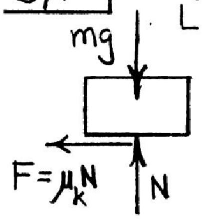
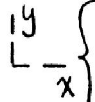
3/1





$$\begin{cases} \Sigma F_y = 0 : N - mg = 0, \quad N = mg \\ \Sigma F_x = ma_{Gx} : -\mu_k mg = ma_{Gx} \end{cases}$$

$$a_{Gx} = -\mu_k g = -(0.4)(9.81) = -3.92 \text{ m/s}$$

Kinematics : $v^2 - v_0^2 = 2a(x - x_0)$

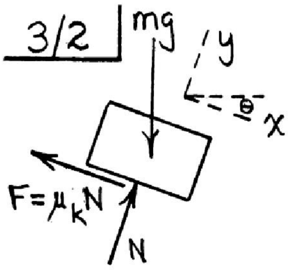
$$0 - 7^2 = 2(-3.92)(x - 0)$$

$$\underline{x = 6.24 \text{ m}}$$

$$v = v_0 + at$$

$$0 = 7 - 3.92t, \quad \underline{t = 1.784 \text{ s}}$$

3/2



$\Sigma F_y = 0: \quad N - mg\cos\theta = 0, \quad N = mg\cos\theta$

$\Sigma F_x = ma_x: \quad mg\sin\theta - \mu_k mg\cos\theta = ma_{Gx}$

$a_{Gx} = g(\sin\theta - \mu_k\cos\theta)$

(a) $\theta = 15°$: $a_{Gx} = 9.81(\sin 15° - 0.4\cos 15°)$
$= -1.251 \text{ m/s}^2$

$v^2 - v_0^2 = 2a(x - x_0)$

$0 - 7^2 = 2(-1.251)(x - 0)$, $\underline{x = 19.58 \text{ m}}$

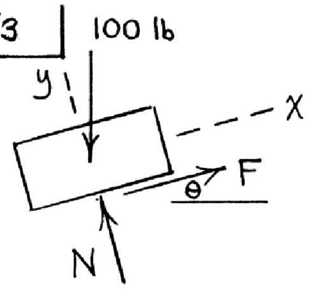
$v = v_0 + at$

$0 = 7 - 1.251t$, $\underline{t = 5.59 \text{ s}}$

(b) $\theta = 30°$: $a_{Gx} = 9.81(\sin 30° - 0.4\cos 30°)$
$= 1.51 \text{ m/s}^2$

<u>Crate does not come to rest.</u>

3/3



$$\theta_{max} = \tan^{-1}\mu_s = \tan^{-1}(0.30)$$
$$= 16.70°$$

$\therefore$ (a) $\theta = 15°$: No motion
$$\underline{a = 0}$$

(b) $\Sigma F_y = 0$: $N = 100 \cos 20° = 94.0$ lb

$$F = \mu_k N = 0.25\,(94.0) = 23.5 \text{ lb}$$

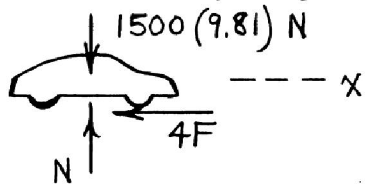
$$\Sigma F_x = ma_x : -100 \sin 20° + 23.5 = \frac{100}{32.2}\,a$$

$$\underline{a = -3.45 \text{ ft/sec}^2}$$

(Block accelerates down plane)

3/4 $v_2^2 - v_1^2 = 2a(x_2 - x_1)$

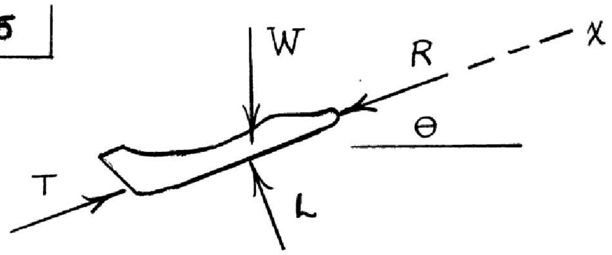
$0^2 - \left(\frac{100}{3.6}\right)^2 = 2a_x(50), \quad a_x = -7.72 \text{ m/s}^2$



$\Sigma F_x = ma_x: \quad -4F = 1500(-7.72)$

$\underline{F = 2890 \text{ N}}$

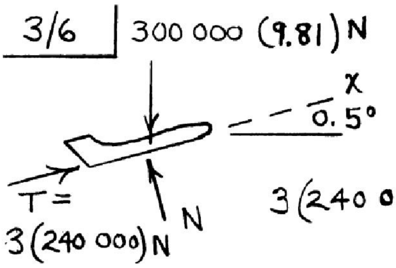
3/5



$$\Sigma F_x = ma_x : \quad T - R - W\sin\theta = \frac{W}{g} a$$

$$n = \frac{T-R}{W} = \underline{\sin\theta + \frac{a}{g}}$$

3/6

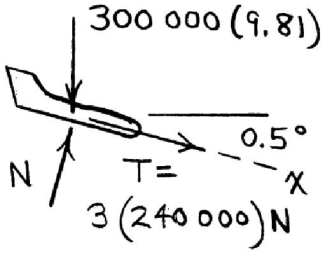


$\Sigma F_x = ma_x$:

$$3(240\,000) - 300\,000(9.81)\sin\tfrac{1}{2}^\circ = 300\,000\, a_x$$

$$a_x = 2.31 \text{ m/s}^2$$

$v^2 = 2a_x s$: $(220/3.6)^2 = 2(2.31)s$, $\underline{s_u = 807 \text{ m}}$



$\Sigma F_x = ma_x$:

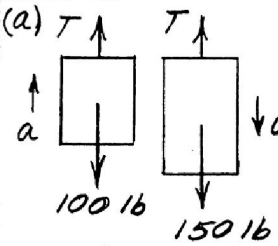
$$3(240\,000) + 300\,000(9.81)\sin\tfrac{1}{2}^\circ = 300\,000\, a_x$$

$$a_x = 2.49 \text{ m/s}^2$$

$v^2 = 2a_x s$: $(220/3.6)^2 = 2(2.49)s$, $\underline{s_d = 751 \text{ m}}$

3/7 $\Sigma F = ma;\quad T - 100 = \frac{100}{32.2}a$

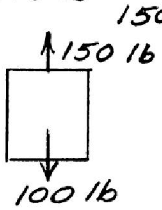
(a)



$$150 - T = \frac{150}{32.2}a$$

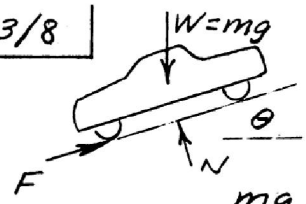
$$50 = \frac{250}{32.2}a\,,\quad a = \frac{32.2}{5} = 6.44\ \frac{ft}{sec^2}$$

(b)



$$150 - 100 = \frac{100}{32.2}a\,,\quad a = \frac{32.2}{2} = 16.10\ \frac{ft}{sec^2}$$

3/8



For $\theta = \theta_1$, accel. = 0

$\Sigma F = 0;\ F = W \sin\theta_1$

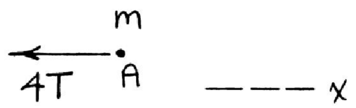
For $\theta = \theta_2$, $\Sigma F = ma$ so

$$mg\sin\theta_1 - mg\sin\theta_2 = ma$$

$$\underline{a = g(\sin\theta_1 - \sin\theta_2)}$$

3/9

$+\leftarrow \Sigma F = ma: \quad 4(40{,}000) = \frac{750{,}000}{32.2}\, a$



$a = 6.87 \text{ ft/sec}^2$

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = -6.87\underline{i} - \underline{0} = \underline{-6.87\underline{i} \text{ ft/sec}^2}$

$v_A = (v_A)_0 + at = 0 + 6.87(10) = 68.7 \text{ ft/sec}$

$v_B = 15\left(\frac{88}{60}\right) = 22 \text{ ft/sec}$

$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = -68.7\underline{i} - 22(\cos 30°\underline{i} + \sin 30°\underline{j})$

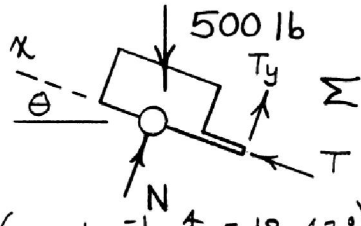
$= \underline{-87.7\underline{i} - 11\underline{j} \text{ ft/sec}}$

3/10 $\overset{+}{\nwarrow}$ $v^2 - v_0^2 = 2a(s - s_0)$

$$0^2 - \left(5\,\frac{5280}{3600}\right)^2 = 2a(-4)$$

$$a = 6.72 \text{ ft/sec}^2$$

FBD of cart (treat as a particle):



(Static)

$\Sigma F_x = 0$: $T_s - 500 \sin 18.43° = 0$

$$T_s = 158.1 \text{ lb}$$

$\left(\theta = \tan^{-1} \frac{4}{12} = 18.43°\right)$

$\Sigma F_x = ma_x$: $T - 500 \sin 18.43° = \dfrac{500}{32.2}(6.72)$

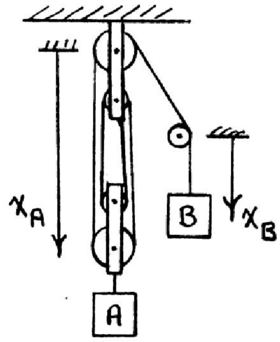
$$T = 262 \text{ lb}$$

Percent increase $n = \dfrac{262 - 158.1}{158.1}(100\%)$

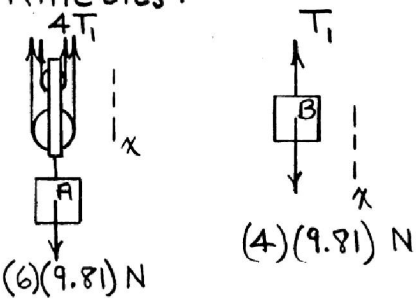
$$= \underline{66.0\%}$$

3/11 Kinematics : $4x_A + x_B = L_{rope} + \text{constant}$

$\therefore \; 4a_A + a_B = 0 \qquad (1)$



Kinetics :



A : $\Sigma F_x = ma_x$: $6(9.81) - 4T_1 = 6a_A \qquad (2)$

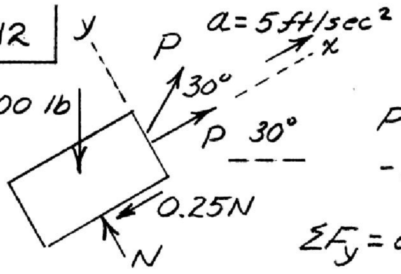
B : $\Sigma F_x = ma_x$: $4(9.81) - T_1 = 4a_B \qquad (3)$

Solution of Eqs. (1) − (3) : $\begin{cases} a_A = -1.401 \text{ m/s}^2 \\ a_B = 5.61 \text{ m/s}^2 \\ T_1 = 16.82 \text{ N} \end{cases}$

Tension in cable above A

$T_2 = 4T_1 = \underline{67.3 \text{ N}}$

3/12



$\Sigma F_x = m a_x$;

$P(1 + \cos 30°) - 0.25N - 100 \sin 30° = \frac{100}{32.2}(5)$

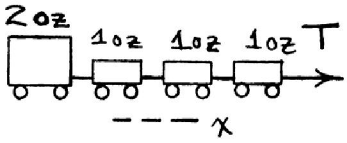
$\Sigma F_y = 0$; $N + P \sin 30° - 100 \cos 30° = 0$

$$\left.\begin{aligned} 1.866P - 0.25N &= 65.53 \\ 0.5P + N &= 86.6 \end{aligned}\right\}$$ solve simultaneously & get $N = 64.7$ lb

$\underline{P = 43.8 \text{ lb}}$

3/13 | Coupler 1 will fail first, because it must accelerate more mass than any other coupler.

Rear part of train:

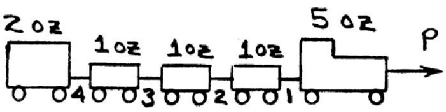


$$\Sigma F_x = ma_x$$

$$T = 0.2 = \left(\frac{5/16}{32.2}\right) a$$

$$a = 20.6 \text{ ft/sec}^2$$

Whole train:

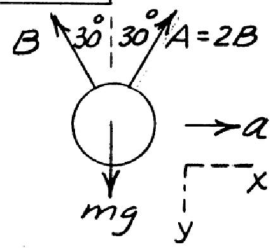


$$\Sigma F_x = ma_x$$

$$P = \left(\frac{10/16}{32.2}\right)(20.6)$$

$$\underline{P = 0.4 \text{ lb}}$$

3/14



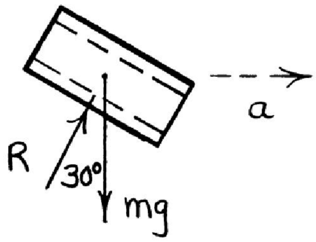
$\Sigma F_x = ma_x$; $2B\sin 30° - B\sin 30° = ma$

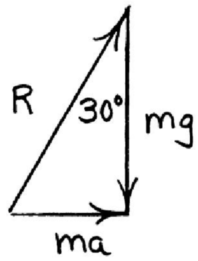
$\Sigma F_y = 0$; $2B\cos 30° + B\cos 30° - mg = 0$

Eliminate B & get $\underline{a = g/3\sqrt{3}}$

3/15

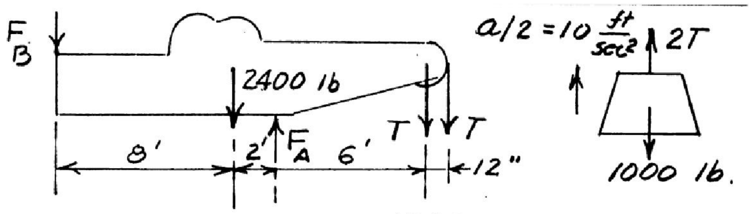
$\Sigma \underline{F} = m\underline{a}$:





$ma = mg\tan 30° \Rightarrow a = g\tan 30° = \underline{5.66\ \text{m/s}^2}$

3/16

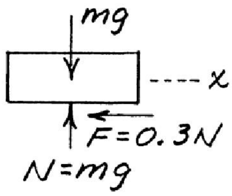


Load; $\Sigma F = ma$; $2T - 1000 = \frac{1000}{32.2}(10)$, $T = 655$ lb

Beam; $\Sigma M_B = 0$; $2400(8) - 10F_A + 655(16 + 17) = 0$

$F_A = \underline{4080 \text{ lb}}$

3/17 Let m = mass of crate



$\Sigma F_x = ma_x$; $-0.3mg = ma_x$

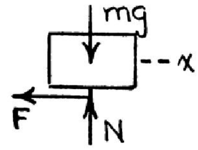
$a_x = -0.3g = -0.3(9.81) = -2.94 \text{ m/s}^2$

$\int_v^0 v\,dv = \int_0^s a_x\,dx$; $-\frac{v^2}{2} = a_x s$

$s = \frac{-(70/3.6)^2/2}{-2.94} = \underline{64.3 \text{ m}}$

3/18 Truck :
$$\begin{cases} v^2 - v_0^2 = 2a_T(x - x_0) \\ 0^2 - (19.44)^2 = 2a_T(50-0) \\ a_T = -3.78 \text{ m/s}^2 \end{cases}$$

Crate :



$$\Sigma F_x = ma_x : \ -F = m(-3.78)$$
$$F = 3.78\,m$$

$$F_{max} = \mu_s N = 0.3(m\,9.81) = 2.94\,m$$

$F > F_{max}$, crate slips, $F = \mu_k N$

$$\therefore \ \Sigma F_x = ma_x : \ -0.25\,mg = ma_c, \ a_c = -2.45 \text{ m/s}^2$$

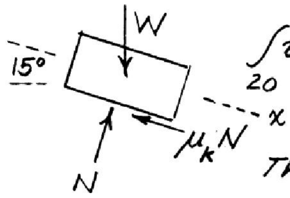
$$a_{c/T} = a_c - a_T = -2.45 - (-3.78) = 1.328 \text{ m/s}^2$$

$$v_{c/T}^2 - v_{c/T_0}^2 = 2a_{c/T}(x_{c/T} - x_{c/T_0})$$
$$v_{c/T}^2 - 0^2 = 2(1.328)(3-0), \quad \underline{v_{c/T} = 2.82 \text{ m/s}}$$

(Truck stopping time = 5.14 s, crate impacts at 2.13 s)

3/19

$\Sigma F_x = ma_x$; $W \sin 15° - \mu_k (W \cos 15°) = \frac{W}{g} a_x$



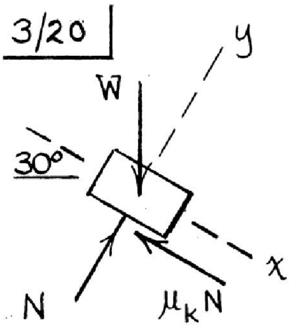
$\int_{20}^{10} v\,dv = \int_{0}^{30} a_x\,dx$, $a_x = \frac{100-400}{2(30)} = -5 \frac{ft}{sec^2}$

constant

Thus $g(\sin 15° - \mu_k \cos 15°) = a_x = -5$

& $\mu_k = \tan 15° + \frac{5}{32.2(0.9659)} = \underline{0.429}$

3/20



$\Sigma F_y = 0: \quad N = W\cos 30°$

$\Sigma F_x = ma_x:$

$W\sin 30° - \mu_k W\cos 30° = \frac{W}{g}a_x$

$a_x = 32.2\left(\frac{1}{2} - \mu_k\frac{\sqrt{3}}{2}\right)$

$v_2^2 - v_1^2 = 2a(x_2 - x_1): \quad 3^2 - 1.2^2 = 2(32.2)\left(\frac{1}{2} - \mu_k\frac{\sqrt{3}}{2}\right)6$

$\mu_k = 0.555$

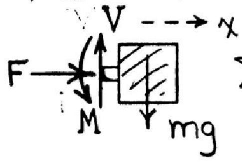
3/21

$$x = X \sin \omega t$$

$$\dot{x} = X \omega \cos \omega t$$

$$\ddot{x} = -X \omega^2 \sin \omega t, \quad \ddot{x}_{max} = X \omega^2$$

FBD of circuit board:



$\Sigma F_x = m a_x$: $\quad F = m(-X\omega^2 \sin \omega t)$

$$\underline{F_{MAX} = m X \omega^2}$$

3/22 $F = ma: \; 2.5 = 70(10^3)\,a, \quad a = 35.7(10^{-6})\ \frac{m}{s^2}$

$\Delta v = \int a\,dt = at, \quad t = \dfrac{(65-40)\,10^3}{35.7(10^{-6})(3600)^2\,24}$

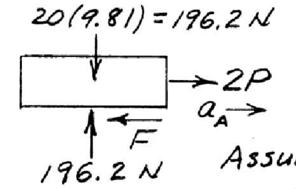
$= 2251$ days or $\underline{6.16 \text{ years}}$

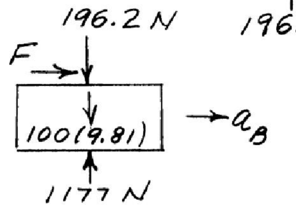
$s = \int v\,dt = v_{AV}\,t = \dfrac{65+40}{2}(10^3)(2251)(24)$

$= \underline{2.84(10^9)\text{ km}}$

3/23

$\mu = 0.5$





(a) $2P = 120$ N

$F_{max} = 0.5(196.2)$
$= 98.1 \text{ N} < 2P$

Assume slipping occurs & $F = 98.1$ N

A; $\Sigma F = ma$; $120 - 98.1 = 20\, a_A$

$\underline{a_A = 1.095 \text{ m/s}^2}$

B; $\Sigma F = ma$; $98.1 = 100\, a_B$

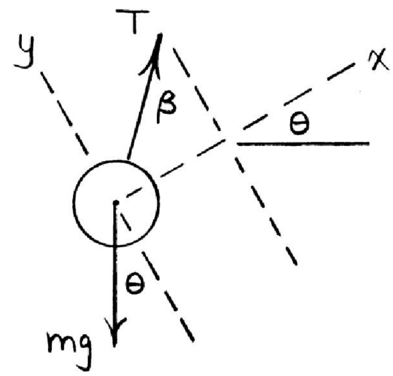
$a_A > a_B$ so assumption OK.

$\underline{a_B = 0.981 \text{ m/s}^2}$

(b) $2P = 80 \text{ N} < F_{max}$ so no slipping occurs & for block & cart combined,

$\Sigma F = ma$; $80 = 120a$, $\underline{a_A = a_B = a = 0.667 \text{ m/s}^2}$

3/24



$\Sigma F_x = ma_x$: $T\sin\beta - mg\sin\theta = ma$

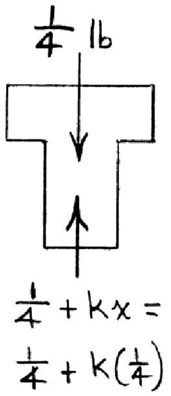
$\Sigma F_y = 0$: $T\cos\beta - mg\cos\theta = 0$

Eliminate T : $\beta = \tan^{-1}\dfrac{a + g\sin\theta}{g\cos\theta}$

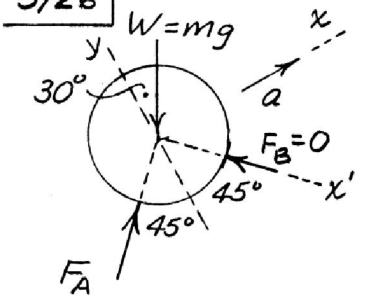
3/25

$$+\uparrow \Sigma F = ma: \quad \frac{1}{4} + \frac{1}{4}k - \frac{1}{4} = \frac{1/4}{g}(5g)$$

$$\underline{k = 5 \text{ lb/in.}}$$



3/26



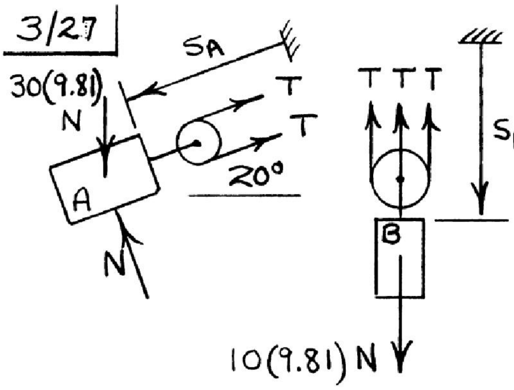
$\Sigma F_{x'} = m a_{x'}$;

$mg \cos(45° + 30°) = m a \cos 45°$

$a = g \dfrac{\cos 75°}{\cos 45°} = 9.81 \dfrac{0.2588}{0.7071}$

$= \underline{0.366g}$

3/27



Kinematic constraint: $L = 2s_A + 3s_B$

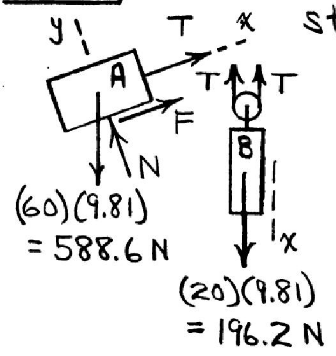
$\Rightarrow \quad 0 = 2a_A + 3a_B \quad (1)$

$\overset{+}{\swarrow}$ $\Sigma F = m_A a_A: \; 30(9.81)\sin 20° - 2T = 30a_A \quad (2)$

$+\downarrow$ $\Sigma F = m_B a_B: \; 10(9.81) - 3T = 10a_B \quad (3)$

Solution of Eqs. (1)-(3): $\begin{cases} \underline{a_A = 1.024 \text{ m/s}^2} \\ \underline{a_B = -0.682 \text{ m/s}^2} \\ T = 35.0 \text{ N} \end{cases}$

3/28



Check for motion by assuming static equilibrium.

