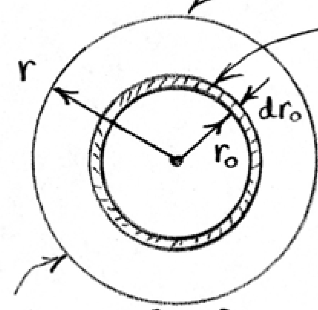
B/1

thickness (depth) = t



$m = \pi r^2 t \rho$

$$\begin{cases} dm = \rho\, dV = \rho\, 2\pi r_0\, dr_0\, t \\ dI = dm\, r_0^{\,2} \end{cases}$$

$$= 2\pi\rho t\, r_0^{\,3}\, dr_0$$

$$I = \int dI = \int_0^r 2\pi \rho t\, r_0^{\,3}\, dr_0^{\,3}$$

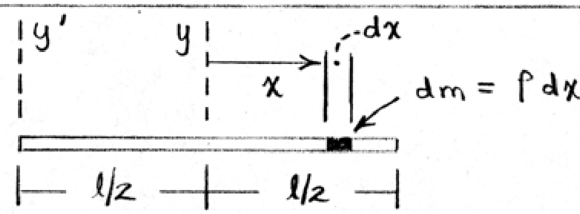
$$= 2\pi\rho t\, \frac{r^4}{4}\left(\frac{m}{\pi r^2 t \rho}\right)$$

$$= \underline{\tfrac{1}{2} m r^2}$$

8/2 Complete sphere of mass $2m$, $I_{zz} = \frac{2}{5}(2m)r^2$

For hemisphere $I_{zz} = I_{xx} = \frac{2}{5}mr^2$

B/3



$dI_{yy} = dm\,x^2 = (\rho\,dx)\,x^2 = \rho x^2 dx$

$I_{yy} = \int dI_{yy} = \int_{-l/2}^{l/2} \rho x^2 dx = \frac{1}{12}\rho l^3$

But the total mass $m = \rho l$

So $I_{yy} = \frac{1}{12}\rho l^3 \left(\frac{m}{\rho l}\right) = \underline{\frac{1}{12} m l^2}$

$I_{y'y'} = I_{yy} + m\left(\frac{l}{2}\right)^2 = \underline{\frac{1}{3} m l^2}$

B/4 Conical shell has same radial distribution of mass as does a circular disk of the same mass and radius. Thus

$$\underline{I_{zz} = \frac{1}{2} m r^2}$$

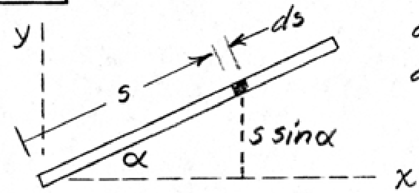
8/5

$$\text{error} = \frac{(\frac{1}{2}mr^2 + md^2) - md^2}{\frac{1}{2}mr^2 + md^2} = \frac{1}{1 + 2(d/r)^2}$$

(a) $d = 10r$, % error $e = \frac{100}{1 + 200} = \underline{0.498\ \%}$

(b) $d = 2r$, % error $e = \frac{100}{1 + 8} = \underline{11.11\ \%}$

B/6



Let ρ = mass per unit length

$dm = \rho\, ds$

$dI_{xx} = (s \sin\alpha)^2 dm$

$I_{xx} = \rho \sin^2\alpha \int_0^b s^2 ds$

$= \frac{1}{3}\rho b^3 \sin^2\alpha$

$= \underline{\frac{1}{3} m b^2 \sin^2\alpha}$

By inspection, $I_{yy} = \underline{\frac{1}{3} m b^2 \cos^2\alpha}$

For negligible z-dimensions, $I_{xx} + I_{yy} = I_{zz}$

$I_{zz} = \frac{1}{3} m b^2 (\sin^2\alpha + \cos^2\alpha) = \underline{\frac{1}{3} m b^2}$

B/7 Error is $\dfrac{md^2 - (\frac{2}{5}mr^2 + md^2)}{\frac{2}{5}mr^2 + md^2}$

$$= \frac{-1}{1 + \frac{5}{2}\left(\frac{d}{r}\right)^2}$$

So percent error $e = \dfrac{-100}{1 + \frac{5}{2}\left(\frac{d}{r}\right)^2}$

(a) $\frac{d}{r} = 2$: $\underline{|e| = 9.09\ \%}$

(b) $\frac{d}{r} = 10$: $\underline{|e| = 0.398\ \%}$

B/8

$$\text{Percent error } e = \frac{\frac{1}{12}ml^2 - \left(\frac{1}{12}ml^2 + \frac{1}{4}mr^2\right)}{\frac{1}{12}ml^2 + \frac{1}{4}mr^2}(100)$$

$$= \frac{-100}{1 + \frac{1/3}{(r/l)^2}} \quad (\text{in percent})$$

Values :

r/l	e
0.01	-0.030 %
0.1	-2.91 %
0.5	-42.9 %

B/9

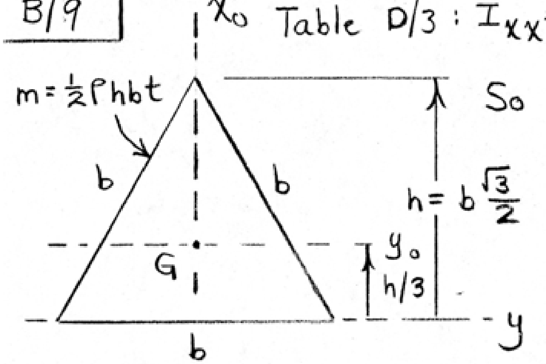


Table D/3 : $I_{xx} = \rho t\left(\frac{bh^3}{12}\right) = \frac{1}{6}mh^2$

So $I_{yy} = \frac{1}{6}m\left(b\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{8}mb^2$

$I_{y_0y_0} = I_{yy} - m\left(\frac{h}{3}\right)^2$

$= \frac{1}{8}mb^2 - m\frac{b^2}{12}$

$= \frac{1}{24}mb^2$

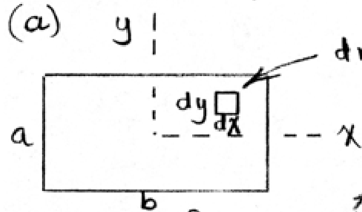
Again from Table D/3:

$I_{x_0x_0} = 2\left(\frac{1}{6}\frac{m}{2}\left(\frac{b}{2}\right)^2\right) = \frac{1}{24}mb^2$

$I_{zz} = I_{x_0x_0} + I_{y_0y_0} = \underline{\frac{1}{12}mb^2}$

B/10 $\quad m = \rho A = \rho ab \quad (\rho = \text{mass/area})$

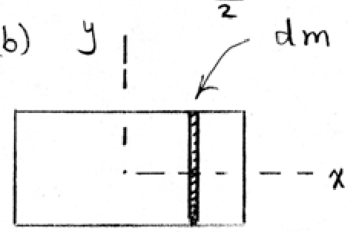
(a)



$$I_{xx} = \int (y^2 + \cancel{z^2}^{\,0})\, dm = \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} y^2 \rho\, dy\, dx$$

$$= \rho \left(\tfrac{1}{12} a^3\right) \int_{-\frac{b}{2}}^{b/2} dx = \tfrac{1}{12} \rho a^3 b \left(\frac{m}{\rho ab}\right) = \underline{\tfrac{1}{12} m a^2}$$

(b)



$dm = \rho a\, dx, \quad dI_{xx} = \frac{1}{12} dm (a^2)$

$$= \tfrac{1}{12} \rho a^3\, dx$$

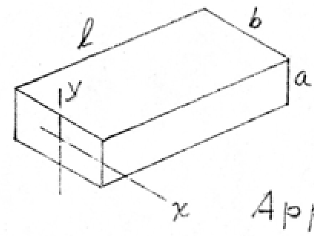
$$I_{xx} = \int dI_{xx} = \int_{-b/2}^{b/2} \tfrac{1}{12} \rho a^3\, dx$$

$$= \tfrac{1}{12} \rho a^3 b \left(\frac{m}{\rho ab}\right) = \underline{\tfrac{1}{12} m a^2}$$

Note that method (b) <u>begins</u> in the <u>middle</u> of method (a). By inspection, $\underline{I_{yy} = \frac{1}{12} m b^2}$

$$I_{zz} = I_{xx} + I_{yy} = \underline{\tfrac{1}{12} m (a^2 + b^2)}$$

B/11 $\ell = 0.3\ m,\ a = 0.03\ m,\ b = 0.2\ m,\ m = 15\ kg$



From Sample Problem B/3

$$I_{yy} = \frac{1}{12} m (b^2 + 4\ell^2)$$

$$I_{xx} = \frac{1}{12} m (a^2 + 4\ell^2)$$

Approx. $I'_{xx} = \frac{1}{3} m \ell^2$

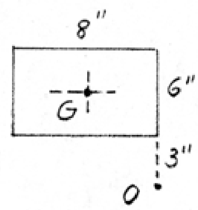
Fractional error $e = (I_{xx} - I'_{xx}) / I_{xx}$

$$= \frac{a^2/12}{a^2/12 + \ell^2/3} = \frac{1}{1 + (2\ell/a)^2}$$

$$I_{yy} = \frac{1}{12}(15)(\overline{0.2}^2 + 4(0.3)^2) = \underline{0.5\ kg \cdot m^2}$$

$$e = \frac{1}{1 + (\frac{2 \times 0.3}{0.03})^2}(100\%) = \frac{100}{401} = \underline{0.25\ \%}$$

B/12 From Sample Problem B/3



$I_G = \frac{1}{12} m(a^2+b^2)$

$= \frac{1}{12} \frac{6(8)(10)}{1728} \frac{489}{32.2} \left(\left[\frac{6}{12}\right]^2 + \left[\frac{8}{12}\right]^2\right)$

$= \frac{1}{12}(4.218)\left(\frac{1}{4} + \frac{4}{9}\right) = 4.218 \frac{25}{12(36)}$

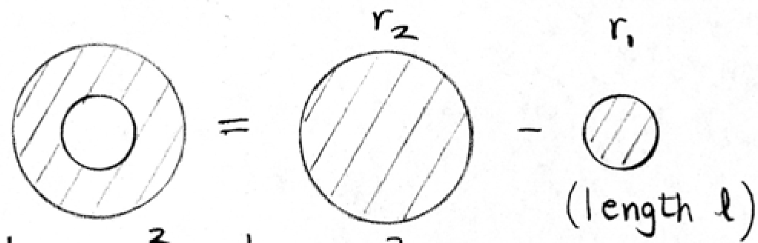
$\overline{OG} = \sqrt{6^2+4^2}$
$= 7.21 \text{ in.}$

$I_O = I_G + md^2$

$= 4.218\left[\frac{25}{12(36)} + \left(\frac{7.21}{12}\right)^2\right]$

$I_{OO} = I_O = 4.218 \frac{60.33}{144} = \underline{1.77 \text{ lb-ft-sec}^2}$

B/13



$$
\begin{aligned}
I_{xx} &= \tfrac{1}{2} m_2 r_2^2 - \tfrac{1}{2} m_1 r_1^2 \\
&= \tfrac{1}{2}\left(\rho \pi r_2^2 l\right) r_2^2 - \tfrac{1}{2}\left(\rho \pi r_1^2 l\right) r_1^2 \\
&= \tfrac{1}{2} \rho \pi l \left(r_2^4 - r_1^4\right) = \tfrac{1}{2} \underbrace{\rho \pi l \left(r_2^2 - r_1^2\right)}_{m} \left(r_2^2 + r_1^2\right) \\
&= \underline{\tfrac{1}{2} m \left(r_2^2 + r_1^2\right)}
\end{aligned}
$$

B/14

$$I = I_{2_0} - I_{1_0} = \frac{1}{2} m_2 r_2^2 - \left[\frac{1}{2} m_1 r_1^2 + m_1 d^2\right]$$
$$= \rho\pi t\left[\frac{r_2^4}{2} - \frac{r_1^4}{2} - r_1^2 d^2\right]$$

With $m = \rho\pi t\left[r_2^2 - r_1^2\right]$,

$$k_0^2 = \frac{I}{m} = \frac{1}{2}(r_2^2 + r_1^2) - \frac{r_1^2 d^2}{(r_2^2 - r_1^2)}$$
$$= \frac{1}{2}(12^2 + 6^2) - \frac{6^2 4^2}{12^2 - 6^2} = 84.7 \text{ in.}^2$$

