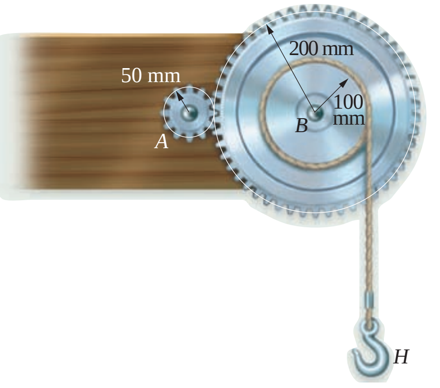
Problem 17.1 In Active Example 17.1, suppose that at a given instant the hook H is moving downward at 2 m/s. What is the angular velocity of gear A at that instant?



Solution: The angular velocity of gear B is

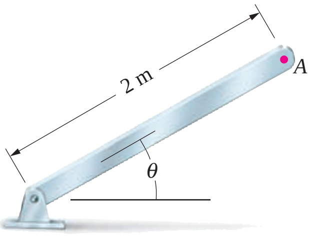
$$\omega_B = \frac{v_H}{r_H} = \frac{2 \text{ m/s}}{0.1 \text{ m}} = 20 \text{ rad/s}.$$

The gears are connected through the common velocity of the contact points

$$r_B\omega_B = r_A\omega_A \Rightarrow \omega_A = \frac{r_B}{r_A}\omega_B = \frac{0.2 \text{ m}}{0.05 \text{ m}}(20 \text{ rad/s}) = 80 \text{ rad/s}.$$

$\boxed{\omega_A = 80 \text{ rad/s counterclockwise.}}$

Problem 17.2 The angle θ (in radians) is given as a function of time by $\theta = 0.2\pi t^2$. At $t = 4$ s, determine the magnitudes of (a) the velocity of point A and (b) the tangential and normal components of acceleration of point A.



Solution: We have

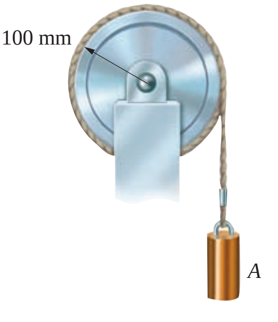
$$\theta = 0.2\pi t^2, \quad \omega = \frac{d\theta}{dt} = 0.4\pi t, \quad \alpha = \frac{d\omega}{dt} = 0.4\pi.$$

Then

(a) $v = r\omega = (2)(0.4\pi)(4) = 10.1 \text{ m/s}.$ $\boxed{v = 10.1 \text{ m/s.}}$

(b) $a_n = r\omega^2 = (2)[(0.4\pi)(4)]^2 = 50.5 \text{ m/s}^2,$
$a_t = r\alpha = (2)(0.4\pi) = 2.51 \text{ m/s}^2.$ $\boxed{a_n = 50.5 \text{ m/s}^2, \; a_t = 2.51 \text{ m/s}^2.}$

Problem 17.3 The mass A starts from rest at $t = 0$ and falls with a constant acceleration of 8 m/s^2. When the mass has fallen one meter, determine the magnitudes of (a) the angular velocity of the pulley and (b) the tangential and normal components of acceleration of a point at the outer edge of the pulley.



Solution: We have

$$a = 8 \text{ m/s}^2, v = \sqrt{2as} = \sqrt{2(8 \text{ m/s}^2)(1 \text{ m})} = 4 \text{ m/s},$$

$$\omega = \frac{v}{r} = \frac{4 \text{ m/s}}{0.1 \text{ m}} = 40 \text{ rad/s},$$

$$\alpha = \frac{a}{r} = \frac{8 \text{ m/s}^2}{0.1 \text{ m}} = 80 \text{ rad/s}^2.$$

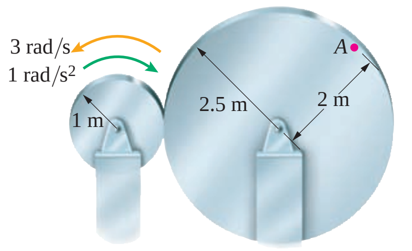
(a) $\boxed{\omega = 40 \text{ rad/s.}}$

(b) $a_t = r\alpha = (0.1 \text{ m})(80 \text{ rad/s}^2) = 8 \text{ m/s}^2,$
$a_n = r\omega^2 = (0.1 \text{ m})(40 \text{ rad/s})^2 = 160 \text{ m/s}^2.$ $\boxed{a_t = 8 \text{ m/s}^2, \; a_n = 160 \text{ m/s}^2.}$

Problem 17.4 At the instant shown, the left disk has an angular velocity of 3 rad/s counterclockwise and an angular acceleration of 1 rad/s^2 clockwise.

(a) What are the angular velocity and angular acceleration of the right disk? (Assume that there is no relative motion between the disks at their point of contact.)

(b) What are the magnitudes of the velocity and acceleration of point A?



Solution:

(a)

$$\boxed{\begin{aligned} r_L\omega_L &= r_R\omega_R \Rightarrow \omega_R = \frac{r_R}{r_L}\omega_L = \frac{1\text{ m}}{2.5\text{ m}}(3\text{ rad/s}) = 1.2\text{ rad/s} \\ r_L\alpha_L &= r_R\alpha_R \Rightarrow \alpha_R = \frac{r_R}{r_L}\alpha_L = \frac{1\text{ m}}{2.5\text{ m}}(1\text{ rad/s}^2) = 0.4\text{ rad/s}^2 \end{aligned}}$$

(b) $v_A = (2\text{ m})(1.2\text{ rad/s}) = 2.4\text{ m/s}$

$$a_{At} = (2\text{ m})(0.4\text{ rad/s}^2) = 0.8\text{ m/s}^2$$

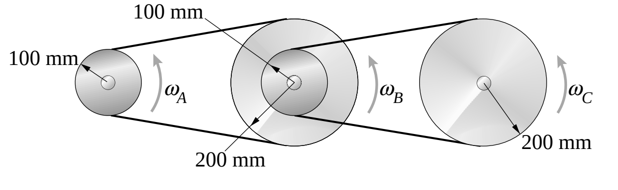
$$a_{An} = (2\text{ m})(1.2\text{ rad/s})^2 = 2.88\text{ m/s}^2$$

$$\Rightarrow \boxed{\begin{aligned} v_A &= 2.4\text{ m/s} \\ a_A &= \sqrt{(0.8)^2 + (2.88)^2}\text{ m/s}^2 = 2.99\text{ m/s}^2 \end{aligned}}$$

Problem 17.5 The angular velocity of the left disk is given as a function of time by $\omega_A = 4 + 0.2t$ rad/s.

(a) What are the angular velocities ω_B and ω_C at $t = 5$ s?

(b) Through what angle does the right disk turn from $t = 0$ to $t = 5$ s?



Solution:

$$\omega_A = (4 + 0.2t)\text{ rad/s}$$

$$r_A\omega_A = r_B\omega_B \;\Rightarrow\; \omega_B = \frac{0.1\text{ m}}{0.2\text{ m}}\omega_A = 0.5\,\omega_A$$

$$r_B\omega_B = r_C\omega_C \;\Rightarrow\; \omega_C = \frac{0.1\text{ m}}{0.2\text{ m}}\omega_B = 0.25\,\omega_A$$

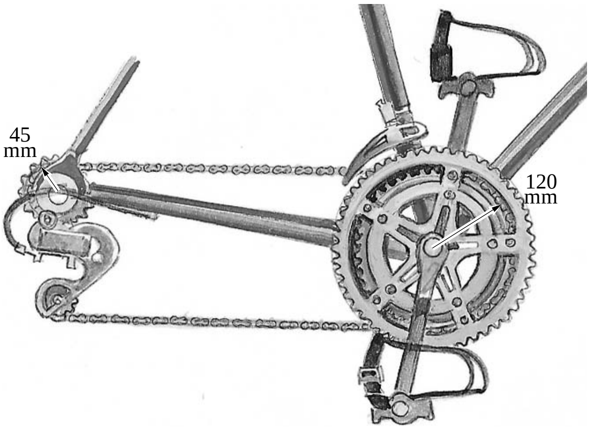
(a) At $t = 5$ s $\quad \omega_A = (4 + 0.2[5])\text{ rad/s} = 5\text{ rad/s}$

$$\boxed{\begin{aligned} \omega_B &= 0.5(5\text{ rad/s}) = 2.5\text{ rad/s} \\ \omega_C &= 0.25(5\text{ rad/s}) = 1.25\text{ rad/s} \end{aligned}}$$

(b) $\omega_C = 0.25\,\omega_A = 0.25(4 + 0.2t)\text{ rad/s} = (1 + 0.05t)\text{ rad/s}$

$$\boxed{\theta_C = \int_0^{5\text{ s}} \omega_C\,dt = [t + 0.025t^2]_0^{5\text{ s}} = 5.625\text{ rad}}$$

Problem 17.6 (a) If the bicycle's 120-mm sprocket wheel rotates through one revolution, through how many revolutions does the 45-mm gear turn? (b) If the angular velocity of the sprocket wheel is 1 rad/s, what is the angular velocity of the gear?



Solution: The key is that the tangential accelerations and tangential velocities along the chain are of constant magnitude

(a) $\theta_B = 2.67$ rev

(b) $v_B = r\omega_B \qquad v_A = r_A\omega_A$

$v_A = v_B$

$v_B = (0.045)\omega_B \qquad v_A = (0.120)(1)$

$$\omega_B = \left(\frac{120}{45}\right) \text{ rad/s} = 2.67 \text{ rad/s}$$

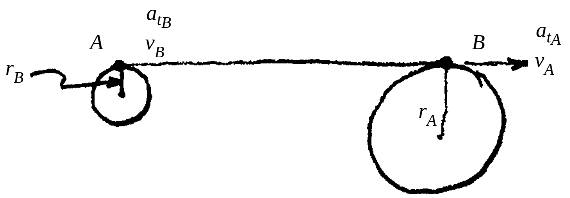
$$r_B\frac{d\theta_B}{dt} = r_A\frac{d\theta_A}{dt}$$

Integrating, we get

$r_B\theta_B = r_A\theta_A \qquad r_A = 0.120$ m

$r_B = 0.045$ m

$$\theta_B = \left(\frac{120}{45}\right)(1) \text{ rev} \qquad \theta_A = 1 \text{ rev.}$$



Problem 17.7 The rear wheel of the bicycle in Problem 17.6 has a 330-mm radius and is rigidly attached to the 45-mm gear. It the rider turns the pedals, which are rigidly attached to the 120-mm sprocket wheel, at one revolution per second, what is the bicycle's velocity?

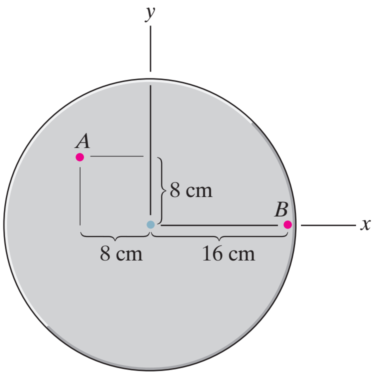
Solution: The angular velocity of the 120 mm sprocket wheel is $\omega = 1$ rev/s $= 2\pi$ rad/s. Use the solution to Problem 17.6. The angular velocity of the 45 mm gear is

$$\omega_{45} = 2\pi\left(\frac{120}{45}\right) = 16.76 \text{ rad/s.}$$

This is also the angular velocity of the rear wheel, from which the velocity of the bicycle is

$v = \omega_{45}(330) = 5.53$ m/s.

Problem 17.8 The disk is rotating about the origin with a constant clockwise angular velocity of 100 rpm. Determine the x and y components of velocity of points A and B (in cm/s).



Solution:

$$\omega = 100 \text{ rpm}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 10.5 \text{ rad/s}.$$

Working with point A

$$r_A = \sqrt{(8\text{ cm})^2 + (8\text{ cm})^2} = 11.3 \text{ cm}, \quad \theta_A = \tan^{-1}\left(\frac{8\text{ cm}}{8\text{ cm}}\right) = 45^\circ$$

$$v_A = r_A\omega = (11.3 \text{ cm})(10.5 \text{ rad/s}) = 118 \text{ cm/s}$$

$$\mathbf{v}_A = v_A(\cos\theta_A\mathbf{i} + \sin\theta_A\mathbf{j}) = (118 \text{ cm/s})(\cos 45^\circ\mathbf{i} + \sin 45^\circ\mathbf{j})$$

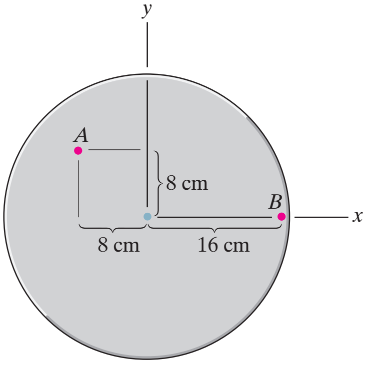
$$\boxed{\mathbf{v}_A = (83.8\mathbf{i} + 83.8\mathbf{j}) \text{ cm/s.}}$$

Working with point B

$$r_B = 16 \text{ in}, \quad v_B = r_B\omega = (16 \text{ cm})(10.5 \text{ rad/s}) = 168 \text{ cm/s}$$

$$\boxed{\mathbf{v}_B = -(168\mathbf{j}) \text{ cm/s.}}$$

Problem 17.9 The disk is rotating about the origin with a constant clockwise angular velocity of 100 rpm. Determine the x and y components of acceleration of points A and B (in cm/s^2).



Solution:

$$\omega = 100 \text{ rpm}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 10.5 \text{ rad/s}.$$

Working with point A

$$r_A = \sqrt{(8\text{ cm})^2 + (8\text{ cm})^2} = 11.3 \text{ cm}, \quad \theta_A = \tan^{-1}\left(\frac{8\text{ cm}}{8\text{ cm}}\right) = 45^\circ$$

$$a_A = r_A\omega^2 = (11.3 \text{ cm})(10.5 \text{ rad/})^2 = 1240 \text{ cm/s}^2$$

$$\mathbf{a}_A = a_A(\cos\theta_A\mathbf{i} - \sin\theta_A\mathbf{j}) = (1240 \text{ cm/s}^2)(\cos 45^\circ\mathbf{i} - \sin 45^\circ\mathbf{j})$$

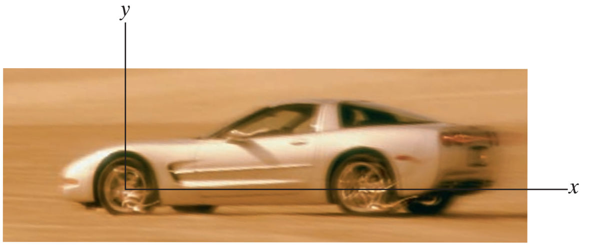
$$\boxed{\mathbf{a}_A = (877\mathbf{i} - 877\mathbf{j}) \text{ cm/s}^2.}$$

Working with point B

$$r_B = 16 \text{ in}, a_B = r_B\omega^2 = (16 \text{ cm})(10.5 \text{ rad/s})^2 = 1750 \text{ cm/s}^2$$

$$\boxed{\mathbf{a}_B = -(1750\mathbf{i}) \text{ cm/s}^2.}$$

Problem 17.10 The radius of the Corvette's tires is 30 cm. It is traveling at 80 km/h when the driver applies the brakes, subjecting the car to a deceleration of 25 m/s^2. Assume that the tires continue to roll, not skid, on the road surface. At that instant, what are the magnitudes of the tangential and normal components of acceleration (in m/s^2) of a point at the outer edge of a tire relative to a nonrotating coordinate system with its origin at the center of the tire?



Solution: We have

$$\omega = \frac{v}{r} = \frac{\left(\dfrac{80 \times 1000}{3600}\right)}{(0.3\ \text{m})} = 74.1\ \text{rad/s},$$

$$\alpha = \frac{a}{r} = \frac{25\ \text{m/s}^2}{(0.3\ \text{m})} = 83.33\ \text{rad/s}^2.$$

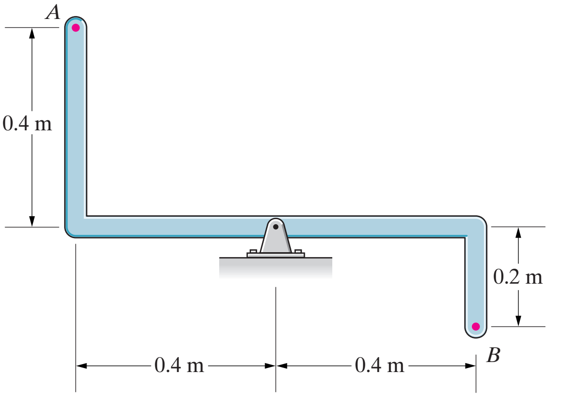
Relative to the given coordinate system (which is not an inertial coordinate system)

$$a_t = r\alpha = (0.3\ \text{m})\,(83.33\ \text{rad/s}^2) = 25\ \text{m/s}^2$$

$$a_n = r\omega^2 = (0.3\ \text{m})\,(74.1\ \text{rad/s})^2 = 1647.2\ \text{m/s}^2.$$

$$\boxed{a_t = 25\ \text{m/s}^2,\quad a_n = 1647.2\ \text{m/s}^2.}$$

Problem 17.11 If the bar has a counterclockwise angular velocity of 8 rad/s and a clockwise angular acceleration of 40 rad/s^2, what are the magnitudes of the accelerations of points A and B?



Solution:

$$\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_{A/O} - \omega^2 \mathbf{r}_{A/O}$$

$$= (-40\ \text{rad/s}^2\mathbf{k}) \times (-0.4\mathbf{i} + 0.4\mathbf{j})\ \text{m} - (8\ \text{rad/s})^2(-0.4\mathbf{i} + 0.4\mathbf{j})\ \text{m}$$

$$= (41.6\mathbf{i} - 9.6\mathbf{j})\ \text{m/s}^2$$

$$\mathbf{a}_B = \boldsymbol{\alpha} \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$= (-40\ \text{rad/s}^2\mathbf{k}) \times (0.4\mathbf{i} - 0.2\mathbf{j})\ \text{m} - (8\ \text{rad/s})^2(0.4\mathbf{i} - 0.2\mathbf{j})\ \text{m}$$

$$= (-33.6\mathbf{i} - 3.2\mathbf{j})\ \text{m/s}^2$$

$$\boxed{\begin{aligned} a_A &= \sqrt{(41.6)^2 + (-9.6)^2}\ \text{m/s}^2 = 42.7\ \text{m/s}^2 \\ a_B &= \sqrt{(-33.6)^2 + (-3.2)^2}\ \text{m/s}^2 = 33.8\ \text{m/s}^2 \end{aligned}}$$

Problem 17.12 Consider the bar shown in Problem 17.11. If $|\mathbf{v}_A| = 3$ m/s and $|\mathbf{a}_A| = 28$ m/s^2, what are $|\mathbf{v}_B|$ and $|\mathbf{a}_B|$?

Solution:

$$v_A = \omega r \Rightarrow 3 \text{ m/s} = \omega\sqrt{(0.4)^2 + (0.4)^2} \text{ m} \Rightarrow \omega = 5.30 \text{ rad/s}$$

$$a_{An} = \omega^2 r = (5.3 \text{ rad/s})^2\sqrt{(0.4)^2 + (0.4)^2} \text{ m} = 15.9 \text{ m/s}^2$$

$$a_{At} = \sqrt{a_A{}^2 - a_{An}{}^2} = \sqrt{(28)^2 - (15.9)^2} \text{ m/s}^2 = 23.0 \text{ m/s}^2$$

$$a_{At} = \alpha r \Rightarrow 23.0 \text{ m/s}^2 = \alpha\sqrt{(0.4)^2 + (0.4)^2} \text{ m} \Rightarrow \alpha = 40.7 \text{ rad/s}$$

$$\boxed{v_B = \omega\sqrt{(0.4)^2 + (-0.2)^2} \text{ m} = 2.37 \text{ m/s}}$$

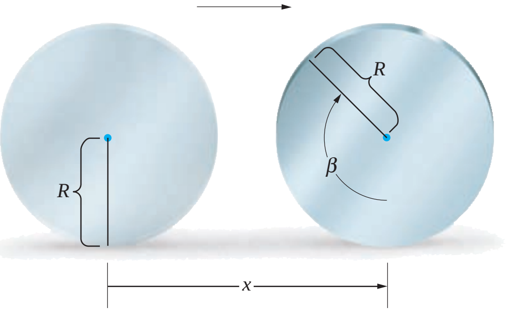
$$a_{Bt} = \alpha\sqrt{(0.4)^2 + (-0.2)^2} = 18.2 \text{ m/s}^2$$

$$a_{Bn} = \omega^2\sqrt{(0.4)^2 + (-0.2)^2} = 12.6 \text{ m/s}^2$$

$$\boxed{a_B = \sqrt{(18.2)^2 + (12.6)^2} \text{ m/s}^2 = 22.1 \text{ m/s}^2}$$

Problem 17.13 A disk of radius $R = 0.5$ m rolls on a horizontal surface. The relationship between the horizontal distance x the center of the disk moves and the angle β through which the disk rotates is $x = R\beta$. Suppose that the center of the disk is moving to the right with a constant velocity of 2 m/s.

(a) What is the disk's angular velocity?
(b) Relative to a nonrotating reference frame with its origin at the center of the disk, what are the magnitudes of the velocity and acceleration of a point on the edge of the disk?

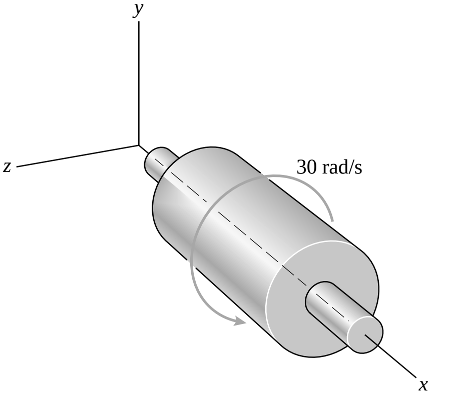


Solution:

(a) $x = R\beta \Rightarrow \dot{x} = R\dot{\beta} \Rightarrow v = R\omega \Rightarrow \boxed{\omega = \dfrac{v}{R} = \dfrac{2 \text{ m/s}}{0.5 \text{ m}} = 4 \text{ rad/s}}$

(b) $$\boxed{\begin{array}{l} v = R\omega = (0.5 \text{ m})(4 \text{ rad/s}) = 2 \text{ m/s} \\ a = a_n = R\omega^2 = (0.5 \text{ m})(4 \text{ rad/s})^2 = 8 \text{ m/s}^2 \end{array}}$$

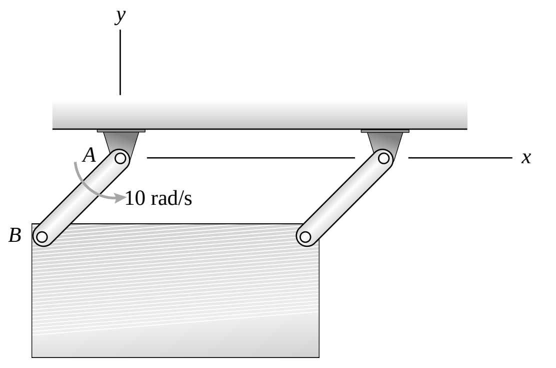
Problem 17.14 The turbine rotates relative to the coordinate system at 30 rad/s about a fixed axis coincident with the x axis. What is its angular velocity vector?



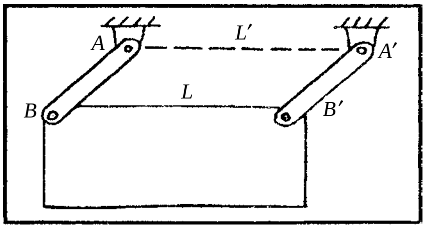
Solution: The angular velocity vector is parallel to the x axis, with magnitude 30 rad/s. By the right hand rule, the positive direction coincides with the positive direction of the x axis.

$\boxed{\boldsymbol{\omega} = 30\mathbf{i}}$ (rad/s).

Problem 17.15 The rectangular plate swings in the $x-y$ plane from arms of equal length. What is the angular velocity of (a) the rectangular plate and (b) the bar AB?



Solution: Denote the upper corners of the plate by B and B', and denote the distance between these points (the length of the plate) by L. Denote the suspension points by A and A', the distance separating them by L'. By inspection, since the arms are of equal length, and since $L = L'$, the figure $AA'B'B$ is a parallelogram. By definition, the opposite sides of a parallelogram remain parallel, and since the fixed side AA' does not rotate, then BB' cannot rotate, so that the plate does not rotate and



$\boxed{\boldsymbol{\omega}_{BB'} = 0}$.

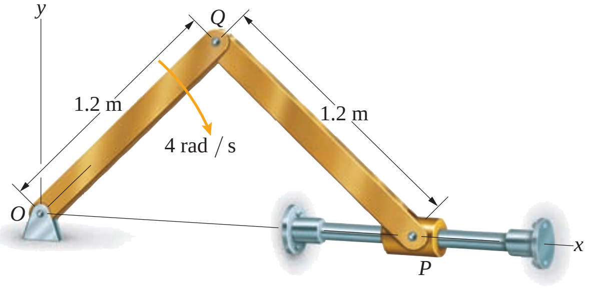
Similarly, by inspection the angular velocity of the bar AB is

$\boxed{\boldsymbol{\omega}_{AB} = 10\mathbf{k}}$ (rad/s),

where by the right hand rule the direction is along the positive z axis (out of the paper).

Problem 17.16 Bar OQ is rotating in the clockwise direction at 4 rad/s. What are the angular velocity vectors of the bars OQ and PQ?

Strategy: Notice that if you know the angular velocity of bar OQ, you also know the angular velocity of bar PQ.

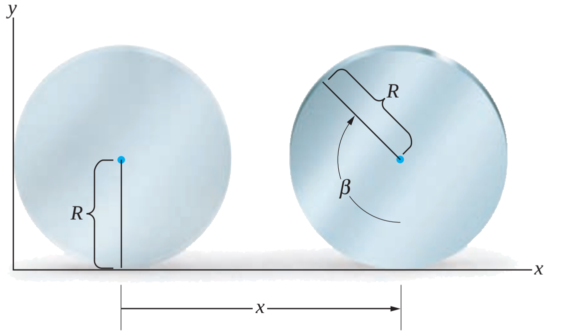


Solution: The magnitudes of the angular velocities are the same. The directions are opposite

$$\boxed{\boldsymbol{\omega}_{OQ} = -(4 \text{ rad/s})\mathbf{k}, \quad \boldsymbol{\omega}_{PQ} = (4 \text{ rad/s})\mathbf{k},}$$

Problem 17.17 A disk of radius $R = 0.5$ m rolls on a horizontal surface. The relationship between the horizontal distance x the center of the disk moves and the angle β through which the disk rotates is $x = R\beta$. Suppose that the center of the disk is moving to the right with a constant velocity of 2 m/s.

(a) What is the disk's angular velocity?
(b) What is the disk's angular velocity vector?



Solution:

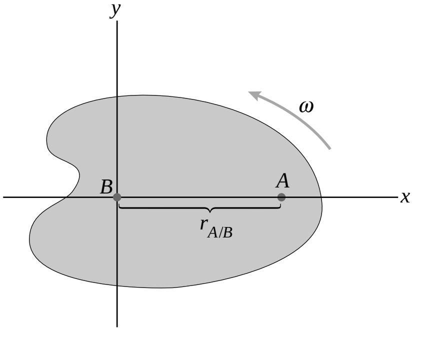
(a) $x = R\beta \Rightarrow \dot{x} = R\dot{\beta} \Rightarrow v = R\omega$

$$\Rightarrow \boxed{\omega = \frac{v}{R} = \frac{2 \text{ m/s}}{0.5 \text{ m}} = 4 \text{ rad/s CW}}$$

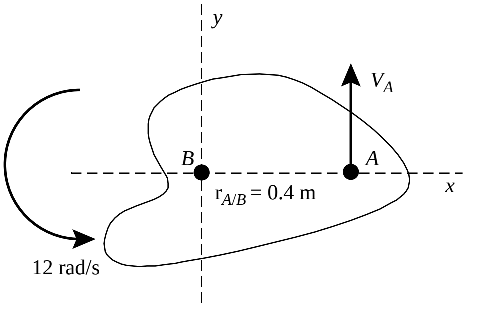
(b) $\boxed{\boldsymbol{\omega} = -(4 \text{ rad/s})\mathbf{k}}$

Problem 17.18 The rigid body rotates with angular velocity $\omega = 12$ rad/s. The distance $r_{A/B} =$ 0.4 m.

(a) Determine the x and y components of the velocity of A relative to B by representing the velocity as shown in Fig. 17.10b.
(b) What is the angular velocity vector of the rigid body?
(c) Use Eq. (17.5) to determine the velocity of A relative to B.



Solution:



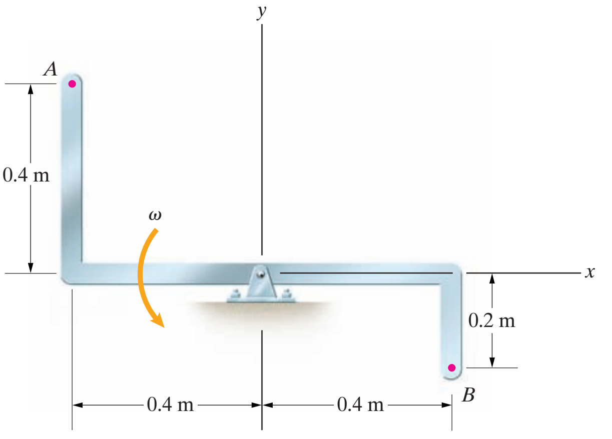
(a) $\boxed{\begin{aligned} v_{Ax} &= 0 \\ v_{Ay} &= (12 \text{ rad/s})(0.4 \text{ m}) = 4.8 \text{ m/s} \end{aligned}}$

(b) $\boxed{\boldsymbol{\omega} = (12 \text{ rad/s})\mathbf{k}}$

(c) $\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_{A/B} = (12 \text{ rad/s})\mathbf{k} \times (0.4 \text{ m})\mathbf{i}$

$\boxed{\mathbf{v}_A = (4.8 \text{ m/s})\mathbf{j}}$

Problem 17.19 The bar is rotating in the counterclockwise direction with angular velocity ω. The magnitude of the velocity of point A is 6 m/s. Determine the velocity of point B.



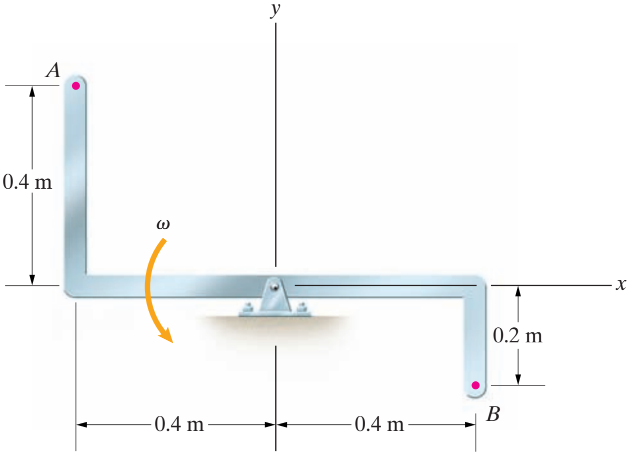
Solution:

$$\omega = \frac{v}{r} = \frac{6 \text{ m/s}}{\sqrt{2}(0.4 \text{ m})} = 10.6 \text{ rad/s}.$$

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_B = (10.6 \text{ rad/s})\mathbf{k} \times (0.4\mathbf{i} - 0.2\mathbf{j}) \text{ m}$$

$$\boxed{\mathbf{v}_B = (2.12\mathbf{i} + 4.24\mathbf{j}) \text{ m/s.}}$$

Problem 17.20 The bar is rotating in the counterclockwise direction with angular velocity ω. The magnitude of the velocity of point A relative to point B is 6 m/s. Determine the velocity of point B.



Solution:

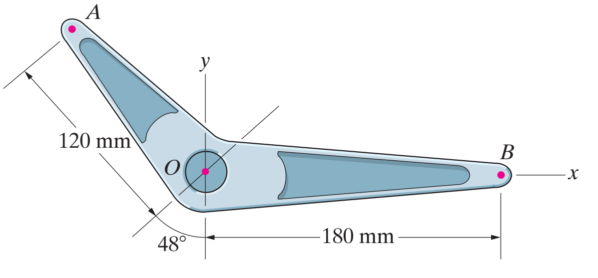
$$r_{A/B} = \sqrt{(0.8 \text{ m})^2 + (0.6 \text{ m})^2} = 1 \text{ m}$$

$$\omega = \frac{v}{r_{A/B}} = \frac{6 \text{ m/s}}{1 \text{ m}} = 6 \text{ rad/s}.$$

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_B = (6 \text{ rad/s})\mathbf{k} \times (0.4\mathbf{i} - 0.2\mathbf{j}) \text{ m}$$

$$\boxed{\mathbf{v}_B = (1.2\mathbf{i} + 2.4\mathbf{j}) \text{ m/s.}}$$

Problem 17.21 The bracket is rotating about point O with counterclockwise angular velocity ω. The magnitude of the velocity of point A relative to point B is 4 m/s. Determine ω.



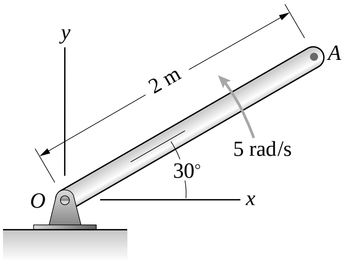
Solution:

$$r_{B/A} = \sqrt{(0.18 + 0.12\cos 48°)^2 + (0.12 \sin 48°)^2} = 0.275 \text{ m}$$

$$\omega = \frac{v_{B/A}}{r_{B/A}} = \frac{4 \text{ m/s}}{0.275 \text{ m}} = 14.5 \text{ rad/s}.$$

$$\boxed{\omega = 14.5 \text{ rad/s.}}$$

Problem 17.22 Determine the x and y components of the velocity of point A.



Solution: The velocity of point A is given by:

$$\mathbf{v}_A = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}_{A/O}.$$

Hence, $\mathbf{v}_A = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5 \\ 2\cos 30^\circ & 2\sin 30^\circ & 0 \end{vmatrix}$

$$= -10 \sin 30^\circ \mathbf{i} + 10 \cos 30^\circ \mathbf{j},$$

or $\mathbf{v}_A = -5\mathbf{i} + 8.66\mathbf{j}$ (m/s).

Problem 17.23 If the angular velocity of the bar in Problem 17.22 is constant, what are the x and y components of the velocity of Point A 0.1 s after the instant shown?

Solution: The angular velocity is given by

$$\omega = \frac{d\theta}{dt} = 5 \text{ rad/s},$$

$$\int_0^\theta d\theta = \int_0^t 5\,dt,$$

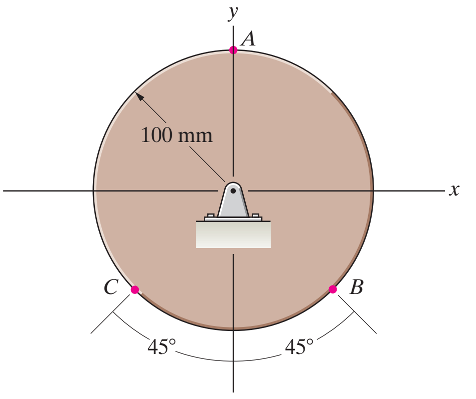
and $\theta = 5t$ rad.

At $t = 0.1$ s, $\theta = 0.5$ rad $= 28.6^\circ$.

$$\mathbf{v}_A = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}_{A/O} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5 \\ 2\cos 28.6^\circ & 2\sin 28.6^\circ & 0 \end{vmatrix}.$$

Hence, $\mathbf{v}_A = -10 \sin 28.6^\circ \mathbf{i} + 10 \cos 28.6^\circ \mathbf{j} = -4.78\mathbf{i} + 8.78\mathbf{j}$.

Problem 17.24 The disk is rotating about the z axis at 50 rad/s in the clockwise direction. Determine the x and y components of the velocities of points A, B, and C.



Solution: The velocity of A is given by $\mathbf{v}_A = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}_{A/O}$, or $\mathbf{v}_A = 0 + (-50\mathbf{k}) \times (0.1\mathbf{j}) = 5\mathbf{i}$ (m/s).

For B, we have

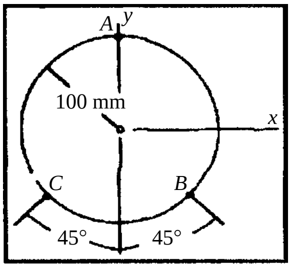
$$\mathbf{v}_B = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}_{B/O} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -50 \\ 0.1\cos 45^\circ & -0.1\sin 45^\circ & 0 \end{vmatrix}$$

$$= -3.54\mathbf{i} - 3.54\mathbf{j} \text{ (m/s)},$$

For C, we have

$$\mathbf{v}_c = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r}_{C/O} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -50 \\ -0.1\cos 45^\circ & -0.1\sin 45^\circ & 0 \end{vmatrix}$$

$$= -3.54\mathbf{i} + 3.54\mathbf{j} \text{ (m/s)}.$$



Problem 17.25 Consider the rotating disk shown in Problem 17.24. If the magnitude of the velocity of point A relative to point B is 4 m/s, what is the magnitude of the disk's angular velocity?

Solution:

$$\mathbf{v}_0 = 0 \quad \boldsymbol{\omega} = \omega\mathbf{k} \quad r = 0.1 \text{ m}$$

$$\mathbf{v}_B = \mathbf{v}_0 + \omega\mathbf{k} \times \mathbf{r}_{OB}$$

$$= \omega\mathbf{k} \times (r\cos 45^\circ\mathbf{i} - r\sin 45^\circ\mathbf{j})$$

$$= (r\omega\cos 45^\circ)\mathbf{j} + (r\omega\sin 45^\circ)\mathbf{i}.$$

$$\mathbf{v}_A = \mathbf{v}_0 + \omega\mathbf{k} \times \mathbf{r}_{OA} = \omega\mathbf{k} \times r\mathbf{j}$$

$$= -r\omega\mathbf{i}.$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B},$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B,$$

$$\mathbf{v}_{A/B} = (-r\omega - r\omega\sin 45^\circ)\mathbf{i} - r\omega\cos 45^\circ\mathbf{j}$$

$$= r\omega(-1 - \sin 45^\circ)\mathbf{i} - r\omega\cos 45^\circ\mathbf{j}.$$

We know

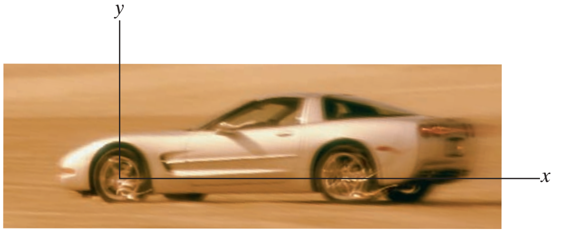
$$|\mathbf{v}_{A/B}| = 4 \text{ m/s}, \quad r = 0.1 \text{ m}$$

$$|\mathbf{v}_{A/B}| = \sqrt{[r\omega(-1 - \sin 45^\circ)]^2 + [-r\omega\cos 45^\circ]^2}$$

Solving for ω, $\omega = 21.6$ rad/s (direction undetermined).

Problem 17.26 The radius of the Corvette's tires is 30 cm. It is traveling at 100 km/h. Assume that the tires roll, not skid, on the road surface.

(a) What is the angular velocity of its wheels?
(b) In terms of the earth-fixed coordinate system shown, determine the velocity (in m/s) of the point of the tire with coordinates (−30 cm, 0,0).



Solution:

(a) $\omega = \dfrac{v}{r} = \dfrac{\left(\dfrac{100 \times 1000}{3600}\right)}{(0.3)} = 92.6 \text{ rad/s.}$ $\boxed{\omega = 92.6 \text{ rad/s.}}$

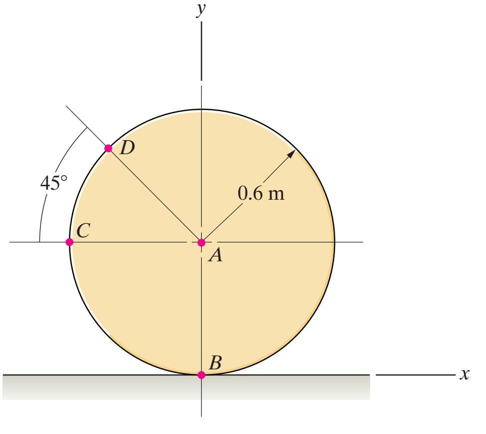
(b)

$$\mathbf{v}_P = \mathbf{v}_O + \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{v}_P = -\left(\frac{100 \times 1000}{3600} \text{ m/s}\right)\mathbf{i} + (92.6 \text{ rad/s})\mathbf{k} \times \left(-[0.3 \text{ m}]\,\mathbf{i}\right)$$

$$\boxed{\mathbf{v}_P = (-27.8\mathbf{i} - 27.8\mathbf{j}) \text{ m/s.}}$$

Problem 17.27 Point A of the rolling disk is moving toward the right. The magnitude of the velocity of point C is 5 m/s. determine the velocities of points B and D.



Solution: Point B is the center of rotation (zero velocity).

$r_{C/B} = \sqrt{2}(0.6 \text{ m}) = 0.849 \text{ m},$

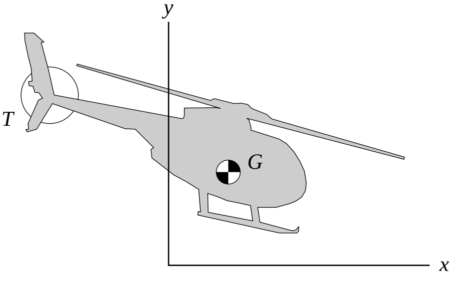
$$\omega = \frac{v_C}{r_{C/B}} = \frac{5 \text{ m/s}}{0.849 \text{ m}} = 5.89 \text{ rad/s}$$

Therefore

$$\mathbf{v}_D = \boldsymbol{\omega} \times \mathbf{r}_{D/B} = -(5.89 \text{ rad/s})\mathbf{k} \times [-0.6\cos 45^\circ \mathbf{i} + (0.6 + 0.6\sin 45^\circ)\mathbf{j}]$$

$$\boxed{\mathbf{v}_D = (6.04\mathbf{i} + 2.5\mathbf{j}) \text{ m/s}, \mathbf{v}_B = 0.}$$

Problem 17.28 The helicopter is in planar motion in the x–y plane. At the instant shown, the position of its center of mass, G, is $x = 2$ m, $y = 2.5$ m, and its velocity is $\mathbf{v}_G = 12\mathbf{i} + 4\mathbf{j}$ (m/s). The position of point T, where the tail rotor is mounted, is $x = -3.5$ m, $y = 4.5$ m. The helicopter's angular velocity is 0.2 (rad/s) clockwise. What is the velocity of point T?



Solution: The position of T relative to G is

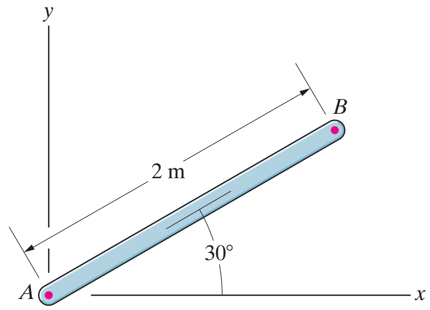
$$\mathbf{r}_{T/G} = (-3.5 - 2)\mathbf{i} + (4.5 - 2.5)\mathbf{j} = -5.5\mathbf{i} + 2\mathbf{j} \text{ (m).}$$

The velocity of T is

$$\mathbf{v}_T = \mathbf{v}_G + \boldsymbol{\omega} \times \mathbf{r}_{T/G} = 12\mathbf{i} + 4\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.2 \\ -5.5 & 2 & 0 \end{vmatrix}$$

$$= 12.4\mathbf{i} + 5.1\mathbf{j} \text{ (m/s)}$$

Problem 17.29 The bar is moving in the $x-y$ plane and is rotating in the counterclockwise direction. The velocity of point A relative to the reference frame is $\mathbf{v}_A = 12\mathbf{i} - 2\mathbf{j}$ (m/s). The magnitude of the velocity of point A relative to point B is 8 m/s. what is the velocity of point B relative to the reference frame?



Solution:

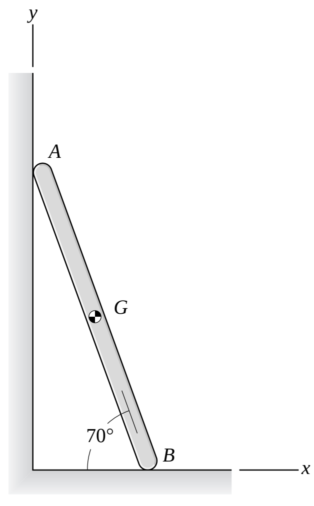
$$\omega = \frac{v}{r} = \frac{8 \text{ m/s}}{2 \text{ m}} = 4 \text{ rad/s},$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A} = (12\mathbf{i} - 2\mathbf{j}) \text{ m/s} + (4 \text{ rad/s})\mathbf{k} \times (2 \text{ m})(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j})$$

$$\boxed{\mathbf{v}_B = (8\mathbf{i} + 4.93\mathbf{j}) \text{ m/s.}}$$

Problem 17.30 Points A and B of the 2-m bar slide on the plane surfaces. Point B is moving to the right at 3 m/s. What is the velocity of the midpoint G of the bar?

Strategy: First apply Eq. (17.6) to points A and B to determine the bar's angular velocity. Then apply Eq. (17.6) to points B and G.



Solution: Take advantage of the constraints (B stays on the floor, A stays on the wall)

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{j} = (3 \text{ m/s})\mathbf{i} + \omega\mathbf{k} \times (2 \text{ m})(-\cos 70^\circ \mathbf{i} + \sin 70^\circ \mathbf{j})$$

$$= (3 - 1.88\ \omega)\mathbf{i} + (-0.684\ \omega)\mathbf{j}$$

Equating $\mathbf{i}$ components we find $3 - 1.88\ \omega = 0 \Rightarrow \omega = 1.60$ rad/s

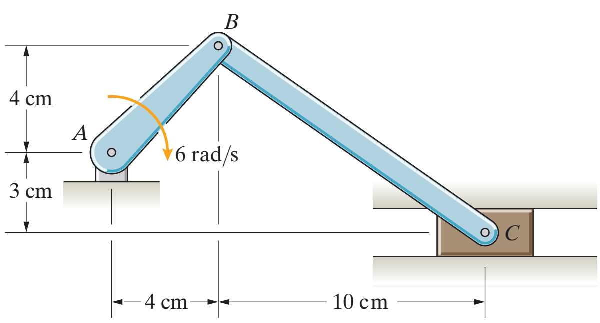
Now find the velocity of point G

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{G/B}$$

$$= (3 \text{ m/s})\mathbf{i} + (1.60 \text{ rad/s})\mathbf{k} \times (1 \text{ m})(-\cos 70^\circ \mathbf{i} + \sin 70^\circ \mathbf{j})$$

$$\boxed{\mathbf{v}_G = (1.5\mathbf{i} - 0.546\mathbf{j}) \text{ m/s}}$$

Problem 17.31 Bar AB rotates at 6 rad/s in the clockwise direction. Determine the velocity (in cm/s) of the slider C.



Solution:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{v}_B = 0 - (6 \text{ rad/s})\mathbf{k} \times (4\mathbf{i} + 4\mathbf{j}) \text{ cm}$$

$$\mathbf{v}_B = (24\mathbf{i} - 24\mathbf{j}) \text{ cm/s.}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$\mathbf{v}_C = (24\mathbf{i} - 24\mathbf{j}) + \omega_{BC}\mathbf{k} \times (10\mathbf{i} - 7\mathbf{j})$$

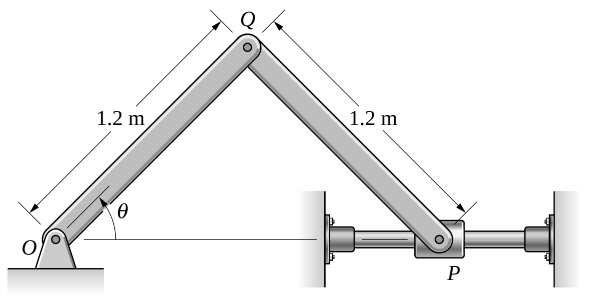
$$\mathbf{v}_C = (24 + 7\ \omega_{BC})\mathbf{i} + (-24 + 10\ \omega_{BC})\mathbf{j}$$

Slider C cannot move in the $\mathbf{j}$ direction, therefore

$$\omega_{BC} = \frac{24}{10} = 2.4 \text{ rad/s.}$$

$$\mathbf{v}_C = (24 + 7(2.4))\mathbf{i} = (43.8 \text{ cm/s})\mathbf{i} \quad \boxed{v_C = 43.8 \text{ cm/s to the right.}}$$

Problem 17.32 If $\theta = 45^\circ$ and the sleeve P is moving to the right at 2 m/s, what are the angular velocities of the bars OQ and PQ?



Solution: From the figure, $\mathbf{v}_0 = 0$, $\mathbf{v}_P = v_P \mathbf{i} = 2\ \mathbf{i}$ (m/s)

$$\mathbf{v}_Q = \mathbf{v}_0 + \omega_{OQ}\mathbf{k} \times (L\cos\theta\mathbf{i} + L\sin\theta\mathbf{j})$$

$$\begin{cases} \mathbf{i}: \quad v_{Q_x} = -\omega_{OQ} L \sin\theta & \textbf{(1)} \\ \mathbf{j}: \quad v_{Q_y} = \omega_{OQ} L \cos\theta & \textbf{(2)} \end{cases}$$

$$\mathbf{v}_P = \mathbf{v}_Q + \omega_{QP}\mathbf{k} \times (L\cos\theta\mathbf{i} - L\sin\theta\mathbf{j})$$

$$\mathbf{i}: \quad 2 = v_{Q_x} + \omega_{QP} L \sin\theta \qquad \textbf{(3)}$$

$$\mathbf{j}: \quad 0 = v_{Q_y} + \omega_{QP} L \cos\theta \qquad \textbf{(4)}$$

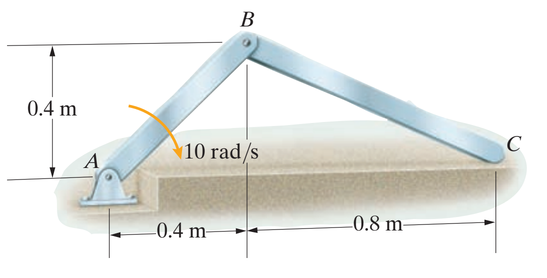
Eqns (1)–(4) are 4 eqns in the 4 unknowns ω_{OQ}, ω_{QP}, v_{Q_x}, v_{Q_y}
Solving, we get

$$v_{Q_x} = 1 \text{ m/s}, \quad v_{Q_y} = -1 \text{ m/s}$$

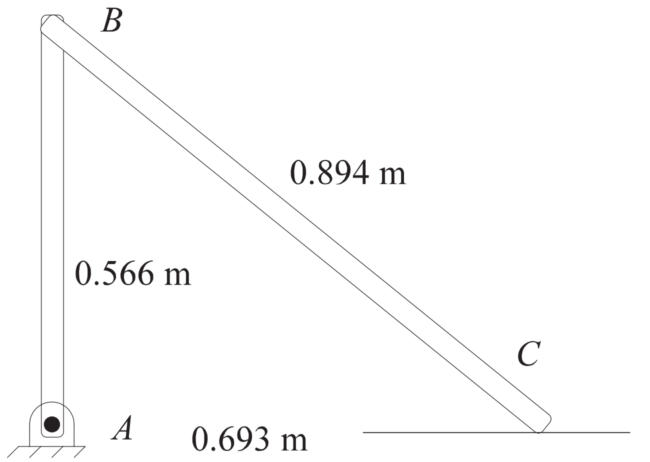
$$\underline{\boldsymbol{\omega}_{OQ} = -1.18\mathbf{k}} \text{ (rad/s)},$$

$$\boldsymbol{\omega}_{QP} = 1.18\mathbf{k} \text{ (rad/s)}.$$

Problem 17.33 In Active Example 17.2, consider the instant when bar AB is vertical and rotating in the clockwise direction at 10 rad/s. Draw a sketch showing the positions of the two bars at that instant. Determine the angular velocity of bar BC and the velocity of point C.



Solution: The sketch is shown. We have



$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$= 0 - (10 \text{ rad/s})\mathbf{k} \times (0.566 \text{ m})\mathbf{j}$$

$$= (5.66 \text{ m/s})\mathbf{i}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{BC}$$

$$= (5.66 \text{ m/s})\mathbf{i}$$

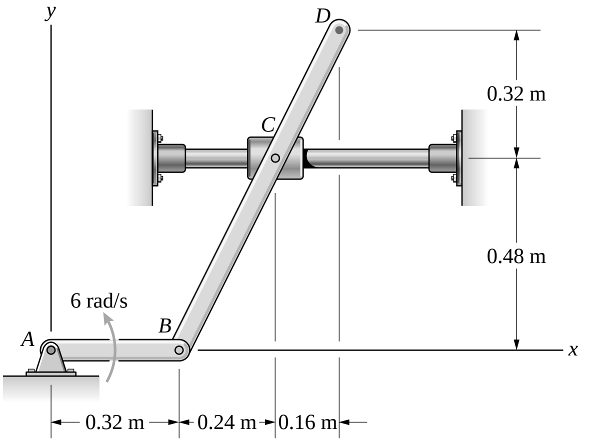
$$+ \omega_{BC}\mathbf{k} \times (0.693\mathbf{i} - 0.566\mathbf{j}) \text{ m}$$

$$= (5.66 + 0.566\ \omega_{BC})\mathbf{i} + (0.693\ \omega_{BC})\mathbf{j}$$

Point C cannot move in the $\mathbf{j}$ direction, therefore

$$\boxed{\omega_{BC} = 0, \mathbf{v}_C = (5.66 \text{ m/s})\mathbf{i}}$$

Problem 17.34 Bar AB rotates in the counterclockwise direction at 6 rad/s. Determine the angular velocity of bar BD and the velocity of point D.



Solution:

$\mathbf{v}_A = 0 \quad \boldsymbol{\omega}_{AB} = 6\,\mathbf{k}$

$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB}\mathbf{k} \times \mathbf{r}_{B/A} = 6\mathbf{k} \times 0.32\mathbf{i} = 1.92\mathbf{j}$ m/s.

$\mathbf{v}_C = v_C\mathbf{i} = \mathbf{v}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{C/B}$:

$v_c\mathbf{i} = 1.92\mathbf{j} + \omega_{BD}\mathbf{k} \times (0.24\mathbf{i} + 0.48\mathbf{j})$.

$$\begin{cases} \mathbf{i}: & v_C = -0.48\omega_{BD} \\ \mathbf{j}: & 0 = 1.92 + 0.24\omega_{BD} \end{cases}$$

Solving, $\omega_{BD} = -8$,

$$\underline{\boldsymbol{\omega}_{BD} = -8\mathbf{k}\left(\frac{\text{rad}}{\text{s}}\right)},$$

$$\underline{\mathbf{v}_C = 3.84\mathbf{i}\ (\text{m/s})}.$$

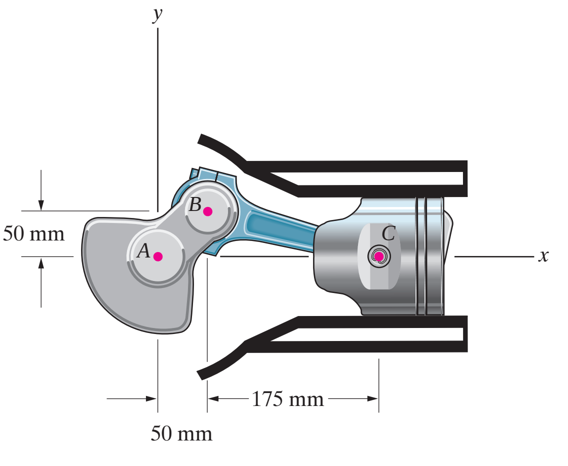
Now for the velocity of point D

$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B}$

$= 1.92\mathbf{j} + (-8\mathbf{k}) \times (0.4\mathbf{i} + 0.8\mathbf{j})$

$\underline{\mathbf{v}_D = 6.40\mathbf{i} - 1.28\mathbf{j}\ (\text{m/s})}$.

Problem 17.35 At the instant shown, the piston's velocity is $\mathbf{v}_C = -14\mathbf{i}$(m/s). What is the angular velocity of the crank AB?



Solution:

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$

$= 0 + \omega_{AB}\mathbf{k} \times (0.05\mathbf{i} + 0.05\mathbf{j})$ m

$= -0.05\omega_{AB}\mathbf{i} + 0.05\omega_{AB}\mathbf{j}$

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}$

$= (-0.05\omega_{AB}\mathbf{i} + 0.05\omega_{AB}\mathbf{j})$

$+ \omega_{BC}\mathbf{k} \times (0.175\mathbf{i} - 0.05\mathbf{j})$

$= (-0.05\omega_{AB} + 0.05\omega_{BC})\mathbf{i} + (0.05\omega_{AB} + 0.175\omega_{BC})\mathbf{j}$

We can now separate components and produce two equations in two unknowns

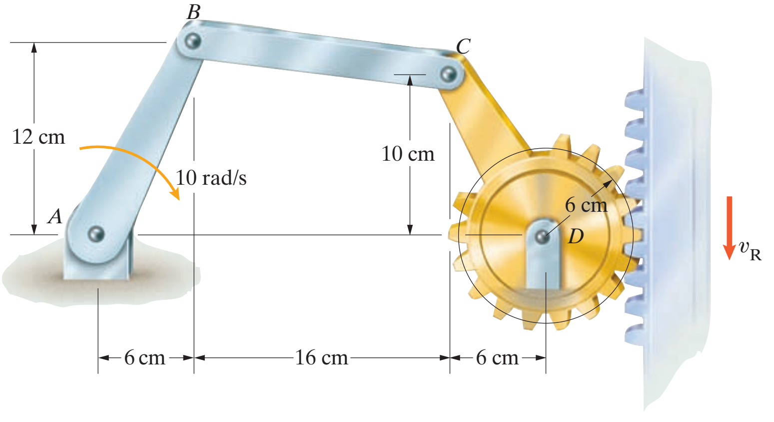
$$-14 = -0.05\omega_{AB} + 0.05\omega_{BC},\ 0 = 0.05\omega_{AB} + 0.175\omega_{BC}$$

Solving we find

$$\omega_{BC} = -62.2 \text{ rad/s},\ \omega_{AB} = 218 \text{ rad/s}.$$

Thus $\boxed{\omega_{AB} = 218 \text{ rad/s counterclockwise.}}$

Problem 17.36 In Example 17.3, determine the angular velocity fo the bar AB that would be necessary so that the downward velocity of the rack $V_R = 120$ cm/s at the instant shown.



Solution: We have

$$\omega_{CD} = \frac{v_R}{r}$$

$$= \frac{120 \text{ cm/s}}{6 \text{ cm}} = 20 \text{ rad/s}.$$

Then

$$\mathbf{V}_B = \mathbf{V}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$= 0 + \omega_{AB}\mathbf{k} \times (6\mathbf{i} + 12\mathbf{j}) = -12\omega_{AB}\mathbf{i} + 6\omega_{AB}\mathbf{j}$$

$$\mathbf{V}_C = \mathbf{V}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= (-12\omega_{AB}\mathbf{i} + 6\omega_{AB}\mathbf{j}) + \omega_{BC}\mathbf{k} \times (16\mathbf{i} - 2\mathbf{j})$$

$$= (-12\omega_{AB} + 2\omega_{BC})\mathbf{i} + (6\omega_{AB} + 16\omega_{BC})\mathbf{j}$$

$$\mathbf{V}_D = \mathbf{V}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$$

$$= [(-12\omega_{AB} + 2\omega_{BC})\mathbf{i} + (6\omega_{AB} + 16\omega_{BC})\mathbf{j}] - (20)\mathbf{k} \times (6\mathbf{i} - 10\mathbf{j})$$

$$= (-12\omega_{AB} + 2\omega_{BC} - 200)\mathbf{i} + (6\omega_{AB} + 16\omega_{BC} - 120)\mathbf{j}$$

Point D cannot move, therefore

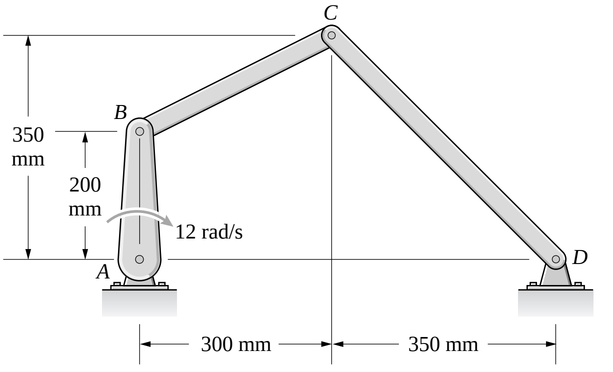
$$-12\omega_{AB} + 2\omega_{BC} - 200 = 0, \ \ 6\omega_{AB} + 16\omega_{BC} - 120 = 0.$$

Solving, we find

$$\omega_{AB} = -14.5 \text{ rad/s}, \quad \omega_{BC} = 12.9 \text{ rad/s}.$$

$$\boxed{\omega_{AB} = 14.5 \text{ rad/s counterclockwise.}}$$

Problem 17.37 Bar AB rotates at 12 rad/s in the clockwise direction. Determine the angular velocities of bars BC and CD.



