8/1

$$k = \frac{W}{\delta_{st}} = \frac{3(9.81)}{0.04} = \underline{736\ \text{N/m}}$$

$$k = 736\ \frac{\text{N}}{\text{m}} \left(\frac{1\ \text{lb/in.}}{175.13\ \text{N/m}} \right) = \underline{4.20\ \text{lb/in.}}$$

$$k = 4.20\ \frac{\text{lb}}{\text{in.}} \left(\frac{12\ \text{in.}}{\text{ft}} \right) = \underline{50.4\ \text{lb/ft}}$$

8/2

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{24(12)}{2}} = \underline{12 \text{ rad/sec}}$$

$$f_n = \left(12 \frac{\text{rad}}{\text{sec}}\right)\left(\frac{1 \text{ cycle}}{2\pi \text{ rad}}\right) = \underline{\frac{6}{\pi} \text{ Hz}}$$

8/3

$$x = x_0 \cos \omega_n t + \frac{\dot{x}_0 \to 0}{\omega_n} \sin \omega_n t$$

$$= \underline{2 \cos 12t \text{ in.}}$$

8/4

$$x = C \sin(\omega_n t + \psi)$$

$$C = \left[x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2\right]^{1/2} = \left[(-2)^2 + \left(\frac{7}{12}\right)^2\right]^{1/2} = \underline{2.08 \text{ in.}}$$

$$\psi = \tan^{-1}\left(\frac{x_0\omega_n}{\dot{x}_0}\right) = \tan^{-1}\left(\frac{(-2)(12)}{7}\right) = -1.287 \text{ rad}$$

$$\therefore \quad \underline{x = 2.08 \sin(12t - 1.287) \text{ in.}}$$

8/5

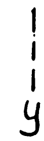
$$\delta_{st} = \frac{W}{k} = \frac{4(9.81)}{144} = \underline{0.273 \text{ m}}$$

$$\omega_n = \sqrt{k/m} = \sqrt{144/4} = 6 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{6} = \underline{\frac{\pi}{3} \text{ s}}$$

$$\dot{x}_{max} = C\omega_n = 0.1(6) = \underline{0.6 \text{ m/s}} = v_{max}$$

8/6 $\omega_n = \sqrt{k/m} = \sqrt{144/4} = 6 \text{ rad/s}$



$$y = y_0 \cos \omega_n t + \frac{\dot{y}_0}{\omega_n} \sin \omega_n t$$

$$= 0.1 \cos 6t \quad (\text{m})$$

$$v = \dot{y} = -0.6 \sin 6t \quad (\text{m/s})$$

$$a = \ddot{y} = -3.6 \cos 6t \quad (\text{m/s}^2)$$

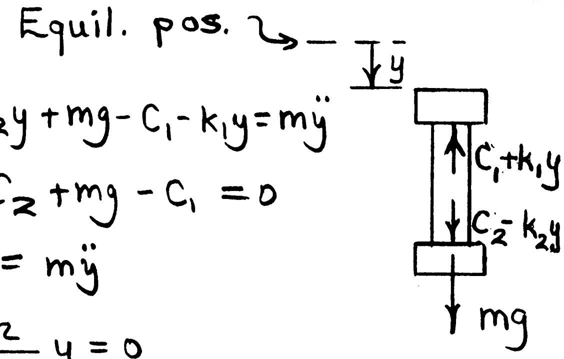
When $t = 3$ s :

$$y = 0.1 \cos(6 \cdot 3) = \underline{0.0660 \text{ m}} \quad (\text{below eq.})$$

$$v = -0.6 \sin(6 \cdot 3) = \underline{0.451 \text{ m/s}} \quad (\text{down})$$

$$a_{max} = \underline{3.6 \text{ m/s}^2}$$

8/7



$\Sigma F_y = m\ddot{y}$: $C_2 - k_2 y + mg - C_1 - k_1 y = m\ddot{y}$

At equilibrium, $C_2 + mg - C_1 = 0$

So $-(k_1 + k_2) y = m\ddot{y}$

$$\ddot{y} + \frac{k_1 + k_2}{m} y = 0$$

$$\omega_n = \sqrt{\frac{k_1 + k_2}{m}} = \sqrt{\frac{3600 + 1800}{2.5}} = 46.5 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{46.5}{2\pi} = \underline{7.40 \text{ Hz}}$$

