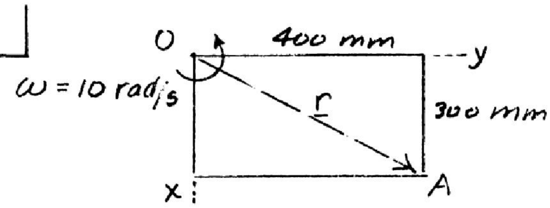
5/1

$$\alpha = \frac{\Delta\omega}{t} = \frac{800-200}{4/60} = 9000 \text{ rev/min}^2$$

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta, \quad 800^2 = 200^2 + 2(9000)\theta \quad (\text{rev/min})^2$$

$$\theta = \frac{800^2 - 200^2}{2(9000)} = \underline{33.3 \text{ rev}} = N$$

5/2



$r = 500$ mm or 0.5 m

(a) Scalar: $v = r\omega = 0.5(10) = \underline{5\ m/s}$

$a = a_n = r\omega^2 = 0.5(10^2) = \underline{50\ m/s^2}$

(b) $\underline{r} = 0.3\underline{i} + 0.4\underline{j}$ m , $\underline{\omega} = 10\underline{k}$ rad/s

$\underline{v} = \underline{\omega} \times \underline{r} = 10\underline{k} \times (0.3\underline{i} + 0.4\underline{j}) = 3\underline{j} + 4(-\underline{i})$

$v = \sqrt{3^2 + (-4)^2} = \underline{5\ m/s}$

$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times \dot{\underline{r}} = 0 + \underline{\omega} \times \underline{v}$

$= \underline{0} + 10\underline{k} \times (3\underline{j} - 4\underline{i}) = -30\underline{i} - 40\underline{j}\ m/s^2$

$|\underline{a}| = \sqrt{30^2 + 40^2} = \underline{50\ m/s^2}$

5/3 | Let $\underline{k}$ be a unit vector out of paper.

(a) $\underline{v}_A = \underline{\omega} \times \underline{r}_{A/O} = 3\underline{k} \times (-0.4\underline{e}_n) = \underline{1.2\underline{e}_t \text{ m/s}}$

$\underline{a}_A = \underline{\alpha} \times \underline{r}_{A/O} - \omega^2 \underline{r}_{A/O} = -14\underline{k} \times (-0.4\underline{e}_n) - 3^2(-0.4\underline{e}_n)$

$= \underline{-5.6\underline{e}_t + 3.6\underline{e}_n \text{ m/s}^2}$

(b) $\underline{v}_B = \underline{\omega} \times \underline{r}_{B/O} = 3\underline{k} \times (-0.4\underline{e}_n + 0.1\underline{e}_t)$

$= \underline{1.2\underline{e}_t + 0.3\underline{e}_n \text{ m/s}}$

$\underline{a}_B = \underline{\alpha} \times \underline{r}_{B/O} - \omega^2 \underline{r}_{B/O}$

$= -14\underline{k} \times (-0.4\underline{e}_n + 0.1\underline{e}_t) - 3^2(-0.4\underline{e}_n + 0.1\underline{e}_t)$

$= \underline{-6.5\underline{e}_t + 2.2\underline{e}_n \text{ m/s}^2}$

5/4

$$\omega_{AB} = \omega_{Cart} = \omega_{OG} = \frac{v_G}{\overline{OG}}, \quad \omega = \frac{14.20}{(56-8)/12} = \underline{3.55 \frac{rad}{sec}}$$

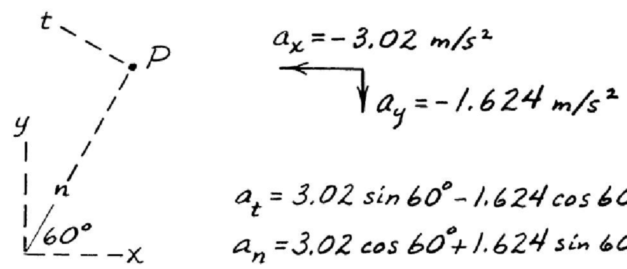
$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\pi/4}{0.638} = \underline{1.231 \text{ rad/sec}}$$

5/5 For $\theta = 90°$, $\underline{a} = -a_t \underline{i} - a_n \underset{\sim}{j}$ so $a_t = r\alpha = 1.8\ m/s^2$,

$$\alpha = \frac{1.8}{0.3} = \underline{6\ rad/s^2}$$

$$\text{\& } a_n = r\omega^2 = 4.8\ m/s^2,\ \omega = \sqrt{4.8/0.3} = \underline{4\ rad/s}$$