Solution: The strategy is analogous to that used in Problem 17.36. The radius vector AB is $\mathbf{r}_{B/A} = 200\mathbf{j}$ (mm). The angular velocity of AB is $\boldsymbol{\omega} = -12\mathbf{k}$ (rad/s). The velocity of point B is

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A} = 0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -12 \\ 0 & 200 & 0 \end{bmatrix} = 2400\mathbf{i} \text{ (mm/s)}.$$

The radius vector BC is $\mathbf{r}_{C/B} = 300\mathbf{i} + (350 - 200)\mathbf{j} = 300\mathbf{i} + 150\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 300 & 150 & 0 \end{bmatrix}$$

$$= (2400 - 150\omega_{BC})\mathbf{i} + \omega_{BC} 300\mathbf{j} \text{ (mm/s)}.$$

The radius vector DC is $\mathbf{r}_{C/D} = -350\mathbf{i} + 350\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_D + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = 0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -350 & 350 & 0 \end{bmatrix}$$

$$= -350\omega_{CD}(\mathbf{i} + \mathbf{j}).$$

Equate the two expressions for $\mathbf{v}_C$, and separate components:

$$(2400 - 150\omega_{BC} + 350\omega_{CD})\mathbf{i} = 0,$$

and $$(300\omega_{BC} + 350\omega_{CD})\mathbf{j} = 0.$$

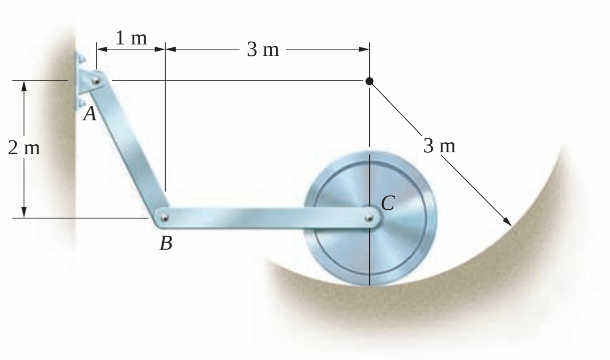
Solve: $\omega_{BC} = 5.33$ rad/s,

$\boxed{\boldsymbol{\omega}_{BC} = 5.33\mathbf{k} \text{ (rad/s)}}$.

$\omega_{CD} = -4.57$ rad/s,

$\boxed{\boldsymbol{\omega}_{CD} = -4.57\mathbf{k} \text{ (rad/s)}}$.

Problem 17.38 Bar AB is rotating at 10 rad/s in the counterclockwise direction. The disk rolls on the circular surface. Determine the angular velocities of bar BC and the disk at the instant shown.



Solution: The point "D" at the bottom of the wheel has zero velocity.

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$= 0 + (10)\mathbf{k} \times (1\mathbf{i} - 2\mathbf{j}) = (20\mathbf{i} + 10\mathbf{j}) \text{ m/s}.$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= (20\mathbf{i} + 10\mathbf{j}) + \omega_{BC}\mathbf{k} \times (3\mathbf{i}) = (20)\mathbf{i} + (10 + 3\omega_{BC})\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$$

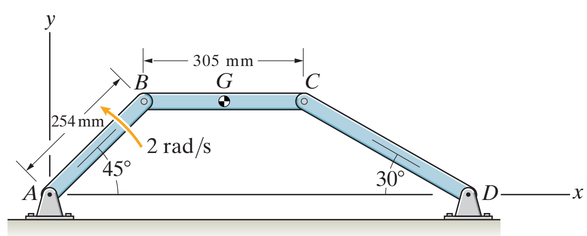
$$= (20)\mathbf{i} + (10 + 3\omega_{BC})\mathbf{j} + \omega_{CD}\mathbf{k} \times (-1\mathbf{j}) = (20 + \omega_{CD})\mathbf{i} + (10 + 3\omega_{BC})\mathbf{j}$$

Since the velocity of D is zero, we can set the components of velocity equal to zero and solve to find

$$\omega_{\text{disk}} = \omega_{CD} = -20 \text{ rad/s}, \quad \omega_{BC} = 3.33 \text{ rad/s}.$$

$$\boxed{\omega_{\text{disk}} = 20 \text{ rad/s clockwise}, \quad \omega_{BC} = 3.33 \text{ rad/s clockwise.}}$$

Problem 17.39 Bar AB rotates at 2 rad/s in the counterclockwise direction. Determine the velocity of the midpoint G of bar BC.



Solution: We first need to find the angular velocities of BC and CD

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (2 \text{ rad/s})\mathbf{k} \times (0.254 \text{ m})(\cos 45^\circ \mathbf{i} + \sin 45^\circ \mathbf{j})$$

$$= (-0.354\mathbf{i} + 0.354\mathbf{j}) \text{ m/s}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = (-0.354\mathbf{i} + 0.354\mathbf{j}) \text{ m/s} + \omega_{BC}\mathbf{k} \times (0.305 \text{ m})\mathbf{i}$$

$$= [(-0.354 \text{ m/s})\mathbf{i} + (0.354 \text{ m/s} + \{0.305 \text{ m}\}\omega_{BC})\mathbf{j}]$$

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$$

$$= [(-0.354 \text{ m/s})\mathbf{i} + (0.354 \text{ m/s} + \{0.305 \text{ m}\}\omega_{BC})\mathbf{j}]$$

$$+ \omega_{CD}\mathbf{k} \times (0.254 \text{ m})[\sin 45^\circ \cot 30^\circ \mathbf{i} - \sin 45^\circ \mathbf{j}]$$

$$= [(-0.354 \text{ m/s} + \{0.18 \text{ m}\}\omega_{CD})\mathbf{i} + (0.354 \text{ m/s}$$

$$+ \{0.305 \text{ m}\}\,\omega_{BC} + \{0.31 \text{ m}\}\omega_{CD})\mathbf{j}]$$

Since D is fixed, we set both components to zero and solve for the angular velocities

$$\begin{array}{l} -0.354 \text{ m/s} + \{0.18 \text{ m}\}\omega_{CD} = 0 \\ 0.354 \text{ m/s} + \{0.305 \text{ m}\}\,\omega_{BC} + \{0.31 \text{ m}\}\omega_{CD} = 0 \end{array} \Rightarrow \begin{array}{l} \omega_{BC} = -3.22 \text{ rad/s} \\ \omega_{CD} = 2.00 \text{ rad/s} \end{array}$$

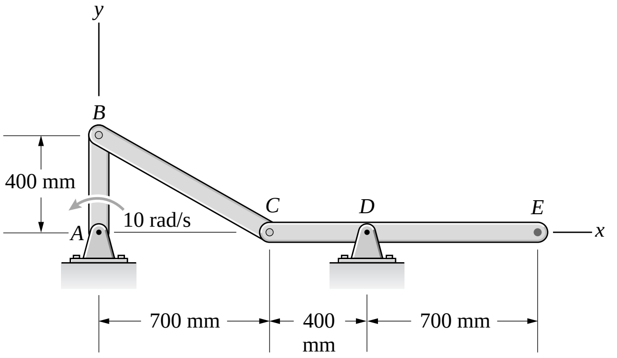
Now we can find the velocity of point G.

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{G/B}$$

$$= (-0.354\mathbf{i} + 0.354\mathbf{j}) \text{ m/s} + (-3.22 \text{ rad/s})\mathbf{k} \times (0.152 \text{ mm})\mathbf{i}$$

$$\boxed{\mathbf{v}_G = (-0.354\,\mathbf{i} - 0.132\,\mathbf{j}) \text{ m/s}}$$

Problem 17.40 Bar AB rotates at 10 rad/s in the counterclockwise direction. Determine the velocity of point E.



Solution: The radius vector AB is $\mathbf{r}_{B/A} = 400\mathbf{j}$ (mm). The angular velocity of bar AB is $\boldsymbol{\omega}_{AB} = 10\mathbf{k}$ (rad/s). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 0 & 400 & 0 \end{bmatrix} = -4000\mathbf{i} \text{ (mm/s)}.$$

The radius vector BC is $\mathbf{r}_{C/B} = 700\mathbf{i} - 400\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = -4000\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 700 & -400 & 0 \end{bmatrix}$$

$$= (-4000 + 400\omega_{BC})\mathbf{i} + 700\omega_{BC}\mathbf{j}.$$

The radius vector CD is $\mathbf{r}_{C/D} = -400\mathbf{i}$ (mm). The point D is fixed (cannot translate). The velocity at point C is

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = (\omega_{CD}(\mathbf{k}) \times (-400\mathbf{i})) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -400 & 0 & 0 \end{bmatrix}$$

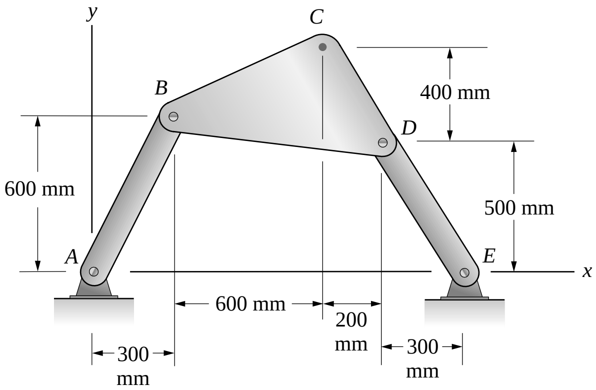
$$= -400\omega_{CD}\mathbf{j}.$$

Equate the two expressions for the velocity at point C, and separate components: $0 = (-4000 + 400\omega_{BC})\mathbf{i}$, $0 = (700\omega_{BC} + 400\omega_{CD})\mathbf{j}$. Solve: $\omega_{BC} = 10$ rad/s, $\omega_{CD} = -17.5$ rad/s. The radius vector DE is $\mathbf{r}_{D/E} = 700\mathbf{i}$ (mm). The velocity of point E is

$$\mathbf{v}_E = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/E} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -17.5 \\ 700 & 0 & 0 \end{bmatrix}$$

$\boxed{\mathbf{v}_E = -12250\mathbf{j} \text{ (mm/s)}}$.

Problem 17.41 Bar AB rotates at 4 rad/s in the counterclockwise direction. Determine the velocity of point C.



Solution: The angular velocity of bar AB is $\boldsymbol{\omega} = 4\mathbf{k}$ (rad/s). The radius vector AB is $\mathbf{r}_{B/A} = 300\mathbf{i} + 600\mathbf{j}$ (mm). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 300 & 600 & 0 \end{bmatrix},$$

from which $\mathbf{v}_B = -2400\mathbf{i} + 1200\mathbf{j}$ (mm/s). The vector radius from B to C is $\mathbf{r}_{C/B} = 600\mathbf{i} + (900 - 600)\mathbf{j} = 600\mathbf{i} + 300\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 600 & 300 & 0 \end{bmatrix}$$

$$= (-2400 - 300\omega_{BC})\mathbf{i} + (1200 + 600\omega_{BC})\mathbf{j} \text{ (mm/s)}.$$

The radius vector from C to D is $\mathbf{r}_{D/C} = 200\mathbf{i} - 400\mathbf{j}$ (mm). The velocity of point D is

$$\mathbf{v}_D = \mathbf{v}_C + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 200 & -400 & 0 \end{bmatrix}$$

$$= \mathbf{v}_C + 400\omega_{BC}\mathbf{i} + 200\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

The radius vector from E to D is $\mathbf{r}_{D/E} = -300\mathbf{i} + 500\mathbf{j}$ (mm). The velocity of point D is

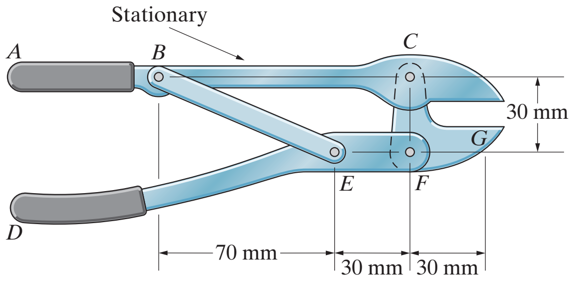
$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DE} \\ -300 & 500 & 0 \end{bmatrix}$$

$$= -500\omega_{DE}\mathbf{i} - 300\omega_{DE}\mathbf{j} \text{ (mm/s)}.$$

Equate the expressions for the velocity of point D; solve for $\mathbf{v}_C$, to obtain one of two expressions for the velocity of point C. Equate the two expressions for $\mathbf{v}_C$, and separate components: $0 = (-500\omega_{DE} - 100\omega_{BC} + 2400)\mathbf{i}$, $0 = (1200 + 300\omega_{DE} + 800\omega_{BC})\mathbf{j}$. Solve $\omega_{DE} = 5.51$ rad/s, $\omega_{BC} = -3.57$ rad/s. Substitute into the expression for the velocity of point C to obtain

$$\boxed{\mathbf{v}_C = -1330\mathbf{i} - 941\mathbf{j} \text{ (mm/s)}.}$$

Problem 17.42 The upper grip and jaw of the pliers ABC is stationary. The lower grip DEF is rotating at 0.2 rad/s in the clockwise direction. At the instant shown, what is the angular velocity of the lower jaw CFG?



Solution:

$$\mathbf{v}_E = \mathbf{v}_B + \boldsymbol{\omega}_{BE} \times \mathbf{r}_{E/B} = 0 + \omega_{BE}\mathbf{k} \times (0.07\mathbf{i} - 0.03\mathbf{j}) \text{ m}$$

$$= \omega_{BE}(0.03\mathbf{i} + 0.07\mathbf{j}) \text{ m}$$

$$\mathbf{v}_F = \mathbf{v}_E + \boldsymbol{\omega}_{DEF} \times \mathbf{r}_{F/E} = \omega_{BE}(0.03\mathbf{i} + 0.07\mathbf{j}) \text{ m}$$

$$+ (-0.2 \text{ rad/s})\mathbf{k} \times (0.03 \text{ m})\mathbf{i}$$

$$= [(0.03 \text{ m})\omega_{BE}\mathbf{i} + (-0.006 \text{ m/s} + \{0.07 \text{ m}\}\omega_{BE})\mathbf{j}]$$

$$\mathbf{v}_C = \mathbf{v}_F + \boldsymbol{\omega}_{CFG} \times \mathbf{r}_{C/G}$$

$$= [(0.03 \text{ m})\omega_{BE}\mathbf{i} + (-0.006 \text{ m/s} + \{0.07 \text{ m}\}\omega_{BE})\mathbf{j}]$$

$$+ \omega_{CFG}\mathbf{k} \times (0.03 \text{ m})\mathbf{j}$$

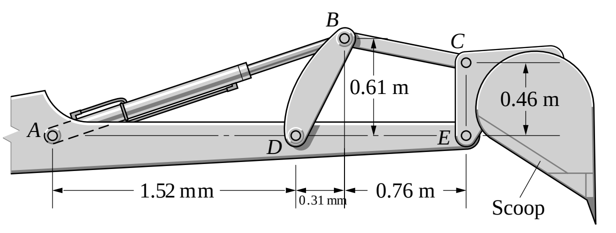
$$= [(0.03 \text{ m})(\omega_{BE} - \omega_{CFG})\mathbf{i} + (-0.006 \text{ m/s} + \{0.07 \text{ m}\}\omega_{BE})\mathbf{j}]$$

Since C is fixed we have

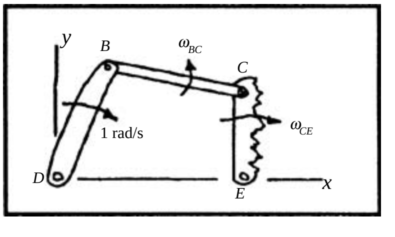
$$\begin{aligned} (0.03 \text{ m})(\omega_{BE} - \omega_{CFG}) &= 0 \\ -0.006 \text{ m/s} + \{0.07 \text{ m}\}\omega_{BE} &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} \omega_{BE} &= 0.0857 \text{ rad/s} \\ \omega_{CFG} &= 0.0857 \text{ rad/s} \end{aligned}$$

So we have $\boxed{\omega_{CFG} = 0.0857 \text{ rad/s CCW}}$

Problem 17.43 The horizontal member ADE supporting the scoop is stationary. If the link BD is rotating in the clockwise direction at 1 rad/s, what is the angular velocity of the scoop?



Solution: The velocity of B is $\mathbf{v}_B = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D}$. Expanding, we get



$$\mathbf{v}_B = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.31 \\ 0.31 & 0.61 & 0 \end{vmatrix} = 0.61\mathbf{i} - 0.31\mathbf{j} \text{ (m/s)}.$$

The velocity of C is

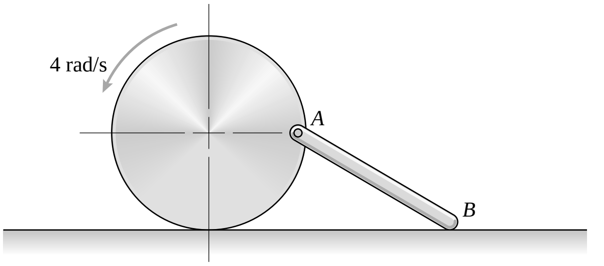
$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = 0.61\mathbf{i} - 0.31\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & +\omega_{BC} \\ 0.76 & -0.15 & 0 \end{vmatrix} \quad \mathbf{(1)}.$$

We can also express the velocity of C as $\mathbf{v}_C = \mathbf{v}_E + \boldsymbol{\omega}_{CE} \times \mathbf{r}_{C/E}$ or

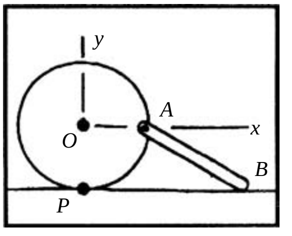
$$\mathbf{v}_C = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & +\omega_{CE} \\ 0 & 0.46 & 0 \end{vmatrix} \quad \mathbf{(2)}.$$

Equating **i** and **j** components in Equations (1) and (2) and solving, we obtain $\omega_{BC} = 0.4$ rad/s and $\omega_{CE} = -1.47$ rad/s.

Problem 17.44 The diameter of the disk is 1 m, and the length of the bar AB is 1 m. The disk is rolling, and point B slides on the plane surface. Determine the angular velocity of the bar AB and the velocity of point B.



Solution: Choose a coordinate system with the origin at O, the center of the disk, with x axis parallel to the horizontal surface. The point P of contact with the surface is stationary, from which



$$\mathbf{v}_P = 0 = \mathbf{v}_0 + \boldsymbol{\omega}_0 \times -\mathbf{R} = \mathbf{v}_0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_0 \\ 0 & -0.5 & 0 \end{bmatrix} = \mathbf{v}_0 + 2\mathbf{i},$$

from which $\mathbf{v}_0 = -2\mathbf{i}$ (m/s). The velocity at A is $\mathbf{v}_A = \mathbf{v}_0 + \boldsymbol{\omega}_0 \times \mathbf{r}_{A/O}$.

$$\mathbf{v}_A = -2\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_0 \\ 0.5 & 0 & 0 \end{bmatrix} = -2\mathbf{i} + 2\mathbf{j} \text{ (m/s)}.$$

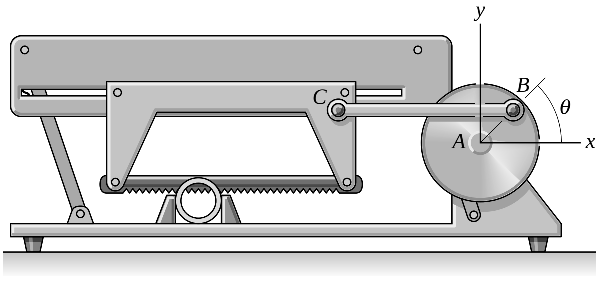
The vector from B to A is $\mathbf{r}_{A/B} = -\mathbf{i}\cos\theta + \mathbf{j}\sin\theta$ (m), where $\theta = \sin^{-1} 0.5 = 30^\circ$. The motion at point B is parallel to the x axis. The velocity at A is

$$\mathbf{v}_A = v_B\mathbf{i} + \boldsymbol{\omega} \times \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -0.866 & 0.5 & 0 \end{bmatrix}$$

$$= (v_B - 0.5\omega_{AB})\mathbf{i} - 0.866\omega_{AB}\mathbf{j} \text{ (m/s)}.$$

Equate and solve: $(-2 - 0.866\omega_{AB})\mathbf{j} = 0$, $(v_B - 0.5\omega_{AB} + 2)\mathbf{i} = 0$, from which $\boldsymbol{\omega}_{AB} = -2.31\mathbf{k}$ (rad/s), $\mathbf{v}_B = -3.15\mathbf{i}$ (m/s).

Problem 17.45 A motor rotates the circular disk mounted at A, moving the saw back and forth. (The saw is supported by a horizontal slot so that point C moves horizontally). The radius AB is 101.6 mm, and the link BC 355.6 mm long. In the position shown, $\theta = 45^\circ$ and the link BC is horizontal. If the angular velocity of the disk is one revolution per second counterclockwise, what is the velocity of the saw?



Solution: The radius vector from A to B is

$$\mathbf{r}_{B/A} = 0.1016\,(\mathbf{i}\cos 45^\circ + \mathbf{j}\sin 45^\circ) = 0.051\sqrt{2}(\mathbf{i}+\mathbf{j})\ (\text{m}).$$

The angular velocity of B is

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A},$$

$$\mathbf{v}_B = 0 + 2\pi(0.051\sqrt{2}) \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = 0.102\pi\sqrt{2}(-\mathbf{i}+\mathbf{j})\ (\text{m/s}).$$

The radius vector from B to C is $\mathbf{r}_{C/B} = (0.1016\ \cos 45^\circ - 0.356)\mathbf{i}. = -\ 0.284\,\mathbf{i}$
The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -0.284-14 & 0 & 0 \end{bmatrix}$$

$$= -0.453\,\mathbf{i} + 0.453\,\mathbf{j} + \mathbf{j}\,(-0.284\,\omega_{BC})$$

$$= -0.453\,\mathbf{i} + (0.453 - 0.284\,\omega_{BC})\,\mathbf{j}$$

The saw is constrained to move parallel to the x axis, hence $0.453 - 0.284\,\omega_{BC} = 0$, and the saw velocity is

$$\boxed{\mathbf{v}_S = -\ 0.453\,\mathbf{i}\ (\text{m/s})}.$$

Problem 17.46 In Problem 17.45, if the angular velocity of the disk is one revolution per second counterclockwise and $\theta = 270^\circ$, what is the velocity of the saw?

Solution: The radius vector from A to B is $\mathbf{r}_{B/A} = -4\mathbf{j}$ (m).
The velocity of B is

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = 2\pi(-0.1016) \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = 0.203\pi\mathbf{i}\ (\text{m/s}).$$

The coordinates of point C are

$$(-0.356\ \cos\beta, +0.1016\sin 45^\circ) = (-0.31,\ 0.072)\ \text{m},$$

$$\text{where}\quad \beta = \sin^{-1}\left(\frac{0.1016\,(1+\sin 45^\circ)}{0.356}\right) = 29.19^\circ.$$

The coordinates of point B are $(0, -0.1016)$ in. The vector from C to B is

$$\mathbf{r}_{C/B} = (-0.31 - 0)\,\mathbf{i} + (0.072 - (-0.1016))\mathbf{j} = -0.31\mathbf{i} + \ 0.173\mathbf{j}\ (\text{m})$$

The velocity at point C is

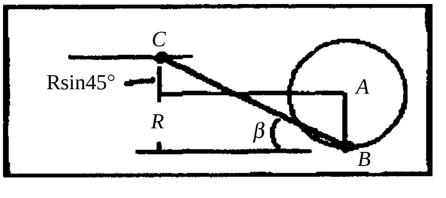
$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -0.31 & 0.173 & 0 \end{bmatrix}$$

$$= (0.203\pi - 0.173\omega_{BC})\mathbf{i} - 0.31\omega_{BC}\mathbf{j}.$$

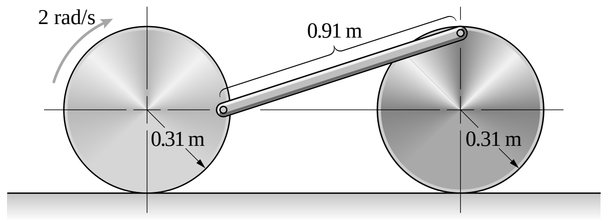
Since the saw is constrained to move parallel to the x axis, $-0.31\omega_{BC}\mathbf{j} = 0$, from which $\omega_{BC} = 0$, and the velocity of the saw is

$$\boxed{\mathbf{v}_C = 0.203\pi\mathbf{i} = 0.638\,\mathbf{i}\ (\text{m/s})}$$

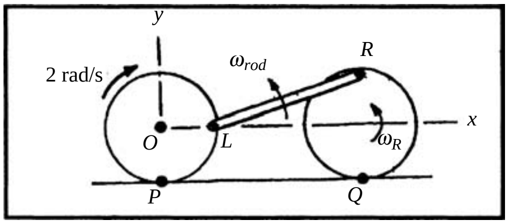
[*Note*: Since the vertical velocity at B reverses direction at $\theta = 270^\circ$, the angular velocity $\omega_{BC} = 0$ can be determined on physical grounds by inspection, simplifying the solution.]



Problem 17.47 The disks roll on a plane surface. The angular velocity of the left disk is 2 rad/s in the clockwise direction. What is the angular velocity of the right disk?



Solution: The velocity at the point of contact P of the left disk is zero. The vector from this point of contact to the center of the left disk is $\mathbf{r}_{O/P} = 0.31\mathbf{j}$ (m). The velocity of the center of the left disk is



$$\mathbf{v}_O = \boldsymbol{\omega} \times \mathbf{r}_{O/P} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -2 \\ 0 & 0.31 & 0 \end{bmatrix} = 0.61\mathbf{i} \text{ (m/s)}.$$

The vector from the center of the left disk to the point of attachment of the rod is $\mathbf{r}_{L/O} = 0.31\mathbf{i}$ (m). The velocity of the point of attachment of the rod to the left disk is

$$\mathbf{v}_L = \mathbf{v}_O + \boldsymbol{\omega} \times \mathbf{r}_{L/O} = 0.61\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -2 \\ 0.31 & 0 & 0 \end{bmatrix}$$

$$= 0.61\mathbf{i} - 0.61\mathbf{j} \text{ (m/s)},$$

The vector from the point of attachment of the left disk to the point of attachment of the right disk is

$$\mathbf{r}_{R/L} = 0.91\,(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) \text{ (m)},$$

where $\theta = \sin^{-1}\left(\dfrac{0.31}{0.91}\right) = 19.47^\circ.$

The velocity of the point on attachment on the right disk is

$$\mathbf{v}_R = \mathbf{v}_L + \boldsymbol{\omega}_{\text{rod}} \times \mathbf{r}_{R/L} = \mathbf{v}_L + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{\text{rod}} \\ 0.863 & 1 & 0 \end{bmatrix}$$

$$= (0.61 - \omega_{\text{rod}})\mathbf{i} + (-0.61 + 0.863\omega_{\text{rod}})\mathbf{j} \text{ (m/s)}.$$

The velocity of point R is also expressed in terms of the contact point Q,

$$\mathbf{v}_R = \boldsymbol{\omega}_{RO} \times \mathbf{r}_{R/O} = \omega_{RO}(2) \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 0.31 & 0 \end{bmatrix}$$

$$= -0.61\omega_{RO}\mathbf{i} \text{ (m/s)}.$$

Equate the two expressions for the velocity $\mathbf{v}_R$ and separate components:

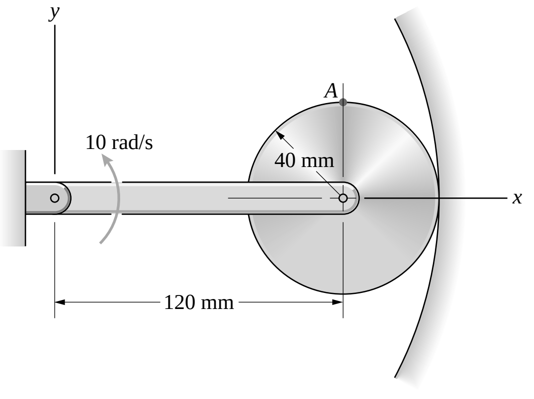
$$(0.61 - \omega_{\text{rod}} + 2\omega_{RO})\mathbf{i} = 0,$$

$$(-0.61 + 0.863\omega_{\text{rod}})\mathbf{j} = 0,$$

from which $\boxed{\boldsymbol{\omega}_{RO} = -0.65\mathbf{k} \text{ (rad/s)}}$

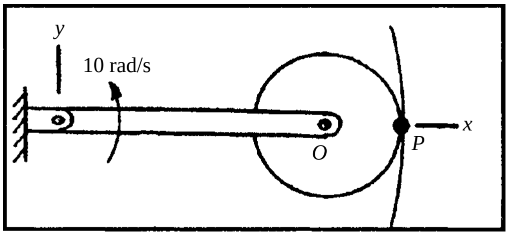
and $\omega_{\text{rod}} = 0.707$ rad/s.

Problem 17.48 The disk rolls on the curved surface. The bar rotates at 10 rad/s in the counterclockwise direction. Determine the velocity of point A.



Solution: The radius vector from the left point of attachment of the bar to the center of the disk is $\mathbf{r}_{\text{bar}} = 120\mathbf{i}$ (mm). The velocity of the center of the disk is

$$\mathbf{v}_O = \boldsymbol{\omega}_{\text{bar}} \times \mathbf{r}_{\text{bar}} = 10(120)\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = 1200\mathbf{j} \text{ (mm/s)}.$$



The radius vector from the point of contact with the disk and the curved surface to the center of the disk is $\mathbf{r}_{O/P} = -40\mathbf{i}$ (m). The velocity of the point of contact of the disk with the curved surface is zero, from which

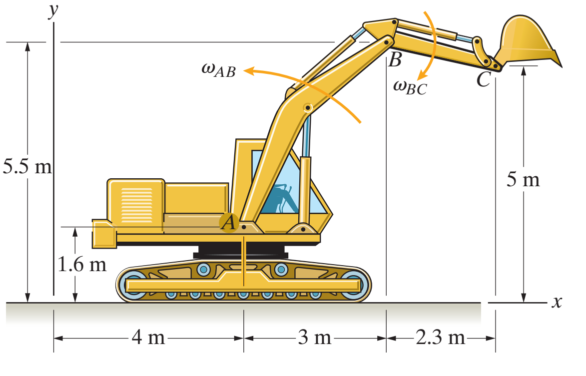
$$\mathbf{v}_O = \boldsymbol{\omega}_O \times \mathbf{r}_{O/P} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_O \\ -40 & 0 & 0 \end{bmatrix} = -40\omega_O\mathbf{j}.$$

Equate the two expressions for the velocity of the center of the disk and solve: $\omega_O = -30$ rad/s. The radius vector from the center of the disk to point A is $\mathbf{r}_{A/O} = 40\mathbf{j}$ (mm). The velocity of point A is

$$\mathbf{v}_A = \mathbf{v}_O + \boldsymbol{\omega}_O \times \mathbf{r}_{A/O} = 1200\mathbf{j} - (30)(40)\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\boxed{= 1200\mathbf{i} + 1200\mathbf{j} \text{ (mm/s)}}$$

Problem 17.49 If $\omega_{AB} = 2$ rad/s and $\omega_{BC} = 4$ rad/s, what is the velocity of point C, where the excavator's bucket is attached?



Solution: The radius vector AB is

$$\mathbf{r}_{B/A} = 3\mathbf{i} + (5.5 - 1.6)\mathbf{j} = 3\mathbf{i} + 3.9\mathbf{j} \text{ (m)}.$$

The velocity of point B is

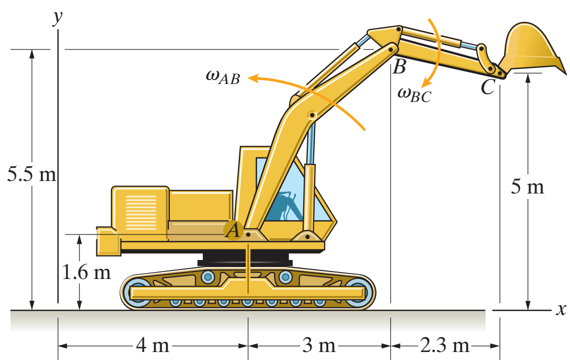
$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 3 & 3.9 & 0 \end{bmatrix} = -7.8\mathbf{i} + 6\mathbf{j} \text{ (m/s)}.$$

The radius vector BC is $\mathbf{r}_{C/B} = 2.3\mathbf{i} + (5 - 5.5)\mathbf{j} = 2.3\mathbf{i} - 0.5\mathbf{j}$ (m). The velocity at point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = -7.8\mathbf{i} + 6\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -4 \\ 2.3 & -0.5 & 0 \end{bmatrix}$$

$$\boxed{= -9.8\mathbf{i} - 3.2\mathbf{j} \text{ (m/s)}}$$

Problem 17.50 In Problem 17.49, if $\omega_{AB} = 2$ rad/s, what clockwise angular velocity ω_{BC} will cause the vertical component of the velocity of point C to be zero? What is the resulting velocity of point C?



Solution: Use the solution to Problem 17.49. The velocity of point B is

$$\mathbf{v}_B = -7.8\mathbf{i} + 6\mathbf{j} \text{ (m/s)}.$$

The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= -7.8\mathbf{i} + 6\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{BC} \\ 2.3 & -0.5 & 0 \end{bmatrix},$$

$$\mathbf{v}_C = (-7.8 - 0.5\omega_{BC})\mathbf{i} + (6 - 2.3\omega_{BC})\mathbf{j} \text{ (m/s)}.$$

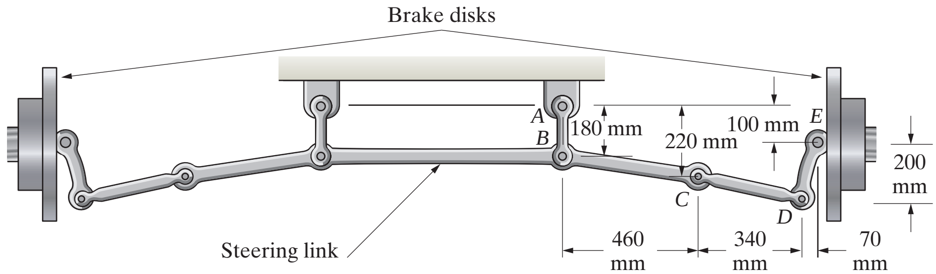
For the vertical component to be zero,

$$\boxed{\omega_{BC} = \frac{6}{2.3} = 2.61 \text{ rad/s clockwise.}}$$

The velocity of point C is

$$\boxed{\mathbf{v}_C = -9.1\mathbf{i} \text{ (m/s)}}$$

Problem 17.51 The steering linkage of a car is shown. Member DE rotates about fixed pin E. The right brake disk is rigidly attached to member DE. The tie rod CD is pinned at C and D. At the instant shown, the Pitman arm AB has a counterclockwise angular velocity of 1 rad/s. What is the angular velocity of the right brake disk?



Solution: Note that the steering link moves in translation only. Thus $\mathbf{v}_B = \mathbf{v}_C$.

$$\mathbf{v}_C = \mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 0 + (1\mathbf{k}) \times (-0.18\mathbf{j}) = 0.18\mathbf{i}$$

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = 0.18\mathbf{i} + \omega_{CD}\mathbf{k} \times (0.34\mathbf{i} - 0.08\mathbf{j})$$

$$= (0.18 + 0.08\omega_{CD})\mathbf{i} + (0.34\omega_{CD})\mathbf{j}$$

$$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{DE} \times \mathbf{r}_{E/D}$$

$$= (0.18 + 0.08\omega_{CD})\mathbf{i} + (0.34\omega_{CD})\mathbf{j} + \omega_{DE}\mathbf{k} \times (0.07\mathbf{i} + 0.2\mathbf{j})$$

$$= (0.18 + 0.08\omega_{CD} - 0.2\omega_{DE})\mathbf{i} + (0.34\omega_{CD} + 0.07\omega_{DE})\mathbf{j}$$

Point E is not moving. Equating the components of the velocity of point E to zero, we have

$$0.18 + 0.08\omega_{CD} - 0.2\omega_{DE} = 0,\ 0.34\omega_{CD} + 0.07\omega_{DE} = 0.$$

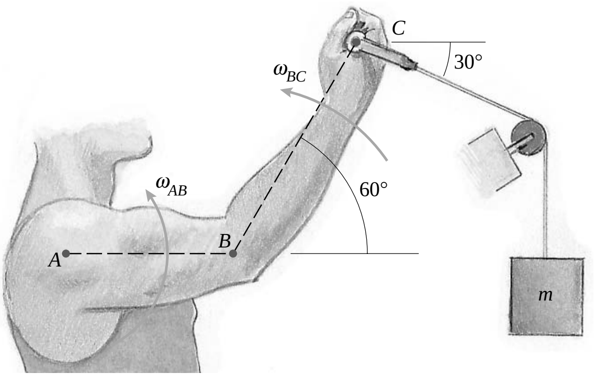
Solving these equations simultaneously, we find that

$$\omega_{CD} = -0.171 \text{ rad/s}, \quad \omega_{DE} = 0.832 \text{ rad/s}.$$

The angular velocity of the right brake disk is then

$$\boxed{\omega_{\text{disk}} = \omega_{DE} = 0.832 \text{ rad/s counterclockwise.}}$$

Problem 17.52 An athlete exercises his arm by raising the mass m. The shoulder joint A is stationary. The distance AB is 300 mm, and the distance BC is 400 mm. At the instant shown, $\omega_{AB} = 1$ rad/s and $\omega_{BC} = 2$ rad/s. How fast is the mass m rising?



Solution: The magnitude of the velocity of the point C parallel to the cable at C is also the magnitude of the velocity of the mass m. The radius vector AB is $\mathbf{r}_{B/A} = 300\mathbf{i}$ (mm). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 300 & 0 & 0 \end{bmatrix} = 300\mathbf{j} \text{ (mm/s)}.$$

The radius vector BC is $\mathbf{r}_{C/B} = 400(\mathbf{i}\cos 60^\circ + \mathbf{j}\sin 60^\circ) = 200\mathbf{i} + 346.4\mathbf{j}$ (mm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = 300\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 200 & 346.4 & 0 \end{bmatrix}$$

$$= -692.8\mathbf{i} + 700\mathbf{j} \text{ (mm/s)}.$$

The unit vector parallel to the cable at C is $\mathbf{e}_C = -\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ = -0.866\mathbf{i} + 0.5\mathbf{j}$. The component of the velocity parallel to the cable at C is

$$\boxed{\mathbf{v}_C \cdot \mathbf{e}_C = 950 \text{ mm/s}},$$

which is the velocity of the mass m.

Problem 17.53 The distance AB is 305 mm, the distance BC is 406.4 mm, $\omega_{AB} = 0.6$ rad/s, and the mass m is rising at 610 mm/s. What is the angular velocity ω_{BC}?

Solution: The radius vector AB is $\mathbf{r}_{B/A} = 0.305\mathbf{i}$ (m). The velocity at point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.6 \\ 0.305 & 0 & 0 \end{bmatrix} = 2.19\,\mathbf{j} \text{ (m/s)}.$$

The radius vector BC is

$$\mathbf{r}_{C/B} = 0.4064\,(\mathbf{i}\cos 60 + \mathbf{j}\sin 60) = 0.2032\,\mathbf{i} + 0.3531\mathbf{j} \text{ (m)}$$

The velocity at C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = 7.2\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0.2032 & 0.3531 & 0 \end{bmatrix}$$

$$= -\,0.3531\omega_{BC}\mathbf{i} + (2.19 + 0.2032\,\omega_{BC})\mathbf{j}.$$

The unit vector parallel to the cable at C is

$$\mathbf{e}_C = -\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 60^\circ = -0.866\mathbf{i} + 0.5\mathbf{j}.$$

The component of the velocity at C parallel to the cable is

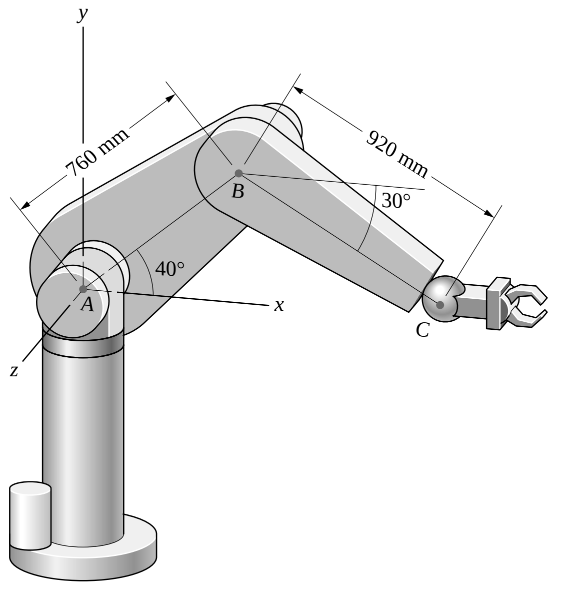
$$|\mathbf{v}_{CP}| = \mathbf{v}_C \cdot \mathbf{e}_C = +\,3.67\omega_{BC} + 1.22\,\omega_{BC} + 1.1 \text{ (m/s)}.$$

This is also the velocity of the rising mass, from which

$$4.89\omega_{BC} + 1.1 = \;0.61,$$

$$\boxed{\omega_{BC} = 1.28 \text{ rad/s}}$$

Problem 17.54 Points B and C are in the $x-y$ plane. The angular velocity vectors of the arms AB and BC are $\omega_{AB} = -0.2\mathbf{k}$ (rad/s), and $\omega_{BC} = 0.4\mathbf{k}$ (rad/s). What is the velocity of point C.



Solution: Locations of Points:

A: $(0, 0, 0)$ m

B: $(0.76 \cos 40^\circ, 0.76 \sin 40^\circ, 0)$ m

C: $(x_B + 0.92 \cos 30^\circ, y_B - 0.92 \sin 30^\circ, 0)$ m

or B: $(0.582, 0.489, 0)$,

C: $(1.379, 0.0285, 0)$ m

$\mathbf{r}_{B/A} = 0.582\mathbf{i} + 0.489\mathbf{j}$ (m)

$\mathbf{r}_{C/B} = 0.797\mathbf{i} - 0.460\mathbf{j}$ (m)

$$\mathbf{v}_A = 0, \omega_{AB} = -0.2\mathbf{k}\left(\frac{\text{rad}}{\text{s}}\right), \quad \omega_{BC} = 0.4\mathbf{k}\left(\frac{\text{rad}}{\text{s}}\right)$$

$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$

$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$

$\mathbf{v}_B = (-0.2\mathbf{k}) \times (0.582\mathbf{i} + 0.489\mathbf{j})$

$\mathbf{v}_B = 0.0977\mathbf{i} - 0.116\mathbf{j}$ (m/s).

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$

$\mathbf{v}_C = \mathbf{v}_B + 0.184\mathbf{i} + 0.319\mathbf{j}$ (m/s)

$\underline{\mathbf{v}_C = 0.282\mathbf{i} + 0.202\mathbf{j} \text{ (m/s)}}$.

Problem 17.55 If the velocity at point C of the robotic arm shown in Problem 17.54 is $\mathbf{v}_C = 0.15\mathbf{i} + 0.42\mathbf{j}$ (m/s), what are the angular velocities of the arms AB and BC?

Solution: From the solution to Problem 17.54,

$$\mathbf{r}_{B/A} = 0.582\mathbf{i} + 0.489\mathbf{j} \text{ (m)}$$

$$\mathbf{r}_{C/B} = 0.797\mathbf{i} - 0.460\mathbf{j} \text{ (m)}$$

$$\mathbf{v}_B = \omega_{AB}\mathbf{k} \times \mathbf{r}_{B/A} \quad (\mathbf{v}_A = 0)$$

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC}\mathbf{k} \times \mathbf{r}_{C/B}$$

We are given

$$\mathbf{v}_C = -0.15\mathbf{i} + 0.42\mathbf{j} + 0\mathbf{k} \text{ (m/s)}.$$

Thus, we know everything in the $\mathbf{v}_C$ equation except ω_{AB} and ω_{BC}.

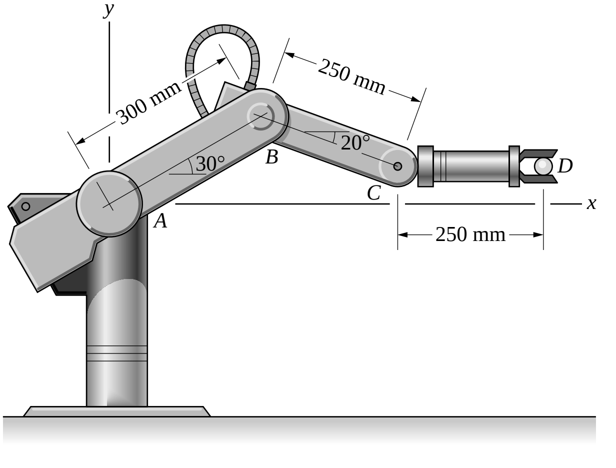
$$\mathbf{v}_C = \omega_{AB}\mathbf{k} \times \mathbf{r}_{B/A} + \omega_{BC}\mathbf{k} \times \mathbf{r}_{C/B}$$

This yields two scalar equations in two unknowns $\mathbf{i}$ and $\mathbf{j}$ components. Solving, we get

$$\underline{\boldsymbol{\omega}_{AB} = 0.476\mathbf{k}} \text{ (rad/s)},$$

$$\underline{\boldsymbol{\omega}_{BC} = 0.179\mathbf{k}} \text{ (rad/s)}.$$

Problem 17.56 The link AB of the robot's arm is rotating at 2 rad/s in the counterclockwise direction, the link BC is rotating at 3 rad/s in the clockwise direction, and the link CD is rotating at 4 rad/s in the counterclockwise direction. What is the velocity of point D?



Solution: The velocity of B is

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A},$$

or $$\mathbf{v}_B = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0.3\cos 30^\circ & 0.3\sin 30^\circ & 0 \end{vmatrix}$$

$$= -0.3\mathbf{i} + 0.520\mathbf{j} \text{ (m/s)}.$$

The velocity of C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

or $$\mathbf{v}_C = -0.3\mathbf{i} + 0.520\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -3 \\ 0.25\cos 20^\circ & -0.25\sin 20^\circ & 0 \end{vmatrix}$$

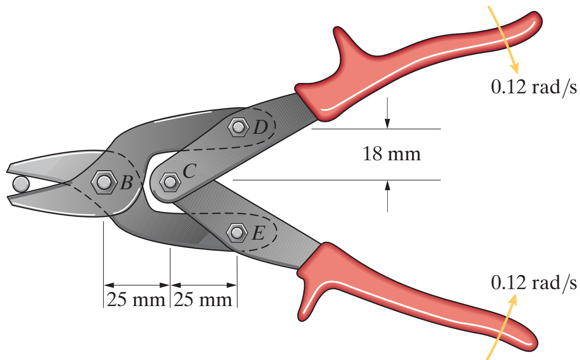
$$= -0.557\mathbf{i} - 0.185\mathbf{j} \text{ (m/s)}.$$

The velocity of D is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = -0.557\mathbf{i} - 0.185\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 0.25 & 0 & 0 \end{vmatrix},$$

or $$\mathbf{v}_D = -0.557\mathbf{i} + 0.815\mathbf{j} \text{ (m/s)}.$$

Problem 17.57 The person squeezes the grips of the shears, causing the angular velocities shown. What is the resulting angular velocity of the jaw *BD*?



Solution:

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = 0 - (0.12 \text{ rad/s})\mathbf{k} \times (0.025\mathbf{i} + 0.018\mathbf{j}) \text{ m}$$

$$= (0.00216\mathbf{i} - 0.003\mathbf{j}) \text{ m/s}$$

$$\mathbf{v}_B = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D}$$

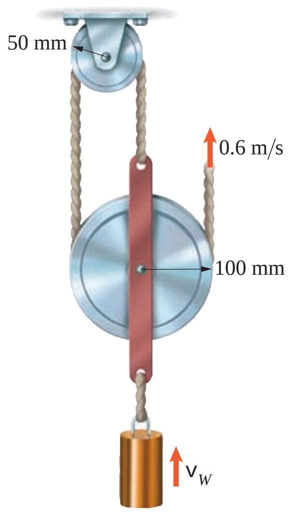
$$= (0.00216\mathbf{i} - 0.003\mathbf{j}) \text{ m/s} + \omega_{BD}\mathbf{k} \times (-0.05\mathbf{i} - 0.018\mathbf{j}) \text{ m}$$

$$= (0.00216 \text{ m/s} + \{0.018 \text{ m}\}\omega_{BD})\mathbf{i} + (-0.003 \text{ m/s} - \{0.05 \text{ m}\}\omega_{BD})\mathbf{j}$$

From symmetry we know that C and B do not move vertically

$$-0.003 \text{ m/s} - \{0.05 \text{ m}\}\omega_{BD} = 0 \Rightarrow \boxed{\begin{array}{l}\omega_{BD} = -0.06 \text{ rad/s} \\ \omega_{BD} = 0.06 \text{ rad/s CW}\end{array}}$$

Problem 17.58 Determine the velocity v_W and the angular velocity of the small pulley.



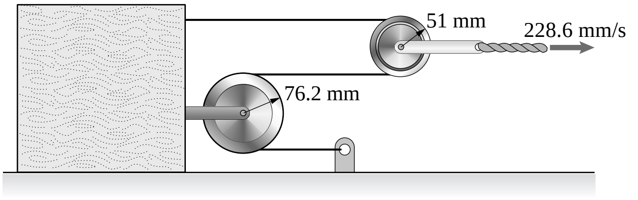
Solution: Since the radius of the bottom pulley is not given, we cannot use Eq (17.6) (or the equivalent). The strategy is to use the fact (derived from elementary principles) that the velocity of the center of a pulley is the mean of the velocities of the extreme edges, where the edges lie on a line normal to the motion, *taking into account the directions of the velocities at the extreme edges*. The center rope from the bottom pulley to the upper pulley moves upward at a velocity of v_W. Since the small pulley is fixed, the velocity of the center is zero, and the rope to the left moves downward at a velocity v_W, from which the left edge of the bottom pulley is moving at a velocity v_W downward. The right edge of the bottom pulley moves upward at a velocity of 0.6 m/s. The velocity of the center of the bottom pulley is the mean of the velocities at the extreme edges, from which $v_W = \dfrac{0.6 - v_W}{2}$.

Solve: $\boxed{v_W = \dfrac{0.6}{3} = 0.2 \text{ m/s}}$.

The angular velocity of the small pulley is

$$\boxed{\omega = \frac{v_W}{r} = \frac{0.2}{0.05} = 4 \text{ rad/s}}$$

Problem 17.59 Determine the velocity of the block and the angular velocity of the small pulley.



Solution: Denote the velocity of the block by v_B. The strategy is to determine the velocities of the extreme edges of a pulley by determining the velocity of the element of rope in contact with the pulley. The upper rope is fixed to the block, so that it moves to the right at the velocity of the block, from which the upper edge of the small pulley moves to the right at the velocity of the block. The fixed end of the rope at the bottom is stationary, so that the bottom edge of the large pulley is stationary. The center of the large pulley moves at the velocity of the block, from which the upper edge of the bottom pulley moves at twice the velocity of the block (since the velocity of the center is equal to the mean of the velocities of the extreme edges, one of which is stationary) from which the bottom edge of the small pulley moves at twice the velocity of the block. The center of the small pulley moves to the right at 9 in/s. The velocity of the center of the small pulley is the mean of the velocities at the extreme edges, from which

$$0.2286 = \frac{0.051 v_B + 0.025 v_B}{0.051} = \frac{0.076}{0.051} v_B,$$

from which

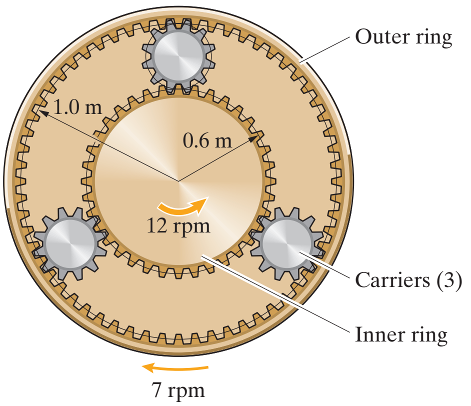
$$\boxed{v_B = \frac{0.051}{0.076}\,0.2286 = 0.152\text{ m/s}}.$$

The angular velocity of small pulley is given by

$$0.2286\,\mathbf{i} = 0.051\mathbf{v}_B\mathbf{i} + \boldsymbol{\omega} \times 0.051\mathbf{j} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ 0 & 0.051 & 0 \end{bmatrix} = 0.051\,v_B\mathbf{i} - 0.051\,\omega\mathbf{i},$$

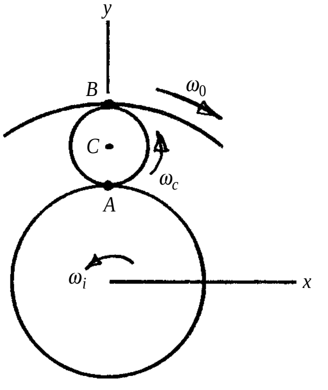
from which $\boxed{\omega = \dfrac{0.305 - 0.2286}{0.051} = 1.5\text{ rad/s}}$

Problem 17.60 The device shown is used in the semiconductor industry to polish silicon wafers. The wafers are placed on the faces of the carriers. The outer and inner rings are then rotated, causing the wafers to move and rotate against an abrasive surface. If the outer ring rotates in the clockwise direction at 7 rpm and the inner ring rotates in the counterclockwise direction at 12 rpm, what is the angular velocity of the carriers?



Solution: The velocity of pt. B is $\mathbf{v}_B = (1 \text{ m})\omega_0 \mathbf{i} = \omega_0 \mathbf{i}$. The velocity of pt. A is $\mathbf{v}_A = -(0.6 \text{ m})\omega_i \mathbf{i}$. Then

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_C \times \mathbf{r}_{B/A} : \omega_0 \mathbf{i} = -0.6\omega_i \mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_C \\ 0 & 0.4 & 0 \end{vmatrix}.$$



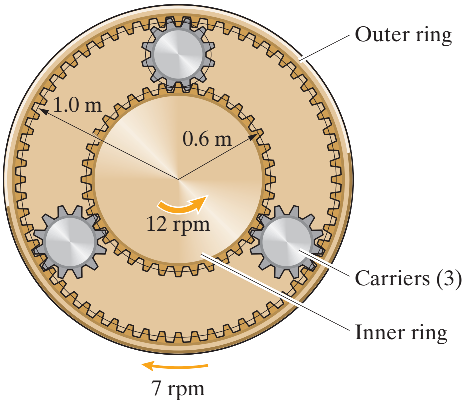
The $\mathbf{i}$ component of this equation is $\omega_0 = -0.6\omega_i - 0.4\omega_C$,

$$\text{so} \quad \omega_C = \frac{-0.6\omega_i - \omega_0}{0.4}$$

$$= \frac{-0.6(12 \text{ rpm}) - 7 \text{ rpm}}{0.4}$$

$$= -35.5 \text{ rpm}.$$

Problem 17.61 In Problem 17.60, suppose that the outer ring rotates in the clockwise direction at 5 rpm and you want the centerpoints of the carriers to remain stationary during the polishing process. What is the necessary angular velocity of the inner ring?



Solution: See the solution of Problem 17.60. The velocity of pt. B is $\mathbf{v}_B = \omega_0 \mathbf{i}$ and the angular velocity of the carrier is

$$\omega_C = \frac{-0.6\omega_i - \omega_0}{0.4}.$$

We want the velocity of pt. C to be zero:

$$\mathbf{v}_C = 0 = \mathbf{v}_B + \boldsymbol{\omega}_C \times \mathbf{r}_{C/B} = \omega_0 \mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_C \\ 0 & -0.2 & 0 \end{vmatrix}.$$

From this equation we see that $\omega_C = -5\omega_0$. Therefore the velocity of pt. A is

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega}_C \times \mathbf{r}_{A/C}$$

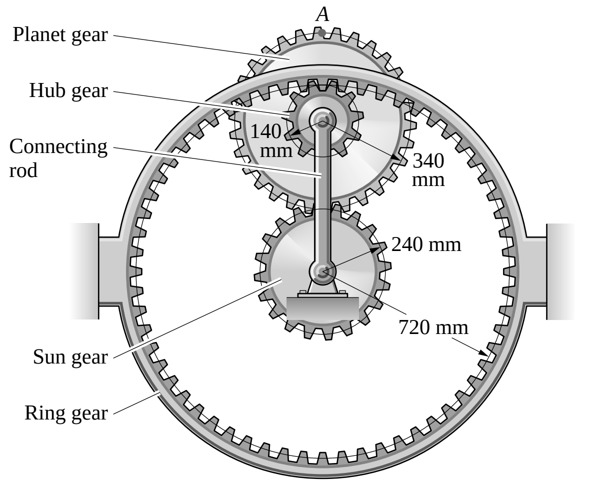
$$= 0 + (-5\omega_0 \mathbf{k}) \times (-0.2\mathbf{j})$$

$$= -\omega_0 \mathbf{i}$$

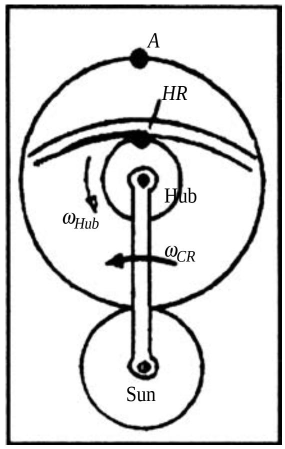
We also know that $\mathbf{v}_A = -(0.6 \text{ m})\omega_i \mathbf{i}$,

$$\text{so} \quad \omega_i = \frac{\omega_0}{0.6} = \frac{5 \text{ rpm}}{0.6} = 8.33 \text{ rpm}.$$

Problem 17.62 The ring gear is fixed and the hub and planet gears are bonded together. The connecting rod rotates in the counterclockwise direction at 60 rpm. Determine the angular velocity of the sun gear and the magnitude of the velocity of point A.



Solution: Denote the centers of the sun, hub and planet gears by the subscripts Sun, Hub, and Planet, respectively. Denote the contact points between the sun gear and the planet gear by the subscript SP and the point of contact between the hub gear and the ring gear by the subscript *HR*. The angular velocity of the connecting rod is $\omega_{CR} = 6.28$ rad/s. The vector distance from the center of the sun gear to the center of the hub gear is $\mathbf{r}_{\text{Hub/Sun}} = (720 - 140)\mathbf{j} = 580\mathbf{j}$ (mm). The velocity of the center of the hub gear is



$$\mathbf{v}_{\text{Hub}} = \boldsymbol{\omega}_{CR} \times \mathbf{r}_{\text{Hub/Sun}} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2\pi \\ 0 & 580 & 0 \end{bmatrix} = -3644\mathbf{i} \text{ (mm/s)}$$

The angular velocity of the hub gear is found from

$$\mathbf{v}_{HR} = 0 = \mathbf{v}_{\text{Hub}} + \boldsymbol{\omega}_{\text{Hub}} \times 140\mathbf{j} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{\text{Hub}} \\ 0 & 140 & 0 \end{bmatrix} + \mathbf{v}_{\text{Hub}}$$

$$= -3644\mathbf{i} - 140\omega_{\text{Hub}}\mathbf{i},$$

from which

$$\omega_{\text{Hub}} = -\frac{3644}{140} = -26.03 \text{ rad/s}.$$

This is also the angular velocity of the planet gear. The linear velocity of point A is

$$\mathbf{v}_A = \boldsymbol{\omega}_{\text{Hub}} \times (340 - 140)\mathbf{j} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -26.03 \\ 0 & 200 & 0 \end{bmatrix}$$

$$\boxed{= 5206\mathbf{i} \text{ (mm/s)}}$$

The velocity of the point of contact with the sun gear is

$$\mathbf{v}_{PS} = \boldsymbol{\omega}_{\text{Hub}} \times (-480\mathbf{j}) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -26.03 \\ 0 & -480 & 0 \end{bmatrix}$$

$$= -12494.6\mathbf{i} \text{ (mm/s)}.$$

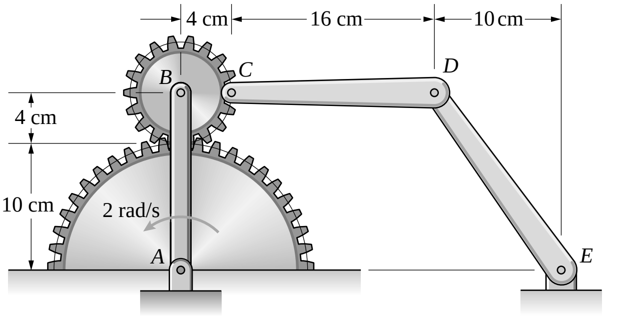
The angular velocity of the sun gear is found from

$$\mathbf{v}_{PS} = -12494.6\mathbf{i} = \boldsymbol{\omega}_{\text{Sun}} \times (240\mathbf{j}) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{\text{Sun}} \\ 0 & 240 & 0 \end{bmatrix}$$

$$= -240\omega_{\text{Sun}}\mathbf{i},$$

from which $\boxed{\omega_{\text{Sun}} = \frac{12494.6}{240} = 52.06 \text{ rad/s}}$

Problem 17.63 The large gear is fixed. Bar AB has a counterclockwise angular velocity of 2 rad/s. What are the angular velocities of bars CD and DE?



Solution: The strategy is to express vector velocity of point D in terms of the unknown angular velocities of CD and DE, and then to solve the resulting vector equations for the unknowns. The vector distance AB is $\mathbf{r}_{B/A} = 14\mathbf{j}$ (cm) The linear velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0 & 14 & 0 \end{bmatrix} = -28\mathbf{i} \text{ (cm/s)}.$$

The lower edge of gear B is stationary. The radius vector from the lower edge to B is $\mathbf{r}_B = 4\mathbf{j}$ (cm), The angular velocity of B is

$$\mathbf{v}_B = \boldsymbol{\omega}_B \times \mathbf{r}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_B \\ 0 & 4 & 0 \end{bmatrix} = -4\omega_B\mathbf{i} \text{ (cm/s)},$$

from which $\omega_B = -\dfrac{v_B}{4} = 7$ rad/s. The vector distance from B to C is $\mathbf{r}_{C/B} = 4\mathbf{i}$ (cm). The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_B \times \mathbf{r}_{C/B} = -28\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 7 \\ 4 & 0 & 0 \end{bmatrix}$$

$$= -28\mathbf{i} + 28\mathbf{j} \text{ (cm/s)}.$$

The vector distance from C to D is $\mathbf{r}_{D/C} = 16\mathbf{i}$ (cm), and from E to D is $\mathbf{r}_{D/E} = -10\mathbf{i} + 14\mathbf{j}$ (cm). The linear velocity of point D is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = -28\mathbf{i} + 28\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 16 & 0 & 0 \end{bmatrix}$$

$$= -28\mathbf{i} + (16\omega_{CD} + 28)\mathbf{j} \text{ (cm/s)}.$$

The velocity of point D is also given by

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DE} \\ -10 & 14 & 0 \end{bmatrix}$$

$$= -14\omega_{DE}\mathbf{i} - 10\omega_{DE}\mathbf{j} \text{ (cm/s)}.$$

Equate components:

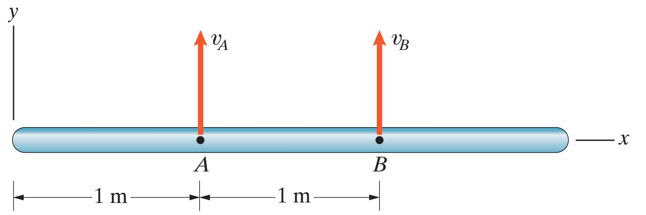
$$(-28 + 14\omega_{DE})\mathbf{i} = 0,$$

$$(16\omega_{CD} + 28 + 10\omega_{DE})\mathbf{j} = 0.$$

Solve: $\boxed{\omega_{DE} = 2 \text{ rad/s}}$, $\boxed{\omega_{CD} = -3 \text{ rad/s}}$.

