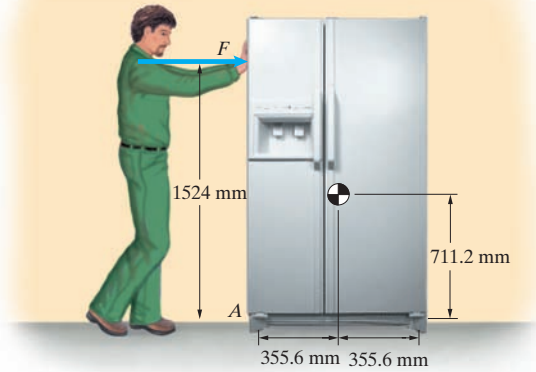


**Problem 18.1** A horizontal force  $F = 133.4 \text{ N}$  is applied to the  $1023 \text{ N}$  refrigerator as shown. Friction is negligible.

- (a) What is the magnitude of the refrigerator's acceleration?  
 (b) What normal forces are exerted on the refrigerator by the floor at  $A$  and  $B$ ?



**Solution:** Assume that the refrigerator rolls without tipping. We have the following equations of motion.

$$\sum F_x : (133.4 \text{ N}) = \left( \frac{1023 \text{ N}}{9.81 \text{ m/s}^2} \right) a$$

$$\sum F_y : A + B - 1023 \text{ N} = 0$$

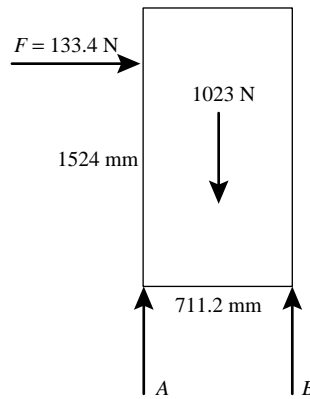
$$\sum M_G : -(133.4 \text{ N})(0.813 \text{ m}) - A(0.356 \text{ m}) + B(0.356 \text{ m}) = 0$$

Solving we find

(a)  $a = 1.28 \text{ m/s}^2$

(b)  $A = 359 \text{ N}, B = 664.1 \text{ N}$

Since  $A > 0$  and  $B > 0$  then our assumption is correct.



**Problem 18.2** Solve Problem 18.1 if the coefficient of kinetic friction at  $A$  and  $B$  is  $\mu_k = 0.1$ .

**Solution:** Assume sliding without tipping

$$\sum F_x : (133.4 \text{ N}) - (0.1)(A + B) = \left( \frac{1023 \text{ N}}{9.81 \text{ m/s}^2} \right) a$$

$$\sum F_y : A + B - 1023 \text{ N} = 0$$

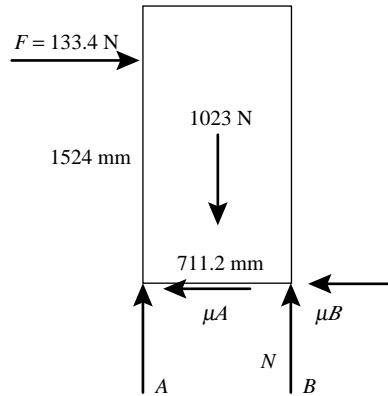
$$\sum M_G : -(133.4 \text{ N})(0.813 \text{ m}) - A(0.356 \text{ m}) + B(0.356 \text{ m})$$

$$- (0.1)(A + B)(0.711 \text{ m}) = 0$$

Solving, we find

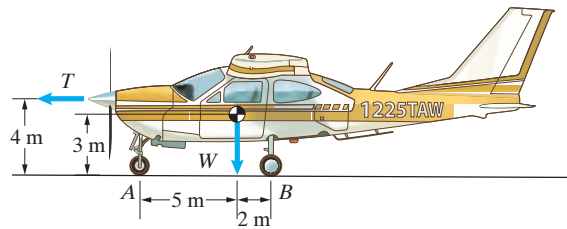
(a)  $a = 0.3 \text{ m/s}^2$

(b)  $A = 256.6 \text{ N}, B = 765.1 \text{ N}$



**Problem 18.3** As the 2800-N airplane begins its take-off run at  $t = 0$ , its propeller exerts a horizontal force  $T = 1000$  N. Neglect horizontal forces exerted on the wheels by the runway.

- (a) What distance has the airplane moved at  $t = 2$  s?  
 (b) what normal forces are exerted on the tires at A and B?



**Solution:** The unknowns are  $N_A$ ,  $N_B$ ,  $a$ .

The equations of motion are:

$$\Sigma F_x : -T = -\frac{W}{g}a,$$

$$\Sigma F_y : N_A + N_B - W = 0$$

$$\Sigma M_G : N_B(2 \text{ m}) - N_A(5 \text{ m})$$

$$+ T(1 \text{ m}) = 0$$

Putting in the numbers for  $T$ ,  $W$ , and  $g$  and solving we find

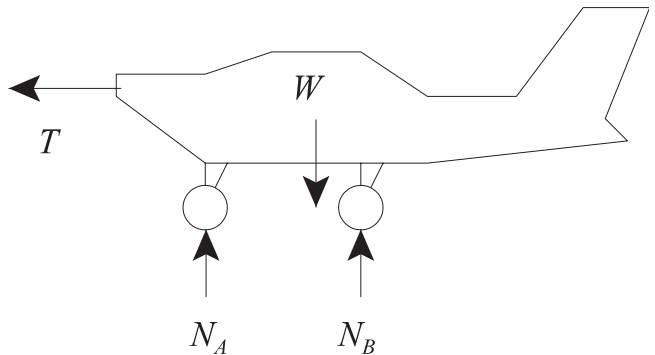
$$N_A = 943 \text{ N}, \quad N_B = 1860 \text{ N}, \quad a = 3.5 \text{ m/s}^2.$$

- (a) The distance is given by  $d = \frac{1}{2}at^2 = \frac{1}{2}(3.5 \text{ m/s}^2)(2 \text{ s})^2 = 7 \text{ m}$

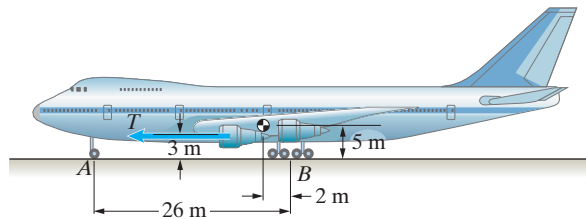
$$d = 7 \text{ m}$$

- (b) The forces were found to be

$$N_A = 943 \text{ N}, \quad N_B = 1860 \text{ N}$$



**Problem 18.4** The Boeing 747 begins its takeoff run at time  $t = 0$ . The normal forces exerted on its tires at A and B are  $N_A = 175$  kN and  $N_B = 2800$  kN. If you assume that these forces are constant and neglect horizontal forces other than the thrust  $T$ , how fast is the airplane moving at  $t = 4$  s? (See Active Example 18.1.)



**Solution:** The unknowns are  $T$ ,  $W$ ,  $a$ . The equations of motion are:

$$\Sigma F_x : -T = -\frac{W}{g}a,$$

$$\Sigma F_y : N_A + N_B - W = 0,$$

$$\Sigma M_G : N_B(2 \text{ m}) - N_A(24 \text{ m})$$

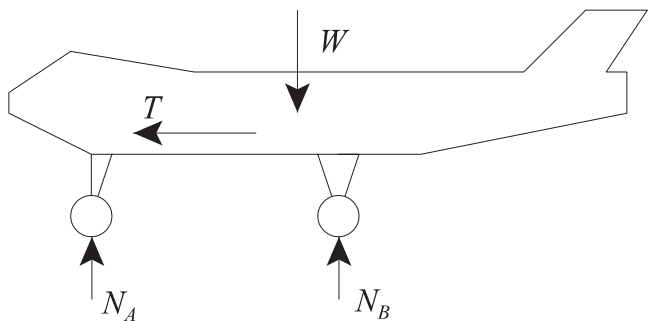
$$- T(2 \text{ m}) = 0.$$

Putting in the numbers for  $N_A$  and  $N_B$  and solving, we find

$$a = 2.31 \text{ m/s}^2, \quad T = 700 \text{ kN}, \quad W = 2980 \text{ kN}.$$

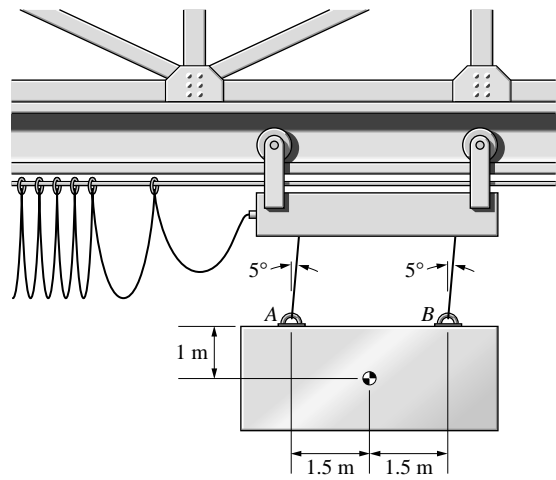
The velocity is then given by

$$v = at = (2.31 \text{ m/s}^2)(4 \text{ s}) = 9.23 \text{ m/s}. \quad v = 9.23 \text{ m/s}.$$



**Problem 18.5** The crane moves to the right with constant acceleration, and the 800-kg load moves without swinging.

- (a) What is the acceleration of the crane and load?  
 (b) What are the tensions in the cables attached at A and B?



**Solution:** From Newton's second law:  $F_x = 800a$  N.  
 The sum of the forces on the load:

$$\sum F_x = F_A \sin 5^\circ + F_B \sin 5^\circ - 800a = 0.$$

$$\sum F_y = F_A \cos 5^\circ + F_B \cos 5^\circ - 800g = 0.$$

The sum of the moments about the center of mass:

$$\sum M_{CM} = -1.5F_A \cos 5^\circ + 1.5F_B \cos 5^\circ$$

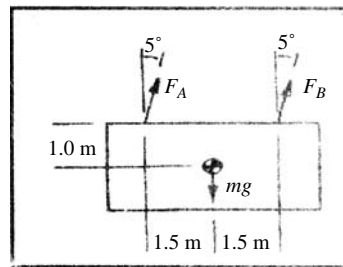
$$- F_A \sin 5^\circ - F_B \sin 5^\circ = 0.$$

Solve these three simultaneous equations:

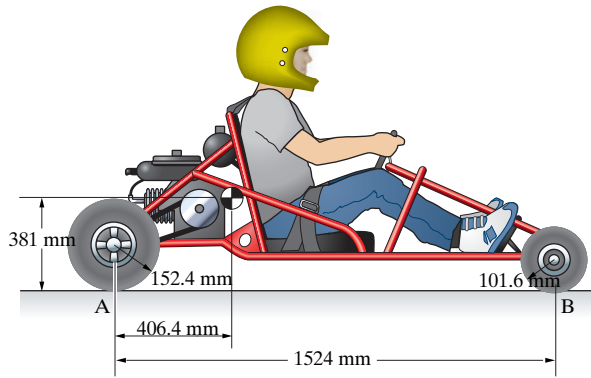
$$a = 0.858 \text{ m/s}^2$$

$$F_A = 3709 \text{ N}$$

$$F_B = 4169 \text{ N}$$



**Problem 18.6** The total weight of the go-cart and driver is 1068 N. The location of their combined center of mass is shown. The rear drive wheels together exert a 106.7 N horizontal force on the track. Neglect the horizontal forces exerted on the front wheels.



- What is the magnitude of the go-cart's acceleration?
- What normal forces are exerted on the tires at A and B?

**Solution:**

$$\sum F_x : (106.7 \text{ N}) = \left( \frac{1068 \text{ N}}{9.81 \text{ m/s}^2} \right) a$$

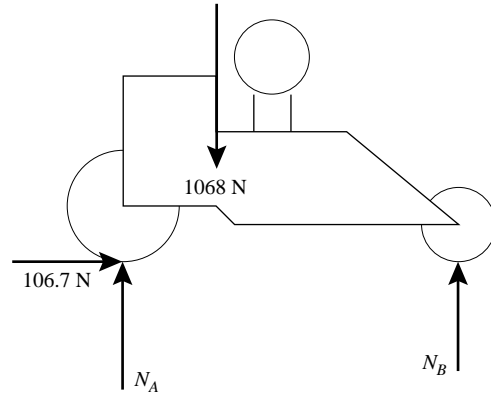
$$\sum F_y : N_A + N_B - (1068 \text{ N}) = 0$$

$$\sum M_G : -N_A(0.406 \text{ m}) + N_B(1.118 \text{ m}) + (106.7 \text{ N})(0.381 \text{ m}) = 0$$

Solving we find

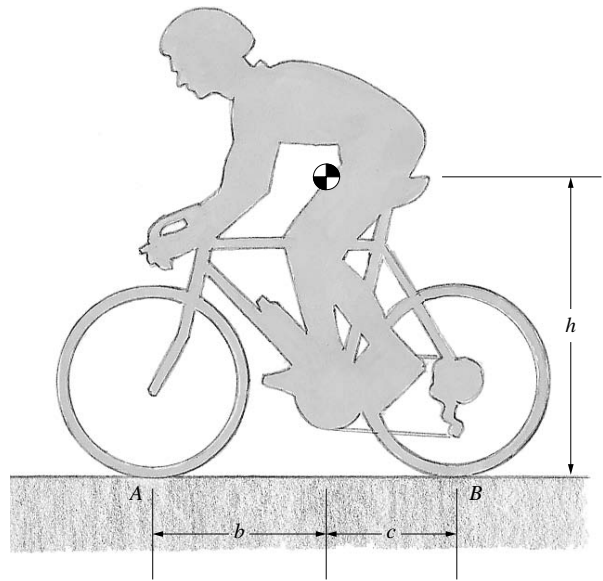
(a)  $a = 0.981 \text{ m/s}^2$

(b)  $N_A = 809.5 \text{ N}, N_B = 258 \text{ N}$



**Problem 18.7** The total weight of the bicycle and rider is 711.7 N. The location of their combined center of mass is shown. The dimensions shown are  $b = 533.4$  mm,  $c = 406.4$  mm, and  $h = 965$  mm. What is the largest acceleration the bicycle can have without the front wheel leaving the ground? Neglect the horizontal force exerted on the front wheel by the road.

**Strategy:** You want to determine the value of the acceleration that causes the normal force exerted on the front wheel by the road to equal zero.



**Solution:** Given:  $b = 0.533$  m.,  $c = 0.406$  m.,  $h = 0.965$  m.

Find:  $a$  so that  $N_A = 0$

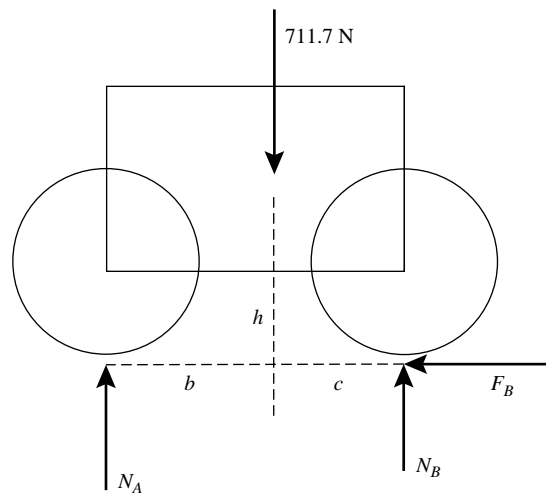
$$\sum F_x : -F_B = -\left(\frac{711.7 \text{ N}}{9.81 \text{ m/s}^2}\right)a$$

$$\sum F_y : N_A + N_B - (711.7 \text{ N}) = 0$$

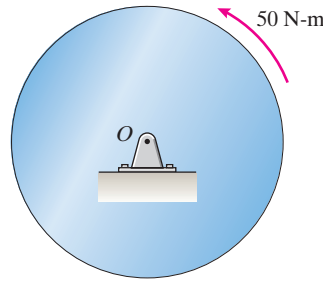
$$\sum M_G : -N_A b + N_B c - F_B h = 0$$

$$N_A = 0$$

Solving we find  $N_B = 711.7$  N,  $F_B = 300$  N,  $a = 4.15 \text{ m/s}^2$



**Problem 18.8** The moment of inertia of the disk about  $O$  is  $I = 20 \text{ kg}\cdot\text{m}^2$ . At  $t = 0$ , the stationary disk is subjected to a constant  $50 \text{ N}\cdot\text{m}$  torque.



- (a) What is the magnitude of the resulting angular acceleration of the disk?  
 (b) How fast is the disk rotating (in rpm) at  $t = 4 \text{ s}$ ?

**Solution:**

(a)  $M = I\alpha \Rightarrow \alpha = \frac{M}{I} = \frac{50 \text{ N}\cdot\text{m}}{20 \text{ kg}\cdot\text{m}^2} = 2.5 \text{ rad/s}^2$ .

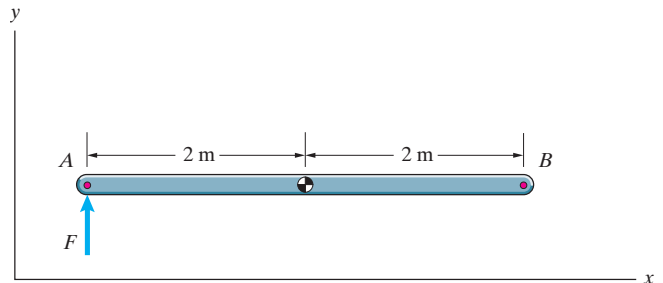
$\alpha = 2.5 \text{ rad/s}^2$ .

- (b) The angular velocity is given by

$$\omega = \alpha t = (2.5 \text{ rad/s}^2)(4 \text{ s}) = 10 \text{ rad/s} \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 95.5 \text{ rpm}.$$

$\omega = 95.5 \text{ rpm}$ .

**Problem 18.9** The  $10\text{-N}$  bar is on a smooth horizontal table. The figure shows the bar viewed from above. Its moment of inertia about the center of mass is  $I = 1 \text{ kg}\cdot\text{m}^2$ . The bar is stationary when the force  $F = 5 \text{ N}$  is applied in the direction parallel to the  $y$  axis. At that instant, determine



- (a) the acceleration of the center of mass, and  
 (b) the acceleration of point  $A$ .

**Solution:**

(a)  $F = ma \Rightarrow a = \frac{F}{m} = \frac{5 \text{ N}}{\left( \frac{10 \text{ N}}{9.81 \text{ m/s}^2} \right)} = 4.905 \text{ m/s}^2$

$\mathbf{a}_G = (4.905 \text{ m/s}^2)\mathbf{j}$ .

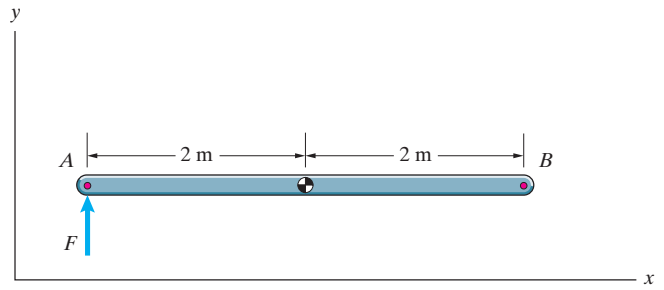
(b)  $\Sigma M_G : -F \frac{l}{2} = I\alpha \Rightarrow \alpha = -\frac{Fl}{2I} = -\frac{(5 \text{ N})(4 \text{ m})}{2(1 \text{ kg}\cdot\text{m}^2)} = -10 \text{ rad/s}^2$ .

$$\mathbf{a}_A = \mathbf{a}_G + \alpha \times \mathbf{r}_{A/G}$$

$$= (4.905 \text{ m/s}^2)\mathbf{j} + (-10 \text{ rad/s}^2)\mathbf{k} \times (-2 \text{ m})\mathbf{i}$$

$\mathbf{a}_A = (24.9 \text{ m/s}^2)\mathbf{j}$ .

**Problem 18.10** The 10-N bar is on a smooth horizontal table. The figure shows the bar viewed from above. Its moment of inertia about the center of mass is  $I = 1 \text{ kg}\cdot\text{m}^2$ . The bar is stationary when the force  $F = 5 \text{ N}$  is applied in the direction parallel to the  $y$  axis. At that instant, determine the acceleration of point  $B$ .



**Solution:**

$$F = ma \Rightarrow a = \frac{F}{m} = \frac{5 \text{ N}}{\left(\frac{10 \text{ N}}{9.81 \text{ m/s}^2}\right)} = 4.905 \text{ m/s}^2$$

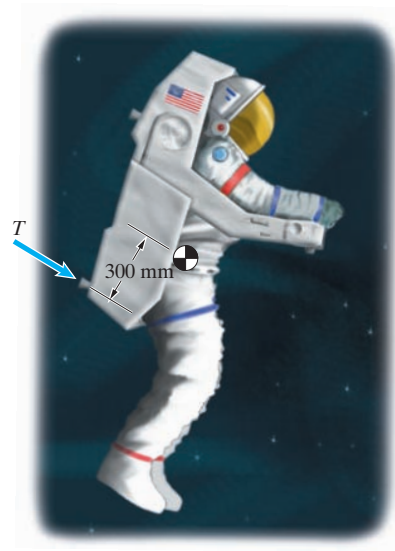
$$\Sigma M_G : -F \frac{l}{2} = I\alpha \Rightarrow \alpha = -\frac{Fl}{2I} = -\frac{(5 \text{ N})(4 \text{ m})}{2(1 \text{ kg}\cdot\text{m}^2)} = -10 \text{ rad/s}^2.$$

$$\mathbf{a}_B = \mathbf{a}_G + \boldsymbol{\alpha} \times \mathbf{r}_{B/G}$$

$$= (4.905 \text{ m/s}^2)\mathbf{j} + (-10 \text{ rad/s}^2)\mathbf{k} \times (2 \text{ m})\mathbf{i}$$

$$\boxed{\mathbf{a}_B = (-15.1 \text{ m/s}^2)\mathbf{j}}$$

**Problem 18.11** The moment of inertia of the astronaut and maneuvering unit about the axis through their center of mass perpendicular to the page is  $I = 40 \text{ kg}\cdot\text{m}^2$ . A thruster can exert a force  $T = 10 \text{ N}$ . For safety, the control system of his maneuvering unit will not allow his angular velocity to exceed  $15^\circ$  per second. If he is initially not rotating, and at  $t = 0$ , he activates the thruster until he is rotating at  $15^\circ$  per second, through how many degrees has he rotated at  $t = 10 \text{ s}$ ?



**Solution:** First find the angular acceleration.

$$\Sigma M_G : Td = I\alpha$$

$$\alpha = \frac{Td}{I} = \frac{(10 \text{ N})(0.3 \text{ m})}{(40 \text{ kg}\cdot\text{m}^2)} = 0.075 \text{ rad/s}^2.$$

To reach maximum angular velocity it takes

$$\omega = \alpha t \Rightarrow t = \frac{\omega}{\alpha} = \frac{(15^\circ/\text{s}) \left(\frac{\pi \text{ rad}}{180^\circ}\right)}{(0.075 \text{ rad/s}^2)} = 3.49 \text{ s}$$

During this time, the astronaut has rotated through

$$\theta_1 = \frac{1}{2}\alpha t^2 = \frac{1}{2}(0.075 \text{ rad/s}^2)(3.49 \text{ s})^2 = 0.457 \text{ rad}.$$

After this time, the astronaut turns at the fixed rate. He rotated an additional angle given by

$$\theta_2 = \omega t = (15^\circ/\text{s}) \left(\frac{\pi \text{ rad}}{180^\circ}\right) (10 \text{ s} - 3.49 \text{ s}) = 1.704 \text{ rad}.$$

The total rotation is then

$$\theta = \theta_1 + \theta_2 = (0.457 + 1.704) \text{ rad} \left(\frac{180^\circ}{\pi \text{ rad}}\right) = 124^\circ.$$

$$\boxed{\theta = 124^\circ}$$

**Problem 18.12** The moment of inertia of the helicopter's rotor is  $420 \text{ N}\cdot\text{m}^2$ . The rotor starts from rest. At  $t = 0$ , the pilot begins advancing the throttle so that the torque exerted on the rotor by the engine (in  $\text{N}\cdot\text{m}$ ) is given as a function of time in seconds by  $T = 200t$ .

- (a) How long does it take the rotor to turn ten revolutions?  
 (b) What is the rotor's angular velocity (in rpm) when it has turned ten revolutions?



**Solution:** Find the angular acceleration

$$T = I\alpha \Rightarrow \alpha = \frac{T}{I} = \frac{200t}{420} = 0.476t$$

Now answer the kinematics questions

$$\alpha = 0.476t, \quad \omega = 0.238t^2, \quad \theta = 0.0794t^3.$$

- (a) When it has turned 10 revolutions,

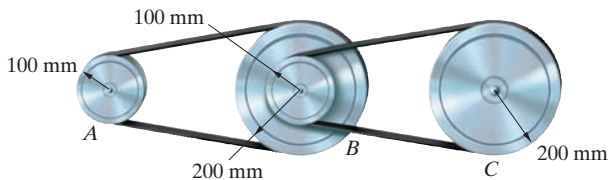
$$(10 \text{ rev}) \left( \frac{2\pi \text{ rad}}{\text{rev}} \right) = 0.0794t^3 \Rightarrow \boxed{t = 9.25 \text{ s.}}$$

- (b) The angular velocity is

$$\omega = 0.238(9.25)^2 = 20.4 \text{ rad/s} \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 195 \text{ rpm}$$

$$\boxed{\omega = 195 \text{ rpm.}}$$

**Problem 18.13** The moments of inertia of the pulleys are  $I_A = 0.0025 \text{ kg}\cdot\text{m}^2$ ,  $I_B = 0.045 \text{ kg}\cdot\text{m}^2$ , and  $I_C = 0.036 \text{ kg}\cdot\text{m}^2$ . A  $5 \text{ N}\cdot\text{m}$  counterclockwise couple is applied to pulley A. Determine the resulting counterclockwise angular accelerations of the three pulleys.



**Solution:** The tension in each belt changes as it goes around each pulley.

The unknowns are  $\Delta T_{AB}$ ,  $\Delta T_{BC}$ ,  $\alpha_A$ ,  $\alpha_B$ ,  $\alpha_C$ .

We will write three dynamic equations and two constraint equations

$$\Sigma M_A : (5 \text{ N}\cdot\text{m}) - \Delta T_{AB}(0.1 \text{ m}) = (0.0025 \text{ kg}\cdot\text{m}^2)\alpha_A$$

$$\Sigma M_B : \Delta T_{AB}(0.2 \text{ m}) - \Delta T_{BC}(0.1 \text{ m}) = (0.045 \text{ kg}\cdot\text{m}^2)\alpha_B$$

$$\Sigma M_C : \Delta T_{BC}(0.2 \text{ m}) = (0.036 \text{ kg}\cdot\text{m}^2)\alpha_C$$

$$(0.1 \text{ m})\alpha_A = (0.2 \text{ m})\alpha_B$$

$$(0.1 \text{ m})\alpha_B = (0.2 \text{ m})\alpha_C.$$

Solving, we find

$$\Delta T_{AB} = 42.2 \text{ N}, \quad \Delta T_{BC} = 14.1 \text{ N},$$

$$\boxed{\alpha_A = 313 \text{ rad/s}^2, \quad \alpha_B = 156 \text{ rad/s}^2, \quad \alpha_C = 78.1 \text{ rad/s}^2.}$$



**Problem 18.14** The moment of inertia of the wind-tunnel fan is  $225 \text{ kg}\cdot\text{m}^2$ . The fan starts from rest. The torque exerted on it by the engine is given as a function of the angular velocity of the fan by  $T = 140 - 0.02\omega^2 \text{ N}\cdot\text{m}$ .

- (a) When the fan has turned 620 revolutions, what is its angular velocity in rpm (revolutions per minute)?  
 (b) What maximum angular velocity in rpm does the fan attain?

**Strategy:** By writing the equation of angular motion, determine the angular acceleration of the fan in terms of its angular velocity. Then use the chain rule:

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \frac{d\theta}{dt} = \frac{d\omega}{d\theta} \omega.$$

**Solution:**

$$\sum M : (140 \text{ N}\cdot\text{m}) - (0.02 \text{ N}\cdot\text{m}/\text{s}^2)\omega^2 = (225 \text{ kg}\cdot\text{m}^2)\alpha$$

$$\alpha = \left( \frac{140}{225} \text{ rad/s}^2 \right) - \left( \frac{0.02}{225} \text{ rad/s}^4 \right) \omega^2$$

$$= (0.622 \text{ rad/s}^2) - (0.0000889 \text{ rad/s}^4)\omega^2$$

(a)  $\alpha = \omega \frac{d\omega}{d\theta} = (0.622 \text{ rad/s}^2) - (0.0000889 \text{ rad/s}^4)\omega^2$

$$\int_0^\omega \frac{\omega d\omega}{(0.622 \text{ rad/s}^2) - (0.0000889 \text{ rad/s}^4)\omega^2} = \int_0^{620(2\pi) \text{ rad}} d\theta$$

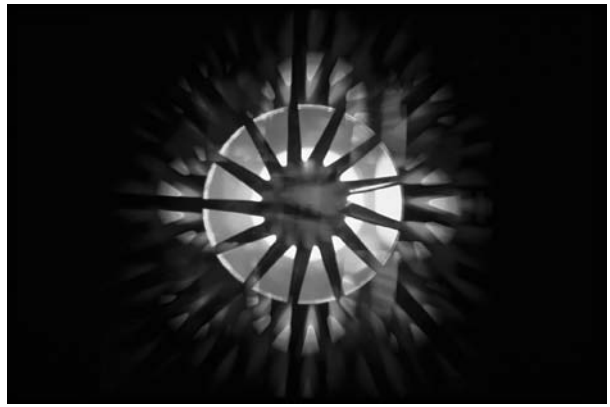
Solving we find

$$\omega = 59.1 \text{ rad/s} \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 565 \text{ rpm}$$

- (b) The maximum angular velocity occurs when the angular acceleration is zero

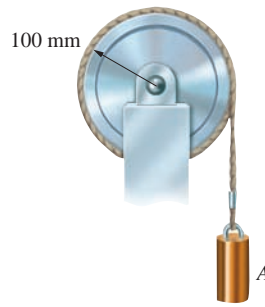
$$\alpha = (0.622 \text{ rad/s}^2) - (0.0000889 \text{ rad/s}^4)\omega^2 = 0$$

$$\omega = 83.7 \text{ rad/s} \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 799 \text{ rpm}$$



**Problem 18.15** The moment of inertia of the pulley about its axis is  $I = 0.005 \text{ kg}\cdot\text{m}^2$ . If the 1-kg mass  $A$  is released from rest, how far does it fall in 0.5 s?

**Strategy:** Draw individual free-body diagrams of the pulley and the mass.



**Solution:** The two free-body diagrams are shown.

The five unknowns are  $T$ ,  $O_x$ ,  $O_y$ ,  $\alpha$ ,  $a$ .

We can write four dynamic equations and one constraint equation, however, we only need to write two dynamic equations and the one constraint equation.

$$\Sigma M_O : -T(0.1 \text{ m}) = -(0.005 \text{ kg}\cdot\text{m}^2)\alpha,$$

$$\Sigma F_y : T - (1 \text{ kg})(9.81 \text{ m/s}^2) = -(1 \text{ kg})a,$$

$$a = (0.1 \text{ m})\alpha.$$

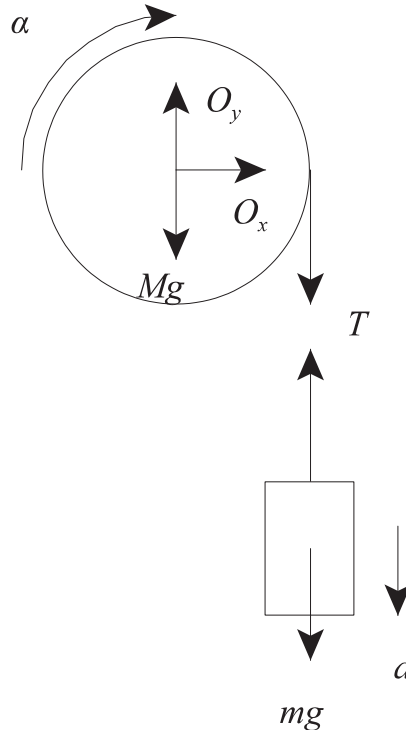
Solving we find

$$T = 3.27 \text{ N}, \quad a = 6.54 \text{ m/s}^2, \quad \alpha = 65.4 \text{ rad/s}^2.$$

Now from kinematics we know

$$d = \frac{1}{2}at^2 = \frac{1}{2}(6.54 \text{ m/s}^2)(0.5 \text{ s})^2$$

$$d = 0.818 \text{ m}.$$



**Problem 18.16** The radius of the pulley is 125 mm and the moment of inertia about its axis is  $I = 0.05 \text{ kg}\cdot\text{m}^2$ . If the system is released from rest, how far does the 20-kg mass fall in 0.5 s? What is the tension in the rope between the 20-kg mass and the pulley?



**Solution:** The free-body diagrams are shown.

We have six unknowns

$$T_1, T_2, O_x, O_y, a, \alpha.$$

We have five dynamic equations and one constraint equation available.

We will use three dynamic equations and the one constraint equation

$$\Sigma M_O : (T_1 - T_2)(0.125 \text{ m}) = -(0.05 \text{ kg}\cdot\text{m}^2)\alpha,$$

$$\Sigma F_{y1} : T_1 - (4 \text{ kg})(9.81 \text{ m/s}^2) = (4 \text{ kg})a,$$

$$\Sigma F_{y2} : T_2 - (20 \text{ kg})(9.81 \text{ m/s}^2) = -(20 \text{ kg})a,$$

$$a = (0.125 \text{ m})\alpha.$$

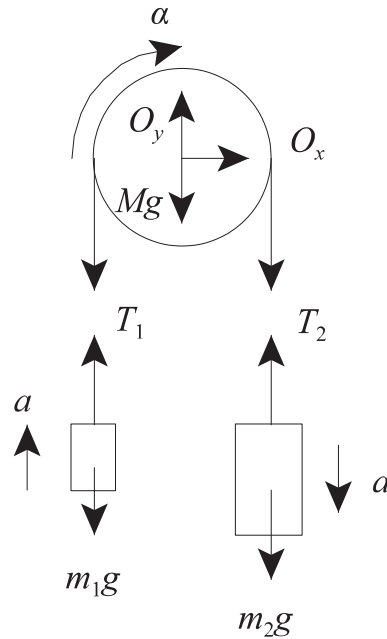
Solving we find

$$T_1 = 62.3 \text{ N}, T_2 = 80.8 \text{ N}, a = 5.77 \text{ m/s}^2, \alpha = 46.2 \text{ rad/s}^2.$$

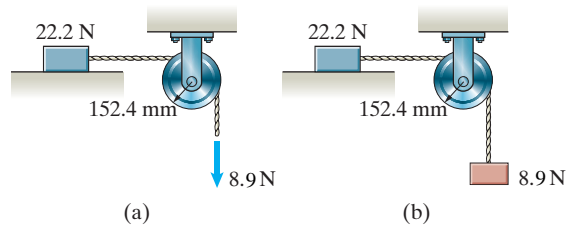
From kinematics we find

$$d = \frac{1}{2}at^2 = \frac{1}{2}(5.77 \text{ m/s}^2)(0.5 \text{ s})^2 = 0.721 \text{ m}.$$

$$d = 0.721 \text{ m}, T_2 = 80.8 \text{ N}.$$



**Problem 18.17** The moment of inertia of the pulley is  $0.54 \text{ kg}\cdot\text{m}^2$ . The coefficient of kinetic friction between the  $22.2 \text{ N}$  weight and the horizontal surface is  $\mu_k = 0.2$ . Determine the magnitude of the acceleration of the  $22.2 \text{ N}$  weight in each case .



**Solution:** The free-body diagrams are shown.

(a)  $T_2 = 8.9 \text{ N}$ .

$$(T_1 - T_2)(0.152 \text{ m}) = -(0.54 \text{ kg}\cdot\text{m}^2)\alpha,$$

$$T_1 - (0.2)(22.2 \text{ N}) = \left(\frac{22.2 \text{ N}}{9.81 \text{ m/s}^2}\right)a,$$

$$a = (0.152 \text{ m})\alpha.$$

Solving we find

$$T_1 = 4.83 \text{ N}, \quad \alpha = 1.14 \text{ rad/s}^2,$$

$$a = 0.174 \text{ m/s}^2$$

(b)  $T_2 \neq 8.9 \text{ N}$ .

$$(T_1 - T_2)(0.152 \text{ m}) = -(0.54 \text{ kg}\cdot\text{m}^2)\alpha, \quad a = (0.152 \text{ m})\alpha,$$

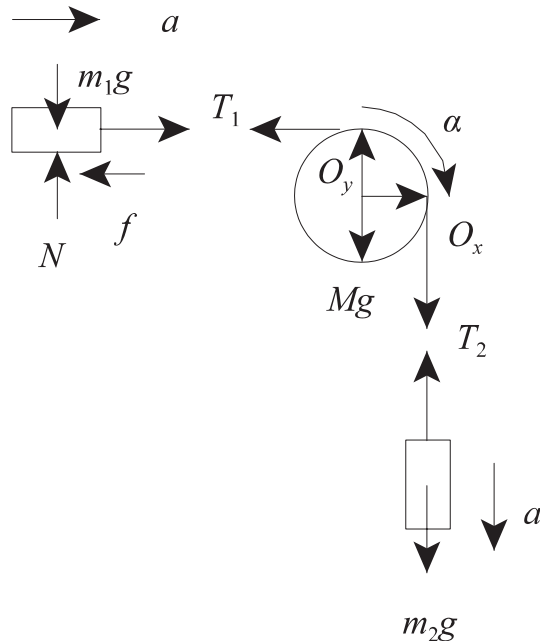
$$T_1 - (0.2)(22.2 \text{ N}) = \left(\frac{22.2 \text{ N}}{9.81 \text{ m/s}^2}\right)a, \quad T_2 - (8.9 \text{ N}) = -\left(\frac{8.9 \text{ N}}{9.81 \text{ m/s}^2}\right)a.$$

Solving we find

$$T_1 = 4.8 \text{ N}, \quad T_2 = 8.75 \text{ N}, \quad \alpha = 1.10 \text{ rad/s}^2,$$

$$a = 0.167 \text{ m/s}^2$$

Note that (b) has more inertia than (a) and therefore has to accelerate more slowly.



**Problem 18.18** The 5-kg slender bar is released from rest in the horizontal position shown. Determine the bar's counterclockwise angular acceleration ( $\alpha$ ) at the instant it is released, and (b) at the instant when it has rotated  $45^\circ$ .

