}{2}\left(\frac{m_A}{m_A + m_B} v_A\right)^2 (m_A + m_B) + \frac{1}{2} m_A v_A^2$$

$$= \frac{1}{2} \frac{m_A m_B}{m_A + m_B} v_A^2 \quad (\text{loss})$$

$$\frac{|\Delta T|}{T} = \frac{1}{2} \frac{m_A m_B}{m_A + m_B} v_A^2 \frac{1}{\frac{1}{2} m_A v_A^2} = \underline{\frac{m_B}{m_A + m_B}}$$

3/259 $v = \sqrt{2gH} = \sqrt{2 \times 32.2 \times 3}$
$= 13.90$ ft/sec



At impact $\Sigma F_x = 0$ so $\Delta G_x = 0$ so

$v' \cos(\beta + 10°) - 13.90 \sin 10° = 0$ -- (a)

$e = 0.7 = \dfrac{v' \sin(\beta + 10°)}{13.90 \cos 10°}$ ------ (b)

Combine & get $\tan(\beta + 10°) = 3.97$

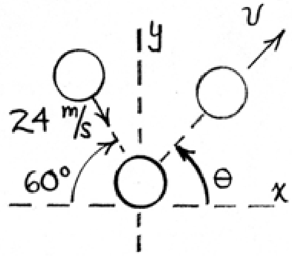
$\beta + 10° = 75.9°$, $\beta = 65.9°$

From Eq. (a) $v' = \dfrac{13.90 \sin 10°}{\cos 75.9°} = 9.88$ ft/sec

From Sample Prob. 2/6, $h = \dfrac{v'^2 \sin^2\beta}{2g} = \dfrac{9.88^2 \sin^2 65.9°}{2 \times 32.2} = \underline{1.263 \text{ ft}}$

$s = \dfrac{v'^2 \sin 2\beta}{2g} = \dfrac{9.88^2 \sin 131.7°}{2 \times 32.2} = \underline{1.132 \text{ ft}}$

3/260



During impact $\Sigma F_x = 0$ so no change in x velocity component.

$$v \cos\theta = 24(0.5) = 12 \text{ m/s}$$

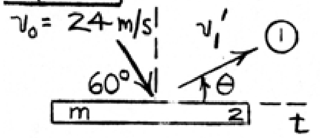
In y-dir., $e = \dfrac{v \sin\theta}{24 \cos 30°} = 0.8$

$$\tan\theta = \frac{16.63}{12} = 1.386$$

$$\underline{\theta = 54.2°}$$

$$v = \frac{12}{\cos 54.2°} = \underline{20.5 \text{ m/s}}$$

3/261



t-momentum conserved:

Ball: $m_1 v_{1_t} = m_1 v_{1t}'$

$v_{1t}' = v_{1t} = 24 \cos 60° = 12$ m/s

Plate: $m_2 v_{2_t} = m_2 v_{2_t}'$, $v_{2_t}' = v_{2_t} = 0$

n-momentum:

$$\cancel{m_1} v_{1n} + \cancel{m_2} v_{2n} = \cancel{m_1} v_{1n}' + \cancel{m_2} v_{2n}'$$

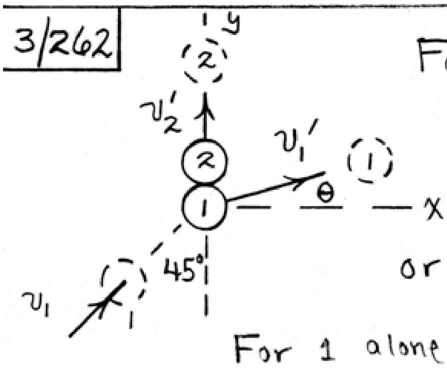
$$-24 \sin 60° = v_{1n}' + v_{2n}'$$

Restitution: $e = \dfrac{v_{2n}' - v_{1n}'}{v_{1n} - v_{2n}}$, $0.8 = \dfrac{v_{2n}' - v_{1n}'}{-24 \sin 60° - 0}$

Solve to find $v_{1n}' = -2.08$ m/s, $\underline{v_{2n}' = -18.71 \text{ m/s}}$

$v_1' = \sqrt{v_{1t}'^2 + v_{1n}'^2} = \underline{12.20 \text{ m/s}}$, $\theta = \tan^{-1}\left(\dfrac{v_{1n}'}{v_{1t}'}\right) = \underline{-9.83°}$

3/262



For 1 & 2 together:

$$mv_1 \cos 45° = mv_1' \sin\theta + mv_2'$$

or $v_1' \sin\theta + v_2' = v_1/\sqrt{2}$ (1)

For 1 alone: $mv_1 \sin 45° = mv_1' \cos\theta$

or $v_1' \cos\theta = v_1/\sqrt{2}$ (2)

Restitution: $v_2' - v_1' \sin\theta = e v_1 \cos 45°$

or $v_2' - v_1' \sin\theta = 0.9 v_1/\sqrt{2}$ (3)

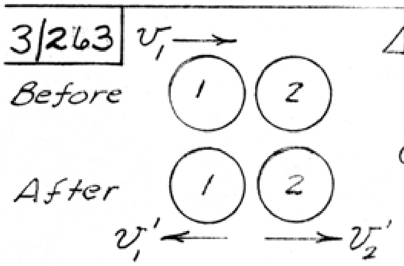
(1) & (3): $v_1' \sin\theta = 0.0354 v_1$; Divide by (2): $\theta = 2.86°$

$$n = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{\frac{1}{2}mv_2'^2 + \frac{1}{2}mv_1'^2}{\frac{1}{2}mv_1^2}$$

$$= 1 - \frac{v_2'^2 + v_1'^2}{v_1^2}$$, where $v_1' = 0.708 v_1$, $v_2' = 0.672 v_1$

So $n = 1 - \dfrac{0.672^2 + 0.708^2}{1} = \underline{0.0475}$

3/263



$$\Delta G = 0;\; m v_1 = -m v_1' + m v_2'$$

$$v_2' = v_1 + v_1'$$

$$e = \frac{v_2' + v_1'}{v_1},\quad v_1' = e v_1 - v_2'$$

Combine & get

$$v_2' = v_1 + e v_1 - v_2'$$

or $$v_2' = \frac{1+e}{2} v_1$$

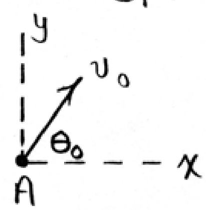
It follows that $$v_3' = \frac{1+e}{2} v_2' = \left(\frac{1+e}{2}\right)^2 v_1$$

$$v_4' = \frac{1+e}{2} v_3' = \left(\frac{1+e}{2}\right)^3 v_1$$

$$\vdots \qquad\qquad \vdots$$

so $$\underline{v_n = \left(\frac{1+e}{2}\right)^{n-1} v_1}$$

$\boxed{3/264}$ Let the launch conditions at A be speed v_0, launch angle θ_0:



The range L_1 is

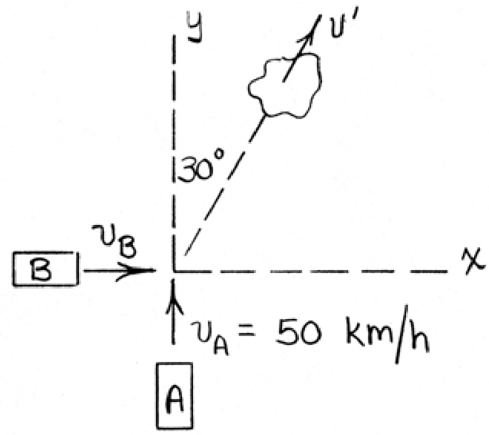
$$L_1 = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}$$

and the velocity components coming into B are $\begin{cases} v_x = v_0 \cos\theta_0 \\ v_y = -v_0 \sin\theta_0 \end{cases}$

The velocity components after impact at B are $v_x = v_0 \cos\theta_0$, $v_y = e v_0 \sin\theta_0$, which result in the range $L_2 = \dfrac{2ev_0^2 \sin\theta_0 \cos\theta_0}{g}$

So $\underline{L_2 = e L_1}$.

3/265



$$G_{1x} = G_{2x}: \quad m_B v_B + 0 = (m_A + m_B) v' \sin 30^\circ$$

$$1600\, v_B = 2800\, v' \left(\tfrac{1}{2}\right) \qquad (1)$$

$$G_{1y} = G_{2y}: \quad m_A v_A + 0 = (m_A + m_B) v' \cos 30^\circ$$

$$1200\,(50) = 2800\, v' (0.866) \qquad (2)$$

From (2): $v' = 24.7$ km/h

From (1): $\underline{v_B = 21.7 \text{ km/h}}$

$\boxed{3/266}$ System linear momentum is conserved:

y, x

$$m_s \underline{v}_s + m_m \underline{v}_m = (m_s + m_m)\underline{v}'$$

$$(1000)(2000\,\underline{i}) + 100(-v_m \underline{j}) = (1000+100)\left[v'(\cos 20°\,\underline{i} - \sin 20°\,\underline{j})\right]$$

Equating coefficients:

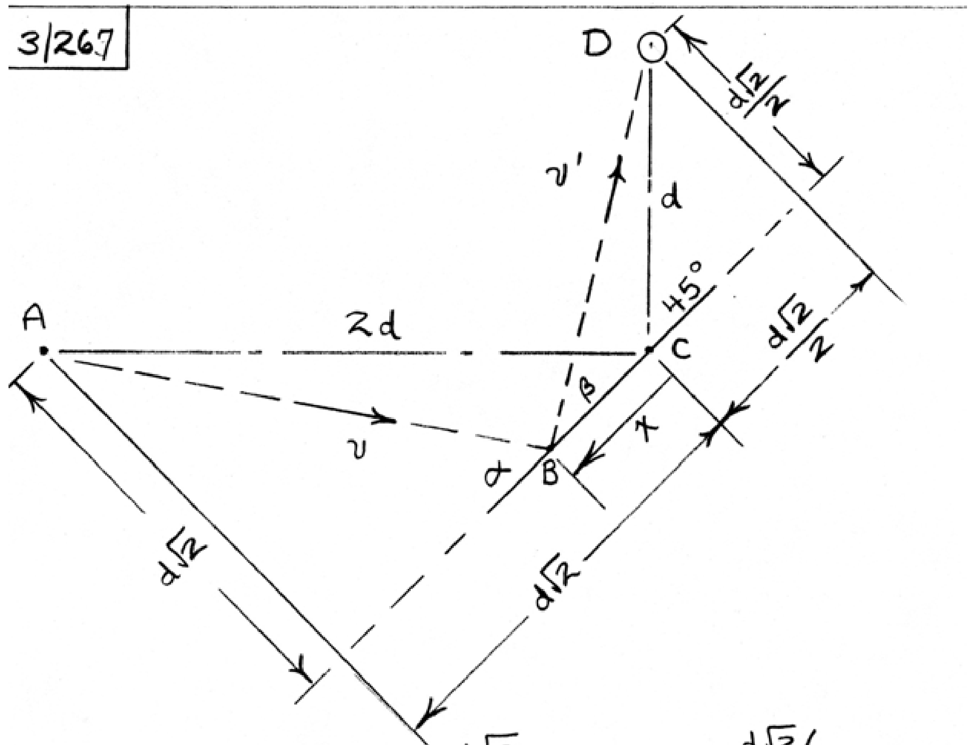
$\underline{i}$: $(1000)(2000) = 1100 \cos 20° \, v'$

$v' = 1935$ m/s

$\underline{j}$: $-100\, v_m = -(1100)(1935) \sin 20°$

$$\underline{v_m = 7280 \text{ m/s}}$$

3/267



$$\tan\alpha = \frac{1}{e}\tan\beta : \quad \frac{d\sqrt{2}}{d\sqrt{2} - x} = \frac{1}{0.8}\,\frac{d\sqrt{2}/2}{\left(\frac{d\sqrt{2}}{2} + x\right)}$$

Solve for $\underline{x = 0.1088d}$

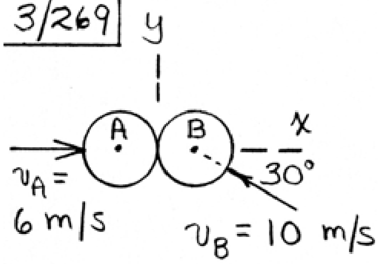
3/268 Let v_s and v_b stand for rebound velocities from steel and brass plates.

Impact speed $= \sqrt{2gh} = \sqrt{2(9.81)(0.15)} = 1.716$ m/s

$$\left.\begin{aligned} 0.6 &= \frac{v_s}{1.716}, \quad v_s = 1.029 \text{ m/s} \\ 0.4 &= \frac{v_s}{1.716}, \quad v_b = 0.686 \text{ m/s} \end{aligned}\right\} \quad \omega = \frac{1.029 - 0.686}{0.60} = 0.572 \text{ rad/s}$$

CCW

3/269



$$v_{A_y}' = v_{A_y} = 0$$

$$v_{B_y}' = v_{B_y} = 10 \sin 30° = 5 \text{ m/s}$$

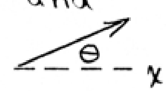
$$m_A v_{A_x} + m_B v_{B_x} = m_A v_{A_x}' + m_B v_{B_x}'$$

$$6 - 10 \cos 30° = v_{A_x}' + v_{B_x}' \qquad (1)$$

$$e = \frac{v_{B_x}' - v_{A_x}'}{v_{A_x} - v_{B_x}} : \quad 0.75 = \frac{v_{B_x}' - v_{A_x}'}{6 - (-10 \cos 30°)} \qquad (2)$$

Solve Eqs. (1) & (2) : $\begin{cases} v_{A_x}' = -6.83 \text{ m/s} \\ v_{B_x}' = 4.17 \text{ m/s} \end{cases}$

Magnitudes and directions



$$\underline{v_A' = 6.83 \ \tfrac{m}{s} \ @ \ \theta_A = 180°}$$

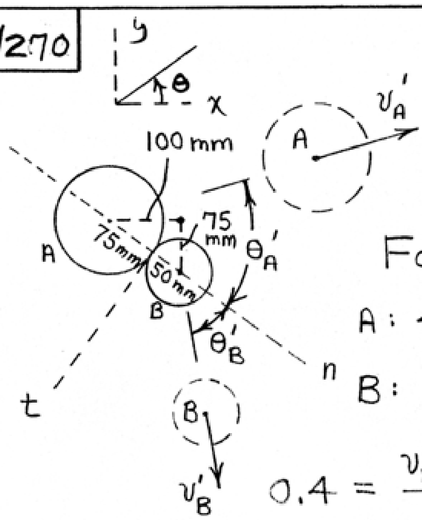
$$\underline{v_B' = 6.51 \ \tfrac{m}{s} \ @ \ \theta_B = 50.2°}$$

Initial: $T_1 = \frac{1}{2} m (6^2 + 10^2) = 68\, m$

Final : $T_2 = \frac{1}{2} m (6.83^2 + 6.51^2) = 44.5\, m$

$$n = \frac{68 - 44.5}{68} (100\%) = \underline{34.6\%}$$

3/270



For system, $\Delta G_n = 0$

$$23 v_A' \cos\theta_A' + 4 v_B' \cos\theta_B' = 23(4)\left(\tfrac{4}{5}\right) - 4(12)\left(\tfrac{4}{5}\right) \quad \text{---(a)}$$

For each sphere, $\Delta G_t = 0$

$$A: \; 4\left(\tfrac{3}{5}\right) = v_A' \sin\theta_A' \quad \text{---(b)}$$

$$B: \; 12\left(\tfrac{3}{5}\right) = v_B' \sin\theta_B' \quad \text{---(c)}$$

$$0.4 = \frac{v_B' \cos\theta_B' - v_A' \cos\theta_A'}{12\left(\frac{4}{5}\right) + 4\left(\frac{4}{5}\right)} \quad \text{--(d)}$$

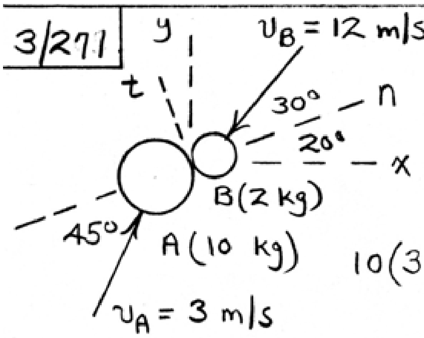
Solve (a), (b), (c), (d): $\underline{v_A' = 2.46 \text{ m/s}}$, $\theta_A' = 77.2°$

$\underline{v_B' = 9.16 \text{ m/s}}$, $\theta_B' = 51.8°$

Relative to the $+x$-axis, the directions of the final velocities are

$$\begin{cases} \theta_A = 77.2 - 36.9 = 40.3° \\ \theta_B = -51.8 - 36.9 = -88.7° \end{cases}$$

3/271



$v_{A_t}' = v_{A_t} = 3 \sin 45° = 2.12 \frac{m}{s}$

$v_{B_t}' = v_{B_t} = -12 \sin 30° = -6 \frac{m}{s}$

$m_A v_{A_n} + m_B v_{B_n} = m_A v_{A_n}' + m_B v_{B_n}'$:

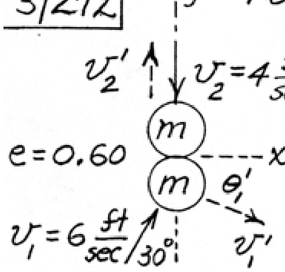
$10(3 \cos 45°) + 2(-12 \cos 30°) =$
$10 v_{A_n}' + 2 v_{B_n}'$ (1)

$$e = \frac{v_{B_n}' - v_{A_n}'}{v_{A_n} - v_{B_n}} \; : \; 0.5 = \frac{v_{B_n}' - v_{A_n}'}{3 \cos 45° - (-12 \cos 30°)} \quad (2)$$

Solve (1) & (2) : $v_{A_n}' = -1.007$ m/s, $v_{B_n}' = 5.25$ m/s

Then
$$\begin{cases} v_{A_x}' = -2.12 \sin 20° - 1.007 \cos 20° = -1.672 \text{ m/s} \\ v_{A_y}' = 2.12 \cos 20° - 1.007 \sin 20° = 1.649 \text{ m/s} \\ v_{B_x}' = -(-6 \sin 20°) + 5.25 \cos 20° = 6.99 \text{ m/s} \\ v_{B_y}' = -6 \cos 20° + 5.25 \sin 20° = -3.84 \text{ m/s} \end{cases}$$

3/272



For system, $\Delta G_y = 0$ so

$$[m(6\cos 30°) - m(4)] - [m(-v_1'\sin\theta_1') + m v_2'] = 0$$

or $v_2' - v_1'\sin\theta_1' = 1.196$ (1)

For each ball $\Delta v_x = 0$ so

$v_1'\cos\theta_1' = 6(\frac{1}{2})$, $v_{2_x}' = 0$

Also $e = 0.60 = \dfrac{v_2' + v_1'\sin\theta_1'}{4 + 6\cos 30°}$, $v_2' + v_1'\sin\theta_1' = 5.518$ (2)

Combine (1) & (2) & get $2v_2' = 1.196 + 5.518$, $\underline{v_2' = 3.36\frac{ft}{sec}}$