The negative sign means a clockwise rotation.

Problem 17.64 If the bar has a clockwise angular velocity of 10 rad/s and $v_A = 20$ m/s, what are the coordinates of its instantaneous center of the bar, and what is the value of v_B?



Solution: Assume that the coordinates of the instantaneous center are (x_C, y_C), $\boldsymbol{\omega} = -\omega\mathbf{k} = -10\mathbf{k}$. The distance to point A is $\mathbf{r}_{A/C} = (1 - x_C)\mathbf{i} + y_C\mathbf{j}$. The velocity at A is

$$\mathbf{v}_A = 20\mathbf{j} = \boldsymbol{\omega} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega \\ 1 - x_C & y_C & 0 \end{bmatrix}$$

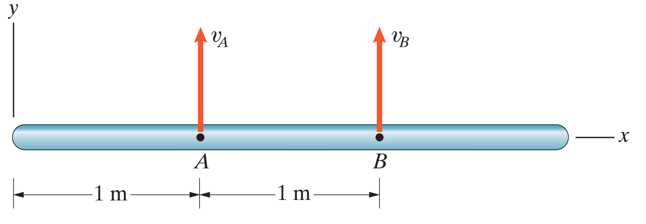
$$= y_C\omega\mathbf{i} - \omega(1 - x_C)\mathbf{j},$$

from which $y_C\omega\mathbf{i} = 0$, and $(20 + \omega(1 - x_C))\mathbf{j} = 0$.

Substitute $\omega = 10$ rad/s to obtain $y_C = 0$ and $x_C = 3$ m. The coordinates of the instantaneous center are $\boxed{(3, 0)\ (\text{m})}$. The vector distance from C to B is $\mathbf{r}_{B/C} = (2 - 3)\mathbf{i} = -\mathbf{i}$ (m). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -10 \\ -1 & 0 & 0 \end{bmatrix} = -10(-\mathbf{j}) \boxed{= 10\mathbf{j}\ (\text{m/s})}$$

Problem 17.65 In Problem 17.64, if $v_A = 24$ m/s and $v_B = 36$ m/s, what are the coordinates of the instantaneous center of the bar, and what is its angular velocity?



Solution: Let (x_C, y_C) be the coordinates of the instantaneous center. The vectors from the instantaneous center and the points A and B are $\mathbf{r}_{A/C} = (1 - x_C)\mathbf{i} + y_C\mathbf{j}$ (m) and $\mathbf{r}_{B/C} = (2 - x_C)\mathbf{i} + y_C\mathbf{j}$. The velocity of A is given by

$$\mathbf{v}_A = 24\mathbf{j} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 1 - x_C & y_C & 0 \end{bmatrix}$$

$$= -\omega_{AB}y_C\mathbf{i} + \omega_{AB}(1 - x_C)\mathbf{j}\ (\text{m/s})$$

The velocity of B is

$$\mathbf{v}_B = 36\mathbf{j} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 2 - x_C & y_C & 0 \end{bmatrix}$$

$$= -y_C\omega_{AB}\mathbf{i} + \omega_{AB}(2 - x_C)\mathbf{j}\ (\text{m/s}).$$

Separate components:

$$24 - \omega_{AB}(1 - x_C) = 0,$$

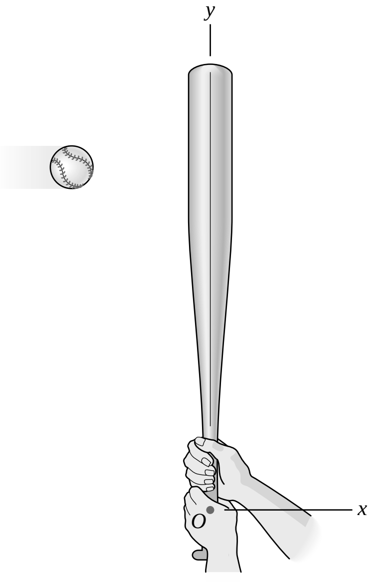
$$36 - \omega_{AB}(2 - x_C) = 0,$$

$$\omega_{AB}y_C = 0.$$

Solve: $\boxed{x_C = -1,}$ $\boxed{y_C = 0,}$

and $\boxed{\omega_{AB} = 12\ \text{rad/s}}$ *counter clockwise*.

Problem 17.66 The velocity of point O of the bat is $\mathbf{v}_O = -1.83\mathbf{i} - 4.27\mathbf{j}$ (m/s), and the bat rotates about the z axis with a counterclockwise angular velocity of 4 rad/s. What are the x and y coordinates of the bat's instantaneous center?



Solution: Let (x_C, y_C) be the coordinates of the instantaneous center. The vector from the instantaneous center to point O is $\mathbf{r}_{O/C} = -x_C\mathbf{i} - y_C\mathbf{j}$ (m). The velocity of point O is

$$\mathbf{v}_0 = -1.83\mathbf{i} - 4.27\mathbf{j} = \boldsymbol{\omega} \times \mathbf{r}_{O/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ -x_C & -y_C & 0 \end{bmatrix}$$

$$= y_C\omega\mathbf{i} - x_C\omega\mathbf{j} \text{ (m/s)}.$$

Equate terms and solve:

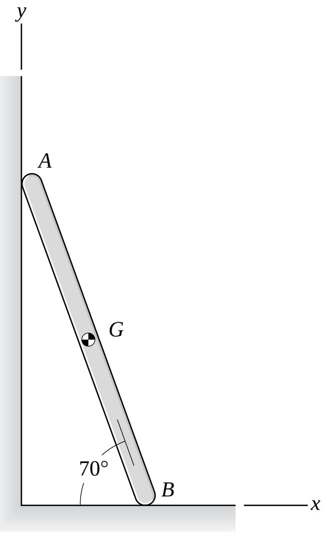
$$y_C = -\frac{1.83}{\omega} = -\frac{1.83}{4} = -0.46 \text{ m},$$

$$x_C = \frac{4.27}{\omega} = \frac{4.27}{4} = 1.07 \text{ m},$$

from which the coordinates are $\boxed{(1.07, -0.46) \text{ m}}$

Problem 17.67 Points A and B of the 1-m bar slide on the plane surfaces. The velocity of B is $\mathbf{v}_B = 2\mathbf{i}$ (m/s).

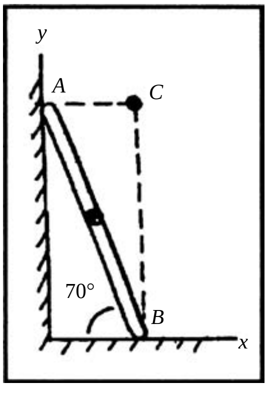
(a) What are the coordinates of the instantaneous center of the bar?
(b) Use the instantaneous center to determine the velocity at A.



Solution:

(a) A is constrained to move parallel to the y axis, and B is constrained to move parallel to the x axis. Draw perpendiculars to the velocity vectors at A and B. From geometry, the perpendiculars intersect at

$$\boxed{(\cos 70^\circ, \sin 70^\circ) = (0.3420, 0.9397)\ \text{m}}.$$



(b) The vector from the instantaneous center to point B is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = 0.3420\mathbf{i} - (0.3420\mathbf{i} + 0.9397\mathbf{j}) = -0.9397\mathbf{j}$$

The angular velocity of bar AB is obtained from

$$\mathbf{v}_B = 2\mathbf{i} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0 & -0.9397 & 0 \end{bmatrix}$$

$$= \omega_{AB}(0.9397)\mathbf{i},$$

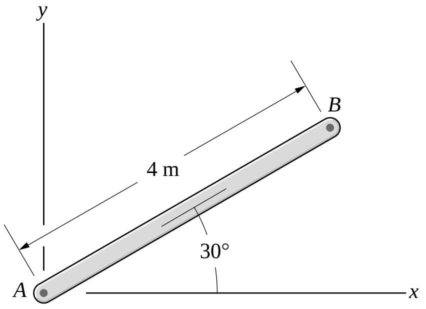
from which $\omega_{AB} = \dfrac{2}{0.9397} = 2.13$ rad/s.

The vector from the instantaneous center to point A is $\mathbf{r}_{A/C} = \mathbf{r}_A - \mathbf{r}_C = -0.3420\mathbf{i}$ (m). The velocity at A is

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.13 \\ -0.3420 & 0 & 0 \end{bmatrix}$$

$$\boxed{= -0.7279\mathbf{j}\ \text{(m/s)}.}$$

Problem 17.68 The bar is in two-dimensional motion in the $x-y$ plane. The velocity of point A is $\mathbf{v}_A = 8\mathbf{i}$ (m/s), and B is moving in the direction parallel to the bar. Determine the velocity of B (a) by using Eq. (17.6) and (b) by using the instantaneous center of the bar.



Solution:

(a) The unit vector parallel to the bar is

$$\mathbf{e}_{AB} = (\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 0.866\mathbf{i} + 0.5\mathbf{j}.$$

The vector from A to B is $\mathbf{r}_{B/A} = 4\mathbf{e}_{AB} = 3.46\mathbf{i} + 2\mathbf{j}$ (m). The velocity of point B is

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 8\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 3.46 & 2 & 0 \end{bmatrix}$$

$$\mathbf{v}_B = (8 - 2\omega_{AB})\mathbf{i} + 3.46\omega_{AB}\mathbf{j}.$$

But $\mathbf{v}_B$ is also moving parallel to the bar,

$$\mathbf{v}_B = v_B\mathbf{e}_{AB} = v_B(0.866\mathbf{i} + 0.5\mathbf{j}).$$

Equate, and separate components:

$$(8 - 2\omega_{AB} - 0.866v_B)\mathbf{i} = 0,$$

$$(3.46\omega_{AB} - 0.5v_B)\mathbf{j} = 0.$$

Solve: $\omega_{AB} = 1$ rad/s, $v_B = 6.93$ m/s, from which

$$\boxed{\mathbf{v}_B = v_B\mathbf{e}_{AB} = 6\mathbf{i} + 3.46\mathbf{j} \text{ (m/s)}}$$

(b) Let (x_C, y_C) be the coordinates of the instantaneous center. The vector from the center to A is

$$\mathbf{r}_{A/C} = \mathbf{r}_A - \mathbf{r}_C = -\mathbf{r}_C = -x_C\mathbf{i} - y_C\mathbf{j} \text{ (m)}.$$

The vector from the instantaneous center to B is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = (3.46 - x_C)\mathbf{i} + (2 - y_C)\mathbf{j}.$$

The velocity of point A is

$$\mathbf{v}_A = 8\mathbf{i} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -x_C & -y_C & 0 \end{bmatrix}$$

$$= \omega_{AB}y_C\mathbf{i} - \omega_{AB}x_C\mathbf{j} \text{ (m/s)}.$$

From which $\boxed{x_C = 0}$, and $\omega_{AB}y_C = 8$. The velocity of point B is

$$\mathbf{v}_B = v_B\mathbf{e}_{AB} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 3.46 - x_C & 2 - y_C & 0 \end{bmatrix}$$

$$= -\omega_{AB}(2 - y_C)\mathbf{i} + \omega_{AB}(3.46 - x_C)\mathbf{j}.$$

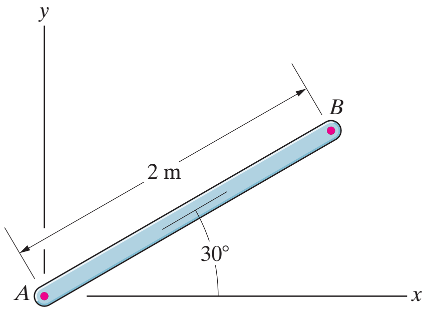
Equate terms and substitute

$\omega_{AB}y_C = 8$, and $x_C = 0$, to obtain: $(0.866v_B + 2\omega_{AB} - 8)\mathbf{i} = 0$, and $(0.5v_C - 3.46\omega_{AB})\mathbf{j} = 0$. These equations are algebraically identical with those obtained in Part (a) above (as can be shown by multiplying all terms by -1). Thus $\omega_{AB} = 1$ rad/s, $v_B = 6.93$ (m/s), and the velocity of B is that obtained in Part (a)

$$\boxed{\mathbf{v}_B = v_B\mathbf{e}_{AB} = 6\mathbf{i} + 3.46\mathbf{j} \text{ (m/s)}}.$$

Problem 17.69 Point A of the bar is moving at 8 m/s in the direction of the unit vector $0.966\mathbf{i} - 0.259\mathbf{j}$, and point B is moving in the direction of the unit vector $0.766\mathbf{i} + 0.643\mathbf{j}$.

(a) What are the coordinates of the bar's instantaneous center?
(b) What is the bar's angular velocity?



Solution: Assume the instantaneous center Q is located at (x, y).

Then

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_{A/Q}, \quad \mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/Q}$$

$$(8\ \text{m/s})(0.966\mathbf{i} - 0.259\mathbf{j}) = \omega\mathbf{k} \times (-x\mathbf{i} - y\mathbf{j})$$

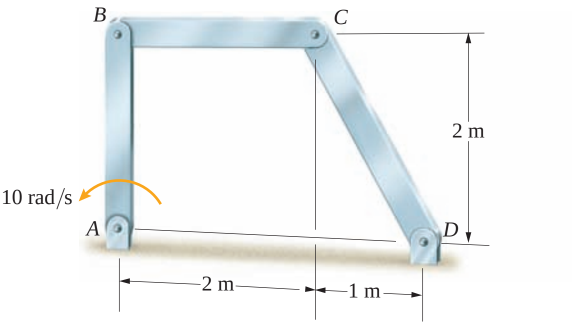
$$v_B(0.766\mathbf{i} + 0.643\mathbf{j}) = \omega\mathbf{k} \times ([\{2\ \text{m}\}\cos 30^\circ - x]\mathbf{i}$$

$$+ [\{2\ \text{m/s}\}\sin 30^\circ - y]\mathbf{j})$$

Expanding we have the four equations

$$\left.\begin{aligned} 7.73\ \text{m/s} &= \omega y \\ -2.07\ \text{m/s} &= -\omega x \\ 0.766 v_B &= \omega(y - 1\ \text{m}) \\ 0.643 v_B &= \omega(1.73\ \text{m} - x) \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} x &= 0.623\ \text{m} \\ y &= 2.32\ \text{m} \end{aligned}}$$

Problem 17.70 Bar AB rotates with a counterclockwise angular velocity of 10 rad/s. At the instant shown, what are the angular velocities of bars BC and CD? (See Active Example 17.4.)



Solution: The location of the instantaneous center for BC is shown, along with the relevant distances. Using the concept of the instantaneous centers we have

$$v_B = (10)(2) = \omega_{BC}(4)$$

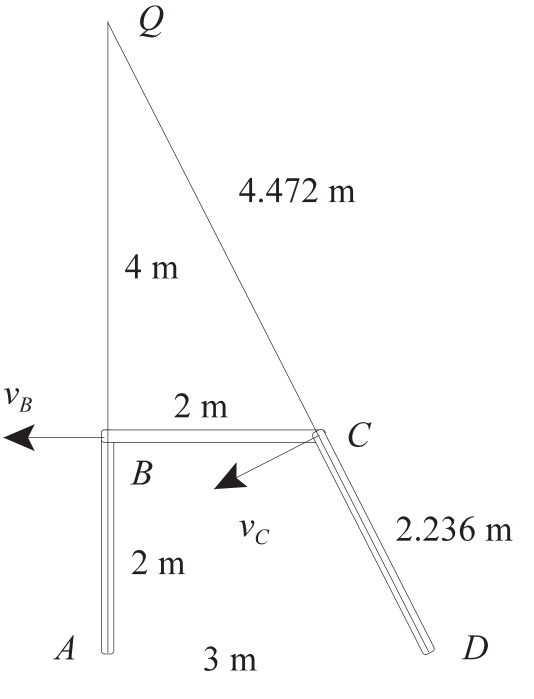
$$\omega_{BC} = 5 \text{ rad/s}$$

$$v_C = (4.472)(5) = 2.236(\omega_{CD})$$

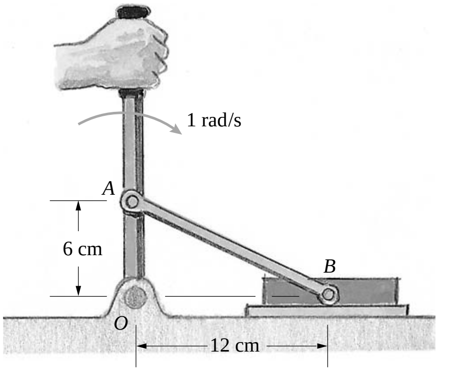
$$\omega_{CD} = 10 \text{ rad/s}$$

We determine the directions by inspection

$$\boxed{\begin{aligned} \omega_{BC} &= 5 \text{ rad/s clockwise.} \\ \omega_{CD} &= 10 \text{ rad/s counterclockwise.} \end{aligned}}$$



Problem 17.71 Use instantaneous centers to determine the horizontal velocity of B.



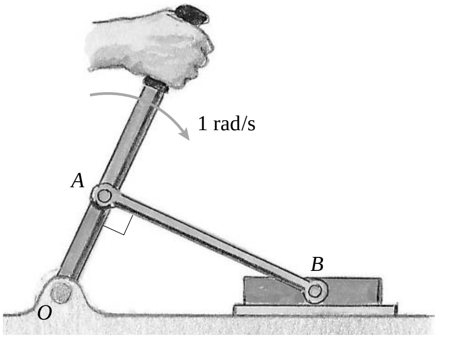
Solution: The instantaneous center of OA lies at O, by definition, since O is the point of zero velocity, and the velocity at point A is parallel to the x-axis:

$$\mathbf{v}_A = \boldsymbol{\omega}_{OA} \times \mathbf{r}_{A/O} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{OA} \\ 0 & 6 & 0 \end{bmatrix} = 6\mathbf{i} \text{ (cm/s)}.$$

A line perpendicular to this motion is parallel to the y axis. The point B is constrained to move on the x axis, and a line perpendicular to this motion is also parallel to the y axis. These two lines will not intersect at any finite distance from the origin, hence *at the instant shown the instantaneous center of bar AB is at infinity and the angular velocity of bar AB is zero*. At the instant shown, the bar AB translates only, from which the horizontal velocity of B is the horizontal velocity at A:

$$\boxed{\mathbf{v}_B = \mathbf{v}_A = 6\mathbf{i} \text{ (cm/s)}}.$$

Problem 17.72 When the mechanism in Problem 17.71 is in the position shown here, use instantaneous centers to determine the horizontal velocity of B.



Solution: The strategy is to determine the intersection of lines perpendicular to the motions at A and B. The velocity of A is parallel to the bar AB. A line perpendicular to the motion at A will be parallel to the bar OA. From the dimensions given in Problem 17.71, the length of bar AB is $r_{AB} = \sqrt{6^2 + 12^2} = 13.42$ cm. Consider the triangle OAB. The interior angle at B is

$$\beta = \tan^{-1}\left(\frac{6}{r_{AB}}\right) = 24.1^\circ,$$

and the interior angle at O is $\theta = 90^\circ - \beta = 65.9^\circ$. The unit vector parallel to the handle OA is $\mathbf{e}_{OA} = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$, and a point on the line is $\mathbf{L}_{OA} = L_{OA}\mathbf{e}_{OA}$, where L_{OA} is the magnitude of the distance of the point from the origin. A line perpendicular to the motion at B is parallel to the y axis. At the intersection of the two lines

$$L_{OA}\cos\theta = \frac{r_{AB}}{\cos\beta},$$

from which $L_{OA} = 36$ cm. The coordinates of the instantaneous center are (14.7, 32.9) (in.).

Check: From geometry, the triangle OAB and the triangle formed by the intersecting lines and the base are similar, and thus the interior angles are known for the larger triangle. From the law of sines

$$\frac{L_{OA}}{\sin 90^\circ} = \frac{r_{OB}}{\sin\beta} = \frac{r_{AB}}{\sin\beta\cos\beta} = 36 \text{ cm},$$

and the coordinates follow immediately from $\mathbf{L}_{OA} = L_{OA}\mathbf{e}_{OA}$. *check*. The vector distance from O to A is $\mathbf{r}_{A/O} = 6(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) = 2.450\mathbf{i} + 5.478\mathbf{j}$ (cm). The angular velocity of the bar AB is determined from the known linear velocity at A.

$$\mathbf{v}_A = \boldsymbol{\omega}_{OA} \times \mathbf{r}_{A/O} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1 \\ 2.450 & 5.477 & 0 \end{bmatrix}$$

$$= 5.48\mathbf{i} - 2.45\mathbf{j} \text{ (cm/s)}.$$

The vector from the instantaneous center to point A is

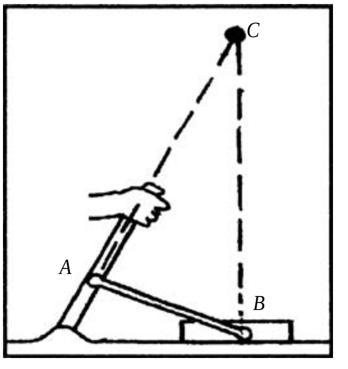
$$\mathbf{r}_{A/C} = \mathbf{r}_{OA} - \mathbf{r}_C = 6\mathbf{e}_{OA} - (14.7\mathbf{i} + 32.86\mathbf{j})$$

$$= -12.25\mathbf{i} - 27.39\mathbf{j} \text{ (cm)}$$

The velocity at point A is

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -12.25 & -27.39 & 0 \end{bmatrix}$$

$$= \omega_{AB}(27.39\mathbf{i} - 12.25\mathbf{j}) \text{ (cm/s)}.$$



Equate the two expressions for the velocity at point A and separate components, $5.48\mathbf{i} = 27.39\omega_{AB}$, $-2.45\mathbf{j} = -12.25\omega_{AB}\mathbf{j}$ (one of these conditions is superfluous) and solve to obtain $\omega_{AB} = 0.2$ rad/s, counterclockwise.

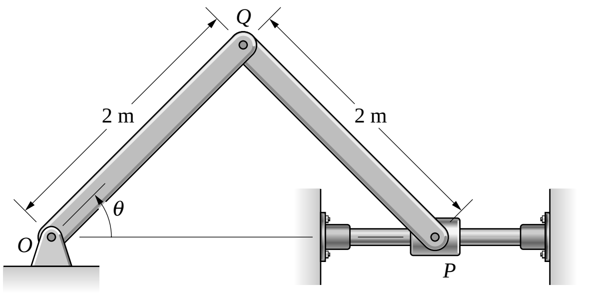
[*Check*: The distance OA is 6 cm. The magnitude of the velocity at A is $\omega_{OA}(6) = (1)(6) = 6$ cm/s. The distance to the instantaneous center from O is $\sqrt{14.7^2 + 32.9^2} = 36$ cm, and from C to A is $(36 - 6) = 30$ cm from which $30\,\omega_{AB} = 6$ cm/s, from which $\omega_{AB} = 0.2$ rad/s. *check*.]. The vector from the instantaneous center to point B is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = 14.7\mathbf{i} - (14.7\mathbf{i} + 32.86\mathbf{j} = -32.86\mathbf{j}) \text{ (cm)}$$

The velocity at point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.2 \\ 0 & -32.86 & 0 \end{bmatrix} \boxed{= 6.57\mathbf{i} \text{ (cm/s)}}$$

Problem 17.73 The angle $\theta = 45^\circ$, and the bar OQ is rotating in the counterclockwise direction at 0.2 rad/s. Use instantaneous centers to determine the velocity of the sleeve P.



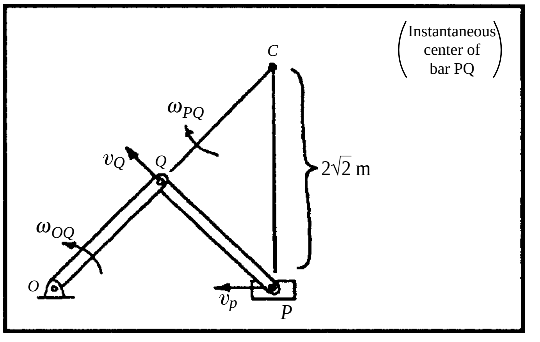
Solution: The velocity of Q is

$v_Q = 2\omega_{0Q} = 2(0.2) = 0.4$ m/s.

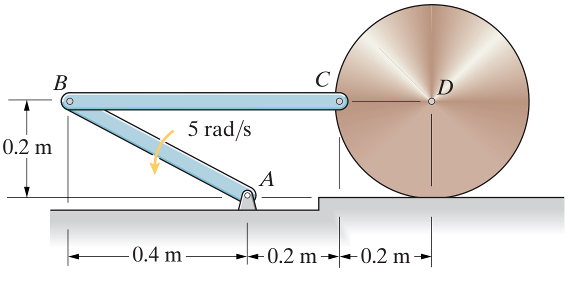
Therefore

$$|\overline{\omega}_{PQ}| = \frac{v_Q}{2\text{ m}} = \frac{0.4}{2} = 0.2 \text{ rad/s}$$

(clockwise) and $|\mathbf{v}_P| = 2\sqrt{2}\omega_{PQ} = 0.566$ m/s ($\mathbf{v}_P$ is to the left).



Problem 17.74 Bar AB is rotating in the counterclockwise direction at 5 rad/s. The disk rolls on the horizontal surface. Determine the angular velocity of bar BC.



Solution: First locate the instantaneous center

From the geometry we have

$$\frac{BC}{AE} = \frac{QB}{QA}$$

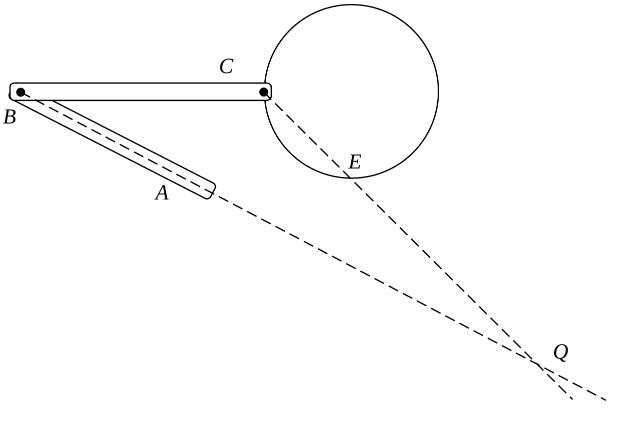
$$\frac{0.6\text{ m}}{0.4\text{ m}} = \frac{QB}{QB - \sqrt{0.2^2 + 0.4^2}\text{ m}}$$

Solving we find $BQ = 1.342$ m

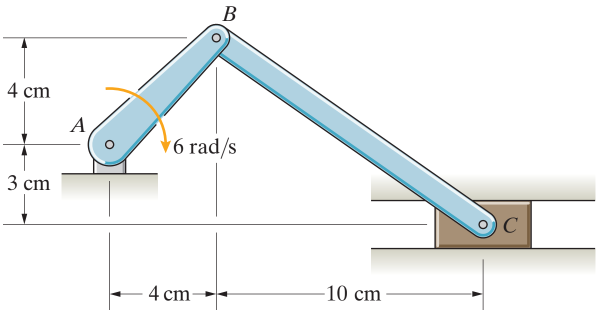
Now

$v_B = \omega_{AB}(AB) = \omega_{BC}(QB)$

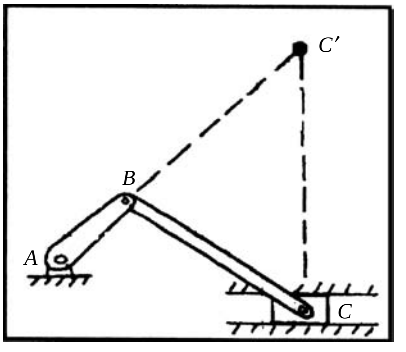
$$\boxed{\omega_{BC} = \frac{AB}{QB}(5\text{ rad/s}) = \frac{\sqrt{0.2^2+0.4^2}\text{ m}}{1.342\text{ m}}(5\text{ rad/s}) = 1.67\text{ rad/s CCW}}$$



Problem 17.75 Bar AB rotates at 6 rad/s in the clockwise direction. Use instantaneous centers to determine the angular velocity of bar BC.



Solution: Choose a coordinate system with origin at A and y axis vertical. Let C' denote the instantaneous center. The instantaneous center for bar AB is the point A, by definition, since A is the point of zero velocity. The vector AB is $\mathbf{r}_{B/A} = 4\mathbf{i} + 4\mathbf{j}$ (cm). The velocity at B is



$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -6 \\ 4 & 4 & 0 \end{bmatrix} = 24\mathbf{i} - 24\mathbf{j} \text{ (cm/s)}.$$

The unit vector parallel to AB is also the unit vector perpendicular to the velocity at B,

$$\mathbf{e}_{AB} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}).$$

The vector location of a point on a line perpendicular to the velocity at B is $\mathbf{L}_{AB} = L_{AB}\mathbf{e}_{AB}$, where L_{AB} is the magnitude of the distance from point A to the point on the line. The vector location of a point on a perpendicular to the velocity at C is $\mathbf{L}_C = (14\mathbf{i} + y\mathbf{j})$ where y is the y-coordinate of the point referenced to an origin at A. When the two lines intersect,

$$\frac{L_{AB}}{\sqrt{2}}\mathbf{i} = 14\mathbf{i},$$

and $y = \dfrac{L_{AB}}{\sqrt{2}} = 14$

from which $L_{AB} = 19.8$ cm, and the coordinates of the instantaneous center are (14, 14) (cm).

[*Check*: The line AC' is the hypotenuse of a right triangle with a base of 14 cm and interior angles of 45°, from which the coordinates of C' are (14, 14) cm *check*.]. The angular velocity of bar BC is determined from the known velocity at B. The vector from the instantaneous center to point B is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = 4\mathbf{i} + 4\mathbf{j} - 14\mathbf{i} - 14\mathbf{j} = -10\mathbf{i} - 10\mathbf{j}.$$

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -10 & -10 & 0 \end{bmatrix}$$

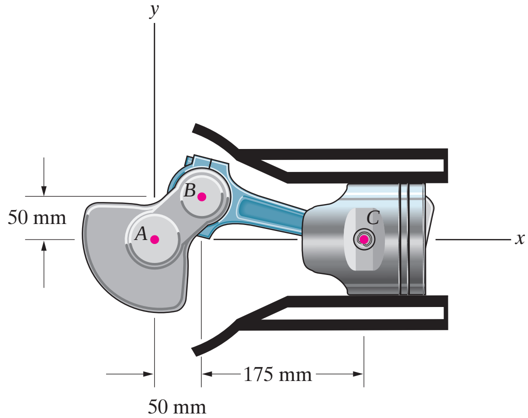
$$= \omega_{BC}(10\mathbf{i} - 10\mathbf{j}) \text{ (cm/s)}.$$

Equate the two expressions for the velocity: $24 = 10\omega_{BC}$, from which

$$\boxed{\omega_{BC} = 2.4 \text{ rad/s}},$$

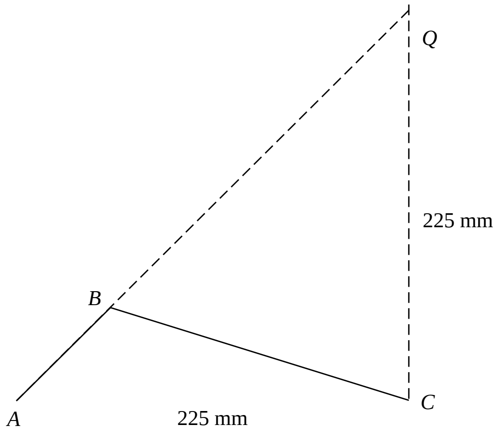
Problem 17.76 The crank AB is rotating in the clockwise direction at 2000 rpm (revolutions per minute).

(a) At the instant shown, what are the coordinates of the instantaneous center of the connecting rod BC?
(b) Use instantaneous centers to determine the angular velocity of the connecting rod BC at the instant shown.



Solution:

$$\omega_{AB} = 2000 \text{ rpm}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60 \text{ s}}\right) = 209 \text{ rad/s}$$



(a) The instantaneous center of BC is located at point Q

$$\boxed{Q = (225, 225) \text{ mm}}$$

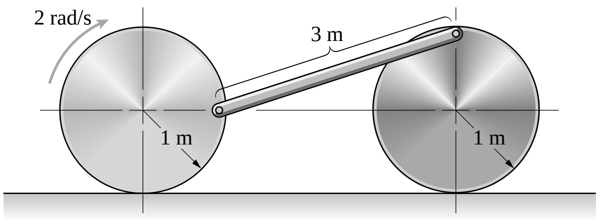
(b) $v_B = \omega_{AB}(AB) = \omega_{BC}(QB)$

$$\omega_{BC} = \frac{AB}{QB}\omega_{AB} = \frac{50\sqrt{2} \text{ mm}}{(225 - 50)\sqrt{2} \text{ mm}}(209 \text{ rad/s}) = 59.8 \text{ rad/s}$$

$$v_C = \omega_{BC}(QC) = (59.8 \text{ rad/s})(0.225 \text{ m}) = 13.5 \text{ m/s}$$

$$\boxed{\mathbf{v}_C = (13.5 \text{ m/s})\mathbf{i}}$$

Problem 17.77 The disks roll on the plane surface. The left disk rotates at 2 rad/s in the clockwise direction. Use the instantaneous centers to determine the angular velocities of the bar and the right disk.



Solution: Choose a coordinate system with the origin at the point of contact of the left disk with the surface, and the x axis parallel to the plane surface. Denote the point of attachment of the bar to the left disk by A, and the point of attachment to the right disk by B. The instantaneous center of the left disk is the point of contact with the surface. The vector distance from the point of contact to the point A is $\mathbf{r}_{A/P} = \mathbf{i} + \mathbf{j}$ (m). The velocity of point A is

$$\mathbf{v}_A = \omega_{LD} \times \mathbf{r}_{A/P} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -2 \\ 1 & 1 & 0 \end{bmatrix} = 2\mathbf{i} - 2\mathbf{j} \text{ (m/s)}.$$

The point on a line perpendicular to the velocity at A is $\mathbf{L}_A = L_A(\mathbf{i} + \mathbf{j})$, where L_A is the distance of the point from the origin. The point B is at the top of the right disk, and the velocity is constrained to be parallel to the x axis. A point on a line perpendicular to the velocity at B is $\mathbf{L}_B = (1 + 3\cos\theta)\mathbf{i} + y\mathbf{j}$ (m), where

$$\theta = \sin^{-1}\left(\frac{1}{3}\right) = 19.5^\circ.$$

At the intersection of these two lines $L_A = 1 + 3\cos\theta = 3.83$ ft, and the coordinates of the instantaneous center of the bar are (3.83, 3.83) (m). The angular velocity of the bar is determined from the known velocity of point A. The vector from the instantaneous center to point A is

$$\mathbf{r}_{A/C} = \mathbf{r}_A - \mathbf{r}_C = \mathbf{i} + \mathbf{j} - 3.83\mathbf{i} - 3.83\mathbf{j} = -2.83\mathbf{i} - 2.83\mathbf{j} \text{ (m)}.$$

The velocity of point A is

$$\mathbf{v}_A = \omega_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -2.83 & -2.83 & 0 \end{bmatrix}$$

$$= \omega_{AB}(2.83\mathbf{i} - 2.83\mathbf{j}) \text{ (m/s)}.$$

Equate the two expressions and solve:

$$\boxed{\omega_{AB} = \frac{2}{2.83} = 0.7071 \text{ (rad/s)}} \text{ counterclockwise.}$$

The vector from the instantaneous center to point B is

$$\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = (1 + 3\cos\theta)\mathbf{i} + 2\mathbf{j} - 3.83\mathbf{i} - 3.83\mathbf{j} = -1.83\mathbf{j}.$$

The velocity of point B is

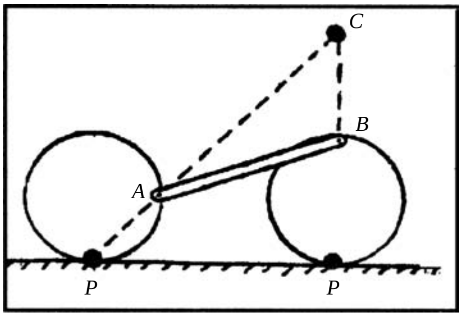
$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.7071 \\ 0 & -1.83 & 0 \end{bmatrix} = 1.294\mathbf{i} \text{ (m/s)}.$$

Using the fixed center at point of contact:

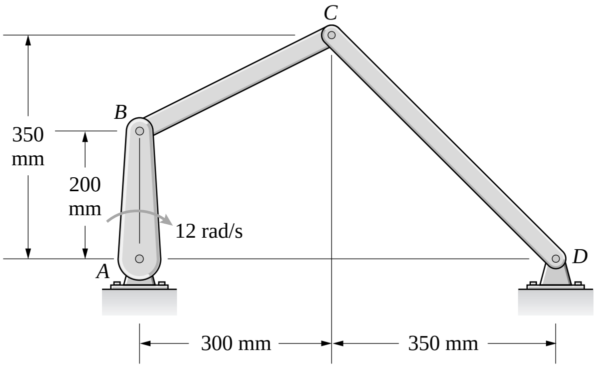
$$\mathbf{v}_B = \omega_{RD} \times \mathbf{r}_{B/P} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{RD} \\ 0 & 2 & 0 \end{bmatrix} = -2\omega_{RD}\mathbf{i} \text{ (m/s)}.$$

Equate the two expressions for $\mathbf{v}_B$ and solve:

$$\boxed{\omega_{RD} = -0.647 \text{ rad/s, clockwise.}}$$



Problem 17.78 Bar AB rotates at 12 rad/s in the clockwise direction. Use instantaneous centers to determine the angular velocities of bars BC and CD.



Solution: Choose a coordinate system with the origin at A and the x axis parallel to AD. The instantaneous center of bar AB is point A, by definition. The velocity of point B is normal to the bar AB. Using the instantaneous center A and the known angular velocity of bar AB the velocity of B is

$$\mathbf{v}_B = \omega \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -12 \\ 0 & 200 & 0 \end{bmatrix} = 2400\mathbf{i} \text{ (mm/s)}.$$

The unit vector perpendicular to the velocity of B is $\mathbf{e}_{AB} = \mathbf{j}$, and a point on a line perpendicular to the velocity at B is $\mathbf{L}_{AB} = L_{AB}\mathbf{j}$ (mm). The instantaneous center of bar CD is point D, by definition. The velocity of point C is constrained to be normal to bar CD. The interior angle at D is 45°, by inspection. The unit vector parallel to DC (and perpendicular to the velocity at C) is

$$\mathbf{e}_{DC} = -\mathbf{i}\cos 45^\circ + \mathbf{j}\sin 45^\circ = \left(\frac{1}{\sqrt{2}}\right)(-\mathbf{i} + \mathbf{j}).$$

The point on a line parallel to DC is

$$\mathbf{L}_{DC} = \left(650 - \frac{L_{DC}}{\sqrt{2}}\right)\mathbf{i} + \frac{L_{DC}}{\sqrt{2}}\mathbf{j} \text{ (mm)}.$$

At the intersection of these lines $\mathbf{L}_{AB} = \mathbf{L}_{DC}$, from which

$$\left(650 - \frac{L_{DC}}{\sqrt{2}}\right) = 0$$

and $L_{AB} = \dfrac{L_{DC}}{\sqrt{2}}$,

from which $L_{DC} = 919.2$ mm, and $L_{AB} = 650$ mm. The coordinates of the instantaneous center of bar BC are (0, 650) (mm). Denote this center by C'. The vector from C' to point B is

$$\mathbf{r}_{B/C'} = \mathbf{r}_B - \mathbf{r}_{C'} = 200\mathbf{j} - 650\mathbf{j} = -450\mathbf{j}.$$

The vector from C' to point C is

$$\mathbf{r}_{C/C'} = 300\mathbf{i} + 350\mathbf{j} - 650\mathbf{j} = 300\mathbf{i} - 300\mathbf{j} \text{ (mm)}.$$

The velocity of point B is

$$\mathbf{v}_B = \omega_{BC} \times \mathbf{r}_{B/C'} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0 & -450 & 0 \end{bmatrix} = 450\omega_{BC}\mathbf{i} \text{ (mm/s)}.$$

Equate and solve: $2400 = 450\omega_{BC}$, from which

$$\boxed{\omega_{BC} = \frac{2400}{450} = 5.33 \text{ (rad/s)}}.$$

The angular velocity of bar CD is determined from the known velocity at point C. The velocity at C is

$$\mathbf{v}_C = \omega_{BC} \times \mathbf{r}_{C/C'} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5.33 \\ 300 & -300 & 0 \end{bmatrix}$$

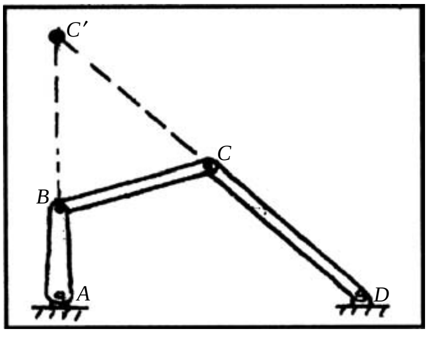
$$= 1600\mathbf{i} + 1600\mathbf{j} \text{ (mm/s)}.$$

The vector from D to point C is $\mathbf{r}_{C/D} = -350\mathbf{i} + 350\mathbf{j}$ (mm). The velocity at C is

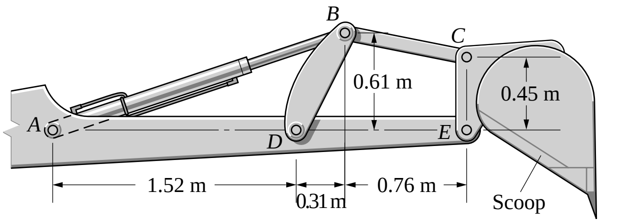
$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -350 & 350 & 0 \end{bmatrix}$$

$$= -350\omega_{CD}\mathbf{i} - 350\omega_{CD}\mathbf{j} \text{ (mm/s)}.$$

Equate and solve: $\boxed{\omega_{CD} = -4.57 \text{ rad/s}}$ clockwise.



Problem 17.79 The horizontal member *ADE* supporting the scoop is stationary. The link *BD* is rotating in the clockwise direction at 1 rad/s. Use instantaneous centers to determine the angular velocity of the scoop.



Solution: The distance from D to B is $r_{BD} = \sqrt{0.31^2 + 0.61^2} = 0.68$ m. The distance from B to H is

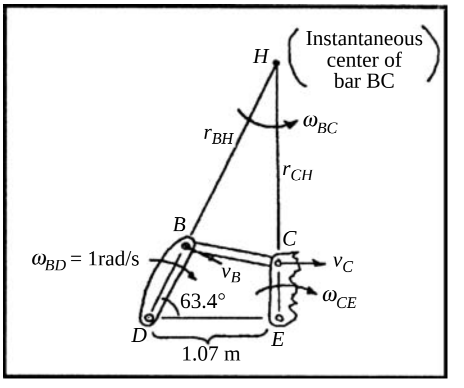
$$r_{BH} = \frac{1.07}{\cos 63.4^\circ} - r_{BD} = 1.7 \text{ m},$$

and the distance from C to H is $r_{CH} = 1.07 \tan 63.4^\circ - r_{CE} = 1.68$ m. The velocity of B is $v_B = r_{BD}\omega_{BD} = (0.68)(1) = 0.68$ m/s. Therefore

$$\omega_{BC} = \frac{v_B}{r_{BH}} = \frac{0.68}{1.7} = 0.4 \text{ rad/s}.$$

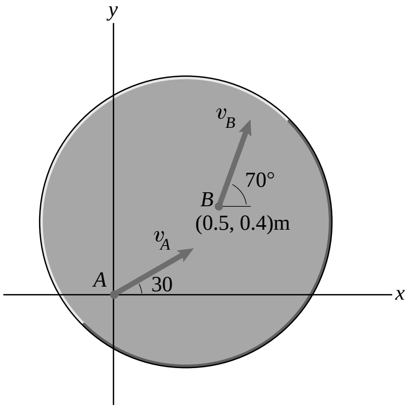
The velocity of C is $v_c = r_{CH}\omega_{BC} = (1.68)(0.4) = 0.67$, so the angular velocity of the scoop is

$$\omega_{CE} = \frac{v_C}{r_{CE}} = \frac{0.67}{0.46} = 1.47 \text{ rad/s}$$

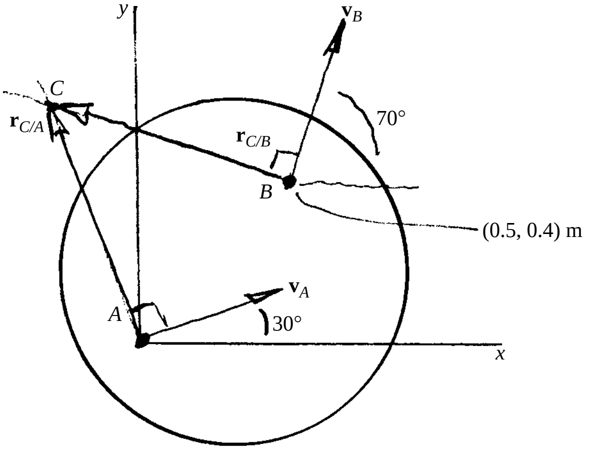


Problem 17.80 The disk is in planar motion. The directions of the velocities of points A and B are shown. The velocity of point A is $v_A = 2$ m/s.

(a) What are the coordinates of the disk's instantaneous center?
(b) Determine the velocity v_B and the disk's angular velocity.



Solution:



$\boldsymbol{\omega} = \omega \mathbf{k}$

$r_{c/A} = x_c \mathbf{i} + y_c \mathbf{j}$

$\mathbf{r}_{c/B} = (x_c - x_B)\mathbf{i} + (y_c - y_B)\mathbf{j}$

The velocity of C, the instantaneous center, is zero.

$\mathbf{v}_c = 0 = \mathbf{v}_A + \omega \mathbf{k} \times \mathbf{r}_{c/A}$

$$\begin{cases} 0 = v_{A_x} - \omega y_c & \textbf{(1)} \\ 0 = v_{A_y} + \omega x_c & \textbf{(2)} \end{cases}$$

where $v_{A_x} = v_A \cos 30^\circ = 2\cos 30^\circ$ m/s

$v_{A_y} = v_A \sin 30^\circ = 1$ m/s

$v_{B_x} = v_B \cos 70^\circ$

$v_{B_y} = v_B \sin 70^\circ$

Also $\mathbf{v}_c = 0 = \mathbf{v}_B + \omega \mathbf{k} \times \mathbf{r}_{c/B}$

$0 = v_B \cos 70^\circ - \omega(y_c - y_B)$ **(3)**

$0 = v_B \sin 70^\circ + \omega(x_c - x_B)$ **(4)**

Eqns (1) $\to$ (4) are 4 eqns in the four unknowns ω, v_B, x_c, and y_c.

Solving,

$\omega = 2.351$ rad/s,

$\underline{\boldsymbol{\omega} = 2.351\mathbf{k} \text{ rad/s}}$,

$\underline{v_B = 2.31 \text{ m/s}}$,

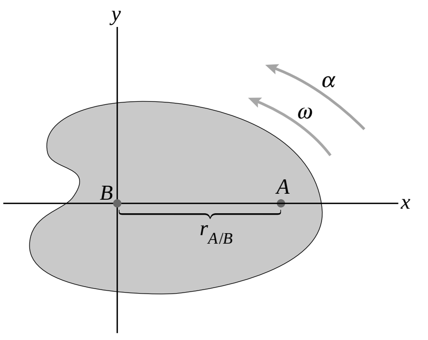
$\underline{x_c = -0.425 \text{ m}}$,

$\underline{y_c = 0.737 \text{ m}}$.

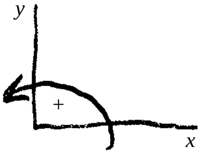
Problem 17.81 The rigid body rotates about the z axis with counterclockwise angular velocity $\omega = 4$ rad/s and counterclockwise angular acceleration $\alpha = 2$ rad/s^2. The distance $r_{A/B} = 0.6$ m.

(a) What are the rigid body's angular velocity and angular acceleration vectors?

(b) Determine the acceleration of point A relative to point B, first by using Eq. (17.9) and then by using Eq. (17.10).



Solution:



(a) By definition,

$$\boldsymbol{\omega} = 4\mathbf{k},$$

$$\boldsymbol{\alpha} = 6\mathbf{k}.$$

(b) $\mathbf{a}_{A/B} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) + \boldsymbol{\alpha} \times \mathbf{r}_{A/B}$

$$\mathbf{a}_{A/B} = 4\mathbf{k} \times (4\mathbf{k} \times 0.6\mathbf{i}) + 2\mathbf{k} \times 0.6\mathbf{i}$$

$$\underline{\mathbf{a}_{A/B} = -9.6\mathbf{i} + 1.2\mathbf{j} \text{ (m/s}^2\text{)}}.$$

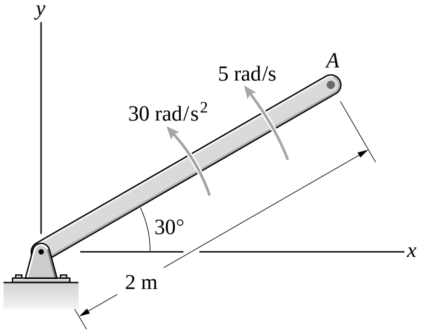
Using Eq. (17.10),

$$\mathbf{a}_{A/B} = \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \omega^2 r_{A/B}$$

$$= 2\mathbf{k} \times 0.6\mathbf{i} - 16(0.6)\mathbf{i}$$

$$\underline{\mathbf{a}_{A/B} = -9.6\mathbf{i} + 1.2\mathbf{j} \text{ (m/s}^2\text{)}}.$$

Problem 17.82 The bar rotates with a counterclockwise angular velocity of 5 rad/s and a counterclockwise angular acceleration of 30 rad/s^2. Determine the acceleration of A (a) by using Eq. (17.9) and (b) by using Eq. (17.10).



Solution:

(a) Eq. (17.9): $\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})$.

Substitute values:

$$\mathbf{a}_B = 0. \qquad \boldsymbol{\alpha} = 30\mathbf{k}\ (\text{rad/s}^2),$$

$$\mathbf{r}_{A/B} = 2(\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 1.732\mathbf{i} + \mathbf{j}\ (\text{m}).$$

$$\boldsymbol{\omega} = 5\mathbf{k}\ (\text{rad/s}).$$

Expand the cross products:

$$\boldsymbol{\alpha} \times \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 1.732 & 1 & 0 \end{bmatrix} = -30\mathbf{i} + 52\mathbf{j}\ (\text{m/s}^2).$$

$$\boldsymbol{\omega} \times \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5 \\ 1.732 & 1 & 0 \end{bmatrix} = -5\mathbf{i} + 8.66\mathbf{j}\ (\text{m/s}).$$

$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5 \\ -5 & 8.66 & 0 \end{bmatrix} = -43.3\mathbf{i} - 25\mathbf{j}\ (\text{m/s}^2).$$

Collect terms: $\boxed{\mathbf{a}_A = -73.3\mathbf{i} + 27\mathbf{j}\ (\text{m/s}^2)}$.

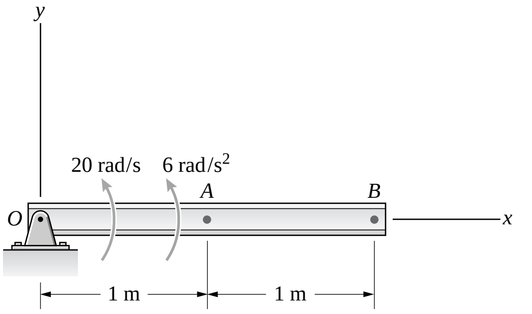
(b) Eq. (17.10): $\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$.

Substitute values, and expand the cross product as in Part (b) to obtain

$$\boxed{\mathbf{a}_A = -30\mathbf{i} + 52\mathbf{j} - (5^2)(1.732\mathbf{i} + \mathbf{j}) = -73.3\mathbf{i} + 27\mathbf{j}\ (\text{m/s}^2)}$$

Problem 17.83 The bar rotates with a counterclockwise angular velocity of 20 rad/s and a counterclockwise angular acceleration of 6 rad/s^2.

(a) By applying Eq. (17.10) to point A and the fixed point O, determine the acceleration of A.
(b) By using the result of part (a) and Eq. (17.10), to points A and B, determine the acceleration point B.



Solution:

(a) $\mathbf{a}_A = \mathbf{a}_0 + \boldsymbol{\alpha} \times \mathbf{r}_{A/O} - \omega^2 \mathbf{r}_{A/O}$

where

$\boldsymbol{\omega} = 20\mathbf{k}$ rad/s

$\boldsymbol{\alpha} = 6\mathbf{k}$ rad/s^2

$\mathbf{r}_{A/O} = 1\mathbf{i}$, and $\mathbf{a}_A = 0$

$\mathbf{a}_A = O + 6\mathbf{k} \times 1\mathbf{i} - 400(1\mathbf{i})$

$\underline{\mathbf{a}_A = -400\mathbf{i} + 6\mathbf{j}\ (\text{m/s}^2)}$.

(b) $\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$

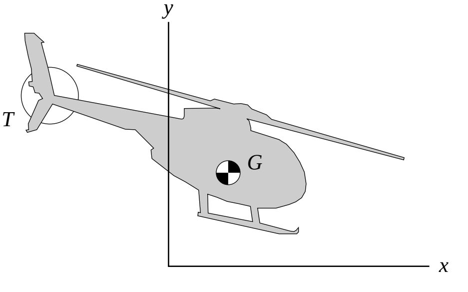
where

$\mathbf{r}_{B/A} = 1\mathbf{i}$

$\mathbf{a}_B = -400\mathbf{i} + 6\mathbf{j} + 6\mathbf{k} \times 1\mathbf{i} - 400(1\mathbf{i})$

$\underline{\mathbf{a}_B = -800\mathbf{i} + 12\mathbf{j}\ (\text{m/s}^2)}$.

Problem 17.84 The helicopter is in planar motion in the x–y plane. At the instant shown, the position of its center of mass G is $x = 2$ m, $y = 2.5$ m, its velocity is $\mathbf{v}_G = 12\mathbf{i} + 4\mathbf{j}$ (m/s), and its acceleration is $a_G = 2\mathbf{i} + 3\mathbf{j}$ (m/s^2). The position of point T where the tail rotor is mounted is $x = -3.5$ m, $y = 4.5$ m. The helicopter's angular velocity is 0.2 rad/s clockwise, and its angular acceleration is 0.1 rad/s^2 counterclockwise. What is the acceleration of point T?



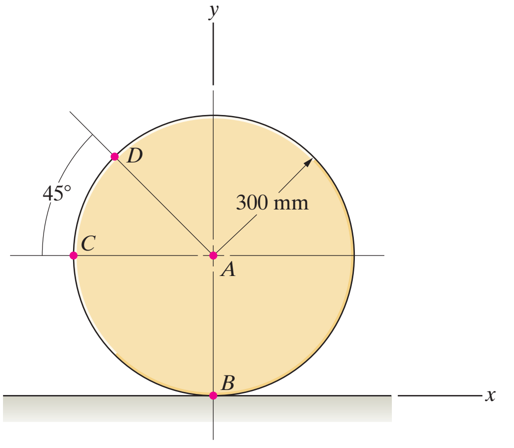
Solution: The acceleration of T is

$\mathbf{a}_T = \mathbf{a}_G + \boldsymbol{\alpha} \times \mathbf{r}_{T/G} - \omega^2 \mathbf{r}_{T/G}$;

$$\mathbf{a}_T = 2\mathbf{i} + 3\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.1 \\ -5.5 & 2 & 0 \end{vmatrix} - (0.2)^2(-5.5\mathbf{i} + 2\mathbf{j})$$

$= 2.02\mathbf{i} + 2.37\mathbf{j}\ (\text{m/s}^2)$.

Problem 17.85 Point A of the rolling disk is moving toward the right and accelerating toward the right. The magnitude of the velocity of point C is 2 m/s, and the magnitude of the acceleration of point C is 14 m/s^2. Determine the acceleration of points B and D. (See Active Example 17.5.)



Solution: First the velocity analysis

$$\mathbf{v}_C = \mathbf{v}_A + \omega \times \mathbf{r}_{C/A}$$

$$= r\omega\mathbf{i} - \omega\mathbf{k} \times (-r\mathbf{i}) = r\omega\mathbf{i} + r\omega\mathbf{j}$$

$$v_c = \sqrt{(r\omega)^2 + (r\omega)^2} = \sqrt{2}r\omega \Rightarrow \omega = \frac{v}{\sqrt{2}r} = \frac{2\text{ m/s}}{\sqrt{2}(0.3\text{ m})} = 4.71\text{ rad/s.}$$

Now the acceleration analysis

$$\mathbf{a}_C = \mathbf{a}_A + \alpha \times \mathbf{r}_{C/A} - \omega^2\mathbf{r}_{C/A}$$

$$= r\alpha\mathbf{i} - \alpha\mathbf{k} \times (-r\mathbf{i}) - \omega^2(-r\mathbf{i}) = (r\alpha + \omega^2 r)\mathbf{i} + (\alpha r)\mathbf{j}$$

$$a_C = \sqrt{(\alpha r + \omega^2 r)^2 + (\alpha r)^2} = r\sqrt{(\alpha + \omega^2)^2 + \alpha^2}$$

$$(14\text{ m/s}^2)^2 = (0.3\text{ m})^2[(\alpha + [4.71]^2)^2 + \alpha^2]$$

Solving this quadratic equation for α we find $\alpha = 20.0$ rad/s^2.

Now

$$\mathbf{a}_B = r\omega^2\mathbf{j} = (0.3\text{ m})(4.71\text{ rad/s})^2\mathbf{j} \quad \boxed{\mathbf{a}_B = 6.67\mathbf{j}\ (\text{m/s}^2)}$$

$$\mathbf{a}_D = \mathbf{a}_A + \alpha \times \mathbf{r}_{D/A} - \omega^2\mathbf{r}_{D/A}$$

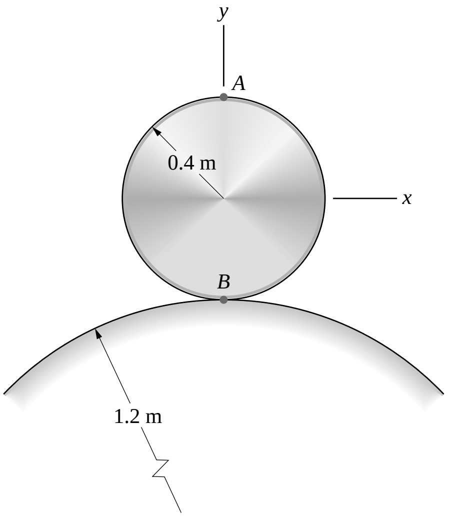
$$= r\alpha\mathbf{i} - \alpha\mathbf{k} \times (-r\cos 45^\circ\mathbf{i} + 4\sin 45^\circ\mathbf{j}) - \omega^2(-r\cos 45^\circ\mathbf{i} + 4\sin 45^\circ\mathbf{j})$$

$$= (r[1 + \sin 45^\circ]\alpha + r\omega^2\cos 45^\circ)\mathbf{i} + (r\alpha\cos 45^\circ - r\omega^2\sin 45^\circ)\mathbf{j}$$

Putting in the numbers we find $\boxed{\mathbf{a}_D = 14.9\mathbf{i} - 0.480\mathbf{j}\ (\text{m/s}^2).}$

Problem 17.86 The disk rolls on the circular surface with a constant clockwise angular velocity of 1 rad/s. What are the accelerations of points A and B?

Strategy: Begin by determining the acceleration of the center of the disk. Notice that the center moves in a circular path and the magnitude of its velocity is constant.



Solution:

$$\mathbf{v}_B = 0$$

$$\mathbf{v}_0 = \mathbf{v}_B + \omega\mathbf{k} \times \mathbf{r}_{O/B} = (-1\mathbf{k}) \times (0.4\mathbf{j})$$

$$\mathbf{v}_0 = 0.4\ \mathbf{i}\ \text{m/s}$$

Point O moves in a circle at constant speed. The acceleration of O is

$$\mathbf{a}_0 = -v_0^2/(R+r)\mathbf{j} = (-0.16)/(1.2+0.4)\mathbf{j}$$

$$\underline{\mathbf{a}_0 = -0.1\mathbf{j}\ (\text{m/s}^2)}.$$

$$\mathbf{a}_B = \mathbf{a}_0 - \omega^2\mathbf{r}_{B/O} = -0.1\mathbf{j} - (1)^2(-0.4)\mathbf{j}$$

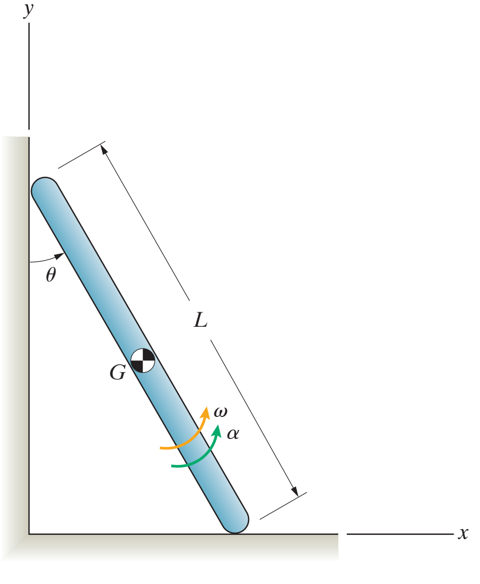
$$\underline{\mathbf{a}_B = 0.3\mathbf{j}\ (\text{m/s}^2)}.$$

$$\mathbf{a}_A = \mathbf{a}_0 - \omega^2\mathbf{r}_{A/O} = -0.1\mathbf{j} - (1)^2(0.4)\mathbf{j}$$

$$\underline{\mathbf{a}_A = -0.5\mathbf{j}\ (\text{m/s}^2)}.$$

Problem 17.87 The length of the bar is $L = 4$ m and the angle $\theta = 30^\circ$. The bar's angular velocity is $\omega = 1.8$ rad/s and its angular acceleration is $\alpha = 6$ rad/s^2. The endpoints of the bar slide on the plane surfaces. Determine the acceleration of the midpoint G.

Strategy: Begin by applying Eq. (17.10) to the endpoints of the bar to determine their accelerations.



