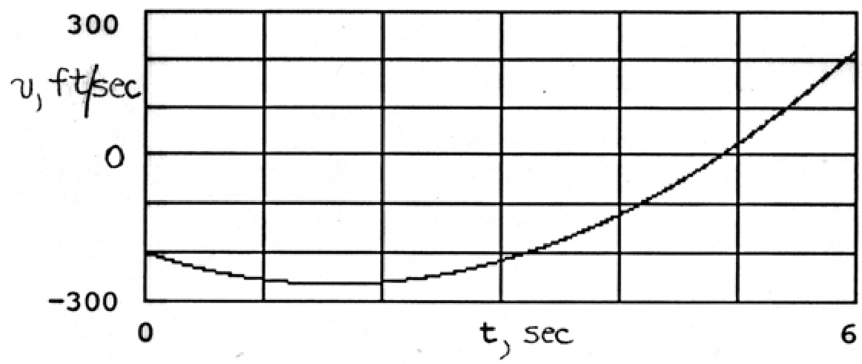
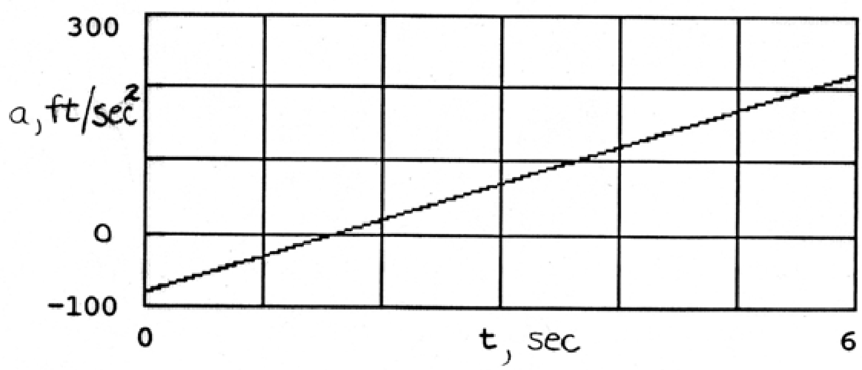
2/1

$\left.\begin{array}{l} v = 25t^2 - 80t - 200 \\ a = \dfrac{dv}{dt} = 50t - 80 \end{array}\right\}$ See plots

$a = 0: \quad 50t - 80 = 0, \quad t = 1.6 \text{ sec}$

At $t = 1.6$ sec, $v = 25(1.6)^2 - 80(1.6) - 200 = \underline{-264 \frac{ft}{sec}}$





2/2 $s = 2t^3 - 40t^2 + 200t - 50$

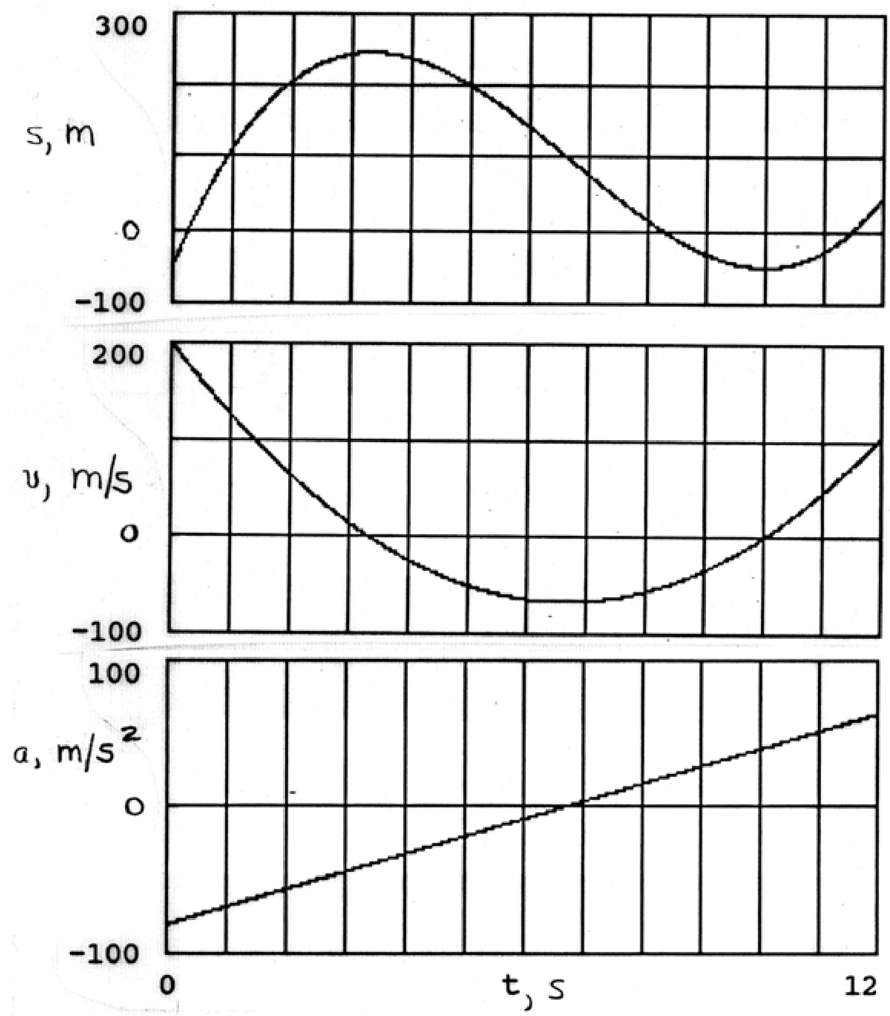
$v = \frac{ds}{dt} = 6t^2 - 80t + 200$

$a = \frac{dv}{dt} = 12t - 80$

See plots

$v = 0: \; 6t^2 - 80t + 200 = 0, \quad t = \frac{80 \pm \sqrt{80^2 - 4(6)(200)}}{-2(6)}$

$\underline{t = 3.33 \text{ s}, \; 10 \text{ s}}$



$\boxed{2/3}$ $v = 2 - 4t + 5t^{3/2}$

$$a = \frac{dv}{dt} = -4 + \frac{15}{2}t^{1/2}$$

$$\frac{ds}{dt} = 2 - 4t + 5t^{3/2}$$

$$\int_{s_0=3}^{s} ds = \int_0^t (2 - 4t + 5t^{3/2})\, dt$$

$$s = 3 + 2t - 2t^2 + 2t^{5/2}$$

At $t = 3\text{s}$: $\left\{ \begin{array}{l} s = 22.2 \text{ m} \\ v = 15.98 \text{ m/s} \\ a = 8.99 \text{ m/s}^2 \end{array} \right.$

$\boxed{2/4}$ $s = (-2 + 3t)e^{-0.5t}$

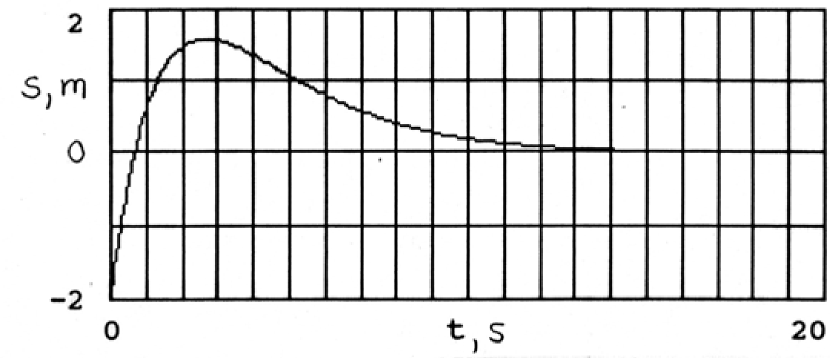
$$v = \frac{ds}{dt} = 3e^{-0.5t} + (-2+3t)(-0.5)e^{-0.5t}$$

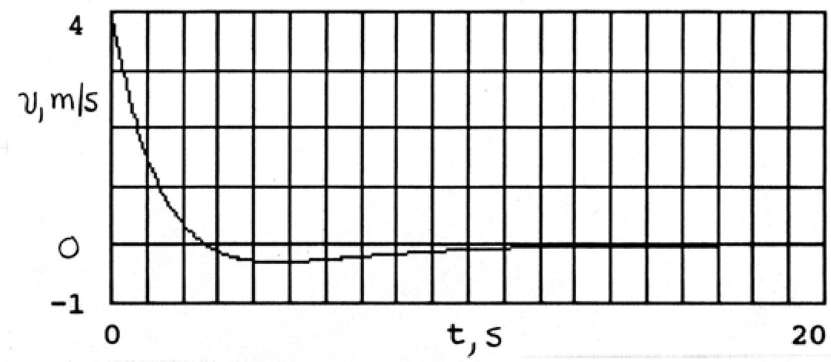
$$= \underline{(4-1.5t)\,e^{-0.5t}}$$

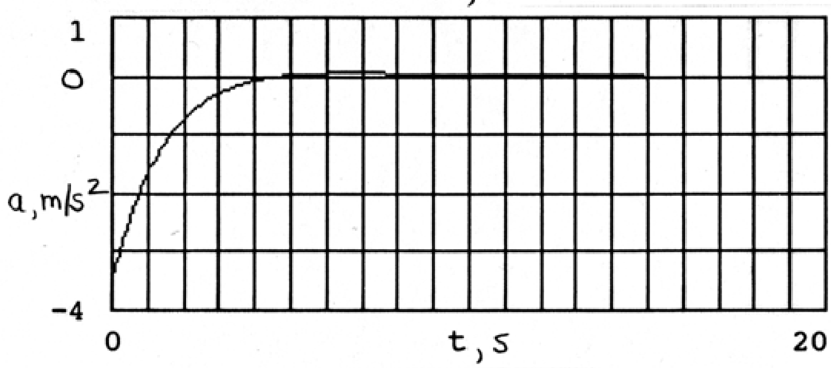
$$a = \frac{dv}{dt} = -1.5e^{-0.5t} + (4-1.5t)(-0.5)e^{-0.5t}$$

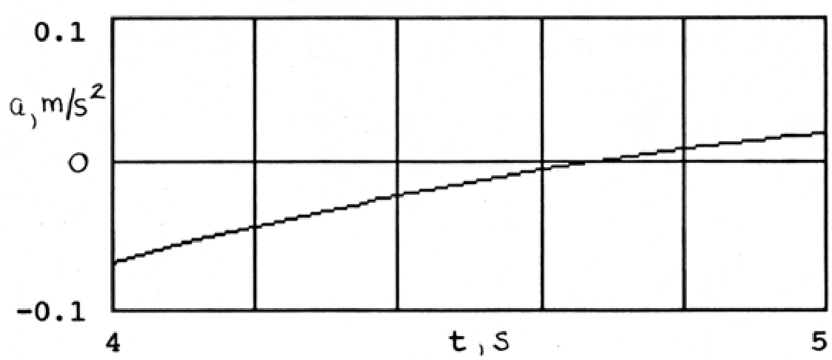
$$= \underline{(-3.5 + 0.75t)\,e^{-0.5t}}$$

$$a = 0: \quad (-3.5 + 0.75t)e^{-0.5t} = 0, \quad \underline{t = 4.67 \text{ s}}$$


2
s, m
0
-2
0
t, s
20


4
v, m/s
0
-1
0
t, s
20


1
0
a, m/s²
-4
0
t, s
20


0.1
a, m/s²
0
-0.1
4
t, s
5

$\boxed{2/5}$ $a = \frac{dv}{dt} = 2t - 10$

$$\int_{v_0=3}^{v} dv = \int_0^t (2t-10)\,dt, \quad \underline{v = 3 - 10t + t^2} \ (m/s)$$

$$\frac{ds}{dt} = 3 - 10t + t^2$$

$$\int_{s_0=-4}^{s} ds = \int_0^t (3 - 10t + t^2)\,dt$$

$$\underline{s = -4 + 3t - 5t^2 + \tfrac{1}{3}t^3} \ (m)$$

2/6 $a = v\dfrac{dv}{ds} = -ks^2$

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} ks^2\,ds \Rightarrow v^2 = v_0^2 - \frac{2}{3}k(s^3 - s_0^3)$$

Taking positive sign: $\underline{v = \left[v_0^2 - \frac{2}{3}k\left(s^3 - s_0^3\right)\right]^{1/2}}$

Numbers: $v = \left[10^2 - \frac{2}{3}(0.1)(5^3 - 3^3)\right]^{1/2}$

$= \underline{9.67 \text{ m/s}}$

2/7

$$a = -kv^{1/2} = \frac{dv}{dt}$$

$$-\int_0^t k\,dt = \int_{v_0}^{v} \frac{dv}{v^{1/2}} \Rightarrow \underline{v = \left(v_0^{1/2} - \frac{1}{2}kt\right)^2}$$

Numbers: $v = \left(7^{1/2} - \frac{1}{2}(0.2)(2)\right)^2 = \underline{5.98 \text{ m/s}}$

Also, $-kv^{1/2} = v\dfrac{dv}{ds}$

$$-\int_{s_0}^{s} k\,ds = \int v^{1/2}\,dv \Rightarrow \underline{v = \left[v_0^{3/2} - \frac{3}{2}k(s-s_0)\right]^{2/3}}$$

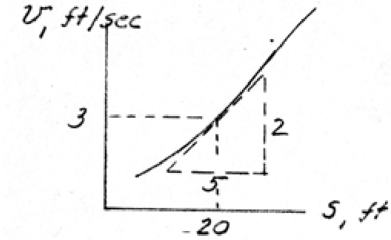
Numbers: $v = \left[7^{3/2} - \frac{3}{2}(0.2)(3-1)\right]^{2/3} = \underline{6.85 \text{ m/s}}$

2/8

$$a = v\frac{dv}{ds} = 10(-3) = \underline{-30 \text{ m/s}^2}$$

2/9

$$a = v\frac{dv}{ds} = 3\left(\frac{2}{5}\right) = \underline{1.2 \text{ ft/sec}^2}$$



2/10

$\uparrow +y$ $\quad y = v_0 t + \frac{1}{2} a t^2$, $y = 80t - \frac{1}{2} 32.2\, t^2$

for $y = -200$ ft,

$$-200 = 80t - 16.1t^2$$

or $16.1t^2 - 80t - 200 = 0$

$$t = \frac{80 \pm \sqrt{(80)^2 + 4(16.1)(200)}}{2(16.1)} = \underline{6.80 \text{ sec}} \quad (\text{or } -1.83 \text{ s})$$

For $\dot{y} = 0$, $v^2 = v_0^2 + 2ay$, $y = h = \dfrac{0 - 80^2}{-2(32.2)} = \underline{99.4 \text{ ft}}$

2/11 For constant acceleration,

$$s = \frac{1}{2}at^2, \quad t = \left(\frac{2s}{a}\right)^{1/2} = \left(\frac{2(30\,000)}{1.5(9.81)}\right)$$

$$= \underline{63.9 \text{ s}}$$

$$v = \sqrt{2as} = \sqrt{2(1.5)(9.81)(30\,000)} = \underline{940 \text{ m/s}}$$

$\boxed{2/12}$ $v^2 - v_0^2 = 2a(s - s_0)$

$$0 - \left[50 \frac{5280}{3600}\right]^2 = 2a(100), \quad a = -26.9 \frac{ft}{sec^2}$$

Then $0 - \left[70 \frac{5280}{3600}\right]^2 = 2(-26.9)\, s$

$\underline{s = 196.0 \text{ ft}}$

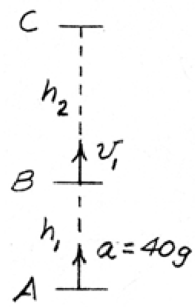
2/13 For $a =$ constant, $v^2 = v_0^2 + 2as$

$$\left[\frac{180\,(5280)}{3600}\right]^2 = 0^2 + 2a\,(300)$$

$a = 116.2\ \text{ft/sec}^2$

or $a = 116.2/32.2 = \underline{3.61g}$

2/14



B to C; $t = 10/2 = 5$ s

$v = v_1 + at$; $0 = v_1 - 9.81(5)$, $v_1 = 49.0$ m/s

$v^2 = v_1^2 + 2as$; $0 = (49.0)^2 + 2(-9.81)h_2$

$h_2 = 122.6$ m

A to B; $v^2 = v_0^2 + 2as$;

$(49.0)^2 = 0 + 2(40)(9.81)h_1$

$h_1 = 3.07$ m

$h = h_1 + h_2 = \underline{125.7 \text{ m}}$

$\boxed{2/15}$ $v^2 = v_0^2 + 2a(s - s_0)$

$$\left(\frac{200}{3.6}\right)^2 = 0^2 + 2(0.4 \cdot 9.81)\, s$$

$$\underline{s = 393 \text{ m}}$$

$v = v_0 + at$: $\left(\frac{200}{3.6}\right) = 0 + 0.4(9.81)\, t$

$$\underline{t = 14.16 \text{ s}}$$

2/16 $\int v\,dv = \int a\,ds$; $\displaystyle\int_{200/3.6}^{30/3.6} v\,dv = a\int_0^{600} ds$

200 km/h

$= 200/3.6$ m/s

30 km/h

$= 30/3.6$ m/s

$$\frac{1}{2(3.6)^2}\left(\overline{30}^2 - \overline{200}^2\right) = 600a$$

$$\underline{a = -2.51\ \text{m/s}^2}$$

2/17

$$v = 20e^{-t/10}$$

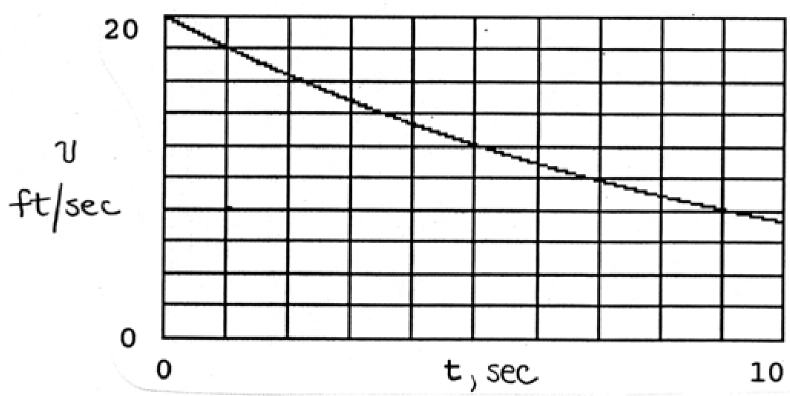
$$a = \dot{v} = -2e^{-t/10}$$

When $t = 10$ sec, $a = -2e^{-10/10} = \underline{-0.736 \text{ ft/sec}^2}$

From $v = \dfrac{ds}{dt} = 20e^{-t/10}$,

$$\int_0^s ds = \int_0^{10} 20e^{-t/10}\,dt$$

$$s = -200e^{-t/10}\Big|_0^{10} = \underline{126.4 \text{ ft}}$$



$\boxed{2/18}$ $v^2 = v_0^2 + 2as$, where $a = g/6$

$v^2 = 2^2 + 2\left(\frac{9.81}{6}\right)5$, $\underline{v = 4.51 \text{ m/s}}$

2/19

$\Delta s =$ area under v-t curve

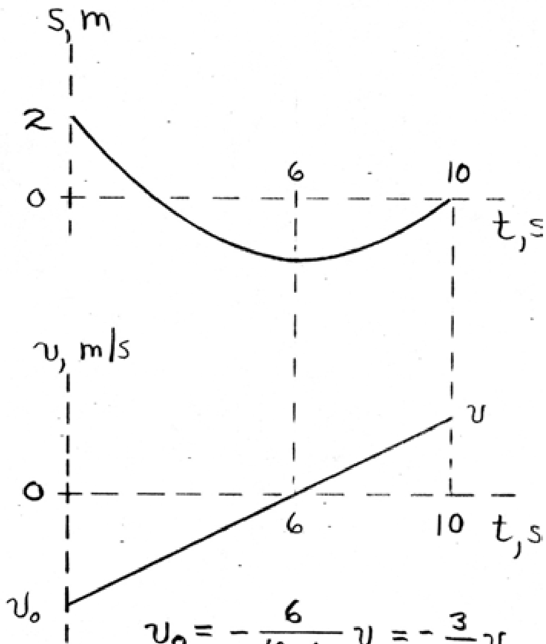
$$0-2 = -\frac{3}{2}v\left(\frac{6}{2}\right) + \frac{1}{2}v(4)$$

$$\underline{v = 0.8 \text{ m/s}}$$

Acceleration

$$a = \frac{dv}{dt} = \frac{0.8}{4}$$

$$= \underline{0.2 \text{ m/s}^2}$$



$$v_0 = -\frac{6}{10-6}v = -\frac{3}{2}v$$

2/20 Acceleration period:

$v = v_0 + at$: $\frac{22}{3.6} = 0 + \frac{9.81}{4} t_a$, $t_a = 2.49$ s

Note that the deceleration time $t_d = t_a$

$v^2 = v_0^2 + 2a\Delta s$: $\left(\frac{22}{3.6}\right)^2 = 0^2 + 2\,\frac{9.81}{4}\,\Delta s_a$

$\Delta s_a = 7.61\text{ m} = \Delta s_d$

Cruise period: $\Delta s_c = 350 - \Delta s_a - \Delta s_d = 335$ m

$\Delta s = v_c t_c$: $335 = \frac{22}{3.6} t_c$, $t_c = 54.8$ s

Total run time $t = t_c + t_a + t_d = \underline{59.8\text{ s}}$

2/21

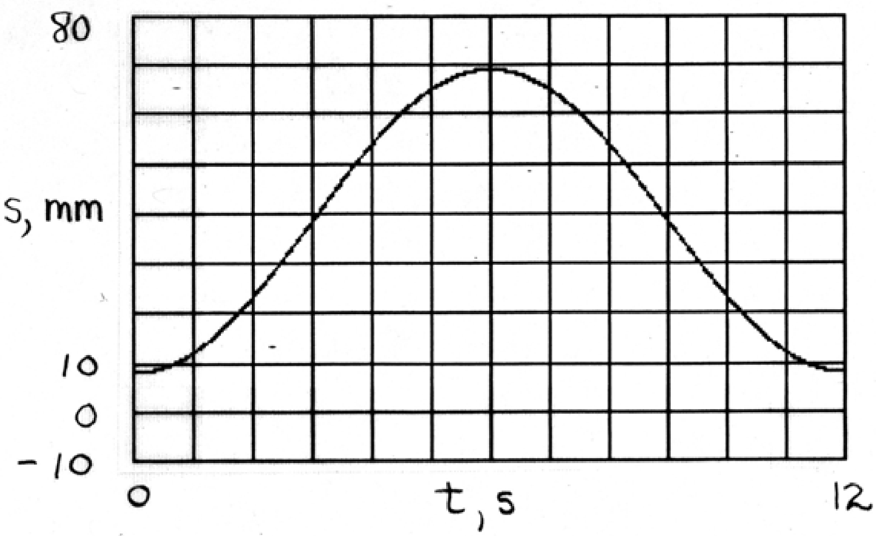
$$v = \frac{ds}{dt} = 16 \sin \frac{\pi t}{6}$$

$$\int_8^s ds = 16 \int_0^t \sin \frac{\pi t}{6} dt$$

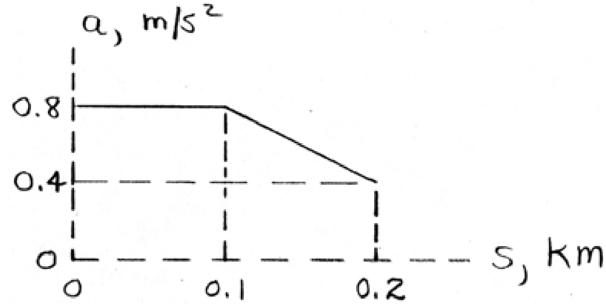
$$s = 8 + 16 \cdot \frac{6}{\pi}\left(-\cos \frac{\pi t}{6}\right)\Big|_0^t = 8 + \frac{96}{\pi}\left[1 - \cos \frac{\pi t}{6}\right]$$

$s = s_{max}$ when $\cos \frac{\pi t}{6} = -1$ or $t = 6$ s

$$s_{max} = 8 + \frac{96}{\pi}\left[1-(-1)\right] = \underline{69.1 \text{ mm}}$$



2/22



From $a = v\dfrac{dv}{ds}$,

$$\int_{v_0}^{v} v\,dv = \int_0^{200} a\,ds = \text{area under } a\text{-}s \text{ curve}$$

$$\frac{v^2}{2} - \frac{(40/3.6)^2}{2} = 0.8(100) + 0.6(100)$$

$\underline{v = 20.1 \text{ m/s}}$ or $\underline{72.3 \text{ km/h}}$

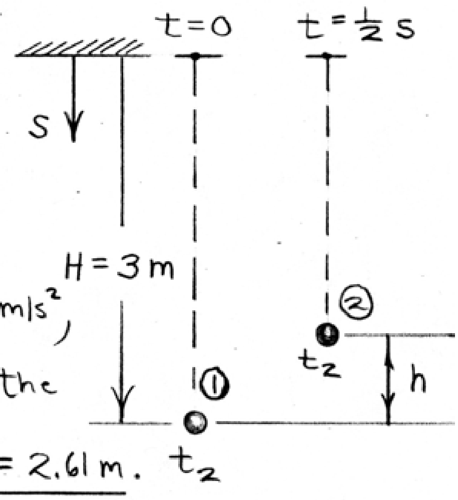
$\boxed{2/23}$ $s = s_0 + v_0 t + \frac{1}{2} g t^2$

Sphere ①: $H = \frac{1}{2} g t_2^{\,2}$

Sphere ②:

$$(H-h) = \frac{1}{2} g \left(t_2 - \frac{1}{2}\right)^2$$

With $H = 3$ m & $g = 9.81$ m/s^2, eliminate t_2 between the 2 equations & obtain $\underline{h = 2.61 \text{ m}}$.



2/24 $\Delta v = \int a\,dt$ = area under a-t curve

For $t = 4\text{s}$, $v_4 - 100 = -4(9.81)\frac{4-2}{2}$, $v_4 = \underline{60.8 \text{ m/s}}$

For $t = 8\text{s}$, $v_8 - 60.8 = -4(9.81)(6-4)$, $v_8 = \underline{-17.72 \text{ m/s}}$

2/25

$v^2 = v_0^2 + 2a(s - s_0)$

$0 = 4^2 + 2\left(-\frac{9.81}{4}\right)(s)$, $\underline{s = 3.26 \text{ m}}$

$v = v_0 + at$: $0 = 4 + \left(-\frac{9.81}{4}\right)t_{up}$, $t_{up} = 1.631 \text{ s}$

$t = 2t_{up} = 2(1.631) = \underline{3.26 \text{ s}}$

$\boxed{2/26}$ $a = \frac{1}{2}\frac{d(v^2)}{ds} = \frac{1}{2}\frac{900-2500}{400-100} = -\frac{8}{3}\frac{ft}{sec^2}$

(constant)

$\Delta v = \int a\,dt$; $v - 50 = -\frac{8}{3}t$

At B: $30 - 50 = -\frac{8}{3}t$, $t = 7.50$ sec

$\Delta s = \int v\,dt = \int_{5.5}^{7.5}\left(50 - \frac{8}{3}t\right)dt = \underline{65.3 \text{ ft}}$

2/27

$a = 400 - kx$, where $k = \frac{400}{6/12}$ sec^{-2}

$a = 400(1 - 2x)$ (x in ft)

$v\,dv = a\,dx$: $\int_0^v v\,dv = 400\int_0^x (1-2x)\,dx$

$v^2 = 800(x - x^2)$, $v = \frac{dx}{dt} = 20\sqrt{2}\sqrt{x - x^2}$

(taking + sign)

$$\int_0^t dt = \int_0^x \frac{dx}{20\sqrt{2}\sqrt{x-x^2}}$$

$$t = -\frac{1}{20\sqrt{2}} \sin^{-1} \frac{1-2x}{\sqrt{1}}\Bigg|_0^x = \frac{1}{20\sqrt{2}}\left[\frac{\pi}{2} - \sin^{-1}(1-2x)\right]$$

(a) $x = \frac{1}{4}$ ft : $t = \frac{1}{20\sqrt{2}}\left[\frac{\pi}{2} - \frac{\pi}{6}\right] = \underline{0.0370 \text{ sec}}$

(b) $x = \frac{1}{2}$ ft : $t = \frac{1}{20\sqrt{2}}\left[\frac{\pi}{2} - 0\right] = \underline{0.0555 \text{ sec}}$

2/28 The area under the a-s curve is

$$\int_0^v v\,dv = \frac{1}{2}v^2$$

$$\text{Area}\Big|_0^{200\text{ m}} = \frac{3+6}{2}(100) + \frac{6+4}{2}(100) = 950\ \text{m}^2/\text{s}^2$$

So $\frac{1}{2}v^2 = 950$, $\underline{v = 43.6\ \text{m/s}}$

$$\frac{dv}{ds} = \frac{a}{v} = \frac{4}{43.6} = \underline{0.0918\ \text{s}^{-1}}$$

2/29 $v_0 = 100/3.6 = 27.8 \text{ m/s}$

$a = -g \sin\theta = -9.81 \sin\left[\tan^{-1}\frac{6}{100}\right] = -0.588 \text{ m/s}^2$

(a) $v = v_0 + at = 27.8 - 0.588(10) = \underline{21.9 \text{ m/s}}$

(b) $v^2 = v_0^2 + 2a(s - s_0) = 27.8^2 + 2(-0.588)(100)$

$v = \underline{25.6 \text{ m/s}}$

2/30 $0 < t < 4\text{ s}$: $a = -3t/4$

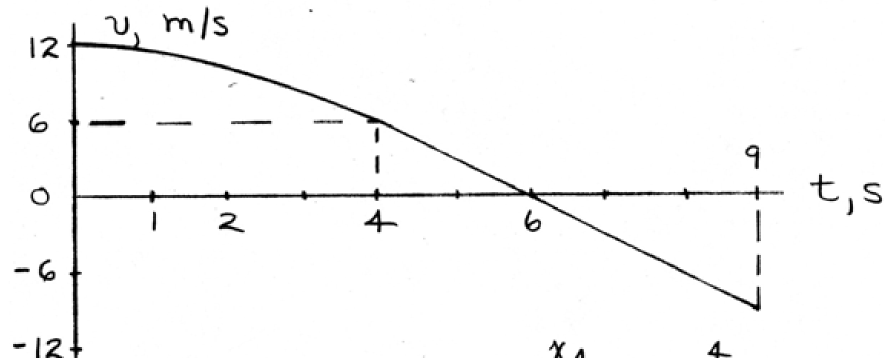
$a = \frac{dv}{dt}$: $\int_{12}^{v} dv = -\int_{0}^{t} \frac{3t}{4}\,dt$

$v = 12 - \frac{3}{8}t^2$, $\underline{v_4 = 6 \text{ m/s}}$

$4 < t < 9\text{ s}$: $a = -3 \text{ m/s}^2 = \text{constant}$

$v = v_4 + a\Delta t = 6 - 3(t-4) = 18 - 3t$

$v_9 = -9 \text{ m/s}$



$0 < t < 4\text{s}$: $dx = v\,dt$: $\int_0^{x_4} dx = \int_0^4 (12 - \frac{3}{8}t^2)\,dt$ $s_4 = 40 \text{ m}$

$4 < t < 9\text{s}$: $x_6 - x_4 = \int_4^6 v\,dt = \frac{1}{2}(6-4)6 = 6 \text{ m}$

$\therefore\ \Delta x = 40 + 6 = \underline{46 \text{ m}}$

$\boxed{2/31}$ $a = v\dfrac{dv}{ds} = \dfrac{1}{2}\dfrac{d(v^2)}{ds} = \dfrac{1}{2}\dfrac{\Delta(v^2)}{\Delta s} = \dfrac{1}{2}\,\dfrac{36-16}{80-30} = \dfrac{1}{5}\ m/s^2$

Counting time from A, $v = v_A + at$, $v = 4 + \frac{1}{5}t$

At B, $6 = 4 + \frac{1}{5}t_B$, $t_B = 10$ sec.

$$\Delta s = \int v\,dt = \int_8^{10}(4 + \tfrac{1}{5}t)\,dt = 4(2) + \frac{1}{10}(100 - 64)$$

$\Delta s = \underline{11.6\ m}$

2/32 $s_{car} = vt = \frac{120}{3.6} t$

$s_{cycle} = v_{av} t_1 + v_{max} t_2 = \frac{1}{2} \frac{150}{3.6} t_1 + \frac{150}{3.6} t_2$

where $t_1 = \frac{v_{max}}{a} = \frac{150}{3.6 \times 6} = 6.94$ s & $t_2 = t - 6.94 - 2$

$s_{car} = s_{cycle}$; $\frac{120}{3.6} t = \frac{75}{3.6} 6.94 + \frac{150}{3.6}(t - 8.94)$

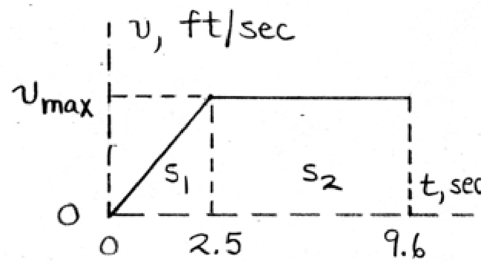
$30t = 820.8$, $t = 27.36$ s

$s = \frac{120}{3.6}(27.36) = \underline{912 \text{ m}}$

2/33

$s_1 = \frac{1}{2}(2.5)v_{max}$

$s_2 = (9.6 - 2.5)v_{max}$

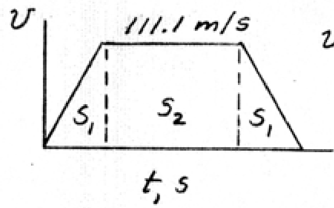


$s_1 + s_2 = (1.25 + 7.1)v_{max} = 100(3)$

$\underline{v_{max} = 35.9 \text{ ft/sec}}$

2/34

$400 \text{ km/h} \equiv \dfrac{400}{3.6} = 111.1 \text{ m/s}$



$v^2 = 2as, \quad s_1 = \dfrac{(111.1)^2}{2(0.6)(9.81)} = 1049 \text{ m}$

$s_2 = 10\,000 - 2(1049)$

$= 7903 \text{ m}$

$t_1 = \dfrac{v}{a} = \dfrac{111.1}{0.6(9.81)} = 18.88 \text{ s}$

$t_2 = \dfrac{s_2}{v} = \dfrac{7903}{111.1} = 71.13 \text{ s}$

$t = 2t_1 + t_2$

$= 2(18.88) + 71.13$

$= \underline{108.9 \text{ s}}$

or $\underline{t = 1.81 \text{ min}}$

2/35

$$a = g - cy = v\frac{dv}{dy}$$

$$\int_0^{y_m} (g - cy)\,dy = \int_{v_0}^{0} v\,dv$$

$$\left(gy - c\frac{y^2}{2}\right)\Big|_0^{y_m} = \frac{v^2}{2}\Big|_{v_0}^{0}$$

$$gy_m - c\frac{y_m^2}{2} = -\frac{v_0^2}{2} \Rightarrow \underline{c = \frac{v_0^2 + 2gy_m}{y_m^2}}$$

2/36 Particle 1 : $a = -kv$

$$-kv = \frac{dv}{dt}$$

$$-k\int_0^t dt = \int_{v_0}^{v} \frac{dv}{v} \Rightarrow \underline{v = v_0 e^{-kt}}$$

Then $\frac{ds}{dt} = v_0 e^{-kt}$

$$\int_{s_0=0}^{s} ds = v_0 \int_{t_0=0}^{t} e^{-kt} dt \Rightarrow \underline{s = \frac{v_0}{k}\left(1 - e^{-kt}\right)}$$

Particle 2 : $a = -kt$

$$-kt = \frac{dv}{dt}$$

$$-k\int_0^t t\,dt = \int_{v_0}^{v} dv \Rightarrow \underline{v = v_0 - \frac{1}{2}kt^2}$$

Then $\frac{ds}{dt} = v_0 - \frac{1}{2}kt^2$

$$\int_0^s ds = \int_0^t \left(v_0 - \frac{1}{2}kt^2\right) dt \Rightarrow \underline{s = v_0 t - \frac{1}{6}kt^3}$$

Particle 3 : $a = -ks$

$$-ks = v\frac{dv}{ds}$$

$$-k\int_0^s s\,ds = \int_{v_0}^{v} v\,dv \Rightarrow \underline{v = \pm\sqrt{v_0^2 - ks^2}}$$

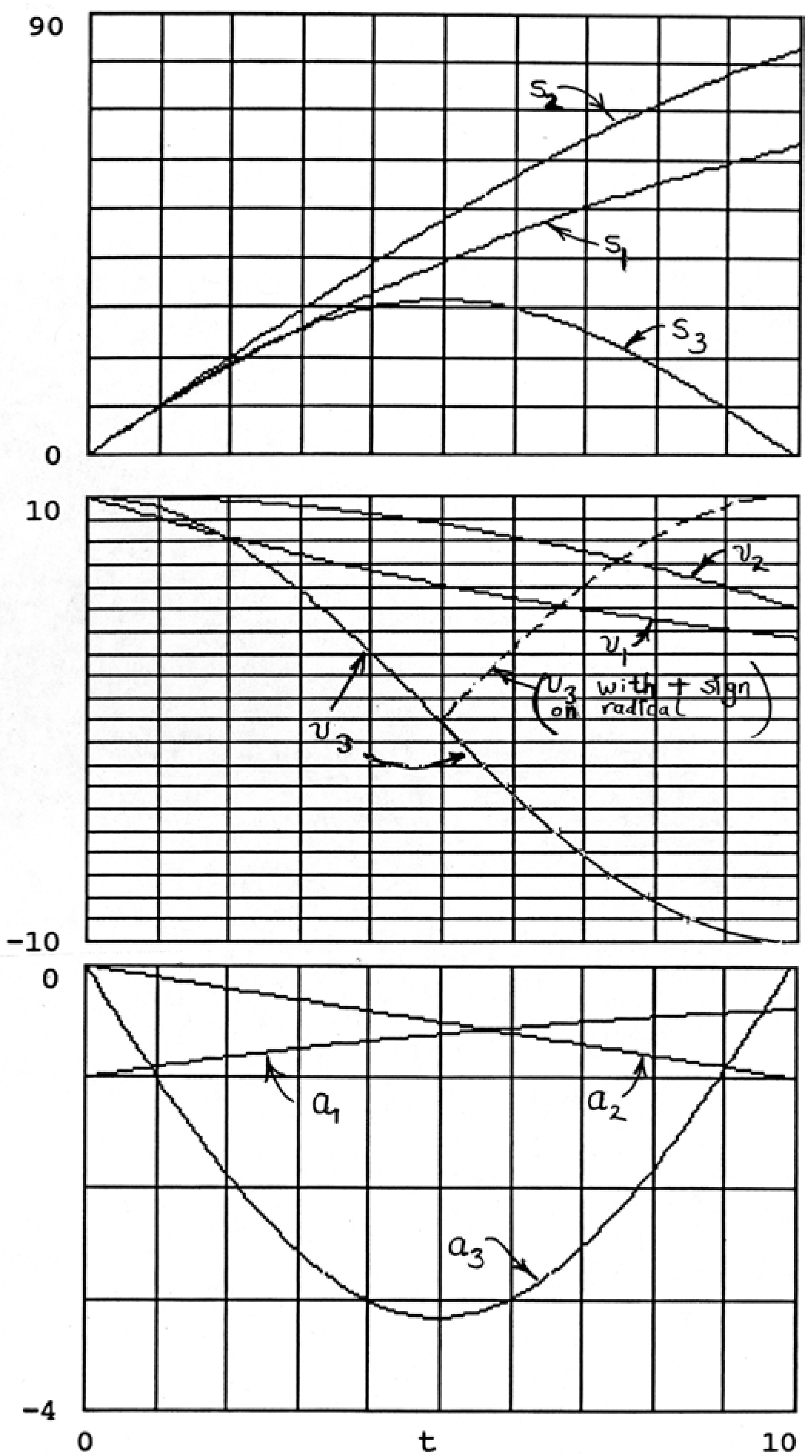
Then

$$\frac{ds}{dt} = \pm\sqrt{v_0^2 - ks^2}$$

Note: Plus sign is chosen until first reversal ($v=0$), thereafter take minus sign, etc.

$$\int_0^s \frac{ds}{\sqrt{v_0^2 - ks^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{k}} \sin^{-1}\left(\frac{\sqrt{k}}{v_0} s\right) = t \Rightarrow \underline{s = \frac{-v_0}{\sqrt{k}} \sin\left(\sqrt{k}\, t\right)}$$


90
0
10
-10
0
-4
0
t
10
s2
s1
s3
v2
v1
v3
(v3 with + sign on radical)
a1
a2
a3

2/37 $a = P/(mv)$, P & m are constant

$v\,dv = a\,ds$; $v\,dv = \frac{P}{mv}\,ds$

$\int_{v_1}^{v_2} m v^2\,dv = P\int_0^s ds$, $\underline{s = \frac{m}{3P}(v_2^3 - v_1^3)}$

$dv = a\,dt$; $dv = \frac{P}{mv}\,dt$

$m\int_{v_1}^{v_2} v\,dv = \int_0^t P\,dt$, $\underline{t = \frac{m}{2P}(v_2^2 - v_1^2)}$

$\boxed{2/38}$ $v\,dv = a\,ds$; $\frac{v\,dv}{-Kv^2} = ds$, $\int_{v_1}^{v_2} \frac{dv}{v} = -K\int_0^s ds$

$\ln\frac{v_2}{v_1} = -Ks$, $K = \frac{1}{s}\ln\frac{v_1}{v_2} = \frac{1}{1500}\ln\frac{100}{20} = \underline{1.073(10^{-3})\ ft^{-1}}$

$a = \frac{dv}{dt}$; $-Kv^2 = \frac{dv}{dt}$, $\int_{v_1}^{v_2} \frac{dv}{v^2} = -Kt$, $t = \frac{1}{K}\left(\frac{1}{v_2} - \frac{1}{v_1}\right)$

$t = \frac{10^3}{1.073}\left(\frac{1}{20} - \frac{1}{100}\right)\frac{30}{44} = \underline{25.4\ sec}$

2/39

$a = -kv = \frac{dv}{dt}$

$$-k\int_0^t dt = \int_{v_0}^{v} \frac{dv}{v} \Rightarrow v = v_0 e^{-kt}$$

Given conditions : $1 = 4e^{-k(2)}$, $k = 0.693\ s^{-1}$

So $v = v_0 e^{-0.693t}$

When $v = \frac{v_0}{10}$: $\frac{v_0}{10} = v_0 e^{-0.693T}$, $\underline{T = 3.32\ s}$

Also, $a = -kv = v\frac{dv}{ds}$

$$-k\int_{s=0}^{s} ds = \int_{v_0}^{v} dv, \quad v = v_0 - ks$$

Given conditions: $\frac{v_0}{10} = v_0 - kD$

With $k = 0.693\ s^{-1}$ and $v_0 = 4$ m/s :

$\frac{4}{10} = 4 - 0.693D$, $\underline{D = 5.19\ m}$

(We note that T is independent of v_0; D is not.)