$\underline{k_0 = 9.20 \text{ in.}}$

8/15 From Sample Problem 8/3, $I_{yy} = \frac{1}{12}m(a^2+4l^2)$

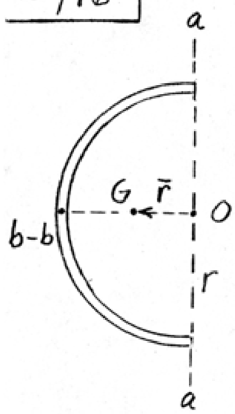
$m = 1300(0.4)(0.36)(0.10) = 18.72$ kg

$I_{yy} = \frac{1}{12}(18.72)\left([0.36]^2+4[0.4]^2\right) = \underline{1.201 \text{ kg-m}^2}$

For I_{xx}, % error $|e| = \dfrac{\frac{1}{12}ma^2}{\frac{1}{12}m(a^2+4l^2)}100\% = \dfrac{100\%}{1+(2l/a)^2}$

where $a = 0.1$ m, $l = 0.4$ m so $|e| = \dfrac{100\%}{1+(0.8/0.1)^2} = \underline{1.538\%}$

B/16



For complete ring of mass $2m$,

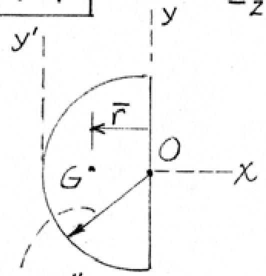
$I_O = (2m)r^2$ & $I_{aa} = \frac{1}{2}(2m)r^2$

So for half ring $\underline{I_{aa} = \frac{1}{2}mr^2}$

$$
\begin{aligned}
I_{bb} &= \bar{I} + (r-\bar{r})^2 m \\
&= I_O - m\bar{r}^2 + (r-\bar{r})^2 m \\
&= I_O + m(r^2 - 2r\bar{r}) \\
&= mr^2 + m(r^2 - 2r\bar{r}) \\
&= 2mr^2\left(1 - \frac{\bar{r}}{r}\right) \quad \text{where } \bar{r} = \frac{2r}{\pi}
\end{aligned}
$$

So $\underline{I_{bb} = 2mr^2\left(1 - \frac{2}{\pi}\right)}$

8/17



$r = 10''$

$\bar{r} = 4r/3\pi$

$$I_{zz} = I_O = \frac{1}{2}mr^2 = \frac{1}{2}\frac{5}{32.2}\left(\frac{10}{12}\right)^2 = 0.0539 \text{ lb-ft-sec}^2$$

$$I_{xx} = I_{yy} = \frac{1}{2}I_{zz} = \underline{0.0270 \text{ lb-ft-sec}^2}$$

$$I_{y'y'} = \bar{I} + m(r-\bar{r})^2$$

$$= I_{yy} - m\bar{r}^2 + m(r-\bar{r})^2$$

$$= I_{yy} + mr(r-2\bar{r}) = I_{yy} + mr^2\left(1-\frac{8}{3\pi}\right)$$

$$= 0.0270 + \frac{5}{32.2}\left(\frac{10}{12}\right)^2\left(1-\frac{8}{3\pi}\right)$$

$$= 0.0270 + 0.0163 = 0.0433 \text{ lb-ft-sec}^2$$

B/18 Disk : $I_{zz} = \frac{1}{2}mr^2$, $I_{xx} = \frac{1}{4}mr^2$

Both rods : $I_{zz} = 0$, $I_{xx} = 2\left(\frac{1}{3}\frac{m}{2}L^2\right) = \frac{1}{3}mL^2$

$I_{zz} = I_{xx}$: $\frac{1}{2}mr^2 + 0 = \frac{1}{4}mr^2 + \frac{1}{3}mL^2$

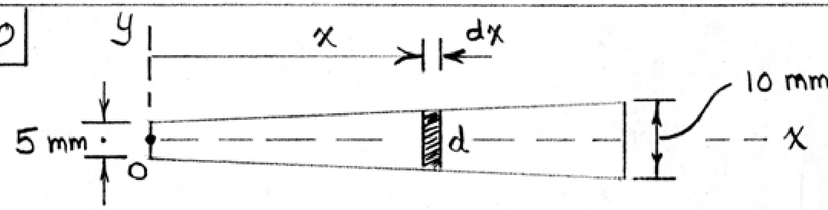
$\frac{3}{4}r^2 = L^2$ or $\underline{L = \frac{r\sqrt{3}}{2}}$

B/19 Handle mass is ρL; frame mass is $\rho\left(\pi \frac{L}{2}\right)$.

$$I_{yy} = \frac{1}{12}\rho L \cdot L^2 + \rho L\left(\frac{3L}{8}\right)^2 + \frac{1}{2}\left(\rho\pi\frac{L}{2}\right)\left(\frac{L}{4}\right)^2 + \rho\pi\frac{L}{2}\left(\frac{9}{8}L\right)^2$$

$$= \underline{\left[\frac{43}{192} + \frac{83}{128}\pi\right]\rho L^3}$$

B/20



Diameter $d = 5 + kx$: $10 = 5 + k(100)$, $k = 0.05$

So $d = 5 + 0.05x$

$dm = \rho dV = \rho \frac{\pi d^2}{4} dx$, $dI_0 = x^2 dm$

$= \frac{\rho\pi}{4}(5 + 0.05x)^2 x^2 dx$

$I_0 = \frac{\rho\pi}{4}\int_0^{100}(25x^2 + 0.5x^3 + 0.0025x^4)\,dx$

$= \frac{\rho\pi}{4}(25.8(10^6))$ kg·mm^2

With $\rho = 7830(10^{-9})$ kg/mm^3 :

$I_0 = 158.9$ kg·mm^2

B/21 Rim: $I = \frac{1}{2} m_2 r_2^2 - \frac{1}{2} m_1 r_1^2 = \frac{1}{2} \rho \pi r_2^4 b - \frac{1}{2} \rho \pi r_1^4 b$

$$= \frac{1}{2} \rho \pi (r_2^4 - r_1^4)(b)$$
$$= \frac{1}{2}(7830)\pi(\overline{0.2}^4 - \overline{0.15}^4)(0.075)$$
$$= 1.009 \text{ kg}\cdot\text{m}^2$$

Hub: $I = \frac{1}{2} \rho \pi (r_2^4 - r_1^4)(b)$

$$= \frac{1}{2}(7830)\pi(\overline{0.05}^4 - \overline{0.025}^4)(0.12)$$
$$= 0.00865 \text{ kg}\cdot\text{m}^2$$

Spokes: $I = 8\left[\frac{m\ell^2}{12} + md^2\right] = 8m\left[\frac{\ell^2}{12} + d^2\right]$

$$= 8(7830)(0.1)(200 \times 10^{-6})\left[\frac{\overline{0.1}^2}{12} + \overline{0.1}^2\right]$$
$$= 0.01357 \text{ kg}\cdot\text{m}^2$$

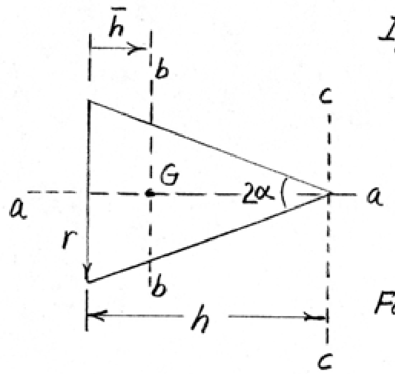
Total $I = 1.009 + 0.00865 + 0.01357$

$$= \underline{1.031 \text{ kg}\cdot\text{m}^2}$$

$$n = \frac{1.009}{1.031}(100) = \underline{97.8\ \%}$$

B/22

$I_{aa} = \frac{3}{10} mr^2$; $I_{cc} = \frac{3}{20} mr^2 + \frac{3}{5} mh^2$ (Table D/4)



$I_{bb} = I_{cc} - m(h - \bar{h})^2$, $\bar{h} = h/4$

$= I_{cc} - m \frac{9}{16} h^2$

$= \frac{3}{20} mr^2 + \frac{3}{5} mh^2 - \frac{9}{16} mh^2$

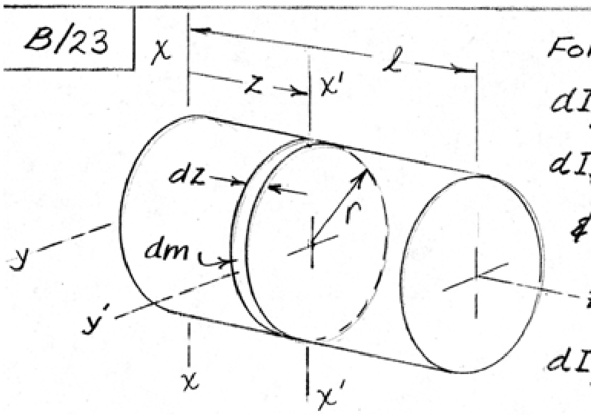
$= \frac{3}{20} mr^2 + \frac{3}{80} mh^2$

For $I_{aa} = I_{bb}$, $\frac{3}{10} mr^2 = \frac{3}{20} mr^2 + \frac{3}{80} mh^2$

$\frac{3}{20} r^2 = \frac{3}{80} h^2$

Thus $\left(\frac{r}{h}\right)^2 = \frac{1}{4}$, $r = \frac{1}{2} h$ & $\tan\alpha = \frac{r}{h} = \frac{1}{2}$, $\underline{\alpha = 26.6°}$

B/23



For slice of mass dm

$dI_{zz} = \frac{1}{2} dm\,(r^2)$

$dI_{x'x'} + dI_{y'y'} = dI_{zz}$

& $dI_{x'x'} = \frac{1}{2} dI_{zz}$

$= \frac{1}{4} r^2 dm$

$dI_{xx} = dI_{x'x'} + z^2 dm$

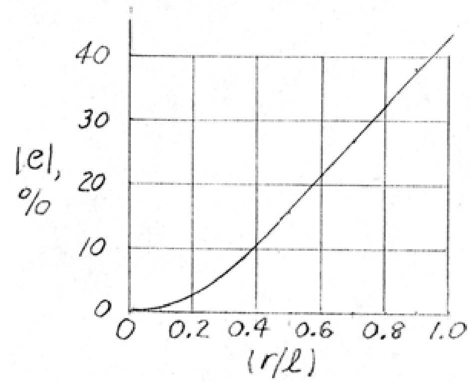
So $dI_{xx} = \frac{1}{4} r^2 dm + z^2 dm$ but $dm = \pi r^2 \rho\, dz$ where ρ = density

& $dI_{xx} = \pi r^2 \rho \left(\frac{r^2}{4} + z^2\right) dz$; $I_{xx} = \pi r^2 \rho \int_0^l \left(\frac{r^2}{4} + z^2\right) dz$

$I_{xx} = \pi r^2 \rho \left(\frac{r^2 l}{4} + \frac{l^3}{3}\right) = \underline{m\left(\frac{r^2}{4} + \frac{l^2}{3}\right)}$

B/24

$$\% \text{ error } |e| = \frac{m\left(\frac{r^2}{4}+\frac{l^2}{3}\right) - m\frac{l^2}{3}}{m\left(\frac{r^2}{4}+\frac{l^2}{3}\right)} 100\% = \frac{100\%}{1 + \frac{4/3}{(r/l)^2}}$$



For $r/l = 0.2$

error $|e| = 2.91\%$

B/25 From the results of Prob. B/23 & the transfer theorem

$I_{xx} = I_{yy} = m\left(\frac{r^2}{4} + \frac{l^2}{3}\right) - m\left(\frac{l}{2}\right)^2 = m\left(\frac{r^2}{4} + \frac{l^2}{12}\right)$

$I_{zz} = \frac{1}{2} m r^2$

For $I_{xx} = I_{yy} = I_{zz}$, $\frac{1}{2} m r^2 = \frac{1}{4} m r^2 + \frac{1}{12} m l^2$, $\underline{l = r\sqrt{3}}$

B/26 The mass distribution is essentially the same as that of a cylindrical shell.

