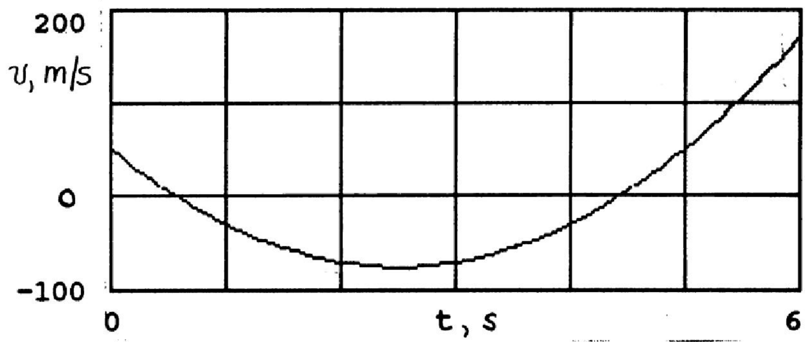
2/1 $\quad v = 20t^2 - 100t + 50$

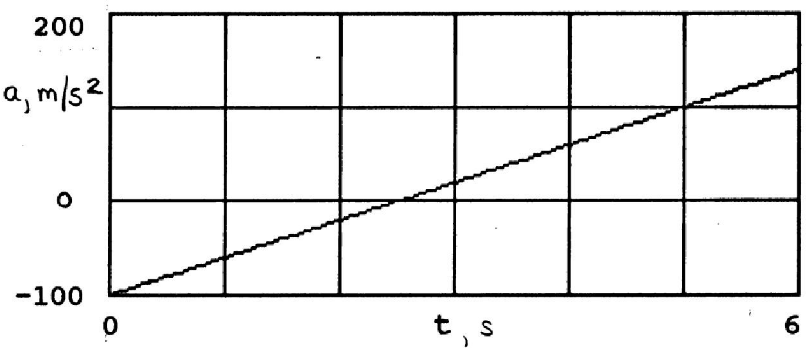
$a = \frac{dv}{dt} = \underline{40t - 100}$ $\Big\}$ See plots

$a = 0: \quad 40t - 100 = 0, \quad t = 2.5\text{ s}$

At $t = 2.5$ s, $\quad v = 20(2.5)^2 - 100(2.5) + 50$

$= \underline{-75\text{ m/s}}$





2/12

$$s = 2t^3 - 30t^2 + 100t - 50$$

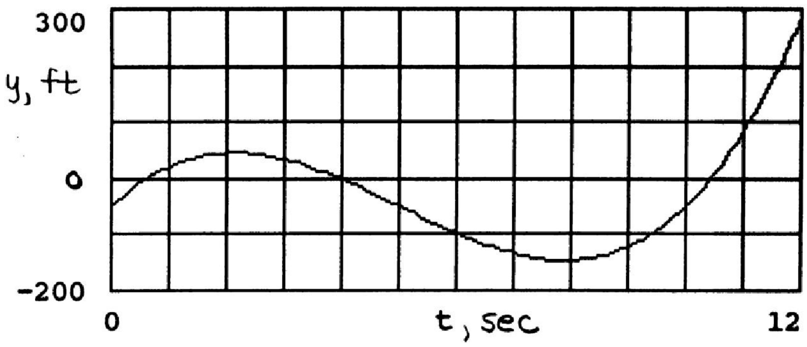
$$v = \frac{ds}{dt} = 6t^2 - 60t + 100$$

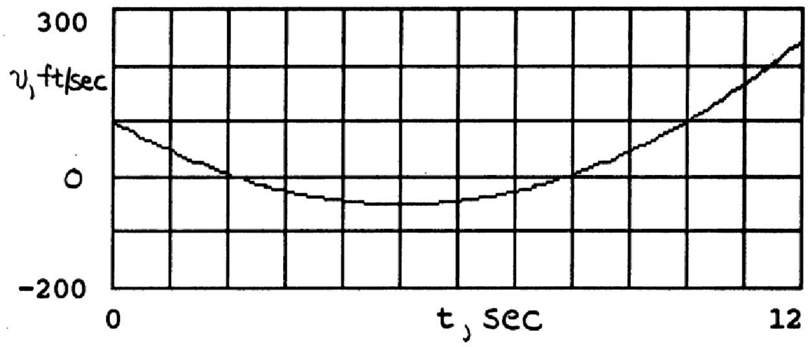
$$a = \frac{dv}{dt} = 12t - 60$$

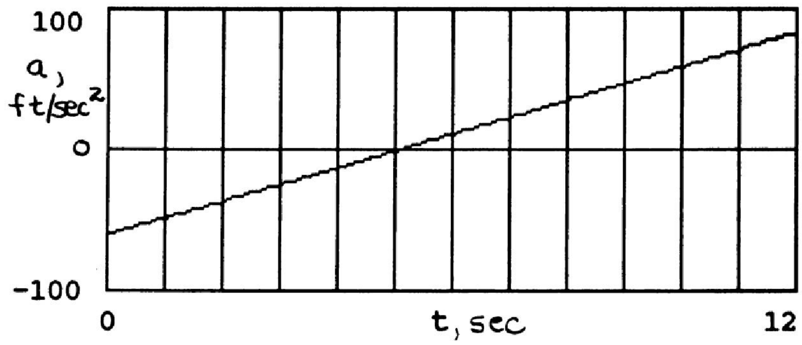
See plots

$$v = 0: \quad 6t^2 - 60t + 100 = 0$$

$$t = \frac{60 \pm \sqrt{60^2 - 4(6)(100)}}{2 \cdot 6} = \underline{2.11 \text{ sec}, \ 7.89 \text{ sec}}$$







2/3

$$v = 2 + 5t^{3/2}$$

$$a = \frac{dv}{dt} = \frac{3}{2} \cdot 5t^{\frac{1}{2}} = \frac{15}{2}\sqrt{t}$$

$$\frac{ds}{dt} = 2 + 5t^{3/2}$$

$$\int_{s_0=0}^{s} ds = \int_{0}^{t} \left(2 + 5t^{3/2}\right) dt$$

$$s = 2t + 2t^{5/2}$$

At $t = 4\text{ s}$: $\begin{cases} s = 72\text{ m}, \quad v = 42\text{ m/s} \\ a = 15\text{ m/s}^2 \end{cases}$

2/4

$$v = 5s^{3/2}$$

$$a = \frac{dv}{dt} = \frac{3}{2} 5 s^{1/2} \frac{ds}{dt} = \frac{15}{2} s^{1/2} v$$

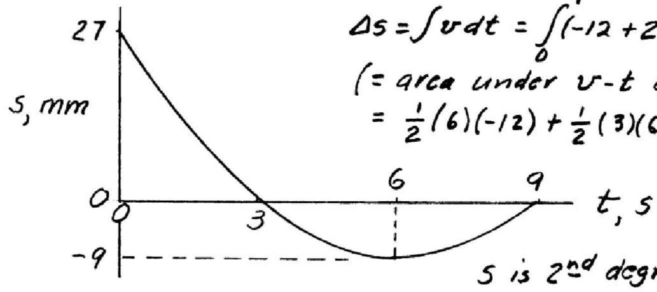
$$= \frac{15}{2} s^{1/2} \left(5 s^{3/2}\right) = \frac{75}{2} s^2$$

When $s = 2$ mm, $a = \frac{75}{2} 2^2 = \underline{150 \text{ mm/s}^2}$

2/5 $s = 27 - 12t + t^2$ mm

$v = \dot{s} = -12 + 2t$ mm/s

$a = \dot{v} = 2$ mm/s^2



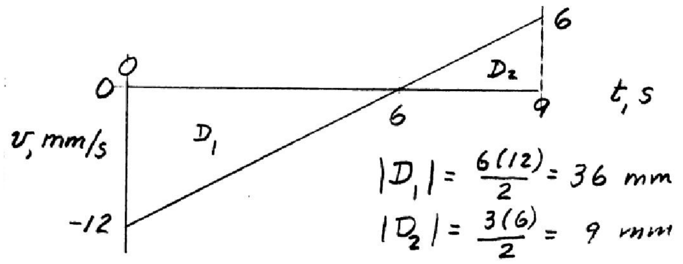
$\Delta s = \int v\,dt = \int_0^9 (-12 + 2t)\,dt = \underline{-27 \text{ mm}}$

(= area under v-t curve

$= \frac{1}{2}(6)(-12) + \frac{1}{2}(3)(6) = -27$ mm)

s is 2nd degree in t so

$a = \ddot{s} = \underline{\text{constant}}$



$|D_1| = \frac{6(12)}{2} = 36$ mm

$|D_2| = \frac{3(6)}{2} = 9$ mm

$D = 36 + 9 = \underline{45 \text{ mm}}$

2/6

$$v = \frac{ds}{dt} = 400 - 16t^2$$

$$\Delta s = \int_0^{\Delta s} ds = \int_0^{6} (400 - 16t^2)\,dt \,, \quad \Delta s = 400t - \frac{16}{3}t^3 \Big]_0^6 =$$

$$= 2400 - 1152 = 1248 \text{ mm}$$

$$\underline{\Delta s = 1.248 \text{ m}}$$

$$D = \Delta s_1 + |\Delta s_2| \,, \quad \Delta s_1 = \int_0^{5} (400 - 16t^2)\,dt = 1333.3 \text{ mm}$$

$$|\Delta s_2| = \left| \int_5^{6} (400 - 16t^2)\,dt \right| = 85.3 \text{ mm}$$

$$D = 1333.3 + 85.3 = 1418.7 \text{ mm}$$

or $\underline{D = 1.419 \text{ m}}$

2/7

$$a = \frac{dv}{dt} = 4t - 30$$

$$\int_{v_0=3}^{v} dv = \int_0^t (4t-30)\,dt\,, \quad \underline{v = 3 - 30t + 2t^2 \text{ m/s}}$$

$$\frac{ds}{dt} = 3 + 2t^2 - 30t$$

$$\int_{s_0=-5}^{s} ds = \int_0^t (3 + 2t^2 - 30t)\,dt$$

$$\underline{s = -5 + 3t - 15t^2 + \frac{2}{3}t^3 \text{ m}}$$

2/8 For constant acceleration,

$s = \frac{1}{2}at^2$, $t = \left(\frac{2s}{a}\right)^{1/2} = \left(\frac{2(30\,000)}{1.5(9.81)}\right)$

$= \underline{63.9\text{ s}}$

$v = \sqrt{2as} = \sqrt{2(1.5)(9.81)(30\,000)} = \underline{940\text{ m/s}}$

2/9

$$v^2 - v_0^2 = 2a(s - s_0)$$

$$0 - \left[50 \frac{5280}{3600}\right]^2 = 2a(100), \quad a = -26.9 \frac{\text{ft}}{\text{sec}^2}$$

Then $$0 - \left[70 \frac{5280}{3600}\right]^2 = 2(-26.9)\, s$$

$$\underline{s = 196.0 \text{ ft}}$$

2/10 For $a =$ constant, $v^2 = v_0^2 + 2as$

$$\left[\frac{180\,(5280)}{3600}\right]^2 = 0^2 + 2a\,(300)$$

$a = 116.2$ ft/sec^2

or $a = 116.2/32.2 = \underline{3.61g}$

2/11 $v^2 = v_0^2 + 2a(s - s_0)$

$$\left(\frac{200}{3.6}\right)^2 = 0^2 + 2(0.4 \cdot 9.81)\, s$$

$$\underline{s = 393 \text{ m}}$$

$v = v_0 + at$: $\left(\frac{200}{3.6}\right) = 0 + 0.4(9.81)\, t$

$$\underline{t = 14.16 \text{ s}}$$

2/12 $\int v\,dv = \int a\,ds\ ;\ \int_{200/3.6}^{30/3.6} v\,dv = a\int_0^{600} ds$

200 km/h
= 200/3.6 m/s

$\frac{1}{2(3.6)^2}\left(\overline{30}^2 - \overline{200}^2\right) = 600a$

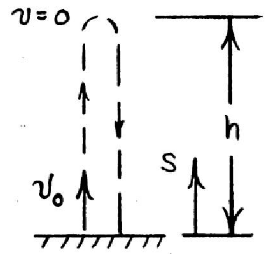
30 km/h
= 30/3.6 m/s

$\underline{a = -2.51\ m/s^2}$

2/13 $v^2 = v_0^2 + 2as$, where $a = g/6$

$v^2 = 2^2 + 2\left(\frac{9.81}{6}\right)5$, $\underline{v = 4.51 \text{ m/s}}$

2/14



$$v^2 = v_0^2 + 2a(s - s_0)$$

Apex: $0^2 = 200^2 + 2(-9.81)\, h$

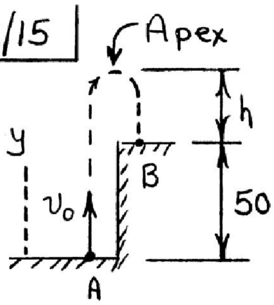
$$\underline{h = 2040 \text{ m}}$$

$$v = v_0 + at$$

Impact : $-200 = 200 - 9.81t$

$$\underline{t = 40.8 \text{ s}}$$

2/15



Evaluate $v^2 = v_0^2 - 2g(y - y_0)$ at apex :

$$0 = 80^2 - 2(32.2)(50 + h - 0)$$

$h = 49.4$ ft

Evaluate $y = y_0 + v_0 t - \frac{1}{2} g t^2$ at B :

$$50 = 0 + 80t - 16.1t^2, \text{ or } 16.1t^2 - 80t + 50 = 0$$

$$t = \frac{80 \pm \sqrt{80^2 - 4(16.1)(50)}}{2(16.1)} = 0.733, \ 4.26 \text{ sec}$$

$t = 4.24$ sec represents the second time at which $y = 50$ ft.

$$v_B = v_0 - gt = 80 - 32.2(4.24) = -56.4 \text{ ft/sec}$$

(or 56.4 ft/sec downward)

$\underline{2/16}$ Acceleration period :

$v = v_0 + at$: $\frac{22}{3.6} = 0 + \frac{9.81}{4} t_a$, $t_a = 2.49$ s

Note that the deceleration time $t_d = t_a$

$v^2 = v_0^2 + 2a\Delta s$: $\left(\frac{22}{3.6}\right)^2 = 0^2 + 2\,\frac{9.81}{4}\,\Delta s_a$

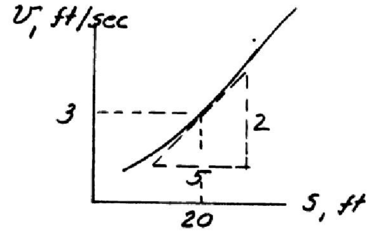
$\Delta s_a = 7.61\text{ m} = \Delta s_d$

Cruise period : $\Delta s_c = 350 - \Delta s_a - \Delta s_d = 335$ m

$\Delta s = v_c t_c$: $335 = \frac{22}{3.6} t_c$, $t_c = 54.8$ s

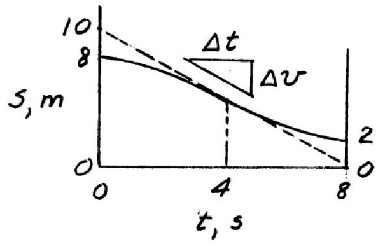
Total run time $t = t_c + t_a + t_d = \underline{59.8 \text{ s}}$

2/17



$$a = v\frac{dv}{ds} = 3\left(\frac{2}{5}\right) = \underline{1.2\ ft/sec^2}$$

2/18



$$v_{AV} = \frac{s_2 - s_1}{\Delta t} = \frac{2-8}{8} = \underline{-0.75 \text{ m/s}}$$

$$v = v_4 = \frac{ds}{dt} = \frac{-10}{8} = \underline{-1.25 \text{ m/s}}$$

2/19 $s = s_0 + v_0 t + \frac{1}{2} g t^2$

Sphere ①: $H = \frac{1}{2} g t_2^{\,2}$

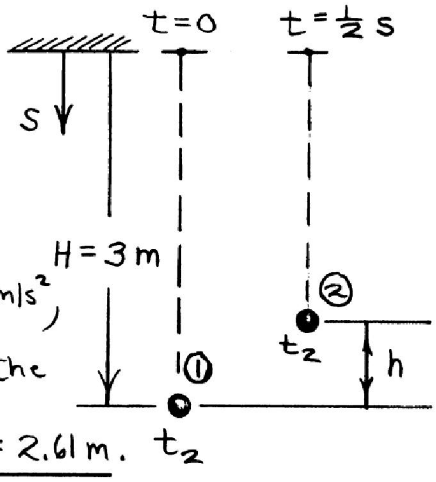
Sphere ②:

$$(H-h) = \frac{1}{2} g \left(t_2 - \frac{1}{2}\right)^2$$

With $H = 3$ m & $g = 9.81$ m/s^2,

eliminate t_2 between the

2 equations & obtain $\underline{h = 2.61 \text{ m}}$.



2/20 $v_C = v_B + a\Delta t_{B-C}$, $a = \dfrac{(60-100)/3.6}{4}$

$= -2.78 \text{ m/s}^2$

$$\Delta s_{B-C} = v_B \Delta t_{B-C} + \tfrac{1}{2} a \Delta t_{B-C}^2$$

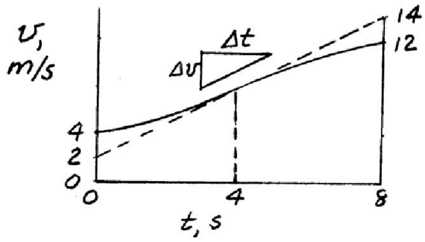
$$= \frac{100}{3.6} 4 + \frac{1}{2}(-2.78)4^2 = 88.9 \text{ m}$$

$$\Delta s_{A-D} = \Delta s_{A-B} + \Delta s_{B-C} + \Delta s_{C-D}$$

$$3000 = \frac{100}{3.6} t + 88.9 + \frac{60}{3.6} t, \quad \underline{t = 65.5 \text{ s}}$$

$$s = \Delta s_{A-B} = \frac{100}{3.6}(65.5) = 1819 \text{ m} \quad \text{or} \quad \underline{s = 1.819 \text{ km}}$$

2/21



$$a_{AV} = \frac{\Delta v}{\Delta t} = \frac{12-4}{8} = 1.00 \text{ m/s}^2$$

$$a_4 = \frac{dv}{dt} = \frac{14-2}{8} = 1.50 \text{ m/s}^2$$

$$\Delta a = a_4 - a_{AV} = 1.50 - 1.00 = \underline{0.50 \ \frac{m}{s^2}}$$

$$\Delta s = \int v\,dt \approx \frac{14+2}{2}(8) = \underline{64 \text{ m}}$$

2/22 $\int_{0.4}^{v} v\,dv = \int a_x\,dx$; $\frac{1}{2}(v^2 - 0.4^2)$ = area under a_x-x curve

Area $= \int a_x\,dx = (a_x)_{av}\Delta x = 3(120-40)10^{-3} = 0.240\,(\text{m/s})^2$

Thus $v^2 = 0.4^2 + 2(0.240) = 0.16 + 0.48 = 0.64$

$v = \sqrt{0.64} = \underline{0.8\ \text{m/s}}$

$\underline{2/23}$ $v^2 = v_0^2 + 2a(s - s_0)$

$0 = 4^2 + 2\left(-\frac{9.81}{4}\right)(s)$, $\underline{s = 3.26 \text{ m}}$

$v = v_0 + at$: $0 = 4 + \left(-\frac{9.81}{4}\right)t_{up}$, $t_{up} = 1.631 \text{ s}$

$t = 2t_{up} = 2(1.631) = \underline{3.26 \text{ s}}$

2/24 $a = 400 - kx$, where $k = \dfrac{400}{6/12}$ sec^{-2}

$$a = 400\,(1 - 2x) \quad (x \text{ in ft})$$

$$v\,dv = a\,dx : \quad \int_0^v v\,dv = 400\int_0^x (1-2x)\,dx$$

$$v^2 = 800(x - x^2), \quad v = \frac{dx}{dt} = 20\sqrt{2}\sqrt{x - x^2}$$

(taking + sign)

$$\int_0^t dt = \int_0^x \frac{dx}{20\sqrt{2}\sqrt{x-x^2}}$$

$$t = -\frac{1}{20\sqrt{2}} \sin^{-1}\frac{1-2x}{\sqrt{1}}\Bigg|_0^x = \frac{1}{20\sqrt{2}}\left[\frac{\pi}{2} - \sin^{-1}(1-2x)\right]$$

(a) $x = \frac{1}{4}$ ft: $t = \dfrac{1}{20\sqrt{2}}\left[\dfrac{\pi}{2} - \dfrac{\pi}{6}\right] = \underline{0.0370 \text{ sec}}$

(b) $x = \frac{1}{2}$ ft: $t = \dfrac{1}{20\sqrt{2}}\left[\dfrac{\pi}{2} - 0\right] = \underline{0.0555 \text{ sec}}$

2/25 $v_0 = 100/3.6 = 27.8$ m/s

$a = -g \sin\theta = -9.81 \sin\left[\tan^{-1}\frac{6}{100}\right] = -0.588$ m/s^2

(a) $v = v_0 + at = 27.8 - 0.588(10) = \underline{21.9 \text{ m/s}}$

(b) $v^2 = v_0^2 + 2a(s - s_0) = 27.8^2 + 2(-0.588)(100)$

$v = \underline{25.6 \text{ m/s}}$

2/26 Train: $v^2 = v_1^2 + 2as$: $60^2 = 80^2 + 2a\left(\frac{1}{2}\right)$

$$a = -2800 \text{ mi/hr}^2$$

$s = v_1 t + \frac{1}{2}at^2$: $1 = 80t - \frac{2800}{2}t^2$

$t = 0.01847$ hr or $t = 0.03867$ hr (disregard)

So $t = 0.01847(60)^2 = 66.5$ sec

Car: $t = 66.5 - 4 = 62.5$ sec

$s = v_1 t + \frac{1}{2}at^2$: $1.3(5280) = 50\frac{44}{30}(62.5) + \frac{a}{2}(62.5)^2$

$\underline{a = 1.168 \text{ ft/sec}^2}$

$v = v_1 + at = 50 + 1.168 \dfrac{3600^2}{5280} \dfrac{62.5}{3600} = \underline{99.8 \dfrac{mi}{hr}}$

2/27 For constant accel.,

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

when $t=0$, $x_0 = 4$ m, $v_0 = 3$ m/s

$t = 4$ s, $\dot{x} = 0$

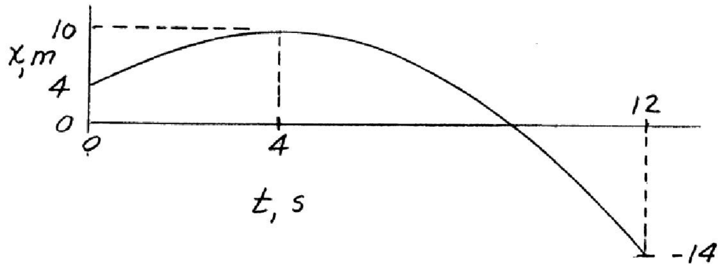
Thus $x = 4 + 3t + \frac{1}{2}at^2$

Also $0 = 3 + 4a$, $a = -3/4$ m/s^2

so $x = 4 + 3t - \frac{3}{8}t^2$

when $t = 12$ s, $x = 4 + 3(12) - \frac{3}{8}(12^2) = \underline{-14 \text{ m}}$

" $t = 4$ s, $x = x_{max} = 4 + 3(4) - \frac{3}{8}(4^2) = \underline{10 \text{ m}}$



2/28 $a = \frac{dv}{dt}$; $v_m = \int a\,dt = at = 6(20) = \underline{120\ m/s}$

Corresponding $h = \frac{1}{2}at^2 = \frac{1}{2}(6)(20)^2 = 1200\ m$

During upward coast, $\int_{v_m}^{0} v\,dv = \int_{0}^{\Delta h} -g\,dy$

$v_m^2 = 2g\Delta h$, $\Delta h = \frac{\overline{120}^2}{2(9.81)} = 734\ m$

Max. $h = 1200 + 734 = 1934\ m$ or $\underline{h = 1.934\ km}$

2/29 $s_{car} = vt = \frac{120}{3.6} t$

$s_{cycle} = v_{av} t_1 + v_{max} t_2 = \frac{1}{2} \frac{150}{3.6} t_1 + \frac{150}{3.6} t_2$

where $t_1 = \frac{v_{max}}{a} = \frac{150}{3.6 \times 6} = 6.94$ s & $t_2 = t - 6.94 - 2$

$s_{car} = s_{cycle}$; $\frac{120}{3.6} t = \frac{75}{3.6} 6.94 + \frac{150}{3.6} (t - 8.94)$

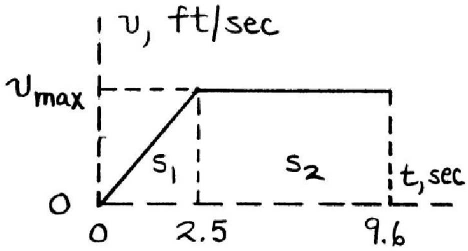
$30t = 820.8$, $t = 27.36$ s

$s = \frac{120}{3.6} (27.36) = \underline{912 \text{ m}}$

2/30

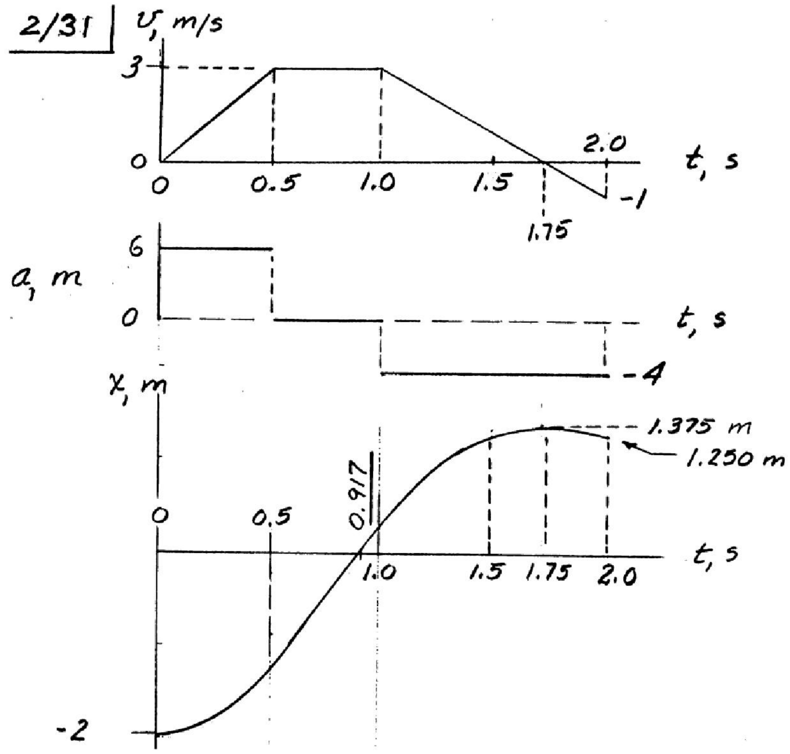
$s_1 = \frac{1}{2}(2.5)v_{max}$

$s_2 = (9.6 - 2.5)v_{max}$

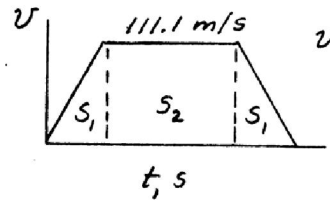


$s_1 + s_2 = (1.25 + 7.1)v_{max} = 100(3)$

$\underline{v_{max} = 35.9 \text{ ft/sec}}$


2/31
v, m/s
3
0
0
0.5
1.0
1.5
2.0
1.75
t, s
-1
6
a, m
0
t, s
-4
x, m
0
0.5
0.917
1.0
1.5
1.75
2.0
t, s
1.375 m
1.250 m
-2

2/32 $400\ km/h \equiv \frac{400}{3.6} = 111.1\ m/s$



$v^2 = 2as, \quad s_1 = \frac{(111.1)^2}{2(0.6)(9.81)} = 1049\ m$

$s_2 = 10\,000 - 2(1049)$
$= 7903\ m$

$t_1 = \frac{v}{a} = \frac{111.1}{0.6(9.81)} = 18.88\ s$

$t_2 = \frac{s_2}{v} = \frac{7903}{111.1} = 71.13\ s$

$t = 2t_1 + t_2$
$= 2(18.88) + 71.13$
$= \underline{108.9\ s}$

or $\underline{t = 1.81\ min}$

$\underline{2/33}$ | $v^2 = \frac{k}{s}$, $v = \dot{s} = \sqrt{k/s}$ where $k = 2^2(9) = 36 \ in^3/sec^2$

$\int_9^s \sqrt{s}\, ds = 6\int_0^t dt$, $\frac{2}{3} s^{3/2}\Big]_9^s = 6(t-0)$,

$s^{3/2} = 27 + 9t$, but $v = \sqrt{k}\, s^{-1/2}$ so

$v = 6(27+9t)^{-1/3}$ & at $t = 3\,sec$, $v = 6(27+27)^{-1/3}$

$= 2^{2/3} = \underline{1.587 \frac{in.}{sec}}$

2/34 Particle 1 : $a = -kv$

$$-kv = \frac{dv}{dt}$$

$$-k\int_0^t dt = \int_{v_0}^{v} \frac{dv}{v} \Rightarrow \underline{v = v_0 e^{-kt}}$$

Then $\frac{ds}{dt} = v_0 e^{-kt}$

$$\int_{s_0=0}^{s} ds = v_0 \int_{t_0=0}^{t} e^{-kt} dt \Rightarrow \underline{s = \frac{v_0}{k}\left(1 - e^{-kt}\right)}$$

Particle 2 : $a = -kt$

$$-kt = \frac{dv}{dt}$$

$$-k\int_0^t t\, dt = \int_{v_0}^{v} dv \Rightarrow \underline{v = v_0 - \frac{1}{2}kt^2}$$

Then $\frac{ds}{dt} = v_0 - \frac{1}{2}kt^2$

$$\int_0^s ds = \int_0^t \left(v_0 - \frac{1}{2}kt^2\right) dt \Rightarrow \underline{s = v_0 t - \frac{1}{6}kt^3}$$

Particle 3 : $a = -ks$

$$-ks = v\frac{dv}{ds}$$

$$-k\int_0^s s\, ds = \int_{v_0}^{v} v\, dv \Rightarrow \underline{v = \pm\sqrt{v_0^2 - ks^2}}$$

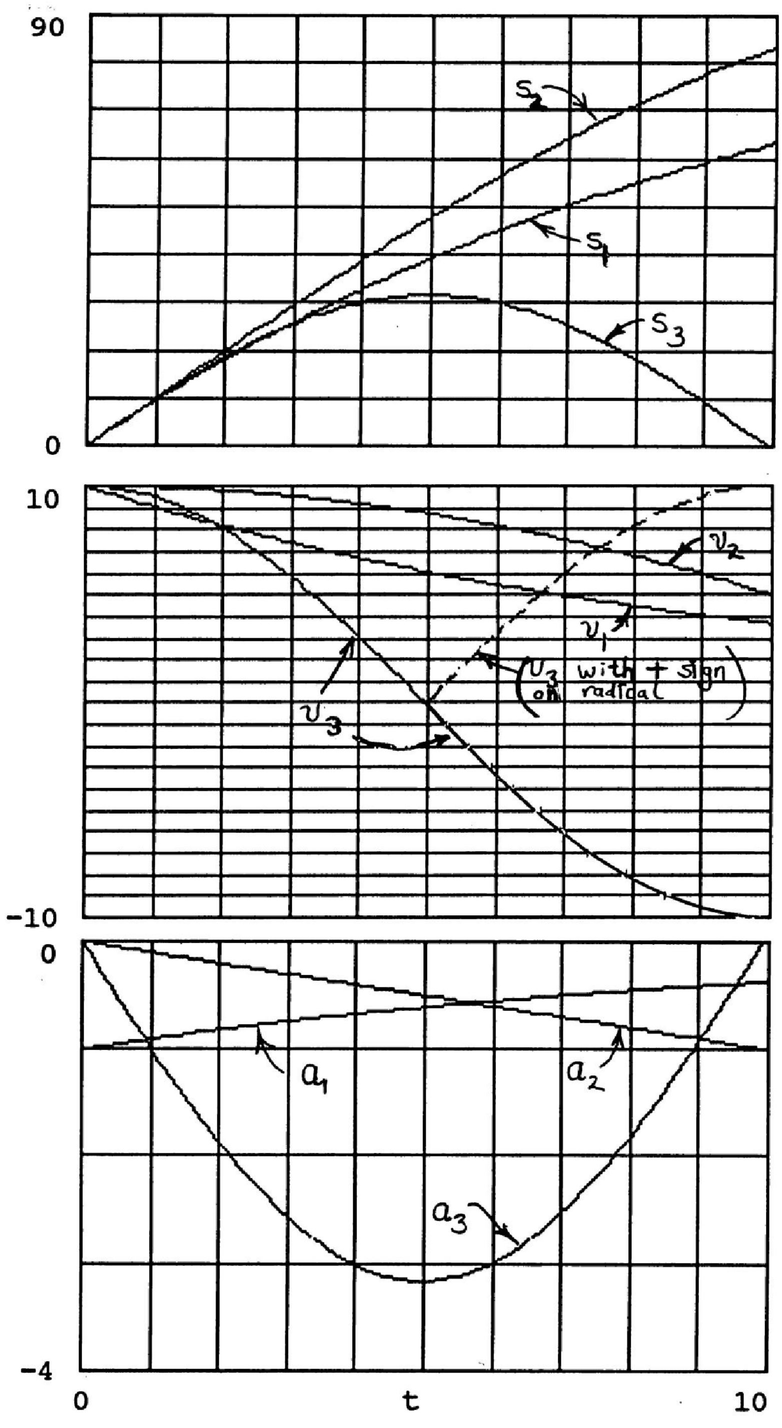
Note: Plus sign is chosen until first reversal ($v=0$), thereafter take minus sign, etc.