& $v_1'\sin\theta_1' = 2.16$; Divide by $v_1'\cos\theta_1' = 3$

& get $\theta_1' = \tan^{-1}0.7203 = 35.8°$ & $v_1' = \dfrac{3}{\cos 35.8°} = \underline{3.70\frac{ft}{sec}}$

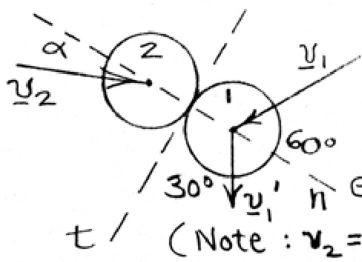
Initial kinetic energy $= \frac{1}{2}m(6^2 + 4^2) = \frac{1}{2}m(52)$

Final " " $= \frac{1}{2}m(\overline{3.70}^2 + \overline{3.36}^2) = \frac{1}{2}m(24.9)$

% loss $= \dfrac{52 - 24.9}{52} = 0.520$ or $\underline{52.0\%}$

3/273 Conservation of n-momentum :

$$m(-v_1 \cos 60°) + m(v_2 \cos\alpha) = mv_{1n}' + mv_{2n}' \qquad (a)$$



Restitution :

$$e = 0.8 = \frac{v_{2n}' - v_{1n}'}{-v_1 \cos 60° - v_2 \cos\alpha} \qquad (b)$$

(Note : $v_2 = v_1$)

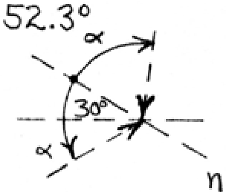
Simultaneous solution of Eqs. (a) and (b) :

$$v_{1n}' = v_1\,[0.9 \cos\alpha - 0.05]$$

$$v_{1t}' = v_{1t} = v_1 \sin 60° = \frac{3}{2} v_1$$

$$\tan 30° = \frac{v_{1n}'}{v_{1t}'} = \frac{v_1\,[0.9\cos\alpha - 0.05]}{\frac{\sqrt{3}}{2} v_1}$$

Solving, $\cos\alpha = 0.611$, $\alpha = \pm 52.3°$



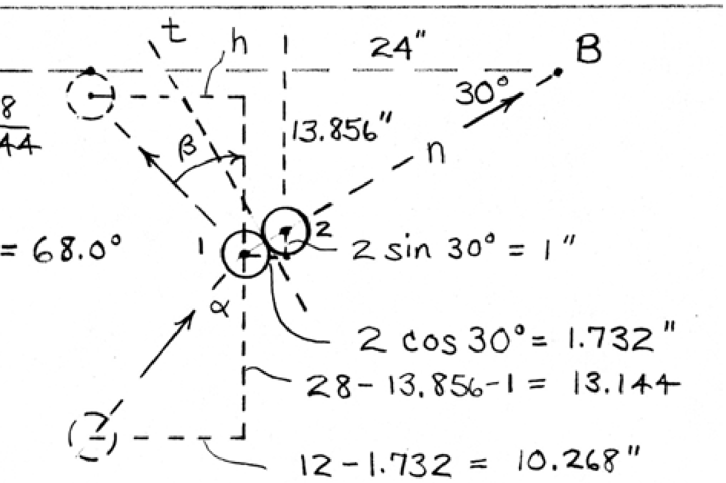
So $\theta = 30° + 52.3° = \underline{82.3°}$

or $\theta = 30° - 52.3° = \underline{-22.3°}$

3/274

$\alpha = \tan^{-1} \dfrac{10.268}{13.144} = 38.0°$

$\theta_1 = \alpha + 30° = 68.0°$



Mom. : $\cancel{m_1}(v_1)_n + \cancel{m_2}(v_2)_n = \cancel{m_1}(v_1')_n + \cancel{m_2}(v_2')_n$

$$v_1 \sin 68.0° = (v_1')_n + (v_2')_n$$

Restitution : $e = \dfrac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n}$

$$0.9 = \frac{(v_2')_n - (v_1')_n}{v_1 \sin 68.0° - 0}$$

Solving, $(v_1')_n = 0.0464 v_1$

Also, $(v_1')_t = (v_1)_t = v_1 \cos 68.0° = 0.375 v_1$

$\tan \theta_1' = \dfrac{(v_1')_n}{(v_1')_t} = \dfrac{0.0464 v_1}{0.375 v_1}$, $\theta_1' = 7.05°$

$\beta = 30° - \theta_1' = 22.95°$, $\tan \beta = \dfrac{h}{13.856 - 1 + 1} = 0.423$

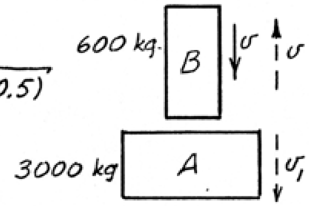
$h = 5.87"$. Then $x = 24 - 1.732 - 5.87 = \underline{16.40 \text{ in.}}$

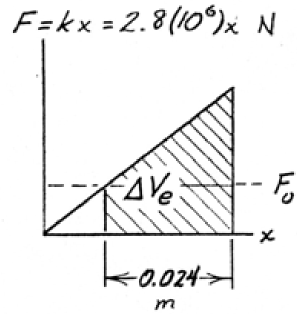
3/275

Hammer: $v = \sqrt{2gh}$

$= \sqrt{2(9.81)(0.5)}$

$= 3.13$ m/s





For anvil: $\Delta T + \Delta V_e + \Delta V_g = 0$

$\Delta T = 0 - \frac{1}{2} 3000 v_1^2$ J

$\Delta V_e = \frac{1}{2}(2.8\times10^6)(0.024)^2 + 29.4(10^3)(0.024)$ J

$F_0 = mg = 3000(9.81) = 29.4(10^3)$ N

$\Delta V_g = -29.4(10^3)(0.024)$ J

Substitute & get $-\frac{1}{2}(3000)v_1^2 + \frac{1}{2}(2.8\times10^6)(0.024)^2 = 0$,

$v_1 = 0.733$ m/s

Hammer & anvil impact: $\Delta G_x = 0$: $3000(0.733) - 600v' - 600(3.13) = 0$

$v' = 0.534$ m/s

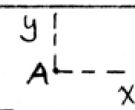
Hammer after impact: $v' = \sqrt{2gh}$, $h = \frac{0.534^2}{2\times9.81} = 0.01453$ m

or $\underline{h = 14.53 \text{ mm}}$

$e = \frac{0.534 - (-0.733)}{3.13} = \underline{0.405}$

▶3/276

$v_{x_A} = 50\cos\alpha, \quad v_{y_A} = 50\sin\alpha$



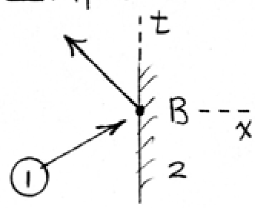
$$t_{AB} = \frac{10}{v_{x_A}} = \frac{10}{50\cos\alpha} = \frac{1}{5\cos\alpha}$$

$$v_{x_B} = v_{x_A} = 50\cos\alpha$$

$$v_{y_B} = v_{y_A} - gt = 50\sin\alpha - \frac{g}{5\cos\alpha}$$

$$y_B = y_A + v_{y_A}t - \tfrac{1}{2}gt^2 = 0 + (50\sin\alpha)\left(\frac{1}{5\cos\alpha}\right) - \frac{g}{2}\left(\frac{1}{25\cos^2\alpha}\right) = 10\tan\alpha - \frac{g}{50\cos^2\alpha}$$

Impact at B:



$$v_{y_B}' = v_{y_B} = 50\sin\alpha - \frac{g}{5\cos\alpha}$$

$$e = \frac{v_{2x}' - v_{1x}'}{v_{1x} - v_{2x}} = \frac{0 - v_{1x}'}{50\cos\alpha - 0} = 0.5$$

$$v_{1x}' = -25\cos\alpha$$

$$t_{BA} = \frac{10}{25\cos\alpha} = \frac{2}{5\cos\alpha}$$

$$y_A = y_B + v_{y_B}'t - \tfrac{1}{2}gt^2$$

$$0 = \left(10\tan\alpha - \frac{g}{50\cos^2\alpha}\right) + \left(50\sin\alpha - \frac{g}{5\cos\alpha}\right)\left(\frac{2}{5\cos\alpha}\right) - \frac{g}{2}\left(\frac{2}{5\cos\alpha}\right)^2$$

Collect terms:

$$30\tan\alpha - \frac{9g}{50}\frac{1}{\cos^2\alpha} = 0$$

Use $\frac{1}{\cos^2\alpha} = (\tan^2\alpha + 1)$ to obtain

$$5.796\tan^2\alpha - 30\tan\alpha + 5.796 = 0$$

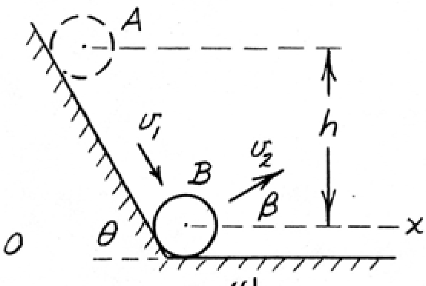
Quadratic solution: $\tan\alpha = 0.201,\ 4.97$

$$\Rightarrow \underline{\alpha = 11.37° \text{ or } 78.6°}$$

▸3/277 From A to B

$\Delta T + \Delta V_g = 0$: $\frac{1}{2} m v_1^2 - mgh = 0$,

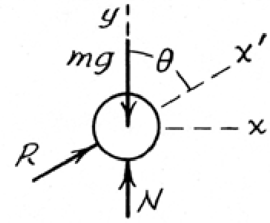
$v_1 = \sqrt{2gh}$



During impact

$\Sigma F_{x'} \geq 0$: $R - mg\cos\theta + N\cos\theta \geq 0$

$R < mg\cos\theta$



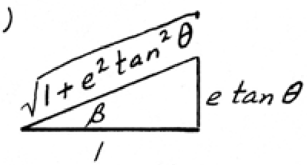
During small time of impact, impulses of R & mg are negligible

so $\int \Sigma F_x \, dt \approx 0$ & $\Delta G_x \approx 0$:

$$m v_1 \cos\theta = m v_2 \cos\beta \qquad (1)$$

$$\int \Sigma F_y \, dt \neq 0 \ \& \ v_2 \sin\beta = e v_1 \sin\theta \qquad (2)$$

Divide (2) by (1) & get $e\tan\theta = \tan\beta$



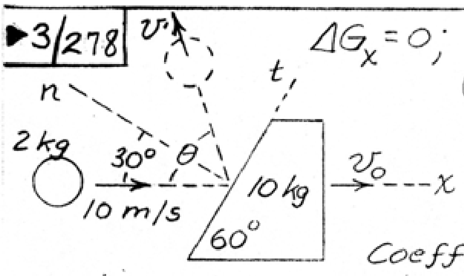
$v_x = v_2\cos\beta = v_1\cos\theta$, $\underline{v_x = \sqrt{2gh}\cos\theta}$

$$n = \frac{|\Delta T|}{T} = \frac{\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2}{\frac{1}{2}mv_1^2} = 1 - \frac{v_2^2}{v_1^2} = 1 - \frac{(e v_1 \sin\theta/\sin\beta)^2}{v_1^2}$$

$$n = 1 - \frac{e^2\sin^2\theta}{\sin^2\beta} = \underline{1 - (\cos^2\theta + e^2\sin^2\theta)} \text{ where } \sin^2\beta = \frac{e^2\tan^2\theta}{1+e^2\tan^2\theta}$$

For a rounded corner of radius greater than that of the sphere, there would be no discontinuity in the magnitude of the velocity and, hence, no impact.

3/278



$\Delta G_x = 0;\ 2(10) + 0 = -2v'\cos\theta + 10v_0$ (For system) (1)

$\Delta G_t = 0$ (for sphere)

$2(10\sin 30°) = 2v'\sin(\theta - 30°)$ (2)

Coefficient of restitution applies to velocity components in n-dir.

$$0.6 = \frac{v_0 \sin 60° + v'\cos(\theta - 30°)}{10\cos 30° + 0} \quad (3)$$

Eq. (3) is $5.196 = 0.866v_0 + 0.866v'\cos\theta + 0.5v'\sin\theta$

Sub. Eq. (1) to eliminate v_0 & get

$5.196 = 0.866(2 + 0.2v'\cos\theta) + 0.866v'\cos\theta + 0.5v'\sin\theta$

or $1.039v'\cos\theta + 0.5v'\sin\theta = 3.464$ (4)

Eq. (2) becomes $0.866v'\sin\theta - 0.5v'\cos\theta = 5$ (5)

Solve (4) & (5) & get $\underline{v' = 6.04 \text{ m/s}}$, $\underline{\theta = 85.9°}$

From Eq. (1) $v_0 = 2.087$ m/s

For carriage $\Delta T + \Delta V_e = 0;\ -\frac{1}{2}10(2.087)^2 + \frac{1}{2}1600\delta^2 = 0$

$\delta^2 = 0.02722$, $\delta = 0.1650$ m or $\underline{\delta = 165.0 \text{ mm}}$

3/279

$$v = \sqrt{\frac{Gm_s}{r}} = \sqrt{\frac{(3.439 \times 10^{-8})(333{,}000)(4.095 \times 10^{23})}{(93 \times 10^{6})(5280)}}$$

$$= 97{,}725 \text{ ft/sec} = \underline{18.51 \text{ mi/sec}}$$

$\boxed{3/280}$ For a circular orbit, $r_{min} = r_{max}$

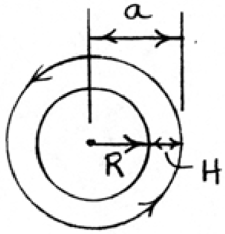
and $a = R+h$, so Eq. 3/48 becomes

$$v = R\sqrt{\frac{g}{R+h}} = 6371(10^3)\sqrt{\frac{9.825}{(6371+590)10^3}}$$

$$= \underline{7569 \text{ m/s}} \text{ or } \underline{27\,250 \text{ km/h}}$$

3/281

Eq. 3/47 with $r = a = R + H$:



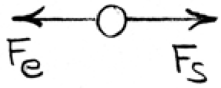
$$v^2 = 2gR^2\left(\frac{1}{a} - \frac{1}{2a}\right) = \frac{gR^2}{R+H}$$

$$= \frac{1.62\,(3476/2)^2}{\frac{3476}{2} + 80}(1000)$$

$\underline{v = 1641 \text{ m/s}}$ or $\underline{5910 \text{ km/h}}$

3/282

Moon m



$$\frac{F_s}{F_e} = \frac{G m_s m / d_{m-s}^2}{G m_e m / d_{m-e}^2} = \left(\frac{d_{m-e}}{d_{m-s}}\right)^2 \frac{m_s}{m_e}$$

$$= \left(\frac{384\,398}{149.6(10^6) - 384\,398}\right)^2 333\,000 = 2.21$$

Therefore, the acceleration of the moon is toward the sun, and thus the path is concave toward the sun!

3/283 From Eq. 3/47 for a circular orbit of altitude H & with $a = r = R+H$

$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2r}\right) = gR^2/(R+H), \quad v = R\sqrt{g/(R+H)}$$

$$v_{escape}^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{\infty}\right) = 2gR^2/(R+H), \quad v = R\sqrt{2g/(R+H)}$$

$$\Delta v = v_{escape} - v = R\sqrt{\frac{g}{R+H}}(\sqrt{2}-1) = (\sqrt{2}-1)v = 0.414\,v$$

For $R = 3959$ mi, $g = 32.23$ ft/sec^2, $H = 200$ mi,

$$v = 3959\sqrt{\frac{32.23/5280}{3959+200}} = 4.80 \text{ mi/sec}, \quad \Delta v = 0.414(4.80) = \underline{1.987 \frac{\text{mi}}{\text{sec}}}$$

$\boxed{3/284}$ $r_{min} = 6371 + 240 = 6611$ km

$r_{max} = 6371 + 400 = 6771$ km

From Eq. 3/43 $\dfrac{r_{min}}{r_{max}} = \dfrac{1-e}{1+e}$

So $(1+e)6611 = (1-e)6771$, $\underline{e = 0.01196}$

From Eq. 3/44 with $a = \frac{1}{2}(r_{max} + r_{min}) = 6691$ km

$$\tau = 2\pi \frac{(6691 \times 10^3)^{3/2}}{(6371 \times 10^3)\sqrt{9.824}} = \underline{5446 \text{ s}} \text{ or}$$

$\underline{\tau = 1 \text{ h } 30 \text{ min } 46 \text{ s}}$

$\boxed{3/285}$ $F = G\,\dfrac{m_1 m_2}{r^2}$

$$= 6.673(10^{-11})\,\frac{1.490(10^{23})\,(1.900)(10^{27})}{(1.070 \times 10^{9})^2} = \underline{16.50(10^{21})\,\text{N}}$$

$$F = mr\omega^2, \quad \omega = \sqrt{\frac{F}{mr}} = \sqrt{\frac{16.50(10^{21})}{1.490(10^{23})\,1.070(10^{9})}}$$

$$= 1.017(10^{-5})\ \text{rad/s}$$

$$\tau = \frac{2\pi}{\omega} = \frac{2\pi}{1.017(10^{-5})} = \underline{6.18(10^{5})\,\text{s}} \ \text{ or } \ \underline{7.17\ \text{days}}$$

$$a_n = \frac{F}{m_1} = \frac{16.50(10^{21})}{1.490(10^{23})} = \underline{110.7(10^{-3})\ \text{m/s}^2}$$

3/286 $r_{min} = 2R, \quad r_{max} = 3R$

$$a = \frac{r_{min} + r_{max}}{2} = 2.5R$$

$$v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}} = R\sqrt{\frac{g}{2.5R}}\sqrt{\frac{3R}{2R}} = \sqrt{\frac{3gR}{5}}$$

The velocity in the original circular orbit is

$$v_c = R\sqrt{\frac{g}{a}} = R\sqrt{\frac{g}{2R}} = \sqrt{\tfrac{1}{2}gR}$$

$$\Delta v = v_p - v_c = \sqrt{gR}\left(\sqrt{\frac{3}{5}} - \sqrt{\frac{1}{2}}\right) = 0.0675\sqrt{gR}$$

Numbers : $\Delta v = 0.0675\sqrt{9.825(6371)(1000)}$

$$= \underline{534 \text{ m/s}}$$

(Δv to occur opposite B)

$\boxed{3/287}$ $r = a = 6371 + 300 = 6671 \text{ km} = 6.671(10^6) \text{ m}$

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi\,(6.671 \times 10^6)^{3/2}}{6.371 \times 10^6 \sqrt{9.825}}$$

$$= 5421 \text{ s}$$

Speed of ground point on equator

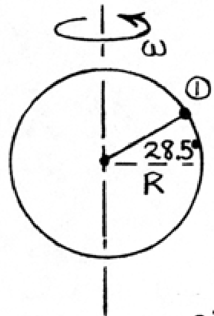
$v_e = R_e \omega_e = (6378)(7.292 \times 10^{-5}) = 0.4651 \text{ km/s}$