B : $2T = 196.2$, $T = 98.1$ N

A : $\Sigma F_x = 0$: $98.1 - 588.6 \sin 30° + F = 0$, $F = 196.2$ N

$F_{max} = \mu_s N = (0.25)(588.6) \cos 30° = 127.4$ N

$F > F_{max} \Rightarrow$ motion $(\swarrow)$

From kinematics, $a_A = 2a_B = 2a$

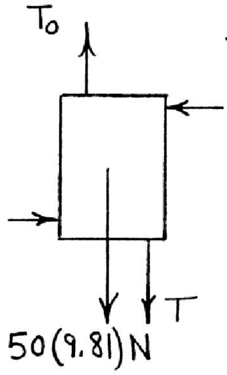
A : $\Sigma F_x = ma_x$: $T + 0.2(588.6 \cos 30°) - 588.6 \sin 30° = 60(2a)$

B : $\Sigma F_x = ma_x$: $-2T + 196.2 = 20a$

Solution : $\underline{a = -0.725 \text{ m/s}^2}$, $\underline{T = 105.4 \text{ N}}$

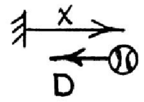
3/29

$T_0 = Te^{\mu\beta} = Te^{0.2(\pi+\pi)} = 3.51T$



$+\downarrow \Sigma F = ma: \quad 50(9.81) + T - 3.51T = 50(1.2)$

$\underline{T = 171.3 \text{ N}}$

3/30 

(Neglect weight for now)

$$\Sigma F_x = ma_x: \ -D = -C_D \tfrac{1}{2}\rho v^2 S = mv\frac{dv}{dx}$$

$$\int_0^x \left(-C_D \tfrac{1}{2}\rho S\right) dx = m\int_{v_0}^{v} \frac{dv}{v}$$

$$\Rightarrow \quad v = v_0 e^{\left(-\frac{1}{2}C_D \rho S x/m\right)}$$

$$= v_0 e^{\left(-\frac{1}{2}(0.3)\left(\frac{0.07530}{32.2}\right)(\pi)\left(\frac{9.125/2\pi}{12}\right)^2 x/\frac{5.125}{16\cdot 32.2}\right)}$$

$$\underline{v = v_0 e^{-1.623(10^{-3})x}}$$

For $v_0 = 90$ mi/hr and $x = 60$ ft: $\underline{v = 81.7 \text{ mi/hr}}$

Comment on y-motion. Assume $v = 90$ mi/hr = constant. Time t to plate is

$$t = \frac{60}{90\left(5280/3600\right)} = 0.455 \text{ sec}$$

$v_y = v_{y_0} - gt = -32.2(0.455) = -14.64$ ft/sec, which would not appreciably change $v = \sqrt{v_x^2 + v_y^2}$.

3/31 $D = kv^2 : \quad 120(10^3) = k\left(300/3.6\right)^2$

$k = 17.28 \ \frac{N \cdot s^2}{m^2}, \qquad D = 17.28\,v^2$

$\Sigma F = ma : \quad -17.28\,v^2 = 5000\,a$

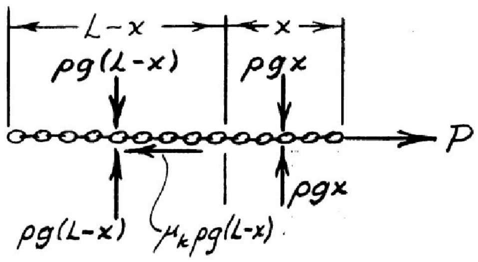
$v\,dv = a\,dx : \quad -\frac{5000}{17.28}\int_{v_1}^{v_2} \frac{v\,dv}{v^2} = \int_0^x dx$

$v_1 = \left(300/3.6\right)\ \text{m/s}, \quad v_2 = v_1/2 = \left(150/3.6\right)\ \text{m/s}$

So $-\frac{5000}{17.28}\ln v \Big|_{300/3.6}^{150/3.6} = x, \qquad \underline{x = 201\ \text{m}}$

3/32

$\Sigma F_x = ma_x$: $P - \mu_k \rho g(L-x) = \rho L \ddot{x}$

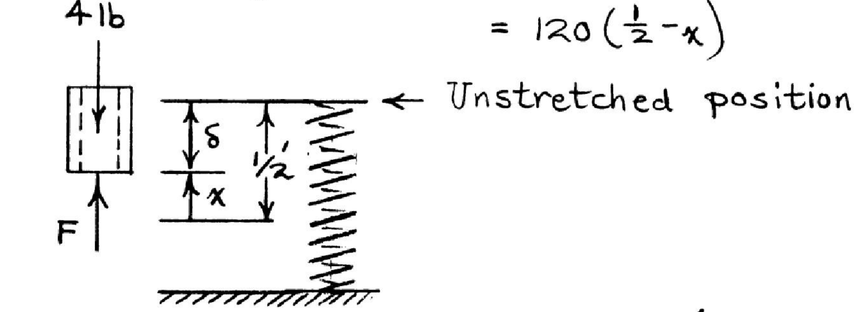


$\int v\,dv = \int \ddot{x}\,dx$: $\int_0^v v\,dv = \int_0^L \left(\frac{P}{\rho L} - \mu_k g + \mu_k g \frac{x}{L}\right) dx$

$$\frac{v^2}{2} = \left(\frac{P}{\rho L} - \mu_k g\right)L + \frac{\mu_k g}{2L}L^2, \quad \underline{v = \sqrt{\frac{2P}{\rho} - \mu_k g L}}$$

3/33 Spring force $F = k\delta = k\left(\frac{1}{2} - x\right)$

$$= 120\left(\frac{1}{2} - x\right)$$



$$\uparrow \Sigma F_x = ma_x: \quad -4 + 120\left(\frac{1}{2} - x\right) = \frac{4}{32.2} a_x$$

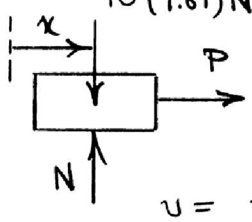
$$\underline{a_x = 32.2\,(14 - 30x)}$$

$$32.2\,(14 - 30x) = v\,\frac{dv}{dx}$$

$$\int_0^{1/2} 32.2\,(14 - 30x)\,dx = \int_0^v v\,dv$$

$$\underline{v = 14.47 \text{ ft/sec}}$$

3/34



$\Sigma F_x = ma_x : P = 10a_x$

$\frac{P}{10} = \frac{dv}{dt}, \quad v = \int_0^t \frac{P}{10} dt$

For $P_1 = 10t$:

$v = t^2/2, \quad s = t^3/6$

At $t = 5$ s, $\underline{v = 12.5 \text{ m/s}}$, $\underline{s = 20.8 \text{ m}}$

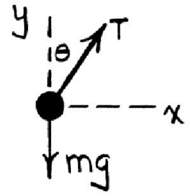
For $P_2 = kt^2$: $50 = k(5)^2$, $k = 2 \text{ N/s}^2$

So $P_2 = 2t^2$

$v = \int_0^t \frac{2t^2}{10} dt = \frac{t^3}{15}, \quad s = t^4/60$

At $t = 5$s, $\underline{v = 8.33 \text{ m/s}}$, $\underline{s = 10.42 \text{ m}}$

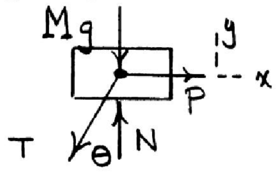
3/35 Mass m:



$$\sum F_y = 0: \; T\cos\theta - mg = 0$$
$$T = mg/\cos\theta$$

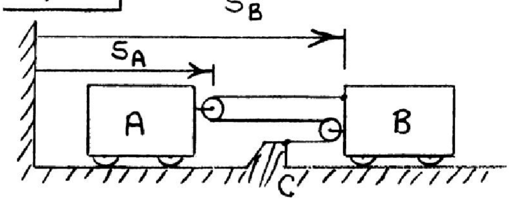
$$\sum F_x = ma_x: \; T\sin\theta = ma$$
$$\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \cancel{m}a, \quad \underline{\theta = \tan^{-1}\left(\frac{a}{g}\right)}$$

Cart M:



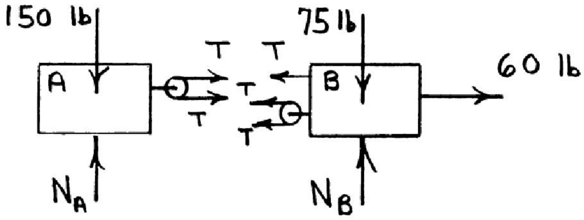
$$\sum F_x = ma_x: \; P - T\sin\theta = Ma$$
$$P = ma + Ma = \underline{(m+M)a}$$

3/36



$$L = 2(s_B - s_A) + (s_B - s_C) + \text{constants}$$

$$\Rightarrow \quad 0 = 3a_B - 2a_A \qquad (1)$$

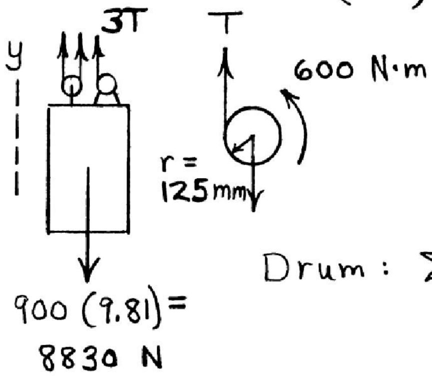


$\overset{+}{\rightarrow}$ $\Sigma F = ma$: (A) $2T = \dfrac{150}{32.2} a_A$ (2)

(B) $60 - 3T = \dfrac{75}{32.2} a_B$ (3)

Solve Eqs. (1)–(3): $\begin{cases} a_A = 7.03 \text{ ft/sec}^2 \\ a_B = 4.68 \text{ ft/sec}^2 \\ T = 16.36 \text{ lb} \end{cases}$

3/37 Case (a) has the higher acceleration because of three (> 2) supporting cables.



Drum: $\Sigma M = 0$: $T(0.125) - 600 = 0$

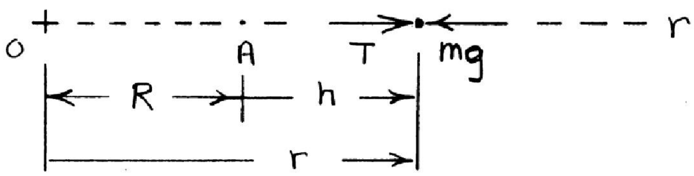
$T = 4800$ N

Elevator: $\Sigma F_y = ma_y$: $3(4800) - 8830 = 900a$

$a = 6.19$ m/s^2

$v = v_0 + at = 0 + 6.19(1.2) = \underline{7.43 \text{ m/s}}$

3/38



$$g = g_0 \frac{R^2}{r^2} \quad \text{(all pertaining to moon)}$$

$$\Sigma F_r = ma_r : \quad T - mg_0 \frac{R^2}{r^2} = mv\frac{dv}{dr}$$

$$\int_R^{2R} \left(\frac{T}{m} - g_0 \frac{R^2}{r^2}\right) dr = \int_0^v v\,dv$$

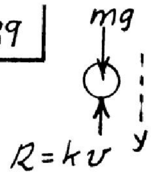
$$\Rightarrow \quad v = \sqrt{\frac{2TR}{m} - g_0 R} = \sqrt{R\left(\frac{2T}{m} - g_0\right)}$$

Numbers :

$$v = \sqrt{\frac{3476(1000)}{2}\left(\frac{2(2500)}{1200} - 1.62\right)}$$

$$= \underline{2100 \text{ m/s}}$$

3/39



$\Sigma F_y = ma_y; \quad mg - kv = ma$

$a = g - \frac{k}{m}v$

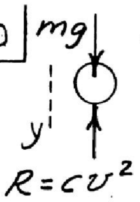
$v\,dv = a\,dy, \quad \int_0^v \frac{v\,dv}{g - \frac{k}{m}v} = \int_0^h dy$

$$\frac{m^2}{k^2}\left[\left(g - \frac{k}{m}v\right) - g\ln\left(g - \frac{k}{m}v\right)\right]_0^v = h$$

$$h = \frac{m^2}{k^2}\left[-\frac{k}{m}v - g\ln\left(1 - \frac{kv}{mg}\right)\right]$$

$$h = \frac{m^2}{k^2}\,g\ln\left(\frac{1}{1 - \frac{kv}{mg}}\right) - \frac{mv}{k}$$

3/40



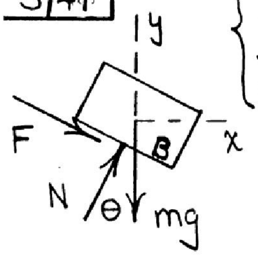
$\Sigma F_y = ma_y;\quad mg - cv^2 = ma$

$a = g - \frac{c}{m}v^2$

$v\,dv = a\,dy,\quad \int_0^v \frac{v\,dv}{g - \frac{c}{m}v^2} = \int_0^h dy$

$-\frac{m}{2c}\ln\left(g - \frac{c}{m}v^2\right)\Big]_0^v = h,\quad h = \frac{m}{2c}\ln\left(\frac{mg}{mg - cv^2}\right)$

3/41



$$\begin{cases} \Sigma F_x = ma_x : & N\sin\theta + F\cos\theta = ma \\ \Sigma F_y = 0 : & N\cos\theta - F\sin\theta - mg = 0 \end{cases}$$

Solving, $\begin{cases} F = m(a\cos\theta - g\sin\theta) \\ N = m(a\sin\theta + g\cos\theta) \end{cases}$

For slip impending, $F = \mu_s N$

or $\cancel{m}(a\cos\theta - g\sin\theta) = \mu_s \cancel{m}(a\sin\theta + g\cos\theta)$

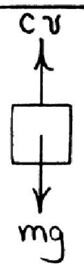
Solve for θ to obtain $\theta = \tan^{-1}\left(\frac{a - \mu_s g}{\mu_s a + g}\right)$

For large a $(a \gg g)$, $\underline{\theta = \tan^{-1}\left(\frac{1}{\mu_s}\right)}$

$$\therefore \underline{\tan^{-1}\left(\frac{1}{\mu_s}\right) \leq \theta \leq \frac{\pi}{2}}$$

3/42

$\Sigma F_y = ma_y$: $\quad mg - cv = ma_y$



$\downarrow y$ (a) For $a_y = 0$, $\quad mg - cv = 0$

$$v_s = \frac{mg}{c} = \frac{100(9.81)}{3000} = \underline{0.327 \text{ m/s}}$$

(b) $mg - cv = m\dfrac{dv}{dt}$

$$\int_0^t dt = \int_0^v \frac{dv}{g - \frac{c}{m}v} = -\frac{m}{c}\int_0^v \frac{-\frac{c}{m}\,dv}{g - \frac{c}{m}v}$$

$$t = -\frac{m}{c}\ln\left(g - \frac{c}{m}v\right)\Big|_0^v = -\frac{m}{c}\ln\left(\frac{g - \frac{c}{m}v}{g}\right)$$

$$\Rightarrow v = \frac{mg}{c}\left[1 - e^{-ct/m}\right] = 0.327\left[1 - e^{-30t}\right]$$

$$\left[1 - e^{-30t}\right] = 0.9 \Rightarrow \underline{t = 0.0768 \text{ s}}$$

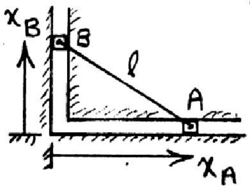
$$v = \frac{dy}{dt} = 0.327\left[1 - e^{-30t}\right]$$

$$\int_0^y dy = \int_0^t 0.327\left[1 - e^{-30t}\right]dt$$

$$y = 0.327\left[t + \tfrac{1}{30}\left(e^{-30t} - 1\right)\right]$$

y when $t = 0.0768$ s is $\underline{y = 0.01529 \text{ m}}$

3/43



$$x_A^2 + x_B^2 = l^2$$

$$\cancel{2} x_A \dot{x}_A + \cancel{2} x_B \dot{x}_B = 0$$

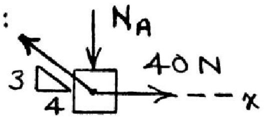
$$x_A \ddot{x}_A + \dot{x}_A^2 + x_B \ddot{x}_B + \dot{x}_B^2 = 0$$

So $\dot{x}_B = -\dfrac{x_A \dot{x}_A}{x_B} = \dfrac{-(0.4)(0.9)}{0.3} = -1.2$ m/s

$$\ddot{x}_B = \frac{-\dot{x}_B^2 - \dot{x}_A^2 - x_A \ddot{x}_A}{x_B} = \frac{-1.2^2 - 0.9^2 - 0.4\ddot{x}_A}{0.3}$$

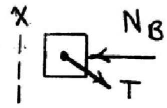
$$= -7.5 - \frac{4}{3}\ddot{x}_A \quad \text{or} \quad a_B = -7.5 - \frac{4}{3} a_A \quad (1)$$

A:



$$\Sigma F_x = ma_x: \; 40 - \frac{4}{5}T = 2a_A \quad (2)$$

B:

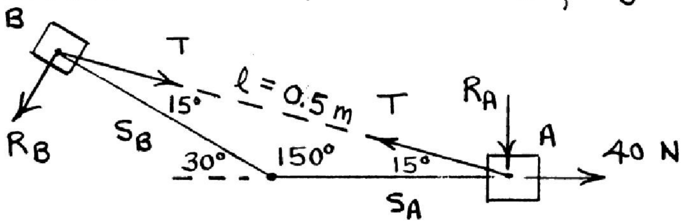


$$\Sigma F_x = ma_x: \; -\frac{3}{5}T = 3a_B \quad (3)$$

Solution of Eqs. (1)-(3): $\begin{cases} a_A = 1.364 \text{ m/s}^2 \\ a_B = -9.32 \text{ m/s}^2 \\ T = 46.6 \text{ N} \end{cases}$

3/44

$$\frac{\sin 150^\circ}{l} = \frac{\sin 15^\circ}{s_B}, \quad s_B = s_A = 0.259 \text{ m}$$



Law of cosines: $l^2 = s_A^2 + s_B^2 - 2 s_A s_B \cos 150^\circ$

$$2 l \dot{l} = 0 = 2 s_A v_A + 2 s_B v_B - 2\left(-\frac{\sqrt{3}}{2}\right)(s_A v_B + s_B v_A)$$

$$s_A v_A + s_B v_B + \frac{\sqrt{3}}{2}(s_A v_B + v_A s_B) = 0^{*}$$

With $s_A = s_B = 0.259$ m, $v_A = 0.4$ m/s : $v_B = -0.4$ m/s

Differentiate $*$: $v_A^2 + s_A a_A + v_B^2 + s_B a_B + \frac{\sqrt{3}}{2}(s_A a_B + v_A v_B + a_A s_B + v_A v_B) = 0$

Numbers: $0.483 a_A + 0.483 a_B + 0.0429 = 0$ (1)

Kinetics:

$\overset{+}{\nwarrow}$ $\Sigma F = m a_B$: $-T \cos 15^\circ = 3 a_B$ (2)

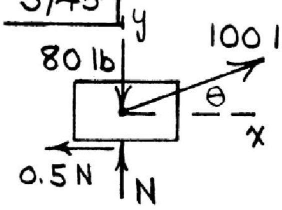
$\xrightarrow{+}$ $\Sigma F = m a_A$: $40 - T \cos 15^\circ = 2 a_A$ (3)

Solution of Eqs. (1)-(3): $T = 25.0$ N

$a_A = 7.95 \text{ m/s}^2$

$a_B = -8.04 \text{ m/s}^2$

3/45



$\Sigma F_x = ma_x: \ 100 \cos\theta - 0.5\,N = \frac{80}{32.2}(26)$

$\Sigma F_y = 0: \ N - 80 + 100 \sin\theta = 0$

Eliminate N to obtain $2\cos\theta + \sin\theta = 2.0919$

Use $\sin\theta = \sqrt{1-\cos^2\theta}$ to obtain

$5\cos^2\theta - 8.3677\cos\theta + 3.3762 = 0$

Solve the quadratic to get $\theta = \underline{5.88°}$, $\underline{47.2°}$

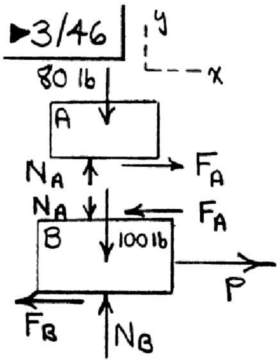
Check that block moves in both cases:

$\theta = 5.88°: \begin{cases} N = 80 - 100 \sin 5.88° = 69.8 \text{ lb} \\ F_{max} = 0.6(69.8) = 41.9 \text{ lb} < 100 \cos 5.88° = 99.5 \text{ lb} \checkmark \end{cases}$

$\theta = 47.2°: \ N = 6.57 \text{ lb}, \ F_{max} = 3.94 \text{ lb}$

$100 \cos 47.2° = 67.9 \text{ lb} \checkmark$

▶3/46



$N_B = 180$ lb

$F_{B\,max} = \mu_s N = (0.15)(180) = 27$ lb

$\therefore$ No motion for $0 \le P \le 27$ lb

$N_A = 80$ lb, $F_{A\,max} = (.2)(80) = 16$ lb

$\Sigma F_x = ma_x$ for A : $16 = \frac{80}{32.2} a_A$

$a_A = 6.44$ ft/sec^2 – max. for no slipping between A and B.

Corresponding P → $\Sigma F_x = ma_x$ for system :

$P - 0.1(180) = \frac{180}{32.2}(6.44)$, $P = 54$ lb.

$27 \le P \le 54$ lb :

$\Sigma F = ma_x$: $P - 0.1(180) = \frac{180}{32.2} a$

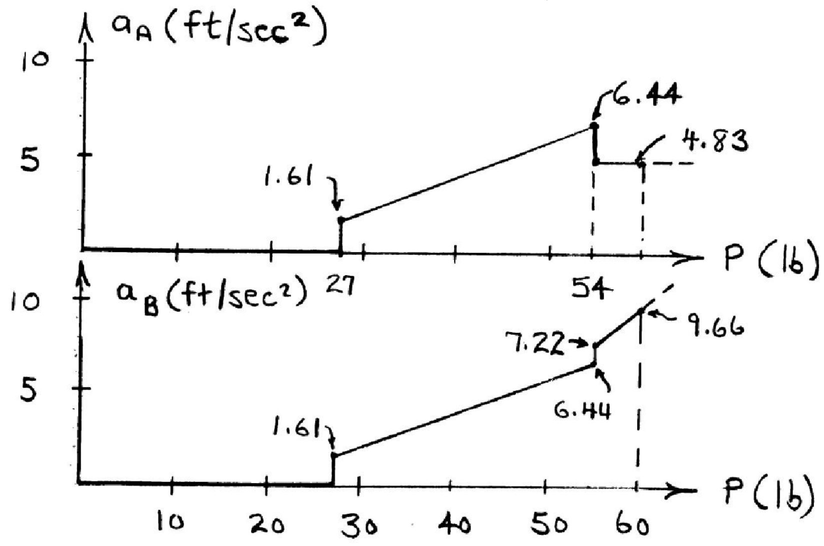
$a_A = a_B = a = 0.1789\,P - 3.22$

$P \ge 54$ lb :

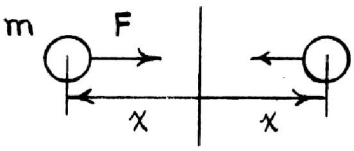
A : $0.15(80) = \frac{80}{32.2} a_A$; $a_A = 4.83$ ft/sec^2

B : $P - (0.1)(180) - (0.15)(80) = \frac{100}{32.2} a_B$

$a_B = 0.322P - 9.66$



▸3/47



$$F = \frac{Gm^2}{x^2}$$

$$m = \rho V = 7210\left(\frac{4}{3}\pi\, 0.05^3\right)$$
$$= 3.775 \text{ kg}$$

$$\Sigma F_x = ma_x: \quad -\frac{Gm^2}{(2x)^2} = mv\frac{dv}{dx}$$

$$-\frac{Gm}{4}\int_{x_0=0.5}^{x}\frac{dx}{x^2} = \int_{v_0=0}^{v} v\,dv$$

$$v = \sqrt{Gm}\sqrt{\frac{1}{2x}-1} = \sqrt{(6.673\times10^{-11})(3.775)}\sqrt{\frac{1}{2(0.05)}-1}$$

$$= \underline{4.76\times10^{-5} \text{ m/s}}$$

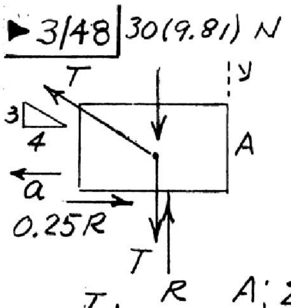
Now, $\dfrac{dx}{dt} = -\sqrt{Gm}\sqrt{\dfrac{\frac{1}{2}-x}{x}}$

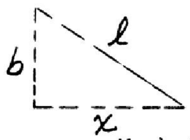
$$\int_{x_0=0.5}^{x=0.05}\frac{\sqrt{x}\,dx}{\sqrt{\frac{1}{2}-x}} = -\sqrt{Gm}\int_0^t dt$$

$$\left[-\sqrt{x}\sqrt{\tfrac{1}{2}-x} + \tfrac{1}{2}\sin^{-1}\sqrt{2x}\right]_{x_0=0.5}^{x=0.05} = -\sqrt{Gm}\,t$$

Solving, $\underline{t = 48{,}800 \text{ s}}$ or $\underline{t = 13 \text{ hr } 33 \text{ min}}$

▶3/48





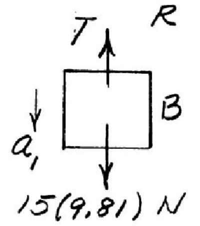
$a = -\ddot{x} \qquad a_1 = -\ddot{\ell}$

$\ell^2 = b^2 + x^2$

$\ell\dot{\ell} = 0 + x\dot{x}$

$\underbrace{\dot{\ell}^2}_{0} + \ell\ddot{\ell} = \underbrace{\dot{x}^2}_{0} + x\ddot{x}\,, \quad \ddot{\ell} = \frac{x}{\ell}\ddot{x}$

or $a_1 = \frac{4}{5}a$



$A;\ \Sigma F_y = 0;\ R + \frac{3}{5}T - 30(9.81) - T = 0$

$R = 0.4T + 294.3$

$\Sigma F_x = ma_x;\ 0.8T - 0.25(0.4T + 294.3) = 30a$

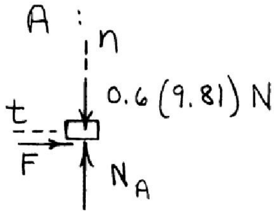
$B;\ \Sigma F_y = ma_y;\ 15(9.81) - T = 15\left(\frac{4}{5}a\right)$

Solve simultaneously to get $\underline{T = 138.0\ \text{N}}$

$\underline{a = 0.766\ \text{m/s}^2}$

3/49

A:

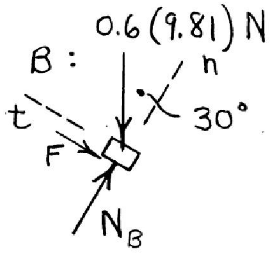


$\Sigma F_n = m\frac{v^2}{\rho}$:

$N_A - 0.6(9.81) = 0.6\frac{5^2}{3}$

$\underline{N_A = 10.89 \text{ N}}$

B:



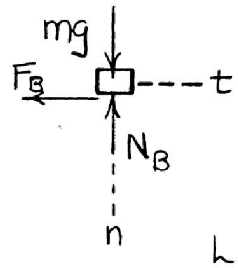
$\Sigma F_n = m\frac{v^2}{\rho}$:

$N_B - 0.6(9.81)\cos 30° = 0.6\frac{4^2}{3}$

$\underline{N_B = 8.30 \text{ N}}$

Note: Friction is along the t-axis and does not affect the above calculations.