Then

$$\frac{ds}{dt} = \pm\sqrt{v_0^2 - ks^2}$$

$$\int_0^s \frac{ds}{\sqrt{v_0^2 - ks^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{k}} \sin^{-1}\left(\frac{\sqrt{k}}{v_0} s\right) = t \Rightarrow \underline{s = \frac{v_0}{\sqrt{k}} \sin\left(\sqrt{k}\, t\right)}$$



90
0
10
-10
0
-4
0
t
10
s2
s1
s3
v2
v1
(v3 with + sign on radical)
v3
a1
a2
a3

2/35 $0 < t < 10\,\mathrm{s}: \quad a = 6 - kt \;, \quad k = \frac{6}{10}\ \mathrm{m/s^3}$

$$a = \frac{dv}{dt} = 6\left(1 - \frac{t}{10}\right), \quad t \text{ in s}, \quad a \text{ in m/s}^2$$

$$\int_0^v dv = \int_0^t 6\left(1 - \frac{t}{10}\right) dt \;, \quad v = 6t - \frac{3}{10}t^2$$

$$v_{10} = 6(10) - \frac{3}{10}(10)^2 = 30\ \mathrm{m/s}$$

$$v = \frac{ds}{dt} \;, \quad s_{10} = \int_0^{10}\left(6t - \frac{3}{10}t^2\right)dt = 200\ \mathrm{m}$$

$$t > 10\,\mathrm{s}: \quad \Delta s = v_{10}\,\Delta t \;, \quad \Delta t = \frac{400-200}{30} = 6.67\,\mathrm{s}$$

$$t = 10 + \Delta t = \underline{16.67\ \mathrm{s}}$$

2/36

$$a = g - cy = v\frac{dv}{dy}$$

$$\int_0^{y_m}(g-cy)\,dy = \int_{v_0}^{0} v\,dv$$

$$\left(gy - c\frac{y^2}{2}\right)\Big|_0^{y_m} = \frac{v^2}{2}\Big|_{v_0}^{0}$$

$$gy_m - c\frac{y_m^2}{2} = -\frac{v_0^2}{2} \Rightarrow c = \frac{v_0^2 + 2gy_m}{y_m^2}$$

2/37 | $v\,dv = a\,ds\,; \quad \dfrac{v\,dv}{-Kv^2} = ds, \quad \displaystyle\int_{v_1}^{v_2} \frac{dv}{v} = -K\int_0^s ds$

$\ln\dfrac{v_2}{v_1} = -Ks\,, \quad K = \dfrac{1}{s}\ln\dfrac{v_1}{v_2} = \dfrac{1}{1500}\ln\dfrac{100}{20} = \underline{1.073(10^{-3})\ \text{ft}^{-1}}$

$a = \dfrac{dv}{dt}\,; \quad -Kv^2 = \dfrac{dv}{dt}\,, \quad \displaystyle\int_{v_1}^{v_2} \frac{dv}{v^2} = -Kt\,, \quad t = \frac{1}{K}\left(\frac{1}{v_2} - \frac{1}{v_1}\right)$

$t = \dfrac{10^3}{1.073}\left(\dfrac{1}{20} - \dfrac{1}{100}\right)\dfrac{30}{44} = \underline{25.4\ \text{sec}}$

2/38 | $v\,dv = a\,dx$, $\int_0^x dx = \int_{v_0}^{v} \frac{v\,dv}{-C_1 - C_2 v^2}$

$$x = \frac{-1}{2C_2} \ln(+C_1 + C_2 v^2)\Big]_{v_0}^{v} = \frac{1}{2C_2} \ln \frac{C_1 + C_2 v_0^2}{C_1 + C_2 v^2}$$

when $v = 0$, $x = D = \underline{\frac{1}{2C_2} \ln\left(1 + \frac{C_2}{C_1} v_0^2\right)}$

2/39 (a) $g_0 = 32.2$ ft/sec^2 = constant

$v^2 = v_0^2 + 2a\,(s - s_0)$: $v^2 = 0^2 + 2(32.2)(500 \cdot 5280)$

$v = \underline{13{,}040 \text{ ft/sec}}$

(b) $a = -g_0 \dfrac{R^2}{r^2} = v\dfrac{dv}{dr}$

$$-g_0 R^2 \int_{R+h}^{R} \frac{dr}{r^2} = \int_{v_0=0}^{v} v\,dv$$

$$-g_0 R^2 \left(-\frac{1}{r}\right)\Bigg|_{R+h}^{R} = \frac{1}{2}v^2\Bigg|_0^v$$

$$\Rightarrow \quad v = \sqrt{\frac{2g_0 Rh}{R+h}} = \sqrt{\frac{2(32.2)(3959)(500)(5280)^2}{(3959+500)(5280)}}$$

$$= \underline{12{,}290 \text{ ft/sec}}$$

2/40 $a = \frac{dv}{dt} = -kv, \quad \int_{v_0}^{v} \frac{dv}{v} = -k \int_0^t dt$

$\ln \frac{v}{v_0} = -kt, \quad \underline{v = v_0 e^{-kt}}$

$v = \frac{dx}{dt} = v_0 e^{-kt}, \quad \int_0^x dx = \int_0^t v_0 e^{-kt} dt$

$$\underline{x = \frac{v_0}{k}\left[1 - e^{-kt}\right]}$$

$v\,dv = a\,dx, \quad \frac{v\,dv}{v} = -k\,dx$

$\int_{v_0}^{v} dv = -k \int_0^x dx, \quad \underline{v = v_0 - kx}$

2/41 (a) $a = 2\ m/s^2 = \text{constant}$

With $v = 250/3.6 = 69.4\ m/s$, we have

$v^2 - v_0^2 = 2a(s - s_0)$: $69.4^2 - 0^2 = 2(2)\, s$

$$\underline{s = 1206\ m}$$

(b) $a = a_0 - kv^2 = v \dfrac{dv}{ds}$

$$\int_0^s ds = \int_0^v \frac{v\, dv}{a_0 - kv^2}$$

$$s = -\frac{1}{2k} \ln\left(a_0 - kv^2\right)\Big|_0^v$$

$$= -\frac{1}{2k} \ln\left[\frac{a_0 - kv^2}{a_0}\right]$$

$$s = -\frac{1}{2(4)(10^{-5})} \ln\left[\frac{2 - 4(10^{-5})(69.4)^2}{2}\right]$$

$$= \underline{1268\ m}$$

2/42 A to B:

$v_B^2 = v_A^2 + 2a\Delta s$: $v_B^2 = 4^2 + 2(0.3)(32.2)(10)$

$v_B = 14.46$ ft/sec

$v_B = v_A + at$: $14.46 = 4 + (0.3)(32.2)t_{AB}$

$t_{AB} = 1.083$ sec

B to C:

$v_C^2 = v_B^2 + 2a\Delta s$: $0 = 14.46^2 + 2a(12)$

$\underline{a = -8.72 \text{ ft/sec}^2}$

$v_C = v_B + at$: $0 = 14.46 - 8.72t_{BC}$

$t_{BC} = 1.659$ sec

$t = t_{AB} + t_{BC} = 1.083 + 1.659 = \underline{2.74 \text{ sec}}$

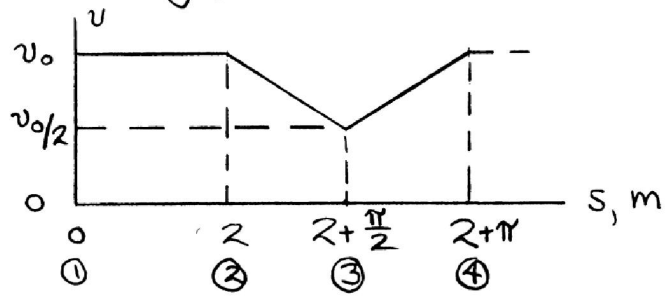
2/43 $v\,dv = a\,dx, \quad \int_0^v v\,dv = \int_0^x \frac{K}{(L-x)^2}\,dx$

$\frac{v^2}{2} = \frac{K}{L-x}\Big]_0^x \, ; \quad v^2 = \frac{2Kx}{L(L-x)}$

For $x = L - D/2$, $\quad \underline{v = 2\sqrt{\frac{K(L-D/2)}{LD}}}$

2/44 Compute the time $t_{1/2}$ required for $\frac{1}{2}$ lap.

Temporarily, let v_0 be the straightaway speed:



Time t_{1-2} to go from ① to ②: $t_{1-2} = \frac{2}{v_0}$

For t_{2-3}: Note that $\frac{dv}{ds} = -c$ (c = pos. const.)

so that $a_t = v\frac{dv}{ds} = -cv = \frac{dv}{dt}$

$-c\int_{t_2}^{t_3} dt = \int_{v_2}^{v_3}\frac{dv}{v}$: $-ct_{2-3} = \ln\left(\frac{v_0/2}{v_0}\right)$

$t_{2-3} = \frac{1}{c}\ln 2$

Likewise, $t_{3-4} = \frac{1}{c}\ln 2$

$c = \frac{v_0/2}{\pi/2} = \frac{v_0}{\pi}$, $t_{\frac{1}{2}} = t_{1-2} + t_{2-3} + t_{3-4} = \frac{6.36}{v_0}$

Lap time $t = 2t_{1/2} = \frac{12.71}{v_0} = \frac{12.71}{0.25} = \underline{50.8\text{ s}}$

2/45 1^{st} interval; $v_8 = at$, $v_8 = 1(8) = 8$ m/s
$\Delta t = 8$ s
$s_8 = \frac{1}{2}at^2$, $s_8 = \frac{1}{2}(1)(8^2) = 32$ m

2^{nd} interval; $\Delta v = \int a\,dt$; $v_{14} - 8 = 2(6)$, $v_{14} = 20$ m/s
$\Delta t = 6$ s
$\Delta s = v_0 t + \frac{1}{2}at^2 = 8(6) + \frac{1}{2}(2)(6^2) = 84$ m
$s_{14} - 32 = 84$, $s_{14} = 116$ m

3^{rd} interval; $\Delta s = vt$, $s_{24} - 116 = 20(10)$, $s_{24} = 316$ m
$\Delta t = 10$ s

Total interval; $\Delta v = \int a\,dt$, $0 - 0 = 8 + 12 + 0 - 2\Delta t$
$\underline{\Delta t = 10\text{ s}}$

4^{th} interval; $\Delta s = v_0 t + \frac{1}{2}at^2 = 20(10) + \frac{1}{2}(-2)(10)^2$
$s_{34} - s_{24} = 100$ m
$s_{34} = s = 100 + 316 = \underline{416\text{ m}}$

2/46 $a = -kv^2$,

$v\,dv = a\,dx$, $\displaystyle\int_{v_0}^{v} \frac{v\,dv}{-kv^2} = \int_0^x dx$, $x = \frac{-1}{k} \ln v \Big]_{v_0}^{v}$

$$x = \frac{1}{k} \ln v_0/v$$

when $v = v_0/2$, $x = D = \frac{1}{k} \ln 2 = \underline{0.693/k}$

$v = \frac{dx}{dt}$ where $kx = \ln v_0/v$, $v = v_0 e^{-kx}$

so $\dfrac{dx}{v_0 e^{-kx}} = dt$ or $\displaystyle\int_0^t dt = \frac{1}{v_0} \int_0^x e^{kx}\,dx$

& $t = \frac{1}{v_0} \frac{1}{k} e^{kx} \Big]_0^x = \frac{1}{kv_0}\left[e^{kx} - 1\right]$

For $x = D$, $e^{kx} = 2$ so $t = \frac{1}{kv_0}[2-1]$, $\underline{t = \frac{1}{kv_0}}$

2/47 $a = \frac{dv}{dt} = -cv^n, \quad \int_{v_0}^{v} \frac{dv}{v^n} = -c\int_0^t dt$

$$-ct = \frac{v^{1-n}}{1-n}\Bigg]_{v_0}^{v}, \quad \underline{v = \left[v_0^{\,1-n} + c(n-1)t\right]^{\frac{1}{1-n}}}$$

2/48 $a = k/x\,, \quad v\,dv = \frac{k}{x}\,dx$

$\int_0^v v\,dv = k\int_{x_1}^{x}\frac{dx}{x}\;; \quad \frac{v^2}{2} = k\ln\frac{x}{x_1}$

Thus $\frac{(600)^2}{2} = k\ln\frac{750}{7.5}\,, \quad k = \frac{0.36}{2(4.605)} = 0.0391\ (km/s)^2$

at $x = 375$ mm, $a = \frac{0.0391}{375(10^{-6})} = \underline{104.2\ km/s^2}$

2/49

$$a = v\frac{dv}{ds} = 3.22 - 0.004v^2$$

$$\int_0^{v_B} \frac{v\,dv}{3.22 - 0.004v^2} = \int_0^{600} ds$$

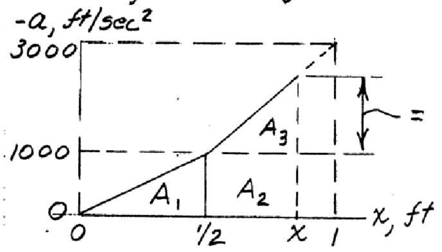
$$\frac{1}{2(-0.004)} \ln\left(3.22 - 0.004v^2\right)\Big|_0^{v_B} = 600$$

$$\ln\left[\frac{3.22 - 0.004v_B^2}{3.22}\right] = 600(2)(-0.004)$$

$$\frac{3.22 - 0.004v_B^2}{3.22} = 0.00823$$

$$\underline{v_B = 28.3 \text{ ft/sec}}$$

2/50 $\int v\,dv = \int a\,dx$ or $\int_{40^2}^{0} dv^2 = -2\begin{pmatrix}\text{area under}\\ a\text{-}x \text{ curve}\end{pmatrix}$



$= \dfrac{x-\frac{1}{2}}{\frac{1}{2}}\,2000 = 4000(x-\tfrac{1}{2})$

$A_1 = \frac{1}{2}\,\frac{1}{2}(1000) = 250 \text{ ft}^2/\text{sec}^2$

$A_2 = (x-\frac{1}{2})1000$

$A_3 = \frac{1}{2}(x-\frac{1}{2})4000(x-\frac{1}{2})$

$A_1 + A_2 + A_3 = 2000x^2 - 1000x + 250$

so $-1600 = -2(2000x^2 - 1000x + 250)$

or $x^2 - 0.5x - 0.275 = 0$

$\underline{x = 0.831 \text{ ft}}$ $(x > 0.5 \text{ ft})$

2/51 Up:

$$a_u = -g - kv^2 = v\frac{dv}{dy}$$

$$\int_0^h dy = -\int_{v_0}^0 \frac{v\,dv}{g+kv^2}$$

$$h = -\frac{1}{2k}\ln\left[g+kv^2\right]_{v_0}^{0} = \frac{1}{2k}\ln\left[\frac{g+kv_0^2}{g}\right]$$

$$h = \frac{1}{2(0.002)}\ln\left[\frac{32.2+0.002(100)^2}{32.2}\right] = \underline{120.8\text{ ft}}$$

Down:

$$a_d = -g + kv^2 = v\frac{dv}{dy}$$

$$\int_h^0 dy = \int_0^{v_f}\frac{v\,dv}{-g+kv^2}$$

$$-h = \frac{1}{2k}\ln\left[-g+kv^2\right]\Big|_0^{v_f} = \frac{1}{2k}\ln\left[\frac{g-kv_f^2}{g}\right]$$

$$\Rightarrow v_f = \sqrt{\frac{g}{k}\left(1-e^{-2kh}\right)}$$

$$= \sqrt{\frac{32.2}{0.002}\left(1-e^{-2(0.002)(120.8)}\right)} = \underline{78.5\ \frac{\text{ft}}{\text{sec}}}$$

$\uparrow y$

2/52 Up : $a_u = -g - kv^2 = \frac{dv}{dt}$

$$\int_0^{t_u} dt = -\int_{v_0}^{0} \frac{dv}{g+kv^2}$$

$$t_u = \frac{1}{\sqrt{gk}} \tan^{-1}\left(\frac{v\sqrt{gk}}{g}\right)\Bigg|_0^{v_0} = \frac{1}{\sqrt{gk}} \tan^{-1}\left(v_0\sqrt{\frac{k}{g}}\right)$$

$$t_u = \frac{1}{\sqrt{32.2(0.002)}} \tan^{-1}\left(100\sqrt{\frac{0.002}{32.2}}\right) = \underline{2.63 \text{ sec}}$$

(Down) : $a_d = -g + kv^2 = \frac{dv}{dt}$

$$\int_0^{t_d} dt = \int_0^{v_f} \frac{dv}{-g+kv^2}$$

$$t_d = \frac{1}{\sqrt{gk}} \tanh^{-1}\left(\frac{v\sqrt{gk}}{g}\right)\Bigg|_0^{v_f} = \frac{1}{\sqrt{gk}} \tanh^{-1}\left(v_f\sqrt{\frac{k}{g}}\right)$$

$$= \frac{1}{\sqrt{32.2(0.002)}} \tanh^{-1}\left(78.5\sqrt{\frac{0.002}{32.2}}\right)$$

$$= \underline{2.85 \text{ sec}}$$

(Refer to solution of Prob. 2/51)

2/53 For an acceleration of form $a = -g - kv^2$, we cite the results from Probs. 2/51 & 2/52

$$\begin{cases} t_u = \dfrac{1}{\sqrt{gk}} \tan^{-1}\left(v_0 \sqrt{\dfrac{k}{g}}\right) \\ h = \dfrac{1}{2k} \ln\left[\dfrac{g + kv_0^2}{g}\right] \end{cases}$$

For the numbers at hand:

$$t_u = \frac{1}{\sqrt{9.81\,(0.0005)}} \tan^{-1}\left(120\sqrt{\frac{0.0005}{9.81}}\right) = 10.11 \text{ s}$$

$$h = \frac{1}{2(0.0005)} \ln\left[\frac{9.81 + 0.0005\,(120)^2}{9.81}\right] = 550 \text{ m}$$

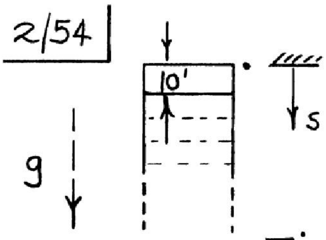
Down (v = constant): $y = y_0 + v_{y_0} t$

$$0 = 550 - 4t_d$$

$$t_d = 137.6 \text{ s}$$

Flight time $t = t_u + t_d = 10.11 + 137.6 = \underline{147.7 \text{ s}}$

2/54



$s = \cancel{s_0} + \cancel{v_0 t} + \frac{1}{2} g t^2$

When $s = 10$ ft,

$10 = \frac{1}{2}(32.2) t_{10'}^2$, $t_{10'} = 0.788$ sec

Time to pass first story from the top is $t_1 = t_{10'} - t_{0'} = 0.788 - 0 = \underline{0.788 \text{ sec}}$

10^{th} story: $90 = \frac{1}{2}(32.2) t_{90'}^2$, $t_{90'} = 2.36$ sec

$100 = \frac{1}{2}(32.2) t_{100'}^2$, $t_{100'} = 2.49$ sec

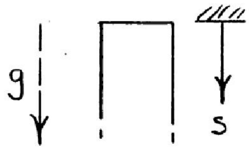
$t_{10} = t_{100'} - t_{90'} = 2.49 - 2.36 = \underline{0.1279 \text{ sec}}$

100^{th} story: $990 = \frac{1}{2}(32.2) t_{990'}^2$

$1000 = \frac{1}{2}(32.2) t_{1000'}^2$

$t_{100} = t_{1000'} - t_{990'} = 7.88 - 7.84 = \underline{0.0395 \text{ sec}}$

2/55



$$a = g - kv^2 = \frac{dv}{dt}$$

$$\int_0^t dt = \int_0^v \frac{dv}{g - kv^2}$$

(see Art. C/10): $t = \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}}\, v \Big|_0^v$

$$= \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}}\, v$$

$$\Rightarrow \quad v = \frac{ds}{dt} = \sqrt{\frac{g}{k}} \tanh\left(\sqrt{gk}\, t\right)$$

$$\int_0^s ds = \sqrt{\frac{g}{k}} \int_0^t \tanh\left(\sqrt{gk}\, t\right)$$

$$s = \frac{1}{k} \ln \cosh \sqrt{gk}\, t$$

or $$t = \frac{\cosh^{-1}\left(e^{sk}\right)}{\sqrt{gk}} = \frac{\cosh^{-1}\left(e^{0.005s}\right)}{0.401}$$

s, ft	t, sec
0	0
10	0.795
90	2.54
100	2.70
990	14.06
1000	14.19

The time t_1 to pass first story is $t_1 = t_{10'} - t_0 = 0.795 - 0 = \underline{0.795 \text{ sec}}$

Similarly,

$$\underline{t_{10} = 0.1592 \text{ sec}}$$

$$\underline{t_{100} = 0.1246 \text{ sec}}$$

▶ 2/56

$$\frac{dv}{dt} = ke^{-bt} - cv - g, \quad \frac{dv}{dt} + cv = ke^{-bt} - g$$

For the standard form for the solution of the first order linear differential equation,

$$e^{-\int c\,dt} = e^{-ct} \quad \&$$

$$v = Ae^{-ct} + e^{-ct}\int (ke^{-bt} - g)e^{ct}\,dt$$

$$= Ae^{-ct} + \frac{k}{c-b}e^{-bt} - \frac{g}{c}$$

when $t = 0$, $v = 0$ so $0 = A + \frac{k}{c-b} - \frac{g}{c}$, $A = \frac{g}{c} - \frac{k}{c-b}$

Thus $v = \left(\frac{g}{c} - \frac{k}{c-b}\right)e^{-ct} + \frac{k}{c-b}e^{-bt} - \frac{g}{c}$

or $\underline{v = \frac{g}{c}(e^{-ct} - 1) + \frac{k}{c-b}(e^{-bt} - e^{-ct})}$

▶2/57 $a = \frac{d^2x}{dt^2} = Kt - k^2x, \quad \frac{d^2x}{dt^2} + k^2x = Kt$

$x = x_c + x_p = A \sin kt + B \cos kt + \frac{K}{k^2}t$

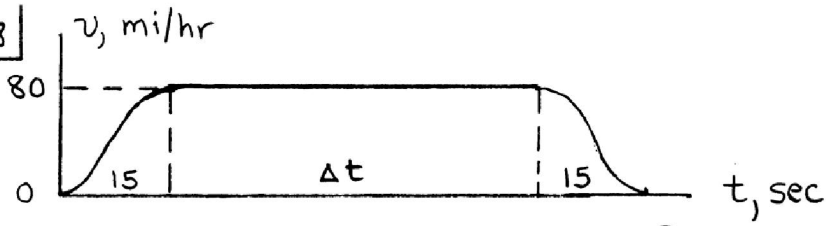
$\dot{x} = Ak \cos kt - Bk \sin kt + \frac{K}{k^2} = 0$ for $t = 0$

Thus $0 = Ak - 0 + \frac{K}{k^2}, \quad A = -K/k^3$

$x = 0$ when $t = 0$, so $0 = 0 + B + 0, \quad B = 0$

Thus $\underline{x = \frac{K}{k^3}(kt - \sin kt)}$

▶2/58



Cubic form : $v = a + bt + ct^2 + dt^3$

$v(0) = a = 0$, $\frac{dv}{dt} = b + 2ct + 3dt^2$

$\left.\frac{dv}{dt}\right|_{t=0} = 0 \Rightarrow b = 0$; $\left.\frac{dv}{dt}\right|_{t=15} = 30c + 675d = 0$

$v(15) = c(15)^2 + d(15)^3 = 80\left(\frac{5280}{3600}\right)$

Solve to obtain $c = 1.564$ ft/sec^3 , $d = -6.95(10^{-2})\frac{ft}{sec^4}$

$\therefore\ v = 1.564t^2 - 0.0695\,t^3$ for $0 \le t \le 15$ sec

$s = \int_0^t (1.564t^2 - 0.0695t^3)\,dt = 0.521t^3 - 0.01738\,t^4$

$s(15) = 880$ ft ; 880 ft also covered during decel.

Distance during cruise : $(2)(5280) - 2(880) = 8800$ ft

$\Delta s = v\Delta t$: $8800 = 80\left(\frac{5280}{3600}\right)\Delta t$, $\Delta t = 75$ sec

$\therefore$ Run time $t = 75 + 2(15) = \underline{105 \text{ sec}}$

$a = \frac{dv}{dt} = 3.13t - 0.209t^2$

$\frac{da}{dt} = 3.13 - 0.417t = 0$, $t = 7.5$ sec

$a_{max} = 3.13(7.5) - 0.209(7.5)^2 = \underline{11.73 \text{ ft/sec}^2}$

2/59 $\underline{v}_{AV} = \frac{\Delta \underline{r}}{\Delta t} = \frac{0.03\underline{i} - 0.05\underline{j}}{0.02} = 1.5\underline{i} - 2.5\underline{j}\ \frac{m}{s}$

$v = |\underline{v}_{AV}| = \sqrt{1.5^2 + 2.5^2} = \underline{2.92\ m/s}$

$\tan\theta = \frac{v_y}{v_x} = \frac{-2.5}{1.5} = -\frac{5}{3}, \quad \underline{\theta = -59.0^\circ}$

2/60 $\underline{a}_{AV} = \frac{\Delta \underline{v}}{\Delta t} = \frac{0.3\underline{i} + 0.4\underline{j}}{0.1} = 3\underline{i} + 4\underline{j}\ \frac{m}{s^2}$

$a = |a_{AV}| = \sqrt{3^2 + 4^2} = \underline{5\ m/s^2}$

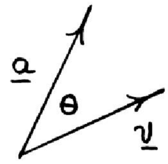
$\tan\theta = \frac{a_y}{a_x} = \frac{4}{3}$, $\underline{\theta = 53.1^\circ}$

2/61

$$\underline{a}_{AV} = \frac{\Delta \underline{v}}{\Delta t}$$

$$4\underline{i} + 6\underline{j} = \left[\underline{v}_{3.67} - (6.12\underline{i} + 3.24\underline{j})\right] / 0.02$$

$$\underline{v} = \underline{v}_{3.67} = \underline{6.20\underline{i} + 3.36\underline{j} \ \ \text{m/s}}$$



$$\cos\theta = \frac{\underline{v}\cdot\underline{a}}{va}$$

$$v = \sqrt{6.20^2 + 3.36^2} = 7.05 \ \text{m/s}$$

$$a = \sqrt{4^2 + 6^2} = 7.21 \ \text{m/s}^2$$

$$\cos\theta = \frac{(6.20\underline{i} + 3.36\underline{j})\cdot(4\underline{i} + 6\underline{j})}{(7.05)(7.21)} = \frac{45.0}{50.9} = 0.884$$

$$\underline{\theta = 27.9^\circ}$$

2/62

$$x = 2t^2 - 4t \qquad y = 3t^2 - \tfrac{1}{3}t^3$$

$$\dot{x} = 4t - 4 \qquad \dot{y} = 6t - t^2$$

$$\ddot{x} = 4 \text{ mm/s}^2 \qquad \ddot{y} = 6 - 2t$$

At $t = 2$ s: $\begin{cases} \dot{x} = 4(2) - 4 = 4 \text{ mm/s} \\ \dot{y} = 6(2) - 2^2 = 8 \text{ mm/s} \end{cases}$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{4^2 + 8^2} = \underline{8.94 \text{ mm/s}}$$

$$\theta_x = \tan^{-1}\frac{\dot{y}}{\dot{x}} = \tan^{-1}\frac{8}{4} = \underline{63.4°}$$

At $t = 2$ s: $\begin{cases} \ddot{x} = 4 \text{ mm/s}^2 \\ \ddot{y} = 6 - 2(2) = 2 \text{ mm/s}^2 \end{cases}$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{4^2 + 2^2} = \underline{4.47 \text{ mm/s}^2}$$

$$\theta_x = \tan^{-1}\frac{\ddot{y}}{\ddot{x}} = \tan^{-1}\frac{2}{4} = \underline{26.6°}$$

2/63 Use coordinate origin at A

Apply $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ at B:

$-4 = 0 + 0 - \frac{1}{2}(9.81) t_B^2$, $t_B = 0.903$ s

Apply $x = x_0 + v_{x_0} t$ at B: $6 = 0 + v_0 (0.903)$

$v_0 = 6.64$ m/s

Apply y-eq. @ C: $-8 = -\frac{1}{2}(9.81) t_C^2$

$t_C = 1.277$ s

Apply x-eq. @ C: $s + 6 = 6.64(1.277)$

$s = 2.49$ m

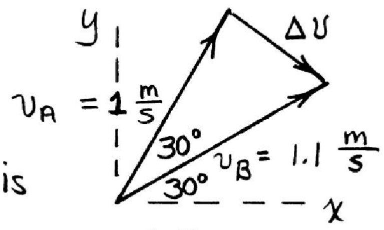
2/64 $v = \dot{s} = \frac{t}{2}$, $v_A = \frac{2}{2} = 1$ m/s, $v_B = \frac{2.2}{2} = 1.1 \frac{m}{s}$

$\Delta v_x = v_{Bx} - v_{Ax} = 1.1 \cos 30° - 1.0 \cos 60° = 0.453 \frac{m}{s}$

$\Delta v_y = v_{By} - v_{Ay} = 1.1 \sin 30° - 1.0 \sin 60° = -0.316 \frac{m}{s}$

$\Delta v = \sqrt{0.453^2 + 0.316^2}$

$= 0.552$ m/s



The average acceleration is

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{0.552}{0.20} = \underline{2.76 \text{ m/s}^2}$$

$$\underline{a}_{av} = \frac{\Delta \underline{v}}{\Delta t} = \frac{0.453\underline{i} - 0.316\underline{j}}{0.20}$$

$$= \underline{2.26\underline{i} - 1.580\underline{j} \text{ m/s}^2}$$

2/65 $\quad a_x = 12t, \quad \int_4^{v_x} dv_x = \int_0^t 12t\,dt, \quad v_x = 4 + 6t^2$

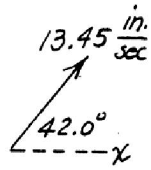
$y = 4t^3 - 3t, \quad v_y = \dot{y} = 12t^2 - 3, \quad a_y = \ddot{y} = 24t$

when $t = 1$ sec, $\quad v_x = 4 + 6(1^2) = 10$ in./sec

$v_y = 12(1^2) - 3 = 9$ in./sec

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{10^2 + 9^2} = \underline{13.45 \text{ in./sec}}$

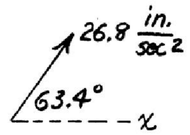
$\theta_x = \tan^{-1} v_y/v_x = \tan^{-1} 9/10 = \underline{42.0^\circ}$



$a_x = 12(1) = 12$ in./sec^2 $\quad a = \sqrt{a_x^2 + a_y^2} = \sqrt{12^2 + 24^2} = \underline{26.8 \frac{\text{in.}}{\text{sec}^2}}$

$a_y = 24(1) = 24$ in./sec^2

$\theta_x = \tan^{-1} a_y/a_x = \tan^{-1} 24/12 = \underline{63.4^\circ}$



2/66

$$\underline{r} = \left(\frac{2}{3}t^3 - \frac{3}{2}t^2\right)\underline{i} + \left(\frac{t^4}{12}\right)\underline{j}$$

$$\underline{v} = \dot{\underline{r}} = \left(2t^2 - 3t\right)\underline{i} + \left(\frac{1}{3}t^3\right)\underline{j}$$

$$\underline{a} = \dot{\underline{v}} = \left(4t - 3\right)\underline{i} + \left(t^2\right)\underline{j}$$

At $t = 2s$

$$\begin{cases} \underline{v} = (2\cdot 2^2 - 3\cdot 2)\underline{i} + \frac{1}{3}2^3\underline{j} = 2\underline{i} + \frac{8}{3}\underline{j} \quad \text{mm/s} \\ \underline{a} = (4\cdot 2 - 3)\underline{i} + 2^2\underline{j} = 5\underline{i} + 4\underline{j} \quad \text{mm/s}^2 \end{cases}$$

$$\cos\theta = \frac{\underline{v}\cdot\underline{a}}{va} = \frac{(2\underline{i} + \frac{8}{3}\underline{j})\cdot(5\underline{i} + 4\underline{j})}{\sqrt{2^2 + \left(\frac{8}{3}\right)^2}\sqrt{5^2 + 4^2}}$$

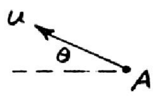
$$\underline{\theta = 14.47^\circ}$$

At $t = 3s$

$$\begin{cases} \underline{v} = (2\cdot 3^2 - 3\cdot 3)\underline{i} + \left(\frac{1}{3}3^3\right)\underline{j} = 9\underline{i} + 9\underline{j} \quad \frac{\text{mm}}{\text{s}} \\ \underline{a} = (4\cdot 3 - 3)\underline{i} + (3^2)\underline{j} = 9\underline{i} + 9\underline{j} \quad \frac{\text{mm}}{\text{s}^2} \end{cases}$$

$$\underline{v} \parallel \underline{a} \Rightarrow \underline{\theta = 0}$$

2/67 From Sample Prob. 2/6



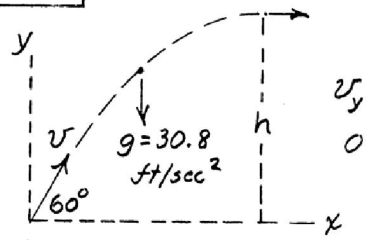
$$2s = \frac{u^2 \sin 2\theta}{g} = \frac{2(u\cos\theta)(u\sin\theta)}{g}$$

But $2s = 22$ ft, $u\cos\theta = 30$ ft/sec, $u\sin\theta = v_y$

So $v_y = \dfrac{2sg}{2u\cos\theta} = \dfrac{22(32.2)}{2(30)} = \underline{11.81 \text{ ft/sec}}$

Also, $h = \dfrac{u^2\sin^2\theta}{2g} = v_y^2/2g = \dfrac{(11.81)^2}{2(32.2)} = \underline{2.16 \text{ ft}}$

2/68 $v = 600\ mi/hr \equiv 880\ ft/sec$



$a_y = -30.8\ ft/sec^2$

$v_y = v_{y_0} + at;$

$0 = 880 \sin 60° - 30.8\, t$

$t = \dfrac{880(0.866)}{30.8} = \underline{24.7\ sec}$

$v_y^2 = v_{y_0}^2 + 2ay; \quad 0 = (880 \sin 60°)^2 - 2(30.8)h$

$h = \dfrac{[880(0.866)]^2}{2(30.8)} = 9429\ ft$

or $\quad h = \dfrac{9429}{5280} = \underline{1.786\ mi}$

2/69 Set up x-y axes at the initial location of G.

$$x = x_0 + v_{x_0} t \quad : \quad 3 = (v_0 \cos\theta)t$$

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \; : \; 3.5 = (v_0 \sin\theta)t - 16.1t^2$$

$$v_y = v_{y_0} - gt \quad : \quad 0 = v_0 \sin\theta - 32.2t$$

Solve simultaneously :

$t = 0.466$ sec

$\underline{v_0 = 16.33 \text{ ft/sec}}$

$\underline{\theta = 66.8^\circ}$

$\underline{2/70}$ $v_y = \dot{y} = 8t$, $y = 4t^2 + C_1$; $y = 2$ ft when $t = 0$

so $C_1 = 2$ ft & $y = 4t^2 + 2$ ft

$a_x = \ddot{x} = 4t$, $\dot{x} = 2t^2 + C_2$; $\dot{x} = 0$ when $t = 0$ so $C_2 = 0$

$x = \frac{2t^3}{3} + C_3$; $x = 0$ when $t = 0$ so $C_3 = 0$

Eliminate t & get $y = 4\sqrt[3]{\frac{9x^2}{4}} + 2$

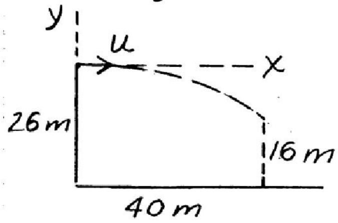
or $\underline{(y-2)^3 = 144x^2}$

When $x = 18$ ft, $\frac{2t^3}{3} = 18$, $t^3 = 27$, $t = 3$ sec

& $\dot{x} = 2(3)^2 = 18 \frac{ft}{sec}$, $\dot{y} = 8(3) = 24 \frac{ft}{sec}$

$v = \sqrt{\overline{18}^2 + \overline{24}^2} = \underline{30 \text{ ft/sec}}$

2/71 $a_y = -g$ so $y = 0 - \frac{1}{2}gt^2$, $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(26-16)}{9.81}}$

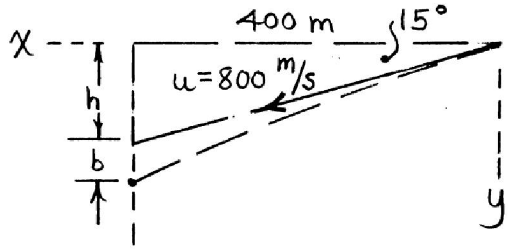


$= 1.428 \text{ s}$

$x = ut$; $u = 40/1.428$

$= \underline{28.0 \text{ m/s}}$

2/72



$h = 400 \tan 15°$

$= 107.2$ m

The equation of the path (Sample Problem 2/6)

is $y = x \tan\theta + \frac{gx^2}{2u^2}\sec^2\theta$ (+ sign here)