From Table D/4 : $I_{zz} = mr^2$

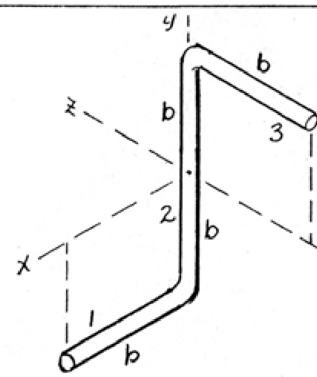
$$I_{xx} = I_{yy} = \frac{1}{2}mr^2 + \frac{1}{12}mL^2$$

So $I_{zz} = \frac{4}{32.2}\left(\frac{3/2}{12}\right)^2 = \underline{0.001941 \text{ lb-ft-sec}^2}$

$$I_{xx} = I_{yy} = \frac{0.001941}{2} + \frac{1}{12}\frac{4}{32.2}\left(\frac{10}{12}\right)^2$$

$$= \underline{0.00816 \text{ lb-ft-sec}^2}$$

B/27



Part 1:

$m_1 = m/4$

$I_{xx} = \frac{m}{4} b^2$

$I_{yy} = \frac{1}{3} \frac{m}{4} b^2 = \frac{1}{12} m b^2$

$I_{zz} = \frac{1}{12} \frac{m}{4} b^2 + \frac{m}{4}\left[\left(\frac{b}{2}\right)^2 + b^2\right]$

$= \frac{1}{3} m b^2$

Part 2:

$m_2 = \frac{1}{2} m$

$I_{xx} = \frac{1}{12} \frac{m}{2} (2b)^2 = \frac{1}{6} m b^2$

$I_{yy} = 0$

$I_{zz} = \frac{1}{6} m b^2$

Part 3:

$m_3 = m/4$

$I_{xx} = \frac{1}{12} \frac{m}{4} b^2 + \frac{m}{4}\left[\left(\frac{b}{2}\right)^2 + b^2\right] = \frac{1}{3} m b^2$

$I_{yy} = \frac{1}{3} \frac{m}{4} b^2 = \frac{1}{12} m b^2$

$I_{zz} = \frac{m}{4} b^2$

Total:

$I_{xx} = m b^2 \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{3}\right) = \underline{\frac{3}{4} m b^2}$

$I_{yy} = m b^2 \left(\frac{1}{12} + 0 + \frac{1}{12}\right) = \underline{\frac{1}{6} m b^2}$

$I_{zz} = m b^2 \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{4}\right) = \underline{\frac{3}{4} m b^2}$

B/28 $\bar{r} = \frac{4r}{3\pi} = \frac{4(100)}{3\pi} = 42.4 \text{ mm}$

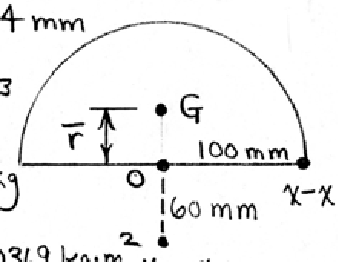


Table D/1: $\rho_{steel} = 7830 \text{ kg/m}^3$

$m = \rho V = 7830 \, \frac{\pi (0.1)^2}{2} (0.060) = 7.38 \text{ kg}$

$I_{oo} = \frac{1}{2} m r^2 = \frac{1}{2}(7.38)(0.1)^2 = 0.0369 \text{ kg}\cdot\text{m}^2$

By symmetry, $I_{xx} = I_{oo} + mr^2 = 0.0369 + 7.38(0.1)^2$

$= 0.1107 \text{ kg}\cdot\text{m}^2$

$I_{x_0 x_0} = I_{GG} + md^2 = (I_{oo} - m\bar{r}^2) + md^2$

$= I_{oo} + m(d^2 - \bar{r}^2)$

$= 0.0369 + 7.38\left[(0.060 + 0.0424)^2 - 0.0424^2\right]$

$= \underline{0.1010 \text{ kg}\cdot\text{m}^2}$

$\boxed{B/29}$ $I_O = \frac{1}{3}ml^2 + 7mx^2 = \underline{m\left(7x^2 + \frac{1}{3}l^2\right)}$

For $x = \frac{3}{4}l$: $I_O = m\left(7\cdot\left(\frac{3}{4}l\right)^2 + \frac{1}{3}l^2\right) = \frac{205}{48}ml^2$

For $x = l$: $I_O = m\left(7l^2 + \frac{1}{3}l^2\right) = \frac{22}{3}ml^2$

$R = \frac{205/48}{22/3} = \underline{0.582}$

B/30

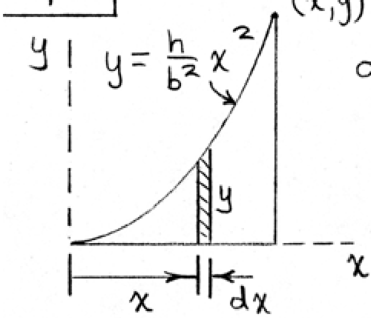


Plate thickness t

$$dm = \rho\, dV = \rho\, y\, dx\, t$$
$$= \rho t \frac{h}{b^2} x^2\, dx$$
$$dI_{xx} = \frac{1}{3}\, dm\, y^2$$
$$= \frac{1}{3}\rho t \frac{h^3}{b^6} x^6\, dx$$
$$dI_{yy} = x^2 dm = \rho t \frac{h}{b^2} x^4\, dx$$

$$m = \int dm = \int_0^b \rho t \frac{h}{b^2} x^2\, dx = \rho t \frac{h}{b^2}\frac{b^3}{3} = \frac{1}{3}\rho t h b$$

$$I_{xx} = \int dI_{xx} = \int_0^b \frac{1}{3}\rho t \frac{h^3}{b^6} x^6\, dx$$
$$= \frac{1}{3}\rho t \frac{h^3}{b^6}\frac{b^7}{7} = \frac{1}{21}\rho t h^3 b\left(\frac{m}{\frac{1}{3}\rho t h b}\right)$$
$$= \underline{\frac{1}{7} m h^2}$$

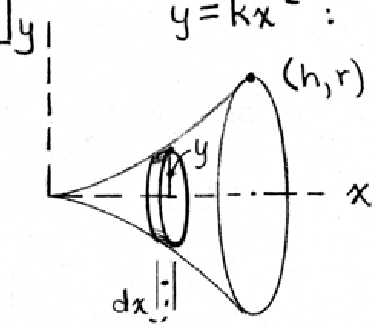
$$I_{yy} = \int dI_{yy} = \int_0^b \rho t \frac{h}{b^2} x^4\, dx$$
$$= \rho t \frac{h}{b^2}\frac{b^5}{5} = \frac{1}{5}\rho t h b^3\left(\frac{m}{\frac{1}{3}\rho t h b}\right) = \underline{\frac{3}{5} m b^2}$$

$$I_{zz} = I_{xx} + I_{yy} = \underline{m\left(\frac{3b^2}{5} + \frac{h^2}{7}\right)}$$

3/31

$y = kx^2$: $r = kh^2 \Rightarrow k = r/h^2$

$\& \; y = \frac{r}{h^2}x^2$



$dm = \rho\, dV = \rho \pi y^2 dx$

$dI_{xx} = \frac{1}{2} dm\, y^2$

$= \frac{1}{2} \rho \pi y^4 dx$

$= \frac{1}{2} \rho \pi \frac{r^4}{h^8} x^8 dx$

$$I_{xx} = \int dI_{xx} = \frac{1}{2} \rho \pi \frac{r^4}{h^8} \int_0^h x^8 dx = \frac{1}{18} \rho \pi r^4 h$$

The mass is $m = \rho V = \int_0^h \rho \pi y^2 dx$

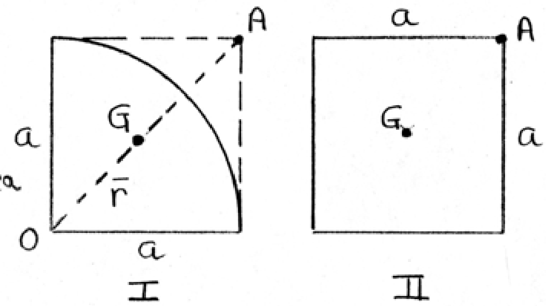
$$= \int_0^h \rho \pi \frac{r^2}{h^4} x^4 dx = \frac{1}{5} \rho \pi r^2 h$$

So $I_{xx} = \frac{1}{18} \rho \pi r^4 h \left(\frac{m}{\frac{1}{5} \rho \pi r^2 h} \right) = \underline{\frac{5}{18} m r^2}$

B/32

$I_A = I_{A_{II}} - I_{A_I}$

Let ρ = mass/unit area



For II:

$I_A = I_G + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{1}{6}ma^2 + \frac{1}{2}ma^2 = \frac{2}{3}ma^2$

$= \frac{2}{3}(\rho a^2)a^2 = 0.667\rho a^4$

For I:

$I_0 = \frac{1}{2}ma^2, \quad I_G = I_0 - m\bar{r}^2 = \frac{1}{2}ma^2 - m\left(\frac{4a}{3\pi}\sqrt{2}\right)^2$

$= 0.1397\,ma^2$

$I_A = I_G + m\left(a\sqrt{2} - \frac{4a}{3\pi}\sqrt{2}\right)^2 = 0.802\,ma^2$

$= 0.802\left(\frac{1}{4}\pi a^2\rho\right)a^2 = 0.630\rho a^4$

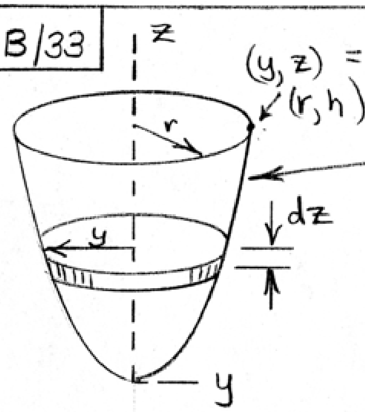
So overall, $I_A = 0.667\rho a^4 - 0.630\rho a^4$

$= 0.0365\rho a^4$

But $m = \rho\left(a^2 - \frac{1}{4}\pi a^2\right) = 0.215\rho a^2$

So $I_A = 0.0365\rho a^4\left(\frac{m}{0.215\rho a^2}\right) = \underline{0.1701\,ma^2}$

B/33



$z = ky^2$: $h = kr^2$, $k = \frac{h}{r^2}$

$\therefore\ z = \frac{h}{r^2} y^2$

$$\begin{cases} dm = \rho\pi y^2 dz \\ dI_{zz} = \frac{1}{2}\rho\pi y^4 dz \end{cases}$$

$$I_{zz} = \int dI_{zz}$$

$$= \int_0^h \frac{1}{2}\rho\pi\left(\frac{r^2}{h}z\right)^2 dz$$

$$I_{zz} = \frac{1}{2}\rho\pi\frac{r^4}{h^2}\cdot\frac{h^3}{3} = \frac{1}{6}\rho\pi r^4 h$$

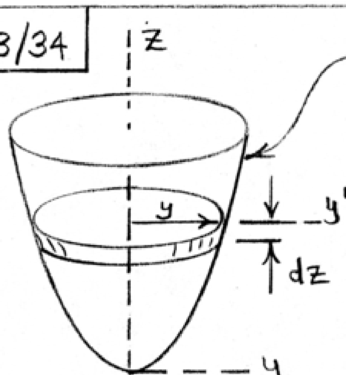
$$m = \rho V = \rho\int_0^h \pi y^2 dz = \rho\pi\int_0^h \frac{r^2}{h} z\, dz$$

$$= \rho\pi\frac{r^2}{h}\frac{h^2}{2} = \frac{1}{2}\rho\pi r^2 h$$

So $I_{zz} = \frac{1}{6}\rho\pi r^4 h\left(\frac{m}{\frac{1}{2}\rho\pi r^2 h}\right) = \frac{1}{3}mr^2$

$$k_z = \sqrt{\frac{I_{zz}}{m}} = \underline{\frac{r}{\sqrt{3}}}$$

B/34



$$z = \frac{h}{r^2} y^2$$

$$dm = \rho \pi y^2 dz$$

$$dI_{yy} = dI_{y'y'} + dm\, z^2$$

$$= \frac{1}{4} dm\, y^2 + dm\, z^2$$

$$= dm\left(\frac{1}{4}\frac{r^2}{h} z + z^2\right)$$

$$= \rho\pi \frac{r^2}{h} z\, dz\left(\frac{1}{4}\frac{r^2}{h} z + z^2\right)$$

$$= \rho\pi \frac{r^2}{h}\left(\frac{1}{4}\frac{r^2}{h} z^2 + z^3\right) dz$$

$$I_{yy} = \int dI_{yy} = \rho\pi \frac{r^2}{h}\int_0^h \left(\frac{1}{4}\frac{r^2}{h} z^2 + z^3\right) dz$$