Solution: Call the top point D and the bottom point B.

$$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{D/B} - \omega^2\mathbf{r}_{D/B}$$

Put in the known constraints

$$a_D\mathbf{j} = a_B\mathbf{i} + \alpha\mathbf{k} \times L(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j}) - \omega^2 L(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j})$$

$$a_D\mathbf{j} = (a_B - \alpha L\cos\theta + \omega^2 L\sin\theta)\mathbf{i} + (-\alpha L\sin\theta - \omega^2 L\cos\theta)\mathbf{j}$$

Equating components we have

$$a_B = \alpha L\cos\theta - \omega^2 L\sin\theta = (6)(4)\cos 30^\circ - (1.8)^2(4)\sin 30^\circ$$
$$= 14.3\ \text{m/s}^2$$

$$a_D = \alpha L\sin\theta - \omega^2 L\cos\theta = (6)(4)\sin 30^\circ - (1.8)^2(4)\cos 30^\circ$$
$$= -23.2\ \text{m/s}^2$$

Now we can use either point as a base point to find the acceleration of point G. We will use B as the base point.

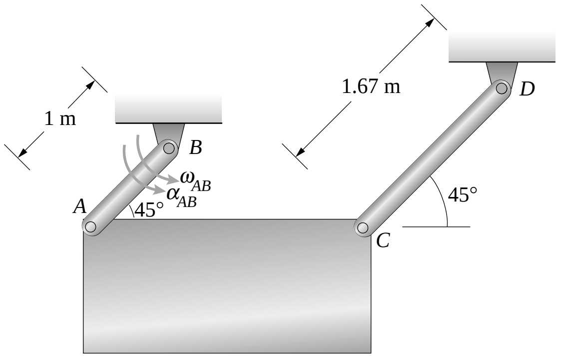
$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{G/B} - \omega^2\mathbf{r}_{GB}$$

$$= 14.3\mathbf{i} + 6\mathbf{k} \times (2)(-\sin 30^\circ\mathbf{i} + \cos 30^\circ\mathbf{j})$$

$$- (1.8)^2(2)(-\sin 30^\circ\mathbf{i} + \cos 30^\circ\mathbf{j})$$

$$\boxed{\mathbf{a}_G = 7.15\mathbf{i} - 11.6\mathbf{j} (\text{m/s}^2.)}$$

Problem 17.88 The angular velocity and angular acceleration of bar AB are $\omega_{AB} = 2$ rad/s and $\alpha_{AB} = 10$ rad/s^2. The dimensions of the rectangular plate are 1 m $\times$ 2 m. What are the angular velocity and angular acceleration of the rectangular plate?



Solution: The instantaneous center for bar AB is point B, by definition. The instantaneous center for bar CD is point D, by definition. The velocities at points A and C are normal to the bars AB and CD, respectively. However, by inspection these bars are parallel at the instant shown, so that lines perpendicular to the velocities at A and C will never intersect—the instantaneous center of the plate AC is at infinity, hence *the plate only translates at the instant shown*, and $\boxed{\omega_{AC} = 0}$. If the plate is not rotating, the velocity at every point on the plate must be the same, and in particular, the vector velocity at A and C must be identical. The vector A/B is

$$\mathbf{r}_{A/B} = -\mathbf{i}\cos 45^\circ - \mathbf{j}\sin 45^\circ = \left(\frac{-1}{\sqrt{2}}\right)(\mathbf{i}+\mathbf{j}) \text{ (m)}.$$

The velocity at point A is

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B} = \frac{-\omega_{AB}}{\sqrt{2}} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \sqrt{2}(\mathbf{i}-\mathbf{j}) \text{ (m/s)}.$$

The vector C/D is

$$\mathbf{r}_{C/D} = (1.67)(-\mathbf{i}\cos 45^\circ - \mathbf{j}\sin 45^\circ) = -1.179(\mathbf{i}+\mathbf{j}) \text{ (m)}.$$

The velocity at point C is

$$\mathbf{v}_C = -1.179\omega_{CD} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = 1.179\omega_{CD}(\mathbf{i}-\mathbf{j}) \text{ (m/s)}.$$

Equate the velocities $\mathbf{v}_C = \mathbf{v}_A$, separate components and solve: $\omega_{CD} = 1.2$ rad/s. Use Eq. (17.10) to determine the accelerations. The acceleration of point A is

$$\mathbf{a}_A = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2 \mathbf{r}_{A/B} = -\frac{10}{\sqrt{2}} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} + \left(\frac{2^2}{\sqrt{2}}\right)(\mathbf{i}+\mathbf{j})$$

$$= 9.9\mathbf{i} - 4.24\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of point C relative to point A is

$$\mathbf{a}_C = \mathbf{a}_A + \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/A} = \mathbf{a}_A + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AC} \\ 2 & 0 & 0 \end{bmatrix}$$

$$= 9.9\mathbf{i} + (2\alpha_{AC} - 4.24)\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of point C relative to point D is $\mathbf{a}_C = \mathbf{a}_D + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2 \mathbf{r}_{C/D}$. Noting $\mathbf{a}_D = 0$,

$$\mathbf{a}_C = -1.179\alpha_{CD} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} + 1.179\omega_{CD}^2(\mathbf{i}+\mathbf{j})$$

$$= (1.179\alpha_{CD} + 1.697)\mathbf{i} + (-1.179\alpha_{CD} + 1.697)\mathbf{j} \text{ (m/s}^2\text{)}.$$

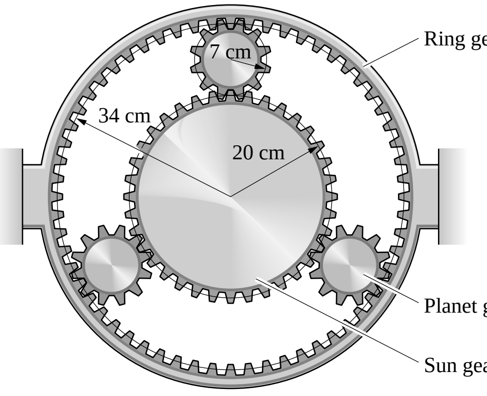
Equate the two expressions for the acceleration at point C and separate components:

$$(-9.9 + 1.179\alpha_{CD} + 1.697)\mathbf{i} = 0,$$

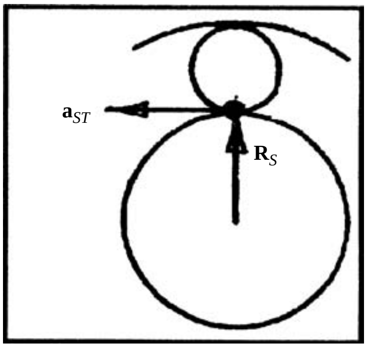
$$(2\alpha_{AC} - 4.24 + 1.179\alpha_{CD} - 1.697)\mathbf{j} = 0.$$

Solve: $\boxed{\alpha_{AC} = -1.13 \text{ (rad/s}^2\text{)}}$ (clockwise), $\alpha_{CD} = 6.96$ (rad/s^2) (counterclockwise).

Problem 17.89 The ring gear is stationary, and the sun gear has an angular acceleration of 10 rad/s^2 in the counterclockwise direction. Determine the angular acceleration of the planet gears.



Solution: The strategy is to use the tangential acceleration at the point of contact of the sun and planet gears, together with the constraint that the point of contact of the planet gear and ring gear is stationary, to determine the angular acceleration of the planet gear. The tangential acceleration of the sun gear at the point of contact with the top planet gear is



$$\mathbf{a}_{ST} = \boldsymbol{\alpha} \times \mathbf{r}_S = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 0 & 20 & 0 \end{bmatrix} = -200\mathbf{i} \text{ (cm/s}^2\text{)}.$$

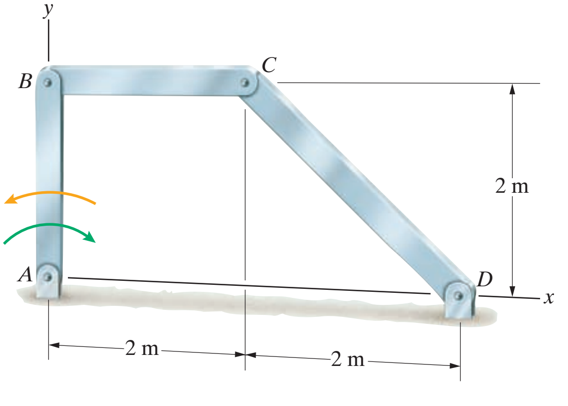
This is also the tangential acceleration of the planet gear at the point of contact. At the contact with the ring gear, the planet gears are stationary, hence the angular acceleration of the planet gear satisfies

$$\boldsymbol{\alpha}_P \times (-2\mathbf{r}_P) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_P \\ 0 & -14 & 0 \end{bmatrix} = -200\mathbf{i}$$

from which

$$\boxed{\alpha_P = -\frac{200}{14} = -14.29 \text{ (rad/s}^2\text{)}} \text{ (clockwise).}$$

Problem 17.90 In Example 17.6, what is the acceleration of the midpoint of bar BC?



Solution: From Example 17.6 we know that

$$\omega_{BC} = -10 \text{ rad/s},$$

$$\alpha_{BC} = 100 \text{ rad/s}^2,$$

$$\omega_{CD} = 10 \text{ rad/s},$$

$$\alpha_{CD} = -100 \text{ rad/s}^2.$$

To find the acceleration of the midpoint "G" of bar BC, we have

$$\mathbf{a}_B = \mathbf{a} + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}{}^2\mathbf{r}_{B/A}$$

$$= 0 - (300 \text{ rad/s}^2)\ \mathbf{k} \times (2\text{ m})\mathbf{j} - (10\text{ rad/s})^2(2\text{ m})\mathbf{j}$$

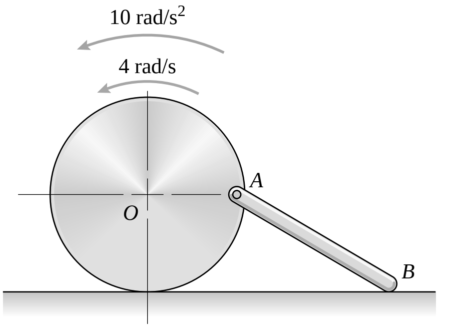
$$= (600\mathbf{i} - 200\mathbf{j}) \text{ m/s}^2$$

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{G/B} - \omega_{BC}{}^2\mathbf{r}_{G/B}$$

$$= (600\mathbf{i} - 200\mathbf{j}) + (100\mathbf{k}) \times (1\mathbf{i}) - (-10)^2(1\mathbf{i})$$

$$\boxed{\mathbf{a}_G = (500\mathbf{i} - 100\mathbf{j}) \text{ m/s}^2.}$$

Problem 17.91 The 1-m-diameter disk rolls, and point B of the 1-m-long bar slides, on the plane surface. Determine the angular acceleration of the bar and the acceleration of point B.



Solution: Choose a coordinate system with the origin at O, the center of the disk, with x axis parallel to the horizontal surface. The point P of contact with the surface is stationary, from which

$$\mathbf{v}_P = 0 = \mathbf{v}_O + \boldsymbol{\omega}_O \times -\mathbf{R} = \mathbf{v}_O + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_O \\ 0 & -0.5 & 0 \end{bmatrix} = \mathbf{v}_O + 2\mathbf{i},$$

from which $\mathbf{v}_O = -2\mathbf{i}$ (m/s). The velocity at A is

$$\mathbf{v}_A = \mathbf{v}_O + \boldsymbol{\omega}_O \times \mathbf{r}_{A/O} = -2\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_O \\ 0.5 & 0 & 0 \end{bmatrix} = -2\mathbf{i} + 2\mathbf{j} \text{ (m/s)}$$

The motion at point B is constrained to be parallel to the x axis. The line perpendicular to the velocity of B is parallel to the y axis. The line perpendicular to the velocity at A forms an angle at 45° with the x axis. From geometry, the line from A to the fixed center is the hypotenuse of a right triangle with base $\cos 30^\circ = 0.866$ and interior angles 45°. The coordinates of the fixed center are $(0.5 + 0.866, 0.866) = (1.366, 0.866)$ in. The vector from the instantaneous center to the point A is $\mathbf{r}_{A/C} = \mathbf{r}_A - \mathbf{r}_C = -0.866\mathbf{i} - 0.866\mathbf{j}$ (m). The angular velocity of the bar AB is obtained from

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -0.866 & -0.866 & 0 \end{bmatrix}$$

$$= 0.866\omega_{AB}\mathbf{i} - 0.866\omega_{AB}\mathbf{j} \text{ (m/s)},$$

from which

$$\omega_{AB} = -\frac{2}{0.866} = -2.31 \text{ (rad/s)}.$$

The acceleration of the center of the rolling disk is $\mathbf{a}_O = -\alpha R\mathbf{i} = -10(0.5)\mathbf{i} = -5\mathbf{i}$ (m/s^2). The acceleration of point A is

$$\mathbf{a}_A = \mathbf{a}_O + \boldsymbol{\alpha}_O \times \mathbf{r}_{A/O} - \omega_O^2 \mathbf{r}_{A/O}$$

$$= -5\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha \\ 0.5 & 0 & 0 \end{bmatrix} - 16(0.5)\mathbf{i}$$

$$= -13\mathbf{i} + 5\mathbf{j} \text{ (m/s}^2\text{)}.$$

The vector B/A is

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = (0.5 + \cos\theta)\mathbf{i} - 0.5\mathbf{j} - 0.5\mathbf{i}$$

$$= 0.866\mathbf{i} - 0.5\mathbf{j} \text{ (m)}.$$

The acceleration of point B is

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2 \mathbf{r}_{A/B}$$

$$= -13\mathbf{i} + 5\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ -\cos\theta & \sin\theta & 0 \end{bmatrix}$$

$$-\omega_{AB}^2(-\mathbf{i}\cos\theta + \mathbf{j}\sin\theta).$$

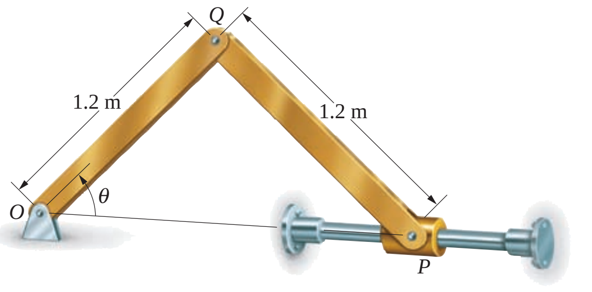
The constraint on B insures that the acceleration of B will be parallel to the x axis. Separate components:

$$a_B = -13 + 0.5\alpha_{AB} - \omega_{AB}^2(0.866),$$

$$0 = 5 + 0.866\alpha_{AB} + 0.5\omega_{AB}^2.$$

Solve: $\boxed{\alpha_{AB} = -8.85 \text{ (rad/s}^2\text{)}}$, where the negative sign means a clockwise rotation. $\boxed{\mathbf{a}_B = -22.04\mathbf{i} \text{ (m/s}^2\text{)}}$

Problem 17.92 If $\theta = 45^\circ$ and sleeve P is moving to the right with a constant velocity of 2 m/s, what are the angular accelerations of the bars OQ and PQ?



Solution:

$\mathbf{v}_0 = \mathbf{a}_0 = 0, \mathbf{v}_p = 2\mathbf{i}, \mathbf{a}_p = 0$

$\mathbf{r}_{Q/o} = 1.2\cos 45^\circ\mathbf{i} + 1.2\sin 45^\circ\mathbf{j}$ m

$\mathbf{r}_{p/Q} = 1.2\cos 45^\circ\mathbf{i} - 1.2\sin 45^\circ\mathbf{j}$ m

$\mathbf{v}_Q = \omega_{oQ}\mathbf{k} \times \mathbf{r}_{Q/o} = \omega_{oQ}\mathbf{k} \times (0.848\mathbf{i} + 0.848\mathbf{j})$

$$\begin{cases} v_{Qx} = -0.848\omega_{oQ} & \textbf{(1)} \\ v_{Qy} = 0.848\omega_{oQ} & \textbf{(2)} \end{cases}$$

$\mathbf{v}_p = \mathbf{v}_Q + \omega_{pQ}\mathbf{k} \times (0.848\mathbf{i} - 0.848\mathbf{j})$

$$\begin{cases} 2 = v_{Qx} + 0.848\omega_{pQ} & \textbf{(3)} \\ O = v_{Qy} + 0.848\omega_{pQ} & \textbf{(4)} \end{cases}$$

Solving eqns. (1)–(4),

$\omega_{oQ} = -1.179$ rad/s, $\omega_{pQ} = 1.179$ rad/s

$v_{Qx} = 1$ m/s $v_{Qy} = -1$ m/s

$\mathbf{a}_Q = \boldsymbol{\alpha}_{oQ} \times \mathbf{r}_{Q/o} - \omega_{oQ}^2\mathbf{r}_{Q/o}$

$$\begin{cases} a_{Qx} = -0.848\alpha_{oQ} - 0.848\omega_{oQ}^2 & \textbf{(5)} \\ a_{Qy} = 0.848\alpha_{oQ} - 0.848\omega_{oQ}^2 & \textbf{(6)} \end{cases}$$

Also,

$\mathbf{a}_p = 0 = \mathbf{a}_Q + \alpha_{pQ}\mathbf{k} \times \mathbf{r}_{p/Q} - \omega_{pQ}^2\mathbf{r}_{p/Q}$

$$\begin{cases} 0 = a_{Qx} + 0.848\alpha_{pQ} - 0.848\omega_{pQ}^2 & \textbf{(7)} \\ 0 = a_{Qy} + 0.848\alpha_{pQ} + 0.848\omega_{pQ}^2 & \textbf{(8)} \end{cases}$$

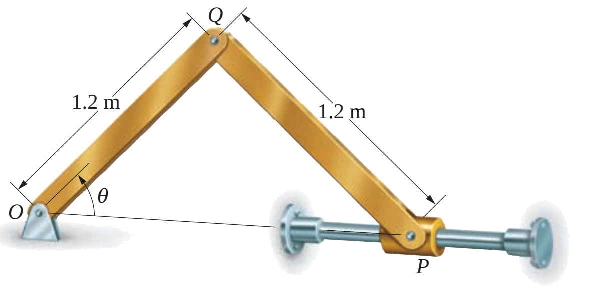
Solving eqns. (5)–(8), we get

$a_{Qx} = 0, a_{Qy} = 0$

$\underline{\alpha_{oQ} = 1.39 \text{ rad/s}^2}$ (clockwise)

$\underline{\alpha_{pQ} = 1.39 \text{ rad/s}^2}$ (counterclockwise)

Problem 17.93 Consider the system shown in Problem 17.92. If $\theta = 50^\circ$ and bar OQ has a constant clockwise angular velocity of 1 rad/s, what is the acceleration of sleeve P?



Solution:

$\boldsymbol{\omega}_{oQ} = -1\mathbf{k}$ rad/s, $\boldsymbol{\alpha}_{oQ} = 0$, $\mathbf{a}_0 = 0$

$$\mathbf{a}_Q = \mathbf{a}_0 + \boldsymbol{\alpha}_{oQ} \times \mathbf{r}_{Q/o} - \omega^2 \mathbf{r}_{Q/o}$$

$$\mathbf{a}_Q = 0 + 0 - (1)^2(1.2\cos 50^\circ \mathbf{i} + 1.2\sin 50^\circ \mathbf{j})$$

$$\mathbf{a}_Q = -0.771\mathbf{i} - 0.919\mathbf{j} \text{ m/s}^2$$

$$\mathbf{a}_p = \mathbf{a}_Q + \alpha_{Qp}\mathbf{k} \times \mathbf{r}_{p/Q} - \omega_{Qp}^2 \mathbf{r}_{p/Q}$$

where $\mathbf{a}_p = a_p \mathbf{i}$

$$\mathbf{r}_{p/Q} = 1.2\cos 50^\circ \mathbf{i} - 1.2\sin 50^\circ \mathbf{j}$$

$\mathbf{i}:$ $a_p = -0.771 + 1.2\alpha_{Qp}\sin 50^\circ - \omega_{Qp}^2(1.2)\cos 50^\circ$ **(1)**

$\mathbf{j}:$ $0 = -0.919 + 1.2\alpha_{Qp}\cos 50^\circ + \omega_{Qp}^2(1.2)\sin 50^\circ$ **(2)**

We have two eqns in three unknowns a_p, α_{Qp}, ω_{Qp}.

We need another eqn. To get it, we use the velocity relationships and determine ω_{QP}. Note $\mathbf{v}_p = v_p\mathbf{i}$.

$$\mathbf{v}_Q = \mathbf{v}_0 + \omega_{oQ} \times r_{Q/o} \quad \mathbf{v}_0 = 0$$

$$= (-1\mathbf{k}) \times [(1.2\cos 50^\circ)\mathbf{i} + (1.2\sin 50)\mathbf{j}]$$

$$= .919\mathbf{i} - 0.771\mathbf{j} \text{ (m/s)}.$$

$$\mathbf{v}_P = \mathbf{v}_Q + \boldsymbol{\omega}_{QP} \times \mathbf{r}_{P/Q}$$

$$= \mathbf{v}_Q + \omega_{QP}\mathbf{k} \times (1.2\cos 50^\circ \mathbf{i} - 1.2\sin 50^\circ \mathbf{j}).$$

$\mathbf{i}:$ $v_P = 0.919 + 1.2\omega_{QP}\sin 50^\circ$

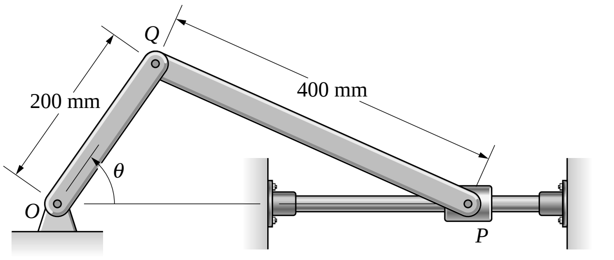
$\mathbf{j}:$ $O = -0.771 + 1.2\omega_{QP}\cos 50^\circ$

Solving, $v_P = 1.839$ m/s, $\omega_{QP} = 1$ rad/s. Now going back to eqns. (1) and (2), we solve to get

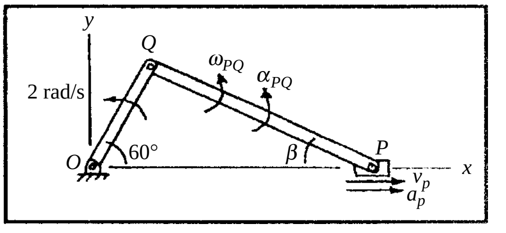
$a_P = -1.54$ m/s^2

$\alpha_{QP} = 0$, (to the left)

Problem 17.94 The angle $\theta = 60^\circ$, and bar OQ has a constant counterclockwise angular velocity of 2 rad/s. What is the angular acceleration of the bar PQ?



Solution: By applying the law of sines, the angle $\beta = 25.7^\circ$ The velocity of Q is $\mathbf{v}_Q = \mathbf{v}_0 + \boldsymbol{\omega}_{0Q} \times \mathbf{r}_{Q/O}$



$$\mathbf{v}_Q = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0.2\cos 60^\circ & 0.2\sin 60^\circ & 0 \end{vmatrix}$$

$$= -0.4\sin 60^\circ \mathbf{i} + 0.4\cos 60^\circ \mathbf{j}.$$

The velocity of P is

$$v_P \mathbf{i} = \mathbf{v}_Q + \boldsymbol{\omega}_{PQ} \times \mathbf{r}_{P/Q}$$

$$= -0.4\sin 60^\circ \mathbf{i} + 0.4\cos 60^\circ \mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{PQ} \\ 0.4\cos\beta & -0.4\sin\beta & 0 \end{vmatrix}.$$

Equating $\mathbf{j}$ components, we get $0 = 0.4\cos 60^\circ + 0.4\omega_{PQ}\cos\beta$, and obtain $\omega_{PQ} = -0.555$ rad/s. The acceleration of Q is

$$\mathbf{a}_Q = \mathbf{a}_0 + \alpha_{0Q} \times \mathbf{r}_{Q/0} - \omega_{0Q}^2 \mathbf{r}_{Q/0},$$

or $\mathbf{a}_Q = 0 + 0 - (2)^2(0.2\cos 60^\circ \mathbf{i} + 0.2\sin 60^\circ \mathbf{j})$

$$= -0.8\cos 60^\circ \mathbf{i} - 0.8\sin 60^\circ \mathbf{j}.$$

The acceleration of P is

$$a_P \mathbf{i} = \mathbf{a}_Q + \boldsymbol{\alpha}_{PQ} \times \mathbf{r}_{P/Q} - \omega_{PQ}^2 \mathbf{r}_{P/Q}$$

$$= -0.8\cos 60^\circ \mathbf{i} - 0.8\sin 60^\circ \mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{PQ} \\ 0.4\cos\beta & -0.4\sin\beta & 0 \end{vmatrix}$$

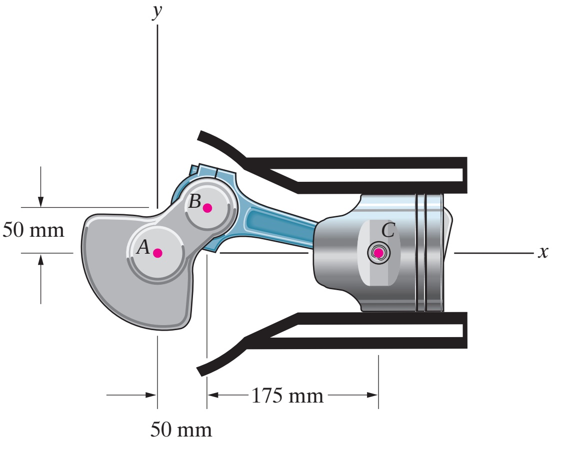
$$- (-0.555)^2(0.4\cos\beta \mathbf{i} - 0.4\sin\beta \mathbf{j}).$$

Equating $\mathbf{j}$ components

$$0 = -0.8\sin 60^\circ + 0.4\alpha_{PQ}\cos\beta + (0.555)^2 0.4\sin\beta.$$

Solving, we obtain $\alpha_{PQ} = 1.77$ rad/s^2.

Problem 17.95 At the instant shown, the piston's velocity and acceleration are $\mathbf{v}_C = -14\mathbf{i}$ (m/s) and $\mathbf{a}_C = -2200\mathbf{i}$ (m/s^2). What is the angular acceleration of the crank AB?



Solution: The velocity analysis:

$$\mathbf{v}_B = \mathbf{V}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$= 0 + \omega_{AB}\mathbf{k} \times (0.05\mathbf{i} + 0.05\mathbf{j})$$

$$= (-0.05\omega_{AB}\mathbf{i} + 0.05\omega_{AB}\mathbf{j})$$

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$

$$= (-0.05\omega_{AB}\mathbf{i} + 0.05\omega_{AB}\mathbf{j}) + \omega_{BC}\mathbf{k} \times (0.175\mathbf{i} - 0.05\mathbf{j})$$

$$= (-0.05\omega_{AB} + 0.05\omega_{AB})\mathbf{i} + (0.05\omega_{AB} + 0.175\omega_{BC})\mathbf{j} = -14\mathbf{i}$$

Equating components and solving we find that $\omega_{AB} = 218$ rad/s, $\omega_{BC} = -62.2$ rad/s. The acceleration analysis:

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}{}^2\mathbf{r}_{B/A}$$

$$= 0 + \alpha_{AB}\mathbf{k} \times (0.05\mathbf{i} + 0.05\mathbf{j}) - (218)^2(0.05\mathbf{i} + 0.05\mathbf{j})$$

$$= (-0.05\alpha_{AB} - 2370)\mathbf{i} + (0.05\alpha_{AB} - 2370)\mathbf{j}$$

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}{}^2\mathbf{r}_{C/B}$$

$$= (-0.05\alpha_{AB} - 2370)\mathbf{i} + (0.05\alpha_{AB} - 2370)\mathbf{j} + \alpha_{BC}\mathbf{k} \times (0.175\mathbf{i} - 0.05\mathbf{j})$$

$$- (-62.2)^2(0.175\mathbf{i} - 0.05\mathbf{j})$$

$$= (-0.05\alpha_{AB} - 2370 + 0.05\alpha_{BC} - 678)\mathbf{i}$$

$$+ (0.05\alpha_{AB} - 2370 + 0.175\alpha_{BC} + 194)\mathbf{j}$$

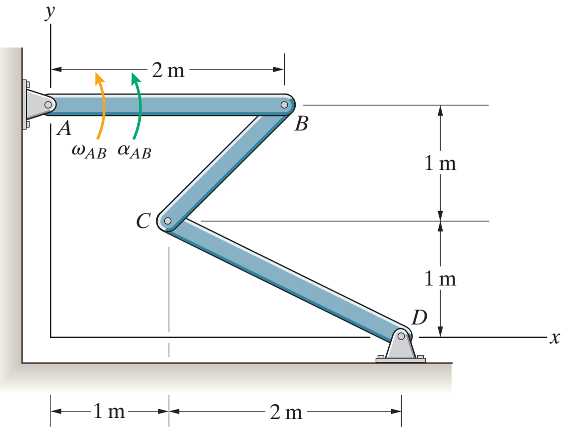
$$= -2200\mathbf{i}$$

Equating components and solving, we find

$$\alpha_{AB} = -3530 \text{ rad/s}^2, \quad \alpha_{BC} = 13{,}500 \text{ rad/s}^2$$

Thus $\boxed{\alpha_{AB} = 3530 \text{ rad/s}^2 \text{ clockwise.}}$

Problem 17.96 The angular velocity and angular acceleration of bar AB are $\omega_{AB} = 4$ rad/s and $\alpha_{AB} = -6$ rad/s^2. Determine the angular accelerations of bars BC and CD.



Solution: From 17.38 we know $\omega_{BC} = 2.67$ rad/s CCW, $\omega_{CD} = 2.67$ rad/s CW

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}{}^2\mathbf{r}_{B/A}$$

$$= 0 + (-6 \text{ rad/s}^2)\mathbf{k} \times (2 \text{ m})\mathbf{i} - (4 \text{ rad/s})^2(2 \text{ m})\mathbf{i} = (-32\mathbf{i} - 12\mathbf{j}) \text{ m/s}^2$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}{}^2\mathbf{r}_{C/B}$$

$$= (-32\mathbf{i} - 12\mathbf{j}) \text{ m/s}^2 + \alpha\mathbf{k} \times (-\mathbf{i} - \mathbf{j}) \text{ m} - (2.67 \text{ rad/s})^2(-\mathbf{i} - \mathbf{j}) \text{ m}$$

$$= [-24.9 \text{ m/s}^2 + \{1 \text{ m}\}\alpha_{BC}]\mathbf{i} + [-4.89 \text{ m/s}^2 - \{1 \text{ m}\}\alpha_{BC}]\mathbf{j}$$

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{D/C} - \omega_{CD}{}^2\mathbf{r}_{D/C}$$

$$= [-24.9 \text{ m/s}^2 + \{1 \text{ m}\}\alpha_{BC}]\mathbf{i} + [-4.89 \text{ m/s}^2 - \{1 \text{ m}\}\alpha_{BC}]\mathbf{j}$$

$$+ \alpha_{CD}\mathbf{k} \times (2\mathbf{i} - \mathbf{j}) \text{ m} - (2.67 \text{ rad/s})^2(2\mathbf{i} - \mathbf{j}) \text{ m}$$

$$= [-39.1 \text{ m/s}^2 + \{1 \text{ m}\}(\alpha_{BC} + \alpha_{CD})]\mathbf{i}$$

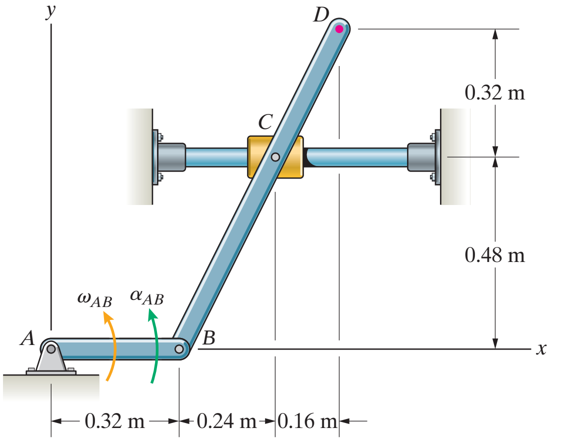
$$+ [2.22 \text{ m/s}^2 - \{1 \text{ m}\}\alpha_{BC} + \{2 \text{ m}\}\alpha_{CD}]\mathbf{j}$$

Since point D is fixed we have

$$\left.\begin{aligned} 2.22 \text{ m/s}^2 - \{1 \text{ m}\}\alpha_{BC} + \{2 \text{ m}\}\alpha_{CD} &= 0 \\ -39.1 \text{ m/s}^2 + \{1 \text{ m}\}(\alpha_{BC} + \alpha_{CD}) &= 0 \end{aligned}\right\}$$

$$\Rightarrow \boxed{\begin{aligned} \alpha_{BC} &= 26.8 \text{ rad/s}^2 \text{ CCW} \\ \alpha_{CD} &= 12.30 \text{ rad/s}^2 \text{ CCW} \end{aligned}}$$

Problem 17.97 The angular velocity and angular acceleration of bar AB are $\omega_{AB} = 2$ rad/s and $\alpha_{AB} = 8$ rad/s^2. What is the acceleration of point D?



Solution: First we must do a velocity analysis to find the angular velocity of BCD

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 0 + (2 \text{ rad/s})\mathbf{k} \times (0.32 \text{ m})\mathbf{i} = (0.64 \text{ m/s})\mathbf{j}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BCD} \times \mathbf{r}_{C/B} = (0.64 \text{ m/s})\mathbf{j} + \omega_{BCD}\mathbf{k} \times (0.24\mathbf{i} + 0.48\mathbf{j}) \text{ m}$$

$$= (-0.48 \text{ m})\omega_{BCD}\mathbf{i} + (0.64 \text{ m/s} + \{0.24 \text{ m}\}\omega_{BCD})\mathbf{j}$$

Since C cannot move in the $\mathbf{j}$ direction we know

$$0.64 \text{ m/s} + \{0.24 \text{ m}\}\omega_{BCD} = 0 \Rightarrow \omega_{BCD} = -2.67 \text{ rad/s}$$

Now we do the acceleration analysis

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}{}^2\mathbf{r}_{B/A}$$

$$= 0 + (8 \text{ rad/s}^2)\mathbf{k} \times (0.32 \text{ m})\mathbf{i} - (2 \text{ rad/s})^2(0.32 \text{ m})\mathbf{i}$$

$$= (-1.28\mathbf{i} + 2.56\mathbf{j}) \text{ m/s}^2$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}{}^2\mathbf{r}_{C/B}$$

$$= (-1.28\mathbf{i} + 2.56\mathbf{j}) \text{ m/s}^2 + \alpha_{BCD}\mathbf{k} \times (0.24\mathbf{i} + 0.48\mathbf{j}) \text{ m}$$

$$- (-2.67 \text{ rad/s})^2(0.24\mathbf{i} + 0.48\mathbf{j}) \text{ m}$$

$$= (-2.99 \text{ m/s}^2 - \{0.48 \text{ m}\}\alpha_{BCD})\mathbf{i}$$

$$+ (-0.853 \text{ m/s}^2 + \{0.24 \text{ m}\}\alpha_{BCD})\mathbf{j}$$

Since C cannot move in the $\mathbf{j}$ direction we know

$$-0.853 \text{ m/s}^2 + \{0.24 \text{ m}\}\alpha_{BCD} = 0 \Rightarrow \alpha_{BCD} = 3.56 \text{ rad/s}^2$$

Now we can find the acceleration of point D

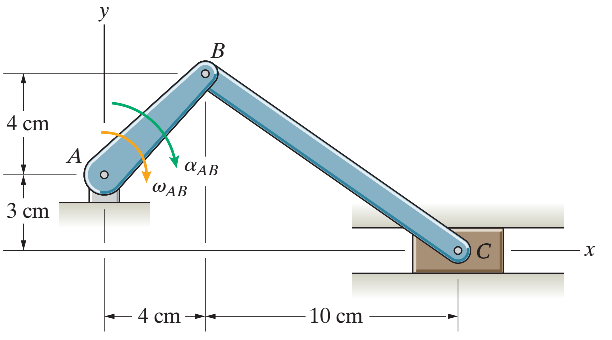
$$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha}_{BCD} \times \mathbf{r}_{D/B} - \omega_{BCD}{}^2\mathbf{r}_{D/B}$$

$$= (-1.28\mathbf{i} + 2.56\mathbf{j}) \text{ m/s}^2 + (3.56 \text{ rad/s}^2)\mathbf{k} \times (0.4\mathbf{i} + 0.8\mathbf{j}) \text{ m}$$

$$- (-2.67 \text{ rad/s})^2(0.4\mathbf{i} + 0.8\mathbf{j}) \text{ m}$$

$$\boxed{\mathbf{a}_D = (-.697\mathbf{i} - 1.71\mathbf{j}) \text{ m/s}^2}$$

Problem 17.98 The angular velocity $\omega_{AB} = 6$ rad/s. If the acceleration of the slider C is zero at the instant shown, what is the angular acceleration α_{AB}?



Solution: The velocity analysis:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$= 0 + (-6)\mathbf{k} \times (4\mathbf{i} + 4\mathbf{j})$$

$$= (24\mathbf{i} - 24\mathbf{j})$$

$$\mathbf{v}_c = \mathbf{v}_B + \boldsymbol{\omega}_{BC} + \mathbf{r}_{C/B}$$

$$= (24\mathbf{i} - 24\mathbf{j}) + \omega_{BC}\mathbf{k} \times (10\mathbf{i} - 7\mathbf{j}) = (24 + 7\omega_{BC})\mathbf{i} + (24 + 10\omega_{BC})\mathbf{j}$$

Since C cannot move in the $\mathbf{j}$ direction, we set the $\mathbf{j}$ component to zero and find that

$$\omega_{BC} = -2.4 \text{ rad/s}.$$

The acceleration analysis:

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - {\omega_{AB}}^2\mathbf{r}_{B/A}$$

$$= 0 - \alpha_{AB}\mathbf{k} \times (4\mathbf{i} + 4\mathbf{j}) - (6)^2(4\mathbf{i} + 4\mathbf{j}) = (4\alpha_{AB} - 144)\mathbf{i} + (-4\alpha_{AB} - 144)\mathbf{j}$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - {\omega_{BC}}^2\mathbf{r}_{C/B}$$

$$= (4\alpha_{AB} - 144)\mathbf{i} + (-4\alpha_{AB} - 144)\mathbf{j} + \alpha_{BC}\mathbf{k} \times (10\mathbf{i} - 7\mathbf{j}) - (-2.4)^2(10\mathbf{i} - 7\mathbf{j})$$

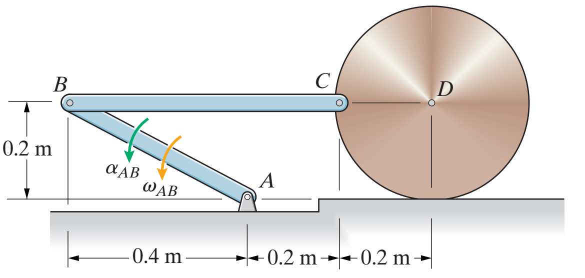
$$= (4\alpha_{AB} - 144 + 7\alpha_{BC} - 57.6)\mathbf{i} + (-4\alpha_{AB} - 144 + 10\alpha_{BC} + 40.3)\mathbf{j}$$

The acceleration of C is zero. Equating both components to zero and solving, we find that

$$\alpha_{AB} = 19.0 \text{ rad/s}^2, \alpha_{BC} = 18.0 \text{ rad/s}^2$$

Thus $\boxed{\alpha_{AB} = 19.0 \text{ rad/s}^2 \text{ clockwise.}}$

Problem 17.99 The angular velocity and angular acceleration of bar AB are $\omega_{AB} = 5$ rad/s and $\alpha_{AB} = 10$ rad/s^2. Determine the angular acceleration of bar BC.



Solution: Do a velocity analysis first to find all of the angular velocities. Let point E be the point on the wheel that is in contact with the ground.

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 0 + (5 \text{ rad/s})\mathbf{k} \times (-0.4\mathbf{i} + 0.2\mathbf{j}) \text{ m}$$

$$= (-\mathbf{i} - 2\mathbf{j}) \text{ m/s}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = (-\mathbf{i} - 2\mathbf{j}) \text{ m/s} + \omega_{BC}\mathbf{k} \times (0.6 \text{ m})\mathbf{i}$$

$$= (-1 \text{ m/s})\mathbf{i} + (-2 \text{ m/s} + \{0.6 \text{ m}\}\omega_{BC})\mathbf{j}$$

$$\mathbf{v}_E = \mathbf{v}_C + \boldsymbol{\omega}_{\text{wheel}} \times \mathbf{r}_{E/C} = (-1 \text{ m/s})\mathbf{i} + (-2 \text{ m/s} + \{0.6 \text{ m}\}\omega_{BC})\mathbf{j}$$

$$+ \omega_{\text{wheel}}\mathbf{k} \times (0.2\mathbf{i} - 0.2\mathbf{j}) \text{ m}$$

$$= (-1 \text{ m/s} + \{0.2 \text{ m}\}\omega_{\text{wheel}})\mathbf{i}$$

$$+ (-2 \text{ m/s} + \{0.6 \text{ m}\}\omega_{BC} + \{0.2 \text{ m}\}\omega_{\text{wheel}})\mathbf{j}$$

Point E is the instantaneous center of the wheel. Therefore

$$\left.\begin{array}{l} -1 \text{ m/s} + \{0.2 \text{ m}\}\omega_{\text{wheel}} = 0 \\ -2 \text{ m/s} + \{0.6 \text{ m}\}\omega_{BC} + \{0.2 \text{ m}\}\omega_{\text{wheel}} = 0 \end{array}\right\}$$

$$\Rightarrow \begin{array}{l} \omega_{BC} = 1.67 \text{ rad/s} \\ \omega_{\text{wheel}} = 5 \text{ rad/s} \end{array}$$

Now we do the acceleration analysis

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}{}^2\mathbf{r}_{B/A}$$

$$= 0 + (10 \text{ rad/s}^2)\mathbf{k} \times (-0.4\mathbf{i} + 0.2\mathbf{j}) \text{ m}$$

$$- (5 \text{ rad/s})^2(-0.4\mathbf{i} + 0.2\mathbf{j}) \text{ m} = (8\mathbf{i} - 9\mathbf{j}) \text{ m/s}^2$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}{}^2\mathbf{r}_{C/B}$$

$$= (8\mathbf{i} - 9\mathbf{j}) \text{ m/s}^2 + \alpha_{BC}\mathbf{k} \times (0.6 \text{ m})\mathbf{i} - (1.67 \text{ rad/s})^2(0.6 \text{ m})\mathbf{i}$$

$$= (6.33 \text{ m/s}^2)\mathbf{i} + (-9 \text{ m/s}^2 + \{0.6 \text{ m}\}\alpha_{BC})\mathbf{j}$$

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{\text{wheel}} \times \mathbf{r}_{D/C} - \omega_{\text{wheel}}{}^2\mathbf{r}_{D/C}$$

$$= (6.33 \text{ m/s}^2)\mathbf{i} + (-9 \text{ m/s}^2 + \{0.6 \text{ m}\}\alpha_{BC})\mathbf{j} + \alpha_{\text{wheel}}\mathbf{k} \times (0.2 \text{ m})\mathbf{i}$$

$$- (5 \text{ rad/s})^2(0.2 \text{ m})\mathbf{i}$$

$$= (1.33 \text{ m/s}^2)\mathbf{i} + (-9 \text{ m/s}^2 + \{0.6 \text{ m}\}\alpha_{BC} + \{0.2 \text{ m}\}\alpha_{\text{wheel}})\mathbf{j}$$

Finally we work down to point E

$$\mathbf{a}_E = \mathbf{a}_D + \boldsymbol{\alpha}_{\text{wheel}} \times \mathbf{r}_{E/D} - \omega_{\text{wheel}}{}^2\mathbf{r}_{E/D}$$

$$= (1.33 \text{ m/s}^2)\mathbf{i} + (-9 \text{ m/s}^2 + \{0.6 \text{ m}\}\alpha_{BC} + \{0.2 \text{ m}\}\alpha_{\text{wheel}})\mathbf{j}$$

$$+ \alpha_{\text{wheel}}\mathbf{k} \times (-0.2 \text{ m})\mathbf{j} - (5 \text{ rad/s})^2(-0.2 \text{ m})\mathbf{j}$$

$$= (1.33 \text{ m/s}^2 + \{0.2 \text{ m}\}\alpha_{\text{wheel}})\mathbf{i}$$

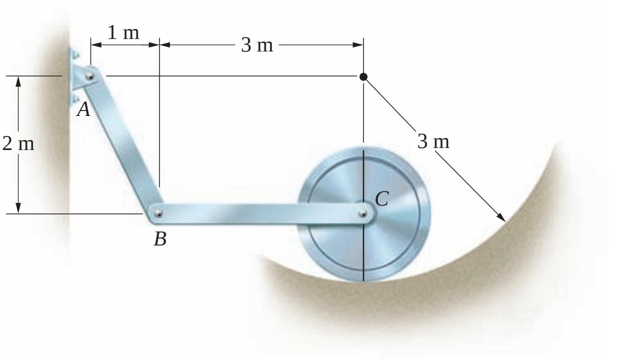
$$+ (-4 \text{ m/s}^2 + \{0.6 \text{ m}\}\alpha_{BC} + \{0.2 \text{ m}\}\alpha_{\text{wheel}})\mathbf{j}$$

D moves horizontally therefore $\mathbf{a}_D \cdot \mathbf{j} = 0$
The wheel does not slip therefore $\mathbf{a}_E \cdot \mathbf{i} = 0$
We have

$$\left.\begin{array}{l} -9 \text{ m/s}^2 + \{0.6 \text{ m}\}\alpha_{BC} + \{0.2 \text{ m}\}\alpha_{\text{wheel}} = 0 \\ 1.33 \text{ m/s}^2 + \{0.2 \text{ m}\}\alpha_{\text{wheel}} = 0 \end{array}\right\}$$

$$\Rightarrow \begin{array}{l} \alpha_{\text{wheel}} = -6.67 \text{ rad/s}^2 \\ \alpha_{BC} = 17.2 \text{ rad/s}^2 \end{array} \quad \boxed{\alpha_{BC} = 17.2 \text{ rad/s}^2 \text{ CCW}}$$

Problem 17.100 At the instant shown, bar AB is rotating at 10 rad/s in the counterclockwise direction and has a counterclockwise angular acceleration of 20 rad/s^2. The disk rolls on the circular surface. Determine the angular accelerations of bar BC and the disk.



Solution: The velocity analysis:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 0 + (10\mathbf{k}) \times (1\mathbf{i} - 2\mathbf{j}) = 20\mathbf{i} + 10\mathbf{j}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = (20\mathbf{i} + 10\mathbf{j}) + \omega_{BC}\mathbf{k} \times (3\mathbf{i}) = (20)\mathbf{i} + (10 + 3\omega_{BC})\mathbf{j}$$

$$\mathbf{v}_C = \boldsymbol{\omega}_{\text{disk}} \times \mathbf{r} = \omega_{\text{disk}}\mathbf{k} \times (1\mathbf{j}) = -\omega_{\text{disk}}\ (1)\mathbf{i}$$

Equating the components of these two expressions for $\mathbf{v}_C$ and solving, we find

$$\omega_{BC} = -3.33 \text{ rad/s}, \quad \omega_{\text{disk}} = -20 \text{ rad/s}, \quad v_C = 20 \text{ m/s}.$$

The acceleration analysis (note that point C is moving in a circle of radius 2 m):

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}{}^2\mathbf{r}$$

$$= 0 + (20\mathbf{k}) \times (1\mathbf{i} - 2\mathbf{j}) - (10)^2(1\mathbf{i} - 2\mathbf{j}) = (-60\mathbf{i} + 220\mathbf{j})$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}{}^2\mathbf{r}_{C/B}$$

$$= (-60\mathbf{i} + 220\mathbf{j}) + \alpha_{BC}\mathbf{k} \times (3\mathbf{i}) - (-3.33)^2(3\mathbf{i}) = (-93.3)\mathbf{i} + (220 + 3\alpha_{BC})\mathbf{j}$$

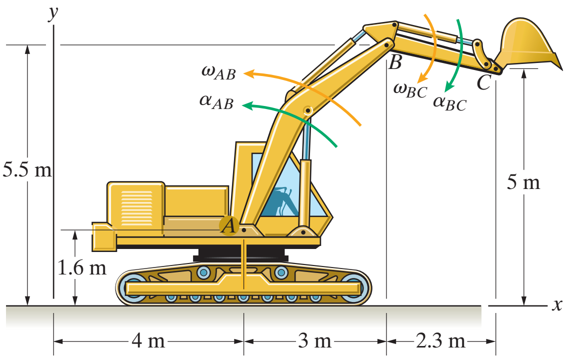
$$\mathbf{a}_C = \boldsymbol{\alpha}_{\text{disk}} \times \mathbf{r} + \frac{v_C^2}{2\text{ m}}\mathbf{j} = \alpha_{\text{disk}}\mathbf{k} \times (1\mathbf{j}) + \frac{(20)^2}{2}\mathbf{j} = -\alpha_{\text{disk}}(1)\mathbf{i} + 200\mathbf{j}$$

Equating the components of these two expressions for $\mathbf{a}_C$ and solving, we find

$\alpha_{BC} = -6.67$ rad/s^2, $\alpha_{\text{disk}} = 93.3$ rad/s^2.

$\boxed{\alpha_{BC} = 6.67 \text{ rad/s}^2 \text{ clockwise}, \ \alpha_{\text{disk}} = 93.3 \text{ rad/s}^2 \text{ counterclockwise.}}$

Problem 17.101 If $\omega_{AB} = 2$ rad/s, $\alpha_{AB} = 2$ rad/s^2, $\omega_{BC} = -1$ rad/s, and $\alpha_{BC} = -4$ rad/s^2, what is the acceleration of point C where the scoop of the excavator is attached?



Solution: The vector locations of points A, B, C are

$$\mathbf{r}_A = 4\mathbf{i} + 1.6\mathbf{j} \text{ (m)},$$

$$\mathbf{r}_B = 7\mathbf{i} + 5.5\mathbf{j} \text{ (m)}.$$

$$\mathbf{r}_C = 9.3\mathbf{i} + 5\mathbf{j} \text{ (m)}.$$

The vectors

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 3\mathbf{i} + 3.9\mathbf{j} \text{ (m)},$$

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = 2.3\mathbf{i} - 0.5\mathbf{j} \text{ (m)}.$$

The acceleration of point B is

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}.$$

$$\mathbf{a}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 3 & 3.9 & 0 \end{bmatrix} - (2^2)(3.0\mathbf{i} + 3.9\mathbf{j}),$$

$$\mathbf{a}_B = +2(-3.9\mathbf{i} + 3\mathbf{j}) - (4)(3\mathbf{i} + 3.9\mathbf{j})$$

$$= -19.8\mathbf{i} - 9.6\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration at point B is

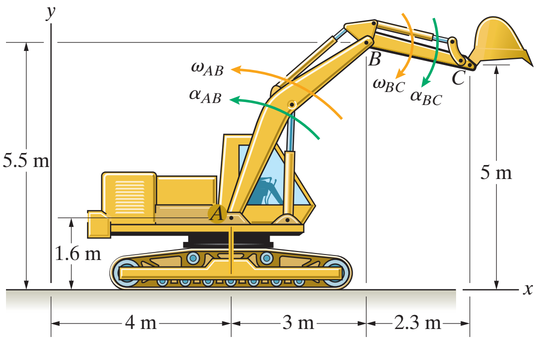
$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2(\mathbf{r}_{C/B})$$

$$= -19.8\mathbf{i} - 9.6\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -4 \\ 2.3 & -0.5 & 0 \end{bmatrix} - 1^2(2.3\mathbf{i} - 0.5\mathbf{j}),$$

$$\mathbf{a}_C = -19.8\mathbf{i} - 9.6\mathbf{j} - 2\mathbf{i} - 9.2\mathbf{j} - 2.3\mathbf{i} + 0.5\mathbf{j}$$

$$\boxed{= -24.1\mathbf{i} - 18.3\mathbf{j} \text{ (m/s}^2\text{)}}$$

Problem 17.102 If the velocity of point C of the excavator in Problem 17.101 is $\mathbf{v}_C = 4\mathbf{i}$ (m/s) and is constant, what are ω_{AB}, α_{AB}, ω_{BC}, α_{BC}?



Solution: The strategy is to determine the angular velocities ω_{AB}, ω_{BC} from the known velocity at point C, and the angular velocities α_{AB}, α_{BC} from the data that the linear acceleration at point C is constant.

The angular velocities: The vector locations of points A, B, C are

$$\mathbf{r}_A = 4\mathbf{i} + 1.6\mathbf{j} \text{ (m)},$$

$$\mathbf{r}_B = 7\mathbf{i} + 5.5\mathbf{j} \text{ (m)},$$

$$\mathbf{r}_C = 9.3\mathbf{i} + 5\mathbf{j} \text{ (m)}.$$

The vectors

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 3\mathbf{i} + 3.9\mathbf{j} \text{ (m)},$$

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = 2.3\mathbf{i} - 0.5\mathbf{j} \text{ (m)}.$$

The velocity of point B is

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 3 & 3.9 & 0 \end{bmatrix} = -3.9\omega_{AB}\mathbf{i} + 3\omega_{AB}\mathbf{j}.$$

The velocity of C in terms of the velocity of B

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$

$$= -3.9\omega_{AB}\mathbf{i} + 3\omega_{AB}\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{BC} \\ 2.3 & -0.5 & 0 \end{bmatrix},$$

$$\mathbf{v}_C = -3.9\omega_{AB}\mathbf{i} + 3\omega_{AB}\mathbf{j} - 0.5\omega_{BC}\mathbf{i} - 2.3\omega_{BC}\mathbf{j} \text{ (m/s)}.$$

Substitute $\mathbf{v}_C = 4\mathbf{i}$ (m/s), and separate components:

$$4 = -3.9\omega_{AB} - 0.5\omega_{BC},$$

$$0 = 3\omega_{AB} - 2.3\omega_{BC}.$$

Solve: $\boxed{\omega_{AB} = -0.8787 \text{ rad/s}}$, $\boxed{\omega_{BC} = -1.146 \text{ rad/s}}$.

The angular accelerations: The acceleration of point B is

$$\mathbf{a}_B = \mathbf{a}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 3 & 3.9 & 0 \end{bmatrix} - \left(\omega_{AB}^2\right)(3\mathbf{i} + 3.9\mathbf{j}),$$

$$\mathbf{a}_B = -3.9\alpha_{AB}\mathbf{i} + 3\alpha_{AB}\mathbf{j} - 3\omega_{AB}^2\mathbf{i} - 3.9\omega_{AB}^2\mathbf{j} \text{ (m/s}^2).$$

The acceleration of C in terms of the acceleration of B is

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\alpha_{BC} \\ 2.3 & -0.5 & 0 \end{bmatrix} - \omega_{BC}^2(2.3\mathbf{i} - 0.5\mathbf{j})$$

$$\mathbf{a}_C = \left(-3.9\alpha_{AB} - 3\omega_{AB}^2\right)\mathbf{i} + \left(3\alpha_{AB} - 3.9\omega_{AB}^2\right)\mathbf{j}$$

$$+ (-0.5\alpha_{BC} - 2.3\omega_{BC}^2)\mathbf{i} + \left(-2.3\alpha_{BC} + 0.5\omega_{BC}^2\right)\mathbf{j}.$$

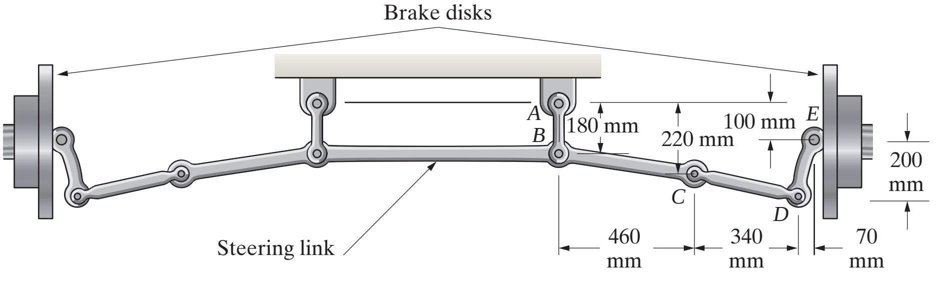
Substitute $\mathbf{a}_C = 0$ from the conditions of the problem, and separate components:

$$0 = -3.9\alpha_{AB} - 0.5\alpha_{BC} - 3\omega_{AB}^2 - 2.3\omega_{BC}^2,$$

$$0 = 3\alpha_{AB} - 2.3\alpha_{BC} - 3.9\omega_{AB}^2 + 0.5\omega_{BC}^2.$$

Solve: $\boxed{\alpha_{BC} = -2.406 \text{ rad/s}^2}$, $\boxed{\alpha_{AB} = -1.06 \text{ rad/s}^2.}$

Problem 17.103 The steering linkage of a car is shown. Member DE rotates about the fixed pin E. The right brake disk is rigidly attached to member DE. The tie rod CD is pinned at C and D. At the instant shown, the Pitman arm AB has a counterclockwise angular velocity of 1 rad/s and a clockwise angular acceleration of 2 rad/s^2. What is the angular acceleration of the right brake disk?



Solution: Note that the steering link translates, but does not rotate. The velocity analysis:

$$\mathbf{v}_C = \mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 0 + (1\mathbf{k}) \times (-0.18\mathbf{j}) = 0.18\mathbf{i}$$

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = 0.18\mathbf{i} + \omega_{CD}\mathbf{k} \times (0.34\mathbf{i} - 0.08\mathbf{j})$$

$$= (0.18 + 0.08\omega_{CD})\mathbf{i} + (0.34\omega_{CD})\mathbf{j}$$

$$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{DE} \times \mathbf{r}_{E/D}$$

$$= (0.18 + 0.08\omega_{CD})\mathbf{i} + (0.34\omega_{CD})\mathbf{j} + \omega_{DE}\mathbf{k} \times (0.07\mathbf{i} + 0.2\mathbf{j})$$

$$= (0.18 + 0.08\omega_{CD} - 0.2\omega_{DE})\mathbf{i} + (0.34\omega_{CD} + 0.07\omega_{DE})\mathbf{j}$$

Since point E is fixed, we can set both components to zero and solve. We find

$$\omega_{CD} = -0.171 \text{ rad/s}, \quad \omega_{DE} = 0.832 \text{ rad/s}.$$

The acceleration analysis:

$$\mathbf{a}_C = \mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{C/B} - \omega_{AB}{}^2\mathbf{r}_{B/A}$$

$$= 0 + (-2\mathbf{k}) \times (-0.18\mathbf{j}) - (1)^2(-0.18\mathbf{j}) = (-0.36\mathbf{i} + 0.18\mathbf{j})$$

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{D/C} - \omega_{CD}{}^2\mathbf{r}_{D/C}$$

$$= (-0.36\mathbf{i} + 0.18\mathbf{j}) + \alpha_{CD}\mathbf{k} \times (0.34\mathbf{i} - 0.08\mathbf{j}) - (-0.171)^2(0.34\mathbf{i} - 0.08\mathbf{j})$$

$$= (-0.370 + 0.08\alpha_{CD})\mathbf{i} + (0.182 + 0.34\alpha_{CD})\mathbf{j}$$

$$\mathbf{a}_E = \mathbf{a}_D + \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{E/D} - \omega_{DE}{}^2\mathbf{r}_{E/D}$$

$$= (-0.370 + 0.08\alpha_{CD})\mathbf{i} + (0.182 + 0.34\alpha_{CD})\mathbf{j}$$

$$+ \alpha_{DE}\mathbf{k} \times (0.07\mathbf{i} + 0.2\mathbf{j}) - (0.832)^2(0.07\mathbf{i} + 0.2\mathbf{j})$$

$$= (-.418 + 0.08\alpha_{CD} - 0.2\alpha_{DE})\mathbf{i} + (0.0436 + 0.34\alpha_{CD} + 0.07\alpha_{DE})\mathbf{j}$$

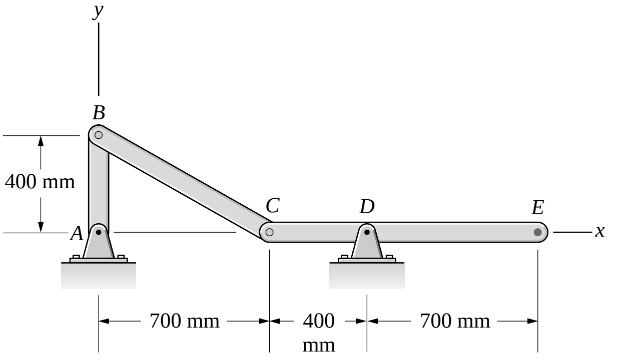
Since point E cannot move, we set both components of $\mathbf{a}_E$ to zero and solve. We find:

$$\alpha_{CD} = 0.278 \text{ rad/s}^2, \quad \alpha_{DE} = -1.98 \text{ rad/s}^2.$$

The angular acceleration of the right brake is the same as the angular acceleration of DE.

$$\boxed{\alpha_{\text{right brake}} = 1.98 \text{ rad/s}^2 \text{ clockwise.}}$$

Problem 17.104 At the instant shown, bar AB has no angular velocity but has a counterclockwise angular acceleration of 10 rad/s^2. Determine the acceleration of point E.



Solution: The vector locations of A, B, C and D are: $\mathbf{r}_A = 0$, $\mathbf{r}_B = 400\mathbf{j}$ (mm), $\mathbf{r}_C = 700\mathbf{i}$ (mm), $\mathbf{r}_D = 1100\mathbf{i}$ (mm). $\mathbf{r}_E = 1800\mathbf{i}$ (mm) The vectors

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 400\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = 700\mathbf{i} - 400\mathbf{j} \text{ (mm)},$$

$$\mathbf{r}_{C/D} = \mathbf{r}_C - \mathbf{r}_D = -400\mathbf{i} \text{ (mm)}$$

(a) *Get the angular velocities* ω_{BC}, ω_{CD}. The velocity of point B is zero. The velocity of C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 700 & -400 & 0 \end{bmatrix}$$

$$= +400\omega_{BC}\mathbf{i} + 700\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of point D

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -400 & 0 & 0 \end{bmatrix}$$

$$= -400\omega_{CD}\mathbf{j} \text{ (mm/s)}.$$

Equate the expressions for $\mathbf{v}_C$ and separate components: $400\omega_{BC} = 0$, $700\omega_{BC} = -400\omega_{CD}$. Solve: $\omega_{BC} = 0$ rad/s, $\omega_{CD} = 0$ rad/s.

