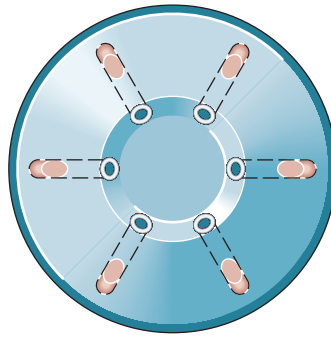Problem 19.1 The moment of inertia of the rotor of the medical centrifuge is $I = 0.2$ kg-m^2. The rotor starts from rest and the motor exerts a constant torque of 0.8 N-m on it.

(a) How much work has the motor done on the rotor when the rotor has rotated through four revolutions?

(b) What is the rotor's angular velocity (in rpm) when it has rotated through four revolutions?

Solution:

(a) $W = (0.8 \text{ N-m})(4 \text{ rev})\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 20.1 \text{ N-m}$ $\boxed{W = 20.1 \text{ N-m}}$

(b) $\dfrac{1}{2}I\omega^2 = W,$ $\dfrac{1}{2}(0.2 \text{ kg-m}^2)\omega^2 = 20.1 \text{ N-m}$ $\Rightarrow$ $\omega = \sqrt{\dfrac{2(20.1 \text{ N-m})}{0.2 \text{ kg-m}^2}}$

$\boxed{\omega = 14.2 \text{ rad/s}}$

Problem 19.2 The 17.8 N slender bar is 0.61 m in length. It started from rest in an initial position relative to the inertial reference frame. When it is in the position shown, the velocity of the end A is $6.71\mathbf{i} + 4.27\mathbf{j}$ (m/s) and the bar has a counterclockwise angular velocity of 12 rad/s. How much work was done on the bar as it moved from its initial position to its present position?
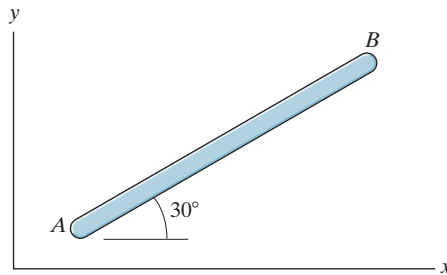
Solution: Work = Change in kinetic energy. To calculate the kinetic energy we will first need to find the velocity of the center of mass

$\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{G/A}$

$= (6.71\mathbf{i} + 4.27\mathbf{j})(\text{m/s}) + (12 \text{ rad/s})\mathbf{k} \times (0.305 \text{ m})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$

$= (-4.27\mathbf{i} + 8.5\mathbf{j}) \text{ m/s}$

Now we can calculate the work, which is equal to the kinetic energy.
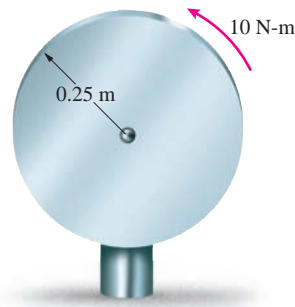
$W = \dfrac{1}{2} mv^2 + \dfrac{1}{2}I\omega^2$

$= \dfrac{1}{2}\left(\dfrac{17.8 \text{ N}}{9.81 \text{ m/s}^2}\right)[(-4.27 \text{ m/s})^2 + (8.5 \text{ m/s})^2]$

$+ \dfrac{1}{2}\left[\dfrac{1}{12}\left(\dfrac{17.8 \text{ N}}{9.81 \text{ m/s}^2}\right)(0.61 \text{ m})^2\right](12 \text{ rad/s})^2$

$= 86.1 \text{ N-m}.$

$\boxed{W = 86.1 \text{ N-m}.}$

Problem 19.3 The 20-kg disk is at rest when the constant 10 N-m counterclockwise couple is applied. Determine the disk's angular velocity (in rpm) when it has rotated through four revolutions (a) by applying the equation of angular motion $\Sigma M = I\alpha$, and (b) by applying the principle of work and energy.

10 N-m

0.25 m

Solution:

(a) First we find the angular acceleration

$$\Sigma M = I\alpha$$

$$(10 \text{ N-m}) = \frac{1}{2}(20 \text{ kg})(0.25 \text{ m})^2\alpha \Rightarrow \alpha = 16 \text{ rad/s}$$

Next we will integrate the angular acceleration to find the angular velocity.

$$\omega\frac{d\omega}{d\theta} = \alpha \Rightarrow \int_0^\omega \omega \, d\omega = \int_0^\theta \alpha \, d\theta \Rightarrow \frac{\omega^2}{2} = \alpha\theta$$

$$\omega = \sqrt{2\alpha\theta} = \sqrt{2(16 \text{ rad/s}^2)(4 \text{ rev})\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)}\left(\frac{60 \text{ s}}{\text{min}}\right) = 271\frac{\text{rev}}{\text{min}}$$

$$\boxed{\omega = 271 \text{ rpm.}}$$
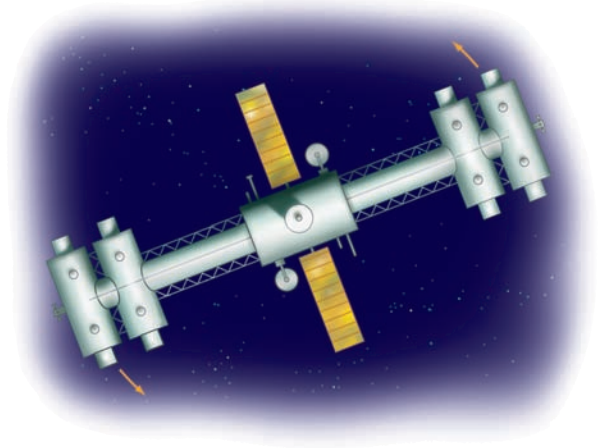
(b) Applying the principle of work energy

$$W = \frac{1}{2}I\omega^2$$

$$(10 \text{ N-m})(4 \text{ rev})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = \frac{1}{2}\left[\frac{1}{2}(20 \text{ kg})(0.25 \text{ m})^2\right]\omega^2$$

$$\omega = 28.4 \text{ rad/s}\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 271 \text{ rpm}$$

$$\boxed{\omega = 271 \text{ rpm.}}$$

Problem 19.4 The space station is initially not rotating. Its reaction control system exerts a constant couple on it until it has rotated 90°, then exerts a constant couple of the same magnitude in the opposite direction so that its angular velocity has decreased to zero when it has undergone a total rotation of 180°. The maneuver takes 6 hours. The station's moment of inertia about the axis of rotation is $I = 1.5 \times 10^{10}$ kg-m². How much work is done in performing this maneuver? In other words, how much energy had to be expended in the form of reaction control fuel?

Solution: We need to solve for the moment that causes a 90° rotation in a 3 hr time period. We will use $\Sigma M = I\alpha$ and the principle of work energy.

$$M = I\alpha = I\frac{d\omega}{dt} \Rightarrow \frac{d\omega}{dt} = \frac{M}{I} \Rightarrow \omega = \frac{M}{I}t$$

$$W = M\theta = \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\frac{2M\theta}{I}}$$

Solving these two equations together, we find

$$M = \frac{2\theta I}{t^2} = \frac{2(90°)\left(\dfrac{\pi \text{ rad}}{180°}\right)(1.5 \times 10^{10} \text{ kg-m}^2)}{\left[(3 \text{ hr})\left(\dfrac{60 \text{ min}}{\text{hr}}\right)\left(\dfrac{60 \text{ min}}{\text{s}}\right)\right]^2} = 404 \text{ N-m}$$

The total work to accomplish the entire maneuver is then

$$W = (404 \text{ N-m})(180°)\left(\frac{\pi \text{ rad}}{180°}\right) = 1270 \text{ N-m}.$$

$\boxed{W = 1270 \text{ N-m.}}$

Problem 19.5 The helicopter's rotor starts from rest. Suppose that its engine exerts a constant 1627 N-m couple on the rotor and aerodynamic drag is negligible. The rotor's moment of inertia is $I = 542$ kg-m^2.

(a) Use work and energy to determine the magnitude of the rotor's angular velocity when it has rotated through five revolutions.

(b) What average power is transferred to the rotor while it rotates through five revolutions?

Solution:

(a) $U = (1627 \text{ N-m})(10\pi \text{ rad}) = \frac{1}{2}(542 \text{ N-s}^2\text{-m})\omega^2$

$\boxed{\omega = 13.7 \text{ rad/s}}$

(b) To find the average power we need to know the time

$1627 \text{ N-m} = (542 \text{ N-s}^2\text{-m})\alpha$

$\alpha = 3 \text{ rad/s}^2, \quad \omega = (3 \text{ rad/s}^2)t, \quad \theta = \frac{1}{2}(3 \text{ rad/s}^2)t^2$

$10\pi \text{ rad} = \frac{1}{2}(3 \text{ rad/s}^2)t^2 \Rightarrow t = 4.58 \text{ s}$

$$\boxed{\begin{array}{l} \text{Power} = \dfrac{U}{t} = \dfrac{(1627 \text{ N})(10\pi \text{ rad})}{4.58 \text{ s}} \\[2mm] \quad = 11171 \text{ N-m/s} = 15.0 \text{ hp} \end{array}}$$

Problem 19.6 The helicopter's rotor starts from rest. The moment exerted on it (in N-m) is given as a function of the angle through which it has turned in radians by $M = 6500 - 20\theta$. The rotor's moment of inertia is $I = 540$ kg-m^2. Determine the rotor's angular velocity (in rpm) when it has turned through 10 revolutions.

Solution: We will integrate to find the work.

$$W = \int M d\theta$$

$$= \int_0^{10(2\pi)} (6500 - 20\theta)\, d\theta = [6500\,\theta - 10\,\theta^2]_0^{10(2\pi)} = 3.69 \times 10^5 \text{ N-m}$$

Using the principle of work energy we can find the angular velocity.

$$W = \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\frac{2W}{I}} = \sqrt{\frac{2(3.69 \times 10^5 \text{ N-m})}{540 \text{ kg-m}^2}} = 37.0 \text{ rad/s}$$

$$\omega = 37.0 \text{ rad/s} \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 353 \text{ rpm}.$$

$\boxed{\omega = 353 \text{ rpm}.}$

Problem 19.7 During extravehicular activity, an astronaut's angular velocity is initially zero. She activates two thrusters of her maneuvering unit, exerting equal and opposite forces $T = 2$ N. The moment of inertia of the astronaut and her equipment about the axis of rotation is 45 kg-m^2. Use the principle of work and energy to determine the angle through which she has rotated when her angular velocity reaches 15° per second.

1.0 m

T

T
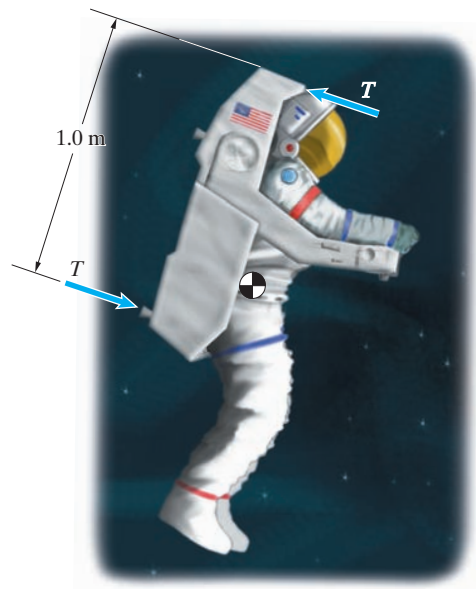
Solution: The moment that is exerted on the astronaut is

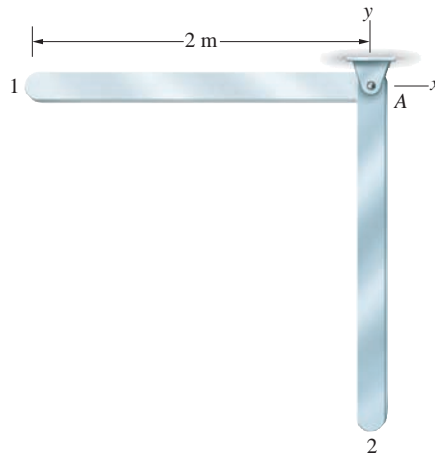$$M = Td = (2 \text{ N})(1 \text{ m}) = 2 \text{ N-m}.$$

Using work energy we have

$$M\theta = \frac{1}{2}I\omega^2 \Rightarrow \theta = \frac{I\omega^2}{2M} = \frac{(45 \text{ kg-m}^2)\left(\dfrac{15°}{180°}\pi \text{ rad/s}\right)^2}{2(2 \text{ N-m})} = 0.771 \text{ rad}.$$

Thus

$$\theta = 0.771 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 44.2°$$

$$\boxed{\theta = 44.2°}$$

Problem 19.8 The 8-kg slender bar is released from rest in the horizontal position 1 and falls to position 2.

(a) How much work is done by the bar's weight as it falls from position 1 to position 2?
(b) How much work is done by the force exerted on the bar by the pin support as the bar falls from position 1 to position 2?
(c) Use conservation of energy to determine the bar's angular velocity when it is in position 2.

Solution:

(a) $W = mgh = (8 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m}) = 78.5$ N-m. $\boxed{W = 78.5 \text{ N-m.}}$
(b) The pin force is applied to point A, which does not move. Therefore $\boxed{W = 0.}$
(c) Using conservation of energy, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 - mgh \Rightarrow \omega = \frac{\sqrt{6gh}}{L} = \frac{\sqrt{6(9.81 \text{ m/s}^2)(1 \text{ m})}}{2 \text{ m}}$$

$$\boxed{\omega = 3.84 \text{ rad/s}}$$

Problem 19.9 The 20-N bar is released from rest in the horizontal position 1 and falls to position 2. In addition to the force exerted on it by its weight, it is subjected to a constant counterclockwise couple $M = 30$ N-m. Determine the bar's counterclockwise angular velocity in position 2.
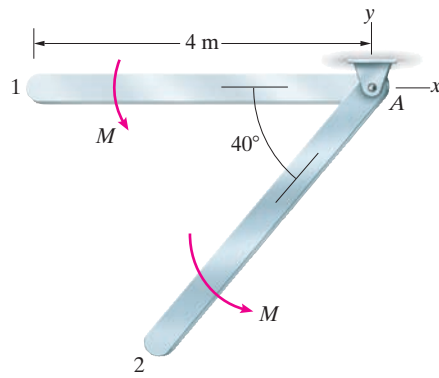
Solution: We will use the energy equation in the form

$$T_1 + V_1 + U_{12} = T_2 + V_2$$

$$0 + 0 + M\theta = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 - mg\frac{L}{2}\sin\theta$$

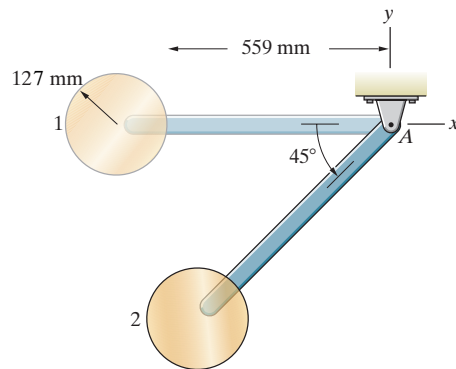$$\Rightarrow \omega = \sqrt{\frac{6M\theta}{mL^2} + \frac{3g}{L}\sin\theta}$$

Thus

$$\omega = \sqrt{\frac{6(30 \text{ N-m})\left(\frac{40^\circ}{180^\circ}\pi \text{ rad}\right)}{\left(\frac{20 \text{ N}}{9.81 \text{ m/s}^2}\right)(4 \text{ m})^2} + \frac{3(9.81 \text{ m/s}^2)}{4 \text{ m}}\sin(40^\circ)} = 2.93 \text{ rad/s}.$$

$$\boxed{\omega = 2.93 \text{ rad/s.}}$$

Problem 19.10 The object consists of an 35.6 N slender bar welded to a circular disk. When the object is released from rest in position 1, its angular velocity in position 2 is 4.6 rad/s. What is the weight of the disk?

Solution: Using conservation of energy we have

$$T_1 + V_1 = T_2 + V_2 \Rightarrow 0 + 0 = T_2 + V_2$$

$$0 = \frac{1}{2}\left(\frac{1}{3}\left[\frac{35.6 \text{ N}}{9.81 \text{ m/s}^2}\right][0.559 \text{ m}]^2 + \frac{1}{2}\left[\frac{W \text{ s}^2}{9.81 \text{ m}}\right][0.127 \text{ m}]^2 + \left[\frac{W \text{ s}^2}{9.81 \text{ m}}\right][0.686]^2\right)(4.6 \text{ rad/s})^2$$

$$- (35.6 \text{ N})(0.280 \text{ m})\sin(45^\circ) - W(0.686)\sin(45^\circ)$$

Solving for W we find

$$W = 98.7 \text{ N}.$$

Problem 19.11 The object consists of an 35.6 N slender bar welded to a 53.4 N circular disk The object is released from rest in position 1. Determine the x and y components of force exerted on the object by the pin support when it is in position 2.
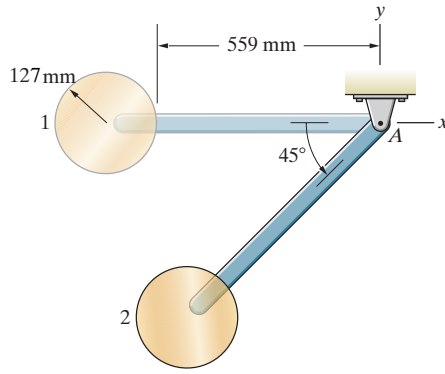
Solution: We first determine the moment of inertia about the fixed point A and the distance from A to the center of mass.

$$I_A = \frac{1}{3}\left(\frac{35.6\ \text{N-s}^2}{9.81\ \text{m}}\right)(0.559)^2 + \frac{1}{2}\left(\frac{53.4\ \text{N-s}^2}{9.81\ \text{m}}\right)(0.127\ \text{m})^2 + \left(\frac{53.4\ \text{N-s}^2}{9.81\ \text{m}}\right)(0.686\ \text{m})^2$$

$$= 2.98\ \text{N-s}^2\text{-m}$$

$$d = \frac{(35.6\ \text{N})(0.280\ \text{m}) + (53.4\ \text{N})(0.686\ \text{m})}{(35.6\ \text{N}) + (53.4\ \text{N})} = 0.523\ \text{m}.$$

We now write the equations of motion and the work energy equation

$$\Sigma F_x : A_x = \left(\frac{89\ \text{N-s}^2}{9.81\ \text{m}}\right)(\alpha\, d \sin 45° + \omega^2 d \cos 45°),$$

$$\Sigma F_y : A_y - (89\ \text{N}) = \left(\frac{89\ \text{N-s}^2}{9.81\ \text{m}}\right)(-\alpha\, d \cos 45° + \omega^2 d \sin 45°),$$

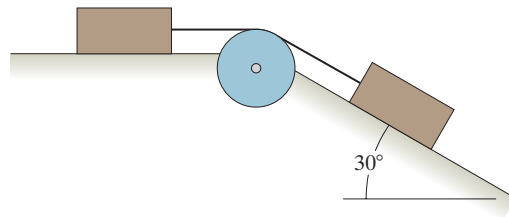$$\Sigma M_A : (89\ \text{N})\, d \cos 45° = I_A \alpha,$$

$$0 + 0 = -(89\ \text{N})\, d \sin 45° + \frac{1}{2} I_A \omega^2.$$

Solving we have

$$\omega = 4.70\ \text{rad/s},\ \ \alpha = 11.0\ \text{rad/s}^2,\ \ A_x = 111.2\ \text{N},\ \ A_y = 125.9\ \text{N}.$$

$\boxed{A_x = 111.2\ \text{N},\ A_y = 125.9\ \text{N}}$

Problem 19.12 The mass of each box is 4 kg. The radius of the pulley is 120 mm and its moment of inertia is 0.032 kg-m^2. The surfaces are smooth. If the system is released from rest, how fast are the boxes moving when the left box has moved 0.5 m to the right?

Solution: Use conservation of energy. The angular velocity of the pulley is v/r.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(4\ \text{kg})v^2 + \frac{1}{2}(4\ \text{kg})v^2 + \frac{1}{2}(0.032\ \text{kg-m}^2)\left(\frac{v}{0.12\ \text{m}}\right)^2$$

$$- (4\ \text{kg})(9.81\ \text{m/s}^2)(0.5\ \text{m}) \sin 30°$$

Solving we have $\boxed{v = 1.39\ \text{m/s}.}$

Problem 19.13 The mass of each box is 4 kg. The radius of the pulley is 120 mm and its moment of inertia is 0.032 kg-m². The coefficient of kinetic friction between the boxes and the surfaces is $\mu_k = 0.12$. If the system is released from rest, how fast are the boxes moving when the left box has moved 0.5 m to the right?


30°
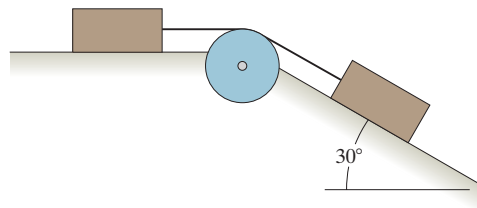
Solution: Use work energy. The angular velocity of the pulley is v/r.

$$T_1 + V_1 + U_{12} = T_2 + V_2$$

$$0 + 0 - (0.12)(8 \text{ kg})(9.81 \text{ m/s}^2)(0.5 \text{ m}) = \frac{1}{2}(8 \text{ kg})v^2 + \frac{1}{2}(0.032 \text{ kg-m}^2)\left(\frac{v}{0.12 \text{ m}}\right)^2$$

$$- (4 \text{ kg})(9.81 \text{ m/s}^2)(0.5 \text{ m}) \sin 30°$$

Solving we have $v = 1.00$ m/s.

Problem 19.14 The 4-kg bar is released from rest in the horizontal position 1 and falls to position 2. The unstretched length of the spring is 0.4 m and the spring constant is $k = 20$ N/m. What is the magnitude of the bar's angular velocity when it is in position 2.

|←—0.6 m—→|←——1 m——→|

1   60°   A

2
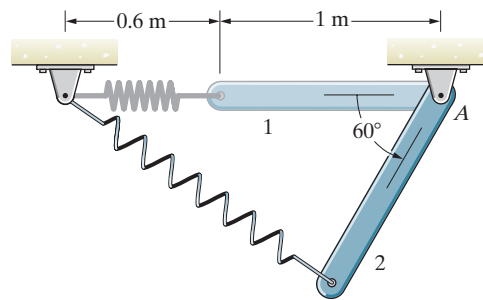
Solution: In position 1 the spring is stretched a distance

$$d_1 = 0.6 \text{ m} - 0.4 \text{ m} = 0.2 \text{ m},$$

while in position 2 the spring is stretched a distance

$$d_2 = \sqrt{(1.6 \text{ m})^2 + (1 \text{ m})^2 - 2(1.6 \text{ m})(1 \text{ m}) \cos 60°} - 0.4 \text{ m} = 1.0 \text{ m}$$

Using conservation of energy we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}kd_1^2 = \frac{1}{2}\left[\frac{1}{3}(4 \text{ kg})(1 \text{ m})^2\right]\omega^2$$

$$- (4 \text{ kg})(9.81 \text{ m/s}^2)(0.5 \text{ m}) \sin 60° + \frac{1}{2}kd_2^2$$

Solving we find $\omega = 3.33$ rad/s.

Problem 19.15 The moments of inertia of gears that can turn freely on their pin supports are $I_A = 0.002$ kg-m^2 and $I_B = 0.006$ kg-m^2. The gears are at rest when a constant couple $M = 2$ N-m is applied to gear B. Neglecting friction, use principle of work and energy to determine the angular velocities of the gears when gear A has turned 100 revolutions.
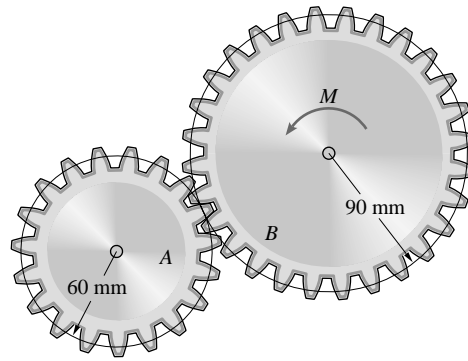
Solution: From the principle of work and energy: $U = T_2 - T_1$, where $T_1 = 0$ since the system starts from rest. The work done is

$$U = \int_0^{\theta_B} M \, d\theta = M\theta_B.$$

Gear B rotates in a positive direction; gear A rotates in a negative direction, $\theta_A = -2\pi(100) = -200\pi$ rad. The angle traveled by the gear B is

$$\theta_B = -\frac{r_A}{r_B}\theta_A = -\left(\frac{0.06}{0.09}\right)(-200\pi) = 418.9 \text{ rad},$$

from which $U = M\theta_B = 2(418.9) = 837.76$ N-m.

The kinetic energy is $T_2 = \left(\frac{1}{2}\right) I_A \omega_A^2 + \left(\frac{1}{2}\right) I_B \omega_B^2$,

where $\omega_B = -\left(\frac{r_A}{r_B}\right)\omega_A$, from which

$$U = \left(\frac{1}{2}\right)\left(0.002 + \left(\frac{0.06}{0.09}\right)^2 (0.006)\right)\omega_A^2 = 837.76 \text{ N-m},$$

from which

$$\boxed{\omega_A = \pm\sqrt{359{,}039} = -599.2 \text{ rad/s}},$$

and $\boxed{\omega_B = -\left(\frac{0.06}{0.09}\right)\omega_A = 399.5 \text{ rad/s}}$

Problem 19.16 The moments of inertia of gears A and B are $I_A = 0.02$ kg-m^2 and $I_B = 0.09$ kg-m^2. Gear A is connected to a torsional spring with constant $k = 12$ N-m/rad. If gear B is given an initial counterclockwise angular velocity of 10 rad/s with the torsional spring unstretched, through what maximum counterclockwise angle does gear B rotate?
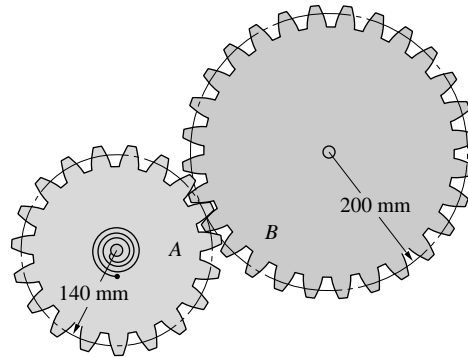
Solution: The counterclockwise velocity of B and clockwise velocity of A are related by $0.2\omega_B = 0.14\omega_A$.

Applying conservation of energy,

$$\frac{1}{2}I_A\omega_{A1}^2 + \frac{1}{2}I_B\omega_{B1}^2 = \frac{1}{2}k\theta_{A2}^2:$$

$$\frac{1}{2}(0.02)\left[\left(\frac{0.2}{0.14}\right)(10)\right]^2 + \frac{1}{2}(0.09)(10)^2 = \frac{1}{2}(12)\theta_{A2}^2$$
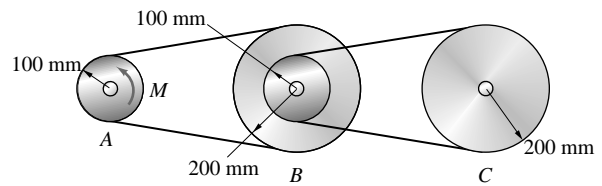
Solving, we obtain

$$\theta_{A2} = 1.04 \text{ rad}$$

so $\theta_{B2} = \frac{0.14}{0.2}\theta_{A2}$

$$= 0.731 \text{ rad} = 41.9°.$$

Problem 19.17 The moments of inertia of three pulleys that can turn freely on their pin supports are $I_A = 0.002$ kg-m^2, $I_B = 0.036$ kg-m^2, and $I_C = 0.032$ kg-m^2. They are stationary when a constant couple $M = 2$ N-m is applied to pulley A. What is the angular velocity of pulley A when it has turned 10 revolutions?

Solution: All pulleys rotate in a positive direction:

$$\omega_C = \left(\frac{0.1}{0.2}\right)\omega_B,$$

$$\omega_B = \left(\frac{0.1}{0.2}\right)\omega_A,$$

from which

$$\omega_C = \left(\frac{0.1}{0.2}\right)^2 \omega_A.$$

From the principle of work and energy: $U = T_2 - T_1$, where $T_1 = 0$ since the pulleys start from a stationary position. The work done is

$$U = \int_0^{\theta_A} M d\theta = 2(2\pi)(10) = 40\pi \text{ N-m}.$$

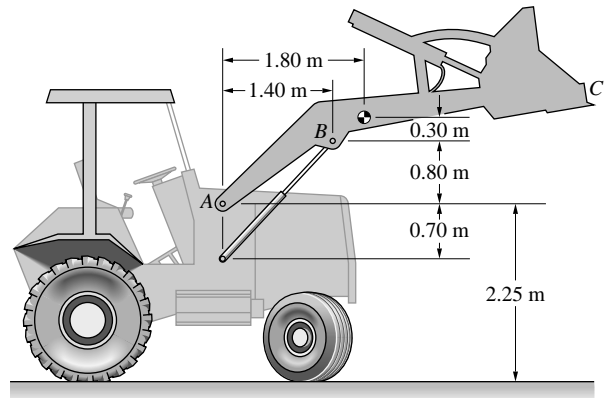The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right)I_A\omega_A^2 + \left(\frac{1}{2}\right)I_B\omega_B^2 + \left(\frac{1}{2}\right)I_C\omega_C^2$$

$$= \left(\frac{1}{2}\right)\left(I_A + \left(\frac{0.1}{0.2}\right)^2 I_B + \left(\frac{0.1}{0.2}\right)^4 I_C\right)\omega_A^2,$$

from which $T_2 = 0.0065\omega_A^2$, and $U = 40\pi = 0.0065\omega_A^2$.

Solve: $\boxed{\omega_A = \sqrt{\dfrac{40\pi}{0.0065}} = 139.0 \text{ rad/s}}$

Problem 19.18 Model the arm ABC as a single rigid body. Its mass is 300 kg, and the moment of inertia about its center of mass is $I = 360$ kg-m^2. Starting from rest with its center of mass 2 m above the ground (position 1), the ABC is pushed upward by the hydraulic cylinders. When it is in the position shown (position 2), the arm has a counterclockwise angular velocity of 1.4 rad/s. How much work do the hydraulic cylinders do on the arm in moving it from position 1 to position 2?

Solution: From the principle of work and energy: $U = T_2 - T_1$, where $T_1 = 0$ since the system starts from rest. The work done is $U = U_{\text{cylinders}} - mg(h_2 - h_1)$, and the kinetic energy is

$$T_2 = \left(\frac{1}{2}\right)I_A\omega^2,$$

from which

$$U_{\text{cylinders}} - mg(h_2 - h_1) = \left(\frac{1}{2}\right)I_A\omega^2$$

In position 1, $h_1 = 2$ m above the ground. In position 2, $h_2 = 2.25 + 0.8 + 0.3 = 3.35$ m. The distance from A to the center of mass is
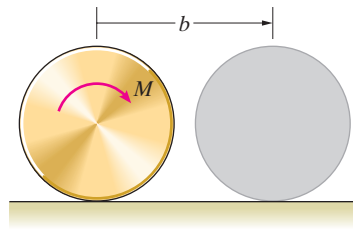
$$d = \sqrt{1.8^2 + 1.1^2} = 2.11 \text{ m},$$

from which

$$I_A = I + md^2 = 1695 \text{ kg-m}^2.$$

Substitute: $U_{\text{cylinders}} - 3973.05 = 1661.1$, from which

$$\boxed{U_{\text{cylinders}} = 5630 \text{ N-m}}$$

Problem 19.19 The mass of the circular disk is 5 kg and its radius is $R = 0.2$ m. The disk is stationary when a constant clockwise couple $M = 10$ N-m is applied to it, causing the disk to roll toward the right. Consider the disk when its center has moved a distance $b = 0.4$ m.

(a) How much work has the couple M done on the disk?
(b) How much work has been done by the friction force exerted on the disk by the surface?
(c) What is the magnitude of the velocity of the center of the disk?

(See Active Example 19.1.)