$\boxed{2/40}$ $a = g - cy^2 = v\frac{dv}{dy}$

$$\int_0^{y_m} (g - cy^2)\,dy = \int_{v_0}^{0} v\,dv$$

$$\left(gy - c\frac{y^3}{3}\right)\Big|_0^{y_m} = \frac{v^2}{2}\Big|_{v_0}^{0}$$

$$gy_m - c\frac{y_m^3}{3} = -\frac{v_0^2}{2} \Rightarrow \underline{c = \frac{3v_0^2 + 6gy_m}{2y_m^3}}$$

2/41 $v\,dv = a\,dx$, $\int_0^x dx = \int_{v_0}^{v} \frac{v\,dv}{-C_1 - C_2 v^2}$

$$x = \frac{-1}{2C_2} \ln\left(+C_1 + C_2 v^2\right)\Big]_{v_0}^{v} = \frac{1}{2C_2} \ln \frac{C_1 + C_2 v_0^2}{C_1 + C_2 v^2}$$

when $v = 0$, $\underline{x = D = \frac{1}{2C_2} \ln\left(1 + \frac{C_2}{C_1} v_0^2\right)}$

$\boxed{2/42}$ (a) $g_0 = 32.2 \text{ ft/sec}^2 = \text{constant}$

$v^2 = v_0^2 + 2a\,(s - s_0)$: $v^2 = 0^2 + 2(32.2)(500 \cdot 5280)$

$v = \underline{13{,}040 \text{ ft/sec}}$

(b) $a = -g_0 \dfrac{R^2}{r^2} = v \dfrac{dv}{dr}$

$$-g_0 R^2 \int_{R+h}^{R} \frac{dr}{r^2} = \int_{v_0=0}^{v} v\,dv$$

$$-g_0 R^2 \left(-\frac{1}{r}\right)\Big|_{R+h}^{R} = \frac{1}{2} v^2 \Big|_0^v$$

$$\Rightarrow \quad v = \sqrt{\frac{2 g_0 R h}{R+h}} = \sqrt{\frac{2(32.2)(3959)(500)(5280)^2}{(3959+500)(5280)}}$$

$$= \underline{12{,}290 \text{ ft/sec}}$$

$\boxed{2/43}$ (a) $(g_m)_0 = 5.32 \text{ ft/sec}^2 = \text{constant}$

$v^2 = v_0^2 + 2(g_m)_0(s - s_0): \quad v^2 = 0^2 + 2(5.32)(750 \cdot 5280)$

$$\underline{v = 6490 \text{ ft/sec}}$$

(b) $a = -(g_m)_0 \dfrac{R_m^2}{r^2} = v\dfrac{dv}{dr}$ (R_m = moon radius)

$$-(g_m)_0 R_m^2 \int_{R_m+h}^{R_m} \frac{dr}{r^2} = \int_{v_0=0}^{v} v\,dv$$

$$-(g_m)_0 R_m^2 \left(-\frac{1}{r}\right)\Bigg|_{R_m+h}^{R_m} = \frac{1}{2}v^2\Bigg|_0^v$$

$$v = \sqrt{\frac{2(g_m)_0 R_m h}{R_m + h}} = \sqrt{\frac{2(5.32)\left(\frac{2160}{2}\right)(750)(5280)^2}{\left(\frac{2160}{2}+750\right)(5280)}}$$

$$= \underline{4990 \text{ ft/sec}}$$

2/44 $a = \frac{dv}{dt} = -kv$, $\int_{v_0}^{v} \frac{dv}{v} = -k\int_0^t dt$

$\ln\frac{v}{v_0} = -kt$, $\underline{v = v_0 e^{-kt}}$

$v = \frac{dx}{dt} = v_0 e^{-kt}$, $\int_0^x dx = \int_0^t v_0 e^{-kt} dt$

$$\underline{x = \frac{v_0}{k}\left[1 - e^{-kt}\right]}$$

$v\,dv = a\,dx$, $\frac{v\,dv}{v} = -k\,dx$

$\int_{v_0}^{v} dv = -k\int_0^x dx$, $\underline{v = v_0 - kx}$

2/45 $a = \frac{dv}{dt}$; $\int_0^v \frac{dv}{g-kv} = \int_0^t dt$, $-\frac{1}{k} \ln (g-kv) \Big]_0^v = t$

$kt = \ln \frac{g}{g-kv}$, $\frac{g}{g-kv} = e^{kt}$, $\underline{v = \frac{g}{k}(1-e^{-kt})}$

$v = \frac{dy}{dt}$; $\int_0^y dy = \frac{g}{k}\int_0^t (1-e^{-kt})\,dt$

$y = \frac{g}{k}\left(t + \frac{1}{k}e^{-kt}\right)_0^t$, $\underline{y = \frac{g}{k}\left[t - \frac{1}{k}(1-e^{-kt})\right]}$

2/46 (a) $a = 2 \text{ m/s}^2 = \text{constant}$

With $v = 250/3.6 = 69.4$ m/s, we have

$$v^2 - v_0^2 = 2a(s - s_0) : 69.4^2 - 0^2 = 2(2)s$$

$$\underline{s = 1206 \text{ m}}$$

(b) $a = a_0 - kv^2 = v\dfrac{dv}{ds}$

$$\int_0^s ds = \int_0^v \frac{v\,dv}{a_0 - kv^2}$$

$$s = -\frac{1}{2k}\ln\left(a_0 - kv^2\right)\Big|_0^v$$

$$= -\frac{1}{2k}\ln\left[\frac{a_0 - kv^2}{a_0}\right]$$

$$s = -\frac{1}{2(4)(10^{-5})}\ln\left[\frac{2 - 4(10^{-5})(69.4)^2}{2}\right]$$

$$= \underline{1268 \text{ m}}$$

2/47 $a = -kv^2$,

$v\,dv = a\,dx$, $\displaystyle\int_{v_0}^{v} \frac{v\,dv}{-kv^2} = \int_0^x dx$, $x = \frac{-1}{k} \ln v \Big]_{v_0}^{v}$

$x = \frac{1}{k} \ln v_0/v$

when $v = v_0/2$, $x = D = \frac{1}{k} \ln 2 = \underline{0.693/k}$

$v = \frac{dx}{dt}$ where $kx = \ln v_0/v$, $v = v_0 e^{-kx}$

so $\frac{dx}{v_0 e^{-kx}} = dt$ or $\displaystyle\int_0^t dt = \frac{1}{v_0} \int_0^x e^{kx} dx$

$t = \frac{1}{v_0} \frac{1}{k} e^{kx} \Big]_0^x = \frac{1}{kv_0} \left[e^{kx} - 1 \right]$

For $x = D$, $e^{kx} = 2$ so $t = \frac{1}{kv_0} [2-1]$, $\underline{t = \frac{1}{kv_0}}$

2/48 $0-60$ mi/hr: $v^2 = v_0^2 + 2as$

$0 < t < t_1$ $\quad (88)^2 = 0 + 2a(200)$, $a = 19.36$ ft/sec^2

$v = v_0 + at$, $v = 0 + 19.36\,t$

$60-0$; $v\,dv = a\,ds$; $a = -kv$ so $\dfrac{v\,dv}{-kv} = ds$

$t_1 < t < t_2$

or $dv = -k\,ds$ & $\displaystyle\int_{88}^{44} dv = -k\int_0^{400} ds$

$44 - 88 = -400k$, $k = 0.11$ 1/sec

& $a = -0.11v$

$a = dv/dt$, $\displaystyle\int_{88}^{v}\frac{dv}{-kv} = \int_{t_1}^{t} dt$, $\dfrac{1}{0.11}\ln\dfrac{88}{v} = t - t_1$

$t_1 = \dfrac{88}{19.36} = 4.55$ sec, $t = \dfrac{1}{0.11}\ln\dfrac{88}{44} + 4.55 = \underline{10.85 \text{ sec}}$

$\boxed{2/49}$ $a = k/x$, $v\,dv = \frac{k}{x}\,dx$

$\int_0^v v\,dv = k\int_{x_1}^{x} \frac{dx}{x}$; $\frac{v^2}{2} = k \ln \frac{x}{x_1}$

Thus $\frac{(600)^2}{2} = k \ln \frac{750}{7.5}$, $k = \frac{0.36}{2(4.605)} = 0.0391\ (km/s)^2$

at $x = 375$ mm, $a = \frac{0.0391}{375(10^{-6})} = \underline{104.2\ km/s^2}$

2/50

$$a = v\frac{dv}{ds} = 3.22 - 0.004v^2$$

$$\int_0^{v_B} \frac{v\,dv}{3.22 - 0.004v^2} = \int_0^{600} ds$$

$$\frac{1}{2(-0.004)} \ln\left(3.22 - 0.004v^2\right)\Big|_0^{v_B} = 600$$

$$\ln\left[\frac{3.22 - 0.004v_B^2}{3.22}\right] = 600(2)(-0.004)$$

$$\frac{3.22 - 0.004v_B^2}{3.22} = 0.00823$$

$$\underline{v_B = 28.3 \text{ ft/sec}}$$

2/51 Up: $a_u = -g - kv^2 = v\frac{dv}{dy}$ $\uparrow y$

$$\int_0^h dy = -\int_{v_0}^0 \frac{v\,dv}{g+kv^2}$$

$$h = -\frac{1}{2k}\ln\left[g+kv^2\right]_{v_0}^0 = \frac{1}{2k}\ln\left[\frac{g+kv_0^2}{g}\right]$$

$$h = \frac{1}{2(0.002)}\ln\left[\frac{32.2+0.002(100)^2}{32.2}\right] = \underline{120.8 \text{ ft}}$$

Down: $a_d = -g + kv^2 = v\frac{dv}{dy}$

$$\int_h^0 dy = \int_0^{v_f}\frac{v\,dv}{-g+kv^2}$$

$$-h = \frac{1}{2k}\ln\left[-g+kv^2\right]\Big|_0^{v_f} = \frac{1}{2k}\ln\left[\frac{g-kv_f^2}{g}\right]$$

$$\Rightarrow v_f = \sqrt{\frac{g}{k}\left(1-e^{-2kh}\right)}$$

$$= \sqrt{\frac{32.2}{0.002}\left(1-e^{-2(0.002)(120.8)}\right)} = \underline{78.5 \frac{\text{ft}}{\text{sec}}}$$

$\boxed{2/52}$ Up : $a_u = -g - kv^2 = \frac{dv}{dt}$

$$\int_0^{t_u} dt = -\int_{v_0}^{0} \frac{dv}{g+kv^2}$$

$$t_u = \frac{1}{\sqrt{gk}} \tan^{-1}\left(\frac{v\sqrt{gk}}{g}\right)\Bigg|_0^{v_0} = \frac{1}{\sqrt{gk}} \tan^{-1}\left(v_0\sqrt{\frac{k}{g}}\right)$$

$$t_u = \frac{1}{\sqrt{32.2(0.002)}} \tan^{-1}\left(100\sqrt{\frac{0.002}{32.2}}\right) = \underline{2.63 \text{ sec}}$$

(Down) : $a_d = -g + kv^2 = \frac{dv}{dt}$

$$\int_0^{t_d} dt = \int_0^{v_f} \frac{dv}{-g+kv^2}$$

$$t_d = \frac{1}{\sqrt{gk}} \tanh^{-1}\left(\frac{v\sqrt{gk}}{g}\right)\Bigg|_0^{v_f} = \frac{1}{\sqrt{gk}} \tanh^{-1}\left(v_f\sqrt{\frac{k}{g}}\right)$$

$$= \frac{1}{\sqrt{32.2(0.002)}} \tanh^{-1}\left(78.5\sqrt{\frac{0.002}{32.2}}\right)$$

$$= \underline{2.85 \text{ sec}}$$

(Refer to solution of Prob. 2/51)

2/53

$$a = c_1 - c_2 v^2 = v\frac{dv}{ds}$$

$$\int_0^s ds = \int_0^v \frac{v\,dv}{c_1 - c_2 v^2} = -\frac{1}{2c_2}\int_0^v \frac{-2c_2 v\,dv}{c_1 - c_2 v^2}$$

$$s = -\frac{1}{2c_2}\ln\left(c_1 - c_2 v^2\right)\Big|_0^v = \frac{1}{2c_2}\ln\left(\frac{c_1}{c_1 - c_2 v^2}\right)$$

When $s = 1320$ ft, $v = 190\left(\frac{5280}{3600}\right) = 279$ ft/sec:

$$1320 = \frac{1}{2(5)(10^{-5})}\ln\left(\frac{c_1}{c_1 - 5(10^{-5})(279)^2}\right)$$

Solve to obtain $\underline{c_1 = 31.4 \text{ ft/sec}^2}$

$\boxed{2/54}$ $a = 31.4 - 5(10^{-5})v^2 = c_1 - c_2 v^2 = \frac{dv}{dt}$

$$\int_0^t dt = \int_0^v \frac{dv}{c_1 - c_2 v^2} = \frac{1}{\sqrt{c_1 c_2}} \tanh^{-1} \sqrt{\frac{c_2}{c_1}}\, v \Big|_0^v$$

$$t = \frac{1}{\sqrt{c_1 c_2}} \tanh^{-1} \sqrt{\frac{c_2}{c_1}}\, v$$

For $v = 190 \left(\frac{5280}{3600}\right) = 279$ ft/sec,

$$t = \frac{1}{\sqrt{31.4(5)(10^{-5})}} \tanh^{-1} \sqrt{\frac{5(10^{-5})}{31.4}}\,(279)$$

$$= \underline{9.27 \text{ sec}}$$

$\boxed{2/55}$ For an acceleration of form $a = -g - kv^2$, we cite the results from Probs. 2/51 & 2/52

$$\begin{cases} t_u = \dfrac{1}{\sqrt{gk}} \tan^{-1}\left(v_0\sqrt{\dfrac{k}{g}}\right) \\ h = \dfrac{1}{2k} \ln\left[\dfrac{g + kv_0^2}{g}\right] \end{cases}$$

For the numbers at hand:

$$t_u = \frac{-1}{\sqrt{9.81\,(0.0005)}} \tan^{-1}\left(120\sqrt{\frac{0.0005}{9.81}}\right) = 10.11\text{ s}$$

$$h = \frac{1}{2(0.0005)} \ln\left[\frac{9.81 + 0.0005\,(120)^2}{9.81}\right] = 550\text{ m}$$

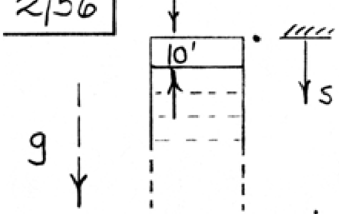
Down (v = constant): $y = y_0 + v_{y_0} t$

$$0 = 550 - 4t_d$$

$$t_d = 137.6\text{ s}$$

Flight time $t = t_u + t_d = 10.11 + 137.6 = \underline{147.7\text{ s}}$

2/56



$$s = \cancel{s_0} + \cancel{v_0 t} + \frac{1}{2} g t^2$$

When $s = 10$ ft,

$$10 = \frac{1}{2}(32.2) t_{10'}^2, \quad t_{10'} = 0.788 \text{ sec}$$

Time to pass first story from the top is $t_1 = t_{10'} - t_{0'} = 0.788 - 0 = \underline{0.788 \text{ sec}}$

10^{th} story: $90 = \frac{1}{2}(32.2) t_{90'}^2, \quad t_{90'} = 2.36$ sec

$$100 = \frac{1}{2}(32.2) t_{100'}^2, \quad t_{100'} = 2.49 \text{ sec}$$

$$t_{10} = t_{100'} - t_{90'} = 2.49 - 2.36 = \underline{0.1279 \text{ sec}}$$

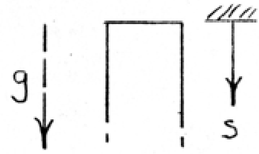
100^{th} story: $990 = \frac{1}{2}(32.2) t_{990'}^2$

$$1000 = \frac{1}{2}(32.2) t_{1000'}^2$$

$$t_{100} = t_{1000'} - t_{990'} = 7.88 - 7.84 = \underline{0.0395 \text{ sec}}$$

2/57

$$a = g - kv^2 = -\frac{dv}{dt}$$



$$\int_0^t dt = \int_0^v \frac{dv}{g - kv^2}$$

(see Art. C/10): $t = \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}}\, v \Big|_0^v$

$$= \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}}\, v$$

$$\Rightarrow \quad v = \frac{ds}{dt} = \sqrt{\frac{g}{k}} \tanh\left(\sqrt{gk}\, t\right)$$

$$\int_0^s ds = \sqrt{\frac{g}{k}} \int_0^t \tanh\left(\sqrt{gk}\, t\right)$$

$$s = \frac{1}{k} \ln \cosh \sqrt{gk}\, t$$

or $$t = \frac{\cosh^{-1}\left(e^{sk}\right)}{\sqrt{gk}} = \frac{\cosh^{-1}\left(e^{0.005s}\right)}{0.401}$$

s, ft	t, sec
0	0
10	0.795
90	2.54
100	2.70
990	14.06
1000	14.19

The time t_1 to pass first story is $t_1 = t_{10'} - t_0 = 0.795 - 0 = \underline{0.795 \text{ sec}}$

Similarly,

$\underline{t_{10} = 0.1592 \text{ sec}}$

$\underline{t_{100} = 0.1246 \text{ sec}}$

2/58

$$a = \frac{d^2x}{dt^2} = Kt - k^2x$$

or $\frac{d^2x}{dt^2} + k^2x = Kt$, a second-order, linear differential equation whose solution is

$$x = x_h + x_p = A\sin kt + B\cos kt + \frac{K}{k^2}t$$

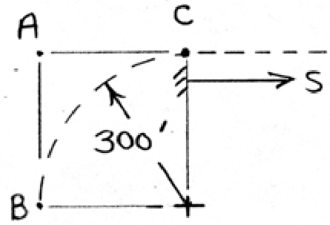
Initial conditions:

$$x(0) = B = 0$$

$$\dot{x}(0) = kA + \frac{K}{k^2} = 0, \quad A = -\frac{K}{k^3}$$

So $x = \frac{K}{k^3}(kt - \sin kt)$

2/59



First, determine B's acceleration time:

$$v = v_0 + at$$

$$65\left(\frac{44}{30}\right) = 25\left(\frac{44}{30}\right) + 3.22t$$

$$t = 18.22 \text{ sec}$$

Distance traveled by A in that time:

$$d_A = 65\left(\frac{44}{30}\right)(18.22) = 1737 \text{ ft}$$

Displacement beyond C: $s_A = 1737 - 300 = 1437$ ft

Distance traveled by B in 18.22 sec:

$$d_B = v_0 t + \frac{1}{2}at^2 = 25\left(\frac{44}{30}\right)(18.22) + \frac{1}{2}(3.22)(18.22)^2$$

$$= 1202 \text{ ft}$$

Displacement beyond C: $s_B = 1202 - \dfrac{\pi(300)}{2} = 731$ ft

So A is ahead of B by $s_A - s_B = 1437 - 731$

$= \underline{706 \text{ ft}}$ (in the steady-state)

2/60 For B, $v_f - v_0 =$ area under a-t curve

$$(65-25)\frac{44}{30} = 3.22t_1 + \frac{1}{2}(3.22)(5)$$

$$t_1 = 15.72 \text{ sec}$$

B reaches 65 mi/hr @ $15.72 + 5 = 20.7 \text{ sec} = t_2$

Speed of B @ t_1: $v_1 = 25\left(\frac{44}{30}\right) + 3.22(15.72)$

$$= 87.3 \text{ ft/sec}$$

The acceleration history $(t_1 < t < t_2)$ is $a = 13.34 - 0.644t$

$$\int_{v_1=87.3}^{v} dv = \int_{t_1=15.72}^{t} (13.34 - 0.644t)\,dt \quad \text{yields}$$

$$v = -42.9 + 13.34t - 0.322t^2$$

Then $\displaystyle\int_{s_1}^{s_2} ds = \int_{t_1=15.72}^{t_2=20.7} (-42.9 + 13.34t - 0.322t^2)\,dt$

yields $(s_2 - s_1) = 463$ ft

Distance traveled by B in 20.7 sec

$$d_B = 25\left(\frac{44}{30}\right)(15.72) + \frac{1}{2}(3.22)(15.72)^2 + 463$$

$$= 1437 \text{ ft}$$

Displacement beyond C: $s_B = 1437 - \frac{\pi(300)}{2} = 966$ ft

Distance traveled by A in 20.7 sec:

$d_A = 65\left(\frac{44}{30}\right)(20.7) = 1975$ ft

Displacement beyond C: $s_A = 1975 - 300 = 1675$ ft

So A is ahead of B by $s_A - s_B = 1675 - 966$

$= \underline{709 \text{ ft}}$ (in the steady-state)

(Not much more than the 706 ft of Prob. 2/59)

2/61

$$\underset{\sim}{a}_{av} = \frac{\Delta \underset{\sim}{v}}{\Delta t} = \frac{(-0.1\underset{\sim}{i} + 1.8\underset{\sim}{j}) - (0.1\underset{\sim}{i} + 2\underset{\sim}{j})}{0.1}$$

$$= -2\underset{\sim}{i} - 2\underset{\sim}{j} \ \text{m/s}$$

$$a_{av} = \sqrt{2^2 + 2^2} = \underline{2.83 \ \text{m/s}^2}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-2}{-2}\right) = \underline{225°}$$

2/162 $x = 3t^2 - 4t$, $\dot{x} = 6t - 4$, $\ddot{x} = 6 \text{ mm/s}^2$

$y = 4t^2 - \frac{1}{3}t^3$, $\dot{y} = 8t - t^2$, $\ddot{y} = 8 - 2t \text{ mm/s}^2$

when $t = 2$ s, $\left.\begin{aligned}\dot{x} &= 12 - 4 = 8 \text{ mm/s} \\ \dot{y} &= 16 - 4 = 12 \text{ mm/s}\end{aligned}\right\}$ $v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{8^2 + 12^2} = \underline{14.42 \frac{mm}{s}}$

$\theta_x = \tan^{-1}\frac{\dot{y}}{\dot{x}} = \tan^{-1}\frac{12}{8} = \underline{56.3°}$

$\ddot{x} = 6 \text{ mm/s}^2$, $\ddot{y} = 8 - 4 = 4 \text{ mm/s}^2$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{6^2 + 4^2} = \underline{7.21 \text{ mm/s}^2}$

$\theta_x = \tan^{-1}\frac{\ddot{y}}{\ddot{x}} = \tan^{-1}\frac{4}{6} = \underline{33.7°}$

2/63

$x = t^2 - 4t + 20$ $\qquad$ $y = 3 \sin 2t$

$\dot{x} = 2t - 4$ $\qquad$ $\dot{y} = 6 \cos 2t$

$\ddot{x} = 2$ $\qquad$ $\ddot{y} = -12 \sin 2t$

At time $t = 3$ sec :

$\dot{x} = 2$ in./sec $\qquad$ $\dot{y} = 5.76$ in./sec

$\ddot{x} = 2$ in./sec^2 $\qquad$ $\ddot{y} = 3.35$ in./sec^2

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{2^2 + 5.76^2} = \underline{6.10 \text{ in./sec}}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{2^2 + 3.35^2} = \underline{3.90 \text{ in./sec}^2}$$

$\underline{v} = 2\underline{i} + 5.76\underline{j}$ in./sec , $\underline{a} = 2\underline{i} + 3.35\underline{j}$ in./sec^2

$$\theta = \cos^{-1} \frac{\underline{v} \cdot \underline{a}}{va} = \cos^{-1}\left(\frac{2(2) + 5.76(3.35)}{(6.10)(3.90)}\right)$$

$$= \underline{11.67°}$$

2/64 $x = y^2/6$ & $\dot{y} = 3$ in./sec

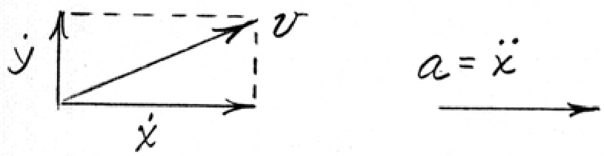
$\dot{x} = \frac{y}{3}\dot{y}$, $\ddot{x} = \frac{\dot{y}^2}{3} + \frac{y}{3}\ddot{y}$ but $\ddot{y} = 0$ & $\dot{y} = 3$ in/sec

Also when $x = 6$ in., $y = \sqrt{36} = 6$ in.

So $\dot{x} = \frac{6}{3}(3) = 6$ in/sec,

Hence $v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{6^2 + 3^2} = \underline{3\sqrt{5}\ \text{in./sec}}$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(3^2/3)^2 + 0} = \underline{3\ \text{in./sec}^2}$



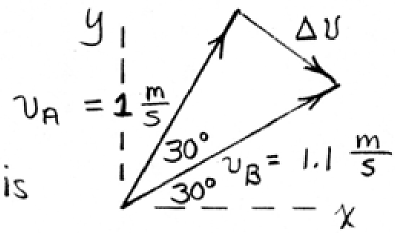
2/65 $v = \dot{s} = \frac{t}{2}$, $v_A = \frac{2}{2} = 1$ m/s, $v_B = \frac{2.2}{2} = 1.1 \frac{m}{s}$

$\Delta v_x = v_{B_x} - v_{A_x} = 1.1 \cos 30° - 1.0 \cos 60° = 0.453 \frac{m}{s}$

$\Delta v_y = v_{B_y} - v_{A_y} = 1.1 \sin 30° - 1.0 \sin 60° = -0.316 \frac{m}{s}$

$\Delta v = \sqrt{0.453^2 + 0.316^2}$

$= 0.552$ m/s



The average acceleration is

$a_{av} = \frac{\Delta v}{\Delta t} = \frac{0.552}{0.20} = \underline{2.76 \text{ m/s}^2}$

$\underline{a}_{av} = \frac{\Delta \underline{v}}{\Delta t} = \frac{0.453\underline{i} - 0.316\underline{j}}{0.20}$

$= \underline{2.26\underline{i} - 1.580\underline{j} \text{ m/s}^2}$

2/66 $x = 20 + \frac{1}{4}t^2, \quad \dot{x} = \frac{1}{2}t, \quad \ddot{x} = \frac{1}{2} \text{ mm/s}^2$

$y = 15 - \frac{1}{6}t^3, \quad \dot{y} = -\frac{1}{2}t^2, \quad \ddot{y} = -t \text{ mm/s}^2$

For $t = 2$ s, $\dot{x} = 1$ mm/s

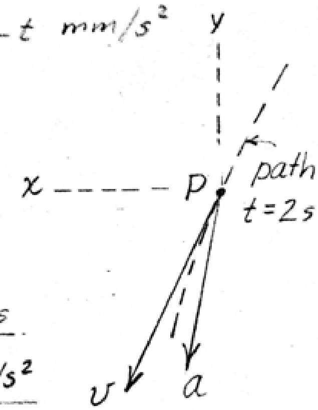
$\dot{y} = -2$ mm/s

$\ddot{x} = \frac{1}{2}$ mm/s^2

$\ddot{y} = -2$ mm/s^2

$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{1^2 + (-2)^2} = \underline{2.24 \text{ mm/s}}$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(1/2)^2 + (-2)^2} = \underline{2.06 \text{ mm/s}^2}$



2/67 $\underline{r} = \left(\frac{2}{3}t^3 - \frac{3}{2}t^2\right)\underline{i} + \left(\frac{t^4}{12}\right)\underline{j}$

$$\underline{v} = \dot{\underline{r}} = \left(2t^2 - 3t\right)\underline{i} + \left(\frac{1}{3}t^3\right)\underline{j}$$

$$\underline{a} = \dot{\underline{v}} = \left(4t - 3\right)\underline{i} + \left(t^2\right)\underline{j}$$

At $t = 2\text{s}$ $\begin{cases} \underline{v} = (2\cdot 2^2 - 3\cdot 2)\underline{i} + \frac{1}{3}2^3\underline{j} = 2\underline{i} + \frac{8}{3}\underline{j} \ \text{mm/s} \\ \underline{a} = (4\cdot 2 - 3)\underline{i} + 2^2\underline{j} = 5\underline{i} + 4\underline{j} \ \text{mm/s}^2 \end{cases}$

$$\cos\theta = \frac{\underline{v}\cdot\underline{a}}{va} = \frac{\left(2\underline{i} + \frac{8}{3}\underline{j}\right)\cdot\left(5\underline{i} + 4\underline{j}\right)}{\sqrt{2^2 + \left(\frac{8}{3}\right)^2}\sqrt{5^2 + 4^2}}$$

$$\underline{\theta = 14.47°}$$

At $t = 3\text{s}$ $\begin{cases} \underline{v} = (2\cdot 3^2 - 3\cdot 3)\underline{i} + \left(\frac{1}{3}3^3\right)\underline{j} = 9\underline{i} + 9\underline{j} \ \frac{\text{mm}}{\text{s}} \\ \underline{a} = (4\cdot 3 - 3)\underline{i} + (3^2)\underline{j} = 9\underline{i} + 9\underline{j} \ \frac{\text{mm}}{\text{s}^2} \end{cases}$

$$\underline{v} \parallel \underline{a} \Rightarrow \underline{\theta = 0}$$

2/68

$$\begin{cases} x = 3\cos 4t; & \dot{x} = -12\sin 4t; & \ddot{x} = -48\cos 4t \\ y = 2\sin 4t; & \dot{y} = 8\cos 4t; & \ddot{y} = -32\sin 4t \end{cases}$$

At time $t = 1.4$ sec :

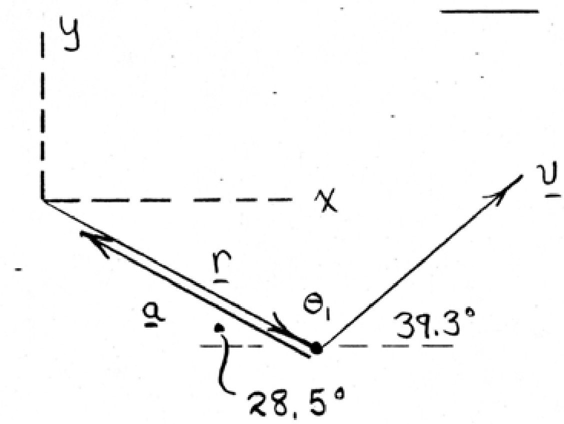
$$\begin{cases} x = 2.33 \text{ ft}; & \dot{x} = 7.58 \text{ ft/sec}; & \ddot{x} = -37.2 \frac{\text{ft}}{\text{sec}^2} \\ y = -1.263 \text{ ft}; & \dot{y} = 6.20 \text{ ft/sec}; & \ddot{y} = 20.2 \frac{\text{ft}}{\text{sec}^2} \end{cases}$$

$r = 2.65$ ft ; $v = 9.79$ ft/sec ; $a = 42.4 \frac{\text{ft}}{\text{sec}^2}$

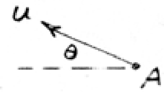
$$\theta_1 = \cos^{-1}\left[\frac{\underline{a}\cdot\underline{v}}{a\,v}\right] = \cos^{-1}\left[\frac{-37.2(7.58)+20.2(6.20)}{42.4(9.79)}\right]$$

$\underline{\theta_1 = 112.2°}$ Similarly, $\theta_2 = \cos^{-1}\left[\frac{\underline{a}\cdot\underline{r}}{a\,r}\right]$

Sketch (not to scale): $= \underline{180°}$



2/69 From Sample Prob. 2/6



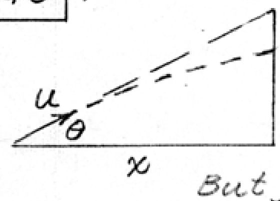
$$2s = \frac{u^2 \sin 2\theta}{g} = \frac{2(u\cos\theta)(u\sin\theta)}{g}$$

But $2s = 22$ ft, $u\cos\theta = 30$ ft/sec, $u\sin\theta = v_y$

so $$v_y = \frac{2sg}{2u\cos\theta} = \frac{22(32.2)}{2(30)} = \underline{11.81 \text{ ft/sec}}$$

Also, $$h = \frac{u^2\sin^2\theta}{2g} = v_y^2/2g = \frac{(11.81)^2}{2(32.2)} = \underline{2.16 \text{ ft}}$$

2/170 $x = 1000$ m $\quad u = 600$ m/s $\quad \theta = 20°$



From Sample Problem 2/6

$$y = x \tan\theta - \frac{g x^2}{2u^2} \sec^2\theta$$

But $y + \delta = x \tan\theta$, so $\delta = \dfrac{9.81(1000)^2}{2(600)^2} \dfrac{1}{(0.9397)^2}$

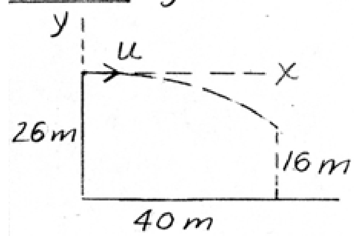
$\underline{\delta = 15.43 \text{ m}}$

$\boxed{2/71}$ Set up x-y axes at the initial location of G.

$$\left.\begin{aligned} x &= x_0 + v_{x_0} t &&: \quad 3 = (v_0 \cos\theta)t \\ y &= y_0 + v_{y_0} t - \tfrac{1}{2} g t^2 &&: \quad 3.5 = (v_0 \sin\theta)t - 16.1 t^2 \\ v_y &= v_{y_0} - g t &&: \quad 0 = v_0 \sin\theta - 32.2 t \end{aligned}\right\}$$

Solve simultaneously : $\left\{\begin{aligned} t &= 0.466 \text{ sec} \\ \underline{v_0} &\underline{= 16.33 \text{ ft/sec}} \\ \underline{\theta} &\underline{= 66.8°} \end{aligned}\right.$

2/72 $a_y = -g$ so $y = 0 - \frac{1}{2}gt^2$, $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(26-16)}{9.81}}$

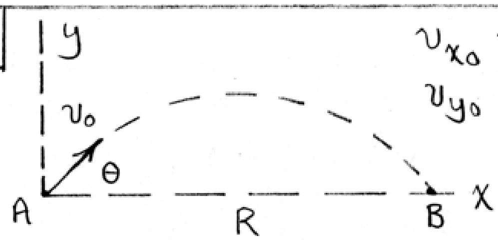


$= 1.428\ s$

$x = ut$; $u = 40/1.428$

$= \underline{28.0\ m/s}$

2/73



$v_{x_0} = v_0 \cos\theta$

$v_{y_0} = v_0 \sin\theta$

$x = x_0 + v_{x_0} t$ @ B: $R = 0 + (v_0 \cos\theta) t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B: $0 = 0 + (v_0 \sin\theta) t_f - \frac{g}{2} t_f^2$ (2)

(2): $t_f = 0, \ \dfrac{2 v_0 \sin\theta}{g}$ ($t = 0$ is launch time)

(1): $R = (v_0 \cos\theta)\left(\dfrac{2 v_0 \sin\theta}{g}\right) = \dfrac{v_0^2 \sin 2\theta}{g}$

$\dfrac{dR}{d\theta} = 0: \ \dfrac{v_0^2}{g} 2 \cos 2\theta = 0 \Rightarrow \underline{\theta = 45^\circ}$

$R_{max} = \dfrac{v_0^2 \sin(2 \cdot 45^\circ)}{g} = \underline{\dfrac{v_0^2}{g}}$

2/74 Use x-y coordinates of the figure.

(a) $v_0 = 45$ ft/sec

$x = x_0 + v_{x_0} t$ @ left wall : $30 = 0 + 45 \cos 60° t$

$t = 1.333$ sec

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$: $y = 5 + 45 \sin 60° (1.333) - 16.1 (1.333)^2$

$= 28.3$ ft (hits wall)

Ans. : $\underline{(x, y) = (30', 28.3')}$

(b) $v_0 = 60$ ft/sec

Repeat above procedure to find $y = 40.9'$ when $x = 30'$, so water clears left wall.