3/50

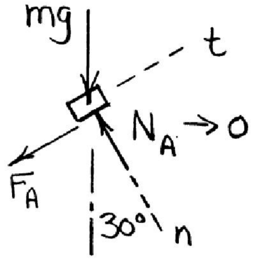


$$\Sigma F_n = m a_n = m \frac{v^2}{\rho}:$$

$$2(9.81) - N = 2\,\frac{3.5^2}{2.4}$$

$$\underline{N_B = 9.41 \text{ N}}$$

Loss of contact at A: $N_A \to 0$

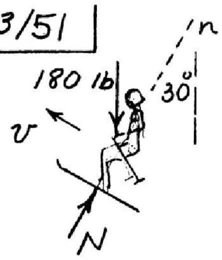


$$\Sigma F_n = m a_n = m \frac{v^2}{\rho}:$$

$$\cancel{m} g \cos 30^\circ = \cancel{m} \frac{v^2}{2.4}$$

$$\underline{v = 4.52 \text{ m/s}}$$

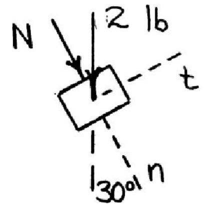
3/51



$$\Sigma F_n = ma_n; \quad N - 180\cos 30^\circ = \frac{180}{32.2}\,\frac{(80)^2}{150}$$

$$N = 180(0.866 + 1.33) = \underline{394 \text{ lb}}$$

3/52



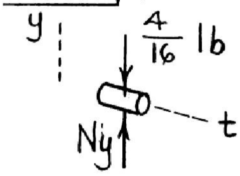
$\Sigma F_n = m\frac{v^2}{\rho}: \; N + 2\cos 30° = \frac{2}{32.2}\,\frac{10^2}{2}$

$\underline{N = 1.374 \text{ lb}}$

$\Sigma F_t = ma_t: \; -2\sin 30° = \frac{2}{32.2}\,\dot{v}$

$\underline{\dot{v} = -16.10 \text{ ft/sec}^2}$

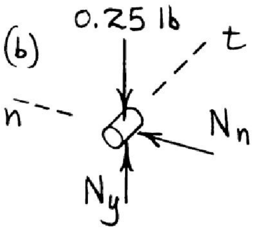
3/53 (a)



$\Sigma F_y = 0: \quad N_y - \frac{4}{16} = 0$

$N_y = R = \underline{0.25 \text{ lb}}$

(b)



$N_y = 0.25$ lb, as in (a)

$\Sigma F_n = m\frac{v^2}{\rho}: \quad N_n = \frac{0.25}{32.2} \frac{3^2}{8/12}$

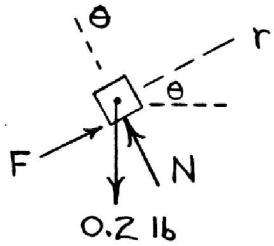
$N_n = 0.1048$ lb

$R = \sqrt{N_y^2 + N_n^2} = \underline{0.271 \text{ lb}}$

3/54

$\sum F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$: $N - 0.2\cos 30°$

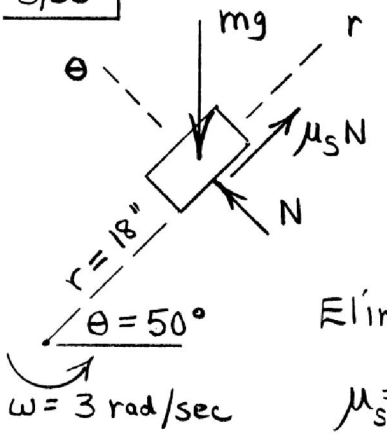
Slider:



$$= \frac{0.2}{32.2}\left(r\cancelto{0}{\ddot{\theta}} + 2(-4)(3)\right)$$

$\underline{N = 0.024 \text{ lb}}$

3/55



$$\Sigma F_\theta = ma_\theta : \; N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

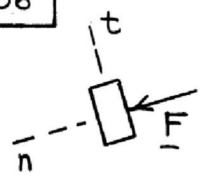
$$\Sigma F_r = ma_r : \; \mu_s N - mg\sin\theta = m(0 - r\omega^2)$$

Eliminate N:

$$\mu_s = \tan\theta - \frac{r\omega^2}{g\cos\theta}$$

Numbers: $\mu_s = \tan 50° - \dfrac{(18/12)(3)^2}{32.2\cos 50°} = \underline{0.540}$

3/56



$$a_n = \frac{v^2}{\rho} = \frac{\left[(35)\left(\frac{5280}{3600}\right)\right]^2}{100}$$

$$= 26.4\ \frac{ft}{sec^2}\left(\frac{1g}{32.2\ ft/sec^2}\right)$$

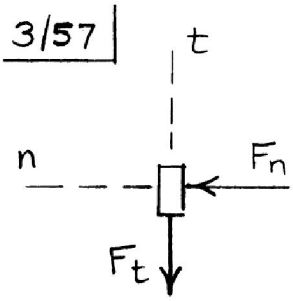
$$= \underline{0.818\ g}$$

$$\Sigma F_n = ma_n : \quad F = \frac{3000}{32.2}(26.4)$$

$$= \underline{2455\ lb}$$

(An average of 614 lb per tire!)

3/57



$$\Sigma F_n = m a_n : \quad F_n = \frac{3000}{32.2} \frac{\left(25 \cdot \frac{5280}{3600}\right)^2}{100}$$

$$F_n = 1253 \text{ lb}$$

$$\sqrt{F_n^{\,2} + F_t^{\,2}} = F_{tot}$$

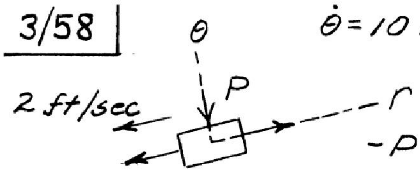
$$1253^2 + F_t^{\,2} = 2400^2$$

$$F_t = 2047 \text{ lb}$$

$$\Sigma F_t = m a_t : \quad -2047 = \frac{3000}{32.2} a_t$$

$$\underline{a_t = -22.0 \text{ ft/sec}^2}$$

3/58



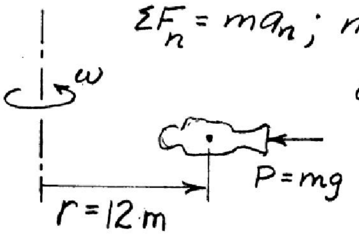
$\dot{\theta} = 10 \text{ rad/sec}, \; \dot{r} = -2 \text{ ft/sec}$

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$-P = \frac{3.22}{32.2}(0 + 2[-2]10)$

$\underline{P = 4 \text{ lb} \quad (\text{side } A)}$

3/59 g = surface gravitational acceleration on earth

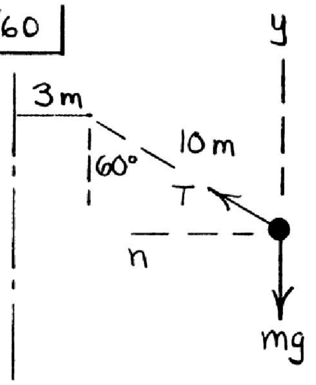


$$\Sigma F_n = ma_n;\quad mg = mr\omega^2,\quad \omega = \sqrt{g/r}$$

$$\omega = \sqrt{\frac{9.81}{12}} = 0.904 \text{ rad/s}$$

so $N = 0.904\left(\frac{60}{2\pi}\right) = \underline{8.63 \text{ rev/min}}$

3/60



$\Sigma F_n = ma_n$:

$T \sin 60° = m[3 + 10 \sin 60°]\,\omega^2$

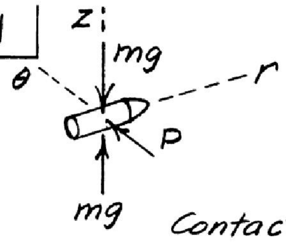
$\Sigma F_y = 0$: $T \cos 60° - mg = 0$

$\Rightarrow \tan 60° = \dfrac{3 + 10 \sin 60°}{9.81}\,\omega^2$

$\omega = 1.207$ rad/s

$N = 1.207\left(\dfrac{60}{2\pi}\right) = \underline{11.53 \text{ rev/min}}$

3/61



$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$P = 0.06\,(0 + 2[600][0.5])$$

$$= 36 \text{ N}$$

Contact is against right-hand side of barrel.

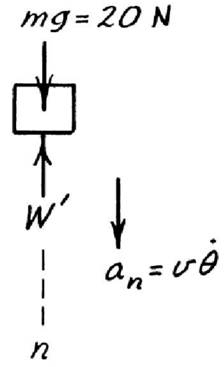
3/62

Make $a_n = g$ $\quad a_n = \frac{v^2}{\rho} = v\dot{\theta}, \quad \dot{\theta} = \frac{a_n}{v} = \frac{g}{v}$

$$\dot{\theta} = \frac{9.81}{\frac{600(1000)}{3600}} = 5.89(10^{-2}) \text{ rad/s} \quad \text{or}$$

$$\underline{\dot{\theta} = 3.37 \text{ deg/s}}$$

3/63



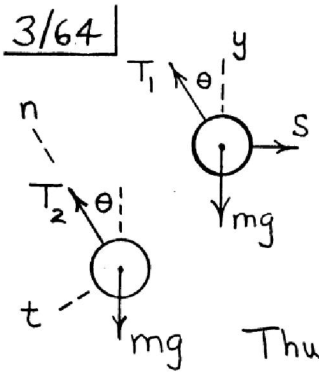
$W = mg$: $m = 20/9.81 = 2.04$ kg

$\Sigma F_n = ma_n$: $20 - W' = 2.04\left(\frac{800 \times 1000}{3600}\,\frac{(1)\pi}{180}\right)$

$= 2.04(3.88) = 7.91$ N

$W' = 20 - 7.91 = \underline{12.09 \text{ N}}$

3/64



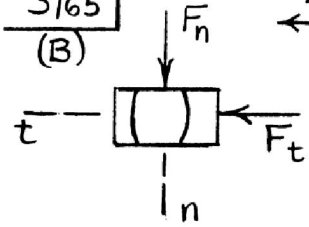
Before: $\sum F_y = 0: T_1 \cos\theta = mg$

After: $\sum F_n = ma_n = m\frac{v^2}{r} = 0$

$T_2 = mg\cos\theta$

Thus $k = T_2/T_1 = \frac{mg\cos\theta}{mg/\cos\theta} = \underline{\cos^2\theta}$

3/65
(B)



$\overset{+}{\leftarrow}$ $v_B^2 = v_A^2 + 2a_t s$

$(+88)^2 = (-66)^2 + 2a_t(-300)$

$a_t = +5.65 \text{ ft/sec}^2$

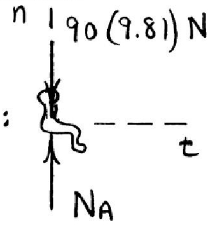
$a_n = v^2/\rho = (66)^2/600 = 7.26 \text{ ft/sec}^2$

$$\begin{cases} \Sigma F_t = ma_t: & F_t = \dfrac{3220}{32.2}(5.65) = 565 \text{ lb} \\ \Sigma F_n = ma_n: & F_n = \dfrac{3220}{32.2}(7.26) = 726 \text{ lb} \end{cases}$$

$F = \sqrt{565^2 + 726^2} = \underline{920 \text{ lb}}$

3/66

A:

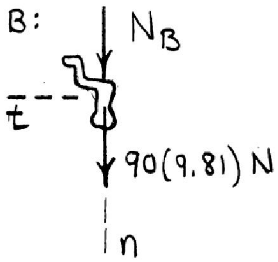


$$\Sigma F_n = ma_n:$$

$$N_A - 90(9.81) = 90\,\frac{(600/3.6)^2}{1000}$$

$$\underline{N_A = 3380\ \text{N}}$$

B:



$$\Sigma F_n = ma_n:$$

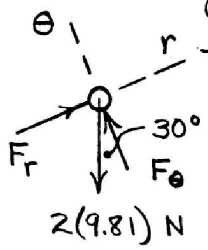
$$N_B + 90(9.81) = 90\,\frac{(600/3.6)^2}{1000}$$

$$\underline{N_B = 1617\ \text{N}}$$

(Note static normal $mg = 90(9.81) = 883$ N)

3/67

F_r and F_θ are the r- and θ-components of the total friction force F.



$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$:

$F_r - 19.62 \sin 30° = 2[0 - 1(-0.873)^2]$

$F_r = 8.29$ N

$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$F_\theta - 19.62 \cos 30° = 2[(1)(3.49) + 2(-0.5)(-0.873)]$

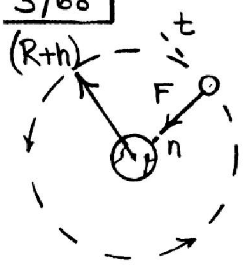
$F_\theta = 25.7$ N

$F = \sqrt{F_r^2 + F_\theta^2} = 27.0$ N

$P = \frac{F/2}{\mu_s} = \frac{27.0/2}{0.5} = \underline{27.0 \text{ N}}$

(Static gripping force $= \underline{19.62 \text{ N}}$)

3/68



$$\Sigma F_n = ma_n : F = \frac{Gm_e m}{(R+h)^2} = m\frac{v^2}{(R+h)}$$

But $v = \frac{s}{t} = \frac{2\pi(R+h)}{(23.944)(3600)}$

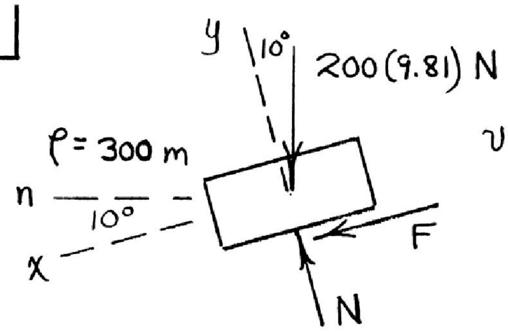
Combining the two equations :

$$v = \frac{2\pi(R+h)}{(23.944)(3600)} = \sqrt{\frac{Gm_e}{(R+h)}}$$

Solve for h to obtain $h = 3.580 \times 10^7$ m

(35,800 km)

3/69



$v = \frac{100}{3.6} = 27.8 \text{ m/s}$

$\Sigma F_x = ma_x : \quad F + 200(9.81)\sin 10° = 200 \frac{27.8^2}{300} \cos 10°$

$\underline{F = 165.9 \text{ N}}$

Check: $\Sigma F_y = ma_y$:

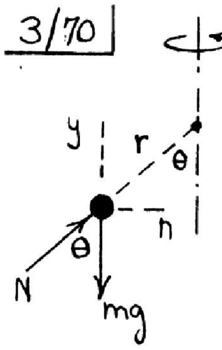
$N - 200(9.81)\cos 10° = 200 \frac{27.8^2}{300} \sin 10°$

$N = 2020 \text{ N}$

$F_{max} = \mu_s N = 0.70(2020) = 1415 \text{ N} > F$

Crate does not slip.

3/70



$\Sigma F_y = 0: \quad N\cos\theta - mg = 0$

$N = mg/\cos\theta$

$\Sigma F_n = ma_n: \quad N\sin\theta = m(r\sin\theta)\omega^2$

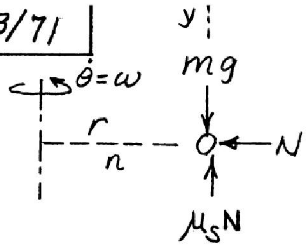
$\left(\frac{\cancel{m}g}{\cos\theta}\right)\cancel{\sin}\theta = \cancel{m}r\cancel{\sin}\theta\,\omega^2$

$\underline{\omega = \sqrt{\frac{g}{r\cos\theta}}}$

Note that $\cos\theta = \frac{g}{r\omega^2} \leq 1$

$\therefore \quad \omega^2 \geq \frac{g}{r}$ is a restriction.

3/71

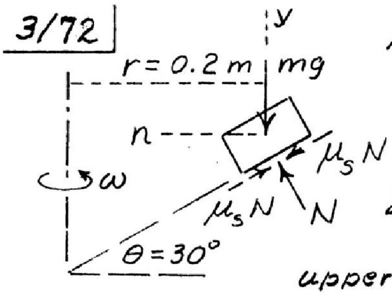


$$\Sigma F_n = ma_n; \quad N = mr\omega^2$$

$$\Sigma F_y = 0; \quad \mu_s(mr\omega^2) = mg$$

$$\omega^2 = \frac{g}{\mu_s r}, \quad \underline{\omega = \sqrt{\frac{g}{\mu_s r}}}$$

3/72



$\mu_s N$ is down for ω_{max}

" " up " ω_{min}

$\Sigma F_y = 0$; $N\cos\theta \mp \mu_s N\sin\theta = mg$

$\Sigma F_n = ma_n$; $N\sin\theta \pm \mu_s N\cos\theta = mr\omega^2$

upper sign for ω_{max}

lower sign for ω_{min}

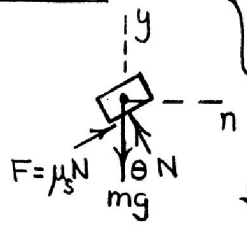
Combine & get $\dfrac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s\sin\theta} = \dfrac{r\omega^2}{g}$

$$\omega = \sqrt{\frac{g}{r}}\sqrt{\frac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s\sin\theta}} = \sqrt{\frac{9.81}{0.2}}\sqrt{\frac{0.5 \pm 0.3(0.866)}{0.866 \mp 0.3(0.5)}}$$

Upper sign $\omega_{max} = \underline{7.21 \text{ rad/s}}$

Lower sign $\omega_{min} = \underline{3.41 \text{ rad/s}}$

3/73

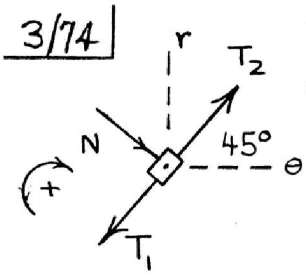


$$\begin{cases} \Sigma F_y = 0: \; N\cos\theta - mg + \mu_s N \sin\theta = 0 \\ \Sigma F_n = ma_n: \; -N\sin\theta + \mu_s N\cos\theta = mr\omega^2 \end{cases}$$

Solving for ω:

$$\omega = \sqrt{\frac{g}{r}\,\frac{(\mu_s\cos\theta - \sin\theta)}{(\cos\theta + \mu_s\sin\theta)}} = \underline{2.73 \text{ rad/s}}$$

3/74



$\omega = \omega_0 + \alpha t = +0.5t$

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1(+0.5t)^2$
$= -0.025t^2$ (m/s^2)

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.1)(+0.5)$
$= 0.05$ m/s^2

Component of total acceleration in direction of wire #2: $0.05 \cos 45° - 0.025t^2 \cos 45°$
Setting this to zero yields $t = 1.414$ s as the time when tension switches from wire #2 to wire #1.