Required distance $d = v_e \tau = (0.4651)(5421)$

$$= \underline{2520 \text{ km}}$$

3/288

① On ground, speed $v_1 = (R \cos 28.5)\omega$



$= 6371(1000)\cos 28.5° (0.7292 \cdot 10^{-4})$

$= 408 \text{ m/s}$

$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(80\,000)\, 408^2$

$= 6.67(10^9) \text{ J}$

$V_1 = -\frac{mgR^2}{R} = -80\,000(9.825)(6371 \cdot 1000) = -5.01(10^{12}) \text{J}$

② In circular orbit: $v_2 = R\sqrt{\frac{g}{r}}$

$= 6371(10^3)\sqrt{\frac{9.825}{(6371+300)(1000)}} = 7.73(10^3) \text{ m/s}$

$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}\, 80\,000\, [7.73(10^3)]^2 = 2.39(10^{12}) \text{ J}$

$V_2 = -\frac{mgR^2}{r} = -\frac{80\,000(9.825)(6371 \cdot 1000)^2}{(6371+300) \cdot 1000} = -4.78(10^{12}) \text{ J}$

$\Delta E = T_2 + V_2 - (T_1 + V_1) = \underline{2.61(10^{12}) \text{ J}}$

3/289 (a) $v = R\sqrt{\dfrac{g}{r}} = 6371(1000)\sqrt{\dfrac{9.825}{(6371+637)(1000)}}$

$= \underline{7544 \text{ m/s}}$

(b) From $r_{min} = a(1-e)$, $a = \dfrac{r_{min}}{1-e} = \dfrac{1.1(6371)}{1-0.1}$

$= 7787 \text{ km}$

$v_P = R\sqrt{\dfrac{g}{a}}\sqrt{\dfrac{1+e}{1-e}} = 6371(1000)\sqrt{\dfrac{9.825}{7787(1000)}}\sqrt{\dfrac{1+0.1}{1-0.1}}$

$= \underline{7912 \text{ m/s} = v}$

(c) $a = \dfrac{r_{min}}{1-e} = \dfrac{1.1(6371)}{1-0.9} = 70\,081 \text{ km}$

$v_P = 6371(1000)\sqrt{\dfrac{9.825}{70\,081(1000)}}\sqrt{\dfrac{1+0.9}{1-0.9}}$

$= \underline{10\,398 \text{ m/s} = v}$

(d) Eq. 3/47 with $a \to \infty$: $v = R\sqrt{\dfrac{2g}{r}}$

This is $\sqrt{2}$ times answer for part (a), so

$v = \sqrt{2}\,(7544) = \underline{10\,668 \text{ m/s}}$

3/290 $v_A = R\sqrt{\frac{g}{r}} = (3959)(5280)\sqrt{\frac{32.23}{(4759)(5280)}}$

$= 23{,}676$ ft/sec

$v_B = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}}$

$= (3959)(5280)\sqrt{\frac{32.23}{\left(\frac{2(3959)+1800}{2}\right)(5280)}}\sqrt{\frac{4959}{4759}}$

$= 23{,}917$ ft/sec

Momentum conservation during impact:

$m_A v_A + m_B v_B = (m_A + m_B) v_C$. But $m_A = m_B$, so

$v_C = \frac{1}{2}(v_A + v_B) = 23{,}796$ ft/sec

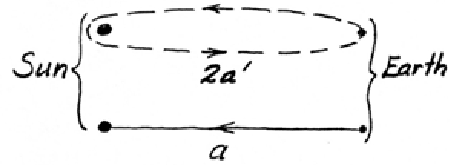
From $v_P = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}}$,

$r_{max} = \dfrac{r_{min}}{\left(\frac{2gR^2}{v_P^2 r_{min}} - 1\right)} = 2.5652 \times 10^7$ ft
(4858 mi)

$h_{max} = r_{max} - R = 4858 - 3959 = \underline{899 \text{ mi}}$

3/291

Radius of actual orbit around the sun is a, which is the major axis $2a'$ of the degenerate ellipse.



R = radius of sun

g = gravitational accel. on surface of sun

Orbital period Eq. 3/44

For actual orbit $\tau = 2\pi \dfrac{a^{3/2}}{R\sqrt{g}}$

For degenerate ellipse $\tau' = 2\pi \dfrac{(a/2)^{3/2}}{R\sqrt{g}}$

so $\dfrac{\tau'}{\tau} = \dfrac{(\frac{1}{2})^{3/2}}{1}$

But time t to fall is $t = \frac{1}{2}\tau' = \frac{1}{2}\left(\frac{1}{2}\right)^{3/2}\tau = \frac{1}{4\sqrt{2}}365.26$

$= \underline{64.6 \text{ days}}$

3/292 The apogee speed at C is

$$v_a = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{min}}{r_{max}}}$$

$$= 6371(10^3)\sqrt{\frac{9.825}{(2\cdot 6371+240+320)1000/2}}\sqrt{\frac{6371+240}{6371+320}}$$

$$= 7697 \text{ m/s}$$

The circular orbit speed at $h = 320$ km is

$$v_{circ} = R\sqrt{g/r_{max}} = 7720 \text{ m/s}$$

$$\Delta v = v_{circ} - v_a = 7720 - 7697 = 23.25 \text{ m/s}$$

$$F\Delta t = m\Delta v: \quad 2(30\,000)(\Delta t) = 85\,000\,(23.25)$$

$$\underline{\Delta t = 32.9 \text{ s}}$$

The burn to increase speed is at C.

3/293 The linear impulses from drag and from the thruster must be equal in magnitude, or

$$Dt = \sum T t_{burn}$$

$t = 10\tau$, where $\tau = 2\pi \dfrac{a^{3/2}}{R\sqrt{g}}$

or $\tau = 2\pi \dfrac{(6.571 \times 10^6)^{3/2}}{6.371 \times 10^6 \sqrt{9.825}} = 5300 \text{ s}$

$t = 10\tau = 53{,}000 \text{ s}$

$D = \dfrac{\sum T t_{burn}}{t} = \dfrac{2(300)}{53{,}000} = \underline{0.01132 \text{ N}}$

3/294 From $r_{min} = a(1-e)$:

$7959 = [4000 + 3959 + 16{,}000](1-e)$, $e = 0.668$

$b = a\sqrt{1-e^2} = 23{,}959\sqrt{1-0.668^2} = \underline{17{,}833 \text{ mi}}$

At B, $r = \sqrt{16{,}000^2 + 17{,}833^2} = 23{,}959$ mi

$v_B{}^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

$= 2(32.23)3959^2(5280)\left[\frac{1}{23{,}959} - \frac{1}{2(23{,}959)}\right]$

$\underline{v_B = 10{,}551 \text{ ft/sec}}$

3/295 Eq. 3/44: $\tau_f = \dfrac{2\pi a^{3/2}}{R\sqrt{g}} = \dfrac{2\pi a^{3/2}}{\sqrt{G m_A}}$

$$= 2\pi \frac{[200(10^9)]^{3/2}}{\sqrt{6.673(10^{-11})\,10^{31}}} = \underline{21{,}760{,}000 \text{ s}}$$

Eq. 3/49b: $\tau_{nf} = \dfrac{2\pi a^{3/2}}{\sqrt{G(m_A + m_B)}}$

$$= 2\pi \frac{[200(10^9)]^{3/2}}{\sqrt{6.673(10^{-11})(10^{31} + 10^{30})}} = \underline{20{,}740{,}000 \text{ s}}$$

(−4.7 percent difference)

$\boxed{3/296}$ Speed in circular orbit is

$$v = R\sqrt{\frac{g}{a}} = (3959)(5280)\sqrt{\frac{32.23}{(4459)(5280)}}$$

$$= 24{,}458 \text{ ft/sec}$$

Time required for B to return to C's burn position : $t = \dfrac{2\pi r - 1000(5280)}{v}$

$$= 5832 \text{ s}$$

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}}, \quad a = \left(\frac{\tau R\sqrt{g}}{2\pi}\right)^{2/3}$$

$$a = \left(\frac{(5832)(3959)(5280)\sqrt{32.23}}{2\pi}\right)^{2/3} = 2.29799\,(10)^7 \text{ ft}$$

At apogee, $v_c = \sqrt{2gR^2\left[\frac{1}{r} - \frac{1}{2a}\right]} = 24{,}156$ ft/sec

$$\Delta v = v - v_c = 24{,}458 - 24{,}156 = \underline{302 \text{ ft/sec}}$$

(Can check to ensure that C does not strike the earth by finding $r_{min} = 2.242 \times 10^7$ ft $> R = 2.090 \times 10^7$ ft !)

3/297 From previous solution, the circular orbit speed is $v = 24{,}458$ ft/sec.

Time required for B to return to C's burn position over almost two circular orbits:

$$t = \frac{4\pi r - (1000)(5280)}{v} = 11{,}881 \text{ s}$$

$$a = \left(\frac{\tau R\sqrt{g}}{2\pi}\right)^{2/3} = \left[\frac{\left(\frac{11{,}881}{2}\right)(3959)(5280)\sqrt{32.23}}{2\pi}\right]^{2/3}$$

$$= 2.32626\,(10^7)\text{ ft}$$

At apogee, $v_c = \sqrt{2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)} = 24{,}309$ ft/sec

$\Delta v = v - v_c = 24{,}458 - 24{,}309 = \underline{148 \text{ ft/sec}}$

3/298 Circular orbit speed

$$v_0 = R\sqrt{\frac{g}{a}} = R\sqrt{\frac{g}{3R}} = \sqrt{\frac{1}{3}gR}$$

Speed at A (apogee) in elliptical orbit:

$$v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{min}}{r_{max}}} = R\sqrt{\frac{g}{2R}}\sqrt{\frac{R}{3R}} = \sqrt{\frac{1}{6}gR}$$

$$v_r = v_0 - v_A = \sqrt{gR}\left[\sqrt{1/3} - \sqrt{1/6}\right] = 0.1691\sqrt{gR}$$

Numbers: $v_r = 0.1691\sqrt{1.62\,\frac{3476}{2}\,(1000)}$

$$= \underline{284 \text{ m/s}} \text{ (directed rearward)}$$

Call the circular orbit period τ_0 and the elliptical orbit period τ_{AB}

$$\tau_0 = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi(3R)^{3/2}}{R\sqrt{g}}\ ; \quad \tau_{AB} = \frac{2\pi(2R)^{3/2}}{R\sqrt{g}}$$

$$\theta = \left(\frac{\tau_{AB}/2}{\tau_0/2}\right)\pi = \left(\frac{2}{3}\right)^{3/2}\pi = 1.710 \text{ rad or } \underline{98.0^\circ}$$

3/299 Circular orbit : $v = R\sqrt{\frac{g}{r}}$

$$v = (3959)(5280)\sqrt{\frac{32.23}{(4159)(5280)}} = 25{,}324 \text{ ft/sec}$$

During burn : $a_t = \frac{F}{m} = \frac{2(6000)}{(175{,}000)/32.2} = 2.208 \frac{ft}{sec^2}$

$$v_a = v - a_t t = 25{,}324 - 2.208(150) = 24{,}993 \frac{ft}{sec}$$

$$v^2 = 2gR^2\left[\frac{1}{r} - \frac{1}{2a}\right]$$

Substitute conditions at B to find $a = 2.1403(10^7)$ ft

Use $v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{1-e}{1+e}}$ to obtain $e = 0.02599$

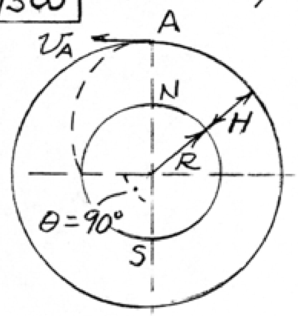
$r = \frac{a(1-e^2)}{1+e\cos\theta}$ utilized at point C :

$$(3959)(5280) = \frac{(2.1403 \times 10^7)(1 - 0.02599^2)}{1 + 0.02599 \cos\theta}$$

$\theta = 26.7°$

$\beta = 180 - \theta = \underline{153.3°}$

3/300 From Eq. 3/43 $\dfrac{1}{r} = \dfrac{1+e\cos\theta}{a(1-e^2)}$



when $\theta = 90°$, $r = R$

" $\theta = 180°$, $r = R+H$

Thus $\dfrac{1}{R} = \dfrac{1}{a(1-e^2)}$

& $\dfrac{1}{R+H} = \dfrac{1-e}{a(1-e^2)} = \dfrac{1}{a(1+e)}$

Solve & get $1-e = \dfrac{R}{R+H}$ & $a(1+e) = R+H$

From Eq. 3/48

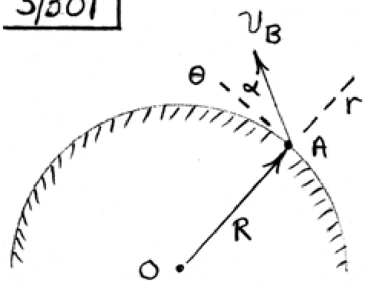
$$v_A = R\sqrt{\frac{g(1-e)}{a(1+e)}} = R\sqrt{g}\sqrt{\frac{R}{R+H}\,\frac{1}{R+H}} = \frac{R\sqrt{gR}}{R+H}$$

For circular orbit

$$v = R\sqrt{\frac{g}{R+H}} \quad \text{so } \Delta v = R\sqrt{\frac{g}{R+H}} - \frac{R\sqrt{gR}}{R+H}$$

$$= R\sqrt{\frac{g}{R+H}}\left(1 - \sqrt{\frac{R}{R+H}}\right)$$

3/301



$v_\theta = v\cos\alpha = 2000\cos 30°$
$= 1732$ m/s

$v_r = v\sin\alpha = 2000\sin 30°$
$= 1000$ m/s

$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$$

Using conditions at B:
$a = 3.2906 \times 10^6$ m

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(2000)^2 = 2\times10^6\,m$$

$$V_B = -\frac{mgR^2}{r} = \frac{-m(9.825)(6.371\times10^6)^2}{6.371\times10^6}$$

$$= -6.2595\times10^7\,m$$

$$E = T_B + V_B = -6.0595\times10^7\,m$$

$$h = rv_\theta = 6.371(10^6)(1732) = 1.1035\times10^{10}$$

Now use $e = \sqrt{1+\frac{2Eh^2}{mg^2R^4}}$ to get $e = 0.9525$

Finally, $r_{max} = a(1+e) = 3.2906(10^6)(1+0.9525)$
$= 6.4249\times10^6$ m

$h_{max} = r_{max} - R = 53\,900$ m or $\underline{53.9\text{ km}}$

3/302 Point A is the apogee, so we have $r_{max} = \frac{3R}{2} = a(1+e)$.

$r = \frac{a(1-e^2)}{1+e\cos\theta}$; At B : $R = \frac{a(1-e^2)}{1+e\cos(135°)}$

Solving, $e = 0.6306$, $a = 0.9199R$

Now, $v_B^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

At A : $v_B^2 = 2(9.825)(6.371 \times 10^6)^2 \times \left(\frac{1}{6.371 \times 10^6} - \frac{1}{2(0.9199)(6.371)(10^6)}\right)$

$\underline{v_B = 7560 \text{ m/s}}$

3/303

$$v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{1-e}{1+e}} = R\sqrt{\frac{9.825}{0.9199R}}\sqrt{\frac{1-0.6306}{1+0.6306}}$$

$$= 1.555\sqrt{R}$$

$$h = r_A v_A = \frac{3}{2}R\left(1.555\sqrt{R}\right) = 2.3332R^{3/2}$$

Conservation of angular momentum requires

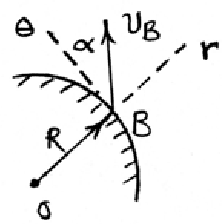
$$h = r_B v_{B\theta} = R v_{B\theta} = 2.3332 R^{3/2}$$

$$v_{B_\theta} = 2.3332 R^{1/2}$$

$$= 2.3332\left(6.371\times 10^6\right)^{1/2} = 5889 \text{ m/s}$$

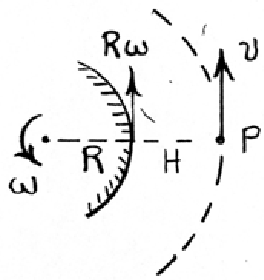
$$v_{B_\theta} = v_B \cos\alpha$$

$$\alpha = \cos^{-1}\left(\frac{v_{B\theta}}{v_B}\right) = \cos^{-1}\left(\frac{5889}{7560}\right) = \underline{38.8°}$$



(Value of v_B from previous solution)

3/304 $a = \frac{1}{2}\left[2(6371) + 150 + 1500\right] = 7196 \text{ km}$



Eq. 3/47 at perigee P:

$$v^2 = 2(9.825)\left[6371\,(10^3)\right]^2 \times \left[\frac{1}{(6371+150)\,10^3} - \frac{1}{2(7196)\,10^3}\right]$$

$v = 8179 \text{ m/s}$

$R\omega = 6371(10^3)(0.7292 \cdot 10^{-4}) = 465 \text{ m/s}$

Absolute dish angular velocity $p_a = \dfrac{v - R\omega}{H}$

Relative dish angular velocity $p = p_a - \omega$

$$p = \frac{v - R\omega}{H} - \omega = \frac{8179 - 465}{150(10^3)} - 0.7292\,(10^{-4})$$

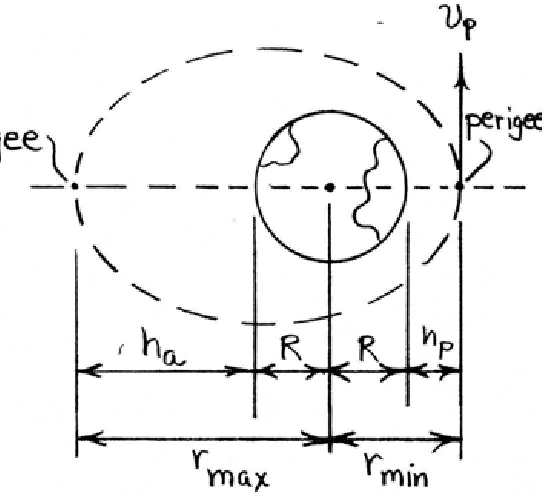
$= \underline{0.0514 \text{ rad/s}}$

3/305

At perigee,

$$a = a_n = \frac{v_p^2}{\rho_p},$$

so $\rho_p = \dfrac{v_p^2}{a_n}$



From Eq. 3/48,

$$v_p^2 = R^2 \frac{g}{a} \frac{r_{max}}{r_{min}}$$

But from Eqs. 3/43: $r_{min} + r_{max} = 2a$, so

$$v_p^2 = gR^2 \frac{r_{max}}{r_{min}} \frac{2}{r_{min} + r_{max}}$$

Also, $a_n = g_{perigee} = g\left(\frac{R}{r_{min}}\right)^2$ (from Chapter 1)

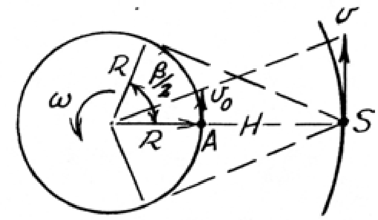
Thus $$\rho_p = 2gR^2 \frac{r_{max}}{r_{min}} \frac{1}{r_{min} + r_{max}} \Big/ g\frac{R^2}{r_{min}^2}$$

$$= 2\frac{r_{max}\, r_{min}}{r_{min} + r_{max}}$$

or $$\underline{\rho_p = 2\frac{(R+h_a)(R+h_p)}{2R + h_a + h_p}}$$

3/306

Path is limited to an equatorial orbit in order to remain above a point A on the equator.