So $b = \frac{gx^2}{2u^2}\sec^2\theta$

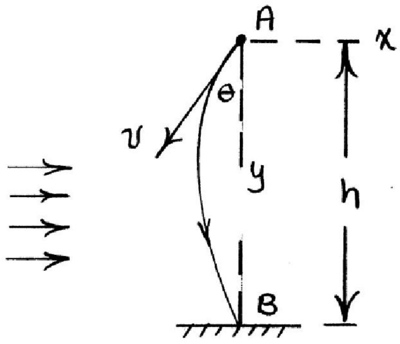
At $x = 400$ m: $b = \frac{9.81(400)^2}{2(800)^2}\sec^2 15°$

$= \underline{1.314 \text{ m}}$

2/73

$a_x = a$

$a_y = g$



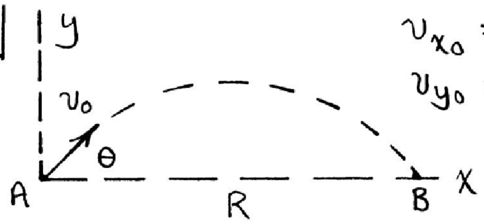
$x = v_{x_0}t + \frac{1}{2}a_x t^2$: $x = -v\sin\theta\, t + \frac{1}{2}at^2$

$y = v_{y_0}t + \frac{1}{2}g\, t^2$: $y = v\cos\theta\, t + \frac{1}{2}gt^2$

At B: $\begin{cases} x = 0 & \text{so } 0 = t\left(-v\sin\theta + \frac{1}{2}at\right),\ t = \frac{2v\sin\theta}{a} \\ y = h & \text{so } h = v\cos\theta\left(\frac{2v\sin\theta}{a}\right) + \frac{1}{2}g\left(\frac{2v\sin\theta}{a}\right)^2 \end{cases}$

$$\underline{h = \frac{2v^2}{a}\sin\theta\left(\cos\theta + \frac{g}{a}\sin\theta\right)}$$

2/74



$v_{x_0} = v_0 \cos\theta$

$v_{y_0} = v_0 \sin\theta$

$x = x_0 + v_{x_0} t$ @ B: $R = 0 + (v_0 \cos\theta) t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B: $0 = 0 + (v_0 \sin\theta) t_f - \frac{g}{2} t_f^2$ (2)

(2): $t_f = 0, \ \dfrac{2 v_0 \sin\theta}{g}$ ($t = 0$ is launch time)

(1): $R = (v_0 \cos\theta)\left(\dfrac{2 v_0 \sin\theta}{g}\right) = \dfrac{v_0^2 \sin 2\theta}{g}$

$\dfrac{dR}{d\theta} = 0: \ \dfrac{v_0^2}{g} 2 \cos 2\theta = 0 \Rightarrow \underline{\theta = 45°}$

$R_{max} = \dfrac{v_0^2 \sin(2 \cdot 45°)}{g} = \underline{\dfrac{v_0^2}{g}}$

2/75 Angle θ for maximum range and, hence, minimum u is $45°$

From Sample Prob. 2/6 range is

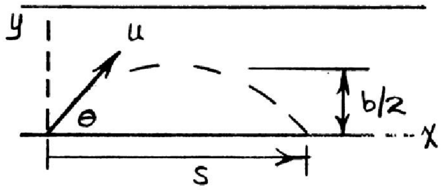
$$2s = \frac{u^2}{g}\sin 2\theta, \quad \text{so} \quad 12(10^3) = \frac{u^2}{9.81}(1)$$

$$u = \sqrt{9.81(12000)} = \underline{343 \text{ m/s}}$$

2/76

$a_y = -\frac{eE}{m}$, constant

$a_x = 0$



$v_y^2 - v_{y_0}^2 = 2ay$: At top, $0 - (u\sin\theta)^2 = 2\left(-\frac{eE}{m}\right)\frac{b}{2}$

$$\underline{E = \frac{mu^2\sin^2\theta}{eb}}$$

$v_y = v_{y_0} + a_y t$: At top, $0 = u\sin\theta - \frac{eE}{m}t$

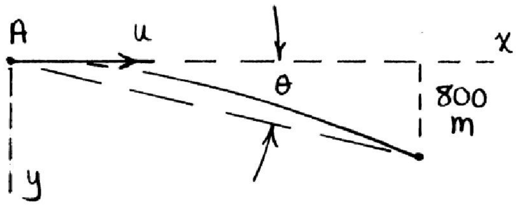
$$t = \frac{mu\sin\theta}{eE}$$

$x = v_{x_0}t$: $s = (u\cos\theta)(2t) = u\cos\theta\left(\frac{2mu\sin\theta}{eE}\right)$

$$= u\cos\theta\left(\frac{2mu\sin\theta}{e\,\frac{mu^2\sin^2\theta}{eb}}\right) = \underline{2b\cot\theta}$$

2/77

$u = \frac{1000}{3.6} = 278 \frac{m}{s}$



y-dir.: $y = v_{y_0} t + \frac{1}{2} g t^2$

$800 = 0 + \frac{1}{2}(9.81) t^2, \quad t = 12.77 \text{ s}$

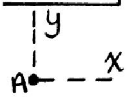
x-dir.: $x = v_{x_0} t + \frac{1}{2} a_x t^2$

$= 278(12.77) + \frac{1}{2}\left(\frac{9.81}{2}\right)(12.77)^2$

$= 3950 \text{ m}$

$\theta = \tan^{-1} \frac{800}{3950} = \underline{11.46°}$

2/78



$v_{x_0} = v_0 \cos\theta = 45 \cos 40^\circ = 34.5$ ft/sec

$v_{y_0} = v_0 \sin\theta = 45 \sin 40^\circ = 28.9$ ft/sec

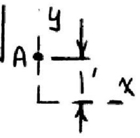
$x = x_0 + v_{x_0} t$ at wall: $60 = 0 + 34.5\,t$

$t = 1.741$ sec

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ at wall:

$y = 1 + 28.9(1.741) - 16.1(1.741)^2 = 2.57$ ft

<u>So water strikes wall 2.57 ft above B.</u>

2/79 

$v_{x_0} = 45 \cos\theta$, $v_{y_0} = 45 \sin\theta$

Strategy: Determine θ for hitting top point of wall; take larger of two values.

$x = x_0 + v_{x_0} t$: $60 = 0 + (45\cos\theta)t$, $t = \frac{60}{45\cos\theta}$

$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$: $3 = 1 + 45\sin\theta\left(\frac{60}{45\cos\theta}\right) - 16.1\left(\frac{60}{45\cos\theta}\right)^2$

Use $\frac{1}{\cos^2\theta} = \sec^2\theta = 1 + \tan^2\theta$ & rearrange:

$$28.6\tan^2\theta - 60\tan\theta + 30.6 = 0$$

$\Rightarrow \begin{cases} \tan\theta = 0.879, & \theta = 41.3° \\ \tan\theta = 1.218, & \theta = 50.6° \end{cases}$ $\therefore$ θ must be just larger than 50.6°

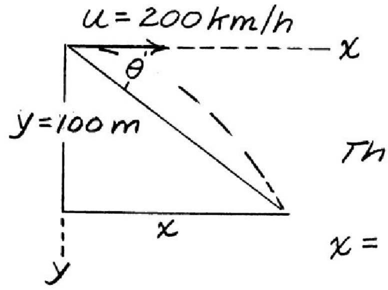
$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ at impact: $0 = 1 + 45\sin 50.6° t - 16.1t^2 \Rightarrow t = 2.19$ sec

$x = x_0 + v_{x_0}t$: $R = (45\cos 50.6°)\,2.19 = 62.5$ ft

$\therefore$ Water lands 2.50 ft to right of B.

2/80 | For y-motion $a_y = g = 9.81\ m/s^2$

$$y = \frac{1}{2} g t^2$$



For x-motion $a_x = 0$ so

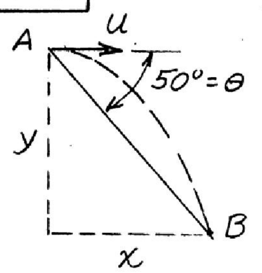
$$x = ut$$

Thus $y = \dfrac{g x^2}{2u^2}$, $x = u\sqrt{\dfrac{2y}{g}}$

$$x = \frac{200}{3.6}\sqrt{\frac{2(100)}{9.81}} = 251\ m$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{100}{251} = \underline{21.7°}$$

2/81



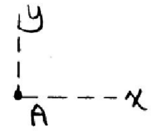
$$y = \frac{1}{2} g t^2, \quad x = ut$$

$$\tan\theta = \frac{y}{x} = \frac{gt}{2u}$$

$$\text{so } u = \frac{gt}{2\tan\theta} = \frac{9.81(3.5)}{2(1.1918)}$$

$$= \underline{14.41 \text{ m/s}}$$

2/82 (a) $v_0 = 140$ ft/sec and $\theta = 8°$:



$x = x_0 + v_{x_0} t$ @ B: $200 = 0 + (140 \cos 8°)t$

$\underline{t = 1.443 \text{ sec}}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B:

$-(7.5 - h) = 0 + 140 \sin 8° (1.443) - \frac{1}{2}(32.2)(1.443)^2$

$\underline{h = 2.10 \text{ ft}}$

(b) $v_0 = 120$ ft/sec and $\theta = 12°$:

$x = x_0 + v_{x_0} t$ @ B: $200 = 0 + (120 \cos 12°)t$

$\underline{t = 1.704 \text{ sec}}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B:

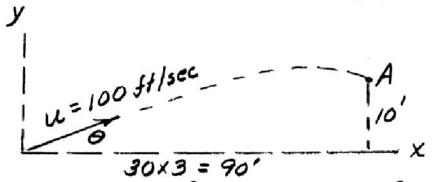
$-(7.5 - h) = 0 + (120 \sin 12°)(1.704) - \frac{1}{2}(32.2)(1.704)^2$

$\underline{h = 3.27 \text{ ft}}$

(In baseball, the time of flight is critical; low trajectories, even with one hop, are better.)

2/83

$$y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$$



Let $m = \tan\theta$

$\sec^2\theta = 1 + \tan^2\theta = 1 + m^2$

$$y = xm - \frac{gx^2}{2u^2}(1+m^2), \quad m^2 - \frac{2u^2}{gx}m + \left(\frac{2u^2y}{gx^2} + 1\right) = 0$$

At A, $m^2 - \frac{2(10^2)^2}{32.2(90)}m + \left(\frac{2(10^2)^2 10}{32.2(90)^2} + 1\right) = 0$

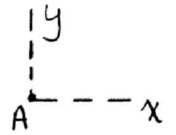
$$m^2 - 6.901m + 1.7668 = 0$$

$$m = \frac{6.901}{2} \pm \frac{1}{2}\sqrt{(6.901)^2 - 4(1.7668)}$$

$$= \frac{6.901 \pm \sqrt{40.56}}{2} = 0.266 \text{ or } 6.635$$

$$\theta = \tan^{-1}m = \underline{14.91^\circ} \qquad (\text{or } 81.4^\circ)$$

2/84 Set up x-y coordinates with origin at A.



$$v_{x_0} = 25 \cos\theta$$

$$v_{y_0} = 25 \sin\theta$$

Evaluate $v_y^2 = v_{y_0}^2 - 2g(y - y_0)$ at ceiling:

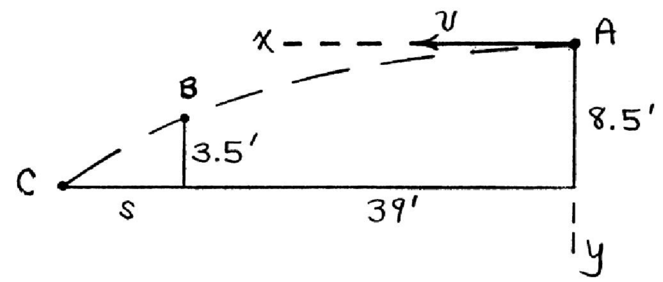
$$0^2 = (25 \sin\theta)^2 - 2(9.81)(5-0)$$

$$\underline{\theta = 23.3^\circ}$$

Range $R = \dfrac{v_0^2 \sin 2\theta}{g} = \dfrac{25^2 \sin(2 \cdot 23.3^\circ)}{9.81}$

$$\underline{R = 46.4 \text{ m}}$$

2/85



$a_x = 0$: $x = v_{x_0} t$, $39 = v t_B$

$a_y = g$: $y = v_{y_0} t + \frac{1}{2} g t^2$

At B : $8.5 - 3.5 = 0 + \frac{1}{2} 32.2 t_B^2$, $t_B = 0.557$ sec

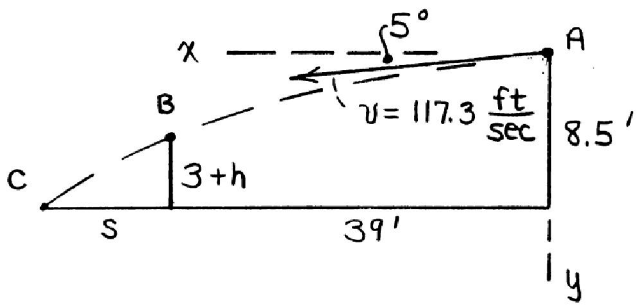
Then $v = \frac{39}{t_B} = \frac{39}{0.557} = 70.0$ ft/sec

$\underline{(47.7 \text{ mi/hr})}$

At C : $8.5 = \frac{1}{2}(32.2) t_c^2$, $t_c = 0.727$ sec

$s + 39 = 70.0(0.727)$, $\underline{s = 11.85 \text{ ft}}$

2/86



$a_x = 0$, $x = v_{x_0} t$: $39 = 117.3 \cos 5°\, t_B$, $t_B = 0.334$ sec

$a_y = g$, $y = v_{y_0} t + \frac{1}{2} g t^2$: At B,

$$5.5 - h = 117.3 \sin 5° (0.334) + \tfrac{1}{2}(32.2)(0.334)^2$$

$$\underline{h = 0.296 \text{ ft}} \text{ or } \underline{h = 3.55 \text{ in.}}$$

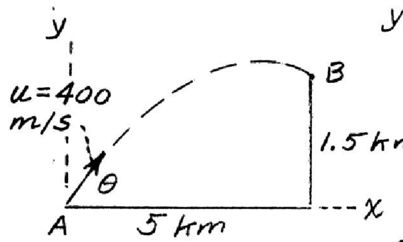
At C: $8.5 = 117.3 \sin 5°\, t_C + 16.1\, t_C^2$

$$t_C = 0.475 \text{ sec}$$

x-equation at C: $39 + s = 117.3 \cos 5° (0.475)$

$$\underline{s = 16.57 \text{ ft}}$$

2/87 Eq. of trajectory (Sample Problem 2/6)



$$y = x \tan\theta - \frac{g x^2}{2u^2} \sec^2\theta$$

$$= x \tan\theta - \frac{g x^2}{2u^2}(1+\tan^2\theta)$$

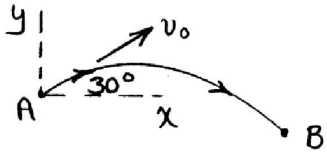
Substitute values & get

$$1500 = 5000 \tan\theta - \frac{9.81(5000)^2}{2(400)^2}(1+\tan^2\theta)$$

or $\tan^2\theta - 6.524 \tan\theta + 2.957 = 0$

solution gives roots $\underline{\theta_1 = 26.1°}$ & $\underline{\theta_2 = 80.6°}$

2/88 Set up x-y axes at A, target at B:



$$\left.\begin{array}{ll} x\text{-eq.}: & x_B = (v_0 \cos 30°)t \\ y\text{-eq.}: & y_B = (v_0 \sin 30°)t - \frac{1}{2}gt^2 \end{array}\right\}$$

For $x_B = 12'$, $y_B = -0.333'$: $\begin{cases} v_0 = 20.6 \text{ ft/sec} \\ t = 0.672 \text{ sec} \end{cases}$

For $x_B = 14'$, $y_B = -0.333'$: $\begin{cases} v_0 = 22.4 \frac{\text{ft}}{\text{sec}} \\ t = 0.723 \text{ sec} \end{cases}$

So the range is $\underline{20.6 \leq v_0 \leq 22.4 \text{ ft/sec}}$

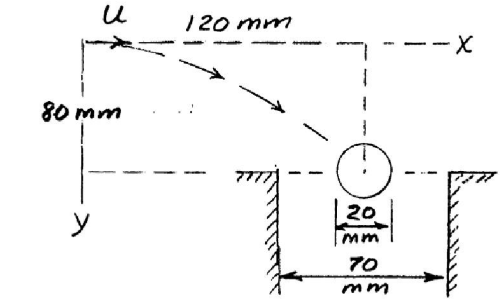
2/89

$x = ut, \quad y = \frac{1}{2}gt^2$

$x = u\sqrt{\frac{2y}{g}}, \quad u = x\sqrt{\frac{g}{2y}}$

$x_{max} = 120 + \frac{70}{2} - \frac{20}{2} = 145 \text{ mm}$

$x_{min} = 120 - \frac{70}{2} + \frac{20}{2} = 95 \text{ mm}$



$$u_{max} = 0.145\sqrt{\frac{9.81}{2(0.080)}} = \underline{1.135 \text{ m/s}}$$

$$u_{min} = 0.095\sqrt{\frac{9.81}{2(0.080)}} = \underline{0.744 \text{ m/s}}$$

2/90 Set up x-y Coordinates at A

x-eq. : $x_B = (36 \cos\theta)t$

y-eq. : $y_B = (36 \sin\theta)t - 16.1t^2$

Solutions :

For $x_B = 40'$, $y_B = -\frac{22}{12}{}'$ (top of stake) :

$\theta = 34.3°$ or $\theta = 53.1°$

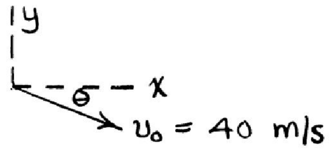
For $x_B = 40'$, $y_B = -3'$ (bottom of stake):

$\theta = 31.0°$ or $\theta = 54.7°$

Ranges : $31.0° \leq \theta \leq 34.3°$

or $53.1° \leq \theta \leq 54.7°$

2/91 Set up x-y coordinates with origin at release point :



$x = x_0 + v_{x_0} t$ at mitt : $20 = (40 \cos\theta) t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ at mitt :

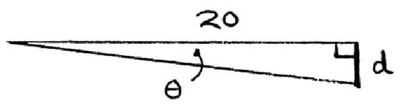
$$-1.8 = 0 + (-40 \sin\theta) t_f - \frac{9.81}{2} t_f^2 \qquad (2)$$

(1) : $t_f = \dfrac{1}{2\cos\theta}$

(2) : $-1.8 = -40 \sin\theta \left(\dfrac{1}{2\cos\theta}\right) - \dfrac{9.81}{2}\left(\dfrac{1}{2\cos\theta}\right)^2$

Use $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$: $1.226 \tan^2\theta + 20 \tan\theta - 0.574 = 0$

$$\Rightarrow \theta = 1.640°$$



$d = 20 \tan 1.640$
$= 0.573'$

$h = (2.2 + 0.6) - (0.573 + 1) = \underline{1.227 \text{ m}}$

2/92 | Use $x-y$ coordinates with origin at the release point :

y

x

$x = x_0 + v_{x_0} t$ @ hoop : $13.75 = 0 + (v_0 \cos 50°) t_f$

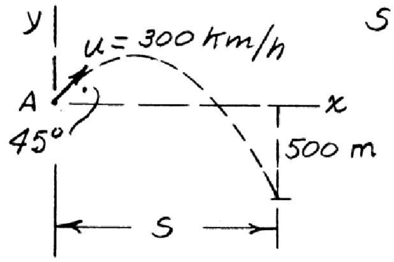
$t_f = 21.4/v_0$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ hoop :

$3 = 0 + v_0 \sin 50° \left(\frac{21.4}{v_0} \right) - 16.1 \left(\frac{21.4}{v_0} \right)^2$

$v_0 = 23.5$ ft/sec

$\underline{2/93}$ $u = 300/3.6 = 83.33 \text{ m/s}$



$s = ut\cos\theta$

$= 83.33(0.7071)t = 58.93\,t$

$y = ut\sin\theta + \frac{1}{2}a_y t^2$

$-500 = 83.33\,t(0.7071) + \frac{1}{2}(-9.81)t^2$

or $t^2 - 12.01\,t - 101.94 = 0$

$$t = \frac{12.01 \pm \sqrt{12.01^2 + 4(101.94)}}{2} = \underline{17.75 \text{ s}}$$

(or -5.74 s)

$s = 58.93(17.75) = 1046.2$ m or $\underline{s = 1.046 \text{ km}}$

▶2/94

$$\underline{a} = -k\underline{v} - g\underline{j}$$

$$a_x\underline{i} + a_y\underline{j} = -k(v_x\underline{i} + v_y\underline{j}) - g\underline{j}$$

$$\left.\right\} \therefore \quad a_x = -kv_x, \quad a_y = -kv_y - g$$

$\underline{x}$:

$$a_x = \frac{dv_x}{dt} = -kv_x$$

$$\int_{v_{x_0}}^{v_x} \frac{dv_x}{v_x} = -\int_0^t k\,dt \Rightarrow v_x = v_{x_0} e^{-kt}$$

$$\text{or } \underline{v_x = (v_0 \cos\theta) e^{-kt}}$$

$$v_x = \frac{dx}{dt} = v_{x_0} e^{-kt}$$

$$\int_0^x dx = \int_0^t v_{x_0} e^{-kt}\,dt$$

$$x = \frac{v_{x_0}}{k}\left[1 - e^{-kt}\right] = \underline{\frac{v_0\cos\theta}{k}\left[1 - e^{-kt}\right]}$$

$\underline{y}$:

$$a_y = \frac{dv_y}{dt} = -kv_y - g$$

$$\int_{v_{y_0}}^{v_y} \frac{dv_y}{kv_y + g} = -\int_0^t dt$$

$$\Rightarrow v_y = \left[v_{y_0} + \frac{g}{k}\right]e^{-kt} - \frac{g}{k} = \underline{\left[v_0 \sin\theta + \frac{g}{k}\right]e^{-kt} - \frac{g}{k}}$$

$$v_y = \frac{dy}{dt} = \left[v_{y_0} + \frac{g}{k}\right]e^{-kt} - \frac{g}{k}$$

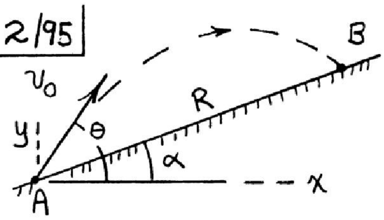
$$\int_0^y dy = \int_0^t \left\{\left[v_{y_0} + \frac{g}{k}\right]e^{-kt} - \frac{g}{k}\right\}dt$$

$$\underline{y = \frac{1}{k}\left[v_0 \sin\theta + \frac{g}{k}\right]\left[1 - e^{-kt}\right] - \frac{g}{k}t}$$

Terminal velocity $(t \to \infty)$: $v_x \to 0$

$$\underline{v_y \to -\frac{g}{k}}$$

▶ 2/95



$x = x_0 + v_{x_0} t$ @ B: $\quad R\cos\alpha = (v_0\cos\theta)t_f$

$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ @ B: $R\sin\alpha = (v_0\sin\theta)t_f - \frac{1}{2}gt_f^2$

x-eq: $\quad t_f = \dfrac{R\cos\alpha}{v_0\cos\theta}$

y-eq: $\quad R\sin\alpha = (v_0\sin\theta)\left(\dfrac{R\cos\alpha}{v_0\cos\theta}\right) - \frac{1}{2}g\left(\dfrac{R\cos\alpha}{v_0\cos\theta}\right)^2$

$$\Rightarrow \quad R = \frac{2v_0^2\cos^2\theta}{g\cos\alpha}(\tan\theta - \tan\alpha)$$

$$\frac{dR}{d\theta} = 0: \quad \frac{4v_0^2\cos\theta(-\sin\theta)}{g\cos\alpha}(\tan\theta - \tan\alpha) + \frac{2v_0^2\cos^2\theta}{g\cos\alpha}\frac{1}{\cos^2\theta} = 0$$

$$\frac{2v_0^2}{g\cos\alpha}\left[2\cos\theta\sin\theta(\tan\alpha - \tan\theta) + 1\right] = 0$$

$$\Rightarrow \quad 2\cos\theta\sin\theta\left(\tan\alpha - \frac{\sin\theta}{\cos\theta}\right) + 1 = 0$$

$$(2\cos\theta\sin\theta)\tan\alpha - 2\sin^2\theta + 1 = 0$$

$$\sin 2\theta\tan\alpha - 2\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta\right) + 1 = 0$$

$$\sin 2\theta\tan\alpha + \cos 2\theta = 0$$

$$\tan 2\theta = -\frac{1}{\tan\alpha}$$

$$2\theta = \tan^{-1}\left(-\frac{1}{\tan\alpha}\right) = 180° - \tan^{-1}\left(\frac{1}{\tan\alpha}\right)$$

$$= 180° - (90° - \alpha) = 90° + \alpha$$

$$\therefore \quad \theta = \underline{\frac{90° + \alpha}{2}}$$

Specific results:
$$\begin{cases} \alpha = 0, & \theta = 45° \\ \alpha = 30°, & \theta = 60° \\ \alpha = 45°, & \theta = 67.5° \end{cases}$$

$$2\theta = \tan^{-1}\left(-\frac{1}{\tan\alpha}\right) = 180^\circ - \tan^{-1}\left(\frac{1}{\tan\alpha}\right)$$

$$= 180^\circ - (90^\circ - \alpha) = 90^\circ + \alpha$$

$$\therefore \quad \theta = \underline{\frac{90^\circ + \alpha}{2}}$$

Specific results :
$$\begin{cases} \alpha = 0, & \theta = 45^\circ \\ \alpha = 30^\circ, & \theta = 60^\circ \\ \alpha = 45^\circ, & \theta = 67.5^\circ \end{cases}$$

2/96 Eq. of path: $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

Let $m = \tan\theta$, $\frac{1}{\cos^2\theta} = \sec^2\theta = 1 + \tan^2\theta = 1+m^2$

Thus $y = mx - \frac{gx^2}{2u^2}(1+m^2)$

or $m^2 - \left(\frac{2u^2}{gx}\right)m + \left(1 + \frac{2u^2 y}{gx^2}\right) = 0$

Roots are equal if discriminant $= 0$

Thus $\left(\frac{2u^2}{gx}\right)^2 - 4\left(1 + \frac{2u^2 y}{gx^2}\right) = 0$

Solve for y & get $y = \frac{u^2}{2g} - \frac{gx^2}{2u^2}$

a vertical parabola

2/97 $a_n = v^2/\rho = (0.6)^2/0.4 = 0.9 \text{ m/s}^2$

(a) $a_t = \dot{v} = 0$, so $\underline{a = a_n = 0.9 \text{ m/s}^2}$

(b) $a_t = \dot{v} = 1.2 \text{ m/s}^2$, so $a = \sqrt{a_t^2 + a_n^2}$

$= \sqrt{1.2^2 + 0.9^2} = \underline{1.5 \text{ m/s}^2}$

2/98 a_1 : speed is increasing, no path curvature.

a_2 : speed increasing, car turning to left.

a_3 : speed stationary, car turning to left.

a_4 : Speed decreasing, car turning to left.

a_5 : Speed decreasing, no path curvature.

a_6 : Speed decreasing, car turning to right.

2/99 $a_n = v^2/\rho = \left(\frac{45}{30}44\right)^2/800 = 5.45$ ft/sec^2

$a = \sqrt{a_n^2 + a_t^2}$; $a_t = \sqrt{a^2 - a_n^2} = \sqrt{10^2 - (5.45)^2} = \underline{8.39 \text{ ft/sec}^2}$

or $a_t = \frac{8.39}{44} 30 = \underline{5.72 \text{ mi/hr each second}}$ increasing or decreasing speed

2/100 $a = a_n = v^2/\rho$, $v = \sqrt{\rho a_n} = \sqrt{(100 - 0.6)0.5(9.81)}$

$= 22.08$ m/s

or $v = 22.08(3.6) = \underline{79.5 \text{ km/h}}$

2/101 $a = a_n = v^2/\rho$, $v^2 = \rho a_n$

$\rho = 98 + 2 = 100$ m, $v^2 = 100(0.4)(9.81)$

$v = 19.81 \frac{m}{s}$ or $71.3 \frac{km}{h}$

2/102 $a = a_n = v\dot{\beta} = \frac{20(1.852)}{3.6}\frac{\pi}{2}\frac{1}{60} = \underline{0.269 \text{ m/s}^2}$

$\boxed{2/103}$ $v = v_0 + a_t t$: $\frac{50}{3.6} = \frac{100}{3.6} + 12 a_t$

$a_t = -1.157 \text{ m/s}^2$

$a = \sqrt{a_t^2 + a_n^2}$: $2 = \sqrt{1.157^2 + a_n^2}$

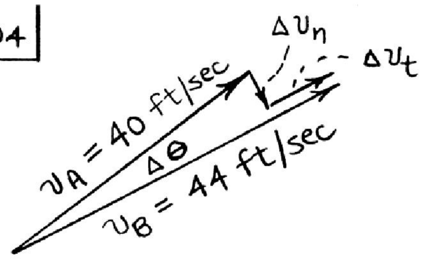
$a_n = 1.631 \text{ m/s}^2$

$v_6 = v_0 + a_t t = \frac{100}{3.6} - 1.157(6) = 20.8 \text{ m/s}$

From $a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{20.8^2}{1.631}$

$= \underline{266 \text{ m}}$

2/104



$$\Delta\theta = \frac{36-26}{180}\pi$$

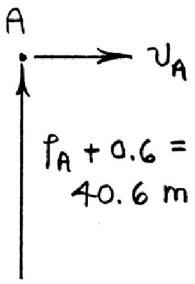
$$= 0.1745 \text{ rad}$$

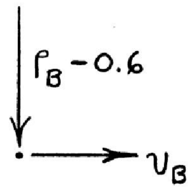
$$v_{AV} = 42 \text{ ft/sec}$$

$$a_n = \frac{\Delta v_n}{\Delta t} = \frac{v\,\Delta\theta}{\Delta t} = \frac{42\,(0.1745)}{3.84-3.64} = \underline{36.7 \frac{\text{ft}}{\text{sec}^2}}$$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{44-40}{0.2} = \underline{20 \text{ ft/sec}^2}$$

2/105





$$v_A = \frac{50}{3.6} = 13.89 \text{ m/s}, \quad v_B = \frac{100}{3.6} = 27.8 \text{ m/s}$$

$$v_B = v_A + a_t t, \quad a_t = \frac{27.8 - 13.89}{10} = 1.389 \text{ m/s}^2$$

At A: $a_n = \dfrac{v^2}{\rho} = \dfrac{13.89^2}{40.6} = 4.75 \text{ m/s}^2$

Total: $a_A = \sqrt{4.75^2 + 1.389^2} = 4.95 \text{ m/s}^2$

At B: For $a_A = a_B = 4.95 \text{ m/s}^2$,

$$4.95 = \sqrt{1.389^2 + a_n^2},$$

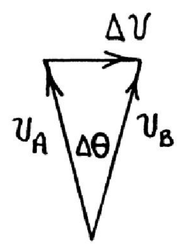
$$a_n = 4.75 = \frac{27.8^2}{\rho_B - 0.6}, \quad \underline{\rho_B = 163.0 \text{ m}}$$

2/106 $v_A = v_B = v = 2$ m/s

$\Delta v = 2v \sin\frac{\Delta\theta}{2} = 4 \sin\frac{\Delta\theta}{2}$ m/s

$\Delta t = \frac{r\,\Delta\theta}{v} = \frac{0.8\,\Delta\theta}{2} = 0.4\,\Delta\theta$ s

$a_{av} = \frac{\Delta v}{\Delta t} = \frac{4\sin\frac{\Delta\theta}{2}}{0.4\,\Delta\theta} = 5\,\frac{\sin\frac{\Delta\theta}{2}}{\Delta\theta/2}$



	$\Delta\theta°$	$\frac{\Delta\theta}{2}°$	$\frac{\Delta\theta}{2}$ rad	$\sin\frac{\Delta\theta}{2}$	a_{av}, m/s^2	% diff.
(a)	30°	15°	0.262	0.259	4.94	1.1
(b)	15°	7.5°	0.1309	0.1305	4.99	0.3
(c)	5°	2.5°	0.0436	0.0436	4.998	0.03

$a_n = \frac{v^2}{\rho} = \frac{2^2}{0.8} = \underline{5 \text{ m/s}^2}$

2/107

$$a_n = v^2/r\,, \quad \text{where} \quad a_n = g = g_0 \frac{R^2}{r^2}$$

So $$v^2 = gr = g_0 \frac{R^2}{r} = g_0 \frac{R^2}{R+h}$$

$$= 9.821 \frac{[6.371(10^6)]^2}{[6.371(10^6) + 0.320(10^6)]}$$

$$v = 7.72(10^3)\ \text{m/s} \quad \text{or} \quad \underline{27.8(10^3)\ \frac{\text{km}}{\text{h}}}$$

2/108 From $a_n = \frac{v^2}{\rho}$, $v = \sqrt{a_n \rho} = \sqrt{0.8g\rho}$

$v_A = \sqrt{0.8g\rho_A} = \sqrt{0.8(9.81)(85)} = \underline{25.8 \text{ m/s}}$

$v_B = \sqrt{0.8g\rho_B} = \sqrt{0.8(9.81)(200)} = \underline{39.6 \text{ m/s}}$

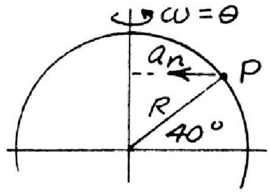
Path BB offers a considerable advantage.