5/6



$a_t = 3.02 \sin 60° - 1.624 \cos 60° = 1.803\ \frac{m}{s^2}$

$a_n = 3.02 \cos 60° + 1.624 \sin 60° = 2.92\ \frac{m}{s^2}$

$a_t = r\alpha$: $\alpha = 1.803/0.3 = \underline{6.01\ rad/s^2}$

$a_n = r\omega^2$: $\omega^2 = 2.92/0.3 = 9.72\ (rad/s)^2$; $\underline{\omega = 3.12\ rad/s}$

5/7

$$\theta = 2t^3 - 3t^2 + 4 \text{ rad.}$$

$$\dot{\theta} = 6t^2 - 6t \text{ rad/s}$$

$$\ddot{\theta} = 12t - 6 \text{ rad/s}^2$$

When $\ddot{\theta} = 42 \text{ rad/s}^2$, $42 = 12t - 6$, $t = 4 \text{ s}$

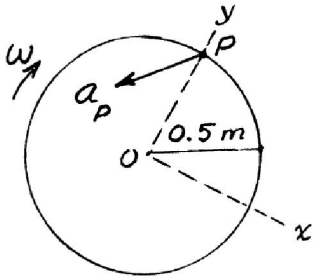
when $\ddot{\theta} = 66 \text{ rad/s}^2$, $66 = 12t - 6$, $t = 6 \text{ s}$

$$\theta_{t=4s} = 2(4^3) - 3(4^2) + 4 = 84 \text{ rad}$$

$$\theta_{t=6s} = 2(6^3) - 3(6^2) + 4 = 328 \text{ rad}$$

$$\Delta\theta = 328 - 84 = \underline{244 \text{ rad}}$$

5/8 For point P, $\underset{\sim}{a}_P = -3\underset{\sim}{i} - 4\underset{\sim}{j}$ m/s²



$a_n = r\omega^2$, $4 = 0.5\,\omega^2$, $\omega = \sqrt{8}$ $\frac{\text{rad}}{\text{s}}$

$\underline{\underset{\sim}{\omega} = -\sqrt{8}\,\underset{\sim}{k} \text{ rad/s}}$

$a_t = r\alpha$, $3 = 0.5\alpha$, $\alpha = 6$ rad/s²

$\underline{\underset{\sim}{\alpha} = 6\,\underset{\sim}{k} \text{ rad/s}^2}$

5/9 $a_B = a_t = r\alpha$

$\omega = \omega_0 + \alpha t$: $300(2\pi)/60 = 0 + \alpha(2)$, $\alpha = 5\pi$ rad/s^2

Thus $5.5 = r(5\pi)$, $r = 0.350$ m

$b = \sqrt{0.350^2 - 0.3^2} = 0.1806$ m or $\underline{b = 180.6 \text{ mm}}$

5/10 $\underline{v}_P = \underline{\omega} \times \underline{r} = 2\underline{k} \times [0.5\underline{i} + 0.2\underline{j} + 0.050\underline{k}]$

$= \underline{-0.4\underline{i} + \underline{j} \ \text{m/s}}$

$\underline{a}_P = \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$

$= -3\underline{k} \times [0.5\underline{i} + 0.2\underline{j} + 0.050\underline{k}]$

$+ 2\underline{k} \times [2\underline{k} \times (0.5\underline{i} + 0.2\underline{j} + 0.050\underline{k})]$

$= \underline{-1.4\underline{i} - 2.3\underline{j} \ \text{m/s}^2}$

Note that $\underline{r}$ could have been taken as $0.5\underline{i} + 0.2\underline{j}$ m

The magnitudes of the above results are

$v_P = 1.077$ m/s and $a_P = 2.69$ m/s^2.