$\frac{v}{R+H} = \omega$ & for circular orbit Eq. 3/47 with $a = r = R+H$ gives $v = R\sqrt{\frac{g}{R+H}}$

$\omega = 0.7292 \times 10^{-4}$ rad/s
$R = 6371$ km

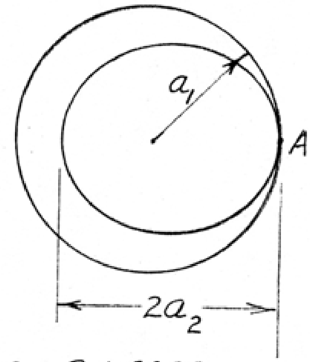
Combine & get $R+H = \sqrt[3]{\frac{gR^2}{\omega^2}}$, $H = \sqrt[3]{\frac{9.825(6371 \times 10^3)^2}{(0.7292 \times 10^{-4})^2}} - 6371 \times 10^3$

$= (42\,170 - 6371)10^3 = 35.8 \times 10^6$ m

or $\underline{H = 35\,800 \text{ km}}$

$\frac{\beta}{2} = \cos^{-1}\frac{R}{R+H} = \cos^{-1}\frac{6371}{42\,170} = 81.3°$, $\underline{\beta = 162.6° \text{ of longitude}}$

3/307 $Ft = m\Delta v$



For circular orbit, $v_1 = R\sqrt{g/a_1}$

$= 6371(10^3)\sqrt{\dfrac{9.825}{12\,371(10^3)}}$

$= 5678$ m/s

For elliptical orbit at apogee A

$v_A = R\sqrt{\dfrac{g}{a_2}}\sqrt{\dfrac{r_{min}}{r_{max}}}$

where $r_{min} = 6371 + 3000 = 9371$ km

$r_{max} = 6371 + 6000 = 12\,371$ km

$a_1 = R + 6000$
$= 6371 + 6000$
$= 12371$ km

$2a_2 = 2a_1 - 3000$
$= 21\,742$ km

$a_2 = 10\,871$ km

So $v_A = 6371(10^3)\sqrt{\dfrac{9.825}{10\,871(10^3)}}\sqrt{\dfrac{9371}{12\,371}}$

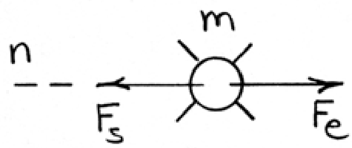
$= 5271$ m/s

Thus $\Delta v = 5678 - 5271 = 406$ m/s

So $2000\,t = 800(406)$

$\underline{t = 162 \text{ s}}$

***3/308**



F_s : force exerted on spacecraft by sun

F_e : force exerted on spacecraft by earth

$$\Sigma F_n = ma_n: \quad F_s - F_e = m\frac{v^2}{\rho} = m\rho\omega^2$$
$$= m(D-h)\left(\frac{2\pi}{T}\right)^2$$

where D is the earth-sun distance and T is the earth orbital period.

$$\frac{Gm_s \cancel{m}}{(D-h)^2} - \frac{Gm_e \cancel{m}}{h^2} = \cancel{m}(D-h)\left(\frac{2\pi}{T}\right)^2$$

With $G = 3.439(10^{-8})\ \frac{\text{ft}^4}{\text{lb-sec}^4}$, $m_s = 333{,}000\ m_e$, $m_e = 4.095(10^{23})$ slugs, $D = 92.96(10^6)(5280)$ ft, and $T = 365.26(24)(3600)$ sec, solve numerically for h as $h = 4.87(10^9)$ ft or $\underline{922{,}000 \text{ mi}}$

▸3/309 For 1, $v_1 = R\sqrt{g/r_1}$; for 2, $v_2 = R\sqrt{g/r_2}$

For transfer ellipse at A, $v_1' = R\sqrt{g/a}\sqrt{r_2/r_1}$

For transfer ellipse at B, $v_2' = R\sqrt{g/a}\sqrt{r_1/r_2}$

$a = \frac{r_1+r_2}{2}$ (Eq. 3/48)

At A, $\Delta v_A = v_1' - v_1 = R\sqrt{g/a}\sqrt{r_2/r_1} - R\sqrt{g/r_1} = \underline{R\sqrt{g/r_1}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)}$

At B, $\Delta v_B = v_2 - v_2' = R\sqrt{g/r_2} - R\sqrt{g/a}\sqrt{r_1/r_2} = \underline{R\sqrt{g/r_2}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right)}$

$$\Delta v_A = 6371(10^3)\sqrt{\frac{9.825(10^3)}{6871}}\left(\sqrt{\frac{2(42171)}{6871+42171}} - 1\right) = \underline{2370 \text{ m/s}}$$

$$\Delta v_B = 6371(10^3)\sqrt{\frac{9.825(10^3)}{42171}}\left(1 - \sqrt{\frac{2(6871)}{6871+42171}}\right) = \underline{1447 \text{ m/s}}$$

▸3/310 Eq. 3/47: $v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

$$7400^2 = 2(9.825)(6371\cdot 1000)^2\left[\frac{1}{7371\cdot 1000} - \frac{1}{2a}\right]$$

$\underline{a = 7462 \text{ km}}$

$$T = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(7400)^2 = 27.38(10^6)m$$

$$V = -\frac{mgR^2}{r} = -\frac{m(9.825)(6371\cdot 1000)^2}{7371(1000)} = -54.1(10^6)m$$

$$E = T + V = -26.7(10^6)m \quad \text{(in Joules)}$$

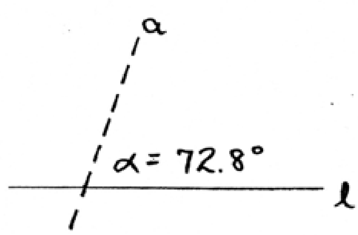
$$h = rv_\theta = 7371(10^3)(7400\cos 2^\circ) = 5.45(10^{10})\ \frac{m^2}{s}$$

$$e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}} = \sqrt{1 + \frac{2(-26.7)10^6 m\ 5.45^2\ 10^{20}}{m(9.825)^2(6371\cdot 1000)^4}}$$

$$= 0.0369$$

From $r = \frac{a(1-e^2)}{1+e\cos\theta}$: $7371 = \frac{7462(1-0.0369^2)}{1+0.0369\cos\theta}$

$\theta = \pm 72.8^\circ$ So $\underline{\alpha' = 72.8^\circ}$

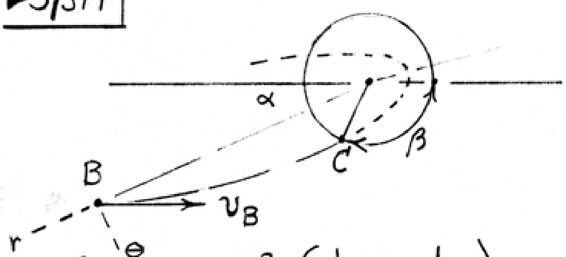


$$r_{min} = a(1-e) = 7462(1-0.0369)$$

$$= 7186 > R = 6371 \text{ m}$$

$\therefore$ Does not strike earth

▶3/311



At B, $r = \sqrt{29}\,R$

$\alpha = \tan^{-1}\left(\frac{2R}{5R}\right)$

$= 21.8°$

$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$$

At B : $3200^2 = 2(9.825)(6.371\times10^6)^2\left[\frac{1}{\sqrt{29}\,6.371(10^6)} - \frac{1}{2a}\right]$

$a = 3.066 \times 10^7$ m

$$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} m\,(3200)^2 = 5.120 \times 10^6 \text{ m}$$

$$V_B = -\frac{mgR^2}{r_B} = -m\,\frac{(9.825)(6.371\times10^6)^2}{\sqrt{29}\,(6.371\times10^6)} = -1.162\times10^7 \text{ m}$$

$$E = T_B + V_B = -6.504 \times 10^6 \text{ m}$$

$$v_\theta = 3200 \sin\alpha = 1188.5 \text{ m/s}$$

$$h = r v_\theta = \sqrt{29}\,(6.371\times10^6)(1188.5) = 4.077\times10^{10} \text{ kg-m}^2\text{/s}$$

$$e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}}$$

$$e = \sqrt{1 + \frac{2(-6.504m)(4.077\times10^{10})^2}{m\,(9.825)^2(6.371\times10^6)^4}}$$

$$= 0.9295$$

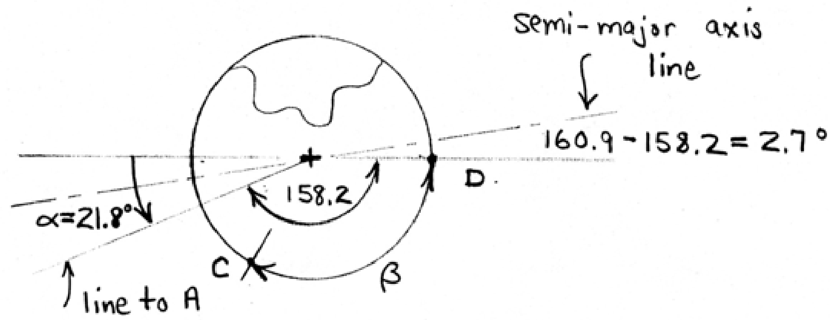
$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

At B : $\sqrt{29}\,(6.371\times10^6) = \dfrac{(3.066\times10^7)(1-0.9295^2)}{1+0.9295\cos\theta}$

$$\theta = 160.9°$$

At C : $6371(10^6) = \dfrac{(3.066\times10^7)(1-0.9295^2)}{1+e\cos\theta}$

$$\theta = 111.8°$$



$$\beta = 111.8 - 2.7 = \underline{109.1°}$$

▶3/312 The speed of the orbiter is

$$v_0 = \sqrt{\frac{Gm_e}{r}} = \sqrt{\frac{6.673(10^{-11})(5.976)(10^{24})}{(6371+200)(1000)}} = 7790 \text{ m/s}$$

The speed of the satellite is

$$v = \sqrt{v_0^2 + v_{s/o}^2} = 7791 \text{ m/s}$$

Eq. 3/47: $v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

$$(7791)^2 = 2(9.825)(6371 \cdot 1000)^2\left[\frac{1}{6571(1000)} - \frac{1}{2a}\right]$$

$$\underline{a = 6572 \text{ km}}$$

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \underline{5301 \text{ s}}$$

Energy $E = \frac{1}{2}mv^2 - \frac{Gm_e m}{r} = -30.3m(10^6) \text{ J}$

$h = rv_\theta = 6571(1000)7790 = 5.12(10^{10}) \text{ m}^2/\text{s}$

Eq. 3/45: $e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}} = \underline{0.01284}$

From $\frac{1}{r} = \frac{1 + e\cos\theta}{a(1-e^2)}$, $\underline{\theta = 90°}$ exactly

(semimajor axis is parallel to x-axis)

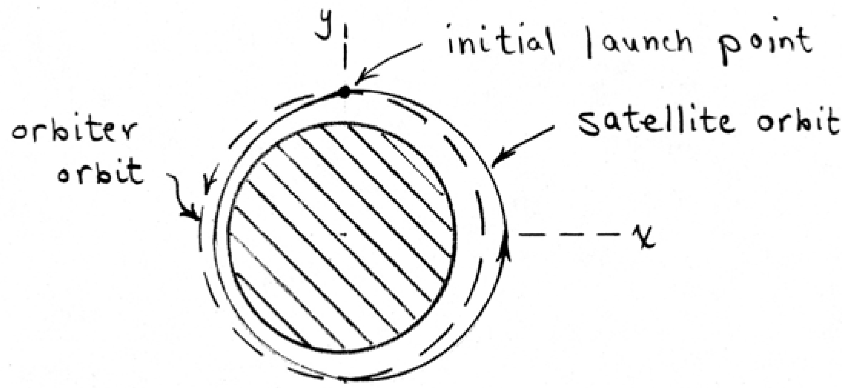
$$r_{min} = a(1-e) = \underline{6.49(10^6)\ \text{m}}$$

$$r_{max} = a(1+e) = \underline{6.66(10^6)\ \text{m}}$$

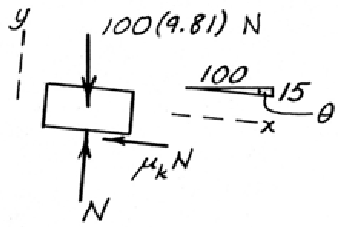
$$v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}} = \underline{7890\ \text{m/s}}$$

$$v_a = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{min}}{r_{max}}} = \underline{7690\ \text{m/s}}$$

Sketch (not to scale):



3/313 Truck bed is a constant-velocity frame of reference so that $U_{rel} = \Delta T_{rel}$ holds.



$\Sigma F_y = 0: \; N - 981 \cos 8.53° = 0$

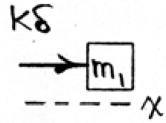
$N = 970$ N

$U_{rel} = \Delta T_{rel}: \; (981 \sin 8.53° - 970\mu_k)2 = \frac{1}{2} 100(0 - 3^2)$

$\theta = \tan^{-1} 0.15 = 8.53°$

$\underline{\mu_k = 0.382}$

3/314



$$\Sigma F_x = ma_x: \quad k\delta = m_1 a_{x_1}$$

$$-k\delta = m_2 a_{x_2}$$

$$a_{1/2} = a_{x_1} - a_{x_2} = \frac{k\delta}{m_1} - \left(-\frac{k\delta}{m_2}\right)$$

or $\underline{a_{1/2} = a_{rel} = k\delta\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$

$\boxed{3/315}$ $v_{rel} = l\dot{\theta} = 0.5(2) = 1 \text{ m/s} \rightarrow$

$v = v + v_{rel} = 2+1 = 3 \text{ m/s} \rightarrow$

$\underline{G} = m\underline{v} = 3(3\underline{i}) = \underline{9\underline{i} \text{ kg}\cdot\text{m/s}}$

$\underline{G}_{rel} = m\underline{v}_{rel} = 3(1\underline{i}) = \underline{3\underline{i} \text{ kg}\cdot\text{m/s}}$

$T = \frac{1}{2}mv^2 = \frac{1}{2}(3)(3)^2 = \underline{13.5 \text{ J}}$

$T_{rel} = \frac{1}{2}mv_{rel}^2 = \frac{1}{2}(3)(1)^2 = \underline{1.5 \text{ J}}$

$\underline{H}_O = -lmv\underline{k} = -(0.5)(3)(3)\underline{k} = \underline{-4.5\underline{k} \ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$

$\underline{H}_{B_{rel}} = -lmv_{rel}\underline{k} = -(0.5)(3)(1)\underline{k} = \underline{-1.5\underline{k} \ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$

$\boxed{3/316}$ Rel. to carrier $U_{rel} = \Delta T_{rel}$

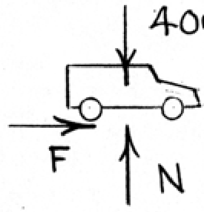
$$(22+P)(10^3)75 = \frac{1}{2}(3)(10^3)\left[(240/3.6)^2 - 0\right]$$

$$22+P = 88.9 \text{ kN}, \quad \underline{P = 66.9 \text{ kN}}$$

3/317 Barge-fixed frame is Newtonian.

$v^2 = v_0^2 + 2a(s - s_0)$: $\left(15\,\frac{5280}{3600}\right)^2 = 2a(80)$

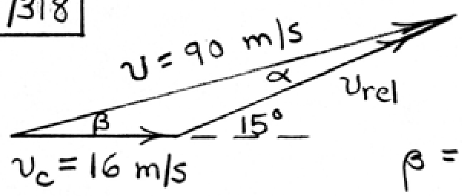
$a = 3.03$ ft/sec^2 $= a_{rel}$



$\xrightarrow{+} \Sigma F = ma$: $F = \frac{4000}{32.2}(3.03)$

$\underline{F = 376 \text{ lb}}$

3/318



$$\frac{\sin 165°}{90} = \frac{\sin \alpha}{16}$$

$$\alpha = 2.64°$$

$$\beta = 180 - 165 - \alpha = 12.36°$$

$$\frac{\sin \beta}{v_{rel}} = \frac{\sin 165°}{90}, \quad v_{rel} = 74.4 \text{ m/s}$$

$$U_{rel} = \Delta T_{rel} : \quad Fd = \frac{1}{2} m (v_{rel}^2 - 0)$$

$$F(100) = \frac{1}{2} 7000 (74.4^2)$$

$$\underline{F = 194\,000 \text{ N}} \text{ or } \underline{194.0 \text{ kN}}$$

3/319 $\text{---} x$

For truck, $a_T = -0.9g$, $t_{stop} = \frac{15 \text{ m/s}}{0.9(9.81 \text{ m/s}^2)} = 1.699 \text{ s}$

For crate, $a_c = -0.7g$

$a_{c/T} = a_c - a_T = -0.7g - (-0.9g) = 0.2g$

(As long as truck is moving)

At $t = t_{stop}$,

$$x_{c/T} = (x_{c/T})_0 + (v_{c/T})_0 t_{stop} + \frac{1}{2} a_{c/T} t_{stop}^2$$

$$= 0 + 0 + \frac{1}{2}(0.2g)(1.699)^2 = 2.83 \text{ m}$$

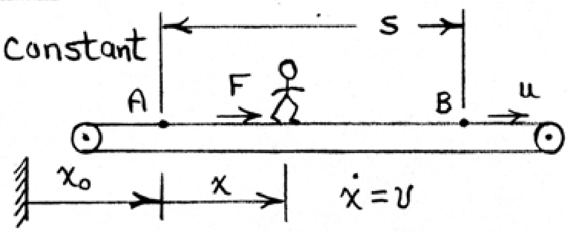
$$v_{c/T} = (v_{c/T})_0 + a_{c/T} t_{stop} = 0 + (0.2g)(1.699) = 3.33 \tfrac{\text{m}}{\text{s}}$$

Then: $v_c^2 = v_{c_0}^2 + 2a_c(x - x_0)$

$$= (3.33)^2 + 2(-0.7g)(3.2 - 2.83)$$

$\underline{v_c = 2.46 \text{ m/s}}$

3/320 $F = \text{constant}$



Absolute: $U = \Delta T$: $F(s + \Delta x_0) = \frac{1}{2}m(u+v)^2 - \frac{1}{2}mu^2$

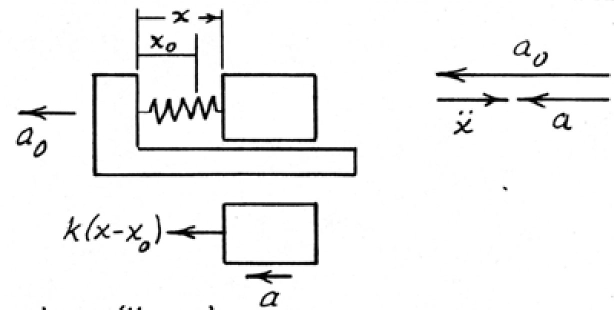
$$Fs + F\Delta x_0 = \frac{1}{2}mv^2 + muv \quad (1)$$

Relative to walkway: $U_{rel} = \Delta T_{rel}$: $Fs = \frac{1}{2}mv^2 - 0$ (2)

Subtract (2) from (1): $F\Delta x_0 = muv$

The term muv represents the work done by force F due only to the movement of the walkway.