**Solution:**

(a) The free-body diagram is shown.

$$\Sigma M_O : mg \frac{L}{2} = \frac{1}{3} mL^2 \alpha$$

$$\alpha = \frac{3g}{2L} = \frac{3(9.81 \text{ m/s}^2)}{2(1.2 \text{ m})} = 12.3 \text{ rad/s}^2.$$

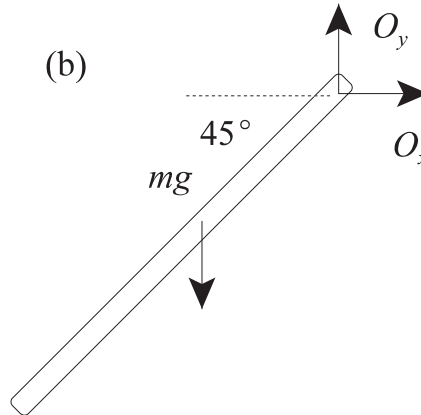
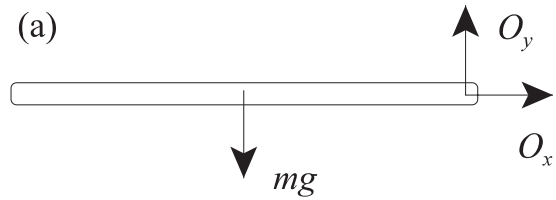
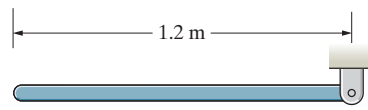
$$\alpha = 12.3 \text{ rad/s}^2.$$

(b) The free-body diagram is shown.

$$\Sigma M_O : mg \frac{L}{2} \cos 45^\circ = \frac{1}{3} mL^2 \alpha$$

$$\alpha = \frac{3g}{2L} \cos 45^\circ = \frac{3(9.81 \text{ m/s}^2)}{2(1.2 \text{ m})} \cos 45^\circ$$

$$\alpha = 8.67 \text{ rad/s}^2.$$



**Problem 18.19** The 5-kg slender bar is released from rest in the horizontal position shown. At the instant when it has rotated  $45^\circ$ , its angular velocity is  $4.16 \text{ rad/s}$ . At that instant, determine the magnitude of the force exerted on the bar by the pin support. (See Example 18.4.)

**Solution:** First find the angular acceleration.

$$\Sigma M_O : mg \frac{L}{2} \cos 45^\circ = \frac{1}{3} mL^2 \alpha$$

$$\alpha = \frac{3g}{2L} \cos 45^\circ = \frac{3(9.81 \text{ m/s}^2)}{2(1.2 \text{ m})} \cos 45^\circ = 8.67 \text{ rad/s}^2$$

Using kinematics we find the acceleration of the center of mass.

$$\mathbf{a}_G = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{G/O} - \omega^2 \mathbf{r}_{G/O}$$

$$\mathbf{a}_G = 0 + (8.67)\mathbf{k} \times (0.6)(-\cos 45^\circ \mathbf{i} - \sin 45^\circ \mathbf{j})$$

$$- (4.16)^2(0.6)(-\cos 45^\circ \mathbf{i} - \sin 45^\circ \mathbf{j})$$

$$= (11.0\mathbf{i} + 3.66\mathbf{j}) \text{ m/s}^2.$$

From Newton's second law we have

$$\Sigma F_x : O_x = ma_x = (5 \text{ kg})(11.0 \text{ m/s}^2) = 55.1 \text{ N}$$

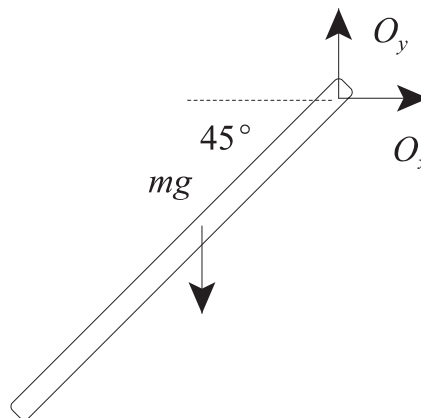
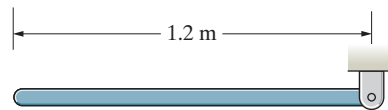
$$\Sigma F_y : O_y - mg = ma_y$$

$$O_y = m(g + a_y) = (5 \text{ kg})(9.81 \text{ m/s}^2 + 3.66 \text{ m/s}^2) = 67.4 \text{ N}$$

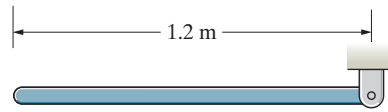
The magnitude of the force in the pin is now

$$O = \sqrt{O_x^2 + O_y^2} = \sqrt{(55.1 \text{ N})^2 + (67.4 \text{ N})^2} = 87.0 \text{ N}.$$

$$O = 87.0 \text{ N}.$$



**Problem 18.20** The 5-kg slender bar is released from rest in the horizontal position shown. Determine the magnitude of its angular velocity when it has fallen to the vertical position.



**Strategy:** Draw the free-body diagram of the bar when it has fallen through an arbitrary angle  $\theta$  and apply the equation of angular motion to determine the bar's angular acceleration as a function of  $\theta$ . Then use the chain rule to write the angular acceleration as

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \frac{d\theta}{dt} = \frac{d\omega}{d\theta} \omega.$$

**Solution:** First find the angular acceleration.

$$\Sigma M_O : mg \frac{L}{2} \cos \theta = \frac{1}{3} mL^2 \alpha \Rightarrow \alpha = \frac{3g}{2L} \cos \theta$$

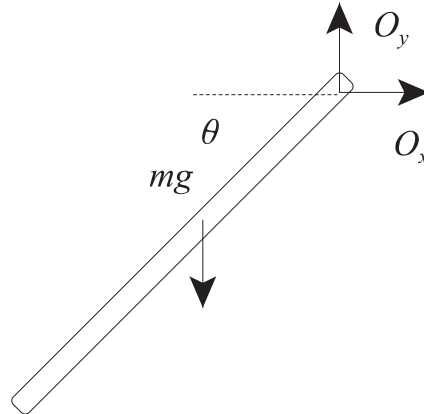
Using the hint we have

$$\alpha = \omega \frac{d\omega}{d\theta} = \frac{3g}{2L} \cos \theta \Rightarrow \int_0^\omega \omega d\omega = \int_0^{90^\circ} \frac{3g}{2L} \cos \theta d\theta$$

$$\frac{1}{2} \omega^2 = \frac{3g}{2L} \sin \theta \Big|_0^{90^\circ} = \frac{3g}{2L}$$

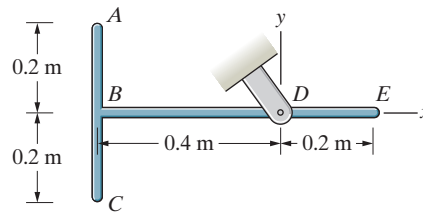
$$\omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3(9.81 \text{ m/s}^2)}{(1.2 \text{ m})}} = 4.95 \text{ rad/s.}$$

$$\omega = 4.95 \text{ rad/s.}$$



**Problem 18.21** The object consists of the 2-kg slender bar  $ABC$  welded to the 3-kg slender bar  $BDE$ . The  $y$  axis is vertical.

- What is the object's moment of inertia about point  $D$ ?
- Determine the object's counterclockwise angular acceleration at the instant shown.



**Solution:** The free-body diagram is shown

(a)

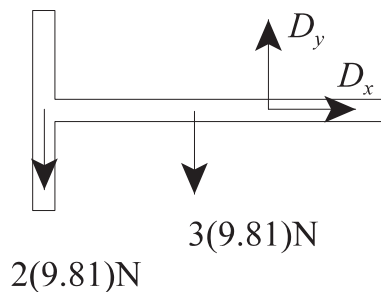
$$I_D = \frac{1}{12} (2 \text{ kg})(0.4 \text{ m})^2 + (2 \text{ kg})(0.4 \text{ m})^2 + \frac{1}{12} (3 \text{ kg})(0.6 \text{ m})^2 + (3 \text{ kg})(0.1 \text{ m})^2$$

$$I_D = 0.467 \text{ kg}\cdot\text{m}^2.$$

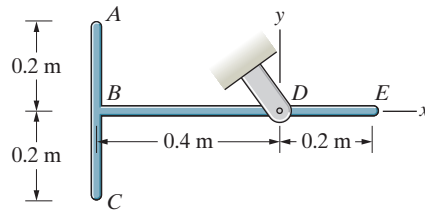
(b)

$$\Sigma M_D : [(2 \text{ kg})(0.4 \text{ m}) + (3 \text{ kg})(0.1 \text{ m})](9.81 \text{ m/s}^2) = (0.467 \text{ kg}\cdot\text{m}^2)\alpha$$

$$\alpha = 23.1 \text{ rad/s}^2.$$



**Problem 18.22** The object consists of the 2-kg slender bar  $ABC$  welded to the 3-kg slender bar  $BDE$ . The  $y$  axis is vertical. At the instant shown, the object has a counterclockwise angular velocity of 5 rad/s. Determine the components of the force exerted on it by the pin support.



**Solution:** The free-body diagram is shown.

The moment of inertia about the fixed point  $D$  is

$$I_D = \frac{1}{12}(2 \text{ kg})(0.4 \text{ m})^2 + (2 \text{ kg})(0.4 \text{ m})^2 + \frac{1}{12}(3 \text{ kg})(0.6 \text{ m})^2 + (3 \text{ kg})(0.1 \text{ m})^2 = 0.467 \text{ kg}\cdot\text{m}^2.$$

The angular acceleration is given by

$$\Sigma M_D : [(2 \text{ kg})(0.4 \text{ m}) + (3 \text{ kg})(0.1 \text{ m})](9.81 \text{ m/s}^2) = (0.467 \text{ kg}\cdot\text{m}^2)\alpha$$

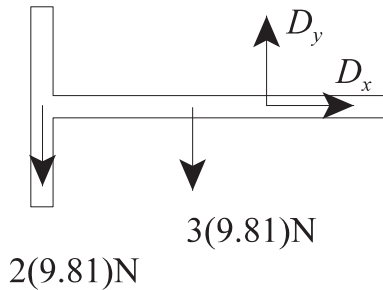
$$\alpha = \frac{10.8 \text{ N}\cdot\text{m}}{0.467 \text{ kg}\cdot\text{m}^2} = 23.1 \text{ rad/s}^2.$$

From Newton's Second Law we have

$$\Sigma F_x : D_x = (2 \text{ kg})(0.4 \text{ m})(5 \text{ rad/s})^2 + (3 \text{ kg})(0.1 \text{ m})(5 \text{ rad/s})^2$$

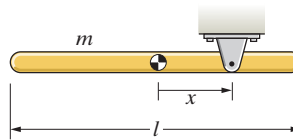
$$\Sigma F_y : D_y - (5 \text{ kg})(9.81 \text{ m/s}^2) = -(2 \text{ kg})(0.4 \text{ m})(23.1 \text{ rad/s}^2) - (3 \text{ kg})(0.1 \text{ m})(23.1 \text{ rad/s}^2)$$

Solving, we find  $D_x = 27.5 \text{ N}, D_y = 23.6 \text{ N}.$



**Problem 18.23** The length of the slender bar is  $l = 4 \text{ m}$  and its mass is  $m = 30 \text{ kg}$ . It is released from rest in the position shown.

- If  $x = 1 \text{ m}$ , what is the bar's angular acceleration at the instant it is released?
- What value of  $x$  results in the largest angular acceleration when the bar is released? What is the angular acceleration?



**Solution:** The moment of inertia about the fixed point is

$$I = \frac{1}{12} ml^2 + mx^2.$$

The angular acceleration can be found

$$\Sigma M_{\text{fixed point}} : mgx = I\alpha = \frac{m}{12}(l^2 + 12x^2)\alpha \Rightarrow \alpha = \frac{12gx}{l^2 + 12x^2}$$

(a) Using the given numbers we have

$$\alpha = \frac{12(9.81 \text{ m/s}^2)(1 \text{ m})}{(4 \text{ m})^2 + 12(1 \text{ m})^2} = 4.20 \text{ rad/s}^2. \quad \alpha = 4.20 \text{ rad/s}^2.$$

- (b) To find the critical value for  $x$  we differentiate and set equal to zero to get

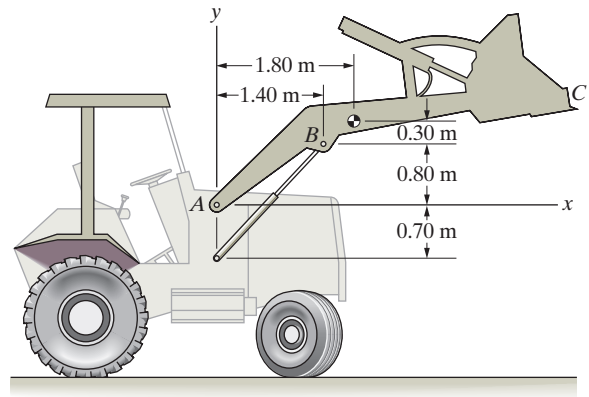
$$\frac{d\alpha}{dx} = \frac{d}{dx} \left( \frac{12gx}{l^2 + 12x^2} \right) = \frac{12g}{l^2 + 12x^2} - \frac{288gx^2}{(l^2 + 12x^2)^2} = \frac{12g(l^2 - 12x^2)}{(l^2 + 12x^2)^2} = 0$$

$$x = \frac{l}{\sqrt{12}} = \frac{(4 \text{ m})}{\sqrt{12}} = 1.15 \text{ m}. \quad x = 1.15 \text{ m}.$$

The corresponding angular acceleration is

$$\alpha = \frac{12(9.81 \text{ m/s}^2)(1.15 \text{ m})}{(4 \text{ m})^2 + 12(1.15 \text{ m})^2} = 4.25 \text{ rad/s}^2 \quad \alpha = 4.25 \text{ rad/s}^2.$$

**Problem 18.24** Model the arm  $ABC$  as a single rigid body. Its mass is  $320\text{ kg}$ , and the moment of inertia about its center of mass is  $I = 360\text{ kg}\cdot\text{m}^2$ . Point  $A$  is stationary. If the hydraulic piston exerts a  $14\text{-kN}$  force on the arm at  $B$  what is the arm's angular acceleration?



**Solution:** The moment of inertia about the fixed point  $A$  is

$$I_A = I_G + md^2 = (360\text{ kg}\cdot\text{m}^2) + (320\text{ kg})[(1.10\text{ m})^2 + (1.80\text{ m})^2]$$

$$= 1780\text{ kg}\cdot\text{m}^2.$$

The angle between the force at  $B$  and the horizontal is

$$\theta = \tan^{-1}\left(\frac{1.5\text{ m}}{1.4\text{ m}}\right) = 47.0^\circ.$$

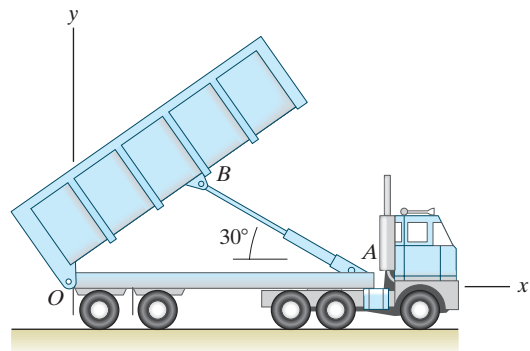
The rotational equation of motion is now

$$\Sigma M_A : (14\text{ kN}) \sin \theta (1.4\text{ m}) - (14\text{ kN}) \cos \theta (0.8\text{ m})$$

$$- (320\text{ kg})(9.81\text{ m/s}^2)(1.80\text{ m}) = (1780\text{ kg}\cdot\text{m}^2) \alpha.$$

Solving, we find  $\alpha = 0.581\text{ rad/s}^2$ .  $\alpha = 0.581\text{ rad/s}^2$  counterclockwise.

**Problem 18.25** The truck's bed weighs  $8000\text{ N}$  and its moment of inertia about  $O$  is  $400000\text{ kg}\cdot\text{m}^2$ . At the instant shown, the coordinates of the center of mass of the bed are  $(3, 4)\text{ m}$  and the coordinates of point  $B$  are  $(5, 3.5)\text{ m}$ . If the bed has a counterclockwise angular acceleration of  $0.2\text{ rad/s}^2$ , what is the magnitude of the force exerted on the bed at  $B$  by the hydraulic cylinder  $AB$ ?



**Solution:** The rotational equation of motion is

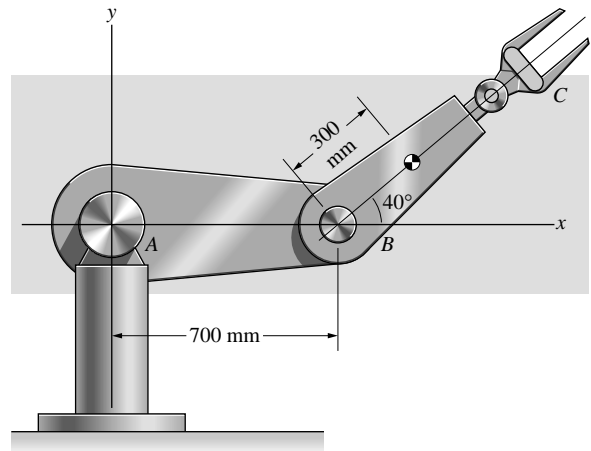
$$\Sigma M_O : F \sin 30^\circ (5\text{ m}) + F \cos 30^\circ (3.5\text{ m}) - (8000\text{ N})(3\text{ m})$$

$$= (400000\text{ kg}\cdot\text{m}^2)(0.2\text{ rad/s}^2)$$

Solving for  $F$  we find  $F = 18,807\text{ N}$ .



**Problem 18.26** Arm  $BC$  has a mass of 12 kg and the moment of inertia about its center of mass is  $3 \text{ kg}\cdot\text{m}^2$ . Point  $B$  is stationary and arm  $BC$  has a constant counterclockwise angular velocity of 2 rad/s. At the instant shown, what are the couple and the components of force exerted on arm  $BC$  at  $B$ ?



**Solution:** Since the angular acceleration of arm  $BC$  is zero, the sum of the moments about the fixed point  $B$  must be zero. Let  $\mathbf{M}_B$  be the couple exerted by the support at  $B$ . Then

$$\mathbf{M}_B + \mathbf{r}_{CM/B} \times m\mathbf{g} = \mathbf{M}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 \cos 40^\circ & 0.3 \sin 40^\circ & 0 \\ 0 & -117.7 & 0 \end{bmatrix} = 0.$$

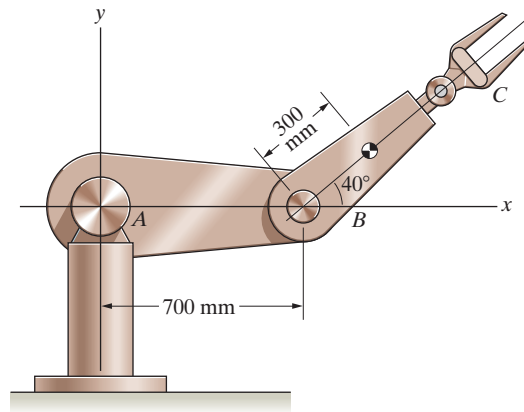
$\mathbf{M}_B = 27.05\mathbf{k} \text{ (N}\cdot\text{m)}$  is the couple exerted at  $B$ . From Newton's second law:  $B_x = ma_x$ ,  $B_y - mg = ma_y$  where  $a_x$ ,  $a_y$  are the accelerations of the center of mass. From kinematics:

$$\begin{aligned} \mathbf{a} &= \boldsymbol{\alpha} \times \mathbf{r}_{CM/O} - \omega^2 \mathbf{r}_{CM/O} \\ &= -(2^2)(\mathbf{i}0.3 \cos 40^\circ + \mathbf{j}0.3 \sin 40^\circ) \\ &= -0.919\mathbf{i} - 0.771\mathbf{j} \text{ (m/s}^2\text{)}, \end{aligned}$$

where the angular acceleration is zero from the problem statement. Substitute into Newton's second law to obtain the reactions at  $B$ :

$$B_x = -11.0 \text{ N}, \quad B_y = 108.5 \text{ N}.$$

**Problem 18.27** Arm  $BC$  has a mass of 12 kg and the moment of inertia about its center of mass is  $3 \text{ kg}\cdot\text{m}^2$ . At the instant shown, arm  $AB$  has a constant clockwise angular velocity of  $2 \text{ rad/s}$  and arm  $BC$  has counter-clockwise angular velocity of  $2 \text{ rad/s}$  and a clockwise angular acceleration of  $4 \text{ rad/s}^2$ . What are the couple and the components of force exerted on arm  $BC$  at  $B$ ?



**Solution:** Because the point  $B$  is accelerating, the equations of angular motion must be written about the center of mass of arm  $BC$ . The vector distances from  $A$  to  $B$  and  $B$  to  $G$ , respectively, are

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 0.7\mathbf{i},$$

$$\begin{aligned} \mathbf{r}_{G/B} &= 0.3 \cos(40^\circ)\mathbf{i} + 0.3 \sin(40^\circ)\mathbf{j} \\ &= 0.2298\mathbf{i} + 0.1928\mathbf{j} \text{ (m)}. \end{aligned}$$

The acceleration of point  $B$  is

$$\mathbf{a}_B = \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} = -\omega_{AB}^2 (0.7\mathbf{i}) \text{ (m/s}^2\text{)}.$$

The acceleration of the center of mass is

$$\begin{aligned} \mathbf{a}_G &= \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{G/B} - \omega_{BC}^2 \mathbf{r}_{G/B} \\ \mathbf{a}_G &= -2.8\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -4 \\ 0.2298 & 0.1928 & 0 \end{bmatrix} - 0.9193\mathbf{i} - 0.7713\mathbf{j} \\ &= -2.948\mathbf{i} - 1.691\mathbf{j} \text{ (m/s}^2\text{)}. \end{aligned}$$

From Newton's second law,

$$B_x = ma_{Gx} = (12)(-2.948) = -35.37 \text{ N}$$

$$B_y - mg = ma_{Gy},$$

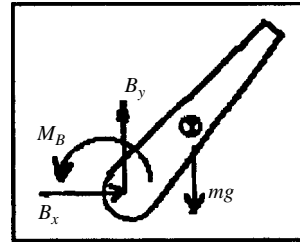
$$B_y = (12)(-1.691) + (12)(9.81) = 97.43 \text{ N}$$

From the equation of angular motion,  $\mathbf{M}_G = \mathbf{I}\boldsymbol{\alpha}_{BC}$ . The moment about the center of mass is

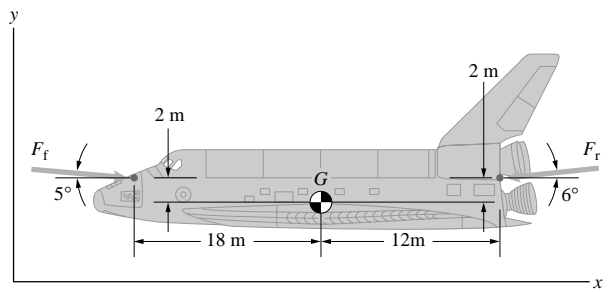
$$\begin{aligned} \mathbf{M}_G &= \mathbf{M}_B + \mathbf{r}_{B/G} \times \mathbf{B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.2298 & -0.1928 & 0 \\ -35.37 & 97.43 & 0 \end{bmatrix} \\ &= M_B \mathbf{k} - 29.21 \mathbf{k} \text{ (N}\cdot\text{m)}. \end{aligned}$$

Note  $I = 3 \text{ kg}\cdot\text{m}^2$  and  $\boldsymbol{\alpha}_{BC} = -4\mathbf{k} \text{ (rad/s}^2\text{)}$ , from which

$$M_B = 29.21 + 3(-4) = 17.21 \text{ N}\cdot\text{m}$$



**Problem 18.28** The space shuttle's attitude control engines exert two forces  $F_f = 8$  kN and  $F_r = 2$  kN. The force vectors and the center of mass  $G$  lie in the  $x$ - $y$  plane of the inertial reference frame. The mass of the shuttle is 54,000 kg, and its moment of inertia about the axis through the center of mass that is parallel to the  $z$  axis is  $4.5 \times 10^6$  kg-m<sup>2</sup>. Determine the acceleration of the center of mass and the angular acceleration. (You can ignore the force on the shuttle due to its weight).



**Solution:** Newton's second law is

$$\sum \mathbf{F} = (F_f \cos 5^\circ - F_r \cos 6^\circ)\mathbf{i} - (F_f \sin 5^\circ + F_r \sin 6^\circ)\mathbf{j} = m\mathbf{a}.$$

Setting  $F_f = 8000$  N,  $F_r = 2000$  N and  $m = 54,000$  kg and solving for  $\mathbf{a}$ , we obtain  $\mathbf{a} = 0.1108\mathbf{i} - 0.0168\mathbf{j}$  (m/s<sup>2</sup>). The equation of angular motion is

$$\begin{aligned} \sum M &= (18)(F_f \sin 5^\circ) - (2)(F_f \cos 5^\circ) \\ &\quad - (12)(F_r \sin 6^\circ) + (2)(F_r \cos 6^\circ) = I\alpha \end{aligned}$$

where  $I = 4.5 \times 10^6$  kg-m<sup>2</sup>. Solving for  $\alpha$  the counterclockwise angular acceleration is  $\alpha = -0.000427$  rad/s<sup>2</sup>.

**Problem 18.29** In Problem 18.28, suppose that  $F_f = 4$  kN and you want the shuttle's angular acceleration to be zero. Determine the necessary force  $F_r$  and the resulting acceleration of the center of mass.

**Solution:** The total moment about the center of mass must equal zero:

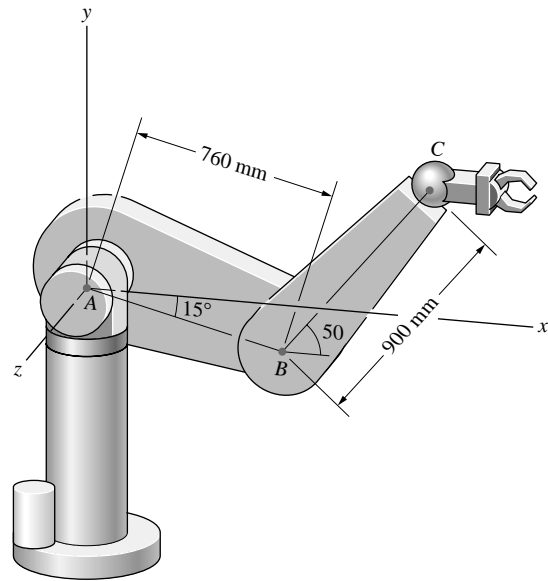
$$\begin{aligned} \sum M &= (18)(F_f \sin 5^\circ) - (2)(F_f \cos 5^\circ) \\ &\quad - (12)(F_r \sin 6^\circ) + (2)(F_r \cos 6^\circ) = 0 \end{aligned}$$

Setting  $F_f = 4000$  N and solving  $F_r = 2306$  N. From Newton's second law

$$\begin{aligned} \sum \mathbf{F} &= (F_f \cos 5^\circ - F_r \cos 6^\circ)\mathbf{i} \\ &\quad - (F_f \sin 5^\circ + F_r \sin 6^\circ)\mathbf{j} = 54,000\mathbf{a}, \end{aligned}$$

we obtain  $\mathbf{a} = 0.0313\mathbf{i} - 0.0109\mathbf{j}$  (m/s<sup>2</sup>).

**Problem 18.30** Points  $B$  and  $C$  lie in the  $x$ - $y$  plane. The  $y$  axis is vertical. The center of mass of the 18-kg arm  $BC$  is at the midpoint of the line from  $B$  to  $C$ , and the moment of inertia of the arm about the axis through the center of mass that is parallel to the  $z$  axis is  $1.5 \text{ kg}\cdot\text{m}^2$ . At the instant shown, the angular velocity and angular acceleration vectors of arm  $AB$  are  $\boldsymbol{\omega}_{AB} = 0.6\mathbf{k}$  (rad/s) and  $\boldsymbol{\alpha}_{AB} = -0.3\mathbf{k}$  (rad/s<sup>2</sup>). The angular velocity and angular acceleration vectors of arm  $BC$  are  $\boldsymbol{\omega}_{BC} = 0.4\mathbf{k}$  (rad/s) and  $\boldsymbol{\alpha}_{BC} = 2\mathbf{k}$  (rad/s<sup>2</sup>). Determine the force and couple exerted on arm  $BC$  at  $B$ .



**Solution:** The acceleration of point  $B$  is  $\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2 \mathbf{r}_{A/B}$  or

$$\mathbf{a}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.3 \\ 0.76 \cos 15^\circ & -0.76 \sin 15^\circ & 0 \end{vmatrix}$$

$$= -(0.6)^2(0.76 \cos 15^\circ \mathbf{i} - 0.76 \sin 15^\circ \mathbf{j})$$

$$= -0.323\mathbf{i} - 0.149\mathbf{j} \text{ (m/s}^2\text{)}$$

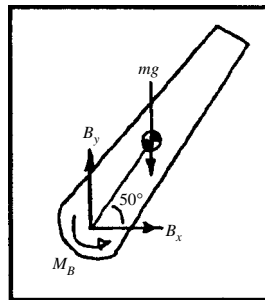
The acceleration of the center of mass  $G$  of arm  $BC$  is

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{G/B} - \omega_{BC}^2 \mathbf{r}_{G/B} \quad \mathbf{a}_B = -0.323\mathbf{i} - 0.149\mathbf{j}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0.45 \cos 50^\circ & 0.45 \sin 50^\circ & 0 \end{vmatrix}$$

$$= -(0.4)^2(0.45 \cos 50^\circ \mathbf{i} + 0.45 \sin 50^\circ \mathbf{j}),$$

or  $\mathbf{a}_G = -1.059\mathbf{i} + 0.374\mathbf{j}$  (m/s<sup>2</sup>). The free body diagram of arm  $BC$  is:



Newton's second law is

$$\sum \mathbf{F} = B_x \mathbf{i} + (B_y - mg) \mathbf{j} = m \mathbf{a}_G:$$

$$B_x \mathbf{i} + [B_y - (18)(9.81)] \mathbf{j} = 18(-1.059\mathbf{i} + 0.374\mathbf{j}).$$

Solving, we obtain  $B_x = -19.1 \text{ N}$ ,  $B_y = 183.3 \text{ N}$ .

The equation of angular motion is

$$\sum M_G = I_{BC} \alpha_{BC}:$$

$$\text{or } (0.45 \sin 50^\circ) B_x - (0.45 \cos 50^\circ) B_y + M_B = (1.5)(2)$$

Solving for  $M_B$ , we obtain  $M_B = 62.6 \text{ N}\cdot\text{m}$ .

**Problem 18.31** Points  $B$  and  $C$  lie in the  $x-y$  plane. The  $y$  axis is vertical. The center of mass of the 18-kg arm  $BC$  is at the midpoint of the line from  $B$  to  $C$ , and the moment of inertia of the arm about the axis through the center of mass that is parallel to the  $z$  axis is  $1.5 \text{ kg}\cdot\text{m}^2$ . At the instant shown, the angular velocity and angular acceleration vectors of arm  $AB$  are  $\boldsymbol{\omega}_{AB} = 0.6\mathbf{k}$  (rad/s) and  $\boldsymbol{\alpha}_{AB} = -0.3\mathbf{k}$  (rad/s<sup>2</sup>). The angular velocity vector of arm  $BC$  is  $\boldsymbol{\omega}_{BC} = 0.4\mathbf{k}$  (rad/s). If you want to program the robot so that the angular acceleration of arm  $BC$  is zero at this instant, what couple must be exerted on arm  $BC$  at  $B$ ?

**Solution:** From the solution of Problem 18.30, the acceleration of point  $B$  is  $\mathbf{a}_B = -0.323\mathbf{i} - 0.149\mathbf{j}$  (m/s<sup>2</sup>). If  $\alpha_{BC} = 0$ , the acceleration of the center of mass  $G$  of arm  $BC$  is

$$\begin{aligned} \mathbf{a}_G &= \mathbf{a}_B - \omega_{BC}^2 \mathbf{r}_{G/B} = -0.323\mathbf{i} - 0.149\mathbf{j} \\ &\quad - (0.4)^2(0.45 \cos 50^\circ \mathbf{i} + 0.45 \sin 50^\circ \mathbf{j}) \\ &= -0.370\mathbf{i} - 0.205\mathbf{j} \text{ (m/s}^2\text{)}. \end{aligned}$$

From the free body diagram of arm  $BC$  in the solution of Problem 18.30. Newton's second law is

$$\sum \mathbf{F} = B_x \mathbf{i} + (B_y - mg)\mathbf{j} = m\mathbf{a}_G:$$

$$B_x \mathbf{i} + [B_y - (18)(9.81)]\mathbf{j} = 18(-0.370\mathbf{i} - 0.205\mathbf{j}).$$

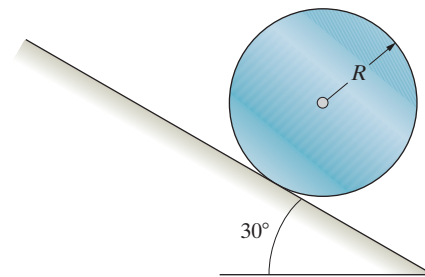
Solving, we obtain  $B_x = -6.65 \text{ N}$ ,  $B_y = 172.90 \text{ N}$ . The equation of angular motion is

$$\sum M_G = I_{BC} \alpha_{BC} = 0:$$

$$(0.45 \sin 50^\circ) B_x - (0.45 \cos 50^\circ) B_y + M_B = 0.$$

Solving for  $M_B$ , we obtain  $M_B = 52.3 \text{ N}\cdot\text{m}$ .

**Problem 18.32** The radius of the 2-kg disk is  $R = 80 \text{ mm}$ . Its moment of inertia is  $I = 0.0064 \text{ kg}\cdot\text{m}^2$ . It rolls on the inclined surface. If the disk is released from rest, what is the magnitude of the velocity of its center two seconds later? (See Active Example 18.2).



**Solution:** There are four unknowns ( $N, f, a, \alpha$ ), three dynamic equations, and one constraint equation. We have

$$\Sigma M_G : -fr = -I\alpha,$$

$$\Sigma F_{\parallel} : mg \sin 30^\circ - f = ma$$

$$a = r\alpha$$

Solving, we find

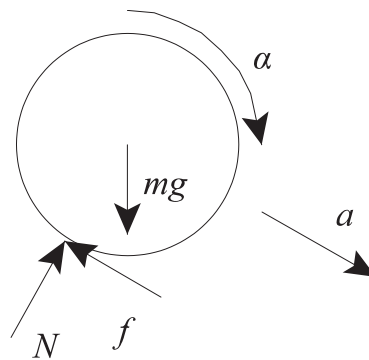
$$a = \frac{mgr^2 \sin 30^\circ}{I + mr^2}$$

$$= \frac{(2 \text{ kg})(9.81 \text{ m/s}^2)(0.08 \text{ m})^2 \sin 30^\circ}{0.0064 \text{ kg}\cdot\text{m}^2 + (2 \text{ kg})(0.08 \text{ m})^2}$$

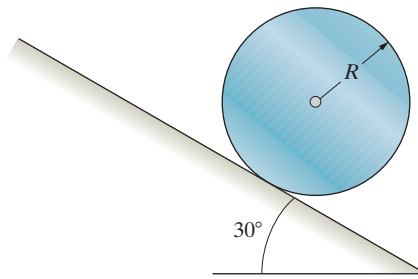
$$= 3.27 \text{ m/s}^2.$$

From the kinematics we have

$$v = at = (3.27 \text{ m/s}^2)(2 \text{ s}) = 6.54 \text{ m/s. } \boxed{v = 6.54 \text{ m/s.}}$$



**Problem 18.33** The radius of the 2-kg disk is  $R = 80$  mm. Its moment of inertia is  $I = 0.0064$  kg·m<sup>2</sup>. What minimum coefficient of static friction is necessary for the disk to roll, instead of slip, on the inclined surface? (See Active Example 18.2.)



**Solution:** There are five unknowns ( $N, f, a, \alpha, \mu_s$ ), three dynamic equations, one constraint equation, and one friction equation. We have

$$\Sigma M_G : -fR = -I\alpha,$$

$$\Sigma F_{\searrow} : mg \sin 30^\circ - f = ma,$$

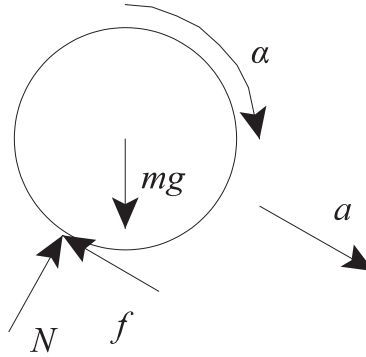
$$\Sigma F_{\nearrow} : N - mg \cos 30^\circ = 0,$$

$$a = R\alpha,$$

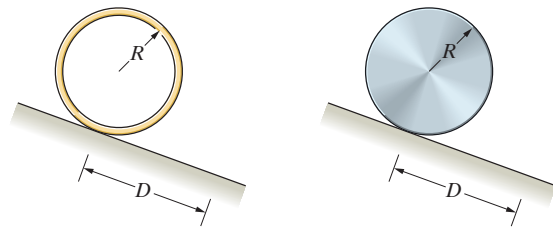
$$f = \mu_s N.$$

Putting in the numbers and solving, we find

$$N = 17.0 \text{ N}, f = 3.27 \text{ N}, a = 3.27 \text{ m/s}^2, \alpha = 40.9 \text{ rad/s}^2, \boxed{\mu_s = 0.192}.$$



**Problem 18.34** A thin ring and a homogeneous circular disk, each of mass  $m$  and radius  $R$ , are released from rest on an inclined surface. Determine the ratio  $v_{\text{ring}}/v_{\text{disk}}$  of the velocities of their centers when they have rolled a distance  $D$ .



**Solution:** There are four unknowns ( $N, f, a, \alpha$ ), three dynamic equations, and one constraint equation. We have

$$\Sigma M_G : -fR = -I\alpha,$$

$$\Sigma F_{\searrow} : mg \sin \theta - f = ma,$$

$$a = R\alpha,$$

$$\text{Solving, we find } a = \frac{mgr^2 \sin \theta}{I + mr^2}$$

$$\text{For the ring } I_{\text{ring}} = mr^2 \Rightarrow a_{\text{ring}} = \frac{g}{2} \sin \theta$$

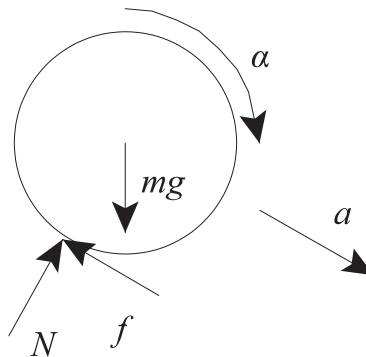
$$\text{For the disk } I_{\text{disk}} = \frac{1}{2}mr^2 \Rightarrow a_{\text{disk}} = \frac{2g}{3} \sin \theta$$

The velocities are then

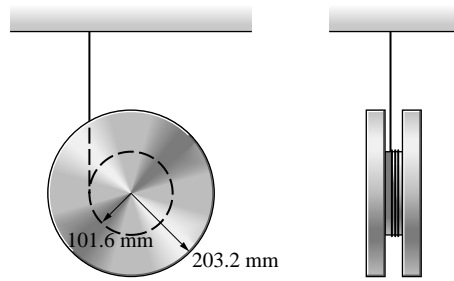
$$v_{\text{ring}} = \sqrt{2a_{\text{ring}}D} = \sqrt{gD \sin \theta}, v_{\text{disk}} = \sqrt{2a_{\text{disk}}D} = \sqrt{\frac{4}{3}gD \sin \theta}$$

The ratio is

$$\frac{v_{\text{ring}}}{v_{\text{disk}}} = \frac{\sqrt{gD \sin \theta}}{\sqrt{\frac{4}{3}gD \sin \theta}} = \sqrt{3/4} \quad \boxed{\frac{v_{\text{ring}}}{v_{\text{disk}}} = \sqrt{3/4}}$$



**Problem 18.35** The stepped disk weighs 178 N and its moment of inertia is  $I = 0.27 \text{ kg}\cdot\text{m}^2$ . If the disk is released from rest, how long does it take its center to fall 0.91 m? (Assume that the string remains vertical.)