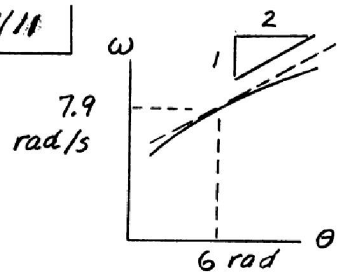
These magnitudes check with

$v_P = r_{xy}\,\omega = \sqrt{0.5^2 + 0.2^2}\,(2) = 1.077$ m/s^2 ✓

and $a_P = \sqrt{a_t^2 + a_n^2} = \sqrt{(r_{xy}\alpha)^2 + (r_{xy}\omega^2)^2}$

$= \sqrt{0.5^2 + 0.2^2}\,\sqrt{3^2 + 2^4} = 2.69$ m/s^2 ✓

5/11



$$\alpha = \omega \frac{d\omega}{d\theta}$$

$$= 7.9\,(1/2) = \underline{3.95\ \text{rad/s}^2}$$

5/12 $\quad \theta = (-1 + 1.5t)e^{-0.5t}$

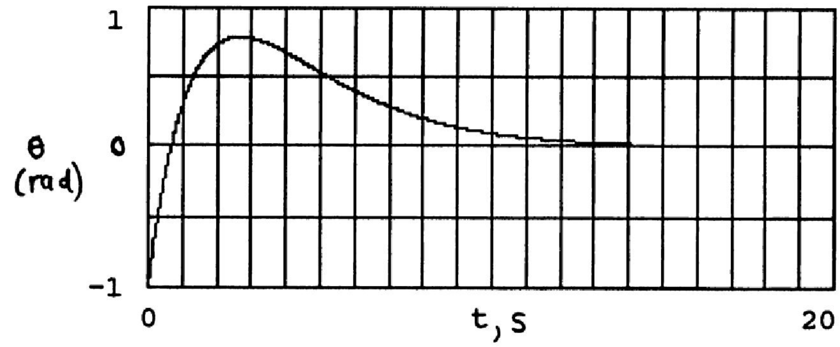
$$\omega = \frac{d\theta}{dt} = -0.5(-1 + 1.5t)e^{-0.5t} + 1.5e^{-0.5t}$$

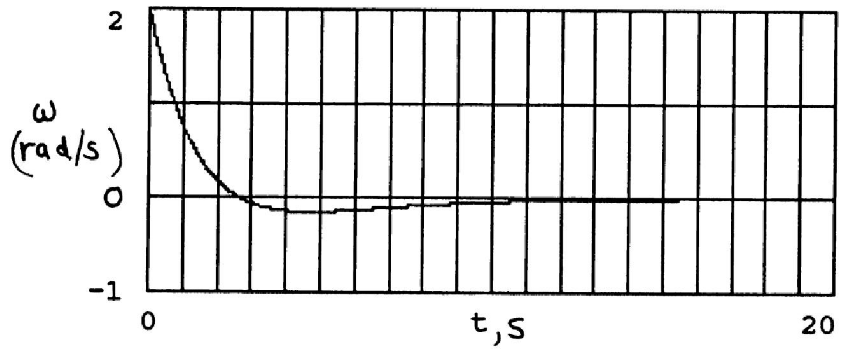
$$= \underline{(2 - 0.75t)e^{-0.5t}}$$

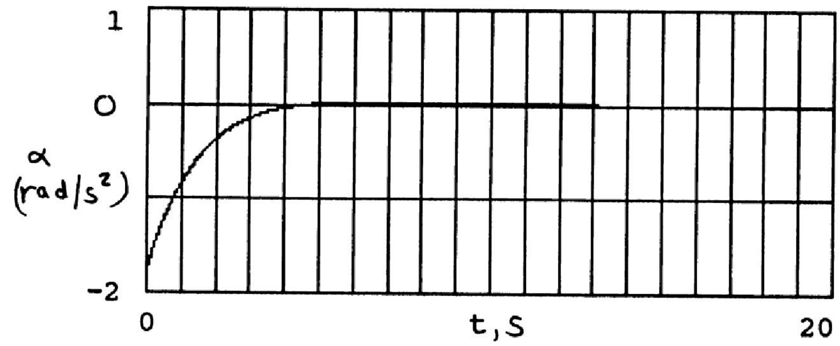
$$\alpha = \frac{d\omega}{dt} = -0.5(2 - 0.75t)e^{-0.5t} - 0.75e^{-0.5t}$$