3/321



$\Sigma F_x = ma_x: \; -k(x-x_0) = m(\ddot{x}-a_0)$

$\dot{x}\,d\dot{x} = \ddot{x}\,dx$ so $\displaystyle\int_0^{\dot{x}} \dot{x}\,d\dot{x} = \int_{x_0}^{x}\left[a_0 - \frac{k}{m}(x-x_0)\right]dx$

$$\frac{1}{2}\dot{x}^2 = \left(a_0 - \frac{kx_0}{m}\right)(x-x_0) - \frac{k}{2m}(x^2-x_0^2)$$

$\dfrac{d}{dx}\left(\dfrac{\dot{x}^2}{2}\right) = a_0 + \dfrac{kx_0}{m} - \dfrac{kx}{m} = 0$ for max. $\dfrac{\dot{x}^2}{2}$ & hence max $\dot{x}$

so $\dfrac{kx}{m} = a_0 + \dfrac{kx_0}{m}, \quad x = x_0 + \dfrac{ma_0}{k}$

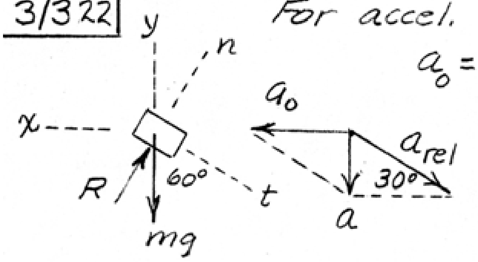
Thus

$$(v_{rel})^2_{max} = \dot{x}^2_{max} = 2\left(a_0 + \frac{kx_0}{m}\right)\left(x_0 + \frac{ma_0}{k} - x_0\right) - \frac{k}{m}\left(x_0^2 + \frac{2ma_0}{k}x_0 + \frac{m^2a_0^2}{k^2} - x_0^2\right)$$

$$= \frac{ma_0^2}{k}$$

$$\underline{(v_{rel})_{max} = a_0\sqrt{m/k}}$$

3/322



For accel. a vertically down,

$a_o = a_{rel} \cos 30°$

$a_x = 0$ so $\Sigma F_x = 0$

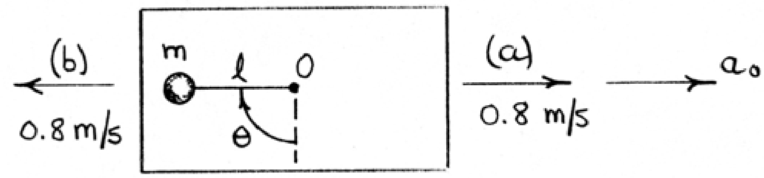
$-R \sin 30° = 0$, $\underline{R = 0}$

$\Sigma F_y = m a_y$, $mg = ma$

$a = g$ & $a_o = a\sqrt{3} = g\sqrt{3}$

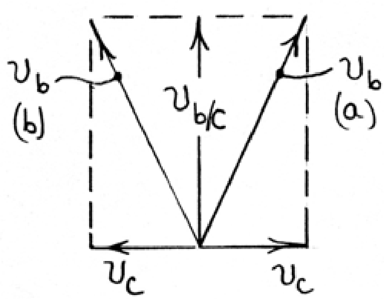
$= 9.81\sqrt{3} = \underline{16.99 \frac{m}{s^2}}$

3/323



$m_b = 10$ kg $\qquad l = 0.8$ m $\qquad \theta = 90°$

$m_c = 250$ kg $\qquad \dot{\theta} = 3$ rad/s



$v_{b/c} = l\dot{\theta} = 0.8(3) = 2.4$ m/s

$T_b = \frac{1}{2} m_b v_b^2 = \frac{1}{2}(10)\left[0.8^2 + 2.4^2\right] = 32$ J

(same for cases (a) and (b))

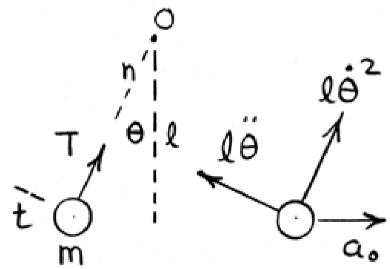
$T_c = \frac{1}{2} m_c v_c^2 = \frac{1}{2}(250)(0.8)^2 = 80$ J

$T = T_b + T_c = 32 + 80 = \underline{112 \text{ J}}$ for both cases

3/324 $\Sigma \underline{F} = m(\underline{a}_o + \underline{a}_{rel})$. In t-dir., $\Sigma F_t = 0$,

so $a_t = l\ddot{\theta} - a_o \cos\theta = 0$

$$\ddot{\theta} = \frac{a_o}{l}\cos\theta \qquad (1)$$



In n-dir., $\Sigma F_n = m a_n$

$$T = m\left(l\dot{\theta}^2 + a_o \sin\theta\right) \quad (2)$$

Integrate (1): $\ddot{\theta} = \dot{\theta}\dfrac{d\dot{\theta}}{d\theta} = \dfrac{a_o}{l}\cos\theta$

$$\int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \int_0^{\theta} \frac{a_o}{l}\cos\theta\, d\theta$$

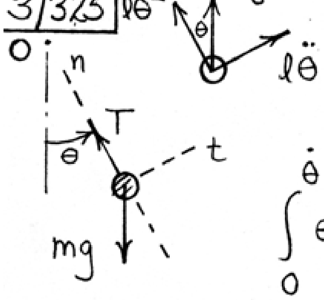
$$\frac{1}{2}\dot{\theta}^2 = \frac{a_o}{l}\sin\theta$$

From (2): $T = m\left[2a_o \sin\theta + a_o \sin\theta\right]$

or $\underline{T = 3ma_o \sin\theta}$

For $\theta = \frac{\pi}{2}$, $T_{\pi/2} = 3ma_o = 3(10)(3) = \underline{90 \text{ N}}$

3/325



$\Sigma F_t = ma_t$:

$$-mg\sin\theta = m(l\ddot{\theta} + a_0 \sin\theta)$$

$$\ddot{\theta} = -\left(\frac{a_0+g}{l}\right)\sin\theta$$

$$\int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = -\int_{\theta_0}^{\theta}\left(\frac{a_0+g}{l}\right)\sin\theta\, d\theta$$

$$\dot{\theta}^2 = 2\left(\frac{a_0+g}{l}\right)(\cos\theta - \cos\theta_0)$$

$\Sigma F_n = ma_n$: $T - mg\cos\theta = m(l\dot{\theta}^2 + a_0\cos\theta)$

$$T = m\left[g(3\cos\theta - 2\cos\theta_0) + a_0(3\cos\theta - 2\cos\theta_0)\right]$$

$$= m(g+a_0)(3\cos\theta - 2\cos\theta_0)$$

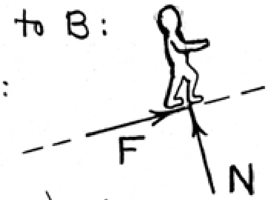
When $\theta = 0$,

$$\underline{T_0 = m(g+a_0)(3 - 2\cos\theta_0)}$$

If $\theta_0 = \pi/2$,

$$\underline{T_0 = 3m(g+a_0)}$$

3/326 For motion from A to B:



Absolute: $U'_{abs} = \Delta T + \Delta V_g$:

$$F(\Delta x_0 + s) = \frac{1}{2}m(v_r+u)^2 - \frac{1}{2}mu^2 + mg(\Delta x_0 + s)\sin\theta$$

$$= \frac{1}{2}mv_r^2 + mv_r u + mg(\Delta x_0 + s)\sin\theta$$

Relative: $U'_{rel} = \Delta T_{rel} + \Delta V_{g_{rel}}$: $Fs = \frac{1}{2}mv_r^2 + mgs\sin\theta$

Work done by walkway: $U'_{abs} - U'_{rel} = mv_r u + mg\Delta x_0 \sin\theta$

$mv_r u$ represents the work done by the belt due only to the motion of the walkway.

For $m = \frac{150}{32.2}$ slugs, $v_r = 2.5$ ft/sec, $u = 2$ ft/sec, $\theta = 10°$, $s = 30$ ft:

$\Sigma F_x = ma_{x_{rel}}$: $a_{x_{rel}} = \frac{v_r^2}{2s} = \frac{2.5^2}{2(30)} = 0.1042 \frac{\text{ft}}{\text{sec}^2}$

$F - 150\sin 10° = \frac{150}{32.2}(0.1042)$, $F = 26.5$ lb

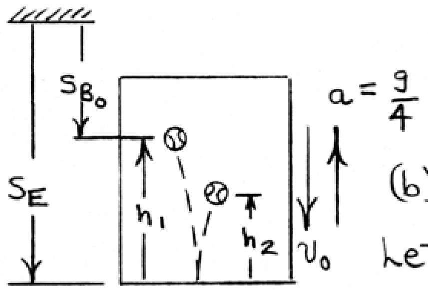
Power by boy: $P_{rel} = Fv_r = 26.5(2.5) = 66.3 \frac{\text{ft-lb}}{\text{sec}}$

or $P_{rel} = 66.3/550 = \underline{0.1206 \text{ hp}}$

▸3/327 (a) $a = 0$, elevator is Newtonian frame

$$v' = ev = e\sqrt{2gh_1} = \sqrt{2gh_2}$$

$$\underline{h_2 = e^2 h_1}$$



(b) $a = \frac{g}{4}$ up

Let B = ball, E = elevator, $\downarrow +$

At impact, $s_B = s_E$: $s_{B_0} + v_{B_0}t + \frac{1}{2}gt^2 = s_{E_0} + v_{E_0}t - \frac{1}{2}\frac{g}{4}t^2$

$$s_{B_0} + v_0 t + \frac{1}{2}gt^2 = (s_{B_0} + h_1) + v_0 t - \frac{1}{8}gt^2, \quad t = 2\sqrt{\frac{2h_1}{5g}}$$

$$v_{B/E} = v_B - v_E = \left(v_0 + g\,2\sqrt{\frac{2h_1}{5g}}\right) - \left(v_0 - \frac{g}{4}2\sqrt{\frac{2h_1}{5g}}\right)$$

$$= \sqrt{\frac{5h_1 g}{2}}$$

After collision, $v'_{B/E_0} = -e\sqrt{\frac{5h_1 g}{2}}$ (up)

$$v'_{B/E} = v'_{B/E_0} + a_{B/E}t = -e\sqrt{\frac{5h_1 g}{2}} + \frac{5}{4}gt$$

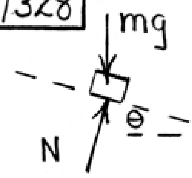
When $v'_{B/E} = 0$, $t = 2e\sqrt{\frac{2h_1}{5g}}$

$$s'_{B/E} = s'_{B/E_0} + v'_{B/E_0}t + \frac{1}{2}\frac{5}{4}gt^2$$

$$= 0 - e\sqrt{\frac{5h_1 g}{2}}\,2e\sqrt{\frac{2h_1}{5g}} + \frac{5}{8}g\,4e^2\frac{2h_1}{5g}$$

$$= -e^2 h_1 \Rightarrow \underline{h_2 = e^2 h_1}$$

►3/328



$$U_{rel} = \Delta T_{rel}$$

$$mg\,l\sin\theta = \frac{1}{2} m v_{rel}^2 - 0$$

$$\underline{v_{rel}^2 = 2gl\sin\theta}$$

$$U = \Delta T: \quad mg\,l\sin\theta + (N\sin\theta)d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

where d is the horizontal distance traveled by the block.

Time to slide from B to C : $l = \frac{1}{2}at^2 = \frac{1}{2}g\sin\theta t^2$

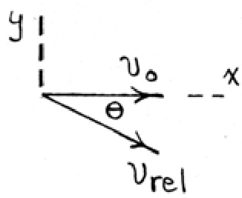
$t = \left(\frac{2l}{g\sin\theta}\right)^{1/2}$. So $d = v_0 t = v_0\sqrt{\frac{2l}{g\sin\theta}}$

Also, $N = mg\cos\theta$

Solving the work-energy equation for v^2:

$$\underline{v_A = \left(v_0^2 + 2gl\sin\theta + 2v_0\cos\theta\sqrt{2lg\sin\theta}\right)^{1/2}}$$

Check :



$$\underline{v}_A = \underline{v}_0 + \underline{v}_{rel}$$

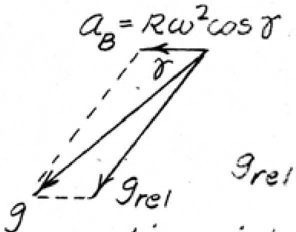
$$= v_0\underline{i} + \sqrt{2gl\sin\theta}\,(\cos\theta\,\underline{i} - \sin\theta\,\underline{j})$$

$$= (v_0 + \sqrt{2gl\sin\theta}\cos\theta)\underline{i} - \sqrt{2gl\sin^3\theta}\,\underline{j}$$

$$v_A^2 = (v_0 + \sqrt{2gl\sin\theta}\cos\theta)^2 + (2gl\sin^3\theta)$$

✓ $v_A^2 = v_0^2 + 2gl\sin\theta + 2v_0\cos\theta\sqrt{2gl\sin\theta}$

▶ 3/329 From law of cosines



$$g_{rel}^2 = g^2 + a_B^2 - 2g\,a_B\cos\gamma$$

$$= g^2\left(1 + \left[\frac{a_B}{g}\right]^2 - 2\,\frac{a_B}{g}\cos\gamma\right)$$

$$g_{rel} = g\left[1 + \frac{a_B}{g}\left(\frac{a_B}{g} - 2\cos\gamma\right)\right]^{1/2}$$

use binomial expansion for 1^{st} two terms

$(1+x)^n = 1 + nx + \cdots$ & get

$$g_{rel} = g\left[1 + \frac{a_B}{g}\left(\frac{a_B}{2g} - \cos\gamma\right) + \cdots\right]$$

$$= g + a_B\left(\frac{a_B}{2g} - \cos\gamma\right) + \cdots$$

$$g_{rel} = g - R\omega^2\cos^2\gamma\left(1 - \frac{R\omega^2}{2g}\right) + \cdots$$

$$R\omega^2 = 6.371(10^6)(0.7292\times10^{-4})^2 = 0.03388 \text{ m/s}^2$$

$$g_{rel} = 9.825 - 0.03388\left(1 - \frac{0.03388}{2\times 9.825}\right)\cos^2\gamma + \cdots$$

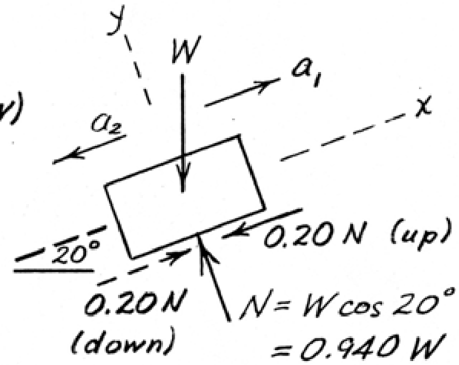
$$= \underline{9.825 - 0.03382\cos^2\gamma} \text{ m/s}^2$$

▸3/330

Case (a): Orbital speed is constant so that $\ddot{x}$ is both the absolute and relative acceleration in the x-direction. Hence $F = m\ddot{x}$ holds.

Case (b): Orbital speed is decreasing in the position shown so that a component of acceleration in the negative x-direction exists so that the true (absolute) acceleration in the x-direction is $\ddot{x}$ minus the tangential orbital deceleration. Consequently $F \neq m\ddot{x}$. Only at the perigee and apogee positions where $\dot{v} = 0$ would $F = m\ddot{x}$ be true.

3/331



(Up) $\Sigma F_x = ma_x: \; -0.20(0.940\,W) - W\sin 20° = \frac{W}{g} a_1$

$a_1 = -17.06$ ft/sec^2

$v^2 = v_1^2 + 2a_1 s:$

$0 = 20^2 + 2(-17.06)s$

$s = 11.72$ ft

(Down) $\Sigma F_x = ma_x: \; -W\sin 20° + 0.20(0.940\,W) = \frac{W}{g}(-a_2)$

$a_2 = -4.96$ ft/sec^2

$v^2 = v_0^2 + 2a_2 s: \; v_2^2 = 0^2 + 2(4.96)(11.72)$

$= 116.3$ (ft/sec)^2

$\underline{v_2 = 10.78 \text{ ft/sec}}$

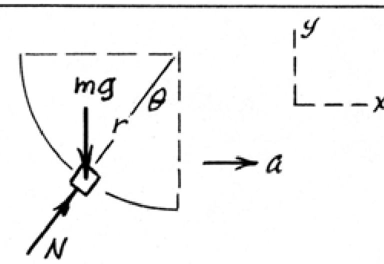
3/332

$\Sigma F_y = 0: N\cos\theta - mg = 0, N\cos\theta = mg$

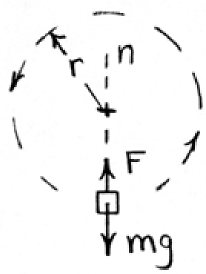
$\Sigma F_x = ma_x: N\sin\theta = ma$

Divide & get $\tan\theta = a/g$

$$\underline{\theta = \tan^{-1}\frac{a}{g}}$$



3/333 Critical condition will occur when weight is at bottom position.