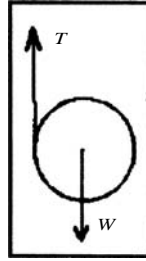
**Solution:** The moment about the center of mass is  $M = -RT$ . From the equation of angular motion:  $-RT = I\alpha$ , from which  $T = -\frac{I\alpha}{R}$ . From the free body diagram and Newton's second law:  $\sum F_y = T - W = ma_y$ , where  $a_y$  is the acceleration of the center of mass. From kinematics:  $a_y = -R\alpha$ . Substitute and solve:

$$a_y = \frac{W}{\left(\frac{I}{R^2} + m\right)}$$

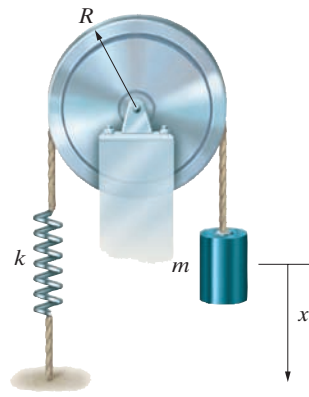
The time required to fall a distance  $D$  is

$$t = \sqrt{\frac{2D}{a_y}} = \sqrt{\frac{2D(I + R^2m)}{R^2W}}$$

For  $D = 0.91 \text{ m}$ ,  $R = 0.102 \text{ m}$ ,  $W = 178 \text{ N}$ ,  $m = \frac{W}{g} = 18.1 \text{ kg}$ ,  
 $I = 0.27 \text{ kg}\cdot\text{m}^2$ ,  $t = 0.676 \text{ s}$



**Problem 18.36** The radius of the pulley is  $R = 100 \text{ mm}$  and its moment of inertia is  $I = 0.1 \text{ kg}\cdot\text{m}^2$ . The mass  $m = 5 \text{ kg}$ . The spring constant is  $k = 135 \text{ N/m}$ . The system is released from rest with the spring unstretched. At the instant when the mass has fallen  $0.2 \text{ m}$ , determine (a) the angular acceleration of the pulley, and (b) the tension in the rope between the mass and the pulley.



**Solution:** The force in the spring is  $kx$ . There are five unknowns ( $O_x$ ,  $O_y$ ,  $T$ ,  $a$ ,  $\alpha$ ), four dynamic equations, and one constraint equation.

$$\Sigma M_O : (kx)R - TR = -I\alpha,$$

$$\Sigma F_y : T - mg = -ma,$$

$$a = R\alpha$$

Solving we find

(a)

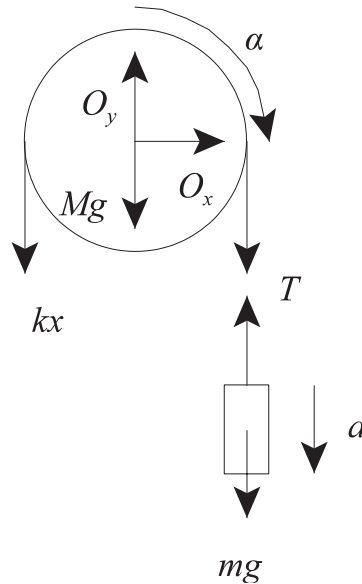
$$\begin{aligned} \alpha &= \frac{R(mg - kx)}{I + mR^2} \\ &= \frac{(0.1 \text{ m})([5 \text{ kg}][9.81 \text{ m/s}^2] - [135 \text{ N/m}][0.2 \text{ m}])}{0.1 \text{ kg}\cdot\text{m}^2 + (5 \text{ kg})(0.1 \text{ m})^2} \end{aligned}$$

$$\alpha = 14.7 \text{ rad/s}^2.$$

(b)

$$T = m(g - R\alpha) = (5 \text{ kg})(9.81 \text{ m/s}^2 - [0.1 \text{ m}][14.7 \text{ rad/s}^2])$$

$$T = 41.7 \text{ N.}$$





**Problem 18.37** The radius of the pulley is  $R = 100$  mm and its moment of inertia is  $I = 0.1$  kg-m<sup>2</sup>. The mass  $m = 5$  kg. The spring constant is  $k = 135$  N/m. The system is released from rest with the spring unstretched. What maximum distance does the mass fall before rebounding?

**Strategy:** Assume that the mass has fallen an arbitrary distance  $x$ . Write the equations of motion of the mass and the pulley and use them to determine the acceleration  $a$  of the mass as a function of  $x$ . Then apply the chain rule:

$$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} v.$$

**Solution:** The force in the spring is  $kx$ . There are five unknowns ( $O_x, O_y, T, a, \alpha$ ), four dynamic equations, and one constraint equation.

$$\Sigma M_O : (kx)R - TR = -I\alpha,$$

$$\Sigma F_y : T - mg = -ma,$$

$$a = R\alpha$$

Solving we find

$$a = \frac{R^2(mg - kx)}{I + mR^2} = v \frac{dv}{dx}$$

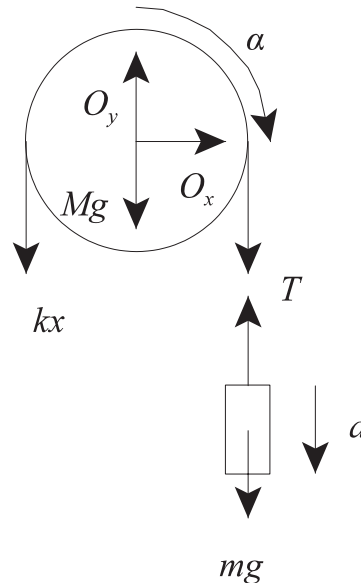
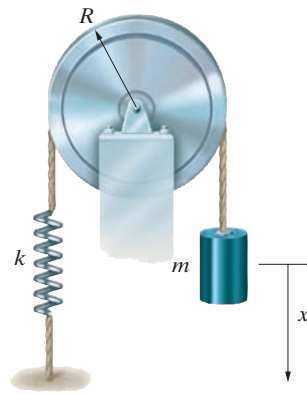
$$\int_0^0 v dv = \int_0^x \frac{R^2(mg - kx)}{I + mR^2} dx = \frac{R^2}{I + mR^2} \int_0^x (mg - kx) dx = 0$$

Thus

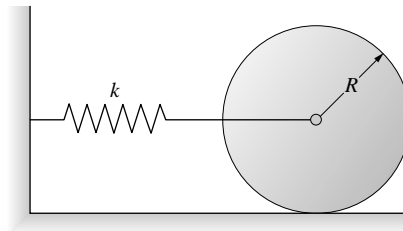
$$mgx - \frac{1}{2}kx^2 = 0 \Rightarrow x = 0 \text{ or } x = \frac{2mg}{k}$$

The maximum distance is

$$x = \frac{2mg}{k} = \frac{2(5 \text{ kg})(9.81 \text{ m/s}^2)}{135 \text{ N/m}} = 0.727 \text{ m} \quad \boxed{x = 0.727 \text{ m.}}$$



**Problem 18.38** The mass of the disk is 45 kg and its radius is  $R = 0.3$  m. The spring constant is  $k = 60$  N/m. The disk is rolled to the left until the spring is compressed 0.5 m and released from rest.



- (a) If you assume that the disk rolls, what is its angular acceleration at the instant it is released?  
 (b) What is the minimum coefficient of static friction for which the disk will not slip when it is released?

**Solution:**

$$x_0 = -0.5$$

$$k = 600 \text{ N/m}$$

$$m = 45 \text{ kg}$$

$$R = 0.3 \text{ m}$$

$$I_0 = \frac{1}{2}mR^2 = 2.025 \text{ N}\cdot\text{m}^2, F_s = kx$$

$$\sum F_x: -F_s - f = ma_{0x}$$

$$\sum F_y: N - mg = 0$$

$$\downarrow \sum M_0: -fR = I_0\alpha$$

Rolling implies  $a_{0x} = -R\alpha$

We have, at  $x = -0.5$  m

$$-kx - f = ma_{0x}$$

$$N - mg = 0$$

$$-Rf = I_0\alpha$$

$$a_{0x} = -R\alpha$$

Four eqns, four unknowns ( $a_{0x}$ ,  $\alpha$ ,  $N$ ,  $f$ )

- (a) Solving  $f = 100$  N,  $N = 441.5$  N

$$\alpha = -14.81 \text{ rad/s}^2 \text{ (clockwise)}$$

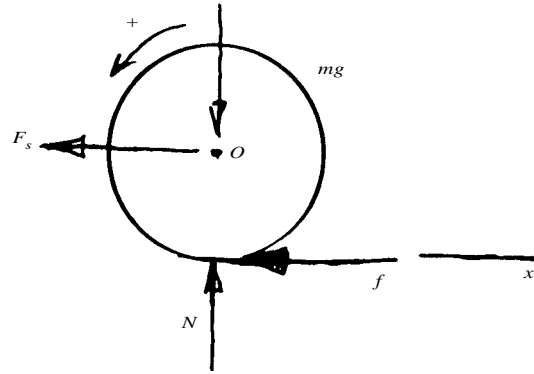
$$a_{0x} = 4.44 \text{ m/s}^2$$

- (b) for impending slip,

$$f = \mu_s N$$

$$\mu_s = f/N = 100/441.5$$

$$\mu_s = 0.227$$



**Problem 18.39** The disk weighs 12 N and its radius is 6 cm. It is stationary on the surface when the force  $F = 10\text{ N}$  is applied.

- If the disk rolls on the surface, what is the acceleration of its center?
- What minimum coefficient of static friction is necessary for the disk to roll instead of slipping when the force is applied?

**Solution:** There are five unknowns ( $N, f, a, \alpha, \mu_s$ ), three dynamic equations, one constraint equation, and one friction equation.

$$\Sigma F_x : F - f = ma,$$

$$\Sigma F_y : N - mg = 0,$$

$$\Sigma M_G : -fr = -\left(\frac{1}{2}mr^2\right)\alpha,$$

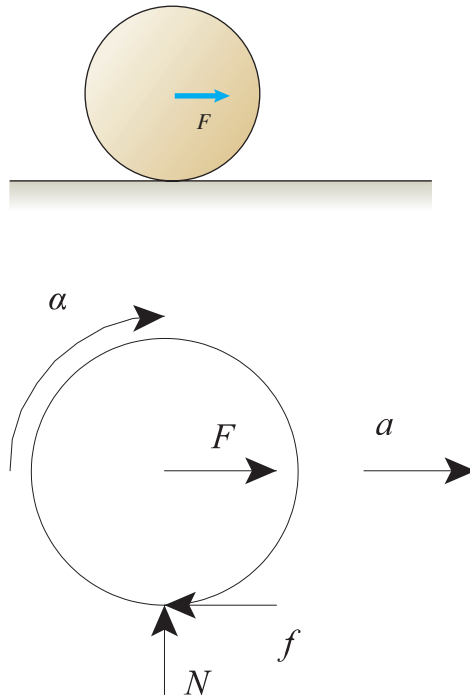
$$a = r\alpha,$$

$$f = \mu_s N.$$

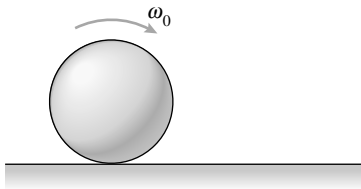
Solving, we find

$$(a) \quad a = \frac{2F}{3m} = \frac{2(10\text{ N})}{3\left(\frac{12\text{ N}}{9.81\text{ m/s}^2}\right)} = 5.45\text{ m/s}^2. \quad \boxed{a = 5.45\text{ m/s}^2.}$$

$$(b) \quad \mu_s = \frac{F}{3mg} = \frac{(10\text{ N})}{3(12\text{ N})} = 0.278 \quad \boxed{\mu_s = 0.278.}$$



**Problem 18.40** A 186.8 N sphere with radius  $R=101.6$  mm is placed on a horizontal surface with initial angular velocity  $\omega_0 = 40$  rad/s. The coefficient of kinetic friction between the sphere and the surface is  $\mu_k = 0.06$ . What maximum velocity will the center of the sphere attain, and how long does it take to reach that velocity?



**Strategy:** The friction force exerted on the spinning sphere by the surface will cause the sphere to accelerate to the right. The friction force will also cause the sphere's angular velocity to decrease. The center of the sphere will accelerate until the sphere is rolling on the surface instead of slipping relative to it. Use the relation between the velocity of the center and the angular velocity of the sphere when it is rolling to determine when the sphere begins rolling.

**Solution:** Given

$$W = 186.8 \text{ N}, g = 9.81 \text{ m/s}^2, m = W/g, R = 0.102 \text{ m}, \mu_k = 0.06$$

We have

$$\sum F_x : \mu_k N = ma$$

$$\sum F_y : N - mg = 0$$

$$\sum M_G : \mu_k N R = \frac{2}{5} m R^2 a$$

Solving we find

$$\alpha = \frac{5\mu_k g}{2R} = 14.49 \text{ rad/s}^2, a = \mu_k g = 0.59 \text{ m/s}^2$$

From kinematics we learn that

$$\alpha = 14.49 \text{ rad/s}^2, \omega = (14.49 \text{ rad/s}^2)t - (40 \text{ rad/s})$$

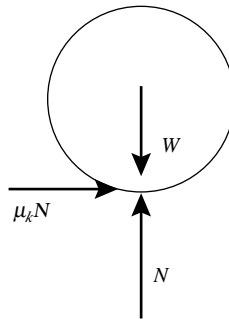
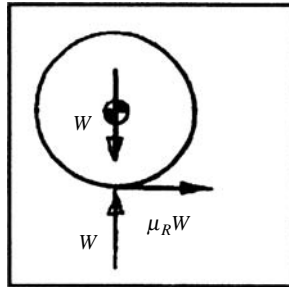
$$a = 0.59 \text{ m/s}^2, v = (0.59 \text{ m/s}^2)t$$

when we reach a steady motion we have

$$v = -R\omega \Rightarrow (0.59 \text{ m/s}^2)t = -(0.102 \text{ m})[(14.49 \text{ rad/s}^2)t - (40 \text{ rad/s})]$$

Solving for the time we find

$$t = 1.97 \text{ s} \Rightarrow v = 1.16 \text{ m/s}$$



**Problem 18.41** A soccer player kicks the ball to a teammate 8 m away. The ball leaves the player's foot moving parallel to the ground at 6 m/s with no angular velocity. The coefficient of kinetic friction between the ball and the grass is  $\mu_k = 0.32$ . How long does it take the ball to reach his teammate? The radius of the ball is 112 mm and its mass is 0.4 kg. Estimate the ball's moment of inertia by using the equation for a thin spherical shell:  $I = \frac{2}{3}mR^2$ .



**Solution:** Given  $\mu = 0.32$ ,  $r = 0.112$  m,  $g = 9.81$  m/s<sup>2</sup>,  $v_0 = 6$  m/s

The motion occurs in two phases.

(a) Slipping.

$$\sum F_x : -\mu N = ma$$

$$\sum F_y : N - mg = 0$$

$$\sum M_G : -\mu N R = \frac{2}{3}mR^2\alpha$$

Solving we find

$$a = -\mu g \Rightarrow v = v_0 - \mu g t, s = v_0 t - \frac{1}{2}\mu g t^2$$

$$\alpha = -\frac{3\mu g}{2R} \Rightarrow \omega = -\frac{3\mu g}{2R} t$$

When it stops slipping we have

$$v = -R\omega \Rightarrow v_0 - \mu g t = \frac{3}{2}\mu g t \Rightarrow t = \frac{2v_0}{5\mu g} = 0.765 \text{ s}$$

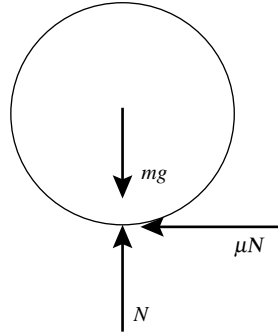
$$v = 3.6 \text{ m/s}, s = 3.67 \text{ m}$$

(b) Rolling—Steady motion

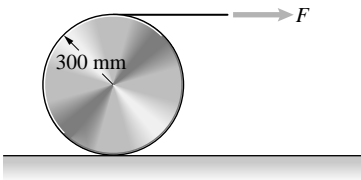
$$a = 0, v = 3.6 \text{ m/s}, s = (3.6 \text{ m/s})(t - 0.765 \text{ s}) + 3.67 \text{ m}$$

When it reaches the teammate we have

$$8 \text{ m} = (3.6 \text{ m/s})(t - 0.765 \text{ s}) + 3.67 \text{ m} \Rightarrow \boxed{t = 1.97 \text{ s}}$$



**Problem 18.42** The 100-kg cylindrical disk is at rest when the force  $F$  is applied to a cord wrapped around it. The static and kinetic coefficients of friction between the disk and the surface equal 0.2. Determine the angular acceleration of the disk if (a)  $F = 500$  N and (b)  $F = 1000$  N.



**Strategy:** First solve the problem by assuming that the disk does not slip, but rolls on the surface. Determine the friction force, and find out whether it exceeds the product of the coefficient of friction and the normal force. If it does, you must rework the problem assuming that the disk slips.

**Solution:** Choose a coordinate system with the origin at the center of the disk in the at rest position, with the  $x$  axis parallel to the plane surface. The moment about the center of mass is  $M = -RF - Rf$ , from which  $-RF - Rf = I\alpha$ . From which

$$f = \frac{-RF - I\alpha}{R} = -F - \frac{I\alpha}{R}.$$

From Newton's second law:  $F - f = ma_x$ , where  $a_x$  is the acceleration of the center of mass. Assume that the disk rolls. At the point of contact  $\mathbf{a}_P = 0$ ; from which  $0 = \mathbf{a}_G + \boldsymbol{\alpha} \times \mathbf{r}_{P/G} - \omega^2 \mathbf{r}_{P/G}$ .

$$\mathbf{a}_G = a_x \mathbf{i} = \boldsymbol{\alpha} \times R\mathbf{j} - \omega^2 R\mathbf{j}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ 0 & R & 0 \end{bmatrix} - \omega^2 R\mathbf{j} = -R\alpha\mathbf{i} - \omega^2 R\mathbf{j},$$

from which  $a_y = 0$  and  $a_x = -R\alpha$ . Substitute for  $f$  and solve:

$$a_x = \frac{2F}{\left(m + \frac{I}{R^2}\right)}.$$

For a disk, the moment of inertia about the polar axis is  $I = \frac{1}{2}mR^2$ , from which

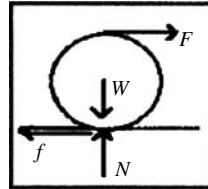
$$a_x = \frac{4F}{3m} = \frac{2000}{300} = 6.67 \text{ m/s}^2.$$

(a) For  $F = 500$  N, the friction force is

$$f = F - ma_x = -\frac{F}{3} = -\frac{500}{3} = -167 \text{ N}.$$

*Note:*  $-\mu_k W = -0.2 \text{ mg} = -196.2 \text{ N}$ , the disk does not slip. The angular velocity is

$$\alpha = -\frac{a_x}{R} = -\frac{6.67}{0.3} = -22.22 \text{ rad/s}^2.$$



(b) For  $F = 1000$  N the acceleration is

$$a_x = \frac{4F}{3m} = \frac{4000}{300} = 13.33 \text{ m/s}^2.$$

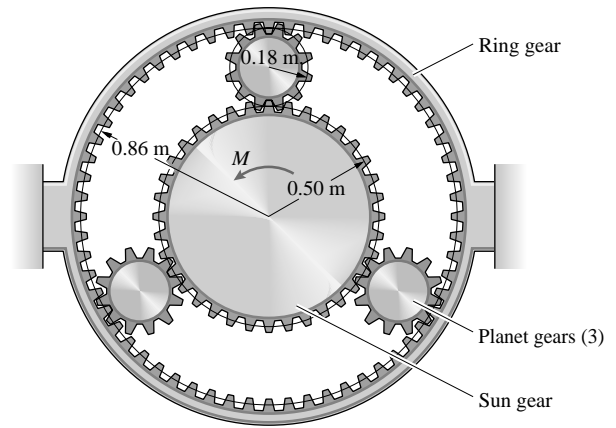
The friction force is

$$f = F - ma_x = 1000 - 1333.3 = -333.3 \text{ N}.$$

*The drum slips.* The moment equation for slip is  $-RF + R\mu_k gm = I\alpha$ , from which

$$\alpha = \frac{-RF + R\mu_k gm}{I} = -\frac{2F}{mR} + \frac{2\mu_k g}{R} = -53.6 \text{ rad/s}^2.$$

**Problem 18.43** The ring gear is fixed. The mass and moment of inertia of the sun gear are  $m_S = 320$  kg and  $I_S = 40$  kg-m<sup>2</sup>. The mass and moment of inertia of each planet gear are  $m_P = 38$  kg and  $I_P = 0.60$  kg-m<sup>2</sup>. If a couple  $M = 200$  N-m is applied to the sun gear, what is the latter's angular acceleration?



**Solution:**

$$M_S = 200 \text{ N-m}$$

$$\text{Sun Gear: } \sum M_O: M_S - 3RF = I_S \alpha_S$$

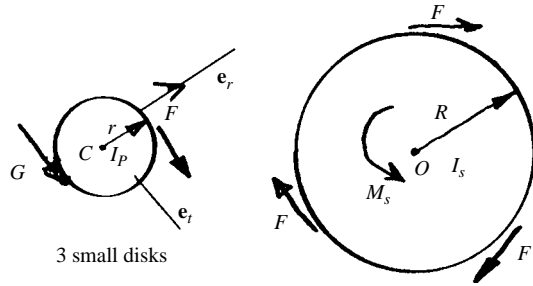
$$\text{Planet Gears: } \sum M_C: Gr - Fr = I_P \alpha_P$$

$$\sum F_t: F + G = m_P a_{ct}$$

From kinematics  $a_{ct} = -r\alpha_P$

$$2\alpha_P r_P = -R\alpha_S$$

We have 5 eqns in 5 unknowns. Solving,  $\alpha_S = 3.95 \text{ rad/s}^2$  (counter-clockwise)



**Problem 18.44** In Problem 18.43, what is the magnitude of the tangential force exerted on the sun gear by each planet gear at their point of contact when the 200 N-m couple is applied to the sun gear?

**Solution:** See the solution to Problem 18.43. Solving the 5 eqns in 5 unknowns yields

$$\alpha_S = 3.95 \text{ rad/s}^2,$$

$$G = 9.63 \text{ N},$$

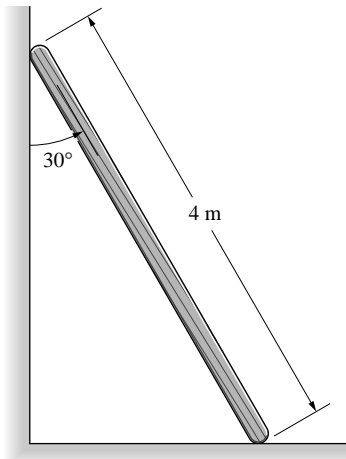
$$a_{Gt} = 0.988 \text{ m/s}^2,$$

$$\alpha_P = -5.49 \text{ rad/s}^2,$$

$$F = 27.9 \text{ N}.$$

$F$  is the required value.

**Problem 18.45** The 18-kg ladder is released from rest in the position shown. Model it as a slender bar and neglect friction. At the instant of release, determine (a) the angular acceleration of the ladder and (b) the normal force exerted on the ladder by the floor. (See Active Example 18.3.)



**Solution:** The vector location of the center of mass is  $\mathbf{r}_G = (L/2) \sin 30^\circ \mathbf{i} + (L/2) \cos 30^\circ \mathbf{j} = 1\mathbf{i} + 1.732\mathbf{j}$  (m). Denote the normal forces at the top and bottom of the ladder by  $P$  and  $N$ . The vector locations of  $A$  and  $B$  are  $\mathbf{r}_A = L \sin 30^\circ \mathbf{i} = 2\mathbf{i}$  (m),  $\mathbf{r}_B = L \cos 30^\circ \mathbf{j} = 3.46\mathbf{j}$  (m). The vectors  $\mathbf{r}_{A/G} = \mathbf{r}_A - \mathbf{r}_G = 1\mathbf{i} - 1.732\mathbf{j}$  (m),  $\mathbf{r}_{B/G} = \mathbf{r}_B - \mathbf{r}_G = -1\mathbf{i} + 1.732\mathbf{j}$  (m). The moment about the center of mass is

$$\mathbf{M} = \mathbf{r}_{B/G} \times \mathbf{P} + \mathbf{r}_{A/G} \times \mathbf{N},$$

$$\mathbf{M} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1.732 & 0 \\ P & 0 & 0 \end{bmatrix} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1.732 & 0 \\ 0 & N & 0 \end{bmatrix}$$

$$= (-1.732P + N)\mathbf{k} \text{ (N-m)}.$$

From the equation of angular motion: (1)  $-1.732 P + N = I\alpha$ . From Newton's second law: (2)  $P = ma_x$ , (3)  $N - mg = ma_y$ , where  $a_x$ ,  $a_y$  are the accelerations of the center of mass.

From kinematics:  $\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$ . The angular velocity is zero since the system was released from rest,

$$\mathbf{a}_G = a_A \mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ -1 & 1.732 & 0 \end{bmatrix} = a_A \mathbf{i} - 1.732\alpha \mathbf{i} - \alpha \mathbf{j}$$

$$= (a_A - 1.732\alpha)\mathbf{i} - \alpha \mathbf{j} \text{ (m/s}^2\text{)},$$

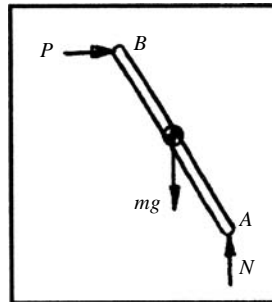
from which  $a_y = -\alpha$ .

Similarly,

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{G/B}, \mathbf{a}_G = \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ 1 & -1.732 & 0 \end{bmatrix}$$

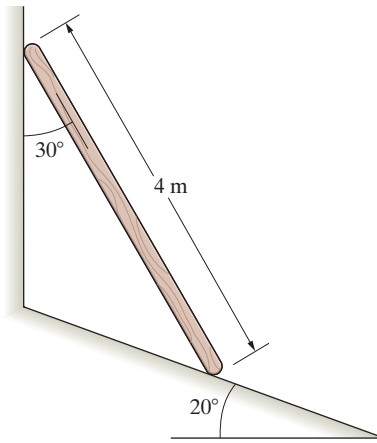
$$= a_B \mathbf{j} + 1.732\alpha \mathbf{i} + \alpha \mathbf{j},$$

from which  $a_x = 1.732\alpha$ . Substitute into (1), (2) and (3) to obtain three equations in three unknowns:  $-1.732P + N = I\alpha$ ,  $P = m(1.732)\alpha$ ,  $N - mg = -m\alpha$ . Solve: (a)  $\alpha = 1.84 \text{ rad/s}^2$ ,  $P = 57.3 \text{ N}$ , (b)  $N = 143.47 \text{ N}$





**Problem 18.46** The 18-kg ladder is released from rest in the position shown. Model it as a slender bar and neglect friction. Determine its angular acceleration at the instant of release.



**Solution:** Given  $m = 18 \text{ kg}$ ,  $L = 4 \text{ m}$ ,  $g = 9.81 \text{ m/s}^2$ ,  $\omega = 0$

First find the kinematic constraints. We have

$$\mathbf{a}_A = \mathbf{a}_G + \boldsymbol{\alpha} \times \mathbf{r}_{A/G}$$

$$= a_x \mathbf{i} + a_y \mathbf{j} + \alpha \mathbf{k} \times \left( \left[ -\frac{L}{2} \sin 30^\circ \right] \mathbf{i} + \left[ \frac{L}{2} \cos 30^\circ \right] \mathbf{j} \right)$$

$$= \left( a_x - \alpha \frac{L}{2} \cos 30^\circ \right) \mathbf{i} + \left( a_y - \alpha \frac{L}{2} \sin 30^\circ \right) \mathbf{j}$$

$$\mathbf{a}_B = \mathbf{a}_G + \boldsymbol{\alpha} \times \mathbf{r}_{B/G}$$

$$= a_x \mathbf{i} + a_y \mathbf{j} + \alpha \mathbf{k} \times \left( \left[ \frac{L}{2} \sin 30^\circ \right] \mathbf{i} + \left[ -\frac{L}{2} \cos 30^\circ \right] \mathbf{j} \right)$$

$$= \left( a_x + \alpha \frac{L}{2} \cos 30^\circ \right) \mathbf{i} + \left( a_y + \alpha \frac{L}{2} \sin 30^\circ \right) \mathbf{j}$$

The constraints are

$$\mathbf{a}_A \cdot \mathbf{i} = a_x - \alpha \frac{L}{2} \cos 30^\circ = 0$$

$$\mathbf{a}_B \cdot (\sin 20^\circ \mathbf{i} + \cos 20^\circ \mathbf{j})$$

$$= \left( a_x + \alpha \frac{L}{2} \cos 30^\circ \right) \sin 20^\circ + \left( a_y + \alpha \frac{L}{2} \sin 30^\circ \right) \cos 20^\circ$$

The dynamic equations:

$$\sum F_x : N_A + N_B \sin 20^\circ = ma_x$$

$$\sum F_y : N_B \cos 20^\circ - mg = ma_y$$

$$\sum M_G : -N_A \left( \frac{L}{2} \cos 30^\circ \right) + N_B \cos 20^\circ \left( \frac{L}{2} \sin 30^\circ \right)$$

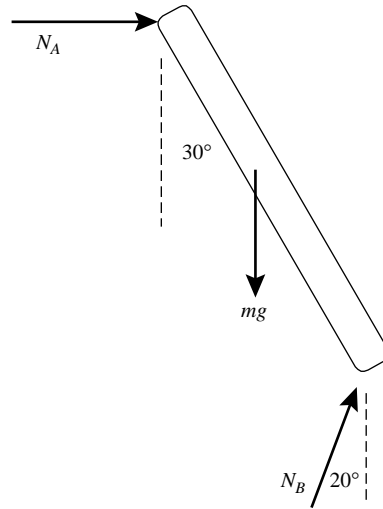
$$+ N_B \sin 20^\circ \left( \frac{L}{2} \cos 30^\circ \right) = \frac{1}{12} mL^2 \alpha$$

Solving five equations in five unknowns we have

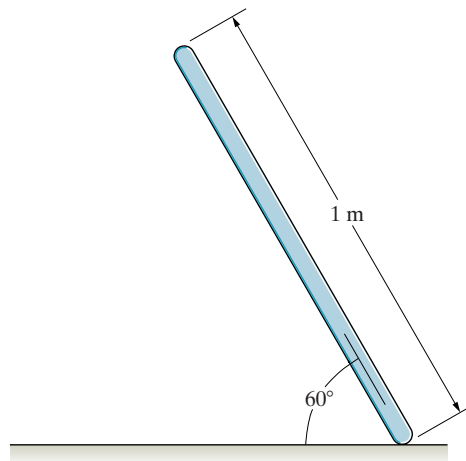
$$\boxed{\alpha = 2.35 \text{ rad/s}^2 \text{ CCW}}$$

Also

$$a_x = 4.07 \text{ ft/s}^2, a_y = -5.31 \text{ ft/s}^2, N_A = 43.7 \text{ N}, N_B = 86.2 \text{ N}$$



**Problem 18.47** The 4-kg slender bar is released from rest in the position shown. Determine its angular acceleration at that instant if (a) the surface is rough and the bar does not slip, and (b) the surface is smooth.



**Solution:**

- (a) The surface is rough. The lower end of the bar is fixed, and the bar rotates around that point.

$$\Sigma M_B : mg \frac{L}{2} \cos \theta = \frac{1}{3} mL^2 \alpha$$

$$\alpha = \frac{3g}{2L} \cos \theta = \frac{3(9.81 \text{ m/s}^2)}{2(1 \text{ m})} \cos 60^\circ$$

$$\alpha = 7.36 \text{ rad/s}^2.$$

- (b) The surface is smooth. There are four unknowns ( $N$ ,  $a_x$ ,  $a_y$ ,  $\alpha$ ), three dynamic equations, and one constraint equation (the  $y$  component of the acceleration of the point in contact with the ground is zero).

$$\Sigma F_x : 0 = ma_x,$$

$$\Sigma F_y : N - mg = ma_y,$$

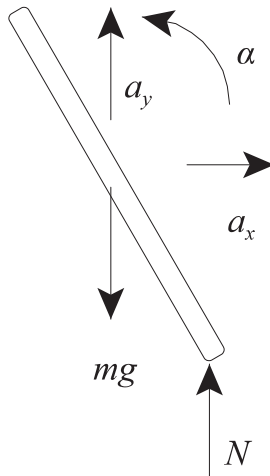
$$\Sigma M_G : N \frac{L}{2} \cos \theta = \frac{1}{12} mL^2 \alpha$$

$$a_y + \alpha \frac{L}{2} \cos \theta = 0$$

Solving, we find

$$\alpha = \frac{6g \cos \theta}{L(1 + 3 \cos^2 \theta)} = \frac{6(9.81 \text{ m/s}^2) \cos 60^\circ}{(1 \text{ m})(1 + 3 \cos^2 60^\circ)} = 16.8 \text{ rad/s}^2.$$

$$\alpha = 16.8 \text{ rad/s}^2.$$



**Problem 18.48** The masses of the bar and disk are 14 kg and 9 kg, respectively. The system is released from rest with the bar horizontal. Determine the bar's angular acceleration at that instant if

- the bar and disk are welded together at A,
- the bar and disk are connected by a smooth pin at A.

**Strategy:** In part (b), draw individual free-body diagrams of the bar and disk.

**Solution:**

(a)  $L = 1.2 \text{ m}$     $R = 0.3 \text{ m}$

$m_B = 14 \text{ kg}$     $m_D = 9 \text{ kg}$

$O$  is a fixed point

For the bar

$$I_G = \frac{1}{12} m_B L^2 = \frac{1}{12} (14)(1.2)^2 = 1.68 \text{ N}\cdot\text{m}^2$$

$$I_{OB} = I_G + m_B \left(\frac{L}{2}\right)^2$$

$$I_{OB} = 6.72 \text{ N}\cdot\text{m}^2$$

For the disk:

$$I_A = \frac{1}{2} m_D R^2 = \frac{1}{2} (9)(0.3)^2 = 0.405 \text{ N}\cdot\text{m}^2$$

$$I_{OD} = I_A + m_D L^2 = 13.37 \text{ N}\cdot\text{m}^2$$

The total moment of inertia of the welded disk and bar about  $O$  is

$$I_T = I_{OB} + I_{OD} = 20.09 \text{ N}\cdot\text{m}^2$$

$$\sum F_x: O_x = O = m a_{Gx}$$

$$\sum F_y: O_y - m_B g - m_D g = (m_B + m_D) a_{Gy}$$

$$\sum M_O: -\left(\frac{L}{2}\right) m_B g - L m_D g = I_T \alpha$$

We can solve the last equation for  $\alpha$  without finding the location and acceleration of the center of mass,  $G$ . Solving,

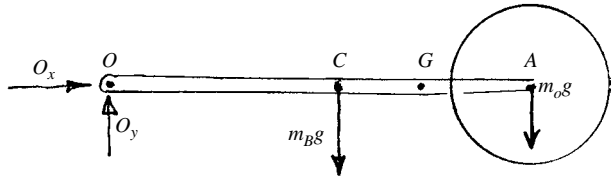
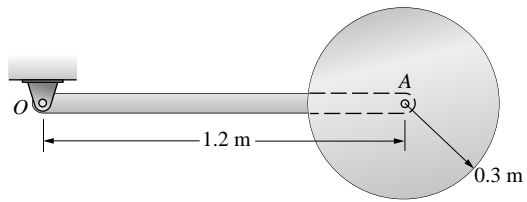
$$\alpha = -9.38 \text{ rad/s}^2 \quad (\text{clockwise})$$

- (b) In this case, only the moment of inertia changes. Since the disk is on a smooth pin, it does not rotate. It acts only as a point mass at a distance  $L$  from point  $O$ .

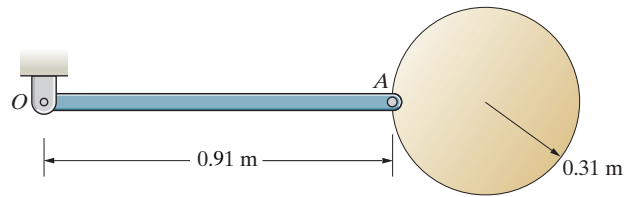
In this case,  $I'_{OD} = m_D L^2$  and  $I'_T = I_{OB} + I'_{OD} = 19.68 \text{ N}\cdot\text{m}^2$   
We now have

$$\sum M_O: -\left(\frac{L}{2}\right) m_B g - L m_D g = I'_T \alpha'$$

$$\text{Solving } \alpha' = -9.57 \text{ rad/s}^2 \quad (\text{clockwise})$$



**Problem 18.49** The 22.2 N horizontal bar is connected to the 44.5 N disk by a smooth pin at A. The system is released from rest in the position shown. What are the angular accelerations of the bar and disk at that instant?