$x = x_0 + v_{x_0} t$ @ right wall : $50 = 0 + 60 \cos 60° t$

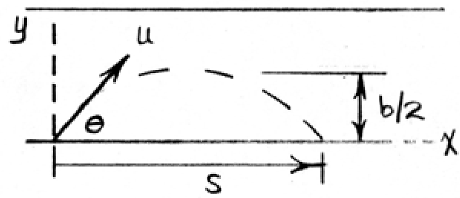
$t = 1.667$ sec

y eq. yields $y = 46.9$ ft @ $t = 1.667$ sec, so water clears building! For horizontal range: From $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ $\underline{y = 0}$, $y_0 = 5$ ft, we find $t = -0.0935$ s & $t = 3.32$ s. From $x = x_0 + v_{x_0} t$: $x = 0 + 60 \cos 60° (3.32) = \underline{99.6 \text{ ft}}$

2/75

$a_y = -\frac{eE}{m}$, constant

$a_x = 0$



$v_y^2 - v_{y_0}^2 = 2ay$: At top, $0 - (u\sin\theta)^2 = 2\left(-\frac{eE}{m}\right)\frac{b}{2}$

$$\underline{E = \frac{mu^2\sin^2\theta}{eb}}$$

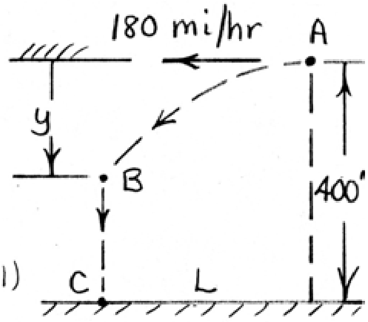
$v_y = v_{y_0} + a_y t$: At top, $0 = u\sin\theta - \frac{eE}{m}t$

$$t = \frac{mu\sin\theta}{eE}$$

$x = v_{x_0}t$: $s = (u\cos\theta)(2t) = u\cos\theta\left(\frac{2mu\sin\theta}{eE}\right)$

$$= u\cos\theta\left(\frac{2mu\sin\theta}{e\,\frac{mu^2\sin^2\theta}{eb}}\right) = \underline{2b\cot\theta}$$

2/76



$t = 0$: package dropped at A

t_B, t_C: times package at point B, C

From A to B: $y = \frac{1}{2} g t_B^2$ (1)

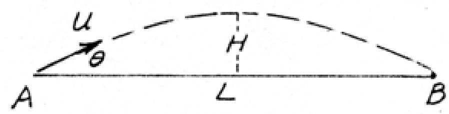
From B to C: $(400 - y) = 6(t_C - t_B)$ (2)

Also, $t_C = 37$ sec. Solve (1) & (2) to obtain $t_B = 3.52$ sec, $y = 199.1$ ft

$L = 180\left(\frac{5280}{3600}\right)(3.52) = \underline{928 \text{ ft}}$

2/77 From Sample Prob. 2/6, $H = \frac{u^2 \sin^2\theta}{2g}$ ---- (a)

$L = 2s = \frac{u^2 \sin 2\theta}{g}$ ---- (b)



So $\frac{H}{L} = \frac{\sin^2\theta}{2(2\sin\theta\cos\theta)}$

$= \frac{1}{4}\tan\theta$

Thus $\underline{\theta = \tan^{-1}(4H/L)}$

From (a) $\sin\theta = \sqrt{2gH}/u$

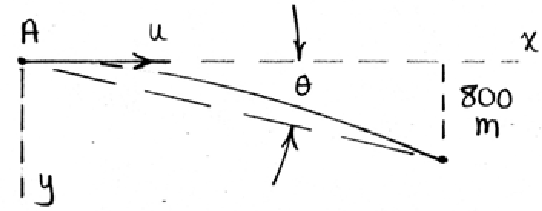
" (b) $Lg/u^2 = 2\sin\theta\cos\theta = 2\sin\theta\sqrt{1-\sin^2\theta}$

$= 2\frac{\sqrt{2gH}}{u}\sqrt{1-\frac{2gH}{u^2}}$

Simplify, solve for u & get $\underline{u = \sqrt{2gH}\sqrt{1+\left(\frac{L}{4H}\right)^2}}$

2/78

$$u = \frac{1000}{3.6} = 278 \ \frac{m}{s}$$



y-dir. : $y = v_{y_0} t + \frac{1}{2} g t^2$

$800 = 0 + \frac{1}{2}(9.81)t^2, \quad t = 12.77 \text{ s}$

x-dir. : $x = v_{x_0} t + \frac{1}{2} a_x t^2$

$= 278(12.77) + \frac{1}{2}\left(\frac{9.81}{2}\right)(12.77)^2$

$= 3950 \text{ m}$

$\theta = \tan^{-1} \frac{800}{3950} = \underline{11.46^\circ}$

2/79 Set up x-y coordinates with origin at A.

$x = x_0 + v_{x_0} t$ @ B: $800 + s \cos 20° = (120 \cos 40°) t$ (1)

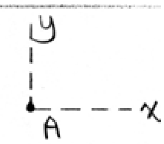
$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B:

$-s \sin 20° = (120 \sin 40°) t - \frac{9.81}{2} t^2$ (2)

Solve (1) & (2) simultaneously to obtain

$\underline{s = 1057 \text{ m}}$, $\underline{t = 19.50 \text{ s}}$

$\boxed{2/80}$ (a) $v_0 = 140$ ft/sec and $\theta = 8°$:



$x = x_0 + v_{x_0} t$ @ B: $200 = 0 + (140 \cos 8°)t$

$\underline{t = 1.443 \text{ sec}}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B:

$-(7.5 - h) = 0 + 140 \sin 8° (1.443) - \frac{1}{2}(32.2)(1.443)^2$

$\underline{h = 2.10 \text{ ft}}$

(b) $v_0 = 120$ ft/sec and $\theta = 12°$:

$x = x_0 + v_{x_0} t$ @ B: $200 = 0 + (120 \cos 12°)t$

$\underline{t = 1.704 \text{ sec}}$

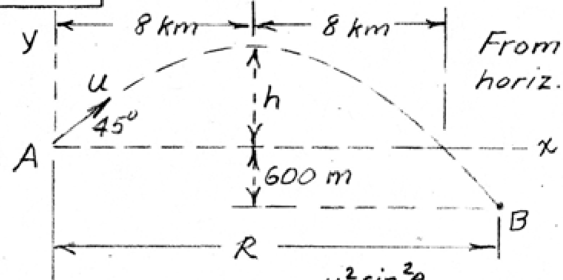
$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B:

$-(7.5 - h) = 0 + (120 \sin 12°)(1.704) - \frac{1}{2}(32.2)(1.704)^2$

$\underline{h = 3.27 \text{ ft}}$

(In baseball, the time of flight is critical; low trajectories, even with one hop, are better.)

2/81



From Sample Prob. 2/6 horiz. range is $2s = \dfrac{u^2 \sin 2\theta}{g}$

so $16(10^3) = \dfrac{u^2 \sin 90°}{9.81}$

$u = \sqrt{157000} = \underline{396\ m/s}$

Max. altitude $h = \dfrac{u^2 \sin^2\theta}{2g}$, $H = h + 600 = \dfrac{(396)^2 \sin^2 45°}{2(9.81)} + 600$

$= 4000 + 600 = \underline{4600\ m}$

$y = ut\sin\theta - \frac{1}{2}gt^2$, $-600 = 396(0.7071)t - \frac{1}{2}(9.81)t^2$

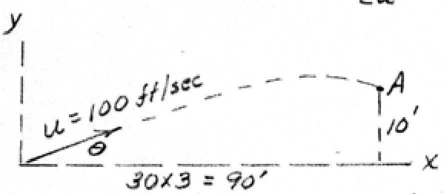
$t^2 - 57.11t - 122.3 = 0$, $t = \frac{57.11}{2} \pm \frac{1}{2}\sqrt{3262 + 489}$

$= 59.18\ s \quad (\text{or } -2.07\ s)$

$x = ut\cos\theta$, $R = 396(59.18)\cos 45° = 16\ 579\ m$

or $R = \underline{16.58\ km}$

2/82 $\quad y = x\tan\theta - \dfrac{gx^2}{2u^2}\sec^2\theta$



Let $m = \tan\theta$

$\sec^2\theta = 1 + \tan^2\theta = 1 + m^2$

$y = xm - \dfrac{gx^2}{2u^2}(1+m^2)$, $\quad m^2 - \dfrac{2u^2}{gx}m + \left(\dfrac{2u^2y}{gx^2} + 1\right) = 0$

At A, $\quad m^2 - \dfrac{2(10^2)^2}{32.2(90)}m + \left(\dfrac{2(10^2)^2 10}{32.2(90)^2} + 1\right) = 0$

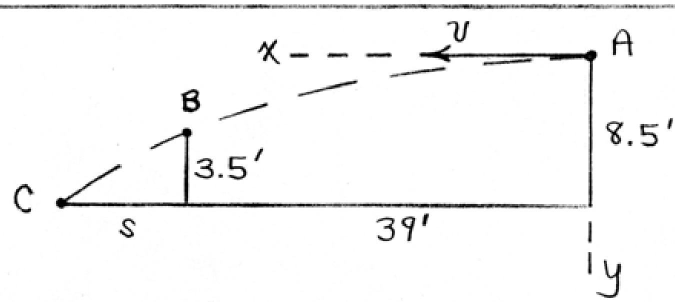
$$m^2 - 6.901\,m + 1.7668 = 0$$

$$m = \frac{6.901}{2} \pm \frac{1}{2}\sqrt{(6.901)^2 - 4(1.7668)}$$

$$= \frac{6.901 \pm \sqrt{40.56}}{2} = 0.266 \text{ or } 6.635$$

$$\theta = \tan^{-1} m = \underline{14.91°} \qquad (\text{or } 81.4°)$$

2/83



$a_x = 0$: $x = v_{x_0} t$, $39 = v t_B$

$a_y = g$: $y = v_{y_0} t + \frac{1}{2} g t^2$

At B : $8.5 - 3.5 = 0 + \frac{1}{2} 32.2 t_B^2$, $t_B = 0.557$ sec

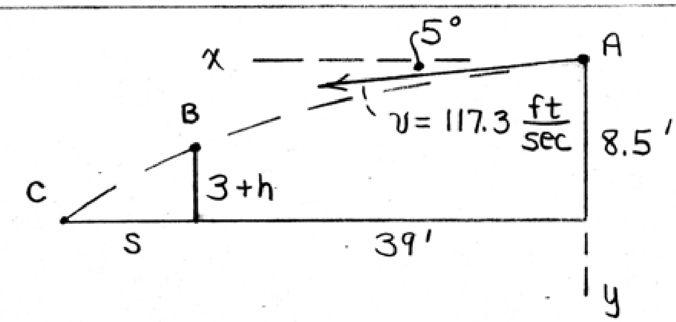
Then $v = \frac{39}{t_B} = \frac{39}{0.557} = 70.0$ ft/sec

(47.7 mi/hr)

At C : $8.5 = \frac{1}{2}(32.2) t_C^2$, $t_C = 0.727$ sec

$s + 39 = 70.0(0.727)$, $s = 11.85$ ft

2/84



$a_x = 0$, $x = v_{x_0} t$: $39 = 117.3 \cos 5° \, t_B$, $t_B = 0.334$ sec

$a_y = g$, $y = v_{y_0} t + \frac{1}{2} g t^2$: At B,

$$5.5 - h = 117.3 \sin 5° (0.334) + \frac{1}{2}(32.2)(0.334)^2$$

$$\underline{h = 0.296 \text{ ft}} \quad \text{or} \quad \underline{h = 3.55 \text{ in.}}$$

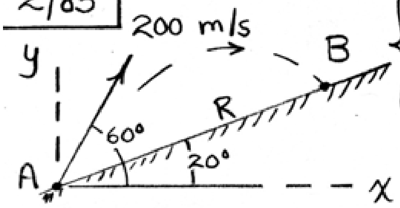
At C: $8.5 = 117.3 \sin 5° \, t_C + 16.1 \, t_C^2$

$$t_C = 0.475 \text{ sec}$$

x-equation at C: $39 + s = 117.3 \cos 5° (0.475)$

$$\underline{s = 16.57 \text{ ft}}$$

2/85



$$\begin{cases} v_{x_0} = 200\cos 60° = 100 \text{ m/s} \\ v_{y_0} = 200 \sin 60° = 173.2 \text{ m/s} \end{cases}$$

t_f = flight time

$x = x_0 + v_{x_0} t$ @ B: $R\cos 20° = 100\, t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B: $R \sin 20° = 173.2\, t_f - \frac{9.81}{2} t_f^2$ (2)

(1): $t_f = 0.00940\, R$

(2): $R \sin 20° = 173.2(0.00940\, R) - \frac{9.81}{2}(0.00940 R)^2$

$\underline{R = 2970 \text{ m}}$

2/86 x-y coordinates with origin at release point :

$$v_{x_0} = v_0 \sin\theta = 12 \sin\theta$$

$$v_{y_0} = v_0 \cos\theta = 12 \cos\theta$$

$v_y = v_{y_0} - gt$ applied at end of flight:

$-12\cos\theta = 12\cos\theta - 9.81 t_f$, $t_f = 2.45\cos\theta$

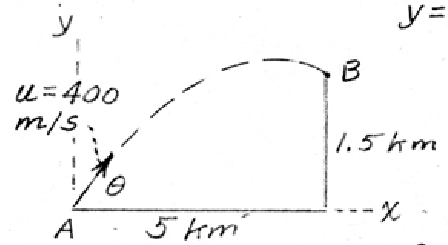
$v_x = v_{x_0} - 0.4t$ applied at end of flight:

$-12\sin\theta = 12\sin\theta - 0.4(2.45\cos\theta)$

$24\sin\theta = 0.979\cos\theta$, $\tan\theta = 0.041$

$\underline{\theta = 2.33°}$

2/87 Eq. of trajectory (Sample Problem 2/6)



$$y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$$

$$= x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$$

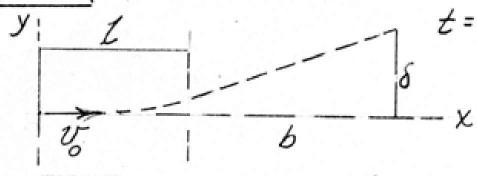
Substitute values & get

$$1500 = 5000\tan\theta - \frac{9.81(5000)^2}{2(400)^2}(1+\tan^2\theta)$$

or $\tan^2\theta - 6.524\tan\theta + 2.957 = 0$

solution gives roots $\underline{\theta_1 = 26.1°}$ & $\underline{\theta_2 = 80.6°}$

2/88 Time to travel through plates is



$t = l/v_0$ since $v_x = v_0 =$ constant

$v_y = a_y t = \frac{eE}{m}\frac{l}{v_0}$ upon emergence from plates

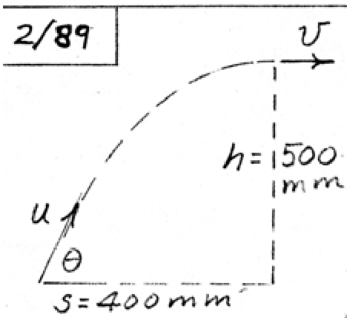
& deflection at this point is

$$y = \frac{1}{2}a_y t^2 = \frac{eE}{2m}\frac{l^2}{v_0^2}$$

Also $\delta - y/b = v_y/v_x$ so $\delta = \frac{eE}{2m}\frac{l^2}{v_0^2} + b\,\frac{eE}{m}\frac{l}{v_0}\Big/v_0$

$$\underline{\delta = \frac{eEl}{mv_0^2}\left(\frac{l}{2} + b\right)}$$

2/89



From results of Sample Problem 2/6

$$h = \frac{u^2 \sin^2\theta}{2g}, \quad s = \frac{u^2 \sin 2\theta}{2g}$$

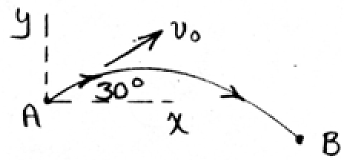
so $\frac{h}{s} = \frac{\sin^2\theta}{\sin 2\theta} = \frac{1}{2}\tan\theta$

$$\theta = \tan^{-1}\frac{2h}{s} = \tan^{-1}\frac{2(500)}{400} = \underline{68.2^\circ}$$

$$u^2 = \frac{2gh}{\sin^2\theta} = \frac{2(9.81)(0.5)}{(0.9285)^2} = 11.38 \text{ m}^2/\text{s}^2, \quad u = 3.37 \text{ m/s}$$

$$v = u\cos\theta = \text{const.}, \quad v = 3.37(0.3714) = \underline{1.253 \text{ m/s}}$$

2/90 Set up x-y axes at A, target at B:



x-eq. : $x_B = (v_0 \cos 30°)t$

y-eq. : $y_B = (v_0 \sin 30°)t - \frac{1}{2}gt^2$

For $x_B = 12'$, $y_B = -0.333'$: $\begin{cases} v_0 = 20.6 \text{ ft/sec} \\ t = 0.672 \text{ sec} \end{cases}$

For $x_B = 14'$, $y_B = -0.333'$: $\begin{cases} v_0 = 22.4 \frac{\text{ft}}{\text{sec}} \\ t = 0.723 \text{ sec} \end{cases}$

So the range is $\underline{20.6 \leq v_0 \leq 22.4 \text{ ft/sec}}$

2/91 Set up x-y coordinates at A

x-eq.: $x_B = (36 \cos\theta)t$

y-eq.: $y_B = (36 \sin\theta)t - 16.1t^2$

Solutions:

For $x_B = 40'$, $y_B = -\frac{22}{12}'$ (top of stake):

$\theta = 34.3^\circ$ or $\theta = 53.1^\circ$

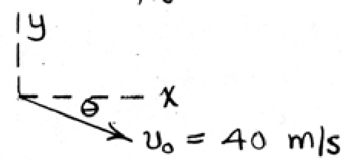
For $x_B = 40'$, $y_B = -3'$ (bottom of stake):

$\theta = 31.0^\circ$ or $\theta = 54.7^\circ$

Ranges: $31.0^\circ \leq \theta \leq 34.3^\circ$

or $\underline{53.1^\circ \leq \theta \leq 54.7^\circ}$

2/92 Set up x-y coordinates with origin at release point :



$x = x_0 + v_{x_0} t$ at mitt : $20 = (40 \cos\theta) t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ at mitt :

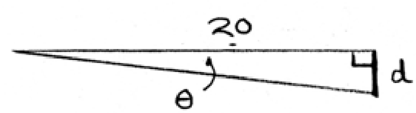
$-1.8 = 0 + (-40 \sin\theta) t_f - \frac{9.81}{2} t_f^{\,2}$ (2)

(1) : $t_f = \dfrac{1}{2\cos\theta}$

(2) : $-1.8 = -40 \sin\theta \left(\dfrac{1}{2\cos\theta}\right) - \dfrac{9.81}{2}\left(\dfrac{1}{2\cos\theta}\right)^2$

Use $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$: $1.226 \tan^2\theta + 20 \tan\theta - 0.574 = 0$

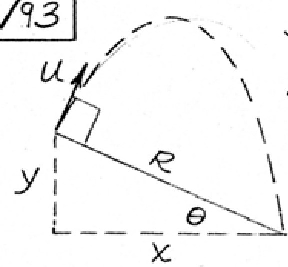
$\Rightarrow \theta = 1.640°$



$d = 20 \tan 1.640$
$= 0.573'$

$h = (2.2 + 0.6) - (0.573 + 1) = \underline{1.227 \text{ m}}$

2/93



$y = R\sin\theta = -ut\cos\theta + \frac{1}{2}gt^2$

$x = R\cos\theta = ut\sin\theta$

Eliminate t and get

$R\sin\theta = -R\frac{\cos\theta}{\sin\theta}\cos\theta + \frac{1}{2}g\left(\frac{R\cos\theta}{u\sin\theta}\right)^2$

$\frac{1}{\sin\theta} = \frac{g}{2}\;\frac{R}{u^2\tan^2\theta}$

$R = \frac{2u^2\tan^2\theta}{g\sin\theta} = \underline{\frac{2u^2}{g}\tan\theta\sec\theta}$

2/94 Use x-y coordinates with origin at the release point : $\begin{array}{l} \vdots y \\ \llcorner - - x \end{array}$

$x = x_0 + v_{x_0} t$ @ hoop : $13.75 = 0 + (v_0 \cos 50°) t_f$

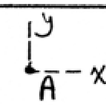
$t_f = 21.4/v_0$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ hoop :

$3 = 0 + v_0 \sin 50° \left(\frac{21.4}{v_0} \right) - 16.1 \left(\frac{21.4}{v_0} \right)^2$

$\underline{v_0 = 23.5 \text{ ft/sec}}$

2/95 Set up x-y coordinates at A :



From $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ evaluated at D,

we have $0 = 0 + v_0 \sin\alpha \, t_D - \frac{1}{2} g t_D^2$ or $t_D = \dfrac{2 v_0 \sin\alpha}{g}$

From $v_y^2 = v_{y_0}^2 - 2g(y - y_0)$ evaluated at apex, we

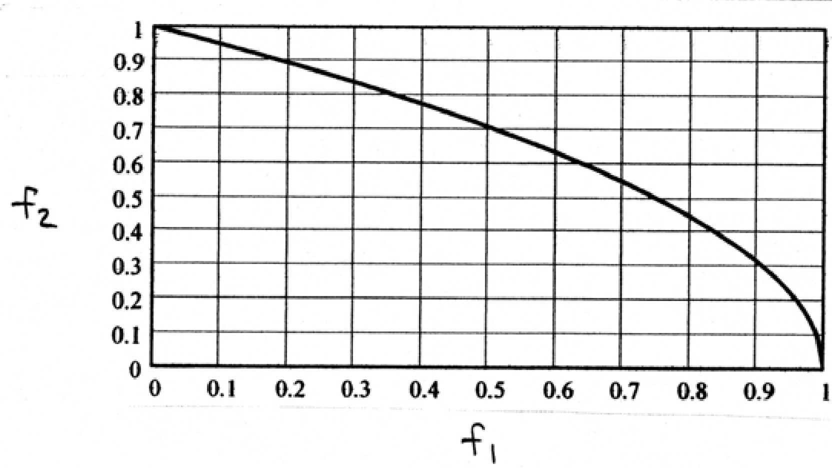
have $0 = v_0^2 \sin^2\alpha - 2gh$ or $h = \dfrac{v_0^2 \sin^2\alpha}{2g}$

Time to $f_1 h$: $f_1 h = f_1 \dfrac{v_0^2 \sin^2\alpha}{2g} = (v_0 \sin\alpha) t - \frac{1}{2} g t^2$

Solve quadratic to obtain
$$\begin{cases} t_B = \dfrac{v_0 \sin\alpha \left(1 - \sqrt{1 - f_1}\right)}{g} \\ t_C = \dfrac{v_0 \sin\alpha \left(1 + \sqrt{1 - f_1}\right)}{g} \end{cases}$$

So $t_{BC} = t_C - t_B = \dfrac{2\sqrt{1 - f_1} \, v_0 \sin\alpha}{g}$

$f_2 = \dfrac{t_{BC}}{t_D} = \sqrt{1 - f_1}$; $f_2 = \frac{1}{2}$ for $f_1 = \frac{3}{4}$.



2/96 With x-y coordinates, origin at A: $\overset{\vdots y}{A\!\bullet\text{- - -}x}$

$x = x_0 + v_{x_0} t$ @ B: $360 = 0 + (100\cos\alpha)t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2}gt^2$ @ B: $-80 = 0 + (100\sin\alpha)t_f - \frac{1}{2}(32.2)t_f^2$ (2)

Simultaneous solutions of (1) & (2):

$$\begin{cases} t_f = 4.03 \text{ sec}, & \alpha = 26.8° \quad (a) \\ t_f = 5.68 \text{ sec}, & \alpha = 50.7° \quad (b) \end{cases}$$

Check at corner $[(x, y) = (280', 0)]$:

(a)

$$t_c = \frac{280}{100\cos 26.8°} = 3.14 \text{ sec}$$

$$y_c = 100\sin 26.8°(3.14) - \frac{32.2}{2}(3.14)^2 = -16.94 \text{ ft}$$

So conditions (a) are not possible.

(b) $$t_c = \frac{280}{100\cos 50.7°} = 4.42 \text{ sec}$$

$$y_c = 100\sin\text{-}50.7°(4.42) - \frac{32.2}{2}(4.42)^2 = 27.5 \text{ ft}$$

Conditions (b) result in clearance at corner

Ans. $\underline{\alpha = 50.7°}$

▶2/97

$$\underline{a} = -k\underline{v} - g\underline{j}$$

$$a_x\underline{i} + a_y\underline{j} = -k(v_x\underline{i} + v_y\underline{j}) - g\underline{j}$$

$\therefore$ $a_x = -kv_x$, $a_y = -kv_y - g$

$\underline{x}$:

$$a_x = \frac{dv_x}{dt} = -kv_x$$

$$\int_{v_{x_0}}^{v_x} \frac{dv_x}{v_x} = -\int_0^t k\,dt \Rightarrow v_x = v_{x_0} e^{-kt}$$

or $\underline{v_x = (v_0 \cos\theta) e^{-kt}}$

$$v_x = \frac{dx}{dt} = v_{x_0} e^{-kt}$$

$$\int_0^x dx = \int_0^t v_{x_0} e^{-kt} dt$$

$$x = \frac{v_{x_0}}{k}\left[1 - e^{-kt}\right] = \underline{\frac{v_0\cos\theta}{k}\left[1 - e^{-kt}\right]}$$

$\underline{y}$:

$$a_y = \frac{dv_y}{dt} = -kv_y - g$$

$$\int_{v_{y_0}}^{v_y} \frac{dv_y}{kv_y + g} = -\int_0^t dt$$

$$\Rightarrow v_y = \left[v_{y_0} + \frac{g}{k}\right]e^{-kt} - \frac{g}{k} = \underline{\left[v_0 \sin\theta + \frac{g}{k}\right]e^{-kt} - \frac{g}{k}}$$

$$v_y = \frac{dy}{dt} = \left[v_{y_0} + \frac{g}{k}\right] e^{-kt} - \frac{g}{k}$$

$$\int_0^y dy = \int_0^t \left\{ \left[v_{y_0} + \frac{g}{k}\right] e^{-kt} - \frac{g}{k} \right\} dt$$

$$\underline{y = \frac{1}{k}\left[v_0 \sin\theta + \frac{g}{k}\right]\left[1 - e^{-kt}\right] - \frac{g}{k} t}$$

Terminal velocity $(t \to \infty)$: $v_x \to 0$

$$\underline{v_y \to -\frac{g}{k}}$$

▸2/98 Use the x-y coordinates of the figure. In the absence of the circular surface B-C, the range over a horizontal surface would be $\frac{v_0^2}{g}\sin 2\theta = \frac{225^2}{32.2}\sin(2\cdot 30°) =$ 1362 ft > 1000 ft, so the impact point is beyond B.

$$x = x_0 + v_{x_0} t \; : \quad x = (225\cos 30°)\, t \qquad (1)$$

$$y = y_0 + v_{y_0} t - \tfrac{1}{2} g t^2 : \; y = (225\sin 30°)t - 16.1t^2 \qquad (2)$$

Surface constraint:

$$(x-1000)^2 + (y-500)^2 = 500^2 \qquad (3)$$

Computer solution of Eqs. (1), (2), & (3):

$$\begin{cases} \underline{x = 1242 \text{ ft}} \\ \underline{y = 62.7 \text{ ft}} \end{cases} \qquad t = 6.38 \text{ sec}$$

(The nature of the solution indicates impact on the circular surface and not above C.)

▸2/99 $a_y = g$, $v_y = gt$, $y = \frac{1}{2}gt^2$

$a_x = 2v_y \omega \cos\gamma = 2gt\omega\cos\gamma$

$v_x = gt^2 \omega \cos\gamma$

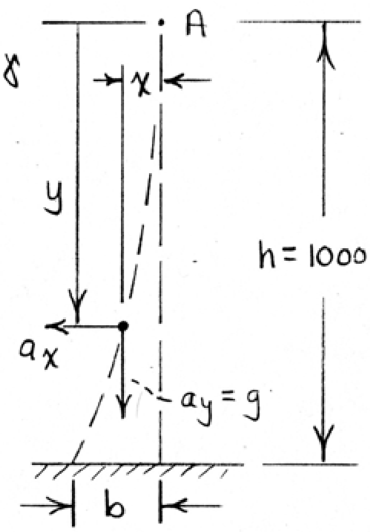
$x = \frac{1}{3} g t^3 \omega \cos\gamma$

When $y = h$, $t = \sqrt{\dfrac{2h}{g}}$

Then

$$x = b = \frac{1}{3} g \left(\frac{2h}{g}\right)^{3/2} \omega \cos\gamma$$

$$b = \frac{2\sqrt{2}}{3} h \sqrt{\frac{h}{g}}\, \omega \cos\gamma$$



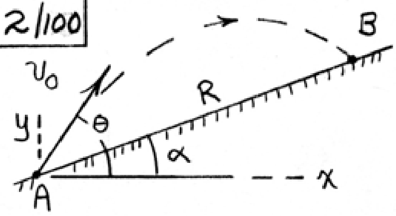
For $h = 1000$ ft, $\omega = 0.7292(10^{-4})$ rad/sec,

and $g = 32.13$ ft/sec^2,

$$b = \frac{2\sqrt{2}}{3}(1000)\sqrt{\frac{1000}{32.13}}\,(0.7292 \cdot 10^{-4}) \cos 30°$$

$= 0.332$ ft or $\underline{3.99 \text{ in.}}$

▶ 2/100



$x = x_0 + v_{x_0} t$ @ B: $\quad R\cos\alpha = (v_0 \cos\theta) t_f$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B: $R \sin\alpha = (v_0 \sin\theta) t_f - \frac{1}{2} g t_f^2$

x-eq: $\quad t_f = \dfrac{R\cos\alpha}{v_0 \cos\theta}$

y-eq: $\quad R\sin\alpha = (v_0 \sin\theta)\left(\dfrac{R\cos\alpha}{v_0\cos\theta}\right) - \dfrac{1}{2} g \left(\dfrac{R\cos\alpha}{v_0\cos\theta}\right)^2$

$\Rightarrow \quad R = \dfrac{2 v_0^2 \cos^2\theta}{g \cos\alpha} (\tan\theta - \tan\alpha)$

$\dfrac{dR}{d\theta} = 0: \quad \dfrac{4 v_0^2 \cos\theta(-\sin\theta)}{g\cos\alpha}(\tan\theta - \tan\alpha) + \dfrac{2 v_0^2 \cos^2\theta}{g\cos\alpha} \dfrac{1}{\cos^2\theta} = 0$

$\dfrac{2 v_0^2}{g\cos\alpha}\left[2\cos\theta\sin\theta(\tan\alpha - \tan\theta) + 1\right] = 0$

$\Rightarrow \quad 2\cos\theta\sin\theta\left(\tan\alpha - \dfrac{\sin\theta}{\cos\theta}\right) + 1 = 0$

$(2\cos\theta\sin\theta)\tan\alpha - 2\sin^2\theta + 1 = 0$

$\sin 2\theta \tan\alpha - 2\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) + 1 = 0$

$\sin 2\theta \tan\alpha + \cos 2\theta = 0$

$\tan 2\theta = -\dfrac{1}{\tan\alpha}$

$$2\theta = \tan^{-1}\left(-\frac{1}{\tan\alpha}\right) = 180^\circ - \tan^{-1}\left(\frac{1}{\tan\alpha}\right)$$

$$= 180^\circ - (90^\circ - \alpha) = 90^\circ + \alpha$$

$$\therefore \quad \theta = \underline{\frac{90^\circ + \alpha}{2}}$$

Specific resutts :

$$\begin{cases} \alpha = 0, & \theta = 45^\circ \\ \alpha = 30^\circ, & \theta = 60^\circ \\ \underline{\alpha = 45^\circ,} & \underline{\theta = 67.5^\circ} \end{cases}$$

2/101 For a_t = constant, $v = \cancel{v_0}^{0} + a_t t$

So $a_t = \dfrac{v}{t} = \dfrac{100\,(1000)}{3600}\Big/10 = 2.78\ \text{m/s}^2$

$v_8 = 2.78(8) = 22.2\ \text{m/s}$

$a_n = \dfrac{v^2}{\rho} = \dfrac{22.2^2}{80} = 6.17\ \text{m/s}^2$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2.78^2 + 6.17^2} = \underline{6.77\ \text{m/s}^2}$

2/102 a_1 : speed v is increasing, no path curvature.

a_2 : speed v increasing, car turning right.

a_3 : speed v stationary, car turning right.

a_4 : speed v decreasing, car turning right.

a_5 : speed v decreasing, no path curvature.

a_6 : speed v decreasing, car turning left.

a_7 : speed v stationary, car turning left.

a_8 : speed v increasing, car turning left.

$\boxed{2/103}$ $a_n = v^2/r = (0.6)^2/0.3 = 1.2 \text{ m/s}^2$

(a) $a_t = \dot{v} = 0$ so $\underline{a = a_n = 1.2 \text{ m/s}^2}$

(b) $a_t = \dot{v} = 0.9 \text{ m/s}^2$ so $a = \sqrt{a_n^2 + a_t^2} = \sqrt{(1.2)^2 + (0.9)^2}$

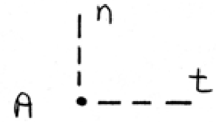
$\underline{a = 1.5 \text{ m/s}^2}$

2/104 $a = a_n = v^2/\rho$, $v = \sqrt{\rho a_n} = \sqrt{(100 - 0.6)0.5(9.81)}$

$= 22.08$ m/s

or $v = 22.08(3.6) = \underline{79.5 \text{ km/h}}$

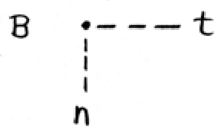
$\boxed{2/105}$



At A: $a_n = \dfrac{v_A^2}{\rho_A}$

$$0.4(9.81) = \frac{v_A^2}{120 - 0.6}$$

$$\underline{v_A = v = 21.6 \text{ m/s}}$$



At B: $a_n = \dfrac{v_B^2}{\rho_B}$

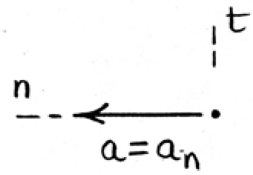
$$0.25\,(9.81) = \frac{21.7^2}{\rho_B + 0.6}$$

$$\underline{\rho_B = 190.4 \text{ m}}$$

2/106 (a) $a_n = \frac{v^2}{\rho} = \frac{1.2^2}{0.6} = 2.4 \text{ m/s}^2$

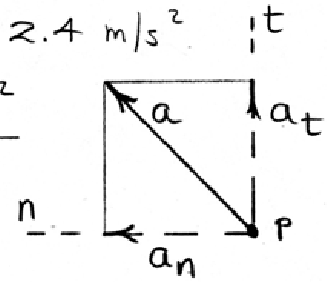
$a_t = 0$

$a = \sqrt{a_n^2 + a_t^2} = \underline{2.4 \text{ m/s}^2}$



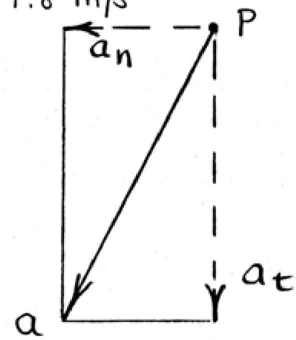
(b) $a_n = 2.4 \text{ m/s}^2, \quad a_t = 2.4 \text{ m/s}^2$

$a = \sqrt{2.4^2 + 2.4^2} = \underline{3.39 \text{ m/s}^2}$



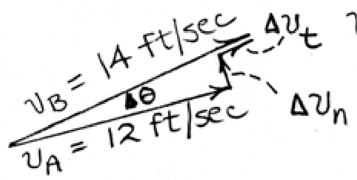
(c) $a_n = 2.4 \text{ m/s}^2, \quad a_t = -4.8 \text{ m/s}^2$

$a = \sqrt{2.4^2 + 4.8^2} = \underline{5.37 \text{ m/s}^2}$



2/107

$$\Delta\theta = (25-15)\frac{\pi}{180} = 0.1745 \text{ rad}$$



$$v_{av} = \frac{12+14}{2} = 13 \text{ ft/sec}$$

$$a_n = \frac{\Delta v_n}{\Delta t} = \frac{v_{av}(\Delta\theta)}{\Delta t} = \frac{13(0.1745)}{2.62-2.4} = \underline{10.31 \frac{\text{ft}}{\text{sec}^2}}$$

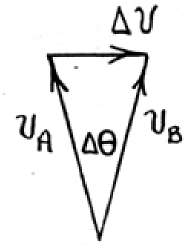
$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{14-12}{0.22} = \underline{9.09 \frac{\text{ft}}{\text{sec}^2}}$$

$\boxed{2/108}$ $v_A = v_B = v = 2$ m/s

$\Delta v = 2v \sin\frac{\Delta\theta}{2} = 4 \sin\frac{\Delta\theta}{2}$ m/s

$\Delta t = \frac{r\Delta\theta}{v} = \frac{0.8\,\Delta\theta}{2} = 0.4\,\Delta\theta$ s

$a_{av} = \frac{\Delta v}{\Delta t} = \frac{4 \sin\frac{\Delta\theta}{2}}{0.4\,\Delta\theta} = 5\,\frac{\sin\frac{\Delta\theta}{2}}{\Delta\theta/2}$



	$\Delta\theta°$	$\frac{\Delta\theta}{2}°$	$\frac{\Delta\theta}{2}$ rad	$\sin\frac{\Delta\theta}{2}$	a_{av}, m/s^2	% diff.
(a)	30°	15°	0.262	0.259	$\underline{4.94}$	1.1
(b)	15°	7.5°	0.1309	0.1305	$\underline{4.99}$	0.3
(c)	5°	2.5°	0.0436	0.0436	$\underline{4.998}$	0.03

$a_n = \frac{v^2}{\rho} = \frac{2^2}{0.8} = \underline{5 \text{ m/s}^2}$

$\boxed{2/109}$ From $a_n = \frac{v^2}{\rho}$, $v = \sqrt{a_n \rho} = \sqrt{0.8 g \rho}$

$v_A = \sqrt{0.8 g \rho_A} = \sqrt{0.8(9.81)(85)} = \underline{25.8 \text{ m/s}}$

$v_B = \sqrt{0.8 g \rho_B} = \sqrt{0.8(9.81)(200)} = \underline{39.6 \text{ m/s}}$

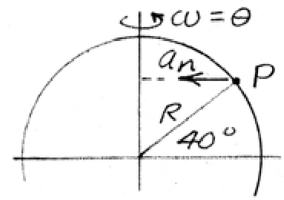
Path BB offers a considerable advantage.