(b) *Get the angular accelerations*. The acceleration of point B is

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 0 & 400 & 0 \end{bmatrix}$$

$$= -4000\mathbf{i} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point B:

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= -4000\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 700 & -400 & 0 \end{bmatrix}.$$

$$\mathbf{a}_C = -4000\mathbf{i} + 400\alpha_{BC}\mathbf{i} + 700\alpha_{BC}\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point D:

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ -400 & 0 & 0 \end{bmatrix}$$

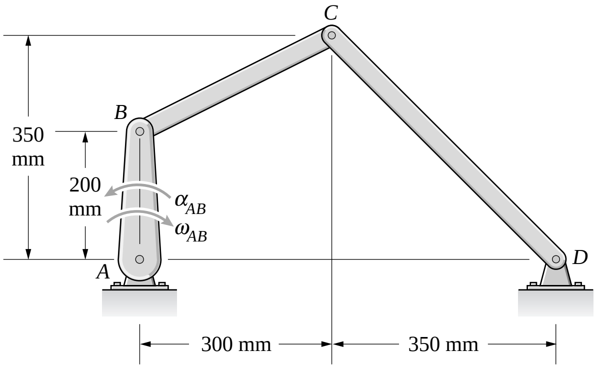
$$= -400\alpha_{CD}\mathbf{j} \text{ (mm/s}^2\text{)}.$$

Equate the expressions and separate components: $-4000 + 400\alpha_{BC} = 0$, $700\alpha_{BC} = -400\alpha_{CD}$.
Solve: $\alpha_{BC} = 10$ rad/s^2, $\alpha_{CD} = -17.5$ rad/s^2, The acceleration of point E in terms of the acceleration of point D is

$$\mathbf{a}_E = \alpha_{CD} \times \mathbf{r}_{E/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -17.5 \\ 700 & 0 & 0 \end{bmatrix}$$

$$\boxed{= -12250\mathbf{j} \text{ (mm/s}^2\text{)}} \text{ (clockwise)}$$

Problem 17.105 If $\omega_{AB} = 12$ rad/s and $\alpha_{AB} = 100$ rad/s^2, what are the angular accelerations of bars BC and CD?



Solution: The vector locations of A, B, C and D are: $\mathbf{r}_A = 0$, $\mathbf{r}_B = 200\mathbf{j}$ (mm), $\mathbf{r}_C = 300\mathbf{i} + 350\mathbf{j}$ (mm), $\mathbf{r}_D = 650\mathbf{i}$ (mm). The vectors

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 200\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = 300\mathbf{i} + 150\mathbf{j} \text{ (mm)},$$

$$\mathbf{r}_{C/D} = \mathbf{r}_C - \mathbf{r}_D = -350\mathbf{i} + 350\mathbf{j} \text{ (mm)}$$

(a) *Get the angular velocities* ω_{BC}, ω_{CD}. The velocity of point B is

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -12 \\ 0 & 200 & 0 \end{bmatrix} = 2400\mathbf{i} \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 300 & 150 & 0 \end{bmatrix}$$

$$= 2400\mathbf{i} - 150\omega_{BC}\mathbf{i} + 300\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of point D

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -350 & 350 & 0 \end{bmatrix}$$

$$= -350\omega_{CD}\mathbf{i} - 350\omega_{CD}\mathbf{j} \text{ (mm/s)}.$$

Equate the expressions for $\mathbf{v}_C$ and separate components: $2400 - 150\omega_{BC} = -350\omega_{CD}$, $300\omega_{BC} = -350\omega_{CD}$. Solve: $\omega_{BC} = 5.33$ rad/s, $\omega_{CD} = -4.57$ rad/s.

(b) *Get the angular accelerations*. The acceleration of point B is

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 100 \\ 0 & 200 & 0 \end{bmatrix}$$

$$- \omega_{AB}^2(200\mathbf{j})$$

$$= -20{,}000\mathbf{i} - 28{,}800\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point B:

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 300 & 150 & 0 \end{bmatrix} - \omega_{BC}^2(300\mathbf{i} + 150\mathbf{j}).$$

$$\mathbf{a}_C = (-20{,}000 - 150\alpha_{BC} - 300\omega_{BC}^2)\mathbf{i}$$

$$+ (-28{,}800 + 300\alpha_{BC} - 150\omega_{BC}^2)\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point D:

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ -350 & 350 & 0 \end{bmatrix} - \omega_{CD}^2(-350\mathbf{i} + 350\mathbf{j})$$

$$\mathbf{a}_C = -350\alpha_{CD}\mathbf{i} - 350\alpha_{CD}\mathbf{j} + 350\omega_{CD}^2\mathbf{i} - 350\omega_{CD}^2\mathbf{j} \text{ (mm/s}^2\text{)}.$$

Equate the expressions and separate components:

$$-20{,}000 - 150\alpha_{BC} - 300\omega_{BC}^2 = -350\alpha_{CD} + 350\omega_{CD}^2,$$

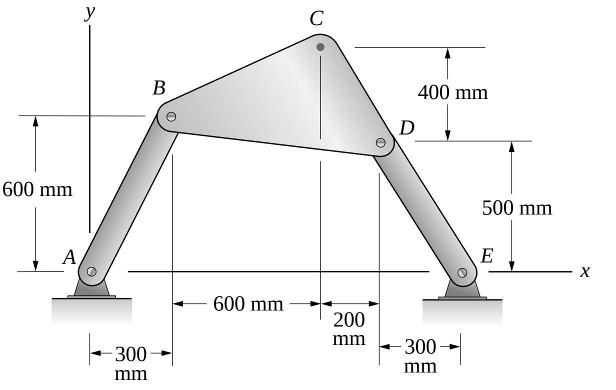
$$-28{,}800 + 300\alpha_{BC} - 150\omega_{BC}^2 = -350\alpha_{CD} - 350\omega_{CD}^2.$$

Solve: $\boxed{\alpha_{BC} = -22.43 \text{ rad/s}^2}$,

$\boxed{\alpha_{CD} = 92.8 \text{ rad/s}^2}$,

where the negative sign means a clockwise acceleration.

Problem 17.106 If $\omega_{AB} = 4$ rad/s counterclockwise and $\alpha_{AB} = 12$ rad/s^2 counterclockwise, what is the acceleration of point C?



Solution: The velocity of B is

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0.3 & 0.6 & 0 \end{vmatrix}$$

$$= -0.6\omega_{AB}\mathbf{i} + 0.3\omega_{AB}\mathbf{j}.$$

The velocity of D is

$$\mathbf{v}_D = \mathbf{v}_B + \omega_{BD} \times \mathbf{r}_{D/B}$$

$$= -0.6\omega_{AB}\mathbf{i} + 0.3\omega_{AB}\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BD} \\ 0.8 & -0.1 & 0 \end{vmatrix}. \quad \textbf{(1)}$$

We can also express the velocity of D as

$$\mathbf{v}_D = \mathbf{v}_E + \omega_{DE} \times \mathbf{r}_{D/E} = \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DE} \\ -0.3 & 0.5 & 0 \end{vmatrix}. \quad \textbf{(2)}$$

Equating **i** and **j** components in Eqns. (1) and (2), we obtain

$$-0.6\omega_{AB} + 0.1\omega_{BD} = -0.5\omega_{DE}, \quad \textbf{(3)}$$

$$0.3\omega_{AB} + 0.8\omega_{BD} = -0.3\omega_{DE}. \quad \textbf{(4)}$$

Solving these two eqns with $\omega_{AB} = 4$ rad/s, we obtain

$$\omega_{BD} = -3.57 \text{ rad/s}, \quad \omega_{DE} = 5.51 \text{ rad/s}.$$

The acceleration of B is

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 0.3 & 0.6 & 0 \end{vmatrix} - \omega_{AB}^2(0.3\mathbf{i} + 0.6\mathbf{j})$$

$$= (-0.6\alpha_{AB} - 0.3\omega_{AB}^2)\mathbf{i} + (0.3\alpha_{AB} - 0.6\omega_{AB}^2)\mathbf{j}.$$

The acceleration of D is

$$\mathbf{a}_D = \mathbf{a}_B + \alpha_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B}$$

$$= (-0.6\alpha_{AB} - 0.3\omega_{AB}^2)\mathbf{i} + (0.3\alpha_{AB} - 0.6\omega_{AB}^2)\mathbf{j}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BD} \\ 0.8 & -0.1 & 0 \end{vmatrix} - \omega_{BD}^2(0.8\mathbf{i} - 0.1\mathbf{j}). \quad \textbf{(5)}$$

We can also express the acceleration of D as

$$\mathbf{a}_D = \mathbf{a}_E + \alpha_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 \mathbf{r}_{D/E}$$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{DE} \\ -0.3 & 0.5 & 0 \end{vmatrix} - \omega_{DE}^2(-0.3\mathbf{i} + 0.5\mathbf{j}). \quad \textbf{(6)}$$

Equating **i** and **j** components in Eqns. (5) and (6), we obtain

$$-0.6\alpha_{AB} - 0.3\omega_{AB}^2 + 0.1\alpha_{BD} - 0.8\omega_{BD}^2$$

$$= -0.5\alpha_{DE} + 0.3\omega_{DE}^2, \quad \textbf{(7)}$$

$$0.3\alpha_{AB} - 0.6\omega_{AB}^2 + 0.8\alpha_{BD} + 0.1\omega_{BD}^2$$

$$= -0.3\alpha_{DE} - 0.5\omega_{DE}^2. \quad \textbf{(8)}$$

Solving these two eqns with $\alpha_{AB} = 12$ rad/s^2, we obtain

$$\alpha_{BD} = -39.5 \text{ rad/s}^2.$$

The acceleration of C is

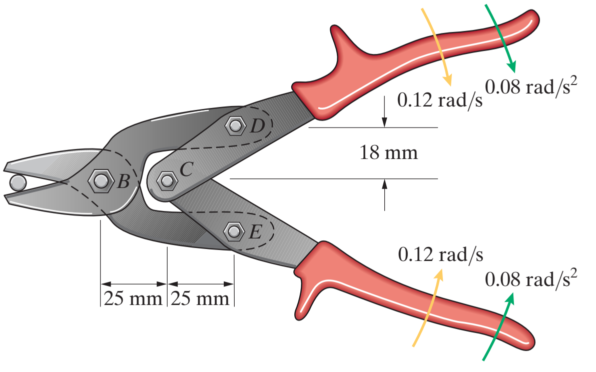
$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BD} \times \mathbf{r}_{C/B} - \omega_{BD}^2 \mathbf{r}_{C/B}$$

$$= (-0.6\alpha_{AB} - 0.3\omega_{AB}^2)\mathbf{i} + (0.3\alpha_{AB} - 0.6\omega_{AB}^2)\mathbf{j}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BD} \\ 0.6 & 0.3 & 0 \end{vmatrix} - \omega_{BD}^2(0.6\mathbf{i} + 0.3\mathbf{j}). \quad \textbf{(9)}$$

$$\mathbf{a}_C = -7.78\mathbf{i} - 33.5\mathbf{j} \text{ (m/s}^2).$$

Problem 17.107 The angular velocities and angular accelerations of the grips of the shears are shown. What is the resulting angular acceleration of the jaw BD?



Solution: From 17.57 we know that $\omega_{BD} = -0.06$ rad/s

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{D/C} - \omega_{CD}{}^2\mathbf{r}_{D/C}$$

$$= 0 - (0.08 \text{ rad/s}^2)\mathbf{k} \times (0.025\mathbf{i} + 0.018\mathbf{j}) \text{ m}$$

$$- (0.12 \text{ rad/s})^2(0.025\mathbf{i} + 0.018\mathbf{j}) \text{ m}$$

$$= (0.00108\mathbf{i} - 0.00226\mathbf{j}) \text{ m/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_D + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{B/D} - \omega_{BD}{}^2\mathbf{r}_{B/D}$$

$$= (0.00108\mathbf{i} - 0.00226\mathbf{j}) \text{ m/s}^2 + \alpha_{BD}\mathbf{k} \times (-0.05\mathbf{i} - 0.018\mathbf{j}) \text{ m}$$

$$- (-0.06 \text{ rad/s})^2(-0.05\mathbf{i} - 0.018\mathbf{j}) \text{ m}$$

$$= (0.00126 \text{ m/s}^2 + \{0.018 \text{ m}\}\alpha_{BD})\mathbf{i}$$

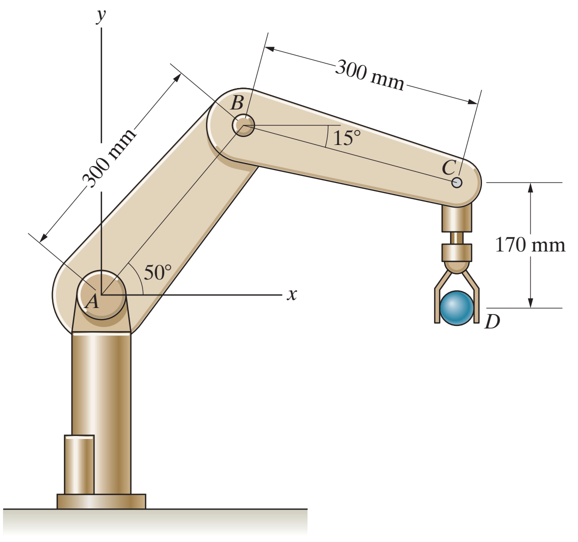
$$+ (-0.00219 \text{ m/s}^2 - \{0.05 \text{ m}\}\alpha_{BD})\mathbf{j}$$

From symmetry, B cannot accelerate in the $\mathbf{j}$ direction. Therefore

$$-0.00219 \text{ m/s}^2 - \{0.05 \text{ m}\}\alpha_{BD} = 0 \Rightarrow \alpha_{BD} = -0.0439 \text{ rad/s}^2$$

$$\boxed{\alpha_{BD} = 0.0439 \text{ rad/s}^2 \text{ CW}}$$

Problem 17.108 If arm AB has a constant clockwise angular velocity of 0.8 rad/s, arm BC has a constant angular velocity of 0.2 rad/s, and arm CD remains vertical, what is the acceleration of part D?



Solution: The constraint that the arm CD remain vertical means that the angular velocity of arm CD is zero. This implies that arm CD translates only, and in a translating, non-rotating element the velocity and acceleration at any point is the same, and the velocity and acceleration of arm CD is the velocity and acceleration of point C. The vectors:

$$\mathbf{r}_{B/A} = 300(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 192.8\mathbf{i} + 229.8\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{C/B} = 300(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 289.78\mathbf{i} - 77.6\mathbf{j} \text{ (mm)}.$$

The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2 \mathbf{r}_{A/B} = -\omega_{AB}^2(192.8\mathbf{i} + 229.8\mathbf{j}) \text{ (mm/s}^2\text{)},$$

since $\alpha_{AB} = 0$. $\mathbf{a}_B = -123.4\mathbf{i} - 147.1\mathbf{j}$ (mm/s). The acceleration of C in terms of the acceleration of B is

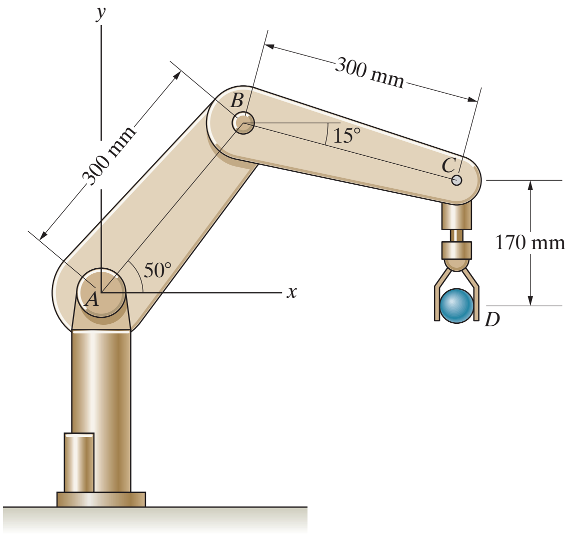
$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$= -123.4\mathbf{i} - 147.1\mathbf{j} - \omega_{BC}^2(289.8\mathbf{i} - 77.6\mathbf{j}),$$

since $\boldsymbol{\alpha}_{BC} = 0$. $\mathbf{a}_C = -135\mathbf{i} - 144\mathbf{j}$ (mm/s^2). Since CD is translating:

$$\boxed{\mathbf{a}_D = \mathbf{a}_C = -135\mathbf{i} - 144\mathbf{j} \text{ (mm/s}^2\text{)}}$$

Problem 17.109 In Problem 17.108, if arm AB has a constant clockwise angular velocity of 0.8 rad/s and you want D to have zero velocity and acceleration, what are the necessary angular velocities and angular accelerations of arms BC and CD?



Solution: Except for numerical values, the solution follows the same strategy as the solution strategy for Problem 17.105. The vectors:

$$\mathbf{r}_{B/A} = 300(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 192.8\mathbf{i} + 229.8\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{C/B} = 300(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 289.78\mathbf{i} - 77.6\mathbf{j} \text{ (mm)},$$

$$\mathbf{r}_{C/D} = 170\mathbf{j} \text{ (mm)}.$$

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.8 \\ 192.8 & 229.8 & 0 \end{bmatrix}$$

$$= 183.8\mathbf{i} - 154.3\mathbf{j} \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 289.8 & -77.6 & 0 \end{bmatrix}.$$

$$\mathbf{v}_C = 183.9\mathbf{i} - 154.3\mathbf{j} + \omega_{BC}(77.6\mathbf{i} + 289.8\mathbf{j}) \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of point D:

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 0 & 170 & 0 \end{bmatrix} = -170\omega_{CD}\mathbf{i} \text{ (mm/s)}.$$

Equate the expressions for $\mathbf{v}_C$ and separate components:

$$183.9 + 77.6\omega_{BC} = -170\omega_{CD},$$

$$-154.3 + 289.8\omega_{BC} = 0.$$

Solve: $\boxed{\omega_{BC} = 0.532 \text{ rad/s}}$, $\boxed{\omega_{CD} = -1.325 \text{ rad/s}}$.

Get the angular accelerations. The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = -\omega_{AB}^2(192.8\mathbf{i} + 229.8\mathbf{j})$$

$$= -123.4\mathbf{i} - 147.1\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point B:

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B} = \mathbf{a}_B - \omega_{BC}^2\mathbf{r}_{C/B}.$$

$$\mathbf{a}_C = -123.4\mathbf{i} - 147.1\mathbf{j} + 77.6\alpha_{BC}\mathbf{i} + 289.8\alpha_{BC}\mathbf{j} - 289.8\omega_{BC}^2\mathbf{i}$$

$$+ 77.6\omega_{BC}^2\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C in terms of the acceleration of point D:

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ 0 & 170 & 0 \end{bmatrix} - \omega_{CD}^2(170\mathbf{j}).$$

$$\mathbf{a}_C = -170\alpha_{CD}\mathbf{i} - 170\omega_{CD}^2\mathbf{j} \text{ (mm/s}^2\text{)}.$$

Equate the expressions and separate components:

$$-123.4 + 77.6\alpha_{BC} - 289.8\omega_{BC}^2 = -170\alpha_{CD},$$

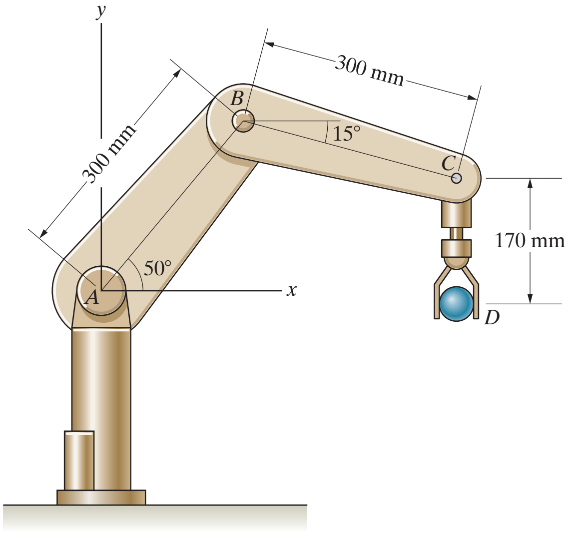
$$-147.1 + 289.8\alpha_{BC} + 77.6\omega_{BC}^2 = -170\omega_{CD}^2.$$

Solve:

$\boxed{\alpha_{BC} = -0.598 \text{ rad/s}^2}$, $\boxed{\alpha_{CD} = 1.482 \text{ rad/s}^2}$,

where the negative sign means a clockwise angular acceleration.

Problem 17.110 In Problem 17.108, if you want arm CD to remain vertical and you want part D to have velocity $\mathbf{v}_D = 1.0\mathbf{i}$ (m/s) and zero acceleration, what are the necessary angular velocities and angular accelerations of arms AB and BC?



Solution: The constraint that CD remain vertical with zero acceleration means that every point on arm CD is translating, without rotation, at a velocity of 1 m/s. This means that the velocity of point C is $\mathbf{v}_C = 1.0\mathbf{i}$ (m/s), and the acceleration of point C is zero. The vectors:

$$\mathbf{r}_{B/A} = 300(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 192.8\mathbf{i} + 229.8\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{C/B} = 300(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 289.78\mathbf{i} - 77.6\mathbf{j} \text{ (mm)}.$$

The angular velocities of AB and BC: The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 192.8 & 229.8 & 0 \end{bmatrix}$$

$$= \omega_{AB}(-229.8\mathbf{i} + 192.8\mathbf{j}) \text{ (mm/s)}.$$

The velocity of C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 289.8 & -77.6 & 0 \end{bmatrix}.$$

$$\mathbf{v}_C = -229.8\omega_{AB}\mathbf{i} + 192.8\omega_{AB}\mathbf{j} + \omega_{BC}(77.6\mathbf{i} + 289.8\mathbf{j}) \text{ (mm/s)}.$$

The velocity of C is known, $\mathbf{v}_C = 1000\mathbf{i}$ (mm/s). Equate the expressions for $\mathbf{v}_C$ and separate components: $1000 = -229.8\omega_{AB} + 77.6\omega_{BC}$, $0 = 192.8\omega_{AB} + 289.8\omega_{BC}$. Solve:

$\boxed{\omega_{AB} = -3.55 \text{ rad/s}}$, $\boxed{\omega_{BC} = 2.36 \text{ rad/s}}$,

where the negative sign means a clockwise angular velocity.

The accelerations of AB and BC: The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 192.8 & 229.8 & 0 \end{bmatrix}$$

$$-\,\omega_{AB}^2(192.8\mathbf{i} + 229.8\mathbf{j} \text{ (mm/s}^2).$$

$$\mathbf{a}_B = \alpha_{AB}(-229.8\mathbf{i} + 192.8\mathbf{j}) - \omega_{AB}^2(192.8\mathbf{i} + 229.8\mathbf{j}) \text{ (mm/s}^2)$$

The acceleration of C in terms of the acceleration of B is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 289.8 & -77.6 & 0 \end{bmatrix} - \omega_{BC}^2(289.8\mathbf{i} - 77.6\mathbf{j}),$$

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC}(77.6\mathbf{i} + 289.8\mathbf{j}) - \omega_{BC}^2(289.8\mathbf{i} - 77.6\mathbf{j}) \text{ (mm/s}^2).$$

The acceleration of point C is known to be zero. Substitute this value for $\mathbf{a}_C$, and separate components:

$$-229.8\alpha_{AB} - 192.8\omega_{AB}^2 + 77.6\alpha_{BC} - 289.8\omega_{BC}^2 = 0,$$

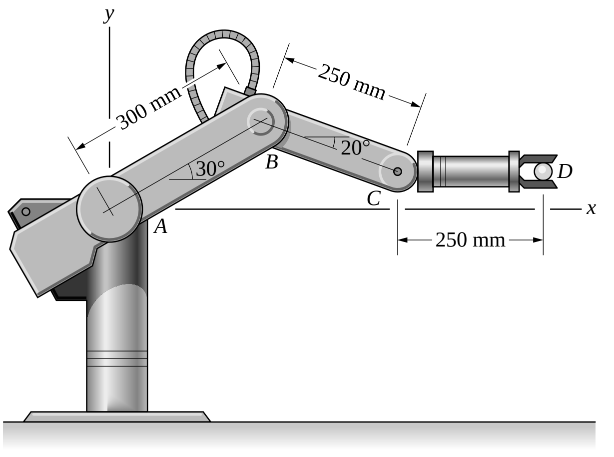
$$192.8\alpha_{AB} - 229.8\omega_{AB}^2 + 289.8\alpha_{BC} + 77.6\omega_{BC}^2 = 0.$$

Solve:

$\boxed{\alpha_{AB} = -12.1 \text{ rad/s}^2}$, $\boxed{\alpha_{BC} = 16.5 \text{ rad/s}^2}$,

where the negative sign means a clockwise angular acceleration.

Problem 17.111 Link AB of the robot's arm is rotating with a constant counterclockwise angular velocity of 2 rad/s, and link BC is rotating with a constant clockwise angular velocity of 3 rad/s. Link CD is rotating at 4 rad/s in the counterclockwise direction and has a counterclockwise angular acceleration of 6 rad/s^2. What is the acceleration of point D?



Solution: The acceleration of B is $\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$. Evaluating, we get

$$\mathbf{a}_B = 0 + 0 - (2)^2(0.3\cos 30^\circ \mathbf{i} + 0.3\sin 30^\circ \mathbf{j})$$

$$= -1.039\mathbf{i} - 0.600\mathbf{j} \text{ (m/s}^2).$$

The acceleration of C is $\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$. Evaluating, we get

$$\mathbf{a}_C = -1.039\mathbf{i} - 0.600\mathbf{j} - (3)^2(0.25\cos 20^\circ \mathbf{i} - 0.25\sin 20^\circ \mathbf{j})$$

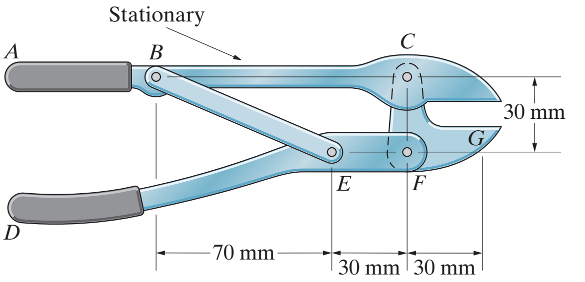
$$= -3.154\mathbf{i} + 0.170\mathbf{j} \text{ (m/s}^2).$$

The acceleration of D is $\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{D/C} - \omega_{CD}^2 \mathbf{r}_{D/C}$. Evaluating, we get

$$\mathbf{a}_D = -3.154\mathbf{i} + 0.170\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 0.25 & 0 & 0 \end{vmatrix} - (4)^2(0.25\mathbf{i})$$

$$= -7.154\mathbf{i} + 1.67\mathbf{j} \text{ (m/s}^2)$$

Problem 17.112 The upper grip and jaw of the pliers ABC is stationary. The lower grip DEF is rotating in the clockwise direction with a constant angular velocity of 0.2 rad/s. At the instant shown, what is the angular acceleration of the lower jaw CFG?



Solution: From 17.42 we know that $\omega_{BE} = \omega_{CFG} = 0.0857$ rad/s.

$$\mathbf{a}_E = \mathbf{a}_B + \boldsymbol{\alpha}_{BE} \times \mathbf{r}_{E/B} - {\omega_{BE}}^2 \mathbf{r}_{E/B}$$

$$= 0 + \alpha_{BE}\mathbf{k} \times (0.07\mathbf{i} - 0.03\mathbf{j}) \text{ m} - (0.0857 \text{ rad/s})^2 (0.07\mathbf{i} - 0.03\mathbf{j}) \text{ m}$$

$$= (-0.000514 \text{ m/s}^2 + \{0.03 \text{ m}\}\alpha_{BE})\mathbf{i}$$

$$+ (0.0002204 \text{ m/s}^2 + \{0.07 \text{ m}\}\alpha_{BE})\mathbf{j}$$

$$\mathbf{a}_F = \mathbf{a}_E + \boldsymbol{\alpha}_{EF} \times \mathbf{r}_{F/E} - {\omega_{EF}}^2 \mathbf{r}_{F/E}$$

$$= (-0.000514 \text{ m/s}^2 + \{0.03 \text{ m}\}\alpha_{BE})\mathbf{i}$$

$$+ (0.0002204 \text{ m/s}^2 + \{0.07 \text{ m}\}\alpha_{BE})\mathbf{j} + 0$$

$$- (0.0857 \text{ rad/s})^2 (0.03 \text{ m})\mathbf{i}$$

$$= (-0.00171 \text{ m/s}^2 + \{0.03 \text{ m}\}\alpha_{BE})\mathbf{i}$$

$$+ (0.0002204 \text{ m/s}^2 + \{0.07 \text{ m}\}\alpha_{BE})\mathbf{j}$$

$$\mathbf{a}_C = \mathbf{a}_F + \boldsymbol{\alpha}_{CFG} \times \mathbf{r}_{C/F} - {\omega_{CFG}}^2 \mathbf{r}_{C/F}$$

$$= (-0.00171 \text{ m/s}^2 + \{0.03 \text{ m}\}\alpha_{BE})\mathbf{i}$$

$$+ (0.0002204 \text{ m/s}^2 + \{0.07 \text{ m}\}\alpha_{BE})\mathbf{j} + \alpha_{CFG}\mathbf{k} \times (0.03 \text{ m})\mathbf{j}$$

$$- (0.0857 \text{ rad/s})^2 (0.03 \text{ m})\mathbf{j}$$

$$= (-0.00171 \text{ m/s}^2 + \{0.03 \text{ m}\}[\alpha_{BE} - \alpha_{CFG}])\mathbf{i} + (0.07 \text{ m})\alpha_{BE}\mathbf{j}$$

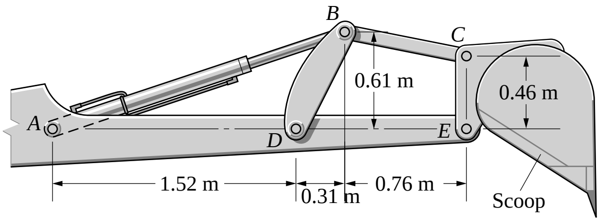
Since C is fixed we have

$$\left.\begin{array}{l} -0.00171 \text{ m/s}^2 + \{0.03 \text{ m}\}[\alpha_{BE} - \alpha_{CFG}] = 0 \\ (0.07 \text{ m})\alpha_{BE} = 0 \end{array}\right\}$$

$$\Rightarrow \begin{array}{l} \alpha_{BE} = 0 \\ \alpha_{CFG} = -0.0571 \text{ rad/s}^2 \end{array}$$

$$\boxed{\begin{array}{r} \alpha_{BE} = 0 \\ \alpha_{CFG} = 0.0571 \text{ rad/s}^2 \text{ CW} \end{array}}$$

Problem 17.113 The horizontal member ADE supporting the scoop is stationary. If the link BD has a clockwise angular velocity of 1 rad/s and a counterclockwise angular acceleration of 2 rad/s^2, what is the angular acceleration of the scoop?



Solution: The velocity of B is

$$\mathbf{v}_B = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1 \\ 0.31 & 0.61 & 0 \end{vmatrix}$$

$$= 0.61\mathbf{i} - 0.31\mathbf{j} \text{ (m/s)}.$$

The velocity of C is

$$\mathbf{v}_c = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = 0.61\mathbf{i} - 0.31\mathbf{j} + 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0.76 & -0.15 & 0 \end{vmatrix} \quad \textbf{(1)}$$

We can also express $\mathbf{v}_c$ as

$$\mathbf{v}_c = \mathbf{v}_E + \boldsymbol{\omega}_{CE} \times \mathbf{r}_{C/E} = 0 + (\omega_{CE}\mathbf{k}) \times (0.46\,\mathbf{j}) = -0.46\,\omega_{CE}\mathbf{i}. \quad \textbf{(2)}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in Equations (1) and (2) we get $0.61 + 0.15\omega_{BC} = -0.46\,\omega_{CE}$, and $-0.31 + 0.76\omega_{BC} = 0$. Solving, we obtain $\omega_{BC} = 0.400$ rad/s and $\omega_{CE} = -1.467$ rad/s.

The acceleration of B is

$$\mathbf{a}_B = \mathbf{a}_D + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{B/D} - \omega_{BD}^2 \mathbf{r}_{B/D},$$

or $$\mathbf{a}_B = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0.31 & 0.61 & 0 \end{vmatrix} - (1)^2\,(0.31\mathbf{i} + 0.61\mathbf{j})$$

$$= -1.52\,\mathbf{i} \text{ (m/s}^2\text{)}.$$

The acceleration of C is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$\mathbf{a}_C = -1.52\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 0.76 & -0.15 & 0 \end{vmatrix} - (0.4)^2(0.76\mathbf{i} - 0.15\mathbf{j}). \quad \textbf{(3)}$$

We can also express $\mathbf{a}_C$ as

$$\mathbf{a}_C = \mathbf{a}_E + \boldsymbol{\alpha}_{CE} \times \mathbf{r}_{C/E} - \omega_{CE}^2 \mathbf{r}_{C/E} = 0 + (\alpha_{CE}\mathbf{k})$$

$$\times (0.46\,\mathbf{j}) - (-1.467)^2(0.46\mathbf{j})$$

$$= -0.46\,\alpha_{CE}\mathbf{i} - 0.98\mathbf{j}. \quad \textbf{(4)}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in Equations (3) and (4), we get

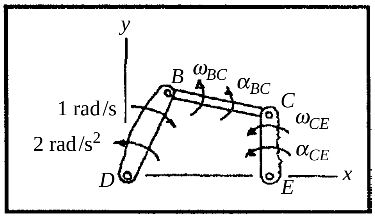
$$-1.52 + 0.15\alpha_{BC} - (0.4)^2(0.76) = -0.46\,\alpha_{CE},$$

and $$0.76\alpha_{BC} + (0.4)^2\,(0.15) = -0.98.$$

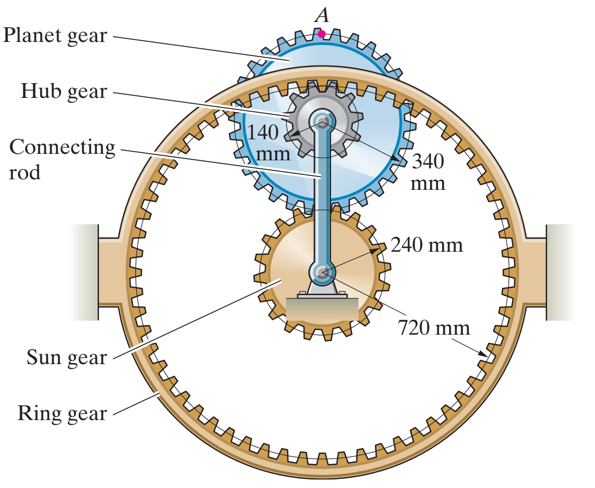
Solving, we obtain

$$\alpha_{BC} = -1.32 \text{ rad/s}^2$$

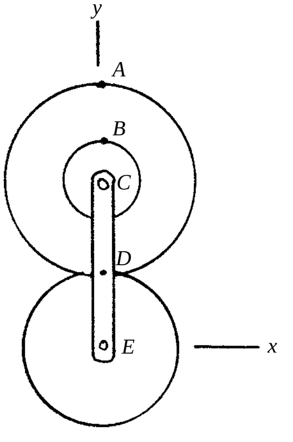
$$\alpha_{CE} = 4.04 \text{ rad/s}^2.$$



Problem 17.114 The ring gear is fixed, and the hub and planet gears are bonded together. The connecting rod has a counterclockwise angular acceleration of 10 rad/s^2. Determine the angular acceleration of the planet and sun gears.



Solution: The x components of the accelerations of pts B and C are



$$a_{Bx} = 0,$$

$$a_{Cx} = -(10 \text{ rad/s}^2)(0.58 \text{ m})$$

$$= -5.8 \text{ m/s}^2.$$

Let α_P and α_S be the angular accelerations of the planet and sun gears.

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_P \times \mathbf{r}_{B/C} - w_P^2 \mathbf{r}_{B/C}$$

$$= \mathbf{a}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_P \\ 0 & 0.14 & 0 \end{vmatrix} - w_P^2(0.14\mathbf{j}).$$

The $\mathbf{i}$ component of this equation is

$$0 = -5.8 - 0.14\alpha_P.$$

We obtain

$$\underline{\alpha_P = -41.4 \text{ rad/s}^2}.$$

Also, $\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_P \times \mathbf{r}_{D/C} - \omega_P^2 \mathbf{r}_{D/C}$

$$= \mathbf{a}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -41.4 \\ 0 & -0.34 & 0 \end{vmatrix} - \omega_P^2(-0.34\mathbf{j}).$$

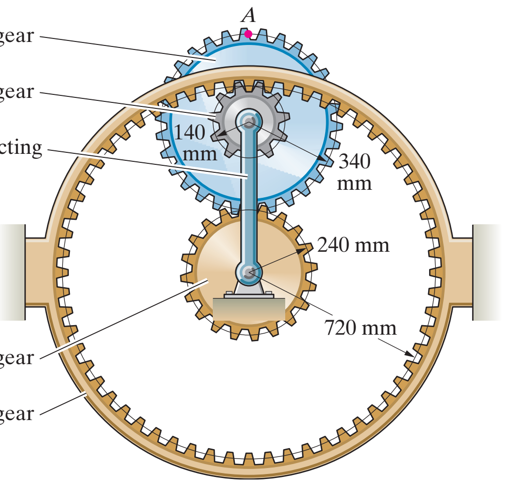
The $\mathbf{i}$ component of this equation is

$$a_{Dx} = -5.8 - (41.4)(0.34) = -19.9 \text{ m/s}^2.$$

Therefore

$$\underline{\alpha_S = \frac{19.9}{0.24} = 82.9 \text{ rad/s}^2}.$$

Problem 17.115 The connecting rod in Problem 17.114 has a counterclockwise angular velocity of 4 rad/s and a clockwise angular acceleration of 12 rad/s^2. Determine the magnitude of the acceleration at point A.



Solution: See the solution of Problem 17.114. The velocities of pts B and C are

$$\mathbf{v}_B = \mathbf{O},\ \mathbf{v}_C = -(4)(0.58)\mathbf{i} = -2.32\mathbf{i}\ \text{(m/s)}.$$

Let ω_P and ω_S be the angular velocities of the planet and sun gears.

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_P \times \mathbf{r}_{B/C}:$$

$$\mathbf{O} = -2.32\mathbf{i} + (\omega_P\mathbf{k}) \times (0.14\mathbf{j})$$

$$= (-2.32 - 0.14\omega_P)\mathbf{i}.$$

We see that $\omega_P = -16.6$ rad/s. Also,

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_P \times \mathbf{r}_{D/C}$$

$$= -2.32\mathbf{i} + (-16.6\mathbf{k}) \times (-0.34\mathbf{j})$$

$$= -7.95\mathbf{i}\ \text{(m/s)},$$

So $\omega_S = \dfrac{7.95}{0.24} = 33.1$ rad/s.

The x components of the accelerations of pts B and C are

$$a_{Bx} = 0,$$

$$a_{Cx} = (12\ \text{rad/s}^2)(0.58\ \text{m})$$

$$= 6.96\ \text{m/s}^2.$$

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_P \times \mathbf{r}_{B/C} - \omega_P^2\mathbf{r}_{B/C}$$

$$= \mathbf{a}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_P \\ 0 & 0.14 & 0 \end{vmatrix} - \omega_P^2(0.14\mathbf{j}).$$

The $\mathbf{i}$ component is

$$0 = 6.96 - 0.14\alpha_P,$$

so $\alpha_P = 49.7$ rad/s^2.

The acceleration of C is

$$\mathbf{a}_C = \mathbf{a}_E + (-12\mathbf{k}) \times \mathbf{r}_{C/E} - (4)^2\mathbf{r}_{C/E}$$

$$= 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -12 \\ 0 & 0.58 & 0 \end{vmatrix} - (4)^2(0.58\mathbf{j})$$

$$= 6.96\mathbf{i} - 9.28\mathbf{j}\ (\text{m/s}^2).$$

Then the acceleration of A is

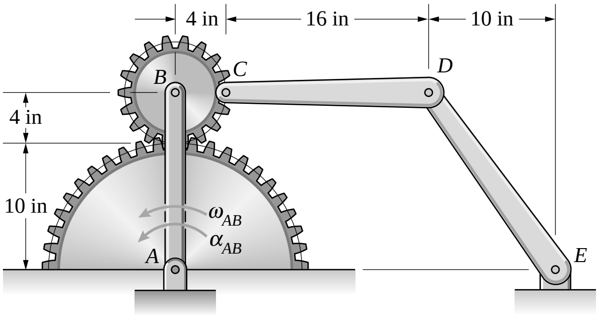
$$\mathbf{a}_A = \mathbf{a}_C + \boldsymbol{\alpha}_P \times \mathbf{r}_{A/C} - \omega_P^2\mathbf{r}_{A/C}$$

$$= 6.96\mathbf{i} - 9.28\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 49.7 \\ 0 & 0.34 & 0 \end{vmatrix} - (-16.6)^2(0.34\mathbf{j})$$

$$= -9.94\mathbf{i} - 102.65\mathbf{j}\ (\text{m/s}^2).$$

$\underline{|\mathbf{a}_A| = 103\ \text{m/s}^2}$.

Problem 17.116 The large gear is fixed. The angular velocity and angular acceleration of the bar *AB* are $\omega_{AB} = 2$ rad/s and $\alpha_{AB} = 4$ rad/s^2. Determine the angular acceleration of the bars *CD* and *DE*.



Solution: The strategy is to express vector velocity of point D in terms of the unknown angular velocities and accelerations of CD and DE, and then to solve the resulting vector equations for the unknowns. *The angular velocities* ω_{CD} and ω_{DE}. (See solution to Problem 17.51). The linear velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 0 & 14 & 0 \end{bmatrix} = -28\mathbf{i} \text{ (in/s)}.$$

The lower edge of gear B is stationary. The velocity of B is also

$$\mathbf{v}_B = \boldsymbol{\omega}_B \times \mathbf{r}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_B \\ 0 & 4 & 0 \end{bmatrix} = -4\omega_B\mathbf{i} \text{ (in/s)}.$$

Equate the velocities $\mathbf{v}_B$ to obtain the angular velocity of B:

$$\omega_B = -\frac{v_B}{4} = 7 \text{ rad/s}.$$

The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_B \times \mathbf{r}_{BC} = -28\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 7 \\ 4 & 0 & 0 \end{bmatrix} = -28\mathbf{i} + 28\mathbf{j} \text{ (in/s)}.$$

The velocity of point D is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{CD} = -28\mathbf{i} + 28\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 16 & 0 & 0 \end{bmatrix}$$

$$= -28\mathbf{i} + (16\omega_{CD} + 28)\mathbf{j} \text{ (in/s)}.$$

The velocity of point D is also given by

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{ED} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DE} \\ -10 & 14 & 0 \end{bmatrix}$$

$$= -14\omega_{DE}\mathbf{i} - 10\omega_{DE}\mathbf{j} \text{ (in/s)}.$$

Equate and separate components:

$$(-28 + 14\omega_{DE})\mathbf{i} = 0, \ (16\omega_{CD} + 28 + 10\omega_{DE})\mathbf{j} = 0.$$

Solve: $\omega_{DE} = 2$ rad/s,

$\omega_{CD} = -3$ rad/s.

The negative sign means a clockwise rotation. *The angular accelerations*. The tangential acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 0 & 14 & 0 \end{bmatrix} = -56\mathbf{i} \text{ (in/s}^2\text{)}.$$

The tangential acceleration at the point of contact between the gears A and B is zero, from which

$$\mathbf{a}_B = \boldsymbol{\alpha}_{BC} \times 4\mathbf{j} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 0 & 4 & 0 \end{bmatrix} = -4\alpha_{BC}\mathbf{i} \text{ (in/s}^2\text{)},$$

from which $\alpha_{BC} = 14$ rad/s^2. The acceleration of point C in terms of the acceleration of point B is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times 4\mathbf{i} - \omega_B^2(4\mathbf{i}) = -56\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 14 \\ 4 & 0 & 0 \end{bmatrix} - 49(4\mathbf{i})$$

$$= -252\mathbf{i} + 56\mathbf{j} \text{ (in/s}^2\text{)}.$$

The acceleration of point D in terms of the acceleration of point C is

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times 16\mathbf{i} - \omega_{CD}^2(16\mathbf{i})$$

$$= \mathbf{a}_C + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ 16 & 0 & 0 \end{bmatrix} - \omega_{CD}^2(16\mathbf{i}),$$

$$\mathbf{a}_D = -396\mathbf{i} + (16\alpha_{CD} + 56)\mathbf{j} \text{ (in/s}^2\text{)}.$$

The acceleration of point D in terms of the acceleration of point E is

$$\mathbf{a}_D = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{DE} \\ -10 & 14 & 0 \end{bmatrix} - \omega_{DE}^2(-10\mathbf{i} + 14\mathbf{j})$$

$$= (40 - 14\alpha_{DE})\mathbf{i} - (10\alpha_{DE} + 56)\mathbf{j} \text{ (in/s}^2\text{)}$$

Equate the expressions for $\mathbf{a}_D$ and separate components:

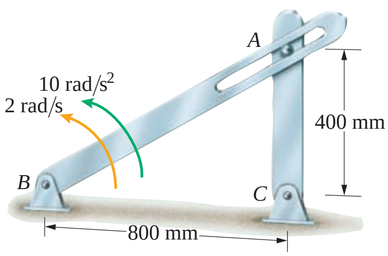
$$-396 = 40 - 14\alpha_{DE}, \ 16\alpha_{CD} + 56 = -10\alpha_{DE} - 56.$$

Solve: $\boxed{\alpha_{DE} = 31.1 \text{ rad/s}^2}$,

$\boxed{\alpha_{CD} = -26.5 \text{ rad/s}^2}$,

where the negative sign means a clockwise angular acceleration.

Problem 17.117 In Active Example 17.7, suppose that the distance from point C to the pin A on the vertical bar AC is 300 mm instead of 400 mm. Draw a sketch of the linkage with its new geometry. Determine the angular velocity of the bar AC and the velocity of the pin A relative to the slot in bar AB.

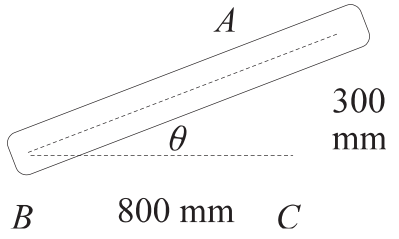


Solution: In the new position $\theta = \tan^{-1}\left(\dfrac{300}{800}\right) = 20.6^\circ$.

The velocity analysis:

$$\mathbf{V}_A = \mathbf{V}_C + \boldsymbol{\omega}_{AC} \times \mathbf{r}_{A/C}$$

$$= 0 + \omega_{AC}\mathbf{k} \times (0.3\mathbf{j}) = -0.3\omega_{AC}\mathbf{i}$$



$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{Arel} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$= 0 + v_{Arel}(\cos\boldsymbol{\theta}\mathbf{i} + \sin\theta\mathbf{j})$$

$$+ (2\mathbf{k}) \times (0.8\mathbf{i} + 0.3\mathbf{j})$$

$$= (v_{Arel}\cos\theta - 0.6)\mathbf{i} + (v_{Arel}\sin\theta + 1.6)\mathbf{j}$$

Equating the components of the two expressions for $\mathbf{v}_A$ we have

$$-0.3\omega_{AC} = v_{Arel}\cos\theta - 0.6,\ 0 = v_{Arel}\sin\theta + 1.6.$$

Solving these two equations, we find

$$v_{Arel} = -4.56 \text{ m/s}, \quad \omega_{AC} = 16.2 \text{ rad/s}.$$

Thus

$$\boxed{\begin{array}{l} A \text{ is moving at } 4.56 \text{ m/s from } B \text{ toward } A, \\ \omega_{AC} = 16.2 \text{ rad/s counterclockwise.} \end{array}}$$

Problem 17.118 The bar rotates with a constant counterclockwise angular velocity of 10 rad/s and sleeve A slides at a constant velocity of 4 m/s relative to the bar. Use Eq. (17.15) to determine the acceleration of A.

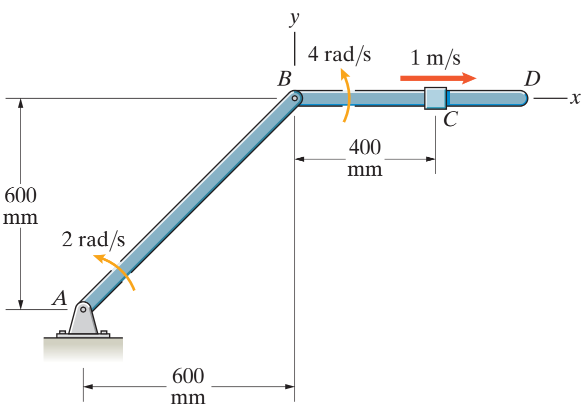
Solution: Eq. (17.15) is

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{\text{Arel}} + 2\omega \times \mathbf{v}_{\text{Arel}} + \alpha \times \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B}.$$

Substitute: $\mathbf{a}_A = 0 + 0 + 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 4 & 0 & 0 \end{bmatrix} + 0 - 100(2\mathbf{i})$

$$\boxed{= -200\mathbf{i} + 80\mathbf{j} \text{ (m/s}^2\text{)}}$$

Problem 17.119 Sleeve C slides at 1 m/s relative to bar BD. Use the body-fixed coordinate system shown to determine the velocity of C.



Solution: The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 600 & 600 & 0 \end{bmatrix} = -1200(\mathbf{i} - \mathbf{j}) \text{ (mm/s)}.$$

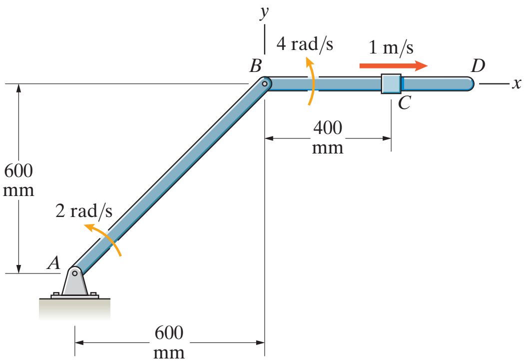
Use Eq. (17.11). The velocity of sleeve C is

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{\text{Arel}} + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{C/B}.$$

$$\mathbf{v}_C = -1200\mathbf{i} + 1200\mathbf{j} + 1000\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 400 & 0 & 0 \end{bmatrix}.$$

$$\boxed{\mathbf{v}_C = -200\mathbf{i} + 2800\mathbf{j} \text{ (mm/s)}}.$$

Problem 17.120 In Problem 17.119, the angular accelerations of the two bars are zero and the sleeve C slides at a constant velocity of 1 m/s relative to bar BD. What is the acceleration of C?



Solution: From Problem 17.119, $\omega_{AB} = 2$ rad/s, $\omega_{BC} = 4$ rad/s. The acceleration of point B is

$$\mathbf{a}_B = -\omega_{AB}^2 \mathbf{r}_{B/A} = -4(600\mathbf{i} + 600\mathbf{j})$$

$$= -2400\mathbf{i} - 2400\mathbf{j} \ (\text{mm/s}^2).$$

Use Eq. (17.15). The acceleration of C is

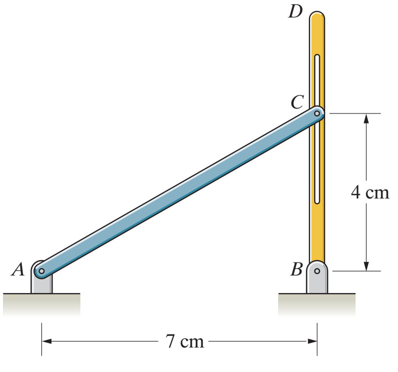
$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C\text{rel}} + 2\boldsymbol{\omega}_{BD} \times \mathbf{v}_{C\text{rel}} + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{C/B} - \omega_{BD}^2 \mathbf{r}_{C/B}.$$

$$\mathbf{a}_C = -2400\mathbf{i} - 2400\mathbf{j} + 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BD} \\ 1000 & 0 & 0 \end{bmatrix} - \omega_{BD}^2(400\mathbf{i}),$$

$$\boxed{\mathbf{a}_C = -8800\mathbf{i} + 5600\mathbf{j} \ (\text{mm/s}^2)}$$

Problem 17.121 Bar AC has an angular velocity of 2 rad/s in the counterclockwise direction that is decreasing at 4 rad/s^2. The pin at C slides in the slot in bar BD.

(a) Determine the angular velocity of bar BD and the velocity of the pin relative to the slot.
(b) Determine the angular acceleration of bar BD and the acceleration of the pin relative to the slot.



Solution: The coordinate system is fixed with respect to the vertical bar.

(a) $$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega}_{AC} \times \mathbf{r}_{C/A} = \mathbf{0} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ 7 & 4 & 0 \end{vmatrix}. \quad \textbf{(1)}$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C\text{rel}} + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{C/B}$$

$$= \mathbf{0} + v_{C\text{rel}}\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BD} \\ 0 & 4 & 0 \end{vmatrix}. \quad \textbf{(2)}$$

Equating **i** and **j** components in Eqs. (1) and (2),

$$-4\omega_{AC} = -4\omega_{BD}, \quad \textbf{(3)}$$

$$7\omega_{AC} = v_{C\text{rel}}, \quad \textbf{(4)}$$

We obtain $\underline{\omega_{BD} = 2 \text{ rad/s}}$, $\underline{v_{C\text{rel}} = 14 \text{ cm/s}}$.

(b) $$\boldsymbol{a}_C = \boldsymbol{a}_A + \boldsymbol{\alpha}_{AC} \times \boldsymbol{r}_{C/A} - \omega_{AC}^2 \boldsymbol{r}_{C/A}$$

$$= \mathbf{0} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AC} \\ 7 & 4 & 0 \end{vmatrix} - \omega_{AC}^2(7\mathbf{i} + 4\mathbf{j}). \quad \textbf{(5)}$$

$$\boldsymbol{a}_C = \boldsymbol{a}_B + \boldsymbol{a}_{C\text{rel}} + 2\boldsymbol{\omega}_{BD} \times \mathbf{v}_{C\text{rel}} + \boldsymbol{\alpha}_{BD} \times \boldsymbol{r}_{C/B} - \omega_{BD}^2 \boldsymbol{r}_{C/B}$$

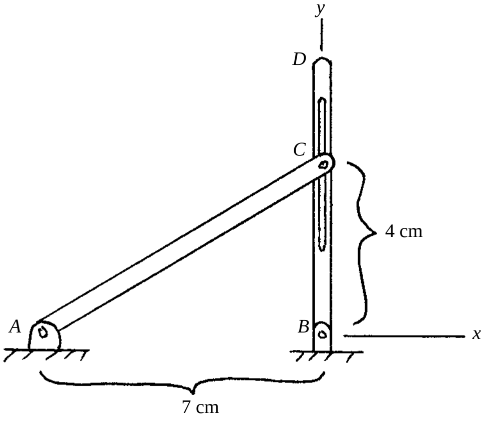
$$= \mathbf{0} + a_{C\text{rel}}\mathbf{j} + 2\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BD} \\ 0 & v_{C\text{rel}} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BD} \\ 0 & 4 & 0 \end{vmatrix} - \omega_{BD}^2(4\mathbf{j}). \textbf{(6)}$$

Equating **i** and **j** components in Eqs. (5) and (6),

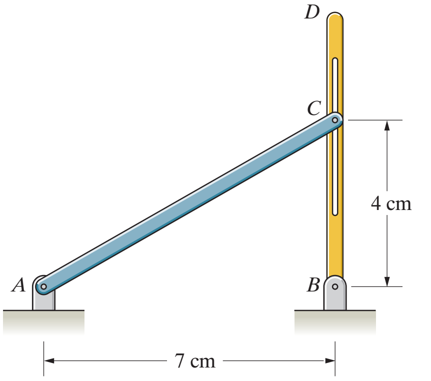
$$-4\alpha_{AC} - 7\omega_{AC}^2 = -2\omega_{BD}v_{C\text{rel}} - 4\alpha_{BD}, \quad \textbf{(7)}$$

$$7\alpha_{AC} - 4\omega_{AC}^2 = a_{C\text{rel}} - 4\omega_{BD}^2, \quad \textbf{(8)}$$

We obtain $\underline{\alpha_{BD} = -11 \text{ rad/s}^2}$, $\underline{a_{C\text{rel}} = -28 \text{ cm/s}^2}$.



Problem 17.122 In the system shown in Problem 17.121, the velocity of the pin C relative to the slot is 21 cm/s upward and is decreasing at 42 cm/s^2. What are the angular velocity and acceleration of bar AC?



Solution: See the solution of Problem 17.121. Solving Eqs. (3), (4), (7), and (8) with $v_{C\text{rel}} = 21$ cm/s and $a_{C\text{rel}} = -42$ cm/s^2, we obtain

$$\underline{\omega_{AC} = 3 \text{ rad/s}},$$

$$\underline{\alpha_{AC} = -6 \text{ rad/s}^2}.$$

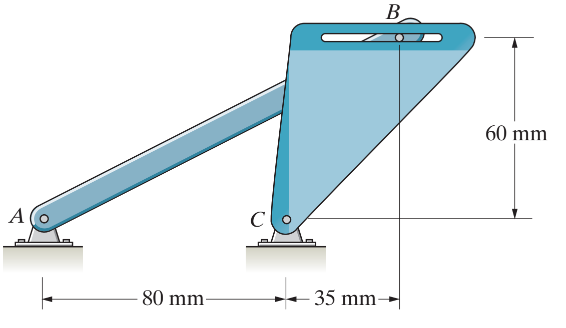
Problem 17.123 In the system shown in Problem 17.121, what should the angular velocity and acceleration of bar AC be if you want the angular velocity and acceleration of bar BD to be 4 rad/s counterclockwise and 24 rad/s^2 counterclockwise, respectively?

Solution: See the solution of Problem 17.121. Solving Eqs. (3), (4), (7), and (8) with $\omega_{BD} = 4$ rad/s^2 and $\alpha_{BD} = 24$ rad/s^2, we obtain

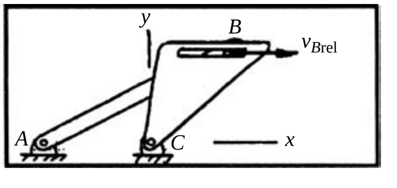
$$\omega_{AC} = 4 \text{ rad/s},$$

$$\alpha_{AC} = 52 \text{ rad/s}^2.$$

Problem 17.124 Bar AB has an angular velocity of 4 rad/s in the clockwise direction. What is the velocity of pin B relative to the slot?



Solution: The velocity of point B is



$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{AB} \\ 115 & 60 & 0 \end{bmatrix} = 240\mathbf{i} - 460\mathbf{j} \text{ (mm/s)}.$$

The velocity of point B is also determined from bar CB

$$\mathbf{v}_B = \mathbf{v}_{B\text{rel}} + \boldsymbol{\omega}_{CB} \times (35\mathbf{i} + 60\mathbf{j}),$$

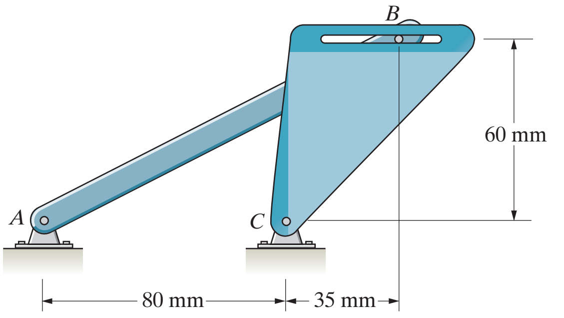
$$\mathbf{v}_B = \mathbf{v}_{B\text{rel}} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CB} \\ 35 & 60 & 0 \end{bmatrix}$$

$$\mathbf{v}_B = v_{B\text{rel}}\mathbf{i} - 60\omega_{CB}\mathbf{i} + 35\omega_{CB}\mathbf{j} \text{ (mm/s)}.$$

Equate like terms: $240 = v_{B\text{rel}} - 60\omega_{CB}$, $-460 = 35\omega_{CB}$ from which

$\omega_{BC} = -13.14$ rad/s, $\boxed{v_{B\text{rel}} = -548.6 \text{ mm/s}}$

Problem 17.125 In the system shown in Problem 17.124, the bar AB has an angular velocity of 4 rad/s in the clockwise direction and an angular acceleration of 10 rad/s^2 in the counterclockwise direction. What is the acceleration of pin B relative to the slot?



Solution: Use the solution to Problem 17.124, from which $\omega_{BC} = -13.14$ rad/s, $v_{B\text{rel}} = -548.6$ mm/s. *The angular acceleration and the relative acceleration.* The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 115 & 60 & 0 \end{bmatrix} - (16)(115\mathbf{i} + 60\mathbf{j}) \text{ (mm/s}^2\text{)},$$

$$\mathbf{a}_B = -600\mathbf{i} + 1150\mathbf{j} - 1840\mathbf{i} - 960\mathbf{j} = -2440\mathbf{i} + 190\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of pin B in terms of bar BC is

$$\mathbf{a}_B = a_{B\text{rel}}\mathbf{i} + 2\boldsymbol{\omega}_{BC} \times v_{B\text{rel}} + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C},$$

$$\mathbf{a}_B = a_{B\text{rel}}\mathbf{i} + 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -13.14 \\ -548.6 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 35 & 60 & 0 \end{bmatrix} - (13.14^2)(35\mathbf{i} + 60\mathbf{j}).$$

$$\mathbf{a}_B = a_{B\text{rel}}\mathbf{i} + 14{,}419.5\mathbf{j} + 35\alpha_{BC}\mathbf{j} - 60\alpha_{BC}\mathbf{i}$$

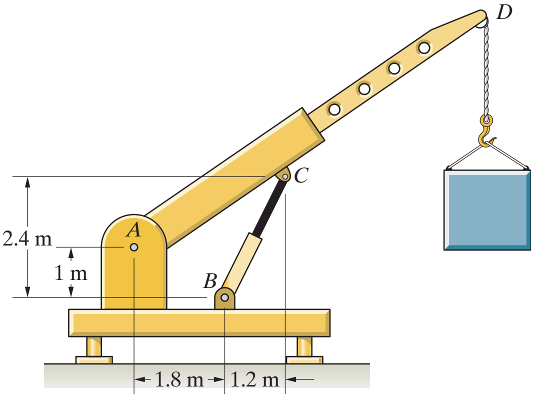
$$- 6045.7\mathbf{i} - 10,364.1\mathbf{j}.$$

Equate expressions for $\mathbf{a}_B$ and separate components: $-2440 = a_{B\text{rel}} - 60\alpha_{BC} - 6045.7$, $190 = 14{,}419.6 + 35\alpha_{BC} - 10364.1$. Solve:

$\boxed{\mathbf{a}_{B\text{rel}} = -3021\mathbf{i} \text{ (mm/s}^2\text{)}}$, $\alpha_{BC} = -110.4$ rad/s^2.