Solution:

(a) $U_{12} = M\theta = M\left(\dfrac{b}{R}\right) = (10 \text{ N-m})\left(\dfrac{0.4 \text{ m}}{0.2 \text{ m}}\right) = 20$ N-m. $\boxed{U_{12} = 20 \text{ N-m.}}$

(b) The friction force is a workless constraint force and does no work. $\boxed{U_{12} = 0}$.

(c) Use work energy

$$U_{12} = \frac{1}{2}\, mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$20 \text{ N-m} = \frac{1}{2}(5 \text{ kg})v^2 + \frac{1}{2}\left(\frac{1}{2}[5 \text{ kg}][0.2 \text{ m}]^2\right)\left(\frac{v}{0.2 \text{ m}}\right)^2$$

Solving we find $\boxed{v = 2.31 \text{ m/s.}}$

Problem 19.20 The mass of the homogeneous cylindrical disk is $m = 5$ kg and its radius is $R = 0.2$ m. The angle $\beta = 15°$. The disk is stationary when a constant clockwise couple $M = 10$ N-m is applied to it. What is the velocity of the center of the disk when it has moved a distance $b = 0.4$ m? (See Active Example 19.1.)
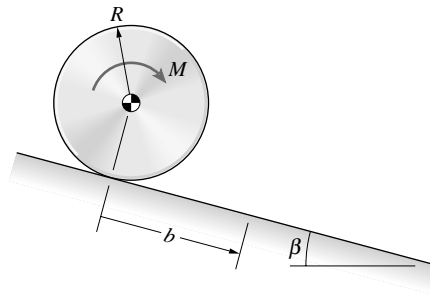
Solution: The angle through which the disk rolls is $\theta = b/R$. The work done by the couple and the disk's weight is
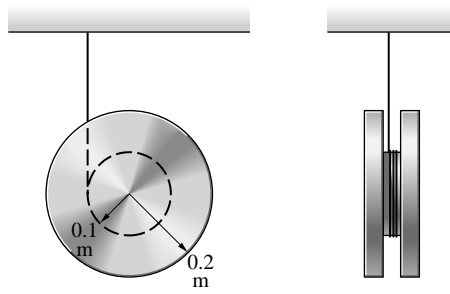
$$v_{12} = M\left(\frac{b}{R}\right) + mgb\sin\beta.$$

Equating the work to the disk's kinetic energy

$$M\left(\frac{b}{R}\right) + mgb\sin\beta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

and using the relation $\omega = v/R$, we obtain

$$v = 2\sqrt{\frac{b}{3}\left(\frac{M}{mR} + g\sin\beta\right)}$$

$$= 2\sqrt{\frac{0.4}{3}\left[\frac{10}{(5)(0.2)} + 9.81\sin 15°\right]}$$

$$= 2.59 \text{ m/s.}$$

Problem 19.21   The mass of the stepped disk is 18 kg and its moment of inertia is 0.28 kg-m^2. If the disk is released from rest, what is its angular velocity when the center of the disk has fallen 1 m?

**Solution:**   The work done by the disk's weight is

$$v_{12} = mgh = (18 \text{ kg}) (9.81 \text{ m/s}^2) (1 \text{ m})$$

$$= 176.6 \text{ N-m}.$$

We equate the work to the final kinetic energy,

$$176.6 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}(18)v^2 + \frac{1}{2}(0.28)\omega^2.$$

Using the relation $v = (0.1)\omega$ and solving for ω, we obtain
$\omega = 27.7 \text{ rad/s}.$

Problem 19.22   The 100-kg homogenous cylindrical disk is at rest when the force $F = 500$ N is applied to a cord wrapped around it, causing the disk to roll. Use the principle of work and energy to determine the angular velocity of the disk when it has turned one revolution.
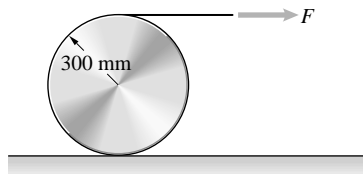
Solution: From the principle of work and energy: $U = T_2 - T_1$, where $T_1 = 0$ since the disk is at rest initially. The distance traveled in one revolution by the center of the disk is $s = 2\pi R = 0.6\pi$ m. As the cord unwinds, the force F acts through a distance of 2 s. The work done is

$$U = \int_0^{2s} F \, ds = 2 \, F(0.6\pi) = 1884.96 \text{ N-m}.$$

The kinetic energy is

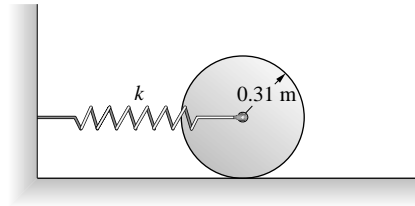$$T_2 = \left(\frac{1}{2}\right) I\omega^2 + \left(\frac{1}{2}\right) mv^2,$$

where $I = \frac{1}{2}mR^2$, and $v = R\omega$, from which

$$T_2 = \frac{3}{4}mR^2\omega^2 = 6.75\omega^2.$$

$U = T_2$, $1884.96 = 6.75\omega^2$,

from which $\omega = -16.7 \text{ rad/s}$ (clockwise).

Problem 19.23 The 15 kg homogenous cylindrical disk is given a clockwise angular velocity of 2 rad/s with the spring unstretched. The spring constant is $k = 43.8$ N/m. If the disk rolls, how far will its center move to the right?

Solution: From the principle of work and energy: $U = T_2 - T_1$, where $T_2 = 0$ since the disk comes to rest at point 2. The work done by the spring is

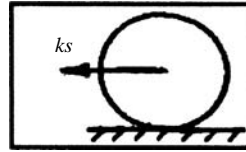$$U = -\frac{1}{2}kS^2.$$

The kinetic energy is

$$T_1 = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2.$$

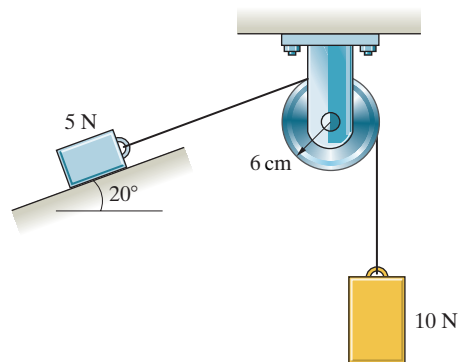By inspection $v = \omega R$, from which

$$T_1 = \left(\frac{1}{2}\right)\left(\frac{m}{2}R^2 + mR^2\right)\omega^2 = \frac{3}{4}mR^2\omega^2 = 0.75(2^2) = 4.07 \text{ N-m},$$

$$U = -T_1, - (43.8)\ S^2 = -4.07,$$

from which $S = \sqrt{0.186} = 0.43$ m.

Problem 19.24 The system is released from rest. The moment of inertia of the pulley is 0.04 kg-m^2. The slanted surface is smooth. Determine the magnitude of the velocity of the 10 N weight when it has fallen 2 m.
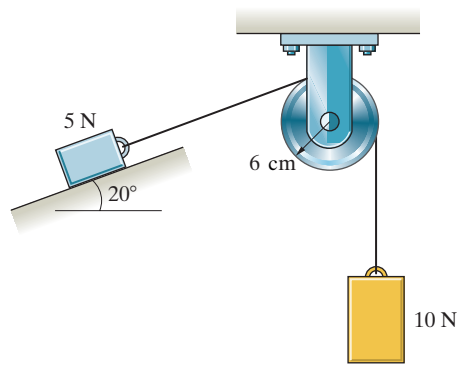

Solution: Use conservation of energy

$$T_1 = 0$$

$$V_1 = 0$$

$$T_2 = \frac{1}{2}\left(\frac{15 \text{ N-s}^2}{9.81 \text{ m}}\right)v^2 + \frac{1}{2}(0.04 \text{ kg-m}^2)\left(\frac{v}{0.06 \text{ m}}\right)^2$$

$$V_2 = -(10 \text{ N})(2 \text{ m}) + (5 \text{ N})(2 \text{ m})\sin 20°$$

$$T_1 + V_1 = T_2 + V_2 \Rightarrow \boxed{v = 1.62 \text{ m/s.}}$$

Problem 19.25 The system is released from rest. The moment of inertia of the pulley is 0.04 kg-m^2. The coefficient of kinetic friction between the 5-N weight and the slanted surface is $\mu_k = 0.3$. Determine the magnitude of the velocity of the 10-N weight when it has fallen 2 m.
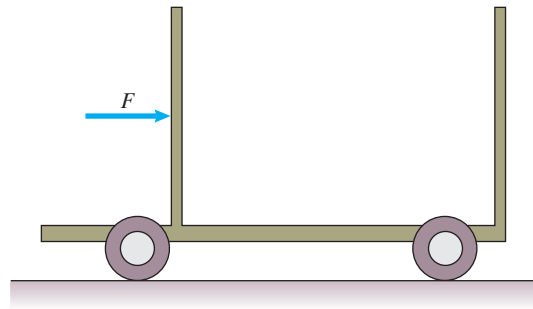
Solution: Use work energy.

$T_1 = 0$

$V_1 = 0$

$U_{12} = -(0.3)(5 \text{ N}) \cos 20° (2 \text{ m})$

$T_2 = \dfrac{1}{2} \left(\dfrac{15 \text{ N-s}^2}{9.81 \text{ m}} \right) v^2 + \dfrac{1}{2} (0.04 \text{ kg-m}^2) \left(\dfrac{v}{0.06 \text{ m}} \right)^2$

$V_2 = -(10 \text{ N})(2 \text{ m}) + (5 \text{ N})(2 \text{ m}) \sin 20°$

$T_1 + V_1 + U_{12} = T_2 + V_2 \Rightarrow \boxed{v = 1.48 \text{ m/s.}}$

Problem 19.26 Each of the cart's four wheels weighs 10 N, has a radius of 5 cm, and has moment of inertia $I = 0.02$ kg-m^2. The cart (not including its wheels) weighs 80 N. The cart is stationary when the constant horizontal force $F = 40$ N is applied. How fast is the cart going when it has moved 2 m to the right?

Solution: Use work energy

$U_{12} = Fd = (40 \text{ N})(2 \text{ m}) = 80 \text{ N-m}$

$T_1 = 0$

$T_2 = \dfrac{1}{2} \left(\dfrac{80 \text{ N-s}^2}{9.81 \text{ m}} \right) v^2 + 4 \left[\dfrac{1}{2} \left(\dfrac{10 \text{ N-s}^2}{9.81 \text{ m}} \right) v^2 + \dfrac{1}{2}(0.02 \text{ kg-m}^2) \left(\dfrac{v}{0.05 \text{ m}} \right)^2 \right]$

$T_1 + U_{12} = T_2 \Rightarrow \boxed{v = 1.91 \text{ m/s.}}$

Problem 19.27 The total moment of inertia of car's two rear wheels and axle is I_R, and the total moment of inertia of the two front wheels is I_F. The radius of the tires is R, and the total mass of the car, including the wheels, is m. The car is moving at velocity v_0 when the driver applies the brakes. If the car's brakes exert a constant retarding couple M on each wheel and the tires do not slip, determine the car's velocity as a function of the distance s from the point where the brakes are applied.
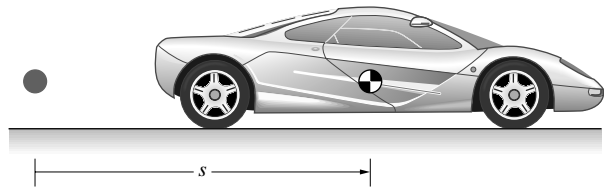
Solution: When the car rolls a distance s, the wheels roll through an angle s/R so the work done by the brakes is $V_{12} = -4M(s/R)$. Let m_F be the total mass of the two front wheels, m_R the mass of the rear wheels and axle, and m_c the remainder of the car's mass. When the car is moving at velocity v its total kinetic energy is

$$T = \tfrac{1}{2}m_c v^2 + \tfrac{1}{2}m_R v^2 + \tfrac{1}{2}I_R(v/R)^2 + \tfrac{1}{2}I_F(v/R)^2$$

$$= \tfrac{1}{2}[m + (I_R + I_F)/R^2]v^2.$$

From the principle of work and energy, $V_{12} = T_2 - T_1$:

$$-4M(s/R) = \tfrac{1}{2}[m + (I_R + I_F)/R^2]v^2 - \tfrac{1}{2}[m + (I_R + I_F)/R^2]v_0^2$$

Solving for v, we get $v = \sqrt{v_0^2 - 8Ms/[Rm + (I_R + I_F)/R]}$.

Problem 19.28 The total moment of inertia of the car's two rear wheels and axle is 0.24 kg-m^2. The total moment of inertia of the two front wheels is 0.2 kg-m^2. The radius of the tires is 0.3 m. The mass of the car, including the wheels, is 1480 kg. The car is moving at 100 km/h. If the car's brakes exert a constant retarding couple of 650 N-m on each wheel and the tires do not slip, what distance is required for the car to come to a stop? (See Example 19.2.)

Solution: From the solution of Problem 19.27, the car's velocity when it has moved a distance s is

$$v = \sqrt{v_0^2 - 8Ms/[Rm + (I_R + I_F)/R]}.$$

Setting $v = 0$ and solving for s we obtain

$$s = [Rm + (I_R + I_F)/R]v_0^2/(8M).$$

The car's initial velocity is

$$v_0 = 100{,}000/3600 = 27.8 \text{ m/s}$$

So, $s = [(0.3)(1480) + (0.24 + 0.2)/0.3](27.8)^2/[8(650)] = 66.1$ m.

Problem 19.29 The radius of the pulley is $R = 100$ mm and its moment of inertia is $I = 0.1$ kg-m^2. The mass $m = 5$ kg. The spring constant is $k = 135$ N/m. The system is released from rest with the spring unstretched. Determine how fast the mass is moving when it has fallen 0.5 m.
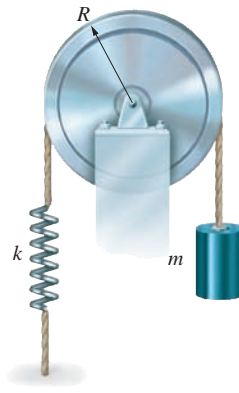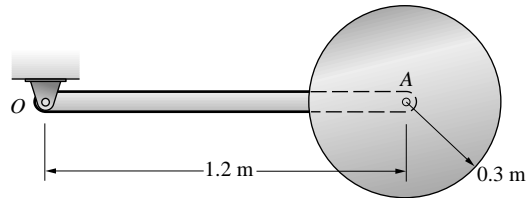

Solution:

$$T_1 = 0, \quad V_1 = 0, \quad T_2 = \frac{1}{2}(5 \text{ kg})v^2 + \frac{1}{2}(0.1 \text{ kg-m}^2)\left(\frac{v}{0.1 \text{ m}}\right)^2$$

$$V_2 = -(5 \text{ kg})(9.81 \text{ m/s}^2)(0.5 \text{ m}) + \frac{1}{2}(135 \text{ N/m})(0.5 \text{ m})^2$$

$$T_1 + V_1 = T_2 + V_2 \quad \Rightarrow \quad \boxed{v = 1.01 \text{ m/s}}$$

Problem 19.30 The masses of the bar and disk are 14 kg and 9 kg, respectively. The system is released from rest with the bar horizontal. Determine the angular velocity of the bar when it is vertical if the bar and disk are welded together at A.

Solution: The work done by the weights of the bar and disk as they fall is

$$U_{12} = m_{\text{bar}}g(0.6 \text{ m}) + m_{\text{disk}}g(1.2 \text{ m})$$

$$= (14)(9.81)(0.6) + (9)(9.81)(1.2)$$

$$= 188.4 \text{ N-m}.$$

The bar's moment of inertia about the pinned end is

$$I_0 = \tfrac{1}{3}m_{\text{bar}}l^2 = \tfrac{1}{3}(14)(1.2)^2$$

$$= 6.72 \text{ kg-m}^2,$$

So the bar's final kinetic energy is

$$T_{\text{bar}} = \tfrac{1}{2}I_0\omega^2$$

$$= 3.36\omega^2.$$

The moment of inertia of the disk about A is

$$I_A = \tfrac{1}{2}m_{\text{disk}}R^2 = \tfrac{1}{2}(9)(0.3)^2$$

$$= 0.405 \text{ kg-m}^2,$$

so the disk's final kinetic energy is

$$T_{\text{disk}} = \tfrac{1}{2}m_{\text{disk}}(l\omega)^2 + \tfrac{1}{2}I_A\omega^2$$

$$= \tfrac{1}{2}[(9)(1.2)^2 + 0.405]\omega^2$$

$$= 6.68\omega^2.$$
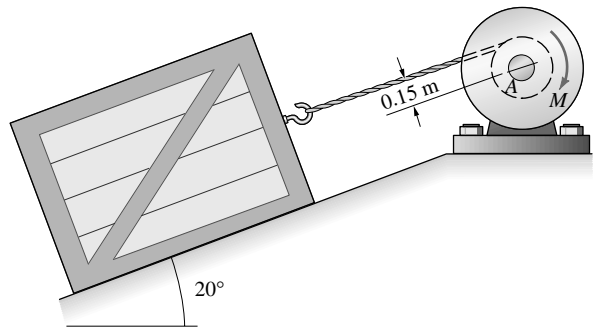
Equating the work to the final kinetic energy,

$$U_{12} = T_{\text{bar}} + T_{\text{disk}}$$

$$188.4 = 3.36\omega^2 + 6.68\omega^2,$$

we obtain

$$\omega = 4.33 \text{ rad/s}.$$

Problem 19.31 The masses of the bar and disk are 14 kg and 9 kg, respectively. The system is released from rest with the bar horizontal. Determine the angular velocity of the bar when it is vertical if the bar and disk are connected by a smooth pin at A.

Solution: See the solution of Problem 19.30. In this case the disk does not rotate, so its final kinetic energy is

$$T_{disk} = \frac{1}{2}m_{disk}(l\omega)^2$$

$$= \frac{1}{2}(9)(1.2)^2\omega^2$$

$$= 6.48\omega^2.$$

Equating the work to the final kinetic energy,

$$U_{12} = T_{bar} + T_{disk}:$$

$$188.4 = 3.36\omega^2 + 6.48\omega^2,$$

we obtain

$$\omega = 4.38 \text{ rad/s}.$$

Problem 19.32 The 45-kg crate is pulled up the inclined surface by the winch. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.4$. The moment of inertia of the drum on which the cable is being wound is $I_A = 4$ kg-m^2. The crate starts from rest, and the motor exerts a constant couple $M = 50$ N-m on the drum. Use the principle of work and energy to determine the magnitude of the velocity of the crate when it has moved 1 m.

Solution: The normal force is

$$N = mg \cos 20°.$$

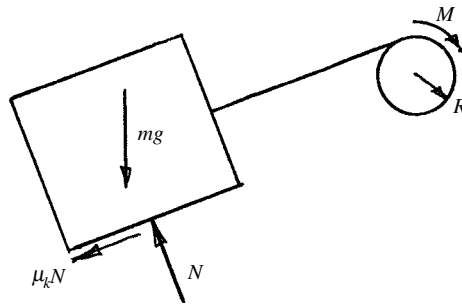As the crate moves 1 m, the drum rotates through an angle $\theta = (1 \text{ m})/R$, so the work done is

$$U_{12} = M\left(\frac{1}{R}\right) - mg \sin 20°(1) - \mu_k(mg \cos 20°)(1).$$

The final kinetic energy is

$$T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Equating the work to the final kinetic energy and using the relation $v = R\omega$, we solve for v, obtaining

$$v = 0.384 \text{ m/s}.$$

Problem 19.33 The 0.61 m slender bars each weigh 17.8 N, and the rectangular plate weighs 89 N. If the system is released from rest in the position shown, what is the velocity of the plate when the bars are vertical?

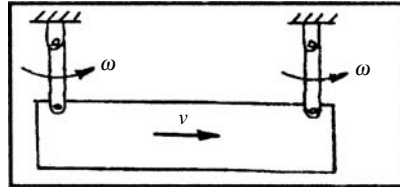**Solution:** The work done by the weights:

$$U = 2W_{bar}h + W_{plate}2h,$$

where $h = \dfrac{L}{2}(1 - \cos 45°) = 0.089$ m

is the change in height, from which $U = 19.06$ N-m. The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right)\left(\frac{W_{plate}}{g}\right)v^2 + 2\left[\frac{1}{2}\left(\frac{W_{bar}L^2}{3g}\right)\right]\omega^2 = 5.1v^2,$$

where $\omega = \dfrac{v}{L}$ has been used.

Substitute into $U = T_2$ and solve: $v = 1.93$ m/s.

Problem 19.34 The mass of the 2-m slender bar is 8 kg. A torsional spring exerts a counterclockwise couple $k\theta$ on the bar, where $k = 40$ N-m/rad and θ is in radians. The bar is released from rest with $\theta = 5°$. What is the magnitude of the bar's angular velocity when $\theta = 60°$?
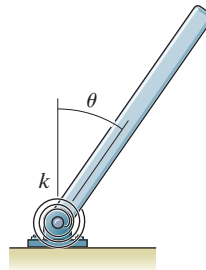
Solution: We will use conservation of energy

$$T_1 = 0$$

$$V_1 = (8 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m}) \cos 5° + \frac{1}{2}\left(40\frac{\text{N-m}}{\text{rad}}\right)\left(\frac{5}{180}\pi \text{ rad}\right)^2$$

$$T_2 = \frac{1}{2}\left[\frac{1}{3}(8 \text{ kg})(2 \text{ m})^2\right]\omega^2$$
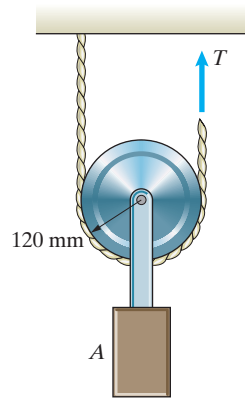
$$V_2 = (8 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m}) \cos 60° + \frac{1}{2}\left(40\frac{\text{N-m}}{\text{rad}}\right)\left(\frac{60}{180}\pi \text{ rad}\right)^2$$

$$T_1 + V_1 = T_2 + V_2 \Rightarrow \boxed{\omega = 1.79 \text{ rad/s.}}$$

Problem 19.35 The mass of the suspended object A is 8 kg. The mass of the pulley is 5 kg, and its moment of inertia is 0.036 kg-m^2. If the force $T = 70N$ is applied to the stationary system, what is the magnitude of the velocity of A when it has risen 0.2 m?


120 mm

T

A

Solution: When the mass A rises 0.2 m, the end of the rope rises 0.4 m.

$$T_1 = 0, \ V_1 = 0, \ T_2 = \frac{1}{2}(13 \text{ kg})v^2 + \frac{1}{2}(0.036 \text{ kg-m}^2)\left(\frac{v}{0.12 \text{ m}}\right)^2$$

$$V_2 = (13 \text{ kg})(9.81 \text{ m/s}^2)(0.2 \text{ m}), \ U = (70 \text{ N})(0.4 \text{ m})$$

$$T_1 + V_1 + U = T_2 + V_2 \ \Rightarrow \ v = \boxed{0.568 \text{ m/s}}$$

Problem 19.36 The mass of the left pulley is 7 kg, and its moment of inertia is 0.330 kg-m^2. The mass of the right pulley is 3 kg, and its moment of inertia is 0.054 kg-m^2. If the system is released from rest, how fast is the 18-kg mass moving when it has fallen 0.1 m?

0.2 m
B
0.3 m A
18 kg
9 kg
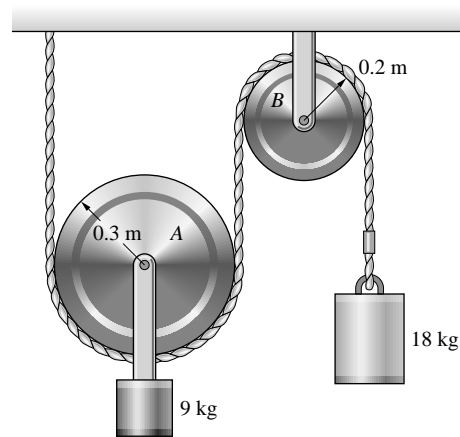
Solution: When mass C falls a distance x, the center of pulley A rises $\frac{1}{2}x$. The potential energy is

$$v = -m_c g x + (m_A + m_D)g\frac{1}{2}x.$$

The angular velocity of pulley B is $w_B = v/R_B$, and the angular velocity of pulley A is $w_A = v/2R_A$. The velocity of the center of pulley A is $w_A R_A = v/2$. The total kinetic energy is

$$T = \frac{1}{2}m_c v^2 + \frac{1}{2}I_B(v/R_B)^2 + \frac{1}{2}(m_A + m_D)(v/2)^2 + \frac{1}{2}I_A(v/2R_A)^2.$$

Applying conservation of energy to the initial and final positions,
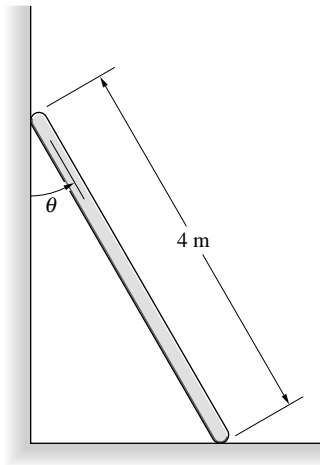
$$O = -m_c g(0.1) + (m_A + m_D)g\frac{1}{2}(0.1) + \frac{1}{2}m_c v^2 + \frac{1}{2}I_B(v/R_B)^2$$

$$+ \frac{1}{2}(m_A + m_D)(v/2)^2 + \frac{1}{2}I_A(v/2R_A)^2.$$

Solving for v, we obtain

$$v = 0.899 \text{ m/s}.$$

ω_B
R_B
ω_A
v
m_A
R_A
m_C
x
v
m_D
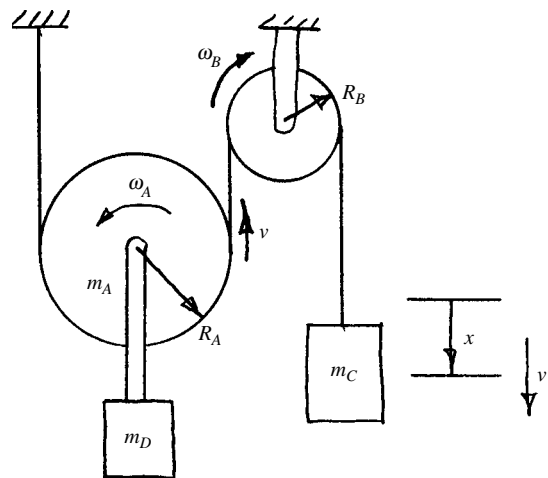
Problem 19.37 The 18-kg ladder is released from rest with $\theta = 10°$. The wall and floor are smooth. Modeling the ladder as a slender bar, use conservation of energy to determine the angular velocity of the bar when $\theta = 40°$.

4 m

Solution: Choose the datum at floor level. The potential energy at the initial position is

$$V_1 = mg\left(\frac{L}{2}\right)\cos 10°.$$

At the final position,

$$V_2 = mg\left(\frac{L}{2}\right)\cos 40°.$$

The instantaneous center of rotation has the coordinates $(L\sin\theta, L\cos\theta)$, where $\theta = 40°$ at the final position. The distance of the center of rotation from the bar center of mass is $\frac{L}{2}$. The angular velocity about this center is $\omega = \left(\frac{2}{L}\right)v$, where v is the velocity of the center of mass of the ladder. The kinetic energy of the ladder is

$$T_2 = \left(\frac{1}{2}\right)mv^2 + \left(\frac{1}{2}\right)\left(\frac{mL^2}{12}\right)\omega^2 = \left(\frac{mL^2}{6}\right)\omega^2,$$

where $v = \left(\frac{L}{2}\right)\omega$ has been used.

From the conservation of energy,

$V_1 = V_2 + T_2$, from which $mg\left(\frac{L}{2}\right)\cos 10°$

$$= mg\left(\frac{L}{2}\right)\cos 40° + \left(\frac{mL^2}{6}\right)\omega^2.$$

Solve: $\omega = 1.269$ rad/s.

Problem 19.38 The 8-kg slender bar is released from rest with $\theta = 60°$. The horizontal surface is smooth. What is the bar's angular velocity when $\theta = 30°$.

Solution: The bar's potential energy is

$$V = mg\tfrac{1}{2}l\sin\theta.$$

No horizontal force acts on the bar, so its center of mass will fall straight down.

The bar's kinetic energy is

$$T = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}Iw^2,$$

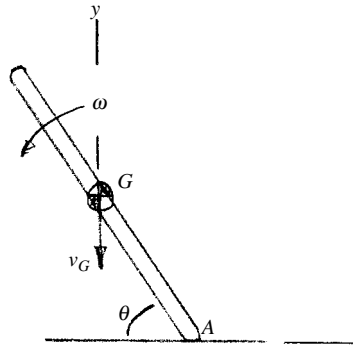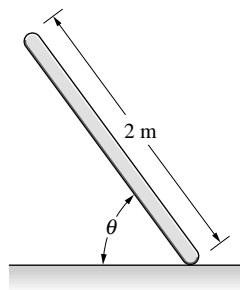where $I = \tfrac{1}{12}ml^2$. Applying conservation of energy to the initial and final states, we obtain

$$mg\tfrac{1}{2}l\sin 60° = mg\tfrac{1}{2}l\sin 30° + \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}Iw^2. \quad \textbf{(1)}$$

To complete the solution, we must determine v_G in terms of ω.

We write the velocity of pt. A in terms of the velocity of pt. G.

$$\mathbf{v}_A = \mathbf{v}_G + \omega \times \mathbf{r}_{A/G}:$$

$$v_A\mathbf{i} = -v_G\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ \dfrac{l}{2}\cos\theta & -\dfrac{l}{2}\sin\theta & 0 \end{vmatrix}.$$

The j component of this equation is

$$0 = -v_G + \omega\tfrac{l}{2}\cos\theta.$$

Setting $\theta = 30°$ in this equation and solving it together with Eq. (1), we obtain $\omega = 2.57$ rad/s.

Problem 19.39 The mass and length of the bar are $m = 4$ kg and $l = 1.2$ m. The spring constant is $k = 180$ N/m. If the bar is released from rest in the position $\theta = 10°$, what is its angular velocity when it has fallen to $\theta = 20°$?

Solution: If the spring is unstretched when $\theta = 0$, the stretch of the spring is

$$S = l(1 - \cos\theta).$$

The total potential energy is

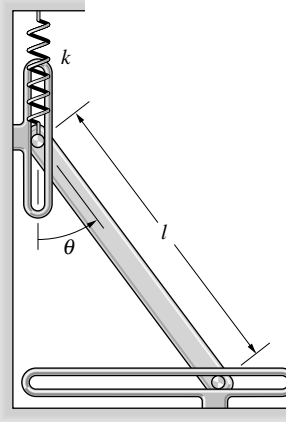$$v = mg(l/2)\cos\theta + \tfrac{1}{2}kl^2(1-\cos\theta)^2.$$

From the solution of Problem 19.37, the bar's kinetic energy is

$$T = \frac{1}{6}ml^2\omega^2.$$

We apply conservation of energy. $T_1 + V_1 = T_2 + V_2$:

$$0 + mg(l/2)\cos 10° + \frac{1}{2}kl^2(1-\cos 10°)^2$$

$$= \frac{1}{6}ml^2\omega_2^2 + mg(l/2)\cos 20° + \frac{1}{2}kl^2(1-\cos 20°)^2.$$

Solving for ω_2, we obtain $\omega_2 = 0.804$ rad/s.