2/110

$a = a_n = r\dot{\theta}^2 = R\cos\gamma\,\dot{\theta}^2$

$= \dfrac{12.742(10^6)}{2}\cos 40^\circ\,(0.729\times10^{-4})^2$

$= \underline{0.0259\ m/s^2}$



$\boxed{2/111}$ $v = v_0 + a_t t = 0 + 1.8(5) = 9$ m/s

$$a_n = \frac{v^2}{\rho} = \frac{9^2}{40} = 2.025 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(1.8)^2 + (2.025)^2} = \underline{2.71 \frac{m}{s^2}}$$

$\boxed{2/112}$ $a_n = v^2/\rho$; $\rho = s/\theta = \dfrac{300}{\pi/4} = 382$ ft

$v = 45\,\dfrac{44}{30} = 66$ ft/sec

$a = a_n = 66^2/382 = \underline{11.40 \text{ ft/sec}^2}$

$\boxed{2/113}$ $a_n = g = \dfrac{v^2}{\rho} = \dfrac{[17,369\,(5280/3600)]^2}{(3959+150)(5280)}$

$= \underline{29.91 \text{ ft/sec}^2}$

Check: $g = g_0\left(\dfrac{R}{R+h}\right)^2 = 32.22\left(\dfrac{3959}{3959+150}\right)^2$

$= \underline{29.91 \text{ ft/sec}^2}$ ✓

2/114 The radius of Jupiter is

$$R = \frac{142\,984}{2}(10^3) = 7.15(10^7)\ \text{m}$$

So $\rho = R+h = 7.15(10^7) + 10^6\ \text{m} = 7.25(10^7)\text{m}$

From the gravitational law

$$a_n = g = g_0 \frac{R^2}{(R+h)^2} = g_0 \frac{R^2}{\rho^2} = 24.85 \frac{[7.15(10^7)]^2}{[7.25(10^7)]^2}$$

$$= 24.2\ \text{m/s}^2$$

From $a_n = \frac{v^2}{\rho}$, $v^2 = a_n \rho = (24.2)(7.25 \cdot 10^7)$

$$\underline{v = 41\,900\ \text{m/s}} \quad \text{or} \quad \underline{v = 150\,700\ \text{km/h}}$$

2/115 $a_t = 20/3.6 = 5.56\ m/s^2$

$a^2 = a_n^2 + a_t^2, \quad a_n^2 = \overline{3(9.81)}^2 - \overline{5.56}^2 = 835.2$

$$a_n = 28.90\ m/s^2$$

$a_n = v^2/\rho, \quad \rho = \dfrac{(800/3.6)^2}{28.90} = \underline{1709\ m}$

$\boxed{2/116}$ $a_t = -0.6 \ \frac{m}{s^2}$, constant

$v_B{}^2 = v_A{}^2 + 2a_t s = 16^2 - 2(0.6)(120)$

$v_B = 10.58$ m/s

$a_n = v_B{}^2/\rho = 10.58^2/60 = 1.867$ m/s^2

$a = \sqrt{a_t{}^2 + a_n{}^2} = \sqrt{0.6^2 + 1.867^2} = \underline{1.961 \ \frac{m}{s^2}}$

2/117 $a_n = v\dot{\beta} = g$, $\dot{\beta} = \dfrac{9.79}{800(10^3)/3600} = 0.04406$ rad/s

or $\dot{\beta} = 0.04406 \dfrac{180}{\pi} = \underline{2.52 \text{ deg/s}}$

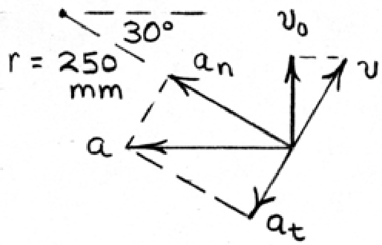
2/118

$v = v_0 / \cos 30° = 2/\cos 30° = 2.31$ m/s

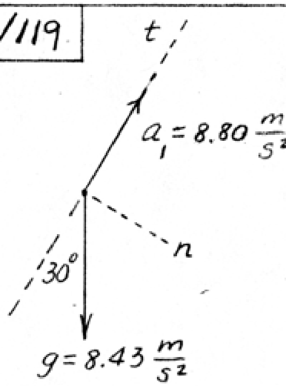
$a_n = v^2/r = 2.31^2/0.250$

$= \underline{21.3 \text{ m/s}^2}$

$a_t = -a_n \tan 30° = \underline{-12.32 \frac{\text{m}}{\text{s}^2}}$



2/119



$a_n = g \sin 30^\circ = 8.43(0.5) = 4.22\ m/s^2$

or $a_n = \frac{4.22}{1000}(3600)^2 = 54630\ km/h^2$

$a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{(30000)^2}{54630} = \underline{16\,480\ km}$

$\dot{v} = a_t = 8.80 - 8.43 \cos 30^\circ = \underline{1.499\ m/s^2}$

$\boxed{2/120}$ $v = r\dot{\theta}$, $v_B = \frac{8}{4}2 = 4$ ft/sec

$a_n = v^2/r$, $a_n = 4^2/_{8/12} = 24$ ft/sec^2

$a_t = r\ddot{\theta}$; $a_t = \frac{8}{4}6 = 12$ ft/sec^2

$a_B = \sqrt{\overline{24}^2 + \overline{12}^2} = \underline{26.8 \text{ ft/sec}^2}$

$\boxed{2/121}$ Relative to space station, $a_n = r\dot{\theta}^2$

where $a_n = 32.17$ ft/sec^2.

Thus $32.17 = (240 + 20)\,\dot{\theta}^2$, $\dot{\theta} = 0.352\ \frac{\text{rad}}{\text{sec}}$

$N = 0.352\left(\frac{60}{2\pi}\right) = \underline{3.36\ \text{rev/min}}$

$\boxed{2/122}$ $a_t = \frac{v_f - v_i}{\Delta t} = \frac{6-3}{2} = 1.5 \text{ m/s}^2$

Halfway through time interval, $v = 4.5$ m/s

$a_{P_1} = \sqrt{a_t^2 + a_n^2} = \sqrt{1.5^2 + \left(\frac{4.5^2}{0.060}\right)^2}$

$= \underline{338 \text{ m/s}^2}$ $(34.4g!)$

$a_{P_2} = a_t = \underline{1.5 \text{ m/s}^2}$

$\boxed{2/123}$ $v^2 = v_0^2 + 2a_t \Delta s$, $18^2 = 2^2 + 2a_t(8)$

$a_t = 20 \text{ m/s}^2$

$a_n = v^2/r = 3^2/0.150 = 60 \text{ m/s}^2$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{\overline{20}^2 + \overline{60}^2} = \underline{63.2 \text{ m/s}^2}$

$\boxed{2/124}$ $v_{x=50'} = \sqrt{2g\left(\frac{50}{20}\right)^2} = \frac{5}{2}\sqrt{2g}\ \frac{\text{ft}}{\text{sec}}$

$a_n = \frac{v^2}{\rho}$, where $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{\left[1+\left(\frac{x}{200}\right)^2\right]^{\frac{3}{2}}}{1/200}$

$\rho_{x=50'} = 219$ ft

$a_n = \frac{25}{4}\, 2(32.2)/219 = \underline{1.838\ \text{ft/sec}^2}$

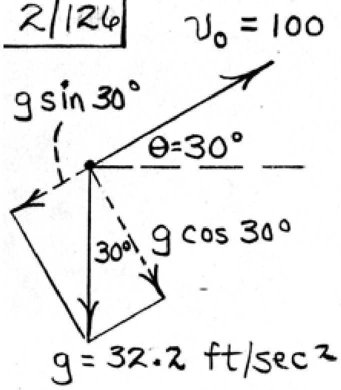
$\boxed{2/125}$ $a_n = v^2/r = 4^2/0.120 = 133.3 \text{ m/s}^2$

$a_t = -a_n \text{ ctn } 4° = -133.3 \text{ ctn } 4° = -1907 \text{ m/s}^2$

With a_t const, $v_f = v_i + a_t t$, $0 = 4 - 1907t$

$t = \frac{4}{1907} = \underline{2.10(10^{-3}) \text{ s}}$

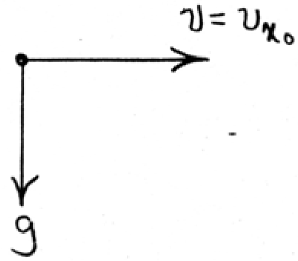
2/126



(a) $a_n = g \cos 30° = \dfrac{v^2}{\rho}$

$\rho = \dfrac{100^2}{g \cos 30°} = \underline{359 \text{ ft}}$

$\dot{v} = -g \sin 30° = \underline{-16.1 \text{ ft/sec}^2}$



(b) $a_n = g = \dfrac{v^2}{\rho}$

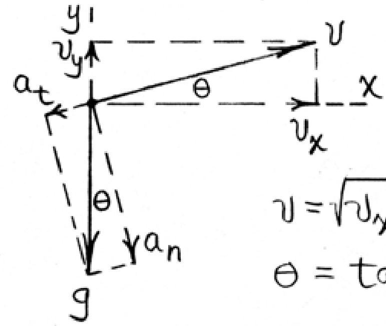
$\rho = \dfrac{(100 \cos 30°)^2}{32.2} = \underline{233 \text{ ft}}$

$\underline{\dot{v} = 0}$

2/127 The time t_{up} to apex is found from

$v_y = v_{y_0} - gt$: $0 = 100 \sin 30° - 32.2\, t_{up}$, $t_{up} = 1.553$ sec

So $t = 1$ sec is before apex and $t = 2.5$ sec is after.



(a) $t = 1$ sec

$v_x = 100 \cos 30° = 86.6$ ft/sec

$v_y = 100 \sin 30° - 32.2(1) = 17.80$ ft/sec

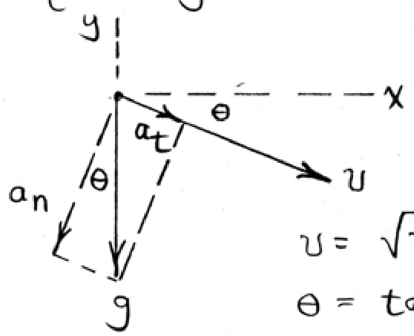
$v = \sqrt{v_x^2 + v_y^2} = 88.4$ ft/sec

$\theta = \tan^{-1} \frac{v_y}{v_x} = 11.61°$

$a_n = g \cos\theta = 32.2 \cos 11.61° = 31.5$ ft/sec^2

$\rho = \frac{v^2}{a_n} = \frac{88.4^2}{31.5} = \underline{248 \text{ ft}}$

$a_t = -g \sin\theta = -32.2 \sin 11.61° = \underline{-6.48 \text{ ft/sec}^2}$



(b) $t = 2.5$ sec

$v_x = 86.6$ ft/sec

$v_y = 100 \sin 30° - 32.2(2.5) = -30.5 \frac{\text{ft}}{\text{sec}}$

$v = \sqrt{v_x^2 + v_y^2} = 91.8$ ft/sec

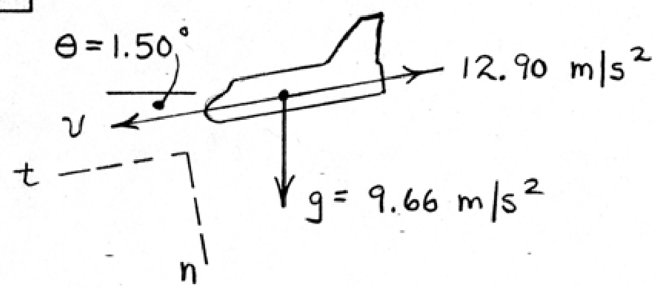
$\theta = \tan^{-1} \frac{|v_y|}{v_x} = 19.40°$

$a_n = g \cos\theta = 32.2 \cos 19.40° = 30.4$ ft/sec^2

$\rho = \frac{v^2}{a_n} = \frac{91.8^2}{30.4} = \underline{278 \text{ ft}}$

$a_t = +g \sin\theta = +32.2 \sin 19.40° = \underline{10.70 \text{ ft/sec}^2}$

2/128



$$\dot{v} = a_t = 9.66 \sin 1.50° - 12.90 = \underline{-12.65 \text{ m/s}^2}$$

$$a_n = g \cos\theta = 9.66 \cos 1.5° = 9.657 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{(15\,450/3.6)^2}{9.657}$$

$$\underline{\rho = 1907 \text{ km}}$$

$\boxed{2/129}$ $a_n = 0.8g = \frac{v^2}{\rho} \Rightarrow v = \sqrt{0.8g\rho}$

Car A : $v_A = \sqrt{0.8\,(9.81)(88)} = 26.3$ m/s

Car B : $v_B = \sqrt{(0.8)(9.81)\,72} = 23.8$ m/s

$$t_A = \frac{s_A}{v_A} = \frac{\pi\,(88)}{26.3} = \underline{10.52 \text{ s}}$$

$$t_B = \frac{s_B}{v_A} = \frac{\pi\,(72) + 2(16)}{23.8} = \underline{10.86 \text{ s}}$$

Car A will win the race!

$\boxed{2/130}$ Evaluate $y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ at C:

$$0 = 150 + 0 - \tfrac{1}{2}(9.81)t^2, \quad t = 3.05 \text{ sec}$$

Evaluate $x = x_0 + v_{x_0}t + \frac{1}{2}a_x t^2$ at C:

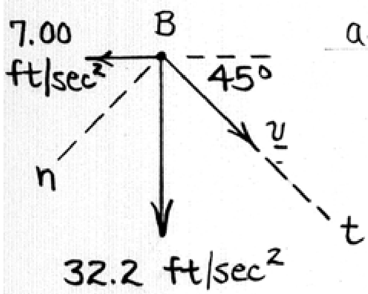
$$120 = 0 + 50(3.08) + \tfrac{1}{2}a_x(3.08)^2, \quad a_x = -7.00 \text{ ft/sec}^2$$

At B: $\begin{cases} v_x = v_{x_0} + a_x t : & v_x = 50 - 7.00t \\ v_y = v_{y_0} - gt : & |v_y| = 32.2t \end{cases}$

Set $v_x = |v_y|$ & obtain $t = 1.275$ sec

So at B: $v_x = |v_y| = 32.2(1.276) = 41.1$ ft/sec

The speed at B is $v = 41.1\sqrt{2} = 58.1$ ft/sec



$$a_n = 32.2\cos 45° + 7.00\cos 45°$$
$$= 27.7 \text{ ft/sec}^2$$

From $a_n = \dfrac{v^2}{\rho}$,

$$\rho = \frac{v^2}{a_n} = \frac{(58.1)^2}{27.7} = \underline{121.7 \text{ ft}}$$

2/131 For a_t = const., $v_C^2 = v_A^2 + 2a_t \Delta s_{A-C}$

$v_A = \frac{250}{3.6}$ m/s, $v_C = \frac{200}{3.6}$ m/s, $a_t = \frac{(200)^2 - (250)^2}{(3.6)^2\, 2(300)} = -2.89$ m/s^2

$v_B^2 = v_A^2 + 2a_t \Delta s_{A-B} = \left(\frac{250}{3.6}\right)^2 + 2(-2.89)(150) = 3954\ (\text{m/s})^2$

$v_B = 62.9$ m/s

at B, $a_n = v_B^2/\rho = \frac{3954}{500} = 7.91$ m/s^2

$a = \sqrt{a_n^2 + a_t^2} = \sqrt{(7.91)^2 + (2.89)^2} = \underline{8.42 \text{ m/s}^2}$

2/132

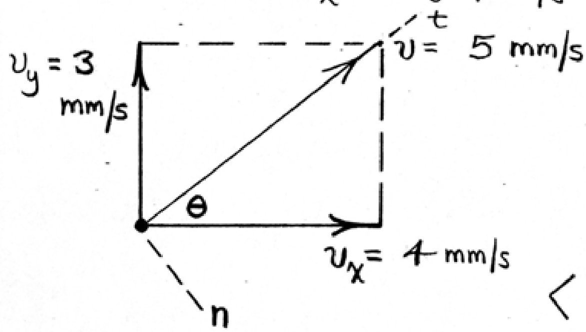
$x = 16 - 12t + 4t^2 \qquad y = 2 + 15t - 3t^2$

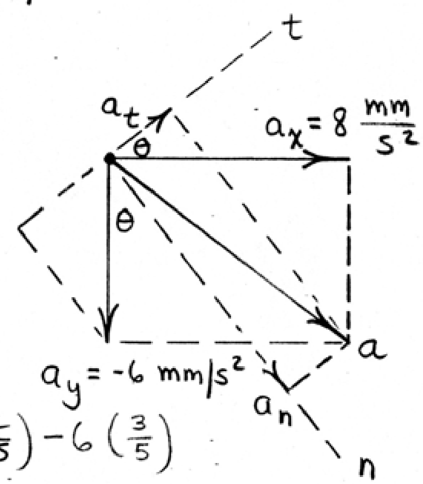
$\dot{x} = 8t - 12 \qquad \dot{y} = 15 - 6t$

$\ddot{x} = 8 \qquad \ddot{y} = -6$

At $t = 2$ s : $\dot{x} = 4$ mm/s $\qquad \dot{y} = 15 - 12 = 3$ mm/s

$\ddot{x} = 8$ mm/s^2 $\qquad \ddot{y} = -6$ mm/s^2





$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$a_t = 8\cos\theta - 6\sin\theta = 8\left(\frac{4}{5}\right) - 6\left(\frac{3}{5}\right)$

$= 2.8$ mm/s^2

$a_n = 6\cos\theta + 8\sin\theta = 6\left(\frac{4}{5}\right) + 8\left(\frac{3}{5}\right)$

$= 9.6$ mm/s^2

$a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{5^2}{9.6} = \underline{2.60 \text{ mm}}$

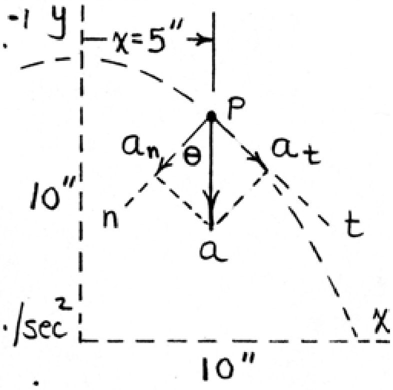
2/133 $y - 10 = kx^2$, $k = -\frac{1}{10}$ in.$^{-1}$

So $y = 10\left(1 - \frac{x^2}{100}\right)$ in.

$\dot{x} = 15$ in./sec , $\ddot{x} = 0$

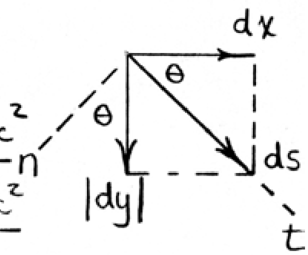
$\dot{y} = -x\dot{x}/5$, $\ddot{y} = -\dot{x}^2/5$

So $a = a_y = \ddot{y} = -\frac{15^2}{5} = -45$ in./sec^2



$\frac{dy}{dx} = -\frac{x}{5} = -\frac{6}{5}$ @ $x = 6$ in.

$\theta = \tan^{-1}\frac{6}{5} = 50.2°$



$a_n = a\cos\theta = 45\cos 50.2° = \underline{28.8 \text{ in./sec}^2}$

$a_t = a\sin\theta = 45\sin 50.2° = \underline{34.6 \text{ in./sec}^2}$

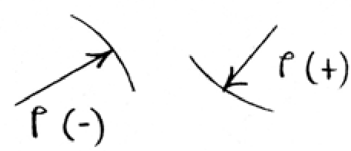
For $x = 6$ in., $\dot{y} = -6(15)/5 = -18$ in./sec

$v = \sqrt{15^2 + 18^2} = 23.4$ in./sec

$a_n = \frac{v^2}{\rho}$, $\rho = \frac{23.4^2}{28.8} = \underline{19.06 \text{ in.}}$

Check : $\rho_{xy} = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{d^2y/dx^2} = \frac{\left[1 + (-6/5)^2\right]^{3/2}}{-1/5}$

$= -19.06$ in. ✓



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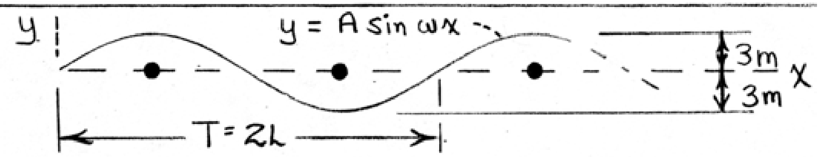


$y = A \sin \omega x$, where $A = 3\text{m}$ & $\omega = \frac{2\pi}{T}$

Radius of curvature $\rho = \dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$\frac{dy}{dx} = A\omega \cos \omega x$, $\frac{d^2y}{dx^2} = -A\omega^2 \sin \omega x$

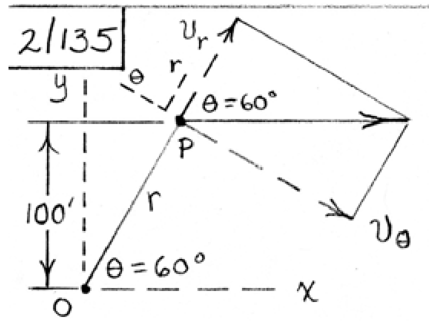
Set $\frac{d\rho}{dx} = 0$ to show that $|\rho|$ is a min @ $x = \frac{T}{4}$ & $x = \frac{3T}{4}$.

$$\rho_{min} = \frac{\left[1 + \left\{A \frac{2\pi}{T} \cos\left(\frac{2\pi}{T} \cdot \frac{T}{4}\right)\right\}^2\right]}{+A\left(\frac{2\pi}{T}\right)^2 \sin\left(\frac{2\pi}{T} \cdot \frac{T}{4}\right)} = \frac{T^2}{4\pi^2 A}$$

$$a_n = \frac{v^2}{\rho} : \quad 0.7\,(9.81) = \frac{(80/3.6)^2}{T^2/(4\pi^2 \cdot 3)}$$

$T = 92.3 \text{ m} = 2L$, $\underline{L = 46.1 \text{ m}}$

$\boxed{2/135}$



$v = 65$ mi/hr

$$\begin{cases} v = 65\left(\frac{5280}{3600}\right) = 95.3 \text{ ft/sec} \\ r = \frac{100}{\sin 60°} = 115.5 \text{ ft} \end{cases}$$

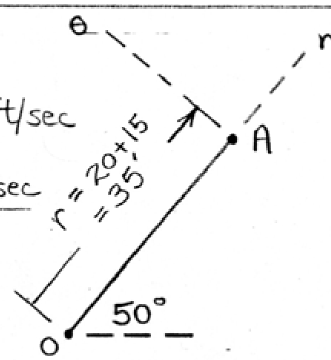
$$v_r = \dot{r} = v\cos\theta = 95.3\cos 60° = \underline{47.7 \text{ ft/sec}}$$

$$v_\theta = r\dot{\theta} = -v\sin\theta$$

$$\dot{\theta} = -\frac{v\sin\theta}{r} = -\frac{95.3\sin 60°}{115.5} = -0.715 \text{ rad/sec}$$

or $\dot{\theta} = -0.715\left(\frac{180}{\pi}\right) = \underline{-41.0 \text{ deg/sec}}$

2/136 $v_r = \dot{r} = 0.5$ ft/sec



$v_\theta = r\dot{\theta} = 35\left(2\frac{\pi}{180}\right) = 1.222$ ft/sec

$v = \sqrt{0.5^2 + 1.222^2} = \underline{1.320 \text{ ft/sec}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 35\left(2\frac{\pi}{180}\right)^2$

$= -0.0426$ ft/sec^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(0.5)\left(2\frac{\pi}{180}\right) = 0.0349 \frac{\text{ft}}{\text{sec}^2}$

$a = \sqrt{0.0426^2 + 0.0349^2} = \underline{0.0551 \text{ ft/sec}^2}$

2/137 $v_r = \dot{r} = 40$ mm/s

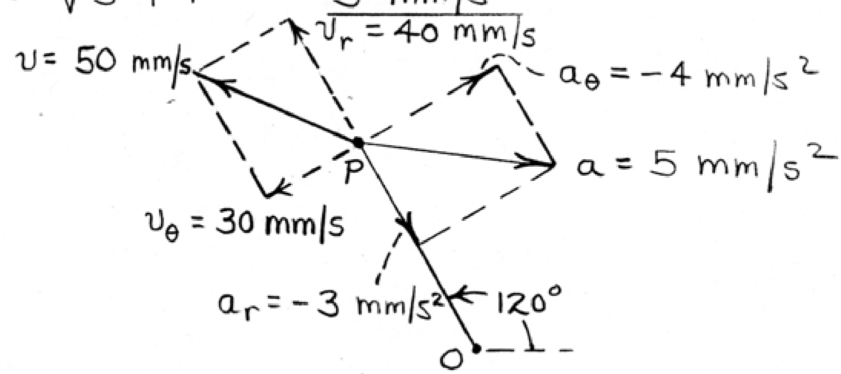
$v_\theta = r\dot{\theta} = 300\,(0.1) = 30$ mm/s

$v = \sqrt{40^2 + 30^2} = \underline{50 \text{ mm/s}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300\,(0.1)^2 = -3 \text{ mm/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300\,(-0.04) + 2\,(40)(0.1) = -4 \text{ mm/s}^2$

$a = \sqrt{3^2 + 4^2} = \underline{5 \text{ mm/s}^2}$



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Position	r	$\dot{r}$	$\ddot{r}$	θ	$\dot{\theta}$	$\ddot{\theta}$
A	+	−	+	+	+	+
B	+	0	+	+	+	0
C	+	+	+	+	+	−

Notes : (1) $r \geq 0$, always , by definition

(2) $\dot{r}$ determined by inspection

(3) $\ddot{r}$ found from $a_r = \ddot{r} - r\dot{\theta}^2 = 0$

(4) $\theta \geq 0$, by definition in figure

(5) $\dot{\theta} > 0$ here , by inspection

(6) $\ddot{\theta}$ found from $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

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$$\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = 1.5\underline{e}_r + (24+7)\left(5\frac{\pi}{180}\right)\underline{e}_\theta$$

$$= \underline{1.5\underline{e}_r + 2.71\underline{e}_\theta \quad \text{ft/sec}}$$

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

$$= \left[-4 - 31\left(5\frac{\pi}{180}\right)^2\right]\underline{e}_r + \left[31\left(2\frac{\pi}{180}\right) + 2(1.5)\left(5\frac{\pi}{180}\right)\right]\underline{e}_\theta$$

$$= \underline{-4.24\,\underline{e}_r + 1.344\,\underline{e}_\theta \quad \text{ft/sec}^2}$$

$\boxed{2/140}$ $\dot{\underline{r}} \neq v$, $\dot{r} \neq \underline{v}$, $\ddot{\underline{r}} \neq a$, and $\ddot{r} \neq \underline{a}$ because scalars are NEVER equal to vectors.

$\dot{r} \neq v$ because $v \neq (v_r = \dot{r})$.

$\ddot{r} \neq a$ because $\ddot{r}$ is only one part of the magnitude of a_r (and therefore of a).

$\dot{\underline{r}} \neq \dot{r}\underline{e}_r$ because $\underline{v} = \dot{\underline{r}}$ also contains an $\underline{e}_\theta$-component.

$\ddot{\underline{r}} \neq \ddot{r}\underline{e}_r$ because $\underline{a} = \ddot{\underline{r}}$ contains another $\underline{e}_r$-component and also an $\underline{e}_\theta$-component.

$\dot{\underline{r}} \neq r\dot{\theta}\underline{e}_\theta$ because $\dot{\underline{r}} = \underline{v}$ also contains an $\underline{e}_r$-component.

2/141

(A) $\underline{v}_A = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = v\underline{e}_r + l\Omega\underline{e}_\theta$

$\underline{a}_A = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

$= -l\Omega^2\underline{e}_r + 2v\Omega\underline{e}_\theta$

(B) $\underline{v}_B = 4v\underline{e}_r + 2l\Omega\underline{e}_\theta$

$\underline{a}_B = -2l\Omega^2\underline{e}_r + 8v\Omega\underline{e}_\theta$

$\boxed{2/142}$ $r = 375 + 125 = 500 \text{ mm}$, $\dot{r} = \dot{l} = -150 \frac{\text{mm}}{\text{s}}$

$\ddot{r} = 0$, $\dot{\theta} = 60\left(\frac{\pi}{180}\right) = \frac{\pi}{3} \text{ rad/s}$, $\ddot{\theta} = 0$

$v_r = \dot{r} = -150 \frac{\text{mm}}{\text{s}}$, $v_\theta = r\dot{\theta} = 500\left(\frac{\pi}{3}\right) = 524 \frac{\text{mm}}{\text{s}}$

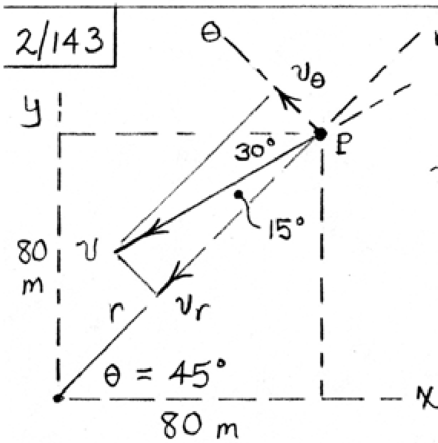
$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-150)^2 + (524)^2} = \underline{545 \frac{\text{mm}}{\text{s}}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 500\left(\frac{\pi}{3}\right)^2 = -548 \text{ mm/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-150)(\pi/3) = -314 \text{ mm/s}^2$

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-548)^2 + (-314)^2} = \underline{632 \text{ mm/s}^2}$

2/143



$$\begin{cases} v = 100 \text{ m/s} \\ r = 80\sqrt{2} = 113.1 \text{ m} \end{cases}$$

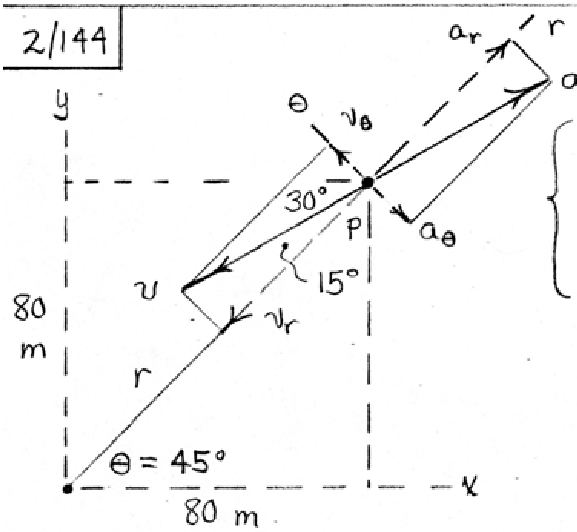
$$v_r = \dot{r} = -v\cos 15° = -100 \cos 15°$$
$$= \underline{-96.6 \text{ m/s}}$$

$$v_\theta = r\dot{\theta} = v \sin 15°$$
$$\dot{\theta} = \frac{100 \sin 15°}{113.1}$$
$$= \underline{0.229 \text{ rad/s}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0, \quad \ddot{r} = r\dot{\theta}^2 = 113.1(0.229)^2 = \underline{5.92 \tfrac{m}{s^2}}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0, \quad \ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r}$$
$$= -\frac{2(-96.6)(0.229)}{113.1} = \underline{-0.391 \tfrac{rad}{s^2}}$$

2/144



From Prob. 2/143,

$$\begin{cases} r = 113.1 \text{ m} \\ \dot{r} = -96.6 \text{ m/s} \\ \dot{\theta} = 0.229 \text{ rad/s} \end{cases}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 20 \cos 15°$$

$$\ddot{r} = 113.1\,(0.229)^2 + 20 \cos 15° = \underline{25.2 \text{ m/s}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -20 \sin 15°$$

$$\ddot{\theta} = \frac{-2(-96.6)(0.229) - 20 \sin 15°}{113.1} = \underline{0.345 \text{ rad/s}^2}$$

2/145 From $\underline{a} = [\ddot{r} - r\dot{\theta}^2]\underline{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\underline{e}_\theta$

we have, for $\ddot{r} = \ddot{\theta} = 0$, $\dot{\theta} = \Omega$, and $r = l$:

$$a = \sqrt{(l\Omega^2)^2 + (2\dot{l}\Omega)^2} = \Omega\sqrt{l^2\Omega^2 + 4\dot{l}^2}$$

$$0.011 = 0.05\sqrt{[4.2(0.05)]^2 + 4\dot{l}^2}$$

Solve for $\dot{l}$: $\underline{\dot{l} = 0.0328 \text{ m/s}}$

or $\underline{\dot{l} = 32.8 \text{ mm/s}}$

2/146

$$\begin{cases} r = r_0 \cosh Kt \\ \dot{r} = r_0 K \sinh Kt \\ \ddot{r} = r_0 K^2 \cosh Kt \end{cases}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = r_0 K^2 \cosh Kt - (r_0 \cosh Kt) K^2 = 0$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2 r_0 K \sinh Kt \,(K)$$

$$= 2 r_0 K^2 \sinh Kt$$

But $\cosh^2 Kt - \sinh^2 Kt = 1$, $\sinh Kt = \sqrt{\cosh^2 Kt - 1}$

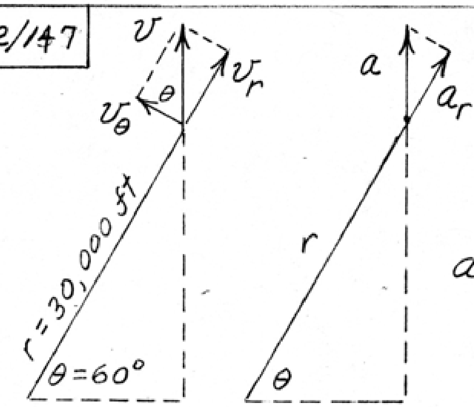
$$= \sqrt{\left(\frac{r}{r_0}\right)^2 - 1}$$

When $r = R$, $\sinh Kt = \sqrt{\left(\frac{R}{r_0}\right)^2 - 1}$

So at the instant of leaving the vane,

$$a = a_\theta = 2 r_0 K^2 \sqrt{\left(\frac{R}{r_0}\right)^2 - 1} = \underline{2K^2\sqrt{R^2 - r_0^2}}$$

2/147



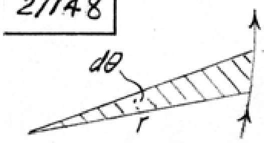
$v_\theta = r\dot{\theta} = 30(10^3)(0.020)$
$= 600$ ft/sec

$v = v_\theta / \cos 60°$
$= 600/0.5 = \underline{1200 \text{ ft/sec}}$

$a_r = \ddot{r} - r\dot{\theta}^2$
$= 70 - 30(10^3)(0.020)^2$
$= 58$ ft/sec^2

$a = a_r / \sin 60°$
$= 58/0.866 = \underline{67.0 \text{ ft/sec}^2}$

2/148

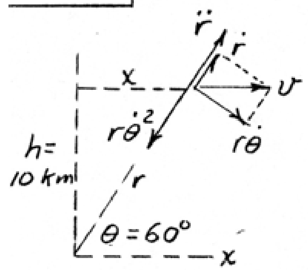


$$dA = \frac{1}{2} r\,d\theta\,(r) = \frac{1}{2} r^2 d\theta$$

$$\dot{A} = \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \dot{\theta} = \text{constant}$$

since $a_\theta = \frac{1}{r} \frac{d}{dt}(r^2\dot{\theta}) = 0$

2/149 Acceleration in all directions is zero, so



$\dot{\theta} = -0.020$ rad/s

$a_r = \ddot{r} - r\dot{\theta}^2 = 0, \quad \ddot{r} = r\dot{\theta}^2$

$r = h/\sin\theta, \quad r = \dfrac{10}{\sqrt{3}/2} = 11.55$ km

$\ddot{r} = 11.55\,(-0.020)^2 = 0.00462$ km/s^2

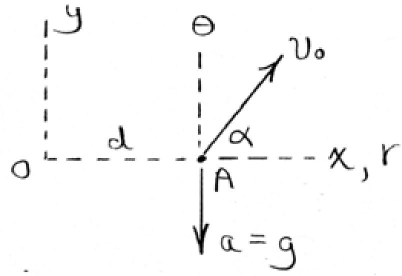
$= \underline{4.62 \text{ m/s}^2}$

$v = |r\dot{\theta}|/\sin\theta = h\dot{\theta}/\sin^2\theta$

$= \dfrac{|10(-0.020)|}{(\sqrt{3}/2)^2} = 0.267$ km/s

or $v = 0.267\,(3600) = \underline{960 \text{ km/h}}$

2/150



$\underline{r = d}$, $\underline{\theta = 0}$

$v_r = \underline{\dot{r} = v_0 \cos\alpha}$

$v_\theta = r\dot{\theta} = v_0 \sin\alpha$, $\quad \underline{\dot{\theta} = \dfrac{v_0 \sin\alpha}{d}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0$, $\quad \underline{\ddot{r} = r\dot{\theta}^2 = \dfrac{v_0^2 \sin^2\alpha}{d}}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -g$, $\quad \ddot{\theta} = -\dfrac{2\dot{r}\dot{\theta} + g}{r}$

$$\text{or } \ddot{\theta} = -\frac{2(v_0\cos\alpha)\left(\frac{v_0\sin\alpha}{r}\right) + g}{r}$$

$$= \underline{-\frac{1}{d}\left(\frac{2v_0^2}{d}\cos\alpha\sin\alpha + g\right)}$$

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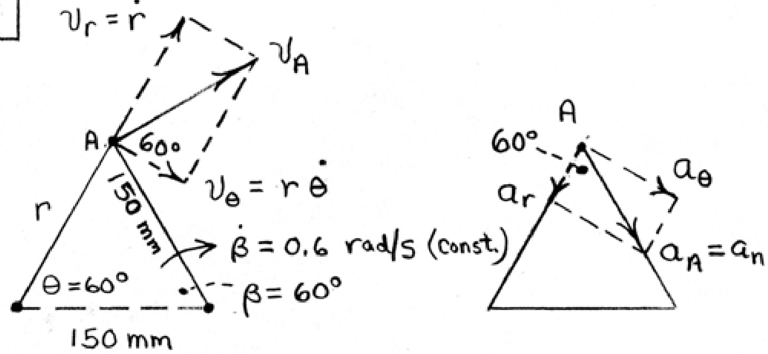

For $\beta = 60°$, $\theta = 60°$, $r = 150$ mm

$v_A = 150(0.6) = 90$ mm/s

$v_\theta = r\dot\theta = -v_A \cos 60°$, $\dot\theta = \dfrac{-90 \cos 60°}{150} = \underline{-0.3 \ \frac{\text{rad}}{\text{s}}}$

$v_r = \dot r = v_A \sin 60° = 90 \sin 60° = \underline{77.9 \text{ mm/s}}$

$a_A = a_n = 150(0.6)^2 = 54$ mm/s^2

$a_r = \ddot r - r\dot\theta^2 : -54 \cos 60° = \ddot r - 150(-0.3)^2$

$\underline{\ddot r = -13.5 \text{ mm/s}^2}$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta : -54 \sin 60° = 150\ddot\theta + 2(77.9)(-0.3)$

$\underline{\ddot\theta = 0}$

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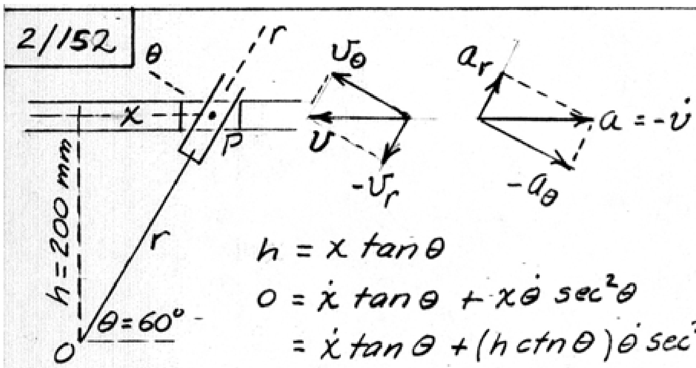


$$h = x \tan\theta$$

$$0 = \dot{x}\tan\theta + x\dot{\theta}\sec^2\theta$$

$$= \dot{x}\tan\theta + (h\,\text{ctn}\,\theta)\dot{\theta}\sec^2\theta$$

$$v = -\dot{x} = h\dot{\theta}\,\text{ctn}\,\theta\sec^2\theta/\tan\theta$$

$$= h\dot{\theta}\csc^2\theta = 200(2)\left(\frac{2}{\sqrt{3}}\right)^2 = \underline{533 \text{ mm/s}}$$

$$v_r = -v\cos\theta = -533(1/2) = \underline{-267 \text{ mm/s}}$$

$$-a = \dot{v} = h\dot{\theta}\,2\csc\theta(-\text{ctn}\,\theta\csc\theta)\dot{\theta} = -2h\dot{\theta}^2\,\text{ctn}\,\theta\csc^2\theta$$

