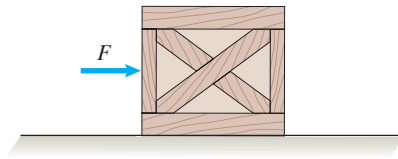**Problem 16.1** The 20-kg crate is stationary at time $t = 0$. It is subjected to a horizontal force given as a function of time (in newtons) by $F = 10 + 2t^2$.

(a) Determine the magnitude of the linear impulse exerted on the crate from $t = 0$ to $t = 4$ s.
(b) Use the principle of impulse and momentum to determine how fast the crate is moving at $t = 4$ s.

Solution:

(a) The impulse

$$I = \int_{t_0}^{t_1} F \, dt = \int_0^4 (10 + 2t^2) \, dt = 10(4) + \frac{2}{3}(4)^3 = 82.7 \text{ N-s}$$

$$\boxed{I = 82.7 \text{ N-s.}}$$

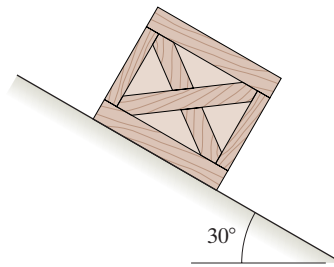(b) Use the principle of impulse and momentum

$$mv_0 + I = mv_1$$

$$0 + 82.7 \text{ N-s} = (20 \text{ kg})v \Rightarrow v = \frac{82.7 \text{ N-s}}{20 \text{ kg}}$$

$$\boxed{v = 4.13 \text{ m/s.}}$$

Problem 16.2 The 100-N crate is released from rest on the inclined surface at time $t = 0$. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.18$.

(a) Determine the magnitude of the linear impulse due to the forces acting on the crate from $t = 0$ to $t = 2$ s.
(b) Use the principle of impulse and momentum to determine how fast the crate is moving at $t = 2$ s.

Solution: We have

$$\Sigma F \nearrow: N - (100 \text{ N})\cos 30^\circ = 0 \Rightarrow N = 86.6 \text{ N}$$

(a) Then, along the slope the impulse is

$$I = (W \sin 30^\circ - \mu_k N)t$$

$$I = ([100 \text{ N}] \sin 30^\circ - [0.18][86.6\text{N}])(2 \text{ s})$$

$$\boxed{I = 68.8 \text{ N-s.}}$$

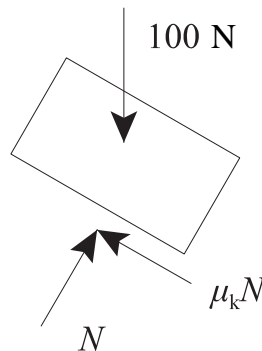(b) Using the principle of impulse-momentum,

$$mv_1 + I = mv_2$$

$$0 + 68.8 \text{ N-s} = \left(\frac{100 \text{ N}}{9.81 \text{ m/s}^2}\right) v_2$$

Solving we find

$$\boxed{v_2 = 6.75 \text{ m/s.}}$$

Problem 16.3 The mass of the helicopter is 9300 kg. It takes off vertically at time $t = 0$. The pilot advances the throttle so that the upward thrust of its engine (in kN) is given as a function of time in seconds by $T = 100 + 2t^2$.

(a) Determine the magnitude of the linear impulse due to the forces acting on the helicopter from $t = 0$ to $t = 3$ s.
(b) Use the principle of impulse and momentum to determine how fast the helicopter is moving at $t = 3$ s.

Solution:

(a) The impulse - using the total force (T and the weight).

$$I = \int_{t_1}^{t_2} F \, dt = \int_{t_1}^{t_2} (T - mg) \, dt$$

$$= \int_0^3 (100 + 2t^2 - 9.3[9.81]) \, dt = (8.77)(3) + \frac{2}{3}(3)^3 = 44.3 \text{ kN-s.}$$

$$\boxed{I = 44.3 \text{ kN-s.}}$$

(b) Using the principle of impulse - momentum,

$$mv_1 + I = mv_2$$

$$0 + 44.3 \text{ kN-s} = (9300 \text{ kg})v_2$$

$$\boxed{v_2 = 4.76 \text{ m/s.}}$$

Problem 16.4 A 150 million-kg cargo ship starts from rest. The total force exerted on it by its engines and hydrodynamic drag (in newtons) can be approximated as a function of time in seconds by $\Sigma F_t = 937,500 - 0.65t^2$. Use the principle of impulse and momentum to determine how fast the ship is moving in 16 minutes.

Solution: The impulse is

$$I = \int_{t_1}^{t_2} F \, dt = \int_0^{16(60)} (937,500 - 0.65t^2) \, dt$$

$$= (937,500)(960) - \frac{1}{3}(0.65)(960)^3 = 7.08 \times 10^8 \text{ N-s.}$$

Using the principle of impulse and momentum, we have

$$mv_1 + I = mv_2$$

$$0 + 7.08 \times 10^8 \text{ N-s} = (150 \times 10^6 \text{ kg})v_2$$

Solving, we find

$$\boxed{v_2 = 4.72 \text{ m/s (9.18 knots).}}$$

Problem 16.5   The combined mass of the motorcycle and rider is 136 kg. The coefficient of kinetic friction between the motorcycle's tires and the road is $\mu_k = 0.6$. The rider starts from rest and spins the rear (drive) wheel. The normal force between the rear wheel and road is 790 N.

(a)   What impulse does the friction force on the rear wheel exert in 2 s?