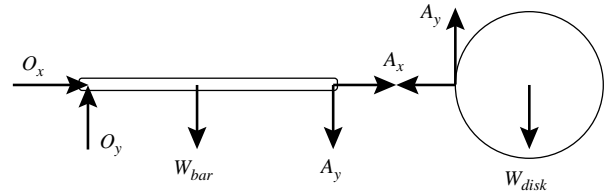
**Solution:** Given

$$g = 9.81 \text{ m/s}^2, W_{bar} = 22.2 \text{ N}, W_{disk} = 44.5 \text{ N},$$

$$m_{bar} = \frac{W_{bar}}{g}, m_{disk} = \frac{W_{disk}}{g}$$

$$L = 0.91 \text{ m}, R = 0.31 \text{ m}$$

The FBDs



The dynamic equations

$$\sum M_O : -m_{bar}g \frac{L}{2} - A_y L = \frac{1}{3} m_{bar} L^2 \alpha_{bar}$$

$$\sum M_{G_{disk}} : -A_y R = \frac{1}{2} m_{disk} R^2 \alpha_{disk}$$

$$\sum F_y : A_y - m_{disk}g = m_{disk} a_{y_{disk}}$$

Kinematic constraint

$$\alpha_{bar} L = a_{y_{disk}} - \alpha_{disk} R$$

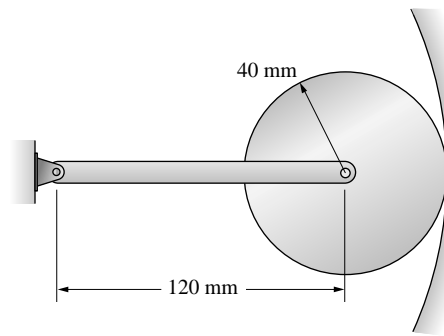
Solving we find

$$\alpha_{disk} = 3.58 \text{ rad/s}^2, \alpha_{bar} = -12.5 \text{ rad/s}^2, a_{y_{disk}} = -34.0 \text{ m/s}^2,$$

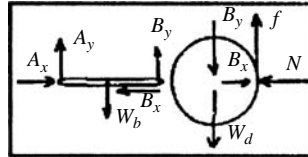
$$A_y = -0.556 \text{ N}$$

$$\text{Thus } \alpha_{disk} = 3.58 \text{ rad/s}^2 \text{ CCW}, \alpha_{bar} = 12.5 \text{ rad/s}^2 \text{ CW}$$

**Problem 18.50** The 0.1-kg slender bar and 0.2-kg cylindrical disk are released from rest with the bar horizontal. The disk rolls on the curved surface. What is the bar's angular acceleration at the instant it is released?



**Solution:** The moment about the center of mass of the disk is  $M = fR$ , from the equation of angular motion,  $Rf = I_d\alpha_d$ . From Newton's second law:  $f - B_y - W_d = m_d a_{dy}$ . Since the disk rolls, the kinematic condition is  $a_{dy} = -R\alpha_d$ . Combine the expressions and rearrange:  $f = I_d\alpha_d/R$ ,  $I_d\alpha_d/R - B_y - W_d = m_d a_{dy}$ , from which  $B_y + W_d = (Rm_d + I_d/R)\alpha_d$ . The moment about the center of mass of the bar is



$$M_b = -\left(\frac{L}{2}\right)A_y + \left(\frac{L}{2}\right)B_y,$$

from which

$$-\left(\frac{L}{2}\right)A_y + \left(\frac{L}{2}\right)B_y = I_b\alpha_b.$$

From Newton's second law  $A_y - W_b + B_y = m_b a_{by}$ , where  $a_{by}$  is the acceleration of the center of mass of the bar. The kinematic condition for the bar is

$$\mathbf{a}_{CM} = \boldsymbol{\alpha}_b \times \left(\left(\frac{L}{2}\right)\mathbf{i}\right) = \left(\frac{L}{2}\right)\alpha_b\mathbf{j},$$

from which

$$a_{by} = \left(\frac{L}{2}\right)\alpha_b.$$

Similarly,  $\mathbf{a}_D = \mathbf{a}_{CM} + \boldsymbol{\alpha}_b \times ((L/2)\mathbf{i})$ , from which  $a_{dy} = L\alpha_b$ .

From which:  $\alpha_d = -L\alpha_b/R$ . Substitute to obtain three equations in three unknowns:

$$B_y + W_d = \left(Rm_d + \frac{I_d}{R}\right)\left(-\frac{L}{R}\right)\alpha_b,$$

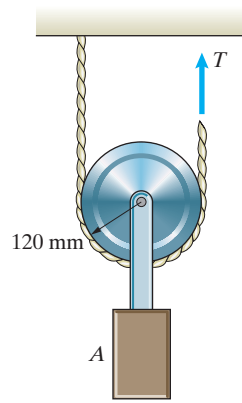
$$-\left(\frac{L}{2}\right)A_y + \left(\frac{L}{2}\right)B_y = I_b\alpha_b,$$

$$A_y - W_b + B_y = m_b\left(\frac{L}{2}\right)\alpha_b.$$

Substitute known numerical values:  $L = 0.12$  m,  $R = 0.04$  m,  $m_b = 0.1$  kg,  $W_b = m_b g = 0.981$  N,  $m_d = 0.2$  kg,  $W_d = m_d g = 1.962$  N,  $I_b = (1/12)m_b(L^2) = 1.2 \times 10^{-4}$  kg-m<sup>2</sup>,  $I_d = (1/2)m_d R^2 = 1.6 \times 10^{-4}$  kg-m<sup>2</sup>. Solve:

$$\alpha_b = -61.3 \text{ rad/s}^2, A_y = 0.368 \text{ N}, B_y = 0.245 \text{ N}.$$

**Problem 18.51** The mass of the suspended object  $A$  is 8 kg. The mass of the pulley is 5 kg, and its moment of inertia is  $0.036 \text{ kg}\cdot\text{m}^2$ . If the force  $T = 70 \text{ N}$ , what is the magnitude of the acceleration of  $A$ ?



**Solution:** Given

$$m_A = 8 \text{ kg}, m_B = 5 \text{ kg}, I_B = 0.036 \text{ kg}\cdot\text{m}^2$$

$$R = 0.12 \text{ m}, g = 9.81 \text{ m/s}^2, T = 70 \text{ N}$$

The FBDs

The dynamic equations

$$\sum F_{yB} : T_2 + T - m_B g - B_y = m_B a_{By}$$

$$\sum F_{yA} : B_y - m_A g = m_A a_{Ay}$$

$$\sum M_B : -T_2 R + T R = I_B \alpha_B$$

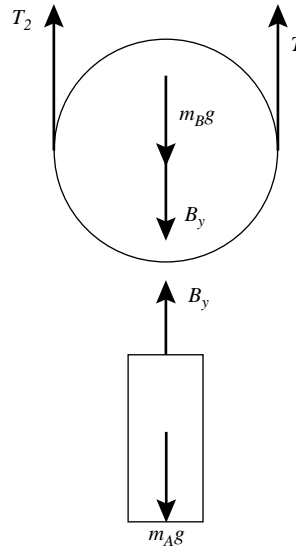
Kinematic constraints

$$a_{By} = a_{Ay}, a_{By} = R \alpha_B$$

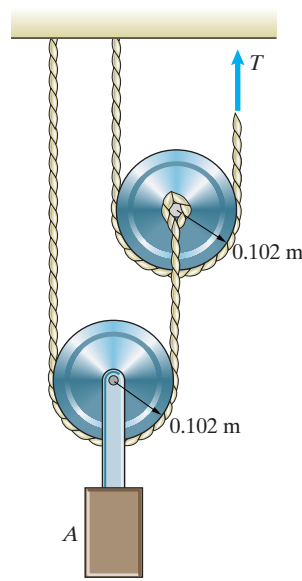
Solving we find  $a_{Ay} = 0.805 \text{ m/s}^2$

We also have

$$a_{By} = 0.805 \text{ m/s}^2, \alpha_B = 6.70 \text{ rad/s}, T_2 = 68.0 \text{ N}, B_y = 84.9 \text{ N}$$



**Problem 18.52** The suspended object  $A$  weighs  $89\text{ N}$ . The pulleys are identical, each weighing  $44.5\text{ N}$  and having moment of inertia  $0.03\text{ kg}\cdot\text{m}^2$ . If the force  $T = 66.7\text{ N}$ , what is the magnitude of the acceleration of  $A$ ?



**Solution:** Given

$$g = 9.81\text{ m/s}^2, W_A = 89\text{ N}, W_{disk} = 44.5\text{ N}, I = 0.03\text{ kg}\cdot\text{m}^2$$

$$m_A = \frac{W_A}{g}, m_{disk} = \frac{W_{disk}}{g}, R = 0.102\text{ m}, T = 66.7\text{ N}$$

The FBDs

The dynamic equations

$$\sum F_{y1} : T_2 + T - T_1 - m_{disk}g = m_{disk}a_1$$

$$\sum F_{y2} : T_4 + T_1 - T_3 - m_{disk}g = m_{disk}a_2$$

$$\sum F_{y3} : T_3 - m_Ag = m_Aa_A$$

$$\sum M_1 : TR - T_2R = I\alpha_1$$

$$\sum M_2 : T_1R - T_4R = I\alpha_2$$

The kinematic constraints

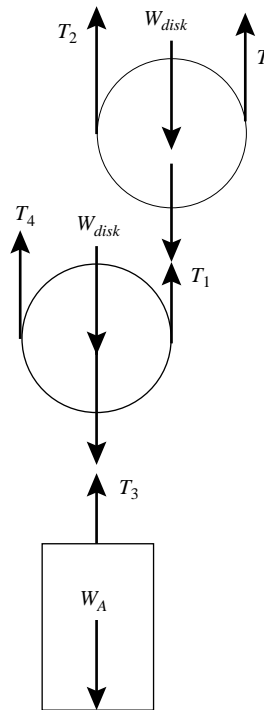
$$a_1 = R\alpha_1, a_2 = R\alpha_2, a_1 = 2R\alpha_2, a_A = a_2$$

Solving we find  $a_A = 0.96\text{ m/s}^2$

We also have

$$a_1 = 1.93\text{ m/s}^2, a_2 = 0.96\text{ m/s}^2, \alpha_1 = 19.0\text{ rad/s}^2, \alpha_2 = 9.48\text{ rad/s}^2$$

$$T_1 = 74.7\text{ N}, T_2 = 60.9\text{ N}, T_3 = 97.9\text{ N}, T_4 = 72.1\text{ N}$$



**Problem 18.53** The 2-kg slender bar and 5-kg block are released from rest in the position shown. If friction is negligible, what is the block's acceleration at that instant? (See Example 18.5.)

**Solution:**  $L = 1 \text{ m}$ ,  $m = 2 \text{ kg}$   $M = 5 \text{ kg}$

Assume directions for  $B_x$ ,  $B_y$ ,  $I_G = \frac{1}{12}m_B L^2$

$$\sum F_x: B_x = ma_{G_x} \quad (1)$$

$$\sum F_y: B_y - mg = m_B a_{G_y} \quad (2)$$

$$\sum M_G: \left(\frac{L}{2} \cos \theta\right) B_y + \left(\frac{L}{2} \sin \theta\right) B_x = I_G \alpha \quad (3)$$

$$\sum F_x: -B_x = M a_{0x} \quad (4)$$

$$\sum F_y: N - B_y - Mg = 0 \quad (5)$$

From kinematics,  $\omega = 0$  (initially)

$$\mathbf{a}_0 = \mathbf{a}_G + \alpha \mathbf{k} \times \mathbf{r}_{0/G}$$

where  $\mathbf{r}_{0/G} = \frac{L}{2} \cos \theta \mathbf{i} - \frac{L}{2} \sin \theta \mathbf{j}$

From the diagram  $\mathbf{a}_0 = a_{0x} \mathbf{i}$

$$\begin{cases} a_{0x} = a_{G_x} + (\alpha L/2) \sin \theta & (6) \\ 0 = a_{G_y} + (\alpha L/2) \cos \theta & (7) \end{cases}$$

We know  $\theta = 55^\circ$ ,  $I_G = 0.167 \text{ kg}\cdot\text{m}^2$ ,  $L = 1 \text{ m}$ ,  $m = 2 \text{ kg}$ ,  $M = 5 \text{ kg}$ . We have 7 eqns in 7 unknowns

$(a_{G_x}, a_{G_y}, a_{0x}, \alpha, B_x, B_y, N)$ ,

Solving, we get

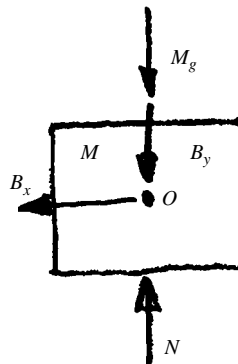
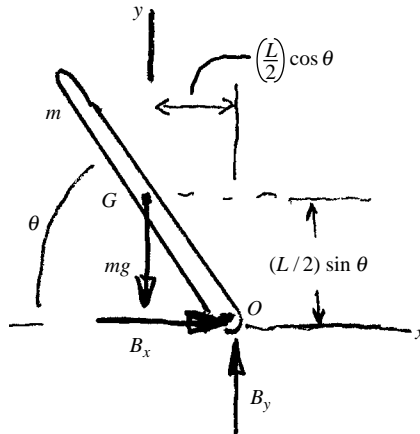
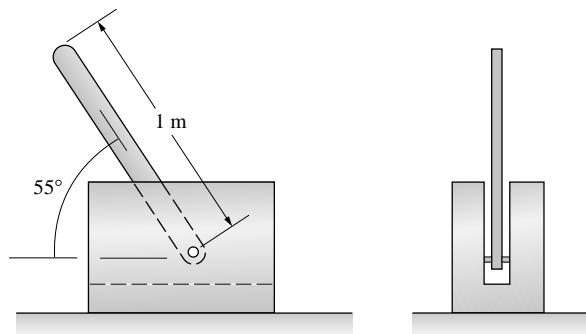
$$B_x = -5.77 \text{ N, (opposite the assumed direction)}$$

$$B_y = 13.97 \text{ N,}$$

$$a_{G_x} = -2.88 \text{ m/s}^2, \quad a_{G_y} = -2.83 \text{ m/s}^2$$

$$\alpha = 9.86 \text{ rad/s}^2, \quad N = 63.0 \text{ N}$$

$$\underline{a_{0x} = 1.15 \text{ m/s}^2. \text{ (to the right)}}$$





**Problem 18.54** The 2-kg slender bar and 5-kg block are released from rest in the position shown. What minimum coefficient of static friction between the block and the horizontal surface would be necessary for the block not to move when the system is released? (See Example 18.5.)

**Solution:** This solution is very similar to that of Problem 18.53. We add a friction force  $f = \mu_s N$  and set  $a_{0x} = 0$ .

$$L = 1 \text{ m} \quad m = 2 \text{ kg}$$

$$M = 5 \text{ kg}$$

$$I_G = \frac{1}{12} mL^2 = 0.167 \text{ kg}\cdot\text{m}^2$$

$$\sum F_x: B_x = ma_{G_x} \quad (1)$$

$$\sum F_y: B_y - mg = ma_{G_y} \quad (2)$$

$$\sum M_G: \left(\frac{L}{2} \cos \theta\right) B_y + \left(\frac{L}{2} \sin \theta\right) B_x = I_G \alpha \quad (3)$$

(These are the same as in Problem 18.53)

Note: In Prob 18.53,  $B_x = -5.77 \text{ N}$  (it was in the opposite direction to that assumed). This resulted in  $a_{0x}$  to the right. Thus, friction must be to the left

$$\sum F_x: -B_x - \mu_s N = ma_{0x} = 0 \quad (4)$$

$$\sum F_y: N - B_y - Mg = 0 \quad (5)$$

From kinematics,

$$\mathbf{a}_0 = \mathbf{a}_G + \boldsymbol{\alpha} \times \mathbf{r}_{0/G} = 0$$

$$0 = a_{G_x} + (\alpha L/2) \sin \theta \quad (6)$$

$$0 = a_{G_y} + (\alpha L/2) \cos \theta \quad (7)$$

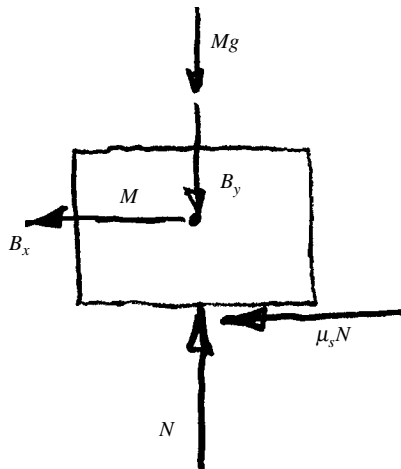
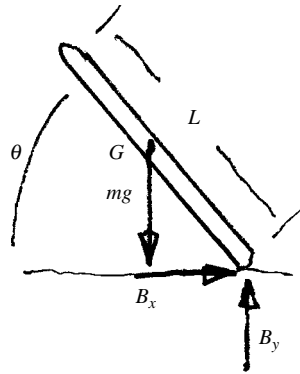
Solving 7 eqns in 7 unknowns, we get

$$B_x = -6.91 \text{ N}, \quad B_y = 14.78 \text{ N},$$

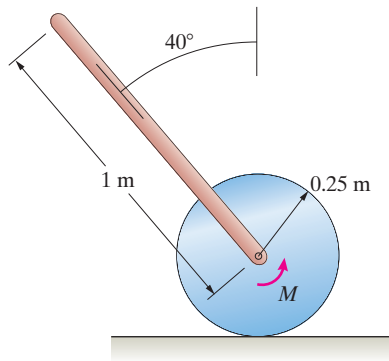
$$a_{G_x} = -3.46 \text{ m/s}^2, \quad a_{G_y} = -2.42 \text{ m/s}^2$$

$$N = 63.8 \text{ N}, \quad \alpha = 8.44 \text{ rad/s}^2$$

$$\mu_s = 0.108$$



**Problem 18.55** As a result of the constant couple  $M$  applied to the 1-kg disk, the angular acceleration of the 0.4-kg slender bar is zero. Determine  $M$  and the counterclockwise angular acceleration of the rolling disk.



**Solution:** There are seven unknowns ( $M, N, f, O_x, O_y, a, \alpha$ ), six dynamic equations, and one constraint equation. We use the following subset of those equations.

$$\Sigma M_{G \text{ rod}} : -O_x(0.5 \text{ m}) \cos 40^\circ$$

$$- O_y(0.5 \text{ m}) \sin 40^\circ = 0,$$

$$\Sigma F_{x \text{ rod}} : -O_x = -(0.4 \text{ kg})a,$$

$$\Sigma F_{y \text{ rod}} : -O_y - (0.4 \text{ kg})(9.81 \text{ m/s}^2) = 0,$$

$$\Sigma M_{G \text{ disk}} : M - f(0.25 \text{ m})$$

$$= \frac{1}{2}(1 \text{ kg})(0.25 \text{ m})^2\alpha,$$

$$\Sigma F_{x \text{ disk}} : O_x - f = -(1 \text{ kg})a,$$

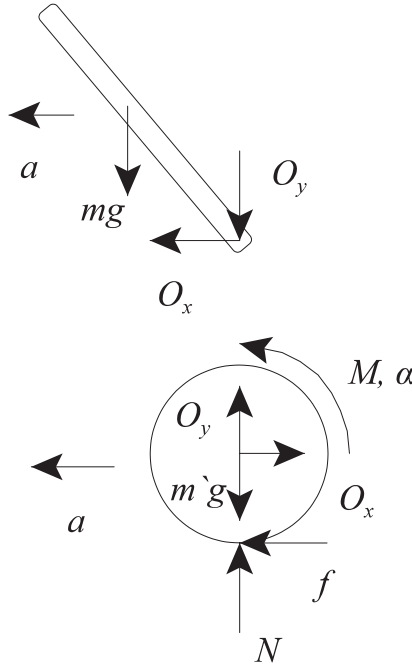
$$a = (0.25 \text{ m})\alpha.$$

Solving, we find

$$O_x = 3.29 \text{ N}, \quad O_y = -3.92 \text{ N},$$

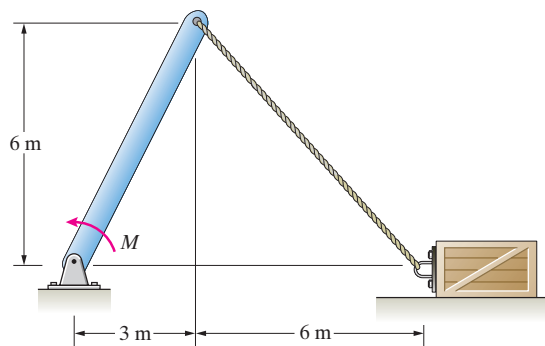
$$f = 11.5 \text{ N}, \quad a = 8.23 \text{ m/s}^2,$$

$$\alpha = 32.9 \text{ rad/s}^2, \quad M = 3.91 \text{ N-m}.$$



$$M = 3.91 \text{ N-m}, \quad \alpha = 32.9 \text{ rad/s}^2.$$

**Problem 18.56** The slender bar weighs 40 N and the crate weighs 80 N. At the instant shown, the velocity of the crate is zero and it has an acceleration of  $14 \text{ m/s}^2$  toward the left. The horizontal surface is smooth. Determine the couple  $M$  and the tension in the rope.



**Solution:** There are six unknowns ( $M, T, N, O_x, O_y, \alpha$ ), five dynamic equations, and one constraint equation. We use the following subset of the dynamic equations.

$$\Sigma M_O : M - (40 \text{ N})(1.5 \text{ m})$$

$$- T \cos 45^\circ (6 \text{ m})$$

$$- T \sin 45^\circ (3 \text{ m})$$

$$= \frac{1}{3} \left( \frac{40 \text{ N}}{9.81 \text{ m/s}^2} \right) (45 \text{ m}^2) \alpha,$$

$$\Sigma F_x : -T \cos 45^\circ = - \left( \frac{80 \text{ N}}{9.81 \text{ m/s}^2} \right) (14 \text{ m/s}^2)$$

The constraint equation is derived from the triangle shown. We have

$$L = \sqrt{45} \text{ m}, \quad d = 6\sqrt{2} \text{ m}, \quad \theta = 63.4^\circ.$$

$$x = L \cos \theta + \sqrt{d^2 - L^2 \sin^2 \theta}$$

$$\dot{x} = \left( -L \sin \theta - \frac{L^2 \cos \theta \sin \theta}{\sqrt{d^2 - L^2 \sin^2 \theta}} \right) \dot{\theta}$$

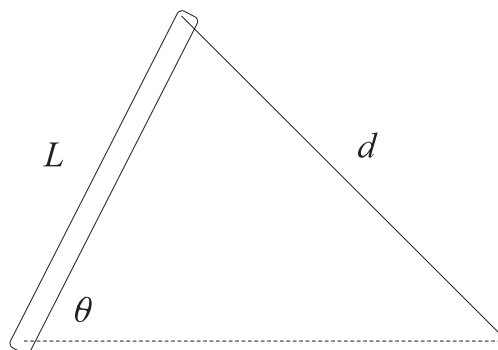
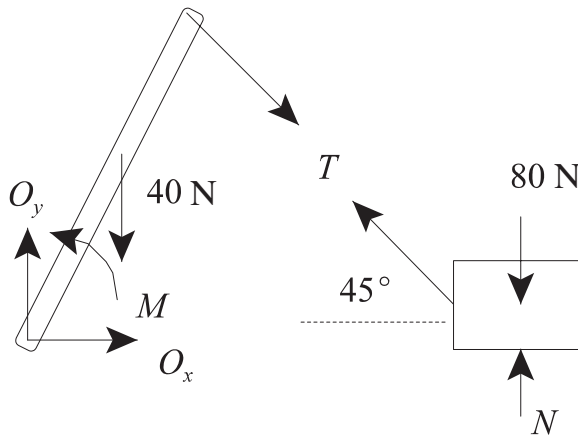
Since the velocity  $\dot{x} = 0$ , then we know that the angular velocity  $\omega = \dot{\theta} = 0$ . Taking one more derivative and setting  $\omega = 0$ , we find

$$\ddot{x} = \left( -L \sin \theta - \frac{L^2 \cos \theta \sin \theta}{\sqrt{d^2 - L^2 \sin^2 \theta}} \right) \ddot{\theta} \Rightarrow -(14 \text{ m/s}^2)$$

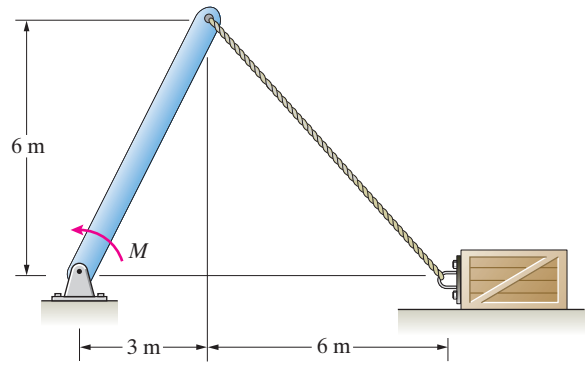
$$= \left( -L \sin \theta - \frac{L^2 \cos \theta \sin \theta}{\sqrt{d^2 - L^2 \sin^2 \theta}} \right) \alpha$$

Solving these equations, we find that

$$\alpha = 1.56 \text{ rad/s}^2, \quad \boxed{M = 1149 \text{ N}\cdot\text{m}, \quad T = 161.5 \text{ N}.}$$



**Problem 18.57** The slender bar weighs 40 N and the crate weighs 80 N. At the instant shown, the velocity of the crate is zero and it has an acceleration of  $14 \text{ m/s}^2$  toward the left. The coefficient of kinetic friction between the horizontal surface and the crate is  $\mu_k = 0.2$ . Determine the couple  $M$  and the tension in the rope.



**Solution:** There are seven unknowns ( $M, T, N, O_x, O_y, \alpha, f$ ), five dynamic equations, one constraint equation, and one friction equation. We use the following subset of the dynamic equations.

$$\Sigma M_O : M - (40 \text{ N})(1.5 \text{ m})$$

$$- T \cos 45^\circ (6 \text{ m})$$

$$- T \sin 45^\circ (3 \text{ m})$$

$$= \frac{1}{3} \left( \frac{40 \text{ N}}{9.81 \text{ m/s}^2} \right) (45 \text{ m}^2) \alpha,$$

$$\Sigma F_x : -T \cos 45^\circ + (0.2)N = - \left( \frac{80 \text{ N}}{9.81 \text{ m/s}^2} \right) (14 \text{ m/s}^2)$$

$$\Sigma F_y : T \sin 45^\circ + N - (80 \text{ N}) = 0.$$

The constraint equation is derived from the triangle shown. We have

$$L = \sqrt{45} \text{ m}, d = 6\sqrt{2} \text{ m}, \theta = 63.4^\circ.$$

$$x = L \cos \theta + \sqrt{d^2 - L^2 \sin^2 \theta}$$

$$\dot{x} = \left( -L \sin \theta - \frac{L^2 \cos \theta \sin \theta}{\sqrt{d^2 - L^2 \sin^2 \theta}} \right) \dot{\theta}$$

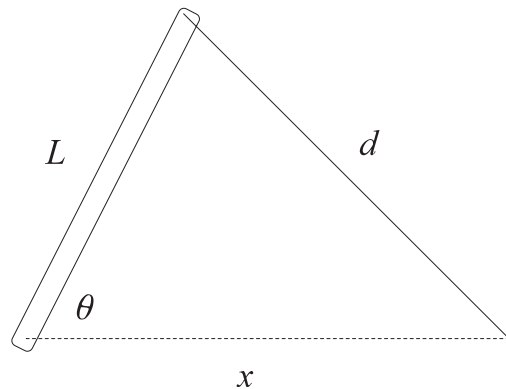
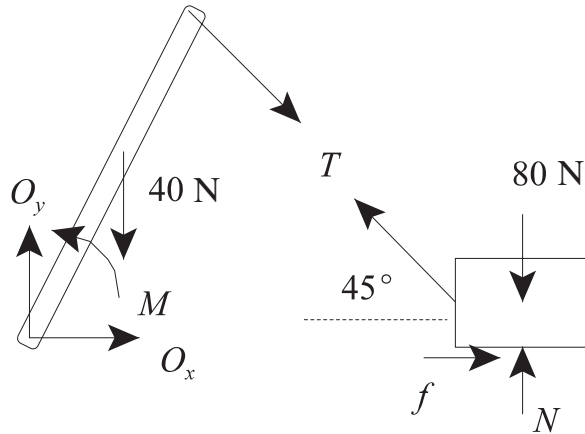
Since the velocity  $\dot{x} = 0$ , then we know that the angular velocity  $\omega = \dot{\theta} = 0$ . Taking one more derivative and setting  $\omega = 0$ , we find

$$\ddot{x} = \left( -L \sin \theta - \frac{L^2 \cos \theta \sin \theta}{\sqrt{d^2 - L^2 \sin^2 \theta}} \right) \ddot{\theta} \Rightarrow -(14 \text{ m/s}^2)$$

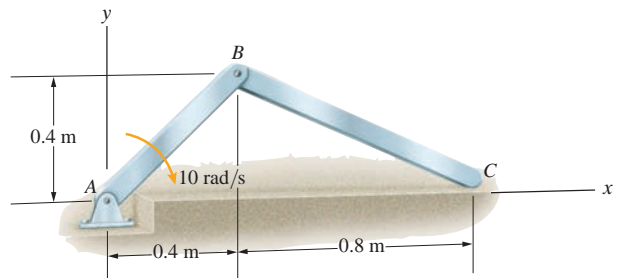
$$= \left( -L \sin \theta - \frac{L^2 \cos \theta \sin \theta}{\sqrt{d^2 - L^2 \sin^2 \theta}} \right) \alpha$$

Solving these equations, we find that

$$\alpha = 1.56 \text{ rad/s}^2, N = -28.5 \text{ N}, \boxed{M = 1094 \text{ N}\cdot\text{m}, T = 152.8 \text{ N}.}$$



**Problem 18.58** Bar  $AB$  is rotating with a constant clockwise angular velocity of  $10 \text{ rad/s}$ . The  $8\text{-kg}$  slender bar  $BC$  slides on the horizontal surface. At the instant shown, determine the total force (including its weight) acting on bar  $BC$  and the total moment about its center of mass.



**Solution:** We first perform a kinematic analysis to find the angular acceleration of bar  $BC$  and the acceleration of the center of mass of bar  $BC$ . First the velocity analysis:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 0 + (-10\mathbf{k}) \times (0.4\mathbf{i} + 0.4\mathbf{j}) = (-4\mathbf{i} + 4\mathbf{j})$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = (-4\mathbf{i} + 4\mathbf{j}) + \omega_{BC}\mathbf{k} \times (0.8\mathbf{i} - 0.4\mathbf{j})$$

$$= (-4 + 0.4\omega_{BC})\mathbf{i} + (4 + 0.8\omega_{BC})\mathbf{j}$$

Since  $C$  stays in contact with the floor, we set the  $\mathbf{j}$  component to zero  $\Rightarrow \omega_{BC} = -5 \text{ rad/s}$ . Now the acceleration analysis.

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= 0 + 0 - (10)^2(0.4\mathbf{i} + 0.4\mathbf{j}) = (-40\mathbf{i} - 40\mathbf{j})$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$= (-40\mathbf{i} - 40\mathbf{j}) + \alpha_{BC}\mathbf{k} \times (0.8\mathbf{i} - 0.4\mathbf{j}) - (-5)^2(0.8\mathbf{i} - 0.4\mathbf{j})$$

$$= (-60 + 0.4\alpha_{BC})\mathbf{i} + (-30 + 0.8\alpha_{BC})\mathbf{j}$$

Since  $C$  stays in contact with the floor, we set the  $\mathbf{j}$  component to zero  $\Rightarrow \alpha_{BC} = 37.5 \text{ rad/s}^2$ . Now we find the acceleration of the center of mass  $G$  of bar  $BC$ .

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{G/B} - \omega_{BC}^2 \mathbf{r}_{G/B}$$

$$= (-40\mathbf{i} - 40\mathbf{j}) + (37.5)\mathbf{k} \times (0.4\mathbf{i} - 0.2\mathbf{j}) - (-5)^2(0.4\mathbf{i} - 0.2\mathbf{j})$$

$$= (-42.5\mathbf{i} - 20\mathbf{j}) \text{ m/s}^2.$$

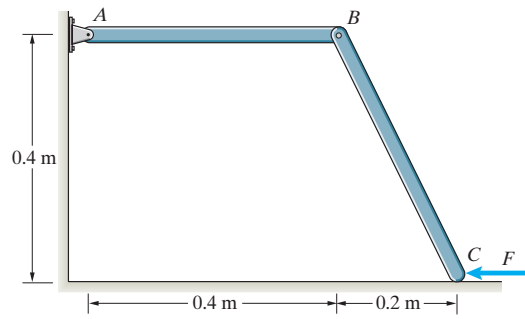
The total force and moment cause the accelerations that we just calculated. Therefore

$$\mathbf{F} = m\mathbf{a}_G = (8 \text{ kg})(-42.5\mathbf{i} - 20\mathbf{j}) \text{ m/s}^2 = (-340\mathbf{i} - 160\mathbf{j}) \text{ N},$$

$$M = I\alpha = \frac{1}{12}(8 \text{ kg})([0.8 \text{ m}]^2 + [0.4 \text{ m}]^2)(37.5 \text{ rad/s}^2) = 20 \text{ N}\cdot\text{m}.$$

$$\mathbf{F} = (-340\mathbf{i} - 160\mathbf{j}) \text{ N}, \quad M = 20 \text{ N}\cdot\text{m} \text{ counterclockwise.}$$

**Problem 18.59** The masses of the slender bars  $AB$  and  $BC$  are 10 kg and 12 kg, respectively. The angular velocities of the bars are zero at the instant shown and the horizontal force  $F = 150$  N. The horizontal surface is smooth. Determine the angular accelerations of the bars.



**Solution:** Given

$$m_{AB} = 10 \text{ kg}, m_{BC} = 12 \text{ kg}, g = 9.81 \text{ m/s}^2$$

$$L_{AB} = 0.4 \text{ m}, L_{BC} = \sqrt{0.4^2 + 0.2^2} \text{ m}, F = 150 \text{ N}$$

The FBDs

The dynamic equations

$$\sum M_A : -m_{AB}g \frac{L_{AB}}{2} + B_y L_{AB} = \frac{1}{3} m_{AB} L_{AB}^2 \alpha_{AB}$$

$$\sum F_{BCx} : -B_x - F = m_{BC} a_{BCx}$$

$$\sum F_{BCy} : -B_y - m_{BC}g + N = m_{BC} a_{BCy}$$

$$\sum M_{BCG} : (B_x - F)(0.2 \text{ m}) + (B_y + N)(0.1 \text{ m}) = \frac{1}{12} m_{BC} L_{BC}^2 \alpha_{BC}$$

The kinematic constraints

$$a_{BCy} = \alpha_{AB} L_{AB} + \alpha_{BC}(0.1 \text{ m})$$

$$a_{BCx} = \alpha_{BC}(0.2 \text{ m})$$

$$\alpha_{AB} L_{AB} + \alpha_{BC}(0.2 \text{ m}) = 0$$

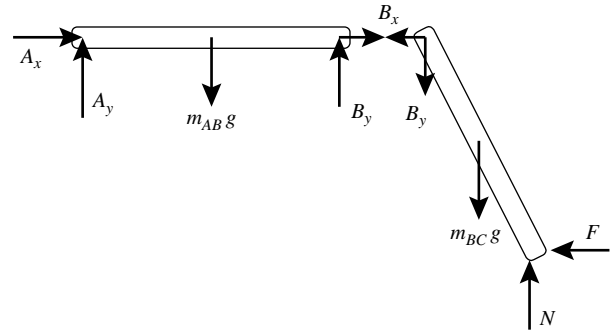
$$\text{Solving we find } \alpha_{AB} = 20.6 \text{ rad/s}^2, \alpha_{BC} = -41.2 \text{ rad/s}^2$$

$$\alpha_{AB} = 20.6 \text{ rad/s}^2 \text{ CCW}, \alpha_{BC} = 41.2 \text{ rad/s}^2 \text{ CW}$$

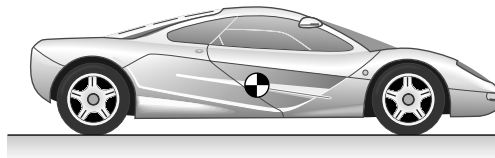
We also find

$$a_{BCx} = -8.23 \text{ m/s}^2, a_{BCy} = 4.12 \text{ m/s}^2$$

$$N = 244 \text{ N}, B_x = -51.2 \text{ N}, B_y = 76.5 \text{ N},$$



**Problem 18.60** Let the total moment of inertia of the car's two rear wheels and axle be  $I_R$ , and let the total moment of inertia of the two front wheels be  $I_F$ . The radius of the tires is  $R$ , and the total mass of the car, including the wheels, is  $m$ . If the car's engine exerts a torque (couple)  $T$  on the rear wheels and the wheels do not slip, show that the car's acceleration is



$$a = \frac{RT}{R^2m + I_R + I_F}.$$

**Strategy:** Isolate the wheels and draw three free-body diagrams.

**Solution:** The free body diagrams are as shown: We shall write three equations of motion for each wheel and two equations of motion for the body of the car: We shall sum moments about the axles on each wheel.

*Rear Wheel:*

$$\sum F_x = F_x + f_R = m_R a,$$

$$\sum F_y = N_R - m_R g - F_y = 0,$$

$$\sum M_{Raxle} = R f_R - T = I_R \alpha = I_R \left(-\frac{a}{R}\right)$$

*Front Wheel:*

$$\sum F_x = G_x + f_F = m_F a,$$

$$\sum F_y = N_F - m_F g - G_y = 0,$$

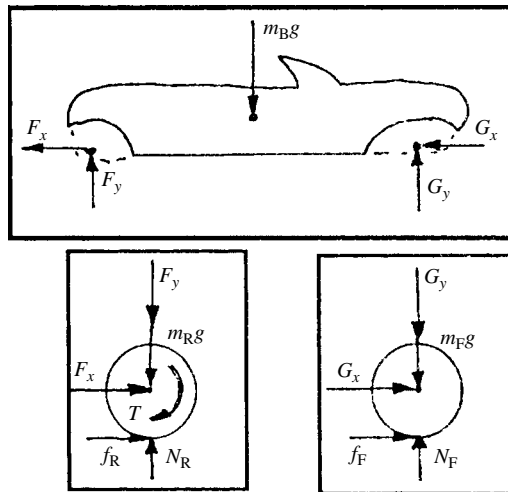
$$\sum M_{Faxle} = R f_F = I_F \alpha = I_F \left(-\frac{a}{R}\right)$$

*Car Body:*

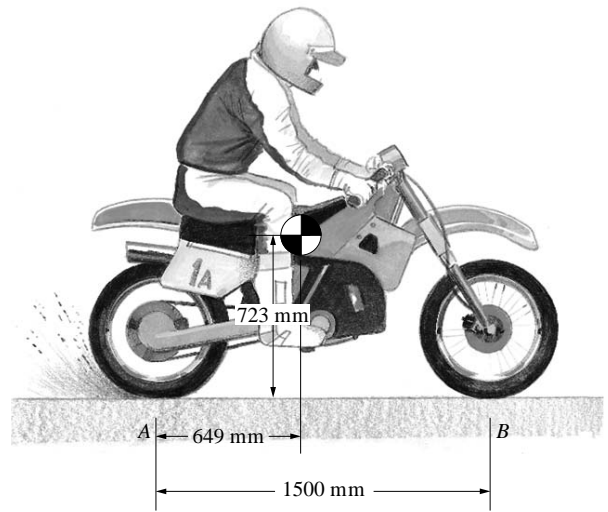
$$\sum F_x = -F_x - G_x = m_B a,$$

$$\sum F_y = F_y + G_y - m_B g = 0.$$

Summing the  $y$  equations for all three bodies, we get  $N_R + N_F = (m_B + m_R + m_F)g = mg$ . Summing the equations for all three bodies in the  $x$  direction, we get  $f_R + f_F = (m_B + m_R + m_F)a = ma$ . (1) From the moment equations for the wheels, we get  $f_F = -I_F a / R^2$  and  $f_R = -I_R a / R^2 + T / R$ . Substituting these into Eq. (1), we get  $a = RT / (mR^2 + I_R + I_F)$  as required.