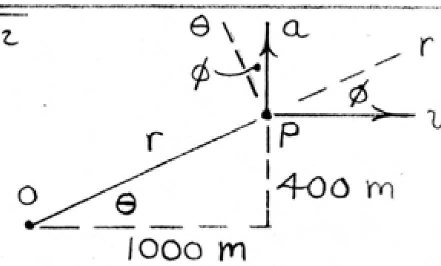
$$a = 2(200)2^2\frac{1}{\sqrt{3}}\left(\frac{2}{\sqrt{3}}\right)^2 = \underline{1232 \text{ mm/s}^2}$$

$$a_r = a\cos\theta = 1232(1/2) = \underline{616 \text{ mm/s}^2}$$

(Alternatively obtain $\dot{r} = v_r$ & $\ddot{r}$ from $r = h\csc\theta$)

2/153 $r = \sqrt{1000^2 + 400^2}$

$= 1077 \text{ m}$



$\theta = \tan^{-1}\frac{400}{1200} = 21.8°$

$v = \frac{600}{3.6} = 166.7 \text{ m/s}$

$a = a_n = \frac{v^2}{\rho} = \frac{166.7^2}{1200} = 23.1 \text{ m/s}^2$

$v_r = \dot{r} = v\cos\theta = 166.7\cos 21.8° = 154.7 \text{ m/s}$

$v_\theta = r\dot{\theta}$: $-166.7\sin 21.8° = 1077\dot{\theta}$, $\dot{\theta} = -0.0575 \frac{\text{rad}}{\text{s}}$

$a_r = \ddot{r} - r\dot{\theta}^2$: $23.1\sin 21.8° = \ddot{r} - 1077(-0.0575)^2$

$\underline{\ddot{r} = 12.15 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$: $23.1\cos 21.8° = 1077\ddot{\theta} + 2(154.7)(-0.0575)$

$\underline{\ddot{\theta} = 0.0365 \text{ rad/s}^2}$

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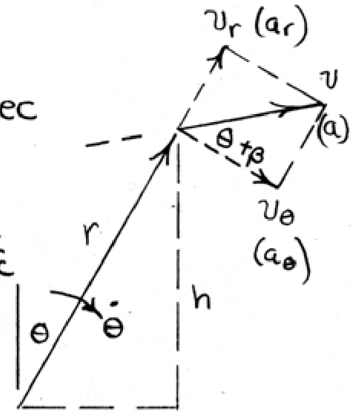


$\dot\theta = 2.20\left(\frac{\pi}{180}\right) = 0.0384$ rad/sec

$v_r = \dot r = 360$ ft/sec

$v_\theta = r\dot\theta = 12{,}000\,(0.0384) = 461\ \frac{ft}{sec}$

$v = \sqrt{v_r^2 + v_\theta^2} = 585$ ft/sec

or $v = \underline{399\ mi/hr}$

$30 + \beta = \tan^{-1}\frac{360}{461} = 38.0°$, $\underline{\beta = 8.00°}$

$h = r\cos 30° = 12{,}000\cos 30° = \underline{10{,}390\ ft}$

$a_r = \ddot r - r\dot\theta^2 = 19.60 - 12{,}000\,(0.0384)^2 = 1.908\ \frac{ft}{sec^2}$

$a = \frac{a_r}{\sin(\theta+\beta)} = \frac{1.908}{\sin 38.0°} = \underline{3.10\ ft/sec^2}$

$a_\theta = \frac{a_r}{\tan(\theta+\beta)} = r\ddot\theta + 2\dot r\dot\theta$

$\frac{1.908}{\tan 38.0°} = 12{,}000\,\ddot\theta + 2(360)(0.0384)$, $\underline{\ddot\theta = -0.00210\ \frac{rad}{sec^2}}$

2/155 $\theta = 0.4 + 0.12t + 0.06t^3$ | $r = 0.8 - 0.1t - 0.05t^2$

$\dot\theta = 0.12 + 0.18t^2$ | $\dot r = -0.1 - 0.1t$

$\ddot\theta = 0.36t$ | $\ddot r = -0.1$

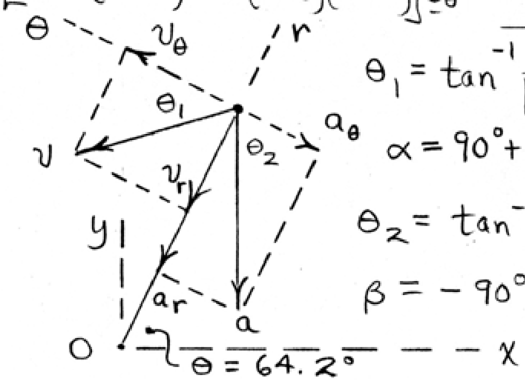
At $t = 2$ s: $\begin{cases} \theta = 1.12 \text{ rad} & r = 0.4 \text{ m} \\ \dot\theta = 0.84 \text{ rad/s} & \dot r = -0.3 \text{ m/s} \\ \ddot\theta = 0.72 \text{ rad/s}^2 & \ddot r = -0.1 \text{ m/s}^2 \end{cases}$

$$\underline{v} = \dot r \underline{e}_r + r\dot\theta \underline{e}_\theta = -0.3\underline{e}_r + 0.4(0.84)\underline{e}_\theta$$

$$= \underline{-0.3\underline{e}_r + 0.336\underline{e}_\theta \text{ m/s}}$$

$$\underline{a} = (\ddot r - r\dot\theta^2)\underline{e}_r + (r\ddot\theta + 2\dot r\dot\theta)\underline{e}_\theta = [-0.1 - 0.4(0.84)^2]\underline{e}_r$$

$$+ [0.4(0.72) + 2(-0.3)(0.84)]\underline{e}_\theta = \underline{-0.382\underline{e}_r - 0.216\underline{e}_\theta \text{ m/s}^2}$$



$\theta_1 = \tan^{-1} \dfrac{|v_r|}{|v_\theta|} = 41.8°$

$\alpha = 90° + \theta + \theta_1 = \underline{195.9°}$

$\theta_2 = \tan^{-1} \dfrac{|a_r|}{|a_\theta|} = 60.5°$

$\beta = -90° + \theta - \theta_2 = \underline{-86.4°}$

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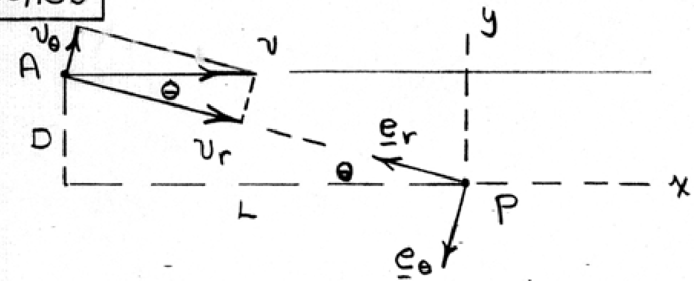
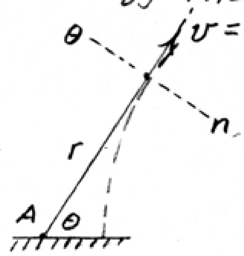


$$v' = |v_r| = v\cos\theta = \underline{v\frac{L}{\sqrt{L^2+D^2}}}$$

$$\text{Numbers: } v' = 70\frac{500}{\sqrt{500^2+20^2}} = \underline{69.9 \text{ mi/hr}}$$

The factor of $\cos\theta$ is the basis for the statement that, kinematically, radar can yield an accurate or low, but not high, speed measurement. As can be seen, however, adherence to the speed limit (not reliance upon $\cos\theta$) is the best policy!

2/157 Radial line r must be tangent to trajectory for $\dot{\theta}=0$. Thus $+\theta$ direction is in the opposite sense to the normal n-direction of the curve.



$r = 35{,}000$ ft, $\dot{r} = 1600$ ft/sec

$\dot{\theta} = 0$, $\ddot{\theta} = -7.20(10^{-3})$ rad/sec^2

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 35{,}000(-7.20)(10^{-3}) + 0$$
$$= -252 \text{ ft/sec}^2$$

$$-a_\theta = a_n = v^2/\rho, \quad \rho = v^2/-a_\theta$$
$$= \frac{(1600)^2}{252} = \underline{10.16(10^3) \text{ ft}}$$

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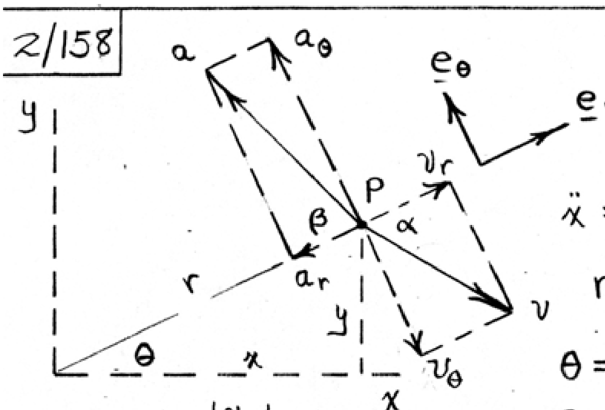

$x = 4\text{ m}, \quad y = 2\text{ m}$

$\dot{x} = 2\sqrt{3}\text{ m/s}, \quad \dot{y} = -2\ \frac{\text{m}}{\text{s}}$

$\ddot{x} = -5\text{ m/s}^2, \quad \ddot{y} = 5\ \frac{\text{m}}{\text{s}^2}$

$r = \sqrt{x^2 + y^2} = \underline{2\sqrt{5}\text{ m}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \underline{26.6°}$

$$\alpha = \tan^{-1}\frac{|v_y|}{|v_x|} + \theta = \tan^{-1}\frac{2}{2\sqrt{3}} + 26.6° = 56.6°$$

$$\beta = \tan^{-1}\left|\frac{a_y}{a_x}\right| + \theta = \tan^{-1}\frac{5}{5} + 26.6° = 71.6°$$

$$v = \sqrt{v_y^2 + v_x^2} = \sqrt{2^2 + (2\sqrt{3})^2} = 4\text{ m/s}$$

$$a = \sqrt{a_y^2 + a_x^2} = \sqrt{5^2 + 5^2} = 7.07\text{ m/s}^2$$

$$\dot{r} = v_r = v\cos\alpha = 4\cos 56.6° = \underline{2.20\text{ m/s}}$$

$$v_\theta = -v\sin\alpha = -4\sin 56.6° = -3.34\text{ m/s}$$

$$v_\theta = r\dot{\theta} : -3.34 = 2\sqrt{5}\,\dot{\theta}, \quad \underline{\dot{\theta} = -0.746\text{ rad/s}}$$

$$a_r = -a\cos\beta = -7.07\cos 71.6° = -2.24\text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 : -2.24 = \ddot{r} - 2\sqrt{5}(0.746)^2, \quad \underline{\ddot{r} = 0.255\ \frac{\text{m}}{\text{s}^2}}$$

$$a_\theta = a\sin\beta = 7.07\sin 71.6° = 6.71\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 6.71 = 2\sqrt{5}\,\ddot{\theta} + 2(2.20)(-0.746), \quad \underline{\ddot{\theta} = 2.24\ \frac{\text{rad}}{\text{s}^2}}$$

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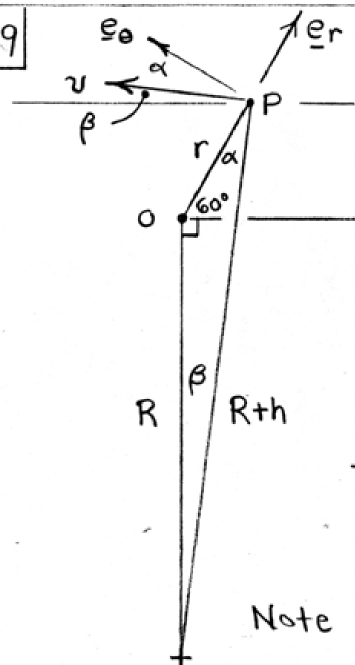

$$\frac{\sin 150^\circ}{R+h} = \frac{\sin\alpha}{R}$$

$$\frac{\sin 150^\circ}{3959+150} = \frac{\sin\alpha}{3959}$$

$$\alpha = 28.8^\circ$$

$$\alpha + \beta + 150^\circ = 180^\circ \Rightarrow \beta = 1.200^\circ$$

$$v_r = \dot{r} = -12{,}272 = -v\sin\alpha$$

$$-12{,}272 = -v\sin 28.8^\circ$$

$$v = \underline{25{,}474\ \text{ft/sec}}$$

Note: Because $\underline{v}$ is nearly parallel to the horizontal at O ($\beta = 1.200^\circ$), one obtains a close approximation ($v = 24{,}544\ \frac{\text{ft}}{\text{sec}}$) to the correct answer by neglecting β (assuming a flat earth).

$\boxed{2/160}$ $r = r_o + b_o \sin 2\pi nt$, $\dot{r} = 2\pi n b_o \cos 2\pi nt$

$\ddot{r} = -4\pi^2 b_o n^2 \sin 2\pi nt$

$\dot{\theta} = \omega$, $\ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2$, $a_r = -4\pi^2 b_o n^2 \sin 2\pi nt - (r_o + b_o \sin 2\pi nt)\omega^2$

$= -(4\pi^2 n^2 + \omega^2) b_o \sin 2\pi nt - r_o \omega^2$

$|a_r|_{max} = \underline{(4\pi^2 n^2 + \omega^2) b_o + r_o \omega^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$, $a_\theta = 0 + 4\pi b_o n\omega \cos 2\pi nt$

$|a_\theta|_{max} = \underline{4\pi b_o n\omega}$

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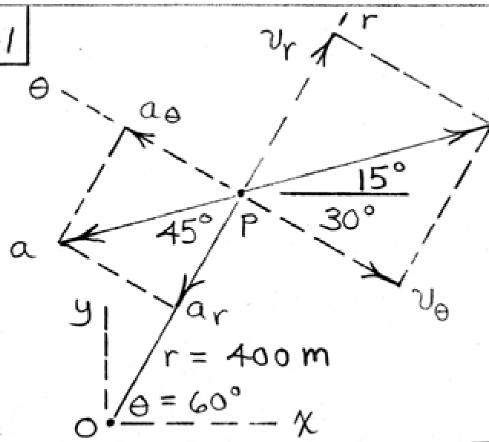

$v = \frac{90}{3.6} = 25$ m/s

$a = 0.5$ m/s^2

$v_r = \dot{r} = v \sin 45° = 25 \frac{\sqrt{2}}{2} = \underline{17.68 \text{ m/s}}$

$v_\theta = r\dot{\theta}$: $-25 \cos 45° = 400\dot{\theta}$, $\underline{\dot{\theta} = -0.0442 \text{ rad/s}}$

$a_r = \ddot{r} - r\dot{\theta}^2$: $-0.5 \cos 45° = \ddot{r} - 400(-0.0442)^2$

$\underline{\ddot{r} = 0.428 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$: $0.5 \sin 45° = 400\ddot{\theta} + 2(17.68)(-0.0442)$

$\underline{\ddot{\theta} = 0.00479 \text{ rad/s}^2}$

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$$\begin{cases} r = 0.75 + 0.5 = 1.25 \text{ m} & \theta = 30° \\ \dot{r} = 0.2 \text{ m/s} & \dot{\theta} = 0.1745 \frac{\text{rad}}{\text{s}} \\ \ddot{r} = -0.3 \text{ m/s}^2 & \ddot{\theta} = 0 \end{cases}$$

$\underline{v} = v_r \underline{e}_r + v_\theta \underline{e}_\theta = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$

$= 0.2\underline{e}_r + 1.25(0.1745)\underline{e}_\theta = 0.2\,\underline{e}_r + 0.218\underline{e}_\theta \ \frac{\text{m}}{\text{s}}$

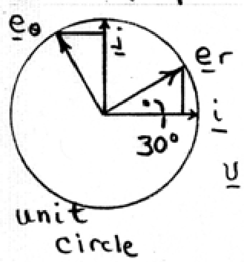
$v = \sqrt{v_r^2 + v_\theta^2} = \underline{0.296 \text{ m/s}}$

$\underline{a} = a_r \underline{e}_r + a_\theta \underline{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

$= [-0.3 - 1.25(0.1745)^2]\underline{e}_r + [1.25(0) + 2(0.2)(0.1745)]\underline{e}_\theta$

$= -0.338\underline{e}_r + 0.0698\underline{e}_\theta \ \text{m/s}^2$

$a = \sqrt{a_r^2 + a_\theta^2} = \underline{0.345 \text{ m/s}^2}$



$\underline{e}_r = \underline{i}\cos 30° + \underline{j}\sin 30°$

$\underline{e}_\theta = -\underline{i}\sin 30° + \underline{j}\cos 30°$

$\underline{v} = 0.2[\underline{i}\,c\,30° + \underline{j}\,s\,30°] + 0.218[-\underline{i}\,s\,30° + \underline{j}\,c\,30°]$

$= \underline{0.064\underline{i} + 0.289\underline{j} \ \text{m/s}}$

$\underline{a} = -0.338[\underline{i}\,c30° + \underline{j}\,s30°] + 0.0698[-\underline{i}\,s\,30° + \underline{j}\,c\,30°]$

$= \underline{-0.328\,\underline{i} - 0.1086\underline{j} \ \text{m/s}^2}$

$\boxed{2/163}$ $r = \overline{BD} = 2R\sin\frac{\theta}{2}$, $\dot{r} = R\dot{\theta}\cos\frac{\theta}{2}$

$\ddot{r} = -\frac{R}{2}\dot{\theta}^2\sin\frac{\theta}{2}$

For $\theta = 30°$: $\begin{cases} r = 2(15)\sin 15° = 7.76 \text{ in.} \\ \dot{r} = 15(4)\cos 15° = 58.0 \text{ in./sec} \\ \ddot{r} = -\frac{15}{2}4^2\sin 15° = -31.1 \text{ in./sec}^2 \end{cases}$

$a_r = \ddot{r} - r\dot{\theta}^2 = -31.1 - 7.76(4)^2 = -155.3 \text{ in./sec}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(58.0)(4) = 464 \text{ in./sec}^2$

$a = \sqrt{a_r^2 + a_\theta^2} = \underline{489 \text{ in./sec}^2}$

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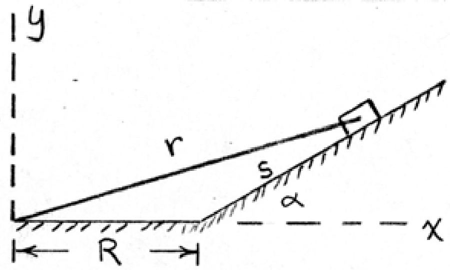

$$x = R + s\cos\alpha = R + (s_0 + v_0 t + \tfrac{1}{2}at^2)\cos\alpha$$

$$= R + \tfrac{1}{2}at^2\cos\alpha$$

$$y = s\sin\alpha$$

$$= \tfrac{1}{2}at^2\sin\alpha$$

$$r = \sqrt{x^2+y^2} = \sqrt{(R+\tfrac{1}{2}at^2\cos\alpha)^2 + (\tfrac{1}{2}at^2\sin\alpha)^2}$$

$$= \sqrt{R^2 + Rat^2\cos\alpha + \tfrac{1}{4}a^2t^4}$$

$$\dot{r} = \tfrac{1}{2}\left(R^2 + Rat^2\cos\alpha + \tfrac{1}{4}a^2t^4\right)^{-1/2}\left[2Rat\cos\alpha + a^2t^3\right]$$

$$= \frac{\tfrac{1}{2}at(2R\cos\alpha + at^2)}{\sqrt{R^2 + Rat^2\cos\alpha + \tfrac{1}{4}a^2t^4}}$$

2/165 From the solution to Prob. 2/164:

$$\begin{cases} x = R + \frac{1}{2}at^2\cos\alpha \\ y = \frac{1}{2}at^2\sin\alpha \end{cases}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left[\frac{\frac{1}{2}at^2\sin\alpha}{R + \frac{1}{2}at^2\cos\alpha}\right]$$

$$\dot{\theta} = \frac{\dfrac{(R + \frac{1}{2}at^2\cos\alpha)(at\sin\alpha) - (\frac{1}{2}at^2\sin\alpha)(at\cos\alpha)}{(R + \frac{1}{2}at^2\cos\alpha)^2}}{1 + \left[\dfrac{\frac{1}{2}at^2\sin\alpha}{R + \frac{1}{2}at^2\cos\alpha}\right]^2}$$

Simplify to

$$\dot{\theta} = \frac{Rat\sin\alpha}{R^2 + Rat^2\cos\alpha + \frac{1}{4}a^2t^4}$$

2/166 $r = 1.6 + 0.3\sin\frac{\pi t}{2}$ m ; $\theta = \frac{\pi}{4} + \frac{\pi}{8}\sin\frac{\pi t}{2}$

$\dot{r} = \frac{0.3\pi}{2}\cos\frac{\pi t}{2}$ m/s ; $\dot{\theta} = \frac{\pi^2}{16}\cos\frac{\pi t}{2}$ rad/s

$\ddot{r} = -\frac{0.3\pi^2}{4}\sin\frac{\pi t}{2}$ m/s^2 ; $\ddot{\theta} = -\frac{\pi^3}{32}\sin\frac{\pi t}{2}$ rad/s^2

$$v_r = \dot{r} = \frac{0.3\pi}{2}\cos\frac{\pi t}{2}$$

$$v_\theta = r\dot{\theta} = \left(1.6 + 0.3\sin\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{0.3\pi^2}{4}\sin\frac{\pi t}{2} - \left(1.6 + 0.3\sin\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \left(1.6 + 0.3\sin\frac{\pi t}{2}\right)\left(-\frac{\pi^3}{32}\sin\frac{\pi t}{2}\right) + 2\left(\frac{0.3\pi}{2}\cos\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)$$

At $t = 1$ s: $\left.\begin{array}{l} v_r = 0 \\ v_\theta = 0 \end{array}\right\}$ $v = \sqrt{v_r^2 + v_\theta^2} = \underline{0}$

$\left.\begin{array}{l} a_r = -0.740 \text{ m/s}^2 \\ a_\theta = -1.841 \text{ m/s}^2 \end{array}\right\}$ $a = \sqrt{0.740^2 + 1.841^2} = \underline{1.984 \text{ m/s}^2}$

At $t = 2$ s: $\left.\begin{array}{l} v_r = -0.471 \text{ m/s} \\ v_\theta = 0.987 \text{ m/s} \end{array}\right\}$ $\underline{v = 1.094 \text{ m/s}}$

$\left.\begin{array}{l} a_r = -0.609 \text{ m/s}^2 \\ a_\theta = 0.581 \text{ m/s}^2 \end{array}\right\}$ $\underline{a = 0.842 \text{ m/s}^2}$

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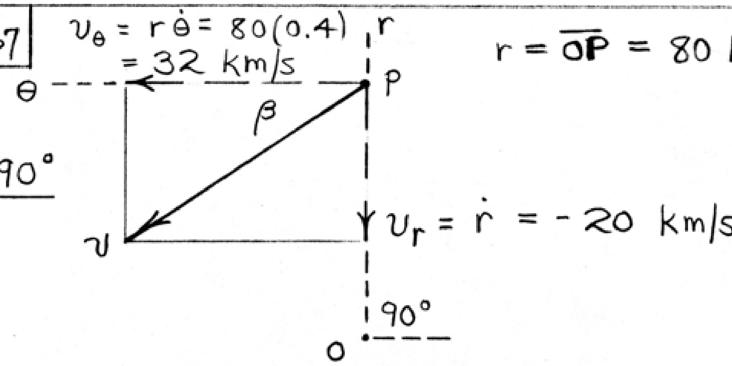
$r = \overline{OP} = 80$ km

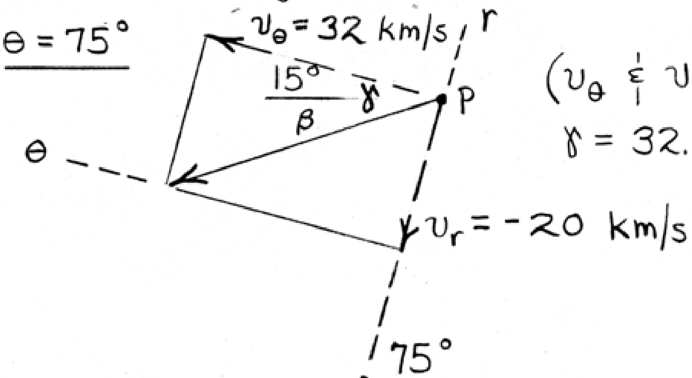
$\underline{\theta = 90^\circ}$



$$v = \sqrt{v_\theta^2 + v_r^2} = \sqrt{32^2 + 20^2} = \underline{37.7 \text{ km/s}}$$

$$\beta = \tan^{-1}\frac{|v_r|}{v_\theta} = \tan^{-1}\frac{20}{32} = \underline{32.0^\circ}$$

$\underline{\theta = 75^\circ}$



(v_θ & v_r unchanged)

$\gamma = 32.0^\circ$

$\underline{v = 37.7 \text{ km/s}}$ (unchanged)

$\beta = \gamma - 15^\circ = 32.0 - 15^\circ = \underline{17.01^\circ}$

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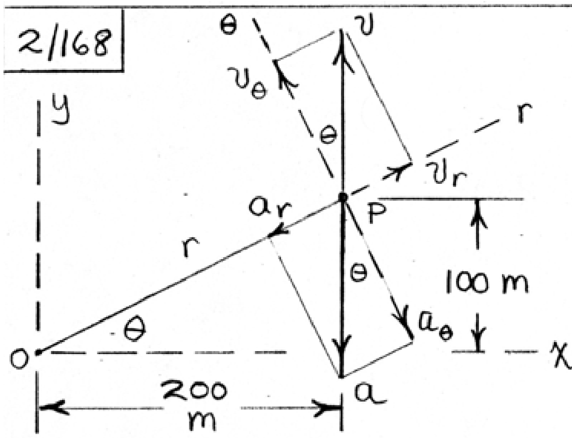

$r = \sqrt{200^2 + 100^2} = \underline{224 \text{ m}}$

$\theta = \tan^{-1} \frac{100}{200} = \underline{26.6°}$

$v_r = \dot{r} = v \sin\theta = 15 \sin 26.6° = \underline{6.71 \text{ m/s}}$

$v_\theta = r\dot{\theta}: \quad 15 \cos 26.6° = 224\dot{\theta}, \quad \dot{\theta} = \underline{0.06 \text{ rad/s}}$

$a = -g - kv^2 = -9.81 - 0.01(15)^2 = -12.06 \text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2: \quad -12.06 \sin 26.6° = \ddot{r} - 224(0.06)^2$

$\underline{\ddot{r} = -4.59 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}: \quad -12.06 \cos 26.6° = 224\ddot{\theta} + 2(6.71)(0.06)$

$\underline{\ddot{\theta} = -0.0518 \text{ rad/s}^2}$

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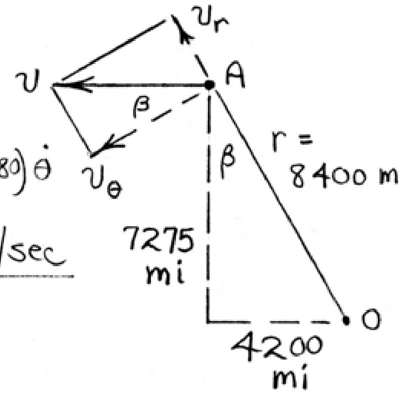

$$v = 12{,}149\left(\frac{5280}{3600}\right) = 17{,}819\ \frac{ft}{sec}$$

$$v_\theta = r\dot\theta : \quad 17{,}819 \cos 30^\circ = 8400(5280)\dot\theta$$

$$\underline{\dot\theta = 3.48(10^{-4})\ \text{rad/sec}}$$

$$v_r = \dot r : \quad 17{,}819 \sin 30^\circ = \dot r$$

$$\underline{\dot r = 8910\ \frac{ft}{sec}}$$

$$\beta = \tan^{-1}\left(\frac{4200}{7275}\right) = 30^\circ$$

$$a_\theta = r\ddot\theta + 2\dot r\dot\theta : \quad 0 = 8400(5280)\ddot\theta + 2(8910)(3.48)(10^{-4})$$

$$\underline{\ddot\theta = -1.398(10^{-7})\ \text{rad/sec}^2}$$

$$a_r = \ddot r - r\dot\theta^2 : \quad -7.159 = \ddot r - 8400(5280)(3.48 \times 10^{-4})^2$$

$$\underline{\ddot r = -1.790\ \text{ft/sec}^2}$$

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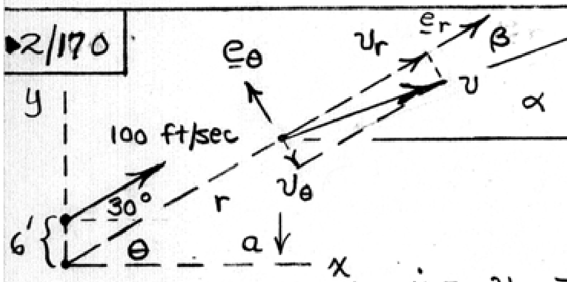

$x = x_0 + v_{x_0} t$

$= 0 + 100 \cos 30° (0.5)$

$= 43.3$ ft

$\dot{x} = v_{x_0} = 100 \cos 30° = 86.6 \frac{ft}{sec}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 = 6 + 100 \sin 30° (0.5) - 16.1 (0.5)^2 = 27.0$ ft

$\dot{y} = v_{y_0} - gt = 100 \sin 30° - 32.2(0.5) = 33.9$ ft/sec

$r = \sqrt{x^2 + y^2} = \sqrt{43.3^2 + 27.0^2} = \underline{51.0 \text{ ft}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(27.0/43.3) = \underline{31.9°}$

$\alpha = \tan^{-1}(v_y/v_x) = \tan^{-1}(33.9/86.6) = 21.4°$

$\beta = \theta - \alpha = 10.54°$, $v = \sqrt{\dot{x}^2 + \dot{y}^2} = 93.0$ ft/sec

$v_r = v \cos\beta = 93.0 \cos 10.54° = \underline{91.4 \text{ ft/sec} = \dot{r}}$

$v_\theta = -v \sin\beta = -93.0 \sin 10.54° = -17.02$ ft/sec

$-17.02 = r\dot{\theta} = 51.0\,\dot{\theta}$, $\underline{\dot{\theta} = -0.334 \text{ rad/sec}}$

$a = \ddot{y} = -32.2$ ft/sec^2

$a_r = -g \sin\theta = -32.2 \sin 31.9° = -17.03$ ft/sec^2

$-17.03 = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - 51.0(-0.334)^2$, $\underline{\ddot{r} = -11.35 \text{ ft/sec}^2}$

$a_\theta = -g \cos\theta = -32.2 \cos 31.9° = -27.3$ ft/sec^2

$-27.3 = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 51.0\,\ddot{\theta} + 2(91.4)(-0.334)$

$\underline{\ddot{\theta} = 0.660 \text{ rad/sec}^2}$

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$$\begin{cases} x = 30\cos 2t; & y = 40\sin 2t; & z = 20t + 3t^2 \\ \dot{x} = -60\sin 2t; & \dot{y} = 80\cos 2t; & \dot{z} = 20 + 6t \\ \ddot{x} = -120\cos 2t; & \ddot{y} = -160\sin 2t; & \ddot{z} = 6 \end{cases}$$

At $t = 2$ s :

$$\begin{cases} x = -19.61 \text{ mm}; & y = -30.3 \text{ mm}; & z = 52 \text{ mm} \\ \dot{x} = 45.4 \text{ mm/s}; & \dot{y} = -52.3 \text{ mm/s}; & \dot{z} = 32 \text{ mm/s} \\ \ddot{x} = 78.4 \text{ mm/s}^2; & \ddot{y} = 121.1 \text{ mm/s}^2; & \ddot{z} = 6 \text{ mm/s}^2 \end{cases}$$

$r = (x^2 + y^2 + z^2)^{1/2} = 63.3$ mm

$v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} = 76.3$ mm/s

$a = (\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2)^{1/2} = 144.4$ mm/s^2

$$\theta_1 = \cos^{-1}\left[\frac{\mathbf{r}\cdot\mathbf{v}}{rv}\right] = \cos^{-1}\left[\frac{-19.61(45.4) - 30.3(-52.3) + 52(32)}{(63.3)(76.3)}\right]$$

$= \underline{60.8°}$

$$\theta_2 = \cos^{-1}\left[\frac{\mathbf{r}\cdot\mathbf{a}}{ra}\right] = \cos^{-1}\left[\frac{-19.61(78.4) - 30.3(121.1) - 52(6)}{(63.3)(144.4)}\right]$$

$= \underline{122.4°}$

2/172 $v_{z_0} = 600 \sin 60° = 520$ ft/sec

$v_{xy_0} = 600 \cos 60° = 300$ ft/sec

$v_z = v_{z_0} - gt = 520 - 32.2(20) = \underline{-124.4 \text{ ft/sec}}$

$v_{xy} = v_{xy_0} = 300$ ft/sec = constant

$v_x = -v_{xy} \sin 20° = -300 \sin 20° = \underline{-102.6 \text{ ft/sec}}$

$v_y = v_{xy} \cos 20° = 300 \cos 20° = \underline{282 \text{ ft/sec}}$

$d_{xy} = v_{xy} t = 300(20) = 6000$ ft

$x = -d_{xy} \sin 20° = -6000 \sin 20° = \underline{-2050 \text{ ft}}$

$y = d_{xy} \cos 20° = 6000 \cos 20° = \underline{5640 \text{ ft}}$

$z = v_{z_0} t - \frac{1}{2} g t^2 = 520(20) - 16.1(20)^2 = \underline{3950 \text{ ft}}$

$\underline{a_x = a_y = 0, \quad a_z = -g = -32.2 \text{ ft/sec}^2}$

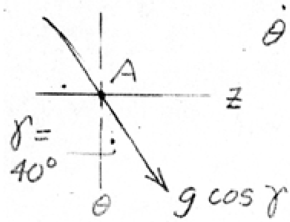
$\boxed{2/173}$ $\underline{v} = 4\underline{i} - 2\underline{j} - \underline{k}$ m/s, $v = \sqrt{4^2 + 2^2 + 1^2} = 4.58 \frac{m}{s}$

$$a_n = a \sin 20° = 8 \sin 20° = 2.74 \text{ m/s}^2$$

From $a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{4.58^2}{2.74} = \underline{7.67 \text{ m}}$

$$\dot{v} = a_t = a \cos 20° = 8 \cos 20° = \underline{7.52 \text{ m/s}^2}$$

2/174 $v_\theta = r\dot{\theta}$ & $v_\theta = v\cos\gamma$ so $\dot{\theta} = \frac{v}{r}\cos\gamma$



$\dot{\theta} = \frac{15}{5}(0.7660) = 2.298$ rad/s

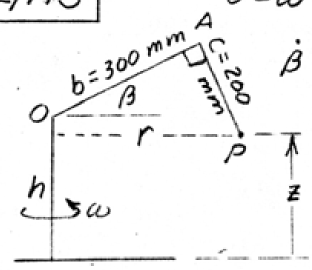
$a_\theta = g\cos^2\gamma = 9.81(0.7660)^2 = 5.76\ m/s^2$

$a_z = g\cos\gamma\sin\gamma = 9.81(0.7660)(0.6428)$
$= 4.83\ m/s^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(2.298)^2 = -26.41\ m/s^2$

$a = \sqrt{26.41^2 + 5.76^2 + 4.83^2} = \underline{27.5\ m/s^2}$

$\boxed{2/175}$



$\dot{\theta} = \omega = \frac{40}{180}\pi = 0.698$ rad/s

$\dot{\beta} = \frac{10}{180}\pi = 0.1745$ rad/s

$\ddot{\beta} = \frac{20}{180}\pi = 0.349$ rad/s^2

Use cylindrical coordinates

$r = b\cos\beta + c\sin\beta$, $\dot{r} = (-b\sin\beta + c\cos\beta)\dot{\beta}$

$\ddot{r} = (-b\cos\beta - c\sin\beta)\dot{\beta}^2 + (-b\sin\beta + c\cos\beta)\ddot{\beta}$

$z = h + b\sin\beta - c\cos\beta$, $\dot{z} = (b\cos\beta + c\sin\beta)\dot{\beta}$

$\ddot{z} = (-b\sin\beta + c\cos\beta)\dot{\beta}^2 + (b\cos\beta + c\sin\beta)\ddot{\beta}$

For $\beta = 30°$, $\dot{r} = (-300\times0.5 + 200\times0.866)(0.1745) = 4.050$ mm/s

$\ddot{r} = (-300\times0.866 - 200\times0.5)(0.1745)^2$
$+(-300\times0.5 + 200\times0.866)(0.349) = -2.860$ mm/s^2

$\ddot{z} = (-300\times0.5 + 200\times0.866)(0.1745)^2$
$+(300\times0.866 + 200\times0.5)(0.349) = 126.30$ mm/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -2.860 - 359.8(0.698)^2 = -178.23$ mm/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(4.049)(0.698) = 5.65$ mm/s^2

$a_z = \ddot{z} = 126.30$ mm/s^2

$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2}$

$= \underline{219}$ mm/s^2

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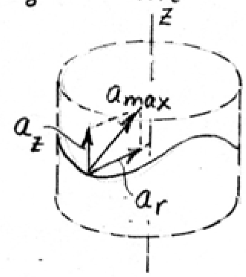
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\omega^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$

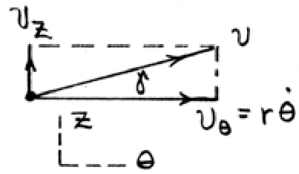
$a_z = \frac{d^2}{dt^2}(z_0 \sin 2\pi nt) = -4n^2\pi^2 z_0 \sin 2\pi nt$

$a = \sqrt{(-r\omega^2)^2 + (-4n^2\pi^2 z_0 \sin 2\pi nt)^2}$

$a_{max} = \sqrt{r^2\omega^4 + 16n^4\pi^4 z_0^2}$



2/177 Helix angle is $\gamma = \tan^{-1} \dfrac{10}{24\pi} = 7.55°$

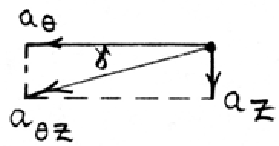


$$v = 15\left(\frac{5280}{3600}\right) = 22 \text{ ft/sec}$$

$$v_z = v \sin\gamma = 22 \sin 7.55° = 2.89 \text{ ft/sec}$$

$$v_\theta = v\cos\gamma = 22\cos 7.55° = 21.8 \text{ ft/sec}$$

From $v_\theta = r\dot{\theta}$, $\dot{\theta} = \dfrac{v_\theta}{r} = \dfrac{21.8}{24} = 0.909$ rad/sec



$$a_{\theta z} = 2\left(\frac{5280}{3600}\right) = 2.93 \text{ ft/sec}^2$$

$$a_\theta = -2.93\cos 7.55° = \underline{-2.91 \text{ ft/sec}^2}$$

$$a_z = -2.93 \sin 7.55° = \underline{-0.386 \text{ ft/sec}^2}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 24(0.909)^2 = \underline{-19.82 \text{ ft/sec}^2}$$

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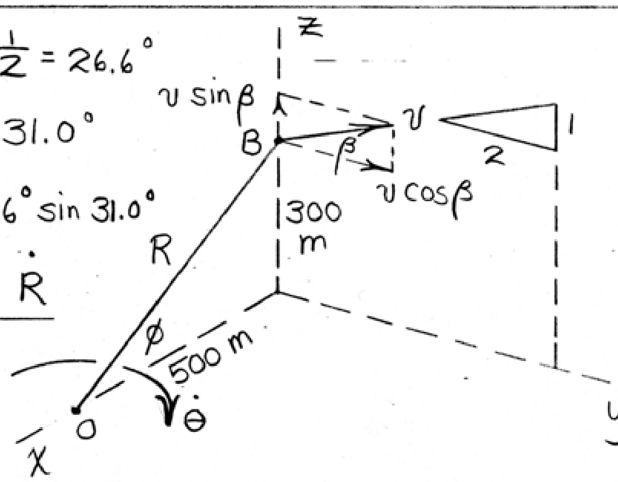
$\beta = \tan^{-1}\frac{1}{2} = 26.6°$

$\phi = \tan^{-1}\frac{3}{5} = 31.0°$

$v_R = 400 \sin 26.6° \sin 31.0°$

$= \underline{92.0 \text{ km/h} = \dot{R}}$



$v_\theta = R\dot{\theta}\cos\phi: \quad \frac{400}{3.6}\cos 26.6° = 500\,\dot{\theta}$

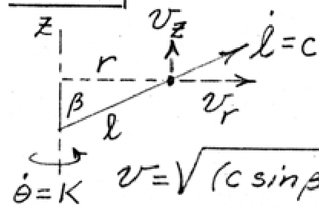
$\underline{\dot{\theta} = 0.1988 \text{ rad/s}}$

$v_\phi = R\dot{\phi}: \quad \frac{400}{3.6}\sin 26.6° \cos 31° = \frac{500}{\cos 31.0°}\,\dot{\phi}$

$\underline{\dot{\phi} = 0.0731 \text{ rad/s}}$

2/179

$v_r = \dot{l}\sin\beta = c\sin\beta$



$v_\theta = r\dot{\theta} = (l\sin\beta)K = Kl\sin\beta$

$v_z = \dot{l}\cos\beta = c\cos\beta$

$$v = \sqrt{(c\sin\beta)^2 + (Kl\sin\beta)^2 + (c\cos\beta)^2}$$

$$= \underline{\sqrt{c^2 + K^2 l^2 \sin^2\beta}}$$

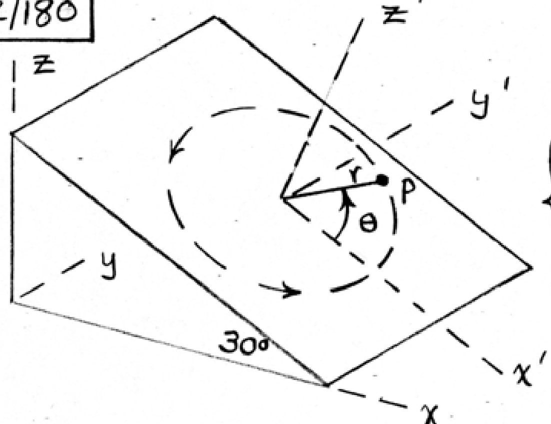
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - l\sin\beta\,(K^2) = -K^2 l\sin\beta$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2c\sin\beta\,(K) = 2Kc\sin\beta$

$a_z = \dot{v}_z = 0$

$$a = \sqrt{(-K^2 l\sin\beta)^2 + (2Kc\sin\beta)^2} = \underline{K\sin\beta\sqrt{K^2l^2 + 4c^2}}$$

2/180



$\theta = \omega t = \frac{v}{r} t$

$\dot{\theta} = \frac{v}{r}$

$\begin{cases} x' = r\cos\theta \\ y' = r\sin\theta \\ z' = 0 \end{cases}$

$\begin{cases} \dot{x}' = -r\dot{\theta}\sin\theta \\ \dot{y}' = r\dot{\theta}\cos\theta \\ \dot{z}' = 0 \end{cases}$

$\ddot{x}' = -r\dot{\theta}^2\cos\theta$, $\ddot{y}' = -r\dot{\theta}^2\sin\theta$, $\ddot{z}' = 0$

Component relationships: (Similar for $\dot{x}$, $\ddot{x}$, etc.)