Problem 17.126 The hydraulic actuator BC of the crane is extending (increasing in length) at a constant rate of 0.2 m/s. At the instant shown, what is the angular velocity of the crane's boom AD?

Strategy: Use Eq. (17.8) to write the velocity of point C in terms of the velocity of point A, and use Eq. (17.11) to write the velocity of point C in terms of the velocity of point B. Then equate your two expressions for the velocity of point C.



Solution: Using Bar ACD

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega}_{AD} \times \mathbf{r}_{C/A} = 0 + \omega_{AD}\mathbf{k} \times (3\mathbf{i} + 1.4\mathbf{j}) \text{ m}$$

$$= \omega_{AD}(-1.4\mathbf{i} + 3\mathbf{j}) \text{ m}$$

Using cylinder BC

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{Brel} + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= 0 + (0.2 \text{ m/s})\left(\frac{1.2\mathbf{i} + 2.4\mathbf{j}}{\sqrt{1.2^2 + 2.4^2}}\right) + \omega_{BC}\mathbf{k} \times (1.2\mathbf{i} + 2.4\mathbf{j}) \text{ m}$$

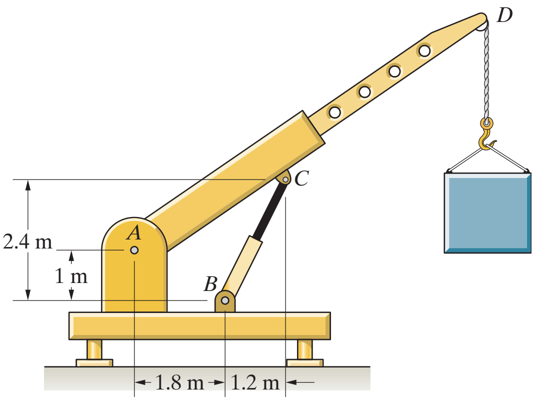
$$= (0.0894 \text{ m/s} - \{2.4 \text{ m}\}\omega_{BC})\mathbf{i} + (0.1789 \text{ m/s} + \{1.2 \text{ m}\}\omega_{BC})\mathbf{j}$$

Equating the components of the two expressions we have

$$\left.\begin{array}{l} (-1.4 \text{ m})\omega_{AD} = 0.0894 \text{ m/s} - (2.4 \text{ m})\omega_{BC} \\ (3 \text{ m})\omega_{AD} = 0.1789 \text{ m/s} + (1.2 \text{ m})\omega_{BC} \end{array}\right\} \Rightarrow \begin{array}{l} \omega_{BC} = 0.0940 \text{ rad/s} \\ \omega_{AD} = 0.0972 \text{ rad/s} \end{array}$$

$\boxed{\omega_{AD} = 0.0972 \text{ rad/s CCW}}$

Problem 17.127 In Problem 17.126, what is the angular acceleration of the crane's boom AD at the instant shown?



Solution: Use the angular velocities from 17.126

Bar ACD

$$\mathbf{a}_C = \mathbf{a}_A + \boldsymbol{\alpha}_{ACD} \times \mathbf{r}_{C/A} - \omega_{ACD}{}^2\mathbf{r}_{C/A}$$

$$= 0 + \alpha_{ACD}\mathbf{k} \times (3\mathbf{i} + 1.4\mathbf{j})\text{ m} - \omega_{ACD}{}^2(3\mathbf{i} + 1.4\mathbf{j})\text{ m}$$

$$= (-\{1.4\text{ m}\}\alpha_{ACD} - \{3\text{ m}\}\omega_{ACD}{}^2)\mathbf{i} + (\{3\text{ m}\}\alpha_{ACD}$$

$$- \{1.4\text{ m}\}\omega_{ACD}{}^2)\mathbf{j}$$

Cylinder BC

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{Brel} + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}{}^2\mathbf{r}_{C/B} + 2\boldsymbol{\omega}_{BC} \times \mathbf{v}_{Brel}$$

$$= 0 + 0 + \alpha_{BC}\mathbf{k} \times (1.2\mathbf{i} + 2.4\mathbf{j})\text{ m} - \omega_{BC}{}^2(1.2\mathbf{i} + 2.4\mathbf{j})\text{ m}$$

$$+ 2\,\omega_{BC}\mathbf{k} \times (0.2\text{ m/s})\left(\frac{1.2\mathbf{i} + 2.4\mathbf{j}}{\sqrt{1.2^2 + 2.4^2}}\right)$$

$$= (-\{2.4\text{ m}\}\alpha_{BC} - \{1.79\text{ m/s}\}\omega_{BC} - \{1.2\text{ m}\}\omega_{BC}{}^2)\mathbf{i}$$

$$+ (\{1.2\text{ m}\}\alpha_{BC} + \{0.894\text{ m/s}\}\omega_{BC} - \{2.4\text{ m}\}\omega_{BC}{}^2)\mathbf{j}$$

Equating the two expressions for the acceleration of C we have

$$-\{1.4\text{ m}\}\alpha_{AD} - \{3\text{ m}\}\omega_{AD}{}^2 = -\{2.4\text{ m}\}\alpha_{BC}$$

$$- \{1.79\text{ m/s}\}\omega_{BC} - \{1.2\text{ m}\}\omega_{BC}{}^2$$

$$\{3\text{ m}\}\alpha_{AD} - \{1.4\text{ m}\}\omega_{AD}{}^2 = \{1.2\text{ m}\}\alpha_{BC}$$

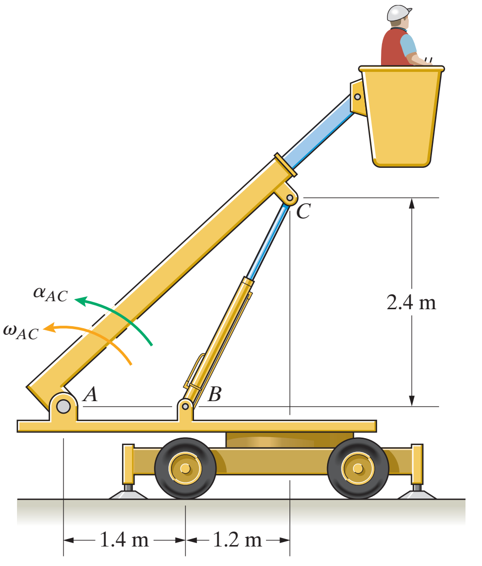
$$+ \{0.894\text{ m/s}\}\omega_{BC} - \{2.4\text{ m}\}\omega_{BC}{}^2$$

Solving (using the angular velocities from 17.126) we find

$\alpha_{BC} = -0.0624$ rad/s, $\alpha_{AD} = 0.000397$ rad/s

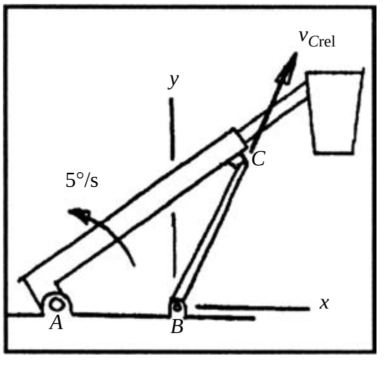
$$\boxed{\alpha_{AD} = 0.000397\text{ rad/s CCW}}$$

Problem 17.128 The angular velocity $\omega_{AC} = 5°$ per second. Determine the angular velocity of the hydraulic actuator BC and the rate at which the actuator is extending.



Solution: The point C effectively slides in a slot in the arm BC. The angular velocity of

$$\omega_{AC} = 5\left(\frac{\pi}{180}\right) = 0.0873 \text{ rad/s}.$$



The velocity of point C with respect to arm AC is

$$\mathbf{v}_C = \omega_{AC} \times \mathbf{r}_{C/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ 2.6 & 2.4 & 0 \end{bmatrix}$$

$$= -0.2094\mathbf{i} + 0.2269\mathbf{j} \text{ (m/s)},$$

The unit vector parallel to the actuator BC is

$$\mathbf{e} = \frac{1.2\mathbf{i} + 2.4\mathbf{j}}{\sqrt{1.2^2 + 2.4^2}} = 0.4472\mathbf{i} + 0.8944\mathbf{j}.$$

The velocity of point C in terms of the velocity of the actuator is

$$\mathbf{v}_C = v_{C\text{rel}}\mathbf{e} + \omega_{BC} \times \mathbf{r}_{C/B}.$$

$$\mathbf{v}_C = v_{C\text{rel}}(0.4472\mathbf{i} + 0.8944\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 1.2 & 2.4 & 0 \end{bmatrix}$$

$$\mathbf{v}_C = v_{C\text{rel}}(0.4472\mathbf{i} + 0.8944\mathbf{j}) + \omega_{BC}(-2.4\mathbf{i} + 1.2\mathbf{j}).$$

Equate like terms in the two expressions:

$$-0.2094 = 0.4472 v_{C\text{rel}} - 2.4\omega_{BC},$$

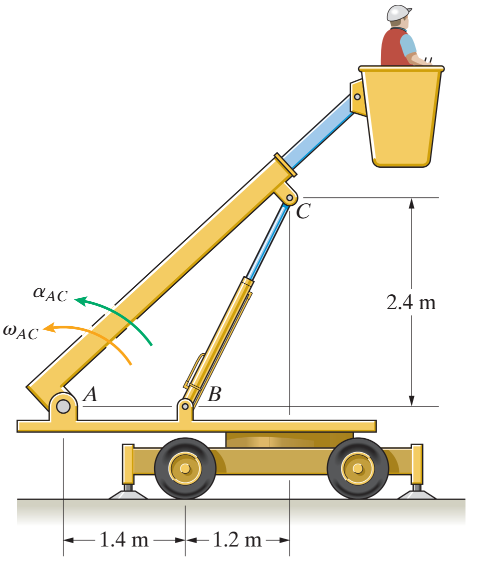
$$0.2269 = 0.8944 v_{C\text{rel}} + 1.2\omega_{BC}.$$

$$\boxed{\omega_{BC} = 0.1076 \text{ rad/s} = 6.17 \text{ deg/s}},$$

$$\boxed{v_{C\text{rel}} = 0.109 \text{ (m/s)}},$$

which is also the velocity of extension of the actuator.

Problem 17.129 In Problem 17.128, if the angular velocity $\omega_{AC} = 5^\circ$ per second and the angular acceleration $\alpha_{AC} = -2^\circ$ per second squared, determine the angular acceleration of the hydraulic actuator BC and the rate of change of the actuator's rate of extension.



Solution: Use the solution to Problem 17.128 for the velocities:

$\omega_{BC} = 0.1076$ rad/s,

$\omega_{AC} = 0.0873$ rad/s

$v_{C\text{rel}} = 0.1093$ (m/s).

The angular acceleration

$$\alpha_{AC} = -2\left(\frac{\pi}{180}\right) = -0.03491 \text{ rad/s}^2.$$

The acceleration of point C is

$$\mathbf{a}_C = \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/A} - \omega_{AC}^2 \mathbf{r}_{C/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AC} \\ 2.6 & 2.4 & 0 \end{bmatrix} - \omega_{AC}^2(2.6\mathbf{i} + 2.4\mathbf{j}),$$

$$\mathbf{a}_C = \alpha_{AC}(-2.4\mathbf{i} + 2.6\mathbf{j}) - \omega_{AC}^2(2.6\mathbf{i} + 2.4\mathbf{j})$$

$$= 0.064\mathbf{i} - 0.109\mathbf{j} \text{ (m/s}^2).$$

The acceleration of point C in terms of the hydraulic actuator is

$$\mathbf{a}_C = a_{C\text{rel}}\mathbf{e} + 2\boldsymbol{\omega}_{BC} \times \mathbf{v}_{C\text{rel}} + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{\mathbf{BC}}^2 \mathbf{r}_{C/B},$$

$$\mathbf{a}_C = a_{C\text{rel}}\mathbf{e} + 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0.4472 v_{C\text{rel}} & 0.8944 v_{C\text{rel}} & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 1.2 & 2.4 & 0 \end{bmatrix} - \omega_{BC}^2(1.2\mathbf{i} + 2.4\mathbf{j})$$

$$\mathbf{a}_C = a_{C\text{rel}}(0.4472\mathbf{i} + 0.8944\mathbf{j}) + 2\omega_{BC}(-0.0977\mathbf{i} + 0.0489\mathbf{j})$$

$$+ \alpha_{BC}(-2.4\mathbf{i} + 1.2\mathbf{j}) - \omega_{BC}^2(1.2\mathbf{i} + 2.4\mathbf{j}).$$

Equate like terms in the two expressions for $\mathbf{a}_C$.

$$0.0640 = 0.4472 a_{C\text{rel}} - 0.0139 - 2.4\alpha_{BC} - 0.0210,$$

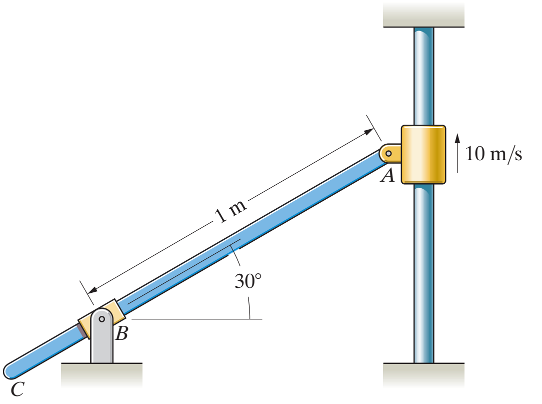
$$-0.1090 = 0.8944 a_{C\text{rel}} - 0.0278 + 1.2\alpha_{BC} + 0.0105.$$

Solve: $\boxed{a_{C\text{rel}} = -0.0378 \text{ (m/s}^2)}$,

which is the rate of change of the rate of extension of the actuator, and

$$\boxed{\alpha_{BC} = -0.0483 \text{ (rad/s}^2) = -2.77 \text{ deg/s}^2}$$

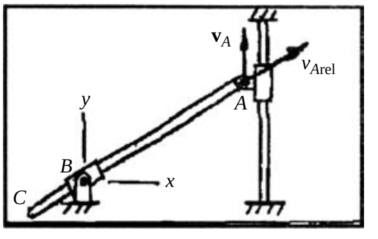
Problem 17.130 The sleeve at A slides upward at a constant velocity of 10 m/s. Bar AC slides through the sleeve at B. Determine the angular velocity of bar AC and the velocity at which the bar slides relative to the sleeve at B. (See Example 17.8.)



Solution: The velocity of the sleeve at A is given to be $\mathbf{v}_A = 10\mathbf{j}$ (m/s). The unit vector parallel to the bar (toward A) is

$$\mathbf{e} = 1(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}) = 0.866\mathbf{i} + 0.5\mathbf{j}.$$

Choose a coordinate system with origin at B that rotates with the bar. The velocity at A is



$$\mathbf{v}_A = \mathbf{v}_B + v_{Arel}\mathbf{e} + \boldsymbol{\omega}_{AC} \times \mathbf{r}_{A/B}$$

$$= 0 + v_{Arel}\mathbf{e} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0.866 & 0.5 & 0 \end{bmatrix}$$

$$\mathbf{v}_A = (0.866\mathbf{i} + 0.5\mathbf{j})v_{Arel} + \omega_{AC}(-0.5\mathbf{i} + 0.866\mathbf{j}) \text{ (m/s)}.$$

The given velocity is $\mathbf{v}_A = 10\mathbf{j}$ (m/s). Equate like components in the two expressions for $\mathbf{v}_A$:

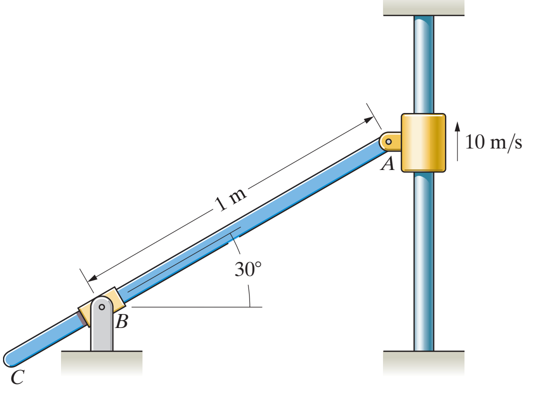
$$0 = 0.866v_{Arel} - 0.5\omega_{AC},$$

$$10 = 0.5v_{Arel} + 0.866\omega_{AC}.$$

Solve: $\boxed{\omega_{AC} = 8.66 \text{ rad/s}}$ (counterclockwise),

$\boxed{v_{Arel} = 5 \text{ m/s}}$ from B toward A.

Problem 17.131 In Problem 17.130, the sleeve at A slides upward at a constant velocity of 10 m/s. Determine the angular acceleration of bar AC and the rate of change of the velocity at which the bar slides relative to the sleeve at B. (See Example 17.8.)



Solution: Use the solution of Problem 17.130:

$$\mathbf{e} = 0.866\mathbf{i} + 0.5\mathbf{j},$$

$$\omega_{AB} = 8.66 \text{ rad/s},$$

$$v_{Arel} = 5 \text{ m/s}.$$

The acceleration of the sleeve at A is given to be zero. The acceleration in terms of the motion of the arm is

$$\mathbf{a}_A = 0 = a_{Arel}\mathbf{e} + 2\boldsymbol{\omega}_{AB} \times v_{Arel}\mathbf{e} + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2 \mathbf{r}_{A/B}.$$

$$\mathbf{a}_A = 0 = a_{Arel}\mathbf{e} + 2v_{Arel}\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 0.866 & 0.5 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 0.866 & 0.5 & 0 \end{bmatrix} - \omega_{AB}^2(0.866\mathbf{i} + 0.5\mathbf{j})$$

$$0 = (0.866\mathbf{i} + 0.5\mathbf{j})a_{Arel} - 43.3\mathbf{i} + 75\mathbf{j}$$

$$+ \alpha_{\mathbf{AB}}(-0.5\mathbf{i} + 0.866\mathbf{j}) - 64.95\mathbf{i} - 37.5\mathbf{j}.$$

Separate components:

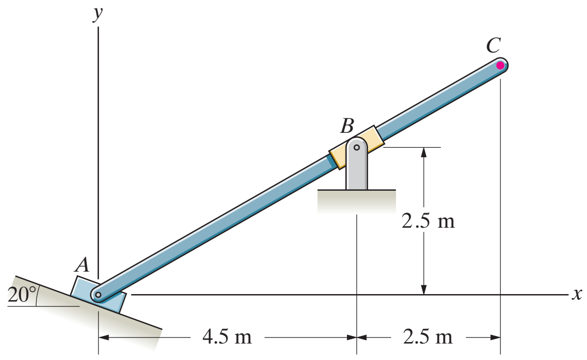
$$0 = 0.866a_{Arel} - 43.3 - 0.5\alpha_{AB} - 64.95,$$

$$0 = 0.5a_{Arel} + 75 + 0.866\alpha_{AB} - 37.5.$$

Solve: $\boxed{a_{Arel} = 75 \text{ (m/s}^2)}$ (toward A).

$\boxed{\alpha_{AB} = -86.6 \text{ rad/s}^2}$, (clockwise).

Problem 17.132 Block A slides up the inclined surface at 2 m/s. Determine the angular velocity of bar AC and the velocity of point C.



Solution: The velocity at A is given to be

$$\mathbf{v}_A = 2(-\mathbf{i}\cos 20^\circ + \mathbf{j}\sin 20^\circ) = -1.879\mathbf{i} + 0.6840\mathbf{j} \text{ (m/s)}.$$

From geometry, the coordinates of point C are

$$\left(7, 2.5\left(\frac{7}{4.5}\right)\right) = (7, 3.89) \text{ (m)}.$$

The unit vector parallel to the bar (toward A) is

$$\mathbf{e} = (7^2 + 3.89^2)^{-1/2}(-7\mathbf{i} - 3.89\mathbf{j}) = -0.8742\mathbf{i} - 0.4856\mathbf{j}.$$

The velocity at A in terms of the motion of the bar is

$$\mathbf{v}_A = v_{A\text{rel}}\mathbf{e} + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B} = v_{A\text{rel}}\mathbf{e} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ -4.5 & -2.5 & 0 \end{bmatrix},$$

$$\mathbf{v_A} = -0.8742 v_{A\text{rel}}\mathbf{i} - 0.4856 v_{A\text{rel}}\mathbf{j} + 2.5\omega_{AC}\mathbf{i} - 4.5\omega_{AC}\mathbf{j} \text{ (m/s)}.$$

Equate the two expressions for $\mathbf{v}_A$ and separate components:

$$-1.879 = -0.8742 v_{A\text{rel}} + 2.5\omega_{AC},$$

$$0.6840 = -0.4856 v_{A\text{rel}} - 4.5\omega_{AC}.$$

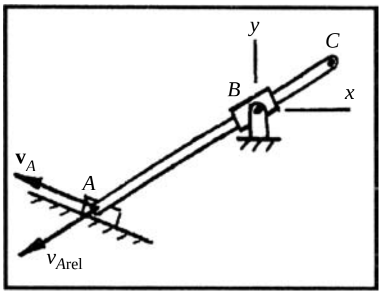
Solve: $v_{A\text{rel}} = 1.311$ m/s,

$\boxed{\omega_{AC} = -0.293 \text{ rad/s}}$ (clockwise).

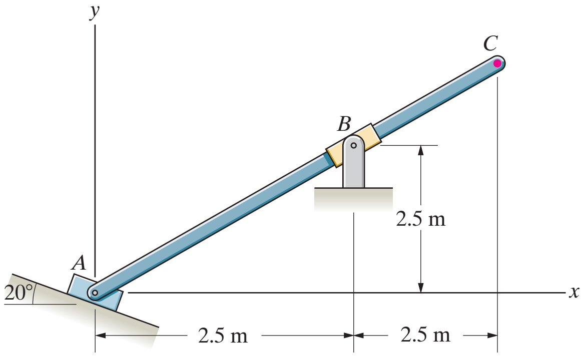
Noting that $\mathbf{v}_A = 2$ m/s, the velocity at point C is

$$\mathbf{v}_C = v_A(-0.8742\mathbf{i} - 0.4856\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.293 \\ 2.5 & 3.89 - 2.5 & 0 \end{bmatrix},$$

$\boxed{\mathbf{v}_C = -0.738\mathbf{i} - 1.37\mathbf{j} \text{ (m/s)}}$.



Problem 17.133 In Problem 17.132, block A slides up the inclined surface at a constant velocity of 2 m/s. Determine the angular acceleration of bar AC and the acceleration of point C.



Solution: *The velocities*: The velocity at A is given to be

$$\mathbf{v}_A = 2(-\mathbf{i}\cos 20^\circ + \mathbf{j}\sin 20^\circ) = -1.879\mathbf{i} + 0.6840\mathbf{j}\ \text{(m/s)}.$$

From geometry, the coordinates of point C are

$$\left(7, 2.5\left(\frac{7}{4.5}\right)\right) = (7, 3.89)\ \text{(m)}.$$

The unit vector parallel to the bar (toward A) is

$$\mathbf{e} = \frac{-7\mathbf{i} - 3.89\mathbf{j}}{\sqrt{7^2 + 3.89^2}} = -0.8742\mathbf{i} - 0.4856\mathbf{j}.$$

The velocity at A in terms of the motion of the bar is

$$\mathbf{v}_A = v_{A\text{rel}}\mathbf{e} + \boldsymbol{\omega}_{\mathbf{AC}} \times \mathbf{r}_{A/B} = v_{A\text{rel}}\mathbf{e} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ -4.5 & -2.5 & 0 \end{bmatrix},$$

$$\mathbf{v}_A = -0.8742 v_{A\text{rel}}\mathbf{i} - 0.4856 v_{A\text{rel}}\mathbf{j} + 2.5\omega_{AC}\mathbf{i} - 4.5\omega_{AC}\mathbf{j}\ \text{(m/s)}.$$

Equate the two expressions and separate components:

$$-1.879 = -0.8742 v_{A\text{rel}} + 2.5\omega_{AC},$$

$$0.6842 = -0.4856 v_{B\text{rel}} - 4.5\omega_{AC}.$$

Solve: $v_{A\text{rel}} = 1.311$ m/s, $\omega_{AC} = -0.293$ rad/s (clockwise).

The accelerations: The acceleration of block A is given to be zero. In terms of the bar AC, the acceleration of A is

$$\mathbf{a}_A = 0 = a_{A\text{rel}}\mathbf{e} + 2\boldsymbol{\omega}_{AC} \times v_{A\text{rel}}\mathbf{e} + \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{A/B} - \omega_{AC}^2\mathbf{r}_{A/B}.$$

$$0 = a_{A\text{rel}}\mathbf{e} + 2\omega_{AC} v_{A\text{rel}} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -0.8742 & -0.4856 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AC} \\ -4.5 & -2.5 & 0 \end{bmatrix} - \omega_{AC}^2(-4.5\mathbf{i} - 2.5\mathbf{j}).$$

$$0 = a_{A\text{rel}}\mathbf{e} + 2\omega_{AC} v_{A\text{rel}}(-e_y\mathbf{i} + e_x\mathbf{j}) + \alpha_{AC}(2.5\mathbf{i} - 4.5\mathbf{j})$$

$$-\omega_{AC}^2(-4.5\mathbf{i} - 2.5\mathbf{j}).$$

Separate components to obtain:

$$0 = -0.8742 a_{A\text{rel}} - 0.3736 + 2.5\alpha_{AC} + 0.3875,$$

$$0 = -0.4856 a_{A\text{rel}} + 0.6742 - 4.5\alpha_{AC} + 0.2153.$$

Solve: $a_{A\text{rel}} = 0.4433\ (\text{m/s}^2)$ (toward A).

$\boxed{\alpha_{AC} = 0.1494\ \text{rad/s}^2}$ (counterclockwise).

The acceleration of point C is

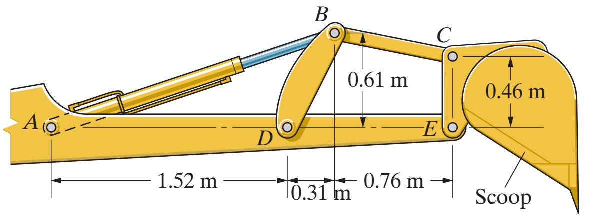
$$\mathbf{a}_C = a_{A\text{rel}}\mathbf{e} + 2\boldsymbol{\omega}_{AC} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/B} - \omega_{AC}^2\mathbf{r}_{C/B}$$

$$\mathbf{a}_C = a_{A\text{rel}}\mathbf{e} + 2\omega_{AC} v_{A\text{rel}} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ e_x & e_y & 0 \end{bmatrix}$$

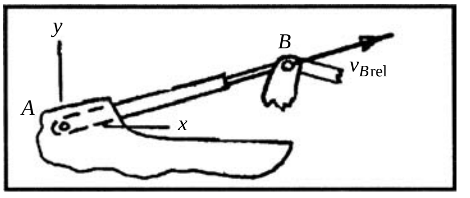
$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AC} \\ 2.5 & 3.89 - 2.5 & 0 \end{bmatrix} - \omega_{AC}^2(2.5\mathbf{i} + (3.89 - 2.5)\mathbf{j}).$$

Substitute numerical values: $\boxed{\mathbf{a}_C = -1.184\mathbf{i} + 0.711\mathbf{j}\ (\text{m/s}^2)}$

Problem 17.134 The angular velocity of the scoop is 1 rad/s clockwise. Determine the rate at which the hydraulic actuator AB is extending.



Solution: The point B slides in the arm AB. The velocity of point C is



$$\mathbf{v}_C = \boldsymbol{\omega}_{\text{scoop}} \times (0.46\mathbf{j}) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 0.46 & 0 \end{bmatrix} = 0.46\mathbf{i} \text{ (m/s)}.$$

Point B is constrained to move normally to the arm DB: The unit vector parallel to DB is

$$\mathbf{e}_{DB} = \frac{0.31\mathbf{i} + 0.61\mathbf{j}}{\sqrt{0.31^2 + 0.61^2}} = 0.4472\mathbf{i} + 0.8944\mathbf{j}.$$

The unit vector normal to $\mathbf{e}_{DB}$ is $\mathbf{e}_{NDB} = 0.8944\mathbf{i} - 0.4472\mathbf{j}$, from which the velocity of C in terms of BC is

$$\mathbf{v}_C = v_B \mathbf{e}_{NBD} + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= v_B(0.8944\mathbf{i} - 0.4472\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0.76 & -0.15 & 0 \end{bmatrix}.$$

$$\mathbf{v}_C = v_B(0.8944\mathbf{i} - 0.4472\mathbf{j}) + \omega_{BC}(0.15\mathbf{i} + 0.76\mathbf{j}).$$

Equate terms in $\mathbf{v}_C$, $0.46 = 0.8944 v_B + 0.15\omega_{BC}$, $O = -0.4472 v_B + 0.76\omega_{BC}$. Solve: $\omega_{BC} = 0.2727$ rad/s, $v_B = 0.465$ m/s, from which $\mathbf{v}_B = v_B \mathbf{e}_{NDB} = 1.364\mathbf{i} - 0.6818\mathbf{j}$ (m/s).

The unit vector parallel to the arm AB is

$$\mathbf{e}_{AB} = \frac{1.83\,\mathbf{i} + 0.61\,\mathbf{j}}{\sqrt{1.83^2 + 0.61^2}} = 0.9487\mathbf{i} + 0.3162\mathbf{j}$$

Choose a coordinate system with origin at A rotating with arm AB. The velocity of point B is

$$\mathbf{v}_B = v_{B\text{rel}} \mathbf{e}_{AB} + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$= v_{B\text{rel}}(0.9487\mathbf{i} + 0.3162\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 1.83 & 0.61 & 0 \end{bmatrix}.$$

$$\mathbf{v}_B = v_{B\text{rel}}(0.9487\mathbf{i} + 0.3162\mathbf{j}) + \omega_{AB}(-0.61\mathbf{i} + 1.83\mathbf{j}).$$

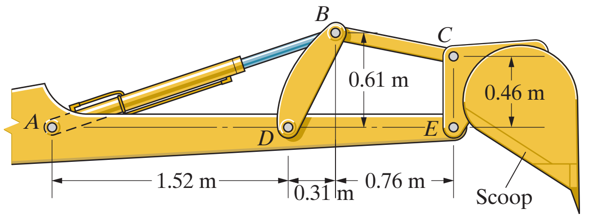
Equate the expressions and separate components:

$$0.416 = 0.9487 v_{B\text{rel}} - 0.61\omega_{AB},$$

$$-0.208 = 0.3162 v_{B\text{rel}} + 1.83\omega_{AB}.$$

Solve: $\omega_{AB} = -0.1704$ rad/s, $\boxed{v_{B\text{rel}} = 0.329 \text{ m/s}}$ which is the rate of extension of the actuator.

Problem 17.135 The angular velocity of the scoop is 1 rad/s clockwise and its angular acceleration is zero. Determine the rate of change of the rate at which the hydraulic actuator AB is extending.



Solution: Choose a coordinate system with the origin at D and the x axis parallel to ADE. The vector locations of points A, B, C, and E are $\mathbf{r}_A = -1.52\mathbf{i}$ m, $\mathbf{r}_B = 0.31\mathbf{i} + 0.61\mathbf{j}$ m, $\mathbf{r}_C = 1.07\mathbf{i} + 0.46\mathbf{j}$ m, $\mathbf{r}_E = 1.07\mathbf{i}$ m. The vector AB is

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 1.83\mathbf{i} + 0.61\mathbf{j} \text{ (ft)},$$

$$\mathbf{r}_{B/D} = \mathbf{r}_B - \mathbf{r}_D = 0.31\mathbf{i} + 0.61\mathbf{j} \text{ (m)}.$$

Assume that the scoop rotates at 1 rad/s about point E. The acceleration of point C is

$$\mathbf{a}_C = \boldsymbol{\alpha}_{\text{Scoop}} \times 4.57\mathbf{j} - \omega^2_{\text{scoop}}(0.46\mathbf{j}) = -0.46\mathbf{j} \text{ (m/s}^2\text{)},$$

since $\alpha_{\text{scoop}} = 0$. The vector from C to B is $\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = -0.76\mathbf{i} + 0.15\mathbf{j}$ (m). The acceleration of point B in terms of point C is

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega^2_{BC}\mathbf{r}_{B/C}$$

$$= 0.46\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -0.76 & 0.15 & 0 \end{bmatrix} - \omega^2_{BC}(-0.76\mathbf{i} + 0.15\mathbf{j}),$$

from which

$$\mathbf{a}_B = -(0.15\alpha_{BC} - 0.76\omega^2_{BC})\mathbf{i} - (0.46 + 0.76\alpha_{BC} + 0.5\omega^2_{BC})\mathbf{j}.$$

The acceleration of B in terms of D is

$$\mathbf{a}_B = \mathbf{a}_D + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{B/D} - \omega^2_{BD}\mathbf{r}_{B/D}$$

$$= \mathbf{a}_D + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BD} \\ 0.31 & 0.61 & 0 \end{bmatrix} - \omega^2_{BD}(0.31\mathbf{i} + 0.61\mathbf{j}).$$

The acceleration of point D is zero, from which $\mathbf{a}_B = -(0.61\alpha_{BD} + 0.31\omega^2_{BD})\mathbf{i} + (0.31\alpha_{BD} - 0.61\omega^2_{BD})\mathbf{j}$. Equate like terms in the two expressions for $\mathbf{a}_B$, $-(0.15\alpha_{BC} - 0.76\omega^2_{BC}) = -(0.61\alpha_{BD} + 0.31\omega^2_{BD})$, $-(0.46 + 0.76\alpha_{BC} + 0.15\omega^2_{BC}) = (0.31\alpha_{BD} - 0.61\omega^2_{BD})$. From the solution to Problem 17.134, $\omega_{BC} = 0.2727$ rad/s, and $v_B = 0.465$ m/s. The velocity of point B is normal to the link BD, from which

$$\omega_{BD} = \frac{v_B}{\sqrt{1^2 + 2^2}} = 0.6818 \text{ rad/s}.$$

Substitute and solve for the angular accelerations: $\alpha_{BC} = -0.1026$ rad/s^2, $\alpha_{BD} = -0.3511$ rad/s^2. From which the acceleration of point B is

$$\mathbf{a}_B = -(2\alpha_{BD} + \omega^2_{BD})\mathbf{i} + (\alpha_{BD} - 2\omega^2_{BD})\mathbf{j}$$

$$= 0.072\mathbf{i} - 0.39\mathbf{j} \text{ (m/s).}^2$$

The acceleration of point B in terms of the arm AB is

$$\mathbf{a}_B = a_{B\text{rel}}\mathbf{e}_{B/A} + 0.61\boldsymbol{\omega}_{AB} \times v_{B\text{rel}}\mathbf{e}_{B/A} + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega^2_{AB}\mathbf{r}_{B/A}.$$

From the solution to Problem 17.134: $\mathbf{e}_{B/A} = 0.9487\mathbf{i} + 0.3162\mathbf{j}$, $v_{B\text{rel}}] = 0.33$ m/s, $\omega_{AB} = -0.1705$ rad/s. From which

$$\mathbf{a}_B = a_{B\text{rel}}(0.9487\mathbf{i} + 0.3162\mathbf{j}) + 0.1162\mathbf{i} - 0.3487\mathbf{j}$$

$$+ \alpha_{AB}(-0.61\mathbf{i} + 1.83\mathbf{j}) - 0.1743\mathbf{i} - 0.0581\mathbf{j}.$$

Equate the accelerations of point B and separate components:

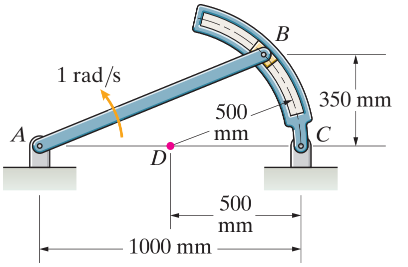
$$0.072 = 0.9487a_{B\text{rel}} - 0.61\alpha_{AB} - 0.018,$$

$$-0.39 = 0.3162a_{B\text{rel}} + 1.83\alpha_{AB} - 0.124.$$

Solve: $\boxed{a_{B\text{rel}} = 0.00116 \text{ m/s}^2}$, which is the rate of change of the rate at which the actuator is extending.

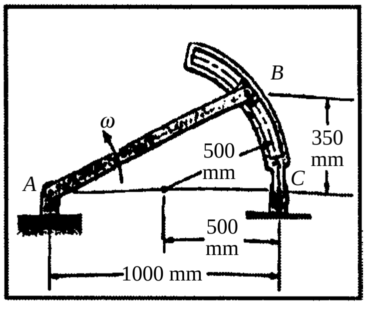
Problem 17.136 Suppose that the curved bar in Example 17.9 rotates with a counterclockwise angular velocity of 2 rad/s.

(a) What is the angular velocity of bar AB?
(b) What is the velocity of block B relative to the slot?



Solution: The angle defining the position of B in the circular slot is

$$\beta = \sin^{-1}\left(\frac{350}{500}\right) = 44.4^\circ.$$



The vectors are

$$\mathbf{r}_{B/A} = (500 + 500\cos\beta)\mathbf{i} + 350\mathbf{j} = 857\mathbf{i} + 350\mathbf{j} \text{ (mm)}.$$

$$\mathbf{r}_{B/C} = (-500 + 500\cos\beta)\mathbf{i} + 350\mathbf{j} \text{ (mm)}.$$

The unit vector tangent to the slot at B is given by

$$\mathbf{e}_B = -\sin\beta\mathbf{i} + \cos\beta\mathbf{j} = -0.7\mathbf{i} + 0.714\mathbf{j}.$$

The velocity of B in terms of AB is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 857 & 350 & 0 \end{bmatrix}$$

$$= \omega_{AB}(-350\mathbf{i} + 857\mathbf{j}) \text{ (mm/s)}.$$

The velocity of B in terms of BC is

$$\mathbf{v}_B = v_{B\text{rel}}\mathbf{e}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$= v_{B\text{rel}}(-0.7\mathbf{i} + 0.714\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -142.9 & 350 & 0 \end{bmatrix},$$

$$\mathbf{v}_B = v_{B\text{rel}}(-0.7\mathbf{i} + 0.714\mathbf{j}) + (-700\mathbf{i} - 285.8\mathbf{j}) \text{ (mm/s)}$$

Equate the expressions for the velocity of B and separate components:

$$-350\omega_{AB} = -0.7v_{B\text{rel}} - 700,$$

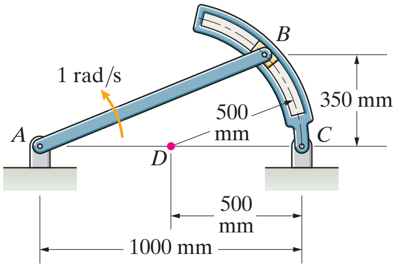
$$-857\omega_{AB} = 0.714v_{B\text{rel}} - 285.8.$$

Solve:

(a) $\boxed{\omega_{AB} = -2 \text{ rad/s}}$ (clockwise).

(b) $\boxed{v_{B\text{rel}} = -2000 \text{ mm/s}}$ (toward C).

Problem 17.137 Suppose that the curved bar in Example 17.9 has a clockwise angular velocity of 4 rad/s and a counterclockwise angular acceleration of 10 rad/s^2. What is the angular acceleration of bar AB?



Solution: Use the solution to Problem 17.118 with new data.

Get the velocities: The angle defining the position of B in the circular slot is

$$\beta = \sin^{-1}\left(\frac{350}{500}\right) = 44.4^\circ.$$

The vectors

$$\mathbf{r}_{B/A} = (500 + 500\cos\beta)\mathbf{i} + 350\mathbf{j} = 857\mathbf{i} + 350\mathbf{j}\ \text{(mm)}.$$

$$\mathbf{r}_{B/C} = (-500 + 500\cos\beta)\mathbf{i} + 350\mathbf{j}\ \text{(mm)}.$$

The unit vector tangent to the slot at B

$$\mathbf{e}_B = -\sin\beta\mathbf{i} + \cos\beta\mathbf{j} = -0.7\mathbf{i} + 0.714\mathbf{j}.$$

The component normal to the slot at B is

$$\mathbf{e}_{NB} = \cos\beta\mathbf{i} + \sin\beta\mathbf{j} = 0.7141\mathbf{i} + 0.7\mathbf{j}.$$

The velocity of B in terms of AB

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 857 & 350 & 0 \end{bmatrix}$$

$$= \omega_{AB}(-350\mathbf{i} + 857\mathbf{j})\ \text{(mm/s)}.$$

The velocity of B in terms of BC is

$$\mathbf{v}_B = v_{B\text{rel}}\mathbf{e}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$= v_{B\text{rel}}(-0.7\mathbf{i} + 0.714\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -\omega_{BC} \\ -142.9 & 350 & 0 \end{bmatrix},$$

$$\mathbf{v}_B = v_{B\text{rel}}(-0.7\mathbf{i} + 0.714\mathbf{j}) + (1400\mathbf{i} + 571.6\mathbf{j})\ \text{(mm/s)}.$$

Equate the expressions for the velocity of B and separate components: $-350\omega_{AB} = -0.7v_{B\text{rel}} + 1400$, $857\omega_{AB} = 0.714v_{B\text{rel}} + 571.6$. Solve: $\omega_{AB} = 4$ rad/s (counterclockwise). $v_{B\text{rel}} = 4000$ mm/s (away from C).

Get the accelerations: The acceleration of point B in terms of the AB is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 857 & 350 & 0 \end{bmatrix} - \omega_{AB}^2(857\mathbf{i} + 350\mathbf{j}),$$

$$\mathbf{a}_B = \alpha_{AB}(-350\mathbf{i} + 857\mathbf{j}) - 138713\mathbf{i} - 5665\mathbf{j}\ \text{(mm/s}^2\text{)}.$$

The acceleration in terms of the arm BC is

$$\mathbf{a}_B = \mathbf{a}_{B\text{rel}} + 2\boldsymbol{\omega}_{BC} \times v_{B\text{rel}}\mathbf{e}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\mathbf{r}_{B/C}.$$

Expanding term by term:

$$\mathbf{a}_{B\text{rel}} = a_{B\text{rel}}\mathbf{e}_B - \left(\frac{v_{B\text{rel}}^2}{500}\right)\mathbf{e}_{NB}$$

$$= a_{B\text{rel}}(-0.7\mathbf{i} + 0.7141\mathbf{j}) - 22{,}852.6\mathbf{i} - 22{,}400\mathbf{j}.$$

Other terms:

$$2\boldsymbol{\omega}_{BC} \times v_{B\text{rel}}\mathbf{e}_B = 22852\mathbf{i} + 22{,}400\mathbf{j},$$

$$\boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} = -3500\mathbf{i} - 1429.3\mathbf{j},$$

$$-\omega_{BC}^2\mathbf{r}_{B/C} = 2286.8\mathbf{i} - 5600\mathbf{j}.$$

Collect terms:

$$\mathbf{a}_B = a_{B\text{rel}}(-0.7\mathbf{i} + 0.7141\mathbf{j}) - 22852.6\mathbf{i} - 22400\mathbf{j} + 22852.6\mathbf{i}$$

$$+ 22400\mathbf{j} - 3500\mathbf{i} - 1429.3\mathbf{j} + 2286.9\mathbf{i} - 5600\mathbf{j}$$

$$\mathbf{a}_B = a_{B\text{rel}}(-0.7\mathbf{i} + 0.7141\mathbf{j}) - 1213\mathbf{i} - 7029.3\mathbf{j}.$$

Equate the two expressions for the acceleration of B to obtain the two equations:

$$-350\alpha_{AB} - 13{,}871 = -0.7a_{B\text{rel}} - 1213.1,$$

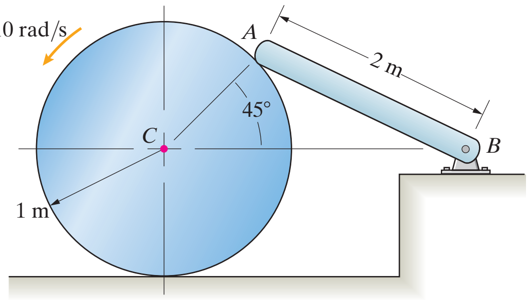
$$857\alpha_{AB} - 5665 = 0.7141a_{B\text{rel}} - 7029.3.$$

Solve:

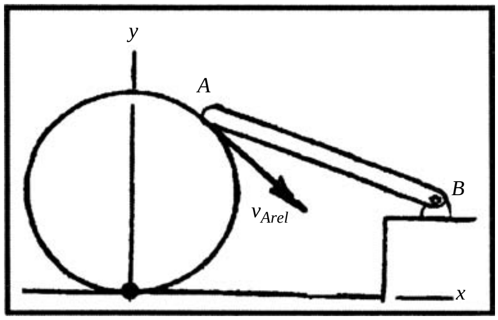
$$a_{B\text{rel}} = 29180\ \text{(mm/s}^2\text{)},$$

$$\boxed{\alpha_{AB} = 22.65\ \text{rad/s}^2}\ \text{(counterclockwise)}.$$

Problem 17.138* The disk rolls on the plane surface with a counterclockwise angular velocity of 10 rad/s. Bar AB slides on the surface of the disk at A. Determine the angular velocity of bar AB.



Solution: Choose a coordinate system with the origin at the point of contact between the disk and the plane surface, with the x axis parallel to the plane surface. Let A be the point of the bar in contact with the disk. The vector location of point A on the disk is



$$\mathbf{r}_A = \mathbf{i}\cos 45^\circ + \mathbf{j}(1 + \sin 45^\circ) = 0.707\mathbf{i} + 1.707\mathbf{j} \text{ (m)}.$$

The unit vector parallel to the radius of the disk is

$$\mathbf{e}_A = \cos 45^\circ\mathbf{i} + \sin 45^\circ\mathbf{j} = 0.707\mathbf{i} + 0.707\mathbf{j}.$$

The unit vector tangent to the surface of the disk at A is

$$\mathbf{e}_{NA} = \mathbf{i}\sin 45^\circ - \mathbf{j}\cos 45^\circ = 0.707\mathbf{i} - 0.707\mathbf{j}.$$

The angle formed by the bar AB with the horizontal is

$$\beta = \sin^{-1}\left(\frac{\sin 45^\circ}{2}\right) = 20.7^\circ.$$

The velocity of point A in terms of the motion of bar AB is

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -2\cos\beta & 2\sin\beta & 0 \end{bmatrix}.$$

$$\mathbf{v}_A = \omega_{AB}(-0.707\mathbf{i} - 1.871\mathbf{j}) \text{ (m/s)}.$$

The velocity of point A in terms of the point of the disk in contact with the plane surface is

$$\mathbf{v}_A = v_{Arel}\mathbf{e}_{NA} + \boldsymbol{\omega}_{\text{disk}} \times \mathbf{r}_A$$

$$= v_{Arel}(0.707\mathbf{i} - 0.707\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{\text{disk}} \\ 0.707 & 1.707 & 0 \end{bmatrix},$$

$$\mathbf{v}_A = v_{Arel}(0.707\mathbf{i} - 0.707\mathbf{j}) + (-17.07\mathbf{i} + 7.07\mathbf{j}).$$

Equate the expressions and separate components:

$$-0.707\omega_{AB} = 0.707v_{Arel} - 17.07,$$

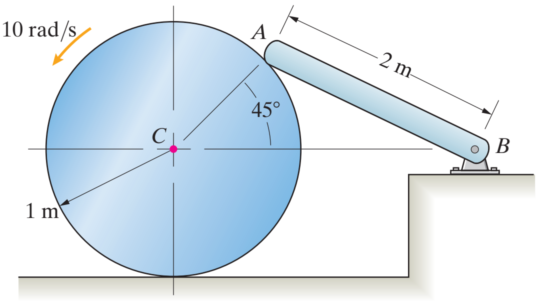
$$-1.871\omega_{AB} = -0.707v_{Arel} + 7.07.$$

Solve:

$$v_{Arel} = 20.3 \text{ m/s},$$

$\boxed{\omega_{AB} = 3.88 \text{ rad/s}}$ (counterclockwise).

Problem 17.139* In Problem 17.138, the disk rolls on the plane surface with a constant counterclockwise angular velocity of 10 rad/s. Determine the angular acceleration of bar *AB*.



Solution: Use the results of the solution to Problem 17.138. Choose a coordinate system with the origin at the point of contact between the disk and the plane surface, with the x axis parallel to the plane surface. The vector location of point A on the disk is

$$\mathbf{r}_A = \mathbf{i}\cos 45^\circ + \mathbf{j}(1 + \sin 45^\circ) = 0.707\mathbf{i} + 1.707\mathbf{j} \text{ (m)}.$$

The unit vector tangent to the surface of the disk at A is

$$\mathbf{e}_{NA} = \mathbf{i}\sin 45^\circ - \mathbf{j}\cos 45^\circ = 0.707\mathbf{i} - 0.707\mathbf{j}.$$

The angle formed by the bar AB with the horizontal is

$$\beta = \sin^{-1}(\sin 45^\circ/2) = 20.7^\circ.$$

Get the velocities: The velocity of point A in terms of the motion of bar AB is

$$\mathbf{v}_A = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ -2\cos\beta & 2\sin\beta & 0 \end{bmatrix}$$

$$= \omega_{AB}(-0.707\mathbf{i} - 1.871\mathbf{j}) \text{ (m/s)}.$$

The acceleration of the center of the disk is zero. The velocity of point A in terms of the center of the disk is

$$\mathbf{v}_A = v_{A\text{rel}}\mathbf{e}_{NA} + \boldsymbol{\omega}_{\text{disk}} \times \mathbf{r}_A$$

$$= v_{A\text{rel}}(0.707\mathbf{i} - 0.707\mathbf{j}) + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{\text{disk}} \\ 0.707 & 1.707 & 0 \end{bmatrix},$$

$$\mathbf{v}_A = v_{A\text{rel}}(0.707\mathbf{i} - 0.707\mathbf{j}) + (-17.07\mathbf{i} + 7.07\mathbf{j}).$$

Equate the expressions and separate components:

$$-0.707\omega_{AB} = 0.707v_{A\text{rel}} - 17.07, \quad -1.871\omega_{AB} = -0.707v_{A\text{rel}} + 7.07.$$

Solve:

$$v_{A\text{rel}} = 20.3 \text{ m/s},$$

$$\omega_{AB} = 3.88 \text{ rad/s} \quad \text{(counterclockwise)}.$$

Get the accelerations: The acceleration of point A in terms of the arm AB is

$$\mathbf{a}_A = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}^2\mathbf{r}_{A/B}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ -1.87 & 0.707 & 0 \end{bmatrix} + 28.15\mathbf{i} - 10.64\mathbf{j} \text{ (m/s}^2\text{)},$$

$$\mathbf{a}_A = \alpha_{AB}(-0.707\mathbf{i} - 1.87\mathbf{j}) + 28.15\mathbf{i} - 10.64\mathbf{j} \text{ (m/s}^2\text{)}.$$

The acceleration of point A in terms of the disk is

$$\mathbf{a}_A = \mathbf{a}_{A\text{rel}} + 2\boldsymbol{\omega}_{\text{disk}} \times v_{A\text{rel}}\mathbf{e}_{NA} + \boldsymbol{\alpha}_{\text{disk}} \times \mathbf{r}_{A/C} - \omega_{\text{disk}}^2\mathbf{r}_{A/C}.$$

Expanding term by term: The acceleration $\mathbf{a}_{A\text{rel}}$ is composed of a tangential component and a radial component:

$$\mathbf{a}_{A\text{rel}} = a_{A\text{rel}}\mathbf{e}_{NA} - \left(\frac{v_{A\text{rel}}^2}{1}\right)\mathbf{e}_A$$

$$= a_{A\text{rel}}(0.707\mathbf{i} - 0.707\mathbf{j}) - 290.3\mathbf{i} - 290.3\mathbf{j}.$$

$$2\boldsymbol{\omega}_{\text{disk}} \times v_{A\text{rel}}\mathbf{e}_{NB} = 286.6\mathbf{i} + 286.6\mathbf{j}, \ \boldsymbol{\alpha}_{\text{disk}} \times \mathbf{r}_A = 0,$$

since the acceleration of the disk is zero.

$$-\omega_{\text{disk}}^2\mathbf{r}_{A/C} = -70.7\mathbf{i} - 70.7\mathbf{j}.$$

Collect terms and separate components to obtain:

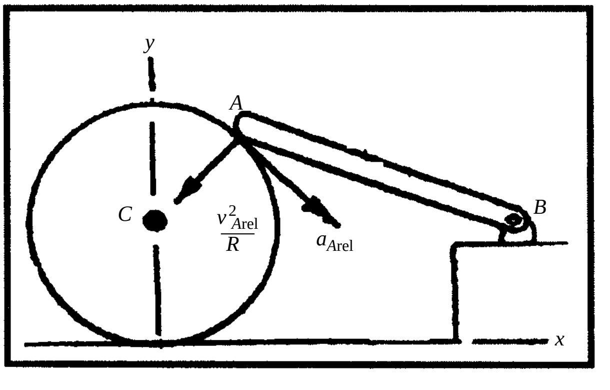
$$-0.707\alpha_{AB} + 28.15 = 0.707a_{A\text{rel}} - 290.3 + 286.6 - 70.7,$$

$$-1.87\alpha_{AB} - 10.64 = -0.707a_{A\text{rel}} - 290.3 + 286.6 - 70.7.$$

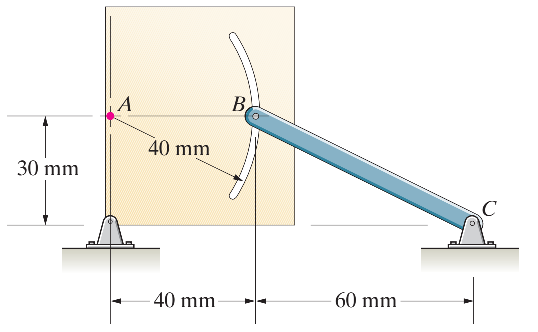
Solve:

$$a_{A\text{rel}} = 80.6 \text{ m/s}^2,$$

$$\boxed{\alpha_{AB} = 64.6 \text{ rad/s}^2} \quad \text{(counterclockwise)}.$$



Problem 17.140* Bar BC rotates with a counterclockwise angular velocity of 2 rad/s. A pin at B slides in a circular slot in the rectangular plate. Determine the angular velocity of the plate and the velocity at which the pin slides relative to the circular slot.



Solution: Choose a coordinate system with the origin O at the lower left pin and the x axis parallel to the plane surface. The unit vector parallel to AB is $\mathbf{e}_{AB} = \mathbf{i}$. The unit vector tangent to the slot at B is $\mathbf{e}_{NAB} = \mathbf{j}$. The velocity of the pin in terms of the motion of BC is $\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$.

$$\mathbf{v}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -60 & 30 & 0 \end{bmatrix} = 2(-30\mathbf{i} - 60\mathbf{j}) = -60\mathbf{i} - 120\mathbf{j} \text{ (mm/s)}.$$

The velocity of the pin in terms of the plate is

$$\mathbf{v}_B = v_{B\text{rel}}\mathbf{j} + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 40 & 30 & 0 \end{bmatrix}$$

$$= v_{B\text{rel}}\mathbf{j} + \omega_{AB}(-30\mathbf{i} + 40\mathbf{j}) \text{ (mm/s)}.$$

Equate the expressions and separate components to obtain

$$-60 = -30\omega_{AB},$$

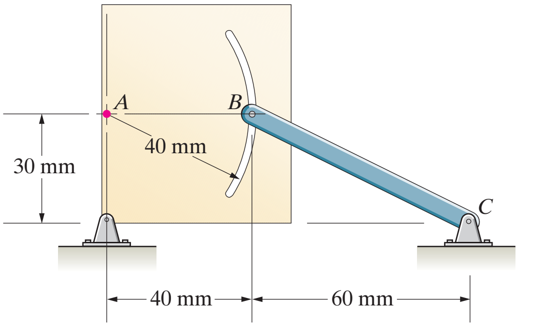
$$-120 = v_{B\text{rel}} + 40\omega_{AB}.$$

Solve:

$$\mathbf{v}_{B\text{rel}} = -200\mathbf{j} \text{ mm/s},$$

$\boxed{\omega_{AB} = 2 \text{ rad/s}}$ (counterclockwise).

Problem 17.141* Bar BC in Problem 17.140 rotates with a constant counterclockwise angular velocity of 2 rad/s. Determine the angular acceleration of the plate.



Solution: Choose the same coordinate system as in Problem 17.140. *Get the velocities*: The unit vector parallel to AB is $\mathbf{e}_{AB} = \mathbf{i}$. The unit vector tangent to the slot at B is $\mathbf{e}_{NAB} = \mathbf{j}$. The velocity of the pin in terms of the motion of BC is

$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -60 & 30 & 0 \end{bmatrix}$$

$$= (-60\mathbf{i} - 120\mathbf{j}) \text{ (mm/s)}.$$

The velocity of the pin in terms of the plate is

$$\mathbf{v}_B = v_{B\text{rel}}\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 40 & 30 & 0 \end{bmatrix}$$

$$= v_{B\text{rel}}\mathbf{e}_{NAB} + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/O}$$

$$= v_{B\text{rel}}\mathbf{j} + \omega_{AB}(-30\mathbf{i} + 40\mathbf{j}) \text{ (mm/s)}.$$

Equate the expressions and separate components to obtain

$$-60 = -30\omega_{AB},$$

$$-120 = v_{B\text{rel}} + 40\omega_{AB}.$$

Solve:

$$\mathbf{v}_{B\text{rel}} = -200\mathbf{j} \text{ mm/s},$$

$$\omega_{AB} = 2 \text{ rad/s} \quad \text{(counterclockwise)}.$$

Get the accelerations: The acceleration of the pin in terms of the arm BC is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\mathbf{r}_{B/C}$$

$$= 0 - 4(-60\mathbf{i} + 30\mathbf{j})$$

$$= 240\mathbf{i} - 120\mathbf{j} \text{ (mm/s}^2).$$

The acceleration of the pin in terms of the plate AB is

$$\mathbf{a}_B = \mathbf{a}_{B\text{rel}} + 2\boldsymbol{\omega}_{AB} \times v_{B\text{rel}}\mathbf{e}_{NAB} + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/O} - \omega_{AB}^2\mathbf{r}_{B/O}.$$

Expand term by term:

$$\mathbf{a}_{B\text{rel}} = a_{B\text{rel}}\mathbf{e}_{NAB} - \left(\frac{v_{B\text{rel}}^2}{40}\right)\mathbf{e}_{AB}$$

$$= a_{B\text{rel}}\mathbf{j} - 1000\mathbf{i} \text{ (mm/s}^2),$$

$$2\boldsymbol{\omega}_{AB} \times v_{B\text{rel}}\mathbf{e}_{NAB} = 800\mathbf{i} \text{ (mm/s}^2).$$

$$\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/O} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 40 & 30 & 0 \end{bmatrix}$$

$$= \alpha_{BA}(-30\mathbf{i} + 40\mathbf{j}) \text{ (mm/s}^2),$$

$$-\omega_{AB}^2(40\mathbf{i} + 30\mathbf{j}) = -160\mathbf{i} - 120\mathbf{j} \text{ (mm/s}^2).$$

Collect terms and separate components to obtain:

$$240 = -1000 + 800 - 30\alpha_{BA} - 160,$$

$$-120 = a_{B\text{rel}} + 40\alpha_{BA} - 120.$$

Solve:

$$a_{B\text{rel}} = 800 \text{ mm/s}^2 \quad \text{(upward)},$$

$$\boxed{\alpha_{AB} = -20 \text{ rad/s}^2}, \text{ (clockwise)}.$$

Problem 17.142* By taking the derivative of Eq. (17.11) with respect to time and using Eq. (17.12), derive Eq. (17.13).

Solution: Eq (17.11) is

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}.$$

Eq (17.12) is

$$\mathbf{v}_{A\text{rel}} = \left(\frac{dx}{dt}\right)\mathbf{i} + \left(\frac{dy}{dt}\right)\mathbf{j} + \mathbf{k}\left(\frac{dz}{dt}\right)\mathbf{k}.$$

Assume that the coordinate system is body fixed and that B is a point on the rigid body, (A is not necessarily a point on the rigid body), such that $\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$, where $\mathbf{r}_{A/B} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and x, y, z are the coordinates of A in body fixed coordinates. Take the derivative of both sides of Eq (17.11):

$$\frac{d\mathbf{v}_A}{dt} = \frac{d\mathbf{v}_B}{dt} + \frac{d\mathbf{v}_{A\text{rel}}}{dt} + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times \frac{d\mathbf{r}_{A/B}}{dt}.$$

By definition,

$$\frac{d\mathbf{v}_A}{dt} = \mathbf{a}_A, \; \frac{d\mathbf{v}_B}{dt} = \mathbf{a}_B, \quad \text{and} \frac{d\boldsymbol{\omega}}{dt} = \boldsymbol{\alpha}.$$

The derivative:

$$\frac{d\mathbf{v}_{A\text{rel}}}{dt} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k} + \frac{dx}{dt}\frac{d\mathbf{i}}{dt} + \frac{dy}{dt}\frac{d\mathbf{j}}{dt} + \frac{dz}{dt}\frac{d\mathbf{k}}{dt}.$$

Using the fact that the derivative of a unit vector represents a rotation of the unit vector,

$$\frac{d\mathbf{i}}{dt} = \boldsymbol{\omega} \times \mathbf{i}, \; \frac{d\mathbf{j}}{dt} = \boldsymbol{\omega} \times \mathbf{j}, \; \frac{d\mathbf{k}}{dt} = \boldsymbol{\omega} \times \mathbf{k}.$$

Substitute into the derivative:

$$\frac{d\mathbf{v}_{A\text{rel}}}{dt} = \mathbf{a}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}}.$$

Noting $\boldsymbol{\omega} \times \frac{d\mathbf{r}_{A/B}}{dt} = \boldsymbol{\omega} \times \left(\frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}\right)$

$$+ \boldsymbol{\omega} \times \left(x\frac{d\mathbf{i}}{dt} + y\frac{d\mathbf{j}}{dt} + z\frac{d\mathbf{k}}{dt}\right)$$

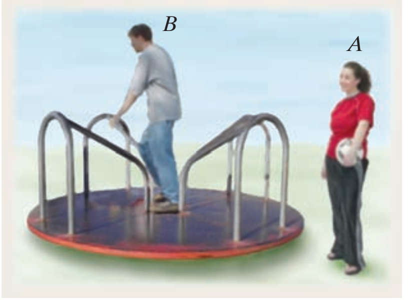
$$= \boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}).$$

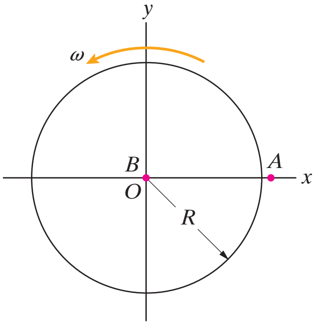
Collect and combine terms: the derivative of Eq (17.11) is

$$\boxed{\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{\text{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})},$$

which is Eq (17.13).