Problem 19.40 The 4-kg slender bar is pinned to a 2-kg slider at A and to a 4-kg homogenous cylindrical disk at B. Neglect the friction force on the slider and assume that the disk rolls. If the system is released from rest with $\theta = 60°$, what is the bar's angular velocity when $\theta = 0$? (See Example 19.3.)
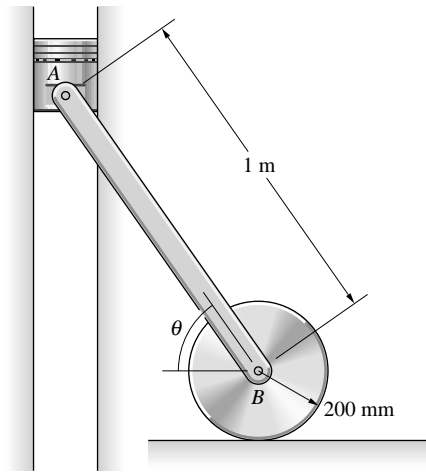
Solution: Choose the datum at $\theta = 0$. The instantaneous center of the bar has the coordinates $(L \cos \theta, L \sin \theta)$ (see figure), and the distance from the center of mass of the bar is $\dfrac{L}{2}$, from which the angular velocity about the bar's instantaneous center is

$$v = \left(\frac{L}{2}\right)\omega,$$

where v is the velocity of the center of mass. The velocity of the slider is

$$v_A = \omega L \cos \theta,$$

and the velocity of the disk is

$$v_B = \omega L \sin \theta$$

The potential energy of the system is

$$V_1 = m_A g L \sin \theta_1 + mg \left(\frac{L}{2}\right) \sin \theta_1.$$

At the datum, $V_2 = 0$. The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right) m_A v_A^2 + \left(\frac{1}{2}\right) m v^2 + \left(\frac{1}{2}\right) \left(\frac{mL^2}{12}\right) \omega^2 + \left(\frac{1}{2}\right) m_B v_B^2$$

$$+ \left(\frac{1}{2}\right) \left(\frac{m_B R^2}{2}\right) \left(\frac{v_B}{R}\right)^2,$$
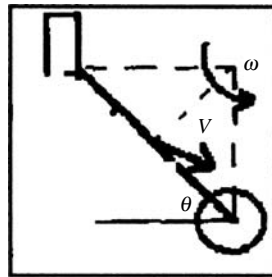
where at the datum

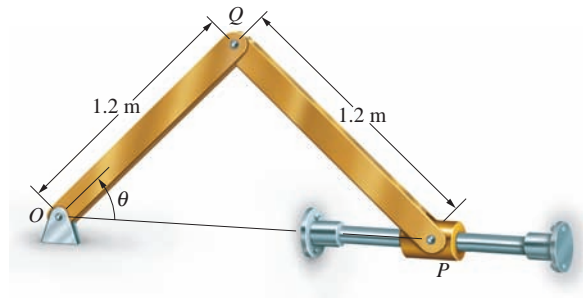$$v_A = \omega L \cos 0° = \omega L,$$

$$v_B = \omega L \sin 0° = 0,$$

$$v = \omega \left(\frac{L}{2}\right).$$

From the conservation of energy: $V_1 = T_2$.

Solve: $\omega = 4.52$ rad/s.

Problem 19.41* The sleeve P slides on the smooth horizontal bar. The mass of each bar is 4 kg and the mass of the sleeve P is 2 kg. If the system is released from rest with $\theta = 60°$, what is the magnitude of the velocity of the sleeve P when $\theta = 40°$?

Solution: We have the following kinematics

$\mathbf{v}_Q = \boldsymbol{\omega}_{OQ} k \times \mathbf{r}_{Q/O}$

$\quad = \omega_{OQ}\mathbf{k} \times (1.2 \text{ m})(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$

$\quad = \omega_{OQ}(1.2 \text{ m})(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j})$

$\mathbf{v}_P = \mathbf{v}_Q + \boldsymbol{\omega}_{PQ} \times \mathbf{r}_{P/Q}$

$\quad = \omega_{OQ}(1.2 \text{ m})(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j}) + \omega_{PQ}\mathbf{k}$

$\quad\quad \times (1.2 \text{ m})(\cos\theta\mathbf{i} - \sin\theta\mathbf{j})$

$\quad = [(\omega_{PQ} - \omega_{OQ})(1.2 \text{ m})\sin\theta]\mathbf{i} + [(\omega_{PQ} + \omega_{OQ})(1.2 \text{ m})\cos\theta]\mathbf{j}$

Point P is constrained to horizontal motion. We conclude that

$$\omega_{PQ} = -\omega_{OQ} \equiv \omega, \ \ v_P = 2\omega(1.2 \text{ m})\sin\theta$$

We also need the velocity of point G

$\mathbf{v}_G = \mathbf{v}_Q + \boldsymbol{\omega}_{PQ} \times \mathbf{r}_{G/Q}$

$\quad = -\omega(1.2 \text{ m})(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j}) + \omega\mathbf{k} \times (0.6 \text{ m})(\cos\theta\mathbf{i} - \sin\theta\mathbf{j})$

$\quad = [(1.8 \text{ m})\omega\sin\theta]\mathbf{i} - [(0.6 \text{ m})\omega\cos\theta]\mathbf{j}$

Now use the energy methods

$T_1 = 0, \ \ V_1 = 2(4 \text{ kg})(9.81 \text{ m/s}^2)(0.6 \text{ m})\sin 60°;$

$T_2 = \dfrac{1}{2}\left(\dfrac{1}{3}[4 \text{ kg}][1.2 \text{ m}]^2\right)\omega^2 + \dfrac{1}{2}\left(\dfrac{1}{12}[4 \text{ kg}][1.2 \text{ m}]^2\right)\omega^2$

$\quad + \dfrac{1}{2}(4 \text{ kg})([(1.8 \text{ m})\omega\sin 40°]^2 + [(0.6 \text{ m})\omega\cos 40°]^2)$

$\quad + \dfrac{1}{2}(2 \text{ kg})(2\omega[1.2 \text{ m}]\sin 40°)^2$

$V_2 = 2(4 \text{ kg})(9.81 \text{ m/s}^2)(0.6 \text{ m})\sin 40°;$

Solving $T_1 + V_1 = T_2 + V_2 \Rightarrow \omega = 1.25 \text{ rad/s} \Rightarrow \boxed{v_P = 1.94 \text{ m/s}}$
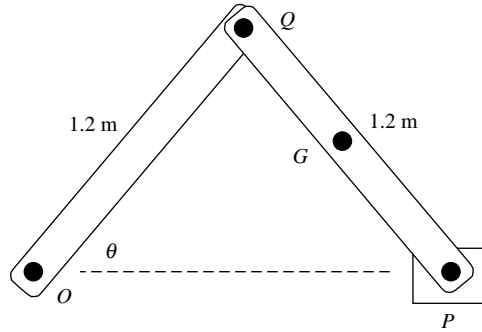
Problem 19.42* The system is in equilibrium in the position shown. The mass of the slender bar ABC is 6 kg, the mass of the slender bar BD is 3 kg, and the mass of the slider at C is 1 kg. The spring constant is $k = 200$ N/m. If a constant 100-N downward force is applied at A, what is the angular velocity of the bar ABC when it has rotated $20°$ from its initial position?

Solution: Choose a coordinate system with the origin at D and the x axis parallel to DC. The equilibrium conditions for the bars: for bar BD,

$$\sum F_x = -B_x + D_x = 0,$$

$$\sum F_y = -B_y - m_{BD}g + D_y = 0.$$

$$\sum M_D = B_x \sin 50° - \left(B_y + \frac{M_{BD}g}{2}\right)\cos 50° = 0.$$

For the bar ABC,

$$\sum F_x = B_x - F = 0, \quad \sum F_y = -F_A + C - m_{ABC}g + B_y = 0.$$

$$\sum M_C = (2F_A - B_y + m_{ABC}g)\cos 50° - B_x \sin 50° = 0.$$

At the initial position $F_A = 0$. The solution:

$B_x = 30.87$ N, $D_x = 30.87$ N,

$B_y = 22.07$ N, $D_y = 51.5$ N,

$F = 30.87$ N, $C = 36.79$ N.

[*Note*: Only the value $F = 30.87$ N is required for the purposes of this problem.] The initial stretch of the spring is

$$S_1 = \frac{F}{k} = \frac{30.87}{200} = 0.154 \text{ m.}$$

The distance D to C is $2\cos\theta$, so that the final stretch of the spring is

$$S_2 = S_1 + (2\cos 30° - 2\cos 50°) = 0.601 \text{ m.}$$

From the principle of work and energy: $U = T_2 - T_1$,

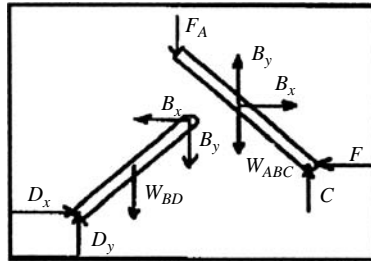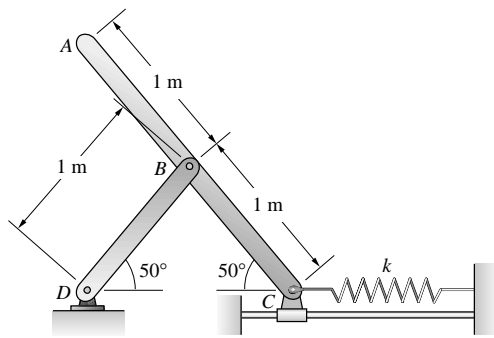where $T_1 = 0$ since the system starts from rest. The work done is

$$U = U_{\text{force}} + U_{ABC} + U_{BD} + U_{\text{spring}}.$$

The height of the point A is $2\sin\theta$, so that the change in height is $h = 2(\sin 50° - \sin 30°)$, and the work done by the applied force is

$$U_{\text{force}} = \int_0^h F_A\, dh = 100(2\sin 50° - 2\sin 30°) = 53.2 \text{ N-m.}$$

The height of the center of mass of bar BD is $\dfrac{\sin\theta}{2}$, so that the work done by the weight of bar BD is

$$U_{BD} = \int_0^h -m_{BD}g\, dh = -\frac{m_{BD}g}{2}(\sin 30° - \sin 50°) = 3.91 \text{ N-m.}$$

The height of the center of mass of bar ABC is $\sin\theta$, so that the work done by the weight of bar ABC is

$$U_{ABC} = \int_0^h -m_{ABC}g\, dh = -m_{ABC}g(\sin 30° - \sin 50°)$$

$$= 15.66 \text{ N-m.}$$

The work done by the spring is

$$U_{\text{spring}} = \int_{S_1}^{S_2} -ks\, ds$$

$$= -\frac{k}{2}(S_2^2 - S_1^2) = -33.72 \text{ N-m.}$$

Collecting terms, the total work:

$$U = 39.07 \text{ N-m.}$$

The bars form an isosceles triangle, so that the changes in angle are equal; by differentiating the changes, it follows that the angular velocities are equal. The distance D to C is $x_{DC} = 2\cos\theta$, from which $v_C = -2\sin\theta\omega$, since D is a stationary. The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right) I_{BD}\omega^2 + \left(\frac{1}{2}\right) I_{ABC}\omega^2 + \left(\frac{1}{2}\right) m_{ABC} v_{ABC}^2$$

$$+ \left(\frac{1}{2}\right) m_C v_C^2 = 5\omega^2,$$
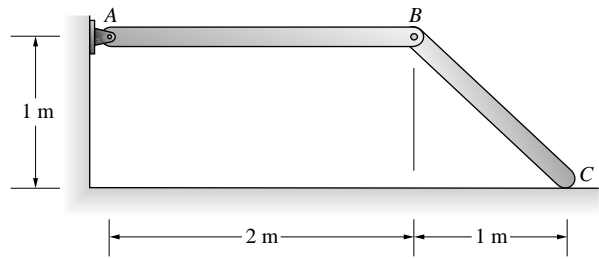
where $I_{BD} = \dfrac{m_{BD}}{3}(1^2)$,

$$I_{ABC} = \frac{m_{ABC}}{12}(2^2),$$

$$v_{ABC} = (1)\omega,$$

$$v_C = -2\sin\theta\omega.$$

Substitute into $U = T_2$ and solve: $\boxed{\omega = 2.80 \text{ rad/s}}$

Problem 19.43* The masses of bars AB and BC are 5 kg and 3 kg, respectively. If the system is released from rest in the position shown, what are the angular velocities of the bars at the instant before the joint B hits the smooth floor?

Solution: The work done by the weights of the bars as they fall is

$$v_{12} = m_{AB}g(0.5 \text{ m}) + m_{BC}g(0.5 \text{ m})$$

$$= (5+3)(9.81)(0.5)$$

$$= 39.24 \text{ N-m.}$$

Consider the two bars just before B hits.

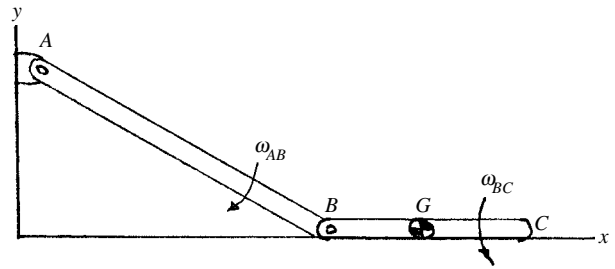The coordinates of pt B are $(\sqrt{3}, 0)$m.

The velocity of pt B is

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{w}_{AB} \times \mathbf{r}_{B/A}$$

$$= 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{AB} \\ \sqrt{3} & -1 & 0 \end{vmatrix}$$

$$= -\omega_{AB}\mathbf{i} - \sqrt{3}\omega_{AB}\mathbf{j}.$$

In terms of pt. C,

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$= v_C\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -\sqrt{2} & 0 & 0 \end{vmatrix}$$

$$= v_C\mathbf{i} - \sqrt{2}\omega_{BC}\mathbf{j}.$$

Equating the two expressions for $\mathbf{v}_B$, we obtain

$$\mathbf{v}_C = -\omega_{AB}\mathbf{i},$$

$$\omega_{BC} = \sqrt{\frac{3}{2}}\omega_{AB}. \quad \textbf{(1)}$$

The velocity of pt G is

$$\mathbf{v}_G = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{G/C}$$

$$= -\omega_{AB}\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -\frac{1}{2}\sqrt{2} & 0 & 0 \end{vmatrix}$$

$$= -\omega_{AB}\mathbf{i} - \frac{1}{2}\sqrt{2}\omega_{BC}\mathbf{j}$$

$$= -\omega_{AB}\mathbf{i} - \frac{1}{2}\sqrt{3}\omega_{AB}\mathbf{j}. \quad \textbf{(2)}$$

We equate the work done to the final kinetic energy of the bars:

$$U_{12} = \frac{1}{2}\left[\frac{1}{3}m_{AB}(2)^2\right]\omega_{AB}^2$$

$$+ \frac{1}{2}m_{BC}|\mathbf{v}_G|^2 + \frac{1}{2}\left[\frac{1}{12}m_{BC}(\sqrt{2})^2\right]\omega_{BC}^2. \quad \textbf{(3)}$$

Substituting Eqs. (1) and (2) into this equation and solving for ω_{AB}, we obtain

$$\omega_{AB} = 2.49 \text{ rad/s.}$$

Then, from Eq. (1), $\omega_{BC} = 3.05$ rad/s.

Problem 19.44* Bar *AB* weighs 22.2 N. Each of the sleeves *A* and *B* weighs 8.9 N. The system is released from rest in the position shown. What is the magnitude of the angular velocity of the bar when sleeve B has moved 76.2 mm to the right?

y

A

228.6 mm

B

—*x*

305 mm

101.6 mm
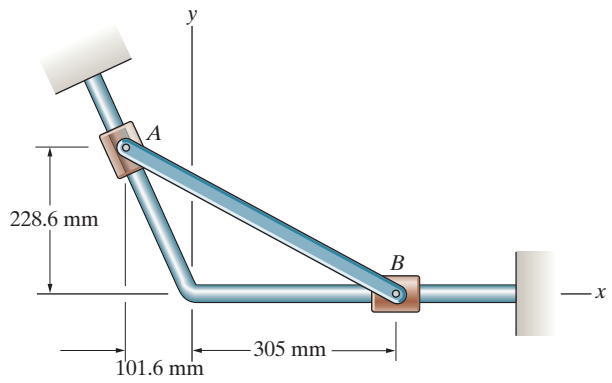
Solution: Find the geometry in position 2

$$(0.407 \text{ m})^2 + (0.229 \text{ m})^2 = (0.381 \text{ m} + x)^2 + \left(\frac{0.229x}{0.102}\right)^2$$

$x = 0.063$ m

Now do the kinematics in position 2

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{i} = v_A\left(\frac{0.102}{0.25}\mathbf{i} - \frac{0.229}{0.25}\mathbf{j}\right)$$

$$+ \omega\mathbf{k} \times (0.445\mathbf{i} - 0.142\mathbf{j}) \text{ m}$$

$$= (0.406v_A + [0.142 \text{ in}]\omega)\mathbf{i} + (-0.914v_A + [0.445 \text{ m}]\omega)\mathbf{j}$$

Equating components we find $v_A = (0.485 \text{ m})\omega$, $v_B = (0.34 \text{ m})\omega$

Now find the velocity of the center of mass G

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{G/B}$$

$$= (0.34 \text{ m})\omega\mathbf{i} + \omega\mathbf{k} \times (-0.222\mathbf{i} + 0.071\mathbf{j}) \text{ m}$$

$$= \omega(0.269\mathbf{i} - 0.222\mathbf{j}) \text{ m}$$

Now we can do work energy

$T_1 = 0$, $V_1 = (22.2 \text{ N})(0.114 \text{ m}) + (8.9 \text{ N})(0.229 \text{ m})$,

$V_2 = (22.2 \text{ N})(0.071 \text{ m}) + (8.9 \text{ N})(0.142 \text{ m})$

$$T_2 = \frac{1}{2}\left(\frac{1}{12}\left[\frac{22.2 \text{ N}}{9.81 \text{ m/s}^2}\right]\left[(0.229 \text{ m})^2 + (0.407 \text{ m})^2\right]\right)\omega^2$$

$$+ \frac{1}{2}\left(\frac{8.9 \text{ N}}{9.81 \text{ m/s}^2}\right)(0.485 \text{ m})^2 \omega^2$$

$$+ \frac{1}{2}\left(\frac{22.2 \text{ N}}{9.81 \text{ m/s}^2}\right)\left[(0.269 \text{ m})^2 + (0.222 \text{ m})^2\right]\omega^2$$

$$+ \frac{1}{2}\left(\frac{8.9 \text{ N}}{9.81 \text{ m/s}^2}\right)(0.34 \text{ m})^2 \omega^2$$

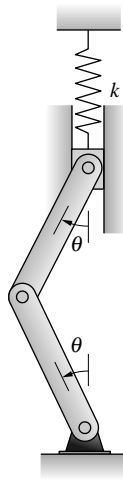Thus $T_1 + V_1 = T_2 + V_2 \implies$ $\boxed{\omega = 2.34 \text{ rad/s}}$

Position 1

228.6 mm

101.6 mm   305 mm

9

4   *A*   *Position 2*

9x/4   *B*

x   381 m
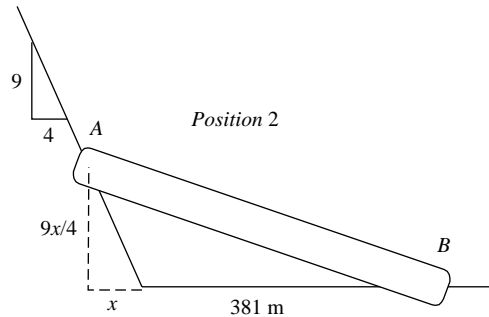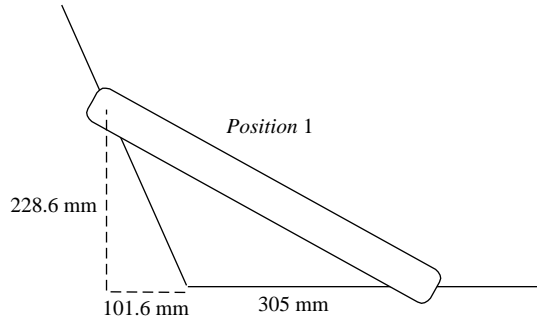
Problem 19.45* Each bar has a mass of 8 kg and a length of 1 m. The spring constant is $k = 100$ N/m, and the spring is unstretched when $\theta = 0$. If the system is released from rest with the bars vertical, what is the magnitude of the angular velocity of the bars when $\theta = 30°$?

Solution: The stretch of the spring is $2l(1 - \cos\theta)$, so the potential energy is

$$v = \frac{1}{2}k[2l(1 - \cos\theta)]^2 + mg\frac{l}{2}\cos\theta + mg\frac{3l}{2}\cos\theta$$

$$= 2kl^2(1 - \cos\theta)^2 + 2mgl\cos\theta.$$

The velocity of pt B is

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ -l\sin\theta & l\cos\theta & 0 \end{vmatrix}$$

$$= -\omega l\cos\theta\mathbf{i} - \omega l\sin\theta\mathbf{j}.$$

The velocity of pt. G is

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{G/B}$$

$$= -\omega l\cos\theta\mathbf{i} - \omega l\sin\theta\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega \\ \dfrac{l}{2}\sin\theta & \dfrac{l}{2}\cos\theta & 0 \end{vmatrix}$$

$$= -\omega\frac{l}{2}\cos\theta\mathbf{i} - \omega\frac{3l}{2}\sin\theta\mathbf{j}.$$

From this expression,

$$|\mathbf{v}_G|^2 = (1 + 8\sin^2\theta)\frac{1}{4}l^2\omega^2.$$

Applying conservation of energy to the initial and final positions,

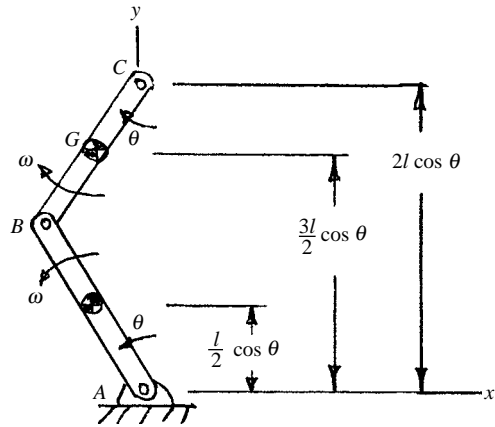$$2mgl = 2kl^2(1 - \cos 30°) + 2mgl\cos 30°$$

$$+ \frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega^2 + \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega^2$$

$$+ \frac{1}{2}m(1 + 8\sin^2 30°)\frac{1}{4}l^2\omega^2.$$

Solving for ω, we obtain

$$\omega = 1.93 \text{ rad/s.}$$

Problem 19.46* The system starts from rest with the crank AB vertical. A constant couple M exerted on the crank causes it to rotate in the clockwise direction, compressing the gas in the cylinder. Let s be the displacement (in meters) of the piston to the right relative to its initial position. The net force toward the left exerted on the piston by atmospheric pressure and the gas in the cylinder is $350/(1 - 10s)$ N. The moment of inertia of the crank about A is 0.0003 kg-m^2. The mass of the connecting rod BC is 0.36 kg, and the center of mass of the rod is at its midpoint. The connecting rod's moment of inertia about its center of mass is 0.0004 kg-m^2. The mass of the piston is 4.6 kg. If the clockwise angular velocity of the crank AB is 200 rad/s when it has rotated 90° from its initial position, what is M? (Neglect the work done by the weights of the crank and connecting rod).
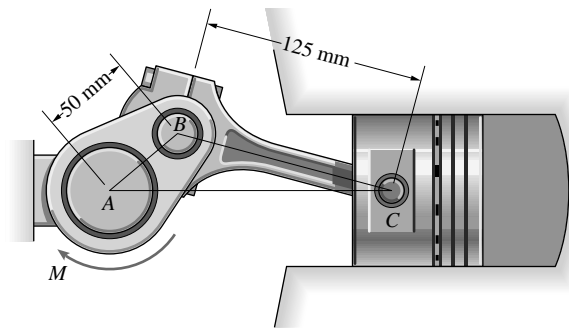
Solution: As the crank rotates through an angle θ the work done by the couple is $M\theta$. As the piston moves to the right a distance s, the work done on the piston by the gas is

$$-\int_0^s \frac{350\,ds}{1 - 10s}.$$

Letting $1 - 10s = 10z$, this is

$$\int_{0.1}^{0.1-s} 35\frac{dz}{z} = 35ln(1 - 10s).$$

The total work is $U_{12} = M\theta + 35ln(1 - 10s)$.

From the given dimensions of the crank and connecting rod, the vector components are

$$r_{B/Ax} = 0.05 \sin\theta, \qquad (1)$$

$$r_{B/Ay} = 0.05 \cos\theta \qquad (2);$$

$$r_{C/Bx} = \sqrt{(0.125)^2 - (0.05 \cos\theta)^2} \qquad (3);$$

$$r_{C/By} = -0.05 \cos\theta \qquad (4).$$

The distance s that the piston moves to the right is

$$s = r_{B/Ax} + r_{C/Bx} - \sqrt{(0.125)^2 - (0.05)^2} \qquad (5).$$

The velocity of B is

$$\mathbf{v}_B = \mathbf{v}_A + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega \\ r_{B/Ax} & r_{B/Ay} & 0 \end{vmatrix} = \omega r_{B/Ay}\mathbf{i} - \omega r_{B/Ax}\mathbf{j}.$$
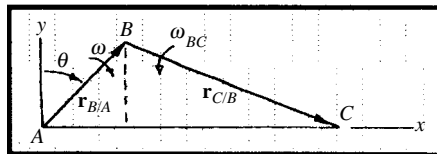
The velocity of C is $\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$:

$$v_C\mathbf{i} = \omega r_{B/Ay}\mathbf{i} - \omega r_{B/Ax}\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ r_{c/Bx} & r_{c/By} & 0 \end{vmatrix}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components, we can solve for v_c and ω_{BC} in terms of ω:

$$v_C = [r_{B/Ay} - r_{C/By}(r_{B/Ax}/r_{C/Bx})]\omega \qquad (6);$$

$$\omega_{BC} = (r_{B/Ax}/r_{C/Bx})\omega \qquad (7).$$

The velocity of the center of mass G (the midpoint) of the connecting rod BC is

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \tfrac{1}{2}\mathbf{r}_{C/B} = \omega r_{B/Ay}\mathbf{i} - \omega r_{B/Ax}\mathbf{j}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ \tfrac{1}{2}r_{C/Bx} & \tfrac{1}{2}r_{C/By} & 0 \end{vmatrix}, \text{ or}$$

$$v_C = \left(\omega r_{B/Ay} - \tfrac{1}{2}\omega_{BC}r_{C/By}\right)\mathbf{i} - \left(\omega r_{B/Ax} - \tfrac{1}{2}\omega_{BC}r_{C/Bx}\right)\mathbf{j} \qquad (8).$$

Let I_A be the moment of inertia of the crank about A, I_{BC} the moment of inertia of the connecting rod about its center of mass, m_{BC} the mass of the connecting rod, and m_p the mass of the piston. The principle of work and energy is: $U_{12} = T_2 - T_1$:

$$M\theta + 35ln(1 - 10s) = \tfrac{1}{2}I_A\omega^2 + \tfrac{1}{2}I_{BC}\omega_{BC}^2 + \tfrac{1}{2}m_{BC}|\mathbf{v}_G|^2$$

$$+ \tfrac{1}{2}m_p v_C^2 \qquad (9).$$

Solving this equation for M and using Equations (1)–(8) with $\omega = 200$ rad/s and $\theta = \pi/2$ rad, we obtain $M = 28.2$ N-m.

Problem 19.47* In Problem 19.46, if the system starts from rest with the crank AB vertical and the couple $M = 40$ N-m, what is the clockwise angular velocity of AB when it has rotated $45°$ from its initial position?

Solution: In the solution to Problem 19.46, we substitute Equations (1)–(8) into Equation (9), set $M = 40$ N-m and $\theta = \pi/4$ rad and solve for ω obtaining $\omega = 49.6$ rad/s.

Problem 19.48 The moment of inertia of the disk about O is 22 kg-m^2. At $t = 0$, the stationary disk is subjected to a constant 50 N-m torque.
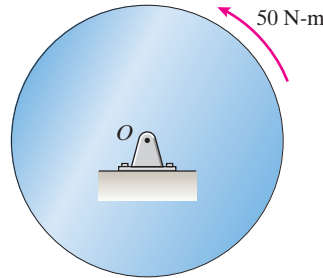
(a) Determine the angular impulse exerted on the disk from $t = 0$ to $t = 5$ s.
(b) What is the disk's angular velocity at $t = 5$ s?

50 N-m

O

Solution:

(a)
$$\text{Angular Impulse} = \int_0^{5 \text{ s}} \sum M \, dt = (50 \text{ N-m})(5 \text{ s})$$
$$= 250 \text{ N-m-s CCW}$$

(b) 250 N-m-s $= (22$ kg-m$^2)\omega$

$\Rightarrow$ $\boxed{\omega = 11.4 \text{ rad/s counterclockwise}}$

Problem 19.49 The moment of inertia of the jet engine's rotating assembly is 400 kg-m^2. The assembly starts from rest. At $t = 0$, the engine's turbine exerts a couple on it that is given as a function of time by $M = 6500 - 125t$ N-m.
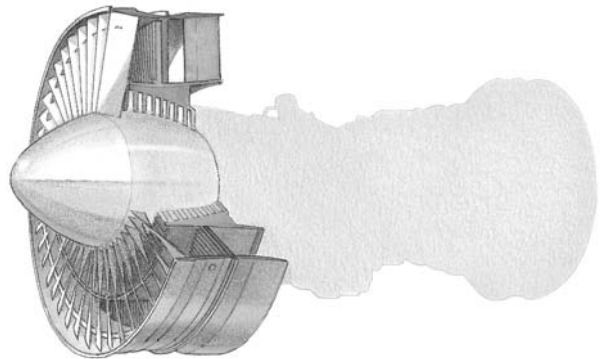
(a) What is the magnitude of the angular impulse exerted on the assembly from $t = 0$ to $t = 20$ s?
(b) What is the magnitude of the angular velocity of the assembly (in rpm) at $t = 20$ s?

Solution:

(a)
$$\text{Angular Impulse} = \int_0^{20 \text{ s}} \sum M \, dt$$
$$= \int_0^{20} (6500 - 125t) \text{ N-m} \, dt = 105 \text{ kN-m-s}$$

(b) 105 kN-m-s $= (400$ kg-m$^2) \, \omega$ $\Rightarrow$ $\boxed{\omega = 263 \text{ rad/s (2510 rpm)}}$

Problem 19.50 An astronaut fires a thruster of his maneuvering unit, exerting a force $T = 2(1 + t)$ N, where t is in seconds. The combined mass of the astronaut and his equipment is 122 kg, and the moment of inertia about their center of mass is 45 kg-m^2. Modeling the astronaut and his equipment as a rigid body, use the principle of angular impulse and momentum to determine how long it takes for his angular velocity to reach 0.1 rad/s.

T

300 mm
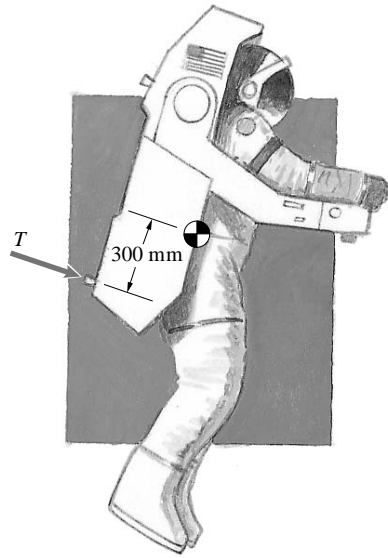
Solution: From the principle of impulse and angular momentum,

$$\int_{t_1}^{t_2} \sum M \, dt = I\omega_2 - I\omega_1,$$

where $\omega_1 = 0$, since the astronaut is initially stationary. The normal distance from the thrust line to the center of mass is $R = 0.3$ m, from which

$$\int_0^{t_2} 2(1 + t)(R) \, dt = I\omega_2.$$

$$0.6 \left(t_2 + \frac{t_2^2}{2} \right) = 45(0.1).$$

Rearrange: $t_2^2 + 2bt_2 + c = 0$, where $b = 1$, $c = -15$. Solve:

$$t_2 = -b \pm \sqrt{b^2 - c} = 3 \text{ or } -5.$$

Since the negative solution has no meaning here, $\boxed{t_2 = 3 \text{ s}}$

Problem 19.51 The combined mass of the astronaut and his equipment is 122 kg, and the moment of inertia about their center of mass is 45 kg-m^2. The maneuvering unit exerts an impulsive force T of 0.2-s duration, giving him a counterclockwise angular velocity of 1 rpm.