$0 < t < 1.414 s$:

$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2)$: $-N\cos 45° + T_2 \cos 45°$
$= 2(-0.1)(+0.5t)^2$

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$: $+N\cos 45° + T_2\cos 45°$
$= 2(0.1)(+0.5)$

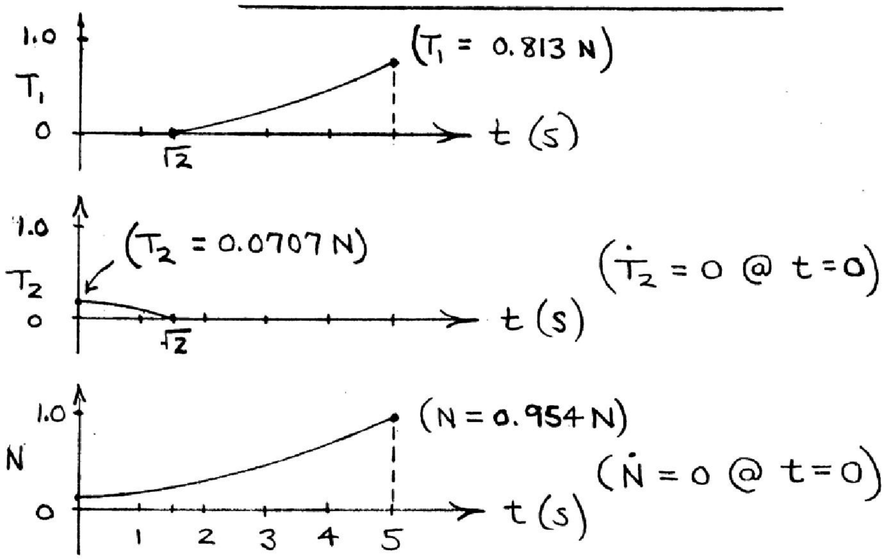
Solving, $\begin{cases} T_2 = 0.0707 - 0.0354t^2 \\ N = 0.0707 + 0.0354t^2 \end{cases}$

$1.414 \le t \le 5$ s

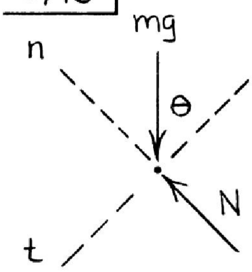
$$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2): -N\cos 45^\circ + T_1 \cos 45^\circ = 2(-0.1)(+0.5t)^2$$

$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}): N\cos 45^\circ - T_1 \cos 45^\circ = 2(0.1)(-0.5)$$

Solving, $\begin{cases} T_1 = -0.0707 + 0.0354t^2 \\ N = 0.0707 + 0.0354t^2 \end{cases}$



3/75



Treat the child as a particle.

$$\begin{cases} \Sigma F_t = ma_t : & mg\cos\theta = ma_t \quad (1) \\ \Sigma F_n = ma_n : & N - mg\sin\theta = m\dfrac{v^2}{R} \quad (2) \end{cases}$$

From (1) : $g\cos\theta = v\dfrac{dv}{ds} = v\dfrac{dv}{R\,d\theta}$

$$\int_{\theta_0 = 20°}^{\theta} Rg\cos\theta\, d\theta = \int_{v_0 = 0}^{v} v\, dv$$

$$v = \left[2Rg\left(\sin\theta - \sin 20°\right)\right]^{1/2}$$

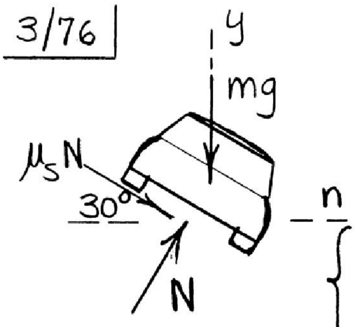
(2) : $N = m\left(g\sin\theta + \dfrac{v^2}{R}\right)$

Numbers ($R = 2.5$ m , $g = 9.81$ m/s^2)

$\theta = 30°$: $\begin{cases} \underline{a_t = 8.50 \text{ m/s}^2} \\ \underline{v = 2.78 \text{ m/s}} \\ \underline{N = 280 \text{ N}} \end{cases}$

$\theta = 90°$: $\begin{cases} \underline{a_t = 0} \\ \underline{v = 5.68 \text{ m/s}} \\ \underline{N = 795 \text{ N}} \end{cases}$

3/76



For no slipping tendency, set $F_{max} = \mu_s N$ to zero in FBD.

$$\begin{cases} \Sigma F_y = 0: & N\cos 30° - mg = 0 \\ \Sigma F_n = m\dfrac{v^2}{\rho}: & N\sin 30° = m\dfrac{v^2}{1200} \end{cases}$$

Solve: $N = 1.155mg$, $v = 149.4$ ft/sec

or $\underline{v = 101.8 \text{ mi/hr}}$

$\underline{v_{min} = 0}$, as $\theta_{max} = \tan^{-1}\mu_s = \tan^{-1}(0.9)$
$= 42.0° > 30°$

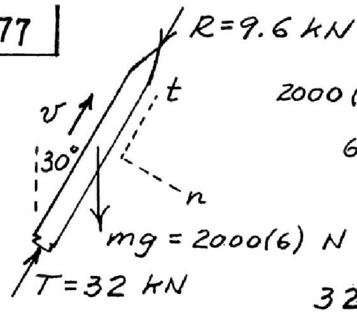
For v_{max}, use F_{max} as shown in FBD.

$$\begin{cases} \Sigma F_y = 0: & N\cos 30° - mg - \mu_s N\sin 30° = 0 \\ \Sigma F_n = m\dfrac{v^2}{\rho}: & \mu_s N\cos 30° + N\sin 30° = m\dfrac{v_{max}^2}{\rho} \end{cases}$$

With $\mu_s = 0.9$: $N = 2.40mg$

$v_{max} = 345$ ft/sec ($\underline{235 \text{ mi/hr}}$).

3/77



$\Sigma F_n = ma_n$;

$$2000(6)\sin 30° = 2000\,\frac{(3000)^2}{\rho}$$

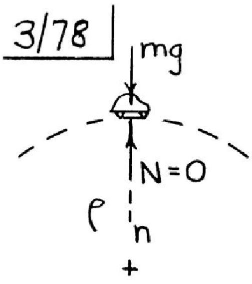
$$6\left(\tfrac{1}{2}\right) = 9(10^6)/\rho\,,\quad \underline{\rho = 3000\ \text{km}}$$

$\Sigma F_t = m\dot{v}$

$$32(10^3) - 2000(6)\cos 30° - 9600 = 2000\dot{v}$$

$$\underline{\dot{v} = 6.00\ \text{m/s}^2}$$

3/78



$\Sigma F_n = ma_n: \quad mg = m\,v^2/\rho\,, \quad v = \sqrt{g\rho}$

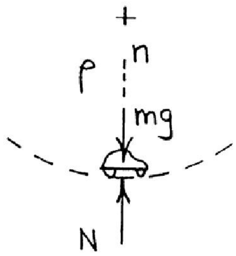
At top of hump, $\frac{dy}{dx} = 0$ and radius of curvature is

$$\rho = \left(\frac{d^2y}{dx^2}\right)^{-1} \text{ where } y = b \sin \frac{2\pi x}{L}$$

$$\frac{dy}{dx} = \frac{2\pi b}{L} \cos \frac{2\pi x}{L}\,, \quad \frac{d^2y}{dx^2} = -\frac{4\pi^2 b}{L^2} \sin \frac{2\pi x}{L}$$

At top of hump, $x = L/4$, $\sin \frac{2\pi x}{L} = 1$,

and $\left|\frac{d^2y}{dx^2}\right| = \frac{1}{\rho} = \frac{4\pi^2 b}{L^2}$, thus $\underline{v = \frac{L}{2\pi}\sqrt{g/b}}$



At bottom of dip:

$\Sigma F_n = ma_n: \quad N - mg = m\frac{g\rho}{\rho}$

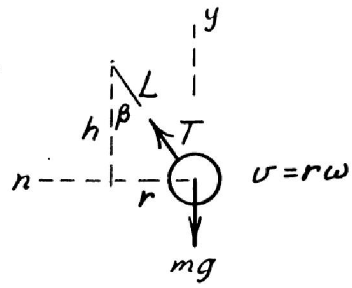
$\underline{N = 2mg}$

3/79

$\Sigma F_y = 0$: $T\cos\beta - mg = 0$, $T\cos\beta = mg$

$\Sigma F_n = ma_n$: $T\sin\beta = mv^2/r$

Divide & get $\tan\beta = \dfrac{v^2}{gr} = \dfrac{r\omega^2}{g}$



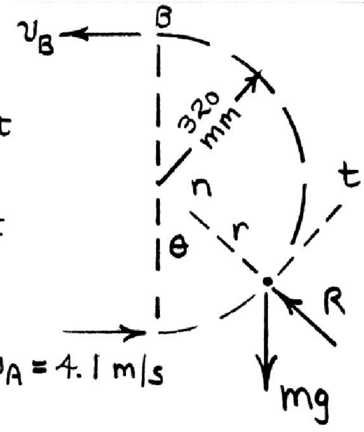
But $r = L\sin\beta$ so $\tan\beta = L\omega^2\sin\beta/g$

or $L\cos\beta = g/\omega^2$

And $h = L\cos\beta$ so $\underline{h = g/\omega^2}$ (depends only on ω & g)

Then $\underline{T} = \dfrac{mg}{\cos\beta} = \dfrac{mg}{h/L} = \dfrac{mgL}{g/\omega^2} = \underline{mL\omega^2}$

3/80



$\Sigma F_t = ma_t$: $-mg\sin\theta = ma_t$

$a_t = -g\sin\theta$

$\Sigma F_n = ma_n$: $R - mg\cos\theta = m\dfrac{v^2}{r}$

$R = m\left(g\cos\theta + \dfrac{v^2}{r}\right)$

$$v\,dv = a_t(r\,d\theta): \int_{4.1}^{v} v\,dv = \int_0^{\theta} (-9.81\sin\theta)(0.320)\,d\theta$$

$v^2 = 10.53 + 6.28\cos\theta$

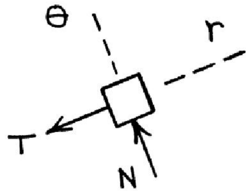
Thus $R = 0.065\left(9.81\cos\theta + \dfrac{10.53 + 6.28\cos\theta}{0.320}\right)$

$= \underline{2.14 + 1.913\cos\theta \quad N}$

For $\theta = 180°$: $v_B^2 = 10.53 + 6.28(-1)$

$\underline{v_B = 2.06 \text{ m/s}}$

3/81



$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2):$$

$$-T = \frac{3}{32.2}\left(0 - \frac{9}{12}6^2\right)$$

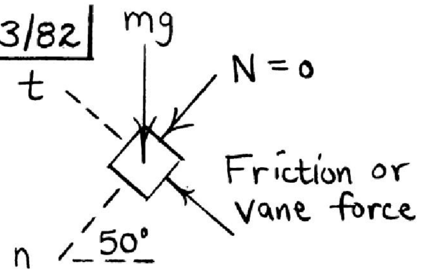
$$\underline{T = 2.52 \text{ lb}}$$

$$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}):$$

$$N = \frac{3}{32.2}\left[\frac{9}{12}(-2) + 2\left(-\frac{2}{12}\right)(6)\right]$$

$\underline{N = -0.326 \text{ lb}}$ (Contact on side B)

3/82

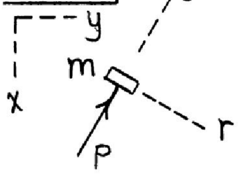


$$\Sigma F_n = ma_n: \quad mg\sin 50° = mr\Omega^2$$

$$\Omega = \sqrt{\frac{g\sin 50°}{r}} = \sqrt{\frac{9.81\sin 50°}{0.330}} = \underline{4.77\ \text{rad/s}}$$

$$(45.6\ \text{rev/min})$$

3/83



(Note : mg & static normal $\perp$ to paper)

$$\Sigma F_r = ma_r: \quad 0 = m\left(\ddot{r} - r\Omega^2\right) \quad (1)$$

$$\Sigma F_\theta = ma_\theta: \quad P = m\left(\underbrace{r\ddot{\theta}}_{0} + 2\dot{r}\Omega\right) \quad (2)$$

(1): $\ddot{r} = \dot{r}\dfrac{d\dot{r}}{dr} = r\Omega^2$

$$\int_{\dot{r}_0}^{\dot{r}} \dot{r}\,d\dot{r} = \int_{r_0}^{r} \Omega^2 r\,dr \Rightarrow \dot{r}^2 = \dot{r}_0^2 + \Omega^2\left(r^2 - r_0^2\right)$$

Numbers : (at end of tube) $\dot{r} = \left[60^2 + 7^2\left(3^2 - \left(\frac{6}{12}\right)^2\right)\right]^{1/2} = 63.5 \ \frac{\text{ft}}{\text{sec}}$

(2): $P = m\left(2\dot{r}\Omega\right) = \dfrac{5/16}{32.2}(2)(63.5)(7)$

$= \underline{8.62 \text{ lb}}$

3/84 The distance traveled from A to C is

$$(s_C - s_A) = 100 + 250\left(30\frac{\pi}{180}\right) = 231 \text{ ft}$$

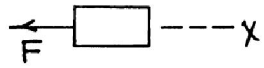
Uniform tangential acceleration : $v_C^2 = v_A^2 + 2a_t(s_C - s_A)$

$$0^2 = \left[60\,\frac{5280}{3600}\right]^2 + 2a_t(231)\,, \quad a_t = -16.77 \text{ ft/sec}^2$$

Speed at B: $v_B^2 = v_A^2 + 2a_t(s_B - s_A)$

$$v_B^2 = \left[60\,\frac{5280}{3600}\right]^2 + 2(-16.77)(100)\,, \quad v_B = 66.3 \text{ ft/sec}$$

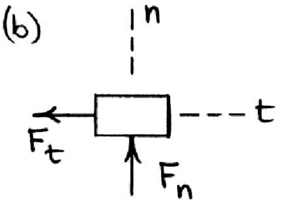
(a)



$$\Sigma F_x = ma_x: \quad -F = \frac{3000}{32.2}(-16.77)$$

$$\underline{F = 1562 \text{ lb}}$$

(b)



$$\Sigma F_t = ma_t: \quad -F_t = \frac{3000}{32.2}(-16.77)$$

$$F_t = 1562 \text{ lb}$$

$$\Sigma F_n = m\frac{v^2}{\rho}: \quad F_n = \frac{3000}{32.2}\,\frac{66.3^2}{250}$$

$$F_n = 1636 \text{ lb}$$

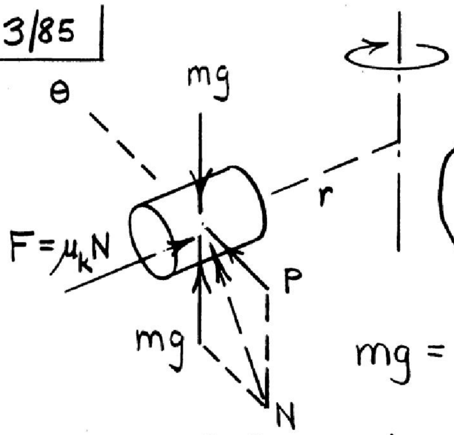
$$F = \sqrt{F_t^2 + F_n^2} = \underline{2260 \text{ lb}}$$

(c) v and therefore F_n go to zero;

$$\underline{F = F_t = 1562 \text{ lb}}$$

(In all FBDs, there is a weight into the paper and a static normal force out of the paper.)

3/85



$\dot{\theta} = 6$ rad/s = constant

(Note: Spring forces not shown on FBD.)

$mg = 0.8(9.81) = 7.85$ N

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

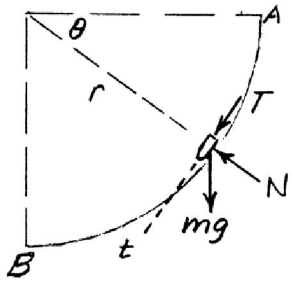
$P = 0.8(0 + 2(0.8)6) = 7.68$ N

$N = \sqrt{7.68^2 + 7.85^2} = 10.98$ N

$F = \mu_k N = 0.4(10.98) = \underline{4.39 \text{ N}}$

3/86 $\Sigma F_t = ma_t$; $T + mg\cos\theta = m a_t$

$a_t = \frac{T}{m} + g\cos\theta$



$v\,dv = a_t(r\,d\theta)$

$$\int_0^v v\,dv = \int_0^{\pi/2}\left(\frac{T}{m} + g\cos\theta\right) r\,d\theta$$

$$\frac{v^2}{2} = \frac{Tr}{m}\frac{\pi}{2} + gr\sin\theta\Big]_0^{\pi/2}$$

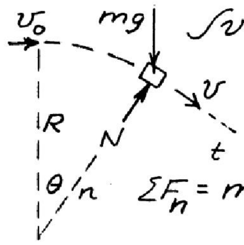
$$= \frac{Tr\pi}{2m} + gr$$

$v^2 = r\left(\frac{\pi T}{m} + 2g\right)$ $\quad \underline{v = \sqrt{r\left(\frac{\pi T}{m} + 2g\right)}}$

At B, $\Sigma F_n = ma_n$; $N - mg = m\frac{v^2}{r}$

$N = mg + T\pi + 2mg$, $\quad \underline{N = T\pi + 3mg}$

3/87 $\Sigma F_t = ma_t$; $mg \sin\theta = ma_t$, $a_t = g\sin\theta$



$\int v\,dv = \int a_t\,ds$; $\int_{v_0}^{v} v\,dv = \int_0^{\theta} g\sin\theta\,(R\,d\theta)$

$$v^2 = v_0^2 + 2gR(1-\cos\theta)$$

$\Sigma F_n = ma_n$; $mg\cos\theta - N = m\dfrac{v^2}{R}$

$$N = mg\cos\theta - \frac{m}{R}v_0^2 - 2mg(1-\cos\theta)$$

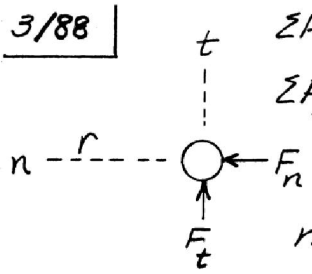
$$= mg\left(3\cos\theta - 2 - \frac{v_0^2}{gR}\right)$$

when $N=0$, $\theta=\beta$ so $3\cos\beta = 2 + \dfrac{v_0^2}{gR}$

$$\beta = \cos^{-1}\left(\frac{2}{3} + \frac{v_0^2}{3gR}\right)$$

For $v_0 = 0$, $\beta = \cos^{-1}\left(\frac{2}{3}\right) = \underline{48.2°}$

3/88



$\Sigma F_t = m a_t$; $F_t = m r \alpha$

$\Sigma F_n = m a_n$; $F_n = m r \omega^2$

$F_{max} = \mu_s N = \mu_s m g$ so

$$m r \sqrt{\alpha^2 + \omega^4} = \mu_s m g$$

& $\omega^2 = \frac{1}{r}\sqrt{\mu_s^2 g^2 - r^2\alpha^2}$ But for constant α,

$\omega^2 = 2\alpha\theta = 2\alpha(2\pi N)$

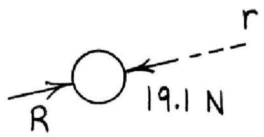
Thus no. of rev. $N = \dfrac{\omega^2}{4\pi\alpha} = \dfrac{1}{4\pi}\sqrt{\left(\dfrac{\mu_s g}{r\alpha}\right)^2 - 1}$

3/89

$$\begin{cases} r = 100 + 10 \sin 6(12)t & \text{mm} \\ \dot{r} = 720 \cos 72t & \text{mm/s} \\ \ddot{r} = -51.84 \sin 72t & \text{m/s}^2 \end{cases}$$

For r a maximum, $\cos 72t = 0$

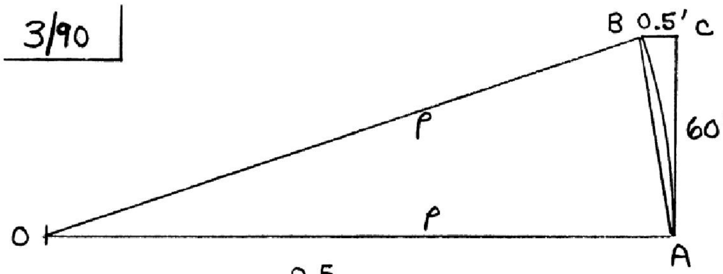
$$a_r = \ddot{r} - r\omega^2 = -51.84 - 0.110(12)^2 = -66.7 \ \frac{m}{s^2}$$



$\Sigma F_r = ma_r : R - 19.1 = 0.1(-67.7)$

$R = 12.33$ N

3/90



$\measuredangle BAC = \tan^{-1} \frac{0.5}{60} = 0.477°$

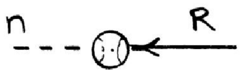
$\measuredangle OBA = \measuredangle OAB = (90 - 0.477) = 89.5°$

$\measuredangle BOA = 180 - 2(89.5) = 0.955° = 2 \measuredangle BAC$

$\overline{AB} = \sqrt{60^2 + 0.5^2} = 60.002'$

$\frac{\sin 0.955°}{60.002'} = \frac{\sin 89.5°}{\rho}$, $\underline{\rho = 3600 \text{ ft}}$

FBD: (horizontal forces)

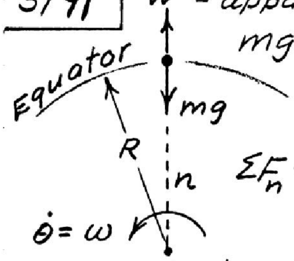


$\Sigma F_n = ma_n : R = \frac{5.125/16}{32.2} \frac{120^2}{3600}$

$\underline{R = 0.0398 \text{ lb}}$

(Note: R = 0.637 oz represents 12.4% of the weight of the baseball)

3/91



W' = apparent weight (force to support m)

mg = true gravitational attraction

where g = absolute acceleration due to gravity

$\Sigma F_n = m a_n$; $mg - W' = mR\omega^2$

$W' = m(g - R\omega^2)$

$$k = \frac{W'}{mg} = 1 - \frac{R\omega^2}{g}$$

For values given $W' = mg\left(1 - \frac{R\omega^2}{g}\right)$

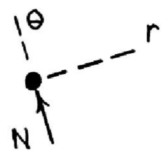
$$= 100\left(1 - \frac{6378(10^3)(0.729)^2(10^{-8})}{9.815}\right) = 100(1 - 0.00345)$$

$$= \underline{99.655\ N}$$

3/92

$\Sigma F_r = ma_r: \quad 0 = m(\ddot{r} - r\dot{\theta}^2)$

Particle:



$$\ddot{r} = r\dot{\theta}^2 = r\omega_0^2$$

$$\dot{r}\,\frac{d\dot{r}}{dr} = r\omega_0^2$$

$$\int_0^{\dot{r}} \dot{r}\,d\dot{r} = \omega_0^2 \int_{r_0}^{r} r\,dr$$

$$\Rightarrow \underline{\dot{r} = \omega_0\sqrt{r^2 - r_0^2} = v_r}$$

$$\frac{dr}{dt} = \omega_0\sqrt{r^2 - r_0^2}$$

$$\int_{r_0}^{r} \frac{dr}{\sqrt{r^2 - r_0^2}} = \omega_0 \int_0^t dt$$

$$\ln\left[r + \sqrt{r^2 - r_0^2}\right]\Big|_{r_0}^{r} = \omega_0 t \Rightarrow \underline{r = \frac{r_0}{2}\left[e^{-\omega_0 t} + e^{\omega_0 t}\right]}$$

$$v_\theta = r\dot{\theta} = r\omega_0 = \underline{\frac{r_0\omega_0}{2}\left[e^{-\omega_0 t} + e^{\omega_0 t}\right]}$$

As a function of t, $\underline{v_r = \frac{r_0\omega_0}{2}\left(e^{\omega_0 t} - e^{-\omega_0 t}\right)}$

In terms of hyperbolic functions,

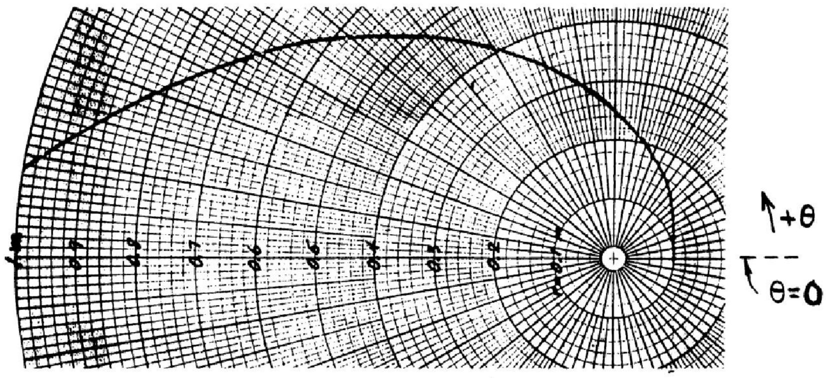
$$v_r = r_0\omega_0 \sinh \omega_0 t$$

$$r = r_0 \cosh \omega_0 t$$

$$\underline{v_\theta = r_0\omega_0 \cosh \omega_0 t}$$

With numbers,

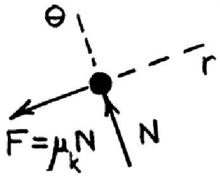
$$\begin{cases} v_r = 0.1 \sinh t \\ r = 0.1 \cosh t \\ v_\theta = 0.1 \cosh t \end{cases}$$



$(r = 1.0 \text{ m} \ @ \ \theta = 171.5°)$

3/93

Particle : $\begin{cases} \Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) : N = m(2\dot{r}\omega_0) \\ \Sigma F_r = m(\ddot{r} - r\dot{\theta}^2) : -\mu_k N = m(\ddot{r} - r\omega_0^2) \end{cases}$



Eliminating N :

$$-\mu_k(2\cancel{m}\dot{r}\,\omega_0) = \cancel{m}(\ddot{r} - r\omega_0^2)$$

$$\ddot{r} + 2\mu_k\omega_0\,\dot{r} - \omega_0^2\, r = 0$$

Assume a solution of form $r = Ce^{st}$

and obtain $s_{1,2} = \omega_0\left[-\mu_k \pm \sqrt{\mu_k^2+1}\,\right]$.