8/8

$$\omega_n = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2(180{,}000)}{100}} = 60 \text{ rad/s}$$

$$x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$$

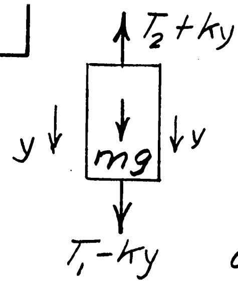
$$= 0 + \frac{0.5}{60} \sin 60t = 8.33(10^{-3}) \sin 60t$$

$$\dot{x} = 60(8.33)(10^{-3}) \cos 60t = 0.5 \cos 60t$$

$$\ddot{x} = -60(0.5) \sin 60t = -30 \sin 60t$$

$$\underline{a_{max} = 30 \text{ m/s}^2}$$

8/9



For equil. position,

$$T_1 + mg - T_2 = 0 \quad \text{-------}(1)$$

$\Sigma F_y = ma_y;$

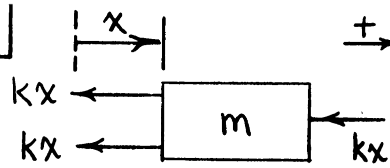
$$T_1 - ky + mg - T_2 - ky = m\ddot{y}$$

Combine with (1) & get

$$0 - 2ky = m\ddot{y} \quad \text{or} \quad \ddot{y} + \frac{2k}{m}y = 0$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2(3000)}{10}} = \underline{3.90\ \text{Hz}}$$

8/10



$\xrightarrow{+} \Sigma F_x = m\ddot{x}$:

$$-3kx = m\ddot{x}$$

$$\ddot{x} + \frac{3k}{m}x = 0$$

$$\omega_n = \sqrt{\frac{3k}{m}} = \sqrt{\frac{3(0.5)(12)}{4/16/32.2}} = 48.1 \text{ rad/sec}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{48.1} = \underline{0.1305 \text{ sec}}$$

$$x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$$

$$= 0.1 \cos(48.1 \cdot 2) + \frac{0.5}{48.1} \sin(48.1 \cdot 2)$$

$$= \underline{-0.0371 \text{ in.}}$$

$$\dot{x} = -x_0 \omega_n \sin \omega_n t + \dot{x}_0 \cos \omega_n t$$

$$= -0.1(48.1) \sin(48.1 \cdot 2) + 0.5 \cos(48.1 \cdot 2)$$

$$= \underline{-4.5 \text{ in./sec}}$$

8/11

$$\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4(17{,}500)}{1000}} = 8.37 \ \frac{\text{rad}}{\text{sec}}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{8.37}{2\pi} = \underline{1.332 \text{ Hz}}$$

We have assumed the unsprung mass (wheels, axles, etc.) to be a small fraction of the total car mass.

8/12 | An astronaut can attach himself or herself to a suitable spring anchored to the orbiter. Upon excitation of simple harmonic motion, one can measure the period, which is indicative of the spring constant (known) and the astronaut mass (the quantity to be measured).

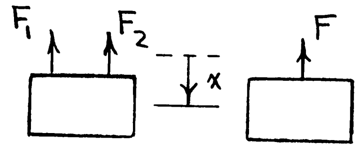
8/13 (a) From Eq. 8/3 frequency $f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

$$3 = \frac{1}{2\pi}\sqrt{\frac{3k}{4000}}, \quad k = 474 \times 10^3 \text{ N/m} \quad \text{or} \quad \underline{k = 474 \text{ kN/m}}$$

(b) For $m = 4000 + 40\,000 = 44\,000$ kg,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3(474 \times 10^3)}{44 \times 10^3}} = \underline{0.905 \text{ Hz}}$$

8/14



(a) $F = F_1 + F_2$

$kx = k_1 x + k_2 x$

$\underline{k = k_1 + k_2}$

(b) $F = F_1 = F_2$

$x_1 = \frac{F_1}{k_1}$, $x_2 = \frac{F_2}{k_2}$, $x = \frac{F}{k}$

From $x = x_1 + x_2$, we have $\frac{F}{k} = \frac{F_1}{k_1} + \frac{F_2}{k_2}$

or $\frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2}$. Thus $\underline{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$