(a) What is the average magnitude of the impulsive force?
(b) What is the magnitude of the resulting change in the velocity of the astronaut's center of mass?
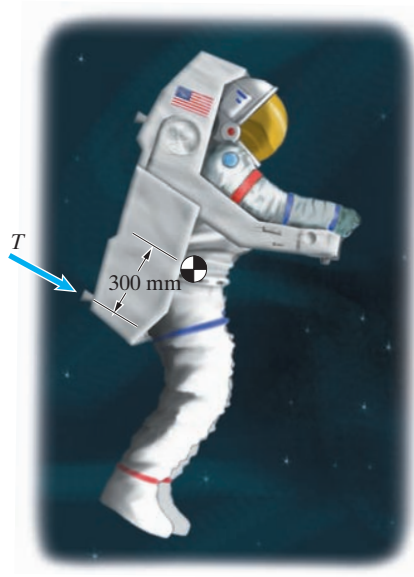
T

300 mm

Solution:

(a) From the principle of moment impulse and angular momentum,

$$\int_{t_1}^{t_2} \sum M \, dt = I\omega_2 - I\omega_1,$$

where $\omega_1 = 0$ since the astronaut is initially stationary. The angular velocity is

$$\omega_2 = \frac{2\pi}{60} = 0.1047 \text{ rad/s}$$

from which $\int_{0}^{0.2} TR \, dt = I\omega_2$,

from which $T(0.3)(0.2) = 45(\omega_2)$,

$$\boxed{T = \frac{45\omega_2}{0.06} = 78.54 \text{ N}}$$

(b) From the principle of impulse and linear momentum

$$\int_{t_1}^{t_2} \sum F \, dt = m(v_2 - v_1)$$

where $v_1 = 0$ since the astronaut is initially stationary.

$$\int_{0}^{0.2} T \, dt = mv_2,$$

from which $\boxed{v_2 = \frac{T(0.2)}{m} = 0.129 \text{ m/s}}$

Problem 19.52 A flywheel attached to an electric motor is initially at rest. At $t = 0$, the motor exerts a couple $M = 200e^{-0.1t}$ N-m on the flywheel. The moment of inertia of the flywheel is 10 kg-m^2.

(a) What is the flywheel's angular velocity at $t = 10$ s?
(b) What maximum angular velocity will the flywheel attain?

Solution:

(a) From the principle of moment impulse and angular momentum,
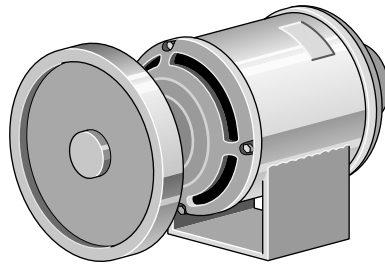
$$\int_{t_1}^{t_2} \sum M \, dt = I\omega_2 - I\omega_1,$$

where $\omega_1 = 0$, since the motor starts from rest.

$$\int_0^{10} 200e^{-0.1t} \, dt = \frac{200}{0.1} \left[-e^{-0.1t}\right]_0^{10} = 2000 \left[1 - e^{-1}\right],$$

$$= 1264.2 \text{ N-m-s.}$$

From which $\boxed{\omega_2 = \dfrac{1264.24}{10} = 126 \text{ rad/s.}}$

(b) An inspection of the angular impulse function shows that the angular velocity of the flywheel is an increasing monotone function of the time, so that the greatest value occurs as $t \to \infty$.

$$\boxed{\omega_{2_{\max}} = \lim_{t \to \infty} \frac{200}{(10)(0.1)} \left[1 - e^{-0.1t}\right] \to 200 \text{ rad/s}}$$

Problem 19.53 A main landing gear wheel of a Boeing 777 has a radius of 0.62 m and its moment of inertia is 24 kg-m^2. After the plane lands at 75 m/s, the skid marks of the wheel's tire is measured and determined to be 18 m in length. Determine the average friction force exerted on the wheel by the runway. Assume that the airplane's velocity is constant during the time the tire skids (slips) on the runway.
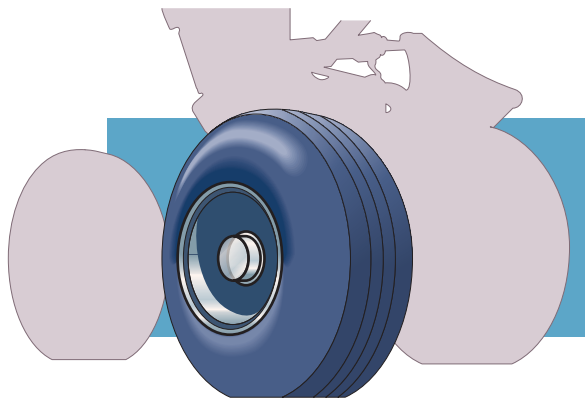
Solution: The tire skids for $t = \dfrac{18 \text{ m}}{75 \text{ m/s}} = 0.24$ s.

When the skidding stops the tire is turning at the rate $\omega = \dfrac{75 \text{ m/s}}{0.62 \text{ m}} = 121$ rad/s

$$Frt = I\omega \implies F = \frac{I\omega}{rt} = \frac{(24 \text{ kg-m}^2)(121 \text{ rad/s})}{(0.62 \text{ m})(0.24 \text{ s})} \quad \boxed{F = 19.5 \text{ kN}}$$

Problem 19.54 The force a club exerts on a 0.045-kg golf ball is shown. The ball is 42 mm in diameter and can be modeled as a homogeneous sphere. The club is in contact with the ball for 0.0006 s, and the magnitude of the velocity of the ball's center of mass after the ball is hit is 36 m/s. What is the magnitude of the ball's angular velocity after it is hit?

Solution: Linear momentum: $Ft = mv \Rightarrow F = \dfrac{mv}{t}$

Angular momentum

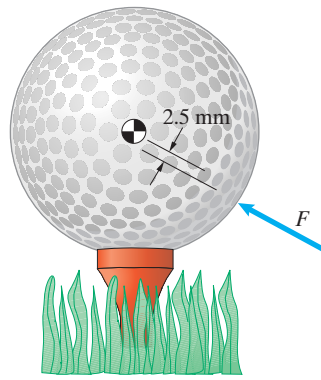$$Ftd = I\omega \Rightarrow \omega = \frac{Ftd}{I} = \frac{mvd}{I} = \frac{(0.045 \text{ kg})(36 \text{ m/s})(0.0025 \text{ m})}{2/5 \ (0.045 \text{ kg})(0.021 \text{ m})^2}$$

Solving $\boxed{\omega = 510 \text{ rad/s}}$

Problem 19.55 Disk A initially has a counterclockwise angular velocity $\omega_0 = 50$ rad/s. Disks B and C are initially stationary. At $t = 0$, disk A is moved into contact with disk B. Determine the angular velocities of the three disks when they have stopped slipping relative to each other. The masses of the disks are $m_A = 4$ kg, $m_B = 16$ kg, and $m_C = 9$ kg. (See Active Example 19.4.)
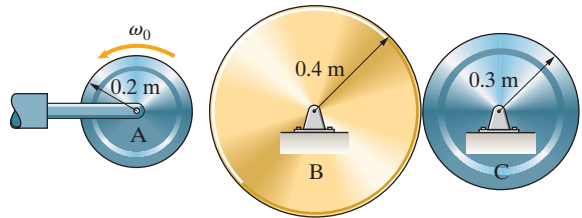
Solution: The FBDs

Given:

$\omega_0 = 50$ rad/s

$m_A = 4$ kg, $r_A = 0.2$ m, $I_A = 1/2 \ m_A r_A^2 = 0.08$ kg-m^2

$m_B = 16$ kg, $r_B = 0.4$ m, $I_B = 1/2 \ m_B r_B^2 = 1.28$ kg-m^2

$m_C = 9$ kg, $r_C = 0.3$ m, $I_C = 1/2 \ m_C r_C^2 = 0.405$ kg-m^2

Impulse Momentum equations:

$-F_1 t r_A = I_A \omega_A - I_A \omega_0, \quad F_1 t r_B - F_2 t r_B = I_B \omega_B, \quad F_2 t r_C = I_C \omega_C$

Constraints when it no longer slips:

$$\omega_A r_A = \omega_B r_B = \omega_C r_C$$

We cannot solve for the slipping time, however, treating $F_1 t$ and $F_2 t$ as two unknowns we have

$$\boxed{\begin{aligned} \omega_A &= 6.90 \text{ rad/s counterclockwise} \\ \omega_B &= 3.45 \text{ rad/s clockwise} \\ \omega_C &= 4.60 \text{ rad/s counterclockwise} \end{aligned}}$$
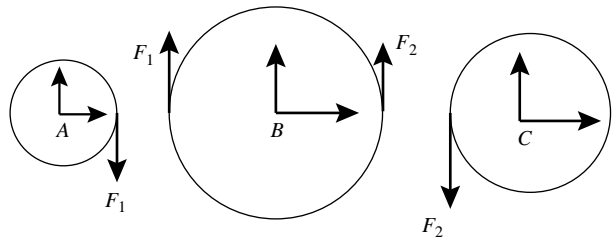
Problem 19.56 In Example 19.5, suppose that in a second test at a higher velocity the angular velocity of the pole following the impact is $\omega = 0.81$ rad/s, the horizontal velocity of its center of mass is $v = 7.3$ m/s, and the duration of the impact is $\Delta t = 0.009$ s. Determine the magnitude of the average force the car exerts on the pole in shearing off the supporting bolts. Do so by applying the principle of angular impulse and momentum in the form given by Eq. (19.32).
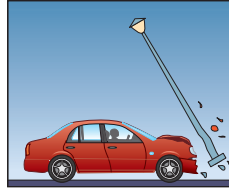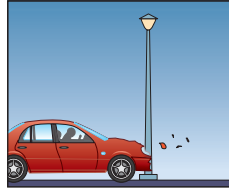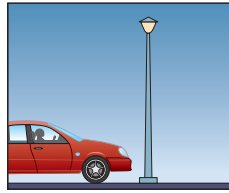
Solution: Using the data from Example 19.5 we write the linear and angular impulse momentum equations for the pole. F is the force of the car and S is the shear force in the bolts

$(F - S)\Delta t = mv$

$F\Delta t \left(\dfrac{L}{2} - h \right) - S\Delta t \dfrac{L}{2} = \dfrac{1}{12} mL^2 \, \omega$

Putting in the numbers we have

$(F - S)(0.009 \text{ s}) = (70 \text{ kg})(7.3 \text{ m/s})$

$F(0.009 \text{ s})(2.5 \text{ m}) - S(0.009 \text{ s})(43 \text{ m}) = \dfrac{1}{12}(70 \text{ kg})(6 \text{ m})^2 \, \omega$

Solving we find $F = 303$ N, $\boxed{S = 246 \text{ N.}}$

Problem 19.57 The force exerted on the cue ball by the cue is horizontal. Determine the value of h for which the ball rolls without slipping. (Assume that the frictional force exerted on the ball by the table is negligible.)
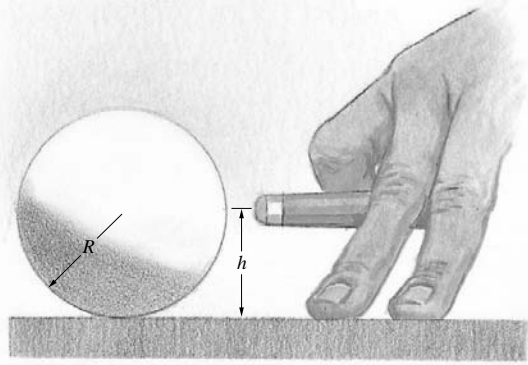
Solution: From the principle of moment impulse and angular momentum,

$$\int_{t_1}^{t_2} (h - R)F \, dt = I(\omega_2 - \omega_1),$$

where $\omega_1 = 0$ since the ball is initially stationary. From the principle of impulse and linear momentum

$$\int_{t_1}^{t_2} F \, dt = m(v_2 - v_1)$$

where $v_1 = 0$ since the ball is initially stationary. *Since the ball rolls*, $v_2 = R\omega_2$, from which the two equations:

$$(h - R)F(t_2 - t_2) = I\omega_2,$$

$$F(t_2 - t_1) = mR\omega_2.$$

The ball is a homogenous sphere, from which

$$I = \tfrac{2}{5}mR^2.$$
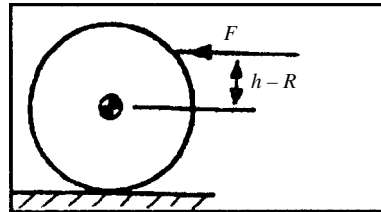
Substitute:

$$(h - R)mR\omega_2 = \frac{2mR^2}{5}\omega_2.$$

Solve: $\boxed{h = \left(\tfrac{7}{5}\right) R}$

Problem 19.58 In Example 19.6, we neglected the moments of inertia of the two masses m about the axes through their centers of mass in calculating the total angular momentum of the person, platform, and masses. Suppose that the moment of inertia of each mass about the vertical axis through its center of mass is $I_M = 0.001$ kg-m^2. If the person's angular velocity with her arms extended to $r_1 = 0.6$ m is $\omega_1 = 1$ revolution per second, what is her angular velocity ω_2 when she pulls the masses inward to $r_2 = 0.2$ m? Compare your result to the answer obtained in Example 19.6.
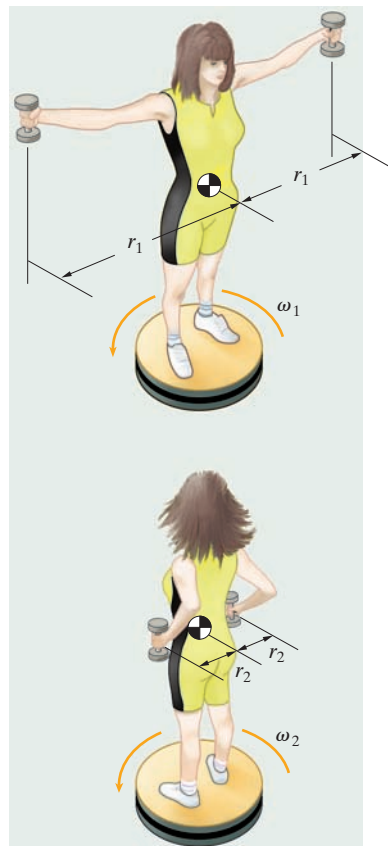
Solution: Using the numbers from Example 19.6, we conserve angular momentum

$$H_{O1} = (I_P + 2mr_1^2 + 2I_M)\omega_1$$

$$= (0.4 \text{ kg-m}^2 + 2[4 \text{ kg}][0.6 \text{ m}]^2 + 2[0.001 \text{ kg-m}^2]) \left(1 \frac{\text{rev}}{\text{s}}\right)$$

$$H_{O2} = (I_P + 2mr_2^2 + 2I_M)\omega_2$$

$$= (0.4 \text{ kg-m}^2 + 2[4 \text{ kg}][0.2 \text{ m}]^2 + 2[0.001 \text{ kg-m}^2])\omega_2$$

$$H_{O1} = H_{O2} \Rightarrow \boxed{\omega = 4.55 \frac{\text{rev}}{\text{s}}.}$$

If we include the moments of inertia of the weights then $\omega = 4.55 \frac{\text{rev}}{\text{s}}$.

Without the moments of inertia (Example 19.6) we found $\omega = 4.56 \frac{\text{rev}}{\text{s}}$.

Problem 19.59 Two gravity research satellites ($m_A = 250$ kg, $I_A = 350$ kg-m^2; $m_B = 50$ kg, $I_B = 16$ kg-m^2) are tethered by a cable. The satellites and cable rotate with angular velocity $\omega_0 = 0.25$ rpm. Ground controllers order satellite A to slowly unreel 6 m of additional cable. What is the angular velocity afterward?
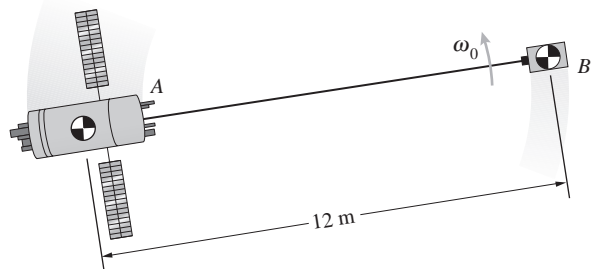
Solution: The initial distance from A to the common center of mass is

$$x_0 = \frac{(0)(250) + (12)(50)}{250 + 50} = 2 \text{ m}.$$

The final distance from A to the common center of mass is

$$x = \frac{(0)(250) + (18)(50)}{250 + 50} = 3 \text{ m}.$$

The total angular momentum about the center of mass is conserved.

$$x_0 m_A (x_0\omega_0) + I_A\omega_0 + (12 - x_0)m_B[(12 - x_0)\omega_0] + I_B\omega_0$$

$$= xm_A(x_\omega) + I_A\omega + (18 - x)m_B[(18 - x)\omega] + I_B\omega;$$

or

$$(2)(250)(2)\omega_0 + 350\omega_0 + (10)(50)(10)\omega_0 + 16\omega_0$$

$$= (3)(250)(3)\omega + 350\omega + (15)(50)(15)\omega + 16\omega.$$

we obtain $\omega = 0.459\omega_0 = 0.115$ rpm.

Problem 19.60 The 2-kg bar rotates *in the horizontal plane* about the smooth pin. The 6-kg collar A slides on the smooth bar. Assume that the moment of inertia of the collar A about its center of mass is negligible; that is, treat the collar as a particle. At the instant shown, the angular velocity of the bar is $\omega_0 = 60$ rpm and the distance from the pin to the collar is $r = 1.8$ m. Determine the bar's angular velocity when $r = 2.4$ m.

Solution: Angular momentum is conserved

$$\left[\frac{1}{3}(2\text{ kg})(3\text{ m})^2 + (6\text{ kg})(1.8\text{ m})^2\right](60\text{ rpm})$$

$$= \left[\frac{1}{3}(2\text{ kg})(3\text{ m})^2 + (6\text{ kg})(2.4\text{ m})^2\right]\omega_2$$

Solving we find $\boxed{\omega_2 = 37.6\text{ rpm}}$

Problem 19.61 The 2-kg bar rotates *in the horizontal plane* about the smooth pin. The 6-kg collar A slides on the smooth bar. The moment of inertia of the collar A about its center of mass is 0.2 kg-m². At the instant shown, the angular velocity of the bar is $\omega_0 = 60$ 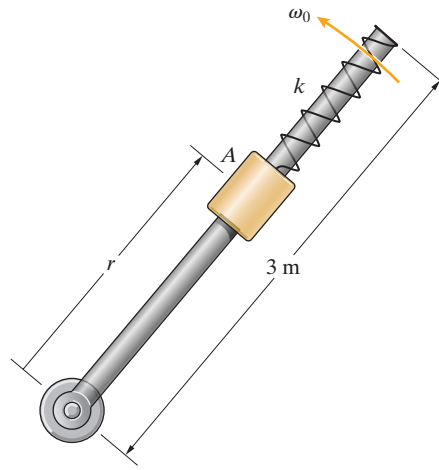rpm and the distance from the pin to the collar is $r = 1.8$ m. Determine the bar's angular velocity when $r = 2.4$ m and compare your answer to that of Problem 19.60.

Solution: Angular momentum is conserved

$$\left[\frac{1}{3}(2\text{ kg})(3\text{ m})^2 + (6\text{ kg})(1.8\text{ m})^2 + (0.2\text{ kg-m}^2)\right](60\text{ rpm})$$

$$= \left[\frac{1}{3}(2\text{ kg})(3\text{ m})^2 + (6\text{ kg})(2.4\text{ m})^2 + (0.2\text{ kg-m}^2)\right]\omega_2$$

Solving we find $\boxed{\omega_2 = 37.7\text{ rpm}}$

Problem 19.62* The 2-kg bar rotates *in the horizontal plane* about the smooth pin. The 6-kg collar A slides on the smooth bar. The moment of inertia of the collar A about its center of mass is 0.2 kg-m². The spring is unstretched when $r = 0$, and the spring constant is $k = 10$ N-m. At the instant shown, the angular velocity of the bar is $\omega_0 = 2$ rad/s, the distance from the pin to the collar is $r = 1.8$ m, and the radial velocity of the collar is zero. Determine the radial velocity of the collar when $r = 2.4$ m.

Solution: Angular momentum is conserved

$$\left[\frac{1}{3}(2\text{ kg})(3\text{ m})^2 + (6\text{ kg})(1.8\text{ m})^2 + (0.2\text{ kg-m}^2)\right](2\text{ rad/s})$$

$$= \left[\frac{1}{3}(2\text{ kg})(3\text{ m})^2 + (6\text{ kg})(2.4\text{ m})^2 + (0.2\text{ kg-m}^2)\right]\omega_2$$

Thus $\omega_2 = 1.258$ rad/s

Energy is conserved

$$T_1 = \frac{1}{2}\left[\frac{1}{3}(2\text{ kg})(3\text{ m})^2 + (6\text{ kg})(1.8\text{ m})^2 + (0.2\text{ kg-m}^2)\right](2\text{ rad/s})^2$$

$$V_1 = \frac{1}{2}(10\text{ N/m})(1.8\text{ m})^2$$

$$T_2 = \frac{1}{2}\left[\frac{1}{3}(2\text{ kg})(3\text{ m})^2 + (6\text{ kg})(2.4\text{ m})^2 + (0.2\text{ kg-m}^2)\right]\omega_2^2$$

$$+ \frac{1}{2}(6\text{ kg})v_r^2$$

$$V_2 = \frac{1}{2}(10\text{ N/m})(2.4\text{ m})^2$$

Solving we find that $\boxed{v_r = 1.46\text{ m/s}}$.

Problem 19.63 The circular bar is welded to the vertical shafts, which can rotate freely in bearings at A and B. Let I be the moment of inertia of the circular bar and shafts about the vertical axis. The circular bar has an initial angular velocity ω_0, and the mass m is released in the position shown with no velocity relative to the bar. Determine the angular velocity of the circular bar as a function of the angle β between the vertical and the position of the mass. Neglect the moment of inertia of the mass about its center of mass; that is, treat the mass as a particle.
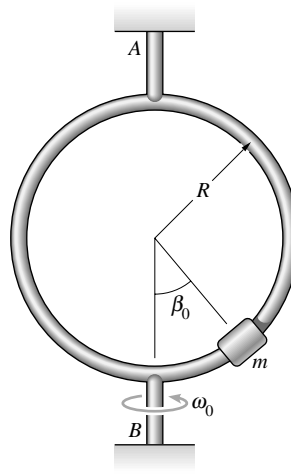
Solution: Angular momentum about the vertical axis is conserved:

$$I\omega_0 + (R\sin\beta_0)m(R\sin\beta_0)\omega_0 = I\omega + (R\sin\beta)m(R\sin\beta)\omega.$$

Solving for

$$\omega, \ \omega = \left(\frac{I + mR^2\sin^2\beta_0}{I + mR^2\sin^2\beta} \right)\omega_0.$$

Problem 19.64 The 10-N bar is released from rest in the $45°$ position shown. It falls and the end of the bar strikes the horizontal surface at P. The coefficient of restitution of the impact is $e = 0.6$. When the bar rebounds, through what angle relative to the horizontal will it rotate?
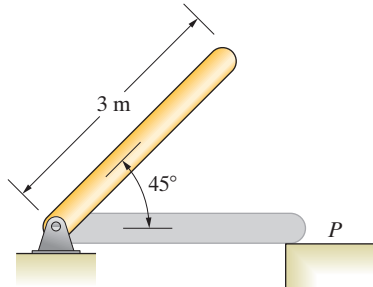
Solution: We solve the problem in three phases.
We start with a work energy analysis to find out how fast the bar is rotating just before the collision

$$T_1 + V_1 = T_2 + V_2,$$

$$0 + (10 \text{ N})(1.5 \text{ m})\sin 45° = \frac{1}{2}\left[\frac{1}{3}\left(\frac{10 \text{ N-s}^2}{9.81 \text{ m}} \right)(3 \text{ m})^2 \right]\omega_2^2 + 0,$$

$$\omega_2 = 2.63 \text{ rad/s}.$$

Next we do an impact analysis to take it through the collision

$$e\omega_2 L = \omega_3 L$$

$$0.6(2.63 \text{ rad/s})(3 \text{ m}) = \omega_3(3 \text{ m}) \Rightarrow \omega_3 = 1.58 \text{ rad/s}.$$

Finally we do another work energy analysis to find the rebound angle.

$$T_3 + V_3 = T_4 + V_4,$$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{10 \text{ N}}{9.81 \text{ m/s}^2} \right)(3 \text{ m})^2 \right]\omega_3^2 + 0 = 0 + (10 \text{ N})(1.5 \text{ m})\sin\theta,$$

$$\boxed{\theta = 14.7°.}$$

Problem 19.65 The 10-N bar is released from rest in the 45° position shown. It falls and the end of the bar strikes the horizontal surface at P. The bar rebounds to a position 10° relative to the horizontal. If the duration of the impact is 0.01 s, what is the magnitude of the average vertical force the horizontal surface exerted on the bar at P?
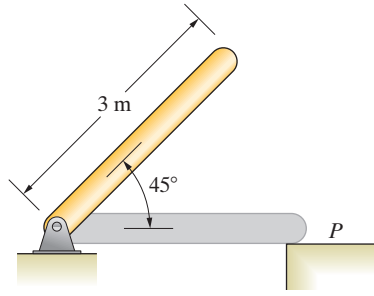
Solution: We solve the problem in three phases.

We start with a work energy analysis to find out how fast the bar is rotating just before the collision

$$T_1 + V_1 = T_2 + V_2,$$

$$0 + (10 \text{ N})(1.5 \text{ m}) \sin 45° = \frac{1}{2} \left[\frac{1}{3} \left(\frac{10 \text{ N-s}^2}{9.81 \text{ m}} \right) (3 \text{ m})^2 \right] \omega_2^2 + 0,$$

$$\omega_2 = 2.63 \text{ rad/s.}$$

Next we do another work energy analysis to find out how fast the bar is rotating just after the collision.

$$T_3 + V_3 = T_4 + V_4,$$

$$\frac{1}{2} \left[\frac{1}{3} \left(\frac{10 \text{ N-s}^2}{9.81 \text{ m}} \right) (3 \text{ m})^2 \right] \omega_3^2 + 0 = 0 + (10 \text{ N})(1.5 \text{ m}) \sin 10°,$$

$$\omega_3 = 1.31 \text{ rad/s.}$$

Now we can use the angular impulse momentum equation about the pivot point to find the force.

$$- I\omega_2 - W\Delta t \frac{L}{2} + F\Delta t L = I\omega_3,$$

$$-\frac{1}{3} \left(\frac{10 \text{ N-s}^2}{9.81 \text{ m}} \right) (3 \text{ m})^2 (2.63 \text{ rad/s}) - (10 \text{ N})(0.01 \text{ s})(1.5 \text{ m})$$

$$+ F(0.01 \text{ s})(3 \text{ m}) = \frac{1}{3} \left(\frac{10 \text{ N-s}^2}{9.81 \text{ m}} \right) (3 \text{ m})^2 (1.31 \text{ rad/s})$$

Solving we find $\boxed{F = 406.6 \text{ N.}}$

Problem 19.66 The 4-kg bar is released from rest in the horizontal position above the fixed projection at A. The distance $b = 0.35$ m. The impact of the bar with the projection is plastic; that is, the coefficient of restitution of the impact is $e = 0$. What is the bar's angular velocity immediately after the impact?

Solution: First we do work energy to find the velocity of the bar just before impact.
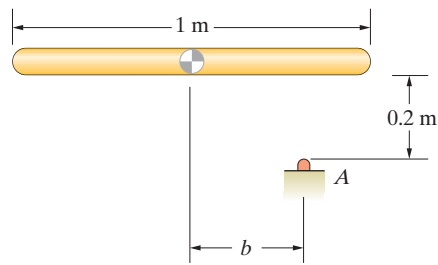
$$T_1 + V_1 = T_2 + V_2$$

$$0 + mgh = \frac{1}{2}\, mv_2^2 + 0 \Rightarrow v_2 = \sqrt{2gh} = \sqrt{2(9.81 \text{ m/s}^2)(0.2 \text{ m})} = 1.98 \text{ m/s}$$

Angular momentum is conserved about the impact point A. After the impact, the bar pivots about the fixed point A.

$$mv_2b = m\left(\frac{L^2}{12} + b^2\right)\omega_3 \Rightarrow \omega_3 = \frac{12bv_2}{L^2 + 12b^2} = \frac{12(0.35 \text{ m})(1.98 \text{ m/s})}{(1 \text{ m})^2 + 12(0.35 \text{ m})^2}$$

$$\boxed{\omega_3 = 3.37 \text{ rad/s.}}$$

Problem 19.67 The 4-kg bar is released from rest in the horizontal position above the fixed projection at A. The coefficient of restitution of the impact is $e = 0.6$. What value of the distance b would cause the velocity of the bar's center of mass to be zero immediately after the impact? What is the bar's angular velocity immediately after the impact?
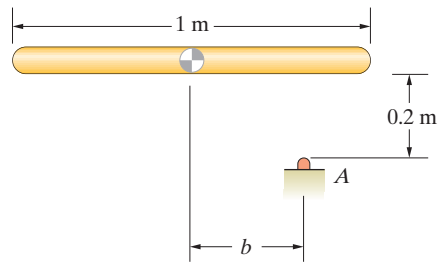
Solution: First we do work energy to find the velocity of the bar just before impact.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + mgh = \frac{1}{2}\, mv_2^2 + 0 \Rightarrow v_2 = \sqrt{2gh} = \sqrt{2(9.81 \text{ m/s}^2)(0.2 \text{ m})} = 1.98 \text{ m/s}$$

Angular momentum is conserved about point A. The coefficient of restitution is used to relate the velocities of the impact point before and after the collision.

$$mv_2b = mv_3b + \frac{1}{12}\, mL^2\, \omega_3, \quad ev_2 = \omega_3b - v_3, \quad v_3 = 0,$$

Solving we have

$$b = \sqrt{\frac{e}{12}}L = \sqrt{\frac{0.6}{12}}(1 \text{ m}) = 0.224 \text{ m,}$$

$$\omega_3 = \frac{12v_2b}{L^2} = \frac{12(1.98 \text{ m/s})(0.224 \text{ m})}{(1 \text{ m})^2} = 5.32 \text{ rad/s.}$$

$$\boxed{b = 0.224 \text{ m,} \quad \omega_3 = 5.32 \text{ rad/s.}}$$

Problem 19.68 The mass of the ship is 544,000 kg, and the moment of inertia of the vessel about its center of mass is 4×10^8 kg-m^2. Wind causes the ship to drift sideways at 0.1 m/s and strike the stationary piling at P. The coefficient of restitution of the impact is $e = 0.2$. What is the ship's angular velocity after the impact?
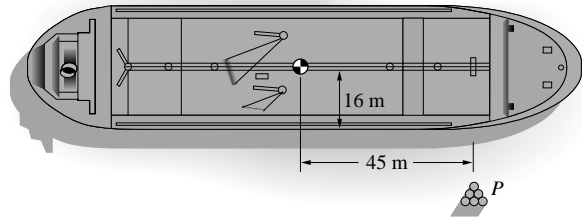
Solution: Angular momentum about P is conserved

$$(45)(mv) = (45)(mv') + I\omega'. \quad \textbf{(1)}$$

The coefficient of restitution is

$$e = \frac{-v'_P}{v}, \quad \textbf{(2)}$$

where v'_P is the vertical component of the velocity of P after the impact. The velocities v'_P and v' are related by

$$v' = v'_P + (45)\omega'. \quad \textbf{(3)}$$

Solving Eqs. (1)–(3), we obtain $\underline{\omega' = 0.00196 \text{ rad/s}}$, $v' = 0.0680$ m/s, and $v'_P = -0.02$ m/s.