$$\therefore \quad r = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

Initial conditions : $\begin{cases} r(0) = r_0 \\ \dot{r}(0) = 0 \end{cases}$

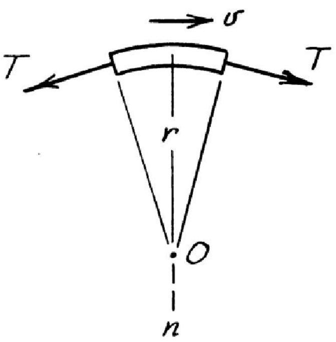
So $\begin{cases} r_0 = C_1 + C_2 \\ 0 = C_1 s_1 + C_2 s_2 \end{cases} \Rightarrow \begin{array}{l} C_1 = -s_2 r_0/(s_1 - s_2) \\ C_2 = s_1 r_0/(s_1 - s_2) \end{array}$

Final Solution :

$$r(t) = \frac{r_0}{2\sqrt{\mu_k^2+1}}\left[\left(\mu_k + \sqrt{\mu_k^2+1}\right)e^{\omega_0\left(-\mu_k + \sqrt{\mu_k^2+1}\right)t} + \left(-\mu_k + \sqrt{\mu_k^2+1}\right)e^{\omega_0\left(-\mu_k - \sqrt{\mu_k^2+1}\right)t}\right]$$

3/94

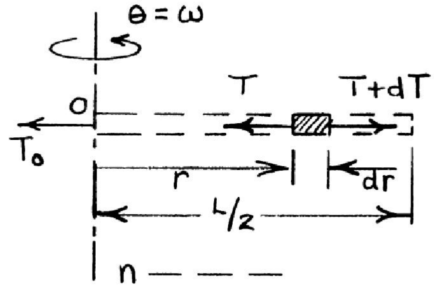
$$\Sigma F_n = m a_n : \ 2T \sin\frac{d\theta}{2} = \rho r d\theta \times \frac{v^2}{r}$$



$$T d\theta = \rho v^2 d\theta, \quad T = \rho v^2$$

so $\sigma_t = T/A = \underline{\rho v^2/A}$

3/95



The mass per unit length is m/L

$$\Sigma F_n = ma_n : \quad T - (T + dT) = \frac{m}{L} dr \, (r\omega^2)$$

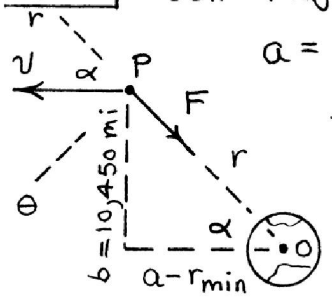
$$\int_{T_0}^{T} -dT = \int_0^r \frac{m\omega^2}{L} r\,dr$$

$$\Rightarrow \quad T = T_0 - \frac{mr^2\omega^2}{2L}$$

When $r = L/2$, $T = 0$: $\quad 0 = T_0 - \dfrac{m(L/2)^2\omega^2}{2L}$

$$T_0 = \frac{mL\omega^2}{8} \quad \Rightarrow \quad \sigma = \frac{T}{A} = \frac{mL\omega^2}{2A}\left(\frac{1}{4} - \frac{r^2}{L^2}\right)$$

3/96 Semi-major axis of ellipse is



$$a = \frac{r_{max} + r_{min}}{2} = \frac{26,259 + 4159}{2} = 15,209 \text{ mi}$$

$$\alpha = \tan^{-1}\frac{b}{a - r_{min}} = \tan^{-1}\frac{10,450}{15,209 - 4159} = 43.4^\circ$$

$$\dot{r} = v_r = v\cos\alpha = 13,244\cos 43.4^\circ = \underline{9620 \text{ ft/sec}}$$

$$r\dot{\theta} = v_\theta = v\sin\alpha, \quad \dot{\theta} = \frac{v\sin\alpha}{r}, \quad \text{where}$$

$$r = \sqrt{b^2 + (a - r_{min})^2} = \sqrt{10,450^2 + (15,209 - 4159)^2}$$

$$= 15,208 \text{ mi}; \text{ so } \dot{\theta} = \frac{13,244 \sin 43.4^\circ}{15,208(5280)} = \underline{1.133(10^{-4})\ \frac{\text{rad}}{\text{sec}}}$$

$$\Sigma F_r = ma_r: \ -\cancel{m}\frac{gR^2}{r^2} = \cancel{m}(\ddot{r} - r\dot{\theta}^2)$$

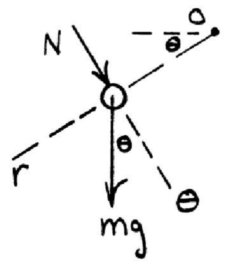
$$\ddot{r} = r\dot{\theta}^2 - \frac{gR^2}{r^2} = 15,208(5280)\left[(1.133)(10^{-4})\right]^2 - \frac{32.23(3959)^2}{15,208^2}$$

$$= \underline{-1.153 \text{ ft/sec}^2}$$

$$\Sigma F_\theta = ma_\theta: \quad 0 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} = -\frac{2(9620)(1.133)(10^{-4})}{15,208(5280)} = \underline{-2.72(10^{-8})\frac{\text{rad}}{\text{sec}^2}}$$

▸3/97



$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2):$$

$$\cancel{m}g \sin\theta = \cancel{m}(\ddot{r} - r\omega_0^2)$$

$$\ddot{r} - \omega_0^2 r = g \sin\omega_0 t$$

Assume $r_h = C e^{st}$ and substitute into equation to obtain $s_1 = -\omega_0$, $s_2 = \omega_0$. Also, assume a particular solution of form $r_p = D \sin\omega_0 t$, substitute, and obtain $D = -g/2\omega_0^2$.

So $r = r_h + r_p = C_1 e^{-\omega_0 t} + C_2 e^{\omega_0 t} - \frac{g}{2\omega_0^2}\sin\omega_0 t$

Initial conditions:

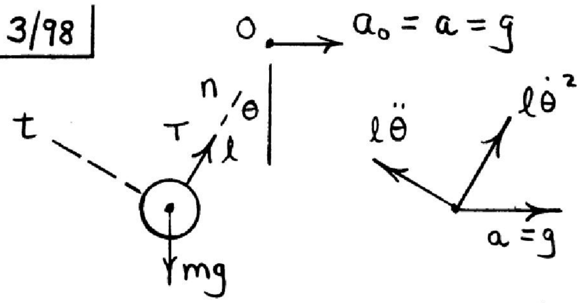
$$\begin{cases} r(0) = C_1 + C_2 = 0 \\ \dot{r}(0) = -\omega_0 C_1 + \omega_0 C_2 - \frac{g}{2\omega_0} = 0 \end{cases}$$

Solve for C_1 and C_2 to obtain

$$r = -\frac{g}{4\omega_0^2} e^{-\theta} + \frac{g}{4\omega_0^2} e^{\theta} - \frac{g}{2\omega_0^2}\sin\theta$$

or $\underline{r = \frac{g}{2\omega_0^2}\left[\sinh\theta - \sin\theta\right]}$

▸ 3/98



$$\underline{a}_m = \underline{a}_o + \underline{a}_{m/o}$$

$$\Sigma F_t = ma_t : \quad -mg\sin\theta = m(\ell\ddot\theta - g\cos\theta)$$

$$\ddot\theta = \frac{g}{\ell}(\cos\theta - \sin\theta) = \dot\theta\frac{d\dot\theta}{d\theta}$$

$$\int_0^\theta \frac{g}{\ell}(\cos\theta - \sin\theta)\,d\theta = \int_0^{\dot\theta}\dot\theta\,d\dot\theta$$

$$\frac{g}{\ell}\left[\sin\theta + \cos\theta - 1\right] = \dot\theta^2/2 \qquad *$$

$\theta = \theta_{max}$ when $\dot\theta = 0$: $\sin\theta + \cos\theta = 1$

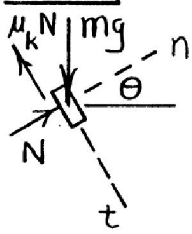
$$\Rightarrow \theta_{min} = 0, \quad \underline{\theta_{max} = \frac{\pi}{2}}$$

$$\Sigma F_n = ma_n : \quad T - mg\cos\theta = m(\ell\dot\theta^2 + g\sin\theta)$$

$$T = m(\ell\dot\theta^2 + g\sin\theta + g\cos\theta)$$

With $*$: $\underline{T = mg(3\sin\theta + 3\cos\theta - 2)}$

▶ 3/99



$$\begin{cases} \Sigma F_n = ma_n: \quad N - mg\sin\theta = m\dfrac{v^2}{r} & (1) \\ \Sigma F_t = ma_t: \quad mg\cos\theta - \mu_k N = ma_t & (2) \end{cases}$$

Solve (1) for N & sub. into (2):

$$g\cos\theta - \mu_k g\sin\theta - \mu_k\frac{v^2}{r} = a_t$$

But $a_t = r\ddot{\theta} = r\dot{\theta}\dfrac{d\dot{\theta}}{d\theta} = r\dfrac{1}{2}\dfrac{d}{d\theta}(\dot{\theta}^2) = \dfrac{1}{2r}\dfrac{d}{d\theta}(r^2\dot{\theta}^2)$

Let $u = (r\dot{\theta})^2 = v^2$ and the EOM becomes

$$\frac{du}{d\theta} + 2\mu_k u = 2gr(\cos\theta - \mu_k\sin\theta)$$

For homogeneous solution, assume $u_H = Ce^{\lambda\theta}$, substitute in to find $u_H = Ce^{-2\mu_k\theta}$

For particular solution, assume $u_p = A\cos\theta + B\sin\theta$, substitute in to find $u_p = \dfrac{2gr}{[1+4\mu_k^2]}\left[+3\mu_k\cos\theta + (1-2\mu_k^2)\sin\theta\right]$

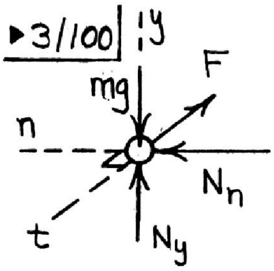
Assemble $u = u_H + u_P$ and apply initial condition $u = v^2 = 0$ @ $\theta = 0$ to obtain

$$u = \frac{-6\mu_k gr}{1+4\mu_k^2}e^{-2\mu_k\theta} + \frac{2gr}{1+4\mu_k^2}\left[3\mu_k\cos\theta + (1-2\mu_k^2)\sin\theta\right]$$

For $\mu_k = 0.2$, $g = 9.81\ \text{m/s}^2$, $r = 3\ \text{m}$, & $\theta = \frac{\pi}{2}$:

$v^2 = 30.4\ \text{m}^2/\text{s}^2$, $\underline{v = 5.52\ \text{m/s}}$

▶3/100



$$\Sigma F_y = 0: \quad N_y = mg$$

$$\Sigma F_n = ma_n: \quad N_n = m\frac{v^2}{r}$$

$$F = \mu_k N_{tot} = \mu_k\sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$$

$$= \frac{\mu_k m}{r}\sqrt{r^2g^2 + v^4}$$

$$\Sigma F_t = ma_t: \quad -\frac{\mu_k \cancel{m}}{r}\sqrt{r^2g^2+v^4} = \cancel{m}v\frac{dv}{ds}$$

$$-\frac{\mu_k}{r}\int_0^s ds = \int_{v_0}^0 \frac{v\,dv}{\sqrt{v^4+r^2g^2}} = \int_{v_0^2}^0 \frac{\frac{1}{2}dx}{\sqrt{x^2+r^2g^2}}$$

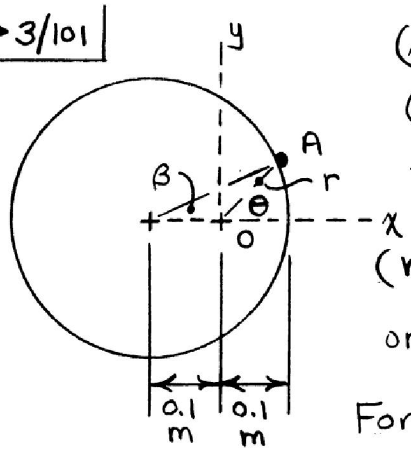
where $x = v^2$, $dx = 2v\,dv$

Integrating,

$$-\frac{\mu_k}{r}s = \frac{1}{2}\ln\left[x + \sqrt{x^2+r^2g^2}\right]_{v_0^2}^0$$

or $$\underline{s = \frac{r}{2\mu_k}\ln\left[\frac{v_0^2 + \sqrt{v_0^4 + r^2g^2}}{rg}\right]}$$

▶3/101



$$(x-x_0)^2+(y-y_0)^2=R_0^2$$

$$(x+0.1)^2+y^2=0.2^2$$

Switch to polar form:

$$(r\cos\theta+0.1)^2+(r\sin\theta)^2=0.2^2$$

or * $r^2+0.2r\cos\theta-0.03=0$

For $\theta=45°$, quadratic formula yields $r=0.1164$ m

Differentiate * with respect to time:

$$2r\dot{r}+0.2\dot{r}\cos\theta-0.2r\dot{\theta}\sin\theta=0 \qquad (1)$$

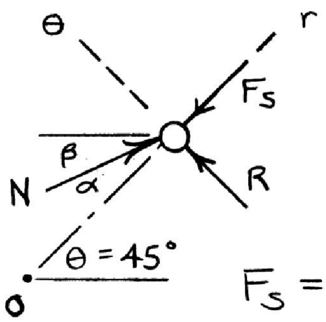
Again: $2\dot{r}^2+2r\ddot{r}+0.2\ddot{r}\cos\theta-0.2\dot{r}\dot{\theta}\sin\theta$

$$-0.2\dot{r}\dot{\theta}\sin\theta-0.2r\cancel{\ddot{\theta}}^{\,0}\sin\theta-0.2r\dot{\theta}^2\cos\theta=0 \quad (2)$$

With $r=0.1164$ m, $\theta=45°$, & $\dot{\theta}=15$ rad/s,

Eqs. (1) & (2) yield $\dot{r}=0.660$ m/s

$\ddot{r}=15.05$ m/s^2



$$\frac{\sin\beta}{0.1164} = \frac{\sin 135°}{0.2}$$

$$\beta = 24.3°$$

$$\alpha = 45° - \beta = 20.7°$$

$$F_s = k\delta = 5000\,(0.1164 - 0.1)$$

$$= 81.9 \text{ N}$$

$$\Sigma F_r = ma_r: \quad N\cos\alpha - F_s = m(\ddot{r} - r\dot{\theta}^2)$$

$$N \cos 20.7° - 81.9 = 0.5\left(15.05 - 0.1164(15^2)\right)$$

$$\underline{N = 81.6 \text{ N}}$$

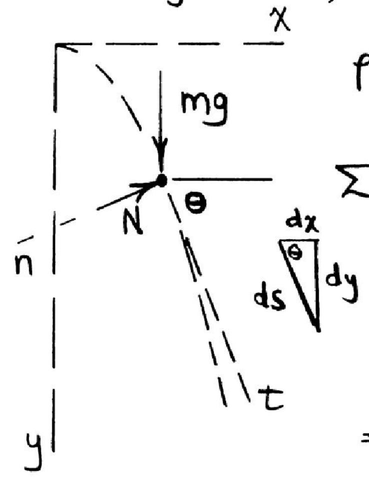
$$\Sigma F_\theta = ma_\theta: \quad R - N\sin\alpha = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$R - 81.6 \sin 20.7 = 0.5\,(0 + 2(0.660)(15))$$

$$\underline{R = 38.7 \text{ N}}$$

▶3/102

$y = kx^2, \quad \tan\theta = \frac{dy}{dx} = 2kx, \quad \frac{d^2y}{dx^2} = 2k$



$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{\left[1 + 4k^2x^2\right]^{3/2}}{2k}$$

$$\Sigma F_t = ma_t: \quad mg\sin\theta = mv\frac{dv}{ds}$$

$$\int_0^s g\sin\theta\, ds = \int_0^v v\,dv$$

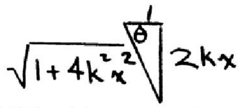
$$g\int_0^y dy = \int_0^v v\,dv$$

$$\Rightarrow v^2 = 2gy$$

$$\Sigma F_n = ma_n: \quad mg\cos\theta - N = m\frac{v^2}{\rho}$$

$$N = m\left[g\cos\theta - \frac{v^2}{\rho}\right] = m\left[g\cos\theta - \frac{2g(kx^2)}{\frac{(1+4k^2x^2)^{3/2}}{2k}}\right]$$

$$\underline{N = \frac{mg}{(1+4k^2x^2)^{3/2}}} \quad \left(\text{Always} > 0, \text{ unless } x \to \infty\right)$$

(Note : 

; $\cos\theta = \frac{1}{\sqrt{1+4k^2x^2}}$)

3/103 (a) spring : $U_{1-2} = \frac{1}{2}k(x_1^2 - x_2^2)$

$U_{1-2} = \frac{1}{2}(4000)[0.1^2 - 0.2^2] = \underline{-60\text{ J}}$

(b) weight : $U_{1-2} = mg(y_1 - y_2)$

$U_{1-2} = 7(9.81)[0.1 \sin 20°] = \underline{2.35\text{ J}}$

3/104 $T_A + U_{A-B} = T_B$: $\frac{1}{2}\cancel{m}v_A^2 - \cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$

$v_B^2 = v_A^2 - 2gh = 5^2 - 2(9.81)(0.8)$

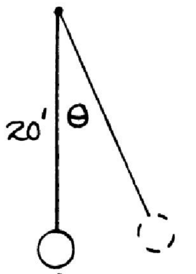
$v_B = 3.05$ m/s

In the absence of friction, only the altitude change, and not the shape of the path followed, is a factor in the work calculation.

3/105 $U = \Delta T$; $64.4(20) + U_f = \frac{1}{2}\frac{64.4}{32.2}(\overline{25}^2 - 3^2)$

$U_f = 616 - 1288 = \underline{-672 \text{ ft-lb}}$

3/106

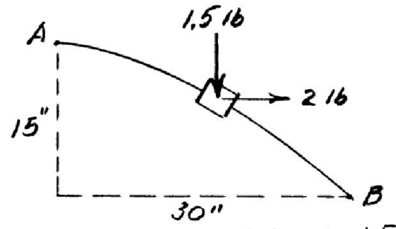


$$T_1 + U_{1-2} = T_2 : \quad \frac{1}{2}\cancel{m}v_1^2 - \cancel{m}gh = 0$$

$$\frac{1}{2}\left[2\,\frac{5280}{3600}\right]^2 - 32.2\left[20\,(1-\cos\theta)\right] = 0$$

$$\underline{\theta = 6.63^\circ}$$

3/107



$U_{1\text{-}2} = \Delta T$; $2(30/12) + 1.5(15/12) = \frac{1}{2}\frac{1.5}{32.2}(v^2 - 0)$

$v^2 = 295.2$, $\underline{v = 17.18 \text{ ft/sec}}$

3/108 For collar, $U_{1-2} = \Delta T = 0$

$$U_{1-2} = 50\left(\frac{50-30}{12}\right) - 30\,\frac{40}{12}\sin 30^\circ - \frac{1}{2}k\left(\frac{6}{12}\right)^2 = 0$$

$\underline{k = 267 \text{ lb/ft}}$

3/109 $U_{1-2} = \Delta T$; $2(\frac{1}{2}kx^2) = \frac{1}{2}mv^2 - 0$

$$k = \frac{1}{2}\frac{mv^2}{x^2} = \frac{1}{2}\frac{3500}{32.2}\left(\frac{5}{30}44\right)^2\frac{1}{(6/12)^2}\frac{1}{12} = \underline{974\ \text{lb/in.}}$$

3/110 $U_{1-2} = \Delta T$: $mgh = \frac{1}{2} m (v^2 - 0)$, $\underline{v = \sqrt{2gh}}$

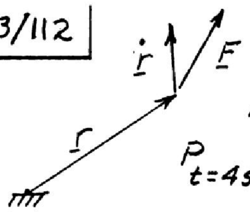
3/111 $U_{1-2} = \Delta T$: $mgh + Q = \frac{1}{2}m(v_B^2 - 0^2)$

$$0.5(9.81)(1.5) + Q = \frac{1}{2}(0.5)(4.70^2 - 0^2)$$

$$\underline{Q = -1.835 \text{ J}} \left(\begin{array}{c}\text{causes loss}\\\text{of energy}\end{array}\right)$$

The lost mechanical energy is transformed to heat energy.

3/112



Power $P = \underline{F} \cdot \dot{\underline{r}}$

$P = (40\underline{i} - 20\underline{j} - 36\underline{k}) \cdot (8\,\underline{i} + 2.4t\underline{j} - 1.5t^2\underline{k})$

$P_{t=4s} = (40\underline{i} - 20\underline{j} - 36\underline{k}) \cdot (8\underline{i} + 9.6\underline{j} - 24\underline{k})$

$= 320 - 192 + 864 = 992$ W

or $\underline{P = 0.992 \text{ kW}}$

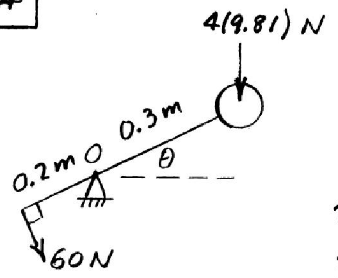
3/113 $P = W\dot{y}$ where $\dot{y} = v\sin\theta$

$\theta = \tan^{-1} 0.05 = 2.86°$, $\sin\theta = 0.0499$

$$P = 200\left(\frac{15}{30}44\right)0.0499 = 219.7 \text{ ft-lb/sec}$$

or $P = \dfrac{219.7}{550} = \underline{0.400 \text{ hp}}$

3/114



For system

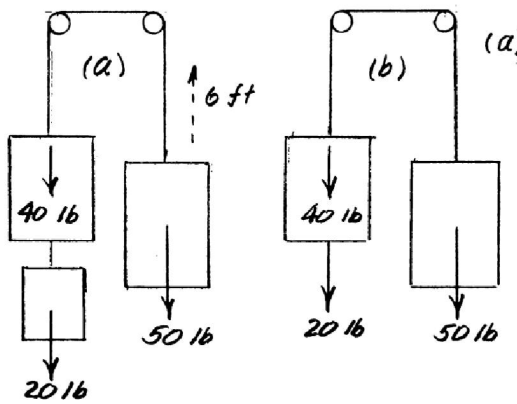
$U_{1-2} = \Delta T$

$60(0.2)(\frac{\pi}{2}) - 4(9.81)(0.3) = \frac{1}{2} 4 v^2$

$v^2 = 3.539\ (m/s)^2$

$\underline{v = 1.881\ m/s}$

3/115 Active-force diagrams for entire system



For sytem $U_{1-2} = \Delta T$

(a)

$$(40+20)6 - 50(6) = \frac{1}{2}\,\frac{40+20+50}{32.2}\,v^2$$

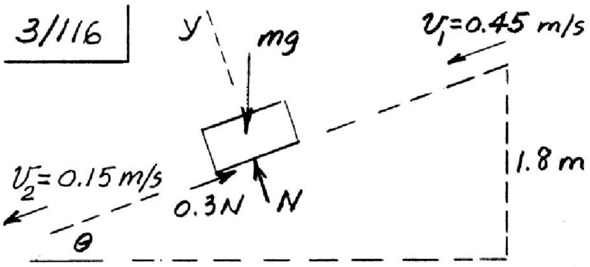
$v = 5.93$ ft/sec

(b)

$$(40+20)6 - 50(6) = \frac{1}{2}\,\frac{40+50}{32.2}\,v^2$$

$v = 6.55$ ft/sec

3/116



$\Sigma F_y = 0, \quad N - mg\cos\theta = 0$

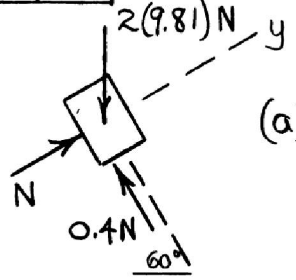
$N = mg\cos\theta$

$$U = \Delta T; \quad (mg\sin\theta - 0.3\,mg\cos\theta)\frac{1.8}{\sin\theta} = \frac{m}{2}(\overline{0.15}^2 - \overline{0.45}^2)$$

$$1.8(9.81)\left(1 - \frac{0.3}{\tan\theta}\right) = -\frac{0.180}{2}$$

$$\tan\theta = 0.2985, \quad \underline{\theta = 16.62^\circ}$$

3/117



$\Sigma F_y = 0: \quad N - 2(9.81)\cos 60° = 0$

$N = 9.81 \text{ N}$

(a) $U_{1\text{-}2} = \Delta T: \quad 2(9.81)(0.5 \sin 60°) - 0.4(9.81)(0.5) = \frac{1}{2}\, 2\, v^2$