$\Sigma F_n = ma_n: \; F - mg = mr\omega^2$

$80 - 0.030(9.81) = 0.030(0.175)\omega^2$

$\omega = 123.2 \ \frac{rad}{s}$

$N = \omega\left(\frac{60}{2\pi}\right) = \underline{1177 \text{ rev/min}}$

3/334 $v^2 = 2gh = 2(9.81)(0.4 + 0.4\cos 30°)$

$= 14.64\ m^2/s^2$

$\Sigma F_n = ma_n:\ T - 2(9.81)\cos 30° = 2\dfrac{14.64}{0.4}$

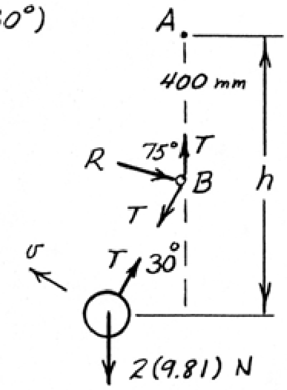
$T = 90.2$ N

Equil. of forces at B:

$R = 2T\cos 75°$

$= 2(90.2)(0.259)$

$= \underline{46.7\text{ N}}$



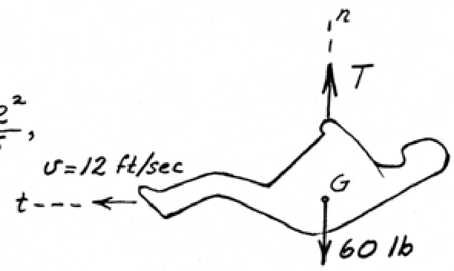
3/335

System:

$\Sigma F_n = m\frac{v^2}{r}$: $T - 60 = \frac{60}{32.2}\frac{12^2}{15}$,

$T = 60(1 + 0.298)$

$= \underline{77.9 \text{ lb}}$

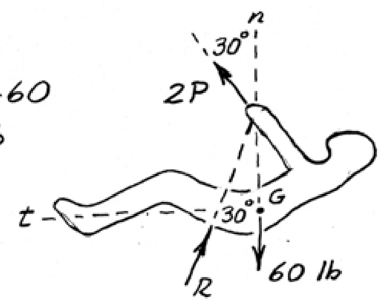


Girl:

$\Sigma F_n = m\frac{v^2}{r}$: $2P\cos 30° + R\cos 30° - 60$

$= \frac{60}{32.2}\frac{12^2}{15} = 17.89 \text{ lb}$

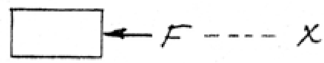
$\Sigma F_t = 0$: $2P\sin 30° - R\sin 30° = 0$



Solve & get

$\underline{P = 22.5 \text{ lb}}$, $\underline{R = 45.0 \text{ lb}}$

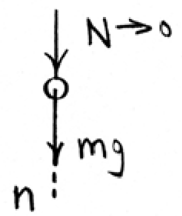
3/336



$$\int \Sigma F_x \, dt = m \Delta v_x$$

$$-\frac{1}{2}(0.4)(8) - \frac{1}{2}(0.4)(10) = 2(v - 4), \quad \underline{v = 2.2 \text{ m/s}}$$

3/337 Dynamics at B (top of loop)



$\Sigma F_n = ma_n: \quad mg = m\dfrac{v_B^2}{R}$

$$v_B^2 = gR$$

Work-kinetic energy from A to B:

$$T_A + U_{A-B} = T_B: \quad 0 + \frac{1}{2}k\delta^2 - mg\mu_k R - mg(2R) = \frac{1}{2}m(gR)$$

$$\delta = \sqrt{\frac{mgR(5+2\mu_k)}{k}}$$

3/338 Possibilities considered $\begin{cases} (a) & 2 \text{ masses with speed } v_1 \\ (b) & 1 \text{ mass with speed } 2v_1 \end{cases}$

Both (a) and (b) conserve system momentum since $2(1mv_1) = 1(2m)v_1$.

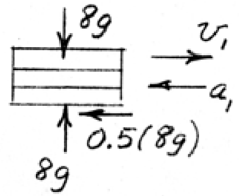
But with $e = 1$, kinetic energy must also be conserved.

Initial : $T = 2(\frac{1}{2}mv_1^2) = mv_1^2$

Final : $\begin{cases} T_a' = 2(\frac{1}{2}mv_1^2) = mv_1^2 \\ T_b' = 1(\frac{1}{2}m(2v_1)^2) = 2mv_1^2 \end{cases}$

So choice (b) is ruled out.

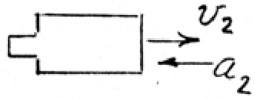
3/339



$F = ma;\quad a_1 = 0.5g$

$v_1 = \sqrt{2a_1 s_1} = \sqrt{2(0.5 \times 9.81) \times 0.8}$

$= 2.80$ m/s



$a_2 = 0.5g$

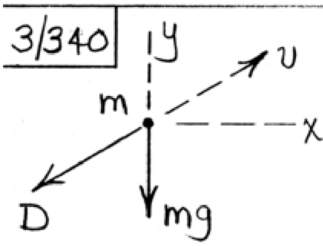
$v_2 = \sqrt{2a_2 s_2} = \sqrt{2(0.5 \times 9.81) \times 1.2}$

$= 3.43$ m/s

$\Delta G = 0;\quad 0.060v + 0 = 8(2.80) + (6 + 0.06)3.43$

$\underline{v = 720 \text{ m/s}}$

3/340



$$\Sigma \underline{F} = m\underline{a} : -C_D \tfrac{1}{2}\rho v^2 S \frac{\underline{v}}{v} - mg\,\underline{j} = m\left(a_x \underline{i} + a_y \underline{j}\right)$$

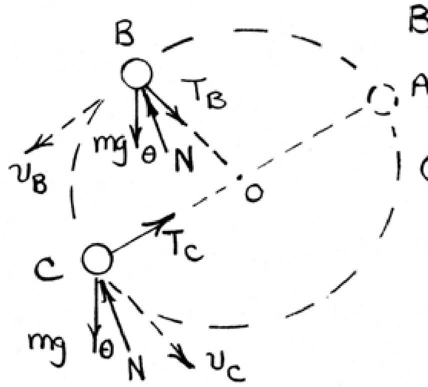
$$-C_D \tfrac{1}{2}\rho v S\left(v_x \underline{i} + v_y \underline{j}\right) - mg\underline{j} = m\left(a_x \underline{i} + a_y \underline{j}\right)$$

$$\text{So} \begin{cases} a_x = -C_D \tfrac{1}{2}\rho S v v_x / m \\ a_y = -C_D \tfrac{1}{2}\rho S v v_y / m - g \end{cases}$$

$$\text{where} \quad v = \sqrt{v_x^2 + v_y^2}$$

The two acceleration expressions are coupled through the speed term. And The expressions are nonlinear.

3/341 $U'_{1-2} = 0$, so $\Delta T + \Delta V_g = 0$



$$B: \tfrac{1}{2}m(v_B^2 - u^2) - mgr\sin\theta = 0$$

$$a_n = \frac{v_B^2}{r} = \frac{u^2}{r} + 2g\sin\theta$$

$$C: \tfrac{1}{2}m(v_C^2 - u^2) - mg(2r\sin\theta) = 0$$

$$a_n = \frac{v_C^2}{r} = \frac{u^2}{r} + 4g\sin\theta$$

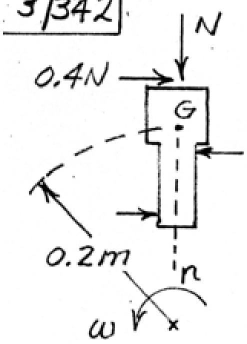
$\Sigma F = ma_n$:

$$B: \underline{T_B = m\left(\frac{u^2}{r} + 2g\sin\theta\right)}$$

$$C: T_C - mg\sin\theta = m\left(\frac{u^2}{r} + 4g\sin\theta\right)$$

$$\underline{T_C = m\left(\frac{u^2}{r} + 5g\sin\theta\right)}$$

3/342

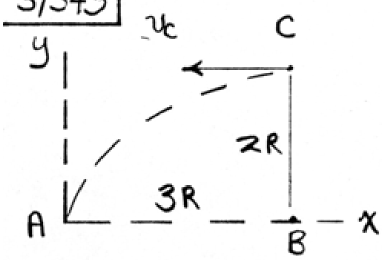


$\omega = \dfrac{3000(2\pi)}{60} = 314.2 \text{ rad/s}$

$\Sigma F_n = ma_n;\quad N = 2(0.2)(314.2)^2$
$= 39.5(10^3) \text{ N}$

$M = 4\mu_k N r = 4(0.4)(39.5)(10^3)(0.3)$
$= \underline{18.96 \text{ kN}\cdot\text{m}}$

3/343



$y_A = y_C + v_{y_C} t - \frac{1}{2} g t^2$:

$0 = 2R + 0\,(t) - \frac{1}{2} g t^2$

$t = 2\sqrt{R/g}$

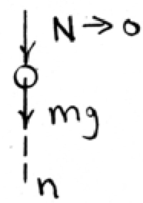
$x_A = x_C + v_{x_C} t$:

$0 = 3R - v_C\, 2\sqrt{R/g}$

$v_C = \frac{3}{2}\sqrt{gR}$

$T_B + V_{B\text{-}C} = T_C$: $\frac{1}{2} m u^2 - mg\,(2R) = \frac{1}{2} m \left[\frac{3}{2}\sqrt{gR}\right]^2$

$\underline{u = \frac{5}{2}\sqrt{gR}}$



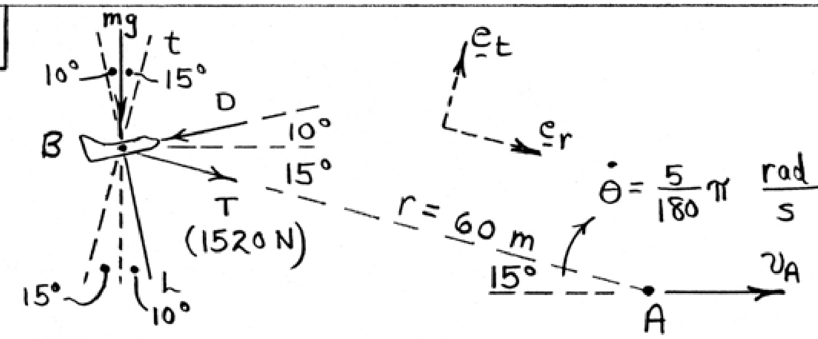
$\Sigma F_n = m a_n$: $mg = m\frac{v_C^2}{R}$, $v_C = \sqrt{gR}$

$x_A = x_C + v_{x_C} t$:

$0 = x_{min} - \sqrt{gR}\; 2\sqrt{R/g}$

$\underline{x_{min} = 2R}$

3/344



$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = \underline{0} + r\dot\theta^2 \underline{e}_r + 0\,\underline{e}_t$$

$$a_B = 60\left[\frac{5\pi}{180}\right]^2 = 0.457 \text{ m/s}^2$$

$$\Sigma F_t = ma_t: \ L\cos 25^\circ - D\sin 25^\circ - 200(9.81)\cos 15^\circ = 0$$

$$\Sigma F_r = ma_r: \ 1520 + 200(9.81)\sin 15^\circ - L\sin 25^\circ - D\cos 25^\circ = 200(0.457)$$

Solve the above two equations to obtain

$$\underline{D = 954 \text{ N}}$$

$$\underline{L = 2540 \text{ N}}$$

3/345 Velocity of plug at bottom is

$$\sqrt{2gh} = \sqrt{2(32.2)6} = 19.66 \text{ ft/sec}$$

$$\Delta G = 0; \quad \frac{2(19.66)}{g} - \frac{(2+4)v}{g} = 0, \quad \underline{v = 6.55 \text{ ft/sec}}$$

$$\Delta T + \Delta V_e = 0; \quad \frac{1}{2}\frac{6}{32.2}(0 - \overline{6.55}^2) + \frac{1}{2}80(x^2 - 0) = 0$$

$$x^2 = 0.100 \text{ ft}^2, \quad \underline{x = 0.316 \text{ ft}}$$

$$n = \frac{\Delta T}{T}; \quad n = \left[\frac{1}{2}\frac{2}{g}(19.66)^2 - \frac{1}{2}\frac{6}{g}(6.55)^2\right] \Big/ \frac{1}{2}\frac{2}{g}(19.66)^2$$

$$= 1 - \frac{6}{2}\left(\frac{6.55}{19.66}\right)^2 = 1 - 3(0.1111) = \underline{0.667}$$

3/346 The method of work-energy cannot handle forces which are functions of time; the impulse-momentum method cannot accept forces which vary with displacement. Newton's Second Law gives the acceleration as

$$a = -\frac{k}{m}x + \frac{F(t)}{m}$$

which is not easily integrated by standard (non-numerical) methods.

3/347 Final Skidding : $U_{1-2} = \Delta T$

(Prime denotes speed after impact) $-\mu_k mgd = 0 - \frac{1}{2}mv'^2$

$v' = \sqrt{2\mu_k g d}$

A : $v_A' = \sqrt{2(0.9)(32.2)(50)} = 53.8$ ft/sec

B : $v_B' = \sqrt{2(0.9)(32.2)(100)} = 76.1$ ft/sec

Collision : $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$$\frac{4000}{g} v_A + 0 = \frac{4000}{g}(53.8) + \frac{2000}{g}(76.1)$$

$$v_A = 91.9 \text{ ft/sec}$$

Initial skidding : $U_{1-2} = \Delta T$

$-\mu_k \cancel{m} g d = \frac{1}{2}\cancel{m}(v_A^2 - v_{A_0}^2)$

$-(0.9)(32.2)(50) = \frac{1}{2}(91.9^2 - v_{A_0}^2)$, $v_{A_0} = 106.5 \frac{ft}{sec}$

(Speed limit was exceeded!) $= 72.6$ mi/hr

3/348 For the system of man and cord for full fall

(a) $U'_{1-2} = 0 = \Delta V_g + \Delta V_e: \; 0 = 80(9.81)(-44) + \frac{1}{2}k(44-20)^2,$

$k = 119.9$ N/m

(b) $U'_{1-2} = 0 = \Delta T + \Delta V_g + \Delta V_e: \; 0 = \frac{1}{2}80v^2 - 80(9.81)(20+y) + \frac{1}{2}119.9\,y^2$

where y = elongation of bungee cord.

$$40\frac{d(v^2)}{dy} = 80(9.81) - 119.9y = 0 \text{ for max } v^2, \; y = 6.55 \text{ m}$$

$$\& \; v^2_{max} = \frac{1}{40}\left\{80(9.81)(20+6.55) - \frac{1}{2}119.9(6.55)^2\right\} = 457 \text{ m}^2/\text{s}^2$$

$$\underline{v_{max} = 21.4 \text{ m/s}}$$

(c) Max. acceleration occurs at bottom where tension is greatest

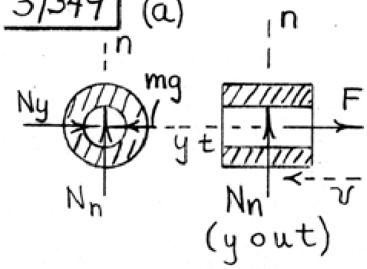
$T_{max} = Ky = 119.9(44-20) = 2880$ N

$\uparrow \Sigma F_y = ma_{max}: \; 2880 - 80(9.81) = 80\,a_{max}$

$$\underline{a_{max} = 26.2 \text{ m/s}^2 \text{ or } \tfrac{8}{3}g}$$

80(9.81) N

3/349 (a)



$\Sigma F_y = 0: \quad N_y = mg$

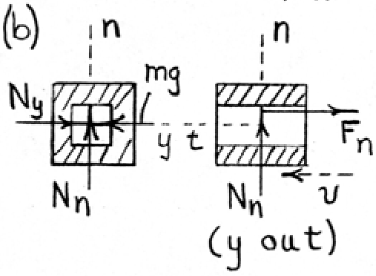
$\Sigma F_n = m\frac{v^2}{\rho}: \quad N_n = m\frac{v^2}{\rho}$

$N_{tot} = \sqrt{N_y^2 + N_n^2} = m\sqrt{g^2 + \frac{v^4}{\rho^2}}$

$F = \mu_k N_{tot} = \mu_k m\sqrt{g^2 + \frac{v^4}{\rho^2}}$

$\Sigma F_t = ma_t: \quad -\mu_k \cancel{m}\sqrt{g^2 + \frac{v^4}{\rho^2}} = \cancel{m}a_t, \quad \underline{a_t = -10.75 \frac{m}{s^2}}$

(b)



As in part (a), $N_y = mg$

and $N_n = m\frac{v^2}{\rho}$.

But $F_y = \mu_k N_y = \mu_k mg$

and $F_n = \mu_k N_n = \mu_k m\frac{v^2}{\rho}$.

$a_t = -\frac{F_y + F_n}{m} = -\mu_k g - \mu_k\frac{v^2}{\rho} = \underline{-14.89 \text{ m/s}^2}$

3/350 (Roman numeral: process; Arabic number: state)

I. Engine moves 1 ft: $U = \Delta T$: $Fd = \frac{1}{2} m (v_2^2 - v_1^2)$

(State ① → State ②) $40,000(1) = \frac{1}{2} \frac{400,000}{32.2} (v_2^2 - 0^2)$

$v_2 = 2.54$ ft/sec

II. "Collision" with A: $m_L v_2 = (m_L + m_A) v_3$

(② → ③) $400,000(2.54) = 600,000 v_3$, $v_3 = 1.692 \frac{\text{ft}}{\text{sec}}$

III. L & A move 1 ft: $40,000(1) = \frac{1}{2} \frac{600,000}{32.2} (v_4^2 - 1.692^2)$

(③ → ④)

$v_4 = 2.67$ ft/sec

IV. "Collision" with B: $(m_L + m_A) v_4 = (m_L + m_A + m_B) v_5$

(④ → ⑤) $600,000(2.67) = 800,000 v_5$

$v_5 = 2.01$ ft/sec

V. L, A, & B move 1 ft: $40,000(1) = \frac{800,000}{32.2} (v_6^2 - 2.01^2)$

(⑤ → ⑥)

$v_6 = 2.69$ ft/sec

VI. "Collision" with C: (⑥ → ⑦)

$$(m_L + m_A + m_B) v_6 = (m_L + m_A + m_B + m_C) v_7$$

$$800,000(2.69) = 1,000,000 v_7$$

(a) $\underline{v_7 = 2.15 \text{ ft/sec} = v}$

With no slack,

$U = \Delta T$: $40,000(3) = \frac{1}{2} \frac{10^6}{32.2} (v'^2 - 0^2)$

(b) $\underline{v' = 2.78 \text{ ft/sec}}$

3/351 D to E : $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$$-\rho = -\frac{1}{2} g t^2 \,, \quad t = \sqrt{\frac{2\rho}{g}}$$

$x = x_0 + v_{x_0} t$: $\quad d = v_D \sqrt{\frac{2\rho}{g}} \,, \quad v_D = d\sqrt{\frac{g}{2\rho}}$

A to D : $\quad U = \Delta T$

$$\frac{1}{2} k \delta^2 - \mu_k m g \rho - m g \rho = \frac{1}{2} m \left(d^2 \frac{g}{2\rho} \right) - 0$$

$$\underline{\delta = \sqrt{\frac{mg}{k}} \sqrt{\frac{d^2}{2\rho} + 2\rho(1+\mu_k)}}$$

But speed at top of hill must be ≥ 0:

$U = \Delta T$: $\quad \frac{1}{2} k \delta^2 - \mu_k m g \rho - 3 m g \rho = \frac{1}{2} m v^2 - 0 \geq 0$

$$\text{or} \quad \delta \geq \sqrt{\frac{2 m g \rho}{k} (3 + \mu_k)}$$

$$\therefore \quad \frac{mg}{k} \left(\frac{d^2}{2\rho} + 2\rho[1+\mu_k] \right) \geq \frac{2 m g \rho}{k} (3 + \mu_k)$$

or $\quad \underline{d \geq 2\sqrt{2}\, \rho}$

3/352 For a minimum escape orbit (parabolic) $e = 1$ & $a \to \infty$

so from Eq. 3/47

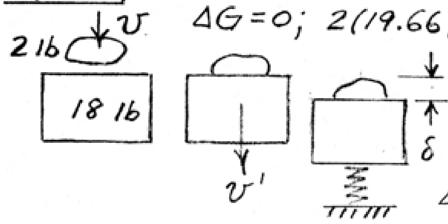
$$v_{esc} = R\sqrt{\frac{2g}{R+H}} = 6371\sqrt{\frac{2 \times 9.825 \times 10^{-3}}{6371 + 2000}} \times 3600$$

$$= 35\ 140 \text{ km/h}$$

Thus $\Delta v = 35140 - 26140 = \underline{9000 \text{ km/h}}$

3/353 $v=\sqrt{2gh}=\sqrt{2(32.2)(6)}=19.66$ ft/sec

$\Delta G=0;\ 2(19.66)+0=(18+2)v',\quad v'=1.966\ \frac{\text{ft}}{\text{sec}}$



Initial spring deflection

$\delta_o = W/(2k) = \frac{18}{2(3)} = 3$ in.