Before impact   After impact
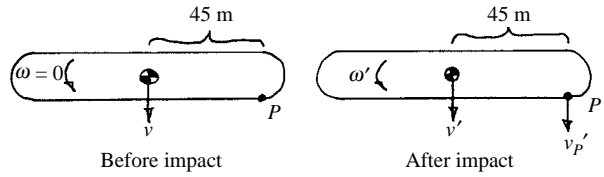
Problem 19.69 In Problem 19.68, if the duration of the ship's impact with the piling is 10 s, what is the magnitude of the average force exerted on the ship by the impact?

Solution: See the solution of Problem 19.68. Let F_p be the average force exerted on the ship by the piling. We apply linear impulse and momentum.

$$-F_p \Delta t = mv' - mv:$$

$$-F_p(10) = (544,000)(0.0680 - 0.1).$$

Solving, we obtain

$$F_p = 1740 \text{ N}.$$

Problem 19.70 In Active Example 19.7, suppose that the ball A weighs 2 N, the bar B weighs 6 N, and the length of the bar is 1 m. The ball is translating at $v_A = 3$ m/s before the impact and strikes the bar at $h=0.6$ m. What is the angular velocity of the bar after the impact if the ball adheres to the bar?
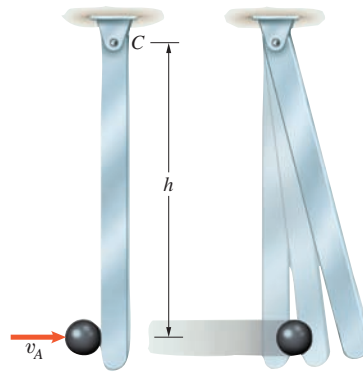
Solution: Angular momentum is conserved about point C.

$$m_A v_A h = \left(\frac{1}{3}m_B L^2 + m_A h^2\right)\omega$$

$$\omega = \frac{m_A v_A h}{\frac{1}{3}m_B L^2 + m_A h^2} = \frac{(2 \text{ N})(3 \text{ m/s})(0.6 \text{ m})}{\frac{1}{3}(6 \text{ N})(1 \text{ m})^2 + (2 \text{ N})(0.6 \text{ m})^2} = 1.32 \text{ rad/s}.$$

$\boxed{\omega = 1.32 \text{ rad/s.}}$

Problem 19.71 The 2-kg sphere A is moving toward the right at 4 m/s when it strikes the end of the 5-kg slender bar B. Immediately after the impact, the sphere A is moving toward the right at 1 m/s. What is the angular velocity of the bar after the impact?
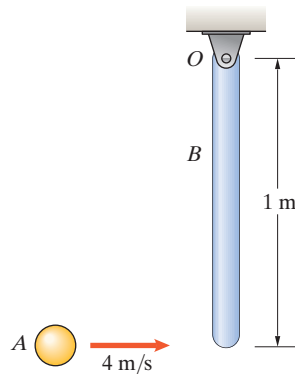
Solution: Angular momentum is conserved about point O.

$$m_A v_{A1} L = m_A v_{A2} L + \frac{1}{3}m_B L^2\, \omega_2$$

$$\omega_2 = \frac{3m_A(v_{A1} - v_{A2})}{m_B L}$$

$$\omega_2 = \frac{3(2 \text{ kg})(4 \text{ m/s} - 1 \text{ m/s})}{(5 \text{ kg})(1 \text{ m})} = 3.6 \text{ m/s}.$$

$\boxed{\omega_2 = 3.6 \text{ m/s counterclockwise.}}$

Problem 19.72 The 2-kg sphere A is moving toward the right at 4 m/s when it strikes the end of the 5-kg slender bar B. The coefficient of restitution is $e = 0.4$. The duration of the impact is 0.002 seconds. Determine the magnitude of the average horizontal force exerted on the bar by the pin support as a result of the impact.

Solution: System angular momentum is conserved about point O. The coefficient of restitution is used to relate the relative velocities before and after the impact. We also use the linear impulse momentum equation for the ball and for the bar.

$$m_A v_{A1} L = m_A v_{A2} L + \frac{1}{3} m_B L^2 \, \omega_2, \quad e v_{A1} = \omega_2 L - v_{A2},$$

$$m_A v_{A1} - F \Delta t = m_A v_{A2}, \quad F \Delta t + R \Delta t = m_B \omega_2 \frac{L}{2}.$$
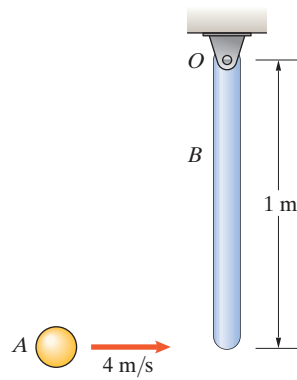
Solving we find

$$R = \frac{(1 + e) m_A m_B v_{A1}}{2(3m_A + m_B) \Delta t} = \frac{1.4(2 \text{ kg})(5 \text{ kg})(4 \text{ m/s})}{2(11 \text{ kg})(0.002 \text{ s})} = 1270 \text{ N}$$

$\boxed{R = 1.27 \text{ kN.}}$

Problem 19.73 The 2-kg sphere A is moving toward the right at 10 m/s when it strikes the unconstrained 4-kg slender bar B. What is the angular velocity of the bar after the impact if the sphere adheres to the bar?
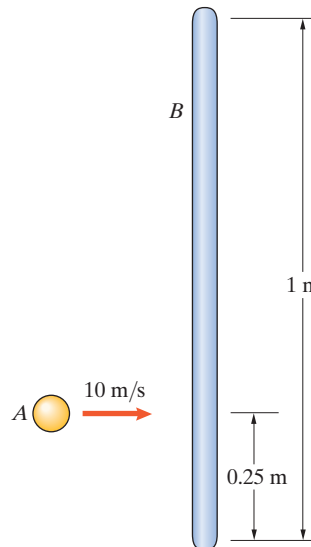
Solution: The coefficient of restitution is $e = 0$. Angular momentum for the system is conserved about the center of mass of the bar. The coefficient of restitution is used to relate the relative velocities before and after the impact. Linear momentum is conserved for the system.

$$m_A v_{A1} \left(\frac{L}{2} - h \right) = m_A v_{A2} \left(\frac{L}{2} - h \right) + \frac{1}{12} m_B L^2 \, \omega_2,$$

$$e v_{A1} = 0 = v_{B2} + \omega_2 \left(\frac{L}{2} - h \right) - v_{A2},$$

$$m_A v_{A1} = m_A v_{A2} + m_B v_{B2}.$$

Solving we find

$$\omega_2 = \frac{6(L - 2h) m_A v_{A1}}{12h^2 m_A - 12h L m_A + L^2 (4m_A + m_B)}$$

$$= \frac{6(1 \text{ m} - 2[0.25 \text{ m}])(2 \text{ kg})(10 \text{ m/s})}{12(0.25 \text{ m})^2 (2 \text{ kg}) - 12(0.25 \text{ m})(1 \text{ m})(2 \text{ kg}) + (1 \text{ m})^2 (12 \text{ kg})}$$

$\boxed{\omega_2 = 8 \text{ rad/s.}}$

Problem 19.74 The 2-kg sphere A is moving to the right at 10 m/s when it strikes the unconstrained 4-kg slender bar B. The coefficient of restitution of the impact is $e = 0.6$. What are the velocity of the sphere and the angular velocity of the bar after the impact?
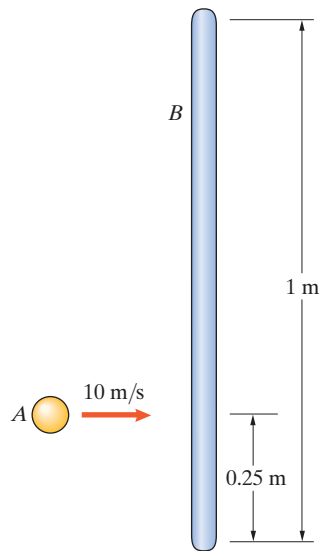
Solution: Angular momentum for the system is conserved about the center of mass of the bar. The coefficient of restitution is used to relate the relative velocities before and after the impact. Linear momentum is conserved for the system.

$$m_A v_{A1} \left(\frac{L}{2} - h \right) = m_A v_{A2} \left(\frac{L}{2} - h \right) + \frac{1}{12} m_B L^2 \, \omega_2,$$

$$e v_{A1} = v_{B2} + \omega_2 \left(\frac{L}{2} - h \right) - v_{A2},$$

$$m_A v_{A1} = m_A v_{A2} + m_B v_{B2}.$$

Solving we find

$$\omega_2 = \frac{6(1 + e)(L - 2h) m_A v_{A1}}{12 h^2 m_A - 12 h L m_A + L^2 (4 m_A + m_B)},$$

$$= \frac{6(1.6)(1 \text{ m} - 2[0.25 \text{ m}])(2 \text{ kg})(10 \text{ m/s})}{12(0.25 \text{ m})^2 (2 \text{ kg}) - 12(0.25 \text{ m})(1 \text{ m})(2 \text{ kg}) + (1 \text{ m})^2 (12 \text{ kg})},$$

$$v_{A2} = \frac{(12 h[h - L] m_A + L^2 [4 m_A - e m_B]) v_{A1}}{12 h^2 m_A - 12 h L m_A + L^2 (4 m_A + m_B)},$$

$$= \frac{(12[0.25 \text{ m}][-0.75 \text{ m}](2 \text{ kg}) + [1 \text{ m}]^2 [5.6 \text{ kg}])(10 \text{ m/s})}{12(0.25 \text{ m})^2 (2 \text{ kg}) - 12(0.25 \text{ m})(1 \text{ m})(2 \text{ kg}) + (1 \text{ m})^2 (12 \text{ kg})}.$$

$$\boxed{\begin{aligned} \omega_2 &= 12.8 \text{ rad/s counterclockwise,} \\ v_{A2} &= 1.47 \text{ m/s to the right.} \end{aligned}}$$

Problem 19.75 The 1.4 N ball is translating with velocity $v_A = 24.4$ m/s perpendicular to the bat just before impact. The player is swinging the 8.6 N bat with angular velocity $\omega = 6\pi$ rad/s before the impact. Point C is the bat's instantaneous center both before and after the impact. The distances $b = 355.6$ mm and $\overline{y} = 660.4$ mm. The bat's moment of inertia about its center of mass is $I_B = 0.045$ kg-m^2. The coefficient of restitution is $e = 0.6$, and the duration of the impact is 0.008 s. Determine the magnitude of the velocity of the ball after the impact and the average force A_x exerted on the bat by the player during the impact if (a) $d = 0$, (b) $d = 76.2$ mm, and (c) $d = 203$ mm.
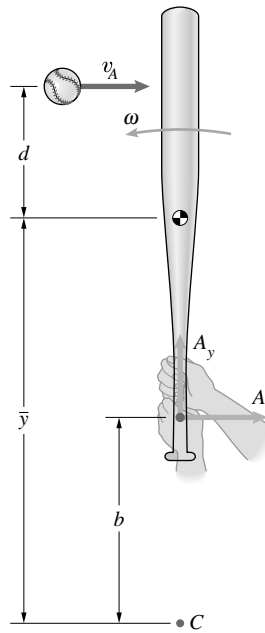
Solution: By definition, the coefficient of restitution is

(1) $\quad e = \dfrac{v'_P - v'_A}{v_A - v_P}$.

The angular momentum about A is conserved:

(2) $\quad m_A v_A (d + \overline{y} - b) + m_B v_B (\overline{y} - b) - I_B \omega$

$\qquad = m_A v'_A (d + \overline{y} - b) + m_B v'_B (\overline{y} - b) - I_B \omega'$.

From kinematics, the velocities about the instantaneous center:

(3) $\quad v_P = -\omega(\overline{y} + d)$,

(4) $\quad v_B = -\omega \overline{y}$,

(5) $\quad v'_P = -\omega'(\overline{y} + d)$,

(6) $\quad v'_B = -\omega \overline{y}$.

Since ω, $\overline{y}$, and d are known, v_B and v_P are determined from (3) and (4), and these six equations in six unknowns reduce to four equations in four unknowns. v'_P, v'_B, v'_A, and ω'. Further reductions may be made by substituting (5) and (6) into (1) and (2); however here the remaining four unknowns were solved by iteration for values of $d = 0$, $d = 0.076$ m, $d = 0.203$ m. The reaction at A is determined from the principle of angular impulse-momentum applied about the point of impact:

(7) $\quad \displaystyle\int_{t_1}^{t_2} A_x (d + \overline{y} - b)\, dt = (dm_B v'_B + I_B \omega')$

$\qquad - (dm_B v_B + I_B \omega)$,

where $t_2 - t_1 = 0.08$ s. Using (4) and (5), the reaction is

$A_x = \dfrac{(I_B - dm_B \mathbf{y})}{(d + \overline{y} - b)(t_2 - t_1)} (\omega' - \omega)$,

where the unknown, ω', is determined from the solution of the first six equations. The values are tabulated:
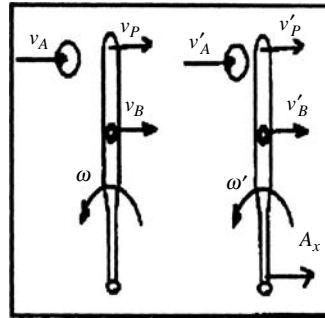
Only the values in the first two columns are required for the problem; the other values are included for checking purposes. *Note:* The reaction reverses between $d = 0.076$ m and $d = 0.203$ m, which means that the point of zero reaction occurs in this interval.

d, m	v'_A, m/s	A_x, N	v'_P, m/s	v'_B, m/s	ω', rad/s
0	−27.83	−186.7	−5.73	−5.73	8.672
0.076	−27.53	−2.12	−4.57	−4.09	6.20
0.203	−26.43	297.3	−2.02	−1.55	2.34

Problem 19.76 In Problem 19.75, show that the force A_x is zero if $d = I_B/(m_B\bar{y})$, where m_B is the mass of the bat.

Solution: From the solution to Problem 19.75, the reaction is

$$A_x = \frac{(I_B - dm_B\bar{y})}{(d + \bar{y} - b)(t_2 - t_1)}(\omega' - \omega).$$

Since $(\omega' - \omega) \neq 0$, the condition for zero reaction is $I_B - dm_B\bar{y} = 0$, from which $d = \dfrac{I_B}{\bar{y}m_B}$.

Problem 19.77 A 10-N slender bar of length $l = 2$ m is released from rest in the horizontal position at a height $h = 2$ m above a peg (Fig. a). A small hook at the end of the bar engages the peg, and the bar swings from the peg (Fig. b). What it the bar's angular velocity immediately after it engages the peg?
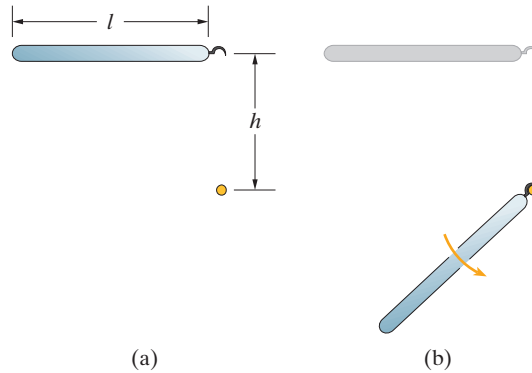
(a)   (b)

Solution: Work energy is used to find the velocity just before impact.

$T_1 + V_1 = T_2 + V_2$

$0 + mgh = \dfrac{1}{2}\, mv_2^2 + 0 \Rightarrow v_2 = \sqrt{2gh}$

Angular momentum is conserved about the peg.

$mv_2\, \dfrac{l}{2} = \dfrac{1}{3}\, ml^2\, \omega_3,$

$\omega_3 = \dfrac{3v_2}{2l} = \dfrac{3\sqrt{2gh}}{2l} = \dfrac{3\sqrt{2(9.81 \text{ m/s}^2)(2 \text{ m})}}{2(2 \text{ m})} = 4.7 \text{ rad/s}.$

$\boxed{\omega_3 = 4.7 \text{ rad/s.}}$

Problem 19.78 A 10-N slender bar of length $l = 2$ m is released from rest in the horizontal position at a height $h = 1$ m above a peg (Fig. a). A small hook at the end of the bar engages the peg, and the bar swings from the peg (Fig. b).

(a) Through what maximum angle does the bar rotate relative to its position when it engages the peg?
(b) At the instant when the bar has reached the angle determined in part (a), compare its gravitational potential energy to the gravitational potential energy the bar had when it was released from rest. How much energy has been lost?

(a)   (b)

Solution: Work energy is used to find the velocity just before impact.

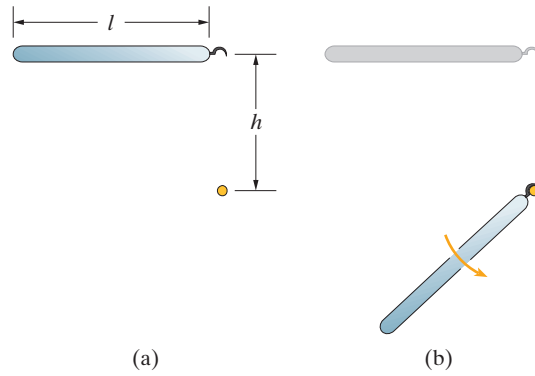$$T_1 + V_1 = T_2 + V_2, 0 + mgh = \frac{1}{2} mv_2^2 + 0 \Rightarrow v_2 = \sqrt{2gh}$$

Angular momentum is conserved about the peg.

$$mv_2 \frac{l}{2} = \frac{1}{3} ml^2 \omega_3, \quad \omega_3 = \frac{3v_2}{2l} = \frac{3\sqrt{2gh}}{2l}$$

Work energy is used to find the angle through which the bar rotates

$$T_3 + V_3 = T_4 + V_4, \frac{1}{2} \left(\frac{1}{3} ml^2 \right) \omega_3^2 - mgh = 0 - mg \left(h + \frac{l}{2} \sin\theta \right)$$

$$\theta = \sin^{-1} \left(-\frac{l\omega_3^2}{3g} \right) = \sin^{-1} \left(-\frac{3h}{2l} \right) = \sin^{-1} \left(-\frac{3}{4} \right) = 229°.$$

The energy that is lost is the difference in potential energies

$$V_1 - V_4 = 0 - \left[-mg \left(h + \frac{l}{2} \sin\theta \right) \right] = (10 \text{ N}) \left(1 \text{ m} + \frac{2 \text{ m}}{2} \sin[229°] \right) = 2.5 \text{ N-m.}$$

(a) $\theta = 229°$, (b) 2.5 N-m.

Problem 19.79 The 14.6 kg disk rolls at velocity $v = 3.05$ m/s toward a 152.4 mm step. The wheel remains in contact with the step and does not slip while rolling up onto it. What is the wheel's velocity once it is on the step?

457.2 mm

10 ft/s

152.4 mm
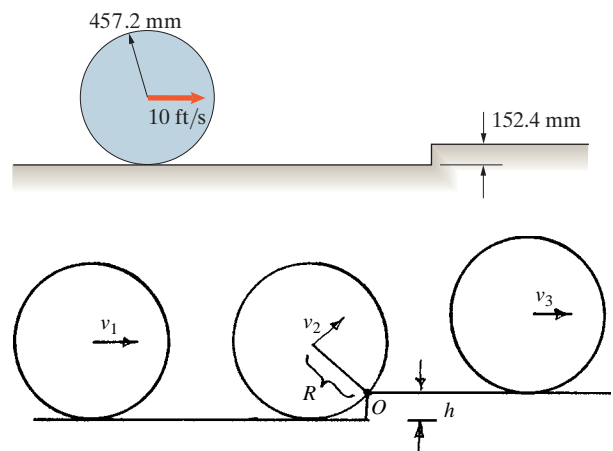
Solution: We apply conservation of angular momentum about 0 to analyze the impact with the step.

$$(R - h)mv_1 + I \left(\frac{v_1}{R} \right) = Rmv_2 + I \left(\frac{v_2}{R} \right). \quad (1)$$

Then we apply work and energy to the "climb" onto the step.

$$-mgh = \left[\frac{1}{2} mv_3^2 + \frac{1}{2} I \left(\frac{v_3}{R} \right)^2 \right] - \left[\frac{1}{2} mv_2^2 + \frac{1}{2} I \left(\frac{v_2}{R} \right)^2 \right]. \quad (2)$$

Solving Eqs. (1) and (2) with $v_1 = 3.05$ m/s and $I = \frac{1}{2} mR^2$, we obtain $v_3 = 1.91$ m/s.

v_1 v_2 v_3 R O h

Problem 19.80   The 14.6 kg disk rolls toward a 152.4 mm step. The wheel remains in contact with the step and does not slip while rolling up onto it. What is the minimum velocity v the disk must have in order to climb up onto the step?

Solution: See the solution of Problem 19.79 solving Eqs. (1) and (2) with $v_3 = 0$, we obtain
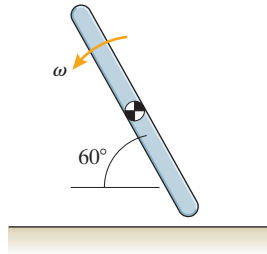
$\underline{v_1 = 1.82 \text{ m/s}}$.

Problem 19.81   The length of the bar is 1 m and its mass is 2 kg. Just before the bar hits the floor, its angular velocity is $\omega = 0$ and its center of mass is moving downward at 4 m/s. If the end of the bar adheres to the floor, what is the bar's angular velocity after the impact?

Solution: Given: $\omega = 0$, $L = 1$ m, $m = 2$ kg, $v_G = 4$ m/s, sticks to floor

Angular momentum about the contact point

$$m v_G \frac{L}{2} \cos 60° = \frac{1}{3} m L^2 \omega' \qquad \boxed{\omega' = 3 \text{ rad/s counterclockwise}}$$

Problem 19.82   The length of the bar is 1 m and its mass is 2 kg. Just before the bar hits the *smooth* floor, its angular velocity is $\omega = 0$ and its center of mass is moving downward at 4 m/s. If the coefficient of restitution of the impact is $e = 0.4$, what is the bar's angular velocity after the impact?

Solution: Given $\omega = 0$, $L = 1$ m, $m = 2$ kg, $v_G = 4$ m/s, $e = 0.4$, smooth floor.

Angular momentum about the contact point

$$m v_G \frac{L}{2} \cos 60° = \frac{1}{12} m L^2 \omega' + m v_G' \frac{L}{2} \cos 60°$$

Coefficient of restitution

$$e v_G = \omega \left(\frac{L}{2} \cos 60° \right) - v_G'$$

Solving we find $\boxed{\omega' = 9.6 \text{ rad/s counterclockwise}}$

Problem 19.83 The length of the bar is 1 m and its mass is 2 kg. Just before the bar hits the *smooth* floor, it has angular velocity ω and its center of mass is moving downward at 4 m/s. The coefficient of restitution of the impact is $e = 0.4$. What value of ω would cause the bar to have no angular velocity after the impact?

Solution: Given $L = 1$ m, $m = 2$ kg, $v_G = 4$ m/s, $e = 0.4$, $\omega' = 0$ smooth floor.

Angular momentum about the contact point

$$mv_G \frac{L}{2} \cos 60° + \frac{1}{12} mL^2 \omega = mv_G' \frac{L}{2} \cos 60°$$

Coefficient of restitution

$$e\left(v_G - \omega \frac{L}{2} \cos 60°\right) = 0 - v_G'$$

Solving we find $\omega = -24$ rad/s $\boxed{\omega = 24 \text{ rad/s clockwise}}$

Problem 19.84 During her parallel-bars routine, the velocity of the 400 N gymnast's center of mass is $1.2\mathbf{i} - 3.05\mathbf{j}$ (m/s) and her angular velocity is zero just before she grasps the bar at A. In the position shown, her moment of inertia about her center of mass is 2.44 kg-m². If she stiffens her shoulders and legs so that she can be modeled as a rigid body, what is the velocity of her center of mass and her angular velocity just after she grasps the bar?

(−203.2, 558.8) mm
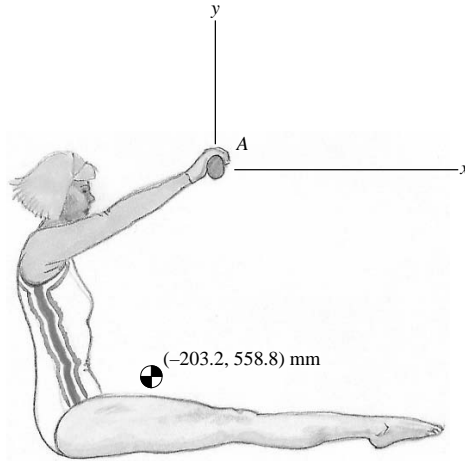
Solution: Let v' and ω' be her velocity and angular velocity after she grasps the bar. The angle $\theta = 20.0°$ and $r = 0.59$ m. Conservation of angular momentum about A is

$$\mathbf{k} \cdot (\mathbf{r} \times m\mathbf{v}) = rmv' + I\omega':$$

$$\mathbf{k} \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ 1.2m & -3.05m & 0 \end{vmatrix} = rmv' + I\omega',$$

$$-3.05mx - 1.2my = rmv' + I\omega'.$$

Solving this equation together with the relation $v' = r\omega'$, we obtain $\underline{\omega' = 3.15 \text{ rad/s}}$, $v' = 1.87$ m/s. Her velocity is

$$\mathbf{v}' = v' \cos\theta\mathbf{i} - v' \sin\theta\mathbf{j}$$

$$= 1.76\mathbf{i} - 0.64\mathbf{j} \text{ (m/s)}.$$
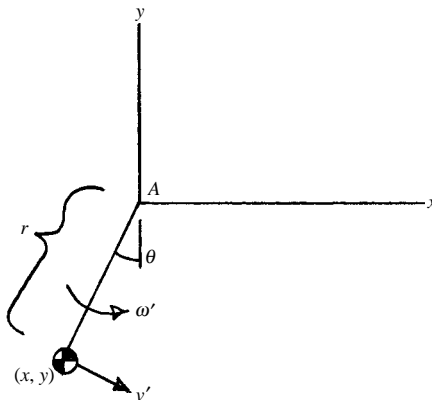
Problem 19.85 The 20-kg homogenous rectangular plate is released from rest (Fig. a) and falls 200 mm before coming to the end of the string attached at the corner A (Fig. b). Assuming that the vertical component of the velocity of A is zero just after the plate reaches the end of the string, determine the angular velocity of the plate and the magnitude of the velocity of the corner B at that instant.

(a)   (b)

Solution: We use work and energy to determine the plate's downward velocity just before the string becomes taut.

$$mg(0.2) = \tfrac{1}{2}mv^2.$$

Solving, $v = 1.98$ m/s. The plate's moment of inertia is

$$I = \frac{1}{12}(20)[(0.3)^2 + (0.5)^2] = 0.567 \text{ kg-m}^2.$$

Angular momentum about A is conserved:

$$0.25(mv) = 0.25(mv') + I\omega'. \quad \textbf{(1)}$$

The velocity of A just after is

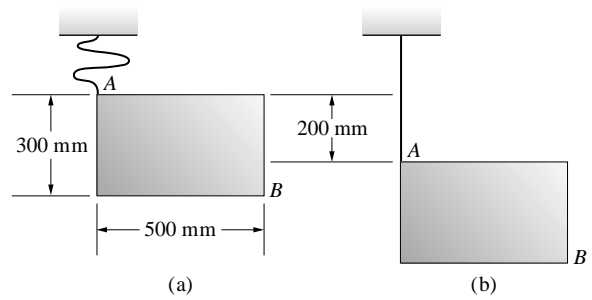$$\mathbf{v}'_A = \mathbf{v}'_G + \boldsymbol{\omega}' \times \mathbf{r}_{A/G}$$

$$= -v'\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega' \\ -0.25 & 0.15 & 0 \end{vmatrix}.$$

The $\mathbf{j}$ component of $\mathbf{v}'_A$ is zero:

$$-v' + 0.25\omega' = 0. \quad \textbf{(2)}$$

Solving Eqs. (1) and (2), we obtain $v' = 1.36$ m/s and $\underline{\omega' = 5.45 \text{ rad/s}}$. The velocity of B is

$$\mathbf{v}'_B = \mathbf{v}'_G + \boldsymbol{\omega}' \times \mathbf{r}_{B/G}$$

$$= -v'\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega' \\ 0.25 & -0.15 & 0 \end{vmatrix}$$

$$\underline{\mathbf{v}'_B = -0.818\mathbf{i} - 2.726\mathbf{j} \text{ (m/s)}.}$$

Just before   Just after
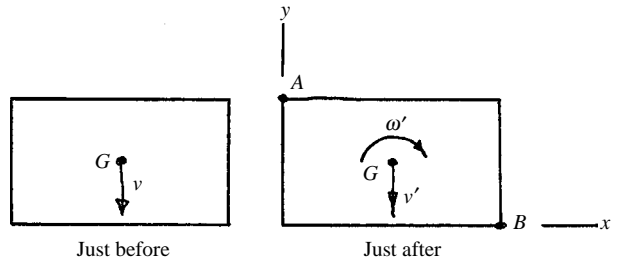
Problem 19.86* Two bars *A* and *B* are each 2 m in length, and each has a mass of 4 kg. In Fig. (a), bar *A* has no angular velocity and is moving to the right at 1 m/s, and bar *B* is stationary. If the bars bond together on impact, (Fig. b), what is their angular velocity ω' after the impact?

(a)   (b)
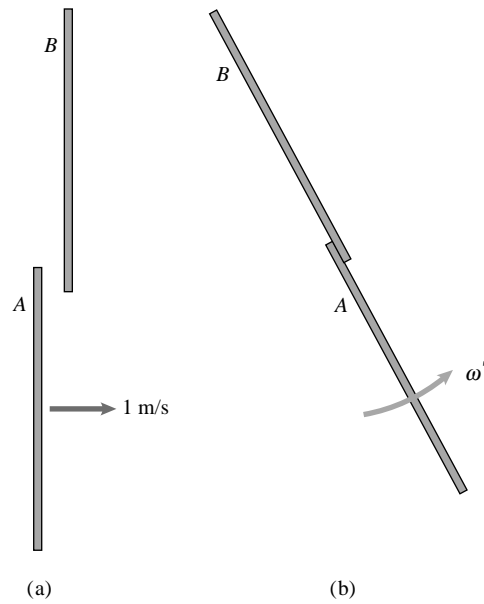
Solution: Linear momentum is conserved:

$$mv_A = mv'_A + mv'_B. \quad \textbf{(1)}$$

Angular momentum about any point is conserved. About *P*,

$$mv_A\left(\frac{l}{2}\right) = mv'_A\left(\frac{l}{2}\right) - mv'_B\left(\frac{l}{2}\right) + 2I\omega', \quad \textbf{(2)}$$

where $I = \frac{1}{12}ml^2$.

The velocities are related by

$$v'_A = v'_B + l\omega'. \quad \textbf{(3)}$$

Solving Eqs. (1)–(3), we obtain

$$\omega' = 0.375 \text{ rad/s}.$$

Before   After
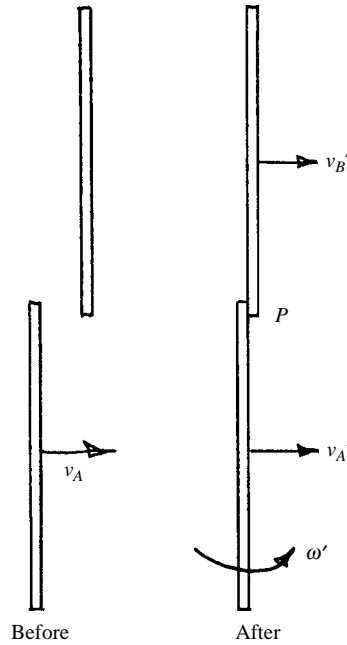
Problem 19.87* In Problem 19.86, if the bars do not bond together on impact and the coefficient of restitution is $e = 0.8$, what are the angular velocities of the bars after the impact?