2/109

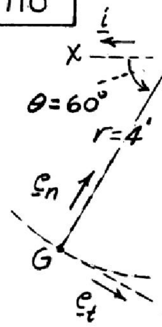
$$a = a_n = r\dot{\theta}^2 = R\cos\gamma\,\dot{\theta}^2$$

$$= \frac{12.742(10^6)}{2}\cos 40^\circ\,(0.729\times 10^{-4})^2$$

$$= \underline{0.0259\ m/s^2}$$



2/110



$$a_n = r\dot{\theta}^2 = 4(2.00)^2 = 16.00 \text{ ft/sec}^2$$

$$a_t = r\ddot{\theta} = 4(4.025) = 16.10 \text{ ft/sec}^2$$

$$\underline{a} = \underline{16.00\underline{e}_n + 16.10\underline{e}_t \text{ ft/sec}^2}$$

$$a_x = -16.00\cos 60° - 16.10\sin 60° = -21.9 \text{ ft/sec}^2$$

$$a_y = 16.10\cos 60° - 16.00\sin 60° = -5.81 \text{ ft/sec}^2$$

$$\underline{a} = \underline{-21.9\underline{i} - 5.81\underline{j} \text{ ft/sec}^2}$$

2/111

$$a_n = g = \frac{v^2}{\rho} = \frac{\left[17{,}369\left(5280/3600\right)\right]^2}{(3959+150)(5280)}$$

$$= \underline{29.91 \text{ ft/sec}^2}$$

Check: $g = g_0\left(\frac{R}{R+h}\right)^2 = 32.22\left(\frac{3959}{3959+150}\right)^2$

$$= \underline{29.91 \text{ ft/sec}^2} \checkmark$$

2/112 $a_t = 20/3.6 = 5.56 \ m/s^2$

$$a^2 = a_n^2 + a_t^2, \quad a_n^2 = \overline{3(9.81)}^2 - \overline{5.56}^2 = 835.2$$

$$a_n = 28.90 \ m/s^2$$

$$a_n = v^2/\rho, \quad \rho = \frac{(800/3.6)^2}{28.90} = \underline{1709 \ m}$$

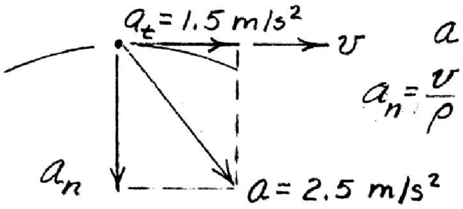
2/113 For P_1 $a_n = v^2/r$, $v = \sqrt{0.1(40)} = \underline{2\ m/s}$

$$a_1 = \sqrt{a_n^2 + a_t^2} = \sqrt{40^2 + 30^2} = \underline{50\ m/s^2}$$

For P_2 $a_n = v^2/r = 2^2/0.05 = 80\ m/s^2$

$$a_2 = \sqrt{a_n^2 + a_t^2} = \sqrt{80^2 + 30^2} = \underline{85.4\ m/s^2}$$

2/114



$a^2 = a_n^2 + a_t^2$

$a_n = \frac{v^2}{\rho} = \sqrt{a^2 - a_t^2}$

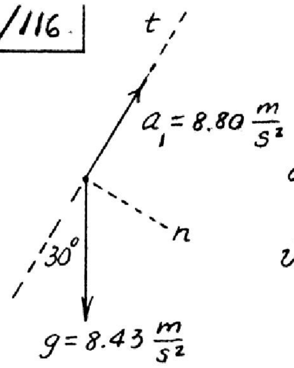
$v^2 = 200\sqrt{2.5^2 - 1.5^2} = 200(2) = 400\ m^2/s^2$

$v = 20\ m/s$ or $20(3.6) = \underline{72\ km/h}$

2/115 $a_n = v\dot{\beta} = g$, $\dot{\beta} = \dfrac{9.79}{800(10^3)/3600} = 0.04406$ rad/s

or $\dot{\beta} = 0.04406 \dfrac{180}{\pi} = \underline{2.52 \text{ deg/s}}$

2/116



$a_n = g \sin 30° = 8.43(0.5) = 4.22 \text{ m/s}^2$

or $a_n = \frac{4.22}{1000}(3600)^2 = 54\,630 \text{ km/h}^2$

$a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{(30000)^2}{54630} = \underline{16\,480 \text{ km}}$

$\dot{v} = a_t = 8.80 - 8.43 \cos 30° = \underline{1.499 \text{ m/s}^2}$

2/117 Relative to space station, $a_n = r\dot{\theta}^2$

where $a_n = 32.17 \text{ ft/sec}^2$.

Thus $32.17 = (240 + 20)\dot{\theta}^2$, $\dot{\theta} = 0.352 \frac{\text{rad}}{\text{sec}}$

$N = 0.352\left(\frac{60}{2\pi}\right) = \underline{3.36 \text{ rev/min}}$

2/118 $\quad a_t = \frac{v_f - v_i}{\Delta t} = \frac{6-3}{2} = 1.5 \text{ m/s}^2$

Halfway through time interval, $v = 4.5$ m/s

$$a_{P_1} = \sqrt{a_t^2 + a_n^2} = \sqrt{1.5^2 + \left(\frac{4.5^2}{0.060}\right)^2}$$

$$= \underline{338 \text{ m/s}^2} \quad (34.4g!)$$

$$a_{P_2} = a_t = \underline{1.5 \text{ m/s}^2}$$

2/119 $\underline{r} = \frac{3}{2}t^2\underline{i} + \frac{2}{3}t^3\underline{j}$ in.

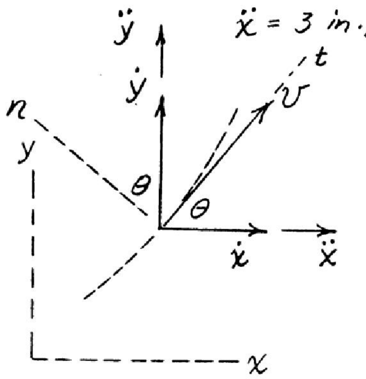
$\underline{v} = \dot{\underline{r}} = 3t\underline{i} + 2t^2\underline{j}$ in./sec

$\underline{a} = \dot{\underline{v}} = 3\underline{i} + 4t\underline{j}$ in./sec^2

For $t = 2$ sec, $\dot{x} = 3(2) = 6$ in./sec

$\dot{y} = 2(2^2) = 8$ in./sec

$\ddot{x} = 3$ in./sec^2, $\ddot{y} = 8$ in./sec^2



$\theta = \tan^{-1}\frac{6}{8} = \tan^{-1}\frac{3}{4}$

$v^2 = \dot{x}^2 + \dot{y}^2 = 6^2 + 8^2 = 100$ (in./sec)^2

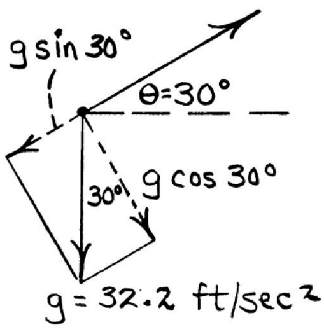
$a_n = \ddot{y}\cos\theta - \ddot{x}\sin\theta$

$= 8(3/5) - 3(4/5) = 12/5$ in./sec^2

$a_n = v^2/\rho$, $\rho = \frac{v^2}{a_n} = \frac{100}{12/5}$

$\underline{\rho = 41.7 \text{ in.}}$

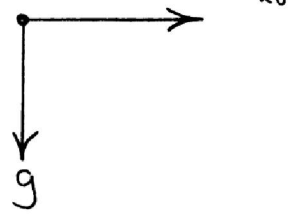
2/120



(a) $a_n = g\cos 30° = \dfrac{v^2}{\rho}$

$$\rho = \frac{100^2}{g\cos 30°} = \underline{359\text{ ft}}$$

$$\dot{v} = -g\sin 30° = \underline{-16.1\text{ ft/sec}^2}$$



(b) $a_n = g = \dfrac{v^2}{\rho}$

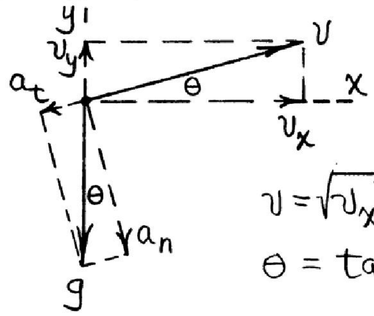
$$\rho = \frac{(100\cos 30°)^2}{32.2} = \underline{233\text{ ft}}$$

$$\underline{\dot{v} = 0}$$

2/121 The time t_{up} to apex is found from

$v_y = v_{y_0} - gt$: $0 = 100 \sin 30° - 32.2\, t_{up}$, $t_{up} = 1.553$ sec

So $t = 1$ sec is before apex and $t = 2.5$ sec is after.



(a) $t = 1$ sec

$v_x = 100 \cos 30° = 86.6$ ft/sec

$v_y = 100 \sin 30° - 32.2(1) = 17.80$ ft/sec

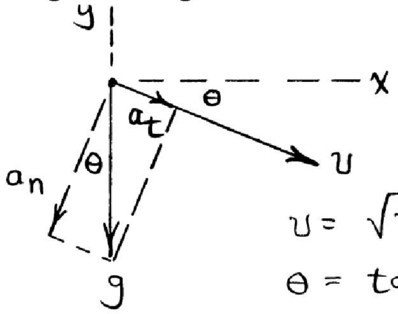
$v = \sqrt{v_x^2 + v_y^2} = 88.4$ ft/sec

$\theta = \tan^{-1} \dfrac{v_y}{v_x} = 11.61°$

$a_n = g \cos\theta = 32.2 \cos 11.61° = 31.5$ ft/sec^2

$\rho = \dfrac{v^2}{a_n} = \dfrac{88.4^2}{31.5} = \underline{248 \text{ ft}}$

$a_t = -g \sin\theta = -32.2 \sin 11.61° = \underline{-6.48 \text{ ft/sec}^2}$



(b) $t = 2.5$ sec

$v_x = 86.6$ ft/sec

$v_y = 100 \sin 30° - 32.2(2.5) = -30.5 \dfrac{\text{ft}}{\text{sec}}$

$v = \sqrt{v_x^2 + v_y^2} = 91.8$ ft/sec

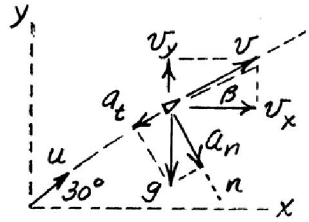
$\theta = \tan^{-1} \dfrac{|v_y|}{v_x} = 19.40°$

$a_n = g \cos\theta = 32.2 \cos 19.40° = 30.4$ ft/sec^2

$\rho = \dfrac{v^2}{a_n} = \dfrac{91.8^2}{30.4} = \underline{278 \text{ ft}}$

$a_t = +g \sin\theta = +32.2 \sin 19.40° = \underline{10.70 \text{ ft/sec}^2}$

2/122 $v = v_0 + at$; $v_x = 1800\,(0.866) + 0 = 1559$ ft/sec



$v_y = 1800\,(0.5) - 32.2\,(10)$

$= 578$ ft/sec

$v = \sqrt{v_x^{\,2} + v_y^{\,2}} = \sqrt{\overline{1559}^2 + \overline{578}^2}$

$= 1663$ ft/sec

$\beta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\dfrac{578}{1559} = 20.3°$

$a_n = g\cos\beta = 32.2\,(0.9376) = 30.19$ ft/sec^2

$a_n = v^2/\rho$, $\rho = \overline{1663}^2/30.19 = \underline{91{,}600 \text{ ft}}$

2/123 $a_n = v^2/\rho = \dfrac{v^2}{R+h}$ where $R = 1738$ km

$h = 200$ km

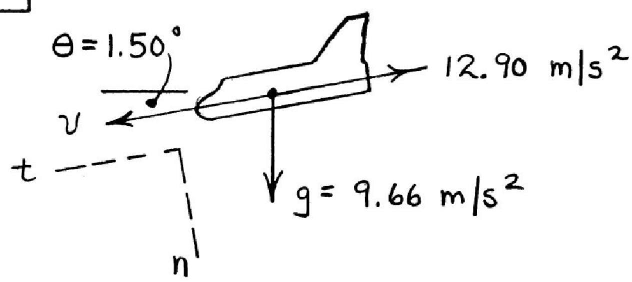
$g_0 = 1.62$ m/s^2 (Appen. B)

$$a_n = g = g_0\left(\frac{R}{R+h}\right)^2 = 1.62\left(\frac{1738}{1738+200}\right)^2 = 1.303 \text{ m/s}^2$$

$$v^2 = (1738+200)\,1.303\,(3600)^2(10^{-3}) = 32.72\,(10^6)\,(\text{km/h})^2$$

$v = \underline{5720 \text{ km/h}}$

2/124



$\dot{v} = a_t = 9.66 \sin 1.50° - 12.90 = \underline{-12.65 \text{ m/s}^2}$

$a_n = g \cos\theta = 9.66 \cos 1.5° = 9.657 \text{ m/s}^2$

$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{(15\,450/3.6)^2}{9.657}$

$\underline{\rho = 1907 \text{ km}}$

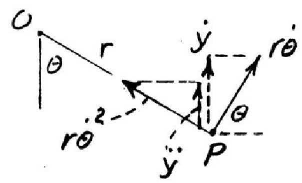
$\underline{2/125}$ (a) $\dot\theta = \omega$, $\ddot\theta = 0$

$\dot y = r\dot\theta \sin\theta$

$= \underline{r\omega \sin\theta}$

$\ddot y = r\dot\theta^2 \cos\theta$

$= \underline{r\omega^2 \cos\theta}$



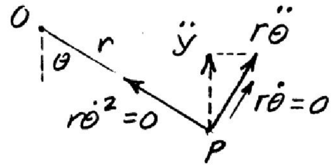
(b) $\dot\theta = 0$, $\ddot\theta = \alpha$

$\dot y = r\dot\theta \sin\theta$

$\underline{= 0}$

$\ddot y = r\ddot\theta \sin\theta$

$= \underline{r\alpha \sin\theta}$



$\boxed{2/126}$ $a_n = 0.8g = \frac{v^2}{\rho} \Rightarrow v = \sqrt{0.8g\rho}$

Car A : $v_A = \sqrt{0.8(9.81)(88)} = 26.3$ m/s

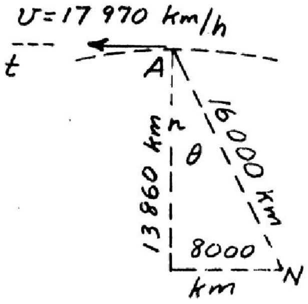
Car B : $v_B = \sqrt{(0.8)(9.81)72} = 23.8$ m/s

$$t_A = \frac{s_A}{v_A} = \frac{\pi(88)}{26.3} = \underline{10.52 \text{ s}}$$

$$t_B = \frac{s_B}{v_A} = \frac{\pi(72) + 2(16)}{23.8} = \underline{10.86 \text{ s}}$$

Car A will win the race!

2/127



Accel. directed from A to N is

$$a = g = g_0 \left(\frac{R}{R+h}\right)^2$$

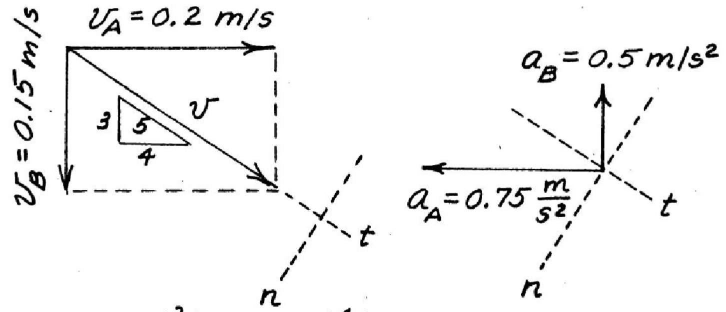
$$= 9.821\left(\frac{6371}{16000}\right)^2 = 1.557 \text{ m/s}^2$$

$$a_n = 1.557 \cos\theta = 1.557(0.866)$$

$$= 1.348 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho}, \quad \rho = \frac{(17\,970)^2}{1.348(10^{-3})(3600)^2} = \underline{18\,480 \text{ km}}$$

▶2/128 $v = \sqrt{0.15^2 + 0.2^2} = 0.25$ m/s (direction of path)



$a_n = 0.75\left(\frac{3}{5}\right) - 0.50\left(\frac{4}{5}\right) = 0.05$ m/s²

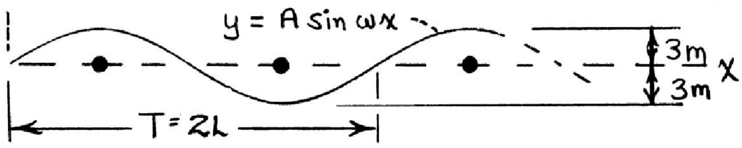
$a_n = v^2/\rho$, $\rho = \overline{0.25}^2/0.05 = \underline{1.25 \text{ m}}$

$a_t = -0.75\left(\frac{4}{5}\right) - 0.5\left(\frac{3}{5}\right) = -0.9$ m/s²

$a_t = \frac{d}{dt}(v) = \frac{d}{dt}(\rho\dot{\theta}) = \dot{\rho}\dot{\theta} + \rho\ddot{\theta} = \dot{\rho}\frac{v}{\rho} + \rho\ddot{\theta}$

so $\dot{\rho}$ cannot be found until $\ddot{\theta}$ is known

▶2/129



$y = A \sin \omega x$, where $A = 3\text{ m}$ & $\omega = \frac{2\pi}{T}$

Radius of curvature $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$\frac{dy}{dx} = A\omega \cos \omega x$, $\frac{d^2y}{dx^2} = -A\omega^2 \sin \omega x$

Set $\frac{d\rho}{dx} = 0$ to show that $|\rho|$ is a min @ $x = \frac{T}{4}$ & $x = \frac{3T}{4}$.

$$\rho_{min} = \frac{\left[1 + \left\{A \frac{2\pi}{T} \cos\left(\frac{2\pi}{T} \cdot \frac{T}{4}\right)\right\}^2\right]}{+A\left(\frac{2\pi}{T}\right)^2 \sin\left(\frac{2\pi}{T} \cdot \frac{T}{4}\right)} = \frac{T^2}{4\pi^2 A}$$

$a_n = \frac{v^2}{\rho}$: $0.7\,(9.81) = \frac{(80/3.6)^2}{T^2/(4\pi^2 \cdot 3)}$

$T = 92.3\text{ m} = 2L$, $\underline{L = 46.1\text{ m}}$

▶ **2/130**

$\dfrac{dy}{dx} = 3x^{1/2}$, $\dfrac{d^2y}{dx^2} = \dfrac{3}{2}x^{-1/2}$, $\rho = \dfrac{[1+9x]^{3/2}}{\frac{3}{2}x^{-1/2}} = \frac{2}{3}\sqrt{x(1+9x)^3}$

$$s = \int ds = \int_0^x \sqrt{1+(dy/dx)^2}\,dx = \int_0^x \sqrt{1+9x}\,dx = \frac{2}{27}\left[\sqrt{(1+9x)^3} - 1\right]$$

When $t = 1$ sec, $s = 2$ in., $2 = \frac{2}{27}\left[\sqrt{(1+9x)^3} - 1\right]$, $x = 0.913$ in.

and $\rho = \frac{2}{3}\sqrt{0.913(1 + 9\times 0.913)^3} = 17.84$ in.

$v = \dot{s} = 6t^2 = 6(1^2) = 6$ in./sec, $a_n = \dfrac{v^2}{\rho} = \dfrac{6^2}{17.84} = 2.02$ in./sec^2

$a_t = \ddot{s} = 12t = 12(1) = 12$ in./sec^2

$a = \sqrt{a_n^2 + a_t^2} = \sqrt{(2.02)^2 + 12^2} = \underline{12.17 \text{ in./sec}^2}$

$\boxed{2/131}$ $\underline{v} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta = 1.5\,\underline{e}_r + (24+7)\left(5\frac{\pi}{180}\right)\underline{e}_\theta$

$$= \underline{1.5\,\underline{e}_r + 2.71\,\underline{e}_\theta \quad \text{ft/sec}}$$

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

$$= \left[-4 - 31\left(5\frac{\pi}{180}\right)^2\right]\underline{e}_r + \left[31\left(2\frac{\pi}{180}\right) + 2(1.5)\left(5\frac{\pi}{180}\right)\right]\underline{e}_\theta$$

$$= \underline{-4.24\,\underline{e}_r + 1.344\,\underline{e}_\theta \quad \text{ft/sec}^2}$$

2/132

Position	r	$\dot{r}$	$\ddot{r}$	θ	$\dot{\theta}$	$\ddot{\theta}$
A	+	−	+	+	+	+
B	+	0	+	+	+	0
C	+	+	+	+	+	−

Notes :
(1) $r \geq 0$, always , by definition
(2) $\dot{r}$ determined by inspection
(3) $\ddot{r}$ found from $a_r = \ddot{r} - r\dot{\theta}^2 = 0$
(4) $\theta \geq 0$, by definition in figure
(5) $\dot{\theta} > 0$ here , by inspection
(6) $\ddot{\theta}$ found from $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

2/133 $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 200(8^2) = -12\,800 \text{ mm/s}^2$

or $a_r = \underline{-12.80 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 200(-20) + 2(-300)(8) = -8800 \text{ m/s}^2$

or $a_\theta = \underline{-8.80 \text{ m/s}^2}$

2/134

(A) $\underline{v}_A = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = \underline{v\underline{e}_r + l\Omega\underline{e}_\theta}$

$\underline{a}_A = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

$= \underline{-l\Omega^2\underline{e}_r + 2v\Omega\underline{e}_\theta}$

(B) $\underline{v}_B = \underline{4v\underline{e}_r + 2l\Omega\underline{e}_\theta}$

$\underline{a}_B = \underline{-2l\Omega^2\underline{e}_r + 8v\Omega\underline{e}_\theta}$

2/135 $r = 375 + 125 = 500\ mm$, $\dot{r} = \dot{l} = -150\ \frac{mm}{s}$

$\ddot{r} = 0$, $\dot{\theta} = 60\left(\frac{\pi}{180}\right) = \frac{\pi}{3}\ rad/s$, $\ddot{\theta} = 0$

$v_r = \dot{r} = -150\ \frac{mm}{s}$, $v_\theta = r\dot{\theta} = 500\left(\frac{\pi}{3}\right) = 524\ \frac{mm}{s}$

$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-150)^2 + (524)^2} = \underline{545\ \frac{mm}{s}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 500\left(\frac{\pi}{3}\right)^2 = -548\ mm/s^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-150)(\pi/3) = -314\ mm/s^2$

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-548)^2 + (-314)^2} = \underline{632\ mm/s^2}$

2/136 From $\underline{a} = [\ddot{r} - r\dot{\theta}^2]\underline{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\underline{e}_\theta$

we have, for $\ddot{r} = \ddot{\theta} = 0$, $\dot{\theta} = \Omega$, and $r = l$:

$$a = \sqrt{(l\Omega^2)^2 + (2\dot{l}\Omega)^2} = \Omega\sqrt{l^2\Omega^2 + 4\dot{l}^2}$$

$$0.011 = 0.05\sqrt{[4.2(0.05)]^2 + 4\dot{l}^2}$$

Solve for $\dot{l}$: $\underline{\dot{l} = 0.0328 \text{ m/s}}$

or $\underline{\dot{l} = 32.8 \text{ mm/s}}$

2/137 $r = t^3/3$, $\dot{r} = t^2$, $\ddot{r} = 2t$

$$\theta = 2\cos\frac{\pi t}{6}, \quad \dot{\theta} = -\frac{\pi}{3}\sin\frac{\pi t}{6}, \quad \ddot{\theta} = -\frac{\pi^2}{18}\cos\frac{\pi t}{6}$$

For $t = 2$ s, $r = 8/3$ m, $\dot{r} = 4$ m/s, $\ddot{r} = 4$ m/s^2

$$\dot{\theta} = -\frac{\pi}{3}\sin\frac{\pi}{3} = -\frac{\pi}{2\sqrt{3}} = -0.907 \text{ rad/s}$$

$$\ddot{\theta} = -\frac{\pi^2}{18}\cos\frac{\pi}{3} = -\frac{\pi^2}{36} = -0.274 \text{ rad/s}^2$$

$v_r = \dot{r} = 4$ m/s, $v_\theta = r\dot{\theta} = (8/3)(-0.907) = -2.42$ m/s

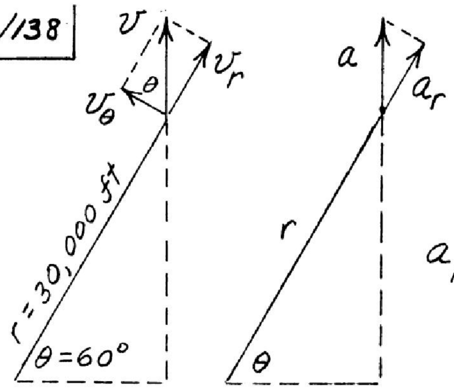
$a_r = \ddot{r} - r\dot{\theta}^2 = 4 - (8/3)(-0.907)^2 = 4 - 2.19 = 1.807$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (8/3)(-0.274) + 2(4)(-0.907) = -7.99$ m/s^2

Thus $\quad \underline{\mathbf{v} = 4\mathbf{e}_r - 2.42\mathbf{e}_\theta \text{ m/s}}$

$\quad \underline{\mathbf{a} = 1.807\mathbf{e}_r - 7.99\mathbf{e}_\theta \text{ m/s}^2}$

2/138



$v_\theta = r\dot{\theta} = 30(10^3)(0.020)$
$= 600$ ft/sec

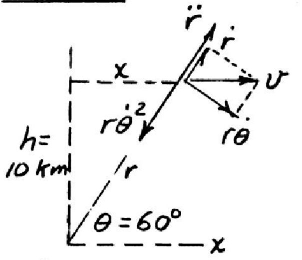
$v = v_\theta / \cos 60°$
$= 600/0.5 = \underline{1200 \text{ ft/sec}}$

$a_r = \ddot{r} - r\dot{\theta}^2$
$= 70 - 30(10^3)(0.020)^2$
$= 58$ ft/sec^2

$a = a_r / \sin 60°$
$= 58/0.866 = \underline{67.0 \text{ ft/sec}^2}$

2/139 $a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = \frac{1}{1/3}\left(-\frac{20-12}{7-4}\right) = -3\,(8/3) = \underline{-8\ m/s^2}$

2/140 Acceleration in all directions is zero, so



$\dot{\theta} = -0.020$ rad/s

$a_r = \ddot{r} - r\dot{\theta}^2 = 0, \quad \ddot{r} = r\dot{\theta}^2$

$r = h/\sin\theta, \quad r = \dfrac{10}{\sqrt{3}/2} = 11.55$ km

$\ddot{r} = 11.55\,(-0.020)^2 = 0.00462$ km/s^2

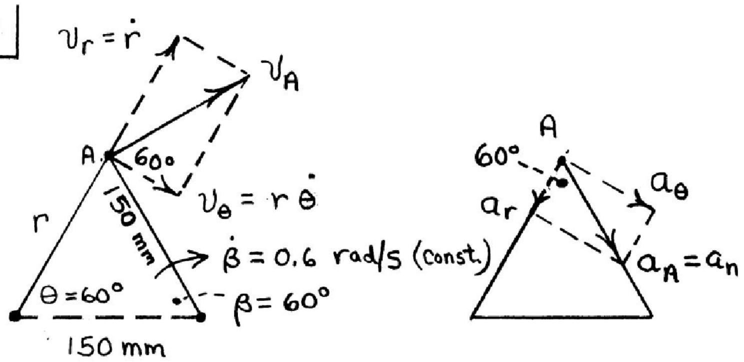
$= \underline{4.62 \text{ m/s}^2}$

$v = |r\dot{\theta}|/\sin\theta = h\dot{\theta}/\sin^2\theta$

$= \dfrac{|10(-0.020)|}{(\sqrt{3}/2)^2} = 0.267$ km/s

or $v = 0.267\,(3600) = \underline{960 \text{ km/h}}$

2/141



For $\beta = 60°$, $\theta = 60°$, $r = 150$ mm

$v_A = 150(0.6) = 90$ mm/s

$v_\theta = r\dot{\theta} = -v_A \cos 60°$, $\dot{\theta} = \dfrac{-90 \cos 60°}{150} = \underline{-0.3 \ \dfrac{\text{rad}}{\text{s}}}$

$v_r = \dot{r} = v_A \sin 60° = 90 \sin 60° = \underline{77.9 \text{ mm/s}}$

$a_A = a_n = 150(0.6)^2 = 54 \text{ mm/s}^2$

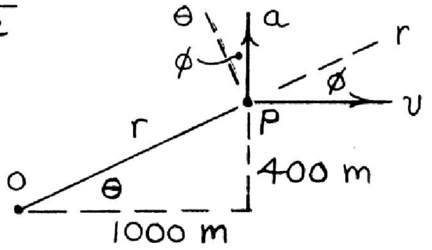
$a_r = \ddot{r} - r\dot{\theta}^2 \ : \ -54 \cos 60° = \ddot{r} - 150(-0.3)^2$

$\underline{\ddot{r} = -13.5 \text{ mm/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : -54 \sin 60° = 150\ddot{\theta} + 2(77.9)(-0.3)$

$\underline{\ddot{\theta} = 0}$

$\boxed{2/142}$ $r = \sqrt{1000^2 + 400^2}$



$= 1077 \text{ m}$

$\theta = \tan^{-1}\frac{400}{1200} = 21.8°$

$v = \frac{600}{3.6} = 166.7 \text{ m/s}$

$a = a_n = \frac{v^2}{\rho} = \frac{166.7^2}{1200} = 23.1 \text{ m/s}^2$

$v_r = \dot{r} = v\cos\theta = 166.7\cos 21.8° = 154.7 \text{ m/s}$

$v_\theta = r\dot{\theta}$: $-166.7\sin 21.8° = 1077\dot{\theta}$, $\dot{\theta} = -0.0575 \frac{\text{rad}}{\text{s}}$

$a_r = \ddot{r} - r\dot{\theta}^2$: $23.1\sin 21.8° = \ddot{r} - 1077(-0.0575)^2$

$\underline{\ddot{r} = 12.15 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$: $23.1\cos 21.8° = 1077\ddot{\theta} + 2(154.7)(-0.0575)$

$\underline{\ddot{\theta} = 0.0365 \text{ rad/s}^2}$

2/143 $a_r = \ddot{r} - r\dot{\theta}^2$ where for $\theta = 0$, $v = r\dot{\theta}$

Also for $\theta = 0$, $a_r = -a_n$ where $(a_n = v^2/\rho)$

So $\ddot{r} - r(v/r)^2 = -v^2/\rho$, $\ddot{r} = v^2\left(\frac{1}{r} - \frac{1}{\rho}\right) = \underline{-v^2\left(\frac{1}{\rho} - \frac{1}{r}\right)}$

2/144 $\theta = 0.4 + 0.12t + 0.06t^3$ | $r = 0.8 - 0.1t - 0.05t^2$

$\dot{\theta} = 0.12 + 0.18t^2$ | $\dot{r} = -0.1 - 0.1t$

$\ddot{\theta} = 0.36t$ | $\ddot{r} = -0.1$

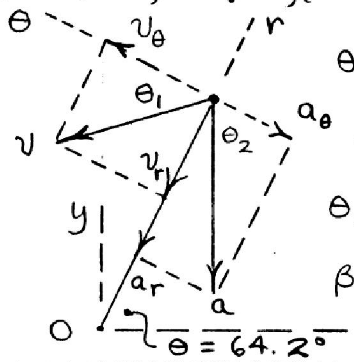
At $t = 2$ s: $\begin{cases} \theta = 1.12 \text{ rad} & r = 0.4 \text{ m} \\ \dot{\theta} = 0.84 \text{ rad/s} & \dot{r} = -0.3 \text{ m/s} \\ \ddot{\theta} = 0.72 \text{ rad/s}^2 & \ddot{r} = -0.1 \text{ m/s}^2 \end{cases}$

$$\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = -0.3\underline{e}_r + 0.4(0.84)\underline{e}_\theta$$

$$= \underline{-0.3\underline{e}_r + 0.336\underline{e}_\theta \text{ m/s}}$$

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta = [-0.1 - 0.4(0.84)^2]\underline{e}_r$$

$$+ [0.4(0.72) + 2(-0.3)(0.84)]\underline{e}_\theta = \underline{-0.382\underline{e}_r - 0.216\underline{e}_\theta \text{ m/s}^2}$$



$\theta_1 = \tan^{-1}\dfrac{|v_r|}{|v_\theta|} = 41.8°$

$\alpha = 90° + \theta + \theta_1 = \underline{195.9°}$

$\theta_2 = \tan^{-1}\dfrac{|a_r|}{|a_\theta|} = 60.5°$

$\beta = -90° + \theta - \theta_2 = \underline{-86.4°}$

2/145 $\theta = 0.8t - \frac{t^2}{20}$ rad $\qquad r = 1.6 - 0.2t$ m

$\dot{\theta} = 0.8 - 0.1t$ rad/s $\qquad \dot{r} = -0.2$ m/s

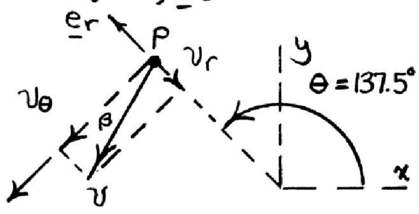
$\ddot{\theta} = -0.1$ rad/s^2 $\qquad \ddot{r} = 0$

At $t = 4$s : $\theta = 2.4$ rad $(137.5°)$, $\dot{\theta} = 0.4$ rad/s

$r = 0.8$ m

$\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = -0.2\underline{e}_r + 0.8(0.4)\underline{e}_\theta$

$= -0.2\,\underline{e}_r + 0.32\,\underline{e}_\theta$ m/s



$\underline{v = 0.377 \text{ m/s}}$

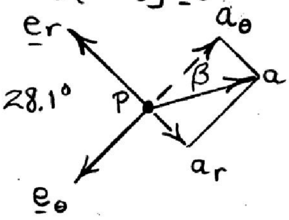
$\beta = \tan^{-1}\left(\frac{0.2}{0.32}\right) = 32.0°$

$\alpha = \theta + 90° + \beta = \underline{260°}$

$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

$= [0 - 0.8(0.4)^2]\underline{e}_r + [0.8(-0.1) + 2(-0.2)(0.4)]\underline{e}_\theta$

$= -0.1280\,\underline{e}_r - 0.240\,\underline{e}_\theta$ m/s^2



$\underline{a = 0.272 \text{ m/s}^2}$, $\beta = \tan^{-1}\frac{0.1280}{0.240} = 28.1°$

$\alpha = \theta - 90 - \beta = \underline{19.44°}$

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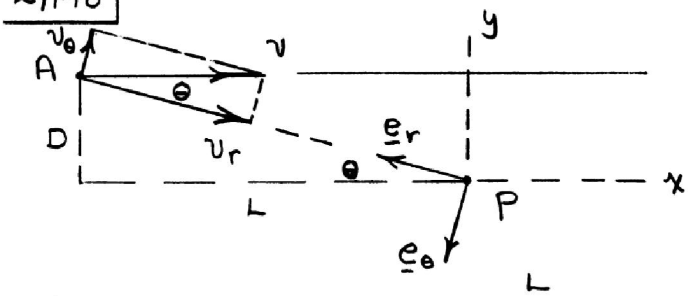

$$v' = |v_r| = v\cos\theta = v\frac{L}{\sqrt{L^2 + D^2}}$$

Numbers : $v' = 70 \dfrac{500}{\sqrt{500^2 + 20^2}} = \underline{69.9 \text{ mi/hr}}$

The factor of $\cos\theta$ is the basis for the statement that, kinematically, radar can yield an accurate or low, but not high, speed measurement. As can be seen, however, adherence to the speed limit (not reliance upon $\cos\theta$) is the best policy!