**Problem 18.61** The combined mass of the motorcycle and rider is 160 kg. Each 9-kg wheel has a 330-mm radius and a moment of inertia  $I = 0.8 \text{ kg}\cdot\text{m}^2$ . The engine drives the rear wheel by exerting a couple on it. If the rear wheel exerts a 400-N horizontal force on the road and you do *not* neglect the horizontal force exerted on the road by the front wheel, determine (a) the motorcycle's acceleration and (b) the normal forces exerted on the road by the rear and front wheels. (The location of the center of mass of the motorcycle *not including* its wheels, is shown.)



**Solution:** In the free-body diagrams shown,  $m_w = 9 \text{ kg}$  and  $m = 160 - 18 = 142 \text{ kg}$ . Let  $a$  be the motorcycle's acceleration to the right and let  $\alpha$  be the wheels' clockwise angular acceleration. Note that

$$a = 0.33\alpha. \quad (1)$$

*Front Wheel:*

$$\sum F_x = B_x + f_F = m_w a, \quad (2)$$

$$\sum F_y = B_y + N_F - m_w g = 0, \quad (3)$$

$$\sum M = -f_F(0.33) = I\alpha. \quad (4)$$

*Rear Wheel:*

$$\sum F_x = A_x + f_R = m_w a, \quad (5)$$

$$\sum F_y = A_y + N_R - m_w g = 0, \quad (6)$$

$$\sum M = M - f_R(0.33) = I\alpha. \quad (7)$$

*Motorcycle:*

$$\sum F_x = -A_x - B_x = ma, \quad (8)$$

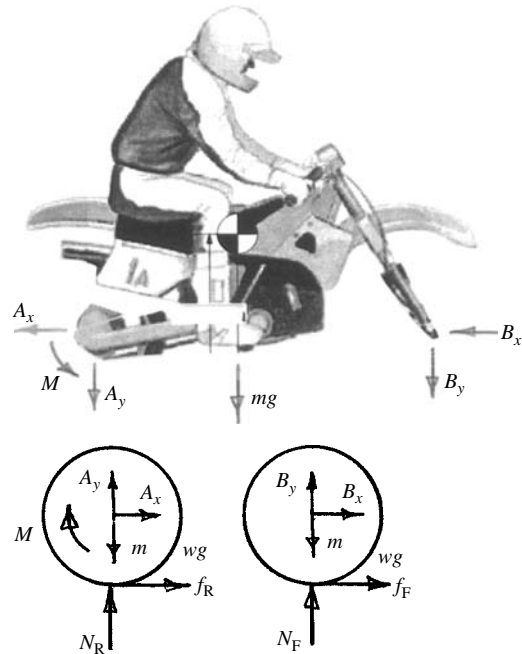
$$\sum F_y = -A_y - B_y - mg = 0, \quad (9)$$

$$\begin{aligned} \sum M = & -M + (A_x + B_x)(0.723 - 0.33) \\ & + B_y(1.5 - 0.649) - A_y(0.649) = 0. \quad (10) \end{aligned}$$

Solving Eqs (1)–(10) with  $f_R = 400 \text{ N}$ , we obtain

$$(a) \quad a = 2.39 \text{ rad/s}^2$$

and (b)  $N_R = 455 \text{ N}$ ,  $N_F = 1115 \text{ N}$ .



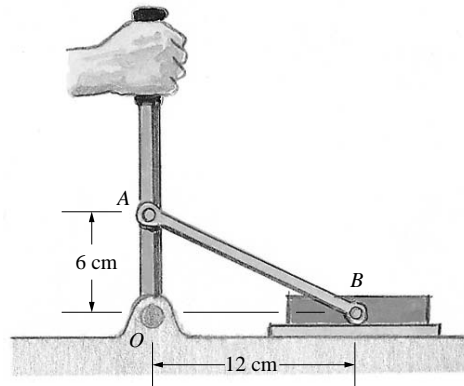


**Problem 18.62** In Problem 18.61, if the front wheel lifts slightly off the road when the rider accelerates, determine (a) the motorcycle's acceleration and (b) the torque exerted by the engine on the rear wheel.

**Solution:** See the solution of Problem 18.61. We set  $N_F = 0$  and replace Eq. (4) by  $f_F = 0$ . Then solving Eqs. (1)–(10), we obtain

- (a)  $a = 9.34 \text{ m/s}^2$ ,
- (b)  $M = 516 \text{ N}\cdot\text{m}$ .

**Problem 18.63** The moment of inertia of the vertical handle about  $O$  is  $0.16 \text{ kg}\cdot\text{m}^2$ . The object  $B$  weighs  $66.7 \text{ N}$  and rests on a smooth surface. The weight of the bar  $AB$  is negligible (which means that you can treat the bar as a two-force member). If the person exerts a  $0.89 \text{ N}$  horizontal force on the handle  $15 \text{ cm}$  above  $O$ , what is the resulting angular acceleration of the handle?



**Solution:** Let  $\alpha$  be the clockwise angular acceleration of the handle. The acceleration of  $B$  is:

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} :$$

$$a_B \mathbf{i} = (6/12)\alpha \mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 1 & -0.5 & 0 \end{vmatrix}$$

we see that  $\alpha_{AB} = 0$  and

$$a_B = (6/12)\alpha \quad (1).$$

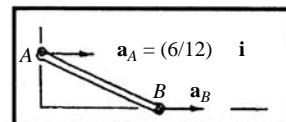
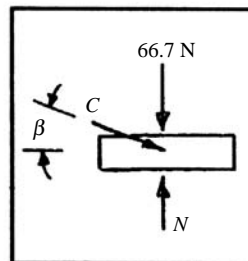
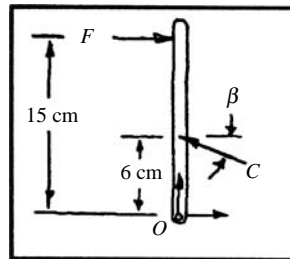
The free body diagrams of the handle and object  $B$  are as shown. Note that  $\beta = \arctan(6/12) = 26.6^\circ$ . Newton's second law for the object  $B$  is

$$C \cos \beta = (0.15/9.81)a_B, \quad (2)$$

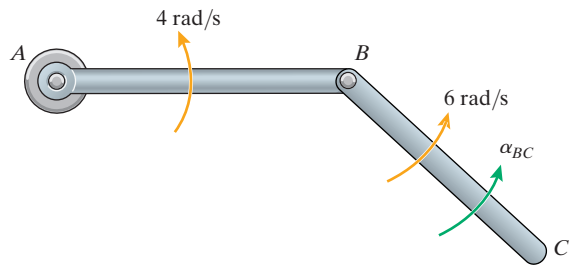
The equation of angular motion for the handle is

$$(15/12)F - (6/12)C \cos \beta = (0.16)\alpha \quad (3).$$

Solving Equations (1)–(3) with  $F = 0.89 \text{ N}$ , we obtain  $\alpha = 6.8 \text{ rad/s}^2$



**Problem 18.64** The bars are each 1 m in length and have a mass of 2 kg. They rotate *in the horizontal plane*. Bar  $AB$  rotates with a constant angular velocity of 4 rad/s in the counterclockwise direction. At the instant shown, bar  $BC$  is rotating in the counterclockwise direction at 6 rad/s. What is the angular acceleration of bar  $BC$ ?



**Solution:** Given  $m = 2 \text{ kg}$ ,  $L = 1 \text{ m}$ ,  $\theta = 45^\circ$

The FBD

The kinematics

$$\begin{aligned} \mathbf{a}_B &= \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\ &= 0 + 0 - (4 \text{ rad/s})^2 (1 \text{ m}) \mathbf{i} = -(16 \text{ m/s}^2) \mathbf{i} \end{aligned}$$

$$\begin{aligned} \mathbf{a}_G &= \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{G/B} - \omega_{BC}^2 \mathbf{r}_{G/B} \\ &= -(16 \text{ m/s}^2) \mathbf{i} + \alpha_{BC} \mathbf{k} \times (0.5 \text{ m})(\cos \theta \mathbf{i} - \sin \theta \mathbf{j}) \\ &\quad - (6 \text{ rad/s})^2 (0.5 \text{ m})(\cos \theta \mathbf{i} - \sin \theta \mathbf{j}) \\ &= (-16 \text{ m/s}^2 + [0.5 \text{ m} \sin \theta] \alpha_{BC} - [18 \text{ m/s}^2] \cos \theta) \mathbf{i} \\ &\quad + ([0.5 \text{ m} \cos \theta] \alpha_{BC} + [18 \text{ m/s}^2] \sin \theta) \mathbf{j} \end{aligned}$$

Our kinematic constraints are

$$a_x = -16 \text{ m/s}^2 + [0.5 \text{ m} \sin \theta] \alpha_{BC} - [18 \text{ m/s}^2] \cos \theta$$

$$a_y = [0.5 \text{ m} \cos \theta] \alpha_{BC} + [18 \text{ m/s}^2] \sin \theta$$

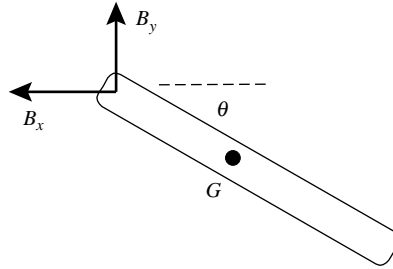
The dynamic equations

$$\sum F_x : -B_x = ma_x$$

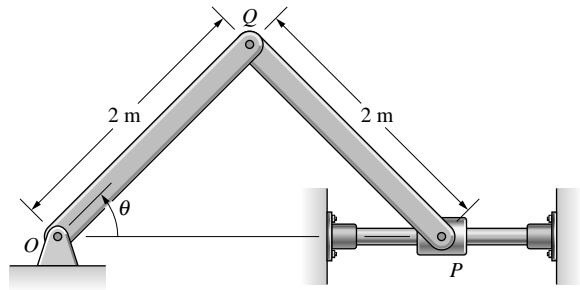
$$\sum F_y : B_y = ma_y$$

$$\sum M_G : B_x (0.5 \text{ m}) \sin \theta - B_y (0.5 \text{ m}) \cos \theta = \frac{1}{12} m (1.0 \text{ m})^2 \alpha_{BC}$$

Solving we find  $\alpha_{BC} = 17.0 \text{ rad/s}^2 \text{ CCW}$



**Problem 18.65** Bars  $OQ$  and  $PQ$  each weigh 6 N. The weight of the collar  $P$  and friction between the collar and the horizontal bar are negligible. If the system is released from rest with  $\theta = 45^\circ$ , what are the angular accelerations of the two bars?



**Solution:** Let  $\alpha_{OQ}$  and  $\alpha_{PQ}$  be the clockwise angular acceleration of bar  $OQ$  and the counterclockwise angular acceleration of bar  $PQ$ . The acceleration of  $Q$  is

$$\mathbf{a}_Q = \mathbf{a}_O + \alpha_{OQ} \times \mathbf{r}_{Q/O} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\alpha_{OQ} \\ 2 \cos 45^\circ & 2 \sin 45^\circ & 0 \end{vmatrix}$$

$$= 2\alpha_{OQ} \sin 45^\circ \mathbf{i} - 2\alpha_{OQ} \cos 45^\circ \mathbf{j}.$$

The acceleration of  $P$  is

$$\mathbf{a}_P = \mathbf{a}_Q + \alpha_{PQ} \times \mathbf{r}_{P/Q}$$

$$a_P \mathbf{i} = 2\alpha_{OQ} \sin 45^\circ \mathbf{i} - 2\alpha_{OQ} \cos 45^\circ \mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{PQ} \\ 2 \cos 45^\circ & 2 \sin 45^\circ & 0 \end{vmatrix}.$$

Equating  $\mathbf{i}$  and  $\mathbf{j}$  components,

$$a_P = 2\alpha_{OQ} \sin 45^\circ - 2\alpha_{PQ} \sin 45^\circ \quad (1)$$

$$0 = -2\alpha_{OQ} \cos 45^\circ + 2\alpha_{PQ} \cos 45^\circ \quad (2).$$

The acceleration of the center of mass of bar  $PQ$  is

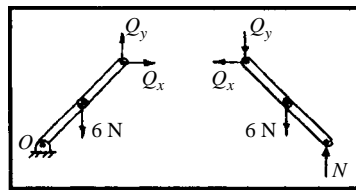
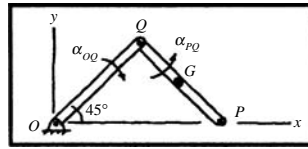
$$\mathbf{a}_G = \mathbf{a}_Q + \alpha_{PQ} \times \mathbf{r}_{G/Q} = 2\alpha_{OQ} \sin 45^\circ \mathbf{i}$$

$$- 2\alpha_{OQ} \cos 45^\circ \mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{PQ} \\ \cos 45^\circ & -\sin 45^\circ & 0 \end{vmatrix}.$$

Hence,

$$a_{Gx} = 2\alpha_{OQ} \sin 45^\circ + \alpha_{PQ} \sin 45^\circ \quad (3);$$

$$a_{Gy} = -2\alpha_{OQ} \cos 45^\circ + \alpha_{PQ} \cos 45^\circ \quad (4).$$



From the diagrams:

The equation of angular motion of bar  $OQ$  is  $\sum M_O = I_O \alpha_{OQ}$ :

$$Q_x(2 \sin 45^\circ) - Q_y(2 \cos 45^\circ) + 6 \cos 45^\circ = \frac{1}{3}(6/9.81)(2)^2 \alpha_{OQ} \quad (5).$$

The equations of motion of bar  $PQ$  are

$$\sum F_x = -Q_x = (6/9.81)a_{Gx} \quad (6)$$

$$\sum F_y = N - Q_y - 6 = (6/9.81)a_{Gy} \quad (7)$$

$$\sum M = (N + Q_y + Q_x)(\cos 45^\circ) = \frac{1}{12}(6/9.81)(2)^2 \alpha_{PQ} \quad (8).$$

Solving Equations (1)–(8), we obtain  $\alpha_{OQ} = \alpha_{PQ} = 6.83 \text{ rad/s}^2$

**Problem 18.66** In Problem 18.65, what are the angular accelerations of the two bars if the collar  $P$  weighs 2 N?

**Solution:** In the solution of Problem 18.65, the free body diagram of bar  $PQ$  has a horizontal component  $P$  to the left where  $P$  is the force exerted on the bar by the collar. Equations (6) and (8) become

$$\sum F_x = -Q_x - P = (6/9.81)a_{G_x}$$

$$\sum M = (N - P + Q_y + Q_x)(\cos 45^\circ) = \frac{1}{12}(6/9.81)(2)^2\alpha_{PQ}$$

and the equation of motion for the collar is  $P = (2/9.81)a_P$  solving equations (1–9), we obtain  $\alpha_{OQ} = \alpha_{PQ} = 4.88 \text{ rad/s}^2$ .

**Problem 18.67** The 4-kg slender bar is pinned to 2-kg sliders at  $A$  and  $B$ . If friction is negligible and the system is released from rest in the position shown, what is the angular acceleration of the bar at that instant?

**Solution:** Express the acceleration of  $B$  in terms of the acceleration of  $A$ ,  $\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A}$ :

$$a_B \cos 45^\circ \mathbf{i} - a_B \sin 45^\circ \mathbf{j} = -a_A \mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 0.5 & -1.2 & 0 \end{vmatrix},$$

$$\text{or } a_B \cos 45^\circ = 1.2\alpha_{AB}, \quad (1);$$

$$\text{and } -a_B \sin 45^\circ = -a_A + 0.5\alpha_{AB}, \quad (2).$$

We express the acceleration of  $G$  in terms of the acceleration of  $A$ ,  $\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{G/A}$ :

$$\mathbf{a}_G = a_{G_x} \mathbf{i} + a_{G_y} \mathbf{j} = -a_A \mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 0.25 & -0.6 & 0 \end{vmatrix},$$

$$\text{or } a_{G_x} = 0.6\alpha_{AB}, \quad (3);$$

$$\text{and } a_{G_y} = -a_A + 0.25\alpha_{AB}, \quad (4);$$

The free body diagrams are as shown. The equations of motion are

$$N - A_x = 0 \quad (5);$$

$$\text{and } (2)(9.81) + A_y = 2a_A, \quad (6);$$

$$\text{Slider } B: P - [B_x + B_y + (2)(9.81)] \cos 45^\circ = 0, \quad (7);$$

$$\text{and } [(2)(9.81) - B_x + B_y] \cos 45^\circ = 2a_B, \quad (8);$$

$$\text{Bar: } A_x + B_x = 4a_{G_x} \quad (9);$$

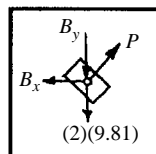
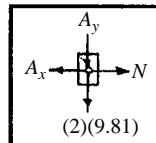
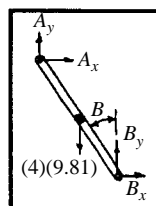
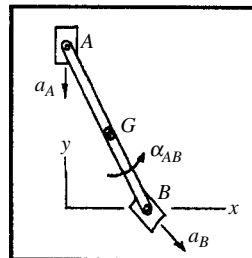
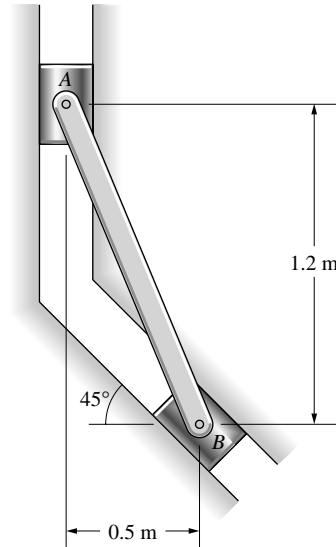
$$\text{and } A_y + B_y - (4)(9.81) = 4a_{G_y} \quad (10);$$

$$(L/2)[(B_x - A_x) \cos \beta + (B_y - A_y) \sin \beta] = \frac{1}{12}(4)L^2\alpha_{AB} \quad (11),$$

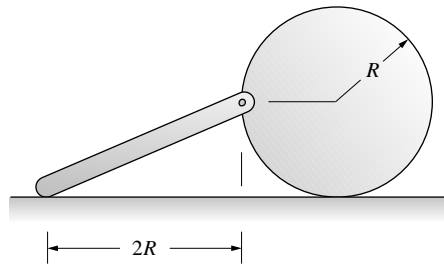
$$\text{where } L = \sqrt{(0.5)^2 + (1.2)^2} \text{ m}$$

$$\text{and } \beta = \arctan(0.5/1.2) = 22.6^\circ.$$

Solving Equations (1)–(11), we obtain  $\alpha_{AB} = 5.18 \text{ rad/s}^2$ .



**Problem 18.68** The mass of the slender bar is  $m$  and the mass of the homogeneous disk is  $4m$ . The system is released from rest in the position shown. If the disk rolls and the friction between the bar and the horizontal surface is negligible, show that the disk's angular acceleration is  $\alpha = 6g/95R$  counterclockwise.



**Solution:** For the bar: The length of the bar is  $L = \sqrt{5}R$ . Apply Newton's second law to the free body diagram of the bar:  $B_x = ma_{G_x}$ ,  $B_y + N_A - mg = ma_{G_y}$ , where  $a_{G_x}$ ,  $a_{G_y}$  are the accelerations of the center of mass of the bar. The moment about the bar center of mass is

$$RB_y - RN_A - \frac{R}{2}B_x = I_B\alpha_{AB}.$$

For the disk: Apply Newton's second law and the equation of angular motion to the free body diagram of the disk.  $f - B_x = 4ma_{D_x}$ ,  $N_D - 4mg - B_y = 0$ ,  $RB_y + Rf = I_D\alpha_D$

From kinematics: Since the system is released from rest,  $\omega_{AB} = \omega_D = 0$ . The acceleration of the center of the disk is  $\mathbf{a}_D = -R\alpha_D\mathbf{i}$ . The acceleration of point B in terms of the acceleration of the center of the disk is

$$\mathbf{a}_B = \mathbf{a}_D + \alpha_D \times \mathbf{r}_{B/D} = \mathbf{a}_D + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_D \\ -R & 0 & 0 \end{bmatrix} = -R\alpha_D\mathbf{i} - R\alpha_D\mathbf{j}.$$

The acceleration of the center of mass of the bar in terms of the acceleration of B is

$$\begin{aligned} \mathbf{a}_G &= \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{G/B} - \omega_{AB}^2 \mathbf{r}_{G/B} = \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ -R & -\frac{R}{2} & 0 \end{bmatrix} \\ &= \mathbf{a}_B + \frac{R\alpha_{AB}}{2}\mathbf{i} - R\alpha_{AB}\mathbf{j}, \end{aligned}$$

$$\mathbf{a}_G = -R\left(\alpha_D - \frac{\alpha_{AB}}{2}\right)\mathbf{i} - R(\alpha_D + \alpha_{AB})\mathbf{j}.$$

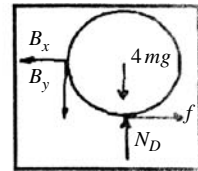
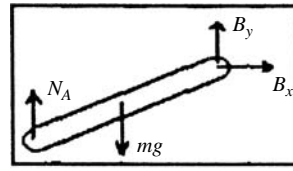
The acceleration of the center of mass of the bar in terms of the acceleration of A is

$$\begin{aligned} \mathbf{a}_G &= \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{G/A} = \mathbf{a}_A + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ R & \frac{R}{2} & 0 \end{bmatrix} \\ &= \mathbf{a}_A - \frac{R\alpha_{AB}}{2}\mathbf{i} + R\alpha_{AB}\mathbf{j}. \end{aligned}$$

From the constraint on the motion,  $\mathbf{a}_A = a_A\mathbf{i}$ . Equate the expressions for  $\mathbf{a}_G$ , separate components and solve:  $\alpha_{AB} = -\frac{\alpha_D}{2}$ . Substitute to obtain  $a_{G_x} = -\frac{5R}{4}\alpha_D$ ,  $a_{G_y} = -\frac{R}{2}\alpha_D$ . Collect the results:

$$(1) \quad B_x = -\frac{5Rm}{4}\alpha_D,$$

$$(2) \quad B_y + N_A - mg = -\frac{Rm}{2}\alpha_D,$$



$$(3) \quad RB_y - RN_A - \frac{R}{2}B_x = -\frac{I_B}{2}\alpha_D,$$

$$(4) \quad f - B_x = -4Rm\alpha_D,$$

$$(5) \quad N_D - 4mg - B_y = 0,$$

$$(6) \quad RB_y + Rf = I_D\alpha_D.$$

From (1), (2), and (3)

$$B_y = \frac{mg}{2} - \left(\frac{9mR}{16} + \frac{I_B}{4R}\right)\alpha_D.$$

From (1), (4) and (6),

$$B_y = \left(\frac{I_D}{R} + \frac{21Rm}{4}\right)\alpha_D.$$

Equate the expressions for  $B_y$  and reduce to obtain

$$\alpha_D = \left(\frac{mg}{2}\right) \frac{1}{\left(\frac{93Rm}{16} + \frac{I_D}{R} + \frac{I_B}{4R}\right)}.$$

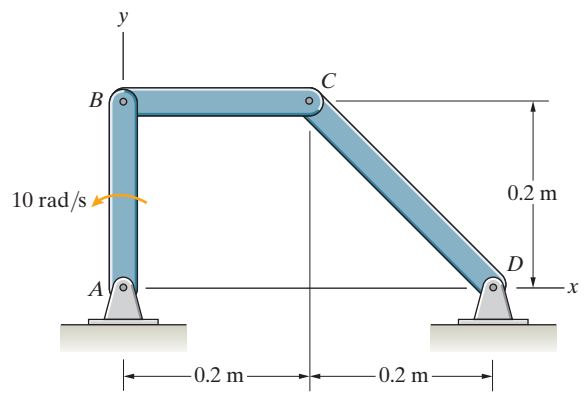
For a homogeneous cylinder of mass  $4m$ ,  $I_D = 2R^2m$ . For a slender bar of mass  $m$  about the center of mass,

$$I_B = \frac{1}{12}mL^2 = \frac{5}{12}mR^2.$$

Substitute and reduce:

$$\boxed{a_D = \frac{6g}{95R}}$$

**Problem 18.69** Bar  $AB$  rotates in the horizontal plane with a constant angular velocity of  $10 \text{ rad/s}$  in the counterclockwise direction. The masses of the slender bars  $BC$  and  $CD$  are  $3 \text{ kg}$  and  $4.5 \text{ kg}$ , respectively. Determine the  $x$  and  $y$  components of the forces exerted on bar  $BC$  by the pins at  $B$  and  $C$  at the instant shown.



**Solution:** First let's do the kinematics

Velocity

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$= 0 + (10 \text{ rad/s})\mathbf{k} \times (0.2 \text{ m})\mathbf{j}$$

$$= -(2 \text{ m/s})\mathbf{i}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= -(2 \text{ m/s})\mathbf{i} + \omega_{BC}\mathbf{k} \times (0.2 \text{ m})\mathbf{i} = -(2 \text{ m/s})\mathbf{i} + (0.2 \text{ m})\omega_{BC}\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$$

$$= -(2 \text{ m/s})\mathbf{i} + (0.2 \text{ m})\omega_{BC}\mathbf{j} + \omega_{CD}\mathbf{k} \times (0.2 \text{ m})(\mathbf{i} - \mathbf{j})$$

$$= (-[2 \text{ m/s}] + [0.2 \text{ m}]\omega_{CD})\mathbf{i} + (0.2 \text{ m})(\omega_{BC} + \omega_{CD})\mathbf{j}$$

Since  $D$  is pinned we find  $\omega_{CD} = 10 \text{ rad/s}$ ,  $\omega_{BC} = -10 \text{ rad/s}$

Acceleration

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= 0 + 0 - (10 \text{ rad/s})^2(0.2 \text{ m})\mathbf{j} = -(20 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$= -(20 \text{ m/s}^2)\mathbf{j} + \alpha_{BC}\mathbf{k} \times (0.2 \text{ m})\mathbf{i} - (-10 \text{ rad/s})^2(0.2 \text{ m})\mathbf{i}$$

$$= -(20 \text{ m/s}^2)\mathbf{j} + ([0.2 \text{ m}]\alpha_{BC} - 20 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{D/C} - \omega_{CD}^2 \mathbf{r}_{D/C}$$

$$= -(20 \text{ m/s}^2)\mathbf{j} + ([0.2 \text{ m}]\alpha_{BC} - 20 \text{ m/s}^2)\mathbf{i} + \alpha_{CD}\mathbf{k}$$

$$\times (0.2 \text{ m})(\mathbf{i} - \mathbf{j}) - (10 \text{ rad/s})^2(0.2 \text{ m})(\mathbf{i} - \mathbf{j})$$

$$= (-40 \text{ m/s}^2 + [0.2 \text{ m}]\alpha_{CD})\mathbf{i} + ([0.2 \text{ m}][\alpha_{BC} + \alpha_{CD}])\mathbf{j}$$

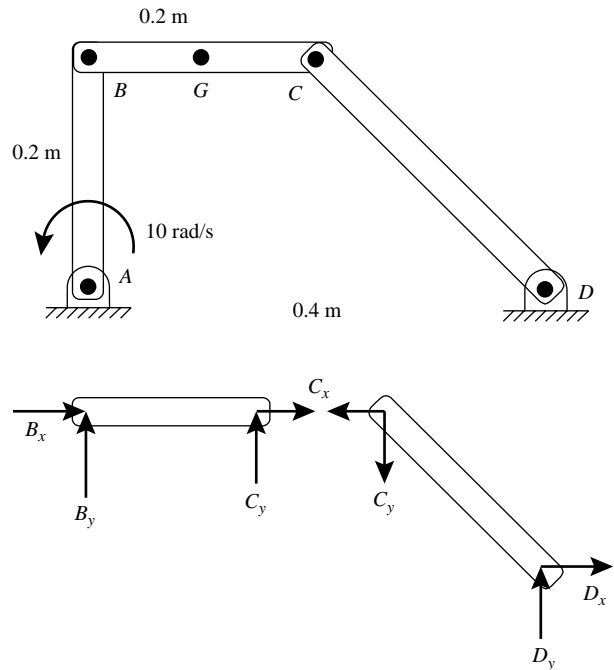
Since  $D$  is pinned we find  $\alpha_{BC} = -200 \text{ rad/s}^2$ ,  $\alpha_{CD} = 200 \text{ rad/s}^2$

Now find the accelerations of the center of mass  $G$ .

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{G1/B} - \omega_{BC}^2 \mathbf{r}_{G1/B}$$

$$= -(20 \text{ m/s}^2)\mathbf{j} + (-200 \text{ rad/s}^2)\mathbf{k} \times (0.1 \text{ m})\mathbf{i} - (-10 \text{ rad/s})^2(0.1 \text{ m})\mathbf{i}$$

$$= (-10\mathbf{i} - 40\mathbf{j}) \text{ m/s}^2$$



The FBDs

The dynamics

$$\sum F_{BCx} : B_x + C_x = (3 \text{ kg})(-10 \text{ m/s}^2)$$

$$\sum F_{BCy} : B_y + C_y = (3 \text{ kg})(-40 \text{ m/s}^2)$$

$$\sum M_{G1} : (C_y - B_y)(0.1 \text{ m}) = \frac{1}{12}(3 \text{ kg})(0.2 \text{ m})^2(-200 \text{ rad/s}^2)$$

$$\sum M_D : C_x(0.2 \text{ m}) + C_y(0.2 \text{ m}) = \frac{1}{3}(4.5 \text{ kg})(\sqrt{2}(0.2 \text{ m}))^2(200 \text{ rad/s}^2)$$

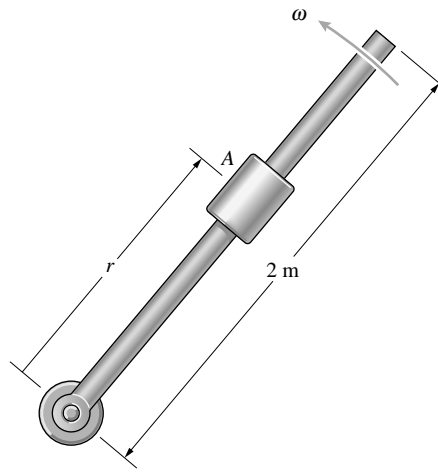
Solving we find

$$B_x = -220 \text{ N}, B_y = -50 \text{ N}$$

$$C_x = 190 \text{ N}, C_y = -70 \text{ N}$$

**Problem 18.70** The 2-kg bar rotates in the horizontal plane about the smooth pin. The 6-kg collar A slides on the smooth bar. At the instant shown,  $r = 1.2$  m,  $\omega = 0.4$  rad/s, and the collar is sliding outward at 0.5 m/s relative to the bar. If you neglect the moment of inertia of the collar (that is, treat the collar as a particle), what is the bar's angular acceleration?

**Strategy:** Draw individual free-body diagrams of the bar and collar and write Newton's second law for the collar in terms of polar coordinates.



**Solution:** Diagrams of the bar and collar showing the force they exert on each other in the horizontal plane are: the bar's equation of angular motion is

$$\sum M_0 = I_0 \alpha: \quad -Nr = \frac{1}{3}(2)(2)^2 \alpha \quad (1)$$

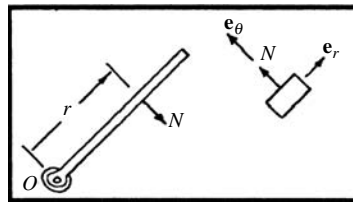
In polar coordinates, Newton's second law for the collar is

$$\sum \mathbf{F} = m\mathbf{a}: \quad N\mathbf{e}_\theta = m \left[ \left( \frac{d^2 r}{dt^2} - r\omega^2 \right) \mathbf{e}_r + \left( r\alpha + 2\frac{dr}{dt}\omega \right) \mathbf{e}_\theta \right].$$

Equating  $\mathbf{e}_\theta$  components,

$$N = m \left( r\alpha + 2\frac{dr}{dt}\omega \right) = (6)[r\alpha + 2(0.5)(0.4)] \quad (2).$$

Solving Equations (1) and (2) with  $r = 1.2$  m gives  $\alpha = -0.255$  rad/s<sup>2</sup>



**Problem 18.71** In Problem 18.70, the moment of inertia of the collar about its center of mass is 0.2 kg·m<sup>2</sup>. Determine the angular acceleration of the bar, and compare your answer with the answer to Problem 18.70.

**Solution:** Let  $C$  be the couple the collar and bar exert on each other: The bar's equation of angular motion is

$$\sum M_0 = I_0 \alpha: \quad -Nr - C = \frac{1}{3}(2)(2)^2 \alpha \quad (1).$$

The collar's equation of angular motion is

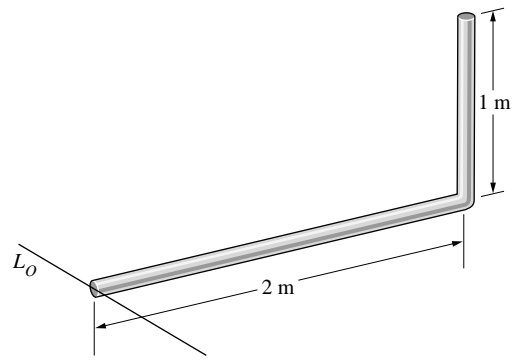
$$\sum M = I\alpha: \quad C = 0.2\alpha \quad (2).$$

From the solution of Problem 18.70, the  $\mathbf{e}_\theta$  component of Newton's second law for the collar is

$$N = (6)[r\alpha + 2(0.5)(0.4)] \quad (3)$$

Solving Equations (1)–(3) with  $r = 1.2$  m gives  $\alpha = -0.250$  rad/s<sup>2</sup>.

**Problem 18.72** The axis  $L_0$  is perpendicular to both segments of the L-shaped slender bar. The mass of the bar is 6 kg and the material is homogeneous. Use integration to determine the moment of inertia of the bar about  $L_0$ .



**Solution:** Let  $A$  be the bar's cross-sectional area. The bar's mass is  $m = 6 \text{ kg} = \rho A(3 \text{ m})$ , so  $\rho A = 2 \text{ kg/m}$ .

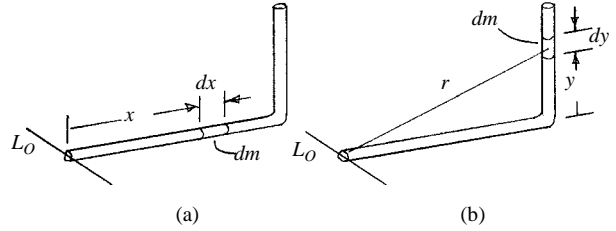
For the horizontal part (Fig. a),

$$I_h = \int_m x^2 dm = \int_0^2 x^2 \rho A dx = \frac{8}{3} \rho A = \frac{16}{3} \text{ kg}\cdot\text{m}^2.$$

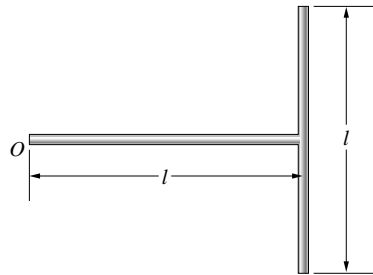
For the vertical part (Fig. b),

$$\begin{aligned} I_v &= \int_m r^2 dm = \int_0^1 (2^2 + y^2) \rho A dy \\ &= \frac{13}{3} \rho A = \frac{26}{3} \text{ kg}\cdot\text{m}^2. \end{aligned}$$

Therefore  $I_0 = I_h + I_v = 14 \text{ kg}\cdot\text{m}^2$ .



**Problem 18.73** Two homogenous slender bars, each of mass  $m$  and length  $l$ , are welded together to form the T-shaped object. Use integration to determine the moment of inertia of the object about the axis through point  $O$  that is perpendicular to the bars.



**Solution:** Divide the object into two pieces, each corresponding to a slender bar of mass  $m$ ; the first parallel to the  $y$ -axis, the second to the  $x$ -axis. By definition

$$I = \int_0^l r^2 dm + \int_m r^2 dm.$$

For the first bar, the differential mass is  $dm = \rho A dr$ . Assume that the second bar is very slender, so that the mass is concentrated at a distance  $l$  from  $O$ . Thus  $dm = \rho A dx$ , where  $x$  lies between the limits  $-\frac{l}{2} \leq x \leq \frac{l}{2}$ . The distance to a differential  $dx$  is  $r = \sqrt{l^2 + x^2}$ . Thus the definition becomes

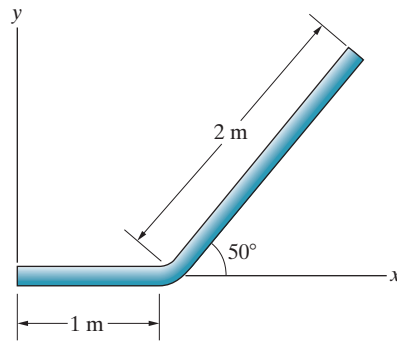
$$I = \rho A \int_0^l r^2 dr + \rho A \int_{-\frac{l}{2}}^{\frac{l}{2}} (l^2 + x^2) dx$$

$$I = \rho A \left[ \frac{r^3}{3} \right]_0^l + \rho A \left[ l^2 x + \frac{x^3}{3} \right]_{-\frac{l}{2}}^{\frac{l}{2}}$$

$$= ml^2 \left( \frac{1}{3} + 1 + \frac{1}{12} \right) = \frac{17}{12} ml^2$$



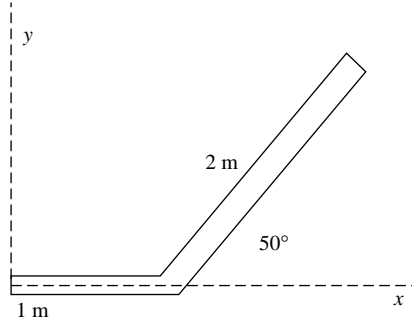
**Problem 18.74** The slender bar lies in the  $x-y$  plane. Its mass is 6 kg and the material is homogeneous. Use integration to determine its moment of inertia about the  $z$  axis.



**Solution:** The density is  $\rho = \frac{6 \text{ kg}}{3 \text{ m}} = 2 \text{ kg/m}$

$$I_z = \int_0^{1 \text{ m}} \rho x^2 dx + \int_0^{2 \text{ m}} \rho [(1 \text{ m} + s \cos 50^\circ)^2 + (s \sin 50^\circ)^2] ds$$

$$I_z = 15.1 \text{ kg}\cdot\text{m}^2$$

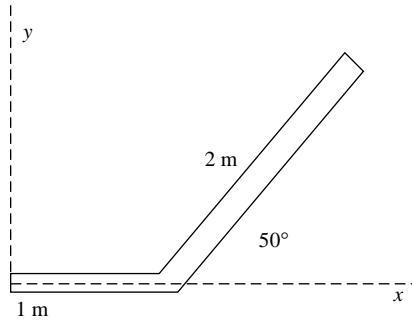


**Problem 18.75** The slender bar lies in the  $x-y$  plane. Its mass is 6 kg and the material is homogeneous. Use integration to determine its moment of inertia about the  $y$  axis.