$\underline{v = 2.56 \text{ m/s}}$

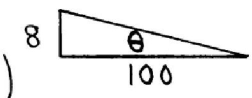
(b) $U_{1\text{-}3} = \Delta T: \quad 2(9.81)(0.5 + x)\sin 60° - 0.4(9.81)(0.5 + x) - \frac{1}{2}(1600)x^2 = 0$

$800x^2 - 13.07x - 6.53 = 0$

$x = 0.0989 \text{ m}$ or $\underline{x = 98.9 \text{ mm}}$

3/118

$$\theta = \tan^{-1}\frac{8}{100} = 4.57^\circ$$



$$U_{1-2} = \Delta T : U_f + mgh = \frac{1}{2} m \left(v_2^2 - v_1^2\right)$$

$$U_f = -1200\,(9.81)\,(500 \sin 4.57^\circ)$$
$$- \frac{1}{2}\,1200\left[\left(\frac{25}{3.6}\right)^2 - \left(\frac{100}{3.6}\right)^2\right]$$

$$U_f = -903\,(10^3)\ \text{J} \quad \text{or} \quad -903\ \text{kJ}$$

Ans. $\underline{Q = 903\ \text{kJ (loss)}}$

3/119 $P = Wh/\Delta t$

or $P = \dfrac{120(9)}{5} \Big/ 550 = \underline{0.393 \text{ hp}}$

Conversions : $h = 9 \text{ ft} \left(\dfrac{0.3048 \text{ m}}{\text{ft}}\right) = 2.74 \text{ m}$

$W = 120 \text{ lb} \left(\dfrac{4.4482 \text{ N}}{\text{lb}}\right) = 534 \text{ N}$

$P = \dfrac{Wh}{\Delta t} = \dfrac{534(2.74)}{5} = \underline{293 \text{ watts}}$

Check : $0.393 \text{hp} \left(\dfrac{745.7 \text{ watts}}{\text{hp}}\right) = 293$ watts ✓

3/120 $U_{1-2} = \Delta T$ applied to system:

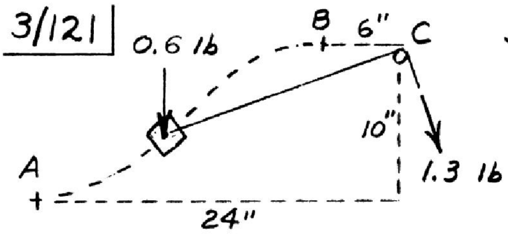
$U_{1-2} = 40\left[\sqrt{0.4^2 + 0.3^2} - 0.1\right] - 0.8(9.81)(0.4)$

$= 12.86$ J

$\Delta T = T_B - T_A = \frac{1}{2}(0.8)v_B^2 - 0$

Thus $12.86 = 0.4 v_B^2$, $\underline{v_B = 5.67 \text{ m/s}}$

3/121



System = slider, cord, & pulley at C.

$$U = -0.6\left(\frac{10}{12}\right) + 1.3\,\frac{20}{12}$$

$$= \frac{5}{3} = 1.667 \text{ ft-lb}$$

$\overline{AC} = 26''$; $\overline{AC} - \overline{BC} = 20''$

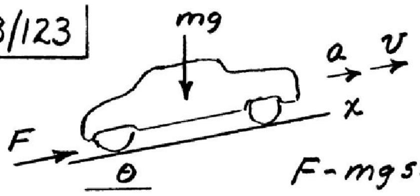
$U = \Delta T$; $1.667 = \frac{1}{2}\,\frac{0.6}{32.2}(v^2 - 0)$, $v^2 = 178.9$, $\underline{v = 13.37 \ \frac{\text{ft}}{\text{sec}}}$

$\underline{3/122}$ Net power required $= 30(140)(24)/33{,}000$

$= 3.05$ hp

Mechanical efficiency $= \dfrac{\text{Power required}}{\text{Power supplied}} = \dfrac{3.05}{4.00} = \underline{0.764}$

3/123



$\theta = \tan^{-1} 0.1, \ \sin\theta = 0.0995$

$\Sigma F_x = ma_x$ where $v^2 = 2ax$

$$F - mg\sin\theta = m\frac{v^2}{2x}$$

$$P = Fv = mgv\sin\theta + \frac{mv^3}{2x}$$

$$= 1500(9.81)\frac{50000}{3600}0.0995 + \frac{1500(50000/3600)^3}{2(100)}$$

$$= 20\,336 + 20\,094 = 40\,430 \text{ W}$$

or $\underline{P = 40.4 \text{ kW}}$

3/124 State ① : release ; state ② : final position

$T_1 + U_{1-2} = T_2$

$$0 + mg(h+d) - \int_0^d kx^4\,dx = 0$$

$$mg(h+d) - \frac{1}{5}kd^5 = 0$$

$$\Rightarrow \quad k = \frac{5mg(h+d)}{d^5}$$

$\boxed{3/125}$ Power output $=$ rate of doing work

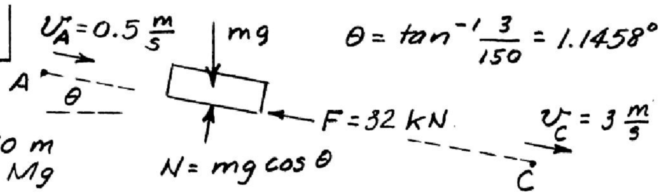
$$= 300(9.81)(2) - 100(9.81)(4)$$

$$= 1962 \text{ J/s } (W)$$

$$= 1.962 \text{ kW}$$

Efficiency $e = \dfrac{\text{Power output}}{\text{Power input}} = \dfrac{1.962}{2.20} = \underline{0.892}$

3/126



$\theta = \tan^{-1}\frac{3}{150} = 1.1458°$

$\overline{AC} = 150$ m

$m = 68$ Mg

$U_{1\text{-}2} = \Delta T$; $\quad 68(10^3)(9.81)(3) - 32(10^3)x = \frac{1}{2}68(10^3)(3^2 - 0.5^2)$

$2001 - 32x = 297.5$, $\quad \underline{x = 53.2\ m}$

3/127 Work done by weight is $mg(r_{\pi/2} - 0)$

$= 0.5(9.81)(0.3 \times \frac{\pi}{2}) = 2.31$ J

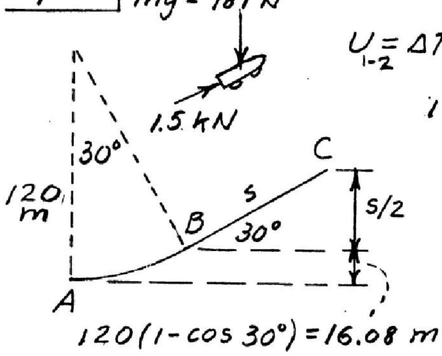
Work done by T is $T(r_{\pi} - r_{\pi/2}) = 10(0.3\pi - 0.3\frac{\pi}{2})$

$= 4.71$ J

$U = \Delta T$; $2.31 + 4.71 = \frac{1}{2}0.5(v^2 - 0)$

$v^2 = 28.10$, $\underline{v = 5.30 \text{ m/s}}$

3/128



$\overline{AB} = r\theta = 120\frac{\pi}{6} = 62.8$ m

$U_{1-2} = \Delta T = 0$ since $T_C = T_A = 0$

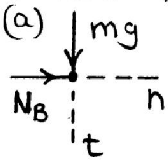
$1500(62.8) - 981(16.08 + \frac{s}{2}) = 0$

$s = 2(94248 - 15771)/981$

$= 160.0$ m

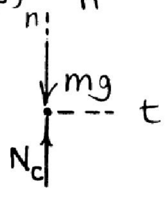
3/129 $T_A + U_{A-B} = T_B$: $0 + 2mgR = \frac{1}{2}mv_B^2$, $v_B^2 = 4gR$

(a)



$\Sigma F_n = ma_n$: $N_B = m\,\frac{4gR}{R} = \underline{4mg}$

(b) $T_A + U_{A-C} = T_C$: $0 + 3mgR = \frac{1}{2}mv_C^2$, $v_C^2 = 6gR$



$\Sigma F_n = ma_n$: $N_C - mg = m\,\frac{6gR}{R}$

$\underline{N_C = 7mg}$

(c) Call stopping point E :

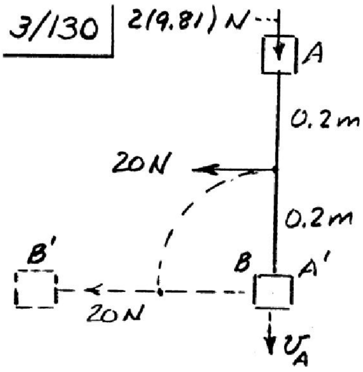
$T_A + U_{A-E} = T_E$

$0 + 2mgR - mg\left(\frac{1}{2}s\right) - \mu_k \frac{\sqrt{3}}{2} mgs = 0$

$\underline{s = \frac{4R}{1+\mu_k\sqrt{3}}}$

$\left(\text{Note: Normal force on incline is } N = mg\cos 30° = \frac{\sqrt{3}}{2}mg\right)$

3/130



$U_{1-2} = \Delta T$

$T_{initial} = 0$, $T_{final} = \frac{1}{2} m v_A^2 = \frac{2}{2} v_A^2$

$(v_B = 0)$

$U_{1-2} = 20(0.2) + 2(9.81)(0.4)$

$= 11.85 \text{ J}$

$11.85 = v_A^2$, $\underline{v_A = 3.44 \text{ m/s}}$

3/131 $U_{1-2} = \Delta T$; $mg(0.8 - 1.2\cos 60°)$

$= \frac{1}{2}m(v_C^2 - 3^2)$

$9.81(0.20) = \frac{1}{2}(v_C^2 - 9)$, $v_C^2 = 12.92$, $\underline{v_C = 3.59\text{ m/s}}$

3/132 $U = \Delta T$; $-\int_0^4 (3x^2 + 60x)\,dx = \frac{1}{2}\frac{48}{32.2}(0 - v^2)12$

$x^3 + 30x^2\Big]_0^4 = \frac{288}{32.2}v^2$, v in ft/sec.

$v^2 = \frac{32.2}{288}(64 + 480) = 60.82\ (\text{ft/sec})^2$, $\underline{v = 7.80 \text{ ft/sec}}$

3/133 The power output of the drivetrain is

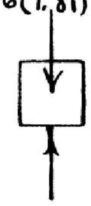
$$P_{out} = Fv = 560\left(\frac{90}{3.6}\right) = 14\,000 \text{ W}$$

The power input to the drivetrain:

$$P_{in} = \frac{P_{out}}{e} = \frac{14\,000}{0.70} = 20\,000 \text{ W}$$

So the motor output $\underline{P = 20 \text{ kW}}$

3/134



$U_{1-2} = \Delta T = 0$:

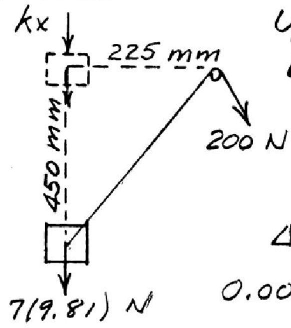
$$6(9.81)(0.1+\delta) - \int_{0.05}^{0.05+\delta} 4000x\,dx = 0$$

$$2000\delta^2 + 141.1\delta - 5.89 = 0$$

$\delta = 0.0294\text{m}$ or $\underline{\delta = 29.4\text{ mm}}$

(Positive result taken from quadratic formula)

3/135 For system of collar & cable



$U_{1-2} = \Delta T$

$U_{1-2} = 200\left[\sqrt{(0.450)^2 + (0.225)^2} - 0.225\right] - 7(9.81)(0.450) - \frac{1}{2}k(0.075)^2$

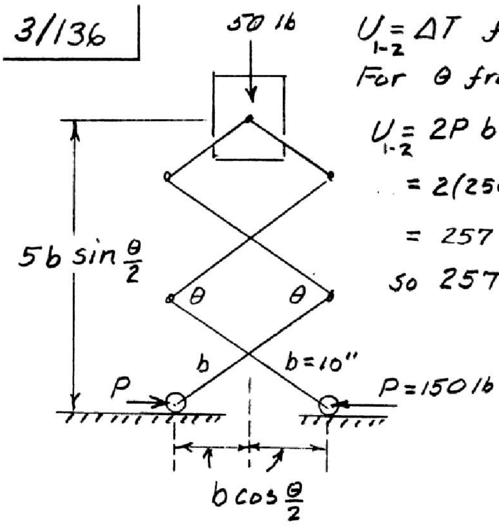
$= 55.6 - 30.9 - 0.00281k$ ✓

$\Delta T = 0$ so

$0.00281k = 55.6 - 30.9$

$k = 8790$ N/m or $\underline{k = 8.79 \text{ kN/m}}$

3/136



$U_{1-2} = \Delta T$ for system

For θ from $60°$ to $180°$,

$U_{1-2} = 2Pb\cos\frac{\theta}{2} - W(5b)(1-\sin\frac{\theta}{2})$

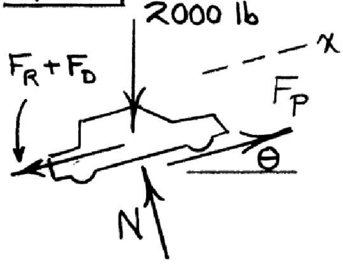
$= 2(250)\frac{10}{12}\cos 30° - 50(\frac{50}{12})(1-\sin 30°)$

$= 257$ ft-lb

so $257 = \frac{1}{2}\frac{50}{32.2}v^2$, $v^2 = 330.6$

$v = 18.18$ ft/sec

3/137 | $mg = 2000$ lb



$\theta = 0$ or $\theta = \tan^{-1}\frac{6}{100} = 3.43°$

$F_D = kv^2$: $50 = k(60)^2$

$\Rightarrow k = 0.01389\ \frac{\text{lb-hr}^2}{\text{mi}^2}$

& $F_D = 0.01389\, v^2$

$\Sigma F_x = 0$; $F_P - F_R - F_D - mg\sin\theta = 0$

$F_P = F_R + F_D + mg\sin\theta$

(a) $\underline{\theta = 0}$: $v = 30$ mi/hr : $F_D = 0.01389(30^2) = 12.50$ lb

$F_P = F_R + F_D = 50 + 12.50 = 62.5$ lb

$P_{30} = Fv = 62.5\left(30\,\frac{5280}{3600}\right)/550 = \underline{5\text{ hp}}$

$v = 60$ mi/hr : $F_D = 50$ lb, $F_P = F_R + F_D = 100$ lb

$P_{60} = Fv = 100\left(60\,\frac{5280}{3600}\right)/550 = \underline{16\text{ hp}}$

(b) $\underline{\theta = 3.43°}$ $F_P = 50 + 50 + 2000\sin 3.43° = 220$ lb

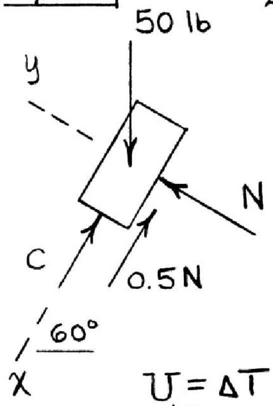
$P_{up} = 220\left(60\,\frac{5280}{3600}\right)/550 = \underline{35.2\text{ hp}}$

Down: $F_P = 50 + 50 - 2000\sin 3.43° = -19.78$ lb

$P_{down} = -19.78\left(60\,\frac{5280}{3600}\right)/550 = \underline{-3.17\text{ hp}}$ (brakes!)

(c) $\Sigma F_x = 0$: $50 + kv^2 - 2000\sin 3.43° = 0$, $\underline{v = 70.9\ \frac{\text{mi}}{\text{hr}}}$

3/138



$\Sigma F_y = 0: \quad N - 50\cos 60° = 0, \quad N = 25 \text{ lb}$

Displacement is $3 + \frac{4}{12} = 3.33$ ft

$$U_{1-2} = (50\sin 60° - 0.5 \cdot 25)\,3.33 - \frac{1}{12}\int_0^{4''} (100x + 9x^2)\,dx$$

$$= 20.0 \quad \text{ft-lb}$$

$U_{1-2} = \Delta T: \quad 20.0 = \frac{1}{2}\frac{50}{32.2}(v^2 - 2^2)$

$\underline{v = 5.46 \text{ ft/sec}}$

3/139 $(U_{1-2})_s = -\int_{x_1}^{x_2} (4x - 120x^3)\,dx$

$= \left(-2x^2 + 30x^4\right)\Big|_{x_1}^{x_2} = -2(x_2^2 - x_1^2) + 30(x_2^4 - x_1^4)$

With $x_1 = 0.1$ m & $x_2 = 0$, $(U_{1-2})_s = 0.017$ kN·m

or $(U_{1-2})_s = 17$ N·m $= 17$ J

$(U_{1-2})_f = -\mu_k mgd = -0.2(10)(9.81)(0.1) = -1.962$ J

$T_1 + U_{1-2} = T_2 : \; 0 + 17 - 1.962 = \frac{1}{2}(10)v^2$

$\underline{v = 1.734 \text{ m/s}}$

For the linear spring, $(U_{1-2})_s = 2(x_1^2 - x_2^2)$

$= 2(0.1)^2 = 0.02$ kN·m $= 20$ J, $\underline{v = 1.899 \text{ m/s}}$

3/140 $P = Fv$; $F = ma$, so $P = mav$

& $a = \frac{P}{mv}$

But $v\,dv = a\,ds$, so $mv^2\,dv = P\,ds$

$$\int_{v_1}^{v_2} mv^2\,dv = \int_0^s P\,ds \;;\quad \frac{m}{3}(v_2^3 - v_1^3) = Ps$$

$$\underline{v_2 = \left(\frac{3Ps}{m} + v_1^3\right)^{1/3}}$$

3/141 $U = \Delta T$; $\int_x^{x_0} kx\,dx = \frac{1}{2}mv^2$, $v^2 = \frac{k}{m}(x_0^2 - x^2)$

Power $P = Fv$, $P^2 = F^2v^2 = (kx)^2\frac{k}{m}(x_0^2 - x^2)$

$\underline{P = kx\sqrt{\frac{k}{m}(x_0^2 - x^2)}}$, $\frac{dP^2}{dx} = \frac{k^3}{m}(2x_0^2x - 4x^3) = 0$ for max P^2

So $x = 0$ $(P = 0)$, $x = x_0/\sqrt{2}$

So $P_{max} = k\frac{x_0}{\sqrt{2}}\sqrt{\frac{k}{m}}\sqrt{x_0^2 - \frac{x_0^2}{2}} = \underline{\frac{k}{2}\sqrt{\frac{k}{m}}\,x_0^2}$ $\underline{\text{at } x = \frac{x_0}{\sqrt{2}}}$

3/142 State ①: release ; state ②: $x = 0.050$ m

$$T_1 + U_{1\text{-}2} = T_2$$

$$\frac{1}{2}mv_1^2 + mgx\sin 20^\circ - \frac{1}{2}(3k)x^2 = \frac{1}{2}mv^2$$

$$10(9.81)(0.050)\sin 20^\circ - \frac{3}{2}(120)(0.050)^2 = \frac{1}{2}(10)v^2$$

(a) $\underline{v = 0.496 \text{ m/s}}$

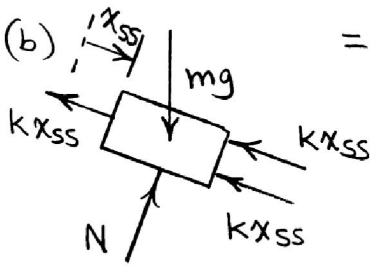
Redefine state ②: $x = x_{max}$, $v = 0$

$T_1 + U_{1\text{-}2} = T_2$: $0 + mg\,x_{max}\sin 20^\circ - \frac{3}{2}k\,x_{max}^2 = 0$

$$x_{max} = 0,\quad x_{max} = \frac{2mg\sin 20^\circ}{3k} = \frac{2(10)(9.81)\sin 20^\circ}{3(120)}$$

$= 0.1864$ m or $\underline{x_{max} = 186.4 \text{ mm}}$

(b)



$\Sigma F_x = 0$:

$$-3kx_{ss} + mg\sin 20^\circ = 0$$

$$x_{ss} = \frac{mg\sin 20^\circ}{3k}$$

With numbers, $\underline{x_{ss} = 93.2 \text{ mm}}$

3/143 Note that $\overline{AO} = \sqrt{18^2 + 30^2} = 35.0$ in.

$$U'_{1-2} = \Delta T + \Delta V_e + \Delta V_g$$

$$\Delta T = \frac{1}{2}\,\frac{2}{32.2}\,v^2 - 0 = \frac{v^2}{32.2}$$

$$\Delta V_e = \frac{1}{2}(1.60)\left[\left(\frac{20-15}{12}\right)^2 - \left(\frac{35.0-15}{12}\right)^2\right]$$

$$= -2.08 \text{ ft-lb}$$

$$\Delta V_g = 2\left(10/12\right) = 1.667 \text{ ft-lb}$$

$$U'_{1-2} = 0$$

So $0 = \dfrac{v^2}{32.2} - 2.08 + 1.667$, $\underline{v = 3.65 \text{ ft/sec}}$

$\boxed{3/144}$ (a) $\Delta T + \Delta V_g = 0; \quad \frac{1}{2}mv^2 - 0 - mgh = 0$

$$v = \sqrt{2gh} = \sqrt{2(9.81)(0.6)} = \underline{3.43 \frac{m}{s}}$$

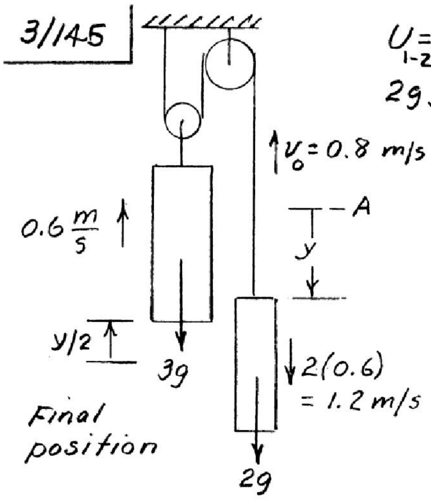
(b) $\Delta V_g + \Delta V_e = 0$ since $\Delta T = 0$

$$-mgh + \frac{1}{2}kx^2 = 0; \quad x^2 = \frac{2mgh}{k} = \frac{2(4)(9.81)(0.6)}{20(10^3)}$$

$$= 0.235(10^{-2})\ m^2$$

$$x = 0.0485\ m \text{ or } \underline{x = 48.5\ mm}$$

3/145



$$U_{1-2} = \Delta T$$

$$2gy - 3g\frac{y}{2} = \frac{1}{2}2([1.2]^2 - [0.8]^2) + \frac{1}{2}3([0.6]^2 - [0.4]^2)$$

$$y = \frac{2}{9.81}(0.80 + 0.3)$$

$$\underline{y = 0.224 \text{ m}}$$

3/146 Establish datum @ A.

(a) $T_A + V_A = T_B + V_B$

$0 + 0 = \frac{1}{2} m v_B^2 - m g h_B$

$v_B = \sqrt{2 g h_B} = \sqrt{2(9.81)(4.5)} = \underline{9.40 \text{ m/s}}$

(b) State F : spring fully compressed

$T_A + V_A = T_F + V_F$

$0 + 0 = 0 - m g h_f + \frac{1}{2} k \delta^2$

$\delta = \sqrt{\frac{2 m g h_f}{k}} = \sqrt{\frac{2(1.2)(9.81)(3)}{24000}} = 0.0542 \text{ m}$

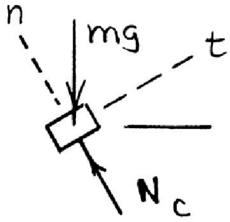
or $\underline{\delta = 54.2 \text{ mm}}$

3/147 Establish datum @ A.