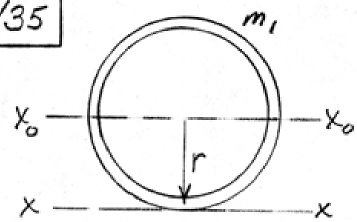
$$= \frac{\rho\pi r^2 h}{4}\left(\frac{r^2}{3} + h^2\right)$$

From solution to Prob. B/33, $m = \frac{1}{2}\rho\pi r^2 h$

So $I_{yy} = \dfrac{\rho\pi r^2 h}{4}\left(\dfrac{r^2}{3} + h^2\right)\left(\dfrac{m}{\frac{1}{2}\rho\pi r^2 h}\right)$

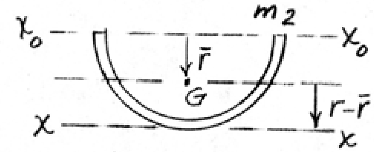
$$= \underline{\frac{1}{2} m\left(h^2 + \frac{r^2}{3}\right)}$$

B/35



Full ring:

$$I_{xx} = I_{x_0x_0} + m_1 r^2$$
$$= \frac{1}{2} m_1 r^2 + m_1 r^2 = \underline{\frac{3}{2} m_1 r^2}$$

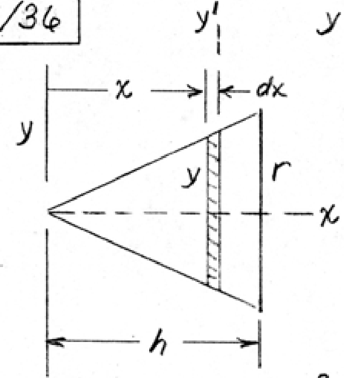


Half ring: $\bar{r} = 2r/\pi$

$$I_{x_0x_0} = \frac{1}{2}\left(\frac{1}{2}\, 2m_2 r^2\right) = \frac{1}{2} m_2 r^2$$
$$I_G = \frac{1}{2} m_2 r^2 - m_2 \bar{r}^2$$
$$= \frac{1}{2} m_2 r^2 - m_2 \frac{4r^2}{\pi^2}$$

$$I_{xx} = I_G + m_2 (r - \bar{r})^2 = \frac{1}{2} m_2 r^2 - m_2 \frac{4r^2}{\pi^2} + m_2 r^2 \left(1 - \frac{2}{\pi}\right)^2$$
$$= \frac{3}{2} m_2 r^2 + m_2 r^2 \left(1 - \frac{4}{\pi} + \frac{4}{\pi^2}\right) - m_2 r^2 \frac{4}{\pi^2}$$
$$= \underline{m_2 r^2 \left(\frac{3}{2} - \frac{4}{\pi}\right)}$$

B/36



$y = \frac{r}{h}x$;

$dI_{xx} = \frac{1}{2} dm\,(y^2) = \frac{1}{2}(\pi y^2 \rho\, dx)\, y^2$

$= \frac{\pi}{2}\rho \frac{r^4}{h^4} x^4 dx$

$I_{xx} = \frac{\pi}{2}\rho \frac{r^4}{h^4}\int_0^h x^4 dx$

$= \frac{\pi}{10}\rho \frac{r^4}{h^4} h^5 = \frac{\pi}{10}\rho h r^4$

But $m = \frac{1}{3}\pi\rho r^2 h$ so $\underline{I_{xx} = \frac{3}{10} m r^2}$

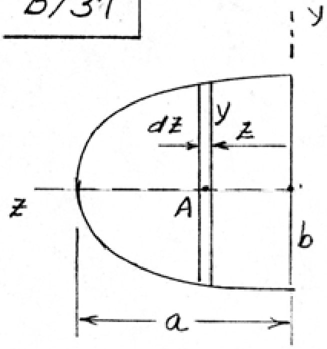
$dI_{yy} = dI_{y'y'} + x^2 dm = \frac{1}{4} dm\, y^2 + x^2 dm = (\frac{y^2}{4} + x^2) dm$

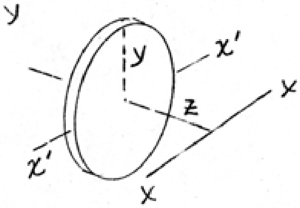
$= (\frac{1}{4}\frac{r^2}{h^2} + 1)\, x^2 \rho\pi y^2 dx = (\frac{r^2}{4h^2} + 1)\rho\pi \frac{r^2}{h^2} x^4 dx$

$I_{yy} = \frac{\rho\pi r^2}{h^2}(\frac{r^2}{4h^2} + 1)\int_0^h x^4 dx = \frac{\rho\pi r^2 h^3}{5}(\frac{r^2}{4h^2} + 1)$

$\underline{I_{yy} = \frac{3}{5} m (\frac{r^2}{4} + h^2)}$

B/37





$$dI_{x'x'} = \frac{1}{4} dm\, y^2$$
$$= \frac{1}{4}\rho\pi y^2 dz\, y^2$$
$$= \frac{1}{4}\rho\pi y^4 dz$$

$$\frac{y^2}{b^2} + \frac{z^2}{a^2} = 1, \quad \text{so } y^2 = b^2\left(1 - \frac{z^2}{a^2}\right)$$

Thus $dI_{xx} = dI_{x'x'} + z^2 dm$

$$dI_{xx} = \frac{1}{4}\rho\pi y^4 dz + (\rho\pi y^2 dz) z^2$$
$$= \rho\pi\left(\frac{1}{4} b^4\left[1 - \frac{z^2}{a^2}\right]^2 + b^2 z^2\left[1 - \frac{z^2}{a^2}\right]\right) dz$$
$$I_{xx} = \int_0^a \rho\pi b^2\left[\frac{b^2}{4} + \left(1 - \frac{b^2}{2a^2}\right) z^2 + \left(\frac{b^2}{4a^2} - 1\right)\frac{z^4}{a^2}\right] dz$$
$$= \rho\pi b^2\left[\frac{ab^2}{4} + \frac{a^3}{3}\left(1 - \frac{b^2}{2a^2}\right) + \frac{a^3}{5}\left(\frac{b^2}{4a^2} - 1\right)\right] = \frac{2}{15}\rho\pi a b^2 (a^2 + b^2)$$

But $m = \int \rho\, dV = \rho\int \pi y^2 dz = \rho\pi\int_0^a b^2\left(1 - \frac{z^2}{a^2}\right) dz = \frac{2}{3}\rho\pi b^2 a$

So $\underline{I_{xx} = \frac{1}{5} m (a^2 + b^2)}$

B/38 Axis 1-1; Shell; $I_{1-1} = mr^2 = \rho\, 2\pi r \ell (r^2) = 2\pi \rho r^3 \ell$

where ρ = mass/unit area

Panels; $I_{1-1} = 2\left\{\frac{1}{12} m (2r)^2 + m(2r)^2\right\}$

$= 8\rho(2r\ell)\left\{\frac{r^2}{12} + r^2\right\} = \frac{52}{3}\rho r^3 \ell$

Total $I_{1-1} = \rho r^3 \ell (2\pi + 52/3) = 23.62 \rho r^3 \ell$

Axis 2-2; Shell; $I_{2-2} = \frac{1}{2} m (r^2 + \frac{1}{6}\ell^2) = \frac{1}{2}\rho(2\pi r\ell)(r^2 + \ell^2/6)$

Panels; $I_{2-2} = 2\left\{\frac{1}{12} m \ell^2\right\} = \frac{1}{6}\rho(2r\ell)\ell^2 = \frac{1}{3}\rho r \ell^3$

Total $I_{2-2} = \rho r \ell (\pi [r^2 + \ell^2/6] + \frac{1}{3}\ell^2) = \rho r \ell (\ell^2[\frac{\pi}{6} + \frac{1}{3}] + \pi r^2)$

For critical condition $I_{1-1} = I_{2-2}$

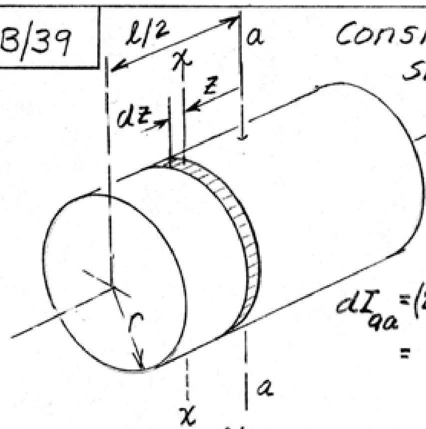
so $23.62 \rho r^3 \ell = \rho r \ell (0.857 \ell^2 + \pi r^2)$

$0.857 \ell^2 = (23.62 - \pi) r^2$

$\ell^2 = 23.89 r^2$, $\ell = 4.89 r$

$I_{1-1} < I_{2-2}$ if $\underline{\ell > 4.89 r}$

B/39



Consider complete cylindrical shell with mass ρ per unit area

$$dI_{xx} = \frac{1}{2} dm\, r^2$$

$$dI_{aa} = dI_{xx} + dm\,(z^2)$$

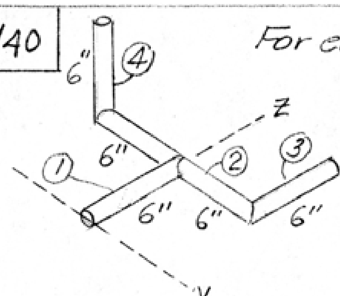
$$dI_{aa} = (2\pi r \rho\, dz)\left(\frac{1}{2} r^2 + z^2\right) = \pi \rho r (r^2 + 2z^2)\, dz$$

$$I_{aa} = \pi \rho r \int_{-l/2}^{l/2} (r^2 + 2z^2)\, dz = \pi \rho r \left[r^2 z + \frac{2z^3}{3} \right]_{-l/2}^{l/2}$$

$$= 2\pi \rho r l \left(\frac{r^2}{2} + \frac{l^2}{12} \right)$$

For half shell of mass $m = \pi \rho r l$, $\underline{I_{aa} = \frac{m}{2}\left(r^2 + \frac{l^2}{6}\right)}$

B/40



For each 6" segment

$m = \frac{0.30}{32.2} = 0.00932$ lb-ft^{-1}-sec^2

Let b = 6-in. length

① $I_{yy} = \frac{1}{3} m l^2 = \frac{1}{3} m b^2$

② $I_{yy} = (2m) b^2 = 2mb^2$

③ $I_{yy} = \frac{1}{12} m b^2 + m\left(b + \frac{b}{2}\right)^2 = \frac{7}{3} m b^2$

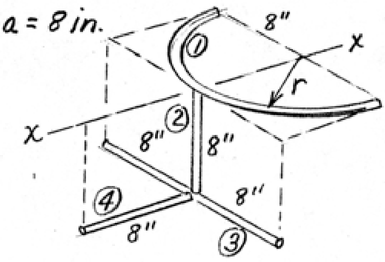
④ $I_{yy} = \frac{1}{12} m b^2 + m\left(b^2 + \left[\frac{b}{2}\right]^2\right) = \frac{4}{3} m b^2$

Thus $I_{yy} = mb^2\left(\frac{1}{3} + 2 + \frac{7}{3} + \frac{4}{3}\right) = 6mb^2 = 6(0.00932)\left(\frac{6}{12}\right)^2$

$= \underline{0.01398 \text{ lb-ft-sec}^2}$

B/41 μ = weight/in. = $\frac{0.667}{12}$ = 0.0556 lb/in.