**Solution:** The density is  $\rho = \frac{6 \text{ kg}}{3 \text{ m}} = 2 \text{ kg/m}$

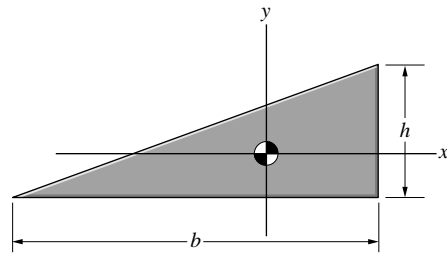
$$I_y = \int_0^{1 \text{ m}} \rho x^2 dx + \int_0^{2 \text{ m}} \rho [(1 \text{ m} + s \cos 50^\circ)^2] ds$$

$$I_y = 12.0 \text{ kg}\cdot\text{m}^2$$



**Problem 18.76** The homogeneous thin plate has mass  $m = 12 \text{ kg}$  and dimensions  $b = 1 \text{ m}$  and  $h = 2 \text{ m}$ . Determine the mass moments of inertia of the plate about the  $x$ ,  $y$ , and  $z$  axes.

**Strategy:** The mass moments of inertia of a thin plate of arbitrary shape are given by Eqs. (18.37)–(18.39) in terms of the moments of inertia of the cross-sectional area of the plate. You can obtain the moments of inertia of the triangular area from Appendix B.



**Solution:**

$$m = 12 \text{ kg}$$

$$\text{Area} = \frac{1}{2}bh$$

$$\rho = \text{mass/Area}$$

$$dm = \rho dA$$

From Appendix B,

$$I_{x_A} = \frac{1}{36}bh^3 \quad I_{y_A} = \frac{1}{36}hb^3$$

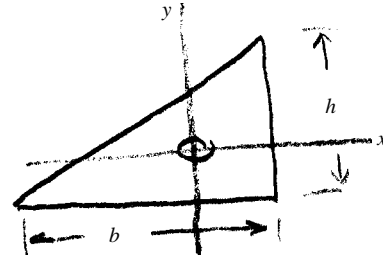
$$\text{Area} = \frac{1}{2}(1)(2) = 1 \text{ m}^2$$

$$\rho = 12 \text{ kg/m}^2$$

$$I_x = \int \rho y^2 dA = \rho \int y^2 dA$$

$$I_x = \rho I_{x_A}, I_y = \rho I_{y_A}$$

$$I_x = 12 \left( \frac{1}{36} \right) (1)(2)^3 = 2.667 \text{ kg}\cdot\text{m}^2$$



$$I_y = \rho I_{y_A} = \frac{12}{36}hb^3 = \frac{1}{3}(2)(1)^3$$

$$I_y = 0.667 \text{ kg}\cdot\text{m}^2$$

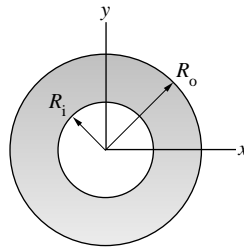
$$I_z = I_x + I_y$$

$$I_z = 2.667 + 0.667 \text{ kg}\cdot\text{m}^2$$

$$I_z = 3.333 \text{ kg}\cdot\text{m}^2$$

**Problem 18.77** The brass washer is of uniform thickness and mass  $m$ .

- (a) Determine its moments of inertia about the  $x$  and  $z$  axes.  
 (b) Let  $R_i = 0$ , and compare your results with the values given in Appendix C for a thin circular plate.



**Solution:**

- (a) The area moments of inertia for a circular area are

$$I_x = I_y = \frac{\pi R^4}{4}.$$

For the plate with a circular cutout,

$$I_x = \frac{\pi}{4}(R_o^4 - R_i^4)$$

The area mass density is  $\frac{m}{A}$ , thus for the plate with a circular cut,

$$\frac{m}{A} = \frac{m}{\pi(R_o^2 - R_i^2)},$$

from which the moments of inertia

$$I_{(x\text{-axis})} = \frac{m(R_o^4 - R_i^4)}{4(R_o^2 - R_i^2)} = \frac{m}{4}(R_o^2 + R_i^2)$$

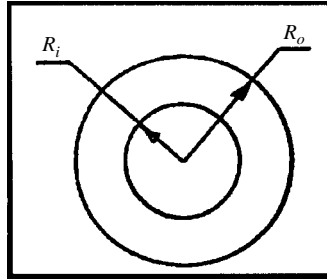
$$I_{(z\text{-axis})} = 2I_{(x\text{-axis})} = \frac{m}{2}(R_o^2 + R_i^2).$$

- (b) Let  $R_i = 0$ , to obtain

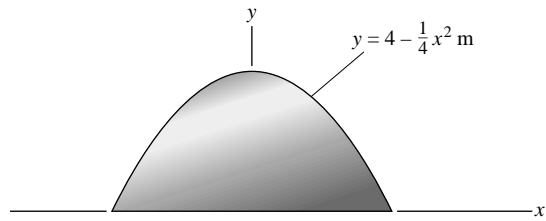
$$I_{x\text{-axis}} = \frac{m}{4}R_o^2,$$

$$I_{(z\text{-axis})} = \frac{m}{2}R_o^2,$$

which agrees with table entries.



**Problem 18.78** The homogenous thin plate is of uniform thickness and weighs 20 N. Determine its moment of inertia about the  $y$  axis.



**Solution:** The definition of the moment of inertia is

$$I = \int_m r^2 dm.$$

The distance from the  $y$ -axis is  $x$ , where  $x$  varies over the range  $-4 \leq x \leq 4$ . Let  $\tau = \frac{m}{A} = \frac{W}{gA}$  be the area mass density. The mass of an element  $y dx$  is  $dm = \frac{W}{gA} y dx$ . Substitute into the definition:

$$\begin{aligned} I_{y\text{-axis}} &= \frac{W}{gA} \int_{-4}^4 x^2 \left(4 - \frac{x^2}{4}\right) dx \\ &= \frac{W}{gA} \left[ \frac{4x^3}{3} - \frac{x^5}{20} \right]_{-4}^4 = \frac{W}{gA} [68.2667]. \end{aligned}$$

The area is

$$A = \int_{-4}^4 \left(4 - \frac{x^2}{4}\right) dx = \left[4x - \frac{x^3}{12}\right]_{-4}^4 = 21.333 \text{ m}^2$$

The moment of inertia about the  $y$ -axis is

$$I_{(y\text{-axis})} = \frac{W}{g} (3.2) = \frac{20}{9.81} (3.2) = 6.52 \text{ kg}\cdot\text{m}^2.$$

**Problem 18.79** Determine the moment of inertia of the plate in Problem 18.78 about the  $x$  axis.

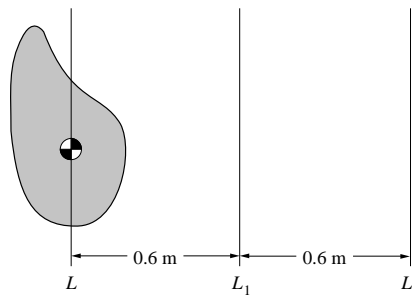
**Solution:** The differential mass is  $dm = \frac{W}{gA} dy dx$ . The distance of a mass element from the  $x$ -axis is  $y$ , thus

$$\begin{aligned} I &= \frac{W}{gA} \int_{-4}^4 dx \int_0^{4-\frac{x^2}{4}} y^2 dy \\ &= \frac{W}{3gA} \int_{-4}^4 \left(4 - \frac{x^2}{4}\right)^3 dx \\ &= \frac{W}{3gA} \left[64x - 4x^3 + \frac{3}{20}x^5 - \frac{x^7}{448}\right]_{-4}^4 \\ &= \frac{W}{3gA} [234.057]. \end{aligned}$$

From the solution to Problem 18.78,  $A = 21.333 \text{ ft}^2$ . Thus the moment of inertia about the  $x$ -axis is

$$I_{x\text{-axis}} = \frac{W}{3g} \frac{(234.057)}{(21.333)} = \frac{W}{g} (3.657) = 7.46 \text{ kg}\cdot\text{m}^2.$$

**Problem 18.80** The mass of the object is 10 kg. Its moment of inertia about  $L_1$  is  $10 \text{ kg}\cdot\text{m}^2$ . What is its moment of inertia about  $L_2$ ? (The three axes are in the same plane.)



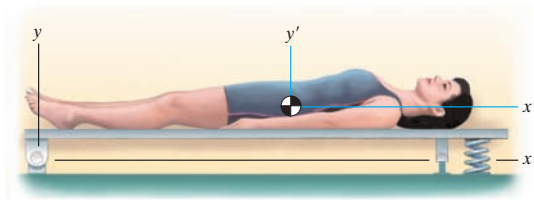
**Solution:** The strategy is to use the data to find the moment of inertia about  $L$ , from which the moment of inertia about  $L_2$  can be determined.

$$I_L = -(0.6)^2(10) + 10 = 6.4 \text{ kg}\cdot\text{m}^2,$$

from which

$$I_{L_2} = (1.2)^2(10) + 6.4 = 20.8 \text{ kg}\cdot\text{m}^2$$

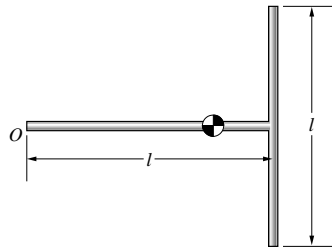
**Problem 18.81** An engineer gathering data for the design of a maneuvering unit determines that the astronaut's center of mass is at  $x = 1.01 \text{ m}$ ,  $y = 0.16 \text{ m}$  and that her moment of inertia about the  $z$  axis is  $105.6 \text{ kg}\cdot\text{m}^2$ . The astronaut's mass is  $81.6 \text{ kg}$ . What is her moment of inertia about the  $z'$  axis through her center of mass?



**Solution:** The distance from the  $z'$  axis to the  $z$  axis is  $d = \sqrt{x^2 + y^2} = 1.02257 \text{ m}$ . The moment of inertia about the  $z'$  axis is

$$\begin{aligned} I_{z'\text{-axis}} &= -d^2m + I_{z\text{-axis}} \\ &= -(1.0457)(81.6) + 105.6 = 20.27 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

**Problem 18.82** Two homogenous slender bars, each of mass  $m$  and length  $l$ , are welded together to form the T-shaped object. Use the parallel-axis theorem to determine the moment of inertia of the object about the axis through point  $O$  that is perpendicular to the bars.



**Solution:** Divide the object into two pieces, each corresponding to a bar of mass  $m$ . By definition  $I = \int_0^l r^2 dm$ . For the first bar, the differential mass is  $dm = \rho A dr$ , from which the moment of inertia about one end is

$$I_1 = \rho A \int_0^l r^2 dr = \rho A \left[ \frac{r^3}{3} \right]_0^l = \frac{ml^2}{3}.$$

For the second bar

$$I_2 = \rho A \int_{-\frac{l}{2}}^{\frac{l}{2}} r^2 dr = \rho A \left[ \frac{r^3}{3} \right]_{-\frac{l}{2}}^{\frac{l}{2}} = \frac{ml^2}{12}$$

is the moment of inertia about the center of the bar. From the parallel axis theorem, the moment of inertia about  $O$  is

$$I_0 = \frac{ml^2}{3} + l^2m + \frac{ml^2}{12} = \frac{17}{12}ml^2$$

**Problem 18.83** Use the parallel-axis theorem to determine the moment of inertia of the T-shaped object in Problem 18.98 about the axis through the center of mass of the object that is perpendicular to the two bars.

**Solution:** The location of the center of mass of the object is

$$x = \frac{m\left(\frac{l}{2}\right) + lm}{2m} = \frac{3}{4}l.$$

Use the results of Problem 18.98 for the moment of inertia of a bar about its center. For the first bar,

$$I_1 = \left(\frac{l}{4}\right)^2 m + \frac{ml^2}{12} = \frac{7}{48}ml^2.$$

For the second bar,

$$I_2 = \left(\frac{l}{4}\right)^2 m + \frac{ml^2}{12} = \frac{7}{48}ml^2.$$

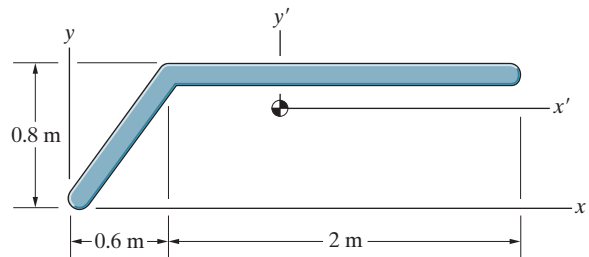
The composite:

$$I_c = I_1 + I_2 = \frac{7}{24}ml^2$$

*Check:* Use the results of Problem 18.98:

$$\begin{aligned} I_c &= -\left(\frac{3l}{4}\right)^2 (2m) + \frac{17}{12}ml^2 \\ &= \left(\frac{-9}{8} + \frac{17}{12}\right)ml^2 = \frac{7}{24}ml^2. \quad \text{check.} \end{aligned}$$

**Problem 18.84** The mass of the homogeneous slender bar is 30 kg. Determine its moment of inertia about the  $z$  axis.



**Solution:** The density is  $\rho = \frac{30 \text{ kg}}{3 \text{ m}} = 10 \text{ kg/m}$

$$\begin{aligned} I_z &= \frac{1}{3}(10 \text{ kg})(1.0 \text{ m})^2 + \frac{1}{12}(20 \text{ kg})(2 \text{ m})^2 \\ &\quad + (20 \text{ kg})[(1.6 \text{ m})^2 + (0.8 \text{ m})^2] \end{aligned}$$

$$I_z = 74 \text{ kg}\cdot\text{m}^2$$

**Problem 18.85** The mass of the homogeneous slender bar is 30 kg. Determine the moment of inertia of the bar about the  $z'$  axis through its center of mass.

**Solution:** First locate the center of mass

$$\bar{x} = \frac{(10 \text{ kg})(0.3 \text{ m}) + (20 \text{ kg})(1.6 \text{ m})}{30 \text{ kg}} = 1.167 \text{ m}$$

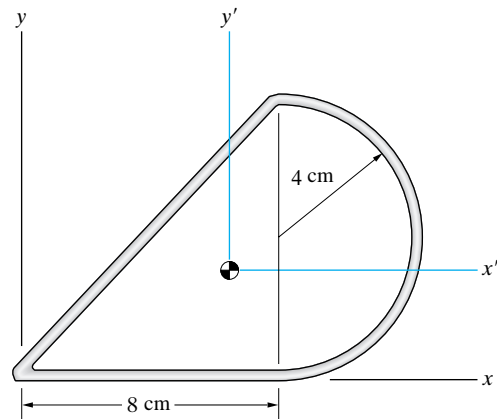
$$\bar{y} = \frac{(10 \text{ kg})(0.4 \text{ m}) + (20 \text{ kg})(0.8 \text{ m})}{30 \text{ kg}} = 0.667 \text{ m}$$

Using the answer to 18.100

$$I_{z'} = (74 \text{ kg}\cdot\text{m}^2) - (30 \text{ kg})(1.167^2 + 0.667^2)\text{m}^2$$

$$I_{z'} = 19.8 \text{ kg}\cdot\text{m}^2$$

**Problem 18.86** The homogeneous slender bar weighs 1.5 N. Determine its moment of inertia about the  $z$  axis.



**Solution:** The Bar's mass is  $m = 0.155 \text{ kg}$ . Its length is  $L = L_1 + L_2 + L_3 = 8 + \sqrt{8^2 + 8^2} + \pi(4) = 31.9 \text{ cm}$ . The masses of the parts are therefore,

$$M_1 = \frac{L_1}{L}m = \left(\frac{8}{31.9}\right)(0.155) = 0.0390 \text{ kg},$$

$$M_2 = \frac{L_2}{L}m = \left(\frac{\sqrt{2}(64)}{31.9}\right)(0.155) = 0.0551 \text{ kg},$$

$$M_3 = \frac{L_3}{L}m = \left(\frac{4\pi}{31.9}\right)(0.155) = 0.0612 \text{ kg}.$$

The center of mass of part 3 is located to the right of its center  $C$  a distance  $2R/\pi = 2(4)/\pi = 2.55 \text{ cm}$ . The moment of inertia of part 3 about  $C$  is

$$\int_{m_3} r^2 dm = m_3 r^2 = (0.0612)(4)^2 = 0.979 \text{ kg}\cdot\text{cm}^2.$$

The moment of inertia of part 3 about the center of mass of part 3 is therefore  $I_3 = 0.979 - m_3(2.55)^2 = 0.582 \text{ kg}\cdot\text{cm}^2$ . The moment of inertia of the bar about the  $z$  axis is

$$\begin{aligned} I_{(z \text{ axis})} &= \frac{1}{3}m_1L_1^2 + \frac{1}{3}m_2L_2^2 + I_3 + m_3[(8 + 2.55)^2 + (4)^2] \\ &= 11.6 \text{ kg}\cdot\text{cm}^2 = 0.00116 \text{ kg}\cdot\text{m}^2. \end{aligned}$$

**Problem 18.87** Determine the moment of inertia of the bar in Problem 18.86 about the  $z'$  axis through its center of mass.

**Solution:** In the solution of Problem 18.86, it is shown that the moment of inertia of the bar about the  $z$  axis is  $I_{(z \text{ axis})} = 11.6 \text{ kg}\cdot\text{cm}^2$ . The  $x$  and  $y$  coordinates of the center of mass coincide with the centroid of the bar:

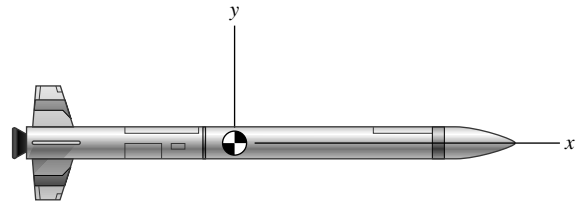
$$\begin{aligned} \bar{x} &= \frac{\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3}{L_1 + L_2 + L_3} \\ &= \frac{(4)(8) + (4)\sqrt{8^2 + 8^2} + \left[8 + \frac{2(4)}{\pi}\right]\pi(4)}{8 + \sqrt{8^2 + 8^2} + \pi(4)} = 6.58 \text{ cm}, \end{aligned}$$

$$\begin{aligned} \bar{y} &= \frac{\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_3 L_3}{L_1 + L_2 + L_3} \\ &= \frac{0 + (4)\sqrt{8^2 + 8^2} + \pi(4)(4)}{8 + \sqrt{8^2 + 8^2} + \pi(4)} = 3.00 \text{ cm}. \end{aligned}$$

The moment of inertia about the  $z'$  axis is

$$I_{(z' \text{ axis})} = I_{(z \text{ axis})} - (\bar{x}^2 + \bar{y}^2) (0.155) = 3.44 \text{ kg}\cdot\text{cm}^2.$$

**Problem 18.88** The rocket is used for atmospheric research. Its weight and its moment of inertia about the  $z$  axis through its center of mass (including its fuel) are 44480 N and 13826  $\text{kg}\cdot\text{m}^2$ , respectively. The rocket's fuel weighs 26688 N, its center of mass is located at  $x = -0.91 \text{ m}$ ,  $y = 0$ , and  $z = 0$ , and the moment of inertia of the fuel about the axis through the fuel's center of mass parallel to  $z$  axis is 2983  $\text{kg}\cdot\text{m}^2$ . When the fuel is exhausted, what is the rocket's moment of inertia about the axis through its new center of mass parallel to  $z$  axis?



**Solution:** Denote the moment of inertia of the empty rocket as  $I_E$  about a center of mass  $x_E$ , and the moment of inertia of the fuel as  $I_F$  about a mass center  $x_F$ . Using the parallel axis theorem, the moment of inertia of the filled rocket is

$$I_R = I_E + x_E^2 m_E + I_F + x_F^2 m_F,$$

about a mass center at the origin ( $x_R = 0$ ).

$$\text{Solve: } I_E = I_R - x_E^2 m_E - I_F - x_F^2 m_F.$$

The objective is to determine values for the terms on the right from the data given. Since the filled rocket has a mass center at the origin, the mass center of the empty rocket is found from

$$0 = m_E x_E + m_F x_F,$$

from which

$$x_E = -\left(\frac{m_F}{m_E}\right) x_F.$$

Using a value of  $g = 9.81 \text{ m/s}^2$ ,

$$m_F = \frac{W_F}{g} = \frac{26688}{9.81} = 2720 \text{ kg},$$

$$m_E = \frac{(W_R - W_F)}{g} = \frac{44480 - 26688}{9.81} = 1812.5 \text{ N}.$$

$$\text{From which } x_E = -\left(\frac{2720}{1812.5}\right) (-0.91) = 1.37 \text{ m}$$

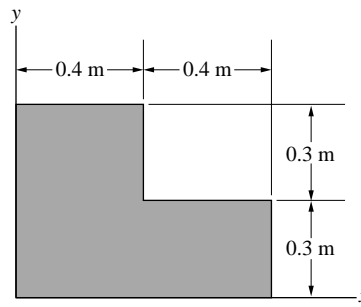
is the new location of the center of mass.

Substitute:

$$\begin{aligned} I_E &= I_R - x_E^2 m_E - I_F - x_F^2 m_F \\ &= 13826 - 3410 - 2983 - 2273 \\ &= 5151 \text{ kg}\cdot\text{m}^2 \end{aligned}$$



**Problem 18.89** The mass of the homogeneous thin plate is 36 kg. Determine the moment of inertia of the plate about the  $x$  axis.



**Solution:** Divide the plate into two areas: the rectangle 0.4 m by 0.6 m on the left, and the rectangle 0.4 m by 0.3 m on the right. The mass density is  $\rho = \frac{m}{A}$ .

The area is

$$A = (0.4)(0.6) + (0.4)(0.3) = 0.36 \text{ m}^2,$$

from which

$$\rho = \frac{36}{0.36} = 100 \text{ kg/m}^2.$$

The moment of inertia about the  $x$ -axis is

$$I_{x\text{-axis}} = \rho \left( \frac{1}{3} \right) (0.4)(0.6^3) + \rho \left( \frac{1}{3} \right) (0.4)(0.3)^3 = 3.24 \text{ kg}\cdot\text{m}^2$$

**Problem 18.90** Determine the moment of inertia of the 36-kg plate in Problem 18.89 about the  $z$  axis.

**Solution:** The basic relation to use is  $I_{z\text{-axis}} = I_{x\text{-axis}} + I_{y\text{-axis}}$ . The value of  $I_{x\text{-axis}}$  is given in the solution of Problem 18.89. The moment of inertia about the  $y$ -axis using the same divisions as in Problem 8.89 and the parallel axis theorem is

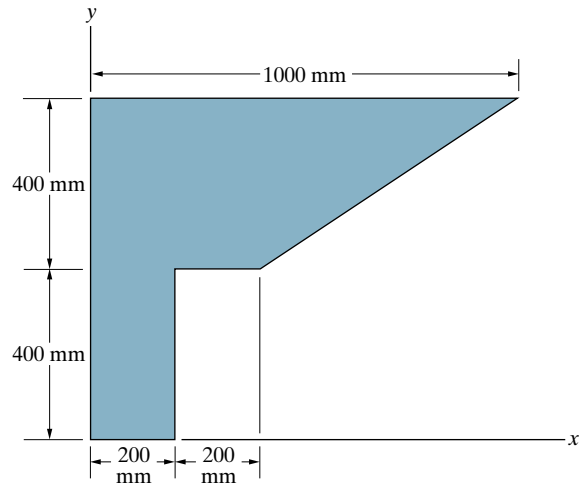
$$I_{y\text{-axis}} = \rho \left( \frac{1}{3} \right) (0.6)(0.4)^3 + \rho \left( \frac{1}{12} \right) (0.3)(0.4)^3$$

$$+ (0.6)^2 \rho (0.3)(0.4) = 5.76 \text{ kg}\cdot\text{m}^2,$$

from which

$$I_{z\text{-axis}} = I_{x\text{-axis}} + I_{y\text{-axis}} = 3.24 + 5.76 = 9 \text{ kg}\cdot\text{m}^2$$

**Problem 18.91** The mass of the homogeneous thin plate is 20 kg. Determine its moment of inertia about the  $x$  axis.



**Solution:** Break the plate into the three regions shown.

$$A = (0.2 \text{ m})(0.8 \text{ m}) + (0.2 \text{ m})(0.4 \text{ m})$$

$$+ \frac{1}{2}(0.4 \text{ m})(0.6 \text{ m}) = 0.36 \text{ m}^2$$

$$\rho = \frac{20 \text{ kg}}{0.36 \text{ m}^2} = 55.6 \text{ kg/m}^2$$

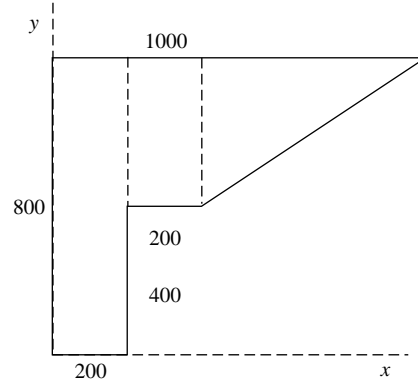
Using the integral tables we have

$$I_x = \frac{1}{3}(0.2 \text{ m})(0.8 \text{ m})^3 + \frac{1}{12}(0.2 \text{ m})(0.4 \text{ m})^3 + (0.2 \text{ m})(0.4 \text{ m})(0.6 \text{ m})^2$$

$$+ \frac{1}{36}(0.6 \text{ m})(0.4 \text{ m})^3 + \frac{1}{2}(0.6 \text{ m})(0.4 \text{ m})(0.667 \text{ m})^2$$

$$= 0.1184 \text{ m}^4$$

$$I_{x\text{-axis}} = (55.6 \text{ kg/m}^2)(0.1184 \text{ m}^4) = 6.58 \text{ kg}\cdot\text{m}^2$$



**Problem 18.92** The mass of the homogeneous thin plate is 20 kg. Determine its moment of inertia about the  $y$  axis.

**Solution:** See the solution to 18.91

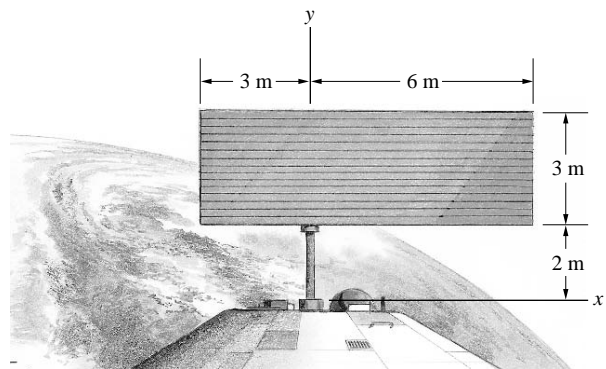
$$I_y = \frac{1}{3}(0.8 \text{ m})(0.2 \text{ m})^3 + \frac{1}{12}(0.4 \text{ m})(0.2 \text{ m})^3 + (0.2 \text{ m})(0.4 \text{ m})(0.3 \text{ m})^2$$

$$+ \frac{1}{36}(0.4 \text{ m})(0.6 \text{ m})^3 + \frac{1}{2}(0.6 \text{ m})(0.4 \text{ m})(0.6 \text{ m})^2$$

$$= 0.0552 \text{ m}^4$$

$$I_{y\text{-axis}} = (55.6 \text{ kg/m}^2)(0.0552 \text{ m}^4) = 3.07 \text{ kg}\cdot\text{m}^2$$

**Problem 18.93** The thermal radiator (used to eliminate excess heat from a satellite) can be modeled as a homogeneous thin rectangular plate. The mass of the radiator is 5 kg. Determine its moments of inertia about the  $x$ ,  $y$ , and  $z$  axes.



**Solution:** The area is  $A = 9(3) = 27 \text{ m}^2$ .

The mass density is

$$\rho = \frac{m}{A} = \frac{5}{27} = 0.1852 \text{ kg/m}^2.$$

The moment of inertia about the centroid of the rectangle is

$$I_{xc} = \rho \left( \frac{1}{12} \right) 9(3^3) = 3.75 \text{ kg}\cdot\text{m}^2,$$

$$I_{yc} = \rho \left( \frac{1}{12} \right) 3(9^3) = 33.75 \text{ kg}\cdot\text{m}^2.$$

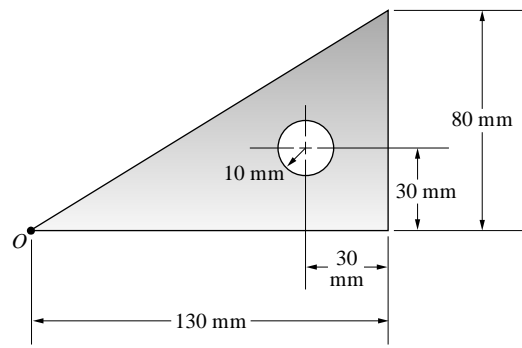
Use the parallel axis theorem:

$$I_{x\text{-axis}} = \rho A(2 + 1.5)^2 + I_{xc} = 65 \text{ kg}\cdot\text{m}^2,$$

$$I_{y\text{-axis}} = \rho A(4.5 - 3)^2 + I_{yc} = 45 \text{ kg}\cdot\text{m}^2.$$

$$I_{z\text{-axis}} = I_{x\text{-axis}} + I_{y\text{-axis}} = 110 \text{ kg}\cdot\text{m}^2$$

**Problem 18.94** The mass of the homogeneous thin plate is 2 kg. Determine the moment of inertia of the plate about the axis through point  $O$  that is perpendicular to the plate.



**Solution:** By determining the moments of inertia of the area about the  $x$  and  $y$  axes, we will determine the moments of inertia of the plate about the  $x$  and  $y$  axes, then sum them to obtain the moment of inertia about the  $z$  axis, which is  $I_0$ .

The areas are

$$A_1 = \frac{1}{2}(130)(80) \text{ mm}^2,$$

$$A_2 = \pi(10)^2 \text{ mm}^2.$$

Using Appendix B,

$$I_x = \frac{1}{12}(130)(80)^3 - \left[ \frac{1}{4}\pi(10)^4 + (30)^2 A_2 \right]$$

$$= 5.26 \times 10^6 \text{ mm}^4,$$

$$I_y = \frac{1}{4}(80)(130)^3 - \left[ \frac{1}{4}\pi(10)^4 + (100)^2 A_2 \right]$$

$$= 40.79 \times 10^6 \text{ mm}^4.$$

Therefore

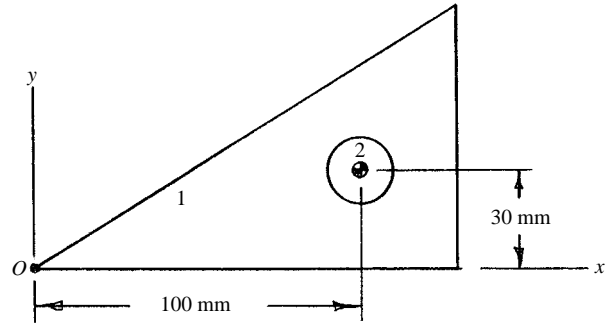
$$I_{(x \text{ axis})} = \frac{m}{A_1 - A_2} I_x = 2150 \text{ kg}\cdot\text{mm}^2,$$

$$I_{(y \text{ axis})} = \frac{m}{A_1 - A_2} I_y = 16700 \text{ kg}\cdot\text{mm}^2.$$

Then

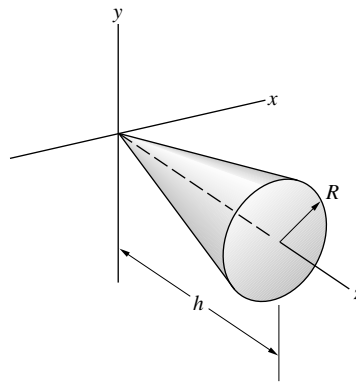
$$I_{(z \text{ axis})} = I_{(x \text{ axis})} + I_{(y \text{ axis})} = 18850 \text{ kg}\cdot\text{mm}^2.$$

$$\underline{I_{(z \text{ axis})} = 0.0188 \text{ kg}\cdot\text{m}^2.}$$



**Problem 18.95** The homogeneous cone is of mass  $m$ . Determine its moment of inertia about the  $z$  axis, and compare your result with the value given in Appendix C. (See Example 18.10.)

**Strategy:** Use the same approach we used in Example 18.10 to obtain the moments of inertia of a homogeneous cylinder.



**Solution:** The differential mass

$$dm = \left(\frac{m}{V}\right) \pi r^2 dz = \frac{3m}{R^2 h} r^2 dz.$$

The moment of inertia of this disk about the  $z$ -axis is  $\frac{1}{2}mr^2$ . The radius varies with  $z$ ,  $r = \left(\frac{R}{h}\right)z$ , from which

$$I_{z\text{-axis}} = \frac{3mR^2}{2h^5} \int_0^h z^4 dz = \frac{3mR^2}{2h^5} \left[\frac{z^5}{5}\right]_0^h = \frac{3mR^2}{10}$$

**Problem 18.96** Determine the moments of inertia of the homogeneous cone in Problem 18.95 about the  $x$  and  $y$  axes, and compare your results with the values given in Appendix C. (See Example 18.10.)

**Solution:** The mass density is  $\rho = \frac{m}{V} = \frac{3m}{\pi R^2 h}$ . The differential element of mass is  $dm = \rho \pi r^2 dz$ . The moment of inertia of this elemental disk about an axis through its center of mass, parallel to the  $x$ - and  $y$ -axes, is  $dI_x = \left(\frac{1}{4}\right)r^2 dm$ . Use the parallel axis theorem,

$$I_x = \int_m \left(\frac{1}{4}\right)r^2 dm + \int_m z^2 dm.$$

Noting that  $r = \frac{R}{h}z$ , then

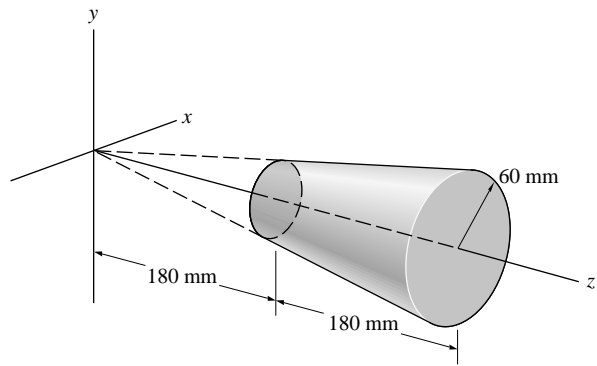
$$r^2 dm = \rho \left(\frac{\pi R^4}{h^4}\right) z^4 dz,$$

and  $z^2 dm = \rho \left(\frac{\pi R^2}{h^2}\right) z^4 dz$ . Substitute:

$$I_x = \rho \left(\frac{\pi R^4}{4h^4}\right) \int_0^h z^4 dz + \rho \left(\frac{\pi R^2}{h^2}\right) \int_0^h z^4 dz,$$

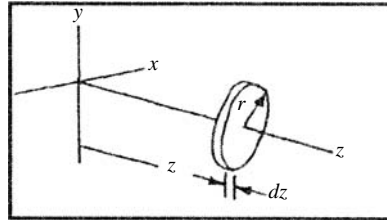
$$I_x = \left(\frac{3mR^2}{4h^5} + \frac{3m}{h^3}\right) \left[\frac{z^5}{5}\right]_0^h = m \left(\frac{3}{20}R^2 + \frac{3}{5}h^2\right) = I_y.$$

**Problem 18.97** The homogeneous object has the shape of a truncated cone and consists of bronze with mass density  $\rho = 8200 \text{ kg/m}^3$ . Determine the moment of inertia of the object about the  $z$  axis.



**Solution:** Consider an element of the cone consisting of a disk of thickness  $dz$ : We can express the radius as a linear function of  $z$   $r = az + b$ . Using the conditions that  $r = 0$  at  $z = 0$  and  $r = 0.06 \text{ m}$  at  $z = 0.36 \text{ m}$  to evaluate  $a$  and  $b$  we find that  $r = 0.167z$ . From Appendix C, the moment of inertia of the element about the  $z$  axis is

$$(I_z)_{\text{element}} = \frac{1}{2}mr^2 = \frac{1}{2}[\rho(\pi r^2)dz]r^2 = \frac{1}{2}\rho\pi(0.167z)^4 dz.$$



We integrate this result to obtain the mass moment of inertia about the  $z$  axis for the cone:

$$\begin{aligned} I_{(z \text{ axis})} &= \int_{0.18}^{0.36} \frac{1}{2}\rho\pi(0.167)^4 \left[ \frac{z^5}{5} \right]_{0.18}^{0.36} dz \\ &= \frac{1}{2}(8200)\pi(0.167)^4 \left[ \frac{z^5}{5} \right]_{0.18}^{0.36} \\ &= 0.0116 \text{ kg}\cdot\text{m}^2. \end{aligned}$$

**Problem 18.98** Determine the moment of inertia of the object in Problem 18.97 about the  $x$  axis.

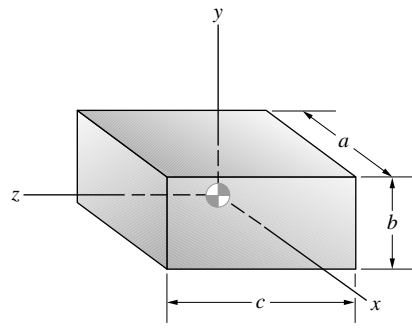
**Solution:** Consider the disk element described in the solution to Problem 18.97. The radius of the laminate is  $r = 0.167z$ . Using Appendix C and the parallel axis theorem, the moment of inertia of the element about the  $x$  axis is

$$\begin{aligned} (I_x)_{\text{element}} &= \frac{1}{4}mr^2 + mz^2 = \frac{1}{4}[\rho(\pi r^2)dz]r^2 + [\rho(\pi r^2)dz]z^2 \\ &= \frac{1}{4}\rho\pi(0.167z)^4 dz + \rho\pi(0.167z)^2 z^2 dz. \end{aligned}$$

Integrating the result,

$$\begin{aligned} I_{(x \text{ axis})} &= \frac{1}{4}\rho\pi(0.167)^4 \int_{0.18}^{0.36} z^4 dz + \rho\pi(0.167)^2 \int_{0.18}^{0.36} z^4 dz \\ &= 0.844 \text{ kg}\cdot\text{m}^2. \end{aligned}$$

**Problem 18.99** The homogeneous rectangular parallelepiped is of mass  $m$ . Determine its moments of inertia about the  $x$ ,  $y$ , and  $z$  axes and compare your results with the values given in Appendix C.