$\begin{cases} x = x'\cos 30° + z'\sin 30° \\ y = y' \\ z = -x'\sin 30° + z'\cos 30° \end{cases}$

Thus

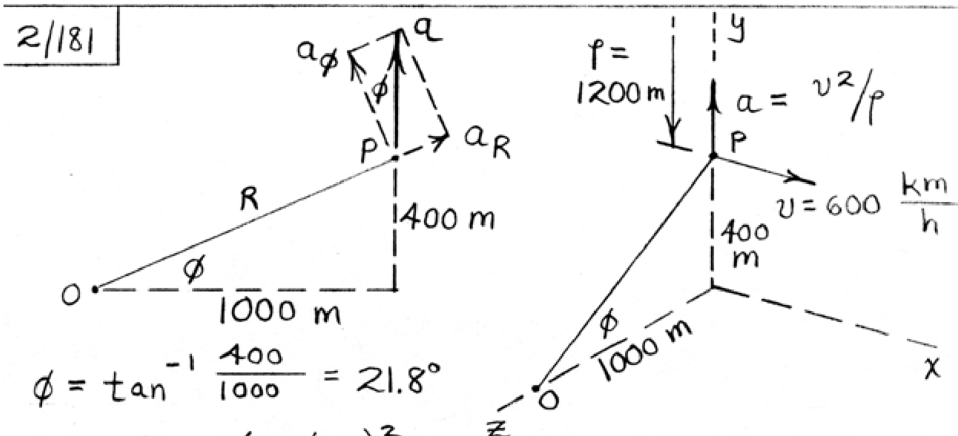
$x = \frac{\sqrt{3}}{2} r\cos\frac{v}{r}t$, $y = r\sin\frac{v}{r}t$, $z = -\frac{1}{2} r\cos\frac{v}{r}t$

$\dot{x} = -\frac{\sqrt{3}}{2} v\sin\frac{v}{r}t$, $\dot{y} = v\cos\frac{v}{r}t$ $\dot{z} = \frac{1}{2} v\sin\frac{v}{r}t$

$\ddot{x} = -\frac{\sqrt{3}}{2}\frac{v^2}{r}\cos\frac{v}{r}t$, $\ddot{y} = -\frac{v^2}{r}\sin\frac{v}{r}t$, $\ddot{z} = \frac{1}{2}\frac{v^2}{r}\cos\frac{v}{r}t$

Note that position relationships do not include the constants associated with the origin positions.

2/181



$\phi = \tan^{-1} \frac{400}{1000} = 21.8°$

$a = \frac{v^2}{\rho} = \frac{(600/3.6)^2}{1200} = 23.1 \text{ m/s}^2$

$\dot{R} = 0, \quad \dot{\phi} = 0, \quad \dot{\theta} = \frac{v_\theta}{R\cos\phi} = \frac{600(1000)/3.6}{1000} = 0.1667 \frac{\text{rad}}{\text{s}}$

Eqs. 2/19: $a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$

$23.1 \sin 21.8° = \ddot{R} - 0 - \frac{1000}{\cos 21.8°}(0.1667)^2 \cos^2 21.8°$

$\underline{\ddot{R} = 34.4 \text{ m/s}^2}$

$a_\phi = \frac{1}{R}\frac{d}{dt}(R^2\dot{\phi}) + R\dot{\theta}^2 \sin\phi\cos\phi = 2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi$

$23.1 \cos 21.8° = 0 + \frac{1000}{\cos 21.8°}\ddot{\phi} + \frac{1000}{\cos 21.8°} 0.1667^2 \sin 21.8° \cos 21.8°$

$\underline{\ddot{\phi} = 0.01038 \text{ rad/s}^2}$

2/182 $R = 0.75 + 0.5 = 1.25\ m$, $\dot{R} = 0.2\ m/s$, $\ddot{R} = -0.3\ \frac{m}{s^2}$

$\phi = 30°$, $\dot{\phi} = 10\left(\frac{\pi}{180}\right)$ rad/s, $\ddot{\phi} = 0$, $\dot{\theta} = 20\left(\frac{\pi}{180}\right)$ rad/s, $\ddot{\theta} = 0$

$$\begin{cases} v_R = \dot{R} = 0.2\ m/s \\ v_\theta = R\dot{\theta}\cos\phi = 1.25\left(20\frac{\pi}{180}\right)\cos 30° = 0.378\ \frac{m}{s} \\ v_\phi = R\dot{\phi} = 1.25\left(10\frac{\pi}{180}\right) = 0.218\ m/s \end{cases}$$

$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{0.480\ m/s}$$

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$$
$$= -0.3 - 1.25\left(10\frac{\pi}{180}\right)^2 - 1.25\left(20\frac{\pi}{180}\right)^2\cos^2 30°$$
$$= -0.4523\ m/s^2$$

$$a_\theta = \cos\phi\left[2\dot{R}\dot{\theta} + R\ddot{\theta}\right] - 2R\dot{\theta}\dot{\phi}\sin\phi$$
$$= \cos 30°\left[2(0.2)\left(20\frac{\pi}{180}\right) + 1.25(0)\right]$$
$$- 2(1.25)\left(10\frac{\pi}{180}\right)\left(20\frac{\pi}{180}\right)\sin 30° = 0.0448\ \frac{m}{s^2}$$

$$a_\phi = 2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi$$
$$= 2(0.2)\left(10\frac{\pi}{180}\right) + 1.25(0) + 1.25\left(20\frac{\pi}{180}\right)^2 0.5\ \frac{\sqrt{3}}{2}$$
$$= 0.1358\ m/s^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.474\ m/s^2}$$

2/183 Spherical coordinates

$v_R = \dot{R} = 0.5$ m/s

$v_\theta = R\dot{\theta}\cos\phi = 15\left(10\frac{\pi}{180}\right)\cos 30° = 2.27$ m/s

$v_\phi = R\dot{\phi} = 15\left(7\frac{\pi}{180}\right) = 1.833$ m/s

$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{2.96 \text{ m/s}}$

$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$

$= 0 - 15\left(7\frac{\pi}{180}\right)^2 - 15\left(10\frac{\pi}{180}\right)^2\cos^2 30° = -0.567$ m/s^2

$a_\theta = \frac{\cos\phi}{R}\left[R^2\ddot{\theta} + 2R\dot{R}\dot{\theta}\right] - 2R\dot{\theta}\dot{\phi}\sin\phi$

$= \frac{\cos 30°}{15}\left[0 + 2(15)(0.5)\left(10\frac{\pi}{180}\right)\right] - 2(15)\left(10\frac{\pi}{180}\right)\left(7\frac{\pi}{180}\right)\sin 30°$

$= -0.1687$ m/s^2

$a_\phi = \frac{1}{R}\left[R^2\ddot{\phi} + 2R\dot{R}\dot{\phi}\right] + R\dot{\theta}^2\sin\phi\cos\phi$

$= \frac{1}{15}\left[0 + 2(15)(0.5)\left(7\frac{\pi}{180}\right)\right] + 15\left(10\frac{\pi}{180}\right)^2\sin 30°\cos 30°$

$= 0.320$ m/s^2

$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.672 \text{ m/s}^2}$

$\boxed{2/184}$ Use Eq. 2/19 where $\dot{\varphi} = -\dot{\beta}$, $R = L$, $\dot{\theta} = \omega$

$a_R = 0 - 1.2(-\frac{3}{2})^2 - 1.2(2)^2\frac{1}{2} = \underline{-5.10 \text{ m/s}^2}$

$a_\theta = \frac{\sin\beta}{L}(2L\dot{L}\omega + 0) + 2L\omega\dot{\beta}\cos\beta = 2\omega(\dot{L}\sin\beta + L\dot{\beta}\cos\beta)$

$= 2(2)(0.9\frac{1}{\sqrt{2}} + 1.2(\frac{3}{2})\frac{1}{\sqrt{2}}) = \frac{10.8}{\sqrt{2}} = \underline{7.64 \text{ m/s}^2}$

$a_\varphi = -2\dot{L}\dot{\beta} + L\omega^2\cos\beta\sin\beta = -2(0.9)\frac{3}{2} + 1.2(2^2)\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$

$= -2.7 + 2.4 = \underline{-0.3 \text{ m/s}^2}$

2/185

$R = 200 + 50 \sin 4\pi t$ $\qquad$ $\dot{\theta} = 120\left(\frac{2\pi}{60}\right) = 4\pi \text{ rad/s}$

$\dot{R} = 200\pi \cos 4\pi t$ $\qquad$ $\ddot{\theta} = 0$

$\ddot{R} = -800\pi^2 \sin 4\pi t$ $\qquad$ $\phi = \frac{\pi}{2} - \beta = 60°, \ \dot{\phi} = \ddot{\phi} = 0$

For $\dot{R}$ maximum, $\cos 4\pi t = 1$ and $\sin 4\pi t = 0$

Eq. 2/19 : $a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2 \cos^2\phi$

$= 0 + (200+0)0 - (200+0)(4\pi)^2 \cos^2 60° = -800\pi^2 \ \frac{\text{mm}}{\text{s}^2}$

$a_\theta = \frac{\cos\phi}{R} \frac{d}{dt}(R^2\dot{\theta}) - 2R\dot{\theta}\dot{\phi}\sin\phi$

$= 2\dot{R}\dot{\theta}\cos\phi - 2R\dot{\theta}\dot{\phi}\sin\phi$

$= 2(200\pi \cdot 1)(4\pi)\cos 60° - 2(200+0)(4\pi)(0)\sin 60°$

$= 800\pi^2 \text{ mm/s}^2$

$a_\phi = \frac{1}{R}\frac{d}{dt}(R^2\dot{\phi}) + R\dot{\theta}^2 \sin\phi\cos\phi$

$= 0 + (200+0)(4\pi)^2 \sin 60° \cos 60° = 800\sqrt{3}\,\pi^2 \text{ mm/s}^2$

$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = 17\,660 \text{ mm/s}^2$ or $\underline{17.66 \text{ m/s}^2}$

▶ 2/186 $R = const$ $\theta = \omega t$ $\sin\phi = z/R$

$$z = \frac{h}{2}(1 - \cos 2\theta) \quad , \quad \dot{z} = \omega h \sin 2\theta \quad \text{where } \dot{\theta} = \omega$$

$$(\cos\phi)\dot{\phi} = \frac{1}{R}\dot{z} \, , \quad \dot{\phi} = \frac{\omega h \sin 2\theta}{R\cos\phi}$$

$$v_R = \dot{R} = 0$$

$$v_\theta = R\dot{\theta}\cos\phi = R\omega\sqrt{1 - \sin^2\phi} = R\omega\sqrt{1 - \left(\frac{h}{2R}[1 - \cos 2\theta]\right)^2}$$

$$v_\phi = R\dot{\phi} = \frac{\omega h \sin 2\theta}{\cos\phi} = h\omega \frac{\sin 2\theta}{\sqrt{1 - \left(\frac{h}{2R}[1 - \cos 2\theta]\right)^2}}$$

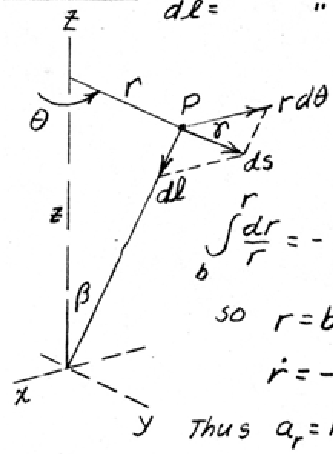
When $\theta = \omega t = \pi/4$, $1 - \cos 2\theta = 1$ so that

$$\underline{v_\theta = R\omega\sqrt{1 - (h/2R)^2}} \, , \quad \underline{v_\phi = \frac{h\omega}{\sqrt{1 - (h/2R)^2}}} \quad , \quad \underline{v_R = 0}$$

▶2/187

ds = differential distance along curve

$d\ell$ = " " in direction of cone element



$r = \frac{b}{h} z, \quad \tan\beta = b/h$

$d\ell = r\,d\theta \tan\gamma$

$dr = -d\ell \sin\beta = -r\,d\theta \tan\gamma \sin\beta$

$$\int_b^r \frac{dr}{r} = -\tan\gamma \sin\beta \int_0^\theta d\theta, \quad \ln r\Big]_b^r = -\tan\gamma \sin\beta\, \theta$$

so $r = be^{-\tan\gamma \sin\beta\,\theta}$, $r = be^{-K\theta}$ where $K = \tan\gamma \sin\beta$

$\dot{r} = -bK\dot{\theta}e^{-K\theta}$, $\ddot{r} = bK^2\dot{\theta}^2 e^{-K\theta}$, $\ddot{\theta} = 0$

Thus $a_r = \ddot{r} - r\dot{\theta}^2 = b\dot{\theta}^2 e^{-K\theta}(K^2 - 1)$

or $\underline{a_r = b\dot{\theta}^2(\tan^2\gamma \sin^2\beta - 1)\, e^{-\theta \tan\gamma \sin\beta}}$

$\underline{\text{where } \beta = \tan^{-1}\frac{b}{h}}$

▶2/188 The terms appearing in Eq. 2/19 are

$R = 50 + 200(1/2)^2 = 100$ mm, $\dot{R} = 400t = 400(1/2) = 200$ mm/s

$\ddot{R} = 400$ mm/s²

$\theta = \omega t = \frac{\pi}{3}(\frac{1}{2}) = \pi/6$ rad, $\dot{\theta} = \pi/3$ rad/s, $\ddot{\theta} = 0$

$\phi = \dot{\phi} t = \frac{2\pi}{3}(\frac{1}{2}) = \pi/3$ rad, $\dot{\phi} = 2\pi/3$ rad/s, $\ddot{\phi} = 0$

$\sin\theta = 1/2$, $\cos\theta = \sqrt{3}/2$, $\sin\phi = \sqrt{3}/2$, $\cos\phi = 1/2$

$$\frac{d}{dt}(R^2\dot{\theta}) = 2R\dot{R}\dot{\theta} + R^2\ddot{\theta} = 2(0.1)(0.2)\pi/3 + 0 = \frac{0.04\pi}{3}\ (m/s)^2$$

$$\frac{d}{dt}(R^2\dot{\phi}) = 2R\dot{R}\dot{\phi} + R^2\ddot{\phi} = 2(0.1)(0.2)2\pi/3 + 0 = \frac{0.08\pi}{3}\ (m/s)^2$$

Thus the components of $\underline{a}$ from Eq. 2/19 become

$$a_R = 0.40 - 0.10(2\pi/3)^2 - 0.10(\pi/3)^2(1/2)^2 = -0.0661\ m/s^2$$

$$a_\theta = \frac{1/2}{0.10}\frac{0.04\pi}{3} - 2(0.10)(\pi/3)(2\pi/3)(\sqrt{3}/2) = -0.1704\ m/s^2$$

$$a_\phi = \frac{1}{0.10}\frac{0.08\pi}{3} + 0.10(\pi/3)^2(\sqrt{3}/2)(1/2) = 0.885\ m/s^2$$

The magnitude of the acceleration is, then,

$$a = \sqrt{(-0.0661)^2 + (-0.1704)^2 + (0.885)^2} = \underline{0.904\ m/s^2}$$

2/189 $\underline{v}_{B/A} = \underline{v}_B - \underline{v}_A = 40\underline{i} - (-80\underline{i}) = 120\underline{i}\ \frac{km}{h}$

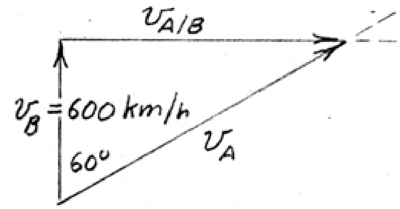
$\underline{a}_{B/A} = \underline{a}_B - \underline{a}_A = \underline{0} - 2\underline{i} = -2\underline{i}\ m/s^2$

— — — — x

2/190 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$v_A = \dfrac{600}{\cos 60^\circ} = \underline{1200 \text{ km/h}}$

$v_{A/B} = 600 \tan 60^\circ$
$= \underline{1039 \text{ km/h}}$

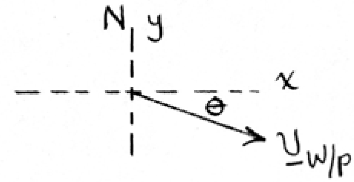


2/191 (a) $\underline{v}_{W/P} = \underline{v}_W - \underline{v}_P$

$$= 3\left(\frac{\sqrt{2}}{2}\underline{i} - \frac{\sqrt{2}}{2}\underline{j}\right) - (-4\underline{i}) = \underline{6.12\underline{i} - 2.12\underline{j}\ \frac{\text{mi}}{\text{hr}}}$$

or $v_{W/P} = (6.12^2 + 2.12^2)^{1/2} = \underline{6.48 \text{ mi/hr}}$

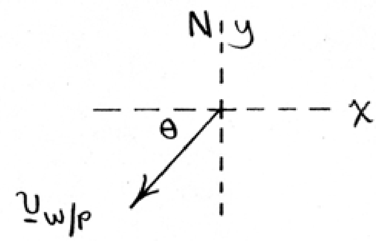
at $\theta = \tan^{-1}\frac{2.12}{6.12} = \underline{19.11^\circ \text{ south of east}}$



(b) $\underline{v}_{W/P} = \underline{v}_W - \underline{v}_P = 3\left(\frac{\sqrt{2}}{2}\underline{i} - \frac{\sqrt{2}}{2}\underline{j}\right) - 4\underline{i} = \underline{-1.879\underline{i} - 2.12\underline{j}\ \frac{\text{mi}}{\text{hr}}}$

or $v_{W/P} = (1.879^2 + 2.12^2)^{1/2} = \underline{2.83 \text{ mi/hr}}$

at $\theta = \tan^{-1}\frac{2.12}{1.879} = \underline{48.5^\circ \text{ south of west}}$



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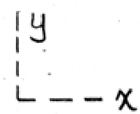
$$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B$$

$$= 120[\cos 15° \underline{i} + \sin 15° \underline{j}] - 90[\cos 60° \underline{i} + \sin 60° \underline{j}]$$

$$= \underline{70.9 \underline{i} - 46.9 \underline{j} \quad \text{km/h}}$$

$$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \underline{0} - 3(-\cos 60° \underline{i} - \sin 60° \underline{j})$$

$$= \underline{1.5 \underline{i} + 2.60 \underline{j} \quad \text{m/s}^2}$$



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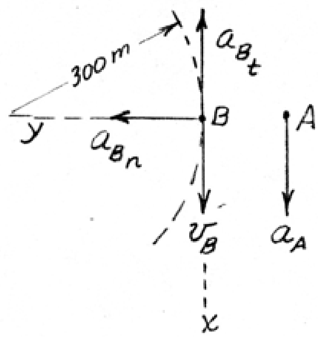
$$a_{B_t} = \frac{8}{3.6} = 2.22\ m/s^2, \quad a_A = \frac{8}{3.6} = 2.22\ m/s^2$$

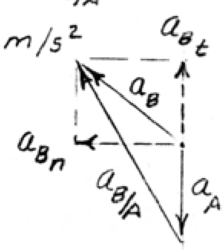
$$a_{B_n} = \frac{v_B^2}{\rho} = \frac{(100/3.6)^2}{300} = 2.57\ m/s^2$$



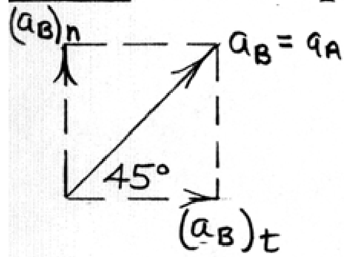
$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

$$-2.22\underline{i} + 2.57\underline{j} = 2.22\underline{i} + \underline{a}_{B/A}$$

$$\underline{a}_{B/A} = \underline{-4.44\underline{i} + 2.57\underline{j}}\ m/s^2$$



$\boxed{2/194}$ With $\underline{a}_{B/A} = \underline{0}$, $\underline{a}_A = \underline{a}_B$



$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{\left(45\,\frac{44}{30}\right)^2}{600}$$

$$= 7.26 \text{ ft/sec}^2$$

$$\dot{v}_B = (a_B)_t = \underline{7.26 \text{ ft/sec}^2}$$

$$a_A = a_B = 7.26\sqrt{2} = \underline{10.27 \frac{\text{ft}}{\text{sec}^2}}$$

$\boxed{2/195}$ $\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B$, $\Omega = 3\,(2\pi/60) = 0.314\ \frac{\text{rad}}{\text{s}}$

$$= \frac{18}{3.6}\,\underline{i} - 9(0.314)(\cos 45°\,\underline{i} - \sin 45°\,\underline{j})$$

$$= \underline{3.00\,\underline{i} + 1.999\,\underline{j}\ \text{m/s}}$$

$$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = 3\,\underline{i} - 9\,(0.314)^2(-\cos 45°\,\underline{i} - \sin 45°\,\underline{j})$$

$$= \underline{3.63\,\underline{i} + 0.628\,\underline{j}\ \text{m/s}^2}$$

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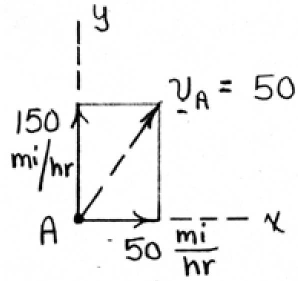


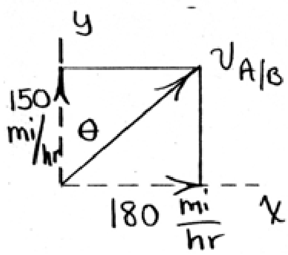
$\underline{v}_A = 50\underline{i} + 150\underline{j}$ mi/hr

$\underline{v}_B = (180-50)(-\underline{i}) = -130\underline{i}\ \frac{mi}{hr}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$: $50\underline{i} + 150\underline{j} = -130\underline{i} + \underline{v}_{A/B}$

$\underline{v}_{A/B} = 180\underline{i} + 150\underline{j}$ mi/hr

$v_{A/B} = \sqrt{180^2 + 150^2} = \underline{234 \text{ mi/hr}}$



$\theta = \tan^{-1}\left(\frac{180}{150}\right) = \underline{50.2°}$

(east of north)

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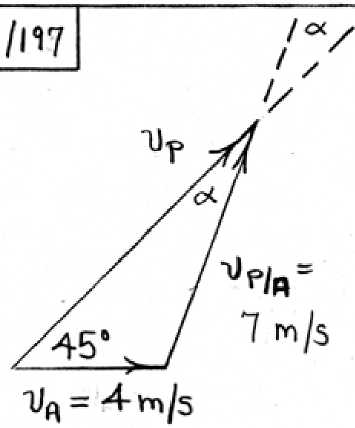


With P being the puck:

$\underline{v}_P = \underline{v}_A + \underline{v}_{P/A}$

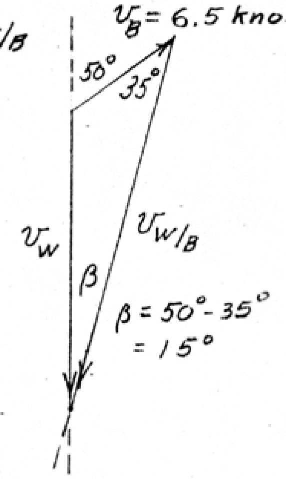
Law of sines :

$$\frac{\sin 45°}{7} = \frac{\sin \alpha}{4}$$

$\underline{\alpha = 23.8°}$

2/198 w = wind
B = boat

$\underline{v}_W = \underline{v}_B + \underline{v}_{W/B}$



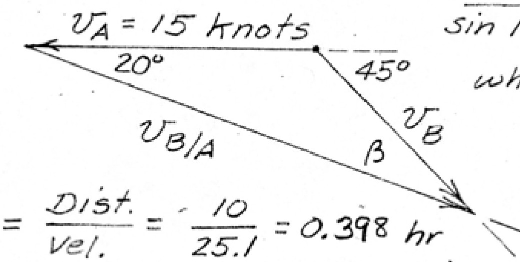
Law of sines,

$$\frac{v_W}{\sin 35^\circ} = \frac{6.5}{\sin 15^\circ}$$

$$v_W = \frac{6.5(0.5736)}{0.2588} = \underline{14.40 \text{ knots}}$$

$\beta = 50^\circ - 35^\circ$
$= 15^\circ$

$\boxed{2/199}$ $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$



$$\frac{v_{B/A}}{\sin 135°} = \frac{15}{\sin\beta}$$

where $\beta = 180 - 20 - 135$
$= 25°$

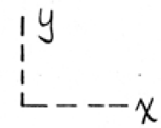
$$v_{B/A} = \frac{0.7071}{0.4226}15$$
$= 25.1$ knots

$$t = \frac{\text{Dist.}}{\text{Vel.}} = \frac{10}{25.1} = 0.398 \text{ hr}$$
or 23.9 min

So collision would occur at $\underline{2{:}24 \text{ pm}}$

2/200 Drop: $v_D = \sqrt{2gh} = \sqrt{2(9.81)(6)} = 10.85$ m/s

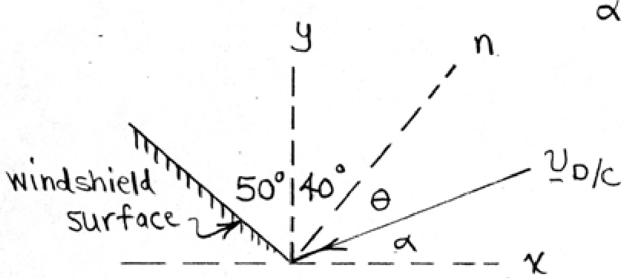
Car: $v_C = 100/3.6 = 27.8$ m/s



$\underline{v}_{D/C} = \underline{v}_D - \underline{v}_C = -10.85\underline{j} - 27.8\underline{i}$ m/s

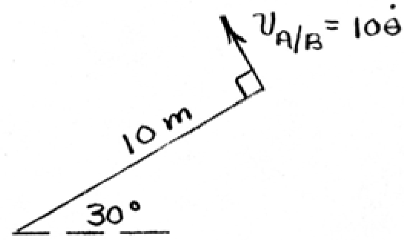
$\alpha = \tan^{-1} \dfrac{10.85}{27.8}$

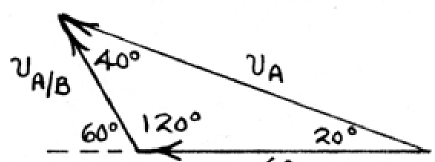
$= 21.3°$



$40° + \theta + \alpha = 90° \Rightarrow \underline{\theta = 28.7° \text{ below normal}}$

2/201 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$





$v_B = \frac{60}{3.6} = 16.67$ m/s

Law of sines : $\frac{16.67}{\sin 40°} = \frac{v_A}{\sin 120°}$

$v_A = 22.5$ m/s or $\underline{v_A = 80.8 \text{ km/h}}$

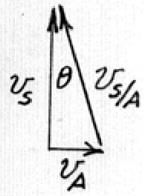
$v_{A/B} = 16.67 \frac{\sin 20°}{\sin 40°} = 10\dot{\theta}$, $\underline{\dot{\theta} = 0.887 \frac{\text{rad}}{\text{s}}}$

2/202 Let s = satellite, A = observer.

$\underline{v}_s = \underline{v}_A + \underline{v}_{s/A}$, $\quad v_A = R\omega$

$= 6378(0.729)(10^{-4})(3600)$

$= 1674$ km/h (East)

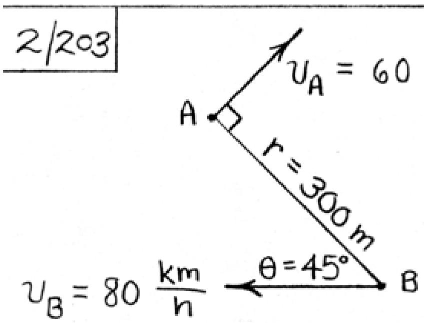


$v_s = 27\,940$ km/h (North)

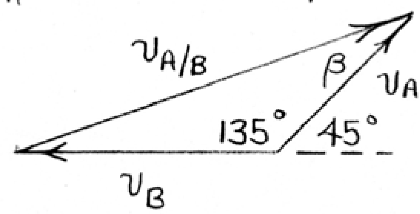
$\theta = \tan^{-1}\dfrac{1674}{27940} = 3.43°$

Satellite appears to travel $\underline{3.43°}$ $\underline{\text{west of north}}$

2/203



$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$



$v_{A/B}^2 = 60^2 + 80^2 - 2(60)(80)\cos 135°$

$v_{A/B} = 129.6$ km/h or $\underline{36.0 \text{ m/s}}$

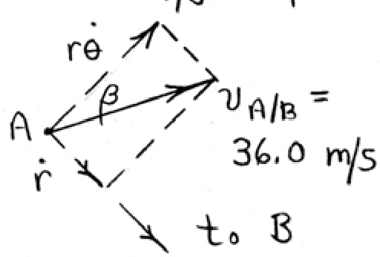
$\frac{36.0}{\sin 135°} = \frac{80/3.6}{\sin\beta}, \quad \beta = 25.9°$

$r\dot{\theta} = v_{A/B}\cos\beta$: $300\,\dot{\theta} = 36.0\cos 25.9°$

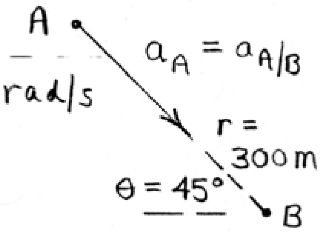
$\underline{\dot{\theta} = 0.1079 \text{ rad/s}}$

$\dot{r} = -v_{A/B}\sin\beta$: $\dot{r} = -36.0\sin 25.9°$

$\underline{\dot{r} = -15.71 \text{ m/s}}$



2/204 From Prob. 2/203,



$\dot{r} = -15.71$ m/s , $\dot{\theta} = 0.1079$ rad/s

$$\underline{a}_A = \overset{\nearrow 0}{\underline{a}_B} + \underline{a}_{A/B}$$

$$a_A = \frac{v_A^2}{\rho} = \frac{(60/3.6)^2}{300}$$

$$= 0.926 \text{ m/s}^2 = a_{A/B}$$

$$(a_{A/B})_r = \ddot{r} - r\dot{\theta}^2 : \ -0.926 = \ddot{r} - 300(0.1079)^2$$

$$\underline{\ddot{r} = 2.57 \text{ m/s}^2}$$

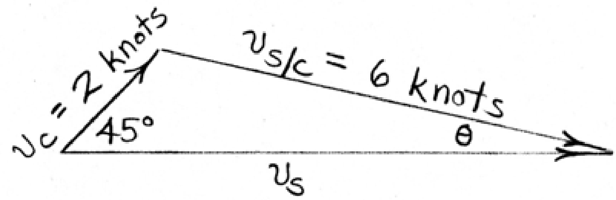
$$(a_{A/B})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : \ 0 = 300\ddot{\theta} + 2(-15.71)(0.1079)$$

$$\underline{\ddot{\theta} = 0.01130 \text{ rad/s}^2}$$

2/205

$\underline{v}_s$: true velocity of ship (A to B)
$\underline{v}_c$: true velocity of current (2 knots NE)
$v_{s/c}$: velocity of ship relative to current (magnitude 6 knots)

$\underline{v}_s = \underline{v}_c + \underline{v}_{s/c}$



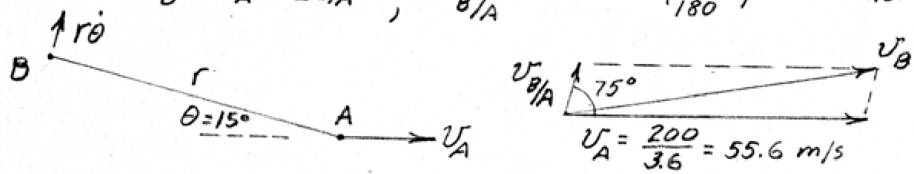
Law of sines : $\dfrac{2}{\sin\theta} = \dfrac{6}{\sin 45°}$, $\theta = 13.63°$

So heading $H = 90° + 13.63° \cong \underline{104°}$

$v_s = 2\cos 45° + 6\cos 13.63° = 7.25$ knots

Time $t = \dfrac{10}{7.25} = 1.380$ hr or $\underline{t = 1 \text{ hr } 23 \text{ min}}$

$\boxed{2/206}$ $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$, $v_{B/A} = r\dot{\theta} = 60(\frac{5}{180}\pi) = 5.24$ m/s



$v_A = \frac{200}{3.6} = 55.6$ m/s

$v_B^2 = (5.24)^2 + (55.6)^2 + 2(5.24)(55.6)\cos 75° = 3264\ (m/s)^2$

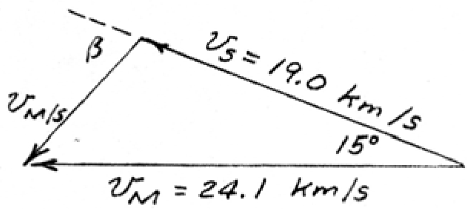
$v_B = 57.1$ m/s or $v_B = 57.1(3.6) = \underline{206 \text{ km/h}}$

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$, $a_A = 0$, $a_{B/A} = r\dot{\theta}^2 = 60(\frac{5\pi}{180})^2 = 0.457$ m/s^2

Thus $a_B = a_{B/A} = \underline{0.457 \text{ m/s}^2}$ from B to A

2/207 Mars will appear to be approaching the spacecraft head on when $v_{M/S}$ is along the line of sight M-S.