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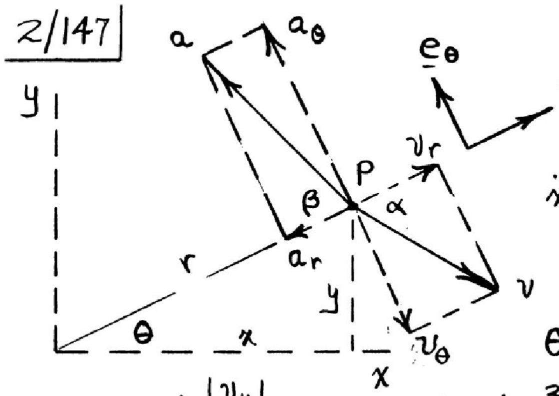

$x = 4\text{m}, \quad y = 2\text{m}$

$\dot{x} = 2\sqrt{3}\ \text{m/s}, \ \dot{y} = -2\ \frac{\text{m}}{\text{s}}$

$\ddot{x} = -5\ \text{m/s}^2, \ \ddot{y} = 5\ \frac{\text{m}}{\text{s}^2}$

$r = \sqrt{x^2+y^2} = \underline{2\sqrt{5}\ \text{m}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \underline{26.6°}$

$\alpha = \tan^{-1}\frac{|v_y|}{|v_x|} + \theta = \tan^{-1}\frac{2}{2\sqrt{3}} + 26.6° = 56.6°$

$\beta = \tan^{-1}\left|\frac{a_y}{a_x}\right| + \theta = \tan^{-1}\frac{5}{5} + 26.6° = 71.6°$

$v = \sqrt{v_y^2 + v_x^2} = \sqrt{2^2 + (2\sqrt{3})^2} = 4\ \text{m/s}$

$a = \sqrt{a_y^2 + a_x^2} = \sqrt{5^2+5^2} = 7.07\ \text{m/s}^2$

$\dot{r} = v_r = v\cos\alpha = 4\cos 56.6° = \underline{2.20\ \text{m/s}}$

$v_\theta = -v\sin\alpha = -4\sin 56.6° = -3.34\ \text{m/s}$

$v_\theta = r\dot{\theta} \ : \ -3.34 = 2\sqrt{5}\,\dot{\theta}, \quad \underline{\dot{\theta} = -0.746\ \text{rad/s}}$

$a_r = -a\cos\beta = -7.07\cos 71.6° = -2.24\ \text{m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 : \ -2.24 = \ddot{r} - 2\sqrt{5}\,(0.746)^2, \quad \underline{\ddot{r} = 0.255\ \frac{\text{m}}{\text{s}^2}}$

$a_\theta = a\sin\beta = 7.07\sin 71.6° = 6.71\ \text{m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : \ 6.71 = 2\sqrt{5}\,\ddot{\theta} + 2(2.20)(-0.746), \quad \underline{\ddot{\theta} = 2.24\ \frac{\text{rad}}{\text{s}^2}}$

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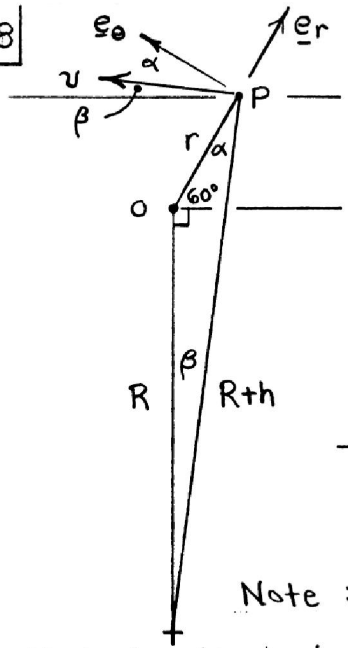


$$\frac{\sin 150°}{R+h} = \frac{\sin \alpha}{R}$$

$$\frac{\sin 150°}{3959+150} = \frac{\sin \alpha}{3959}$$

$$\alpha = 28.8°$$

$$\alpha + \beta + 150° = 180° \Rightarrow \beta = 1.200°$$

$$v_r = \dot{r} = -12{,}272 = -v \sin \alpha$$

$$-12{,}272 = -v \sin 28.8°$$

$$v = \underline{25{,}474 \text{ ft/sec}}$$

Note: Because $\underline{v}$ is nearly parallel to the horizontal at O ($\beta = 1.200°$), one obtains a close approximation ($v = 24{,}544 \frac{ft}{sec}$) to the correct answer by neglecting β (assuming a flat earth).

2/149 $r = K\theta$, $\dot{r} = K\dot{\theta}$, $\ddot{r} = K\ddot{\theta} = K\alpha$

$$\dot{\theta}^2 = 2\int_{\pi/4}^{3\pi/4} \alpha \, d\theta = 2\alpha\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) = \alpha\pi$$

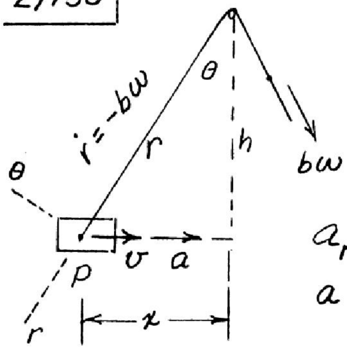
$$a_r = \ddot{r} - r\dot{\theta}^2 = K\alpha - K\theta(\alpha\pi) = K\alpha\left(1 - \frac{3\pi^2}{4}\right)$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = K\frac{3\pi}{4}\alpha + 2K\sqrt{\alpha\pi}\sqrt{\alpha\pi} = \frac{11}{4}K\alpha\pi$$

$$a = \sqrt{a_r^2 + a_\theta^2} = K\alpha\sqrt{\left(1 - \frac{3\pi^2}{4}\right)^2 + \left(\frac{11}{4}\pi\right)^2} = K\alpha\sqrt{1 + \frac{97}{16}\pi^2 + \frac{9}{16}\pi^4}$$

$$= \underline{10.76\,K\alpha}$$

2/150



$$v\sin\theta = -\dot r = b\omega \quad , \quad \ddot r = 0$$

$$\underline{v = b\omega/\sin\theta}$$

$$v_\theta = r\dot\theta = -v\cos\theta = -b\omega\,\mathrm{ctn}\,\theta$$

$$\dot\theta = -\frac{b\omega}{r}\,\mathrm{ctn}\,\theta$$

$$a_r = \ddot r - r\dot\theta^2 = -a\sin\theta$$

$$a = \frac{1}{\sin\theta}\left(r\dot\theta^2 - \ddot r\right) = \underline{\frac{b^2\omega^2}{h}\,\mathrm{ctn}^3\theta}$$

Alternatively:

$$x^2 + h^2 = r^2, \quad 2x\dot x + 0 = 2r\dot r$$

$$v = -\dot x = -\frac{r\dot r}{x} = \frac{b\omega}{\sin\theta} \quad \text{(check)}$$

$$x\ddot x + \dot x^2 = \dot r^2 + 0$$

$$a = -\ddot x = \frac{1}{x}\left(\dot x^2 - \dot r^2\right) = \frac{1}{h\tan\theta}\left(\frac{b^2\omega^2}{\sin^2\theta} - b^2\omega^2\right)$$

$$= \frac{b^2\omega^2}{h}\left[\frac{1}{\tan\theta}\,\frac{\cos^2\theta}{\sin^2\theta}\right] = \frac{b^2\omega^2}{h}\,\mathrm{ctn}^3\theta \quad \text{(check)}$$

2/151 $r = r_0 + b_0 \sin 2\pi nt, \quad \dot{r} = 2\pi n b_0 \cos 2\pi nt$

$$\ddot{r} = -4\pi^2 b_0 n^2 \sin 2\pi nt$$

$$\dot{\theta} = \omega, \quad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \quad a_r = -4\pi^2 b_0 n^2 \sin 2\pi nt - (r_0 + b_0 \sin 2\pi nt)\omega^2$$

$$= -(4\pi^2 n^2 + \omega^2) b_0 \sin 2\pi nt - r_0\omega^2$$

$$|a_r|_{max} = \underline{(4\pi^2 n^2 + \omega^2) b_0 + r_0 \omega^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}, \quad a_\theta = 0 + 4\pi b_0 n\omega \cos 2\pi nt$$

$$|a_\theta|_{max} = \underline{4\pi b_0 n \omega}$$

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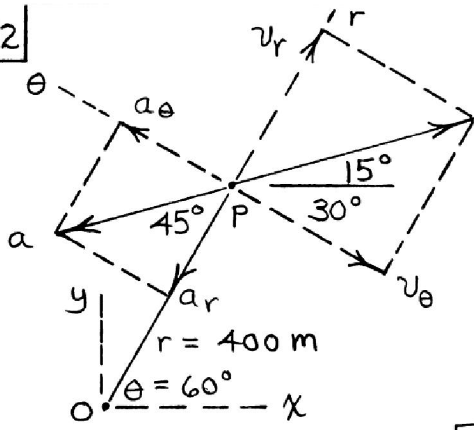


$v = \frac{90}{3.6} = 25$ m/s

$a = 0.5$ m/s^2

$v_r = \dot{r} = v \sin 45° = 25 \frac{\sqrt{2}}{2} = \underline{17.68 \text{ m/s}}$

$v_\theta = r\dot{\theta}$: $-25 \cos 45° = 400\dot{\theta}$, $\underline{\dot{\theta} = -0.0442 \text{ rad/s}}$

$a_r = \ddot{r} - r\dot{\theta}^2$: $-0.5 \cos 45° = \ddot{r} - 400(-0.0442)^2$

$\underline{\ddot{r} = 0.428 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$: $0.5 \sin 45° = 400\ddot{\theta} + 2(17.68)(-0.0442)$

$\underline{\ddot{\theta} = 0.00479 \text{ rad/s}^2}$

2/153

$$\begin{cases} r = 0.75 + 0.5 = 1.25\text{ m} \\ \dot{r} = 0.2\text{ m/s} \\ \ddot{r} = -0.3\text{ m/s}^2 \end{cases} \qquad \begin{aligned} \theta &= 30° \\ \dot{\theta} &= 0.1745\ \tfrac{\text{rad}}{\text{s}} \\ \ddot{\theta} &= 0 \end{aligned}$$

$$\underline{v} = v_r \underline{e}_r + v_\theta \underline{e}_\theta = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$$

$$= 0.2\underline{e}_r + 1.25(0.1745)\underline{e}_\theta = 0.2\underline{e}_r + 0.218\underline{e}_\theta\ \frac{\text{m}}{\text{s}}$$

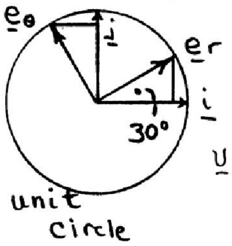
$$v = \sqrt{v_r^2 + v_\theta^2} = \underline{0.296\text{ m/s}}$$

$$\underline{a} = a_r\underline{e}_r + a_\theta\underline{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

$$= [-0.3 - 1.25(0.1745)^2]\underline{e}_r + [1.25(0) + 2(0.2)(0.1745)]\underline{e}_\theta$$

$$= -0.338\underline{e}_r + 0.0698\underline{e}_\theta\ \text{m/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \underline{0.345\text{ m/s}^2}$$



$$\underline{e}_r = \underline{i}\cos 30° + \underline{j}\sin 30°$$

$$\underline{e}_\theta = -\underline{i}\sin 30° + \underline{j}\cos 30°$$

$$\underline{v} = 0.2[\underline{i}\,c30° + \underline{j}\,s30°] + 0.218[-\underline{i}\,s30° + \underline{j}\,c30°]$$

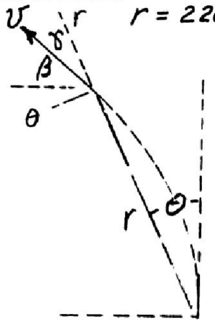
$$= \underline{0.064\underline{i} + 0.289\underline{j}\ \text{m/s}}$$

$$\underline{a} = -0.338[\underline{i}\,c30° + \underline{j}\,s30°] + 0.0698[-\underline{i}\,s30° + \underline{j}\,c30°]$$

$$= \underline{-0.328\underline{i} - 0.1086\underline{j}\ \text{m/s}^2}$$

2/154 $\theta = 22^\circ$, $\dot\theta = 0.0788$ rad/s, $\ddot\theta = -0.0341$ rad/s^2

$r = 2200$ m, $\dot r = 500$ m/s, $\ddot r = 4.66$ m/s^2



$v_r = \dot r$, $v_\theta = r\dot\theta = 2200(0.0788) = 173.4$ m/s

$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(500)^2 + (173.4)^2} = \underline{529 \text{ m/s}}$

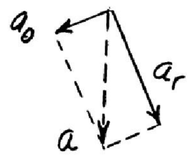
$\gamma = \tan^{-1} v_\theta / v_r = \tan^{-1}\dfrac{173.4}{500} = 19.12^\circ$

$\beta = 90 - \gamma - \theta = 90 - 19.12 - 22 = \underline{48.9^\circ}$

$a_r = \ddot r - r\dot\theta^2 = 4.66 - 2200(0.0788)^2 = -9.00$ m/s^2

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 2200(-0.0341) + 2(500)(0.0788) = 3.78$ m/s^2

$a = \sqrt{a_\theta^2 + a_r^2} = \sqrt{(3.78)^2 + (9.00)^2} = \underline{9.76 \text{ m/s}^2}$



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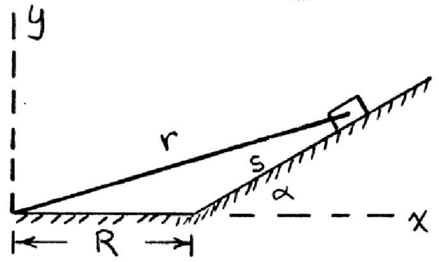

$$x = R + s\cos\alpha = R + (s_0 + v_0 t + \tfrac{1}{2}at^2)\cos\alpha$$

$$= R + \tfrac{1}{2}at^2\cos\alpha$$

$$y = s\sin\alpha$$

$$= \tfrac{1}{2}at^2\sin\alpha$$

$$r = \sqrt{x^2+y^2} = \sqrt{(R + \tfrac{1}{2}at^2\cos\alpha)^2 + (\tfrac{1}{2}at^2\sin\alpha)^2}$$

$$= \sqrt{R^2 + Rat^2\cos\alpha + \tfrac{1}{4}a^2t^4}$$

$$\dot{r} = \tfrac{1}{2}(R^2 + Rat^2\cos\alpha + \tfrac{1}{4}a^2t^4)^{-1/2}\left[2Rat\cos\alpha + a^2t^3\right]$$

$$= \frac{\tfrac{1}{2}at(2R\cos\alpha + at^2)}{\sqrt{R^2 + Rat^2\cos\alpha + \tfrac{1}{4}a^2t^4}}$$

2/156 From the solution to Prob. 2/155:

$$\begin{cases} x = R + \frac{1}{2}at^2 \cos\alpha \\ y = \frac{1}{2}at^2 \sin\alpha \end{cases}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left[\frac{\frac{1}{2}at^2\sin\alpha}{R + \frac{1}{2}at^2\cos\alpha}\right]$$

$$\dot{\theta} = \frac{\dfrac{(R+\frac{1}{2}at^2\cos\alpha)(at\sin\alpha) - (\frac{1}{2}at^2\sin\alpha)(at\cos\alpha)}{(R+\frac{1}{2}at^2\cos\alpha)^2}}{1 + \left[\dfrac{\frac{1}{2}at^2\sin\alpha}{R+\frac{1}{2}at^2\cos\alpha}\right]^2}$$

Simplify to

$$\underline{\dot{\theta} = \frac{Rat\sin\alpha}{R^2 + Rat^2\cos\alpha + \frac{1}{4}a^2t^4}}$$

2/157

$r = 1.6 + 0.3\sin\frac{\pi t}{2}$ m ; $\theta = \frac{\pi}{4} + \frac{\pi}{8}\sin\frac{\pi t}{2}$

$\dot{r} = \frac{0.3\pi}{2}\cos\frac{\pi t}{2}$ m/s ; $\dot{\theta} = \frac{\pi^2}{16}\cos\frac{\pi t}{2}$ rad/s

$\ddot{r} = -\frac{0.3\pi^2}{4}\sin\frac{\pi t}{2}$ m/s^2 ; $\ddot{\theta} = -\frac{\pi^3}{32}\sin\frac{\pi t}{2}$ rad/s^2

$$v_r = \dot{r} = \frac{0.3\pi}{2}\cos\frac{\pi t}{2}$$

$$v_\theta = r\dot{\theta} = \left(1.6+0.3\sin\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{0.3\pi^2}{4}\sin\frac{\pi t}{2} - \left(1.6+0.3\sin\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \left(1.6+0.3\sin\frac{\pi t}{2}\right)\left(-\frac{\pi^3}{32}\sin\frac{\pi t}{2}\right) + 2\left(\frac{0.3\pi}{2}\cos\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)$$

At $t = 1$ s : $\left.\begin{aligned} v_r &= 0 \\ v_\theta &= 0 \end{aligned}\right\}$ $v = \sqrt{v_r^2 + v_\theta^2} = \underline{0}$

$\left.\begin{aligned} a_r &= -0.740 \text{ m/s}^2 \\ a_\theta &= -1.841 \text{ m/s}^2 \end{aligned}\right\}$ $a = \sqrt{0.740^2 + 1.841^2} = \underline{1.984 \text{ m/s}^2}$

At $t = 2$ s : $\left.\begin{aligned} v_r &= -0.471 \text{ m/s} \\ v_\theta &= 0.987 \text{ m/s} \end{aligned}\right\}$ $\underline{v = 1.094 \text{ m/s}}$

$\left.\begin{aligned} a_r &= -0.609 \text{ m/s}^2 \\ a_\theta &= 0.581 \text{ m/s}^2 \end{aligned}\right\}$ $\underline{a = 0.842 \text{ m/s}^2}$

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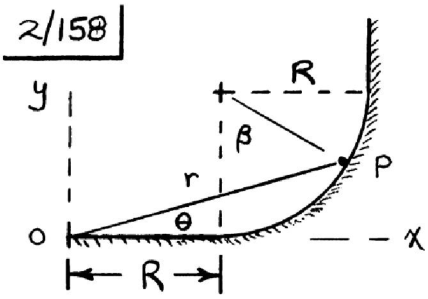


$s = R + R\beta = vt$:

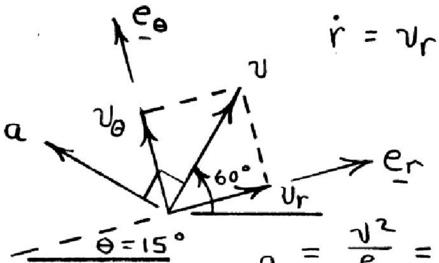
$1.2 + 1.2\beta = 0.6\left(2\left(1+\frac{\pi}{3}\right)\right)$

$\beta = \frac{\pi}{3}$ or $60°$

$x = R + R\sin\beta = 1.2 + 1.2\sin 60° = 2.24$ m

$y = R - R\cos\beta = 1.2 - 1.2\cos 60° = 0.6$ m

$r = \sqrt{x^2+y^2} = \underline{2.32\text{ m}}$, $\theta = \tan^{-1}\frac{y}{x} = \underline{15°}$



$\dot{r} = v_r = v\cos 45° = 0.6\frac{\sqrt{2}}{2} = \underline{0.424\frac{m}{s}}$

$v_\theta = r\dot{\theta} = 2.32\dot{\theta} = 0.6\frac{\sqrt{2}}{2}$

$\underline{\dot{\theta} = 0.1830 \text{ rad/s}}$

$a = \frac{v^2}{\rho} = \frac{0.6^2}{1.2} = 0.3 \text{ m/s}^2$

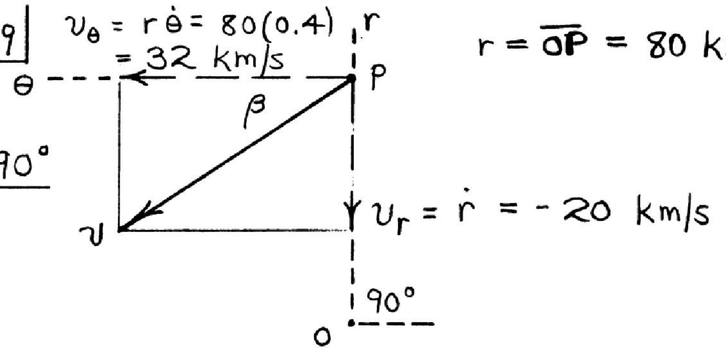
$a_r = -a\frac{\sqrt{2}}{2} = \ddot{r} - r\dot{\theta}^2 : -0.3\frac{\sqrt{2}}{2} = \ddot{r} - 2.32(0.1830)^2$

$\underline{\ddot{r} = -0.1345 \text{ m/s}^2}$

$a_\theta = a\frac{\sqrt{2}}{2} = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 0.3\frac{\sqrt{2}}{2} = 2.32\ddot{\theta} + 2(0.424) \times (0.1830)$

$\underline{\ddot{\theta} = 0.025 \text{ rad/s}^2}$

2/159



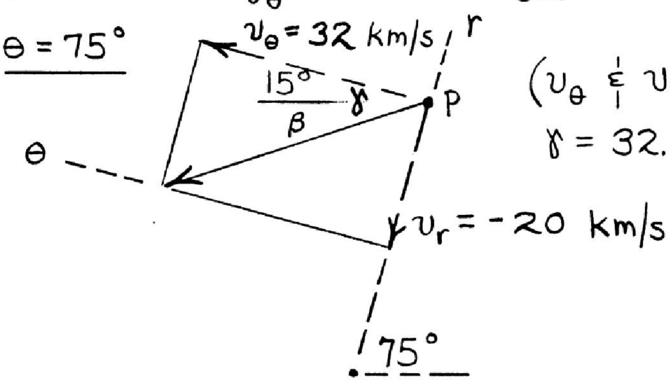
$r = \overline{OP} = 80$ km

$\underline{\theta = 90°}$

$$v = \sqrt{v_\theta^2 + v_r^2} = \sqrt{32^2 + 20^2} = \underline{37.7 \text{ km/s}}$$

$$\beta = \tan^{-1}\frac{|v_r|}{v_\theta} = \tan^{-1}\frac{20}{32} = \underline{32.0°}$$

$\underline{\theta = 75°}$



(v_θ & v_r unchanged)

$\gamma = 32.0°$

$\underline{v = 37.7 \text{ km/s}}$ (unchanged)

$$\beta = \gamma - 15° = 32.0 - 15° = \underline{17.01°}$$

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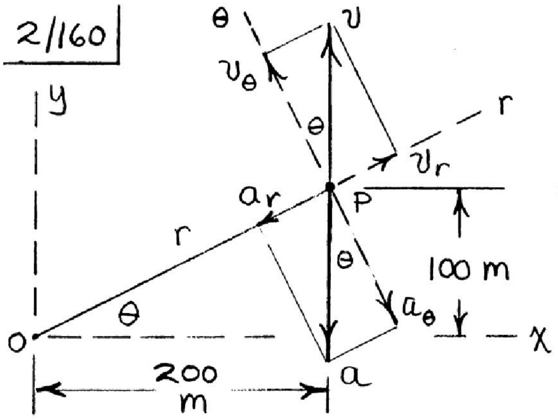


$r = \sqrt{200^2 + 100^2}$

$= \underline{224 \text{ m}}$

$\theta = \tan^{-1} \frac{100}{200}$

$= \underline{26.6^\circ}$

$v_r = \dot{r} = v \sin\theta = 15 \sin 26.6^\circ = \underline{6.71 \text{ m/s}}$

$v_\theta = r\dot{\theta}$: $15 \cos 26.6^\circ = 224\dot{\theta}$, $\dot{\theta} = \underline{0.06 \text{ rad/s}}$

$a = -g - kv^2 = -9.81 - 0.01(15)^2 = -12.06 \text{ m/s}^2$

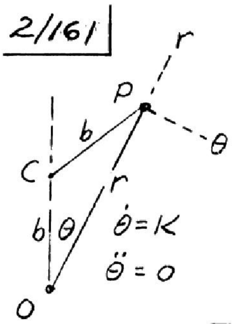
$a_r = \ddot{r} - r\dot{\theta}^2$: $-12.06 \sin 26.6^\circ = \ddot{r} - 224(0.06)^2$

$\underline{\ddot{r} = -4.59 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$: $-12.06 \cos 26.6^\circ = 224\ddot{\theta} + 2(6.71)(0.06)$

$\underline{\ddot{\theta} = -0.0518 \text{ rad/s}^2}$

2/161



$r = 2b\cos\theta$, $\dot{r} = -2b\dot{\theta}\sin\theta = -2bK\sin\theta$

$\ddot{r} = -2bK^2\cos\theta$

$a_r = \ddot{r} - r\dot{\theta}^2 = -2bK^2\cos\theta - 2b\cos\theta\,(K^2)$

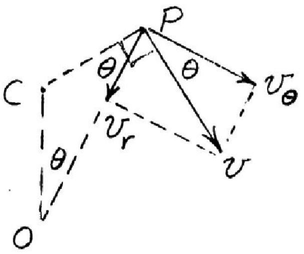
$= -4bK^2\cos\theta$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-2bK\sin\theta)K$

$= -4bK^2\sin\theta$

$$a = \sqrt{a_r^2 + a_\theta^2} = 4bK^2\sqrt{\sin^2\theta + \cos^2\theta} = \underline{4bK^2}$$

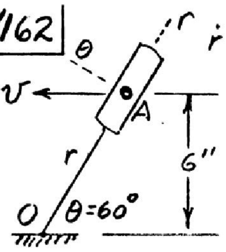
$v_r = \dot{r} = -2bK\sin\theta$, $v_\theta = r\dot{\theta} = 2bK\cos\theta$



$v = \sqrt{v_r^2 + v_\theta^2} = \underline{2bK \text{ const.}}$

(hence acceleration is directed from P to C and is $a_n = v^2/b$)

$\boxed{2/162}$



$$\dot{r} = v_r = -v\cos\theta = -3\cos 60^\circ = \underline{-1.5 \text{ ft/sec}}$$

$$r\dot{\theta} = v\sin\theta = 3\sin 60^\circ = 2.60 \text{ ft/sec}$$

$$\dot{\theta} = \frac{2.60}{\frac{6}{12}/\sin 60^\circ} = 2.6\sqrt{3} = \underline{4.50 \text{ rad/sec}}$$

accel. $= 0$ so $a_r = \ddot{r} - r\dot{\theta}^2 = 0$, $\ddot{r} = r\dot{\theta}^2 = \frac{1}{\sqrt{3}}(4.50)^2 = \underline{11.69 \frac{\text{ft}}{\text{sec}^2}}$

& $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$, $\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} = -\frac{2(-1.5)(4.50)}{1/\sqrt{3}} = \underline{23.38 \frac{\text{rad}}{\text{sec}^2}}$

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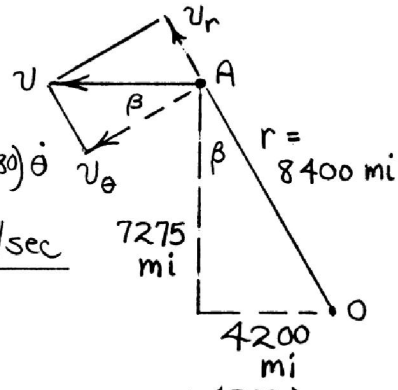
$$v = 12{,}149\left(\frac{5280}{3600}\right) = 17{,}819\ \frac{ft}{sec}$$



$$v_\theta = r\dot\theta : \quad 17{,}819\cos 30^\circ = 8400(5280)\dot\theta$$

$$\underline{\dot\theta = 3.48(10^{-4})\ \text{rad/sec}}$$

$$v_r = \dot r : \quad 17{,}819 \sin 30^\circ = \dot r$$

$$\underline{\dot r = 8910\ \frac{ft}{sec}}$$

$$\beta = \tan^{-1}\left(\frac{4200}{7275}\right) = 30^\circ$$

$$a_\theta = r\ddot\theta + 2\dot r\dot\theta : \quad 0 = 8400(5280)\ddot\theta + 2(8910)(3.48)(10^{-4})$$

$$\underline{\ddot\theta = -1.398(10^{-7})\ \text{rad/sec}^2}$$

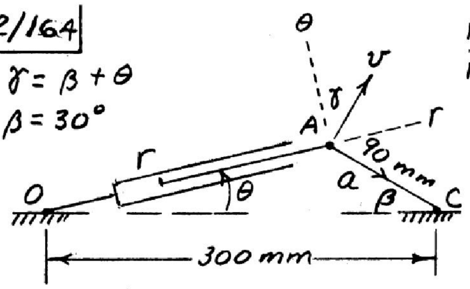
$$a_r = \ddot r - r\dot\theta^2 : \quad -7.159 = \ddot r - 8400(5280)(3.48\times10^{-4})^2$$

$$\underline{\ddot r = -1.790\ \text{ft/sec}^2}$$

2/164

$\gamma = \beta + \theta$

$\beta = 30°$



$r\cos\theta + 90\cos 30° = 300$

$r\sin\theta = 90\sin 30°$

solve & get $r = 226.6$ mm

$\theta = 11.46°$

$\gamma = 30 + 11.46 = 41.46°$

$v = \overline{AC}\,\dot\beta = 90(60) = 5400$ mm/s or 5.40 m/s

$a = a_n = \overline{AC}\,\dot\beta^2 = 90(60)^2 = 324(10^3)$ mm/s^2 or 324 m/s^2

$\cdot \quad a_t = \overline{AC}\,\ddot\beta = 0$

$v_r = \dot r = v\sin\gamma = 5.40\sin 41.46° = \underline{3.58 \text{ m/s}}$

$v_\theta = r\dot\theta = v\cos\gamma = 5.40\cos 41.46° = 4.05$ m/s, $\dot\theta = \dfrac{4.05}{0.2266} = \underline{17.86 \dfrac{\text{rad}}{\text{s}}}$

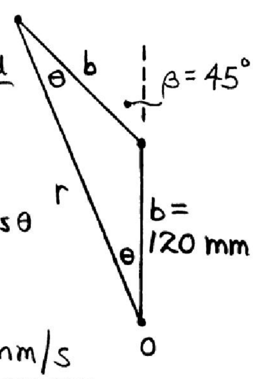
$a_r = \ddot r - r\dot\theta^2 = a\cos\gamma$, $\ddot r = 0.2266(17.86)^2 + 324\cos 41.46°$

$= 72.29 + 242.83 = \underline{315 \text{ m/s}^2}$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = -a\sin\gamma$, $\ddot\theta = \dfrac{-2\dot r\dot\theta}{r} - \dfrac{a\sin\gamma}{r}$

$= -\dfrac{2(3.58)(17.86)}{0.2266} - \dfrac{324\sin 41.46°}{0.2266} = \underline{-1510 \dfrac{\text{rad}}{\text{s}^2}}$

▶2/165 $2\theta = \beta = 45°, \quad \theta = 22.5°$



Because $\theta = \frac{\beta}{2}, \quad \dot{\theta} = \frac{\dot{\beta}}{2} = \frac{4}{2} = 2\ \frac{\text{rad}}{\text{s}}$

Also, $\ddot{\theta} = \frac{\ddot{\beta}}{2} = 0$

$r = 2b\cos\theta, \quad \dot{r} = -2b\dot{\theta}\sin\theta, \quad \ddot{r} = -2b\dot{\theta}^2\cos\theta$

$r = 2(120)\cos 22.5° = 222\ \text{mm}$

$\dot{r} = -2(120)(2)\sin 22.5° = \underline{-183.7\ \text{mm/s}}$

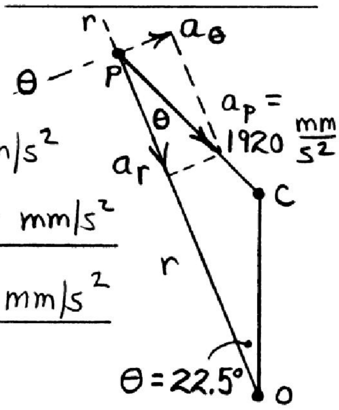
$\ddot{r} = -2(120)(2)^2\cos 22.5° = \underline{-887\ \text{mm/s}^2}$

$a_r = \ddot{r} - r\dot{\theta}^2 = -887 - 222(2)^2 = -1774\ \text{mm/s}^2$ or $\underline{-1.774\ \frac{\text{m}}{\text{s}^2}}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 222(0) + 2(-183.7)(2) = -735\ \text{mm/s}^2$

or $\underline{a_\theta = -0.735\ \text{m/s}^2}$

Second Solution:



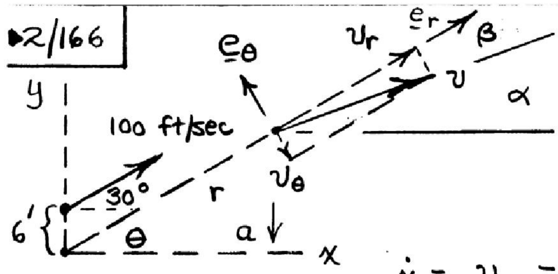
$(a_P)_t = b\ddot{\beta} = 0$

$(a_P)_n = b\dot{\beta}^2 = 120(4)^2 = 1920\ \text{mm/s}^2$

$a_r = -1920\cos 22.5° = \underline{-1774\ \text{mm/s}^2}$

$a_\theta = -1920\sin 22.5° = \underline{-735\ \text{mm/s}^2}$

▸2/166



$x = x_0 + v_{x_0} t$

$= 0 + 100 \cos 30° (0.5)$

$= 43.3$ ft

$\dot{x} = v_{x_0} = 100 \cos 30° = 86.6 \ \frac{ft}{sec}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 = 6 + 100 \sin 30° (0.5) - 16.1 (0.5)^2 = 27.0$ ft

$\dot{y} = v_{y_0} - gt = 100 \sin 30° - 32.2(0.5) = 33.9$ ft/sec

$r = \sqrt{x^2 + y^2} = \sqrt{43.3^2 + 27.0^2} = \underline{51.0 \text{ ft}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(27.0/43.3) = \underline{31.9°}$

$\alpha = \tan^{-1}(v_y/v_x) = \tan^{-1}(33.9/86.6) = 21.4°$

$\beta = \theta - \alpha = 10.54°$, $v = \sqrt{\dot{x}^2 + \dot{y}^2} = 93.0$ ft/sec

$v_r = v \cos\beta = 93.0 \cos 10.54° = \underline{91.4 \text{ ft/sec} = \dot{r}}$

$v_\theta = -v \sin\beta = -93.0 \sin 10.54° = -17.02$ ft/sec

$-17.02 = r\dot{\theta} = 51.0 \ \dot{\theta}$, $\underline{\dot{\theta} = -0.334 \text{ rad/sec}}$

$a = \ddot{y} = -32.2$ ft/sec^2

$a_r = -g \sin\theta = -32.2 \sin 31.9° = -17.03$ ft/sec^2

$-17.03 = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - 51.0(-0.334)^2$, $\underline{\ddot{r} = -11.35 \text{ ft/sec}^2}$

$a_\theta = -g \cos\theta = -32.2 \cos 31.9° = -27.3$ ft/sec^2

$-27.3 = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 51.0\ddot{\theta} + 2(91.4)(-0.334)$

$\underline{\ddot{\theta} = 0.660 \text{ rad/sec}^2}$

2/167 $\underline{v} = 4\underline{i} - 2\underline{j} - \underline{k}$ m/s, $v = \sqrt{4^2+2^2+1^2} = 4.58 \frac{m}{s}$

$a_n = a \sin 20° = 8 \sin 20° = 2.74$ m/s^2

From $a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{4.58^2}{2.74} = \underline{7.67 \text{ m}}$

$\dot{v} = a_t = a \cos 20° = 8 \cos 20° = \underline{7.52 \text{ m/s}^2}$

2/168 $v_{z_0} = 500 \sin 60° = 433$ ft/sec

$v_{xy_0} = 500 \cos 60° = 250$ ft/sec

$v_z = v_{z_0} - gt = 433 - 32.2(20) = \underline{-211 \text{ ft/sec}}$

$v_{xy} = v_{xy_0} = \text{constant} = 250$ ft/sec

$v_x = v_{xy} \cos 20° = 250 \cos 20° = \underline{235 \text{ ft/sec}}$

$v_y = v_{xy} \sin 20° = 250 \sin 20° = \underline{85.5 \text{ ft/sec}}$

$d_{xy} = v_{xy} t = 250(20) = 5000$ ft

$x = d_{xy} \cos 20° = 5000 \cos 20° = \underline{4700 \text{ ft}}$

$y = d_{xy} \sin 20° = 5000 \sin 20° = \underline{1710 \text{ ft}}$

$z = v_{z_0} t - \frac{1}{2} g t^2 = 433(20) - 16.1(20)^2 = \underline{2220 \text{ ft}}$