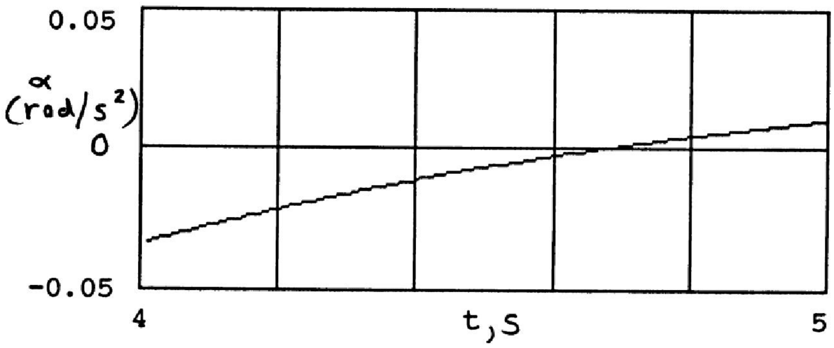
$$= \underline{(-1.75 + 0.375t)e^{-0.5t}}$$

$\alpha = 0$ when $-1.75 + 0.375t = 0$, $\underline{t = 4.67 \text{ s}}$


1
θ
(rad)
0
-1
0
t, s
20


2
ω
(rad/s)
0
-1
0
t, s
20


1
0
α
(rad/s²)
-2
0
t, s
20


0.05
α
(rad/s²)
0
-0.05
4
t, s
5

5/13 $\underline{\omega}_{OA} = \underline{\omega}_{BC} = -6\underline{k}$ rad/s

$\underline{r}_A = 0.3\underline{i} + 0.28\underline{j}$ m

$\underline{v}_A = \underline{\omega} \times \underline{r}_A = -6\underline{k} \times (0.3\underline{i} + 0.28\underline{j}) = -1.8\underline{j} + 1.68\underline{i}$ m/s

$\underline{v}_A = 1.68\underline{i} - 1.8\underline{j}$ m/s

$\underline{a}_A = \dot{\underline{\omega}} \times \underline{r}_A + \underline{\omega} \times \underline{v}_A = 0 + (-6\underline{k}) \times (1.68\underline{i} - 1.8\underline{j})$

$= -10.08\underline{j} - 10.8\underline{i}$

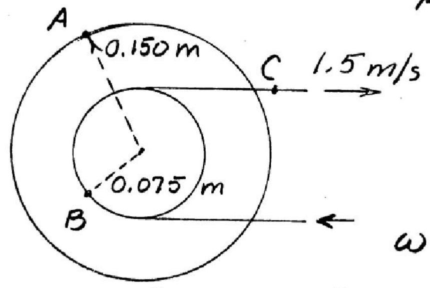
$\underline{a}_A = -10.8\underline{i} - 10.08\underline{j}$ m/s^2

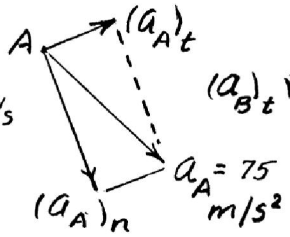
5/14 At B, $v = \frac{50}{30}44 = 73.3$ ft/sec, $r = 180 - \frac{18}{12} = 178.5$ ft

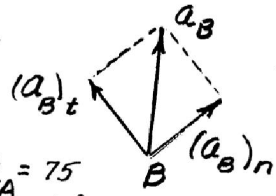
$\omega = v/r = 73.3/178.5 = \underline{0.411 \text{ rad/sec}}$

Between A & B $\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{30}{180}\pi/1.52 = \underline{0.344 \text{ rad/sec}}$

5/15







$$\omega = v/r = \frac{1.5}{0.075} = 20 \text{ rad/s}$$

$$(a_A)_n = r\omega^2 = 0.15(20)^2 = 60 \text{ m/s}^2$$

$$(a_A)_t = \sqrt{(75)^2 - (60)^2} = 45 \text{ m/s}^2$$

$$\alpha = a_t/r = 45/0.15 = \underline{300 \text{ rad/s}^2}$$

$$(a_B)_n = 0.075(20)^2 = 30 \text{ m/s}^2$$

$$(a_B)_t = r\alpha = 0.075(300) = 22.5 \text{ m/s}^2$$

$$a_B = \sqrt{(30)^2 + (22.5)^2} = \underline{37.5 \text{ m/s}^2}$$

$$a_C = a_{B_t} = \underline{22.5 \text{ m/s}^2}$$

5/16

$a = a_n = \frac{v^2}{r}, \quad \left(\frac{v^2}{r}\right)_A = \frac{4}{3}\left(\frac{v^2}{4}\right)_B$