$\Delta T+\Delta V_g+\Delta V_e=0$

$\Delta T = 0-\frac{1}{2}\frac{20}{32.2}(1.966)^2=-1.200$ ft-lb

$\Delta V_g=-20\ \delta/12=-1.667\delta$ ft-lb where δ is in inches

$\Delta V_e=\frac{1}{2}(2)(3)\left[(3+\delta)^2-3^2\right]\frac{1}{12}=\frac{(3+\delta)^2-9}{4}$ ft-lb

Thus $-1.200-1.667\delta+\frac{(3+\delta)^2-9}{4}=0$

or $\delta^2-0.667\delta-4.800=0$

$\delta=\frac{0.667}{2}\pm\frac{1}{2}\sqrt{0.444+19.20}=0.333\pm2.216$

$\underline{\delta=2.55\text{ in.}}$ (or $\delta=-1.88$ in.)

3/354 $\xrightarrow{+}$ $G_1 = G_2$

$$\cancel{m}v_A + \cancel{m}v_B = 2\cancel{m}v'$$

$$3 - 5 = 2v', \quad \underline{v' = -1 \text{ m/s} \quad (\text{left})}$$

$$H_{G_1} = H_{G_2} : \curvearrowleft^{+} \; \cancel{m}\cancel{r}v_A + \cancel{m}\cancel{r}v_B = 2\cancel{m}r^{\cancel{2}}\dot{\theta}'$$

$$\dot{\theta}' = \frac{v_A + v_B}{2r} = \frac{3+5}{2(0.4)}$$

$$= \underline{10 \text{ rad/s (CCW)}}$$

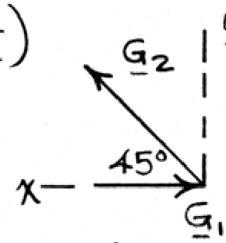
$\boxed{3/355}$ From S.P. 2/6, $\quad 2s = \dfrac{v^2 \sin 2\theta}{g}$

$$350 = \frac{v^2 \sin 90^\circ}{32.2}, \qquad v = 106.2 \text{ ft/sec}$$

$$\underline{G}_1 = m\underline{v}_1 = \frac{5/16}{32.2}\left(90\,\frac{5280}{3600}\right)(-\underline{i}) = -1.281\,\underline{i} \text{ lb-sec}$$

$$\underline{G}_2 = m\underline{v} = \frac{5/16}{32.2}\,106.2\left(\frac{\underline{i}}{\sqrt{2}} + \frac{\underline{j}}{\sqrt{2}}\right)$$

$$= 0.729\,(\underline{i} + \underline{j}) \text{ lb-sec}$$



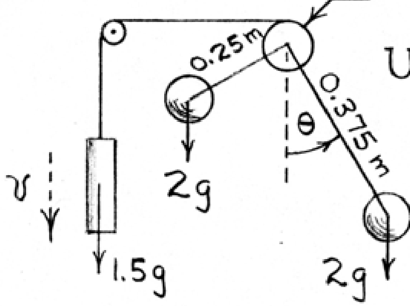
$$\underline{F}_{av}\,\Delta t = \underline{G}_2 - \underline{G}_1 : \underline{F}_{av}(0.005) = 0.729(\underline{i}+\underline{j}) - (-1.281\,\underline{i})$$

$$\underline{F}_{av} = 402\,\underline{i} + 145.7\,\underline{j} \text{ lb}$$

$$F_{av} = \sqrt{402^2 + 145.7^2} = \underline{428 \text{ lb}}$$

(Note: The weight of the baseball is ignored during its impact with the bat. With the weight included, F_{av} still rounds to 428 lb!)

3/356



Work done during angular displacement θ is

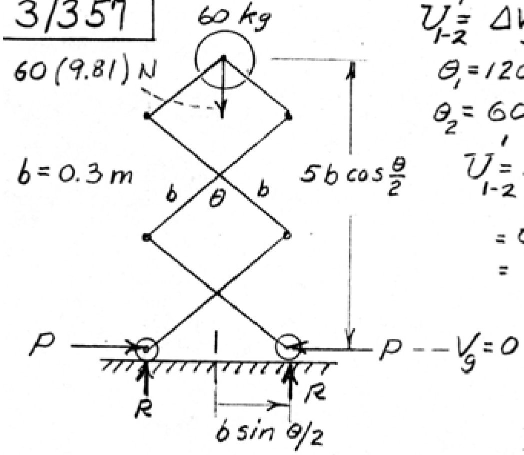
$$U = 1.5g\,(0.025\,\theta) + 2g\,(0.25\sin\theta) - 2g\,(0.375)\,(1-\cos\theta)$$

For $\theta = 30°$, $U = 1.659$ J

$$\Delta T = \frac{1}{2}\left[1.5\,v^2 + 2\left(\frac{0.25}{0.025}v\right)^2 + 2\left(\frac{0.375}{0.025}v\right)^2\right]$$

$= 325.8\,v^2$. $U_{1\text{-}2} = \Delta T$ yields $\underline{0.0714\ \text{m/s}\ (71.4\ \frac{\text{mm}}{\text{s}})}$

3/357



$U'_{1-2} = \Delta V_g + \Delta T$

$\theta_1 = 120°, \quad v_1 = 0$

$\theta_2 = 60°, \quad v_2 = 3 \text{ m/s}$

$U'_{1-2} = 2Pb\left(\sin\frac{\theta_1}{2} - \sin\frac{\theta_2}{2}\right)$

$= 0.6P(\sin 60° - \sin 30°)$

$= 0.2196P \quad J$

$\Delta V_g = mg\,\Delta h$

$= mg(5b)\left(\cos\frac{\theta_2}{2} - \cos\frac{\theta_1}{2}\right)$

$= 60(9.81)(1.5)(\cos 30° - \cos 60°)$

$= 323.2 \quad J$

$\Delta T = \frac{1}{2}60(3)^2 - 0 = 270 \text{ J}$

Thus $0.2196P = 323.2 + 270$, $\quad P = \dfrac{323.2 + 270}{0.2196} = 2701 \text{ N}$

or $\underline{P = 2.70 \text{ kN}}$

For complete system

$\Sigma F = ma; \quad 2R - 60(9.81) = 60(20)$

$\underline{R = 894 \text{ N}}$

3/358 Drop of A (state ① → state ②) :

$T_1 + U_{1-2} = T_2$: $0 + m_A g\, 1.8(1-\cos 60^\circ) = \frac{1}{2} m_A v_{A_2}^2$

$v_{A_2} = 4.20$ m/s

Collision (② → ③):

$$\begin{cases} m_A v_{A_2} + m_B \cancel{v_{B_2}}^{\,0} = m_A v_{A_3} + m_B v_{B_3} & (1) \\ v_{B_3} - v_{A_3} = 0.7(v_{A_2} - \cancel{v_{B_2}}^{\,0}) & (2) \end{cases}$$

Solution : $v_{A_3} = 2.42$ m/s, $v_{B_3} = 5.36$ m/s

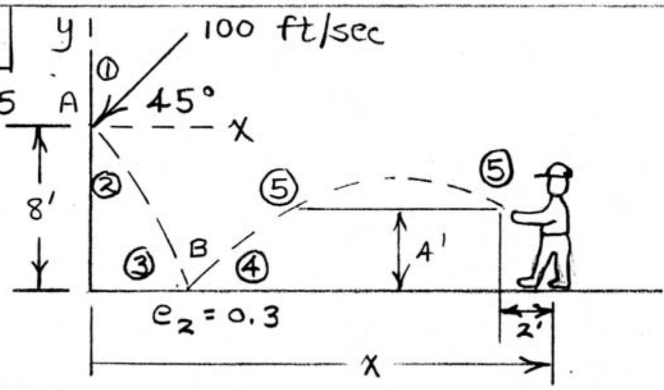
Rise of B (③ → ④) :

$T_3 + U_{3-4} = T_4$:

$\frac{1}{2} m_B (5.36)^2 - m_B (9.81)[2.4(1-\cos 30^\circ) + s \sin 30^\circ] = 0$

$\underline{s = 2.28 \text{ m}}$

3/359



Use coordinates & states ① - ⑤ shown.

$v_{1x} = -100 \cos 45° = -70.7$ ft/sec

$v_{1y} = -100 \sin 45° = -70.7$ ft/sec

$v_{2x} = -e_1 v_{1x} = -0.5(-70.7) = 35.4$ ft/sec

$v_{2y} = v_{1y} = -70.7$ ft/sec

$v_{3x} = v_{2x} = 35.4$ ft/sec

$v_{3y} = -\sqrt{v_{2y}^2 + 2g(8)} = -\sqrt{70.7^2 + 2(32.2)(8)} = -74.3 \frac{ft}{sec}$

$v_{3y} = v_{2y} - gt_3 : -74.3 = -70.7 - 32.2\, t_3, \quad t_3 = 0.1104$ sec

$v_{4x} = v_{3x} = 35.4$ ft/sec

$v_{4y} = -e_2 v_{3y} = -0.3(-74.3) = 22.3$ ft/sec

$y_5 = y_4 + v_{y_4} t_5 - \frac{1}{2} g t_5^2 :$

$-4 = -8 + 22.3\, t_5 - 16.1\, t_5^2 : \quad t_5 = 0.212, \; 1.172$ sec

Then $x = x_3 + v_{4x} t_5 + 2$, where $x_3 = v_{2x} t_3$

$= 35.4(0.1104) = 3.73$ ft

Thus $x = 3.73 + 35.4(0.212) + 2 = \underline{13.40 \text{ ft}}$

or $x = 3.73 + 35.4(1.172) + 2 = \underline{47.3 \text{ ft}}$

3/360 Results of Prob. 3/309:

$$\Delta v_A = R\sqrt{\frac{g}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)$$

Nominally,

$$(\Delta v_A)_n = (3959)(5280)\sqrt{\frac{32.23}{(3959+170)(5280)}} \times$$

$$\left(\sqrt{\frac{2(3959+22{,}300)}{(3959+170)+(3959+22{,}300)}} - 1\right) = 7997 \ \frac{\text{ft}}{\text{sec}}$$

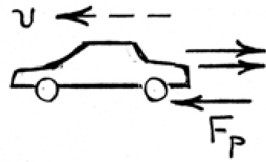
Actually,

$$(\Delta v_A)_a = (3959)(5280)\sqrt{\frac{32.23}{(3959+170)(5280)}} \times$$

$$\left(\sqrt{\frac{2(3959+700)}{(3959+170)+(3959+700)}} - 1\right) = 755 \ \frac{\text{ft}}{\text{sec}}$$

$$\frac{(\Delta v_A)_a}{(\Delta v_A)_n} = \frac{t'}{t}, \quad t' = \frac{(\Delta v_A)_a}{(\Delta v_A)_n}\, t = \frac{755}{7997}(90) = \underline{8.50 \text{ sec}}$$

▶3/361



$F_D = C_D\left(\frac{1}{2}\rho v^2\right)S$

$F_R = 200$ lb

$\overset{+}{\leftarrow}\Sigma F = 0: \quad F_P = F_D + F_R$

Undamaged: $F_P = 0.3\left[\frac{1}{2}\,\frac{0.07530}{32.2}\left(200\cdot\frac{5280}{3600}\right)^2\right]30$

$+ 200 = 1105$ lb

Power required: $P = F_P \cdot v = 1105\cdot\left(200\,\frac{5280}{3600}\right)$

$= 324(10^3)$ ft-lb/sec

Damaged (power available is unchanged)

$P = F_P' \cdot v': \; 324(10^3) = \left[0.4\left(\frac{1}{2}\,\frac{0.07530}{32.2}\,v'^2\right)30\right.$

$\left. + 200\right]v'$

Solve cubic: $v' = 268$ ft/sec or $\underline{182.9 \text{ mi/hr}}$

$\blacktriangleright$ 3/362 $\begin{cases} F_R = -k_1 v \,, & k_1 = 0.833 \ \frac{\text{lb-hr}}{\text{mi}} = 0.5682 \ \frac{\text{lb-sec}}{\text{ft}} \\ F_D = -k_2 v^2 , & k_2 = 0.0139 \ \frac{\text{lb-hr}^2}{\text{mi}^2} = 0.006457 \ \frac{\text{lb-sec}^2}{\text{ft}^2} \end{cases}$

(a) $P_{30} = Fv = \left[0.833(30) + 0.0139(30)^2\right]\left[30\left(\frac{5280}{3600}\right)\right]$

$= 1650 \ \frac{\text{ft-lb}}{\text{sec}} = \underline{3 \text{ hp}}$

$P_{60} = Fv = \left[0.833(60) + 0.0139(60)^2\right]\left[60\left(\frac{5280}{3600}\right)\right]$

$= 8800 \ \frac{\text{ft-lb}}{\text{sec}} = \underline{16 \text{ hp}}$

(b) $-k_1 v - k_2 v^2 = m \frac{dv}{dt}$

$$\int_0^t dt = -m \int_{v_1}^{v_2} \frac{dv}{v(k_1 + k_2 v)}$$

$$t = -\frac{m}{k_1} \ln\left[\frac{v_2 (k_1 + k_2 v_1)}{v_1 (k_1 + k_2 v_2)}\right]$$

$$t = -\frac{2000/32.2}{0.5682} \ln\left[\frac{7.33(0.5682 + 0.006457(88))}{88\,(0.5682 + 0.006457(7.33))}\right]$$

$= \underline{205 \text{ sec}}$

$-k_1 v - k_2 v^2 = m v \frac{dv}{ds}$

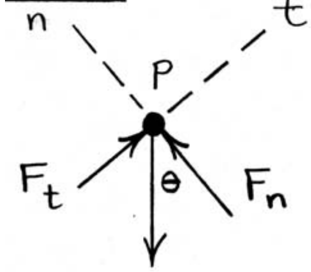
$$\int_0^s ds = -m \int_{v_1}^{v_2} \frac{dv}{k_1 + k_2 v}$$

$$s = -\frac{m}{k_2} \ln(k_1 + k_2 v)\Big|_{v_1}^{v_2}$$

$$= -\frac{m}{k_2} \ln\left[\frac{k_1 + k_2 v_2}{k_1 + k_2 v_1}\right] = \underline{5898 \text{ ft}}$$

*3/363

$\theta = t^2, \quad \dot{\theta} = 2t, \quad \ddot{\theta} = 2 \text{ rad/s}^2$



$0.4(9.81) = 3.924 \text{ N}$

$\Sigma F_n = ma_n: \quad F_n - 3.924 \cos t^2 = 0.4(1.5)(2t)^2$

$F_n = 3.924 \cos t^2 + 2.4 t^2 \quad (\text{N})$

$\Sigma F_t = ma_t: \quad F_t - 3.924 \sin t^2 = 0.4(1.5)(2)$

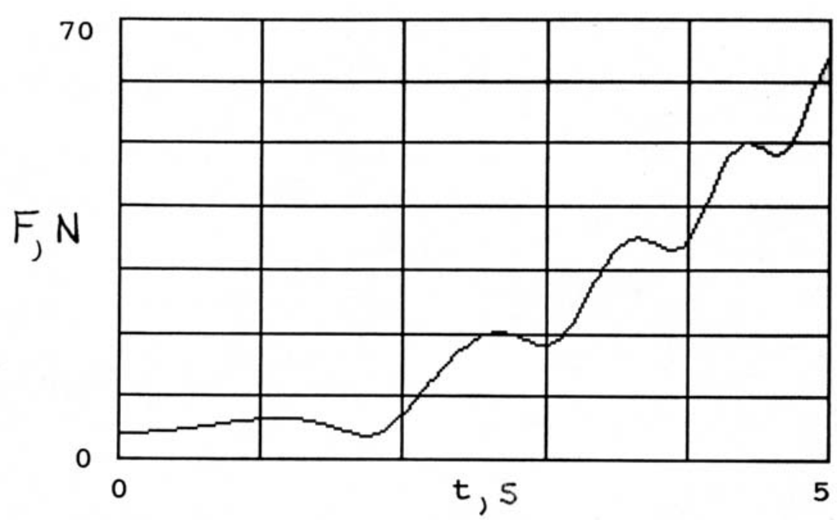
$F_t = 3.924 \sin t^2 + 1.2 \quad (\text{N})$

$F = \sqrt{F_n^2 + F_t^2}$; simplify to

$F = \sqrt{16.84 + 18.84 t^2 \cos t^2 + 9.42 \sin t^2 + 5.76 t^4}$

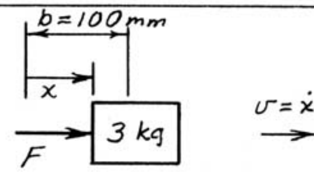
When $F = 30 \text{ N}$, $\underline{t = 3.40 \text{ s}}$ (numerically)