Problem 17.143 In Active Example 17.10, suppose that the merry-go-round has counterclockwise angular velocity ω and counterclockwise angular acceleration α. The person A is standing still on the ground. Determine her acceleration relative to your reference frame at the instant shown.





Solution: First the velocity analysis

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{Arel} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$0 = 0 + \mathbf{v}_{Arel} + \omega\mathbf{k} \times (R\mathbf{i})$$

$$\mathbf{v}_{Arel} = -\omega R\mathbf{j}$$

Now the acceleration analysis

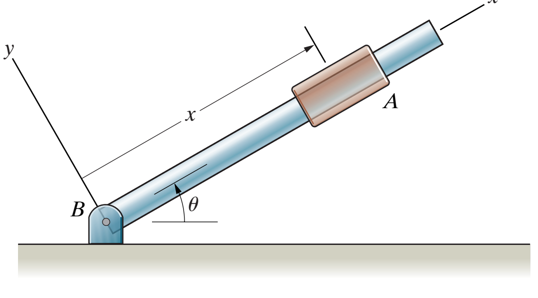
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{Arel} + \mathbf{a} \times \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B} + 2\boldsymbol{\omega} \times \mathbf{v}_{Arel}$$

$$0 = 0 + \mathbf{a}_{Arel} + \alpha\mathbf{k} \times (R\mathbf{i}) - \omega^2(R\mathbf{i}) + 2(\omega\mathbf{k}) \times (-\omega R\mathbf{i})$$

$$\mathbf{a}_{Arel} = -\alpha R\mathbf{j} + \omega^2 R\mathbf{i} + 2\omega^2 R\mathbf{i}$$

$$\boxed{\mathbf{a}_{Arel} = -\omega^2 R\mathbf{i} - \alpha R\mathbf{j}}$$

Problem 17.144 The $x-y$ coordinate system is body fixed with respect to the bar. The angle θ (in radians) is give as a function of time by $\theta = 0.2 + 0.04t^2$. The x coordinate of the sleeve A (in metre) is given as a function of time by $x = 1 + 0.03t^3$. Use Eq. (17.16) to determine the velocity of the sleeve at $t = 4$ s relative to a nonrotating reference frame with its origin at B. (Although you are determining the velocity of A relative to a nonrotating reference frame, your answer will be expressed in components in terms of the body-fixed reference frame.)



Solution: We have

$$\theta = 0.2 + 0.04t^2, \quad \dot{\theta} = 0.08t$$

$$x = 1 + 0.03t^3, \quad \dot{x} = 0.09t^2$$

At the instant $t = 4$s, $\dot{\theta} = 0.32$, $x = 2.92$, $\dot{x} = 1.44$

Thus

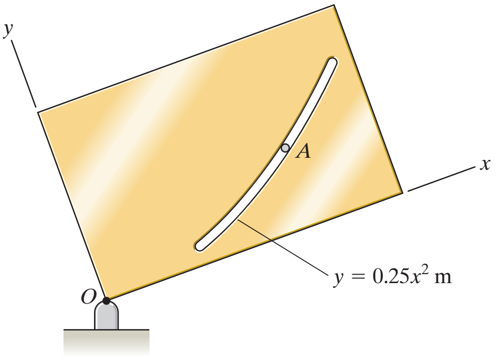
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{Arel} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$= 0 + \dot{x}\mathbf{i} + \dot{\theta}\mathbf{k} \times (x\mathbf{i}) = \dot{x}\mathbf{i} + x\dot{\theta}\mathbf{j}$$

$$= (1.44)\mathbf{i} + (2.92)(0.32)\mathbf{j}$$

$$\boxed{\mathbf{v}_A = (1.44\mathbf{i} + 0.934\mathbf{j}) \text{ m/s.}}$$

Problem 17.145 The metal plate is attached to a fixed ball-and-socket support at O. The pin A slides in a slot in the plate. At the instant shown, $x_A = 1$ m, $dx_A/dt = 2$ m/s, and $d^2x_A/dt^2 = 0$, and the plate's angular velocity and angular acceleration are $\boldsymbol{\omega} = 2\mathbf{k}$ (rad/s) and $\boldsymbol{\alpha} = 0$. What are the x, y, and z components of the velocity and acceleration of A relative to a nonrotating reference frame with its origin at O?



Solution: The velocity is $\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/O}$. The relative velocity is

$$\mathbf{v}_{A\text{rel}} = \left(\frac{dx}{dt}\right)\mathbf{i} + \left(\frac{dy}{dt}\right)\mathbf{j} + \left(\frac{dz}{dt}\right)\mathbf{k},$$

where $\dfrac{dx}{dt} = 2$ m/s, $\dfrac{dy}{dt} = \dfrac{d}{dt}0.25x^2 = 0.5x\dfrac{dx}{dt} = 1$ m/s, $\dfrac{dz}{dt} = 0$, and $\mathbf{r}_{A/O} = x\mathbf{i} + y\mathbf{j} + z\mathbf{j} = \mathbf{i} + 0.25\mathbf{j} + 0$, from which

$$\boxed{\begin{aligned}\mathbf{v}_A &= 2\mathbf{i} + \mathbf{j} + \omega(\mathbf{k} \times (\mathbf{i} + 0.25\mathbf{j})) = 2\mathbf{i} + \mathbf{j} + 2(-0.25\mathbf{i} + \mathbf{j}) \\ &= 1.5\mathbf{i} + 3\mathbf{j} \text{ (m/s)}.\end{aligned}}$$

The acceleration is

$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A\text{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/O} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/O}).$$

Noting

$$\mathbf{a}_{A\text{rel}} = \left(\frac{d^2x}{dt^2}\right)\mathbf{i} + \left(\frac{d^2y}{dt^2}\right)\mathbf{j} + \left(\frac{d^2z}{dt^2}\right)\mathbf{k},$$

where

$$\left(\frac{d^2x}{dt^2}\right) = 0, \ \frac{d^2y}{dt^2} = \frac{d^2}{dt^2}0.25x^2 = 0.5\left(\frac{dx}{dt}\right)^2 = 2, \ \left(\frac{d^2z}{dt^2}\right) = 0.$$

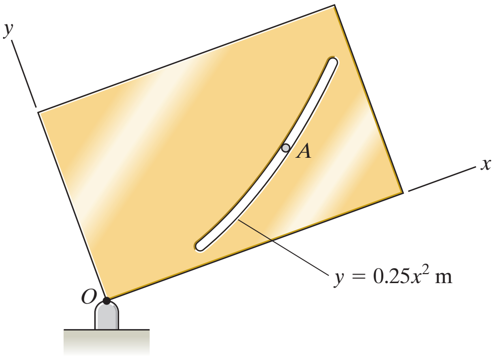
Substitute:

$$\mathbf{a}_A = 2\mathbf{j} + 2\omega(\mathbf{k} \times (2\mathbf{i} + \mathbf{j})) + \omega^2(\mathbf{k} \times (\mathbf{k} \times (\mathbf{i} + 0.25\mathbf{j})))$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 2 & 1 & 0 \end{bmatrix} + 4\mathbf{k} \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0.25 & 0 \end{bmatrix}$$

$$\mathbf{a}_A = 2\mathbf{j} - 4\mathbf{i} + 4\omega\mathbf{j} + 4\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -0.25 & 1 & 0 \end{bmatrix} \quad \boxed{= -8\mathbf{i} + 9\mathbf{j} \text{ (m/s}^2)}$$

Problem 17.146 Suppose that at the instant shown in Problem 17.145, $x_A = 1$ m, $dx_A/dt = -3$ m/s, $d^2x_A/dt^2 = 4$ m/s^2, and the plate's angular velocity and angular acceleration are $\boldsymbol{\omega} = -4\mathbf{j} + 2\mathbf{k}$ (rad/s), and $\boldsymbol{\alpha} = 3\mathbf{i} - 6\mathbf{j}$ (rad/s^2). What are the x, y, z components of the velocity and acceleration of A relative to a nonrotating reference frame that is stationary with respect to O?



Solution: The velocity is $\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/O}$. The relative velocity is

$$\mathbf{v}_{A\text{rel}} = \left(\frac{dx}{dt}\right)\mathbf{i} + \left(\frac{dy}{dt}\right)\mathbf{j} + \left(\frac{dz}{dt}\right)\mathbf{k},$$

where $\dfrac{dx}{dt} = -3$ m/s, $\dfrac{dy}{dt} = \dfrac{d}{dt}0.25x^2 = 0.5x\dfrac{dx}{dt} = -15$ m/s, $\dfrac{dz}{dt} = 0$, and $\mathbf{r}_{A/O} = x\mathbf{i} + y\mathbf{j} + z\mathbf{j} = \mathbf{i} + 0.25\mathbf{j} + 0$, from which $\mathbf{v}_A = -3\mathbf{i} - 1.5\mathbf{j} + \boldsymbol{\omega} \times (\mathbf{i} + 0.25\mathbf{j})$.

$$\mathbf{v}_A = -3\mathbf{i} - 1.5\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 2 \\ 1 & 0.25 & 0 \end{bmatrix} = -3\mathbf{i} - 1.5\mathbf{j} - 0.5\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$$

$$\boxed{= -3.5\mathbf{i} + 0.5\mathbf{j} + 4\mathbf{k} \text{ (m/s)}}$$

The acceleration is $\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A\text{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/O} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/O})$. Noting

$$\mathbf{a}_{A\text{rel}} = \left(\frac{d^2x}{dt^2}\right)\mathbf{i} + \left(\frac{d^2y}{dt^2}\right)\mathbf{j} + \left(\frac{d^2z}{dt^2}\right)\mathbf{k},$$

where

$$\left(\frac{d^2x}{dt^2}\right) = 4 \text{ m/s}^2,$$

$$\frac{d^2y}{dt^2} = \frac{d^2}{dt^2}0.25x^2 = 0.5\left(\frac{dx}{dt}\right)^2 + 0.5x\left(\frac{d^2x}{dt^2}\right) = 6.5 \text{ (m/s}^2\text{)},$$

$$\left(\frac{d^2z}{dt^2}\right) = 0, \boldsymbol{\alpha} = 3\mathbf{i} - 6\mathbf{j} \text{ (rad/s}^2\text{)}, \mathbf{v}_{A\text{rel}} = -3\mathbf{i} - 1.5\mathbf{j},$$

and from above: $\boldsymbol{\omega} \times \mathbf{r}_{A/O} = -0.5\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. Substitute:

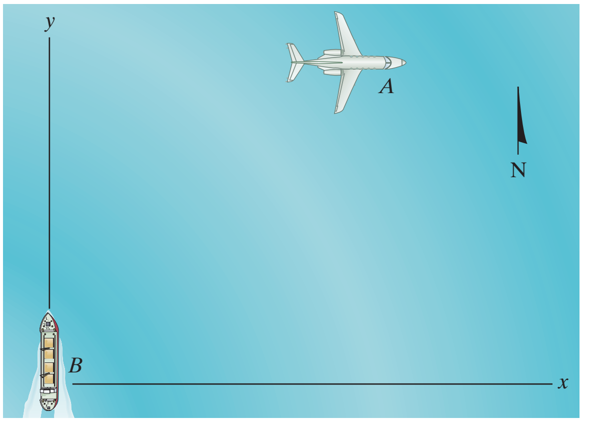
$$\mathbf{a}_A = 4\mathbf{i} + 6.5\mathbf{j} + 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 2 \\ -3 & -1.5 & 0 \end{bmatrix} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -6 & 0 \\ 1 & 0.25 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 2 \\ -0.5 & 2 & 4 \end{bmatrix}.$$

$$\mathbf{a}_A = 4\mathbf{i} + 6.5\mathbf{j} + 2(3\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}) + (6.75\mathbf{k}) + (-20\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\boxed{\mathbf{a}_A = -10\mathbf{i} - 6.5\mathbf{j} - 19.25\mathbf{k} \text{ (m/s}^2\text{)}}$$

Problem 17.147 The coordinate system is fixed relative to the ship B. At the instant shown, the ship is sailing north at 5 m/s relative to the earth, and its angular velocity is 0.26 rad/s counterclockwise. Using radar, it is determined that the position of the airplane is $1080\mathbf{i} + 1220\mathbf{j} + 6300\mathbf{k}$ (m) and its velocity relative to the ship's coordinate system is $870\mathbf{i} - 45\mathbf{j} - 21\mathbf{k}$ (m/s). What is the airplane's velocity relative to the earth? (See Example 17.11.)



Solution:

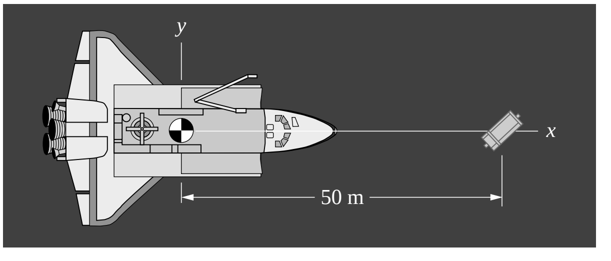
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{Arel} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$= 5\mathbf{j} + (870\mathbf{i} - 45\mathbf{j} - 21\mathbf{k}) + (0.26\mathbf{k}) \times (1080\mathbf{i} + 1220\mathbf{j} + 6300\mathbf{k})$$

$$= (870\mathbf{i} - 40\mathbf{j} - 21\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.26 \\ 1080 & 1220 & 6300 \end{vmatrix}$$

$$\boxed{\mathbf{v}_A = (553\mathbf{i} + 241\mathbf{j} - 21\mathbf{k})\ \text{m/s}.}$$

Problem 17.148 The space shuttle is attempting to recover a satellite for repair. At the current time, the satellite's position relative to a coordinate system fixed to the shuttle is $50\mathbf{i}$ (m). The rate gyros on the shuttle indicate that its current angular velocity is $0.05\mathbf{j} + 0.03\mathbf{k}$ (rad/s). The Shuttle pilot measures the velocity of the satellite relative to the body-fixed coordinate system and determines it to be $-2\mathbf{i} - 1.5\mathbf{j} + 2.5\mathbf{k}$ (rad/s). What are the x, y, and z components of the satellite's velocity relative to a nonrotating coordinate system with its origin fixed to the shuttle's center of mass?



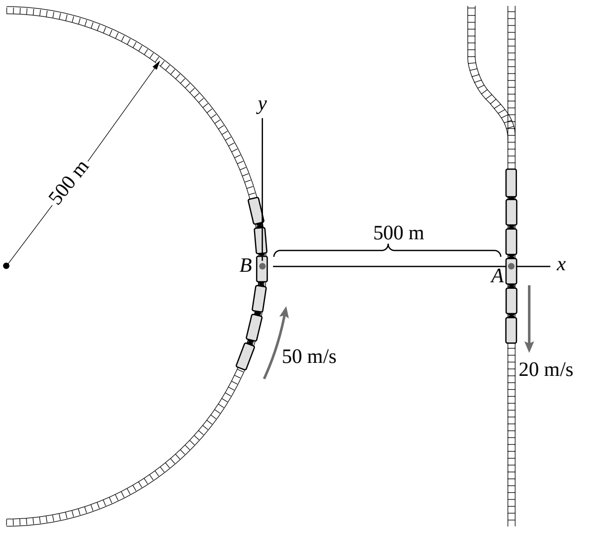
Solution: The velocity of the satellite is

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$= 0 - 2\mathbf{i} - 1.5\mathbf{j} + 2.5\mathbf{k} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.05 & 0.03 \\ 50 & 0 & 0 \end{bmatrix}$$

$$\boxed{= -2\mathbf{i} + 1.5\mathbf{j} + 2.5\mathbf{k} - 1.5\mathbf{j} - 2.5\mathbf{k} = -2\mathbf{i}\ (\text{m/s})}$$

Problem 17.149 The train on the circular track is traveling at a constant speed of 50 m/s in the direction shown. The train on the straight track is traveling at 20 m/s in the direction shown and is increasing its speed at 2 m/s^2. Determine the velocity of passenger A that passenger B observes relative to the given coordinate system, which is fixed to the car in which B is riding.



Solution:

The angular velocity of B is $\omega = \dfrac{50}{500} = 0.1$ rad/s.

The velocity of A is $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$.

At the instant shown, $\mathbf{v}_A = -20\mathbf{j}$ (m/s), $\mathbf{v}_B = +50\mathbf{j}$ (m/s),

and $\mathbf{r}_{A/B} = 500\mathbf{i}$ (m), from which

$$\mathbf{v}_{A\text{rel}} = -20\mathbf{j} - 50\mathbf{j} - \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.1 \\ 500 & 0 & 0 \end{bmatrix} = -20\mathbf{j} - 50\mathbf{j} - 50\mathbf{j},$$

$$\boxed{\mathbf{v}_{A\text{rel}} = -120\mathbf{j} \text{ (m/s)}}$$

Problem 17.150 In Problem 17.149, determine the acceleration of passenger A that passenger B observes relative to the coordinate system fixed to the car in which B is riding.

Solution:

Use the solution to Problem 17.149:

$\mathbf{v}_{A\text{rel}} = -120\mathbf{j}$ (m/s),

$$\omega = \frac{50}{500} = 0.1 \text{ rad/s},$$

$\mathbf{r}_{A/B} = 500\mathbf{i}$ (m).

The acceleration of A is $\mathbf{a}_A = -2\mathbf{j}$ (m/s),

The acceleration of B is

$\mathbf{a}_B = -500(\omega^2)\mathbf{i} = -5\mathbf{i}$ m/s^2,

and $\boldsymbol{\alpha} = 0$, from which

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A\text{rel}} + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}).$$

Rearrange:

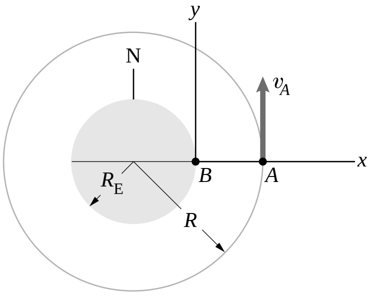
$$\mathbf{a}_{A\text{rel}} = \mathbf{a}_A - \mathbf{a}_B - 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}).$$

$$\mathbf{a}_{\text{Re}l} = -2\mathbf{j} + 5\mathbf{i} - 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ 0 & -120 & 0 \end{bmatrix} - \omega^2 \left(\mathbf{k} \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 500 & 0 & 0 \end{bmatrix} \right).$$

$$\mathbf{a}_{\text{Re}l} = -2\mathbf{j} + 5\mathbf{i} - 2(120\omega)\mathbf{i} - \omega^2 \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 500 & 0 \end{bmatrix},$$

$$\boxed{\mathbf{a}_{A\text{rel}} = -14\mathbf{i} - 2\mathbf{j} \text{ (m/s}^2\text{)}}.$$

Problem 17.151 The satellite A is in a circular polar orbit (a circular orbit that intersects the earth's axis of rotation). The radius of the orbit is R, and the magnitude of the satellite's velocity relative to a non-rotating reference frame with its origin at the center of the earth is v_A. At the instant shown, the satellite is above the equator. An observer B on the earth directly below the satellite measures its motion using the earth-fixed coordinate system shown. What are the velocity and acceleration of the satellite relative to B's earth-fixed coordinate system? The radius of the earth is R_E and the angular velocity of the earth is ω_E. (See Example 17.12.)



Solution: From the sketch, in the coordinate system shown, the location of the satellite in this system is $\mathbf{r}_A = (R - R_E)\mathbf{i}$, from which $\mathbf{r}_{A/B} = \mathbf{r}_A - 0 = (R - R_E)\mathbf{i}$. The angular velocity of the observer is $\boldsymbol{\omega}_E = -\omega_E\mathbf{k}$. The velocity of the observer is $\mathbf{v}_B = -\omega_E R_E\mathbf{k}$. The velocity of the satellite is $\mathbf{v}_A = v_A\mathbf{j}$.
The relative velocity is

$$\mathbf{v}_{A\text{rel}} = \mathbf{v}_A - \mathbf{v}_B - \boldsymbol{\omega}_E \times \mathbf{r}_{A/B},$$

$$\mathbf{v}_{A\text{rel}} = v_A\mathbf{j} + \omega_E R_E\mathbf{k} - (\omega_E)\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ R - R_E & 0 & 0 \end{bmatrix}$$

$$\boxed{= v_A\mathbf{j} + R\omega_E\mathbf{k}}.$$

From Eqs (17.26) and (17.27)

$$\mathbf{a}_{A\text{rel}} = \mathbf{a}_A - \mathbf{a}_B - 2\boldsymbol{\omega}_E \times \mathbf{v}_{A\text{rel}} - \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \boldsymbol{\omega}_E \times (\boldsymbol{\omega}_E \times \mathbf{r}_{A/B}).$$

The accelerations:

$$\mathbf{a}_A = -\omega_A^2 R\mathbf{i} = -\left(\frac{v_A^2}{R}\right)\mathbf{i},\ \mathbf{a}_B = -\omega_E^2 R_E\mathbf{i},\ \boldsymbol{\alpha} = 0, \text{ from which}$$

$$\mathbf{a}_{A\text{rel}} = -\left(\frac{v_A^2}{R}\right) + \mathbf{i}\omega_E^2 R_E - 2\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_E & 0 \\ 0 & v_A & R\omega_E \end{bmatrix}$$

$$- \boldsymbol{\omega}_E \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_E & 0 \\ R - R_E & 0 & 0 \end{bmatrix}.$$

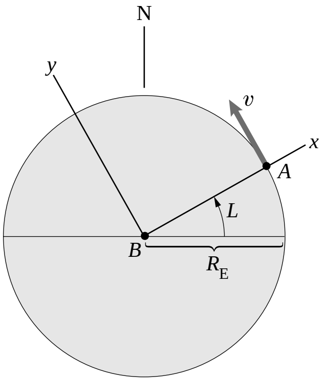
$$\mathbf{a}_{A\text{rel}} = -\omega_A^2 R\mathbf{i} + \omega_E^2 R_E\mathbf{i}$$

$$- 2\omega_E^2 R\mathbf{i} - \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_E & 0 \\ 0 & 0 & -\omega_E(R - R_E) \end{bmatrix}$$

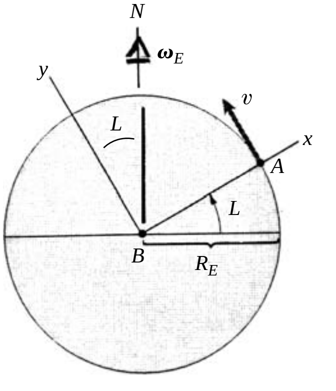
$$= -\left(\frac{v_A^2}{R}\right)\mathbf{i} + \omega_E^2 R_E\mathbf{i} - 2\omega_E^2 R\mathbf{i} + \omega_E^2(R - R_E)\mathbf{i}$$

$$\boxed{\mathbf{a}_{A\text{rel}} = -\left(\left(\frac{v_A^2}{R}\right) + \omega_E^2 R\right)\mathbf{i}}$$

Problem 17.152 A car A at north latitude L drives north on a north–south highway with constant speed v. The earth's radius is R_E, and the earth's angular velocity is ω_E. (The earth's angular velocity vector points north.) The coordinate system is earth fixed, and the x axis passes through the car's position at the instant shown. Determine the car's velocity and acceleration (a) relative to the earth-fixed coordinate system and (b) relative to a nonrotating reference frame with its origin at the center of the earth.



Solution:



(a) In earth fixed coords,

$$\mathbf{v}_{\text{rel}} = v\mathbf{j},$$

$$\mathbf{a}_{\text{rel}} = -v^2/R_E\mathbf{i}. \text{ (motion in a circle)}$$

(b) $\mathbf{v}_A = \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega}_E \times \mathbf{r}_{A/B} + \mathbf{v}_B (\mathbf{v}_B = 0)$

$$= v\mathbf{j} + (\omega_E \sin L\mathbf{i} + \omega_E \cos L\mathbf{j}) \times R_E\mathbf{i}$$

$$\underline{\mathbf{v}_A = v\mathbf{j} - \omega_E R_E \cos L\mathbf{k}}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A\text{rel}} + 2\boldsymbol{\omega}_E \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B}$$

$$+ \boldsymbol{\omega}_E \times (\boldsymbol{\omega}_E \times \mathbf{r}_{A/B})$$

where $\boldsymbol{\omega}_E = \omega_E \sin L\mathbf{i} + \omega_E \cos L\mathbf{j}$

and $\mathbf{r}_{A/B} = R_E\mathbf{i}$

$$\mathbf{a}_A = 0 - \frac{v^2}{R_E}\mathbf{i} + 2v\omega_E \sin L\mathbf{k}$$

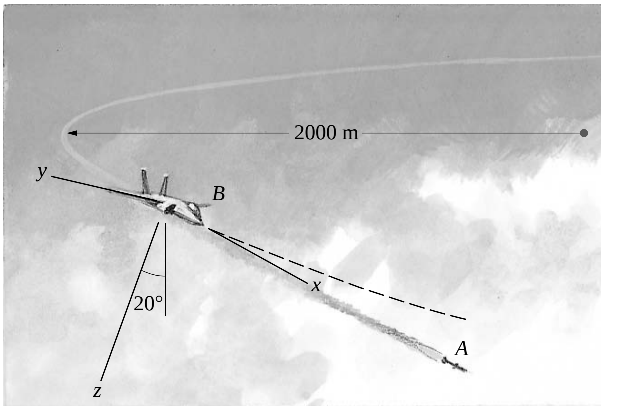
$$+ (\omega_E \sin L\mathbf{i} + \omega_E \cos L\mathbf{j}) \times (-\omega_E R_E \cos L\mathbf{k})$$

$$\mathbf{a}_A = -\left(\frac{v^2}{R_E} + \omega_E^2 R_E \cos^2 L\right)\mathbf{i}$$

$$+ (\omega_E^2 R_E \sin L \cos L)\mathbf{j}$$

$$+ 2v\omega_E \sin L\mathbf{k}$$

Problem 17.153 The airplane B conducts flight tests of a missile. At the instant shown, the airplane is traveling at 200 m/s relative to the earth in a circular path of 2000-m radius *in the horizontal plane*. The coordinate system is fixed relative to the airplane. The x axis is tangent to the plane's path and points forward. The y axis points out the plane's right side, and the z axis points out the bottom of the plane. The plane's bank angle (the inclination of the z axis from the vertical) is constant and equal to 20°. *Relative to the airplane's coordinate system*, the pilot measures the missile's position and velocity and determines them to be $\mathbf{r}_{A/B} = 1000\mathbf{i}$ (m) and $\mathbf{v}_{A/B} = 100.0\mathbf{i} + 94.0\mathbf{j} + 34.2\mathbf{k}$ (m/s).



(a) What are the x,y, and z components of the airplane's angular velocity vector?
(b) What are the x, y, and z components of the missile's velocity relative to the earth?

Solution:

(a) The bank angle is a rotation about the x axis; assume that the rotation is counterclockwise, so that the z axis is rotated toward the positive y axis. The magnitude of the angular velocity is

$$\omega = \frac{200}{2000} = 0.1 \text{ rad/s.}$$

In terms of airplane fixed coordinates,

$$\boldsymbol{\omega} = 0.1(\mathbf{i}\sin 20^\circ - \mathbf{j}\cos 20^\circ) \text{ (rad/s).}$$

$$\boxed{\boldsymbol{\omega} = 0.03242\mathbf{j} - 0.0940\mathbf{k} \text{ rad/s}}$$

(b) The velocity of the airplane in earth fixed coordinates is

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$= 200\mathbf{i} + 100\mathbf{i} + 94.0\mathbf{j} + 34.2\mathbf{k}$$

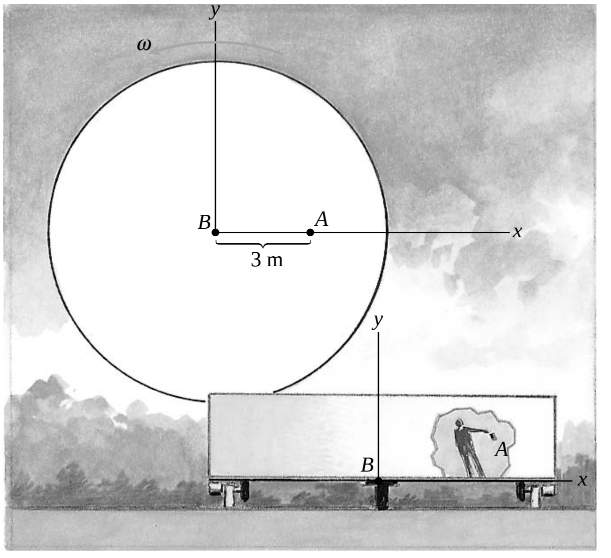
$$+ \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.0342 & -0.0940 \\ 1000 & 0 & 0 \end{bmatrix}$$

$$\boxed{\mathbf{v}_A = 300\mathbf{i} + 94.0\mathbf{j} + 34.2\mathbf{k} - 94.0\mathbf{j} - 34.2\mathbf{k} = 300\mathbf{i} \text{ (m/s)}}$$

Problem 17.154 To conduct experiments related to long-term spaceflight, engineers construct a laboratory on earth that rotates about the vertical axis at B with a constant angular velocity ω of one revolution every 6 s. They establish a laboratory-fixed coordinate system with its origin at B and the z axis pointing upward. An engineer holds an object stationary relative to the laboratory at point A, 3 m from the axis of rotation, and releases it. At the instant he drops the object, determine its acceleration relative to the laboratory-fixed coordinate system,

(a) assuming that the laboratory-fixed coordinate system is inertial and
(b) not assuming that the laboratory-fixed coordinate system is inertial, but assuming that an earth-fixed coordinate system with its origin at B is inertial.

(See Example 17.13.)



Solution: (a) If the laboratory system is inertial, Newton's second law is $\mathbf{F} = m\mathbf{a}$. The only force is the force of gravity; so that as the object free falls the acceleration is

$$\boxed{-g\mathbf{k} = -9.81\mathbf{k}\ (\text{m/s}^2)}.$$

If the earth fixed system is inertial, the acceleration observed is the centripetal acceleration and the acceleration of gravity:

$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_B) - \mathbf{g},$$

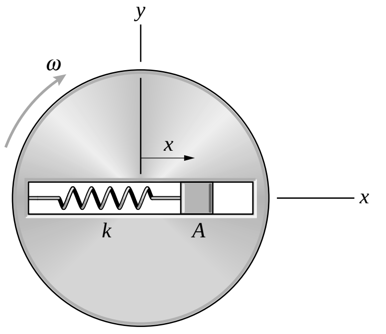
where the angular velocity is the angular velocity of the coordinate system relative to the inertial frame.

$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) - \mathbf{g} = -\left(\frac{2\pi}{6}\right)^2 \left(\mathbf{k} \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 3 & 0 & 0 \end{bmatrix}\right) - 9.81\mathbf{k}$$

$$= \left(\frac{\pi^2}{3}\right)\mathbf{i} - 9.81\mathbf{k}$$

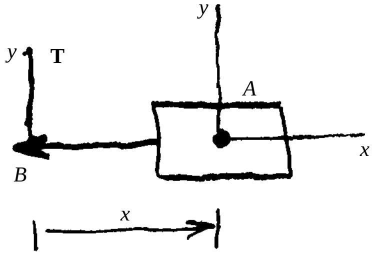
$$\boxed{= 3.29\mathbf{i} - 9.81\mathbf{k}\ (\text{m/s}^2)}$$

Problem 17.155 The disk rotates *in the horizontal plane* about a fixed shaft at the origin with constant angular velocity $w = 10$ rad/s. The 2-kg slider A moves in a smooth slot in the disk. The spring is unstretched when $x = 0$ and its constant is $k = 400$ N/m. Determine the acceleration of A relative to the body-fixed coordinate system when $x = 0.4$ m.



Strategy: Use Eq. (17.30) to express Newton's second law for the slider in terms of the body-fixed coordinate system.

Solution:



$$\mathbf{T} = -kx = (-400)(0.4)$$

$$\mathbf{T} = -160\mathbf{i} \text{ Newtons}$$

$$\sum \mathbf{F} = -160\mathbf{i} = m\mathbf{a}_{A\text{rel}} + m[\cancel{\mathbf{a}}_B^0 + 2\cancel{\boldsymbol{\omega}}^0 \times \mathbf{v}_{\text{rel}}$$

$$+ \cancel{\boldsymbol{\alpha}}^0 \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})]$$

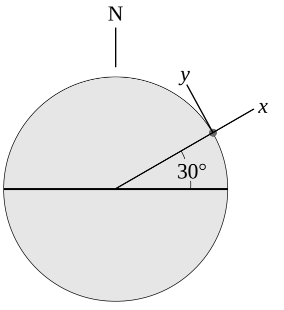
where $\boldsymbol{\omega} = -10\mathbf{k}$, $\mathbf{r}_{A/B} = 0.4\mathbf{i}$, $m = 2$

$$-160\mathbf{i} = 2\mathbf{a}_{A\text{rel}} - 80\mathbf{i}$$

$$\underline{\mathbf{a}_{A\text{rel}} = -40\mathbf{i}\ (\text{m/s}^2)}.$$

Problem 17.156* Engineers conduct flight tests of a rocket at 30° north latitude. They measure the rocket's motion using an earth-fixed coordinate system with the x axis pointing upward and the y axis directed northward. At a particular instant, the mass of the rocket is 4000 kg, the velocity of the rocket relative to the engineers' coordinate system is $2000\mathbf{i} + 2000\mathbf{j}$ (m/s), and the sum of the forces exerted on the rocket by its thrust, weight, and aerodynamic forces is $400\mathbf{i} + 400\mathbf{j}$ (N). Determine the rocket's acceleration relative to the engineers' coordinate system,

(a) assuming that their earth-fixed coordinate system is inertial and
(b) not assuming that their earth-fixed coordinate system is inertial.



Solution: Use Eq. (17.22):

$$\sum \mathbf{F} - m[\mathbf{a}_B + 2\boldsymbol{\omega} \times \mathbf{v}_{A\mathrm{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})]$$

$$= m\mathbf{a}_{A\mathrm{rel}}.$$

(a) If the earth fixed coordinate system is assumed to be inertial, this reduces to $\sum \mathbf{F} = m\mathbf{a}_{A\mathrm{rel}}$, from which

$$\mathbf{a}_{A\mathrm{rel}} = \frac{1}{m}\sum \mathbf{F} = \frac{1}{4000}(400\mathbf{i} + 400\mathbf{j})$$

$$\boxed{= 0.1\mathbf{i} + 0.1\mathbf{j}\ (\mathrm{m/s^2})}$$

(b) If the earth fixed system is not assumed to be inertial, $\mathbf{a}_B = -R_E\omega_E^2 \cos^2\lambda\mathbf{i} + R_E\omega_E^2 \cos\lambda \sin\lambda\mathbf{j}$, the angular velocity of the rotating coordinate system is $\boldsymbol{\omega} = \omega_E \sin\lambda\mathbf{i} + \omega_E \cos\lambda\mathbf{j}$ (rad/s). The relative velocity in the earth fixed system is $\mathbf{v}_{A\mathrm{rel}} = 2000\mathbf{i} + 2000\mathbf{j}$ (m/s), and $\mathbf{r}_{A/B} = R_E\mathbf{i}$ (m).

$$2\boldsymbol{\omega} \times \mathbf{v}_{A\mathrm{rel}} = 2\omega_E \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sin\lambda & \cos\lambda & 0 \\ 2000 & 2000 & 0 \end{bmatrix}$$

$$= 4000\omega_E(\sin\lambda - \cos\lambda)\mathbf{k}$$

$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) = \boldsymbol{\omega} \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_E \sin\lambda & \omega_E \cos\lambda & 0 \\ R_E & 0 & 0 \end{bmatrix}$$

$$= \boldsymbol{\omega} \times (-R_E\omega_E \cos\lambda)\mathbf{k}$$

$$\boldsymbol{\omega} \times (-R_E\omega_E \cos\lambda)\mathbf{k} = R_E\omega_E^2 \cos\lambda \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sin\lambda & \cos\lambda & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= (-R_E\omega_E^2 \cos^2\lambda)\mathbf{i} + (R_E\omega_E^2 \cos\lambda \sin\lambda)\mathbf{j}.$$

Collect terms,

$$\mathbf{a}_{A\mathrm{rel}} = +(R_E\omega_E^2 \cos^2\lambda)\mathbf{i} - (R_E\omega_E^2 \cos\lambda \sin\lambda)\mathbf{j}$$

$$- 4000\omega_E(\sin\lambda - \cos\lambda)\mathbf{k} + 0.1\mathbf{i} + 0.1\mathbf{j}.$$

Substitute values:

$$R_E = 6336 \times 10^3 \text{ m}, \omega_E = 0.73 \times 10^{-4} \text{ rad/s}, \lambda = 30°,$$

$$\boxed{\mathbf{a}_{A\mathrm{rel}} = 0.125\mathbf{i} + 0.0854\mathbf{j} + 0.1069\mathbf{k}\ (\mathrm{m/s^2})}$$

Note: The last two terms in the parenthetic expression for $\mathbf{a}_A$ in

$$\sum \mathbf{F} - m[\mathbf{a}_B + 2\boldsymbol{\omega} \times \mathbf{v}_{A\mathrm{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})]$$

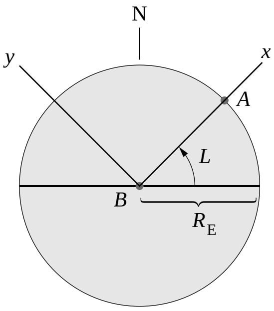
$$= m\mathbf{a}_{A\mathrm{rel}}$$

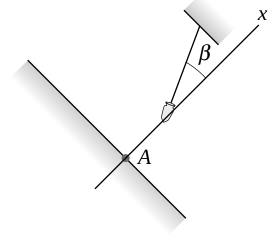
can be neglected without significant change in the answers.

Problem 17.157* Consider a point A on the surface of the earth at north latitude L. The radius of the earth is R_E and its angular velocity is ω_E. A plumb bob suspended just above the ground at point A will hang at a small angle β relative to the vertical because of the earth's rotation. Show that β is related to the latitude by

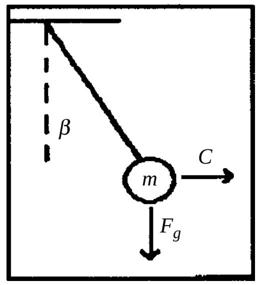
$$\tan\beta = \frac{\omega_E^2 R_E \sin L \cos L}{g - \omega_E^2 R_E \cos^2 L}.$$

Strategy: Using the earth-fixed coordinate system shown, express Newton's second law in the form given by Eq. (17.22).





Solution: Use Eq. (17.25). $\sum \mathbf{F} - m[\mathbf{a}_B + 2\boldsymbol{\omega} \times \mathbf{v}_{A\text{rel}} + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})] = m\mathbf{a}_{A\text{rel}}$. The bob is stationary, so that $\mathbf{v}_{A\text{rel}} = 0$. The origin of the coordinate system is stationary, so that $\mathbf{a}_B = 0$. The external force is the weight of the bob $\sum \mathbf{F} = m\mathbf{g}$. The relative acceleration is the apparent acceleration due to gravity, $m\mathbf{a}_{A\text{rel}} = m\mathbf{g}_{\text{Apparent}}$. Substitute:



$$\mathbf{g}_{\text{Apparent}} = \mathbf{g} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})$$

$$= g\mathbf{i} - \boldsymbol{\omega} \times \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_E \sin L & \omega_E \cos L & 0 \\ R_E & 0 & 0 \end{bmatrix}$$

$$\mathbf{g}_{\text{Apparent}} = g\mathbf{i} - \boldsymbol{\omega} \times (-R_E\omega_E \cos L)\mathbf{k}$$

$$= g\mathbf{i} + R_E\omega_E^2 \cos L \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sin L & \cos L & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\mathbf{g}_{\text{Apparent}} = g\mathbf{i} - (R_E\omega_E^2 \cos^2 L)\mathbf{i} + (R_E\omega_E^2 \cos L \sin L)\mathbf{j}.$$

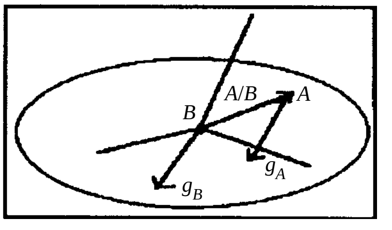
The vertical component of the apparent acceleration due to gravity is $g_{\text{vertical}} = g - R_E\omega_E^2 \cos^2 L$. The horizontal component of the apparent acceleration due to gravity is $g_{\text{horizontal}} = R_E\omega_E^2 \cos L \sin L$. From equation of angular motion, the moments about the bob suspension are $M_{\text{vertical}} = (\lambda \sin\beta)mg_{\text{vertical}}$ and $M_{\text{horizontal}} = (\lambda \cos\beta)mg_{\text{horizontal}}$, where λ is the length of the bob, and m is the mass of the bob. In equilibrium, $M_{\text{vertical}} = M_{\text{horizontal}}$, from which $g_{\text{vertical}} \sin\beta = g_{\text{horizontal}} \cos\beta$. Substitute and rearrange:

$$\boxed{\tan\beta = \frac{g_{\text{horizontal}}}{g_{\text{vertical}}} = \frac{R_E\omega_E^2 \cos L \sin L}{g - R_E\omega_E^2 \cos^2 L}}.$$

Problem 17.158* Suppose that a space station is in orbit around the earth and two astronauts on the station toss a ball back and forth. They observe that the ball appears to travel between them in a straight line at constant velocity.

(a) Write Newton's second law for the ball as it travels between the astronauts in terms of a nonrotating coordinate system with its origin fixed to the station. What is the term $\sum F$? Use the equation you wrote to explain the behavior of the ball observed by the astronauts.

(b) Write Newton's second law for the ball as it travels between the astronauts in terms of a nonrotating coordinate system with its origin fixed to the center of the earth. What is the term $\sum \mathbf{F}$? Explain the difference between this equation and the one you obtained in part (a).

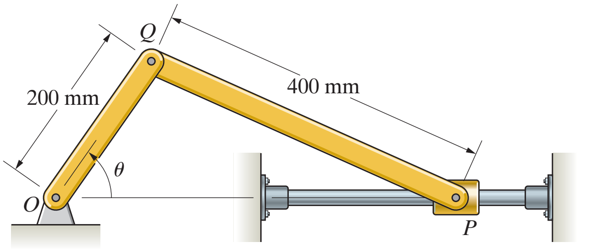


Solution: An earth-centered, non-rotating coordinate system can be treated as inertial for analyzing the motions of objects near the earth (See Section 17.2.) Let O be the reference point of this reference frame, and let B be the origin of the non-rotating reference frame fixed to the space station, and let A denote the ball. The orbiting station and its contents and the station-fixed non-rotating frame are in free fall about the earth (they accelerate relative to the earth due to the earth's gravitational attraction), so that the forces on the ball in the fixed reference frame *exclude the earth's gravitational attraction*. Let $\mathbf{g}_B$ be the station's acceleration, and let $\mathbf{g}_A$ be the ball's acceleration relative to the earth due to the earth's gravitational attraction. Let $\sum F$ be the sum of all forces on the ball, *not including the earth's gravitational attraction*. Newton's second law for the ball of mass m is $\sum \mathbf{F} + m\mathbf{g}_A = m\mathbf{a}_A = m(\mathbf{a}_B + \mathbf{a}_{A/B}) = m\mathbf{g}_B + m\mathbf{a}_{A/B}$. Since the ball is within a space station whose dimensions are small compared to the distance from the earth, $\mathbf{g}_A$ is equal to $\mathbf{g}_B$ within a close approximation, from which $\sum \mathbf{F} = m\mathbf{a}_{A/B}$. The sum of the forces on the ball *not including the force exerted by the earth's gravitational attraction* equals the mass times the ball's acceleration relative to a reference frame fixed with respect to the station. As the astronauts toss the ball back and forth, the only other force on it is aerodynamic drag. Neglecting aerodynamic drag, $\mathbf{a}_{A/B} = 0$, from which the ball will travel in a straight line with constant velocity.

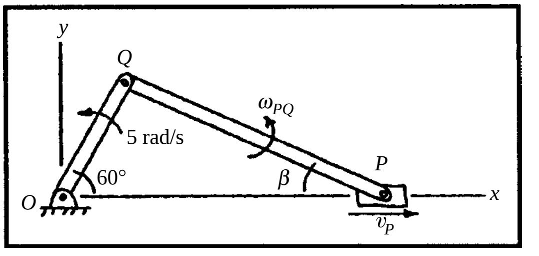
(b) Relative to the earth-centered non-rotating reference frame, Newton's second law for the ball is $\sum \mathbf{F} = m\mathbf{a}_A$ where $\sum \mathbf{F}$ is the sum of all forces on the ball, including aerodynamic drag *and the force due to the earth's gravitational attraction*. Neglect drag, from which $\mathbf{a}_A = \mathbf{g}_A$; the ball's acceleration is its acceleration due to the earth's gravitational attraction, because in this case we are determining the ball's motion relative to the earth.

Note: An obvious unstated assumption is that the time of flight of the ball as it is tossed between the astronauts is much less than the period of an orbit. Thus the very small acceleration differences $\mathbf{g}_A - \mathbf{g}_B$ will have a negligible effect on the path of the ball over the short time interval.

Problem 17.159 If $\theta = 60^\circ$ and bar OQ rotates in the counterclockwise direction at 5 rad/s, what is the angular velocity of bar PQ?



Solution: By applying the law of sines, $\beta = 25.7^\circ$. The velocity of Q is



$\mathbf{v}_Q = \mathbf{v}_0 + \boldsymbol{\omega}_{OQ} \times \mathbf{r}_{Q/O}$ or

$$\mathbf{v}_Q = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 5 \\ 0.2\cos 60^\circ & 0.2\sin 60^\circ & 0 \end{vmatrix} = -\sin 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j}.$$

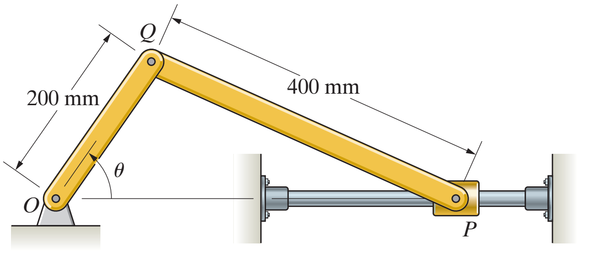
The velocity of P is

$$v_p \mathbf{i} = \mathbf{v}_Q + \boldsymbol{\omega}_{PQ} \times \mathbf{r}_{P/Q}$$

$$= -\sin 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{PQ} \\ 0.4\cos\beta & -0.4\sin\beta & 0 \end{vmatrix}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components $v_P = -\sin 60^\circ + 0.4\omega_{PQ}\sin\beta$, and $0 = \cos 60^\circ + 0.4\omega_{PQ}\cos\beta$. Solving, we obtain $v_P = -1.11$ m/s and $\omega_{PQ} = -1.39$ rad/s.

Problem 17.160 Consider the system shown in Problem 17.159. If $\theta = 55^\circ$ and the sleeve P is moving to the left at 2 m/s, what are the angular velocities of bars OQ and PQ?



Solution: By applying the law of sines, $\beta = 24.2^\circ$ The velocity of Q is

$$\mathbf{v}_Q = \mathbf{v}_0 + \boldsymbol{\omega}_{OQ} \times \mathbf{r}_{Q/O} = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{OQ} \\ 0.2\cos 55^\circ & 0.2\sin 55^\circ & 0 \end{vmatrix}$$

$$= -0.2\omega_{OQ}\sin 55^\circ \mathbf{i} + 0.2\omega_{OQ}\cos 55^\circ \mathbf{j} \qquad \textbf{(1)}$$

We can also express $\mathbf{v}_Q$ as

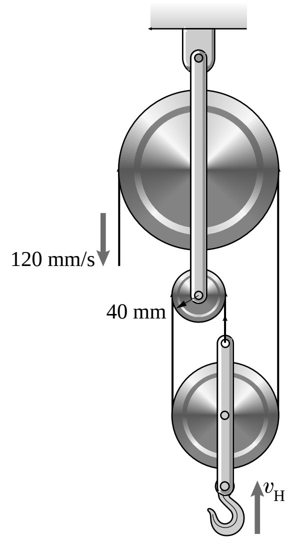
$$\mathbf{v}_Q = \mathbf{v}_P + \boldsymbol{\omega}_{PQ} \times \mathbf{r}_{Q/P}$$

$$= -2\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{PQ} \\ -0.4\cos\beta & 0.4\sin\beta & 0 \end{vmatrix}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in Equations (1) and (2), we get $-0.2\omega_{OQ}\sin 55^\circ = -2 - 0.4\omega_{PQ}\sin\beta$, and $0.2\omega_{OQ}\cos 55^\circ = -0.4\omega_{PQ}\cos\beta$. Solving, we obtain

$$\omega_{OQ} = 9.29 \text{ rad/s} \omega_{PQ} = -2.92 \text{ rad/s}.$$

Problem 17.161 Determine the vertical velocity v_H of the hook and the angular velocity of the small pulley.



Solution: The upper pulley is fixed so that it cannot move, from which the upward velocity of the rope on the right is equal to the downward velocity on the left, and the upward velocity of the rope on the right of the lower pulley is 120 mm/s. The small pulley is fixed so that it does not move. The upward velocity on the right of the small pulley is v_H mm/s, from which the downward velocity on the left is v_H mm/s. The upward velocity of the center of the bottom pulley is the mean of the difference of the velocities on the right and left, from which

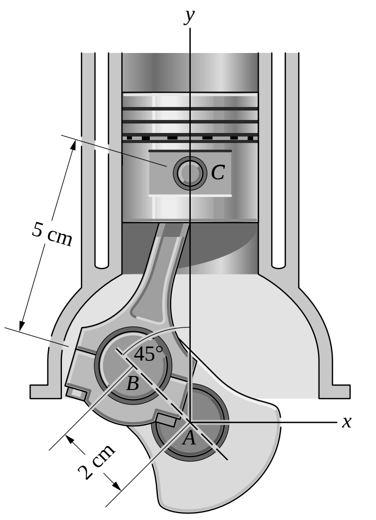
$$v_H = \frac{120 - v_H}{2}.$$

Solve, $\boxed{v_H = 40 \text{ mm/s}}$.

The angular velocity of the small pulley is

$$\boxed{\omega = \frac{v_H}{R} = \frac{40}{40} = 1 \text{ rad/s}}.$$

Problem 17.162 If the crankshaft AB is turning in the counterclockwise direction at 2000 rpm, what is the velocity of the piston?



Solution: The angle of the crank with the vertical is 45°. The angular velocity of the crankshaft is

$$\omega = 2000\left(\frac{2\pi}{60}\right) = 209.44 \text{ rad/s.}$$

The vector location of point B (the main rod bearing) $\mathbf{r}_B = 2(-\mathbf{i}\sin 45^\circ + \mathbf{j}\cos 45^\circ) = 1.414(-\mathbf{i}+\mathbf{j})$ cm. The velocity of point B (the main rod bearing) is

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = 1.414\omega \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$= -296.2(\mathbf{i}+\mathbf{j}) \text{ (cm/s).}$$

From the law if sines the interior angle between the connecting rod and the vertical at the piston is obtained from $\dfrac{2}{\sin\theta} = \dfrac{5}{\sin 45^\circ}$, from which

$$\theta = \sin^{-1}\left(\frac{2\sin 45^\circ}{5}\right) = 16.43^\circ.$$

The location of the piston is $\mathbf{r}_C = (2\sin 45^\circ + 5\cos\theta)\mathbf{j} = 6.21\mathbf{j}$ (cm). The vector $\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = -1.414\mathbf{i} - 4.796\mathbf{j}$ (cm). The piston is constrained to move along the y axis. In terms of the connecting rod the velocity of the point B is

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = v_C\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 1.414 & -4.796 & 0 \end{bmatrix}$$

$$= v_C\mathbf{j} + 4.796\omega_{BC}\mathbf{i} - 1.414\omega_{BC}\mathbf{j} \text{ (cm/s).}$$

Equate expressions for $\mathbf{v}_B$ and separate components:

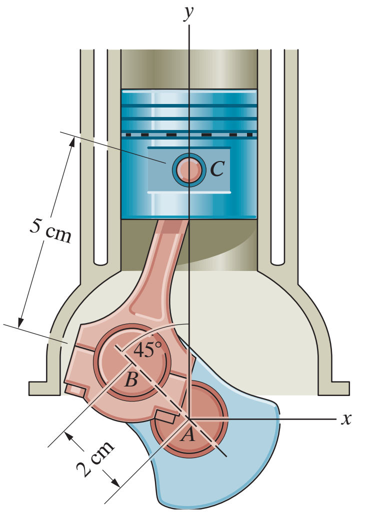
$$-296.2 = 4.796\omega_{BC},$$

$$-296.2 = v_C - 1.414\omega_{BC}.$$

Solve: $\boxed{\mathbf{v}_C = -383.5\mathbf{j} \text{ (cm/s)} = -3.835\mathbf{j} \text{ (m/s)}}$,

$$\omega_{BC} = -61.8 \text{ rad/s.}$$

Problem 17.163 In Problem 17.162, if the piston is moving with velocity $\mathbf{v}_C = 240\mathbf{j}$ (cm/s), what are the angular velocities of the crankshaft AB and the connecting rod BC?



Solution: Use the solution to Problem 17.162. The vector location of point B (the main rod bearing) $\mathbf{r}_B = 1.414(-\mathbf{i}+\mathbf{j})$ cm. From the law if sines the interior angle between the connecting rod and the vertical at the piston is

$$\theta = \sin^{-1}\left(\frac{2\sin 45^\circ}{5}\right) = 16.43^\circ.$$

The location of the piston is $\mathbf{r}_C = (2\sin 45^\circ + 5\cos\theta)\mathbf{j} = 6.21\mathbf{j}$ (cm). The piston is constrained to move along the y axis. In terms of the connecting rod the velocity of the point B is

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = 240\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -1.414 & -4.796 & 0 \end{bmatrix}$$

$$\mathbf{v}_B = 240\mathbf{j} + 4.796\omega_{BC}\mathbf{i} - 1.414\omega_{BC}\mathbf{j} \text{ (cm/s)}.$$

In terms of the crank angular velocity, the velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 1.414\omega_{AB}\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$= -1.414\omega_{AB}(\mathbf{i}+\mathbf{j}) \text{ (cm/s)}.$$

Equate expressions and separate components:

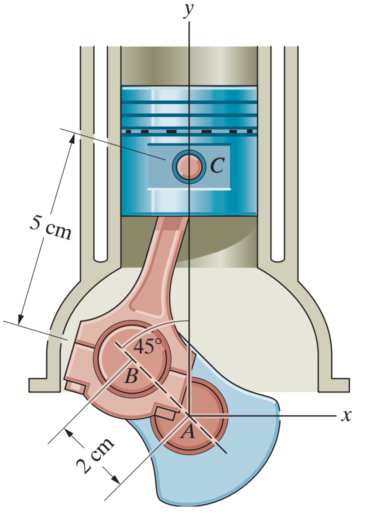
$$4.796\omega_{BC} = -1.414\omega_{BC} = -1.414\omega_{AB}.$$

Solve:

$\boxed{\omega_{BC} = 38.65 \text{ rad/s}}$ (counterclockwise).

$\boxed{\omega_{AB} = 131.1 \text{ rad/s} = -12515 \text{ rpm}}$ (clockwise).

Problem 17.164 In Problem 17.162, if the piston is moving with velocity $\mathbf{v}_C = 240\mathbf{j}$ (cm/s), and its acceleration is zero, what are the angular accelerations of crankshaft AB and the connecting rod BC?



Solution: Use the solution to Problem 17.163. $\mathbf{r}_{B/A} = 1.414(-\mathbf{i} + \mathbf{j}$ cm, $\omega_{AB} = -131.1$ rad/s, $\mathbf{r}_{B/C} = \mathbf{r}_B - \mathbf{r}_C = -1.414\mathbf{i} - 4.796\mathbf{j}$ (cm), $\omega_{BC} = 38.65$ rad/s. For point B,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

$$= 1.414\alpha_{AB}\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} - 1.414\omega_{AB}^2(-\mathbf{i}+\mathbf{j}),$$

$$\mathbf{a}_B = -1.414\alpha_{AB}(\mathbf{i}+\mathbf{j}) + 24291(\mathbf{i}-\mathbf{j}) \text{ (cm/s}^2\text{)}.$$

In terms of the angular velocity of the connecting rod,

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\mathbf{r}_{B/C},$$

$$\mathbf{a}_B = \alpha_{BC}\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -1.414 & -4.796 & 0 \end{bmatrix} - \omega_{BC}^2(-1.414\mathbf{i} - 4.795\mathbf{j}) \text{ (cm/s}^2\text{)},$$

$$\mathbf{a}_B = 4.796\alpha_{BC}\mathbf{i} - 1.414\alpha_{BC}\mathbf{j} + 2112.3\mathbf{i} + 71630\mathbf{j} \text{ (cm/s}^2\text{)}.$$

Equate expressions and separate components:

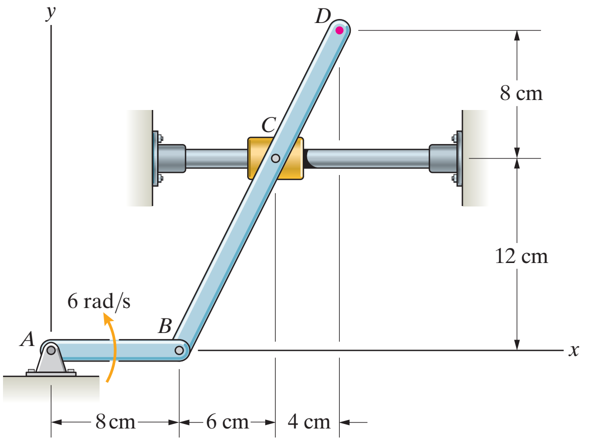
$$-1.414\alpha_{AB} + 24291 = 4.796\alpha_{BC} + 2112.3,$$

$$-1414\alpha_{AB} - 24291 = 1.414\alpha_{BC} + 7163.$$

Solve: $\boxed{\alpha_{AB} = -13{,}605 \text{ rad/s}^2}$ (clockwise).

$\boxed{\alpha_{BC} = 8636.5 \text{ rad/s}^2}$ (counterclockwise).

Problem 17.165 Bar AB rotates at 6 rad/s in the counterclockwise direction. Use instantaneous centers to determine the angular velocity of bar BCD and the velocity of point D.



Solution: The strategy is to determine the angular velocity of bar BC from the instantaneous center; using the constraint on the motion of C. The vector $\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = (8\mathbf{i} + 4\mathbf{j}) - 4\mathbf{j} = 8\mathbf{i}$ (cm). The velocity of point B is $\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \omega_{AB}(\mathbf{k} \times 8\mathbf{i}) = 48\mathbf{j}$ (cm/s). The velocity of point B is normal to the x axis, and the velocity of C is parallel to the x axis. The instantaneous center of bar BC has the coordinates (14, 0). The vector

$$\mathbf{r}_{B/IC} = \mathbf{r}_B - \mathbf{r}_{IC} = (8\mathbf{i} + 4\mathbf{j}) - (14\mathbf{i} - 4\mathbf{j}) = -6\mathbf{i} \text{ (cm)}.$$

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/IC} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -6 & 0 & 0 \end{bmatrix} = -6\omega_{BC}\mathbf{j} = 48\mathbf{j},$$

from which

$$\boxed{\omega_{BC} = -\frac{48}{6} = -8 \text{ rad/s}}.$$

The velocity of point C is

$$\mathbf{v}_C = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/IC} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 0 & 12 & 0 \end{bmatrix} = 96\mathbf{i} \text{ (cm/s)}.$$

The velocity of point D is normal to the unit vector parallel to BCD

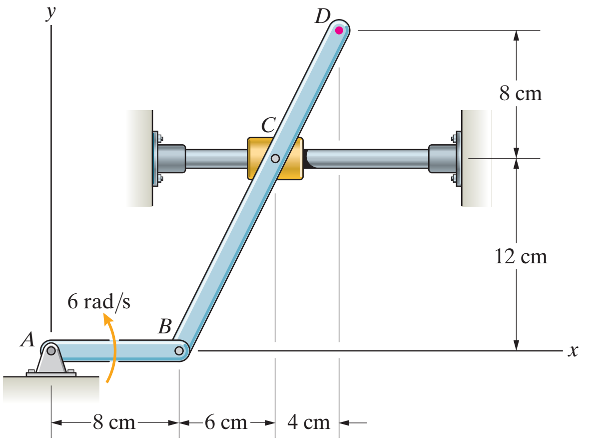
$$\mathbf{e} = \frac{6\mathbf{i} + 12\mathbf{j}}{\sqrt{6^2 + 12^{12}}} = 0.4472\mathbf{i} + 0.8944\mathbf{j}.$$

The intersection of the projection of this unit vector with the projection of the unit vector normal to velocity of C is occurs at point C, from which the coordinates of the instantaneous center for the part of the bar CD are (14, 12). The instantaneous center is translating at velocity $\mathbf{v}_C$, from which the velocity of point D is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{D/ICD} = 96\mathbf{i} - 8(\mathbf{k} \times (4\mathbf{i} + 8\mathbf{j}))$$

$$= 160\mathbf{i} - 32\mathbf{j} \text{ (cm/s)}.$$

Problem 17.166 In Problem 17.165, bar AB rotates with a constant angular velocity of 6 rad/s in the counterclockwise direction. Determine the acceleration of point D.