$r = \frac{4}{4/3} = \underline{3 \text{ in.}}$

5/17 $\quad \underline{v}_A = \underline{\omega} \times \underline{r}_A \; ; \quad 8\underline{j} = \omega \underline{k} \times 4\underline{i} \, , \quad \omega = 2 \text{ rad/sec}$

$$\underline{\underline{\omega} = 2\underline{k} \text{ rad/sec}}$$

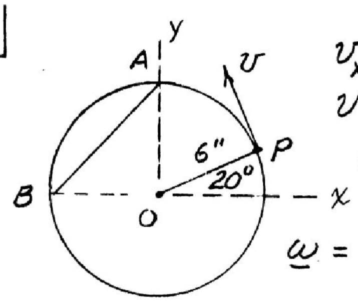
$(\underline{a}_B)_t = \underline{\alpha} \times \underline{r}_B , \quad 6\underline{i} = \alpha \underline{k} \times 4\underline{j} \, , \quad \alpha = -3/2 \text{ rad/sec}^2$

$$\underline{\underline{\alpha} = -\tfrac{3}{2}\underline{k} \text{ rad/sec}^2}$$

$\underline{r}_C = \frac{4}{\sqrt{2}}(\underline{i} - \underline{j}) \text{ in.}$

$$\begin{aligned}
\underline{a}_C &= \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times (\underline{\omega} \times \underline{r}_C) \\
&= -\tfrac{3}{2}\underline{k} \times \frac{4}{\sqrt{2}}(\underline{i} - \underline{j}) + 2\underline{k} \times \left(2\underline{k} \times \frac{4}{\sqrt{2}}[\underline{i} - \underline{j}]\right) \\
&= \frac{6}{\sqrt{2}}(-\underline{i} - \underline{j}) + \frac{16}{\sqrt{2}}(-\underline{i} + \underline{j}) = \underline{\sqrt{2}(-11\underline{i} + 5\underline{j})} \text{ in./sec}^2
\end{aligned}$$

5/18



$v_x = -4.2$ ft/sec

$v = 4.2/\sin 20° = 12.28$ ft/sec

$\omega = \frac{v}{r} = \frac{12.28}{6/12} = 24.6 \frac{\text{rad}}{\text{sec}}$

CCW

$\underset{\sim}{\omega} = \underset{\sim}{\omega}_{OP} = \underline{+24.6\underset{\sim}{k} \text{ rad/sec}}$

Element BC remains parallel to z-axis so has no angular velocity

5/19

$$\alpha = \frac{d\omega}{dt} = 2 - kt = 2 - 0.2t$$

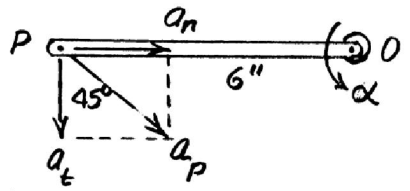
$$\int_{\omega_0}^{\omega} d\omega = \int_0^t (2-0.2t)\,dt, \quad \omega = \omega_0 + 2t - 0.1t^2$$

$$\omega_0 = 200 \times 2\pi/60 = 20.9 \text{ rad/s}$$

For $t = 5$ s, $\omega = 20.9 + 2(5) - 0.1(5^2) = 28.4$ rad/s

$$N = 28.4 \times 60/2\pi = \underline{272 \text{ rev/min}}$$

5/20



$$\alpha = \frac{600(2\pi)}{60}\,\frac{1}{2} = 10\pi \text{ rad/sec}^2$$

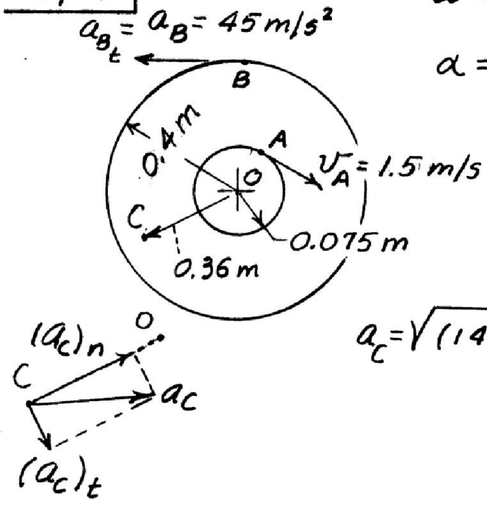
$a_t = r\alpha = 6(10\pi) = 60\pi$ in./sec^2