$r = a = 8$ in.



mass per unit length is

$$\rho = \frac{0.0556}{32.2(12)} = 1.438(10^{-4})\ \frac{\text{lb-sec}^2}{\text{in.}^2}$$

$m_1 = \pi r \rho = \pi a \rho$

$m_2 = a\rho$

$m_3 = 2a\rho$

$m_4 = a\rho$

① $I_{xx} = \frac{1}{2} m_1 r^2 = \frac{1}{2}\pi a \rho r^2 = \frac{1}{2}\pi \rho a^3$

② $I_{xx} = \frac{1}{3} m_2 a^2 = \frac{1}{3}\rho a^3$

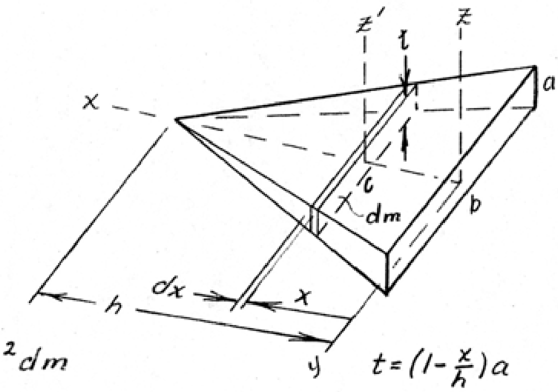
③ $I_{xx} = \frac{1}{12} m_3 (2a)^2 + m_3 a^2 = \frac{4}{3}(2a\rho)a^2 = \frac{8}{3}\rho a^3$

④ $I_{xx} = m_4 a^2 = \rho a^3$

Total $I_{xx} = \rho a^3\left(\frac{\pi}{2} + \frac{1}{3} + \frac{8}{3} + 1\right) = 5.57\,\rho a^3$

$= 5.57(1.438)(10^{-4})(8^3) = \underline{0.410\ \text{lb-in.-sec}^2}$

B/42



For small a, $I_{yy} = \int x^2 dm$

$t = (1-\frac{x}{h})a$

$c = (1-\frac{x}{h})b$

$$dm = \rho dV = \rho t c dx = \rho(1-\frac{x}{h})a(1-\frac{x}{h})b\,dx$$
$$= \rho ab(1-\frac{x}{h})^2 dx, \quad \rho = \text{density}$$

$$m = \rho ab\int_0^h (1-\frac{x}{h})^2 dx = \rho ab\left(x - \frac{x^2}{h} + \frac{x^3}{3h^2}\right)_0^h = \frac{1}{3}\rho abh$$

$$I_{yy} = \rho ab\int_0^h (1-\frac{x}{h})^2 x^2 dx = \rho ab\left(\frac{x^3}{3} - \frac{2x^4}{4h} + \frac{x^5}{5h^2}\right)_0^h = \frac{1}{30}\rho abh^3,$$

$$\underline{I_{yy} = \frac{1}{10}mh^2}$$

B/43 From the figure in the solution for Prob. B/42 & the expression for dm, $\rho ab(1-\frac{x}{h})^2 dx$, the moment of inertia of dm about the z-axis is $dI_{zz} = dI_{z'z'} + x^2 dm$ by the transfer-of-axis theorem. Also, from the results of Prob. B/42 or Table D/4

$$dI_{z'z'} = \frac{1}{12} dm\, c^2 = \frac{1}{12} dm \left(1-\frac{x}{h}\right)^2 b^2$$

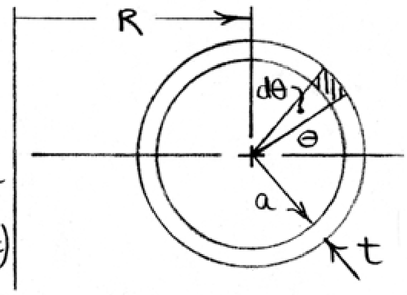
so $dI_{zz} = \left[\frac{1}{12} b^2 \left(1-\frac{x}{h}\right)^2 + x^2\right] dm = \rho ab\left[\frac{1}{12} b^2\left(1-\frac{x}{h}\right)^4 + x^2\left(1-\frac{x}{h}\right)^2\right] dx$

$$I_{zz} = \rho ab \int_0^h \left[\frac{1}{12} b^2 \left(1-\frac{x}{h}\right)^4 + x^2\left(1-\frac{x}{h}\right)^2\right] dx$$

$$= \rho ab\left[\frac{b^2 h}{60} + \frac{h^3}{30}\right] = \frac{\rho abh}{30}\left(\frac{b^2}{2} + h^2\right)$$

& from solution to Prob. B/42, $m = \frac{1}{3}\rho abh$, so $\underline{I_{zz} = \frac{1}{10} m\left(\frac{b^2}{2} + h^2\right)}$

B/44



Let ρ = mass per unit surface area. For elemental ring of cross section $a\,d\theta(t)$ and circumference $2\pi(R + a\cos\theta)$:

$$dm = \rho\,(a\,d\theta)\,2\pi\,(R + a\cos\theta)$$

$$dI = (R + a\cos\theta)^2\,dm = 2\pi\rho\,(R + a\cos\theta)^3\,a\,d\theta$$

$$\text{So } m = 2\pi\rho a\int_0^{2\pi}(R + a\cos\theta)\,d\theta = 4\pi^2\rho a R$$

$$I = 2\pi\rho a\int_0^{2\pi}(R^3 + 3R^2 a\cos\theta + 3Ra^2\cos^2\theta + a^3\cos^3\theta)\,d\theta$$

$$= 2\pi\rho a\,[\,①+②+③+④\,]$$

$$① = \int_0^{2\pi} R^3\,d\theta = 2\pi R^3$$

$$② = 3R^2 a\int_0^{2\pi}\cos\theta\,d\theta = 0$$

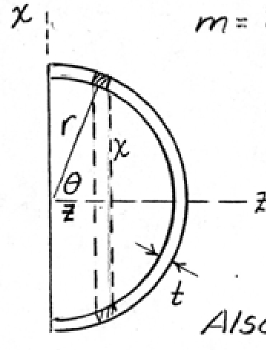
$$③ = 3Ra^2\int_0^{2\pi}\cos^2\theta = 3Ra^2\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{2\pi} = 3\pi R a^2$$

$$④ = a^3\int_0^{2\pi}\cos^3\theta\,d\theta = \frac{a^3}{3}\left[\sin\theta\,(\cos^2\theta + 2)\right]_0^{2\pi} = 0$$

$$\text{So } I = 2\pi^2\rho a R\,(2R^2 + 3a^2)\left(\frac{m}{4\pi^2\rho a R}\right)$$

$$= \underline{\tfrac{1}{2}m\,(2R^2 + 3a^2)}$$

B/45



$$dm = \rho\, dV = \rho t\, 2\pi x\, r\, d\theta = 2\pi\rho t r^2 \sin\theta\, d\theta$$

$$m = 2\pi\rho t r^2 \int_0^{\pi/2} \sin\theta\, d\theta = 2\pi\rho t r^2$$

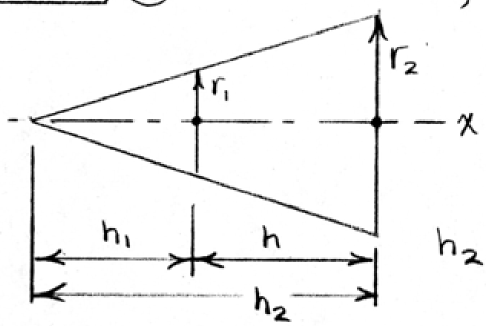
$$I_{zz} = \int x^2 dm = 2\pi\rho t r^2 \int_0^{\pi/2} r^2 \sin^3\theta\, d\theta$$

$$= 2\pi\rho t r^4 \left[-\frac{\cos\theta}{3}(2 + \sin^2\theta) \right]_0^{\pi/2}$$

$$= \frac{4}{3}\pi\rho t r^4 = \underline{\frac{2}{3} m r^2}$$

Also, $\underline{I_{xx} = I_{zz} = \frac{2}{3} m r^2}$ since each is half that for whole shell of mass $2m$

B/46 ② : Entire cone ; ① : "Missing" top



$$\frac{h_1}{r_1} = \frac{h_1+h}{r_2}$$

$$h_1 = \frac{hr_1}{r_2-r_1}$$

$$h_2 = h_1+h = \frac{hr_1}{r_2-r_1} + h = \frac{hr_2}{r_2-r_1}$$

$$I_2 = \frac{3}{10} m_2 r_2^2 = \frac{3}{10}\left(\rho \frac{1}{3}\pi r_2^2 \frac{hr_2}{r_2-r_1}\right) r_2^2 \quad \left(\text{See Table D/4}\right)$$

$$= \frac{1}{10}\rho\pi \frac{hr_2^5}{r_2-r_1}$$

$$I_1 = \frac{3}{10} m_1 r_1^2 = \frac{3}{10}\left(\rho \frac{1}{3}\pi r_1^2 \frac{hr_1}{r_2-r_1}\right) r_1^2$$

$$= \frac{1}{10}\rho\pi \frac{hr_1^5}{r_2-r_1}$$

Frustum mass $m = \rho\frac{1}{3}\pi\left[r_2^2 \frac{hr_2}{r_2-r_1} - r_1^2 \frac{hr_1}{r_2-r_1}\right]$

$$= \frac{1}{3}\rho\pi h \frac{r_2^3-r_1^3}{r_2-r_1}$$

So $I = I_2 - I_1 = \frac{1}{10}\rho\pi h \frac{r_2^5-r_1^5}{r_2-r_1}\left(\frac{m}{\frac{1}{3}\rho\pi h \frac{r_2^3-r_1^3}{r_2-r_1}}\right)$

$$= \underline{\frac{3}{10} m \frac{r_2^5-r_1^5}{r_2^3-r_1^3}}$$

*B/47 Let ρ = mass per unit area

Panels: $\begin{cases} I_{xx} = 2\left\{\frac{1}{12}m(2r)^2 + m(2r)^2\right\} = \frac{104}{3}\rho r^4 \\ I_{yy} = 2\left\{\frac{1}{12}m(2r)^2\right\} = \frac{2}{3}mr^2 = \frac{8}{3}\rho r^4 \\ I_{zz} = 2\left\{\frac{1}{6}m(2r)^2 + m(2r)^2\right\} = \frac{28}{3}mr^2 = \frac{112}{3}\rho r^4 \end{cases}$

($m = \rho(2r)^2$ = mass of each panel

Cylindrical shell (see Table D/4):

$$\begin{cases} I_{xx} = I_{yy} = \frac{1}{2}mr^2 + \frac{1}{12}mL^2 = \frac{m}{2}\left(r^2 + \frac{L^2}{6}\right) \\ \qquad\qquad = \pi r L \rho\left(r^2 + \frac{L^2}{6}\right) \\ I_{zz} = mr^2 = 2\pi r^3 L\rho \end{cases}$$

Complete model:

$$\begin{cases} I_{xx} = \frac{104}{3}\rho r^4 + \pi r L\rho\left(r^2 + \frac{L^2}{6}\right) \\ I_{yy} = \frac{8}{3}\rho r^4 + \pi r L\rho\left(r^2 + \frac{L^2}{6}\right) \\ I_{zz} = \frac{112}{3}\rho r^4 + 2\pi r^3 L\rho \end{cases}$$

Since $I_{yy} < I_{xx}$, I_{zz} must be less than I_{yy}:

$$\left(\frac{112}{3}\rho r^4 + 2\pi r^3 L\rho\right) < \left(\frac{8}{3}\rho r^4 + \pi r L\rho\left(r^2 + \frac{L^2}{6}\right)\right)$$

or $\frac{\pi}{6}\left(\frac{L}{r}\right)^3 - \pi\frac{L}{r} - \frac{104}{3} > 0$

Solve cubic for value of $\frac{L}{r}$ making left side zero to obtain $\frac{L}{r} = 4.54$

Inequality is satisfied if $\underline{L > 4.54r}$

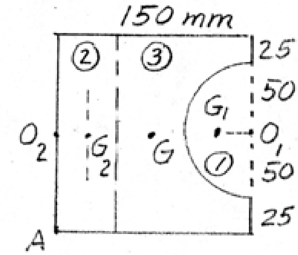
▶8/48 Groove ① $I_{aa} = \bar{I} + m(\overline{G_1A}^2)$

$= I_{O_1O_1} - m(\overline{G_1O_1}^2) + m(\overline{G_1A}^2)$

$= m(\frac{1}{2}r^2 - \overline{G_1O_1}^2 + \overline{G_1A}^2)$

$= (11\,370)\frac{\pi(0.05)^2(0.15)}{2}\left[\frac{\overline{0.05}^2}{2} - \overline{0.0212}^2 + 0.0222\right]$

$= 0.1541\ kg \cdot m^2$ (negative)



(a-a)