Solution: Linear momentum is conserved:

$$mv_A = mv'_A + mv'_B. \quad \textbf{(1)}$$

Angular momentum of *each* bar about the point of contact is conserved:

$$mv_A\left(\frac{l}{2}\right) = mv'_A\left(\frac{l}{2}\right) + I\omega'_A, \quad \textbf{(2)}$$

$$0 = -mv'_B\left(\frac{l}{2}\right) + I\omega'_B. \quad \textbf{(3)}$$

Coefficient of restitution:

$$e = \frac{v'_{BP} - v'_{AP}}{v_A}. \quad \textbf{(4)}$$

The velocities are related by

$$v'_A = v'_{AP} + \left(\frac{l}{2}\right)\omega'_A, \quad \textbf{(5)}$$

$$v'_{BP} = v'_B + \left(\frac{l}{2}\right)\omega'_B. \quad \textbf{(6)}$$

Before   After
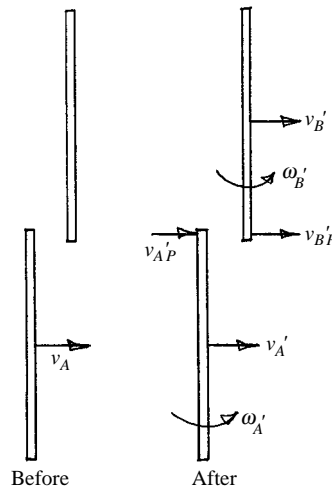
Solving Eqs. (1)–(6), we obtain

$$\underline{\omega'_A = \omega'_B = 0.675 \text{ rad/s.}}$$

Problem 19.88* Two bars A and B are each 2 m in length, and each has a mass of 4 kg. In Fig. (a), bar A has no angular velocity and is moving to the right at 1 m/s, and B is stationary. If the bars bond together on impact (Fig. b), what is their angular velocity ω' after the impact?

(a)   (b)

Solution: Linear momentum is conserved:

$$x - \text{DIR:} \quad m_A v_A = m_A v'_{Ax} + m_B v'_{Bx}, \quad \textbf{(1)}$$

$$y - \text{DIR:} \quad 0 = m_A v'_{Ay} + m_B v'_{By}. \quad \textbf{(2)}$$

Total angular momentum is conserved. Calculating it about 0,

$$0 = -\frac{l}{2}m_A v'_{Ay} + I_A\omega' - \frac{l}{2}m_B v'_{Bx} + I_B\omega'. \quad \textbf{(3)}$$
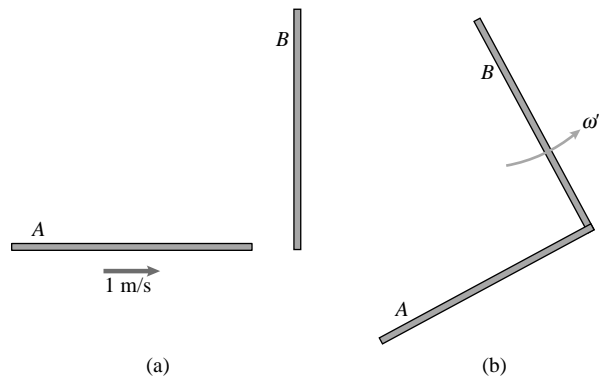
We also have the kinematic relation

$$\mathbf{v}'_B = \mathbf{v}'_A + \boldsymbol{\omega}' \times \mathbf{r}_{B/A}:$$

$$v'_{Bx}\mathbf{i} + v'_{By}\mathbf{j} = v'_{Ax}\mathbf{i} + v'_{Ay}\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega' \\ \dfrac{l}{2} & \dfrac{l}{2} & 0 \end{vmatrix},$$

Before impact   After impact
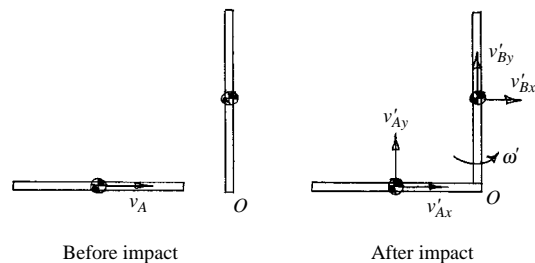
$$v'_{Bx} = v'_{Ax} - \frac{l}{2}\omega', \quad \textbf{(4)}$$

$$v'_{By} = v'_{Ay} + \frac{l}{2}\omega'. \quad \textbf{(5)}$$

Solving Eqs. (1)–(5), we obtain $\omega' = 0.3$ rad/s.

Problem 19.89* The horizontal velocity of the landing airplane is 50 m/s, its vertical velocity (rate of descent) is 2 m/s, and its angular velocity is zero. The mass of the airplane is 12 Mg, and the moment of inertia about its center of mass is 1×10^5 kg-m². When the rear wheels touch the runway, they remain in contact with it. Neglecting the horizontal force exerted on the wheels by the runway, determine the airplane's angular velocity just after it touches down. (See Example 19.8.)

1.8 m

0.3 m

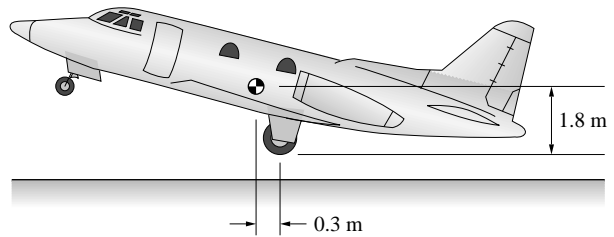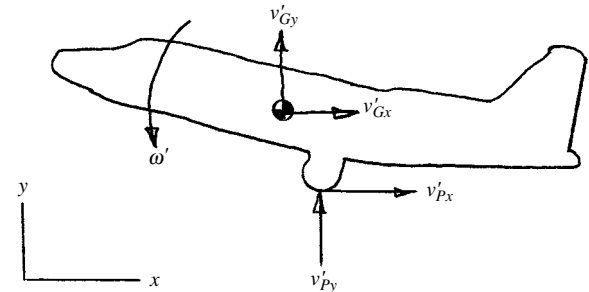**Solution:** Use a reference frame that moves to the left with the airplane's horizontal velocity. Before touchdown, the velocity of the center of mass is $\mathbf{v}_G = -2\mathbf{j}$ (m/s). Because there is no horizontal force, $v'_{Gx} = 0$ (see Fig.), and it is given that $v'_{Py} = 0$, where P is the point of contact.

Angular momentum about P is conserved:

$$m(2 \text{ m/s})(0.3 \text{ m}) = -mv'_{Gy}(0.3 \text{ m}) + I\omega'. \quad (1)$$

The velocities are related by

$$\mathbf{v}'_G = \mathbf{v}'_P + \boldsymbol{\omega}' \times \mathbf{r}_{G/P}:$$

$$v'_{Gy}\mathbf{j} = v'_{Px}\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega' \\ -0.3 & 1.8 & 0 \end{vmatrix}. \quad (2)$$

The **j** component of this equation is

$$v'_{Gy} = -0.3\omega'. \quad (3)$$

Solving Eqs. (1) and (3), we obtain

$\underline{\omega' = 0.0712 \text{ rad/s.}}$

Problem 19.90* Determine the angular velocity of the airplane in Problem 19.89 just after it touches down if its wheels don't stay in contact with the runway and the coefficient of restitution of the impact is $e = 0.4$. (See Example 19.8.)

Solution: See the solution of Problem 19.89. In this case v'_{Py} is not zero but is determined by

$$e = -\frac{v'_{Py}}{v_{Py}}:$$

$$0.4 = -\frac{v'_{Py}}{(-2)}$$

We see that $v'_{Py} = 0.8$ m/s. From Eqs. (1) and (2) of the solution of Problem 19.89,

$$0.6m = -0.3mv'_{Gy} + I\omega',$$

$$v'_{Gy} = v'_{Py} - 0.3\omega'.$$

Solving these two equations, we obtain

$\underline{\omega' = 0.0997 \text{ rad/s.}}$

Problem 19.91*   While attempting to drive on an icy street for the first time, a student skids his 1260-kg car (A) into the university president's unoccupied 2700-kg Rolls-Royce Corniche (B). The point of impact is P. Assume that the impacting surfaces are smooth and parallel to the y axis, and the coefficient of restitution of the impact is $e = 0.5$. The moments of inertia of the cars about their centers of mass are $I_A = 2400$ kg-m^2 and $I_B = 7600$ kg-m^2. Determine the angular velocities of the cars and the velocities of their centers of mass after the collision. (See Example 19.9.)
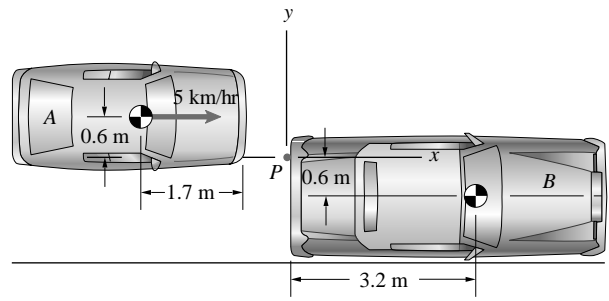
Solution: Car A's initial velocity is

$$v_A = \frac{5000}{3600} = 1.39 \text{ m/s}$$

The force of the impact is parallel to the x-axis so the cars are moving in the x direction after the collision. Linear momentum is conserved:

$$m_A v_A = m_A v'_A + m_B v'_B \quad \textbf{(1)}$$

The angular momentum of each car about the point of impact is conserved.

$$-0.6 m_A v_A = -0.6 m_A v'_A + I_A \omega'_A \quad \textbf{(2)}$$

$$0 = 0.6 m_B v'_B + I_B \omega'_B \quad \textbf{(3)}.$$

The velocity of car A at the point of impact after the collision is

$$\mathbf{v}'_{AP} = \mathbf{v}'_A + \boldsymbol{\omega}'_A \times \mathbf{r}_{P/A} = v'_A \mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega'_A \\ 1.7 & -0.6 & 0 \end{vmatrix}$$

$$\mathbf{v}'_{AP} = (v'_A + 0.6\omega'_A)\mathbf{i} + 1.7\omega'_A \mathbf{j}.$$

The corresponding equation for car B is

$$\mathbf{v}'_{BP} = \mathbf{v}'_B + \boldsymbol{\omega}'_B \times \mathbf{r}_{P/B} = v'_B \mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega'_B \\ -3.2 & 0.6 & 0 \end{vmatrix}, \quad \text{or}$$

$$v'_{BP} = (v'_B - 0.6\omega'_B)\mathbf{i} - 3.2\omega'_B \mathbf{j}$$

The x-components of the velocities at P are related by the coefficient of restitution:

$$e = \frac{(v'_B - 0.6\omega'_B) - (v'_A + 0.6\omega'_A)}{v_A} \quad \textbf{(4)}.$$

Solving Equations (1)–(4), we obtain

$$v'_A = 0.174 \text{ m/s}, \ v'_B = 0.567 \text{ m/s}$$

$$\omega'_A = -0.383 \text{ rad/s}, \ \omega'_B = -0.12 \text{ rad/s}.$$

Problem 19.92*   The student in Problem 19.91 claimed he was moving at 5 km/h prior to the collision, but police estimate that the center of mass of the Rolls-Royce was moving at 1.7 m/s after the collision. What was the student's actual speed? (See Example 19.9.)

Solution: Setting $v'_B = 1.7$ m/s in Equations (1)–(4) of the solution of Problem 19.91 and treating v_A as an unknown, we obtain

$$\underline{v_A = 4.17 \text{ m/s} = 15.0 \text{ km/h}.}$$

Problem 19.93 Each slender bar is 48 cm long and weighs 20 N. Bar A is released in the horizontal position shown. The bars are smooth, and the coefficient of restitution of their impact is $e = 0.8$. Determine the angle through which B swings afterward.
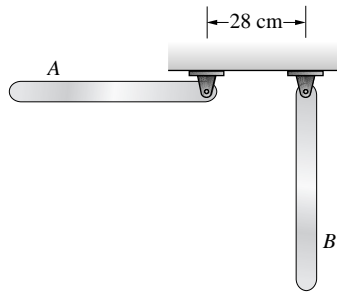
Solution: Choose a coordinate system with the x axis parallel to bar A in the initial position, and the y axis positive upward. The strategy is (a) from the principle of work and energy, determine the angular velocity of bar A the instant before impact with B; (b) from the definition of the coefficient of restitution, determine the value of e in terms of the angular velocities of the two bars at the instant after impact; (c) from the principle of angular impulse-momentum, determine the relation between the angular velocities of the two bars after impact; (d) from the principle of work and energy, determine the angle through which bar B swings.

The angular velocity of bar A before impact: The angle of swing of bar A is

$$\beta = \sin^{-1}\left(\frac{28}{48}\right) = 35.69°.$$

Denote the point of impact by P. The point of impact is

$$-h = -L\cos\beta = -3.25 \text{ cm}$$

The change in height of the center of mass of bar A is

$$-\frac{L}{2}\cos\beta = -\frac{h}{2}.$$

From the principle of work and energy, $U = T_2 - T_1$, where $T_1 = 0$, since the bar is released from rest. The work done by the weight of bar A is

$$U = \int_0^{-\frac{h}{2}} -W\,dh = \frac{Wh}{2}.$$

The kinetic energy the bar is $T_2 = \left(\frac{1}{2}\right)I_A\omega_A^2$,

from which $\omega_A = \sqrt{\dfrac{Wh}{I_A}} = \sqrt{\dfrac{3g\cos\beta}{L}} = 4.43$ rad/s,

where $I_A = \dfrac{mL^2}{3}$.

The angular velocities at impact: By definition, the coefficient of restitution is

$$e = \frac{v'_{BPx} - v'_{APx}}{v_{APx} - v_{BPx}}.$$
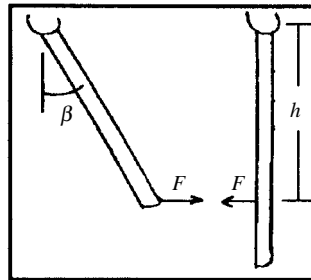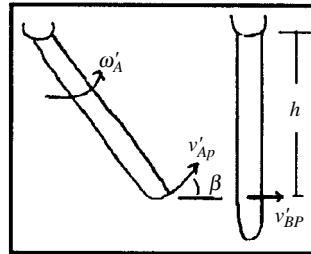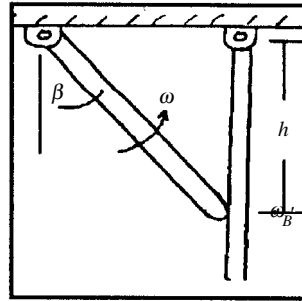
Bar B is at rest initially, from which

$$v_{BPx} = 0, v'_{BPx} = v'_{BP}, \text{ and}$$

$$v'_{APx} = v'_{AP}\cos\beta, v_{APx} = v_{AP}\cos\beta,$$

from which

$$e = \frac{v'_{BP} - v'_{AP}\cos\beta}{v_{AP}\cos\beta}.$$

The angular velocities are related by

$$v'_{BP} = h\omega'_B, \quad v'_{AP} = L\omega'_A, \quad v_{AP} = L\omega_A,$$

from which

$$e = \frac{h\omega'_B - (L\cos\beta)\omega'_A}{(L\cos\beta)\omega_A} = \frac{\omega_B - \omega'_A}{\omega_A},$$

from which

(1) $\quad \omega'_B - \omega'_A = e\omega_A$

The force reactions at P: From the principle of angular impulse-momentum,

$$\int_{t_1}^{t_2} Fh\, dt = Fh(t_2 - t_1) = I_B(\omega'_B - 0), \quad \text{and}$$

$$\int_{t_1}^{t_2} -F(L\cos\beta)\, dt = -F(L\cos\beta)(t_2 - t_1)$$

$$= I_A(\omega'_A - \omega_A).$$

Divide the second equation by the first:

$$\frac{-L\cos\beta}{h} = -1 = \frac{\omega'_A - \omega_A}{\omega'_B},$$

from which

(2) $\quad \omega'_B + \omega'_A = \omega_A.$

Solve (1) and (2):

$$\omega'_A = \frac{(1-e)}{2}\omega_A,$$

$$\omega'_B = \frac{(1+e)}{2}\omega_A.$$

The principle of work and energy: From the principle of work and energy, $U = T_2 - T_1$, where $T_2 = 0$, since the bar comes to rest after rotating through an angle γ. The work done by the weight of bar B as its center of mass rotates through the angle γ is

$$U = \int_{-\frac{L}{2}}^{-\frac{L\cos\gamma}{2}} -W_B\, dh = -W\left(\frac{L}{2}\right)(1 - \cos\gamma).$$

The kinetic energy is

$$T_1 = \left(\frac{1}{2}\right) I_B \omega'^2_B = \left(\frac{1}{8}\right) I_B (1+e)^2 \omega_A^2$$

$$= \frac{I_B(1+e)^2(3g\cos\beta)}{8L} = \frac{WL(1+e)^2\cos\beta}{8}.$$

Substitute into $U = -T_1$ to obtain

$$\cos\gamma = 1 - \frac{(1+e)^2\cos\beta}{4} = 0.3421,$$

from which $\boxed{\gamma = 70°}$

Problem 19.94* The *Apollo* CSM (*A*) approaches the *Soyuz* Space Station (*B*). The mass of the *Apollo* is $m_A = 18$ Mg, and the moment of inertia about the axis through the center of mass parallel to the z axis is $I_A = 114$ Mg-m². The mass of the *Soyuz* is $m_B = 6.6$ Mg, and the moment of inertia about the axis through its center of mass parallel to the z axis is $I_B = 70$ Mg-m². The *Soyuz* is stationary relative to the reference frame shown and the CSM approaches with velocity $\mathbf{v}_A = 0.2\mathbf{i} + 0.05\mathbf{j}$ (m/s) and no angular velocity. What is the angular velocity of the attached spacecraft after docking?

(A)   (B)
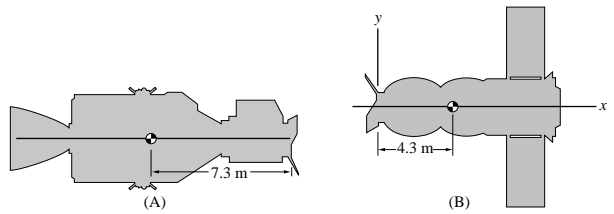
Solution: The docking port is at the origin on the *Soyuz*, and the configuration the instant after contact is that the centers of mass of both spacecraft are aligned with the x axis. Denote the docking point of contact by *P*. (*P* is a point on each spacecraft, and by assumption, lies on the x-axis.) The linear momentum is conserved:

$$m_A \mathbf{v}_{GA} = m_A \mathbf{v}'_{GA} + m_B v'_{GB},$$

from which

$$m_A(0.2) = m_A v'_{GAx} + m_B v'_{GBx},$$

and

(1) $m_A(0.05) = m_A v'_{GAy} + m_B v'_{GBy}.$

Denote the vectors from *P* to the centers of mass by

$$\mathbf{r}_{P/GA} = -7.3\mathbf{i} \text{ (m), and}$$

$$\mathbf{r}_{P/GB} = +4.3\mathbf{i} \text{ (m)}.$$

The angular momentum about the origin is conserved:

$$\mathbf{r}_{P/GA} \times m_A \mathbf{v}_{GA} = \mathbf{r}_{P/GA} \times m_A \mathbf{v}'_{GA} + I_A \boldsymbol{\omega}'_A$$

$$+ \mathbf{r}_{P/GB} \times m_B \mathbf{v}'_{GB}.$$

Denote the vector distance from the center of mass of the *Apollo* to the center of mass of the *Soyuz* by

$$\mathbf{r}_{B/A} = 11.6\mathbf{i} \text{ (m)}.$$

From kinematics, the instant after contact:

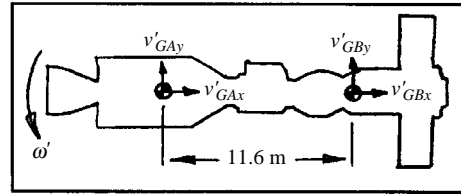$$\mathbf{v}'_{GB} = \mathbf{v}'_{GA} + \boldsymbol{\omega}' \times \mathbf{r}_{B/A}.$$

Reduce:

$$\mathbf{v}'_{GB} = \mathbf{v}'_{GA} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega' \\ 11.6 & 0 & 0 \end{bmatrix}$$

$$= v'_{GAx}\mathbf{i} + (v'_{GAy} + 11.6\omega')\mathbf{j},$$

from which $v'_{GBx} = v'_{GAx}$,

(2) $v'_{GBy} = v'_{GAy} + 11.6\omega' \cdot \mathbf{r}_{P/GA} \times m_A \mathbf{v}_{GA}$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7.3 & 0 & 0 \\ 0.21m_A & 0.05m_A & 0 \end{bmatrix} = (-0.365m_A)\mathbf{k},$$

$$\mathbf{r}_{P/GA} \times m_A \mathbf{v}'_{GA} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7.3 & 0 & 0 \\ m_A v'_{GAx} & m_A v'_{GAy} & 0 \end{bmatrix}$$

$$= (-7.3m_A v'_{GAy})\mathbf{k}.$$

$$\mathbf{r}_{P/GB} \times m_B \mathbf{v}'_{GB} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4.3 & 0 & 0 \\ m_B v'_{GAx} & m_A(v'_{GAy} + 11.6\omega') & 0 \end{bmatrix}$$

$$= 4.3m_B(v'_{GAy} + 11.6\omega')\mathbf{k}.$$

Collect terms and substitute into conservation of angular momentum expression, and reduce:

$$-0.365m_A = (-7.3m_A + 4.3m_B)v'_{GAy}$$

$$+ (49.9m_B + I_A + I_B)\omega'.$$

From (1) and (2),

$$v'_{GAy} = \frac{0.05m_A - 11.6m_B\omega'}{m_A + m_B}.$$

These two equations in two unknowns can be further reduced by substitution and algebraic reduction, but here they have been solved by iteration:
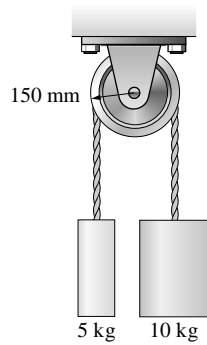
$$\boxed{\omega' = -0.003359 \text{ rad/s}},$$

$$v'_{GAy} = 0.04704 \text{ m/s},$$

from which

$$v'_{GBy} = v'_{GAy} + 11.6\omega' = 0.00807 \text{ m/s}$$

Problem 19.95 The moment of inertia of the pulley is 0.2 kg-m². The system is released from rest. Use the principle of work and energy to determine the velocity of the 10-kg cylinder when it has fallen 1 m.

150 mm

5 kg   10 kg

Solution: Choose a coordinate system with the y axis positive upward. Denote $m_L = 5$ kg, $m_R = 10$ kg, $h_R = -1$ m, $R = 0.15$ m. From the principle of work and energy, $U = T_2 - T_1$ where $T_1 = 0$ since the system is released from rest. The work done by the left hand weight is

$$U_L = \int_0^{h_L} -m_L g \, dh = -m_L g h_L.$$

The work done by the right hand weight is

$$U_L = \int_0^{h_R} -m_R g \, dh = -m_R g h_R.$$

Since the pulley is one-to-one, $h_L = -h_R$, from which

$$U = U_L + U_R = (m_L - m_R) g h_R.$$

The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right) I_P \omega^2 + \left(\frac{1}{2}\right) m_L v_L^2 + \left(\frac{1}{2}\right) m_R v_R^2.$$

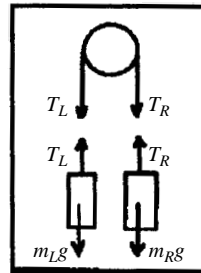Since the pulley is one-to-one, $v_L = -v_R$. From kinematics

$$\omega = \frac{v_R}{R},$$

from which

$$T_2 = \left(\frac{1}{2}\right) \left(\frac{I_P}{R^2} + m_L + m_R\right) v_R^2.$$

Substitute and solve:

$$v_R = \sqrt{\frac{2(m_L - m_R) g h_R}{\left(\dfrac{I_P}{R^2} + m_L + m_R\right)}} = 2.026\ldots = 2.03 \text{ m/s}$$

Problem 19.96 The moment of inertia of the pulley is 0.2 kg-m². The system is released from rest. Use momentum principles to determine the velocity of the 10-kg cylinder 1 s after the system is released.

Solution: Use the coordinate system and notations of Problem 19.95. From the principle of linear impulse-momentum for the left hand weight:

$$\int_{t_1}^{t_2} (T_L - m_L g)\, dt = m_L(v_{L2} - v_{L1}) = m_L v_{L2},$$

since $v_{L1} = 0$, from which

(1) $T_L(t_2 - t_1) = m_L g(t_2 - t_1) + m_L v_2.$

For the right hand weight,

$$\int_{t_1}^{t_2} (T_R - m_R g)\, dt = m_R v_{R2},$$

from which

(2) $T_R(t_2 - t_1) = m_R g(t_2 - t_1) + m_R v_{R2}.$

From the principle of angular impulse-momentum for the pulley:

$$\int_{t_1}^{t_2} (T_L - T_R) R\, dt = I_P \omega_2,$$

from which

(3) $(T_L - T_R)(t_2 - t_1) = \dfrac{I_P}{R}\omega_2.$

Substitute (1) and (2) into (3):

$$(m_L - m_R)g(t_2 - t_1) + m_L v_{L2} - m_R v_{R2} = \frac{I_P}{R}\omega_2.$$

Since the pulley is one to one, $v_{L2} = -v_{R2}$. From kinematics:

$$\omega_2 = \frac{v_{R2}}{R},$$

from which

$$\boxed{v_{R2} = -\frac{(m_R - m_L)g(t_2 - t_1)}{\dfrac{I_P}{R^2} + m_L + m_R} = -2.05 \text{ m/s}}$$

Problem 19.97 Arm BC has a mass of 12 kg, and the moment of inertia about its center of mass is 3 kg-m². Point B is stationary. Arm BC is initially aligned with the (horizontal) x axis with zero angular velocity, and a constant couple M applied at B causes the arm to rotate upward. When it is in the position shown, its counterclockwise angular velocity is 2 rad/s. Determine M.

Solution: Assume that the arm BC is initially stationary. Denote $R = 0.3$ m. From the principle of work and energy, $U = T_2 - T_1$, where $T_1 = 0$. The work done is

$$U = \int_0^\theta M\, d\theta + \int_0^{R\sin\theta} -mg\, dh = M\theta - mgR\sin\theta.$$

The angle is

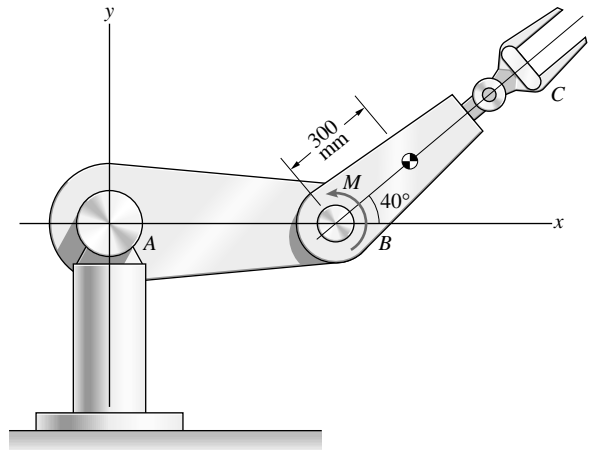$$\theta = 40\left(\frac{\pi}{180}\right) = 0.6981 \text{ rad}.$$

The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right)mv^2 + \left(\frac{1}{2}\right)I_{BC}\omega^2.$$

From kinematics, $v = R\omega$, from which

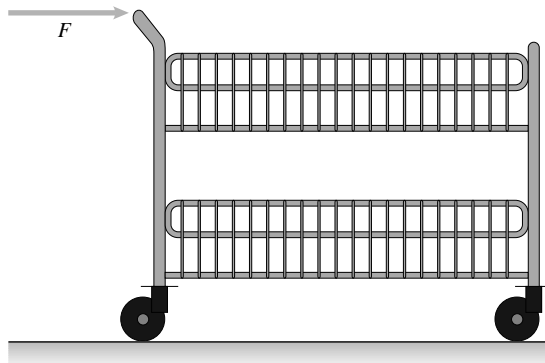$$T_2 = \left(\frac{1}{2}\right)(mR^2 + I_{BC})\omega^2.$$

Substitute into $U = T_2$ and solve:

$$\boxed{M = \frac{(mR^2 + I_{BC})\omega^2 + 2mgR\sin\theta}{2\theta} = 44.2 \text{ N-m}}$$

Problem 19.98 The cart is stationary when a constant force F is applied to it. What will the velocity of the cart be when it has rolled a distance b? The mass of the body of the cart is m_C, and each of the four wheels has mass m, radius R, and moment of inertia I.

Solution: From the principle of work and energy, $U = T_2 - T_1$, where $T_1 = 0$. The work done is

$$U = \int_0^b F\,dx = Fb.$$

The kinetic energy is

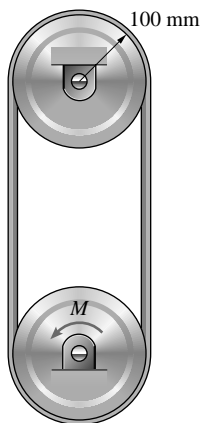$$T_2 = \left(\frac{1}{2}\right) m_C v^2 + 4\left(\frac{1}{2}\right) m v^2 + 4\left(\frac{1}{2}\right) I \omega^2.$$

From kinematics, $v = R\omega$, from which

$$T_2 = \left(\frac{m_C}{2} + 2m + 2\frac{I}{R^2}\right) v^2.$$

Substitute into $U = T_2$ and solve:

$$\boxed{v = \sqrt{\dfrac{2Fb}{m_C + 4m + 4\left(\dfrac{I}{R^2}\right)}}}$$

Problem 19.99 Each pulley has moment of inertia $I = 0.003$ kg-m^2, and the mass of the belt is 0.2 kg. If a constant couple $M = 4$ N-m is applied to the bottom pulley, what will its angular velocity be when it has turned 10 revolutions?

Solution: Assume that the system is initially stationary. From the principle of work $U = T_2 - T_1$, where $T_1 = 0$. The work done is

$$U = \int_0^\theta M\,d\theta = M\theta,$$

where the angle is $\theta = 10(2\pi) = 62.83$ rad. The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right) m_{\text{belt}} v^2 + 2\left(\frac{1}{2}\right) I_{\text{pulley}} \omega^2.$$

From kinematics, $v = R\omega$, where $R = 0.1$ m, from which

$$T_2 = \left(\frac{1}{2}\right)(R^2 m_{\text{belt}} + 2 I_{\text{pulley}}) \omega^2.$$

Substitute into $U = T_2$ and solve:

$$\boxed{\omega = \sqrt{\dfrac{2M\theta}{(R^2 m_{\text{belt}} + 2 I_{\text{pulley}})}} = 250.7 \text{ rad/s}}$$

Problem 19.100 The ring gear is fixed. The mass and moment of inertia of the sun gear are $m_S = 321$ kg and $I_S = 5962$ kg-m^2. The mass and moment of inertia of each planet gear are $m_P = 39.4$ kg and $I_P = 88.1$ kg-m^2. A couple $M = 814$ N-m is applied to the sun gear. Use work and energy to determine the angular velocity of the sun gear after it has turned 100 revolutions.

Ring gear

177.8 mm

863.6 mm

M

508 mm

Planet gears (3)
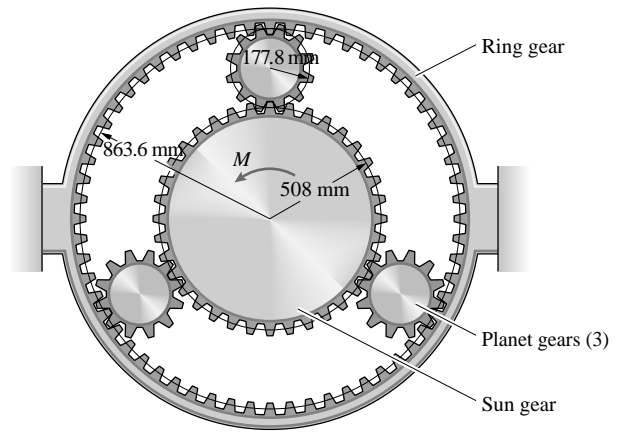
Sun gear

Solution: Denote the radius of planetary gear, $R_P = 0.178$ m, the radius of sun gear $R_S = 0.508$ m, and angular velocities of the sun gear and planet gear by ω_S, ω_P. Assume that the system starts from rest. From the principle of work and energy $U = T_2 - T_1$, where $T_1 = 0$. The work done is

$$U = \int_0^\theta M d\theta = M\theta,$$

where the angle is

$\theta = (100)(2\pi) = 200\pi = 628.3$ rad.