$\underline{v}_M = \underline{v}_S + \underline{v}_{M/S}$

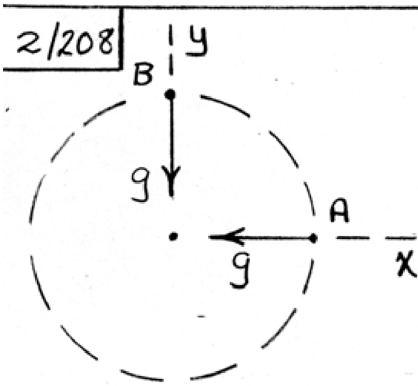


$(v_{M/S})^2 = (19.0)^2 + (24.1)^2 - 2(19.0)(24.1)\cos 15° = 57.2\ (\text{km/s})^2$

$v_{M/S} = 7.56$ km/s, $\dfrac{24.1}{\sin(\pi - \beta)} = \dfrac{7.56}{\sin 15°}$

$\sin(\pi - \beta) = \sin\beta = 0.8246$, $\underline{\beta = 55.6°}$

2/208



$$g = g_0 \left(\frac{R}{R+h}\right)^2$$

$$= 9.825\left(\frac{6371}{6371+1500}\right)^2$$

$$= 6.44 \text{ m/s}^2$$

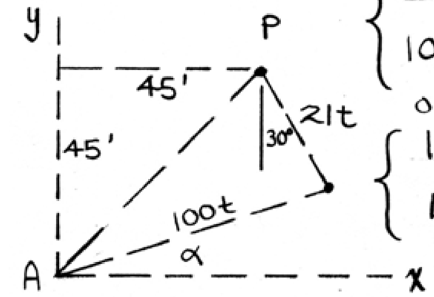
$$\underline{a}_{B/A} = \underline{a}_B - \underline{a}_A$$

$$= -6.44\underline{j} - (-6.44\underline{i})$$

$$= 6.44\underline{i} - 6.44\underline{j} \text{ m/s}^2$$

$$a_{B/A} = 6.44\sqrt{2} = \underline{9.10 \text{ m/s}^2}$$

2/209



$$\begin{cases} 100t\cos\alpha = 45 + 21t\sin 30° \\ 100t\sin\alpha = 45 - 21t\cos 30° \end{cases}$$

or

$$\begin{cases} 100\cos\alpha = \frac{45}{t} + 21\sin 30° \\ 100\sin\alpha = \frac{45}{t} - 21\cos 30° \end{cases}$$

Subtract $2^{\underline{nd}}$ from $1^{\underline{st}}$:

$100(\cos\alpha - \sin\alpha) = 21\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$ or $\cos\alpha - \sin\alpha = 0.287$

$\cos\alpha = \sqrt{1-\sin^2\alpha} = 0.287 + \sin\alpha$

SQBS & rearrange : $2\sin^2\alpha + 0.574\sin\alpha - 0.918 = 0$

Positive solution : $\underline{\alpha = 33.3°}$; then $t = 0.616$ sec

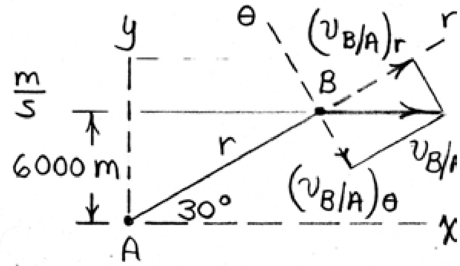
$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 100[\cos\alpha\,\underline{i} + \sin\alpha\,\underline{j}]$

$- 21[\sin 30°\,\underline{i} - \cos 30°\,\underline{j}]$

$= \underline{73.1\,\underline{i} + 73.1\,\underline{j} \text{ ft/sec}}$

▶2/210 $v_{B/A} = v_B - v_A$

$= (1500-1000)/3.6 = 138.9\ \frac{m}{s}$



$(v_{B/A})_r = \dot{r} = 138.9\ \cos 30°$

$= 120.3\ m/s$

$(v_{B/A})_\theta = r\dot{\theta} : -138.9 \sin 30° = \frac{6000}{\sin 30°}\ \dot{\theta}$

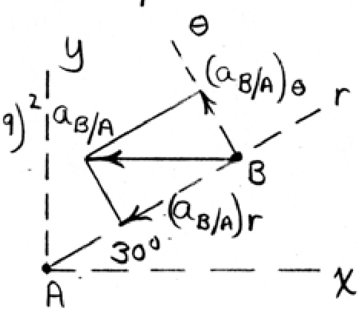
$\dot{\theta} = -0.00579\ rad/s$

$a_{B/A} = a_B - a_A = 0 - 1.2 = -1.2\ m/s^2$

$(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2$

$-1.2 \cos 30° = \ddot{r} - 12000\,(-0.00579)^2$

$\underline{\ddot{r} = -0.637\ m/s^2}$



$(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$1.2 \sin 30° = 12000\,\ddot{\theta} + 2(120.3)(-0.00579)$

$\underline{\ddot{\theta} = 0.1660\,(10^{-3})\ rad/s^2}$

▸ 2/211 Find flight time t :

$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$: $7 = 3 + 100\sin 30^\circ t - 16.1t^2$

Solve to obtain 0.0822 sec (discard) & $t = 3.02$ sec

Range $R = x_0 + v_{x_0}t = 0 + 100\cos 30^\circ (3.02)$
$= 262$ ft

Fielder must run $262 - 220 = 41.8$ ft
in $(3.02 - 0.25)$ sec $\Rightarrow v_B = \frac{41.8}{2.77} = 15.08$ ft/sec

Velocity components of ball when caught:

$v_x = v_{x_0} = 100\cos 30^\circ = 86.6$ ft/sec

$v_y = v_{y_0} - gt = 100\sin 30^\circ - 32.2(3.02) = -47.4\ \frac{ft}{sec}$

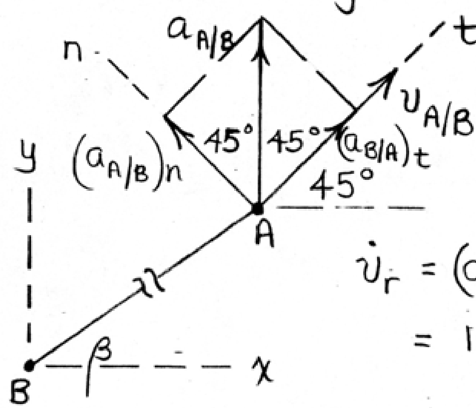
$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = (86.6\underline{i} - 47.4\underline{j}) - 15.08\underline{i}$

$= \underline{71.5\underline{i} - 47.4\underline{j}\ \ \text{ft/sec}}$

▶2/212 (a) $\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 50\underline{i} - (-50\underline{j}) = \underline{50\underline{i} + 50\underline{j} \text{ m/s}}$

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \frac{v_A^2}{\rho_A}\underline{j} - \underline{0} = \frac{50^2}{2000}\underline{j} = \underline{1.250\underline{j} \text{ m/s}^2}$

(b) Use the results of part (a) for a normal-tangential analysis:



$v_{A/B} = \sqrt{50^2 + 50^2} = 70.7 \text{ m/s}$

$\dot{v}_r = (a_{B/A})_t = a_{A/B}\cos 45°$

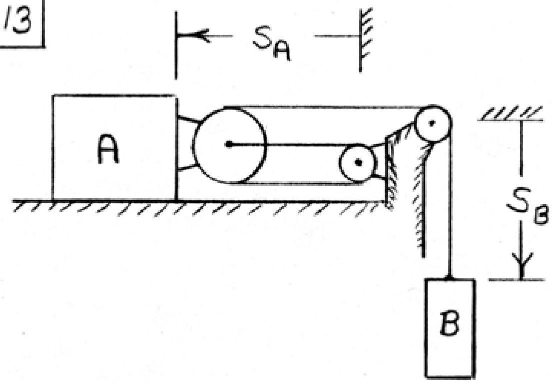
$= 1.250\,\frac{\sqrt{2}}{2} = \underline{0.884 \text{ m/s}^2}$

$(a_{B/A})_n = a_{B/A}\sin 45° = \frac{v_{A/B}^2}{\rho_r}$

$1.250\,\frac{\sqrt{2}}{2} = \frac{70.7^2}{\rho_r}$, $\underline{\rho_r = 5660 \text{ m}}$

(in n-direction from A)

2/213



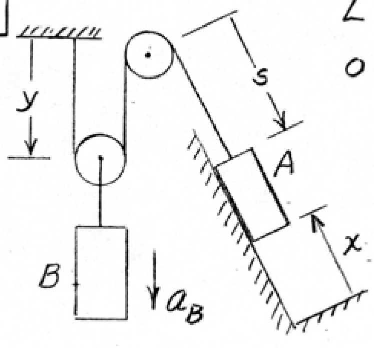
Length of cable $L = 3s_A + s_B + \text{constant}$

Differentiate: $0 = 3v_A + v_B$

$$v_B = -3v_A = -3(-3.6)$$

$$= \underline{10.8 \text{ ft/sec (down)}}$$

2/214



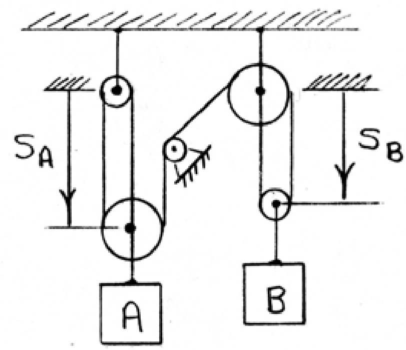
$L = 2y + s + \text{const.}$

$0 = 2\dot{y} + \dot{s}$, $0 = 2\ddot{y} + \ddot{s}$

$a_B = \ddot{y} = -\frac{\ddot{s}}{2} = +\frac{\ddot{x}}{2}$

$a_B = \frac{0.044}{2} = \underline{0.022 \text{ m/s}^2}$

2/215



The length of the main cable is

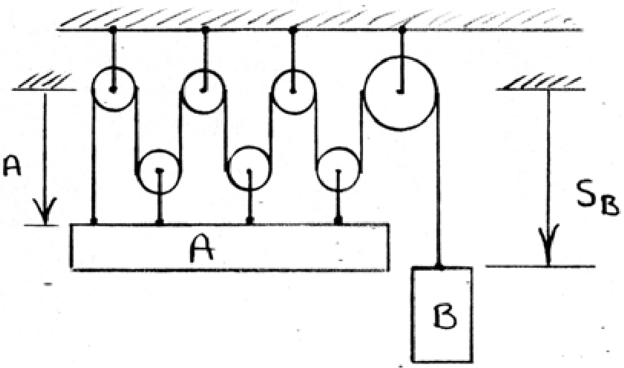
$L = 3s_A + 2s_B + \text{constants}$

$\Rightarrow 0 = 3v_A + 2v_B \;;\quad 0 = 3a_A + 2a_B$

So $v_B = -\frac{3}{2}v_A = -\frac{3}{2}(0.8) = \underline{-1.2 \text{ m/s}}$ (up)

and $a_B = -\frac{3}{2}a_A = -\frac{3}{2}(-2) = \underline{3 \text{ m/s}^2}$ (down)

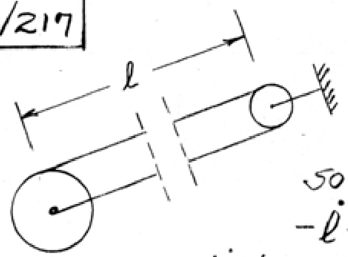
2/216



$L = 7s_A + s_B +$ Constants

$\underline{0 = 7a_A + a_B}$ (for coordinates shown)

2/217



Length of cable is

$L = 2\ell$

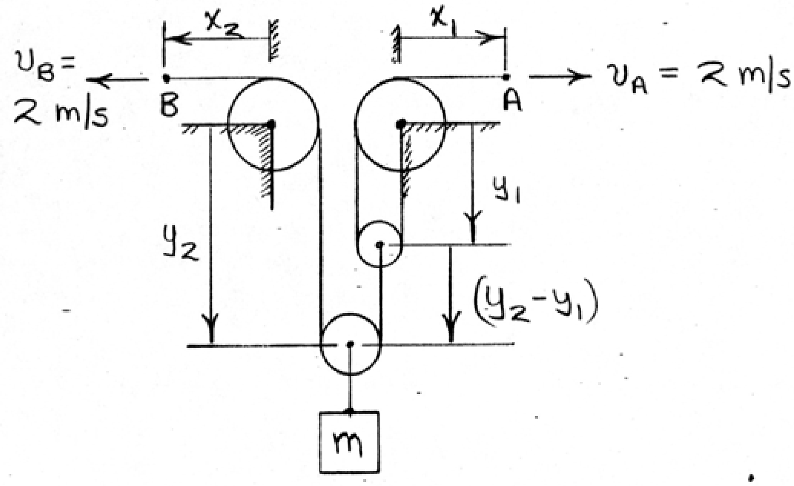
$\dot{L} = 2\dot{\ell}$

so velocity of truck is

$-\dot{\ell} = \frac{1}{2}(-\dot{L}) = \frac{1}{2}(40) = 20$ mm/s

time $t = \frac{\text{distance}}{\text{velocity}} = \frac{4(10^3)}{20} = 200$ s or 3 min 20 s

2/218



$x_1 + 2y_1 =$ constant; $\dot{x}_1 + 2\dot{y}_1 = 0,\quad \dot{y}_1 = -\dfrac{\dot{x}_1}{2}$

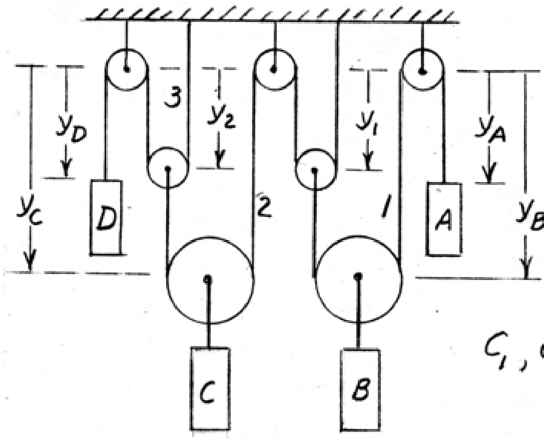
$x_2 + y_2 + (y_2 - y_1) =$ constant; $\dot{x}_2 + 2\dot{y}_2 - \dot{y}_1 = 0$

$\dot{x}_2 + 2\dot{y}_2 - \left(-\dfrac{\dot{x}_1}{2}\right) = 0$

$\dot{x}_2 + 2\dot{y}_2 + \dfrac{\dot{x}_1}{2} = 0,\quad \dot{y}_2 = -\dfrac{\dot{x}_2}{2} - \dfrac{\dot{x}_1}{4} = -\dfrac{2}{2} - \dfrac{2}{4} = -1.5$ m/s

or 1.5 m/s up

2/219



3 degrees of freedom

C_1, C_2, C_3 are constant lengths

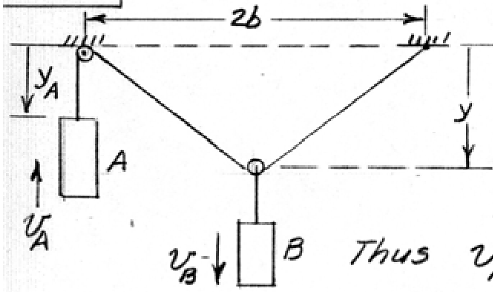
$L_1 = y_B + y_A + (y_B - y_1) + C_1$; $\quad 0 = 2\dot{y}_B + \dot{y}_A - \dot{y}_1$

$L_2 = y_C + 2y_1 + (y_C - y_2) + C_2 \quad 0 = 2\dot{y}_C + 2\dot{y}_1 - \dot{y}_2$

$L_3 = 2y_2 + y_D + C_3, \quad 0 = 2\dot{y}_2 + \dot{y}_D$

Eliminate $\dot{y}_1$ & $\dot{y}_2$ & get $\underline{4v_A + 8v_B + 4v_C + v_D = 0}$

2/220



Cable length is

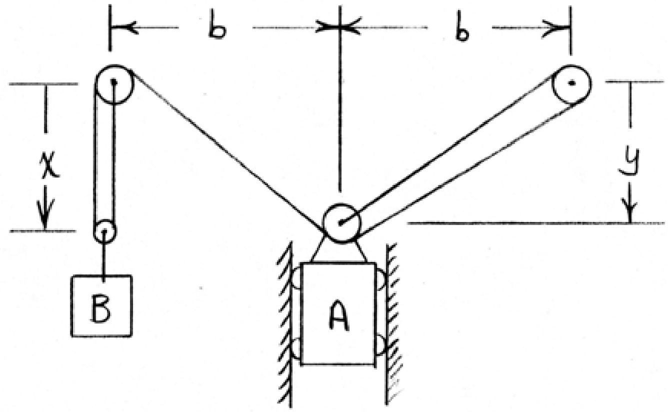
$$L = y_A + 2\sqrt{y_B^2 + b^2}$$

$$0 = \dot{y}_A + \frac{2y\dot{y}}{\sqrt{y^2 + b^2}}$$

But $v_A = -\dot{y}_A$, $v_B = \dot{y}$

Thus $$\underline{v_A = \frac{2y}{\sqrt{y^2 + b^2}}\, v_B}$$

2/221



The total length of the cable is

$$L = 2x + 3\sqrt{y^2+b^2} + \text{constant}.$$

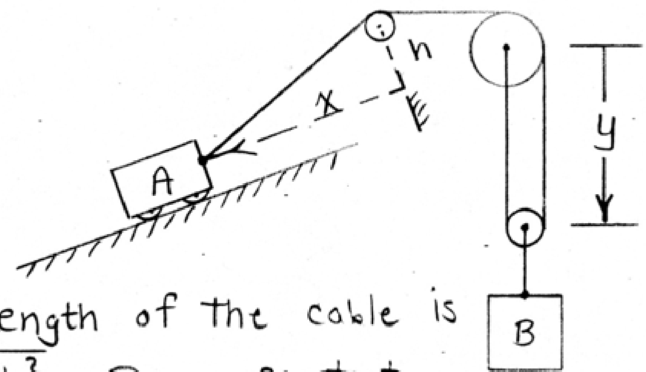
Differentiate to obtain

$$\dot{L} = 0 = 2\dot{x} + 3\frac{y\dot{y}}{\sqrt{y^2+b^2}}$$

With $\dot{x} = v_B$ & $\dot{y} = v_A$, we have

$$v_B = -\frac{3y}{2\sqrt{y^2+b^2}}v_A$$

2/222



The total length of the cable is

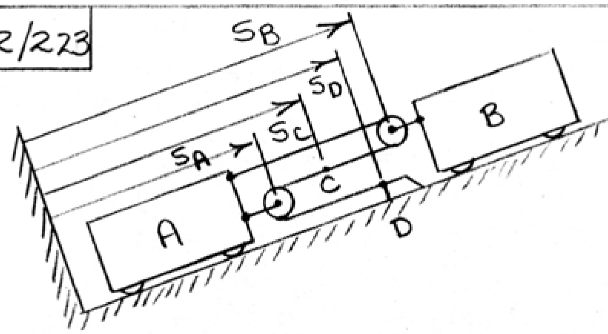
$$L = \sqrt{x^2+h^2} + 2y + \text{constant}$$

$$\dot{L} = 0 = \frac{1}{2}\frac{2x\dot{x}}{\sqrt{x^2+h^2}} + 2\dot{y}$$

With $v_A = \dot{x}$ & $v_B = -\dot{y}$:

$$\underline{v_A = +\frac{2\sqrt{x^2+h^2}}{x}v_B}$$

2/223



The cable length is $L = 2(s_B - s_A) + s_D - s_A$ + Constants

Differentiating:

$0 = 2v_B - 3v_A \; ; \quad 0 = 2a_B - 3a_A$

So $v_A = \frac{2}{3} v_B = \frac{2}{3}(3) = 2$ ft/sec

$a_A = \frac{2}{3} a_B = \frac{2}{3}(6) = 4$ ft/sec^2

$v_{B/A} = v_B - v_A = 3 - 2 = \underline{1 \text{ ft/sec}}$

$a_{B/A} = a_B - a_A = 6 - 4 = \underline{2 \text{ ft/sec}^2}$

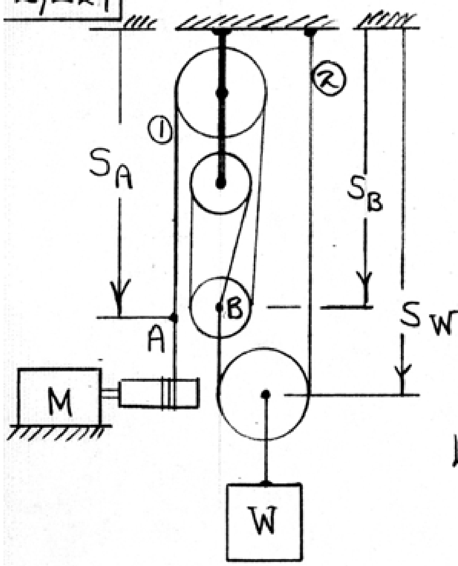
The length of cable between A and C is

$L' = (s_B - s_A) + (s_B - s_C) = 2s_B - s_A - s_C$ + constants

$\Rightarrow 0 = 2v_B - v_A - v_C \; ; \; v_C = 2v_B - v_A = 2(3) - 2 = \underline{4 \text{ ft/sec}}$

(All answers are quantities directed up incline.)

2/224



Let A be a point on cable 1, and let B be the indicated pulley.

$$L_1 = S_A + 3S_B + \text{Constant}$$

$$0 = v_A + 3v_B \qquad (1)$$

$$L_2 = S_W - S_B + S_W$$

$$= 2S_W - S_B$$

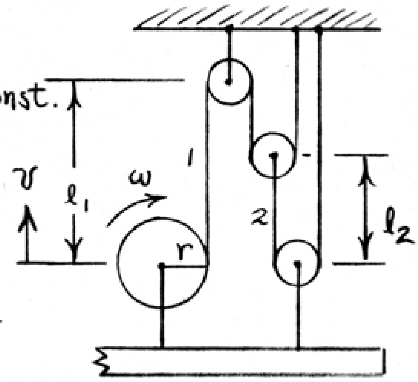
$$0 = 2v_W - v_B \qquad (2)$$

Combine (1) & (2) : $v_A + 6v_W = 0$

So $v_W = -\frac{1}{6} v_A$

Hence W rises $\frac{1}{6}(180)(10) = \underline{300 \text{ mm} = h}$

2/225



Length $L_1 = l_1 + 2(l_1 - l_2) + \text{const.}$

$\dot{L}_1 = -r\omega = 3\dot{l}_1 - 2\dot{l}_2$

Length $L_2 = l_2 + l_1 + \text{const.}$

$\dot{L}_2 = 0 = \dot{l}_2 + \dot{l}_1, \quad -\dot{l}_1 = \dot{l}_2$

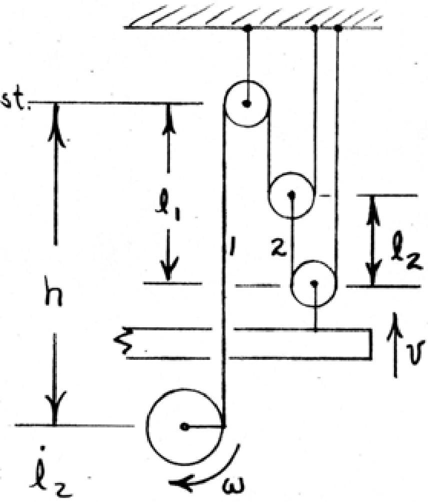
But $v = -\dot{l}_1$, so

$-r\omega = 3(-v) - 2v, \quad r\omega = 5v$

$$v = \frac{r\omega}{5} = \frac{0.1(40)(2\pi/60)}{5} = 0.0838 \ \frac{m}{s}$$

or $\underline{v = 83.8 \text{ mm/s}}$

2/226



Length $L_1 = h + 2(l_1 - l_2) + \text{const.}$

$\dot{L}_1 = -r\omega = 0 + 2\dot{l}_1 - 2\dot{l}_2$

But $v = -\dot{l}_1$

So $-r\omega = -2v - 2\dot{l}_2$

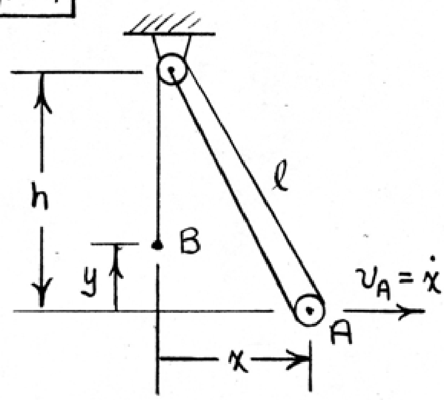
Length $L_2 = l_1 + l_2 + \text{const.}$

$\dot{L}_2 = 0 = \dot{l}_1 + \dot{l}_2$, $v = \dot{l}_2$

$\therefore r\omega = 2v + 2v$, $v = \dfrac{r\omega}{4} = \dfrac{0.1(40)(2\pi/60)}{4}$

$v = 0.1047$ m/s or $\underline{v = 104.7 \dfrac{mm}{s}}$

2/227



Length of cable is

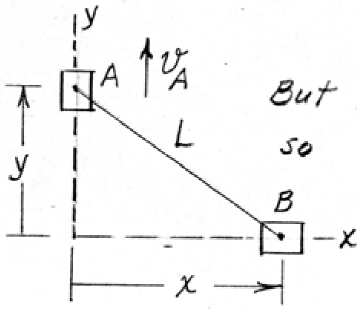
$$L = h - y + 2\ell + \text{constant} = h - y + 2\sqrt{x^2+h^2} + \text{const.}$$

$$\dot{L} = 0 = -\dot{y} + \frac{2x\dot{x}}{\sqrt{x^2+h^2}}$$

Substitute v_A for $\dot{x}$ and v_B for $\dot{y}$:

$$v_B = \frac{2x}{\sqrt{x^2+h^2}}\, v_A$$ (4 times as fast as with Sample Prob. 2/15)

2/228



$x^2 + y^2 = L^2$; $x\dot{x} + y\dot{y} = 0$

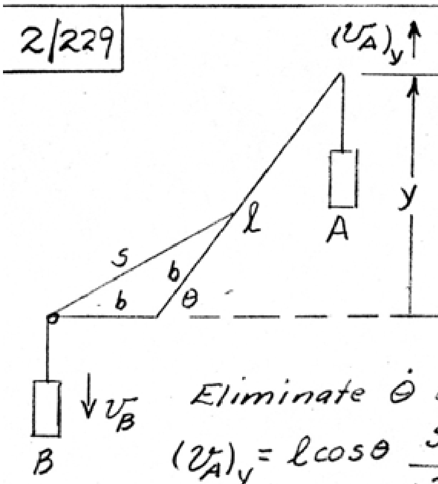
& $\dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0$

But $\dot{y} = v_A$, $\ddot{y} = 0$

so $a_x = \ddot{x} = -\dfrac{\dot{x}^2 + \dot{y}^2}{x} = -\dfrac{\dot{y}^2 \dfrac{y^2}{x^2} + \dot{y}^2}{x}$

$a_x = -\dfrac{L^2}{x^3}\dot{y}^2 = -\underline{\dfrac{L^2 v_A^2}{(L^2 - y^2)^{3/2}}}$

2/229



$s^2 = 2b^2 + 2b^2\cos\theta$

$= 2b^2(1+\cos\theta)$

$2s\dot{s} = 2b^2(-\dot{\theta}\sin\theta)$

$v_B = -\dot{s} = \frac{b^2}{s}\dot{\theta}\sin\theta$

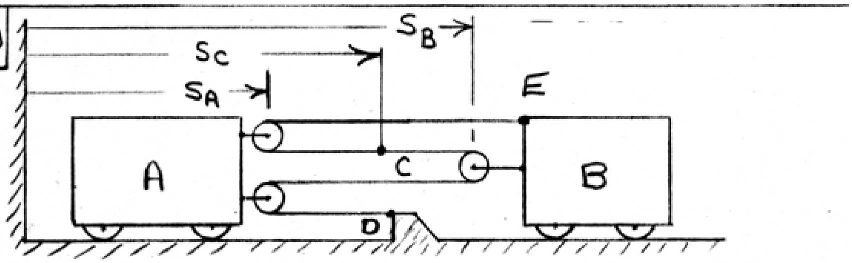
$y = \ell\sin\theta$

$(v_A)_y = \dot{y} = \ell\dot{\theta}\cos\theta$

Eliminate $\dot{\theta}$ & get

$$(v_A)_y = \ell\cos\theta\,\frac{s v_B}{b^2\sin\theta} = \frac{\ell\sqrt{2(1+\cos\theta)}}{b\tan\theta}\,v_B$$

▸2/230



Cable length $L = 3(s_B - s_A) + (s_D - s_A)$

$0 = 3v_B - 4v_A$, $0 = 3a_B - 4a_A$

$v_A = \frac{3}{4} v_B = \frac{3}{4}(2) = 1.5$ m/s

$a_A = \frac{3}{4} a_B = \frac{3}{4}(3) = 2.25$ m/s^2

$v_{B/A} = v_B - v_A = 2 - (1.5) = \underline{0.5 \text{ m/s}}$

$a_{B/A} = a_B - a_A = 3 - (2.25) = \underline{0.75 \text{ m/s}^2}$

Length of cable between points E and C:

$L' = (s_B - s_A) + (s_C - s_A) + \text{constants}$

$0 = v_B - 2v_A + v_C \Rightarrow v_C = 2v_A - v_B$

or $v_C = 2(1.5) - 2 = \underline{1 \text{ m/s}}$

(All answers are quantities directed to <u>right</u>)

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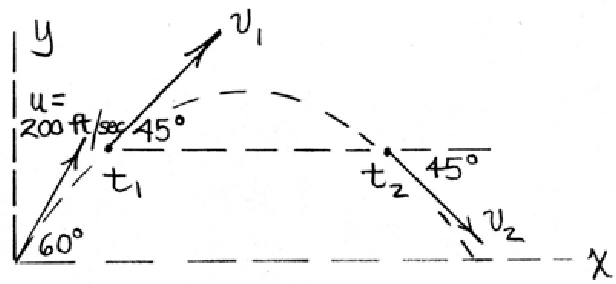


$\dot{x} = u\cos\theta = 200\cos 60° = 100$ ft/sec

$\dot{y} = u\sin\theta - gt = 200\sin 60° - 32.2t = 173.2 - 32.2t$

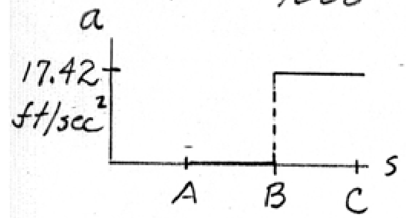
At t_1: $\dot{x} = \dot{y}$: $100 = 173.2 - 32.2t_1$, $\underline{t_1 = 2.27 \text{ sec}}$

At t_2: $\dot{x} = -\dot{y}$: $100 = -173.2 + 32.2t_2$, $\underline{t_2 = 8.48 \text{ sec}}$

2/232 From A to B $a = 0$

From B to C $a = a_n = v^2/\rho$

$$a_n = \frac{(90 \times 44/30)^2}{1000} = \underline{17.42 \text{ ft/sec}^2}$$



Abrupt acceleration would cause abrupt forces which would be uncomfortable for passengers. A transition section to change curvature gradually over an interval of track would be required.

2/233

$$r = r_0 + b\sin\frac{2\pi t}{\tau}, \quad \dot{r} = \frac{2\pi}{\tau} b\cos\frac{2\pi t}{\tau}$$

$$\ddot{r} = -\frac{4\pi^2}{\tau^2} b\sin\frac{2\pi t}{\tau}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{4\pi^2}{\tau^2} b\sin\frac{2\pi t}{\tau} - r\dot{\theta}^2 = 0$$

$$\Rightarrow \quad r = r_0 \frac{1}{1 + \left(\frac{\tau\dot{\theta}}{2\pi}\right)^2}$$

$\boxed{2/234}$ $\dot{x} = 20$ mm/s , $\ddot{x} = 0$

$y = x^2/160$, $\dot{y} = x\dot{x}/80$, $\ddot{y} = (\dot{x}^2 + x\ddot{x})/80$

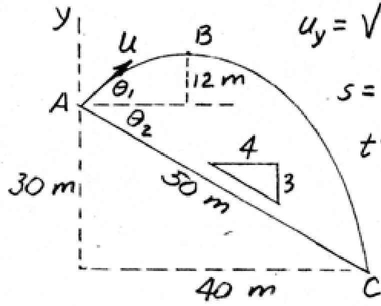
$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{\dot{x}^2 + (x\dot{x}/80)^2} = \dot{x}\sqrt{1 + (x/80)^2}$

For $x = 60$ mm

$v = 20\sqrt{1 + (60/80)^2} = \underline{25 \text{ mm/s}}$

$a = \ddot{y} = \dot{x}^2/80$ since $\ddot{x} = 0$, $a = (20)^2/80 = \underline{5 \text{ mm/s}^2}$

$\boxed{2/235}$ y-dir., $v^2 = v_1^2 + 2as$; $0 = u_y^2 - 2g(12)$ A to B



$u_y = \sqrt{2(9.81)(12)} = 15.34$ m/s

$s = v_1 t + \frac{1}{2}at^2$; $-30 = 15.34t - \frac{9.81}{2}t^2$

$t^2 - 3.128t - 6.116 = 0$,

$t = 4.49$ s $(t = -1.36$ s$)$

x-dir, $40 = u_x(4.49)$, $u_x = 8.91$ m/s

$u = \sqrt{(8.91)^2 + (15.34)^2} = \underline{17.74 \text{ m/s}}$

$\theta_1 = \tan^{-1}\frac{15.34}{8.91} = 59.86°$; $\theta_2 = \tan^{-1}\frac{3}{4} = 36.87°$

$\theta = \theta_1 + \theta_2 = 59.86 + 36.87 = \underline{96.7°}$

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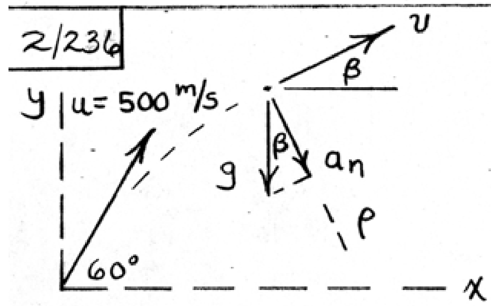

$v_x = 500 \cos 60° = 250 \text{ m/s}$

$v_y = v_{y_0} - gt = 500 \sin 60° - 9.81(30) = 138.7 \frac{m}{s}$

$v^2 = v_x^2 + v_y^2 = 250^2 + 138.7^2 = 81.7(10^3) \text{ m}^2/\text{s}^2$

$\beta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{138.7}{250} = 29.0°$

$a_n = g \cos\beta = 9.81 \cos 29.0° = 8.58 \text{ m/s}^2$

$a_n = v^2/\rho \Rightarrow \rho = \frac{v^2}{a_n} = \frac{81.7(10^3)}{8.58} = 9529 \text{ m}$

or $\underline{\rho = 9.53 \text{ km}}$

$\boxed{2/237}$ $\theta = 4\left[t + 30e^{-0.03t} - 30\right]$ (rad)

$\dot{\theta} = 4\left[1 - 0.9e^{-0.03t}\right]$ (rad/sec)

$\ddot{\theta} = 0.1080\,e^{-0.03t}$ (rad/sec^2)

$r\dot{\theta}^2 = 30\left[4\left(1 - 0.9\,e^{-0.03t}\right)\right]^2 = 32.2(10)$

$\left(1 - 0.9\,e^{-0.03t}\right)^2 = 0.671$

$\left(1 - 0.9\,e^{-0.03t}\right) = \pm 0.819$

Take (+) as (−) will result in $t < 0$:

$\left(1 - 0.9\,e^{-0.03t}\right) = 0.819 \Rightarrow$ $\underline{t = 53.5 \text{ sec}}$

$\ddot{\theta} = 0.1080\,e^{-0.03(53.5)} = 0.0217$ rad/sec^2

$r\ddot{\theta} = 30(0.0217) = 0.651$ ft/sec^2 $(0.020g)$

Thus a_t can be neglected.