$\overline{G_1O_1} = \overline{G_2O_2} = \frac{4(0.05)}{3\pi}$

$= 0.0212\ m$

$\overline{G_1A}^2 = (0.15 - 0.0212)^2 + (0.075)^2$

$= 0.0222\ m^2$

Groove ② $I_{aa} = \frac{1}{2}$ that for complete cyl. by symmetry

So from Table D/4, $I_{aa} = \frac{1}{2}\frac{2m}{12}(3r^2 + 4l^2)$

$= \frac{(11\,370)}{2}\frac{\pi(0.05)^2(0.15)}{12}\left[3(0.05)^2 + 4(0.15)^2\right]$

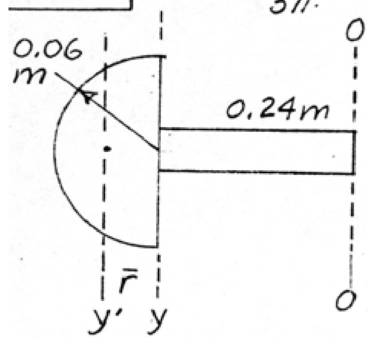
$= 0.0544\ kg \cdot m^2$ (negative)

③ $I_{aa} = \frac{1}{12}m(a^2 + a^2) + m(\frac{a^2}{4} + \frac{a^2}{4}) = \frac{2}{3}ma^2$

$= \frac{2}{3}(11\,370)(0.15)^3(0.15)^2 = 0.576\ kg \cdot m^2$

Total $I_{aa} = 0.576 - 0.0544 - 0.1541 = \underline{0.367\ kg \cdot m^2}$

▸ B/49 $\bar{r} = \frac{4r}{3\pi} = \frac{4(0.06)}{3\pi} = 0.0255\ m$



From Table D/4 for semicylinder

$I_{yy} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$

$= \frac{1}{4}0.8(0.06)^2 + \frac{1}{12}0.8(0.120)^2$

$= 7.20(10^{-4}) + 9.60(10^{-4})$

$= 16.80(10^{-4})\ kg \cdot m^2$

$I_{y'y'} = I_{yy} - m\bar{r}^2 =$

$= 16.80(10^{-4}) - 0.8(0.0255)^2$

$= 11.61(10^{-4})\ kg \cdot m^2$

$I_{oo} = I_{y'y'} + md^2 = 11.61(10^{-4}) + 0.8(0.24 + 0.0255)^2$

$= 0.0575\ kg \cdot m^2$

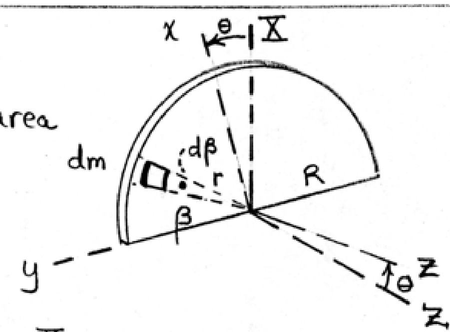
Handle: $I_{oo} \approx \frac{1}{3}ml^2 = \frac{1}{3}0.5(0.24)^2 = 0.0096\ kg \cdot m^2$

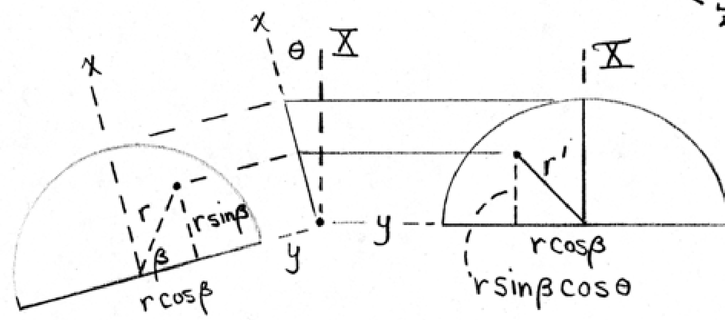
Total $I_{oo} = 0.0575 + 0.0096 = \underline{0.0671\ \ kg \cdot m^2}$

▶ B/50

Let ρ = mass per unit area

$dm = \rho dA = \rho r\,dr\,d\beta$





r' = distance from dm to Z-axis

$$r'^2 = r^2(\cos^2\theta \sin^2\beta + \cos^2\beta)$$

$$dI_{ZZ} = r'^2\,dm = \rho r^3(\cos^2\theta \sin^2\beta + \cos^2\beta)\,dr\,d\beta$$

$$I_{ZZ} = \rho \frac{R^4}{4}\int_0^{\pi} (\cos^2\theta \sin^2\beta + \cos^2\beta)\,d\beta$$

$$= \rho \frac{R^4}{4}\left[\frac{\pi}{2}(1+\cos^2\theta)\right] \cdot \frac{m}{\rho\pi R^2/2}$$

$$= \underline{\frac{1}{4}mR^2(1+\cos^2\theta)}$$

B/51

$I_{xy} = \underline{0}$

$I_{xz} = m(-l)(2l) = \underline{-2ml^2}$

$I_{yz} = m(l)(-l) + m(-l)(l) = \underline{-2ml^2}$

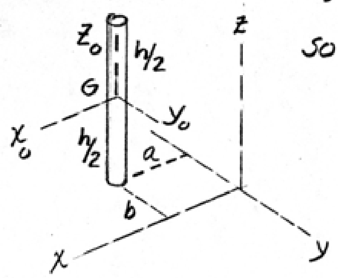
B/52

$I_{xy} = m(l)(-l) + m(-l)(l) = \underline{-2ml^2}$

$I_{xz} = m(2l)(-l) + m(-2l)(l) = \underline{-4ml^2}$

$I_{yz} = \underline{0}$

B/53



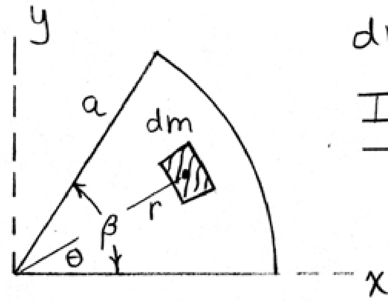
$\bar{I}_{xy} = \bar{I}_{yz} = \bar{I}_{xz} = 0$

so $I_{xy} = 0 + ma(-b) = \underline{-mab}$

$I_{yz} = 0 + m(-b)\frac{h}{2} = \underline{-\frac{1}{2}mbh}$

$I_{xz} = 0 + ma\frac{h}{2} = \underline{\frac{1}{2}mah}$

B/54



$dm = \rho t r\,dr\,d\theta$

$\underline{I_{xz} = I_{yz} = 0}$

$$I_{xy} = \int xy\,dm = \int_0^\beta\int_0^a (r\cos\theta)(r\sin\theta)\rho t\,r\,dr\,d\theta$$

$$= \rho t \int_0^\beta\int_0^a r^3 \cos\theta \sin\theta\,dr\,d\theta$$

$$= \rho t \int_0^\beta \frac{a^4}{4} \cos\theta \sin\theta\,d\theta$$

$$= \frac{\rho t a^4}{4}\,\frac{\sin^2\beta}{2} = \frac{\rho t a^4 \sin^2\beta}{8}\left(\frac{m}{\rho t \beta a^2/2}\right)$$

$$= \underline{\underline{\frac{m a^2 \sin^2\beta}{4\beta}}}$$

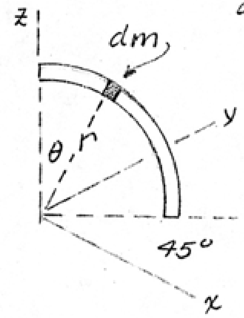
$\boxed{B/55}$ $I_{xy} = -\left(\frac{b}{4}\right)\left(\frac{b}{4}\right)\left(\rho\pi\left(\frac{b}{8}\right)^2\right)$

$$-\left(-\frac{b}{4}\right)\left(-\frac{b}{4}\right)\left(\rho\pi\left(\frac{b}{8}\right)^2\right) = -\underline{\frac{\rho\pi b^4}{512}}$$

$\underline{I_{xz} = I_{yz} = 0}$

B/56 Let ρ = mass per unit length of rod

$$dm = \rho r \, d\theta$$



$$I_{xy} = \int_0^{\pi/2} (r \sin\theta \cos 45^\circ)^2 \rho r \, d\theta$$

$$= \rho r^3 \frac{1}{2} \int_0^{\pi/2} \sin^2\theta \, d\theta = \frac{1}{8} \pi \rho r^3$$

$$= \underline{\frac{1}{4} m r^2}$$

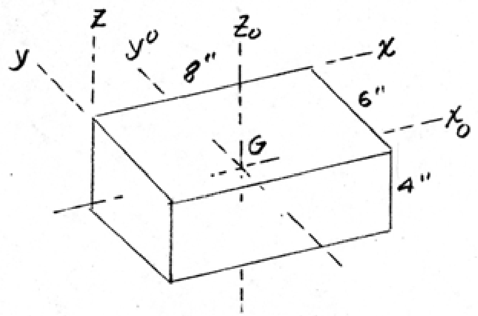
$$I_{xz} = I_{yz} = \int yz \, dm = \int_0^{\pi/2} (r \sin\theta \sin 45^\circ)(r \cos\theta) \rho r \, d\theta$$

$$= \rho r^3 \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta = \frac{\rho r^3}{\sqrt{2}} \left[\frac{-\cos 2\theta}{4} \right]_0^{\pi/2}$$

$$= \frac{1}{2\sqrt{2}} \rho r^3 = \underline{\frac{1}{\pi\sqrt{2}} m r^2}$$

B/57

$m = \frac{50}{32.2}\frac{1}{12} = 0.1294$ lb-in^{-1}-sec^2

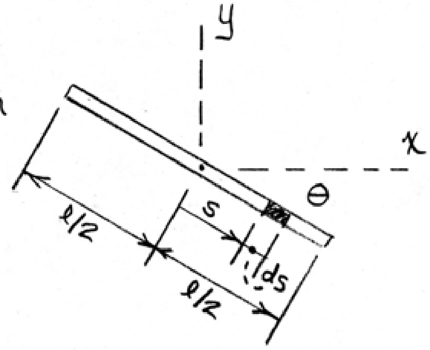


$I_{xy} = I_{x_0y_0} + m d_x d_y = 0 + 0.1294(4)(-3) = \underline{-1.553 \text{ lb-in.-sec}^2}$

$I_{yz} = I_{y_0z_0} + m d_y d_z = 0 + 0.1294(-3)(-2) = \underline{0.776 \text{ lb-in.-sec}^2}$

$I_{xz} = I_{x_0z_0} + m d_x d_z = 0 + 0.1294(4)(-2) = \underline{-1.035 \text{ lb-in.-sec}^2}$

B/58



Let ρ = mass/unit length

$dm = \rho\, ds$; $m = \rho l$

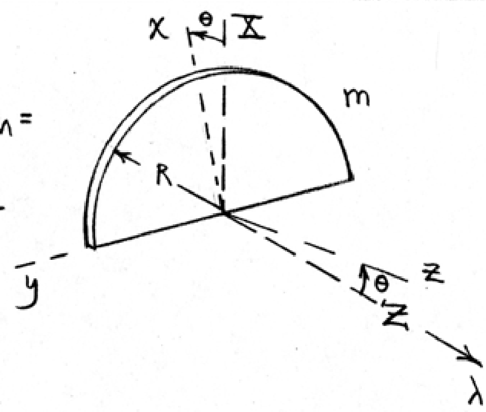
$dI_{xy} = xy\, dm$

$= (s\cos\theta)(-s\sin\theta)\rho\, ds$

$= -\frac{1}{2}\rho\sin 2\theta\, s^2 ds$

$I_{xy} = -\frac{1}{2}\rho\sin 2\theta \int_{-l/2}^{l/2} s^2 ds = \underline{-\frac{1}{24} m l^2 \sin 2\theta}$

B/59



Use Eq. B/10: $I_{ZZ} = I_M =$

$I_{xx} l^2 + I_{yy} m^2 + I_{zz} n^2$

$-2I_{xy} lm - 2I_{xz} ln$

$-2I_{yz} mn$

Use the unit vector $\underline{\lambda} = l\underline{i} + m\underline{j} + n\underline{k}$

with $l = -\sin\theta$, $m = 0$, $n = \cos\theta$

$I_{xx} = \frac{1}{4}mR^2$, $I_{yy} = \frac{1}{4}mR^2$, $I_{zz} = \frac{1}{2}mR^2$

$I_{xy} = I_{xz} = I_{yz} = 0$

So $I_{ZZ} = \frac{1}{4}mR^2(\sin^2\theta) + \frac{1}{4}mR^2(0) + \frac{1}{2}mR^2(\cos^2\theta)$