$a_n = r\omega^2 = 60\pi$ in./sec^2 for 45°

so $\omega^2 = 60\pi/6 = 10\pi$, $\omega = 5.60$ rad/s

$\omega = \omega_0 + \alpha t$: $5.60 = 0 + 10\pi t$, $\underline{t = 0.1784 \text{ sec}}$

5/21



$\omega = v/r = \dfrac{1.5}{0.075} = 20\ rad/s$

$\alpha = a_t/r = \dfrac{45}{0.4} = 112.5\ rad/s^2$

$(a_C)_n = r\omega^2 = 0.36(20)^2 = 144\ m/s^2$

$(a_C)_t = r\alpha = 0.36(112.5) = 40.5\ m/s^2$

$a_C = \sqrt{(144)^2 + (40.5)^2} = \underline{149.6\ m/s^2}$

5/22 $\alpha = 1.8 - k\theta$ rev/s^2 , θ in revolutions

$0.6 = 1.8 - k(20)$, $k = 0.06$ 1/s^2

so $\alpha = 1.8 - 0.06\theta$ rev/s^2 ; $\omega_0 = 300/60 = 5$ rev/s

$\omega \, d\omega = \alpha \, d\theta$: $\int_5^{\omega} \omega \, d\omega = \int_0^{20} (1.8 - 0.06\theta) \, d\theta$

$\omega^2 = 5^2 + 2\left[1.8\theta - 0.03\theta^2\right]_0^{20} = 25 + 48 = 73$ (rev/s)^2

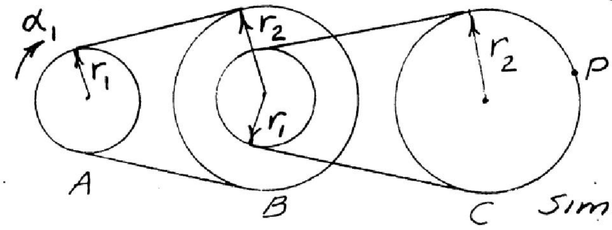
$\omega = \sqrt{73} = 8.54$ rev/s or $N = 8.54(60) = \underline{513 \text{ rev/min}}$

$\underline{5/23}$ For gear A, $\Delta\omega = \int_2^6 \alpha_A \, dt$, $N_A = 2N_B$

$$(N_A - 600)\frac{2\pi}{60} = \frac{4+8}{2}(6-2), \; N_A = 600 + 229 = 829 \text{ rev/min}$$

so at $t = 6$ s, $N_B = \frac{829}{2} = \underline{415 \text{ rev/min}}$

▶5/24



$r_2\omega_C = r_1\omega_B$

$r_2\omega_B = r_1\omega_A$

so that

$\omega_C = \left(\frac{r_1}{r_2}\right)^2\omega_A$

Similarly $\alpha_C = \left(\frac{r_1}{r_2}\right)^2\alpha_1$

$\omega_A = \alpha_1 t$ so that $\underline{\omega_C = \left(\frac{r_1}{r_2}\right)^2\alpha_1 t}$

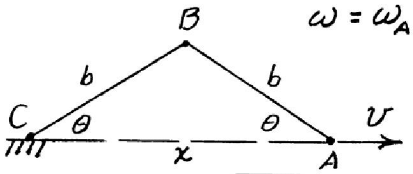
For P, $a_n = r_2\omega_C^2 = r_2\left[\left(\frac{r_1}{r_2}\right)^2\alpha_1 t\right]^2$

$a_t = r_2\alpha_C = r_2\left(\frac{r_1}{r_2}\right)^2\alpha_1$

$a_P = \sqrt{a_n^2 + a_t^2} = \underline{\frac{r_1^2}{r_2}\alpha_1\sqrt{1 + \left(\frac{r_1}{r_2}\right)^4\alpha_1^2 t^4}}$

5/25 $x = 2b\cos\theta$, $\dot{x} = -2b\dot{\theta}\sin\theta$, $v = \dot{x}$

$\omega = \omega_{AB} = \dot{\theta}$ so $\omega = \dfrac{-v}{2b\sin\theta}$ CW



For $a = \ddot{x}$ const., $\dot{x}^2 = 2ax$

$v = \sqrt{2ax}$

so $\omega = \dfrac{\sqrt{2ax}}{2b\sqrt{1-\cos^2\theta}} = \dfrac{\sqrt{2ax}}{\sqrt{4b^2 - x^2}}$