**Solution:** Consider a rectangular slice normal to the  $x$ -axis of dimensions  $b$  by  $c$  and mass  $dm$ . The area density of this slice is  $\rho = \frac{dm}{bc}$ . The moment of inertia about the  $y$  axis of the centroid of a thin plate is the product of the area density and the area moment of inertia of the plate:  $dI_y = \rho \left( \frac{1}{12} \right) bc^3$ , from which  $dI_y = \left( \frac{1}{12} \right) c^2 dm$ . By symmetry, the moment of inertia about the  $z$  axis is

By symmetry, the argument can be repeated for each coordinate, to obtain

$$dI_z = \left( \frac{1}{12} \right) b^2 dm.$$

$$I_y = \frac{m}{12} (a^2 + c^2) \quad I_z = \frac{m}{12} (b^2 + a^2)$$

Since the labeling of the  $x$ -  $y$ - and  $z$ -axes is arbitrary,

$$dI_x = dI_z + dI_y,$$

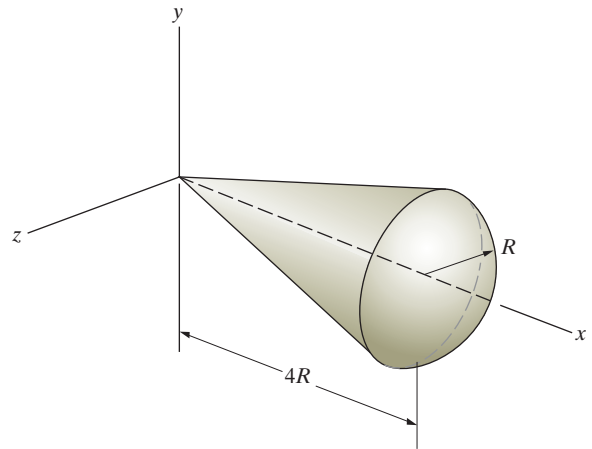
where the  $x$ -axis is normal to the area of the plate. Thus

$$dI_x = \left( \frac{1}{12} \right) (b^2 + c^2) dm,$$

from which

$$I_x = \left( \frac{1}{12} \right) (b^2 + c^2) \int_m dm = \frac{m}{12} (b^2 + c^2).$$

**Problem 18.100** The sphere-capped cone consists of material with density  $7800 \text{ kg/m}^3$ . The radius  $R = 80 \text{ mm}$ . Determine its moment of inertia about the  $x$  axis.



**Solution:** Given  $\rho = 7800 \text{ kg/m}^3$ ,  $R = 0.08 \text{ m}$

Using the tables we have

$$I_x = \frac{3}{10} \left( \rho \frac{1}{3} \pi R^2 [4R] \right) R^2 + \frac{2}{5} \left( \rho \frac{2}{3} \pi R^3 \right) R^2$$

$$I_x = 0.0535 \text{ kg-m}^2$$

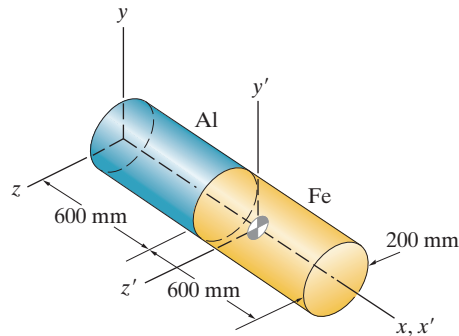
**Problem 18.101** Determine the moment of inertia of the sphere-capped cone described in Problem 18.100 about the  $y$  axis.

**Solution:** The center of mass of a half-sphere is located a distance  $\frac{3R}{8}$  from the geometric center of the circle.

$$I_y = \left(\frac{1}{3}\pi R^2[4R]\right)\left(\frac{3}{5}[4R]^2 + \frac{3}{20}R^2\right) + \frac{2}{5}\left(\rho\frac{2}{3}\pi R^3\right)R^2 - \left(\rho\frac{2}{3}\pi R^3\right)\left(\frac{3R}{8}\right)^2 + \left(\rho\frac{2}{3}\pi R^3\right)\left(4R + \frac{3R}{8}\right)^2$$

$$I_y = 2.08 \text{ kg}\cdot\text{m}^2$$

**Problem 18.102** The circular cylinder is made of aluminum (Al) with density  $2700 \text{ kg/m}^3$  and iron (Fe) with density  $7860 \text{ kg/m}^3$ . Determine its moment of inertia about the  $x'$  axis.



**Solution:**

$$I_x = \frac{1}{2}[(2700 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(0.6 \text{ m})](0.1 \text{ m})^2 + \frac{1}{2}[(7860 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(0.6 \text{ m})](0.1 \text{ m})^2$$

$$I_x = 0.995 \text{ kg}\cdot\text{m}^2$$

**Problem 18.103** Determine the moment of inertia of the composite cylinder in Problem 18.102 about the  $y'$  axis.

**Solution:** First locate the center of mass

$$\bar{x} = \frac{[(2700 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(0.6 \text{ m})](0.3 \text{ m}) + [(7860 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(0.6 \text{ m})](0.9 \text{ m})}{(2700 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(0.6 \text{ m}) + (7860 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(0.6 \text{ m})}$$

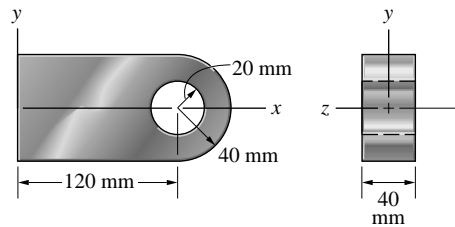
$$\bar{x} = 0.747 \text{ m}$$

$$I_y = [(2700 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(0.6 \text{ m})]\left[\frac{1}{12}(0.6 \text{ m})^2 + \frac{1}{4}(0.1 \text{ m})^2\right] + [(2700 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(0.6 \text{ m})](\bar{x} - 0.3 \text{ m})^2 + [(7860 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(0.6 \text{ m})]\left[\frac{1}{12}(0.6 \text{ m})^2 + \frac{1}{4}(0.1 \text{ m})^2\right] + [(7860 \text{ kg/m}^3)\pi(0.1 \text{ m})^2(0.6 \text{ m})](0.9 \text{ m} - \bar{x})^2$$

$$I_y = 20.1 \text{ kg}\cdot\text{m}^2$$



**Problem 18.104** The homogeneous machine part is made of aluminum alloy with mass density  $\rho = 2800 \text{ kg/m}^3$ . Determine the moment of inertia of the part about the  $z$  axis.



**Solution:** We divide the machine part into the 3 parts shown: (The dimension into the page is 0.04 m) The masses of the parts are

$$m_1 = (2800)(0.12)(0.08)(0.04) = 1.075 \text{ kg},$$

$$m_2 = (2800)\frac{1}{2}\pi(0.04)^2(0.04) = 0.281 \text{ kg},$$

$$m_3 = (2800)\pi(0.02)^2(0.04) = 0.141 \text{ kg}.$$

Using Appendix C and the parallel axis theorem the moment of inertia

of part 1 about the  $z$  axis is

$$\begin{aligned} I_{(z \text{ axis})1} &= \frac{1}{12}m_1[(0.08)^2 + (0.12)^2] + m_1(0.06)^2 \\ &= 0.00573 \text{ kg}\cdot\text{m}^2. \end{aligned}$$

The moment of inertia of part 2 about the axis through the center  $C$

that is parallel to the  $z$  axis is

$$\frac{1}{2}m_2R^2 = \frac{1}{2}m_2(0.04)^2$$

The distance along the  $x$  axis from  $C$  to the center of mass of part 2 is  $4(0.04)/(3\pi) = 0.0170 \text{ m}$ . Therefore, the moment of inertia of part 2 about the  $z$  axis through its center of mass that is parallel to the axis is

$$\frac{1}{2}m_2(0.04)^2 - m_2(0.0170)^2 = 0.000144 \text{ kg}\cdot\text{m}^2.$$

Using this result, the moment of inertia of part 2 about the  $z$  axis is

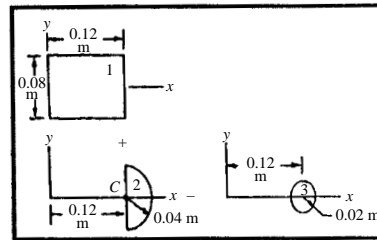
$$I_{(z \text{ axis})2} = 0.000144 + m_2(0.12 + 0.017)^2 = 0.00544 \text{ kg}\cdot\text{m}^2.$$

The moment of inertia of the material that would occupy the hole 3 about the  $z$  axis is

$$I_{(z \text{ axis})3} = \frac{1}{2}m_3(0.02)^2 + m_3(0.12)^2 = 0.00205 \text{ kg}\cdot\text{m}^2.$$

Therefore,

$$I_{(z \text{ axis})} = I_{(z \text{ axis})1} + I_{(z \text{ axis})2} - I_{(z \text{ axis})3} = 0.00911 \text{ kg}\cdot\text{m}^2.$$



**Problem 18.105** Determine the moment of inertia of the machine part in Problem 18.104 about the  $x$  axis.

**Solution:** We divide the machine part into the 3 parts shown in the solution to Problem 18.104. Using Appendix C and the parallel axis theorem, the moments of inertia of the parts about the  $x$  axis are:

$$I_{(x \text{ axis})_1} = \frac{1}{12} m_1 [(0.08)^2 + (0.04)^2]$$

$$= 0.0007168 \text{ kg}\cdot\text{m}^2$$

$$I_{(x \text{ axis})_2} = m_2 \left[ \frac{1}{12} (0.04)^2 + \frac{1}{4} (0.04)^2 \right]$$

$$= 0.0001501 \text{ kg}\cdot\text{m}^2$$

$$I_{(x \text{ axis})_3} = m_3 \left[ \frac{1}{12} (0.04)^2 + \frac{1}{4} (0.02)^2 \right]$$

$$= 0.0000328 \text{ kg}\cdot\text{m}^2.$$

Therefore,

$$I_{(x \text{ axis})} = I_{(x \text{ axis})_1} + I_{(x \text{ axis})_2} - I_{(x \text{ axis})_3}$$

$$= 0.000834 \text{ kg}\cdot\text{m}^2.$$

**Problem 18.106** The object shown consists of steel of density  $\rho = 7800 \text{ kg/m}^3$  of width  $w = 40 \text{ mm}$ . Determine the moment of inertia about the axis  $L_0$ .

**Solution:** Divide the object into four parts:

Part (1): The semi-cylinder of radius  $R = 0.02 \text{ m}$ , height  $h_1 = 0.01 \text{ m}$ .

Part (2): The rectangular solid  $L = 0.1 \text{ m}$  by  $h_2 = 0.01 \text{ m}$  by  $w = 0.04 \text{ m}$ .

Part (3): The semi-cylinder of radius  $R = 0.02 \text{ m}$ ,  $h_1 = 0.01 \text{ m}$

Part (4): The cylinder of radius  $R = 0.02 \text{ m}$ , height  $h = 0.03 \text{ m}$ .

$$\text{Part (1)} \quad m_1 = \frac{\rho \pi R^2 h_1}{2} = 0.049 \text{ kg},$$

$$I_1 = \frac{m_1 R^2}{4} = 4.9 \times 10^{-6} \text{ kg}\cdot\text{m}^2,$$

$$\text{Part (2)} \quad m_2 = \rho w L h_2 = 0.312 \text{ kg},$$

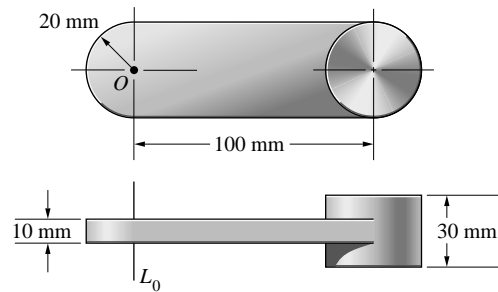
$$I_2 = (1/12) m_2 (L^2 + w^2) + m_2 (L/2)^2$$

$$= 0.00108 \text{ kg}\cdot\text{m}^2.$$

$$\text{Part (3)} \quad m_3 = m_1 = 0.049 \text{ kg},$$

$$I_3 = - \left( \frac{4R}{3\pi} \right)^2 m_2 + I_1 + m_3 \left( L - \frac{4R}{3\pi} \right)^2$$

$$= 0.00041179 \text{ kg}\cdot\text{m}^2.$$



$$\text{Part (4)} \quad m_4 = \rho \pi R^2 h = 0.294 \text{ kg},$$

$$I_4 = \left( \frac{1}{2} \right) m_4 (R^2) + m_4 L^2 = 0.003 \text{ kg}\cdot\text{m}^2.$$

The composite:

$$I_{L0} = I_1 + I_2 - I_3 + I_4 = 0.00367 \text{ kg}\cdot\text{m}^2$$

**Problem 18.107** Determine the moment of inertia of the object in Problem 18.106 about the axis through the center of mass of the object parallel to  $L_0$ .

**Solution:** The center of mass is located relative to  $L_0$  is given by

$$\bar{x} = \frac{m_1 \left( -\frac{4R}{3\pi} \right) + m_2(0.05) - m_3 \left( 0.1 - \frac{4R}{3\pi} \right) + m_4(0.1)}{m_1 + m_2 - m_3 + m_4}$$

$$= 0.066 \text{ m,}$$

$$I_c = -\bar{x}^2 m + I_{L_0} = -0.00265 + 0.00367 = 0.00102 \text{ kg-m}^2$$

**Problem 18.108** The thick plate consists of steel of density  $\rho = 7729 \text{ kg/m}^3$ . Determine the moment of inertia of the plate about the  $z$  axis.

**Solution:** Divide the object into three parts: Part (1) the rectangle 8 cm by 16 cm, Parts (2) & (3) the cylindrical cut outs.

Part (1):  $m_1 = \rho 8(16)(4) = 3.96 \text{ kg.}$

$$I_1 = (1/12)m_1(16^2 + 8^2) = 105.6 \text{ kg-cm}^2.$$

Part (2):  $m_2 = \rho \pi (2^2)(4) = 0.388 \text{ kg,}$

$$I_2 = \frac{m_2(2^2)}{2} + m_2(4^2) = 7 \text{ kg-cm}^2.$$

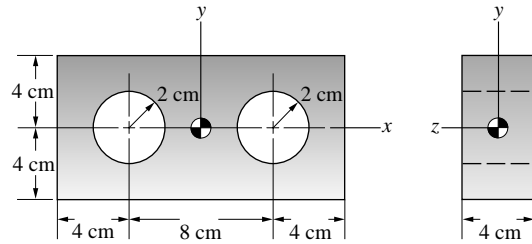
Part (3):  $m_3 = m_2 = 0.388 \text{ kg,}$

$$I_3 = I_2 = 7 \text{ kg-cm}^2.$$

The composite:

$$I_{z\text{-axis}} = I_1 - 2I_2 = 91.6 \text{ kg-cm}^2$$

$$I_{z\text{-axis}} = 0.00916 \text{ kg-m}^2$$



**Problem 18.109** Determine the moment of inertia of the object in Problem 18.108 about the  $x$  axis.

**Solution:** Use the same divisions of the object as in Problem 18.108.

Part (1):  $I_{1x\text{-axis}} = \left( \frac{1}{12} \right) m_1(8^2 + 4^2) = 26.4 \text{ kg-cm}^2,$

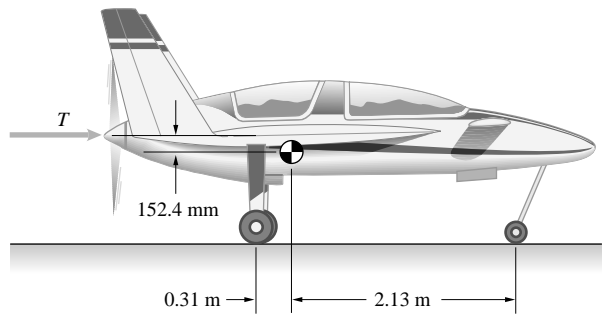
Part (2):  $I_{2x\text{-axis}} = (1/12)m_2(3(2^2) + 4^2) = 0.91 \text{ kg-cm}^2.$

The composite:

$$I_{x\text{-axis}} = I_{1x\text{-axis}} - 2I_{2x\text{-axis}} = 24.6 \text{ kg-cm}^2$$

$$= 0.00246 \text{ kg-m}^2$$

**Problem 18.110** The airplane is at the beginning of its takeoff run. Its weight is 4448 N and the initial thrust  $T$  exerted by its engine is 1334 N. Assume that the thrust is horizontal, and neglect the tangential forces exerted on its wheels.



- (a) If the acceleration of the airplane remains constant, how long will it take to reach its takeoff speed of 128.7 km/h  
 (b) Determine the normal force exerted on the forward landing gear at the beginning of the takeoff run.

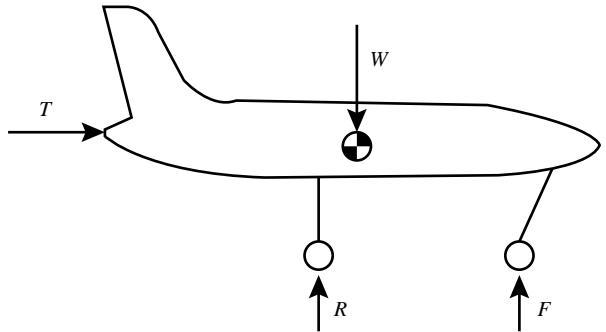
**Solution:** The acceleration under constant thrust is

$$a = \frac{T}{m} = \frac{1334(9.81)}{4448} = 2.94 \text{ m/s}^2.$$

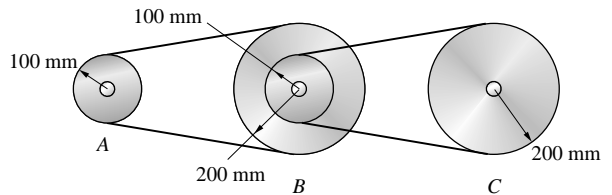
The time required to reach 128.7 km/h = 35.8 m/s is

$$t = \frac{v}{a} = \frac{35.8}{2.94} = 12.1 \text{ s}$$

The sum of the vertical forces:  $\sum F_y = R + F - W = 0$ . The sum of the moments:  $\sum M = 2.13F - 0.152T - 0.31R = 0$ . Solve:  $R = 3809 \text{ N}$ ,  $F = 639 \text{ N}$



**Problem 18.111** The pulleys can turn freely on their pin supports. Their moments of inertia are  $I_A = 0.002 \text{ kg}\cdot\text{m}^2$ ,  $I_B = 0.036 \text{ kg}\cdot\text{m}^2$ , and  $I_C = 0.032 \text{ kg}\cdot\text{m}^2$ . They are initially stationary, and at  $t = 0$  a constant  $M = 2 \text{ N}\cdot\text{m}$  is applied at pulley A. What is the angular velocity of pulley C and how many revolutions has it turned at  $t = 2 \text{ s}$ ?



**Solution:** Denote the upper and lower belts by the subscripts  $U$  and  $L$ . Denote the difference in the tangential component of the tension in the belts by

$$\Delta T_A = T_{LA} - T_{UA},$$

$$\Delta T_B = T_{LB} - T_{UB}.$$

From the equation of angular motion:  $M + R_A \Delta T_A = I_A \alpha_A$ ,  $-R_{B1} \Delta T_A + R_{B2} \Delta T_B = I_B \alpha_B$ ,  $-R_C \Delta T_B = I_C \alpha_C$ . From kinematics,  $R_A \alpha_A = R_{B1} \alpha_B$ ,  $R_{B2} \alpha_B = R_C \alpha_C$ , from which

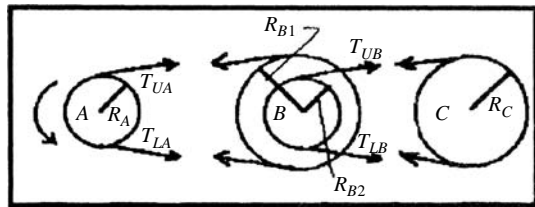
$$\alpha_A = \frac{R_{B1} R_C}{R_A R_{B2}} \alpha_C = \frac{(0.2)(0.2)}{(0.1)(0.1)} \alpha_C = 4\alpha_C,$$

$$\alpha_B = \frac{R_C}{R_{B2}} \alpha_C = \frac{0.2}{0.1} \alpha_C = 2\alpha_C.$$

Substitute and solve:  $\alpha_C = 38.5 \text{ rad/s}^2$ , from which

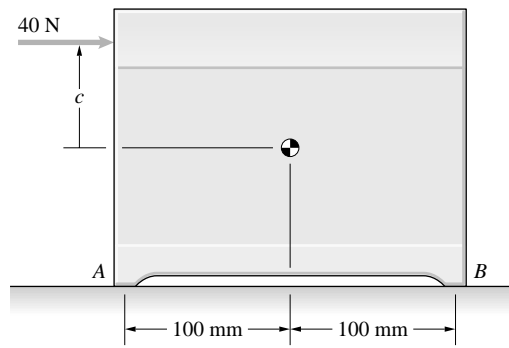
$$\omega_C = \alpha_C t = 76.9 \text{ rad/s}$$

$$N = \theta \left( \frac{1}{2\pi} \right) = \frac{\alpha_C}{4\pi} (t^2) = 12.2 \text{ revolutions}$$



**Problem 18.112** A 2 kg box is subjected to a 40-N horizontal force. Neglect friction.

- If the box remains on the floor, what is its acceleration?
- Determine the range of values of  $c$  for which the box will remain on the floor when the force is applied.



**Solution:**

- From Newton's second law,  $40 = (2)a$ , from which

$$a = \frac{40}{2} = 20 \text{ m/s}^2$$

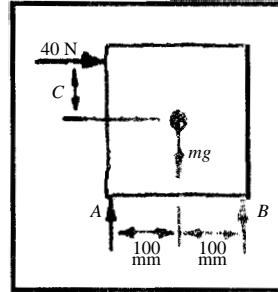
- The sum of forces:  $\sum F_y = A + B - mg = 0$ . The sum of the moments about the center of mass:  $\sum M = 0.1B - 0.1A - 40c = 0$ . Substitute the value of B from the first equation into the second equation and solve for  $c$ :

$$c = \frac{(0.1)mg - (0.2)A}{40}$$

The box leg at A will leave the floor as  $A \leq 0$ , from which

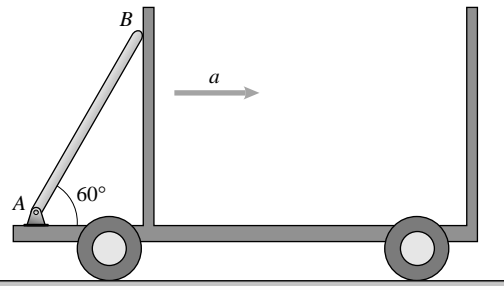
$$c \leq \frac{(0.1)(2)(9.81)}{40} \leq 0.0491 \text{ m}$$

for values of  $A \geq 0$ .



**Problem 18.113** The slender, 2-kg bar  $AB$  is 3 m long. It is pinned to the cart at  $A$  and leans against it at  $B$ .

- (a) If the acceleration of the cart is  $a = 20 \text{ m/s}^2$ , what normal force is exerted on the bar by the cart at  $B$ ?  
 (b) What is the largest acceleration  $a$  for which the bar will remain in contact with the surface at  $B$ ?



**Solution:** Newton's second law applied to the center of mass of the bar yields

$$-B + A_x = ma_{G_x}, \quad A_y - W = ma_{G_y},$$

$$-A_y \left( \frac{L \cos \theta}{2} \right) + (B + A_x) \left( \frac{L \sin \theta}{2} \right) = I_G \alpha,$$

where  $a_{G_x}$ ,  $a_{G_y}$  are the accelerations of the center of mass. From kinematics,

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{G/A} - \omega_{AB}^2 \mathbf{r}_{G/A} = 20\mathbf{i} \text{ m/s}^2$$

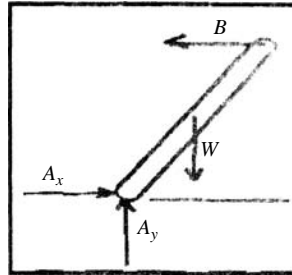
where  $\alpha = 0$ ,  $\omega_{AB} = 0$  so long as the bar is resting on the cart at  $B$  and is pinned at  $A$ . Substitute the kinematic relations to obtain three equations in three unknowns:

$$-B + A_x = ma, \quad A_y - W = 0,$$

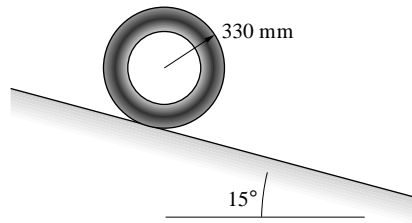
$$-A_y \left( \frac{L \cos \theta}{2} \right) + (B + A_x) \left( \frac{L \sin \theta}{2} \right) = 0.$$

Solve:  $B = \frac{W \cot \theta}{2} - \frac{ma}{2}$ . For  $W = mg = 19.62 \text{ N}$ ,  $\theta = 60^\circ$ ,  $m = 2 \text{ kg}$ , and  $a = 20 \text{ m/s}^2$ ,  $B = -14.34 \text{ N}$ , from which the bar has moved away from the cart at point  $B$ . (b) The acceleration that produces a zero normal force is

$$a = g \cot \theta = 5.66 \text{ m/s}^2.$$



**Problem 18.114** To determine a 4.5-kg tire's moment of inertia, an engineer lets the tire roll down an inclined surface. If it takes the tire 3.5 s to start from rest and roll 3 m down the surface, what is the tire's moment of inertia about its center of mass?



**Solution:** From Newton's second law and the angular equation of motion,

$$mg \sin 15^\circ - f = ma,$$

$$Rf = I\alpha.$$

From these equations and the relation  $a = R\alpha$ , we obtain

$$a = \frac{mg \sin 15^\circ}{m + I/R^2}. \quad (1)$$

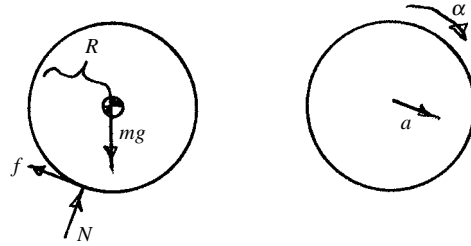
We can determine the acceleration from

$$s = \frac{1}{2} at^2 :$$

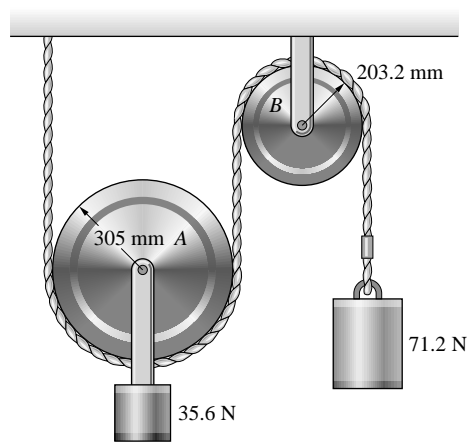
$$3 = \frac{1}{2} a(3.5)^2,$$

obtaining  $a = 0.490 \text{ m/s}^2$ . Then from Eq. (1) we obtain

$$\underline{I = 2.05 \text{ kg}\cdot\text{m}^2}.$$



**Problem 18.115** Pulley A weighs 17.8 N,  $I_A = 0.081 \text{ kg}\cdot\text{m}^2$ , and  $I_B = 0.019 \text{ kg}\cdot\text{m}^2$ . If the system is released from rest, what distance does the 71.2 N weight fall in 0.5 s?



**Solution:** The strategy is to apply Newton's second law and the equation of angular motion to the free body diagrams. Denote the rightmost weight by  $W_R = 71.2 \text{ N}$ , the mass by  $m_R = 7.26 \text{ kg}$ , and the leftmost weight by  $W_L = 17.8 + 35.6 = 53.4 \text{ N}$ , and the mass by  $m_L = 1.66 \text{ kg}$ .  $R_B = 0.203 \text{ m}$  is the radius of pulley B,  $I_B = 0.019 \text{ kg}\cdot\text{m}^2$ , and  $R_A = 0.305 \text{ m}$  is the radius of pulley A, and  $I_A = 0.081 \text{ kg}\cdot\text{m}^2$ . Choose a coordinate system with the  $y$  axis positive upward.

The 71.2 N weight: (1)  $T_1 - W_R = m_R a_{Ry}$ .

Pulley B: The center of the pulley is constrained against motion, and the acceleration of the rope is equal (except for direction) on each side of the pulley. (2)  $-R_B T_1 + R_B T_2 = I_B \alpha_B$ . From kinematics, (3)  $a_{Ry} = R_B \alpha_B$ . Combine (1), (2) and (3) and reduce:

$$(4) T_2 = W_R + \left( \frac{I_B}{R_B^2} + m_R \right) a_{Ry}$$

Pulley A: (5)  $T_2 + T_3 - W_L = m_L a_{Ay}$ , where  $a_{Ay}$  is the acceleration of the center of the pulley. (6)  $-R_A T_3 + R_A T_2 = I_A \alpha_A$ . From the kinematics of pulley A, the acceleration of the left side of the pulley is zero, so that the acceleration of the right side relative to the left side is

$$\mathbf{a}_{\text{right}} = -a_{Ry} \mathbf{j} = \mathbf{a}_{\text{left}} + \boldsymbol{\alpha}_A \times (2R_A \mathbf{i})$$

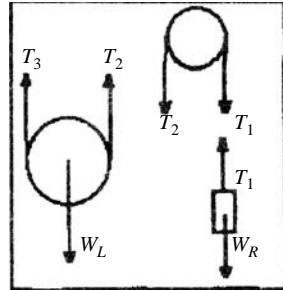
$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_A \\ 2R_A & 0 & 0 \end{bmatrix} = 0 + 2R_A \alpha_A \mathbf{j},$$

from which (7)  $a_{Ry} = -2R_A \alpha_A$ , where the change in direction of the acceleration of the 71.2 N weight across pulley B is taken into account. Similarly, the acceleration of the right side relative to the acceleration of the center of the pulley is

$$\mathbf{a}_{\text{right}} = -a_{Ry} \mathbf{j} = \mathbf{a}_A + \boldsymbol{\alpha}_A \times (R_A \mathbf{i}) = \mathbf{a}_A + R_A \alpha_A \mathbf{j},$$

from which (8)  $a_{Ay} = -\frac{a_{Ry}}{2}$ . Combine (5), (6), (7) and (8) and reduce

$$\text{to obtain (9) } T_2 = \frac{W_A}{2} - \left( \frac{I_A}{4R_A^2} + \frac{m_A}{4} \right) a_y.$$



The total system: Equate (4) and (9) (the two expressions for  $T_2$ ) and solve:

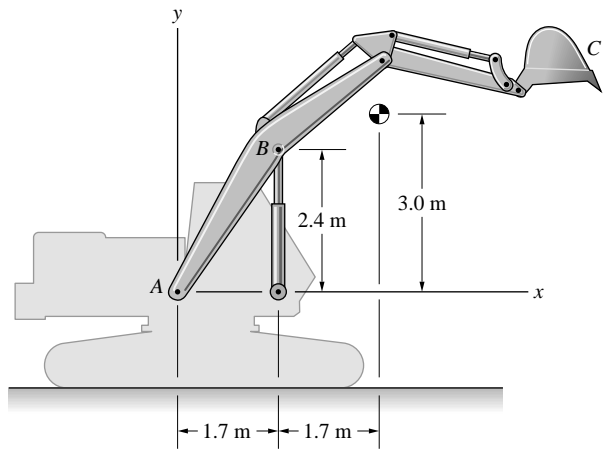
$$a_{Ry} = \frac{\left( \frac{W_L}{2} - W_R \right)}{\left( + \frac{I_B}{R_B^2} + m_R + \frac{I_A}{4R_A^2} + \frac{m_L}{4} \right)}.$$

Substitute numerical values:  $a_{Ry} = -4.78 \text{ m/s}^2$ . The distance that the 71.2 N weight will fall in one-half second is

$$s = \frac{a_{Ry}}{2} t^2 = \frac{-4.78}{8} = -0.6 \text{ m}$$



**Problem 18.116** Model the excavator's arm  $ABC$  as a single rigid body. Its mass is 1200 kg, and the moment of inertia about its center of mass is  $I = 3600 \text{ kg}\cdot\text{m}^2$ . If point  $A$  is stationary, the angular velocity of the arm is zero, and the angular acceleration is  $1.0 \text{ rad/s}^2$  counterclockwise, what force does the vertical hydraulic cylinder exert on the arm at  $B$ ?



**Solution:** The distance from  $A$  to the center of mass is

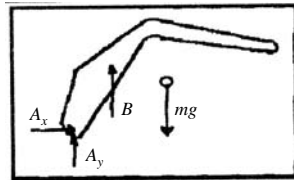
$$d = \sqrt{(3.4)^2 + (3)^2} = 4.53 \text{ m.}$$

The moment of inertia about  $A$  is

$$I_A = I + d^2m = 28,270 \text{ kg}\cdot\text{m}^2.$$

From the equation of angular motion:  $1.7B - 3.4mg = I_A\alpha$ .

Substitute  $\alpha = 1.0 \text{ rad/s}^2$ , to obtain  $B = 40,170 \text{ N}$ .



**Problem 18.117** Model the excavator's arm  $ABC$  as a single rigid body. Its mass is 1200 kg, and the moment of inertia about its center of mass is  $I = 3600 \text{ kg}\cdot\text{m}^2$ . The angular velocity of the arm is  $2 \text{ rad/s}$  counterclockwise and its angular acceleration is  $1 \text{ rad/s}^2$  counterclockwise. What are the components of the force exerted on the arm at  $A$ ?

**Solution:** The acceleration of the center of mass is

$$\mathbf{a}_G = \boldsymbol{\alpha} \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ 3.4 & 3 & 0 \end{bmatrix} - \omega^2(3.4\mathbf{i} + 3\mathbf{j})$$

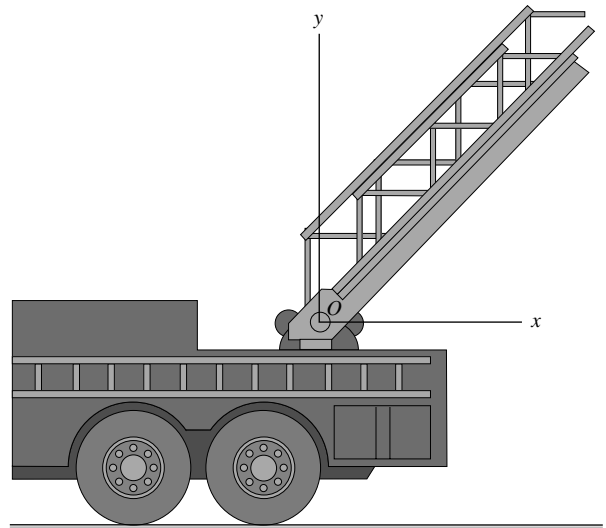
$$= -16.6\mathbf{i} - 8.6\mathbf{j} \text{ m/s}^2.$$

From Newton's second law:

$$A_x = ma_{Gx} = -19,900 \text{ N}, \quad A_y + B - mg = ma_{Gy}.$$

From the solution to Problem 18.132,  $B = 40,170 \text{ N}$ , from which  $A_y = -38,720 \text{ N}$

**Problem 18.118** To decrease the angle of elevation of the stationary 200-kg ladder, the gears that raised it are disengaged, and a fraction of a second later a second set of gears that lower it are engaged. At the instant the gears that raised the ladder are disengaged, what is the ladder's angular acceleration and what are the components of force exerted on the ladder by its support at  $O$ ? The moment of inertia of the ladder about  $O$  is  $I_0 = 14,000 \text{ kg}\cdot\text{m}^2$ , and the coordinates of its center of mass at the instant the gears are disengaged are  $x = 3 \text{ m}$ ,  $y = 4 \text{ m}$ .



**Solution:** The moment about  $O$ ,  $-mgx = I_0\alpha$ , from which

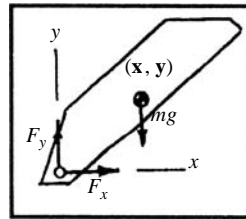
$$\alpha = -\frac{(200)(9.81)(3)}{14,000} = -0.420 \text{ rad/s}^2.$$

The acceleration of the center of mass is

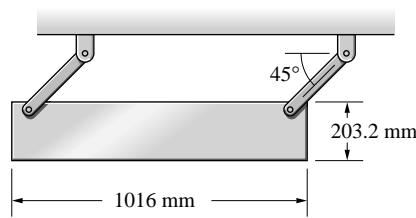
$$\mathbf{a}_G = \boldsymbol{\alpha} \times \mathbf{r}_{G/O} - \omega^2 \mathbf{r}_{G/O} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ 3 & 4 & 0 \end{bmatrix} = -4\alpha\mathbf{i} + 3\alpha\mathbf{j}$$

$$\mathbf{a}_G = 1.68\mathbf{i} - 1.26\mathbf{j} \text{ (m/s}^2\text{)}.$$

From Newton's second law:  $F_x = ma_{Gx} = 336 \text{ N}$ ,  $F_y - mg = ma_{Gy}$ , from which  $F_y = 1710 \text{ N}$



**Problem 18.119** The slender bars each weigh 17.8 N and are 254 mm long. The homogenous plate weighs 44.5 N. If the system is released from rest in the position shown, what is the angular acceleration of the bars at that instant?



**Solution:** From geometry, the system is a parallelogram, so that the plate translates without rotating, so that the acceleration of every point on the plate is the same.

Newton's second law and the equation of angular motion applied to the plate:  $-F_{Ax} - F_{Bx} = m_p a_{PGx}$ ,  $F_{Ay} + F_{By} - W_p = m_p a_{PGy}$ . The motion about the center of mass:

$$-F_{Ay}(0.508) + F_{Ax}(0.102) + F_{Bx}(0.102) + F_{By}(0.508) = I_p \alpha = 0.$$

Newton's second law for the bars:  $-F_{Ay} + A_y - W_B = m_B a_{BGy}$ ,  $F_{Ax} + A_x = m_B a_{BGx}$ .  $-F_{By} + B_y - W_B = m_B a_{BGy}$ .  $F_{Bx} + B_x = m_B a_{BGx}$ . The angular acceleration about the center of mass:

$$F_{Ax}(0.127) \cos \theta + F_{Ay}(0.127) \sin \theta - A_x(0.127) \cos \theta + A_y(0.127) \sin \theta = I_B \alpha,$$

$$F_{Bx}(0.127) \cos \theta + F_{By}(0.127) \sin \theta - B_x(0.127) \cos \theta + B_y(0.127) \sin \theta = I_B \alpha.$$

From kinematics: the acceleration of the center of mass of the bars in terms of the acceleration at point A is

$$\mathbf{a}_{BG} = \boldsymbol{\alpha} \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ -0.127 \cos \theta & -0.127 \sin \theta & 0 \end{bmatrix}$$

$$= 0.127 \sin \theta \alpha \mathbf{i} - 0.127 \cos \theta \alpha \mathbf{j} \text{ (m/s}^2\text{)}.$$

From which

$$a_{BGx} = (0.127) \sin \theta \alpha, \quad a_{BGy} = -(0.127) \cos \theta \alpha,$$

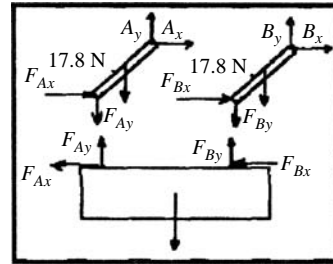
since  $\omega = 0$  upon release.