$= \frac{1}{4}mR^2(\sin^2\theta + 2\cos^2\theta)$

$= \underline{\frac{1}{4}mR^2(1 + \cos^2\theta)}$

B/60

Part 1: $m_1 = m/4$

$$I_{xy} = \frac{m}{4}\left(\frac{b}{2}\right)(-b) = -\frac{1}{8}mb^2$$

$$I_{yz} = 0, \ I_{xz} = 0$$

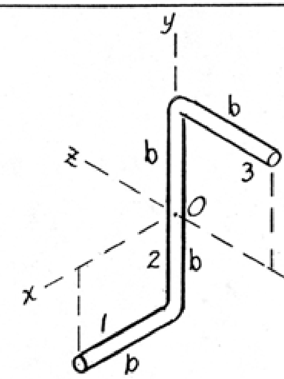
Part 2: $m_2 = m/2$

$$I_{xy} = 0, I_{yz} = 0, I_{xz} = 0$$

Part 3: $m_3 = m/4$

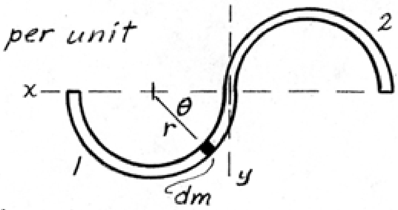
$$I_{xy} = 0, I_{xz} = 0$$

$$I_{yz} = \frac{m}{4}(b)\left(-\frac{b}{2}\right) = -\frac{1}{8}mb^2$$



Combined: $\underline{I_{xy} = -\frac{1}{8}mb^2}$, $\underline{I_{xz} = 0}$, $\underline{I_{yz} = -\frac{1}{8}mb^2}$

B/61 $I_{xy_1} = I_{xy_2}$; Let ρ = mass per unit length



$m_1 = \pi r\rho$, $m = 2\pi r\rho$

$$I_{xy_1} = \int xy\,dm = \int_0^{\pi} r(1-\cos\theta)(r\sin\theta)\rho r\,d\theta$$

$$= \rho r^3 \int_0^{\pi} (\sin\theta - \sin\theta\cos\theta)\,d\theta = \rho r^3\left[-\cos\theta - \tfrac{1}{2}\sin^2\theta\right]_0^{\pi} = 2\rho r^3$$

$$I_{xy} = 2I_{xy_1} = 4\rho r^3 = \underline{2mr^2/\pi}, \quad \underline{I_{xz} = I_{yz} = 0}$$

B/62

$I_{xy} = \int xy\,dm$

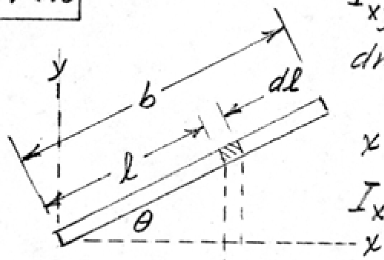
$dm = \rho h\,d\ell$ where ρ = mass per unit of plate area

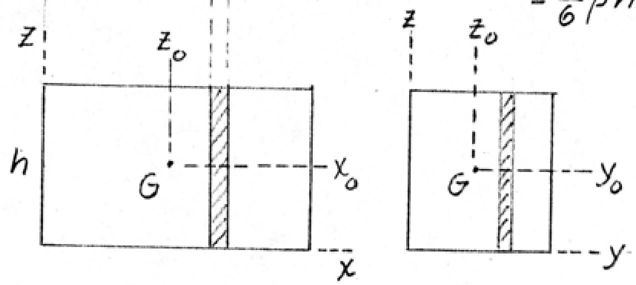
$x = \ell\cos\theta,\ y = \ell\sin\theta$

$I_{xy} = \rho h \sin\theta\cos\theta \int_0^b \ell^2 d\ell$

$= \frac{1}{6}\rho h b^3 \sin 2\theta$

$= \underline{\frac{1}{6} m b^2 \sin 2\theta}$





$I_{xz} = \bar{I}_{xz} + m d_x d_z = 0 + m\left(\frac{b}{2}\cos\theta\right)\left(\frac{h}{2}\right) = \underline{\frac{1}{4} m b h \cos\theta}$

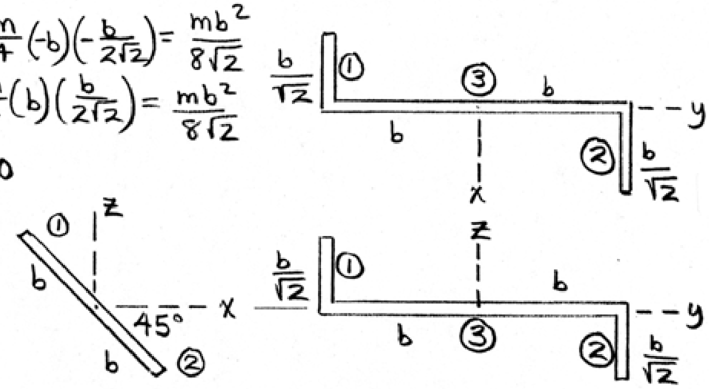
$I_{yz} = \bar{I}_{yz} + m d_y d_z = 0 + m\left(\frac{h}{2}\right)\left(\frac{b}{2}\sin\theta\right) = \underline{\frac{1}{4} m b h \sin\theta}$

B/63

Part ①: $I_{xy} = \frac{m}{4}(-b)\left(-\frac{b}{2\sqrt{2}}\right) = \frac{mb^2}{8\sqrt{2}}$

②: $I_{xy} = \frac{m}{4}(b)\left(\frac{b}{2\sqrt{2}}\right) = \frac{mb^2}{8\sqrt{2}}$

③: $I_{xy} = 0$



For Parts ① & ② of combined mass $m/2$, it can be shown via integration that $I_{xz} = -\frac{1}{12}mb^2$.

Part ③: $I_{xz} = 0$

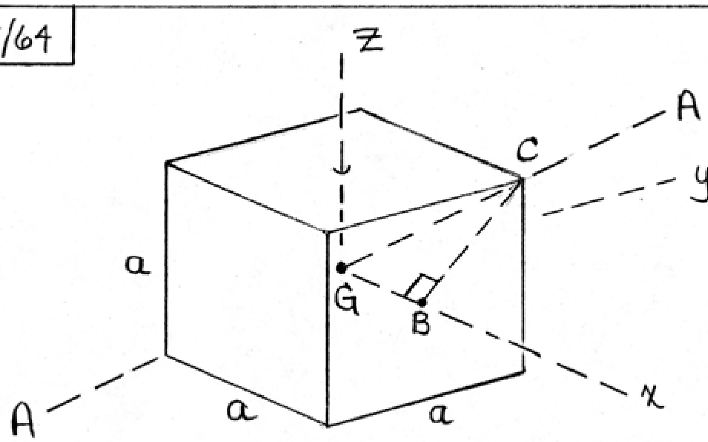
Part ①: $I_{yz} = \frac{m}{4}(-b)\left(\frac{b}{2\sqrt{2}}\right) = -\frac{mb^2}{8\sqrt{2}}$

②: $I_{yz} = \frac{m}{4}(b)\left(-\frac{b}{2\sqrt{2}}\right) = -\frac{mb^2}{8\sqrt{2}}$

③: $I_{yz} = 0$

Totals: $\begin{cases} I_{xy} = \dfrac{mb^2}{4\sqrt{2}} \\ I_{xz} = -\dfrac{1}{12}mb^2 \\ I_{yz} = -\dfrac{mb^2}{4\sqrt{2}} \end{cases}$

B/64



Choose origin of x-y-z axes at center G

By symmetry, the direction cosines of AA

are $l = m = n = \cos \measuredangle BGC$

$\overline{GB} = \frac{a}{2}$, $\overline{BC} = \frac{a\sqrt{2}}{2}$, $\overline{GC} = \frac{a}{2}\sqrt{3}$, so $l = m = n = \frac{1}{\sqrt{3}}$

$I_{xx} = I_{yy} = I_{zz} = \frac{1}{6} ma^2$, $I_{xy} = I_{xz} = I_{yz} = 0$

From Eq. B/10: $I_{AA} = 3\left(\frac{1}{6} ma^2\right)\left(\frac{1}{\sqrt{3}}\right)^2$

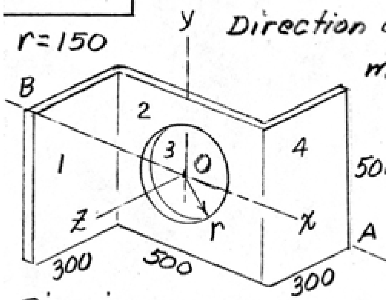
$$= \underline{\frac{ma^2}{6}}$$

B/65 $\overline{OB} = \sqrt{(250)^2 + (250)^2 + (300)^2} = 464 \text{ mm}$

$r = 150$

Direction cosines of OB are $l = -\frac{250}{464} = -0.539$

$m = \frac{250}{464} = 0.539, \quad n = \frac{300}{464} = 0.647$



Masses are

$m_1 = m_4 = 7830(0.3)(0.5)(0.015) = 17.62 \text{ kg}$

$m_3 = -7830(\pi)(0.150)^2(0.015) = -8.30 \text{ kg}$

$m_2 = 7830(0.5)^2(0.015) = 29.36 \text{ kg}$

Dim. in mm

	1	2	3	4	Totals ($kg \cdot m^2$)
I_{xx}	0.896	0.612	−0.047	0.896	2.356
I_{yy}	1.630	0.612	−0.047	1.630	3.825
I_{zz}	1.468	1.223	−0.093	1.468	4.067
I_{xz}	−0.661	0	0	−0.661	−1.321

Substitute into Eq. B/10 & get

$$I_{AB} = 2.356(-0.539)^2 + 3.825(0.539)^2 + 4.067(0.647)^2 - 2(-1.321)(-0.539)(0.647) \quad \text{where } I_{xy} = I_{yz} = 0$$

$$I_{AB} = 2.58 \text{ kg} \cdot \text{m}^2$$

B/66 $I_{xx} = I_{yy} = I_{zz} = 2\left(\frac{2}{5}mr^2 + mb^2\right) + \frac{2}{5}mr^2$

$$= m\left(\frac{6}{5}r^2 + 2b^2\right) = I$$

$I_{xy} = I_{yz} = I_{xz} = 0$

Thus for any axis OM through O, Eq. B/10 gives $I_M = I(l^2 + m^2 + n^2) = I$ independent of l, m, n.