$\underline{a_x = a_y = 0, \quad a_z = -g = -32.2 \text{ ft/sec}^2}$

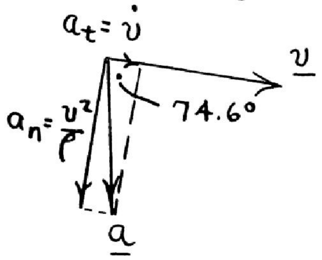
2/169 $\underline{v} = 6\underline{i} - 3\underline{j} + 2\underline{k}$ m/s , $v = \sqrt{6^2+3^2+2^2} = 7\,\frac{m}{s}$

$\underline{a} = 3\underline{i} - \underline{j} - 5\underline{k}$ m/s^2, $a = \sqrt{3^2+1^2+5^2} = \sqrt{35}\,\frac{m}{s^2}$

$\theta = \cos^{-1}\left[\frac{\underline{v}\cdot\underline{a}}{va}\right] = \cos^{-1}\left[\frac{6(3)-3(-1)-2(5)}{7\sqrt{35}}\right]$

$= \underline{74.6°}$

In osculating plane :



$\dot{v} = a\cos 74.6° = \underline{1.571 \text{ m/s}^2}$

$a_n = a\sin 74.6° = 5.70 \text{ m/s}^2$

$= \frac{v^2}{\rho}$

$\therefore \rho = \frac{v^2}{a_n} = \frac{7^2}{5.70} = \underline{8.59 \text{ m}}$

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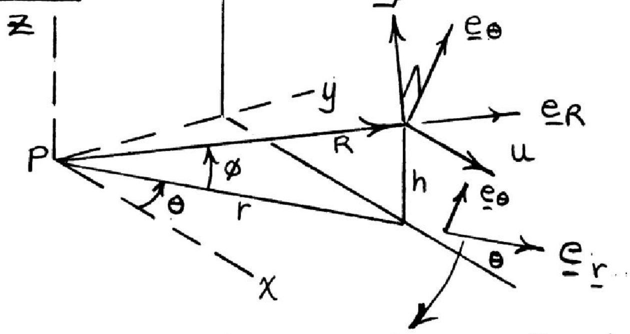


Resolve u along $\underline{e}_\theta$ & $\underline{e}_r$ directions :

$\underline{e}_\theta$: $\underline{-u\sin\theta} = v_\theta$

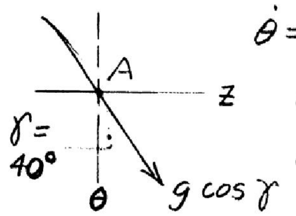
$\underline{e}_r$: $u\cos\theta = v_r$

Now resolve v_r along $\underline{e}_R$ & $\underline{e}_\phi$ directions :

$\underline{e}_R$: $v_r\cos\phi = \underline{u\cos\theta\cos\phi} = v_R$

e_ϕ : $-v_r\sin\phi = \underline{-u\cos\theta\sin\phi} = v_\phi$

2/171 $v_\theta = r\dot{\theta}$ & $v_\theta = v\cos\gamma$ so $\dot{\theta} = \frac{v}{r}\cos\gamma$



$\dot{\theta} = \frac{15}{5}(0.7660) = 2.298$ rad/s

$a_\theta = g\cos^2\gamma = 9.81(0.7660)^2 = 5.76\ m/s^2$

$a_z = g\cos\gamma\sin\gamma = 9.81(0.7660)(0.6428)$
$= 4.83\ m/s^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(2.298)^2 = -26.41\ m/s^2$

$a = \sqrt{26.41^2 + 5.76^2 + 4.83^2} = \underline{27.5\ m/s^2}$

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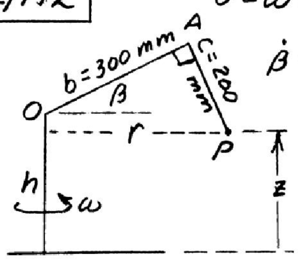

$\dot{\theta} = \omega = \frac{40}{180}\pi = 0.698 \text{ rad/s}$

$\dot{\beta} = \frac{10}{180}\pi = 0.1745 \text{ rad/s}$

$\ddot{\beta} = \frac{20}{180}\pi = 0.349 \text{ rad/s}^2$

Use cylindrical coordinates

$r = b\cos\beta + c\sin\beta$, $\dot{r} = (-b\sin\beta + c\cos\beta)\dot{\beta}$

$\ddot{r} = (-b\cos\beta - c\sin\beta)\dot{\beta}^2 + (-b\sin\beta + c\cos\beta)\ddot{\beta}$

$z = h + b\sin\beta - c\cos\beta$, $\dot{z} = (b\cos\beta + c\sin\beta)\dot{\beta}$

$\ddot{z} = (-b\sin\beta + c\cos\beta)\dot{\beta}^2 + (b\cos\beta + c\sin\beta)\ddot{\beta}$

For $\beta = 30°$, $\dot{r} = (-300\times0.5 + 200\times0.866)(0.1745) = 4.050 \text{ mm/s}$

$\ddot{r} = (-300\times0.866 - 200\times0.5)(0.1745)^2$
$+ (-300\times0.5 + 200\times0.866)(0.349) = -2.860 \text{ mm/s}^2$

$\ddot{z} = (-300\times0.5 + 200\times0.866)(0.1745)^2$
$+ (300\times0.866 + 200\times0.5)(0.349) = 126.30 \text{ mm/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -2.860 - 359.8(0.698)^2 = -178.23 \text{ mm/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(4.049)(0.698) = 5.65 \text{ mm/s}^2$

$a_z = \ddot{z} = 126.30 \text{ mm/s}^2$

$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2}$

$= \underline{218.5 \text{ mm/s}^2}$

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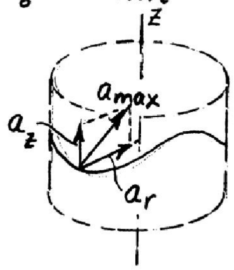
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\omega^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$$

$$a_z = \frac{d^2}{dt^2}(z_0 \sin 2\pi n t) = -4n^2\pi^2 z_0 \sin 2\pi n t$$

$$a = \sqrt{(-r\omega^2)^2 + (-4n^2\pi^2 z_0 \sin 2\pi n t)^2}$$

$$a_{max} = \sqrt{r^2\omega^4 + 16n^4\pi^4 z_0^2}$$



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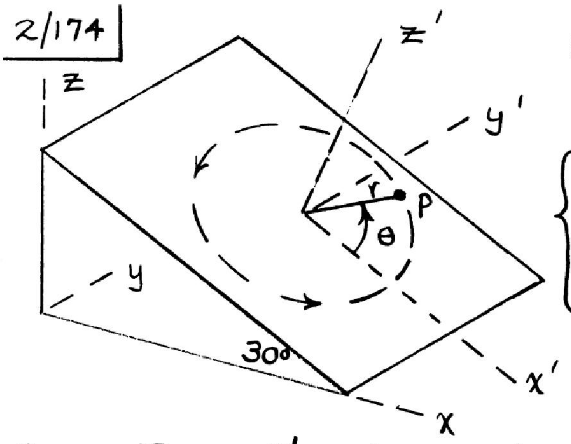

$\theta = \omega t = \frac{v}{r} t$

$\dot\theta = \frac{v}{r}$

$$\begin{cases} x' = r\cos\theta \\ y' = r\sin\theta \\ z' = 0 \end{cases} \qquad \begin{cases} \dot x' = -r\dot\theta\sin\theta \\ \dot y' = r\dot\theta\cos\theta \\ \dot z' = 0 \end{cases}$$

$\ddot x' = -r\dot\theta^2\cos\theta$, $\ddot y' = -r\dot\theta^2\sin\theta$, $\ddot z' = 0$

Component relationships:
(Similar for $\dot x$, $\ddot x$, etc.)

$$\begin{cases} x = x'\cos 30° + z'\sin 30° \\ y = y' \\ z = -x'\sin 30° + z'\cos 30° \end{cases}$$

Thus

$x = \frac{\sqrt3}{2} r\cos\frac{v}{r}t$, $y = r\sin\frac{v}{r}t$, $z = -\frac{1}{2} r\cos\frac{v}{r}t$

$\dot x = -\frac{\sqrt3}{2} v\sin\frac{v}{r}t$, $\dot y = v\cos\frac{v}{r}t$, $\dot z = \frac{1}{2} v\sin\frac{v}{r}t$

$\ddot x = -\frac{\sqrt3}{2}\frac{v^2}{r}\cos\frac{v}{r}t$, $\ddot y = -\frac{v^2}{r}\sin\frac{v}{r}t$, $\ddot z = \frac{1}{2}\frac{v^2}{r}\cos\frac{v}{r}t$

Note that position relationships do not include the constants associated with the origin positions.

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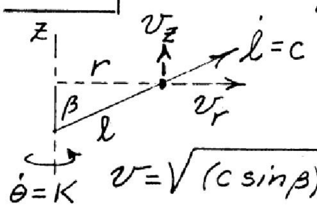

$$v_r = \dot{l}\sin\beta = c\sin\beta$$

$$v_\theta = r\dot\theta = (l\sin\beta)K = Kl\sin\beta$$

$$v_z = \dot{l}\cos\beta = c\cos\beta$$

$$v = \sqrt{(c\sin\beta)^2 + (Kl\sin\beta)^2 + (c\cos\beta)^2}$$

$$= \underline{\sqrt{c^2 + K^2 l^2 \sin^2\beta}}$$

$$a_r = \ddot{r} - r\dot\theta^2 = 0 - l\sin\beta\,(K^2) = -K^2 l\sin\beta$$

$$a_\theta = r\ddot\theta + 2\dot{r}\dot\theta = 0 + 2c\sin\beta\,(K) = 2Kc\sin\beta$$

$$a_z = \dot{v}_z = 0$$

$$a = \sqrt{(-K^2 l\sin\beta)^2 + (2Kc\sin\beta)^2} = \underline{K\sin\beta\sqrt{K^2 l^2 + 4c^2}}$$

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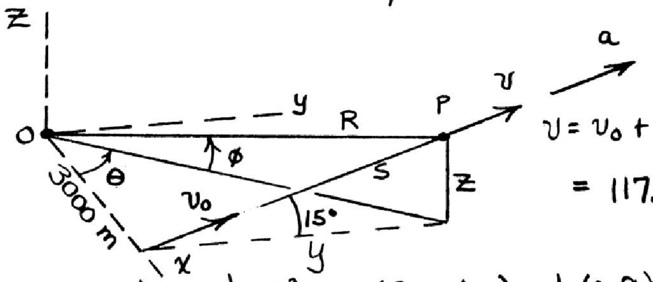
$a = 0.8\ m/s^2$, $v_0 = \frac{250}{3.6} = 69.4\ m/s^2$



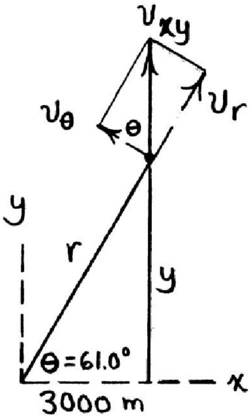
$v = v_0 + at = 69.4 + 0.8(60)$

$= 117.4\ m/s$

$s = v_0 t + \frac{1}{2}at^2 = 69.4(60) + \frac{1}{2}(0.8)(60)^2 = 5610\ m$

$y = 5610 \cos 15° = 5420\ m$

$\theta = \tan^{-1} \frac{5420}{3000} = 61.0°$



$r = \sqrt{3000^2 + 5420^2} = 6190\ m$

$v_{xy} = 117.4 \cos 15° = 113.4\ m/s$

$v_r = \dot{r} = 113.4 \sin 61.0° = \underline{99.2\ m/s}$

$v_\theta = r\dot{\theta} = 113.4 \cos 61.0°$

$\dot{\theta} = \frac{113.4 \cos 61.0°}{6190} = \underline{8.88(10^{-3})\ rad/s}$

$\dot{z} = v_z = v \sin 15° = 117.4 \sin 15° = \underline{30.4\ m/s}$

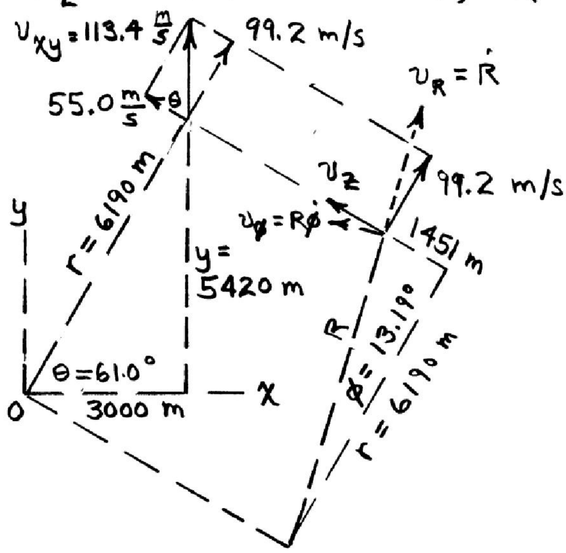
2/177 From the solution to Prob. 2/176,

$v = 117.4$ m/s, $\theta = 61.0°$, $y = 5420$ m, $r = 6190$ m

$z = y \tan 15° = 5420 \tan 15° = 1451$ m

$\phi = \tan^{-1}\frac{1451}{6190} = 13.19°$, $v_{xy} = 117.4 \cos 15° = 113.4$ m/s

$v_z = v \sin 15° = 30.4$ m/s, $R = \sqrt{1451^2 + 6190^2} = 6360$ m



$v_R = \dot{R} = 99.2 \cos 13.19° + 30.4 \sin 13.19° = \underline{103.6 \text{ m/s}}$

$v_\theta = R\dot{\theta}\cos\phi = r\dot{\theta}$, $\dot{\theta} = \frac{55.0}{6190} = \underline{8.88(10^{-3}) \text{ rad/s}}$

$v_\phi = R\dot{\phi} = 30.4 \cos 13.19° - 99.2 \sin 13.19° = 6.95$ m/s

$\dot{\phi} = \frac{6.95}{6360} = \underline{1.093\,(10^{-3}) \text{ rad/s}}$

2/178 $R = 0.75 + 0.5 = 1.25 \text{ m}$, $\dot{R} = 0.2 \text{ m/s}$, $\ddot{R} = -0.3 \frac{m}{s^2}$

$\phi = 30°$, $\dot{\phi} = 10\left(\frac{\pi}{180}\right)$ rad/s, $\ddot{\phi} = 0$, $\dot{\theta} = 20\left(\frac{\pi}{180}\right)$ rad/s, $\ddot{\theta} = 0$

$$\begin{cases} v_R = \dot{R} = 0.2 \text{ m/s} \\ v_\theta = R\dot{\theta}\cos\phi = 1.25\left(20\frac{\pi}{180}\right)\cos 30° = 0.378 \frac{m}{s} \\ v_\phi = R\dot{\phi} = 1.25\left(10\frac{\pi}{180}\right) = 0.218 \text{ m/s} \end{cases}$$

$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{0.480 \text{ m/s}}$$

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$$
$$= -0.3 - 1.25\left(10\frac{\pi}{180}\right)^2 - 1.25\left(20\frac{\pi}{180}\right)^2\cos^2 30°$$
$$= -0.4523 \text{ m/s}^2$$

$$a_\theta = \cos\phi\left[2\dot{R}\dot{\theta} + R\ddot{\theta}\right] - 2R\dot{\theta}\dot{\phi}\sin\phi$$
$$= \cos 30°\left[2(0.2)\left(20\frac{\pi}{180}\right) + 1.25(0)\right]$$
$$- 2(1.25)\left(10\frac{\pi}{180}\right)\left(20\frac{\pi}{180}\right)\sin 30° = 0.0448 \frac{m}{s^2}$$

$$a_\phi = 2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi$$
$$= 2(0.2)\left(10\frac{\pi}{180}\right) + 1.25(0) + 1.25\left(20\frac{\pi}{180}\right)^2 0.5\,\frac{\sqrt{3}}{2}$$
$$= 0.1358 \text{ m/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.474 \text{ m/s}^2}$$

2/179

$$R = (r^2+h^2)^{1/2}, \qquad \dot{R} = \frac{1}{2}\frac{2r\dot{r}}{\sqrt{r^2+h^2}} = \frac{r\dot{r}}{\sqrt{r^2+h^2}}$$

$$r = 2b\sin\frac{\beta}{2}, \qquad \dot{r} = b\dot{\beta}\cos\frac{\beta}{2}$$

$$\theta = \beta/2, \qquad \dot{\theta} = \dot{\beta}/2$$

$$u = b\dot{\beta}, \qquad \dot{\beta} = \text{constant}$$

$$v_R = \dot{R} = \frac{2b\sin\frac{\beta}{2}\, b\dot{\beta}\cos\frac{\beta}{2}}{\sqrt{4b^2\sin^2\frac{\beta}{2}+h^2}} = \frac{b^2\dot{\beta}\sin\beta}{\sqrt{4b^2\sin^2\frac{\beta}{2}+h^2}}$$

$$= \underline{\frac{bu\sin\beta}{\sqrt{4b^2\sin^2\frac{\beta}{2}+h^2}}}$$

$$v_\theta = R\dot{\theta}\cos\phi = r\dot{\theta} = 2b\sin\frac{\beta}{2}\,\frac{u}{2b} = \underline{u\sin\frac{\beta}{2}}$$

$$v_\phi = R\dot{\phi}, \quad \sin\phi = \frac{h}{R}, \quad \cos\phi\,\dot{\phi} = \frac{d}{dt}\left(\frac{h}{R}\right) = -\frac{h}{R^2}\dot{R}$$

$$\dot{\phi} = -\frac{h}{R^2}\dot{R}\,\frac{R}{r} = -\frac{h\dot{R}}{rR}$$

$$\text{So } v_\phi = \frac{-h\dot{R}}{r} = \frac{-hr\dot{r}}{\sqrt{r^2+h^2}}\,\frac{1}{r} = \frac{-hb\dot{\beta}\cos\frac{\beta}{2}}{\sqrt{4b^2\sin^2\frac{\beta}{2}+h^2}} = \underline{\frac{-hu\cos\frac{\beta}{2}}{\sqrt{4b^2\sin^2\frac{\beta}{2}+h^2}}}$$

2/180 Spherical coordinates

$v_R = \dot{R} = 0.5 \text{ m/s}$

$v_\theta = R\dot{\theta}\cos\phi = 15\left(10\frac{\pi}{180}\right)\cos 30° = 2.27 \text{ m/s}$

$v_\phi = R\dot{\phi} = 15\left(7\frac{\pi}{180}\right) = 1.833 \text{ m/s}$

$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{2.96 \text{ m/s}}$

$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$

$= 0 - 15\left(7\frac{\pi}{180}\right)^2 - 15\left(10\frac{\pi}{180}\right)^2\cos^2 30° = -0.567 \text{ m/s}^2$

$a_\theta = \frac{\cos\phi}{R}\left[R^2\ddot{\theta} + 2R\dot{R}\dot{\theta}\right] - 2R\dot{\theta}\dot{\phi}\sin\phi$

$= \frac{\cos 30°}{15}\left[0 + 2(15)(0.5)\left(10\frac{\pi}{180}\right)\right] - 2(15)\left(10\frac{\pi}{180}\right)\left(7\frac{\pi}{180}\right)\sin 30°$

$= -0.1687 \text{ m/s}^2$

$a_\phi = \frac{1}{R}\left[R^2\ddot{\phi} + 2\dot{R}\dot{\phi}\right] + R\dot{\theta}^2\sin\phi\cos\phi$

$= \frac{1}{15}\left[0 + 2(15)(0.5)\left(7\frac{\pi}{180}\right)\right] + 15\left(10\frac{\pi}{180}\right)^2\sin 30°\cos 30°$

$= 0.320 \text{ m/s}^2$

$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.672 \text{ m/s}^2}$

$\boxed{2/181}$ $R = 24\,m$ const, $\dot{\theta} = \omega = \frac{2(2\pi)}{60} = \frac{\pi}{15}$ rad/s, $\ddot{\theta} = 0$

$\beta = 30^\circ$, $\varphi = \frac{\pi}{2} - \beta$, $\dot{\varphi} = -\dot{\beta} = -0.10$ rad/s, $\ddot{\varphi} = -\ddot{\beta} = 0$

$v_R = \dot{R} = 0$, $v_\theta = R\dot{\theta}\cos\varphi = 24\left(\frac{\pi}{15}\right)\frac{1}{2} = 2.51$ m/s

$v_\varphi = R\dot{\varphi} = 24(-0.10) = -2.4$ m/s

$v = \sqrt{(2.51)^2 + (2.4)^2} = \underline{3.48 \text{ m/s}}$

$a_R = \ddot{R} - R\dot{\varphi}^2 - R\dot{\theta}^2\cos^2\varphi = 0 - 24(-0.10)^2 - 24\left(\frac{\pi}{15}\right)^2\left(\frac{1}{2}\right)^2 = -0.503\ \frac{m}{s^2}$

$a_\theta = \frac{\cos\varphi}{R}\frac{d}{dt}(R^2\dot{\theta}) - 2R\dot{\theta}\dot{\varphi}\sin\varphi = 0 - 2(24)\frac{\pi}{15}(-0.10)\frac{\sqrt{3}}{2} = 0.871\ \frac{m}{s^2}$

$a_\varphi = \frac{1}{R}\frac{d}{dt}(R^2\dot{\varphi}) + R\dot{\theta}^2\sin\varphi\cos\varphi = 0 + 24\left(\frac{\pi}{15}\right)\frac{\sqrt{3}}{2}\frac{1}{2} = 0.456\ m/s^2$

$a = \sqrt{(-0.503)^2 + (0.871)^2 + (0.456)^2} = \underline{1.104\ m/s^2}$

2/182 Use Eq. 2/19 where $\dot{\varphi} = -\dot{\beta}$, $R = L$, $\dot{\theta} = \omega$

$$a_R = 0 - 1.2\left(-\tfrac{3}{2}\right)^2 - 1.2(2)^2\tfrac{1}{2} = \underline{-5.10 \text{ m/s}^2}$$

$$a_\theta = \frac{\sin\beta}{L}(2L\dot{L}\omega + 0) + 2L\omega\dot{\beta}\cos\beta = 2\omega(\dot{L}\sin\beta + L\dot{\beta}\cos\beta)$$

$$= 2(2)\left(0.9\tfrac{1}{\sqrt{2}} + 1.2\left(\tfrac{3}{2}\right)\tfrac{1}{\sqrt{2}}\right) = \frac{10.8}{\sqrt{2}} = \underline{7.64 \text{ m/s}^2}$$

$$a_\varphi = -2\dot{L}\dot{\beta} + L\omega^2\cos\beta\sin\beta = -2(0.9)\tfrac{3}{2} + 1.2(2^2)\tfrac{1}{\sqrt{2}}\tfrac{1}{\sqrt{2}}$$

$$= -2.7 + 2.4 = \underline{-0.3 \text{ m/s}^2}$$

▶2/183 $R = \text{const}$ $\qquad \theta = \omega t \qquad \sin\phi = z/R$

$$z = \frac{h}{2}(1-\cos 2\theta)\,, \quad \dot z = \omega h \sin 2\theta \quad \text{where } \dot\theta = \omega$$

$$(\cos\phi)\,\dot\phi = \frac{1}{R}\dot z\,, \quad \dot\phi = \frac{\omega h \sin 2\theta}{R\cos\phi}$$

$$v_R = \dot R = 0$$

$$v_\theta = R\dot\theta\cos\phi = R\omega\sqrt{1-\sin^2\phi} = R\omega\sqrt{1-\left(\frac{h}{2R}[1-\cos 2\theta]\right)^2}$$

$$v_\phi = R\dot\phi = \frac{\omega h \sin 2\theta}{\cos\phi} = h\omega \frac{\sin 2\theta}{\sqrt{1-\left(\frac{h}{2R}[1-\cos 2\theta]\right)^2}}$$

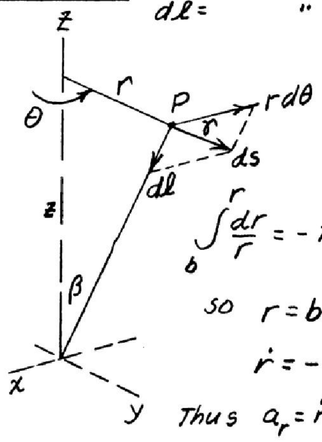
When $\theta = \omega t = \pi/4$, $1-\cos 2\theta = 1$ so that

$$\underline{v_\theta = R\omega\sqrt{1-(h/2R)^2}}\,, \quad \underline{v_\phi = \frac{h\omega}{\sqrt{1-(h/2R)^2}}}\,, \quad \underline{v_R = 0}$$

▶2/184

ds = differential distance along curve

$d\ell$ = " " in direction of cone element



$$r = \frac{b}{h}z, \quad \tan\beta = b/h$$

$$d\ell = r\,d\theta \tan\gamma$$

$$dr = -d\ell \sin\beta = -r\,d\theta \tan\gamma \sin\beta$$

$$\int_b^r \frac{dr}{r} = -\tan\gamma \sin\beta \int_0^\theta d\theta, \quad \ln r\Big]_b^r = -\tan\gamma \sin\beta\ \theta$$

so $r = be^{-\tan\gamma \sin\beta\,\theta}$, $r = be^{-K\theta}$ where $K = \tan\gamma \sin\beta$

$$\dot r = -bK\dot\theta e^{-K\theta}, \quad \ddot r = bK^2\dot\theta^2 e^{-K\theta}, \quad \ddot\theta = 0$$

Thus $a_r = \ddot r - r\dot\theta^2 = b\dot\theta^2 e^{-K\theta}(K^2-1)$

or $\underline{a_r = b\dot\theta^2(\tan^2\gamma \sin^2\beta - 1)\,e^{-\theta\tan\gamma\sin\beta}}$

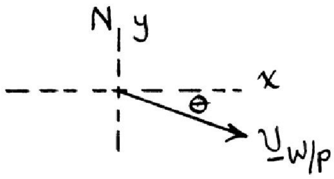
$\underline{\text{where } \beta = \tan^{-1}\frac{b}{h}}$

2/185 (a) $\underline{v}_{W/P} = \underline{v}_W - \underline{v}_P$

$$= 3\left(\frac{\sqrt{2}}{2}\underline{i} - \frac{\sqrt{2}}{2}\underline{j}\right) - (-4\underline{i}) = \underline{6.12\underline{i} - 2.12\underline{j}\ \frac{\text{mi}}{\text{hr}}}$$

or $v_{W/P} = (6.12^2 + 2.12^2)^{1/2} = \underline{6.48\ \text{mi/hr}}$

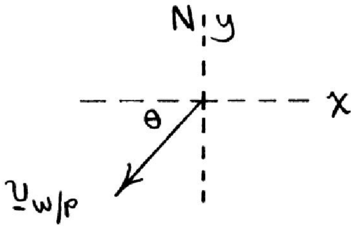
at $\theta = \tan^{-1}\dfrac{2.12}{6.12} = \underline{19.11^\circ \text{ south of east}}$



(b) $\underline{v}_{W/P} = \underline{v}_W - \underline{v}_P = 3\left(\frac{\sqrt{2}}{2}\underline{i} - \frac{\sqrt{2}}{2}\underline{j}\right) - 4\underline{i} = \underline{-1.879\underline{i} - 2.12\underline{j}\ \frac{\text{mi}}{\text{hr}}}$

or $v_{W/P} = (1.879^2 + 2.12^2)^{1/2} = \underline{2.83\ \text{mi/hr}}$

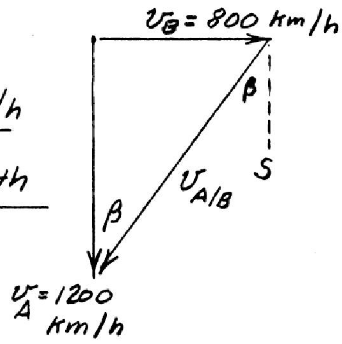
at $\theta = \tan^{-1}\dfrac{2.12}{1.879} = \underline{48.5^\circ \text{ south of west}}$



2/186 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$v_{A/B} = \sqrt{(1200)^2 + (800)^2} = \underline{1442 \text{ km/h}}$

$\beta = \tan^{-1} \frac{800}{1200} = \underline{33.7° \text{ west of south}}$



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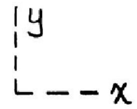
$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= 120[\cos 15^\circ \mathbf{i} + \sin 15^\circ \mathbf{j}] - 90[\cos 60^\circ \mathbf{i} + \sin 60^\circ \mathbf{j}]$$

$$= \underline{70.9\,\mathbf{i} - 46.9\,\mathbf{j} \quad \text{km/h}}$$

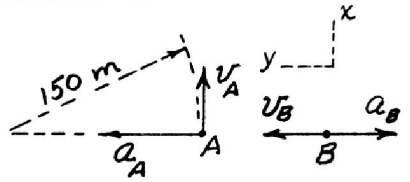
$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B = \mathbf{0} - 3(-\cos 60^\circ \mathbf{i} - \sin 60^\circ \mathbf{j})$$

$$= \underline{1.5\,\mathbf{i} + 2.60\,\mathbf{j} \quad \text{m/s}^2}$$



2/188 | If $\frac{d}{dt}(\overline{AB}) = |v_{A/B}|$ then $\underline{v}_{A/B}$ does not change direction, which requires that $|\underline{v}_A| / |\underline{v}_B| = \text{const}$

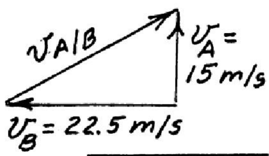
2/189 $v_A = 54/3.6 = 15\ m/s$, $v_B = 81/3.6 = 22.5\ m/s$



$a_A = \dfrac{v_A^2}{\rho} = \dfrac{15^2}{150} = 1.5\ m/s^2$

$a_B = 3\ m/s^2$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

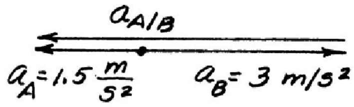


$v_{A/B} = \sqrt{(22.5)^2 + (15)^2}$

$= 27.0\ m/s$

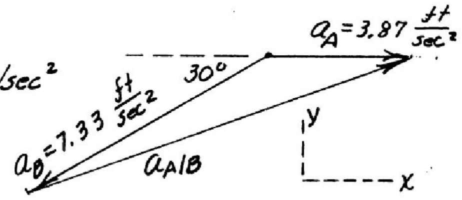
$\underline{v}_{A/B} = 15\underline{i} - 22.5\underline{j}\ m/s$

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$



$\underline{a}_{A/B} = 4.5\underline{j}\ m/s^2$

2/190 $\quad \underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$



$a_A = v^2/\rho = (44)^2/500 = 3.87\ ft/sec^2$

$a_B = \frac{5}{30}\,44 = 7.33\ ft/sec^2$

$\underline{a}_{A/B} = (7.33\cos 30° + 3.87)\underline{i} + 7.33\sin 30°\,\underline{j}$

$= \underline{10.22\underline{i} + 3.67\underline{j}}\quad ft/sec^2$

2/191

$$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B \;, \quad \Omega = 3\,(2\pi/60) = 0.314\ \frac{\text{rad}}{\text{s}}$$

$$= \frac{18}{3.6}\,\underline{i} - 9(0.314)(\cos 45°\,\underline{i} - \sin 45°\,\underline{j})$$

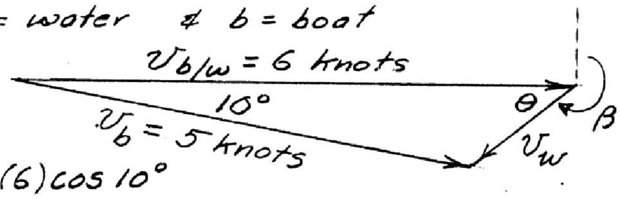
$$= \underline{3.00\,\underline{i} + 2.00\,\underline{j}\ \ \text{m/s}}$$

$$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = 3\,\underline{i} - 9\,(0.314)^2(-\cos 45°\,\underline{i} - \sin 45°\,\underline{j})$$

$$= \underline{3.63\,\underline{i} + 0.628\,\underline{j}\ \ \text{m/s}^2}$$

2/192 Let w = water & b = boat

$\underline{v}_b = \underline{v}_w + \underline{v}_{b/w}$



$v_w^2 = 5^2 + 6^2 - 2(5)(6)\cos 10°$

$= 1.91$, $\underline{v_w = 1.38 \text{ knots}}$

$5/\sin\theta = 1.38/\sin 10°$, $\sin\theta = 0.6280$, $\theta = 38.9°$

$\beta = 270 - 38.9 = \underline{231°}$

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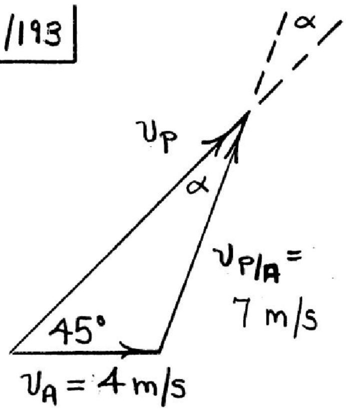


With P being the puck:

$$\underline{v}_P = \underline{v}_A + \underline{v}_{P/A}$$

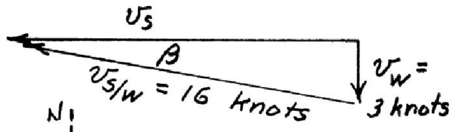
Law of sines:

$$\frac{\sin 45°}{7} = \frac{\sin \alpha}{4}$$

$$\underline{\alpha = 23.8°}$$

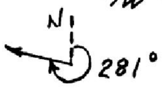
2/194 | w = water, s = ship

$$\underline{v}_s = \underline{v}_w + \underline{v}_{s/w}$$



$\beta = \sin^{-1} 3/16 = 10.8°$

Heading $= 270 + 10.8 = \underline{281°}$

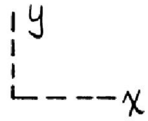


time $t = \dfrac{\text{Dist.}}{\text{vel.}} = \dfrac{24}{\sqrt{(16)^2 - 3^2}} = \underline{1.527 \text{ hr}}$

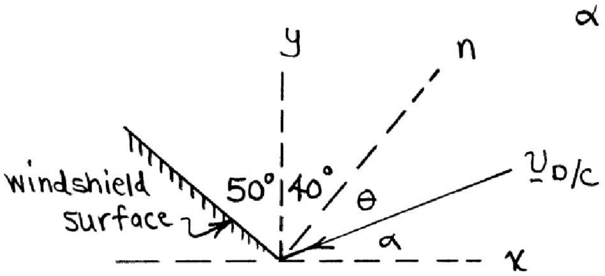
2/195 Drop: $v_D = \sqrt{2gh} = \sqrt{2(9.81)(6)} = 10.85$ m/s

Car: $v_C = 100/3.6 = 27.8$ m/s

$\underline{v}_{D/C} = \underline{v}_D - \underline{v}_C = -10.85\underline{j} - 27.8\underline{i}$ m/s



$\alpha = \tan^{-1} \dfrac{10.85}{27.8} = 21.3°$



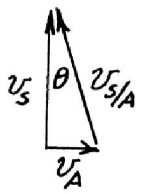
$40° + \theta + \alpha = 90° \Rightarrow \underline{\theta = 28.7° \text{ below normal}}$

2/196 | Let s = satellite , A = observer

$\underline{v}_s = \underline{v}_A + \underline{v}_{s/A}$, $v_A = R\omega$

$= 6378(0.729)(10^{-4})(3600)$

$= 1674$ km/h (East)



$v_s = 27\,940$ km/h (North)

$\theta = \tan^{-1}\dfrac{1674}{27940} = 3.43°$

Satellite appears to travel $\underline{3.43°}$ <u>west of north</u>

2/197 Use $g = g_0 \left(\frac{R}{R+h}\right)^2$

For A, $g_A = 32.23 \left(\frac{3959}{3959+200}\right)^2 = 29.2 \text{ ft/sec}^2$

For B, $g_B = 32.23 \left(\frac{3959}{3959+22{,}300}\right)^2 = 0.733 \text{ ft/sec}^2$

$$\underline{a}_{B/A} = \underline{a}_B - \underline{a}_A = +0.733\underline{i} - (-29.2)\underline{j}$$

$$= \underline{+0.733\underline{i} + 29.2\underline{j} \quad \text{ft/sec}^2}$$

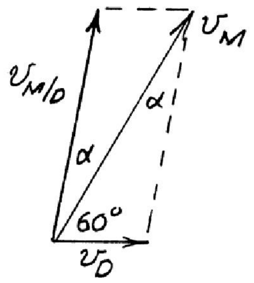
2/198 | Let v_D = velocity of destroyer $= 30(1.852)$
$= 55.6$ km/h

or $v_D = \dfrac{55.6}{3.6} = 15.43$ m/s

Let v_M = horiz. velocity of missile

$(v_{M/D})$ = " " " " rel. to ship

$\underline{v}_M = \underline{v}_D + \underline{v}_{M/D}$ $= 75 \cos 30° = 65.0$ m/s



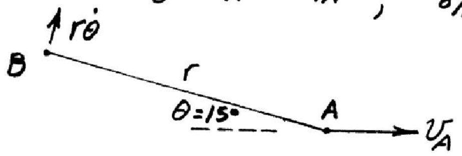
Law of sines,

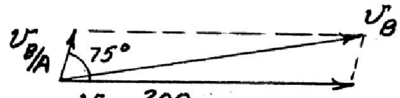
$\sin\alpha = \dfrac{v_D}{v_{M/D}} \sin 60° = \dfrac{15.43}{65.0}\dfrac{\sqrt{3}}{2} = 0.2058$

$\underline{\alpha = 11.88°}$

2/199

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$, $v_{B/A} = r\dot{\theta} = 60(\frac{5}{180}\pi) = 5.24$ m/s





$v_A = \frac{200}{3.6} = 55.6$ m/s

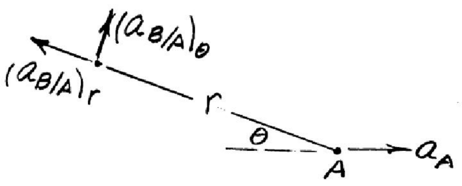
$v_B^2 = (5.24)^2 + (55.6)^2 + 2(5.24)(55.6)\cos 75° = 3264\ (m/s)^2$

$v_B = 57.1$ m/s or $v_B = 57.1(3.6) = \underline{206 \text{ km/h}}$

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$, $a_A = 0$, $a_{B/A} = r\dot{\theta}^2 = 60(\frac{5\pi}{180})^2 = 0.457$ m/s²

Thus $a_B = a_{B/A} = \underline{0.457 \text{ m/s}^2}$ from B to A

2/200 $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

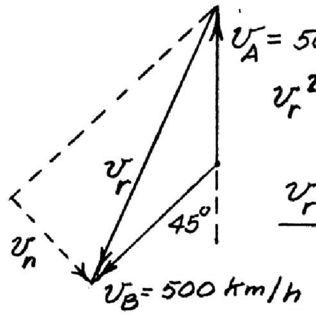


$(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0 = 0$

$(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$

Thus $a_B = a_A = \frac{5}{3.6} = \underline{1.389 \text{ m/s}^2}$

2/201 $\underline{v}_B = \underline{v}_A + \underline{v}_r$ where $\underline{v}_r = \underline{v}_{B/A}$



$$v_r^2 = (500)^2 + (500)^2 + 2(500)(500)\cos 45°$$
$$= 854(10^3) \quad (\text{km/h})^2$$
$$\underline{v_r = 924 \text{ km/h}}$$

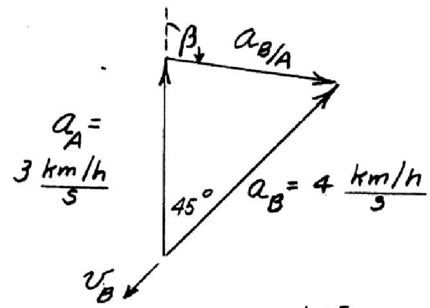
$$v_n = 500 \cos 45° = \underline{354 \text{ km/h}}$$

Let h = possible constant difference in altitude

$\underline{k}$ = unit vector in vertical direction

Since $\frac{d}{dt}(h\underline{k}) = 0$, vertical separation has no influence on relative-velocity equations.