The kinetic energy is

$$T_2 = \left(\tfrac{1}{2}\right) I_S \omega_S^2 + 3\left(\left(\tfrac{1}{2}\right) m_P v_P^2 + \left(\tfrac{1}{2}\right) I_P \omega_P^2\right).$$

The velocity of the outer radius of the sun gear is $v_S = R_S \omega_S$. The velocity of the center of mass of the planet gears is the average velocity of the velocity of the sun gear contact and the ring gear contact,

$$v_P = \frac{v_S + v_R}{2} = \frac{v_S}{2},$$

since $v_R = 0$. The angular velocity of the planet gears is

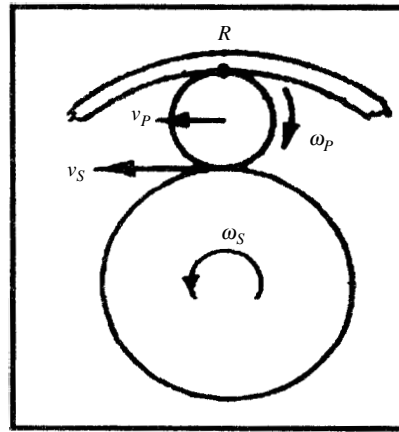$$\omega_P = \frac{v_P}{R_P}.$$

Collect terms:

$$\omega_P = \left(\frac{R_S}{R_P}\right) \frac{\omega_S}{2},$$

$$v_P = \frac{R_S}{2} \omega_S.$$

Substitute into $U = T_2$ and solve:

$$\omega_S = \sqrt{\frac{2M\theta}{\left(I_S + \frac{3}{4}\left(m_P R_S^2 + I_P \left(\frac{R_S}{R_P}\right)^2\right)\right)}} = 12.5 \text{ rad/s}$$

Problem 19.101 The moments of inertia of gears A and B are $I_A = 0.019$ kg-m^2, and $I_B = 0.136$ kg-m^2. Gear A is connected to a torsional spring with constant $k = 0.27$ N-m/rad. If the spring is unstretched and the surface supporting the 22.2 N weight is removed, what is the velocity of the weight when it has fallen 76.2 mm?

Solution: Denote $W = 22.2$ N, $s = 0.762$ m is the distance the weight falls, $r_B = 0.254$ m, $r_A = 0.152$ m, $r'_B = 0.762$ m, are the radii of the gears and pulley. Choose a coordinate system with y positive upward. From the conservation of energy $T + V = $ const. Choose the datum at the initial position, such that $V_1 = 0$, $T_1 = 0$, from which $T_2 + V_2 = 0$ at any position 2. The gear B rotates in a negative direction and the gear A rotates in a positive direction. By inspection,

$$\theta_B = -\frac{s}{r'_B} = -\frac{0.762}{0.762} = -1 \text{ rad},$$

$$\theta_A = -\left(\frac{r_B}{r_A}\right)\theta_B = 1.667 \text{ rad},$$

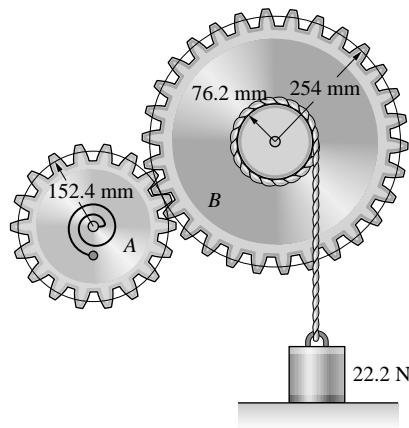$$v = r'_B\omega_B = 0.762\omega_B,$$

$$\omega_A = -\left(\frac{r_B}{r_A}\right)\omega_B = 1.667\omega_B.$$

The moment exerted by the spring is negative, from which the potential energy in the spring is

$$V_{\text{spring}} = -\int_0^{\theta_A} M d\theta = \int_0^{\theta_A} k\theta d\theta = \frac{1}{2}k\theta_A^2 = 0.377 \text{ N-m}.$$

The force due to the weight is negative, from which the potential energy of the weight is

$$V_{\text{weight}} = -\int_0^{-s} (-W)\, dy = -Ws = -1.695 \text{ N-m}.$$

76.2 mm   254 mm

152.4 mm B

A

22.2 N

The kinetic energy of the system is

$$T_2 = \left(\frac{1}{2}\right)I_A\omega_A^2 + \left(\frac{1}{2}\right)I_B\omega_B^2 + \left(\frac{1}{2}\right)\left(\frac{W}{g}\right)v^2.$$

Substitute: $T_2 = 17.3v^2$, from which

$$V_{\text{spring}} + V_{\text{weight}} + 17.3\,v^2 = 0.$$

Solve $\boxed{v = 0.276 \text{ m/s}}$ downward.

Problem 19.102 Consider the system in Problem 19.101.

(a) What maximum distance does the 22.2 N weight fall when the supporting surface is removed?
(b) What maximum velocity does the weight achieve?

Solution: Use the solution to Problem 19.101:

$$V_{\text{spring}} + V_{\text{weight}} + 17.3v^2 = 0.$$

$$V_{\text{spring}} = -\int_0^{\theta_A} M d\theta = \int_0^{\theta_A} k\theta\, d\theta = \frac{1}{2}k\theta_A^2 = 64.5s^2,$$

$$V_{\text{weight}} = -\int_0^{-s} (-W)\, dy = -Ws,$$

from which $64.5s^2 - 22.2s + 17.3v^2 = 0$.

(a) The maximum travel occurs when $v = 0$, from which

$$\boxed{s_{\text{max}} = \frac{22.2}{64.5} = 0.344 \text{ m}}$$

(where the other solution $s_{\text{max}} = 0$ is meaningless here).

(b) The maximum velocity occurs at

$$\frac{dv^2}{ds} = 0 = 2\frac{64.5}{17.3}s - \frac{22.2}{17.3} = 0,$$

from which

$$s_{v-\text{max}} = 0.17 \text{ m}.$$

This is indeed a maximum, since

$$\frac{d^2(v^2)}{ds^2} = 2\left(\frac{64.5}{17.3}\right) > 0,$$

and $\boxed{|v_{\text{max}}| = 0.332 \text{ m/s}}$ (downward).

Problem 19.103 Each of the go-cart's front wheels weighs 22.2 N and has a moment of inertia of 0.0136 kg-m². The two rear wheels and rear axle form a single rigid body weighing 178 N and having a moment of inertia of 0.136 kg-m². The total weight of the rider and go-cart, including its wheels, is 1067 N. The go-cart starts from rest, its engine exerts a constant torque of 20.3 N-m on the rear axle, and its wheels do not slip. Neglecting friction and aerodynamic drag, how fast is the go-cart moving when it has traveled 15.2 m?


152.4 mm   101.6 mm

1524 mm
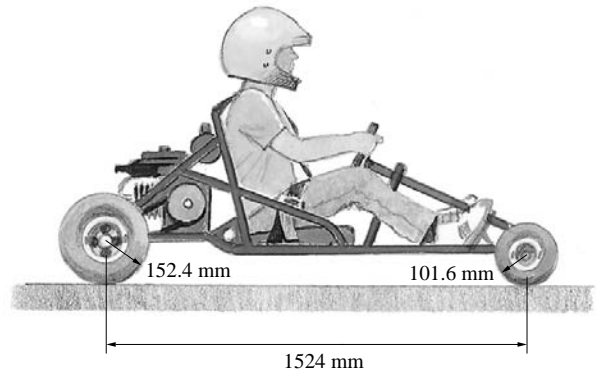
Solution: From the principle of work and energy: $U = T_2 - T_1$, where $T_1 = 0$, since the go-cart starts from rest. Denote the rear and front wheels by the subscripts A and B, respectively. The radius of the rear wheels $r_A = 0.152$ m. The radius of the front wheels is $r_B = 0.102$ m. The rear wheels rotate through an angle $\theta_A = \dfrac{15.2}{0.152} = 100$ rad, from which the work done is

381 mm   A
152.4 mm   101.6 mm   B
406.4 mm   1524 mm

$$U = \int_0^{\theta_A} M d\theta = M\theta_A = 20.3(100) \text{ N-m.}$$

The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right)\frac{W}{g}v^2 + \left(\frac{1}{2}\right)I_A\omega_A^2 + 2\left(\frac{1}{2}\right)I_B\omega_B^2$$

(for two front wheels). The angular velocities are related to the go-cart velocity by $\omega_A = \dfrac{v}{r_A} = 2v$, $\omega_B = \dfrac{v}{r_B} = 3v$, from which $T_2 = 1.23v^2$ N-m. Substitute into $U = T_2$ and solve: $v = 5.89$ m/s.
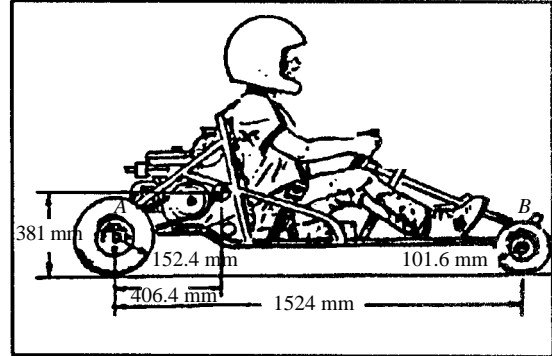
Problem 19.104 Determine the maximum power and the average power transmitted to the go-cart in Problem 19.103 by its engine.

Solution: The maximum power is $P_{\max} = M\omega_{A\max}$, where $\omega_{A\max} = \dfrac{v_{\max}}{R}$. From which $P_{\max} = \dfrac{Mv_{\max}}{R}$. Under constant torque, the acceleration of the go-cart is constant, from which the maximum velocity is the greatest value of the velocity, which will occur at the end of the travel. From the solution to Problem 19.103, $v_{\max} = 19.32$ ft/s, from which

$$P_{\max} = \frac{Mv_{\max}}{r_A} = \frac{20.3(5.89)}{0.152} = 786.6 \text{ N-m/s.}$$

The average power is $P_{\text{ave}} = \dfrac{U}{t}$. From the solution to Problem 19.103, $U = 2030$ N-m. Under constant acceleration, $v = at$, and $s = \frac{1}{2}at^2$, from which $s = \frac{1}{2}vt$, and $t = \dfrac{2s}{v} = \dfrac{30.48}{5.89} = 5.177$ s, from which

$$P_{\text{ave}} = \frac{U}{t} = \frac{2030}{5.177} = 392 \text{ N-m/s.}$$

Problem 19.105 The system starts from rest with the 4-kg slender bar horizontal. The mass of the suspended cylinder is 10 kg. What is the angular velocity of the bar when it is in the position shown?
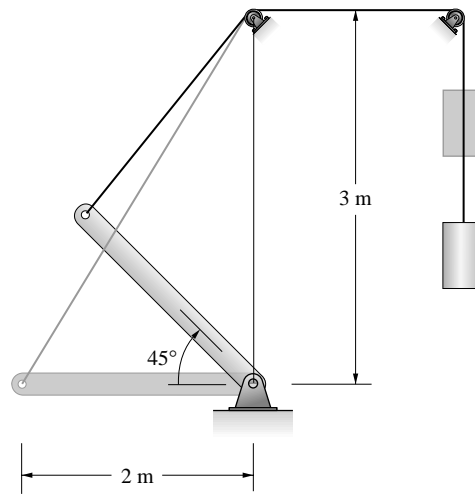
Solution: From the principle of work and energy: $U = T_2 - T_1$, where $T_1 = 0$ since the system starts from rest. The change in height of the cylindrical weight is found as follows: By inspection, the distance between the end of the bar and the pulley when the bar is in the horizontal position is $d_1 = \sqrt{2^2 + 3^2} = 3.61$ m. The law of cosines is used to determine the distance between the end of the bar and the pulley in the position shown:

$$d_2 = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 45°} = 2.125 \text{ m},$$

from which $h = d_1 - d_2 = 1.481$ m. The work done by the cylindrical weight is

$$U_{\text{cylinder}} = \int_0^{-h} -m_c g \, ds = m_c g h = 145.3 \text{ N-m}.$$

The work done by the weight of the bar is

$$U_{\text{bar}} = \int_0^{\cos 45°} -m_b g \, dh = -m_b g \cos 45° = -27.75 \text{ N-m},$$

from which $U = U_{\text{cylinder}} + U_{\text{bar}} = 117.52$ N-m. From the sketch, (which shows the final position) the component of velocity normal to the bar is $v \sin \beta$, from which $2\omega = v \sin \beta$. From the law of sines:

$$\sin \beta = \left(\frac{3}{d_2}\right) \sin 45° = 0.9984.$$

The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right) m_c v^2 + \left(\frac{1}{2}\right) I \omega_b^2,$$

from which $T_2 = 22.732\omega^2$, where $v = \left(\frac{2}{\sin \beta}\right)\omega$ and $I = \frac{m(2^2)}{3}$ has been used. Substitute into $U = T_2$ and solve: $\omega = 2.274$ rad/s.
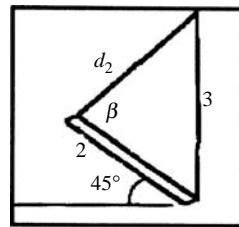
Problem 19.106 The 0.1-kg slender bar and 0.2-kg cylindrical disk are released from rest with the bar horizontal. The disk rolls on the curved surface. What is the angular velocity of the bar when it is vertical?

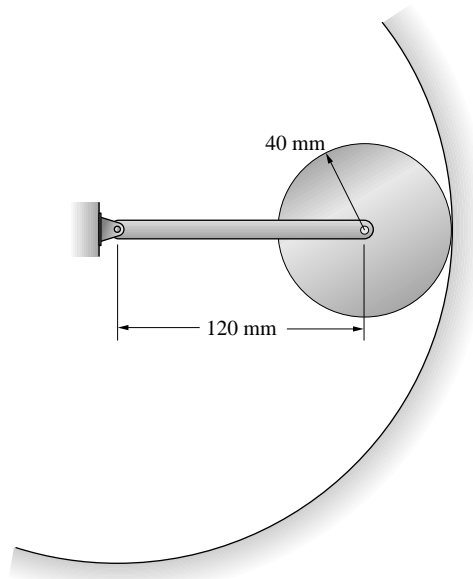**Solution:** From the principle of work and energy, $U = T_2 - T_1$, where $T_1 = 0$. Denote $L = 0.12$ m, $R = 0.04$ m, the angular velocity of the bar by ω_B, the velocity of the disk center by v_D, and the angular velocity of the disk by ω_D. The work done is

$$U = \int_0^{-\frac{L}{2}} -m_B g \, dh + \int_0^{-L} -m_D g \, dh = \left(\frac{L}{2}\right) m_B g + L m_D g.$$

From kinematics, $v_D = L \omega_B$, and $\omega_D = \dfrac{v_D}{R}$. The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right) I_B \omega_B^2 + \left(\frac{1}{2}\right) m_D v_D^2 + \left(\frac{1}{2}\right) I_D \omega_D^2.$$

Substitute the kinematic relations to obtain

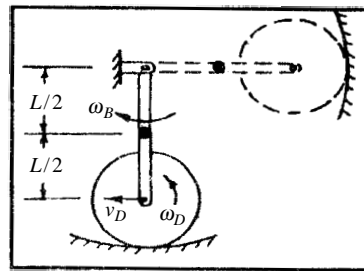$$T_2 = \left(\frac{1}{2}\right) \left(I_B + m_D L^2 + I_D \left(\frac{L}{R}\right)^2 \right) \omega_B^2,$$

where $I_B = \dfrac{m_B L^2}{3}$,

$$I_D = \frac{m_D R^2}{2},$$
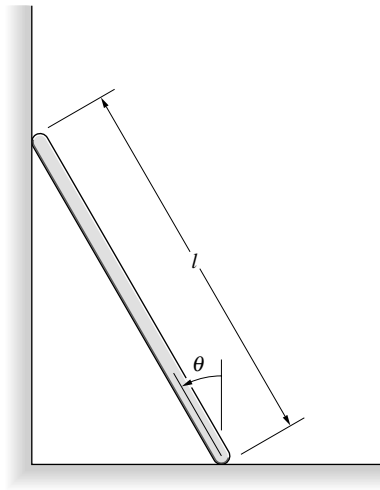
from which $T_2 = \left(\dfrac{1}{2}\right) \left(\dfrac{m_B}{3} + \dfrac{3 m_D}{2}\right) L^2 \omega_B^2$.

Substitute into $U = T_2$ and solve:

$$\boxed{\omega_B = \sqrt{\frac{6 \, g(m_B + 2 m_D)}{(2 m_B + 9 m_D) L}} = 11.1 \text{ rad/s}.}$$

Problem 19.107 A slender bar of mass m is released from rest in the vertical position and allowed to fall. Neglecting friction and assuming that it remains in contact with the floor and wall, determine the bar's angular velocity as a function of θ.

Solution: The strategy is

(a) to use the kinematic relations to determine the relation between the velocity of the center of mass and the angular velocity about the instantaneous center, and

(b) the principle of work and energy to obtain the angular velocity of the bar. *The kinematics*: Denote the angular velocity of the bar about the instantaneous center by ω. The coordinates of the instantaneous center of rotation of the bar are $(L\sin\theta, L\cos\theta)$. The coordinates of the center of mass of the bar are

$$\left(\frac{L}{2}\sin\theta, \frac{L}{2}\cos\theta\right).$$

The vector distance from the instantaneous center to the center of mass is

$$\mathbf{r}_{G/C} = -\frac{L}{2}(\mathbf{i}\sin\theta + \mathbf{j}\cos\theta).$$

The velocity of the center of mass is

$$\mathbf{v}_G = \boldsymbol{\omega} \times \mathbf{r}_{G/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ -\dfrac{L}{2}\sin\theta & -\dfrac{L}{2}\cos\theta & 0 \end{bmatrix}$$

$$= \frac{\omega L}{2}(\mathbf{i}\cos\theta - \mathbf{j}\sin\theta),$$

from which $|\mathbf{v}_G| = \dfrac{\omega L}{2}$.

The principle of work and energy: $U = T_2 - T_1$ where $T_1 = 0$. The work done by the weight of the bar is

$$U = mg\left(\frac{L}{2}\right)(1 - \cos\theta).$$
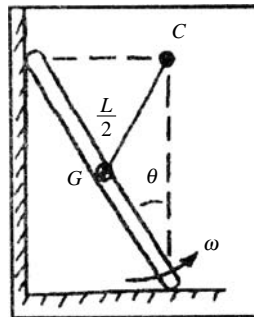
The kinetic energy is

$$T_2 = \left(\frac{1}{2}\right)mv_G^2 + \left(\frac{1}{2}\right)I_B\omega^2.$$
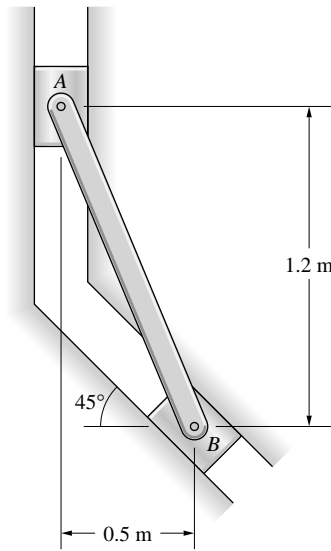
Substitute $v_G = \dfrac{\omega L}{2}$ and $I_B = \dfrac{mL^2}{12}$ to obtain $T_2 = \dfrac{mL^2\omega^2}{6}$. Substitute into $U = T_2$ and solve:

$$\boxed{\omega = \sqrt{\frac{3g(1 - \cos\theta)}{L}}.}$$

Problem 19.108 The 4-kg slender bar is pinned to 2-kg sliders at A and B. If friction is negligible and the system starts from rest in the position shown, what is the bar's angular velocity when the slider at A has fallen 0.5 m?

Solution: Choose a coordinate system with the origin at the initial position of A and the y axis positive upward. The strategy is

(a) to determine the distance that B has fallen and the center of mass of the bar has fallen when A falls 0.5 m,

(b) use the coordinates of A, B, and the center of mass of the bar and the constraints on the motion of A and B to determine the kinematic relations, and

(c) use the principle of work and energy to determine the angular velocity of the bar.

The displacement of B: Denote the length of the bar by $L = \sqrt{1.2^2 + 0.5^2} = 1.3$ m. Denote the horizontal and vertical displacements of B when A falls 0.5 m by d_x and d_y, which are in the ratio $\dfrac{d_y}{d_x} = \tan 45° = 1$, from which $d_x = d_y = d$. The vertical distance between A and B is reduced by the distance 0.5 m and increased by the distance d_y, and the horizontal distance between A and B is increased by the distance d_x, from which $L^2 = (1.2 - 0.5 + d_y)^2 + (0.5 + d_x)^2$. Substitute $d_x = d_y = d$ and $L = 1.3$ m and reduce to obtain $d^2 + 2bd + c = 0$, where $b = 0.6$, and $c = -0.475$. Solve: $d_{1,2} = -b \pm \sqrt{b^2 - c} = 0.3138$ m, or $= -1.514$ m, from which only the positive root is meaningful.
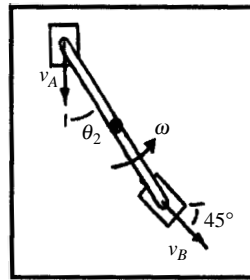
The final position coordinates: The coordinates of the initial position of the center of mass of the bar are

$$(x_{G1}, y_{G1}) = \left(\frac{L}{2} \sin \theta_1, -\frac{L}{2} \cos \theta_1 \right) = (0.25, -0.6) \text{ (m)},$$

where $\theta_1 = \sin^{-1} \left(\dfrac{0.5}{L} \right) = 22.61°$

is the angle of the bar relative to the vertical. The coordinates of the final position of the center of mass of the bar are

$$(x_{G2}, y_{G2}) = \left(\frac{L}{2} \sin \theta_2, -0.5 - \frac{L}{2} \cos \theta_2 \right) = (0.4069, -1.007),$$

where $\theta_2 = \sin^{-1} \left(\dfrac{0.5 + d}{L} \right) = 38.75°$.

The vertical distance that the center of mass falls is $h = y_{G2} - y_{G1} = -0.4069$ m. The coordinates of the final positions of A and B are, respectively $(x_{A2}, y_{A2}) = (0, -0.5)$, and $(x_{B2}, y_{B2}) = (0.5 + d, -(1.2 + d)) = (0.8138, -1.514)$. The vector distance from A to B is

$$\mathbf{r}_{B/A} = (x_{B2} - x_{A2})\mathbf{i} + (y_{B2} - y_{A2})\mathbf{j}$$

$$= 0.8138\mathbf{i} - 1.014\mathbf{j} \text{ (m)}.$$

Check: $|\mathbf{r}_{B/A}| = L = 1.3$ m. *check*. The vector distance from A to the center of mass is

$$\mathbf{r}_{G/A} = (x_{G2} - x_{A2})\mathbf{i} + (y_{G2} - y_{A2})\mathbf{j}$$

$$= 0.4069\mathbf{i} - 0.5069\mathbf{j} \text{ (m)}.$$

Check: $\mathbf{r}_{G/A} = \dfrac{L}{2} = 0.65$ m. *check*.

The kinematic relations: From kinematics, $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$. The slider A is constrained to move vertically, and the slider B moves at a $45°$ angle, from which $\mathbf{v}_A = -v_A\mathbf{j}$ (m/s), and

$$\mathbf{v}_B = (v_B \cos 45°)\mathbf{i} - (v_B \sin 45°)\mathbf{j}.$$

$$\mathbf{v}_B = \mathbf{v}_A + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ 0.8138 & -1.014 & 0 \end{bmatrix}$$

$$= \mathbf{i}(1.014\omega) + \mathbf{j}(-v_A + 0.8138\omega),$$

from which (1) $v_B = \left(\dfrac{1.014}{\cos 45°}\right)\omega = 1.434\omega$,

and (2) $v_A = v_B \sin 45° + 0.8138\omega = 1.828\omega$.

The velocity of the center of mass of the bar is

$$\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{G/A} = -v_A\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ 0.4069 & -0.5069 & 0 \end{bmatrix}$$

$$= (0.5069\omega)\mathbf{i} + (-v_A + 0.4069\omega)\mathbf{j},$$

$$\mathbf{v}_G = 0.5069\omega\mathbf{i} - 1.421\omega\mathbf{j} \text{ (m/s)},$$

from which (3) $v_G = 1.508\omega$

The principle of work and energy: From the principle of work and energy, $U = T_2 - T_1$, where $T_1 = 0$. The work done is

$$U = \int_0^{-d_1} -m_B g\, dh + \int_0^{-0.5} -m_A g\, dh + \int_0^{-h} -m_{\text{bar}} g\, dh,$$

$$U = m_B g(0.3138) + m_A g(0.5) + m_{\text{bar}} g(0.4069) = 31.93 \text{ N-m}.$$

The kinetic energy is

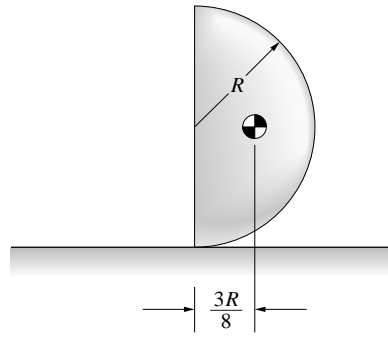$$T_2 = \left(\frac{1}{2}\right) m_B v_B^2 + \left(\frac{1}{2}\right) m_A v_A^2 + \left(\frac{1}{2}\right) m_{\text{bar}} v_G^2 + \left(\frac{1}{2}\right) I_{\text{bar}} \omega^2,$$

substitute $I_{\text{bar}} = \dfrac{m_{\text{bar}} L^2}{12}$, and (1), (2) and (3) to obtain $T_2 = 10.23\omega^2$. Substitute into $U = T_2$ and solve:

$$\boxed{\omega = \sqrt{\frac{31.93}{10.23}} = 1.77 \text{ rad/s}}$$

Problem 19.109 A homogeneous hemisphere of mass m is released from rest in the position shown. If it rolls on the horizontal surface, what is its angular velocity when its flat surface is horizontal?

Solution: The hemisphere's moment of inertia about O is $\frac{2}{5}mR^2$, so its moment of inertia about G is

$$I = \frac{2}{5}mR^2 - \left(\frac{3}{8}R\right)^2 m = \frac{83}{320}mR^2.$$
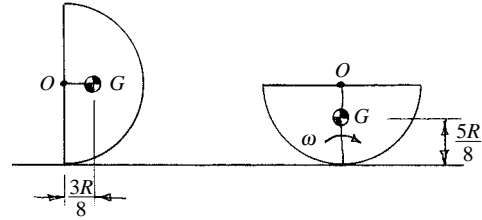
The work done is

$$mg\left(R - \frac{5}{8}R\right) = \frac{3}{8}mgR.$$

Work and energy is

$$\frac{3}{8}mgR = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m\left[\left(\frac{5R}{8}\right)\omega\right]^2 + \frac{1}{2}\left(\frac{83}{320}mR^2\right)\omega^2.$$

Solving for ω, we obtain

$$\omega = \sqrt{\frac{15g}{13R}}.$$

Problem 19.110 The homogeneous hemisphere of mass m is released from rest in the position shown. It rolls on the horizontal surface. What normal force is exerted on the hemisphere by the horizontal surface at the instant the flat surface of the hemisphere is horizontal?

Solution: See the solution of Problem 19.109. The acceleration of G is

$$\mathbf{a}_G = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{G/O} - \omega^2 \mathbf{r}_{G/O}$$

$$= a_O\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ 0 & -h & 0 \end{vmatrix} - \omega^2(-h\mathbf{j}),$$

so $a_{Gy} = \omega^2 h$. Therefore $N - mg = ma_{Gy} = m\omega^2 h$. Using the result from the solution of Problem 19.109,

$$N = mg + m\left(\frac{15g}{13R}\right)\left(\frac{3R}{8}\right) = 1.433 \ mg.$$
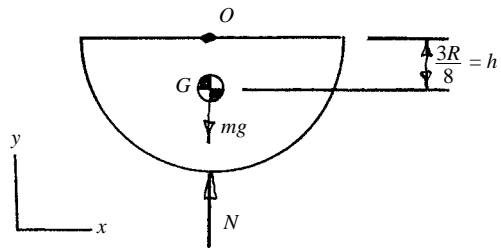
Problem 19.111 The slender bar rotates freely *in the horizontal plane* about a vertical shaft at O. The bar weighs 89 N and its length is 1.83 m. The slider A weighs 8.9 N. If the bar's angular velocity is $\omega = 10$ rad/s and the radial component of the veloc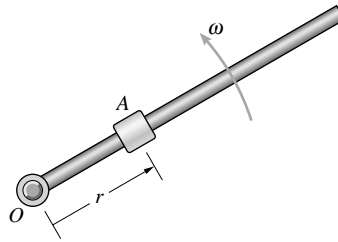ity of A is zero when $r = 0.31$ m, what is the angular velocity of the bar when $r = 1.22$ m? (The moment of inertia of A about its center of mass is negligible; that is, treat A as a particle.)