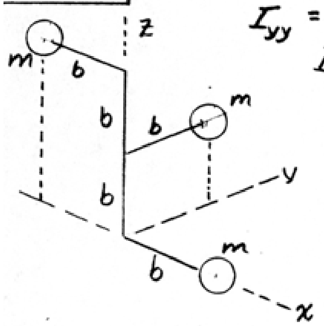
*** B/67**

$I_{xx} = m(2b^2) + m(2b)^2 = 6mb^2$

$I_{yy} = m(b^2 + b^2 + b^2 + (2b)^2) = 7mb^2$

$I_{zz} = m(b^2 + b^2 + b^2) = 3mb^2$

$I_{xy} = 0, \; I_{yz} = mb^2, \; I_{xz} = -2mb^2$



Let $I_o = I/mb^2$ so Eq. B/11 becomes

$$\begin{vmatrix} 6-I_o & 0 & +2 \\ 0 & 7-I_o & -1 \\ +2 & -1 & 3-I_o \end{vmatrix} mb^2 = 0$$

Expansion gives $I_o^3 - 16I_o^2 + 86I_o - 92 = 0$

Solution by computer or algebraic formula gives

$I_1 = 7.53\ mb^2$

$I_2 = 6.63\ mb^2$

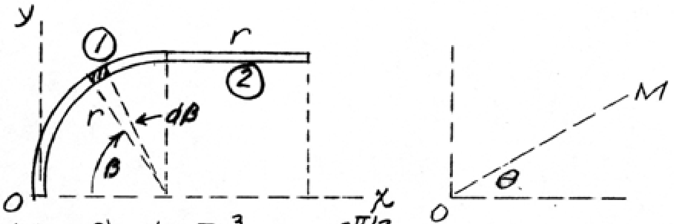
$\underline{I_3 = 1.844\ mb^2}$

For I_1 direction cosines substitute in Eq. B/12 & get

$$\left.\begin{aligned} (6-7.525)l_1 - 0 + 2n_1 &= 0 \\ 0 + (7-7.525)m_1 - n_1 &= 0 \\ 2l_1 - m_1 + (3-7.525)n_1 &= 0 \end{aligned}\right\} \text{with } l_1^2 + m_1^2 + n_1^2 = 1$$

solve & get $\underline{l_1 = 0.521,\ m_1 = -0.756,\ n_1 = 0.397}$

***B/68**



① $I_x = \frac{1}{4}\frac{1}{2}(4mr^2) = \frac{1}{4}\rho\pi r^3$, $I_y = \int_0^{\pi/2}[r(1-\cos\beta)]^2 \rho r\,d\beta = \rho r^3(\frac{3\pi}{4} - 2)$

$I_{xy} = \int_0^{\pi/2} r(1-\cos\beta)\, r\sin\beta\, \rho r\,d\beta = \rho r^3(-\cos\beta - \frac{1}{2}\sin^2\beta)_0^{\pi/2} = \frac{1}{2}\rho r^3$

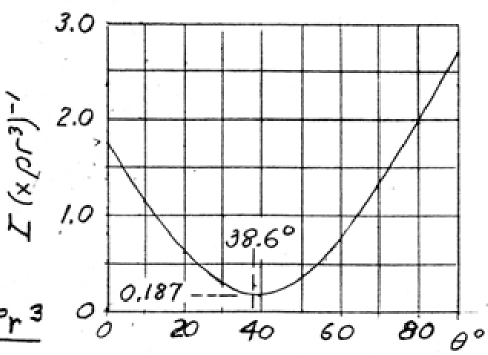
② $I_x = \rho r^3$, $I_y = \rho r(\frac{r^2}{12} + [\frac{3r}{2}]^2) = \frac{7}{3}\rho r^3$, $I_{xy} = \rho r(\frac{3r}{2})r = \frac{3}{2}\rho r^3$

Totals, $I_x = \rho r^3(\frac{\pi}{4} + 1) = 1.785\rho r^3$, $I_y = \rho r^3(\frac{3\pi}{4} - 2 + \frac{7}{3}) = 2.69\rho r^3$

$I_{xy} = \rho r^3(\frac{1}{2} + \frac{3}{2}) = 2\rho r^3$

Eq. A/9, $I_{OM} = I = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$

Set up computer program & solve for $0 < \theta < 90°$



check: Eqs. A/10 & A/11

$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4}{2.69 - 1.785}$

$2\alpha = 77.3°$, $\underline{\alpha = 38.6°}$

$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$= (2.24 - 2.05)\rho r^3 = \underline{\mathbf{0.1870\,\rho r^3}}$

*B/69

$$I_{xx} = m(\sqrt{2}\ell)^2 + m(\sqrt{2}\ell)^2 + m(2\ell)^2$$
$$= 8m\ell^2$$

$$I_{yy} = m\ell^2 + m\ell^2 + m(\sqrt{5}\ell)^2 = 7m\ell^2$$

$$I_{zz} = m\ell^2 + m\ell^2 + m\ell^2 = 3m\ell^2$$

Eq. B/11, with $I_0 = \dfrac{I}{m\ell^2}$:

$$m\ell^2 \begin{vmatrix} (8-I_0) & 0 & +2 \\ 0 & (7-I_0) & +2 \\ +2 & +2 & (3-I_0) \end{vmatrix} = 0$$

Notes: $I_{xy} = 0$; $I_{xz} = I_{yz} = -2m\ell^2$ from Prob. B/51

Numerical solution of cubic:

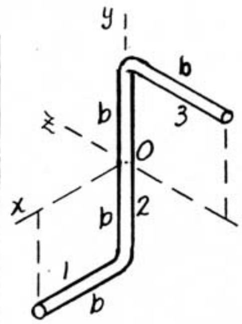
$\underline{I_1 = 9m\ell^2}$, $\underline{I_2 = 7.37\,m\ell^2}$, $\underline{I_3 = 1.628\,m\ell^2}$

For I_1, solution of Eqs. B/12 along with $l_1^2 + m_1^2 + n_1^2 = 1$ yields

$$\begin{cases} l_1 = 0.816 \\ m_1 = 0.408 \\ n_1 = 0.408 \end{cases}$$

*B/70

		Part 1	2	3	Total
Prob. B/27	I_{xx}	$\frac{1}{4}mb^2$	$\frac{1}{6}mb^2$	$\frac{1}{3}mb^2$	$\frac{3}{4}mb^2$
	I_{yy}	$\frac{1}{12}mb^2$	0	$\frac{1}{12}mb^2$	$\frac{1}{6}mb^2$
	I_{zz}	$\frac{1}{3}mb^2$	$\frac{1}{6}mb^2$	$\frac{1}{4}mb^2$	$\frac{3}{4}mb^2$
Prob. B/60	I_{xy}	$-\frac{1}{8}mb^2$	0	0	$-\frac{1}{8}mb^2$
	I_{xz}	0	0	0	0
	I_{yz}	0	0	$-\frac{1}{8}mb^2$	$-\frac{1}{8}mb^2$



Substitute in Eq. B/11 and let $I = I_0 mb^2$

$$\begin{vmatrix} \frac{3}{4}-I_0 & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{6}-I_0 & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{3}{4}-I_0 \end{vmatrix} = 0$$

Expand & get

$$I_0^3 - \frac{5}{3}I_0^2 + \frac{25}{32}I_0 - \frac{9}{128} = 0$$

Solve by computer program or by algebraic formula. (In this case expansion of the determinant yields a common factor $(\frac{3}{4}-I_0)$ so the cubic becomes

$$(\tfrac{3}{4}-I_0)([\tfrac{1}{6}-I_0][\tfrac{3}{4}-I_0]-\tfrac{1}{32})=0 \text{ or } (\tfrac{3}{4}-I_0)(I_0^2-\tfrac{11}{12}I_0+\tfrac{3}{32})=0$$

so $I_{0_1} = 0.750$ or $\underline{I_1 = 0.750\ mb^2}$

$I_{0_2} = 0.799$ $\underline{I_2 = 0.799\ mb^2}$

$I_{0_3} = 0.1173$ $\underline{I_3 = 0.1173\ mb^2}$

For $I_3 = 0.1173\ mb^2$ (minimum moment of inertia) the direction cosines satisfy Eq. B/12 & $\ell^2+m^2+n^2=1$ so

$$\left.\begin{aligned} 0.633\ell &+ 0.125m + 0 &= 0 \\ 0.125\ell &+ 0.0494m + 0.125n &= 0 \\ 0 &+ 0.125m + 0.633n &= 0 \end{aligned}\right\}$$

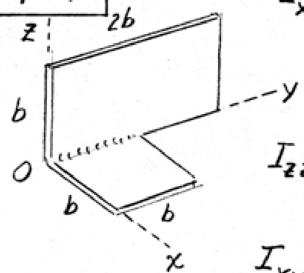
Sol. gives $\ell = m = n = 0$ and also

$\underline{\ell = 0.1903,}$

$\underline{m = -0.963,}$

$\underline{n = 0.1903}$

***B/71**



$$I_{xx} = \frac{1}{3}\rho b^4 + \frac{1}{12}\rho(2b^2)(b^2+4b^2) + \rho(2b^2)\left(\frac{b^2}{4}+b^2\right)$$
$$= \frac{11}{3}\rho b^4$$

$$I_{yy} = \frac{1}{3}\rho b^4 + \frac{1}{3}\rho(2b^2)b^2 = \rho b^4$$

$$I_{zz} = \frac{1}{12}\rho b^2(b^2+b^2) + \rho b^2\left(\frac{b^2}{4}+\frac{b^2}{4}\right) + \frac{1}{3}\rho(2b^2)(2b)^2$$
$$= \frac{10}{3}\rho b^4$$

$$I_{xy} = \rho b^2\left(\frac{b}{2}\,\frac{b}{2}\right) = \frac{1}{4}\rho b^4,\ I_{xz} = 0,\ I_{yz} = \rho b^4$$

Substitute in Eq. B/11 letting $I = I_o \rho b^4$ & get

$$\begin{vmatrix} \frac{11}{3} - I_o & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 - I_o & -1 \\ 0 & -1 & \frac{10}{3} - I_o \end{vmatrix} = 0,$$

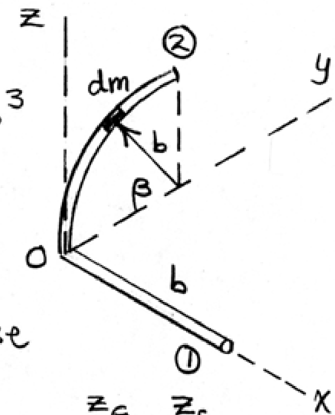
Expand & get

$$I_o^3 - 8I_o^2 + 18.160 I_o - 8.347 = 0$$

Solve by computer program or algebraic formula & get

$$\boxed{\begin{aligned} I_1 &= 3.78\rho b^4 \\ I_2 &= 0.612\rho b^4 \\ I_3 &= 3.61\rho b^4 \end{aligned}}$$

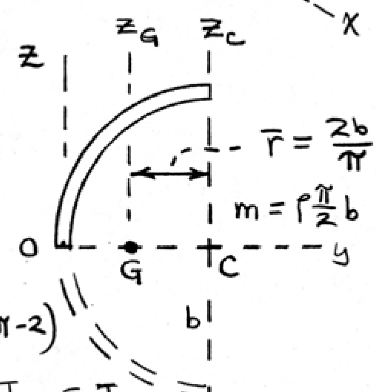
*B/72



Part ①: $I_{xx} = 0$, $I_{yy} = I_{zz} = \frac{1}{3}\rho b^3$

$I_{xy} = I_{xz} = I_{yz} = 0$

Part ②: I_{xx}, I_{yy}, & I_{zz} by symmetry are one-half those of semicircular ring.



$I_O = I_G + m(b-\bar{r})^2$

$= I_C - m\bar{r}^2 + m(b-\bar{r})^2$

$= mb^2 + m(b^2 - 2b\bar{r})$

$= \rho\frac{\pi}{2}b^3\left[1 + 1 - \frac{4}{\pi}\right] = \rho b^3(\pi - 2)$

$I_{yy} = \frac{1}{2}mb^2 = \frac{1}{4}\rho\pi b^3$, $I_{zz} + I_{yy} = I_O$, so

$I_{zz} = \rho b^3\left(\frac{3\pi}{4} - 2\right)$

$I_{xy} = 0$, $I_{xz} = 0$, $I_{yz} = \int yz\,dm$

$I_{yz} = \int_0^{\pi/2}(b - b\cos\beta)(b\sin\beta)\rho b\,d\beta = \frac{1}{2}\rho b^3$

For complete rod :

$I_{xx} = \rho b^3(\pi-2) = 1.1416\,\rho b^3$, $I_{xy} = 0$

$I_{yy} = \rho b^3\left(\frac{\pi}{4}+\frac{1}{3}\right) = 1.1187\,\rho b^3$, $I_{xz} = 0$

$I_{zz} = \rho b^3\left(\frac{3\pi}{4}-\frac{5}{3}\right) = 0.6895\,\rho b^3$, $I_{yz} = \frac{1}{2}\rho b^3$

Substitute in Eq. B/11 & obtain for $\frac{I}{\rho b^3} = I'$:

$$\begin{vmatrix} (1.1416 - I') & -0 & -0 \\ -0 & (1.1187 - I') & -0.5 \\ -0 & -0.5 & (0.6895 - I') \end{vmatrix} = 0$$

Expand: $I'^3 - 2.950 I'^2 + 2.586 I' - 0.5952 = 0$

Numerical solution (or by cubic formula)

$\underline{I_1 = 1.448\,\rho b^3}$, $\underline{I_2 = 0.360\,\rho b^3}$, $\underline{I_3 = 1.142\,\rho b^3}$

From Eq. B/12, the direction cosines for I_2-axis are

$(1.1416 - 0.360)\,l \quad -(0)m - (0)n = 0 \qquad (1)$

$-(0)l \quad + (1.1187 - 0.360)\,m - 0.5n = 0 \qquad (2)$

$-0(l) \quad - 0.5m + (0.6895 - 0.360)n = 0 \qquad (3)$

Solution : $\underline{l = 0}$, $\underline{m = 0.5503}$, $\underline{n = 0.8350}$