2/202 $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$



$(a_{B/A})^2 = 3^2 + 4^2 - 2(3)(4)\cos 45°$

$= 8.03\ \left(\frac{km/h}{s}\right)^2$

$a_{B/A} = 2.83\ \frac{km/h}{s}$

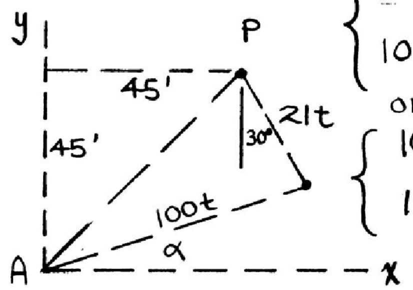
$= \frac{2.83}{3.6} = \underline{0.787\ m/s^2}$

$4^2 = 3^2 + (2.83)^2 + 2(3)(2.83)\cos\beta$, $\cos\beta = \frac{-1.030}{17.00}$

$= -0.0606$

$\underline{\beta = 93.5°}$

2/203



$$\begin{cases} 100t\cos\alpha = 45 + 21t\sin 30° \\ 100t\sin\alpha = 45 - 21t\cos 30° \end{cases}$$

or

$$\begin{cases} 100\cos\alpha = \frac{45}{t} + 21\sin 30° \\ 100\sin\alpha = \frac{45}{t} - 21\cos 30° \end{cases}$$

Subtract 2^{nd} from 1^{st}:

$100(\cos\alpha - \sin\alpha) = 21\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$ or $\cos\alpha - \sin\alpha = 0.287$

$\cos\alpha = \sqrt{1-\sin^2\alpha} = 0.287 + \sin\alpha$

SQ BS & rearrange: $2\sin^2\alpha + 0.574\sin\alpha - 0.918 = 0$

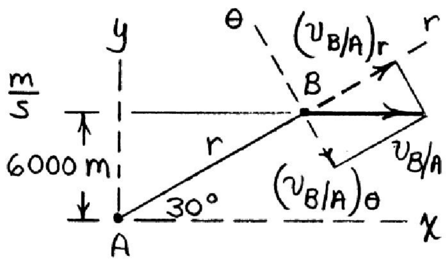
Positive solution: $\underline{\alpha = 33.3°}$; Then $t = 0.616$ sec

$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 100[\cos\alpha\,\underline{i} + \sin\alpha\,\underline{j}]$

$\qquad - 21[\sin 30°\,\underline{i} - \cos 30°\,\underline{j}]$

$= \underline{73.1\,\underline{i} + 73.1\,\underline{j}\ \text{ft/sec}}$

▶2/204



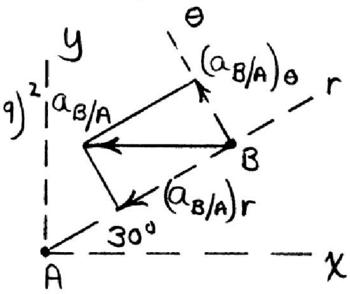
$$v_{B/A} = v_B - v_A$$
$$= (1500-1000)/3.6 = 138.9 \ \frac{m}{s}$$

$$(v_{B/A})_r = \dot{r} = 138.9 \cos 30^\circ$$
$$= 120.3 \ m/s$$

$$(v_{B/A})_\theta = r\dot{\theta} : -138.9 \sin 30^\circ = \frac{6000}{\sin 30^\circ}\dot{\theta}$$

$$\dot{\theta} = -0.00579 \ rad/s$$

$$a_{B/A} = a_B - a_A = 0 - 1.2 = -1.2 \ m/s^2$$



$$(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2$$
$$-1.2\cos 30^\circ = \ddot{r} - 12000(-0.00579)^2$$
$$\underline{\ddot{r} = -0.637 \ m/s^2}$$

$$(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$1.2\sin 30^\circ = 12000\ddot{\theta} + 2(120.3)(-0.00579)$$
$$\underline{\ddot{\theta} = 0.1660(10^{-3}) \ rad/s^2}$$

▶ 2/205 Find flight time t :

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$: $7 = 3 + 100 \sin 30° t - 16.1 t^2$

Solve to obtain 0.0822 sec (discard) & $t = 3.02$ sec

Range $R = x_0 + v_{x_0} t = 0 + 100 \cos 30° (3.02)$

$= 262$ ft

Fielder must run $262 - 220 = 41.8$ ft

in $(3.02 - 0.25)$ sec $\Rightarrow v_B = \frac{41.8}{2.77} = 15.08$ ft/sec

Velocity components of ball when caught :

$v_x = v_{x_0} = 100 \cos 30° = 86.6$ ft/sec

$v_y = v_{y_0} - gt = 100 \sin 30° - 32.2(3.02) = -47.4 \frac{ft}{sec}$

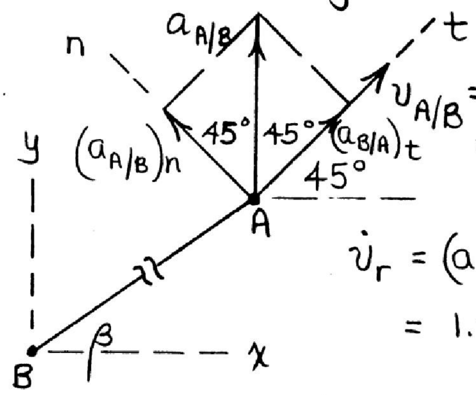
$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = (86.6\underline{i} - 47.4\underline{j}) - 15.08\underline{i}$

$= \underline{71.5\,\underline{i} - 47.4\,\underline{j} \quad \text{ft/sec}}$

▶2/206 (a) $\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 50\underline{i} - (-50\underline{j}) = \underline{50\underline{i} + 50\underline{j} \text{ m/s}}$

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \frac{v_A^2}{\rho_A}\underline{j} - \underline{0} = \frac{50^2}{2000}\underline{j} = \underline{1.250\underline{j} \text{ m/s}^2}$

(b) Use the results of part (a) for a normal-tangential analysis:



$v_{A/B} = \sqrt{50^2 + 50^2} = 70.7 \text{ m/s}$

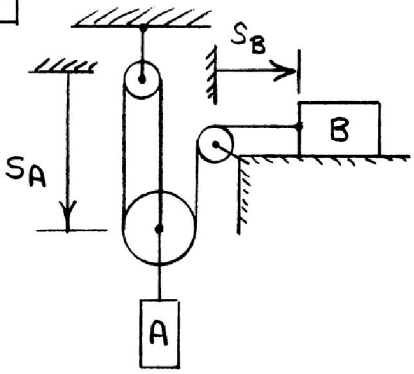
$\dot{v}_r = (a_{B/A})_t = a_{A/B} \cos 45° = 1.250 \frac{\sqrt{2}}{2} = \underline{0.884 \text{ m/s}^2}$

$(a_{B/A})_n = a_{B/A} \sin 45° = \frac{v_{A/B}^2}{\rho_r}$

$1.250 \frac{\sqrt{2}}{2} = \frac{70.7^2}{\rho_r}, \quad \underline{\rho_r = 5660 \text{ m}}$

(in n-direction from A)

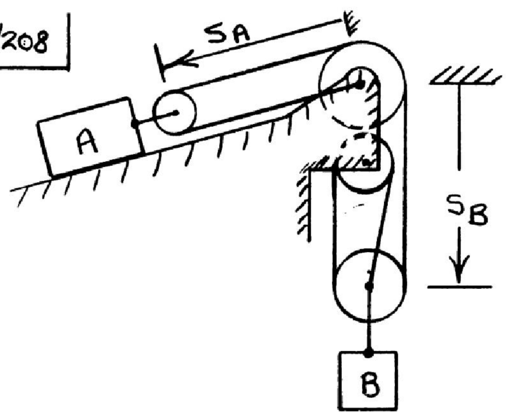
2/207



Length of cable $L = s_B + 3s_A$ + constants

$0 = v_B + 3v_A$, $v_A = -\frac{v_B}{3} = -\left(\frac{-1.2}{3}\right) = 0.4\ \frac{m}{s}$ (down)

2/208



Cable length $L = 2s_A + 3s_B +$ constants

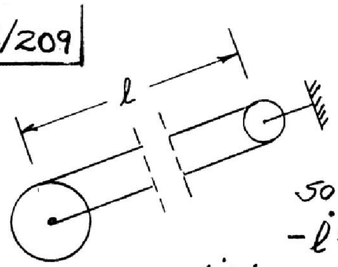
$0 = 2v_A + 3v_B\,, \quad 0 = 2a_A + 3a_B$

$v_A = -\frac{3}{2}v_B = -\frac{3}{2}(2) = \underline{-3 \text{ ft/sec}}$

$a_A = -\frac{3}{2}a_B = -\frac{3}{2}(-0.5) = \underline{+0.75 \text{ ft/sec}^2}$

or $\begin{cases} v_A = 3 \text{ ft/sec up the incline} \\ a_A = 0.75 \text{ ft/sec}^2 \text{ down the incline} \end{cases}$

2/209



Length of cable is

$L = 2\ell$

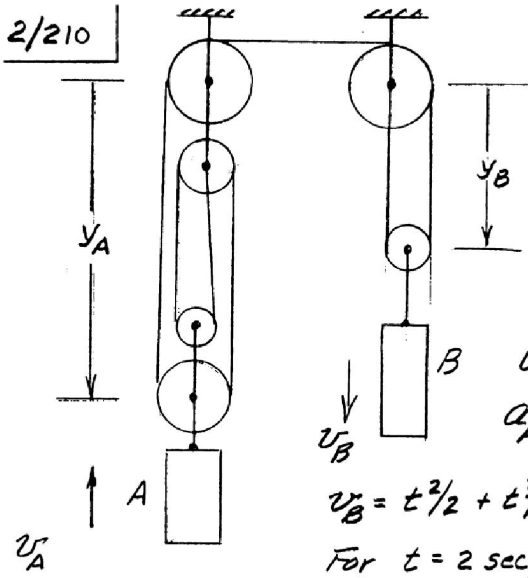
$\dot{L} = 2\dot{\ell}$

so velocity of truck is

$-\dot{\ell} = \frac{1}{2}(-\dot{L}) = \frac{1}{2}(40) = 20$ mm/s

time $t = \dfrac{\text{distance}}{\text{velocity}} = \dfrac{4(10^3)}{20} = 200$ s or 3 min 20 s

2/210



Total length of cable to within constants is

$L = 4y_A + 2y_B$

$0 = 4\dot{y}_A + 2\dot{y}_B$

$0 = 4\ddot{y}_A + 2\ddot{y}_B$

Upward accel. of A is

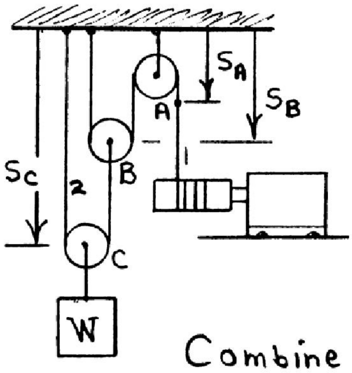
$a_A = -\ddot{y}_A = \frac{1}{2}\ddot{y}_B = \frac{1}{2}a_B$

$v_B = t^2/2 + t^3/6, \quad a_B = \dot{v}_B = t + t^2/2$

For $t = 2$ sec, $a_B = 2 + 4/2 = 4$ ft/sec^2

Thus $a_A = \frac{1}{2}a_B = \underline{2 \text{ ft/sec}^2}$

2/211



Let A be a point on cable 1:

$$L_1 = s_A + 2s_B$$

$$0 = v_A + 2v_B \qquad (1)$$

$$L_2 = s_C + (s_C - s_B)$$

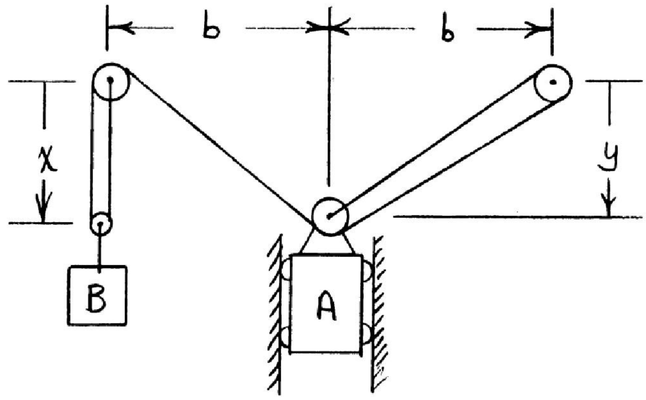
$$0 = 2v_C - v_B \qquad (2)$$

Combine (1) & (2) to obtain

$$v_C = -\frac{1}{4}v_A = -\frac{1}{4}(320) = -80 \text{ mm/s}$$

So G (and W) rises $h = 80(5) = \underline{400 \text{ mm}}$

2/212



The total length of the cable is

$$L = 2x + 3\sqrt{y^2+b^2} + \text{constant}$$

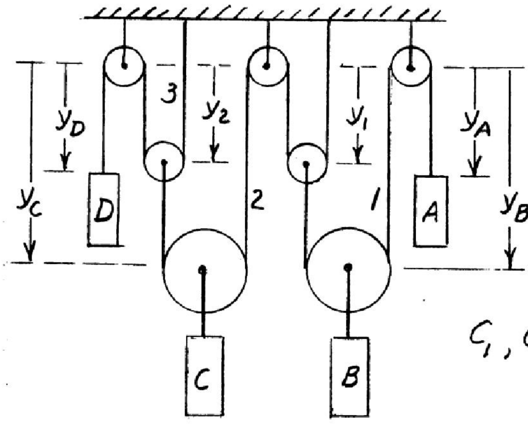
Differentiate to obtain

$$\dot{L} = 0 = 2\dot{x} + 3\frac{y\dot{y}}{\sqrt{y^2+b^2}}$$

With $\dot{x} = v_B$ & $\dot{y} = v_A$, we have

$$v_B = -\frac{3y}{2\sqrt{y^2+b^2}}v_A$$

2/213



3 degrees of freedom

C_1, C_2, C_3 are constant lengths

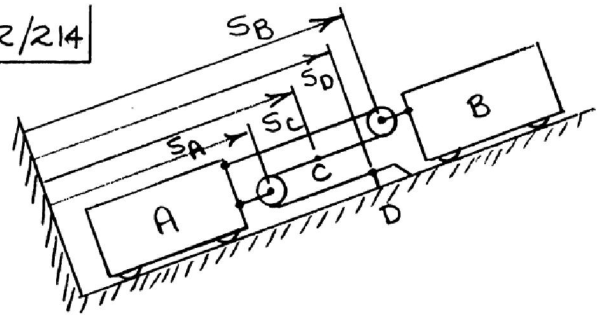
$L_1 = y_B + y_A + (y_B - y_1) + C_1$; $0 = 2\dot{y}_B + \dot{y}_A - \dot{y}_1$

$L_2 = y_C + 2y_1 + (y_C - y_2) + C_2$ $\quad 0 = 2\dot{y}_C + 2\dot{y}_1 - \dot{y}_2$

$L_3 = 2y_2 + y_D + C_3$, $\quad 0 = 2\dot{y}_2 + \dot{y}_D$

Eliminate $\dot{y}_1$ & $\dot{y}_2$ & get $4v_A + 8v_B + 4v_C + v_D = 0$

2/214



The cable length is $L = 2(s_B - s_A) + s_D - s_A$
+ Constants

Differentiating:

$$0 = 2v_B - 3v_A \; ; \qquad 0 = 2a_B - 3a_A$$

So $v_A = \frac{2}{3} v_B = \frac{2}{3}(3) = 2 \text{ ft/sec}$

$$a_A = \frac{2}{3} a_B = \frac{2}{3}(6) = 4 \text{ ft/sec}^2$$

$$v_{B/A} = v_B - v_A = 3 - 2 = \underline{1 \text{ ft/sec}}$$

$$a_{B/A} = a_B - a_A = 6 - 4 = \underline{2 \text{ ft/sec}^2}$$

The length of cable between A and C is

$$L' = (s_B - s_A) + (s_B - s_C) = 2s_B - s_A - s_C + \text{constants}$$

$\Rightarrow 0 = 2v_B - v_A - v_C \; ; \; v_C = 2v_B - v_A = 2(3) - 2 = \underline{4 \text{ ft/sec}}$

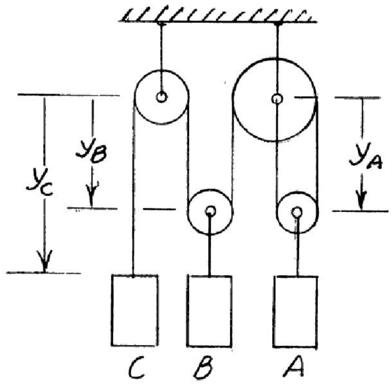
(All answers are quantities directed up incline.)

2/215 Length of cable $L = 2y_A + 2y_B + y_C + \text{const.}$

$$0 = 2\dot{y}_A + 2\dot{y}_B + \dot{y}_C$$

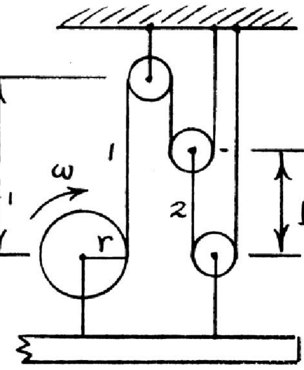
$$0 = 2\ddot{y}_A + 2\ddot{y}_B + \ddot{y}_C$$

so $\underline{2a_A + 2a_B + a_C = 0}$



<u>2 degrees of freedom</u>

2/216



Length $L_1 = l_1 + 2(l_1 - l_2) + \text{const.}$

$\dot{L}_1 = -r\omega = 3\dot{l}_1 - 2\dot{l}_2$

Length $L_2 = l_2 + l_1 + \text{const.}$

$\dot{L}_2 = 0 = \dot{l}_2 + \dot{l}_1, \quad -\dot{l}_1 = \dot{l}_2$

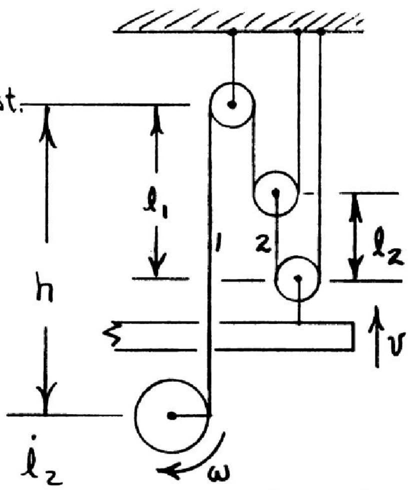
But $v = -\dot{l}_1$, so

$-r\omega = 3(-v) - 2v, \qquad r\omega = 5v$

$$v = \frac{r\omega}{5} = \frac{0.1(40)(2\pi/60)}{5} = 0.0838 \ \frac{\text{m}}{\text{s}}$$

or $\underline{v = 83.8 \text{ mm/s}}$

2/217



Length $L_1 = h + 2(\ell_1 - \ell_2) + \text{const.}$

$\dot{L}_1 = -r\omega = 0 + 2\dot{\ell}_1 - 2\dot{\ell}_2$

But $v = -\dot{\ell}_1$

So $-r\omega = -2v - 2\dot{\ell}_2$

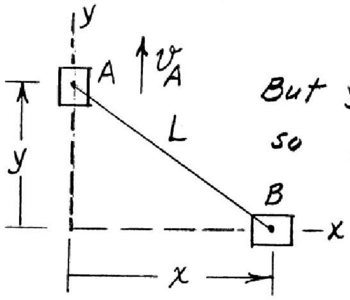
Length $L_2 = \ell_1 + \ell_2 + \text{Const.}$

$\dot{L}_2 = 0 = \dot{\ell}_1 + \dot{\ell}_2$, $v = \dot{\ell}_2$

$\therefore r\omega = 2v + 2v$, $v = \dfrac{r\omega}{4} = \dfrac{0.1(40)(2\pi/60)}{4}$

$v = 0.1047$ m/s or $\underline{v = 104.7 \ \dfrac{mm}{s}}$

2/218



$x^2+y^2=L^2$; $x\dot{x}+y\dot{y}=0$

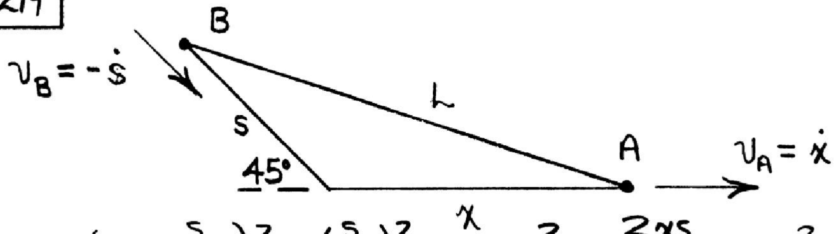
& $\dot{x}^2+x\ddot{x}+\dot{y}^2+y\ddot{y}=0$

But $\dot{y}=v_A$, $\ddot{y}=0$

so $a_x=\ddot{x}=-\dfrac{\dot{x}^2+\dot{y}^2}{x}=-\dfrac{\dot{y}^2\frac{y^2}{x^2}+\dot{y}^2}{x}$

$a_x=-\dfrac{L^2}{x^3}\dot{y}^2=-\dfrac{L^2 v_A^2}{(L^2-y^2)^{3/2}}$

2/219



$$L^2 = \left(x + \frac{s}{\sqrt{2}}\right)^2 + \left(\frac{s}{\sqrt{2}}\right)^2 = x^2 + \frac{2xs}{\sqrt{2}} + s^2$$

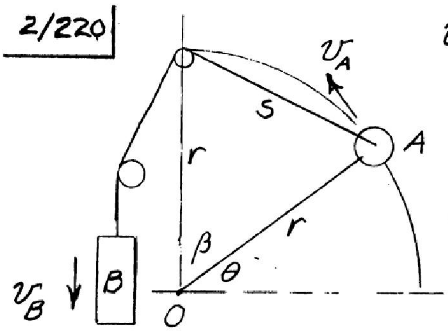
$$0 = 2x\dot{x} + \frac{2\dot{x}s}{\sqrt{2}} + \frac{2x\dot{s}}{\sqrt{2}} + 2s\dot{s} = 0$$

$$0 = x v_A + \frac{s}{\sqrt{2}} v_A - \frac{x}{\sqrt{2}} v_B - s v_B$$

$$v_B\left(s + \frac{x}{\sqrt{2}}\right) = v_A\left(x + \frac{s}{\sqrt{2}}\right) \text{ so } v_B = \frac{x + s/\sqrt{2}}{s + x/\sqrt{2}} v_A$$

$$\text{or } \underline{v_B = \frac{s + \sqrt{2}\,x}{x + \sqrt{2}\,s} v_A}$$

2/220



$v_B = -\dot{s}$, $v_A = r\dot{\theta} = -r\dot{\beta}$

$s = 2r\sin\beta/2$

$\dot{s} = r\dot{\beta}\cos\beta/2$

so $-v_B = -v_A\cos\beta/2$

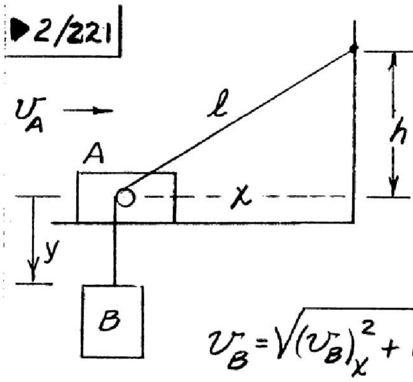
But $\cos\frac{\beta}{2} = \cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$= \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$

Thus $v_B = v_A\frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$

$$v_A = \frac{v_B\sqrt{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}$$

▶2/221



$$\ell^2 = x^2 + h^2$$

$$\ell\dot{\ell} = x\dot{x}\ , \quad \dot{y} = -\dot{\ell} = -\frac{x}{\ell}\dot{x}$$

But $v_A = -\dot{x}$

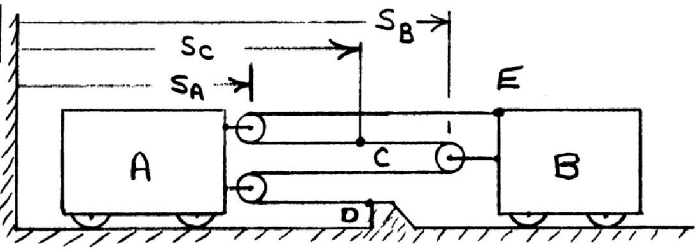
so $(v_B)_y = \dot{y} = \frac{x}{\ell} v_A$

$$(v_B)_x = \dot{x} = -v_A$$

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = v_A\sqrt{1 + \frac{x^2}{\ell^2}}$$

$$= \underline{v_A\sqrt{\frac{2x^2 + h^2}{x^2 + h^2}}}$$

▶2/222



Cable length $L = 3(s_B - s_A) + (s_D - s_A)$

$0 = 3v_B - 4v_A\ ,\quad 0 = 3a_B - 4a_A$

$v_A = \frac{3}{4}v_B = \frac{3}{4}(2) = 1.5 \text{ m/s}$

$a_A = \frac{3}{4}a_B = \frac{3}{4}(3) = 2.25 \text{ m/s}^2$

$v_{B/A} = v_B - v_A = 2 - (1.5) = \underline{0.5 \text{ m/s}}$

$a_{B/A} = a_B - a_A = 3 - (2.25) = \underline{0.75 \text{ m/s}^2}$

Length of cable between points E and C:

$L' = (s_B - s_A) + (s_C - s_A) + \text{constants}$

$0 = v_B - 2v_A + v_C \Rightarrow v_C = 2v_A - v_B$

or $v_C = 2(1.5) - 2 = \underline{1 \text{ m/s}}$

(All answers are quantities directed to $\underline{\text{right}}$)

2/223 $\quad s = 8e^{-0.4t} - 6t + t^2$

$v = \frac{ds}{dt} = -3.2e^{-0.4t} - 6 + 2t$

$a = \frac{dv}{dt} = 1.28e^{-0.4t} + 2$

$a = 3 \text{ m/s}^2$ when $1.28e^{-0.4t} + 2 = 3$

$1.28e^{-0.4t} = 1, \quad e^{-0.4t} = 0.781$

$-0.4t = -0.247, \quad t = 0.617 \text{ s}$

$v = -3.2e^{-0.4(0.617)} - 6 + 2(0.617)$

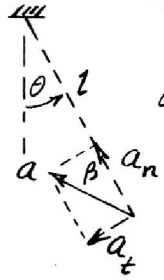
$= \underline{-7.27 \text{ m/s}}$

$\underline{2/224}$ $+\uparrow$ $v_y^2 = v_{y_0}^2 - 2g(y - y_0)$

$$0^2 = v_0^2 - 2(32.2)(3)$$

$$\underline{v_0 = 13.90 \text{ ft/sec}}$$

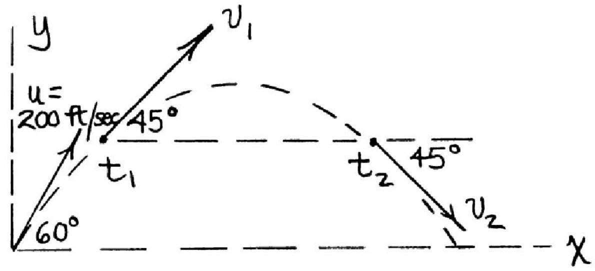
2/225



$a_t = -l\ddot{\theta} = a \sin\beta$, $\ddot{\theta} = -\frac{a}{l}\sin\beta$

$a_n = l\dot{\theta}^2 = a\cos\beta$, $\dot{\theta} = \pm\sqrt{\frac{a}{l}\cos\beta}$

2/226



$\dot{x} = u\cos\theta = 200\cos 60° = 100$ ft/sec

$\dot{y} = u\sin\theta - gt = 200\sin 60° - 32.2t = 173.2 - 32.2t$

At t_1: $\dot{x} = \dot{y}$: $100 = 173.2 - 32.2t_1$, $\underline{t_1 = 2.27 \text{ sec}}$

At t_2: $\dot{x} = -\dot{y}$: $100 = -173.2 + 32.2t_2$, $\underline{t_2 = 8.48 \text{ sec}}$

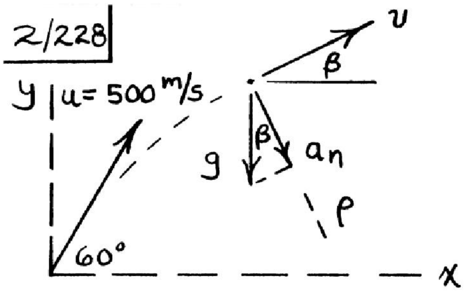
$\underline{2/227}$ $r = r_0 + b\sin\frac{2\pi t}{\tau}$, $\dot{r} = \frac{2\pi}{\tau} b\cos\frac{2\pi t}{\tau}$

$$\ddot{r} = -\frac{4\pi^2}{\tau^2} b\sin\frac{2\pi t}{\tau}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{4\pi^2}{\tau^2} b\sin\frac{2\pi t}{\tau} - r\dot{\theta}^2 = 0$$

$$\Rightarrow \quad \underline{r = r_0 \frac{1}{1+\left(\frac{\tau\dot{\theta}}{2\pi}\right)^2}}$$

2/228



$$v_x = 500 \cos 60° = 250 \text{ m/s}$$

$$v_y = v_{y_0} - gt = 500 \sin 60° - 9.81(30) = 138.7 \ \frac{m}{s}$$

$$v^2 = v_x^2 + v_y^2 = 250^2 + 138.7^2 = 81.7(10^3) \ m^2/s^2$$

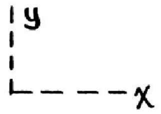
$$\beta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{138.7}{250} = 29.0°$$

$$a_n = g \cos \beta = 9.81 \cos 29.0° = 8.58 \ m/s^2$$

$$a_n = v^2/\rho \Rightarrow \rho = \frac{v^2}{a_n} = \frac{81.7(10^3)}{8.58} = 9529 \ m$$

or $\underline{\rho = 9.53 \text{ km}}$

2/229



$$\underline{v}_A = \underline{v}_W + \underline{v}_{A/W} = -48\underline{i} + 220\,\underline{i} = 172\underline{i}\ \frac{\text{km}}{\text{h}}$$

On descent : $\underline{v}_A = 172(\cos 10°\,\underline{i} - \sin 10°\,\underline{j})$ km/h

$$\underline{v}_{A/C} = \underline{v}_A - \underline{v}_C = 172(\cos 10°\,\underline{i} - \sin 10°\,\underline{j}) - 30\underline{i}$$

$$= 139.4\underline{i} - 29.9\,\underline{j}\ \text{km/h}$$

$$\beta = \tan^{-1}\left(\frac{29.9}{139.4}\right) = \underline{12.09°}$$

2/230 $\ddot{y} = -g, \quad \dot{y} = u \sin 45° - gt, \quad y = ut \sin 45° - \frac{1}{2} g t^2$