8/15

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{k/m_{Tot}}$$

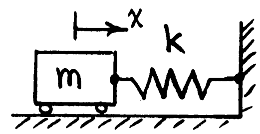
$$\frac{1}{0.75} = \frac{1}{2\pi}\sqrt{600/(m+6)} \;,\; \underline{m = 2.55 \text{ kg}}$$

$$\omega_n = \sqrt{k/m_{Tot}} = \sqrt{600/(6+2.55)} = 8.38 \text{ rad/s}$$

$$a_{max} = \omega_n^2 C = 8.38^2(0.050) = 3.51 \text{ m/s}^2$$

$$a_{max} = \mu_s g : \quad 3.51 = \mu_s (9.81) \;,\; \underline{\mu_s = 0.358}$$

8/16 Equivalent system :



$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{(3000)(12)}{2500/32.2}} = 21.5 \text{ rad/sec}$$

$$x = \cancel{x_0}^{\,0} \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$$

$$= \frac{5(5280/3600)}{21.5} \sin 21.5t = 0.341 \sin 21.5t$$

$x_{max} = 0.341$ ft or $\underline{4.09 \text{ in.}}$

$v = (0.341)(21.5) \cos 21.5t = \underline{7.33 \cos 21.5t}$
(in ft/sec)

or $\underline{v = 88.0 \cos 21.5t \text{ in./sec}}$

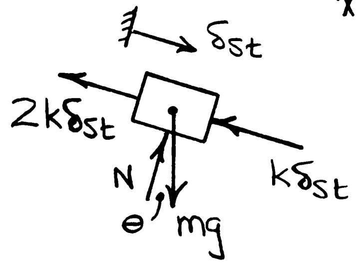
8/17

$$k = \frac{W}{\delta_{st}} = \frac{120}{0.9} = 133.3 \text{ lb/in.}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{(133.3)(12)}{120/32.2}} = 20.7 \text{ rad/sec}$$

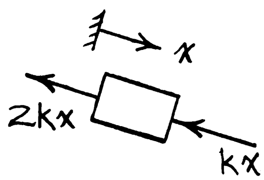
$$f_n = \frac{\omega_n}{2\pi} = \underline{3.30 \text{ Hz}}$$

8/18.



$$\Sigma F_x = 0: \; -3k\delta_{st} + mg\sin\theta = 0$$

$$\underline{\delta_{st} = \frac{mg\sin\theta}{3k}}$$



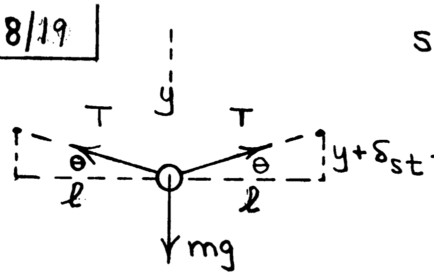
$$\Sigma F_x = m\ddot{x}: \; -3kx = m\ddot{x}$$

$$\ddot{x} + \frac{3k}{m}x = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{3k}{m}}} = \underline{2\pi\sqrt{\frac{m}{3k}}}$$

8/19

$$\sin\theta \cong \theta \cong \frac{y+\delta_{st}}{l}$$



δ_{st} = static deflection
y = dynamic deflection

$$\Sigma F_y = m\ddot{y}: \quad -2T\sin\theta + mg = m\ddot{y}$$

$$-2T\left(\frac{y+\delta_{st}}{l}\right) + mg = m\ddot{y}$$

$$-2T\frac{y}{l} - 2T\frac{\delta_{st}}{l} + mg = m\ddot{y}$$

$$\ddot{y} + \left(\frac{2T}{ml}\right)y = 0, \quad \underline{\omega_n = \sqrt{\frac{2T}{ml}}}$$

Although done above, the inclusion of the forces $+mg$ and $-2T\frac{\delta_{st}}{l}$, which sum to zero, is not necessary.