$T_A + V_A = T_C + V_C : 0 + 0 = \frac{1}{2}mv_C^2 - mgh_C$

$v_C = \sqrt{2gh_C} = \sqrt{2(9.81)(3 + 1.5\cos 30^\circ)}$

$= 9.18 \text{ m/s}$



(a) $\Sigma F_n = m\frac{v^2}{\rho} : N_C - 1.2(9.81)\cos 30^\circ = 1.2\frac{9.18^2}{1.5}$

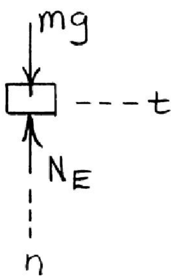
$\underline{N_C = 77.7 \text{ N}}$

(b) $\Sigma F_n = 0 : N_C - 1.2(9.81)\cos 30^\circ = 0$

$\underline{N_C = 10.19 \text{ N}}$

$T_A + V_A = T_E + V_E : 0 + 0 = \frac{1}{2}mv_E^2 - mgh_E$

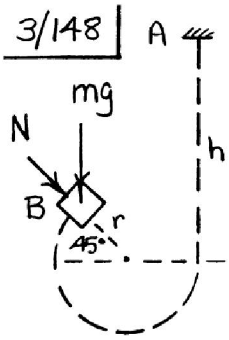
$v_E = \sqrt{2gh_E} = \sqrt{2(9.81)(3)} = 7.67 \text{ m/s}$



$\Sigma F_n = m\frac{v^2}{\rho} : -N_E + 1.2(9.81) = 1.2\frac{7.67^2}{1.5}$

$\underline{N_E = -35.3 \text{ N (down)}}$

3/148



$$U'_{1-2} = \Delta T + \Delta V_g = 0$$

$$\frac{1}{2}mv^2 - mg\left(h - \frac{r}{\sqrt{2}}\right) = 0$$

$$v^2 = 2g\left(h - \frac{r}{\sqrt{2}}\right)$$

$$\Sigma F_n = ma_n : \quad N + \frac{mg}{\sqrt{2}} = m\frac{v^2}{r}$$

$$\Rightarrow N = mg\left[\left(\frac{h}{r} - \frac{1}{\sqrt{2}}\right)2 - \frac{1}{\sqrt{2}}\right]$$

$$= mg\left[2\frac{h}{r} - \frac{3}{\sqrt{2}}\right]$$

With $m = 0.25$ kg, $r = 0.15$ m, & $h = 0.6$m,

$N = 14.42$ N

3/149 $T_A + V_A = T_B + V_B$, datum @ B

$$0 + mgR + \frac{1}{2}k\left[R\sqrt{2} - R\right]^2 = \frac{1}{2}mv_B^2 + 0$$

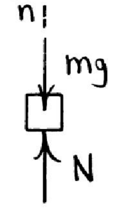
$$\underline{v_B = \sqrt{2gR + \frac{kR^2}{m}\left(3 - 2\sqrt{2}\right)}}$$

$T_A + V_A = T_C + V_C$, datum @ C

$$0 + 2mgR + \frac{1}{2}k\left[R\sqrt{2} - R\right]^2 = \frac{1}{2}mv_C^2 + 0$$

$$\underline{v_C = \sqrt{4gR + \frac{kR^2}{m}\left(3 - 2\sqrt{2}\right)}}$$

Kinetics at C:



$$\Sigma F_n = ma_n: \quad N - mg = m\frac{v_C^2}{R}$$

$$\Rightarrow \underline{N = m\left[5g + \frac{kR}{m}\left(3 - 2\sqrt{2}\right)\right]}$$

3/150 For the system, $U'_{1-2} = 0$ so $\Delta V_g = 0$

$\Delta V_{g_{100}} = -100(7) = -700$ ft-lb

$\Delta V_{g_W} = W\left[2(12\sqrt{2}) - 2\sqrt{5^2+12^2}\right] = 7.94W$

Thus $7.94W - 700 = 0$, $\underline{W = 88.1 \text{ lb}}$

3/151 (a) $\Delta T + \Delta V_g = 0$

$$\frac{1}{2}\frac{5}{32.2}v^2 + \frac{1}{2}\frac{10}{32.2}\left(\frac{12}{18}v\right)^2 + 5\frac{18}{12}\sin 60^\circ - 10\frac{12}{12}\sin 60^\circ = 0$$

$0.1467 v^2 = 2.165$, $v^2 = 14.76\ (ft/sec)^2$

$\underline{v = 3.84\ ft/sec}$

(b) For entire interval $\Delta T = 0$, $\Delta V_g + \Delta V_e = 0$

$-2.165(12) + \frac{1}{2}(200)x^2 = 0$, $x^2 = 0.2598\ (in)^2$

$\underline{x = 0.510\ in.}$

3/152

$$\Delta T = \frac{1}{2}(1.5)(3^2 - 2^2) = 3.75 \text{ J}$$

$$\Delta V_e = \frac{1}{2}(800)\left([0.4-0.3]^2 - [0.5-0.3]^2\right) = -12 \text{ J}$$

$$\Delta V_g = 1.5(9.81)(0.4) = 5.89 \text{ J}$$

$$U_f = U'_{1-2} = \Delta T + \Delta V_e + \Delta V_g$$

$$U_f = 3.75 - 12 + 5.89 = \underline{-2.36 \text{ J}}$$

$$|U_f| = F_{av}\,s \;,\quad F_{av} = \frac{2.36}{0.7} = \underline{3.38 \text{ N}}$$

3/153 Let m be the mass of the car

$$U'_{1-2} = \Delta T + \Delta V_g : \quad 0 = \frac{1}{2} m (v^2 - v_0^2) + mgy$$

$$v^2 = v_0^2 - 2gy$$

$$a_n = \frac{v^2}{\rho} : \quad \frac{v_0^2}{\rho_0} = \frac{v_0^2 - 2gy}{\rho} , \quad \underline{\rho = \rho_0 \left(1 - \frac{2gy}{v_0^2}\right)}$$

For car to remain in contact with the track at the top, $a_n > g$, so for constant a_n, $v_0^2/\rho_0 > g$ so $\underline{v_{0_{min}} \doteq \sqrt{\rho_0 g}}$

3/154 Establish datum at release point.

$$T_A + V_A = T_B + V_B$$

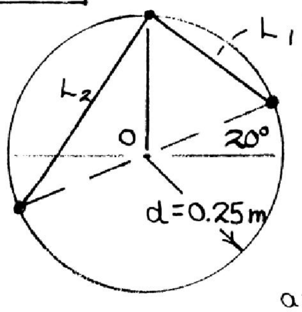
$$0 + \frac{1}{2}k_A x_A^2 = 0 + mg(x_A + d + x_B) + \frac{1}{2}k_B x_B^2$$

$$\frac{1}{2}(48)(12)\left(\frac{5}{12}\right)^2 = 14\left(\frac{5+14+x_B}{12}\right) + \frac{1}{2}(10)(12)\left(\frac{x_B}{12}\right)^2$$

$$\underline{x_B = 6.89 \text{ in.}}$$

The fact that $x_B > x_A$ is due to the difference in spring stiffnesses (along with the particular value $d = 20 - 6 = 14''$). Note that $d = 14''$ is the distance which the collar moves when out of contact with the springs.

3/155



$$\begin{cases} L_1 = 2d\sin(90°-20°)/2 = 0.287\text{ m} \\ \delta_1 = 0.25\sqrt{2} - L_1 = 0.0668\text{ m} \\ L_2 = 2d\sin\left(\frac{90°+20°}{2}\right) = 0.410\text{ m} \\ \delta_2 = L_2 - 0.25\sqrt{2} = 0.0560\text{ m} \end{cases}$$

We may ignore the equal and opposite potential energy changes associated with two of the masses,

$T_1 + V_1 = T_2 + V_2$, datum at O.

$$0 - mgd\cos 20° + \tfrac{1}{2}k\delta_1^2 + \tfrac{1}{2}k\delta_2^2 = 3\left(\tfrac{1}{2}md^2\dot{\theta}^2\right) - mgd$$

$$0 - 3(9.81)(0.25)\cos 20° + \tfrac{1}{2}1200(0.0668)^2$$
$$+\tfrac{1}{2}1200(0.0560)^2 = \tfrac{3}{2}3(0.25)^2\dot{\theta}^2 - 3(9.81)(0.25)$$

Solving, $\underline{\dot{\theta} = 4.22\text{ rad/s}}$

3/156 The system is conservative so

$\Delta T + \Delta V_e + \Delta V_g = 0$; Spring stretch = 13-12=1 in.

$\Delta T = \frac{1}{2}\frac{3}{32.2}(v_B^2 - 8^2) = 0.0466 v_B^2 - 2.981$ ft-lb

$\Delta V_e = 2\left(\frac{1}{2}\,10 \times 1^2 - 0\right)\frac{1}{12} = 0.833$ ft-lb

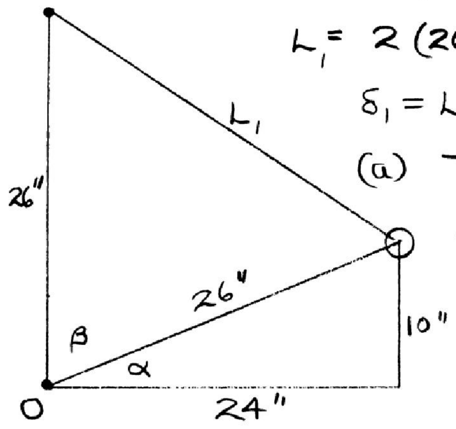
$\Delta V_g = 3(-5/12) = -1.250$ ft-lb

Thus $0.0466\, v_B^2 - 2.981 + 0.833 - 1.250 = 0$

$0.0466\, v_B^2 = 3.398$, $v_B^2 = 72.94$ $(ft/sec)^2$

$\underline{v_B = 8.54 \text{ ft/sec}}$

3/157



$\alpha = \tan^{-1}\frac{10}{24} = 22.6°$, $\beta = 90 - \alpha = 67.4°$

$L_1 = 2(26)\sin\frac{\beta}{2} = 28.8''$

$\delta_1 = L_1 - 25'' = 3.84''$

(a) $T_1 + V_1 = T_2 + V_2$, datum @ O

$$0 + 9\left(\frac{10}{12}\right) + \frac{1}{2}1.2(12)\left(\frac{3.84}{12}\right)^2$$
$$= \frac{1}{2}\frac{9}{32.2}v^2 + \frac{1}{2}(1.2)(12)\times$$
$$\left(\left[2(26)\sin\frac{90°}{2} - 25\right]/12\right)^2$$

$\underline{v = 3.06 \text{ ft/sec}}$

(b) $T_1 + V_1 = T_3 + V_3$, datum @ O

$$0 + 9\left(\frac{10}{12}\right) + \frac{1}{2}1.2(12)\left(\frac{3.84}{12}\right)^2 = \frac{1}{2}\frac{9}{32.2}v^2$$
$$-9\,\frac{26}{12}\sin(35° - \alpha) + \frac{1}{2}(1.2)(12)\left[\frac{2(26)\sin\frac{\beta + 35°}{2} - 25}{12}\right]^2$$

$\underline{v = 1.641 \text{ ft/sec}}$

3/158 A force analysis reveals that A will move down & B will move up.

Kinematics : $3v_A = 2v_B$ (speeds)

$T_1 + V_1 = T_2 + V_2$, datum @ initial position

$$0 + 0 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B \left(\frac{3}{2} v_A\right)^2 + m_B g h_B - m_A g h_A$$

$$0 = \frac{1}{2}(40) v_A^2 + \frac{1}{2} 8 \frac{9}{4} v_A^2 + 8(9.81)(1) - 40(9.81)\left(\frac{2}{3}(1)\sin 20°\right)$$

$\underline{v_A = 0.616 \text{ m/s}}$, $\underline{v_B = \frac{3}{2} v_A = 0.924 \text{ m/s}}$

3/159 Establish datum at $\theta = 0$ (state A); state B is $\theta = 30°$.

$$T_A + V_A = T_B + V_B$$

$$0 + 0 = 2\left(\frac{1}{2} m v_B^2\right) - 2mgL(1 - \cos\theta) + \frac{1}{2} k \left[2 \frac{L}{2} \sin\theta\right]^2$$

Numbers: $0 = 1.5v^2 - 2(1.5)(9.81)(0.48)(1 - \cos 30°) + \frac{1}{2} 60 \left[0.48 \sin 30°\right]^2$

Solving, $\underline{v = 0.331 \text{ m/s}}$

3/160 | $\Delta T + \Delta V_g = 0$

$$\frac{1}{2}m(v^2 - v_0^2) - \frac{mgR^2}{r} + \frac{mgR^2}{R} = 0$$

Let $v = 0$ for $r = \infty$ & get

$$-\frac{v_0^2}{2} + gR, \quad v_0 = \sqrt{2gR}$$

$$= \sqrt{2 \times 9.825 \times 6371 \times 10^3}$$

$$= 11\,190 \text{ m/s} = \underline{11.19 \text{ km/s}}$$

3/161 | Constant total energy is $E = T_A + V_{g_A} = T_P + V_{g_A}$

Thus $\frac{1}{2} m v_A^2 - \frac{mgR^2}{r_A} = \frac{1}{2} m v_P^2 - \frac{mgR^2}{r_P}$

$v_A^2 = v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right)$, $\underline{v_A = \sqrt{v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right)}}$

3/162 $\Delta T = \frac{1}{2} m (v_B^2 - v_A^2) = \frac{1}{2} \frac{48}{32.2} [\overline{7600}^2 - \overline{8000}^2] \left(\frac{5280}{3600}\right)^2$

$$= -10.00(10^6) \text{ ft-lb}$$

$\Delta V_g = -mgR^2\left(\frac{1}{r_B} - \frac{1}{r_A}\right) = -48(3959)^2(5280)\left(\frac{1}{4008} - \frac{1}{3983}\right)$

$$= 6.221\,(10^6) \text{ ft-lb}$$

$U = -P(250)(5280) = -1.320\,P(10^6)$ ft-lb

$U = \Delta T + \Delta V_g$; $-1.320\,P = -10.00 + 6.221$, $\underline{P = 2.86 \text{ lb}}$

$\underline{3/163}$ | $\Delta T + \Delta V_g = 0 \quad , \quad V_g = -\dfrac{mgR^2}{r}$

Mean radius of earth is $R = 6371$ km

$g = 9.825\,(3600)^2/1000 = 127.3\,(10^3)$ km/h^2

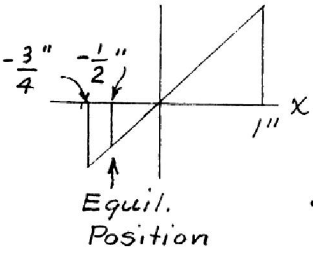
Thus

$$\frac{1}{2}m\left(v_B^2 - [24\,000]^2\right) + 127.3(10^3)(6371)^2 m\left(-\frac{1}{6500} + \frac{1}{7000}\right) = 0$$

$$\frac{1}{2}v_B^2 - 288(10^6) + 5167(10^9)(-0.01099)(10^{-3}) = 0$$

$$v_B^2 = 2\left[288 + 56.8\right]10^6 = 690\,(10^6), \quad \underline{v_B = 26\,300 \text{ km/h}}$$

3/164 $F = 10x$



Let x = stretch of spring from position of zero force

$$\Delta V_g + \Delta V_e + \Delta T = 0$$

$$\Delta V_g = -Wh = -5\frac{1.75}{12} = -0.729 \text{ ft-lb}$$

$$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{10}{2}\left(\left[\frac{3}{4}\right]^2 - \left[1\right]^2\right)\frac{1}{12}$$

$$= -0.1823 \text{ ft-lb}$$

$$\Delta T = \frac{1}{2}\frac{W}{g}v^2 = \frac{1}{2}\frac{5}{32.2}v^2 = 0.0776v^2$$

Thus $0.0776v^2 - 0.729 - 0.1823 = 0$, $v^2 = 11.75$, $\underline{v = 3.43 \frac{ft}{sec}}$

3/165 $U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$ for system

$U'_{1-2} = 0$

$\Delta T = \frac{1}{2} m v^2 = \frac{1}{2} 5 v^2 = 2.5 v^2$ J

$\Delta V_g = mgh = -5 \times 9.81(0.100 + x)$ J, x in meters

$\Delta V_e = \frac{1}{2} k (x_2^2 - x_1^2) = \frac{1}{2} 1.8(10^3) x^2 = 900 x^2$ J

For x_{max}, $\Delta T = 0$ so $0 = 0 - 5 \times 9.81(0.100 + x_{max}) + 900 x_{max}^2$

$$x_{max}^2 - 0.0545 x_{max} - 0.00545 = 0$$

$$x_{max} = 0.1059 \text{ m} \quad (\text{or } -0.0514)$$

or $\underline{x_{max} = 105.9 \text{ mm}}$

For v_{max}, $0 = 2.5 v^2 - 5 \times 9.81(0.100 + x) + 900 x^2$

$$v^2 = 1.962 + 19.62 x - 360 x^2$$

$\frac{d(v^2)}{dx} = 19.62 - 720 x = 0$ for max v^2 & hence max v

$x = 0.0272$ m or $\underline{x = 27.2 \text{ mm}}$

$$v_{max} = \sqrt{1.962 + 19.62(0.0272) - 360(0.0272^2)}$$

$= \underline{1.493 \text{ m/s}}$

3/166 For interval from impact to maximum deformation of spring, energy is conserved for system of spring & cylinder

so $\Delta E = \Delta T + \Delta V_g + \Delta V_e = 0$

$\Delta T = 0 - \frac{1}{2}mv^2, \quad \Delta V_g = -mg\delta$

Initial stretch of spring is the static deflection under constant-velocity (equil.) condition δ_{ST}

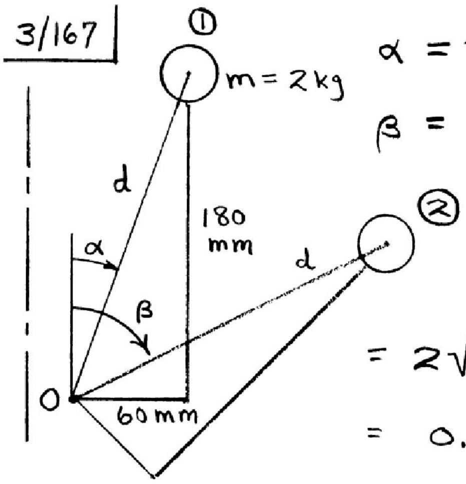
$\delta_{ST} = mg/k$.

$$\Delta V_e = \frac{1}{2}k(\delta_{ST}+\delta)^2 - \frac{1}{2}k\delta_{ST}^2 = k\delta_{ST}\delta + \frac{1}{2}k\delta^2$$
$$= mg\delta + \frac{1}{2}k\delta^2$$

Thus $0 = -\frac{1}{2}mv^2 - mg\delta + mg\delta + \frac{1}{2}k\delta^2$

$mv^2 = k\delta^2, \quad \underline{\delta = v\sqrt{m/k}}$

3/167



$\alpha = \tan^{-1}\frac{60}{180} = 18.43°$

$\beta = \alpha + 45° = 63.4°$

Stretch in spring is $\delta = 2d(\sin\beta - \sin\alpha)$

$= 2\sqrt{0.060^2 + 0.180^2}\,(\sin 63.4° - \sin 18.43°)$

$= 0.219$ m

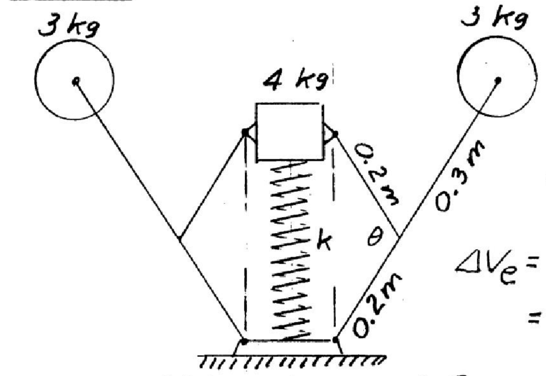
$T_1 + V_1 = T_2 + V_2$, datum at O

$0 + 2mgd\cos\alpha = 0 + 2mgd\cos\beta + \frac{1}{2}k\delta^2$

$2(2)(9.81)(0.1897)\cos 18.43° = 2(2)(9.81)(0.1897)\cos 63.4° + \frac{1}{2}k(0.219)^2$

Solving, $\underline{k = 155.1 \text{ N/m}}$

3/168



$\Delta V_e + \Delta V_g + \Delta T = 0$

$\Delta T = 0$

$\Delta V_e = \frac{1}{2}(900)\left[0.4(1 - \sin\frac{\theta}{2})\right]^2$
$= 72(1 - \sin\theta/2)^2$ J

$\Delta V_g = -2(3)(9.81)(0.5)(1 - \sin\frac{\theta}{2}) - 4(9.81)(0.4)(1 - \sin\frac{\theta}{2})$
$= -45.13(1 - \sin\frac{\theta}{2})$

Thus $72(1 - \sin\frac{\theta}{2})^2 = 45.13(1 - \sin\frac{\theta}{2})$

$\sin\frac{\theta}{2} = 1 - \frac{45.13}{72} = 0.3733$, $\underline{\theta = 43.8°}$

3/169 Ellipse eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$

$= \sqrt{1 - \frac{0.6^2}{0.8^2}} = 0.661$

$r_{min} = a(1-e) = 0.8(1-0.661) = 0.271$ m

$r_{max} = a(1+e) = 0.8(1+0.661) = 1.329$ m

$T_A + V_A = T_C + V_C$

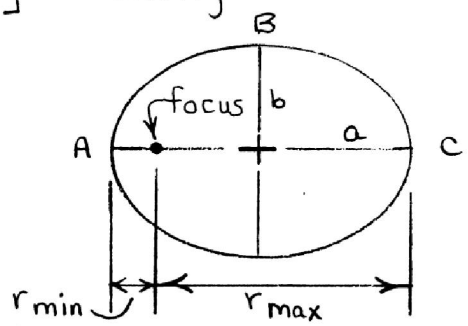
$\frac{1}{2} m v_A^2 + 0 = 0 + \frac{1}{2} k x_C^2$

$\frac{1}{2}(0.4) v_A^2 = \frac{1}{2}(3)\left[1.329 - 0.271\right]^2$, $v_A = 2.90$ m/s

Then $T_A + V_A = T_B + V_B$

$\frac{1}{2}(0.4)(2.90)^2 + 0 = \frac{1}{2}(0.4) v_B^2 + \frac{3}{2}\left\{\left[(0.8-0.271)^2 + (0.6)^2\right]^{1/2} - 0.271\right\}^2$

$\underline{v_B = 2.51 \text{ m/s}}$



3/170 $U'_{1-2} = 0$ so $T_1 + V_{g_1} = T_2 + V_{g_2}$

Take datum $V_g = 0$ at ground level.