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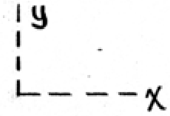

$$\underline{v}_A = \underline{v}_W + \underline{v}_{A/W} = -48\underline{i} + 220\,\underline{i} = 172\underline{i}\ \frac{km}{h}$$

On descent : $\underline{v}_A = 172(\cos 10^\circ \underline{i} - \sin 10^\circ \underline{j})$ km/h

$$\underline{v}_{A/c} = \underline{v}_A - \underline{v}_c = 172(\cos 10^\circ \underline{i} - \sin 10^\circ \underline{j}) - 30\underline{i}$$

$$= 139.4\underline{i} - 29.9\underline{j} \ \text{km/h}$$

$$\beta = \tan^{-1}\left(\frac{29.9}{139.4}\right) = \underline{12.09^\circ}$$

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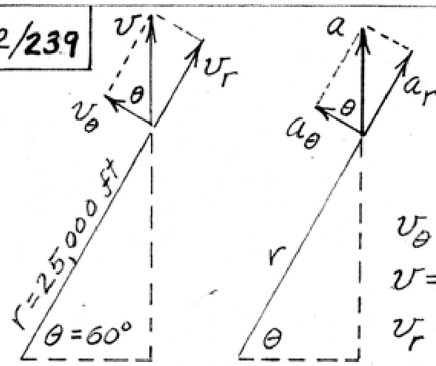


Given $r = 25{,}000$ ft

$\dot{\theta} = 0.03$ rad/sec

$\theta = 60°$

$a = 64$ ft/sec^2

$v_\theta = r\dot{\theta} = 25(10^3)(0.03) = 750$ ft/sec

$v = 750/\cos 60° = 1500$ ft/sec

$v_r = \dot{r} = 1500 \cos 30° = 1299$ ft/sec

$a_\theta = a \cos 60° = 64(0.5) = 32$ ft/sec^2

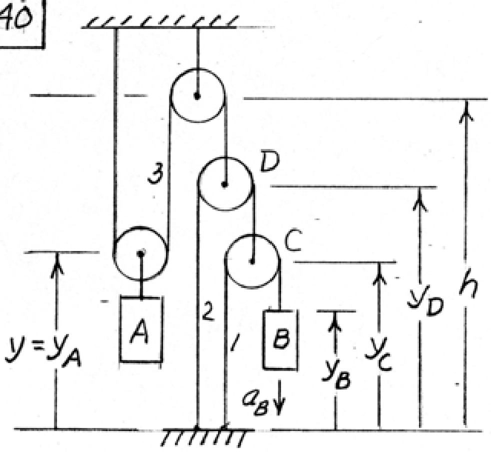
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$, so $r\ddot{\theta} = 32 - 2(1299)(0.03) = -45.94$

$$\ddot{\theta} = -\frac{45.94}{25{,}000} = \underline{-1.838(10^{-3})\text{ rad/sec}^2}$$

$a_r = a \sin 60° = 64(\sqrt{3}/2) = 55.43$ ft/sec^2

$a_r = \ddot{r} - r\dot{\theta}^2$, so $\ddot{r} = 55.43 + 25(10^3)(0.03)^2 = \underline{77.9\text{ ft/sec}^2}$

2/240



$y = y_A = t^2/4$ m

$\dot{y}_A = t/2$ m/s

$a_A = \ddot{y}_A = 1/2$ m/s^2

one degree of freedom

Cable lengths

$L_1 = y_C + (y_C - y_B) + C_1 \qquad 0 = 2\ddot{y}_C - \ddot{y}_B$ where $a_B = -\ddot{y}_B$

$L_2 = y_D + (y_D - y_C) + C_2 \qquad 0 = 2\ddot{y}_D - \ddot{y}_C$

$L_3 = 2(h - y_A) + h - y_D + C_3, \quad 0 = -2\ddot{y}_A - \ddot{y}_D$

Eliminate $\ddot{y}_C$ & $\ddot{y}_D$ & get $\ddot{y}_B = -8\ddot{y}_A$ or $a_B = 8\ddot{y}_A = \underline{4 \text{ m/s}^2}$

(By inspection of pulley displacements
$-dy_B = 8\,dy_A$, so $a_B = 8a_A = 8(1/2) = 4$ m/s^2)

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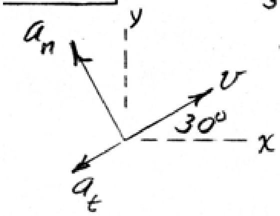


$v = \frac{1000}{3.6} = 278$ m/s , $a_t = \frac{15}{3.6} = 4.17$ m/s^2

$a_n = v^2/\rho = (278)^2/1500 = 51.4$ m/s^2

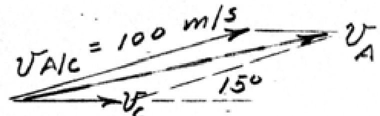
$\ddot{x} = -51.4 \sin 30° - 4.17 \cos 30° = \underline{-29.3 \text{ m/s}^2}$

$\ddot{y} = 51.4 \cos 30° - 4.17 \sin 30° = \underline{42.5 \text{ m/s}^2}$

$\boxed{2/242}$ Carrier deck has a constant velocity, so may be used as an inertial coordinate base.
Velocity of aircraft relative to carrier is

$$v_{A/C}^2 = 2as = 2(50)100 = 10\,000\,(m/s)^2, \quad v_{A/C} = 100 \text{ m/s}$$

$$\underline{v}_A = \underline{v}_C + \underline{v}_{A/C}$$

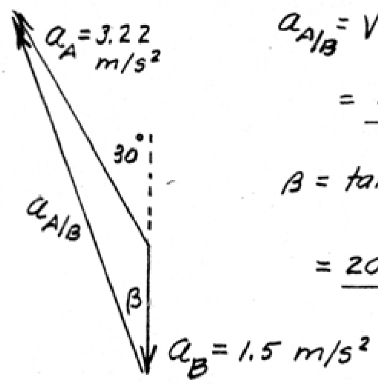


$$v_C = 30(1.852)/3.6 = 15.43 \text{ m/s}$$

$$v_A^2 = (100)^2 + (15.43)^2 + 2(100)(15.43)\cos 15^\circ = 13\,220 \;(m/s)^2$$

$$v_A = 115.0 \text{ m/s or } v_A = v = 115.0(3.6) = \underline{414 \text{ km/h}}$$

2/243 $\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$, $a_A = v_A^2/\rho = (50/3.6)^2/60 = 3.22\ m/s^2$



$$a_{A/B} = \sqrt{\left(3.22\frac{\sqrt{3}}{2} + 1.5\right)^2 + \left(3.22\,[1/2]\right)^2}$$

$$= \underline{4.58\ m/s^2}$$

$$\beta = \tan^{-1}\frac{1.608}{4.28} = \tan^{-1} 0.3752$$

$$= \underline{20.6^\circ \text{ west of north}}$$

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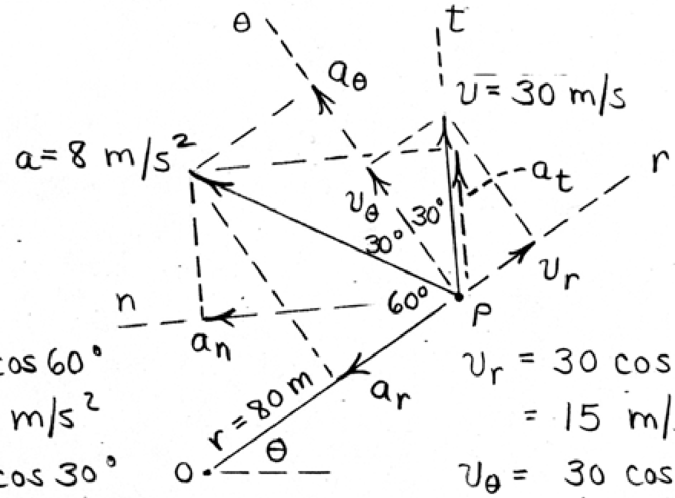

$a_r = -8\cos 60° = -4 \text{ m/s}^2$

$a_\theta = 8\cos 30° = 6.93 \text{ m/s}^2$

$v_r = 30\cos 60° = 15 \text{ m/s}$

$v_\theta = 30\cos 30° = 26.0 \text{ m/s}$

$[r\text{-}\theta]$ $\quad v_r = \dot{r} = \underline{15 \text{ m/s}}$

$v_\theta = r\dot{\theta}: \; 26.0 = 80\dot{\theta}, \; \underline{\dot{\theta} = 0.325 \text{ rad/s}}$

$a_r = \ddot{r} - r\dot{\theta}^2: \; -4 = \ddot{r} - 80(0.325)^2, \; \underline{\ddot{r} = 4.44 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}: \; 6.93 = 80\ddot{\theta} + 2(15)(0.325), \; \underline{\ddot{\theta} = -0.0352 \text{ rad/s}^2}$

$[n\text{-}t]$: $\quad a_n = 8\cos 30° = \underline{6.93 \text{ m/s}^2}$

$a_t = 8\cos 60° = \underline{4 \text{ m/s}^2}$

$a_n = \dfrac{v^2}{\rho}, \; \rho = \dfrac{v^2}{a_n} = \dfrac{30^2}{6.93} = \underline{129.9 \text{ m}}$

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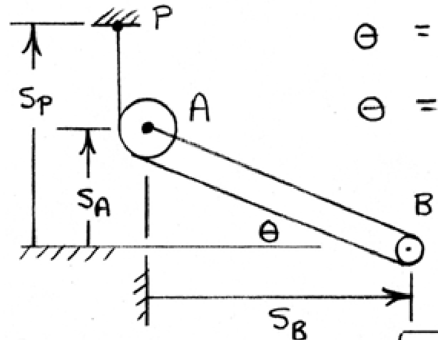

$\theta = 45°$: $s_B/s_A = 1$

$\theta = 30°$: $s_B/s_A = \sqrt{3}$

$\theta = 15°$: $s_B/s_A = 3.73$

$$L = (s_P - s_A) + 2\sqrt{s_A^2 + s_B^2}$$

$$0 = -v_A + 2(s_A v_A + s_B v_B)\Big/\sqrt{s_A^2 + s_B^2}$$

$$\Rightarrow v_B = \left[\sqrt{1 + (s_B/s_A)^2} - 2\right] \frac{v_A}{2(s_B/s_A)}$$

$\theta = 45°$: $v_B = -0.293 v_A = -0.293(-1) = \underline{0.293 \frac{m}{s}}$

$\theta = 30°$: $v_B = \underline{0}$

$\theta = 15°$: $v_B = 0.250 v_A = 0.250(-1) = \underline{-0.250 \frac{m}{s}}$

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$$\begin{cases} x = 50 \text{ ft}, & \dot{x} = -10 \text{ ft/sec}, & \ddot{x} = -10 \text{ ft/sec}^2 \\ y = 25 \text{ ft}, & \dot{y} = 10 \text{ ft/sec}, & \ddot{y} = 5 \text{ ft/sec}^2 \end{cases}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(-10)^2 + 10^2} = \underline{10\sqrt{2} \text{ ft/sec}}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(-10)^2 + 5^2} = \underline{11.18 \text{ ft/sec}^2}$$

$$\underline{e}_t = \underline{v}/v = (-10\underline{i} + 10\underline{j})/10\sqrt{2} = \underline{\frac{\sqrt{2}}{2}(-\underline{i} + \underline{j})}$$

$$a_t = \underline{a} \cdot \underline{e}_t = (-10\underline{i} + 5\underline{j}) \cdot \frac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) = \underline{10.61 \text{ ft/sec}^2}$$

$$\underline{a}_t = a_t \underline{e}_t = 10.61 \frac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) = \underline{-7.5\underline{i} + 7.5\underline{j} \text{ ft/sec}^2}$$

$$\underline{a}_n = \underline{a} - \underline{a}_t = (-10\underline{i} + 5\underline{j}) - (-7.5\underline{i} + 7.5\underline{j}) = \underline{-2.5(\underline{i} + \underline{j}) \text{ ft/sec}^2}$$

$$a_n = \sqrt{2.5^2 + 2.5^2} = \underline{3.54 \text{ ft/sec}^2}$$

$$\rho = v^2/a_n = (10\sqrt{2})^2/3.54 = \underline{56.6 \text{ ft}}$$

$$\underline{e}_n = \underline{a}_n/a_n = -2.5(\underline{i} + \underline{j})/3.54 = \underline{-\frac{\sqrt{2}}{2}(\underline{i} + \underline{j})}$$

$$\underline{e}_r = \underline{r}/r = 50\underline{i} + 25\underline{j}/\sqrt{50^2 + 25^2} = \underline{0.894\underline{i} + 0.447\underline{j}}$$

$$\underline{e}_\theta = \underline{e}_r \text{ rotated CCW } 90° = \underline{-0.447\underline{i} + 0.894\underline{j}}$$

$$v_r = \underline{v} \cdot \underline{e}_r = (-10\underline{i} + 10\underline{j}) \cdot (0.894\underline{i} + 0.447\underline{j}) = \underline{-4.47 \text{ ft/sec}}$$

$$\underline{v}_r = v_r \underline{e}_r = -4.47(0.894\underline{i} + 0.447\underline{j}) = \underline{-4\underline{i} - 2\underline{j} \text{ ft/sec}}$$

$$v_\theta = \underline{v} \cdot \underline{e}_\theta = (-10\underline{i} + 10\underline{j}) \cdot (-0.447\underline{i} + 0.894\underline{j}) = \underline{13.42 \text{ ft/sec}}$$

$$\underline{v}_\theta = v_\theta \underline{e}_\theta = 13.42(-0.447\underline{i} + 0.894\underline{j}) = \underline{-6\underline{i} + 12\underline{j} \text{ ft/sec}}$$

$$a_r = \underline{a} \cdot \underline{e}_r = (-10\underline{i} + 5\underline{j}) \cdot (0.894\underline{i} + 0.447\underline{j}) = \underline{-6.71 \text{ ft/sec}^2}$$

$$\underline{a}_r = a_r \underline{e}_r = -6.71(0.894\underline{i} + 0.447\underline{j}) = \underline{-6\underline{i} - 3\underline{j} \text{ ft/sec}^2}$$

$$a_\theta = \underline{a}\cdot\underline{e}_\theta = (-10\underline{i} + 5\underline{j})\cdot(-0.447\underline{i} + 0.894\underline{j}) = \underline{8.94 \text{ ft/sec}^2}$$

$$\underline{a}_\theta = a_\theta \underline{e}_\theta = 8.94(-0.447\underline{i} + 0.894\underline{j}) = \underline{-4\underline{i} + 8\underline{j} \quad \text{ft/sec}^2}$$

$$r = \sqrt{x^2+y^2} = \sqrt{50^2+25^2} = \underline{55.9 \text{ ft}}$$

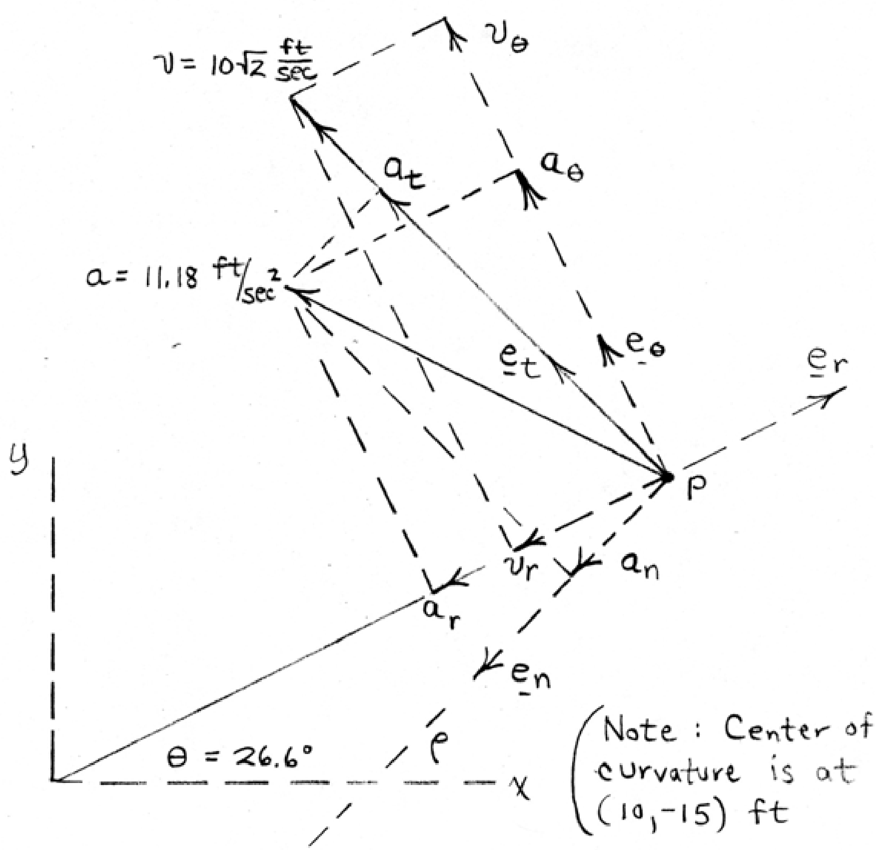
$$\dot{r} = v_r = \underline{-4.47 \text{ ft/sec}}$$

$$v_\theta = r\dot{\theta}, \quad \dot{\theta} = v_\theta/r = 13.42/55.9 = \underline{0.240 \text{ rad/sec}}$$

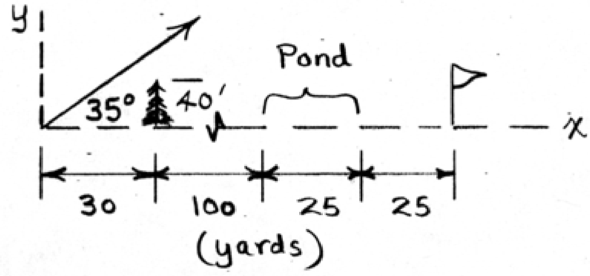
$$a_r = \ddot{r} - r\dot{\theta}^2, \quad \ddot{r} = a_r + r\dot{\theta}^2 = -6.71 + 55.9\,(0.240)^2$$
$$= \underline{-3.49 \text{ ft/sec}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}, \quad \ddot{\theta} = \frac{1}{r}(a_\theta - 2\dot{r}\dot{\theta})$$
$$= \frac{1}{55.9}\left[8.94 - 2(-4.47)(0.240)\right] = \underline{0.1984 \text{ rad/sec}^2}$$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(25/50) = \underline{26.6^\circ}$$



$\boxed{2/247}$ $v_0 = 125$ ft/sec



Time to tree: $x = x_0 + v_{x_0} t$: $90 = 0 + 125 \cos 35° t$

$t = 0.879$ sec

Altitude : $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$y = 0 + 125 \sin 35° (0.879) - 16.1 (0.879)^2 = 50.6$ ft

So ball clears (slender) tree.

Flight time (y-eq.) : $0 = 0 + 125 \sin 35° t_f - 16.1 t_f^2$

$t_f = 0$ (launch time) or $t = 4.45$ sec (impact time)

Range (x-eq.) : $R = 0 + 125 \cos 35° (4.45)$

$= 456$ ft or $\underline{152.0 \text{ yd}}$

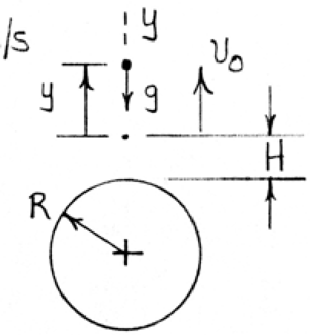
Ball lands in water hazard!

2/248 $v_0 = 27000/3.6 = 7500 \text{ m/s}$

$H = 35(10^4) \text{ m}$

$g_0 = 9.832 \text{ m/s}^2$ (Fig. 1/1)

$R = 6371 \text{ km}$ or $6.371(10^6) \text{ m}$



$$g = g_0 \left(\frac{R}{R+H+y}\right)^2$$

From $a = v\dfrac{dv}{dy}$, we have

$$\int_{v_0}^{0} v\,dv = \int_0^h -g_0\left(\frac{R}{R+H+y}\right)^2 dy$$

$$-\frac{1}{2}v_0^2 = g_0R^2 \left.\frac{1}{R+H+y}\right|_0^h$$

$$v_0^2 = 2g_0R^2 \frac{h}{(R+H)(R+H+h)}$$

Solve for h: $h = \dfrac{(R+H)^2 v_0^2}{2g_0R^2 - (R+H)v_0^2}$

Substitute numerical values and obtain

$h = 6048(10^3) \text{ m}$ or $\underline{h = 6048 \text{ km}}$

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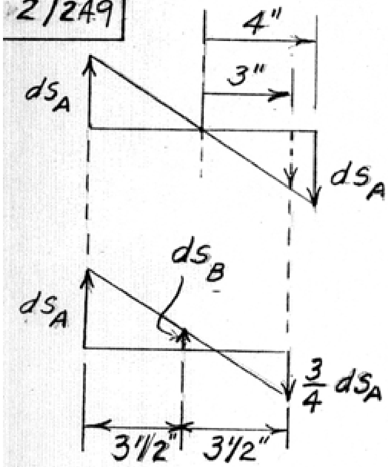


ds_B = average of ds_A and $-\frac{3}{4} ds_A$

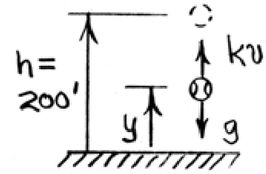
$$= \frac{ds_A + (-\frac{3}{4} ds_A)}{2} = \frac{1}{8} ds_A$$

so $a_B = \frac{1}{8} a_A$

$$= \frac{2}{8} = \underline{0.25 \text{ ft/sec}^2}$$

***2/250**

$$a = v\frac{dv}{dy} = -g + kv^2$$



$$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_h^y dy$$

$$\frac{1}{2k}\ln\left[-g+kv^2\right]_0^v = y\Big|_h^y$$

$$\frac{1}{2k}\ln\left[\frac{-g+kv^2}{-g}\right] = y-h \Rightarrow v = \sqrt{\frac{g}{k}\left[1-e^{2k(y-h)}\right]}$$

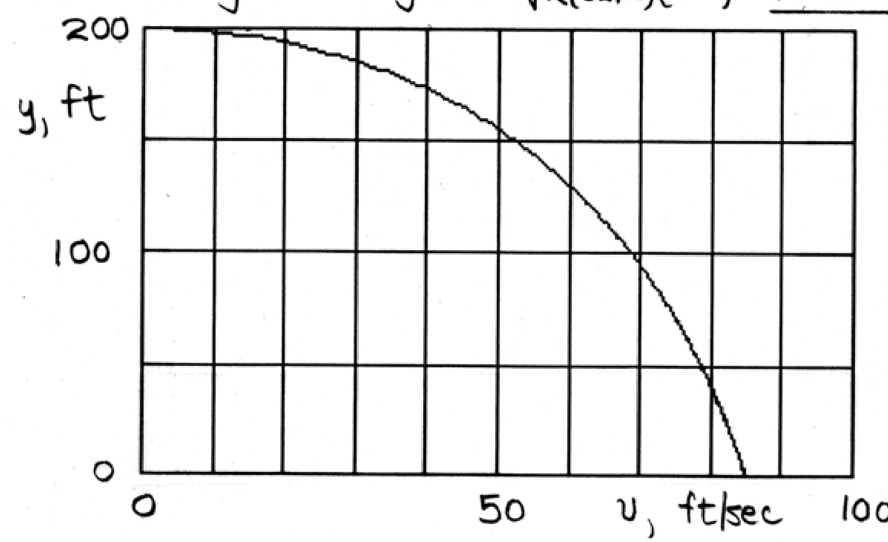
Given numbers : $85 = \sqrt{\frac{32.2}{k}\left[1-e^{2k(0-200)}\right]}$

Numerical solution : $\underline{k = 0.00323\ \text{ft}^{-1}}$

Terminal speed : $g = kv^2$: $32.2 = 0.00323\,v_t^2$

$$\underline{v_t = 99.8\ \text{ft/sec}}$$

Without drag : $v' = \sqrt{2gh} = \sqrt{2(32.2)(200)} = \underline{113.5\ \text{ft/sec}}$



*2/251 $a_t = \frac{dv}{dt} = g\cos\theta - \frac{k}{m}v$

With $v = r\dot{\theta}$: $\frac{d}{dt}(r\dot{\theta}) = g\cos\theta - \frac{k}{m}(r\dot{\theta})$

or $\frac{d^2\theta}{dt^2} + \frac{k}{m}\frac{d\theta}{dt} - \frac{g}{r}\cos\theta = 0$

This is a nonlinear, second-order differential equation, so a numerical integration is in order. To switch to first order form, we let

$x_1 = \theta$ & $x_2 = \dot{\theta}$

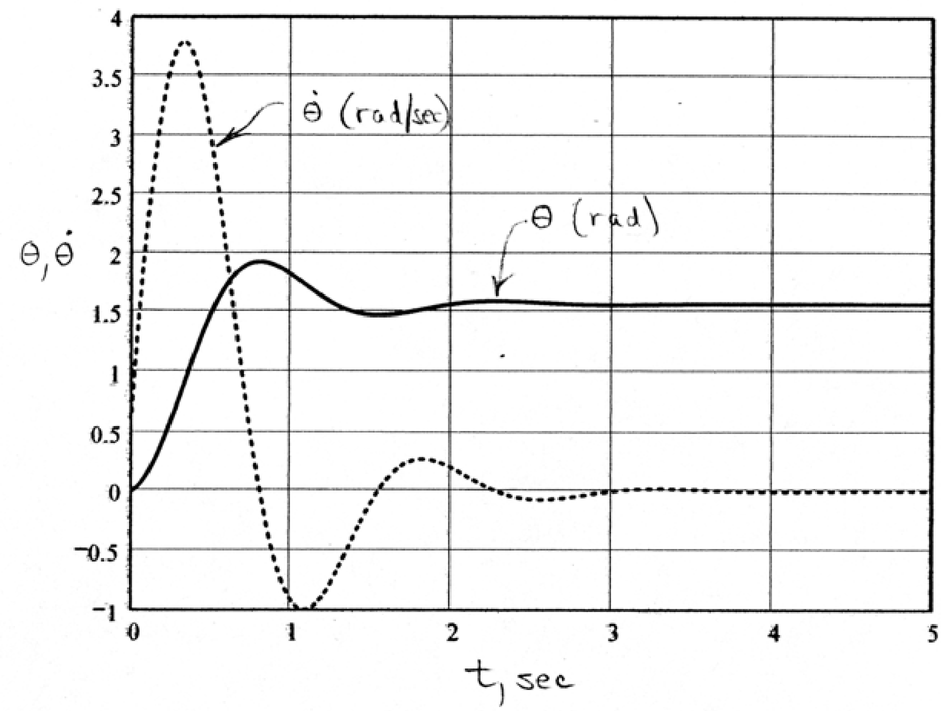
So $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m}x_2 + \frac{g}{r}\cos x_1 \end{cases}$ $\Bigg\|$ $\begin{matrix} x_{10} = \theta_0 = 0 \\ x_{20} = \dot{\theta}_0 = \frac{v_0}{r} \end{matrix}$

The plots below show θ and $\dot{\theta}$ as functions of t.

$\theta_{max} = 110.4°$ @ $t = 0.802$ sec

$\dot{\theta}_{max} = 3.79$ rad/sec @ $t = 0.324$ sec

$\theta = 90°$ @ $t = 0.526$ sec



4
3.5
3
2.5
2
1.5
1
0.5
0
−0.5
−1
0
1
2
3
4
5
$\theta, \dot{\theta}$
$\dot{\theta}$ (rad/sec)
θ (rad)
t, sec

*2/252

$$\ddot{\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta} = \frac{g}{l}\cos\theta$$

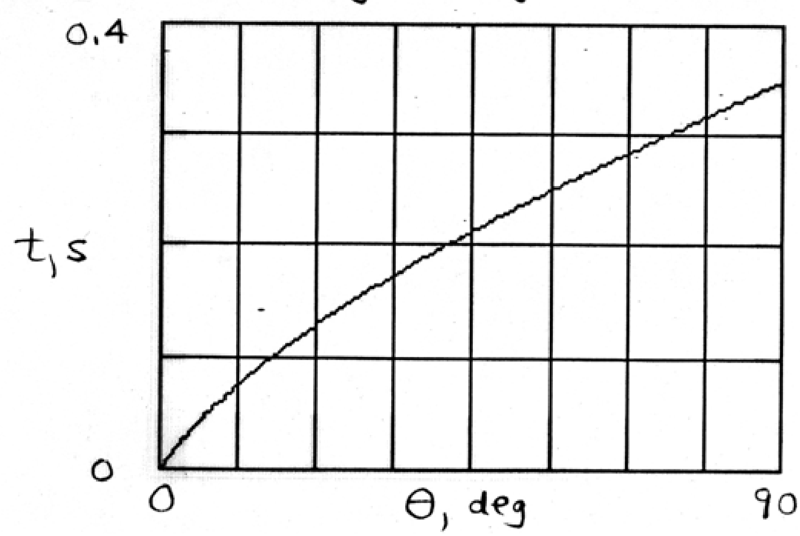
$$\int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \frac{g}{l}\int_0^{\theta} \cos\theta\, d\theta$$

$$\dot{\theta} = \left[\dot{\theta}_0^2 + \frac{2g}{l}\sin\theta\right]^{1/2}$$

Then $\dot{\theta} = \frac{d\theta}{dt} = \left[\dot{\theta}_0^2 + \frac{2g}{l}\sin\theta\right]^{1/2}$

$$t = \int_0^{\theta} \frac{d\theta}{\sqrt{\dot{\theta}_0^2 + \frac{2g}{l}\sin\theta}}$$

With $\dot{\theta}_0 = 2$ rad/s, $l = 0.6$ m, $g = 9.81$, & $\theta = \frac{\pi}{2}$, a numerical integration yields $\underline{t' = 0.349 \text{ s}}$.



$\boxed{*2/253}$ $v\,dv = a\,ds$, $\quad a = \dfrac{T - 4.50v^2}{m}$

$$\int_0^v \frac{m\,v\,dv}{T - 4.50v^2} = \int_0^s ds\,; \quad s = \frac{m}{9.00}\ln\frac{T}{T - 4.50v^2}$$

$$e^{9.00s/m} = \frac{T}{T - 4.50v^2}\,, \quad v = \sqrt{\frac{T}{4.50}\left(1 - e^{-9.00s/m}\right)}$$

where $T = 250$ kN, s = distance in meters, $m = 16000$ tons (metric)

(1 metric ton = 1000 kg)

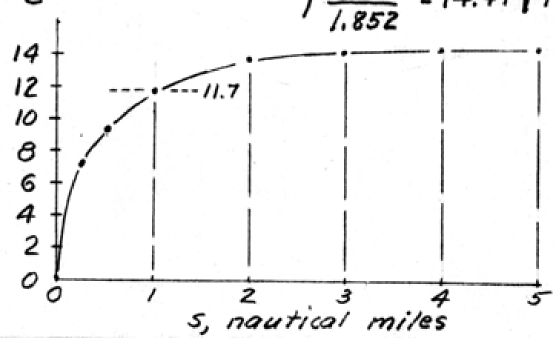
v = speed in m/s

or if v = speed in knots & s = distance in nautical miles, then

$$v = \sqrt{\frac{250}{4.50}\left(1 - e^{-9.00\times1852s/16000}\right)}\,\frac{3.6}{1.852} = 14.49\sqrt{1 - e^{-1.042s}}$$

$\underline{v_{1\,mi} = 11.66 \text{ knots}}$

$\underline{v_{max} = 14.49 \text{ knots}}$



$\boxed{*2/254}$ Let $\omega_n = \sqrt{g/l}$: $\begin{cases} \theta = \theta_0 \sin \omega_n t \\ \dot{\theta} = \theta_0 \omega_n \cos \omega_n t \\ \ddot{\theta} = -\theta_0 \omega_n^2 \sin \omega_n t \end{cases}$

$a_t = l\ddot{\theta} = -l\theta_0 \omega_n^2 \sin \omega_n t = = g\theta_0 \sin \omega_n t$

$a_n = l\dot{\theta}^2 = l\theta_0^2 \omega_n^2 \cos^2 \omega_n t = g\theta_0^2 \cos^2 \omega_n t$

$a = \sqrt{a_t^2 + a_n^2} = g\theta_0 \sqrt{\sin^2 \omega_n t + \theta_0^2 \cos^4 \omega_n t}$

a^2 (and therefore a) is an extreme when

$\frac{da^2}{d\theta} = 0 = g^2\theta_0^2 \left[2 \sin\omega_n t\,(\cos \omega_n t) + \theta_0^2\, 4\cos^3 \omega_n t\,(-\sin \omega_n t)\right]$

$\Rightarrow \left[1 - 2\theta_0^2 \cos^2 \omega_n t\right] = 0$

$\left[1 - 2\left(\frac{\pi}{3}\right)^2 \cos^2 \omega_n t\right] = 0, \quad \omega_n t = 0.830 \text{ rad}$

With $\omega_n = \sqrt{g/l} = \sqrt{\frac{9.81}{0.8}} = 3.50 \text{ s}^{-1}, \; t = 0.237 \text{ s}$

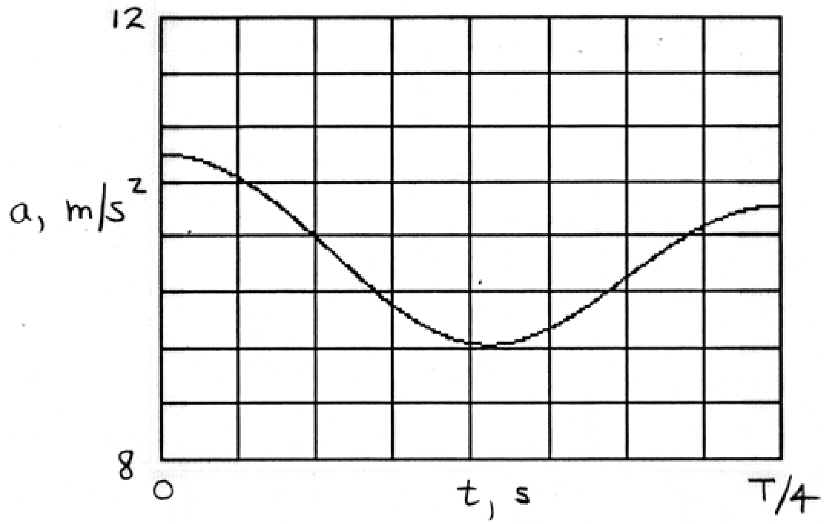
$\theta = \frac{\pi}{3} \sin(0.830) = 0.772 \text{ rad} \quad (44.3°)$

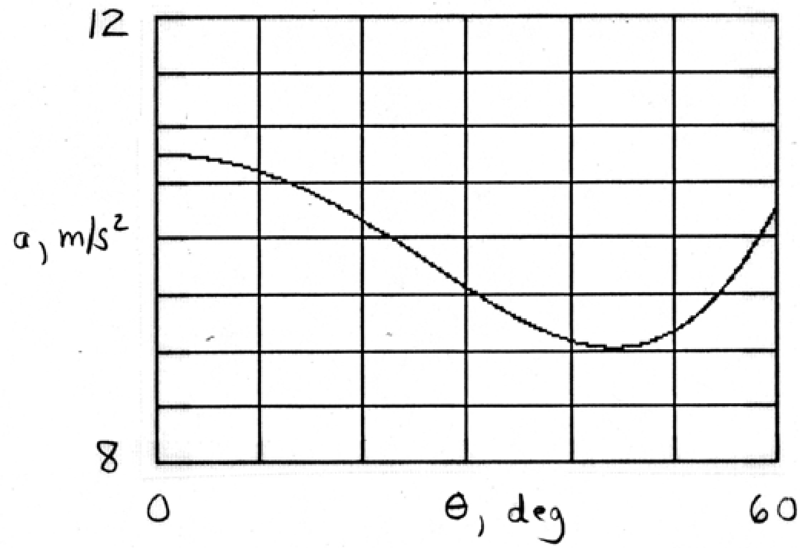
As can be seen from the plots below,

the above represents a minimum:

$\underline{a_{min} = 9.03 \text{ m/s}^2 \text{ @ } \theta = 44.3° \;\&\; t = 0.237 \text{ s}}$

$\underline{a_{max} = 10.76 \text{ m/s}^2 \text{ @ } \theta = 0 \;\&\; t = 0}$





Note : Period $T = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.50} = 1.794 \text{ s}$

$$\frac{T}{4} = 0.449 \text{ s}$$

$\boxed{*2/255}$ $a = \frac{dv}{dt} = c_1 - c_2 v^2$

$$\int_0^t dt = \int_0^v \frac{dv}{c_1 - c_2 v^2} = \frac{1}{\sqrt{c_1 c_2}} \tanh^{-1}\sqrt{\frac{c_2}{c_1}}\, v \Big|_0^v$$

$$t = \frac{1}{\sqrt{c_1 c_2}} \tanh^{-1}\sqrt{\frac{c_2}{c_1}}\, v$$

Then $v = \frac{ds}{dt} = \sqrt{\frac{c_1}{c_2}} \tanh\sqrt{c_1 c_2}\, t$

$$\int_0^s ds = \sqrt{\frac{c_1}{c_2}} \int_0^t \tanh\sqrt{c_1 c_2}\, t\, dt$$

$$s = \sqrt{\frac{c_1}{c_2}} \cdot \frac{1}{\sqrt{c_1 c_2}} \ln\left(\cosh\sqrt{c_1 c_2}\, t\right)\Big|_0^t$$

$$= \frac{1}{c_2} \ln\left(\cosh\sqrt{c_1 c_2}\, t\right)$$

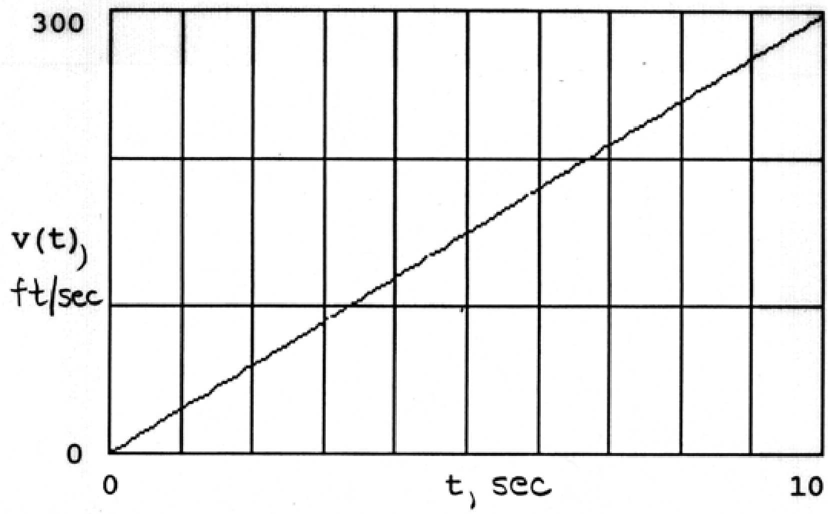
With $s = 1320$ ft, $c_1 = 30$ ft/sec^2, & $t = 9.4$ sec:

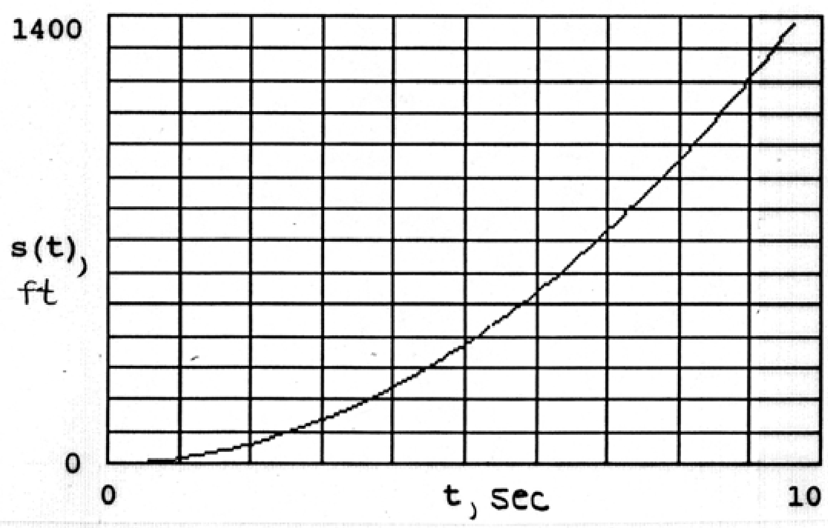
$$1320 = \frac{1}{c_2} \ln\left(\cosh\sqrt{30 c_2} \cdot 9.4\right)$$

$$= \frac{1}{c_2} \ln\left(\cosh 51.5\sqrt{c_2}\right)$$

Numerical solution: $\underline{c_2 = 9.28\,(10^{-6})\ \text{ft}^{-1}}$

(On plots below, note that v vs. t appears linear, but it is not!)


300
v(t),
ft/sec
0
0
t, sec
10


1400
s(t),
ft
0
0
t, sec
10

*2/256

$$a = v\frac{dv}{dy} = -g - kv^2$$

$$\int_{v_0}^{0} \frac{v\,dv}{-g-kv^2} = \int_0^h dy$$

Let $u = g + kv^2$, $du = 2kv\,dv$: $\displaystyle\int_{v=v_0}^{v=0} -\frac{1}{2k}\frac{du}{u} = h$

$$h = -\frac{1}{2k}\ln\left(g+kv^2\right)\Big|_{v_0}^{0} = -\frac{1}{2k}\ln\left(\frac{g}{g+kv_0^2}\right)$$

$2kh = \ln\left(1+\dfrac{kv_0^2}{g}\right)$: $\quad 2(5280)k = \ln\left(1+\dfrac{2000^2 k}{32.2}\right)$

Solve numerically to obtain $\underline{k = 3.63\,(10^{-4})\text{ ft}^{-1}}$

***2/257** $y = x^2/4$, x & y in inches ; $x = 4 \sin 2t$, t in seconds

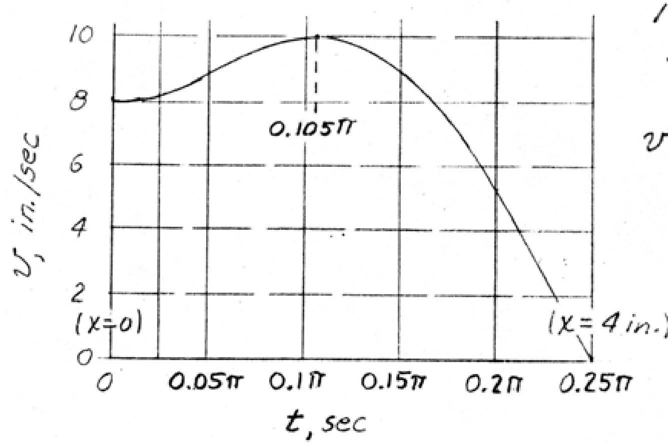
$\dot{y} = \frac{x\dot{x}}{2} = 2 \sin 2t\,(8 \cos 2t) = 16 \sin 2t \cos 2t$ in./sec

$\dot{x} = 8 \cos 2t$

$v^2 = \dot{x}^2 + \dot{y}^2 = 64 \cos^2 2t + 256 \sin^2 2t \cos^2 2t$

$= 64 \cos^2 2t\,(1 + 4 \sin^2 2t)$ (in./sec)^2

$v = 8 \cos 2t \sqrt{1 + 4 \sin^2 2t}$



$\frac{dv}{dt} = 0$ gives

$1 - 2\sin^2 2t = 1/4$

$\sin 2t = \sqrt{3/8}$

$\cos 2t = \sqrt{5/8}$

$v = 8\sqrt{\frac{5}{8}}\sqrt{1 + 4(3/8)}$

$= \underline{10 \text{ in./sec}}$

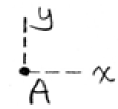
$2t = 0.659$ rad

$t = \underline{0.330 \text{ sec}}$

$(= 0.105\pi \text{ sec})$

@ $\underline{x = 2.45 \text{ in.}}$

***2/258** Set up x-y coordinates @ A :



Let coordinates of B be (R, h).

$x = x_0 + v_{x_0} t$ @ B : $\quad R = 0 + (v_0 \cos\alpha) t_f \qquad (1)$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B : $\quad h = 0 + (v_0 \sin\alpha) t_f - \frac{1}{2} g t_f^2 \qquad (2)$

Solve (1) & (2) for R & t_f :

$$\left.\begin{aligned} R &= \frac{v_0 \cos\alpha}{g}\left[v_0 \sin\alpha + \sqrt{v_0^2 \sin^2\alpha - 2gh}\right] \\ t_f &= \frac{1}{g}\left[v_0 \sin\alpha + \sqrt{v_0^2 \sin^2\alpha - 2gh}\right] \end{aligned}\right\}$$

where the + sign has been taken to maximize R.

Set $\frac{dR}{d\alpha} = 0$ (done by computer) to find

$\underline{\alpha = 48.5°}$ $\quad$ (for $h = 10$ m, $v_0 = 30$ m/s, and $g = 9.81$ m/s^2)

The corresponding value of R is

$\underline{R = 81.1 \text{ m}}$

So the 10-m plateau is indeed achieved, as assumed (because $R > 50$ m).