Solution: From the definition of angular momentum, only *the radial position* of the slider need be taken into account in applying the principle of the conservation of angular momentum; that is, *the radial velocity* of the slider at $r = 1.22$ m does not change the angular momentum of the bar. From the conservation of angular momentum:

$$I_{\text{bar}}\omega_1 + r_{1A}^2\left(\frac{W_A}{g}\right)\omega_1 = I_{\text{bar}}\omega_2 + r_{2A}^2\left(\frac{W_A}{g}\right)\omega_2.$$

Substitute numerical values:

$$I_{\text{bar}} + r_{1A}^2\left(\frac{W_A}{g}\right) = \frac{W_{\text{bar}}}{3g}L^2 + r_{1A}^2\left(\frac{W_A}{g}\right) = 10.2 \text{ kg-m}^2$$

$$I_{\text{bar}} + r_{21A}^2\left(\frac{W_A}{g}\right) = \frac{W_{\text{bar}}}{3g}L^2 + r_{2A}^2\left(\frac{W_A}{g}\right) = 11.5 \text{ kg-m}^2,$$

from which $\boxed{\omega_2 = \left(\frac{10.2}{11.5}\right)\omega_1 = 8.90 \text{ rad/s}}$

Problem 19.112 A satellite is deployed with angular velocity $\omega = 1$ rad/s (Fig. a). Two internally stored antennas that span the diameter of the satellite are then extended, and the satellite's angular velocity decreases to ω' (Fig. b). By modeling the satellite as a 500-kg sphere of 1.2-m radius and each antenna as a 10-kg slender bar, determine ω'.
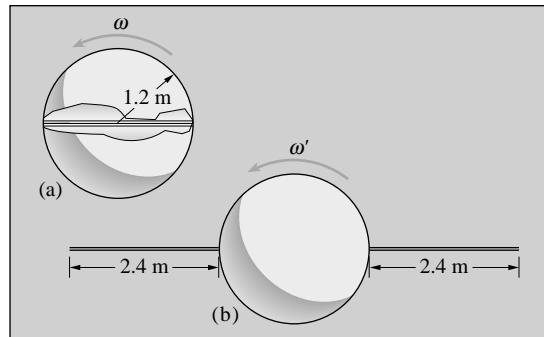
Solution: Assume (I) in configuration (a) the antennas are folded inward, each lying on a line passing so near the center of the satellite that the distance from the line to the center can be neglected; (II) when extended, the antennas are entirely external to the satellite. Denote $R = 1.2$ m, $L = 2R = 2.4$ m. The moment of inertia of the antennas about the center of mass of the satellite in configuration (a) is

$$I_{\text{ant-folded}} = 2\left(\frac{mL^2}{12}\right) = 9.6 \text{ kg-m}^2.$$

The moment of inertia of the antennas about the center of mass of the satellite in configuration (b) is

$$I_{\text{ant-ext}} = 2\left(\frac{mL^2}{12}\right) + 2\left(R + \frac{L}{2}\right)^2 m = 124.8 \text{ kg-m}^2.$$

The moment of inertia of the satellite is

$$I_{\text{sphere}} = \frac{2}{5}m_{\text{sphere}}R^2 = 288 \text{ kg-m}^2.$$

The angular momentum is conserved,

$$I_{\text{sphere}}\omega + I_{\text{ant-folded}}\omega = I_{\text{sphere}}\omega' + I_{\text{ant-ext}}\omega',$$

where $I_{\text{ant-folded}}, I_{\text{ant-ext}}$ *are for both antennas*, from which $297.6\omega = 412.8\omega'$. Solve $\boxed{\omega' = 0.721 \text{ rad/s}}$

Problem 19.113 An engineer decides to control the angular velocity of a satellite by deploying small masses attached to cables. If the angular velocity of the satellite in configuration (a) is 4 rpm, determine the distance d in configuration (b) that will cause the angular velocity to be 1 rpm. The moment of inertia of the satellite is $I = 500$ kg-m^2 and each mass is 2 kg. (Assume that the cables and masses rotate with the same angular velocity as the satellite. Neglect the masses of the cables and the mass moments of inertia of the masses about their centers of mass.)
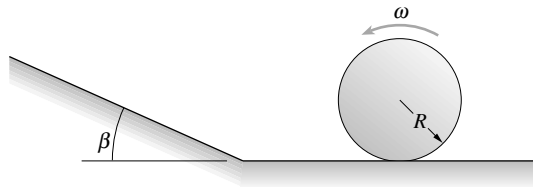

(a)   (b)
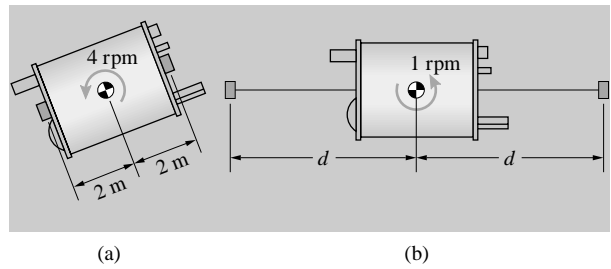
Solution: From the conservation of angular momentum,

$$(I + 2m(2^2))\omega_1 = (I + 2md^2)\omega_2.$$

Solve:
$$d = \sqrt{\frac{(I + 8m)\left(\dfrac{\omega_1}{\omega_2}\right) - I}{2m}} = 19.8 \text{ m}$$
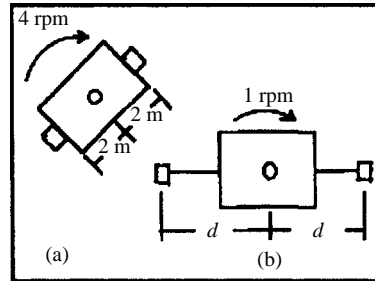
Problem 19.114 The homogenous cylindrical disk of mass m rolls on the horizontal surface with angular velocity ω. If the disk does not slip or leave the slanted surface when it comes into contact with it, what is the angular velocity ω' of the disk immediately afterward?

**Solution:** The velocity of the center of mass of the disk is parallel to the surface before and after contact. The angular momentum about the point of contact is conserved $mvR\cos\beta + I\omega = mv'R + I\omega'$. From kinematics, $v = R\omega$ and $v' = R\omega'$. Substitute into the angular moment condition to obtain:

$$(mR^2\cos\beta + I)\omega = (mR^2 + I)\omega'.$$

Solve: $\boxed{\omega' = \dfrac{(2\cos\beta + 1)}{3}\omega}$

Problem 19.115 The 44.5 N slender bar falls from rest in the vertical position and hits the smooth projection at B. The coefficient of restitution of the impact is $e = 0.6$, the duration of the impact is 0.1 s, and $b = 0.31$ m. Determine the average force exerted on the bar at B as a result of the impact.

0.91 m

A B

$\leftarrow\!\!-\!\!-\,b\,-\!\!-\!\!\rightarrow$
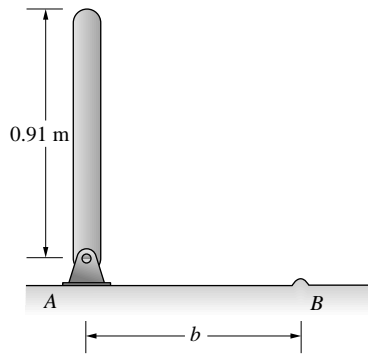
Solution: Choose a coordinate system with the origin at A and the x axis parallel to the plane surface, and y positive upward. The strategy is to

(a) use the principle of work and energy to determine the velocity before impact,

(b) the coefficient of restitution to determine the velocity after impact,

(c) and the principle of angular impulse-momentum to determine the average force of impact.

From the principle of work and energy, $U = T_2 - T_1$, where $T_1 = 0$. The center of mass of the bar falls a distance $h = \dfrac{L}{2}$. The work done by the weight of the bar is $U = mg\left(\dfrac{L}{2}\right)$. The kinetic energy is $T_2 = \left(\frac{1}{2}\right) I\omega^2$, where $I = \dfrac{mL^2}{3}$. Substitute into $U = T_2$ and solve:

$\omega = -\sqrt{\dfrac{3g}{L}}$, where the negative sign on the square root is chosen to be consistent with the choice of coordinates. By definition, the coefficient of restitution is $e = \dfrac{v'_B - v'_A}{v_A - v_B}$, where v_A, v'_A are *the velocities of the bar at a distance b from A* before and after impact. Since the projection B is stationary before and after the impact, $v_B = v'_B = 0$, from which $v'_A = -ev_A$. From kinematics, $v_A = b\omega$, and $v'_A = b\omega'$, from which $\omega' = -e\omega$. The principle of angular impulse-momentum about the point A is

$$\int_0^{t_2-t_1} bF_B\, dt = (I\omega' - I\omega) = \frac{mL^2}{3}(\omega' - \omega),$$

where F_B is the force exerted on the bar by the projection at B, from which

$$bF_B(t_2 - t_1) = -\frac{mL^2}{3}(1 + e)\omega.$$

Solve: $\boxed{F_B = \dfrac{mL^2(1+e)}{3b(t_2-t_1)}\sqrt{\dfrac{3g}{L}} = 376.4 \text{ N.}}$

Problem 19.116 The 44.5 N bar falls from rest in the vertical position and hits the smooth projection at B. The coefficient of restitution of the impact is $e = 0.6$ and the duration of the impact is 0.1 s. Determine the distance b for which the average force exerted on the bar by the support A as a result of the impact is zero.

Solution: From the principle of linear impulse-momentum,

$$\int_{t_1}^{t_2} \sum F\, dt = m(v'_G - v_G),$$

where v_G, v'_G are the velocities of the center of mass of the bar before and after impact, and $\sum F = F_A + F_B$ are the forces exerted on the bar at A and B. From kinematics, $v'_G = \left(\dfrac{L}{2}\right)\omega'$, $v_G = \left(\dfrac{L}{2}\right)\omega$. From the solution to Problem 19.115, $\omega' = -e\omega$, from which

$$F_A + F_B = -\frac{mL(1+e)}{2(t_2-t_1)}\omega.$$
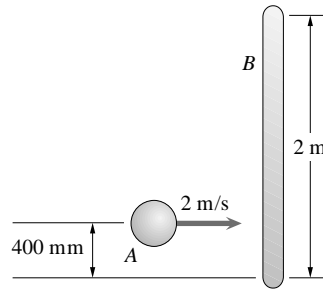
If the reaction at A is zero, then

$$F_B = -\frac{mL(1+e)}{2(t_2-t_1)}\omega.$$

From the solution to Problem 19.115,

$$F_B = -\frac{mL^2(1+e)}{3b(t_2-t_1)}\omega.$$

Substitute and solve: $\boxed{b = \dfrac{2}{3}L = 0.61 \text{ m}}$

Problem 19.117   The 1-kg sphere A is moving at 2 m/s when it strikes the end of the 2-kg stationary slender bar B. If the velocity of the sphere after the impact is 0.8 m/s to the right, what is the coefficient of restitution?

B

2 m

2 m/s

400 mm

A

Solution: Denote the distance of the point of impact P from the end of the bar by $d = 0.4$ m. The linear momentum is conserved: (1) $m_A v_A = m_A v_A' + m_B v_{CM}'$, where v_{CM}' is the velocity of the center of mass of the bar after impact, and v_A, v_A' are the velocities of the sphere before and after impact. By definition, the coefficient of restitution is $e = \dfrac{v_P' - v_A'}{v_A - v_P}$, where v_P, v_P' are the velocities of the bar at the point of impact. The point P is stationary before impact, from which (2) $v_P' - v_A' = e v_A$. From kinematics,
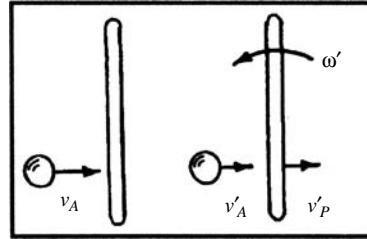
(3) $v_{CM}' = v_P' - \left(\dfrac{L}{2} - d\right)\omega'$.

Substitute (2) into (3) to obtain

(4) $v_{CM}' = v_A' + e v_A - \left(\dfrac{L}{2} - d\right)\omega'$.

Substitute (4) into (1) to obtain

(5) $(m_A - e m_B)v_A = (m_A + m_B)v_A' - \left(\dfrac{L}{2} - d\right)m_B\omega'$.

The angular momentum of the bar about the point of impact is conserved:

(6) $0 = -\left(\dfrac{L}{2} - d\right)m_B v_{CM}' + I_{CM}\omega'$.

Substitute (3) into (6) to obtain,

(7) $\left(\dfrac{L}{2} - d\right)m_B(v_A' + e v_A) = \left(I_{CM} + \left(\dfrac{L}{2} - d\right)^2 m_B\right)\omega'$,

where $I_{CM} = \dfrac{m_B L^2}{12}$.

Solve the two equations (5) and (7) for the two unknowns to obtain: $\omega' = 1.08$ rad/s and $\boxed{e = 0.224.}$

Problem 19.118 The slender bar is released from rest in the position shown in Fig (a) and falls a distance $h = 0.31$ mm. When the bar hits the floor, its tip is supported by a depression and remains on the floor (Fig. b). The length of the bar is 0.31 m and its weight is 1.1 N. What is angular velocity ω of the bar just after it hits the floor?

(a)   (b)
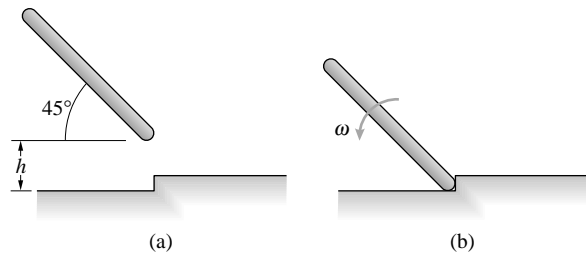
Solution: Choose a coordinate system with the x axis parallel to the surface, with the y axis positive upward. The strategy is to use the principle of work and energy to determine the velocity just before impact, and the conservation of angular momentum to determine the angular velocity after impact. From the principle of work and energy, $U = T_2 - T_1$, where $T_1 = 0$ since the bar is released from rest. The work done is $U = -mgh$, where $h = 0.31$ m. The kinetic energy is $T_2 = \left(\frac{1}{2}\right) mv^2$. Substitute into $U = T_2 - T_1$ and solve: $v = \sqrt{-2gh} = 2.44$ m/s. The conservation of angular momentum about the point of impact is $(\mathbf{r} \times m\mathbf{v}) = (\mathbf{r} \times m\mathbf{v}') + I_B \boldsymbol{\omega}'$. At the instant before the impact, the perpendicular distance from the point of impact to the center of mass velocity vector is $\left(\frac{L}{2}\right) \cos 45°$. After impact, the center of mass moves in an arc of radius $\left(\frac{L}{2}\right)$ about the point of impact so that the perpendicular distance from the point of impact to the velocity vector is $\left(\frac{L}{2}\right)$. From the definition of the cross product, for motion in the x, y plane,

$$(\mathbf{r} \times m\mathbf{v}) \cdot \mathbf{k} = \left(\frac{L}{2} \cos 45°\right) mv,$$
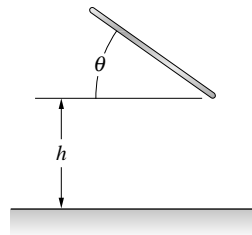
$$(\mathbf{r} \times m\mathbf{v}') \cdot \mathbf{k} = \left(\frac{L}{2}\right) mv',$$

and $I_B \boldsymbol{\omega}' \cdot \mathbf{k} = I_B \omega'$.

From kinematics, $v' = \frac{L}{2} \omega'$. Substitute to obtain:

$$\boxed{\omega' = \frac{mv(\cos 45°)}{\left(\dfrac{mL}{2} + \dfrac{2I_B}{L}\right)} = \frac{3v}{2\sqrt{2}L} = \frac{3\sqrt{-gh}}{2L} = 8.51 \text{ rad/s}}$$

Problem 19.119 The slender bar is released from rest with $\theta = 45°$ and falls a distance $h = 1$ m onto the smooth floor. The length of the bar is 1 m and its mass is 2 kg. If the coefficient of restitution of the impact is $e = 0.4$, what is the angular velocity of the bar just after it hits the floor?

Solution: Choose a coordinate system with the x axis parallel to the plane surface and y positive upward. The strategy is to

(a) use the principle of work and energy to obtain the velocity the instant before impact,

(b) use the definition of the coefficient of restitution to find the velocity just after impact,

(c) get the angular velocity-velocity relations from kinematics and

(d) use the principle of the conservation of angular momentum about the point of impact to determine the angular velocity.

From the principle of work and energy, $U = T_2 - T_1$, where $T_1 = 0$. The center of mass of the bar also falls a distance h before impact. The work done is $U = \int_0^{-h} -mg \, dh = mgh$. The kinetic energy is $T_2 = \left(\frac{1}{2}\right) m v_{Gy}^2$. Substitute into $U = T_2$ and solve: $v_{Gy} = -\sqrt{2gh}$, where the negative sign on the square root is chosen to be consistent with the choice of coordinates.

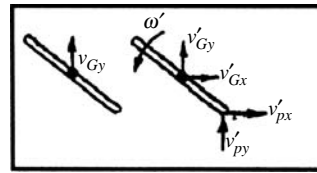Denote the point of impact on the bar by P. From the definition of the coefficient of restitution,

$$e = \frac{v'_{Py} - v'_{By}}{v_{By} - v_{Py}}.$$

Since the floor is stationary before and after impact, $v_B = v'_B = 0$, and
(1) $v'_{Py} = -ev_{Py} = -ev_{Gy}$. From kinematics: $\mathbf{v}'_P = \mathbf{v}'_G + \boldsymbol{\omega}' \times \mathbf{r}_{P/G}$, where $\mathbf{r}_{P/G} = \left(\frac{L}{2}\right)(\mathbf{i}\cos\theta - \mathbf{j}\sin\theta)$, from which

$$(2)\ \mathbf{v}'_{Py} = \mathbf{v}'_{Gy} + \left(\frac{L}{2}\right) \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega' \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}$$

$$= v'_{Gy}\mathbf{j} + \left(\frac{L}{2}\right)(\mathbf{i}\omega'\sin\theta + \mathbf{j}\omega'\cos\theta).$$

Substitute (1) into (2) to obtain

$$(3)\ v'_{Gy} = -ev_{Gy} - \left(\frac{L}{2}\right)\omega'\cos\theta.$$

The angular momentum is conserved about the point of impact:

$$(4)\ -\left(\frac{L}{2}\right)\cos\theta \, mv_{Gy} = -\left(\frac{L}{2}\right)\cos\theta \, mv'_{Gy} + I_G\omega'.$$

Substitute (3) into (4) to obtain

$$-\frac{\cdot L}{2}m\cos\theta(1 + e)v_G = \left(I_G + \left(\frac{L}{2}\right)^2 m\cos^2\theta\right)\omega'.$$

Solve:

$$\omega' = -\frac{mL(1 + e)\cos\theta}{2\left(I_G + \left(\frac{L\cos\theta}{2}\right)^2 m\right)}v_{Gy},$$

$$\boxed{\omega' = -\frac{6(1 + e)\cos\theta}{(1 + 3\cos^2\theta)}(-\sqrt{2gh}) = 10.52 \text{ rad/s}},$$

where $I_G = \frac{mL^2}{12}$ has been used.

Problem 19.120 The slender bar is released from rest and falls a distance $h = 1$ m onto the smooth floor. The length of the bar is 1 m and its mass is 2 kg. The coefficient of restitution of the impact is $e = 0.4$. Determine the angle θ for which the angular velocity of the bar after it hits the floor is a maximum. What is the maximum angular velocity?

Solution: From the solution to Problem 19.119,

$$\omega' = \frac{6(1 + e)\cos\theta}{(1 + 3\cos^2\theta)}\sqrt{2gh}.$$
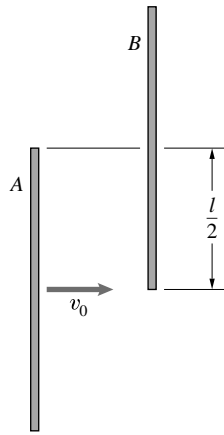
Take the derivative:

$$\frac{d\omega'}{d\theta} = 0 = \frac{-6(1 + e)\sin\theta}{(1 + 3\cos^2\theta)}\sqrt{2gh} + \frac{6(1 + e)(6)\cos^2\theta\sin\theta}{(1 + 3\cos^2\theta)^2}\sqrt{2gh}$$

$$= 0,$$

from which $3\cos^2\theta_{\max} - 1 = 0$,

$$\cos\theta_{\max} = \sqrt{\frac{1}{3}}, \boxed{\theta_{\max} = 54.74°},$$

and $\boxed{\omega'_{\max} = 10.7 \text{ rad/s}}$.

Problem 19.121 A nonrotating slender bar A moving with velocity v_0 strikes a stationary slender bar B. Each bar has mass m and length l. If the bars adhere when they collide, what is their angular velocity after the impact?

Solution: From the conservation of linear momentum, $mv_0 = mv'_A + mv'_B$. From the conservation of angular momentum about the mass center of A

$$0 = I_B\omega'_A + I_B\omega'_B + (\mathbf{r} \times m\mathbf{v}'_B)$$

$$= (I_B\omega'_A + I_B\omega'_B)\mathbf{k} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \dfrac{L}{2} & 0 \\ v'_B & 0 & 0 \end{bmatrix},$$

$$0 = (I_B\omega'_A + I_B\omega'_B)\mathbf{k} - m\left(\frac{L}{2}\right)v'_B)\mathbf{k}.$$

Since the bars adhere, $\omega'_A = \omega'_B$, from which $2I_B\omega' = m\left(\dfrac{L}{2}\right)v'_B$. From kinematics

$$\mathbf{v}'_B = \mathbf{v}'_A + \boldsymbol{\omega}' \times \mathbf{r}_{AB} = v'_A\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega' \\ 0 & \dfrac{L}{2} & 0 \end{bmatrix}$$

$$= \left(v'_A - \frac{L}{2}\omega'\right)\mathbf{i},$$

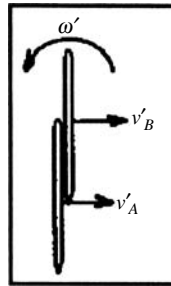from which $v'_B = v'_A - \left(\dfrac{L}{2}\right)\omega'$.

Substitute into the expression for conservation of linear momentum to obtain

$$v'_B = \frac{v_0 - \left(\dfrac{L}{2}\right)\omega'}{2}.$$

Substitute into the expression for the conservation of angular momentum to obtain:

$$\omega' = \frac{\left(\dfrac{mL}{2}\right)v_0}{4I_B + \dfrac{mL^2}{4}} \cdot I_B = \frac{mL^2}{12},$$

from which $\boxed{\omega' = \dfrac{6v_0}{7L}}$

Problem 19.122 An astronaut translates toward a non-rotating satellite at 1.0**i** (m/s) relative to the satellite. Her mass is 136 kg, and the moment of inertia about the axis through her center of mass parallel to the z axis is 45 kg-m^2. The mass of the satellite is 450 kg and its moment of inertia about the z axis is 675 kg-m^2. At the instant the astronaut attaches to the satellite and begins moving with it, the position of her center of mass is $(-1.8, -0.9, 0)$ m. The axis of rotation of the satellite after she attaches is parallel to the z axis. What is their angular velocity?
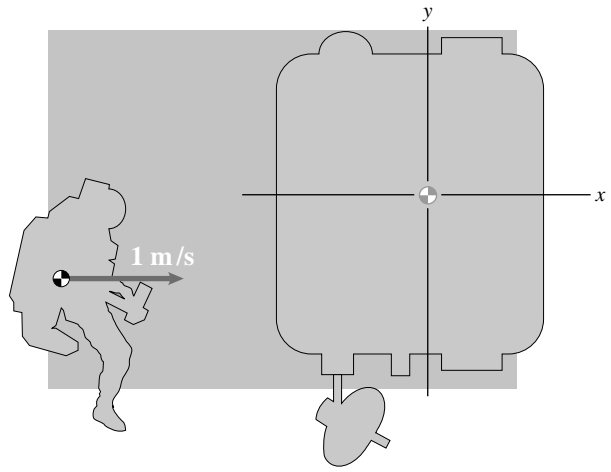
Solution: Choose a coordinate system with the origin at the center of mass of the satellite, and the y axis positive upward. The linear momentum is conserved:

(1) $\quad m_A v_{Ax} = m_A v'_{Ax} + m_S v'_{Sx}$,

(2) $\quad 0 = m_A v'_{Ay} + m_S v'_{Sy}$. The angular momentum about the center of mass of the satellite is conserved: $\mathbf{r}_{A/S} \times m_A \mathbf{v}_A = \mathbf{r}_{A/S} \times m_A \mathbf{v}'_A + (I_A + I_S)\boldsymbol{\omega}'$, where $\mathbf{r}_{A/S} = -1.8\mathbf{i} - 0.9\mathbf{j}$ (m), and $\mathbf{v}_A = 1.0\mathbf{i}$ (m/s).
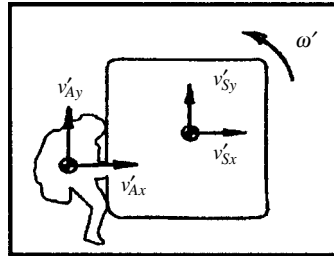
$$\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.8 & -0.9 & 0 \\ m_A v_{Ax} & 0 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.8 & -0.9 & 0 \\ m_A v'_{Ax} & m_A v'_{Ay} & 0 \end{bmatrix}$$

$$+ (I_A + I_S)\boldsymbol{\omega}', \text{ from which}$$

(3) $\quad 0.9 m_A v_{Ax} = -1.8 m_A v'_{Ay} + 0.9 m_A v'_{Ax} + (I_A + I_S)\omega'$.

From kinematics:

$$\mathbf{v}'_A = \mathbf{v}'_S + \boldsymbol{\omega}' \times \mathbf{r}_{A/S} = \mathbf{v}_S + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega' \\ -1.8 & -0.9 & 0 \end{bmatrix}$$

$$= (v'_{Sx} + 0.9\omega')\mathbf{i} + (v'_{Sy} - 1.8\omega')\mathbf{j} \text{ from which}$$

(4) $\quad v'_{Ax} = v'_{Sx} + 0.9\omega'$,

(5) $\quad v'_{Ay} = v'_{Sy} - 1.8\omega'$.

With $v_{Ax} = 1.0$ m/s, these are five equations in five unknowns. The number of equations can be reduced further, but here they are solved by iteration (using **TK Solver Plus**) to obtain: $v'_{Ax} = 0.289$ m/s, $v'_{Sx} = 0.215$ m/s, $v'_{Ay} = -0.114$ m/s, $v'_{Sy} = 0.0344$ m/s,

$\boxed{\omega' = 0.0822 \text{ rad/s}}$

Problem 19.123 In Problem 19.122, suppose that the design parameters of the satellite's control system require that the angular velocity of the satellite not exceed 0.02 rad/s. If the astronaut is moving parallel to the x axis and the position of her center of mass when she attaches is $(-1.8, -0.9, 0)$ m, what is the maximum relative velocity at which she should approach the satellite?

Solution: From the solution to Problem 19.122, the five equations are:

(1) $m_A v_{Ax} = m_A v'_{Ax} + m_S v'_S$,

(2) $0 = m_A v'_{Ay} + m_S v'_{Sy}$

(3) $0.9 m_A v_{Ax} = -1.8 m_A v'_{Ay} + 0.9 m_A v'_{Ax} + (I_A + I_S)\omega'$

(4) $v'_{Ax} = v'_{Sx} + 0.9\omega'$,

(5) $v'_{Ay} = v'_{Sy} - 1.8\omega'$.

With $\omega' = 0.02$ rad/s, these five equations have the solutions:

$$\boxed{v_{Ax} = 0.243 \text{ m/s}}, \; v'_{Ax} = 0.070 \text{ m/s}, \; v'_{Sx} = 0.052 \text{ m/s},$$

$$v'_{Ay} = -0.028 \text{ m/s}, \; v'_{Sy} = 0.008 \text{ m/s}.$$

Problem 19.124 A 12454 N car skidding on ice strikes a concrete abutment at 4.83 km/h. The car's moment of inertia about its center of mass is 2439 kg-m^2. Assume that the impacting surfaces are smooth and parallel to the y axis and that the coefficient of restitution of the impact is $e = 0.8$. What are the angular velocity of the car and the velocity of its center of mass after the impact?

Solution: Let P be the point of impact on with the abutment. (P is located on the vehicle.) Denote the vector from P to the center of mass of the vehicle by $\mathbf{r} = a\mathbf{i} + 0.61\mathbf{j}$, where a is unknown. The velocity of the vehicle is

$$v_{Gx} = 4.83 \text{ km/h} = 1.34 \text{ m/s}.$$

From the conservation of linear momentum in the y direction, $mv_{Gy} = mv'_{Gy}$, from which $v'_{Gy} = v_{Gy} = 0$. Similarly, $v_{Ay} = v'_{Ay} = 0$. By definition,

$$e = \frac{v'_{Px} - v'_{Ax}}{v_{Ax} - v_{Px}}.$$

Assume that the abutment does not yield under the impact, $v_{Ax} = v'_{Ax} = 0$, from which

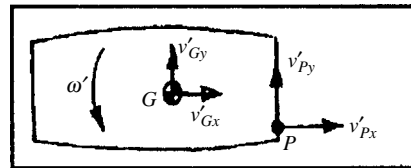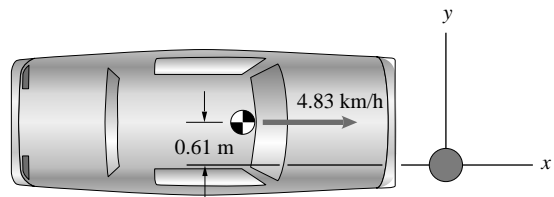$$v'_{Px} = -ev_{Px} = -ev_{Gx} = -(0.8)(1.34) = -1.07 \text{ m/s}$$

The conservation of angular momentum about P is $\mathbf{r} \times m\mathbf{v}_G = \mathbf{r} \times m\mathbf{v}'_G + I\omega'$. From kinematics, $\mathbf{v}'_G = \mathbf{v}'_P + \omega' \times \mathbf{r}$. Reduce:

$$\mathbf{v}'_G = v'_{Px}\mathbf{i} + \omega' \times \mathbf{r} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega' \\ a & 0.61 & 0 \end{bmatrix}$$

$$= (v'_{Px} - 0.61\omega')\mathbf{i} + a\omega'\mathbf{j} = (-ev_{Gx} - 0.61\omega')\mathbf{i} + a\omega'\mathbf{j},$$

from which $v'_{Gx} = -(ev_{Gx} + 0.61\omega')$, $\mathbf{r} \times m\mathbf{v}_G = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & 0.61 & 0 \\ 1.34\,m & 0 & 0 \end{bmatrix}$

$$= -0.82m\mathbf{k}.$$

$$\mathbf{r} \times m\mathbf{v}'_G = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & 0.61 & 0 \\ -(ev_{Gx} + 0.61\,\omega')m & 0 & 0 \end{bmatrix}$$

$$= 0.61(ev_{Gx} + 0.61\omega')m\mathbf{k}, \; I\omega' = I\omega'\mathbf{k}.$$

Substitute into the expression for the conservation of angular momentum to obtain

$$\boxed{\omega' = \frac{-1.47}{\left(0.61^2 + \dfrac{I}{m}\right)} = \frac{-1.47}{(0.372 + 1.92)} = -0.641 \text{ rad/s}}$$

The velocity of the center of mass is

$$\mathbf{v}'_G = v'_{Gx}\mathbf{i} = -(ev_{Gx} + 0.61\,\omega')\mathbf{i} = -2.24\mathbf{i} \text{ (m/s)}$$

Problem 19.125 A 756 N receiver jumps vertically to receive a pass and is stationary at the instant he catches the ball. At the same instant, he is hit at P by a 801 N linebacker moving horizontally at 4.6 m/s. The wide receiver's moment of inertia about his center of mass is 9.5 kg-m^2. If you model the players as rigid bodies and assume that the coefficient of restitution is $e = 0$, what is the wide receiver's angular velocity immediately after the impact?

355.6 mm
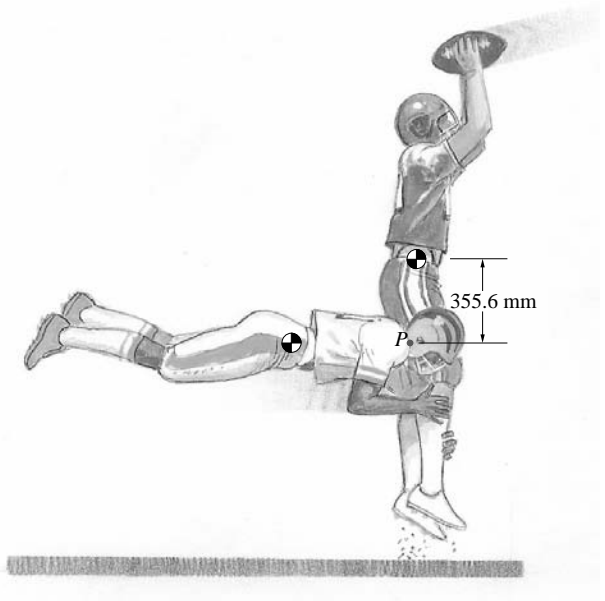
Solution: Denote the receiver by the subscript B, and the tacker by the subscript A. Denote $d = 0.356$ m. The conservation of linear momentum: (1) $m_A v_A = m_A v'_A + m_B v'_B$. From the definition, $e = \dfrac{v'_{BP} - v'_A}{v_A - v_{BP}}$. Since $v_{BP} = 0$, (2) $e v_A = v'_{BP} - v'_A$. The angular momentum is conserved about point P: (3) $0 = -m_B \, d v'_B + I_B \omega'_B$. From kinematics: (4) $v'_{BP} = v'_B + d\omega'_B$.
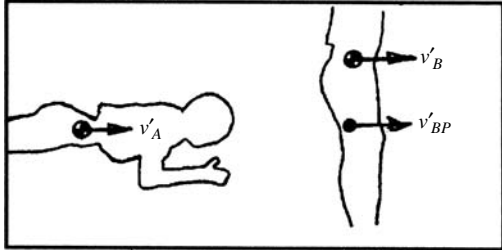

The solution: For $e = 0$, from (1),

$$v'_A = v_A - \left(\frac{m_B}{m_A}\right) v'_B.$$

From (2) and (4) $v'_A = v'_B + d\omega'_B$. From

(3) $v'_B = \left(\dfrac{I_B}{d m_B}\right) \omega'.$

Combine these last two equations and solve:

$$\boxed{\omega'_B = \frac{v_A}{\dfrac{I_B}{d m_B}\left(1 + \dfrac{m_B}{m_A}\right) + d} = 4.445\ldots = 4.45 \text{ rad/s.}}$$