$T_1 = \frac{1}{2}\,\frac{175+10}{32.2}\,v^2 = 2.87 v^2$, $T_2 = 0$

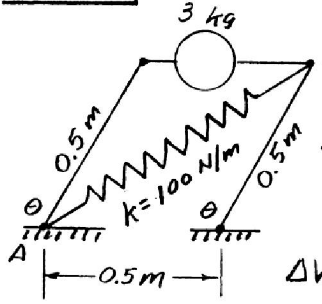
$V_{g_1} = (175+10)\,\frac{42}{12} = 648$ ft-lb

$V_{g_2} = 175(18) + 10(8) = 3230$ ft-lb

So $2.87 v^2 + 648 = 0 + 3230$

$\underline{v = 30.0 \text{ ft/sec}}$ or $\underline{20.4 \text{ mi/hr}}$

3/171 Spring deformation is



$$\delta = \overline{AB} - 0.5\sqrt{2}$$

$$\overline{AB} = \sqrt{2(0.5)^2 - 2(0.5)^2\cos\theta}$$

$$= 0.5\sqrt{2}\sqrt{1-\cos 135°}$$

$$\delta = 0.5\sqrt{2}\left[\sqrt{1+0.7071} - 1\right]$$

$$= 0.217 \text{ m}$$

$$\Delta V_e = \frac{1}{2}k\delta^2 = \frac{1}{2}(100)(0.217)^2 = 2.35 \text{ J}$$

$$\Delta V_g = -mg\Delta h = -3(9.81)(0.5)(1-\sin 135°) = -4.31 \text{ J}$$

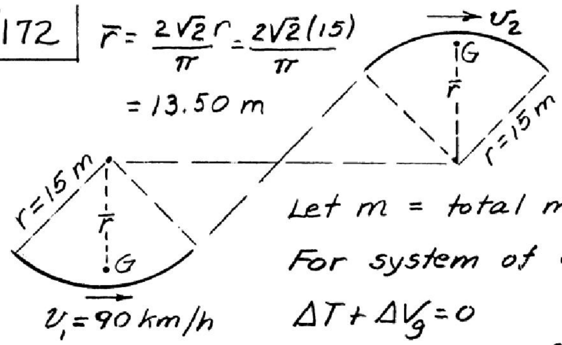
$$\Delta T + \Delta V_g + \Delta V_e = 0; \quad \frac{1}{2}3v^2 - 4.31 + 2.35 = 0$$

$$v^2 = 1.307, \quad \underline{v = 1.143 \text{ m/s}}$$

3/172

$\bar{r} = \frac{2\sqrt{2}\,r}{\pi} = \frac{2\sqrt{2}(15)}{\pi}$

$= 13.50$ m



Let m = total mass of train

For system of cars

$\Delta T + \Delta V_g = 0$

$\frac{1}{2}m(v_2^2 - v_1^2) + mg(2\bar{r}) = 0 \;,\; v_2^2 = v_1^2 - 4g\bar{r}$

$v_2^2 = \left[\frac{90(1000)}{3600}\right]^2 - 4(9.81)(13.50) = 625 - 529.9 = 95.07 \;(\text{m/s})^2$

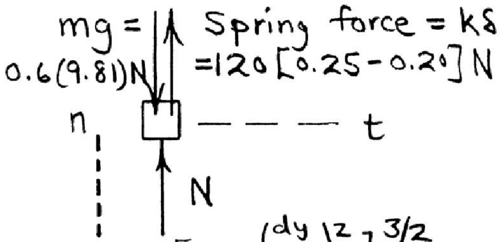
$v_2 = 9.75$ m/s or $\underline{v_2 = 35.1 \text{ km/h}}$

3/173 $T_A + V_A = T_B + V_B$, datum @ B.

$$0 + 0.6(9.81)(0.5) + \frac{1}{2}120\left[\sqrt{0.25^2+0.5^2} - 0.2\right]^2$$
$$= \frac{1}{2}(0.6)v_B^2 + \frac{1}{2}120\left[0.25-0.20\right]^2$$

$$\underline{v_B = 5.92 \text{ m/s}}$$

Kinetics at B:



Radius of curvature $\rho = \dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$y = kx^2$; $0.5 = k(0.5)^2 \Rightarrow k = 2$

$y = 2x^2$, $\dfrac{dy}{dx} = 4x$, $\dfrac{d^2y}{dx^2} = 4$

When $x = 0$, $\rho = \dfrac{[1+0^2]^{3/2}}{4} = 0.25$ m

$\Sigma F_n = ma_n$: $N + 120(0.05) - 0.6(9.81)$
$$= 0.6\,\frac{5.92^2}{0.25}$$

$$\underline{N = 84.1 \text{ N}}$$

3/174 $x^2 + y^2 = 0.9^2$, $x\dot{x} + y\dot{y} = 0$, $v_A = -\dot{y} = \frac{x}{y}\dot{x} = \frac{x}{y} v_B$

$\Delta T + \Delta V_g = 0$; $\frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mg\left(y - \frac{0.9}{\sqrt{2}}\right) = 0$

$\dot{x}^2\left(1 + \frac{x^2}{y^2}\right) = 2(9.81)\left(\frac{0.9}{\sqrt{2}} - y\right)$, $\dot{x}^2 \frac{x^2+y^2}{y^2} = 19.62\left(\frac{0.9}{\sqrt{2}} - y\right)$

$0.9^2\dot{x}^2 = 19.62\left(\frac{0.9}{\sqrt{2}}y^2 - y^3\right)$

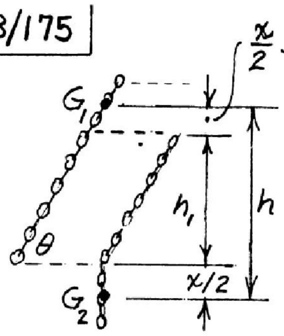
For max. $\dot{x}$, $\frac{d(\dot{x}^2)}{dy} = \frac{19.62}{0.81}\left(\frac{1.8}{\sqrt{2}}y - 3y^2\right) = 0$

So $y\left(\frac{1.8}{\sqrt{2}} - 3y\right) = 0$, $y = 0.6/\sqrt{2}$ m

& $\dot{x}^2 = \frac{19.62}{0.81}\left(\frac{0.9}{\sqrt{2}}\frac{0.36}{2} - \frac{0.108}{\sqrt{2}}\right) = \frac{19.62\sqrt{2}}{30}$

$v_{B_{max}} = \dot{x} = \sqrt{\frac{19.62\sqrt{2}}{30}} = \underline{0.962 \text{ m/s}}$

3/175



$h_1 = (L-x)\sin\theta$

$h = (L-x)\sin\theta + \frac{x}{2}\sin\theta + \frac{x}{2}$

$= L\sin\theta + \frac{x}{2}(1-\sin\theta)$

Let ρ = mass per unit length

$\Delta V_g + \Delta T = 0$

ΔV_g is that of the length x dropping a distance h

$\Delta V_g = -\rho g x h = -\rho g\left[Lx\sin\theta + \frac{x^2}{2}(1-\sin\theta)\right]$

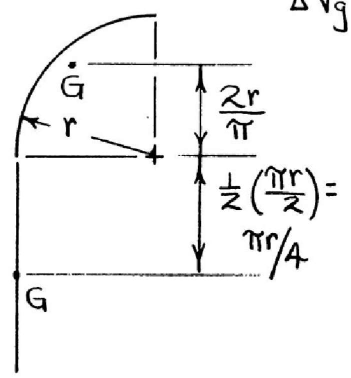
$\Delta T = \frac{1}{2}\rho L v^2$

Thus $-\rho g\left[Lx\sin\theta + \frac{x^2}{2}(1-\sin\theta)\right] + \frac{1}{2}\rho L v^2 = 0$

$$v = \sqrt{2gx\left[\sin\theta - \frac{x}{2L}(1-\sin\theta)\right]}$$

3/176 For entire chain, $U'_{1-2} = \Delta(T + V_g)$

$U'_{1-2} = 0$, $\Delta T = \frac{1}{2} m v^2 = \frac{1}{2}\rho\left(\frac{\pi r}{2}\right)v^2 = \frac{1}{4}\rho\pi r v^2$



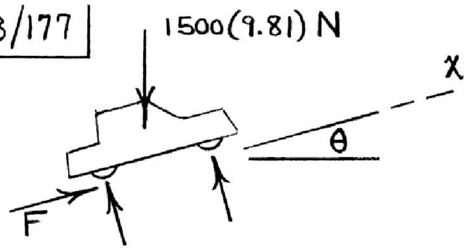
$$\Delta V_g = -\rho\left(\frac{\pi r}{2}\right)g\left[\frac{\pi r}{4} + \frac{2r}{\pi}\right]$$

$$= -\frac{1}{2}\rho g r^2 \pi\left(\frac{\pi}{4} + \frac{2}{\pi}\right)$$

So $\frac{1}{4}\rho\pi r v^2 - \frac{1}{2}\rho g r^2 \pi\left(\frac{\pi}{4} + \frac{2}{\pi}\right) = 0$

$$\underline{v = \sqrt{gr\left(\frac{\pi}{2} + \frac{4}{\pi}\right)}}$$

3/177



$\theta = \tan^{-1}\frac{1}{10} = 5.71°$

$\int \Sigma F_x \, dt = \Delta G_x : \left[F - 1500(9.81)\sin 5.71°\right] 8 = 1500(60-30)\frac{1000}{3600}$

$F = 3030$ N or $\underline{F = 3.03 \text{ kN}}$

3/178

$$\begin{cases} \underline{v} = 1.5t^3\underline{i} + (2.4 - 3t^2)\underline{j} + 5\underline{k} & (\text{m/s}) \\ \dot{\underline{v}} = 4.5t^2\underline{i} - 6t\underline{j} & (\text{m/s}^2) \end{cases}$$

At $t = 2\text{s}$:
$$\begin{cases} \underline{v} = 12\underline{i} - 9.6\underline{j} + 5\underline{k} \text{ m/s} \\ \dot{v} = 18\underline{i} - 12\underline{j} \text{ m/s}^2 \end{cases}$$

Then $\underline{G} = m\underline{v} = 1.2(12\underline{i} - 9.6\underline{j} + 5\underline{k})$

$$= \underline{14.40\underline{i} - 11.52\underline{j} + 6\underline{k} \text{ kg}\cdot\text{m/s}}$$

$$G = \sqrt{14.40^2 + 11.52^2 + 6^2} = \underline{19.39 \text{ kg}\cdot\text{m/s}}$$

$$\Sigma\underline{F} = \dot{\underline{G}} : \underline{R} = m\dot{\underline{v}} = 1.2(18\underline{i} - 12\underline{j})$$

$$= \underline{21.6\underline{i} - 14.4\underline{j} \text{ N}}$$

3/179 | Conservation of system linear momentum:

$\xrightarrow{+}$ $0.075(600) = 50.075\, v_f$, $v_f = 0.899$ m/s

Initial energy $T_1 = \frac{1}{2}(0.075)(600)^2 = 13\ 500$ J

Final energy $T_2 = \frac{1}{2}(50.075)(0.899)^2 = 20.2$ J

Absolute energy loss $|\Delta E| = T_1 - T_2 = \underline{13\ 480 \text{ J}}$

Percent lost : $n = \frac{|\Delta E|}{T_1}(100\%) = \underline{99.9\ \%}$

$\underline{3/180}$

$$\int \Sigma F\,dt = m\Delta v; \quad (16\,000 - \Delta R)9 = 10\,000(1050-1000)/3.6$$

$$\underline{\Delta R = 568 \text{ N}}$$

3/181 $\Delta G = 0$; $150{,}000 \times 2 + 120{,}000 \times 3$

$= (150{,}000 + 120{,}000)\, v$, $\underline{v = 2.44 \text{ mi/hr}}$

$$|\Delta E| = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 - \frac{1}{2}(m_A + m_B) v^2$$

$$= \frac{1}{2(32.2)} \left(\frac{44}{30}\right)^2 \left[150{,}000 \times 2^2 + 120{,}000 \times 3^2 - 270{,}000 \times \overline{2.44}^2\right]$$

$$\underline{= 2230 \text{ ft-lb loss}}$$

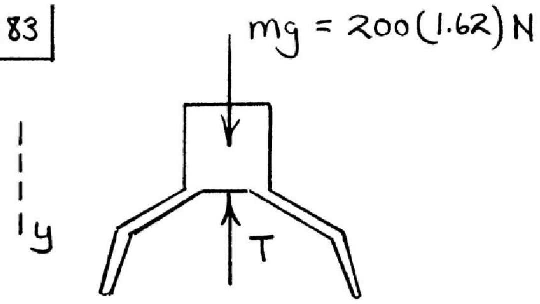
$\underline{3/182}$ No difference between cases (a) & (b).

$G_1 = G_2 : \quad mv = (3m)v', \quad \underline{v' = \frac{v}{3}}$

$T = \frac{1}{2}mv^2, \quad T' = \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \frac{1}{6}mv^2$

$$n = \frac{T - T'}{T} = \frac{\frac{1}{2}mv^2 - \frac{1}{6}mv^2}{\frac{1}{2}mv^2} = \underline{\frac{2}{3}}$$

3/183



$\int \Sigma F_y \, dt = m \Delta v_y$:

$200(1.62)(5) - \left[\frac{1}{2} 2(800) + 2(800)\right] = 200(v - 6)$

$\underline{v = 2.10 \text{ m/s}}$

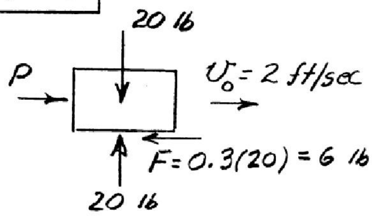
3/184 | 6 th rocket burns out after 7 sec so total impulse on sled during the 10 sec is

$6(8600) - 10R$.

$\int \Sigma F dt = m \Delta v$; $6(8600) - 10R = \frac{5200}{32.2}\left(200 \frac{44}{30} - 0\right)$

$\underline{R = 423 \text{ lb}}$

3/185

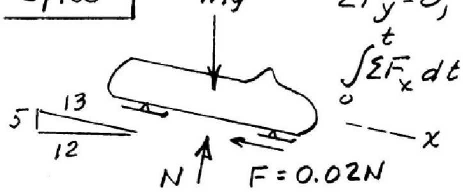


$$\int_0^t \Sigma F\,dt = m\Delta v$$

$$16(0.2) + 8(0.2) - 6(0.4) = \frac{20}{32.2}(v - 2)$$

$$\underline{v = 5.86 \text{ ft/sec}}$$

3/186



$\Sigma F_y = 0;\quad N = mg\,\frac{12}{13}$

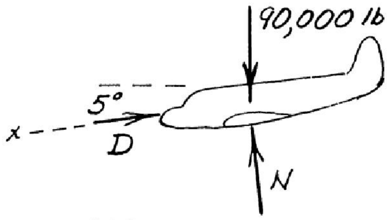
$\int_0^t \Sigma F_x\,dt = m\Delta v_x$

$\left(mg\frac{5}{13} - 0.02\,mg\frac{12}{13}\right)t$

$= m\left(\frac{100\times1000}{3600} - 0\right)$

$\frac{9.81}{13}(5 - 0.02[12])\,t = 27.78,\quad \underline{t = 7.73\text{ s}}$

3/187



$\int \Sigma F_x \, dt = \Delta G_x$:

$$(90{,}000 \sin 5° - D)\,120 = \frac{90{,}000}{32.2}(360 - 400)\frac{5280}{3600}$$

$\underline{D = 9210 \text{ lb}}$

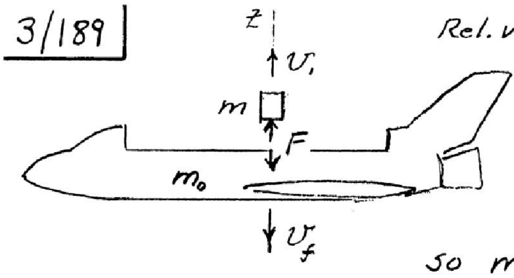
$\boxed{3/188}$ $\int F\,dt = m\Delta v$

$$(50{,}000 \cos 20°)\,t = \frac{150{,}000 \times 2240}{32.2}\;\frac{1 \times 1.151}{1}\;\frac{44}{30}$$

$$46{,}985\,t = 17.62 \times 10^6$$

$t = 375$ sec or $\underline{t = 6.25 \text{ min}}$

3/189



Rel. velocity is

$v_1 + v_f = 0.3$ m/s --(1)

$\int F\,dt = m v_1$

$\int -F\,dt = m_f(-v_f)$

so $m v_1 = m_0 v_f$

$800\, v_1 = 90\,000\, v_f$ -----(2)

Solve (1) & (2) & get $v_f = 0.3 - \frac{90000}{800} v_f$

$\underline{v_f = 0.00264 \text{ m/s}}$

So $F_{av}\int_0^4 dt = 90\,000\,(0.00264)$, $F_{av} = \frac{90(2.64)}{4} = \underline{59.5 \text{ N}}$

3/190 $G_1 = G_2$: $\quad mv = (m + pm)v'$, $\underline{v' = \dfrac{v}{1+p}}$

$$\bar{a}_A = \frac{v'-v}{\Delta t} = \frac{\frac{v}{1+p} - v}{\Delta t} = \underline{\frac{-vp}{\Delta t(1+p)}}$$

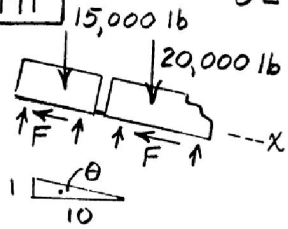
$$\bar{a}_B = \frac{v'-0}{\Delta t} = \underline{\frac{v}{\Delta t(1+p)}} \quad \text{------} \oplus$$

For $p = \frac{1}{2}$, $\quad v' = \dfrac{v}{1+1/2} = \underline{\dfrac{2}{3}v}$

$$\bar{a}_A = \frac{-v\left(\frac{1}{2}\right)}{\Delta t\left(1+\frac{1}{2}\right)} = \underline{-\frac{1}{3}\frac{v}{\Delta t}}$$

$$\bar{a}_B = \frac{v}{\Delta t\left(1+\frac{1}{2}\right)} = \underline{\frac{2}{3}\frac{v}{\Delta t}}$$

3/191



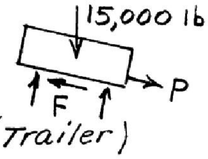
$\theta = \tan^{-1} 0.1 = 5.71°$, $\sin\theta = 0.0995$

$\int \Sigma F_x \, dt = m \Delta v_x$

$[35{,}000 \times 0.0995 - 2F]5$

$= \frac{35{,}000}{32.2}\left(0 - 20\frac{44}{30}\right)$

$F = 4930$ lb

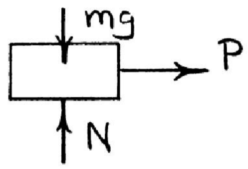


$[P - 4930 + 15{,}000 \times 0.0995]5$

$= \frac{15{,}000}{32.2}\left(0 - 20\frac{44}{30}\right)$

$P = 704$ lb (tension)

3/192



$\xrightarrow{+}$ $mv_1 + \int \Sigma F \, dt = mv_2$:

$$0 + \int_0^t F_0 e^{-bt} \, dt = mv$$

$$v = \frac{F_0}{mb}\left(1 - e^{-bt}\right), \quad v \to \frac{F_0}{mb} \text{ as } t \to \infty$$

$$\frac{ds}{dt} = \frac{F_0}{mb}\left(1 - e^{-bt}\right)$$

$$\int_{s_0=0}^{s} ds = \int \frac{F_0}{mb}\left(1 - e^{-bt}\right) dt$$

$$s = \frac{F_0}{mb}\left[t + \frac{1}{b}\left(e^{-bt} - 1\right)\right]$$

3/193 $F = kt^2: \quad 20 = k(2)^2, \quad k = 5\ \text{N/s}^2$

$F = 5t^2 \qquad (0 \le t \le 2\,\text{s})$

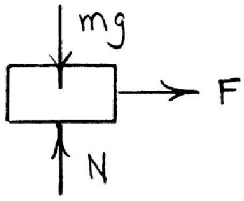
$t = 1\,\text{s}: \quad \cancelto{0}{mv_0} + \int_0^1 F\,dt = mv_1$

$v_1 = \frac{1}{4}\int_0^1 5t^2\,dt = \frac{1}{4}\,5\,\frac{t^3}{3}\Big|_0^1 = \underline{0.417\ \text{m/s}}$

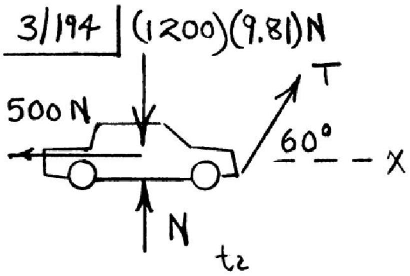
$t = 3\,\text{s}: \quad \cancelto{0}{mv_0} + \int_0^3 F\,dt = mv_3$

$v_3 = \frac{1}{4}\left[\int_0^2 5t^2\,dt + \frac{3-2}{2}(25-20) + (3-2)(20)\right]$

$= \underline{8.96\ \text{m/s}}$



3/194



$$mv_{x_1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = mv_{x_2}:$$

$$1200\left(\frac{30}{3.6}\right) + \left[-500 + T\cos 60°\right]15 = 1200\left(\frac{70}{3.6}\right)$$

$\underline{T = 2780 \text{ N}}$

3/195 Impact velocity $v_0 = \sqrt{2gh} = \sqrt{2(9.81)(1.4)}$
$= 5.24$ m/s

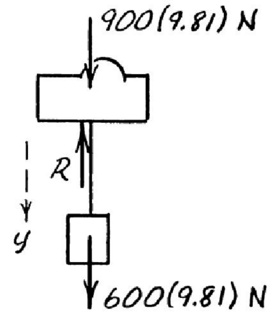
$\Delta G = 0$; $450(5.24) + 0 = (450 + 240)v$
$v = 3.42$ m/s

Impulse of weights is negligible compared with impulse of impact forces.

3/196 Entire system: $\int \Sigma F_y \, dt = \Delta G_y$

$$[(600+900)9.81 - R]6 = 600(0.5-3)$$

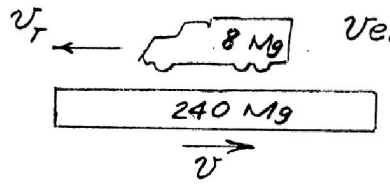
$$\underline{R = 14.96 \text{ kN}}$$



3/197 | $\Delta G = 0; \quad 12(20) + 0 = (350 + 12)v$

$\underline{v = 0.663 \text{ km/h}}$

3/198



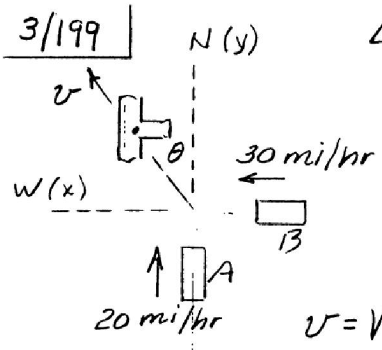
Vel. truck $v_T = v_{rel} - v$

$= 6 - v$ km/h

$\Delta G = 0$, $G_{initial} = G_{final} = 0$

$8(6 - v) = 240 v$, $v = 0.1935$ km/h

3/199



$$\Delta G_x = 0; \quad \frac{3200}{g}(30) = \frac{(3200+3400)}{g} v_x$$

$$v_x = 14.55 \text{ mi/hr}$$

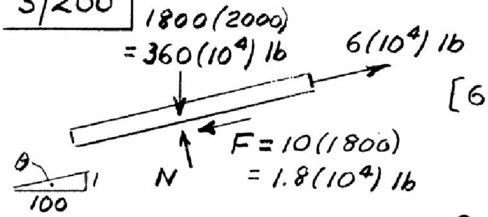
$$\Delta G_y = 0; \quad \frac{3400(20)}{g} = \frac{(3200+3400)}{g} v_y$$

$$v_y = 10.30 \text{ mi/hr}.$$

$$v = \sqrt{(14.55)^2 + (10.30)^2} = \underline{17.82 \text{ mi/hr}}$$

$$\theta = \tan^{-1}\frac{v_x}{v_y} = \tan^{-1}\frac{14.55}{10.30} = \underline{54.7^\circ}$$

3/200



$\sin\theta = 0.0100$

$$\int \Sigma F\,dt = m\Delta v$$

$$[6 - 1.8 - 360(0.0100)]10^4 t = \frac{360(10^4)}{32.2}(30-20)\frac{44}{30}$$

$$0.600\,t = 163.98$$

$t = 273.2$ sec or $\underline{t = 4 \text{ min } 33 \text{ sec}}$

3/201 $\underline{G}_1 = \underline{G}_2$: $m_s \underline{v}_s + m_m \underline{v}_m = (m_s + m_m)\underline{v}$

$$1000\,(2000)\underline{j} + 10\,(5000)\left[\frac{+5\underline{i} - 4\underline{j} - 2\underline{k}}{\sqrt{5^2+4^2+2^2}}\right] = (1000+10)\,\underline{v}$$

$$\underline{v} = 36.9\underline{i} + 1951\underline{j} - 14.76\underline{k} \quad \text{m/s}$$

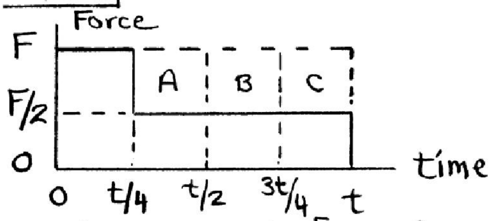
The angle between $\underline{v}_s$ and $\underline{v}$ is

$$\beta = \cos^{-1}\frac{\underline{v}\cdot\underline{v}_s}{v v_s}$$

$$= \cos^{-1}\left[\frac{(36.9\underline{i} + 1951\underline{j} - 14.76\underline{k})\cdot 2000\underline{j}}{\sqrt{36.9^2 + 1951^2 + 14.76^2}\;2000}\right]$$

$$= 1.167^\circ$$

3/202



Solid area is 5/8 of nominal area, so $\underline{\eta = 62.5\%}$

In order to compensate, areas A, B, & C must be added after time t, so the extra time $\underline{t' = \frac{3}{4} t}$.