$\theta = 3.40^2 = \underline{11.57 \text{ rad}}$ or $\theta = \underline{663°}$



$\boxed{*3/364}$ Power $P = Fv$ where $F = k(b-x)$

$$F = 1.8(10^3)(0.1-x) \text{ N}$$



$$U = \Delta T: \quad U = \int_0^x F\,dx = \int_0^x 1.8(10^3)(0.1-x)\,dx$$

$$= 1.8(10^3)\left(0.1x - \frac{x^2}{2}\right) \text{ J}$$

$$\Delta T = \frac{1}{2}mv^2 - 0 = \frac{1}{2}3v^2 \text{ J}$$

Thus $v^2 = \frac{2}{3}1.8(10^3)\left(0.1x - \frac{x^2}{2}\right)$

$$P^2 = F^2v^2 = (1.8)^2(10^6)(0.1-x)^2 \times \frac{2}{3}1.8(10^3)\left(0.1x - \frac{x^2}{2}\right)$$

$$= 3.89(10^9)(0.1-x)^2\left(0.1x - \frac{x^2}{2}\right) \text{ watts}^2 \text{ with } x \text{ in meters}$$

$$\frac{d(P^2)}{dx} = 3.89(10^9)\left\{2(0.1-x)(-1)\left(0.1x - \frac{x^2}{2}\right) + (0.1-x)^2(0.1-x)\right\}$$

$$= 3.89(10^9)(-0.2x + x^2 + 0.01 - 0.2x + x^2)(0.1-x)$$

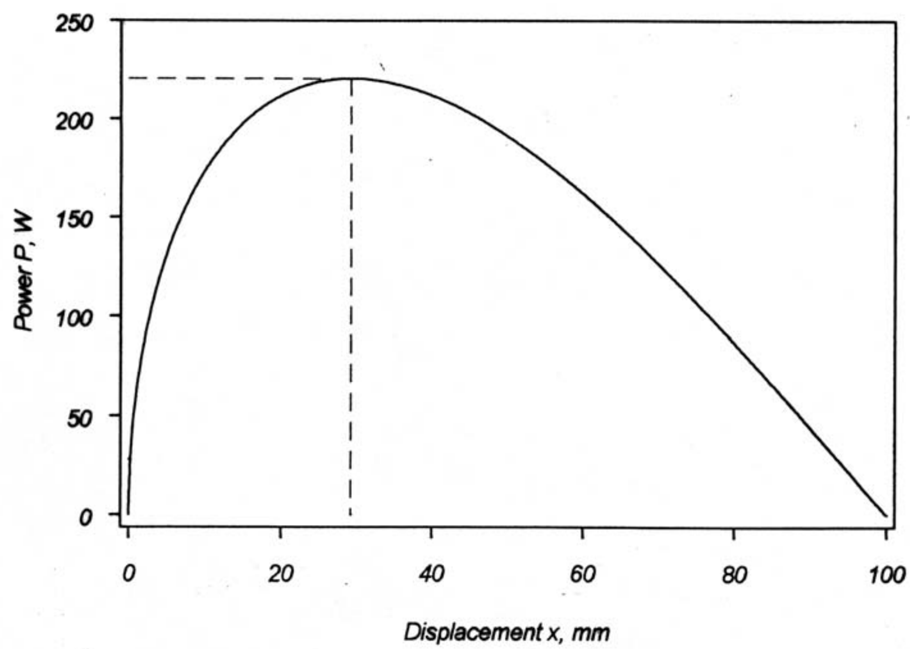
$$= 3.89(10^9)(0.1-x)(x^2 - 0.2x + 0.005) \times 2 = 0 \text{ for max or min}$$

so $x = 0.1$ or $x = 0.1 \pm 0.0707 = 0.1707$ m or $\underline{x = 0.0293 \text{ m}}$

$$\underline{P = \sqrt{38.9(10^8)}\,(0.1-x)\sqrt{0.1x - 0.5x^2} \text{ W}}$$

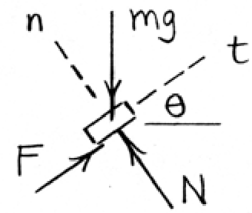
Substitute x from above & get

$\underline{P_{max} = 220 \text{ W}}$



***3/365**

$$a_n = r\Omega^2 = \frac{13}{12}(7.5)^2 = 60.9\ \frac{ft}{sec^2}$$



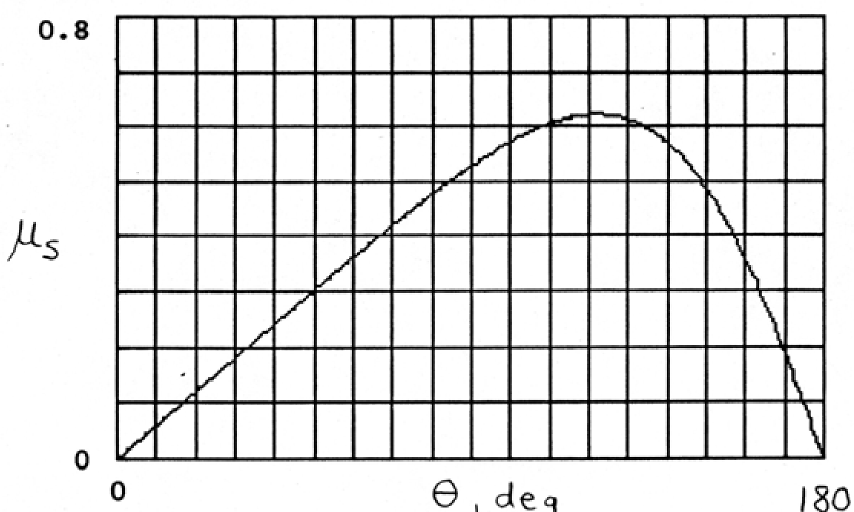
$$\begin{cases} \Sigma F_n = ma_n : N - mg\cos\theta = 60.9\,m \\ \Sigma F_t = ma_t : F - mg\sin\theta = 0 \end{cases}$$

Slipping impends when $F = F_{max} = \mu_s N$

Simultaneous solution : $\mu_s = \dfrac{32.2\sin\theta}{60.9 + 32.2\cos\theta}$

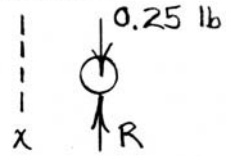
See plot of μ_s vs. θ below. Set $\dfrac{d\mu_s}{d\theta} = 0$ or numerically determine that

$\underline{\mu_{min} = 0.622}$ @ $\underline{\theta = 121.9°}$



Note that it is impossible to see slippage <u>first</u> occur at any angle greater than $\theta = 121.9°$!

***3/366**



$\Sigma F_x = ma_x: \quad 0.25 - R = \dfrac{0.25}{32.2} a$

$(0.25 - R)\,dx = \dfrac{0.25}{32.2} a\,dx$

But $a\,dx = v\,dv$, so

$(0.25 - R)\,dx = \dfrac{0.25}{32.2} v\,dv = 0.00388\, d(v^2)$

For small intervals : $(0.25 - R)\,\Delta x = 0.00388\,\Delta(v^2)$

or $\Delta(v^2) = (64.4 - 258R)\,\Delta x$

Set up program to produce the following table :

x ft	Δx ft	R lb	$64.4-258R$ ft/sec²	Δv^2 (ft/sec)²	v^2 (ft/sec)²	v ft/sec
0		0	64.4		0	0
	1			64.4		
1		0.04	54.1		64.4	8.02
	1			54.1		
2		0.08	43.7		118.5	10.9
	1			43.7		
3		0.13	30.9		162.2	12.7
	1			30.9		
4		0.16	23.1		193.1	13.9
	1			23.1		
5		0.19	15.4		216.2	14.7
	1			15.4		
6		0.21	10.2		231.6	15.2
	1			10.2		
7		0.22	7.6		241.8	15.6
	1			7.6		
8		0.23	5.1		249.4	15.8
	1			5.1		
9		0.25	0		254.5	16.0
	1			0		
10					254.5	16.0

So $\underline{v = 16.0 \text{ ft/sec}}$.

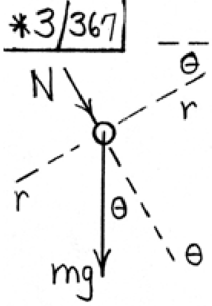
For $R = kv^2 \qquad W - kv^2 = \dfrac{W}{g} a$

$$\int_0^x \frac{g}{W}\,dx = \int_0^v \frac{v\,dv}{W - kv^2}$$

$$\Rightarrow v = \sqrt{\frac{W}{k}\left(1 - e^{-2gkx/W}\right)}$$

With numbers, $\underline{v = 16.3 \text{ ft/sec}}$

***3/367**



$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$:

$mg \sin\theta = m(\ddot{r} - r\omega_0^2)$

$\ddot{r} - \omega_0^2 r = g \sin \omega_0 t$

Assume $r_h = Ce^{st}$ to obtain $s_1 = -\omega_0$, $s_2 = \omega_0$. Assume particular solution of form $r_p = D \sin \omega_0 t$ and find $D = -\frac{g}{2\omega_0^2}$. So

$$r = r_h + r_p = C_1 e^{-\omega_0 t} + C_2 e^{\omega_0 t} - \frac{g}{2\omega_0^2} \sin \omega_0 t$$

Use the initial conditions $r(0) = \dot{r}(0) = 0$ to find C_1 and C_2, allowing us to write the solution as

$$r = \frac{g}{4\omega_0^2}\left(-e^{-\theta} + e^{\theta} - 2\sin\theta\right)$$

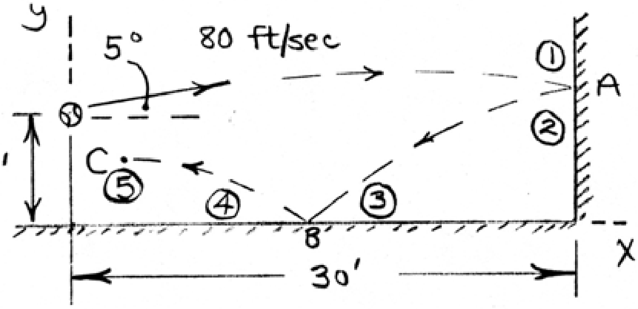
Now, set $r = 1$ m and $\omega_0 = 0.5$ rad/s and use Newton's method to solve for θ as $\theta = 0.535$ rad, or $\underline{30.6°}$. From $\theta = \omega_0 t$, $t = \frac{0.535}{0.5} = \underline{1.069 \text{ s}}$.

***3/368**

Define states

①-⑤ as shown.

$\begin{cases} v_0 = 80 \text{ ft/sec} \\ \theta = 5° \end{cases}$



$v_{0_x} = v_0 \cos\theta = 80 \cos 5° = 79.7 \text{ ft/sec}$

$v_{0_y} = v_0 \sin\theta = 80 \sin 5° = 6.97 \text{ ft/sec}$

$t_{01} = \frac{30}{79.7} = 0.376 \text{ sec}$

$y_1 = 3 + 6.97(0.376) - 16.1(0.376)^2 = 3.34 \text{ ft}$

$v_{1x} = v_{0x} = 79.7 \text{ ft/sec}$

$v_{1y} = v_{0y} - g t_{01} = 6.97 - 32.2(0.376) = -5.15 \text{ ft/sec}$

Now program the following numbered equations:

$v_{2x} = -e v_{1x}$ (1)

$v_{2y} = v_{1y}$ (2)

$v_{3x} = v_{2x}$ (3)

$v_{3y} = -\sqrt{v_{2y}^2 + 2 g y_1}$ (4)

$t_{23} = (v_{2y} - v_{3y})/g$ (5)

$$x_3 = 30 + v_{2x} t_{23} \tag{6}$$

$$v_{4x} = v_{3x} \tag{7}$$

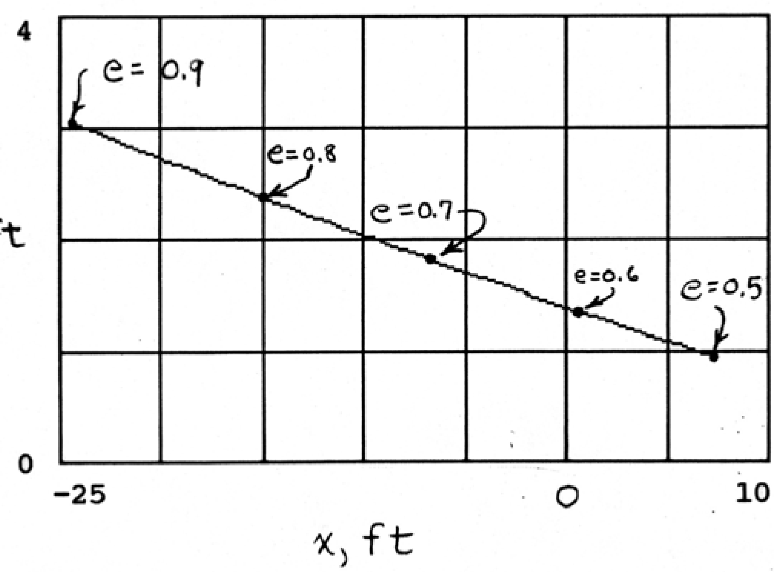
$$v_{4y} = -e v_{3y} \tag{8}$$

$$t_{45} = v_{4y}/g \tag{9}$$

$$x_5 = x_3 + v_{4x} t_{45} = x \tag{10}$$

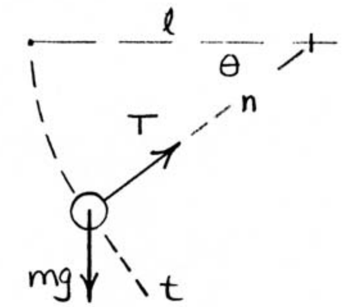
$$y_5 = v_{4y} t_{45} - \tfrac{1}{2} g t_{45}^2 = y \tag{11}$$

Solve Eqs. (1)–(11) for $0.5 \le e \le 0.9$ to obtain the following plot.



For $x = 0$, $\underline{e = 0.610,\ y = 1.396 \text{ ft}}$

*3/369



$$v_0 = l\dot{\theta}_0 = 0.5\,(0.2) = 0.1 \text{ m/s}$$

$$\Sigma F_t = ma_t : \quad mg\cos\theta = m l\ddot{\theta}, \quad \ddot{\theta} = \frac{g}{l}\cos\theta$$

$$v\,dv = a_t\,ds : \quad v\,dv = l\ddot{\theta}\,(l\,d\theta) = g l\cos\theta\,d\theta$$

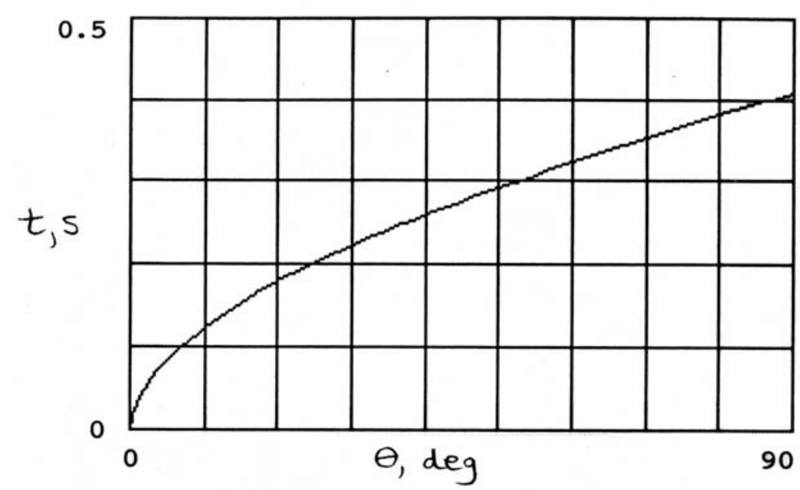
$$\int_{v_0=0.1}^{v} v\,dv = \int_{\theta_0=0}^{\theta} g\,l\cos\theta\,d\theta, \quad v^2 = 2gl\sin\theta + v_0^2$$

$$v = \frac{ds}{dt} = \frac{l\,d\theta}{dt} : \quad \sqrt{2gl\sin\theta + v_0^2} = l\,\frac{d\theta}{dt}$$

Rearrange to $\displaystyle\int_{t_0=0}^{t} dt \quad \int_{\theta_0=0}^{\theta} l\,\frac{d\theta}{\sqrt{2gl\sin\theta + v_0^2}}$

So $\displaystyle t = 0.5\int_0^{\theta}\frac{d\theta}{\sqrt{9.81\sin\theta + 0.01}}$

Set up a numerical integration scheme (see Appendix C/12) and integrate the above for various upper limits $(0 \le \theta \le \pi/2)$

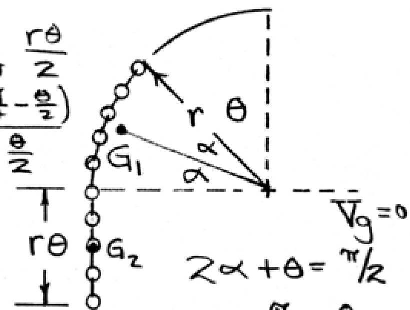


When $\theta = 90°$, $\underline{t = 0.409 \text{ s}}$.

*3/370 For $\theta \neq 0$,

$$V_g = \rho g \left(\frac{\pi r}{2} - r\theta\right) \bar{r} \sin\alpha - \rho g r\theta \, \frac{r\theta}{2}$$

where $\bar{r} = \dfrac{r \sin\alpha}{\alpha} = r \dfrac{\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\frac{\pi}{4} - \frac{\theta}{2}}$



$$V_g = \rho g r^2 \left\{ \left(\frac{\pi}{2} - \theta\right) \frac{\sin^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\frac{\pi}{4} - \frac{\theta}{2}} - \frac{\theta^2}{2} \right\}$$

$2\alpha + \theta = \pi/2$

For $\theta = 0$, $V_g = \rho g \dfrac{\pi r}{2} \dfrac{2r}{\pi} = \rho g r^2$

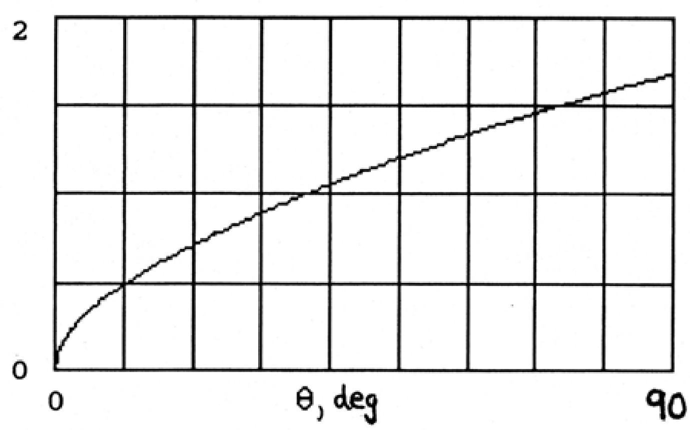
$\alpha = \dfrac{\pi}{4} - \dfrac{\theta}{2}$

Thus $\Delta V_g = (V_g)_\theta - (V_g)_{\theta=0}$

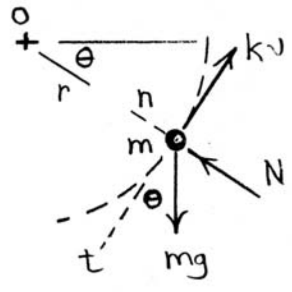
$$= 3\rho g r^2 \left\{ 1 + \frac{\theta^2}{2} - \left(\frac{\pi}{2} - \theta\right) \frac{\sin^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\frac{\pi}{4} - \frac{\theta}{2}} \right\}$$

$U'_{1-2} = 0 = \Delta T + \Delta V_g$; with $\Delta T = \frac{1}{2}\rho \frac{\pi r}{2} v^2$, we get

$$v' = \frac{v}{\sqrt{gr}} = \sqrt{2} \sqrt{\frac{1}{\pi}\left(2 + \theta^2\right) - \left(1 - \frac{2\theta}{\pi}\right) \frac{\sin^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\frac{\pi}{4} - \frac{\theta}{2}}}$$



*3/371



$\Sigma F_t = ma_t$:

$$mg\cos\theta - kv = mv\frac{dv}{ds}$$

But $ds = r\,d\theta$, so

$$mg\cos\theta - kv = mv\frac{dv}{r\,d\theta}$$

$$\frac{dv}{d\theta} = \frac{gr\cos\theta}{v} - \frac{kr}{m}$$

It is not possible to separate variables, so we numerically integrate to obtain v as a function of θ.

$\Sigma F_n = m\frac{v^2}{\rho}$: $\quad N - mg\sin\theta = m\frac{v^2}{r}$

$$N = m\left[g\sin\theta + \frac{v^2}{r}\right]$$

where v is available from the previously mentioned numerical integration. Plots of both v and N as functions of θ are shown below.

The maxima are

$$\underline{v_{max} = 5.69 \text{ ft/sec} \ @ \ \theta = 50.8°}$$

$$\underline{N_{max} = 2.75 \text{ lb} \ @ \ \theta = 66.2°}$$