Solution: Use the solution to Problem 17.165. The accelerations are determined from the angular velocity, the known accelerations of B, and the constraint on the motion of C. The vector $\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = 8\mathbf{i}$ (cm). The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{A/B} = 0 - 36(8\mathbf{i}) = -288\mathbf{i}\ (\text{cm/s}^2).$$

From the solution to Problem 17.165, $\omega_{BC} = -8$ rad/s. (clockwise).

$$\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{v}_B = (14\mathbf{i} + 12\mathbf{j}) - (8\mathbf{i}) = 6\mathbf{i} + 12\mathbf{j}\ (\text{cm}).$$

The vector $\mathbf{r}_{B/C} = -\mathbf{r}_{C/B}$. The acceleration of point B in terms of the angular acceleration of point C is

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{BC} - \omega_{BC}^2 \mathbf{r}_{B/C}$$

$$= \mathbf{a}_C\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -6 & -12 & 0 \end{bmatrix} - 64(-6\mathbf{i} - 12\mathbf{j}).$$

$$\mathbf{a}_B = a_C\mathbf{i} + 12\alpha_{BC}\mathbf{i} - 6\alpha_{BC}\mathbf{j} + 384\mathbf{i} + 768\mathbf{j}.$$

Equate the expressions and separate components:

$$-288 = a_C + 12\alpha_{BC} + 384, \quad 0 = -6\alpha_{BC} + 768.$$

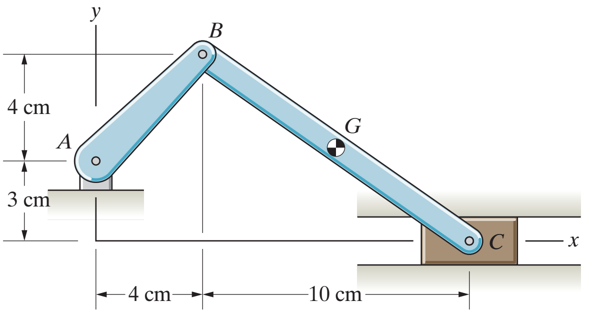
Solve $\alpha_{BC} = 128$ rad/s^2 (counterclockwise), $a_C = -2208$ cm/s^2. The acceleration of point D is

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{D/C} - \omega_{BC}^2 \mathbf{r}_{D/C}$$

$$= -2208\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 4 & 8 & 0 \end{bmatrix} - \omega_{BC}^2(4\mathbf{i} + 8\mathbf{j}).$$

$$\boxed{\begin{aligned}\mathbf{a}_D &= -2208\mathbf{i} + (128)(-8\mathbf{i} + 4\mathbf{j}) - (64)(4\mathbf{i} + 8\mathbf{j}) \\ &= -3490\mathbf{i}\ (\text{cm/s}^2)\end{aligned}}.$$

Problem 17.167 Point C is moving to the right at 20 cm/s. What is the velocity of the midpoint G of bar BC?



Solution:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{w}_{AB} \times \mathbf{r}_{B/A} = \mathbf{0} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 4 & 4 & 0 \end{vmatrix}.$$

Also,

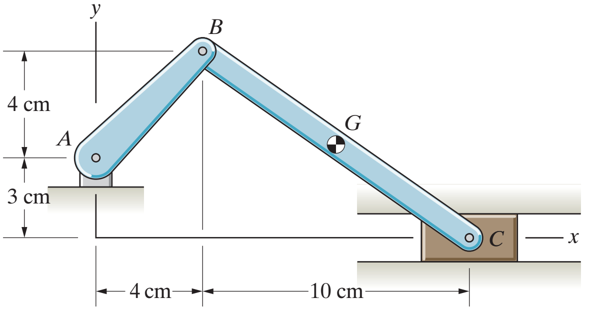
$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{w}_{BC} \times \mathbf{r}_{B/C} = 20\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -10 & 7 & 0 \end{vmatrix}.$$

Equating **i** and **j** components in these two expressions, $-4\omega_{AB} = 20 - 7\omega_{BC}$, $4\omega_{AB} = -10\omega_{BC}$, and solving, we obtain $\omega_{AB} = -2.94$ rad/s, $\omega_{BC} = 1.18$ rad/s. Then the velocity of G is

$$\mathbf{v}_G = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{G/C} = 20\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -5 & 3.5 & 0 \end{vmatrix}$$

$$= \underline{15.88\,\mathbf{i} - 5.88\,\mathbf{j}\ (\text{cm/s})}.$$

Problem 17.168 In Problem 17.167, point C is moving to the right with a constant velocity of 20 cm/s. What is the acceleration of the midpoint G of bar BC?



Solution: See the solution of Problem 17.167.

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 4 & 4 & 0 \end{vmatrix} - \omega_{AB}^2(4\mathbf{i} + 4\mathbf{j}).$$

Also, $\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C}$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -10 & 7 & 0 \end{vmatrix} - \omega_{BC}^2(-10\mathbf{i} + 7\mathbf{j}).$$

Equating **i** and **j** components in these two expressions,

$$-4\alpha_{AB} - 4\omega_{AB}^2 = -7\alpha_{BC} + 10\omega_{BC}^2,$$

$$4\alpha_{AB} - 4\omega_{AB}^2 = -10\alpha_{BC} - 7\omega_{BC}^2,$$

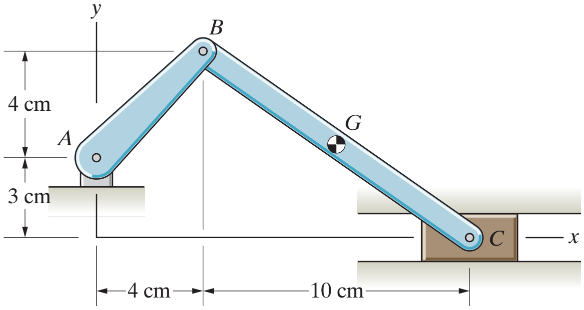
and solving yields $\alpha_{AB} = -4.56$ rad/s^2, $\alpha_{BC} = 4.32$ rad/s^2.

Then $\mathbf{a}_G = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{G/C} - \omega_{BC}^2 \mathbf{r}_{G/C}$

$$= \mathbf{O} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -5 & 3.5 & 0 \end{vmatrix} - \omega_{BC}^2(-5\mathbf{i} + 3.5\mathbf{j})$$

$$= \underline{-8.18\mathbf{i} - 26.4\mathbf{j}\ (\text{cm/s}^2)}.$$

Problem 17.169 In Problem 17.167, if the velocity of point C is $\mathbf{v}_C = 1.0\mathbf{i}$ (cm/s), what are the angular velocity vectors of arms AB and BC?



Solution: Use the solution to Problem 17.167: The velocity of the point B is determined from the known velocity of point C and the known velocity of C:

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = 1.0\mathbf{i} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -10 & 7 & 0 \end{bmatrix}$$

$$= 1.0\mathbf{i} - 7\omega_{BC}\mathbf{i} - 10\omega_{BC}\mathbf{j}.$$

The angular velocity of bar AB is determined from the velocity of B.

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 4 & 4 & 0 \end{bmatrix} = -4\omega_{\mathbf{AB}}(\mathbf{i} - \mathbf{j}) \text{ (cm/s)}$$

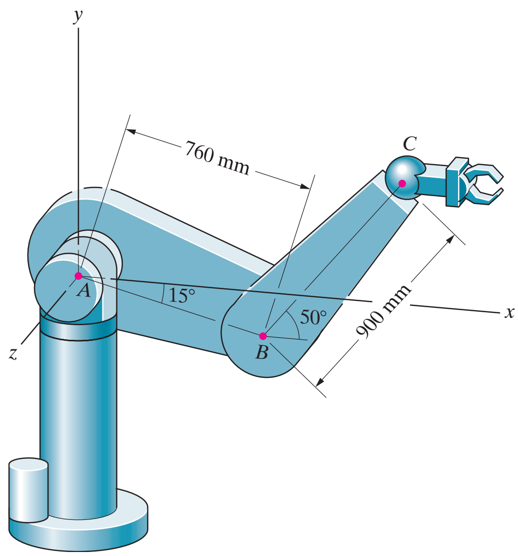
Equate expressions, separate components,

$$1.0 - 7\omega_{BC} = -4\omega_{AB}, \quad -10\omega_{BC} = 4\omega_{AB}.$$

Solve: $\omega_{AB} = -0.147$ rad/s, $\omega_{BC} = 0.0588$ rad/s, from which

$\boxed{\boldsymbol{\omega}_{AB} = -0.147\mathbf{k} \text{ (rad/s)}}$, $\boxed{\boldsymbol{\omega}_{BC} = 0.0588\mathbf{k} \text{ (rad/s)}}$

Problem 17.170 Points B and C are in the $x-y$ plane. The angular velocity vectors of arms AB and BC are $\boldsymbol{\omega}_{AB} = -0.5\mathbf{k}$ (rad/s) and $\boldsymbol{\omega}_{BC} = -2.0\mathbf{k}$ (rad/s). Determine the velocity of point C.



Solution: The vector

$$\mathbf{r}_{B/A} = 760(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 734.1\mathbf{i} - 196.7\mathbf{j} \text{ (mm)}.$$

The vector

$$\mathbf{r}_{C/B} = 900(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 578.5\mathbf{i} + 689.4\mathbf{j} \text{ (mm)}.$$

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = -0.5 \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 734.1 & -196.7 & 0 \end{bmatrix}$$

$$\mathbf{v}_B = -98.35\mathbf{i} - 367.1\mathbf{j} \text{ (mm/s)}.$$

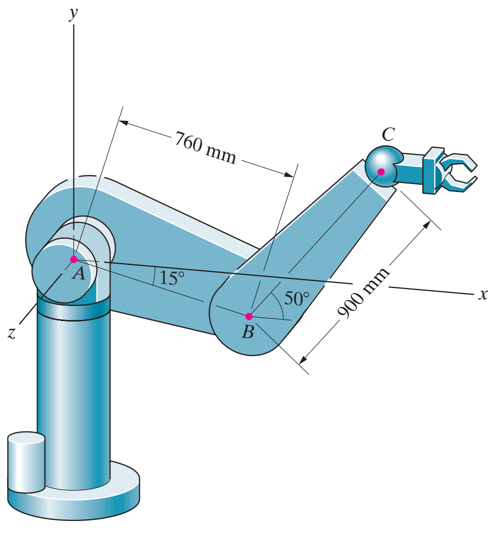
The velocity of point C

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= -98.35\mathbf{i} - 367.1\mathbf{j} + (2)(\mathbf{k} \times (578.5\mathbf{i} + 689.4\mathbf{j})),$$

$$\boxed{\begin{aligned} \mathbf{v}_C &= -98.35\mathbf{i} - 367.1\mathbf{j} - 1378.9\mathbf{i} + 1157.0\mathbf{j} \\ &= -1477.2\mathbf{i} + 790\mathbf{j} \text{ (mm/s)} \end{aligned}}$$

Problem 17.171 In Problem 17.170, if the velocity vector of point C is $\mathbf{v}_C = 1.0\mathbf{i}$ (m/s), what are the angular velocity vectors of arms AB and BC?



Solution: Use the solution to Problem 17.170.

The vector

$\mathbf{r}_{B/A} = 760(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 734.1\mathbf{i} - 196.7\mathbf{j}$ (mm).

The vector

$\mathbf{r}_{C/B} = 900(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 578.5\mathbf{i} + 689.4\mathbf{j}$ (mm).

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 734.1 & -196.7 & 0 \end{bmatrix}$$

$$= 196.7\omega_{AB}\mathbf{i} + 734.1\omega_{AB}\mathbf{j} \text{ (mm/s)}.$$

The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= 196.7\omega_{AB}\mathbf{i} + 734.1\omega_{AB}\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 578.5 & 687.4 & 0 \end{bmatrix},$$

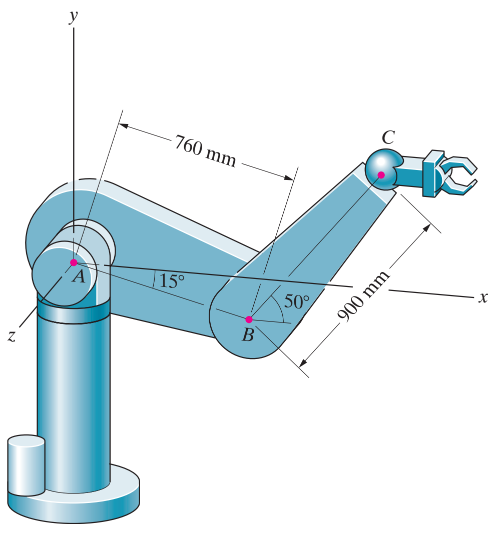
$$1000\mathbf{i} = 196.7\omega_{AB}\mathbf{i} + 734.1\omega_{AB}\mathbf{j} - 687.4\omega_{BC}\mathbf{i} + 578.5\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

Separate components:

$$1000 = 196.7\omega_{AB} - 687.4\omega_{BC},\ 0 = 734.1\omega_{AB} + 578.5\omega_{BC}.$$

Solve: $\boxed{\boldsymbol{\omega}_{AB} = 0.933\mathbf{k} \text{ (rad/s)}}$, $\boxed{\boldsymbol{\omega}_{BC} = -1.184\mathbf{k} \text{ (rad/s)}}$

Problem 17.172 The angular velocity vectors of arms AB and BC are $\boldsymbol{\omega}_{AB} = -0.5\mathbf{k}$ (rad/s) and $\boldsymbol{\omega}_{BC} = 2.0\mathbf{k}$ (rad/s), and their angular accelerations are $\boldsymbol{\alpha}_{AB} = 1.0\mathbf{k}$ (rad/s^2), and $\alpha_{BC} = 1.0\mathbf{k}$ (rad/s^2). What is the acceleration of point C?



Solution: Use the solution to Problem 17.170.

The vector

$\mathbf{r}_{B/A} = 760(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 734.1\mathbf{i} - 196.7\mathbf{j}$ (mm).

The vector

$\mathbf{r}_{C/B} = 900(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 578.5\mathbf{i} + 689.4\mathbf{j}$ (mm).

The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 734.1 & -196.7 & 0 \end{bmatrix}$$

$$- (0.5^2)(734.1\mathbf{i} - 196.7\mathbf{j}) \text{ (mm/s}^2\text{)}.$$

$$\mathbf{a}_B = 196.7\mathbf{i} + 734.1\mathbf{j} - 183.5\mathbf{i} + 49.7\mathbf{j} = 13.2\mathbf{i} + 783.28\mathbf{j} \text{ (mm/s}^2\text{)}.$$

The acceleration of point C is

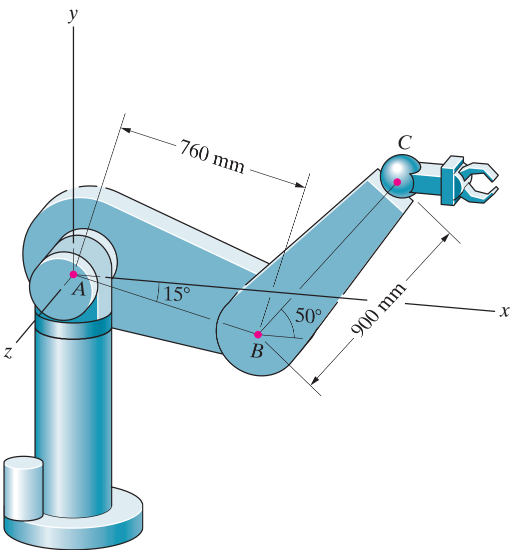
$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 578.5 & 689.4 & 0 \end{bmatrix} - (2^2)(578.5\mathbf{i} + 689.4\mathbf{j})$$

$$\mathbf{a}_C = 13.2\mathbf{i} + 783.3\mathbf{j} - 689.4\mathbf{i} + 578.5\mathbf{j} - 2314\mathbf{i} - 2757.8\mathbf{j} \text{ (mm/s}^2\text{)}.$$

$$\boxed{\mathbf{a}_C = -2990\mathbf{i} - 1396\mathbf{j} \text{ (mm/s}^2\text{)}}$$

Problem 17.173 The velocity of point C is $\mathbf{v}_C = 1.0\mathbf{i}$ (m/s) and $\mathbf{a}_C = 0$. What are the angular velocity and angular acceleration vectors of arm BC?



Solution: Use the solution to Problem 17.171. The vector $\mathbf{r}_{B/A} = 760(\mathbf{i}\cos 15^\circ - \mathbf{j}\sin 15^\circ) = 734.1\mathbf{i} - 196.7\mathbf{j}$ (mm). The vector $\mathbf{r}_{C/B} = 900(\mathbf{i}\cos 50^\circ + \mathbf{j}\sin 50^\circ) = 578.5\mathbf{i} + 689.4\mathbf{j}$ (mm). The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 734.1 & -196.7 & 0 \end{bmatrix}$$

$$= 196.7\omega_{AB}\mathbf{i} + 734.1\omega_{AB}\mathbf{j} \text{ (mm/s)}.$$

The velocity of point C is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= 196.7\omega_{AB}\mathbf{i} + 734.1\omega_{AB}\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 578.5 & 689.4 & 0 \end{bmatrix}$$

$$1000\mathbf{i} = 196.7\omega_{AB}\mathbf{i} + 734.1\omega_{AB}\mathbf{j} - 689.4\omega_{BC}\mathbf{i}$$

$$+ 578.5\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

Separate components:

$$1000 = 196.7\omega_{AB} - 689.4\omega_{BC},$$

$$0 = 734.1\omega_{AB} + 578.5\omega_{BC}.$$

Solve: $\boldsymbol{\omega}_{AB} = 0.933\mathbf{k}$ (rad/s), $\boxed{\boldsymbol{\omega}_{BC} = -1.184\mathbf{k} \text{ (rad/s)}}$.

The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 734.1 & -196.7 & 0 \end{bmatrix}$$

$$- (\omega_{AB}^2)(734.1\mathbf{i} - 196.7\mathbf{j}) \text{ (mm/s)}^2.$$

$$\mathbf{a}_B = 196.7\alpha_{AB}\mathbf{i} + 734.1\alpha_{AB}\mathbf{j} - 639.0\mathbf{i} + 171.3\mathbf{j} \text{ (mm/s}^2\text{)}$$

The acceleration of point C is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

$$= \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 578.5 & 689.4 & 0 \end{bmatrix} - (\omega_{BC}^2)(578.5\mathbf{i} + 689.4)$$

$$\mathbf{a}_C = 0 = \alpha_{AB}(196.7\mathbf{i} + 734.1\mathbf{j}) + \alpha_{BC}(-689.4\mathbf{i} + 578.5\mathbf{j})$$

$$- 811.0\mathbf{i} - 966.5\mathbf{j} - 639.0\mathbf{i} + 171.3\mathbf{j} \text{ (mm/s}^2\text{)}$$

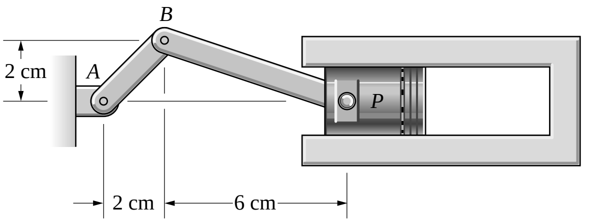
Separate components:

$$196.7\alpha_{AB} - 689.4\alpha_{BC} - 811.3 - 639.0 = 0,$$

$$734.1\alpha_{AB} + 578.5\alpha_{BC} - 966.5 + 171.3 = 0.$$

Solve: $\alpha_{AB} = 2.24$ rad/s^2, $\boxed{\alpha_{BC} = -1.465 \text{ (rad/s}^2\text{)}}$

Problem 17.174 The crank AB has a constant clockwise angular velocity of 200 rpm. What are the velocity and acceleration of the piston P?



Solution:

200 rpm $= 200(2\pi)/60 = 20.9$ rad/s.

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \mathbf{0} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -20.9 \\ 2 & 2 & 0 \end{vmatrix}.$$

Also, $$\mathbf{v}_B = \mathbf{v}_P + \boldsymbol{\omega}_{BP} \times \mathbf{r}_{B/P} = v_P\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BP} \\ -6 & 2 & 0 \end{vmatrix}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components in these two expressions,

$-(-20.9)(2) = v_P - 2\omega_{BP}$, $(-20.9)(2) = -6\omega_{BP}$,

we obtain $\omega_{BP} = 6.98$ rad/s and $\underline{v_P = 55.9 \text{ cm/s}}$.

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

$$= \mathbf{0} + \mathbf{0} - (-20.9)^2(2\mathbf{i} + 2\mathbf{j}).$$

Also, $$\mathbf{a}_B = \mathbf{a}_P + \boldsymbol{\alpha}_{BP} \times \mathbf{r}_{B/P} - \omega_{BP}^2\mathbf{r}_{B/P}$$

$$= a_P\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BP} \\ -6 & 2 & 0 \end{vmatrix} - \omega_{BP}^2(-6\mathbf{i} + 2\mathbf{j}).$$

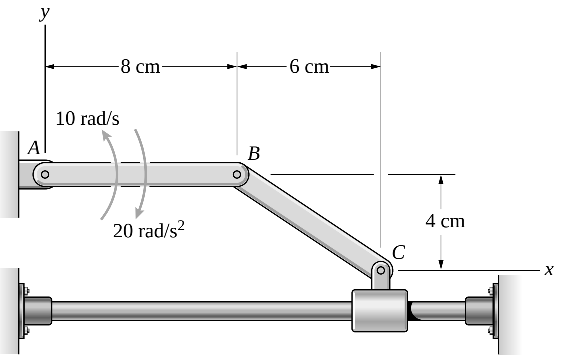
Equating $\mathbf{i}$ and $\mathbf{j}$ components,

$$-2(20.9)^2 = a_P - 2\alpha_{BP} + 6\omega_{BP}^2,$$

$$-2(20.9)^2 = -6\alpha_{BP} - 2\omega_{BP}^2,$$

and solving, we obtain $\underline{a_P = -910 \text{ cm/s}^2}$.

Problem 17.175 Bar AB has a counterclockwise angular velocity of 10 rad/s and a clockwise angular acceleration of 20 rad/s^2. Determine the angular acceleration of bar BC and the acceleration of point C.



Solution: Choose a coordinate system with the origin at the left end of the horizontal rod and the x axis parallel to the horizontal rod. The strategy is to determine the angular velocity of bar BC from the instantaneous center; using the angular velocity and the constraint on the motion of C, the accelerations are determined.

The vector $\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A = (8\mathbf{i} + 4\mathbf{j}) - 4\mathbf{j} = 8\mathbf{i}$ (cm).

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 8 & 0 & 0 \end{bmatrix} = 80\mathbf{j} \text{ (cm/s)}.$$

The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{A/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -20 \\ 8 & 0 & 0 \end{bmatrix} - 100(8\mathbf{i})$$

$$= -800\mathbf{i} - 160\mathbf{j} \text{ (cm/s}^2).$$

The velocity of point B is normal to the x axis, and the velocity of C is parallel to the x axis. The line perpendicular to the velocity at B is parallel to the x-axis, and the line perpendicular to the velocity at C is parallel to the y axis. The intercept is at (14, 4), which is the instantaneous center of bar BC. Denote the instantaneous center by C''.

The vector $\mathbf{r}_{B/C''} = \mathbf{r}_B - \mathbf{r}_{C''} = (8\mathbf{i} + 4\mathbf{j}) - (14\mathbf{i} - 4\mathbf{j}) = -6\mathbf{i}$ (cm).

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/IC} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -6 & 0 & 0 \end{bmatrix} = -6\omega_{BC}\mathbf{j} = 80\mathbf{j},$$

from which $\omega_{BC} = -\dfrac{80}{6} = -13.33$ rad/s.

The vector $\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = (14\mathbf{i}) - (8\mathbf{i} + 4\mathbf{j}) = 6\mathbf{i} - 4\mathbf{j}$ (cm).

The acceleration of point C is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$= -800\mathbf{i} - 160\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 6 & -4 & 0 \end{bmatrix} - 1066.7\mathbf{i} + 711.1\mathbf{j} \text{ (cm/s}^2).$$

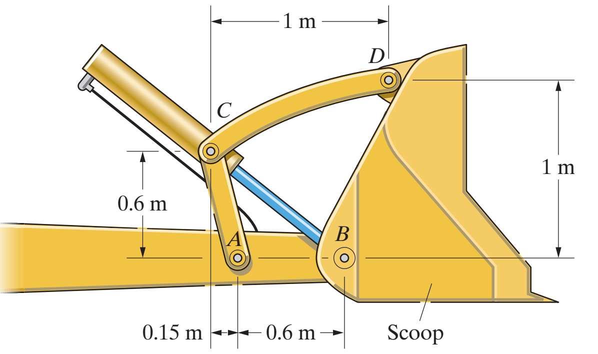
The acceleration of point C is constrained to be parallel to the x axis. Separate components:

$$a_C = -800 + 4\alpha_{BC} - 1066.7, \quad 0 = -160 + 6\alpha_{BC} + 711.1.$$

Solve:

$\boxed{\mathbf{a}_C = -2234\mathbf{i} \text{ (cm/s}^2)}$, $\boxed{\alpha_{BC} = -91.9 \text{ rad/s}^2}$ (clockwise).

Problem 17.176 The angular velocity of arm AC is 1 rad/s counterclockwise. What is the angular velocity of the scoop?



Solution: Choose a coordinate system with the origin at A and the y axis vertical. The vector locations of B, C and D are $\mathbf{r}_B = 0.6\mathbf{i}$ (m), $\mathbf{r}_C = -0.15\mathbf{i} + 0.6\mathbf{j}$ (m), $\mathbf{r}_D = (1 - 0.15)\mathbf{i} + 1\mathbf{j} = 0.85\mathbf{i} + \mathbf{j}$ (m), from which $\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C = 1\mathbf{i} + 0.4\mathbf{j}$ (m), and $\mathbf{r}_{D/B} = \mathbf{r}_D - \mathbf{r}_B = 0.25\mathbf{i} + \mathbf{j}$ (m). The velocity of point C is

$$\mathbf{v}_C = \boldsymbol{\omega}_{AC} \times \mathbf{r}_{C/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ -0.15 & 0.6 & 0 \end{bmatrix}$$

$$= -0.6\mathbf{i} - 0.15\mathbf{j} \text{ (m/s)}.$$

The velocity of D in terms of the velocity of C is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = -0.6\mathbf{i} - 0.15\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 1 & 0.4 & 0 \end{bmatrix}$$

$$= -0.6\mathbf{i} - 0.15\mathbf{j} + \omega_{CD}(-0.4\mathbf{i} + \mathbf{j}).$$

The velocity of point D in terms of the angular velocity of the scoop is

$$\mathbf{v}_D = \boldsymbol{\omega}_{DB} \times \mathbf{r}_{D/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DB} \\ 0.25 & 1 & 0 \end{bmatrix} = \omega_{DB}(-\mathbf{i} + 0.25\mathbf{j}).$$

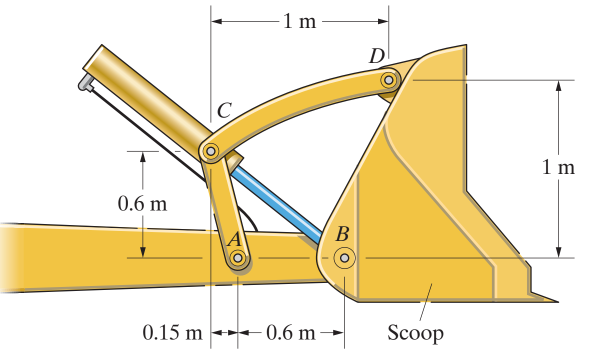
Equate expressions and separate components:

$$-0.6 - 0.4\omega_{CD} = -\omega_{DB}, \; -0.15 + \omega_{CD} = 0.25\omega_{DB}.$$

Solve:

$\omega_{CD} = 0.333$ rad/s, $\boxed{\omega_{DB} = 0.733 \text{ rad/s}}$ (counterclockwise).

Problem 17.177 The angular velocity of arm AC in Problem 17.176 is 2 rad/s counterclockwise, and its angular acceleration is 4 rad/s^2 clockwise. What is the angular acceleration of the scoop?



Solution: Use the solution to Problem 17.176. Choose a coordinate system with the origin at A and the y axis vertical. The vector locations of B, C and D are $\mathbf{r}_B = 0.6\mathbf{i}$ (m), $\mathbf{r}_C = -0.15\mathbf{i} + 0.6\mathbf{j}$ (m), $\mathbf{r}_D = (1 - 0.15)\mathbf{i} + 1\mathbf{j} = 0.85\mathbf{i} + \mathbf{j}$ (m), from which $\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C = 1\mathbf{i} + 0.4\mathbf{j}$ (m), and $\mathbf{r}_{D/B} = \mathbf{r}_D - \mathbf{r}_B = 0.25\mathbf{i} + \mathbf{j}$ (m). The velocity of point C is

$$\mathbf{v}_C = \boldsymbol{\omega}_{AC} \times \mathbf{r}_{C/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AC} \\ -0.15 & 0.6 & 0 \end{bmatrix} = -1.2\mathbf{i} - 0.3\mathbf{j} \text{ (m/s)}.$$

The velocity of D in terms of the velocity of C is

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C} = -1.2\mathbf{i} - 0.3\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 1 & 0.4 & 0 \end{bmatrix}$$

$$= -1.2\mathbf{i} - 0.3\mathbf{j} + \omega_{CD}(-0.4\mathbf{i} + \mathbf{j}).$$

The velocity of point D in terms of the angular velocity of the scoop is

$$\mathbf{v}_D = \boldsymbol{\omega}_{DB} \times \mathbf{r}_{D/B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{DB} \\ 0.25 & 1 & 0 \end{bmatrix} = \omega_{DB}(-\mathbf{i} + 0.25\mathbf{j}).$$

Equate expressions and separate components:

$$-1.2 - 0.4\omega_{CD} = -\omega_{DB}, \quad -0.3 + \omega_{CD} = 0.25\omega_{DB}.$$

Solve: $\omega_{CD} = 0.667$ rad/s, $\omega_{DB} = 1.47$ rad/s. The angular acceleration of the point C is

$$\mathbf{a}_C = \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/A} - \omega_{AC}^2 \mathbf{r}_{C/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \boldsymbol{\alpha}_{AC} \\ -0.15 & 0.6 & 0 \end{bmatrix} - \omega_{AC}^2(-0.15\mathbf{i} + 0.6\mathbf{j}),$$

$$\mathbf{a}_C = 2.4\mathbf{i} + 0.6\mathbf{j} + 0.6\mathbf{i} - 2.4\mathbf{j} = 3\mathbf{i} - 1.8\mathbf{j} \text{ (m/s}^2).$$

The acceleration of point D in terms of the acceleration of point C is

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{D/C} - \omega_{CD}^2 \mathbf{r}_{D/C}$$

$$= 3\mathbf{i} - 1.8\mathbf{j} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ 1 & 0.4 & 0 \end{bmatrix} - \omega_{CD}^2(\mathbf{i} + 0.4\mathbf{j}).$$

$$\mathbf{a}_C = \alpha_{CD}(-0.4\mathbf{i} + \mathbf{j}) + 2.56\mathbf{i} - 1.98\mathbf{j} \text{ (m/s}^2).$$

The acceleration of point D in terms of the angular acceleration of point B is

$$\mathbf{a}_D = \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ 0.25 & 1 & 0 \end{bmatrix} - \omega_{BD}^2(0.25\mathbf{i} + \mathbf{j}).$$

$$\mathbf{a}_D = \alpha_{BD}(-\mathbf{i} + 0.25\mathbf{j}) - 0.538\mathbf{i} - 2.15\mathbf{j}.$$

Equate expressions for $\mathbf{a}_D$ and separate components:

$$-0.4\alpha_{CD} + 2.56 = -\alpha_{BD} - 0.538,$$

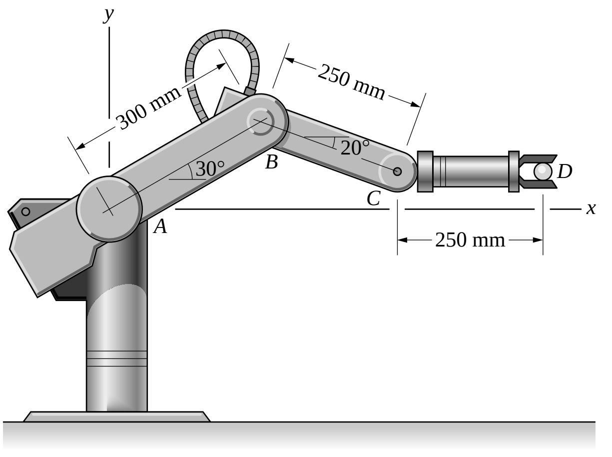
$$\alpha_{CD} - 1.98 = 0.25\alpha_{BD} - 2.15.$$

Solve:

$$\alpha_{CD} = -1.052 \text{ rad/s}^2, \ \boxed{\alpha_{BD} = -3.51 \text{ rad/s}^2},$$

where the negative sign means a clockwise acceleration.

Problem 17.178 If you want to program the robot so that, at the instant shown, the velocity of point D is $\mathbf{v}_D = 0.2\mathbf{i} + 0.8\mathbf{j}$ (m/s) and the angular velocity of arm CD is 0.3 rad/s counterclockwise, what are the necessary angular velocities of arms AB and BC?



Solution: The position vectors are:

$$\mathbf{r}_{B/A} = 300(\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 259.8\mathbf{i} + 150\mathbf{j} \text{ (mm)},$$

$$\mathbf{r}_{C/B} = 250(\mathbf{i}\cos 20^\circ - \mathbf{j}\sin 20^\circ) = 234.9\mathbf{i} - 85.5\mathbf{j} \text{ (mm)},$$

$$\mathbf{r}_{C/D} = -250\mathbf{i} \text{ (mm)}.$$

The velocity of the point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 259.8 & 150 & 0 \end{bmatrix}$$

$$\mathbf{v}_B = -150\omega_{AB}\mathbf{i} + 259.8\omega_{AB}\mathbf{j}.$$

The velocity of point C in terms of the velocity of B is

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 234.9 & -85.5 & 0 \end{bmatrix}$$

$$= -150\omega_{AB}\mathbf{i} + 259.8\omega_{AB}\mathbf{j} + 85.5\omega_{BC}\mathbf{i} + 234.9\omega_{BC}\mathbf{j} \text{ (mm/s)}.$$

The velocity of point C in terms of the velocity of point D is

$$\mathbf{v}_C = \mathbf{v}_D + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = 200\mathbf{i} + 800\mathbf{j} + 0.3\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -250 & 0 & 0 \end{bmatrix}$$

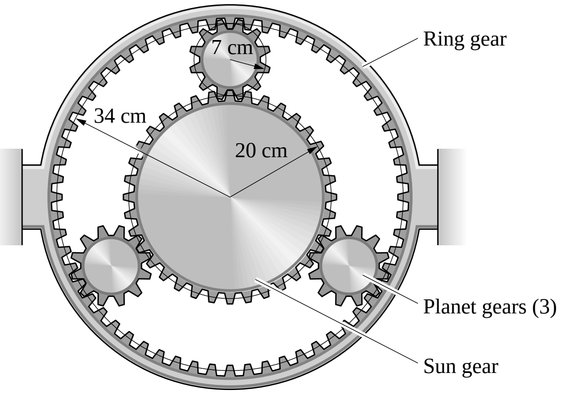
$$= 200\mathbf{i} + 725\mathbf{j} \text{ (mm/s)}.$$

Equate the expressions for $\mathbf{v}_C$ and separate components:

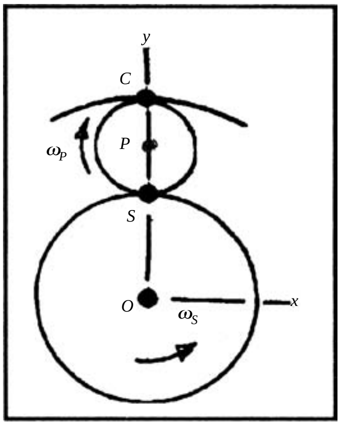
$$-150\omega_{AB} + 85.5\omega_{BC} = 200, \text{ and } 259.8\omega_{AB} + 234.9\omega_{BC} = 725.$$

Solve: $\boxed{\omega_{AB} = 0.261 \text{ rad/s}}$, $\boxed{\omega_{BC} = 2.80 \text{ rad/s}}$.

Problem 17.179 The ring gear is stationary, and the sun gear rotates at 120 rpm (revolutions per minute) in the counterclockwise direction. Determine the angular velocity of the planet gears and the magnitude of the velocity of their centerpoints.



Solution: Denote the point O be the center of the sun gear, point S to be the point of contact between the upper planet gear and the sun gear, point P be the center of the upper planet gear, and point C be the point of contact between the upper planet gear and the ring gear. The angular velocity of the sun gear is



$$\omega_S = \frac{120(2\pi)}{60} = 4\pi \text{ rad/s},$$

from which $\boldsymbol{\omega}_S = 4\pi\mathbf{k}$ (rad/s). At the point of contact between the sun gear and the upper planet gear the velocities are equal. The vectors are: from center of sun gear to S is $\mathbf{r}_{P/S} = 20\mathbf{j}$ (cm), and from center of planet gear to S is $\mathbf{r}_{S/P} = -7\mathbf{j}$ (cm). The velocities are:

$$\mathbf{v}_{S/O} = \mathbf{v}_O + \boldsymbol{\omega}_S \times (20\mathbf{j}) = 0 + \omega_S(20)(\mathbf{k} \times \mathbf{j})$$

$$\mathbf{v}_{S/O} = 20\omega_S \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = -20(\omega_S)$$

$$\mathbf{i} = -251.3\mathbf{i} \text{ (cm/s)}.$$

From equality of the velocities, $\mathbf{v}_{S/P} = \mathbf{v}_{S/O} = -251.3\mathbf{i}$ (cm/s). The point of contact C between the upper planet gear and the ring gear is stationary, from which

$$\mathbf{v}_{S/P} = -251.3\mathbf{i} = \mathbf{v}_C + \boldsymbol{\omega}_P \times \mathbf{r}_{C/S}$$

$$= 0 + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_P \\ -14 & 0 & 0 \end{bmatrix} = 14\omega_P\mathbf{i} = -251.3\mathbf{i}$$

from which $\boxed{\omega_P = 17.95 \text{ rad/s.}}$

The velocity of the centerpoint of the top most planet gear is

$$\mathbf{v}_P = \mathbf{v}_{S/P} + \boldsymbol{\omega}_P \times \mathbf{r}_{P/S} = -251.3\mathbf{i} + (-17.95)(-7)(\mathbf{k} \times \mathbf{j})$$

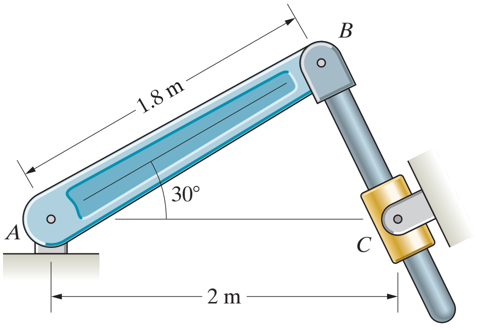
$$= -251.3\mathbf{i} + 125.65 \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{v}_P = -125.7\mathbf{i} \text{ (cm/s)}$$

The magnitude is $\boxed{v_{PO} = 125.7 \text{ cm/s}}$

By symmetry, the magnitudes of the velocities of the centerpoints of the other planetary gears is the same.

Problem 17.180 Arm AB is rotating at 10 rad/s in the clockwise direction. Determine the angular velocity of the arm BC and the velocity at which the arm slides relative to the sleeve at C.



Solution: The position vectors are

$$\mathbf{r}_{B/A} = 1.8(\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 1.56\mathbf{i} + 0.9\mathbf{j}\ \text{(m)}.$$

$$\mathbf{r}_{B/C} = \mathbf{r}_{B/A} - 2\mathbf{i} = -0.441\mathbf{i} + 0.9\mathbf{j}\ \text{(m)}.$$

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -10 \\ 1.56 & 0.9 & 0 \end{bmatrix} = 9\mathbf{i} - 15.6\mathbf{j}\ \text{(m/s)}$$

The unit vector from B to C is

$$\mathbf{e}_{BC} = \frac{-\mathbf{r}_{B/C}}{|\mathbf{r}_{B/C}|} = 0.4401\mathbf{i} - 0.8979\mathbf{j}.$$

The relative velocity is parallel to this vector:

$$\mathbf{v}_{C\text{rel}} = v_{C\text{rel}}\mathbf{e}_{BC} = v_{C\text{rel}}(0.4401\mathbf{i} - 0.8979\mathbf{j})\ \text{(m/s)}$$

The velocity of B in terms of the velocity of C is

$$\mathbf{v}_B = \mathbf{v}_{\text{rel}} + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = \mathbf{v}_{\text{rel}} + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -0.441 & 0.9 & 0 \end{bmatrix},$$

$$\mathbf{v}_B = 0.4401 v_{C\text{rel}}\mathbf{i} - 0.8979 v_{C\text{rel}}\mathbf{j} - 0.9\omega_{BC}\mathbf{i} - 0.441\omega_{BC}\mathbf{j}\ \text{(m/s)}.$$

Equate the expressions for $\mathbf{v}_B$ and separate components:

$$9 = 0.4401 v_{C\text{rel}} - 0.9\omega_{BC}, \text{ and}$$

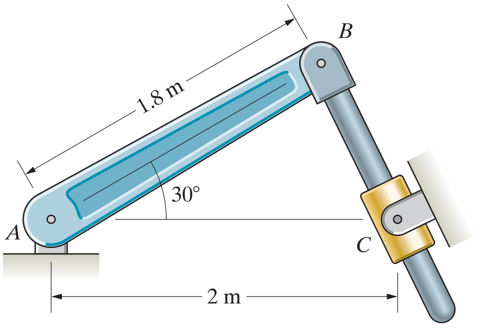
$$-15.6 = -0.8979 v_{C\text{rel}} - 0.441\omega_{BC}.$$

Solve:

$\boxed{v_{C\text{rel}} = 17.96 \text{ m/s}}$ (toward C).

$\boxed{\omega_{BC} = -1.22 \text{ rad/s}}$ (clockwise)

Problem 17.181 In Problem 17.180, arm AB is rotating with an angular velocity of 10 rad/s and an angular acceleration of 20 rad/s^2, both in the clockwise direction. Determine the angular acceleration of arm BC.



Solution: Use the solution to 17.180. The vector

$$\mathbf{r}_{B/A} = 1.8(\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 1.56\mathbf{i} + 0.9\mathbf{j} \text{ (m)}.$$

$$\mathbf{r}_{B/C} = \mathbf{r}_{B/A} - 2\mathbf{i} = -0.441\mathbf{i} + 0.9\mathbf{j} \text{ (m)}.$$

The angular velocity:

$$\omega_{BC} = -1.22 \text{ rad/s},$$

and the relative velocity is $v_{C\text{rel}} = 17.96$ m/s.

The unit vector parallel to bar BC is $\mathbf{e} = 0.4401\mathbf{i} - 0.8979\mathbf{j}$

The acceleration of point B is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{AB} \\ 1.56 & 0.9 & 0 \end{bmatrix} - \omega_{AB}^2(1.56\mathbf{i} + 0.9\mathbf{j}),$$

$$\mathbf{a}_B = 18\mathbf{i} - 31.2\mathbf{j} - 155.9\mathbf{i} - 90\mathbf{j} = -137.9\mathbf{i} - 121.2\mathbf{j} \text{ (m/s}^2).$$

The acceleration of point B in terms of the acceleration of bar BC is

$$\mathbf{a}_B = \mathbf{a}_{C\text{rel}} + 2\boldsymbol{\omega}_{BC} \times \mathbf{v}_{C\text{rel}} + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C}.$$

Expanding term by term:

$$\mathbf{a}_{C\text{rel}} = a_{C\text{rel}}(0.4401\mathbf{i} - 0.8979\mathbf{j}) \text{ (m/s}^2),$$

$$2\boldsymbol{\omega}_{BC} \times \mathbf{v}_{C\text{rel}} = 2v_{C\text{rel}}\omega_{BC} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 0.440 & -0.8979 & 0 \end{bmatrix}$$

$$= -39.26\mathbf{i} - 19.25\mathbf{j} \text{ (m/s}^2),$$

$$\boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -0.4411 & 0.9 & 0 \end{bmatrix}$$

$$= \alpha_{BC}(-0.9\mathbf{i} - 0.4411\mathbf{j}) \text{ (m/s}^2)$$

$$- \omega_{BC}^2(-0.4411\mathbf{i} + 0.9\mathbf{j})$$

$$= 0.6539\mathbf{i} - 1.334\mathbf{j} \text{ (m/s}^2).$$

Collecting terms,

$$\mathbf{a}_B = a_{C\text{rel}}(0.4401\mathbf{i} - 0.8979\mathbf{j}) - \alpha_{BC}(0.9\mathbf{i} + 0.4411\mathbf{j})$$

$$- 38.6\mathbf{i} - 20.6\mathbf{j} \text{ (m/s}^2).$$

Equate the two expressions for $\mathbf{a}_B$ and separate components:

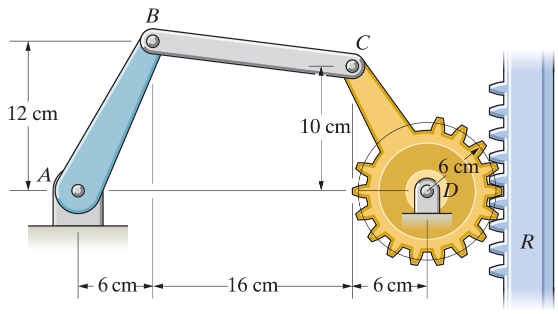
$$-137.9 = 0.4401a_{C\text{rel}} - 0.9\alpha_{BC} - 38.6,$$

and $-121.2 = -0.8979a_{C\text{rel}} - 0.4411\alpha_{BC} - 20.6.$

Solve: $a_{C\text{rel}} = 46.6$ m/s^2 (toward C)

$$\boxed{\alpha_{BC} = 133.1 \text{ rad/s}^2}$$

Problem 17.182 Arm AB is rotating with a constant counterclockwise angular velocity of 10 rad/s. Determine the vertical velocity and acceleration of the rack R of the rack-and-pinion gear.



Solution: The vectors:

$\mathbf{r}_{B/A} = 6\mathbf{i} + 12\mathbf{j}$ (cm). $\mathbf{r}_{C/B} = 16\mathbf{i} - 2\mathbf{j}$ (cm).

The velocity of point B is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ 6 & 12 & 0 \end{bmatrix} = -120\mathbf{i} + 60\mathbf{j} \text{ (cm/s)}.$$

The velocity of point C in terms of the velocity of point B is

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B} = \mathbf{v}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 16 & -2 & 0 \end{bmatrix}$$

$$= -120\mathbf{i} + 60\mathbf{j} + \omega_{BC}(2\mathbf{i} + 16\mathbf{j}) \text{ (cm/s)}$$

The velocity of point C in terms of the velocity of the gear arm CD is

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ -6 & 10 & 0 \end{bmatrix}$$

$$= -10\omega_{CD}\mathbf{i} - 6\omega_{CD}\mathbf{j} \text{ (cm/s)}.$$

Equate the two expressions for $\mathbf{v}_C$ and separate components:

$$-120 + 2\omega_{BC} = -10\omega_{CD}, \quad 60 + 16\omega_{BC} = -6\omega_{CD}.$$

Solve: $\omega_{BC} = -8.92$ rad/s, $\omega_{CD} = 13.78$ rad/s,

where the negative sign means a clockwise rotation. The velocity of the rack is

$$\mathbf{v}_R = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{R/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{CD} \\ 6 & 0 & 0 \end{bmatrix} = 6\omega_{CD}\mathbf{j},$$

$$\boxed{\mathbf{v}_R = 82.7\mathbf{j} \text{ (cm/s)} = 0.827\mathbf{j} \text{ (m/s)}}$$

The angular acceleration of point B is

$$\mathbf{a}_B = -\omega_{AB}^2 \mathbf{r}_{B/A} = -100(6\mathbf{i} + 12\mathbf{j}) = -600\mathbf{i} - 1200\mathbf{j} \text{ (cm/s}^2\text{)}.$$

The acceleration of point C is

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B},$$

$$\mathbf{a}_C = \mathbf{a}_B + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 16 & -2 & 0 \end{bmatrix} - \omega_{BC}^2(16\mathbf{i} - 2\mathbf{j})$$

$$= \mathbf{a}_B + 2\alpha_{BC}\mathbf{i} + 16\alpha_{BC}\mathbf{j} - \omega_{BC}^2(16\mathbf{i} - 2\mathbf{j}).$$

Noting

$$\mathbf{a}_B - \omega_{BC}^2(16\mathbf{i} - 2\mathbf{j}) = -600\mathbf{i} - 1200\mathbf{j} - 1272.7\mathbf{i} + 159.1\mathbf{j}$$

$$= -1872.7\mathbf{i} - 1040.9\mathbf{j},$$

from which $\mathbf{a}_C = +\alpha_{BC}(2\mathbf{i} + 16\mathbf{j}) - 1873\mathbf{i} - 1041\mathbf{j}$ (cm/s^2)

The acceleration of point C in terms of the gear arm is

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2 \mathbf{r}_{C/D}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ -6 & 10 & 0 \end{bmatrix} - \omega_{CD}^2(-6\mathbf{i} + 10\mathbf{j}) \text{ (cm/s}^2\text{)},$$

$$\mathbf{a}_C = -10\alpha_{CD}\mathbf{i} - 6\alpha_{CD}\mathbf{j} + 1140\mathbf{i} - 1900\mathbf{j} \text{ (cm/s}^2\text{)}.$$

Equate expressions for $\mathbf{a}_C$ and separate components:

$$2\alpha_{BC} - 1873 = -10\alpha_{CD} + 1140,$$

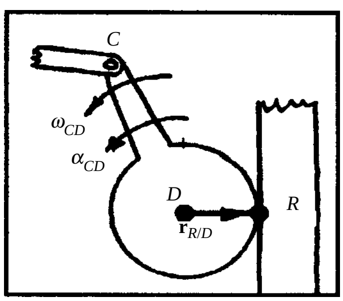
$$16\alpha_{BC} - 1041 = -6\alpha_{CD} - 1900.$$

Solve: $\alpha_{CD} = 337.3$ rad/s^2, and $\alpha_{BC} = -180.2$ rad/s^2.

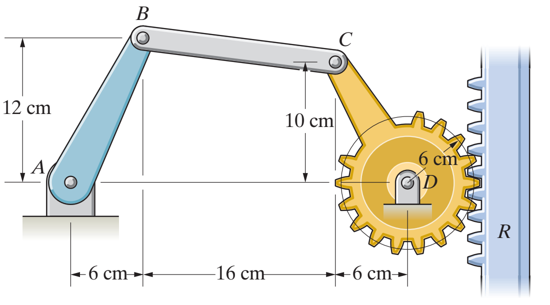
The acceleration of the rack R is the tangential component of the acceleration of the gear at the point of contact with the rack:

$$\mathbf{a}_R = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{R/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{CD} \\ 6 & 0 & 0 \end{bmatrix} = 6\alpha_{CD}\mathbf{j} \text{ (cm/s}^2\text{)}.$$

$$\boxed{\mathbf{a}_R = 2024\mathbf{j} \text{ (cm/s}^2\text{)} = 20.24\mathbf{j} \text{ (m/s}^2\text{)}}$$



Problem 17.183 The rack R of the rack-and-pinion gear is moving upward with a constant velocity of 120 cm/s. What are the angular velocity and angular acceleration of bar BC?



Solution: The constant velocity of the rack R implies that the angular acceleration of the gear is zero, and the angular velocity of the gear is $\omega_{CD} = \dfrac{120}{6} = 20$ rad/s. The velocity of point C in terms of the gear angular velocity is

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 20 \\ -6 & 10 & 0 \end{bmatrix} = -200\mathbf{i} - 120\mathbf{j} \text{ (cm/s)}.$$

The velocity of point B in terms of the velocity of point C is

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = \mathbf{v}_C + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ -16 & 2 & 0 \end{bmatrix},$$

$$\mathbf{v}_B = -200\mathbf{i} - 120\mathbf{j} - 2\omega_{BC}\mathbf{i} - 16\omega_{BC}\mathbf{j} \text{ (cm/s)}.$$

The velocity of point B in terms of the angular velocity of the arm AB is

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{AB} \\ 6 & 12 & 0 \end{bmatrix}$$

$$= -12\omega_{AB}\mathbf{i} + 6\omega_{AB}\mathbf{j} \text{ (cm/s)}.$$

Equate the expressions for $\mathbf{v}_B$ and separate components

$$-200 - 2\omega_{BC} = -12\omega_{AB}, -120 - 16\omega_{BC} = 6\omega_{AB}.$$

Solve: $\omega_{AB} = 14.5$ rad/s, $\boxed{\omega_{BC} = -12.94 \text{ rad/s}}$, where the negative sign means a clockwise rotation. The angular acceleration of the point C in terms of the angular velocity of the gear is

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D} = 0 - \omega_{CD}^2(-6\mathbf{i} + 10\mathbf{j})$$

$$= 2400\mathbf{i} - 4000\mathbf{j} \text{ (cm/s}^2\text{)}.$$

The acceleration of point B in terms of the acceleration of C is

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\mathbf{r}_{B/C}$$

$$= \mathbf{a}_C + \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ -16 & 2 & 0 \end{bmatrix} - \omega_{BC}^2(-16\mathbf{i} + 2\mathbf{j}).$$

$$\mathbf{a}_B = \alpha_{BC}(-2\mathbf{i} - 16\mathbf{j}) + 2400\mathbf{i} - 4000\mathbf{j} + 2680\mathbf{i} - 335\mathbf{j}.$$

$$\mathbf{a}_B = -2\alpha_{BC}\mathbf{i} - 16\alpha_{BC}\mathbf{j} + 5080\mathbf{i} - 433.5\mathbf{j} \text{ (cm/s}^2\text{)}.$$

The acceleration of point B in terms of the angular acceleration of arm AB is

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A}$$

$$= \alpha_{AB}(\mathbf{k} \times (6\mathbf{i} + 12\mathbf{j})) - \omega_{AB}^2(6\mathbf{i} + 12\mathbf{j}) \text{ (cm/s}^2\text{)}$$

$$\mathbf{a}_B = \alpha_{AB}(-12\mathbf{i} + 6\mathbf{j}) - 1263.2\mathbf{i} - 2526.4\mathbf{j} \text{ (cm/s}^2\text{)}.$$

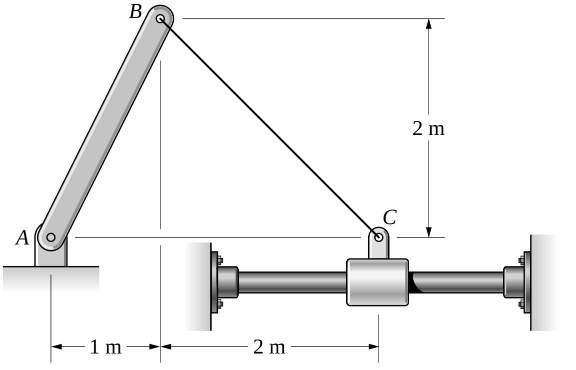
Equate the expressions for $\mathbf{a}_B$ and separate components:

$$-2\alpha_{BC} + 5080 = -12\alpha_{AB} - 1263.2,$$

$$-16\alpha_{BC} - 4335 = 6\alpha_{AB} - 2526.4.$$

Solve: $\alpha_{AB} = -515.2$ rad/s^2, $\boxed{\alpha_{BC} = 80.17 \text{ rad/s}^2}$

Problem 17.184 Bar AB has a constant counterclockwise angular velocity of 2 rad/s. The 1-kg collar C slides on the smooth horizontal bar. At the instant shown, what is the tension in the cable BC?



Solution:

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A} = \mathbf{0} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix} = -4\mathbf{i} + 2\mathbf{j} \text{ (m/s)}.$$

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B} :$$

$$v_C\mathbf{i} = -4\mathbf{i} + 2\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{BC} \\ 2 & -2 & 0 \end{vmatrix}.$$

From the **i** and **j** components of this equation,

$$v_C = -4 + 2\omega_{BC},$$

$$0 = 2 + 2\omega_{BC},$$

we obtain $\omega_{BC} = -1$ rad/s.

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= \mathbf{0} + \mathbf{0} - (2)^2(\mathbf{i} + 2\mathbf{j})$$

$$= -4\mathbf{i} - 8\mathbf{j} \text{ (m/s}^2).$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B} :$$

$$a_C\mathbf{i} = -4\mathbf{i} - 8\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_{BC} \\ 2 & -2 & 0 \end{vmatrix} - (-1)^2(2\mathbf{i} - 2\mathbf{j}).$$

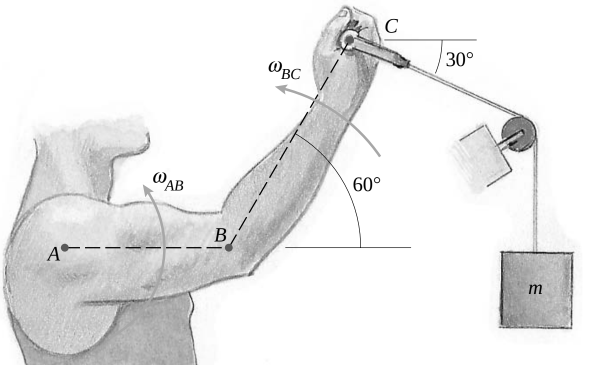
From the **i** and **j** components of this equation,

$$a_C = -4 + 2\alpha_{BC} - 2,$$

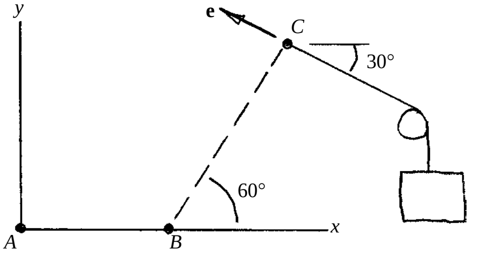
$$0 = -8 + 2\alpha_{BC} + 2,$$

we obtain $a_C = 0$. The force exerted on the collar at this instant is zero, so $\underline{T_{BC} = 0}$.

Problem 17.185 An athlete exercises his arm by raising the 8-kg mass m. The shoulder joint A is stationary. The distance AB is 300 mm, the distance BC is 400 mm, and the distance from C to the pulley is 340 mm. The angular velocities $\omega_{AB} = 1.5$ rad/s and $\omega_{BC} = 2$ rad/s are constant. What is the tension in the cable?



Solution:



$$\mathbf{a}_B = -\omega_{AB}^2 r_{B/A}\mathbf{i} = -(1.5)^2(0.3)\mathbf{i}$$

$$= -0.675\mathbf{i} \text{ (m/s}^2\text{)}.$$

$$\mathbf{a}_C = \mathbf{a}_B - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$= -0.675\mathbf{i} - (2)^2(0.4\cos 60^\circ\mathbf{i} + 0.4\sin 60^\circ\mathbf{j})$$

$$= -1.475\mathbf{i} - 1.386\mathbf{j} \text{ (m/s}^2\text{)}.$$

$$\mathbf{a}_C \cdot \mathbf{e} = (-1.475)(-\cos 30^\circ) + (-1.386)(\sin 30^\circ)$$

$$= 0.585 \text{ m/s}^2.$$

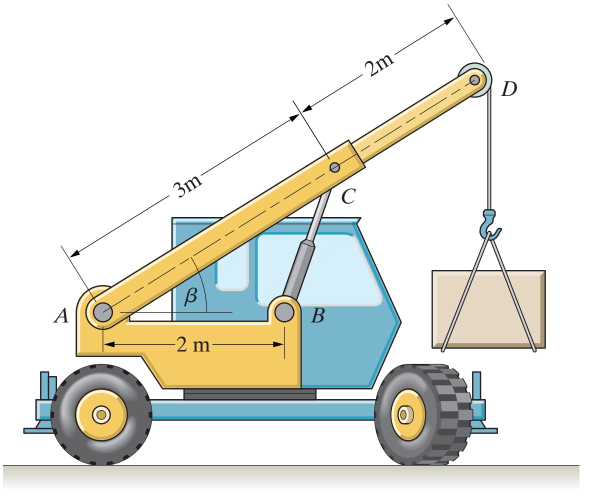
This is the upward acceleration of the mass, so

$$T - mg = m(0.585),$$

$$T = (8)(9.81 + 0.585)$$

$$= 83.2 \text{ N}.$$

Problem 17.186 The hydraulic actuator BC of the crane is extending (increasing in length) at a constant rate of 0.2 m/s. When the angle $\beta = 35^o$, what is the angular velocity of the crane's boom AD?



Solution: Using AD we have

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega}_{AD} \times \mathbf{r}_{C/A}$$

$$= 0 + \omega_{AD}\mathbf{k} \times (3\text{ m})(\cos 35^\circ \mathbf{i} + \sin 35^\circ \mathbf{j})$$

$$= (3\text{ m})\omega_{AD}(-\sin 35^\circ \mathbf{i} + \cos 35^\circ \mathbf{j})$$

Using cylinder BC

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{Crel} + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$= 0 + (0.2\text{ m/s})\left(\frac{[3\cos 35^\circ - 2]\mathbf{i} + [3\sin 35^\circ]\mathbf{j}}{\sqrt{[3\cos 35^\circ - 2]^2 + [3\sin 35^\circ]^2}}\right)$$

$$+ \omega_{BC}\mathbf{k} \times [3\cos 35^\circ - 2]\mathbf{i} + [3\sin 35^\circ]\mathbf{j}$$

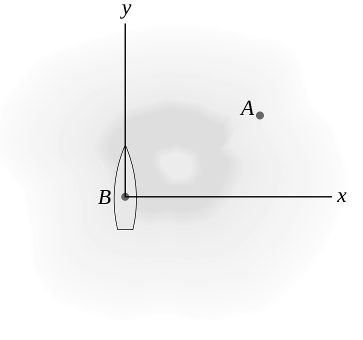
$$= (0.0514\text{ m/s} - \{1.72\text{ m}\}\omega_{BC})\mathbf{i} + (0.193\text{ m/s} - \{0.457\text{ m}\}\omega_{BC})\mathbf{j}$$

Equating components and solving we find

$$\omega_{BC} = 0.133\text{ rad/s}, \quad \omega_{AD} = 0.103\text{ rad/s}$$

$$\boxed{\omega_{AD} = 0.103\text{ rad/s}}$$

Problem 17.187 The coordinate system shown is fixed relative to the ship B. The ship uses its radar to measure the position of a stationary buoy A and determines it to be $400\mathbf{i} + 200\mathbf{j}$ (m). The ship also measures the velocity of the buoy relative to its body-fixed coordinate system and determines it to be $2\mathbf{i} - 8\mathbf{j}$ (m/s). What are the ship's velocity and angular velocity relative to the earth? (Assume that the ship's velocity is in the direction of the y axis).



Solution:

$$\mathbf{v}_A = 0 = \mathbf{v}_B + \mathbf{v}_{A\text{rel}} + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$= v_B\mathbf{j} + 2\mathbf{i} - 8\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega \\ 400 & 200 & 0 \end{vmatrix}.$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components to zero, $0 = 2 - 200\omega$ $\;0 = v_B - 8 + 400\omega$ we obtain $\omega = 0.01$ rad/s and $v_B = 4$ m/s.