The acceleration of the plate:

$$\mathbf{a}_P = \boldsymbol{\alpha} \times \mathbf{r}_{P/A} - \omega^2 \mathbf{r}_{P/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ -0.254 \cos \theta & -0.254 \sin \theta & 0 \end{bmatrix}$$

$$= 0.254 \sin \theta \alpha \mathbf{i} - 0.254 \cos \theta \alpha \mathbf{j} \text{ (m/s}^2\text{)}.$$

From which  $a_{Px} = (0.254) \sin \theta \alpha$ ,  $a_{Py} = -(0.254) \cos \theta \alpha$ .



Substitute to obtain the nine equations in nine unknowns:

- (1)  $-F_{Ax} - F_{Bx} = 0.254 m_p \sin \theta \alpha,$
- (2)  $F_{Ay} + F_{By} - W_p = -0.254 m_p \cos \theta \alpha,$
- (3)  $-0.508 F_{Ay} + 0.102 F_{Ax} + 0.508 F_{By} + 0.102 F_{Bx} = 0,$
- (4)  $-F_{Ay} + A_y - W_B = -(0.127) m_B \cos \theta \alpha,$
- (5)  $F_{Ax} + A_x = (0.127) m_B \sin \theta \alpha,$
- (6)  $F_{Ax} \sin \theta + F_{Ay} \cos \theta - A_x \sin \theta + A_y \cos \theta = (7.87) I_B \alpha,$
- (7)  $F_{Bx} + B_x = (0.127) m_B \sin \theta \alpha,$
- (8)  $-F_{By} + B_y - W_B = -(0.127) m_B \cos \theta \alpha,$
- (9)  $F_{Bx} \cos \theta + F_{By} \sin \theta - B_x \cos \theta + B_y \sin \theta = (7.87) I_B \alpha.$  The number of equations and number of unknowns can be reduced by combining equations, but here the choice is to solve the system by iteration using **TK Solver Plus**. The results:  $F_{Ax} = -9.83 \text{ N}$ ,  $F_{Ay} = 7.47 \text{ N}$ ,  $F_{Bx} = -14.77 \text{ N}$ ,  $A_x = 14.77 \text{ N}$ ,  $A_y = 20.37 \text{ N}$ ,  $B_x = 19.7 \text{ N}$ ,  $B_y = 25.3 \text{ N}$ .  $\alpha = 30.17 \text{ rad/s}^2$ .

**Problem 18.120** A slender bar of mass  $m$  is released from rest in the position shown. The static and kinetic friction coefficients of friction at the floor and the wall have the same value  $\mu$ . If the bar slips, what is its angular acceleration at the instant of release?

**Solution:** Choose a coordinate system with the origin at the intersection of wall and floor, with the  $x$  axis parallel to the floor. Denote the points of contact at wall and floor by  $P$  and  $N$  respectively, and the center of mass of the bar by  $G$ . The vector locations are

$$\mathbf{r}_N = iL \sin \theta, \mathbf{r}_P = jL \cos \theta, \mathbf{r}_G = \frac{L}{2}(i \sin \theta + j \cos \theta).$$

From Newton's second law:

$$P - \mu N = ma_{Gx}, N + \mu P - mg = ma_{Gy},$$

where  $a_{Gx}$ ,  $a_{Gy}$  are the accelerations of the center of mass. The moment about the center of mass is

$$\mathbf{M}_G = \mathbf{r}_{P/G} \times (P\mathbf{i} + \mu P\mathbf{j}) + \mathbf{r}_{N/G} \times (N\mathbf{j} - \mu N\mathbf{i}) :$$

$$\mathbf{M}_G = \frac{PL}{2} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\sin \theta & \cos \theta & 0 \\ 1 & \mu & 0 \end{bmatrix} + \frac{NL}{2} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sin \theta & -\cos \theta & 0 \\ -\mu & 1 & 0 \end{bmatrix}.$$

$$\mathbf{M}_G = -\left(\frac{PL}{2}\right)(\cos \theta + \mu \sin \theta)\mathbf{k} + \left(\frac{NL}{2}\right)(\sin \theta - \mu \cos \theta)\mathbf{k}$$

From the equation of angular motion,

$$-\left(\frac{PL}{2}\right)(\cos \theta + \mu \sin \theta) + \left(\frac{NL}{2}\right)(\sin \theta - \mu \cos \theta) = I_B \alpha$$

From kinematics: Assume that at the instant of slip the angular velocity  $\omega = 0$ . The acceleration of the center of mass in terms of the acceleration at point  $N$  is

$$\mathbf{a}_G = \mathbf{a}_N + \alpha \times \mathbf{r}_{G/N} - \omega^2 \mathbf{r}_{G/N}$$

$$= a_N \mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ -\frac{L \sin \theta}{2} & \frac{L \cos \theta}{2} & 0 \end{bmatrix}$$

$$\mathbf{a}_G = \left(a_N - \frac{\alpha L \cos \theta}{2}\right)\mathbf{i} + \left(-\frac{\alpha L \sin \theta}{2}\right)\mathbf{j},$$

$$\text{from which } a_{Gy} = -\frac{L \sin \theta}{2}\alpha.$$

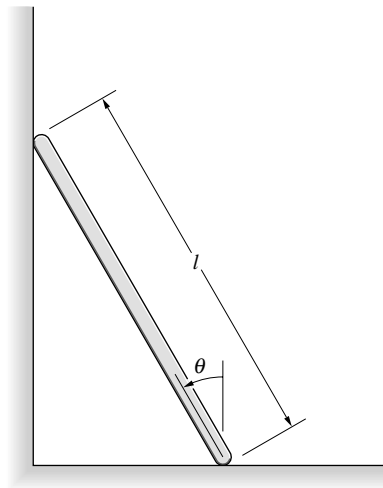
The acceleration of the center of mass in terms of the acceleration at point  $P$  is  $\mathbf{a}_G = \mathbf{a}_P + \alpha \times \mathbf{r}_{G/P}$ .

$$\mathbf{a}_G = \mathbf{a}_P + \alpha \times \mathbf{r}_{G/P} - \omega^2 \mathbf{r}_{G/P}$$

$$= a_P \mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ \frac{L \sin \theta}{2} & -\frac{L \cos \theta}{2} & 0 \end{bmatrix},$$

$$\mathbf{a}_G = \left(\frac{\alpha L \cos \theta}{2}\right)\mathbf{i} + \left(a_P + \frac{\alpha L \sin \theta}{2}\right)\mathbf{j},$$

$$\text{from which } a_{Gx} = \frac{L \cos \theta}{2}\alpha.$$



Substitute to obtain the three equations in three unknowns,

- (1)  $P - \mu N = \frac{mL \cos \theta}{2}\alpha,$
- (2)  $\mu P + N = -\frac{mL \sin \theta}{2}\alpha + mg.$
- (3)  $-\frac{PL}{2}(\cos \theta + \mu \sin \theta) + \frac{NL}{2}(\sin \theta - \mu \cos \theta) = I_B \alpha.$

Solve the first two equations for  $P$  and  $N$ :

$$P = \frac{mL}{2(1 + \mu^2)}(\cos \theta - \mu \sin \theta)\alpha + \frac{\mu mg}{(1 + \mu^2)}.$$

$$N = -\frac{mL}{2(1 + \mu^2)}(\sin \theta + \mu \cos \theta)\alpha + \frac{mg}{(1 + \mu^2)}.$$

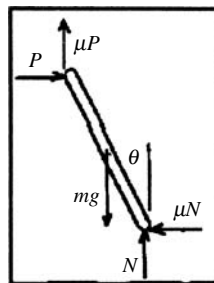
Substitute the first two equations into the third, and reduce to obtain

$$\alpha \left[ I_B + \frac{mL^2}{4} \left( \frac{1 - \mu^2}{1 + \mu^2} \right) \right] = \frac{mgL}{2} \left( \frac{1 - \mu^2}{1 + \mu^2} \right)$$

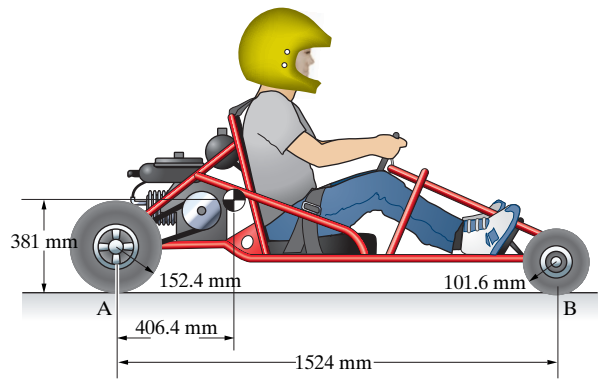
$$\sin \theta - mgL \left( \frac{\mu}{1 + \mu^2} \right) \cos \theta.$$

Substitute  $I_B = \left(\frac{1}{12}\right)mL^2$ , reduce, and solve:

$$\alpha = \frac{(3(1 - \mu^2) \sin \theta - 6\mu \cos \theta)g}{(2 - \mu^2)L}$$



**Problem 18.121** Each of the go-cart's front wheels weighs 22.2 N and has a moment of inertia of  $0.014 \text{ kg}\cdot\text{m}^2$ . The two rear wheels and rear axle form a single rigid body weighing 177.9 N and having a moment of inertia of  $0.136 \text{ kg}\cdot\text{m}^2$ . The total weight of the go-cart and driver is 1067 N. (The location of the center of mass of the go-cart and driver, *not including* the front wheels or the rear wheels and rear axle, is shown.) If the engine exerts a torque of 16.3 N-m on the rear axle, what is the go-cart's acceleration?



**Solution:** Let  $a$  be the cart's acceleration and  $\alpha_A$  and  $\alpha_B$  the wheels' angular accelerations. Note that

$$a = (0.152)\alpha_A, \quad (1)$$

$$a = (0.102)\alpha_B. \quad (2)$$

Front wheel:

$$\sum F_x = B_x + f_B = (44.5/9.81)a, \quad (3)$$

$$\sum F_y = B_y + N_B - 10 = 0, \quad (4)$$

$$\sum M = -f_B(0.102) = (0.028)\alpha_B. \quad (5)$$

Rear Wheel:

$$\sum F_x = A_x + f_A = (177.9/9.81)a, \quad (6)$$

$$\sum F_y = A_y + N_A - 177.9 = 0, \quad (7)$$

$$\sum M = 16.3 - f_A(0.152) = (0.136)\alpha_A. \quad (8)$$

Cart:

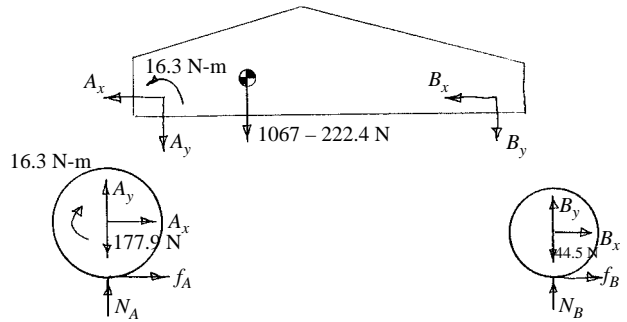
$$\sum F_x = -A_x - B_x = (844.6/9.81)a, \quad (9)$$

$$\sum F_y = -A_y - B_y - 844.6 = 0, \quad (10)$$

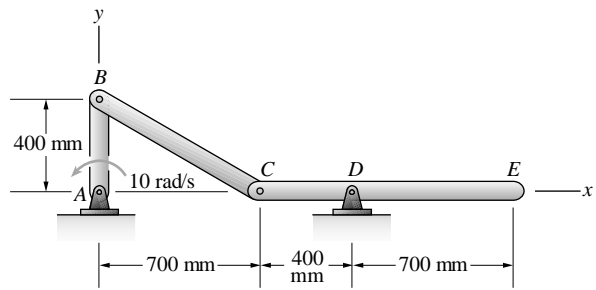
$$\sum M = B_x[(0.381 - 0.102)] + B_y[(1.524 - 0.406)] + A_x[(0.381 - 0.152)] - A_y(0.406) - 16.3 = 0. \quad (11)$$

Solving Eqs. (1)–(11), we obtain

$$a = 0.91 \text{ m/s}^2.$$



**Problem 18.122** Bar  $AB$  rotates with a constant angular velocity of 10 rad/s in the counterclockwise direction. The masses of the slender bars  $BC$  and  $CDE$  are 2 kg and 3.6 kg, respectively. The  $y$  axis points upward. Determine the components of the forces exerted on bar  $BC$  by the pins at  $B$  and  $C$  at the instant shown.



**Solution:** The velocity of point  $B$  is

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 0 & 0.4 & 0 \end{bmatrix}$$

$$= -0.4(10)\mathbf{i} = -4\mathbf{i} \text{ (m/s).}$$

The velocity of point  $C$  is

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B} = -4\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0.7 & -0.4 & 0 \end{bmatrix}$$

$$= -4\mathbf{i} + 0.4\omega_{BC}\mathbf{i} + 0.7\omega_{BC}\mathbf{j} \text{ (m/s).}$$

From the constraint on the motion at point  $C$ ,  $\mathbf{v}_C = v_C\mathbf{j}$ . Equate components:  $0 = -4 + 0.4\omega_{BC}$ ,  $v_C = 0.7\omega_{BC}$ , from which  $\omega_{BC} = 10$  rad/s,  $v_C = 7$  m/s. The velocity at  $C$  in terms of the angular velocity  $\omega_{CDE}$ ,

$$\mathbf{v}_C = \mathbf{v}_D + \omega_{CDE} \times \mathbf{r}_{C/D}$$

$$= 0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CDE} \\ -0.4 & 0 & 0 \end{bmatrix} = -0.4\omega_{CDE}\mathbf{j},$$

$$\text{from which } \omega_{CDE} = -\frac{7}{0.4} = -17.5 \text{ rad/s.}$$

The acceleration of point  $B$  is

$$\mathbf{a}_B = -\omega_{AB}^2 \mathbf{r}_B = -(10^2)(0.4)\mathbf{j} = -40\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration at point  $C$  is  $\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$ .

$$\mathbf{a}_C = -40\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 0.7 & -0.4 & 0 \end{bmatrix} - \omega_{BC}^2(0.7\mathbf{i} - 0.4\mathbf{j}) \text{ (m/s}^2\text{)}.$$

$$\mathbf{a}_C = +(0.4\alpha_{BC} - 0.7\omega_{BC}^2)\mathbf{i} + (-40 + 0.7\alpha_{BC} + 0.4\omega_{BC}^2)\mathbf{j} \text{ (m/s}^2\text{)}.$$

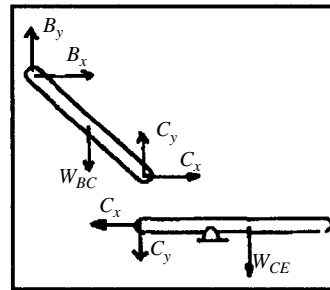
The acceleration in terms of the acceleration at  $D$  is

$$\mathbf{a}_C = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CDE} \\ -0.4 & 0 & 0 \end{bmatrix} - \omega_{CDE}^2(-0.4\mathbf{i})$$

$$= -0.4\alpha_{CDE}\mathbf{j} + 0.4\omega_{CDE}^2\mathbf{i}.$$

Equate components and solve:

$$\alpha_{BC} = 481.25 \text{ rad/s}^2, \alpha_{CDE} = -842.19 \text{ rad/s}^2.$$



The acceleration of the center of mass of  $BC$  is

$$\mathbf{a}_G = -40\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 0.35 & -0.2 & 0 \end{bmatrix} - \omega_{BC}^2(0.35\mathbf{i} - 0.2\mathbf{j}),$$

from which  $\mathbf{a}_G = 61.25\mathbf{i} + 148.44\mathbf{j} \text{ (m/s}^2\text{)}$

The equations of motion:  $B_x + C_x = m_{BC}a_{Gx}$ ,  $B_y + C_y - m_{BC}g = m_{BC}a_{Gy}$ , where the accelerations  $a_{Gx}$ ,  $a_{Gy}$  are known. The moment equation,  $0.35C_y + 0.2C_x - 0.2B_x - 0.35B_y = I_{BC}\alpha_{BC}$ , where  $\alpha_{BC}$ , is known, and

$$I_{BC} = \left(\frac{1}{12}\right)m_{BC}L_{BC}^2 = 0.1083 \text{ kg}\cdot\text{m}^2, 0.4C_y - 0.15m_{CE}g = I_D\alpha_{CE},$$

$$\text{where } I_D = \left(\frac{1}{12}\right)m_{CE}L_{CE}^2 + (0.15)^2m_{CE} = 0.444 \text{ kg}\cdot\text{m}^2,$$

is the moment of inertia about the pivot point  $D$ , and 0.15 m is the distance between the point  $D$  and the center of mass of bar  $CDE$ . Solve these four equations in four unknowns by iteration:

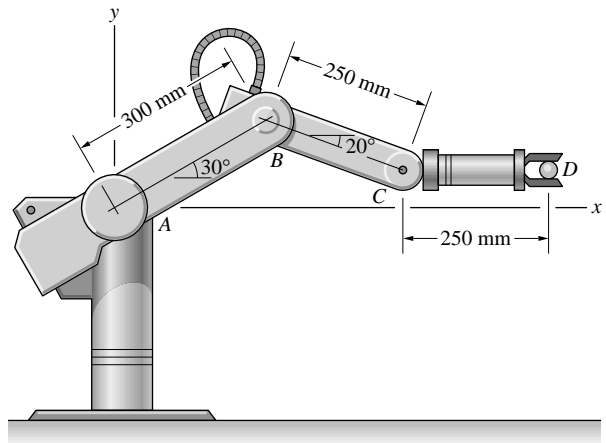
$$B_x = -1959 \text{ N,}$$

$$B_y = 1238 \text{ N,}$$

$$C_x = 2081 \text{ N,}$$

$$C_y = -922 \text{ N.}$$

**Problem 18.123** At the instant shown, the arms of the robotic manipulator have the constant counterclockwise angular velocities  $\omega_{AB} = -0.5 \text{ rad/s}$ ,  $\omega_{BC} = 2 \text{ rad/s}$ , and  $\omega_{CD} = 4 \text{ rad/s}$ . The mass of arm  $CD$  is  $10 \text{ kg}$ , and the center of mass is at its midpoint. At this instant, what force and couple are exerted on arm  $CD$  at  $C$ ?



**Solution:** The relative vector locations of  $B$ ,  $C$ , and  $D$  are

$$\begin{aligned} \mathbf{r}_{B/A} &= 0.3(\mathbf{i} \cos 30^\circ + \mathbf{j} \sin 30^\circ) \\ &= 0.2598\mathbf{i} + 0.150\mathbf{j} \text{ (m)}, \end{aligned}$$

$$\begin{aligned} \mathbf{r}_{C/B} &= 0.25(\mathbf{i} \cos 20^\circ - \mathbf{j} \sin 20^\circ) \\ &= 0.2349\mathbf{i} - 0.08551\mathbf{j} \text{ (m)}, \end{aligned}$$

$$\mathbf{r}_{D/C} = 0.25\mathbf{i} \text{ (m)}.$$

The acceleration of point  $B$  is

$$\mathbf{a}_B = -\omega_{AB}^2 \mathbf{r}_{B/A} = -(0.5^2)(0.3 \cos 30^\circ \mathbf{i} + 0.3 \sin 30^\circ \mathbf{j}),$$

$$\mathbf{a}_B = -0.065\mathbf{i} - 0.0375\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration at point  $C$  is

$$\mathbf{a}_C = \mathbf{a}_B - \omega_{BC}^2 \mathbf{r}_{C/B} = \mathbf{a}_B - \omega_{BC}^2 (0.2349\mathbf{i} - 0.08551\mathbf{j}).$$

$$\mathbf{a}_C = -1.005\mathbf{i} + 0.3045\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of the center of mass of  $CD$  is

$$\mathbf{a}_G = \mathbf{a}_C - \omega_{CD}^2 (0.125\mathbf{i}) \text{ (m/s}^2\text{)},$$

from which

$$\mathbf{a}_G = -3.005\mathbf{i} + 0.3045\mathbf{j} \text{ (m/s}^2\text{)}.$$

For the arm  $CD$  the three equations of motion in three unknowns are

$$C_y - m_{CD}g = m_{CD}a_{Gy}, \quad C_x = m_{CD}a_{Gx}, \quad M - 0.125C_y = 0,$$

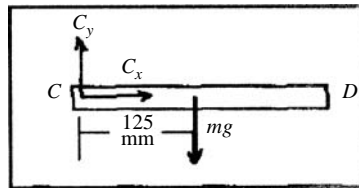
which have the direct solution:

$$C_y = 101.15 \text{ N},$$

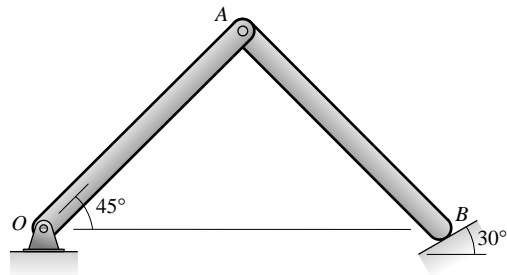
$$C_x = -30.05 \text{ N}.$$

$$M = 12.64 \text{ N}\cdot\text{m},$$

where the negative sign means a direction opposite to that shown in the free body diagram.



**Problem 18.124** Each bar is 1 m in length and has a mass of 4 kg. The inclined surface is smooth. If the system is released from rest in the position shown, what are the angular accelerations of the bars at that instant?



**Solution:** For convenience, denote  $\theta = 45^\circ$ ,  $\beta = 30^\circ$ , and  $L = 1$  m. The acceleration of point A is

$$\mathbf{a}_A = \alpha_{OA} \times \mathbf{r}_{A/O} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{OA} \\ L \cos \theta & L \sin \theta & 0 \end{bmatrix}$$

$$\mathbf{a}_A = \alpha_{OA}(-\mathbf{i}L \sin \theta + \mathbf{j}L \cos \theta) \text{ (m/s}^2\text{)}.$$

The acceleration of A is also given by

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B}.$$

$$\mathbf{a}_A = \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ -L \cos \theta & L \sin \theta & 0 \end{bmatrix}$$

$$\mathbf{a}_A = \mathbf{a}_B - \mathbf{i}\alpha_{AB}L \sin \theta - \mathbf{j}\alpha_{AB}L \cos \theta \text{ (m/s}^2\text{)}.$$

From the constraint on the motion at B, Equate the expressions for the acceleration of A to obtain the two equations:

$$(1) -\alpha_{OA}L \sin \theta = a_B \cos \beta - \alpha_{AB}L \sin \theta,$$

$$\text{and (2) } \alpha_{OA}L \cos \theta = a_B \sin \beta - \alpha_{AB}L \cos \theta.$$

The acceleration of the center of mass of AB is

$$\mathbf{a}_{GAB} = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{GAB/A}$$

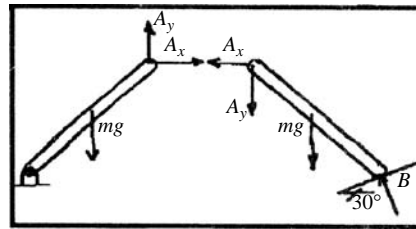
$$= \mathbf{a}_A + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ \frac{L \cos \theta}{2} & -\frac{L \sin \theta}{2} & 0 \end{bmatrix}$$

$$\mathbf{a}_{GAB} = \mathbf{a}_A + \frac{L\alpha_{AB}}{2} \sin \theta \mathbf{i} + \frac{\alpha_{AB}L}{2} \cos \theta \mathbf{j} \text{ (m/s}^2\text{)},$$

from which

$$(3) a_{GABx} = -\alpha_{OA}L \sin \theta + \frac{L\alpha_{AB}}{2} \sin \theta \text{ (m/s}^2\text{)},$$

$$(4) a_{GABy} = \alpha_{OA}L \cos \theta + \frac{L\alpha_{AB}}{2} \cos \theta.$$



The equations of motion for the bars: for the pin supported left bar:

$$(5) A_y L \cos \theta - A_x L \sin \theta - mg \left(\frac{L}{2}\right) \cos \theta = I_{OA} \alpha_{OA},$$

$$\text{where } I_{OA} = \left(\frac{mL^2}{3}\right) = \frac{4}{3} \text{ kg}\cdot\text{m}^2.$$

The equations of motion for the right bar:

$$(6) -A_x - B \sin \beta = ma_{GABx},$$

$$(7) -A_y - mg + B \cos \beta = ma_{GABy},$$

$$(8) A_y \left(\frac{L}{2}\right) \cos \theta + A_x \left(\frac{L}{2}\right) \sin \theta + B \left(\frac{L}{2}\right) \sin \theta \cos \beta$$

$$- B \left(\frac{L}{2}\right) \cos \theta \sin \beta = I_{GAB} \alpha_{AB},$$

$$\text{where } I_{GAB} = \left(\frac{1}{12}\right) mL^2 = \left(\frac{1}{3}\right) \text{ kg}\cdot\text{m}^2.$$

These eight equations in eight unknowns are solved by iteration:  $A_x = -19.27$  N,  $A_y = 1.15$  N,  $\alpha_{OA} = 0.425$  rad/s<sup>2</sup>,  $\alpha_{AB} = -1.59$  rad/s<sup>2</sup>,  $B = 45.43$  N,  $a_{GABx} = -0.8610$  m/s<sup>2</sup>,  $a_{GABy} = -0.2601$  m/s<sup>2</sup>



**Problem 18.125** Each bar is 1 m in length and has a mass of 4 kg. The inclined surface is smooth. If the system is released from rest in the position shown, what is the magnitude of the force exerted on bar  $OA$  by the support at  $O$  at that instant?

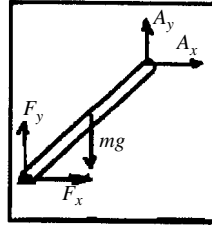
**Solution:** The acceleration of the center of mass of the bar  $OA$  is

$$\mathbf{a}_{GOA} = \alpha_{OA} \times \mathbf{r}_{G/OA} = \mathbf{a}_A + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{OA} \\ \frac{L \cos \theta}{2} & \frac{L \sin \theta}{2} & 0 \end{bmatrix},$$

$$\mathbf{a}_{GOA} = -\frac{L \sin \theta}{2} \alpha_{OA} \mathbf{i} + \frac{L \cos \theta}{2} \alpha_{OA} \mathbf{j} \text{ (m/s}^2\text{)}.$$

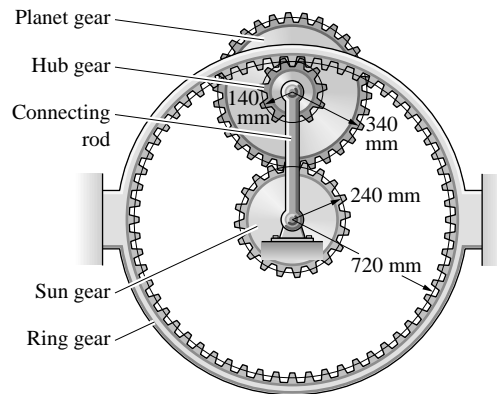
The equations of motion:

$$F_x + A_x = m a_{GOAx}, F_y + A_y - mg = m a_{GOAy}.$$



Use the solution to Problem 18.140:  $\theta = 45^\circ$ ,  $\alpha_{GA} = 0.425 \text{ rad/s}^2$ ,  $A_x = -19.27 \text{ N}$ ,  $m = 4 \text{ kg}$ , from which  $F_x = 18.67 \text{ N}$ ,  $F_y = 38.69 \text{ N}$ , from which  $|\mathbf{F}| = \sqrt{F_x^2 + F_y^2} = 42.96 \text{ N}$

**Problem 18.126** The fixed ring gear lies in the horizontal plane. The hub and planet gears are bonded together. The mass and moment of inertia of the combined hub and planet gears are  $m_{HP} = 130 \text{ kg}$  and  $I_{HP} = 130 \text{ kg}\cdot\text{m}^2$ . The moment of inertia of the sun gear is  $I_s = 60 \text{ kg}\cdot\text{m}^2$ . The mass of the connecting rod is 5 kg, and it can be modeled as a slender bar. If a 1 kN-m counterclockwise couple is applied to the sun gear, what is the resulting angular acceleration of the bonded hub and planet gears?



**Solution:** The moment equation for the sun gear is

$$(1) M - 0.24F = I_s \alpha_s.$$

For the hub and planet gears:

$$(2) (0.48)\alpha_{HP} = -0.24\alpha_s,$$

$$(3) F - Q - R = m_{HP}(0.14)(-\alpha_{HP}),$$

$$(4) (0.14)Q + 0.34F - I_{HP}(-\alpha_{HP}).$$

For the connecting rod:

$$(5) (0.58)R = I_{CR}\alpha_{CR},$$

$$\text{where } I_{CR} = \left(\frac{1}{3}\right)m_{GR}(0.58^2) = 0.561 \text{ kg}\cdot\text{m}^2.$$

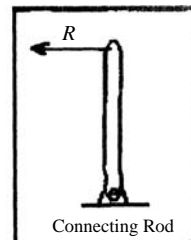
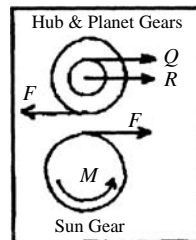
$$(6) (0.58)\alpha_{CR} = -(0.14)\alpha_{HP}.$$

These six equations in six unknowns are solved by iteration:

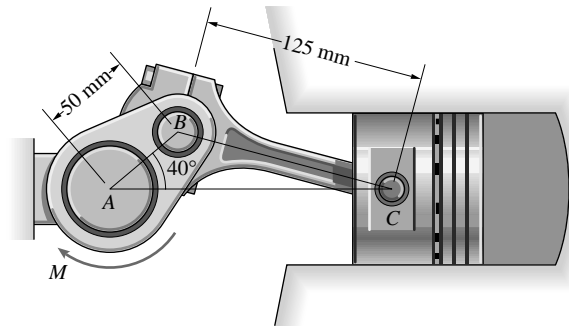
$$F = 1482.7 \text{ N}, \alpha_s = 10.74 \text{ rad/s}^2,$$

$$\alpha_{HP} = -5.37 \text{ rad/s}^2, Q = 1383.7 \text{ N},$$

$$R = 1.25 \text{ N}, \alpha_{CR} = 1.296 \text{ rad/s}^2.$$



**Problem 18.127** The system is stationary at the instant shown. The net force exerted on the piston by the exploding fuel-air mixture and friction is 5 kN to the left. A clockwise couple  $M = 200$  N-m acts on the crank  $AB$ . The moment of inertia of the crank about  $A$  is  $0.0003$  kg-m<sup>2</sup>. The mass of the connecting rod  $BC$  is  $0.36$  kg, and its center of mass is  $40$  mm from  $B$  on the line from  $B$  to  $C$ . The connecting rod's moment of inertia about its center of mass is  $0.0004$  kg-m<sup>2</sup>. The mass of the piston is  $4.6$  kg. What is the piston's acceleration? (Neglect the gravitational forces on the crank and connecting rod.)



**Solution:** From the law of sines:

$$\frac{\sin \beta}{0.05} = \frac{\sin 40^\circ}{0.125},$$

from which  $\beta = 14.9^\circ$ . The vectors

$$\begin{aligned} \mathbf{r}_{B/A} &= 0.05(\mathbf{i} \cos 40^\circ + \mathbf{j} \sin 40^\circ) \mathbf{r}_{B/A} \\ &= 0.0383\mathbf{i} + 0.0321\mathbf{j} \text{ (m)}. \end{aligned}$$

$$\mathbf{r}_{B/C} = 0.125(-\mathbf{i} \cos \beta + \mathbf{j} \sin \beta) \text{ (m)}.$$

$$\mathbf{r}_{B/C} = -0.121\mathbf{i} + 0.0321\mathbf{j} \text{ (m)}.$$

The acceleration of point  $B$  is

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A},$$

$$\mathbf{a}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 0.0383 & 0.0321 & 0 \end{bmatrix}$$

$$- \omega_{AB}^2(0.0383\mathbf{i} + 0.0321\mathbf{j}) \text{ (m/s}^2\text{)}.$$

The acceleration of point  $B$  in terms of the acceleration of point  $C$  is

$$\mathbf{a}_B = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{B/C} = a_C \mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -0.121 & 0.0321 & 0 \end{bmatrix}$$

$$- \omega_{BC}^2(-0.121\mathbf{i} + 0.0321\mathbf{j}) \text{ (m/s}^2\text{)}.$$

Equate the two expressions for the acceleration of point  $B$ , note  $\omega_{AB} = \omega_{BC} = 0$ , and separate components:

$$(1) -0.05\alpha_{AB} \sin 40^\circ = a_C - 0.125\alpha_{BC} \sin \beta,$$

$$(2) 0.05\alpha_{AB} \cos 40^\circ = -0.125\alpha_{BC} \cos \beta.$$

The acceleration of the center of mass of the connecting rod is

$$\mathbf{a}_{GCR} = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{GCR/C} - \omega_{BC}^2 \mathbf{r}_{GCR/C},$$

$$\mathbf{a}_{GCR} = a_C \mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -0.085 \cos \beta & 0.085 \sin \beta & 0 \end{bmatrix}$$

$$- \omega_{BC}^2(-0.085 \cos \beta \mathbf{i} + 0.085 \sin \beta \mathbf{j}) \text{ (m/s}^2\text{)},$$

from which

$$(3) a_{GCRx} = a_C - 0.085\alpha_{BC} \sin \beta \text{ (m/s}^2\text{)},$$

$$(4) a_{GCRy} = -0.085\alpha_{BC} \cos \beta \text{ (m/s}^2\text{)}.$$

The equations of motion for the crank:

$$(5) B_y(0.05 \cos 40^\circ) - B_x(0.05 \sin 40^\circ) - M = I_A \alpha_{AB}$$

For the connecting rod:

$$(6) -B_x + C_x = m_{CR} a_{GCRx}$$

$$(7) -B_y + C_y = m_{CR} a_{GCRy}$$

$$(8) C_y(0.085 \cos \beta) + C_x(0.085 \sin \beta) + B_x(0.04 \sin \beta) + B_y(0.04 \cos \beta) = I_{GCR} \alpha_{BC}$$

For the piston:

$$(9) -C_x - 5000 = m_P a_C.$$

These nine equations in nine unknowns are solved by iteration:

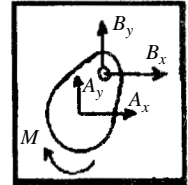
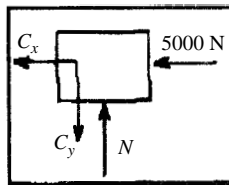
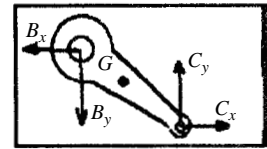
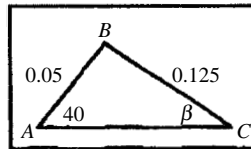
$$\alpha_{AB} = 1255.7 \text{ rad/s}^2, \alpha_{BC} = -398.2 \text{ rad/s}^2,$$

$$a_{GCRx} = -44.45 \text{ m/s}^2, a_{GCRy} = 32.71 \text{ m/s}^2,$$

$$B_y = 1254.6 \text{ N}, B_x = -4739.5 \text{ N},$$

$$C_x = -4755.5 \text{ N}, C_y = 1266.3 \text{ N},$$

$$a_C = -53.15 \text{ m/s}^2.$$



**Problem 18.128** If the crank  $AB$  in Problem 18.127 has a counterclockwise angular velocity of 2000 rpm at the instant shown, what is the piston's acceleration?

**Solution:** The angular velocity of  $AB$  is

$$\omega_{AB} = 2000 \left( \frac{2\pi}{60} \right) = 209.44 \text{ rad/s.}$$

The angular velocity of the connecting rod  $BC$  is obtained from the expressions for the velocity at point  $B$  and the known value of  $\omega_{AB}$ :

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0.05 \cos 40^\circ & 0.05 \sin 40^\circ & 0 \end{bmatrix}.$$

$$\mathbf{v}_B = -0.05 \sin 40^\circ \omega_{AB} \mathbf{i} + 0.05 \cos 40^\circ \omega_{AB} \mathbf{j} \text{ (m/s).}$$

$$\mathbf{v}_B = v_C \mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -0.125 \cos \beta & 0.125 \sin \beta & 0 \end{bmatrix},$$

$$\mathbf{v}_B = v_C \mathbf{i} - 0.125 \sin \beta \omega_{BC} \mathbf{i} - 0.125 \cos \beta \omega_{BC} \mathbf{j} \text{ (m/s).}$$

From the  $\mathbf{j}$  component,  $0.05 \cos 40^\circ \omega_{AB} = -0.125 \cos \beta \omega_{BC}$ , from which  $\omega_{BC} = -66.4$  rad/s. The nine equations in nine unknowns obtained in the solution to Problem 18.127 are

$$(1) -0.05\alpha_{AB} \sin 40^\circ - 0.05\omega_{AB}^2 \cos 40^\circ$$

$$= a_C - 0.125\alpha_{BC} \sin \beta + 0.125\omega_{BC}^2 \cos \beta,$$

$$(2) 0.05\alpha_{AB} \cos 40^\circ - 0.05\omega_{AB}^2 \sin 40^\circ$$

$$= -0.125\alpha_{BC} \cos \beta - 0.125\omega_{BC}^2 \sin \beta,$$

$$(3) a_{GCRx} = a_C - 0.085\alpha_{BC} \sin \beta + 0.085\omega_{BC}^2 \cos \beta \text{ (m/s}^2\text{),}$$

$$(4) a_{GCRy} = -0.085\alpha_{BC} \cos \beta - 0.085\omega_{BC}^2 \sin \beta \text{ (m/s}^2\text{),}$$

$$(5) B_y(0.05 \cos 40^\circ) - B_x(0.05 \sin 40^\circ) - M = I_A \alpha_{AB},$$

$$(6) -B_x + C_x = m_{CR} a_{GCRx},$$

$$(7) -B_y + C_y = m_{CR} a_{GCRy},$$

$$(8) C_y(0.085 \cos \beta) + C_x(0.085 \sin \beta)$$

$$+ B_x(0.04 \sin \beta) + B_y(0.04 \cos \beta) = I_{GCR} \alpha_{BC}.$$

$$(9) -C_x - 5000 = m_P a_C.$$

These nine equations in nine unknowns are solved by iteration:

$$\alpha_{AB} = -39,386.4 \text{ rad/s}^2, \alpha_{BC} = 22,985.9 \text{ rad/s}^2,$$

$$a_{GCRx} = -348.34 \text{ m/s}^2, a_{GCRy} = -1984.5 \text{ m/s}^2,$$

$$B_y = 1626.7 \text{ N}, B_x = -3916.7 \text{ N},$$

$$C_x = -4042.1 \text{ N}, C_y = 912.25 \text{ N},$$

$$a_c = -208.25 \text{ (m/s}^2\text{)}$$