$\ddot{x} = 0, \quad \dot{x} = u \cos 45°, \quad x = ut \cos 45°$

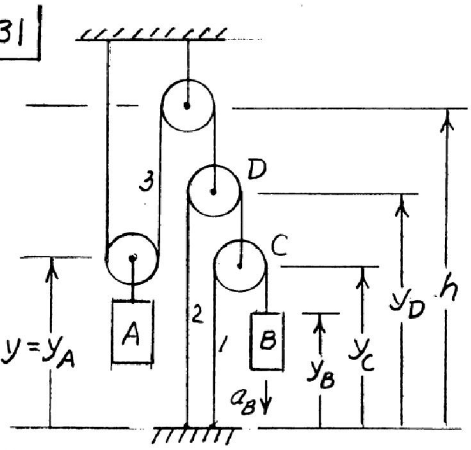
$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = 1 - \frac{gt}{u}\sqrt{2} = 1 - \frac{2gx}{u^2}, \quad x = \frac{u^2}{2g}\left(1 - \frac{dy}{dx}\right)$

$\frac{dy}{dx} = \tan 20° = 0.3640, \quad x = \frac{(15)^2(10^6)^2}{(3600)^2(2)(9)(10^{-3})}(1 - 0.364) = 613 \text{ km}$

$t = \frac{x}{u \cos 45°} = \frac{613}{15000}\sqrt{2} = 0.0578 \text{ h}$ or $\underline{3 \text{ min } 28 \text{ sec}}$

$h = y = \frac{15(10^3)(0.0578)}{\sqrt{2}} - \frac{9(10^{-3})}{2}(0.0578)^2(3600)^2 = \underline{418 \text{ km}}$

2/231



$y = y_A = t^2/4$ m

$\dot{y}_A = t/2$ m/s

$a_A = \ddot{y}_A = 1/2$ m/s^2

one degree of freedom

Cable lengths

$L_1 = y_C + (y_C - y_B) + C_1$ $\qquad 0 = 2\ddot{y}_C - \ddot{y}_B$ where $a_B = -\ddot{y}_B$

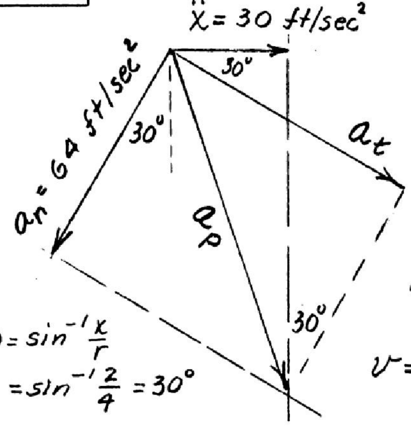
$L_2 = y_D + (y_D - y_C) + C_2$ $\qquad 0 = 2\ddot{y}_D - \ddot{y}_C$

$L_3 = 2(h - y_A) + h - y_D + C_3,$ $\qquad 0 = -2\ddot{y}_A - \ddot{y}_D$

Eliminate $\ddot{y}_C$ & $\ddot{y}_D$ & get $\ddot{y}_B = -8\ddot{y}_A$ or $a_B = 8\ddot{y}_A = \underline{4 \text{ m/s}^2}$

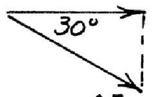
(By inspection of pulley displacements
$-dy_B = 8\,dy_A$, so $a_B = 8a_A = 8(1/2) = 4$ m/s^2)

2/232



$\theta = \sin^{-1}\frac{x}{r}$

$= \sin^{-1}\frac{2}{4} = 30°$



$v = 4/\frac{\sqrt{3}}{2} = \frac{8}{\sqrt{3}}$ ft/sec

$a_n = v^2/r = \left(\frac{8}{\sqrt{3}}\right)^2/\frac{4}{12} = 64$ ft/sec^2

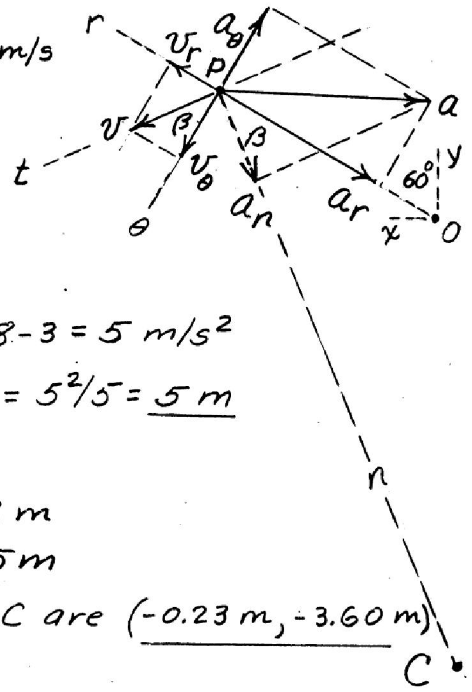
$a_t = \frac{30}{\sqrt{3}/2} + \frac{64}{\sqrt{3}} = \frac{124}{\sqrt{3}} = 71.6$ ft/sec^2

$v = r\dot{\theta}, \quad \dot{\theta} = \frac{v}{r} = \frac{8/\sqrt{3}}{4/12} = \underline{13.86 \text{ rad/sec}}$

$a_t = r\ddot{\theta}, \quad \ddot{\theta} = \frac{a_t}{r} = \frac{71.6}{4/12} = \underline{215 \text{ rad/sec}^2}$

2/233 | $v_r = 3\ m/s$, $v_\theta = 4\ m/s$ $\}$ $v = 5\ m/s$

$\tan\beta = 3/4$



$a_r = -10\ m/s^2$

$a_\theta = -5\ m/s^2$

$a_n = |a_r|\cos\beta - |a_\theta|\sin\beta$

$= 10\,(4/5) - 5(3/5) = 8 - 3 = 5\ m/s^2$

$a_n = v^2/\rho, \quad \rho = v^2/a_n = 5^2/5 = \underline{5\ m}$

$\overline{OP} = 2\ m$

$\overline{CP} = 5\ m$

x-y Coordinates of C are $\underline{(-0.23\ m, -3.60\ m)}$

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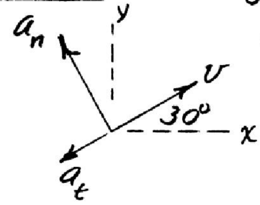


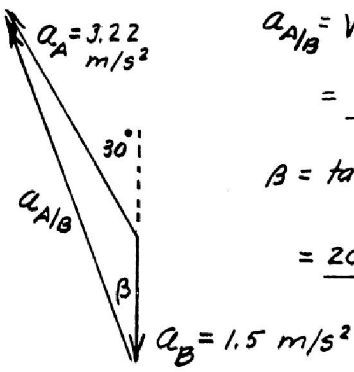
$v = \frac{1000}{3.6} = 278$ m/s , $a_t = \frac{15}{3.6} = 4.17$ m/s^2

$a_n = v^2/\rho = (278)^2/1500 = 51.4$ m/s^2

$\ddot{x} = -51.4 \sin 30° - 4.17 \cos 30° = \underline{-29.3 \text{ m/s}^2}$

$\ddot{y} = 51.4 \cos 30° - 4.17 \sin 30° = \underline{42.5 \text{ m/s}^2}$

2/235 $\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$, $a_A = v_A^2/\rho = (50/3.6)^2/60 = 3.22\ m/s^2$



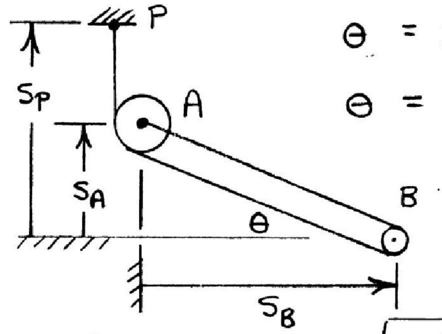
$$a_{A/B} = \sqrt{\left(3.22\frac{\sqrt{3}}{2} + 1.5\right)^2 + \left(3.22\,[1/2]\right)^2}$$

$$= \underline{4.58\ m/s^2}$$

$$\beta = \tan^{-1}\frac{1.608}{4.28} = \tan^{-1} 0.3752$$

$$= \underline{20.6° \text{ west of north}}$$

2/236



$\theta = 45°$: $s_B/s_A = 1$

$\theta = 30°$: $s_B/s_A = \sqrt{3}$

$\theta = 15°$: $s_B/s_A = 3.73$

$$L = (s_P - s_A) + 2\sqrt{s_A^2 + s_B^2}$$

$$0 = -v_A + 2(s_A v_A + s_B v_B)\Big/\sqrt{s_A^2 + s_B^2}$$

$$\Rightarrow v_B = \left[\sqrt{1 + (s_B/s_A)^2} - 2\right]\frac{v_A}{2(s_B/s_A)}$$

$\theta = 45°$: $v_B = -0.293 v_A = -0.293(-1) = \underline{0.293 \frac{m}{s}}$

$\theta = 30°$: $v_B = \underline{0}$

$\theta = 15°$: $v_B = 0.250 v_A = 0.250(-1) = \underline{-0.250 \frac{m}{s}}$

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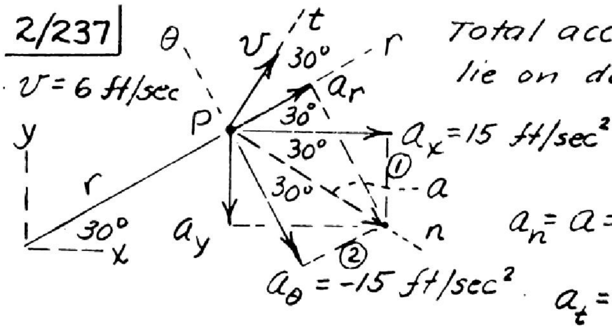
$v = 6$ ft/sec



Total acceleration must lie on dotted lines ① & ②

From 30-60 triangles,

$a_n = a = 15/\cos 30° = 10\sqrt{3}\ \dfrac{\text{ft}}{\text{sec}^2}$

$a_\theta = -15$ ft/sec² $a_t = 0$

$a_y = -15 \tan 30° = -5\sqrt{3}$ ft/sec², $a_r = 10\sqrt{3} \sin 30°$
$= 5\sqrt{3}$ ft/sec²

$a_n = v^2/\rho$, $\rho = \dfrac{v^2}{a_n} = \dfrac{6^2}{10\sqrt{3}} = \dfrac{6\sqrt{3}}{5}$ ft

2/238

$$\begin{cases} x = 50 \text{ ft}, & \dot{x} = -10 \text{ ft/sec}, & \ddot{x} = -10 \text{ ft/sec}^2 \\ y = 25 \text{ ft}, & \dot{y} = 10 \text{ ft/sec}, & \ddot{y} = 5 \text{ ft/sec}^2 \end{cases}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(-10)^2 + 10^2} = \underline{10\sqrt{2} \text{ ft/sec}}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(-10)^2 + 5^2} = \underline{11.18 \text{ ft/sec}^2}$$

$$\underline{e}_t = \underline{v}/v = (-10\underline{i} + 10\underline{j})/10\sqrt{2} = \underline{\tfrac{\sqrt{2}}{2}(-\underline{i} + \underline{j})}$$

$$a_t = \underline{a} \cdot \underline{e}_t = (-10\underline{i} + 5\underline{j}) \cdot \tfrac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) = \underline{10.61 \text{ ft/sec}^2}$$

$$\underline{a}_t = a_t \underline{e}_t = 10.61 \tfrac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) = \underline{-7.5\underline{i} + 7.5\underline{j} \text{ ft/sec}^2}$$

$$\underline{a}_n = \underline{a} - \underline{a}_t = (-10\underline{i} + 5\underline{j}) - (-7.5\underline{i} + 7.5\underline{j}) = \underline{-2.5(\underline{i} + \underline{j}) \text{ ft/sec}^2}$$

$$a_n = \sqrt{2.5^2 + 2.5^2} = \underline{3.54 \text{ ft/sec}^2}$$

$$\rho = v^2/a_n = (10\sqrt{2})^2/3.54 = \underline{56.6 \text{ ft}}$$

$$\underline{e}_n = \underline{a}_n/a_n = -2.5(\underline{i} + \underline{j})/3.54 = \underline{-\tfrac{\sqrt{2}}{2}(\underline{i} + \underline{j})}$$

$$\underline{e}_r = \underline{r}/r = 50\underline{i} + 25\underline{j} \Big/ \sqrt{50^2 + 25^2} = \underline{0.894\underline{i} + 0.447\underline{j}}$$

$$\underline{e}_\theta = \underline{e}_r \text{ rotated CCW } 90^\circ = \underline{-0.447\underline{i} + 0.894\underline{j}}$$

$$v_r = \underline{v} \cdot \underline{e}_r = (-10\underline{i} + 10\underline{j}) \cdot (0.894\underline{i} + 0.447\underline{j}) = \underline{-4.47 \text{ ft/sec}}$$

$$\underline{v}_r = v_r \underline{e}_r = -4.47(0.894\underline{i} + 0.447\underline{j}) = \underline{-4\underline{i} - 2\underline{j} \text{ ft/sec}}$$

$$v_\theta = \underline{v} \cdot \underline{e}_\theta = (-10\underline{i} + 10\underline{j}) \cdot (-0.447\underline{i} + 0.894\underline{j}) = \underline{13.42 \text{ ft/sec}}$$

$$\underline{v}_\theta = v_\theta \underline{e}_\theta = 13.42(-0.447\underline{i} + 0.894\underline{j}) = \underline{-6\underline{i} + 12\underline{j} \text{ ft/sec}}$$

$$a_r = \underline{a} \cdot \underline{e}_r = (-10\underline{i} + 5\underline{j}) \cdot (0.894\underline{i} + 0.447\underline{j}) = \underline{-6.71 \text{ ft/sec}^2}$$

$$\underline{a}_r = a_r \underline{e}_r = -6.71(0.894\underline{i} + 0.447\underline{j}) = \underline{-6\underline{i} - 3\underline{j} \text{ ft/sec}^2}$$

$$a_\theta = \underline{a}\cdot\underline{e}_\theta = (-10\underline{i} + 5\underline{j})\cdot(-0.447\underline{i} + 0.894\underline{j}) = \underline{8.94 \ \text{ft/sec}^2}$$

$$\underline{a}_\theta = a_\theta \underline{e}_\theta = 8.94(-0.447\underline{i} + 0.894\underline{j}) = \underline{-4\underline{i} + 8\underline{j} \ \ \text{ft/sec}^2}$$

$$r = \sqrt{x^2+y^2} = \sqrt{50^2+25^2} = \underline{55.9 \ \text{ft}}$$

$$\dot{r} = v_r = \underline{-4.47 \ \text{ft/sec}}$$

$$v_\theta = r\dot{\theta}, \quad \dot{\theta} = v_\theta/r = 13.42/55.9 = \underline{0.240 \ \text{rad/sec}}$$

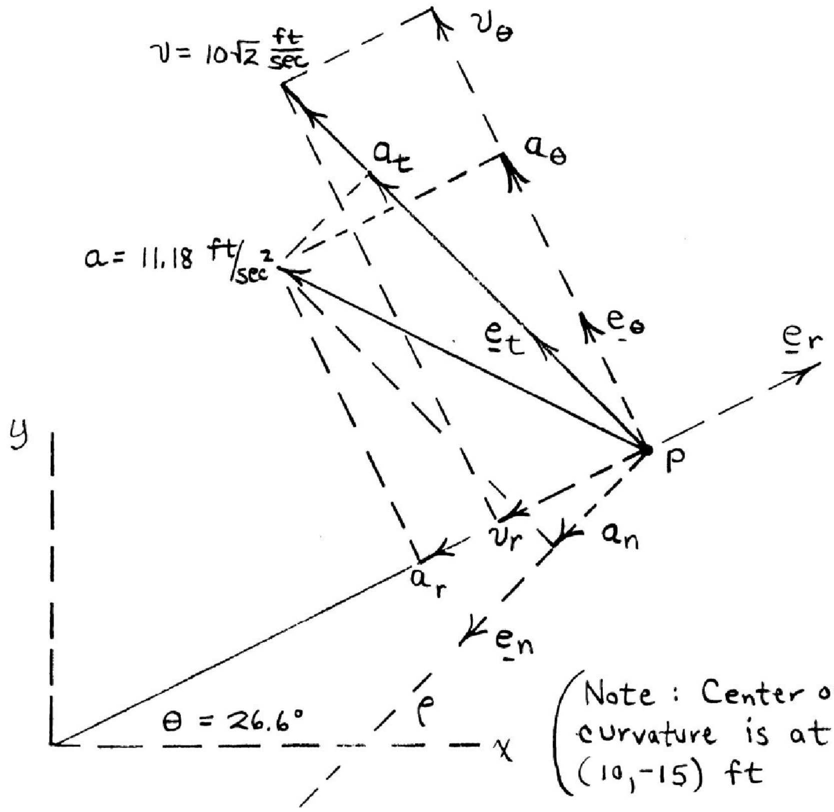
$$a_r = \ddot{r} - r\dot{\theta}^2, \quad \ddot{r} = a_r + r\dot{\theta}^2 = -6.71 + 55.9\,(0.240)^2$$

$$= \underline{-3.49 \ \text{ft/sec}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}, \quad \ddot{\theta} = \frac{1}{r}(a_\theta - 2\dot{r}\dot{\theta})$$

$$= \frac{1}{55.9}\left[8.94 - 2(-4.47)(0.240)\right] = \underline{0.1984 \ \text{rad/sec}^2}$$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(25/50) = \underline{26.6^\circ}$$



(Note : Center of curvature is at (10, −15) ft

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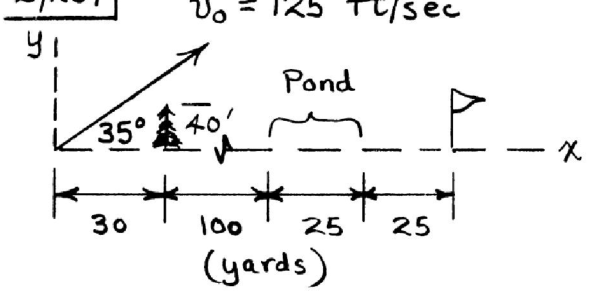


Time to tree: $x = x_0 + v_{x_0} t$: $90 = 0 + 125 \cos 35° t$

$t = 0.879$ sec

Altitude : $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$y = 0 + 125 \sin 35° (0.879) - 16.1 (0.879)^2 = 50.6$ ft

So ball clears (slender) tree.

Flight time (y-eq.) : $0 = 0 + 125 \sin 35° t_f - 16.1 t_f^2$

$t_f = 0$ (launch time) or $t = 4.45$ sec (impact time)

Range (x-eq.) : $R = 0 + 125 \cos 35° (4.45)$

$= 456$ ft or $\underline{152.0 \text{ yd}}$

Ball lands in water hazard!

▶2/240 $\theta = \theta_0 \cos\omega t$, $\dot\theta = -\theta_0\omega \sin\omega t$, $\ddot\theta = -\theta_0\omega^2 \cos pt$

$\dot\phi = K$, $\ddot\phi = 0$, $R = b$, $\dot R = \ddot R = 0$

From Eq. 2/19

$a_R = 0 - bK^2 - b\theta_0^2\omega^2 \sin^2\omega t \cos^2\phi$

$a_\theta = b\cos\phi\,(-\theta_0\omega^2 \cos\omega t) - 2b\,(-\theta_0\omega \sin\omega t)K \sin\phi$

$a_\phi = 0 + b(\theta_0\omega \sin\omega t)^2 \sin\phi \cos\phi$

At A $\cos\omega t = -1$, $\sin\omega t = 0$

$a_R = -bK^2$, $a_\theta = b\omega^2\theta_0 \cos\phi$, $a_\phi = 0$

So $\underline{a = b\sqrt{K^4 + \omega^4\theta_0^2\cos^2\phi}}$

At B $\cos\omega t = 0$, $\sin\omega t = 1$, $\phi = \pi/2$

So $\underline{a = bK\sqrt{K^2 + 4\omega^2\theta_0^2}}$

*2/241 $x_A = x_B$: $0.16 \sin \frac{\pi t}{2} = 0.08t$

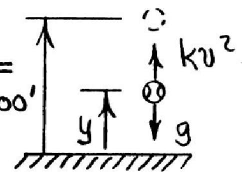
or $2 \sin \frac{\pi t}{2} - t = 0$

Solve numerically to obtain $\underline{t = 1.473 \text{ s}}$

Then $x = 0.08(1.473) = \underline{0.1178 \text{ m}}$

***2/242**

$$a = v\frac{dv}{dy} = -g + kv^2$$



$$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_h^y dy$$

$$\frac{1}{2k}\ln\left[-g+kv^2\right]_0^v = y\Big|_h^y$$

$$\frac{1}{2k}\ln\left[\frac{-g+kv^2}{-g}\right] = y-h \Rightarrow v = \sqrt{\frac{g}{k}\left[1-e^{2k(y-h)}\right]}$$

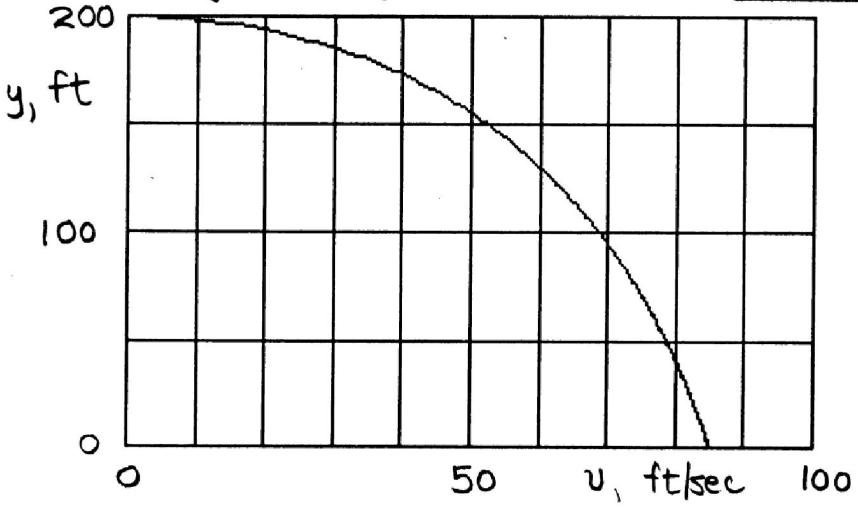
Given numbers : $85 = \sqrt{\frac{32.2}{k}\left[1-e^{2k(0-200)}\right]}$

Numerical solution : $\underline{k = 0.00323 \text{ ft}^{-1}}$

Terminal speed : $g = kv^2$: $32.2 = 0.00323\,v_t^2$

$$\underline{v_t = 99.8 \text{ ft/sec}}$$

Without drag : $v' = \sqrt{2gh} = \sqrt{2(32.2)(200)} = \underline{113.5 \text{ ft/sec}}$



*2/243 $r = 100 - 50\cos\theta$, $\dot{\theta} = 2$ rad/s , $\ddot{\theta} = 0$

$\dot{r} = 50\dot{\theta}\sin\theta = 100\sin\theta$

$\ddot{r} = 100\dot{\theta}\cos\theta = 200\cos\theta$

$v_r = \dot{r} = 100\sin\theta$

$v_\theta = r\dot{\theta} = (100 - 50\cos\theta)2 = 200 - 100\cos\theta$

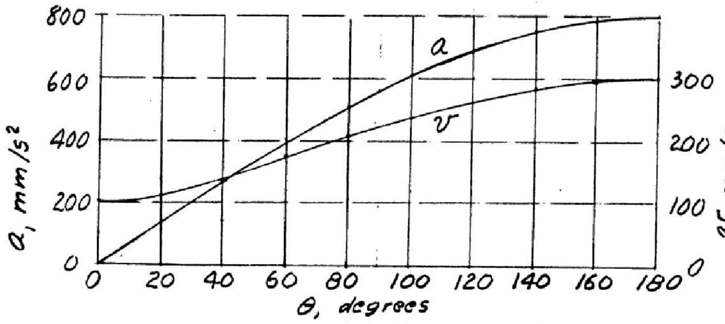
$v = \sqrt{v_r^2 + v_\theta^2} = 100\sqrt{5 - 4\cos\theta}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 200\cos\theta - 4(100 - 50\cos\theta) = 400(\cos\theta - 1)$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(100\sin\theta)2 = 400\sin\theta$

$a = \sqrt{a_r^2 + a_\theta^2} = 400\sqrt{2}\sqrt{1 - \cos\theta}$

[At $\theta = 0$, $a = a_r = 0$ since $\ddot{r}$ & $r\dot{\theta}^2$ cancel for $b = 2c$]



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$$\ddot{\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta} = \frac{g}{l}\cos\theta$$

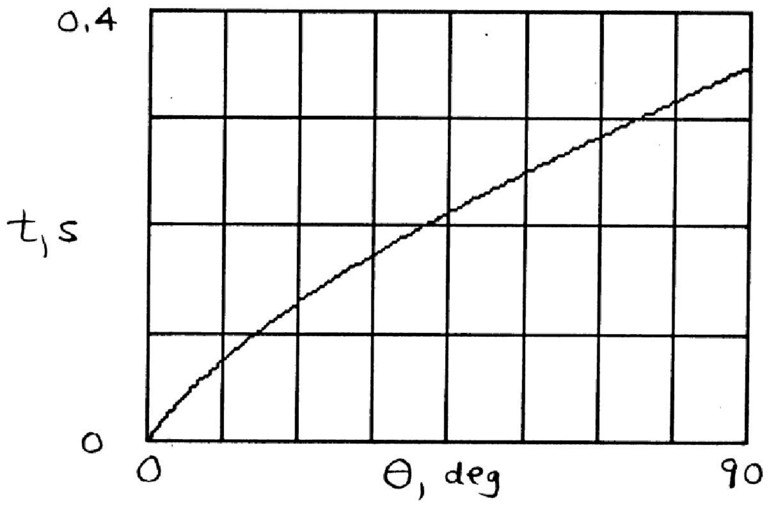
$$\int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \frac{g}{l}\int_0^{\theta} \cos\theta\, d\theta$$

$$\dot{\theta} = \left[\dot{\theta}_0^{\,2} + \frac{2g}{l}\sin\theta\right]^{1/2}$$

Then $\dot{\theta} = \frac{d\theta}{dt} = \left[\dot{\theta}_0^{\,2} + \frac{2g}{l}\sin\theta\right]^{1/2}$

$$t = \int_0^{\theta} \frac{d\theta}{\sqrt{\dot{\theta}_0^{\,2} + \frac{2g}{l}\sin\theta}}$$

With $\dot{\theta}_0 = 2$ rad/s, $l = 0.6$ m, $g = 9.81$, & $\theta = \frac{\pi}{2}$, a numerical integration yields $t' = 0.349$ s.



***2/245** Let $\omega_n = \sqrt{g/l}$:

$$\begin{cases} \theta = \theta_0 \sin \omega_n t \\ \dot\theta = \theta_0 \omega_n \cos \omega_n t \\ \ddot\theta = -\theta_0 \omega_n^2 \sin \omega_n t \end{cases}$$

$$a_t = l\ddot\theta = -l\theta_0 \omega_n^2 \sin \omega_n t = -g\theta_0 \sin \omega_n t$$

$$a_n = l\dot\theta^2 = l\theta_0^2 \omega_n^2 \cos^2 \omega_n t = g\theta_0^2 \cos^2 \omega_n t$$

$$a = \sqrt{a_t^2 + a_n^2} = g\theta_0 \sqrt{\sin^2 \omega_n t + \theta_0^2 \cos^4 \omega_n t}$$

a^2 (and therefore a) is an extreme when

$$\frac{da^2}{d\theta} = 0 = g^2\theta_0^2 \left[2 \sin \omega_n t (\cos \omega_n t) + \theta_0^2 \, 4 \cos^3 \omega_n t (-\sin \omega_n t) \right]$$

$$\Rightarrow \left[1 - 2\theta_0^2 \cos^2 \omega_n t \right] = 0$$

$$\left[1 - 2\left(\tfrac{\pi}{3}\right)^2 \cos^2 \omega_n t \right] = 0, \quad \omega_n t = 0.830 \text{ rad}$$

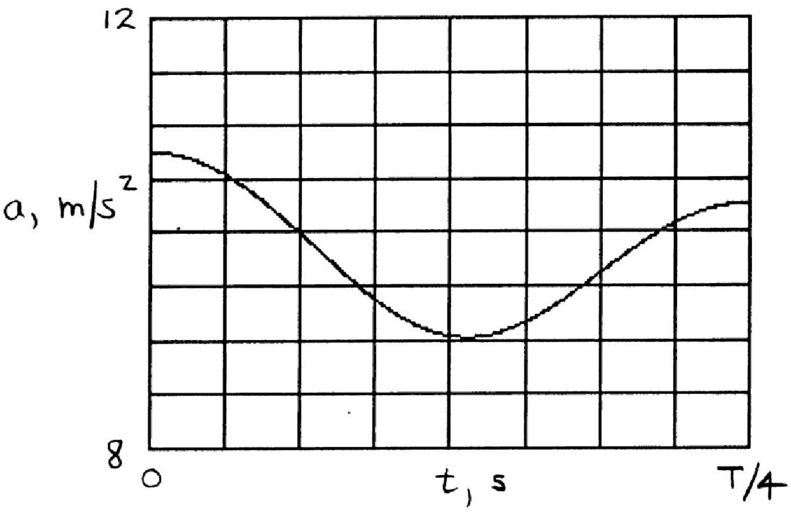
With $\omega_n = \sqrt{g/l} = \sqrt{\frac{9.81}{0.8}} = 3.50 \text{ s}^{-1}$, $t = 0.237$ s

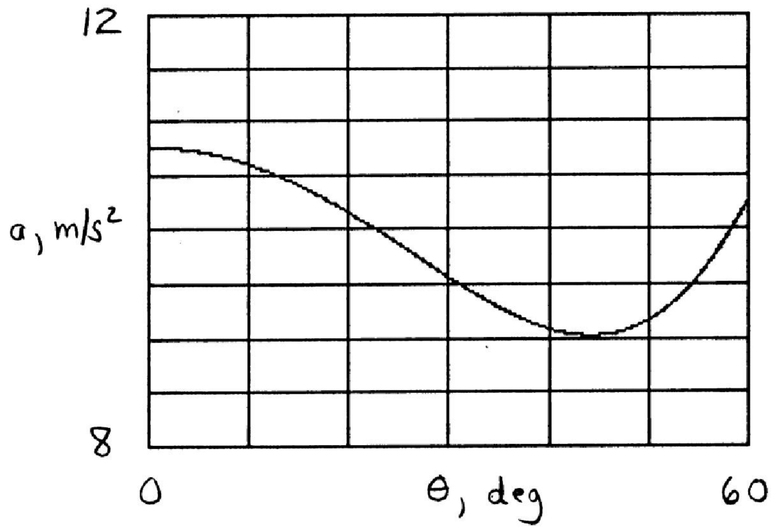
$$\theta = \frac{\pi}{3} \sin(0.830) = 0.772 \text{ rad } (44.3°)$$

As can be seen from the plots below, the above represents a minimum :

$a_{min} = 9.03 \text{ m/s}^2$ @ $\theta = 44.3°$ & $t = 0.237$ s

$a_{max} = 10.76 \text{ m/s}^2$ @ $\theta = 0$ & $t = 0$





Note : Period $T = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.50} = 1.794\text{ s}$

$\frac{T}{4} = 0.449\text{ s}$

*2/246 $\theta = \theta_0 \sin\frac{2\pi t}{\tau} = \frac{\pi}{2}\sin\pi t$, $\dot\theta = \frac{\pi^2}{2}\cos\pi t$, $\ddot\theta = -\frac{\pi^3}{2}\sin\pi t$

$a_n = \ell\dot\theta^2 = \ell\frac{\pi^4}{4}\cos^2\pi t$, $a_t = \ell\ddot\theta = -\ell\frac{\pi^3}{2}\sin\pi t$

$a = \sqrt{a_n^2 + a_t^2} = \frac{\ell\pi^3}{2}\sqrt{\frac{\pi^2}{4}\cos^4\pi t + \sin^2\pi t}$

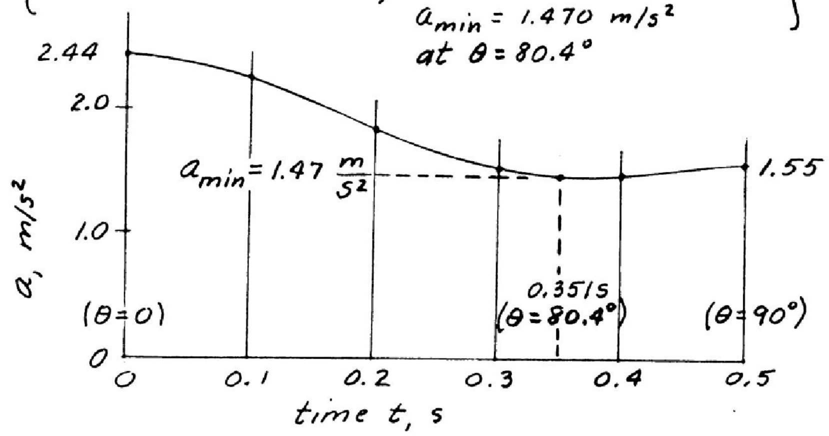
Program & solve for range $0 < t < 0.5$ s

$$\left\{ \frac{da^2}{dt} = \frac{\ell^2\pi^7}{8}\sin 2\pi t\,(2 - \pi^2\cos^2\pi t) = 0 \text{ for max \& min.} \right.$$

$\cos\pi t = \sqrt{2}/\pi$, $\pi t = 1.104$ rad, $t = 0.351$ s

$a_{min} = 1.470$ m/s^2

at $\theta = 80.4°$ $\}$



*2/247 $a = v\dfrac{dv}{dy} = -g - kv^2$

$$\int_{v_0}^{0} \frac{v\,dv}{-g-kv^2} = \int_0^h dy$$

Let $u = g + kv^2$, $du = 2kv\,dv$: $\displaystyle\int_{v=v_0}^{v=0} -\frac{1}{2k}\frac{du}{u} = h$

$$h = -\frac{1}{2k}\ln\left(g+kv^2\right)\Big|_{v_0}^{0} = -\frac{1}{2k}\ln\left(\frac{g}{g+kv_0^2}\right)$$

$$2kh = \ln\left(1+\frac{kv_0^2}{g}\right):\quad 2(5280)k = \ln\left(1+\frac{2000^2 k}{32.2}\right)$$

Solve numerically to obtain $\underline{k = 3.63\,(10^{-4})\ \text{ft}^{-1}}$

$\boxed{*2/248}$ $y = x^2/4$, x & y in inches ; $x = 4 \sin 2t$, t in seconds

$\dot{y} = \frac{x\dot{x}}{2} = 2 \sin 2t\,(8 \cos 2t) = 16 \sin 2t \cos 2t$ in./sec

$\dot{x} = 8 \cos 2t$

$v^2 = \dot{x}^2 + \dot{y}^2 = 64 \cos^2 2t + 256 \sin^2 2t \cos^2 2t$

$= 64 \cos^2 2t\,(1 + 4 \sin^2 2t)$ $(\text{in./sec})^2$

$v = 8 \cos 2t \sqrt{1 + 4 \sin^2 2t}$

$\frac{dv}{dt} = 0$ gives

$1 - 2\sin^2 2t = 1/4$

$\sin 2t = \sqrt{3/8}$

$\cos 2t = \sqrt{5/8}$

$v = 8\sqrt{\frac{5}{8}}\sqrt{1 + 4(3/8)}$

$= \underline{10 \text{ in./sec}}$

$2t = 0.659$ rad

$t = \underline{0.330 \text{ sec}}$

$(= 0.105\pi$ sec$)$

@ $\underline{x = 2.45 \text{ in.}}$