(b)   If you neglect other horizontal forces, what velocity is attained by the motorcycle in 2 s?

Solution:

$m = 136$ kg

$g = 9.81$ m/s^2

$N_R = 790$ N
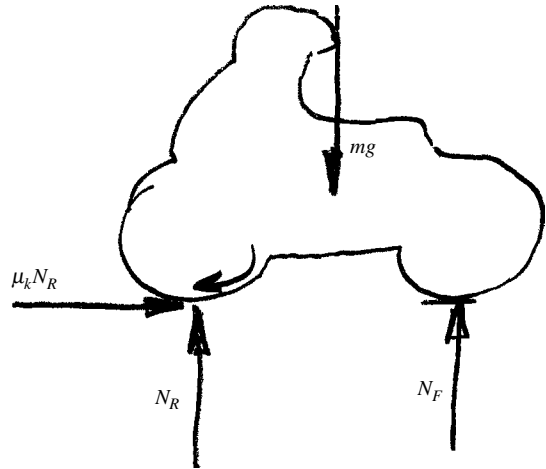
$$\sum F_x = \mu_k N_R = m\frac{dv_x}{dt}$$

$$\text{Imp} = \int_0^2 \mu_k N_R \, dt = \mu_k N_R t \Big|_0^2$$

(a) $\underline{\text{Imp} = (0.6 \times 790 \times 2) = 948 \text{ N-s}}$

(b) $\int_0^2 \mu_k N_R \, dt = m\int_0^v dv = \text{Imp}$

$948 \text{ N-s} = mv$

$\underline{v = 948 \text{ N-s}/136 \text{ kg} = 6.97 \text{ m/s}}$

Problem 16.6 A bioengineer models the force generated by the wings of the 0.2-kg snow petrel by an equation of the form $F = F_0(1 + \sin \omega t)$, where F_0 and ω are constants. From video measurements of a bird taking off, he estimates that $\omega = 18$ and determines that the bird requires 1.42 s to take off and is moving at 6.1 m/s when it does. Use the principle of impulse and momentum to determine the constant F_0.
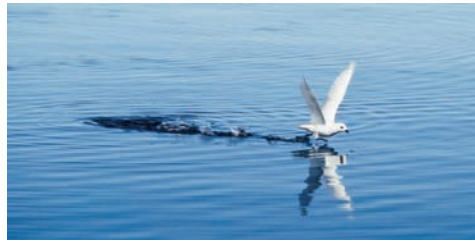
Solution:

$$\int_0^t F_0(1 + \sin \omega t)\, dt = mv$$

$$F_0 \left(t - \frac{1}{\omega} \cos \omega t \right)_0^t = F_0 \left(t + \frac{1}{\omega}[1 - \cos \omega t] \right) = mv$$

$$F_0 = \frac{mv}{t + \dfrac{1}{\omega}(1 - \cos \omega t)} = \frac{(0.2\ \text{kg})\,(6.1\ \text{m/s})}{(1.42\ \text{s}) + \dfrac{1}{18\ \text{rad/s}}(1 - \cos[(18)(1.42)])}$$
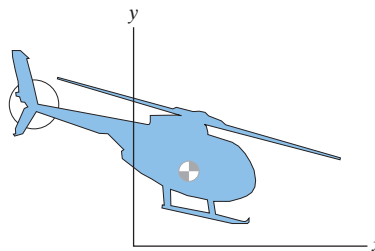
$\boxed{F_0 = 0.856\ \text{N.}}$

Problem 16.7 In Active Example 16.1, what is the average total force acting on the helicopter from $t = 0$ to $t = 10$ s?

Solution: From Active Example 16.1 we know the total impulse that occurs during the time. Then

$$\mathbf{F}\Delta t = \mathbf{I} \Rightarrow \mathbf{F} = \frac{\mathbf{I}}{\Delta t}$$

$$\mathbf{F} = \frac{(36{,}000\mathbf{i} + 3600\mathbf{j})\ \text{N-s}}{10\ \text{s}}$$

$\boxed{\mathbf{F} = (3600\mathbf{i} + 360\mathbf{j})\ \text{N.}}$

Problem 16.8 At time $t = 0$, the velocity of the 15-kg object is $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ (m/s). The total force acting on it from $t = 0$ to $t = 4$ s is

$$\Sigma\mathbf{F} = (2t^2 - 3t + 7)\mathbf{i} + 5t\mathbf{j} + (3t + 7)\mathbf{k}\ \text{(N)}.$$

Use the principle of impulse and momentum to determine its velocity at $t = 4$ s.
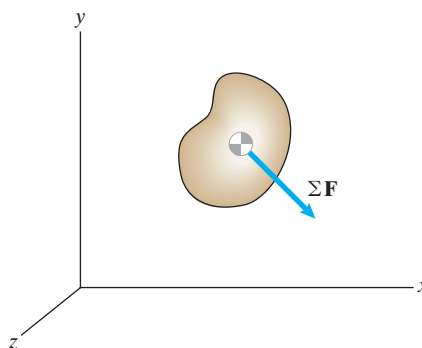
Solution: Working in components we have

$$(15)(2) + \int_0^4 (2t^2 - 3t + 7)\, dt = (15)v_{2x}$$

$$(15)(3) + \int_0^4 5t\, dt = (15)v_{2y}$$

$$(15)(-5) + \int_0^4 (3t + 7)\, dt = (15)v_{2z}$$

Solving we find $v_{2x} = 5.11$ m/s, $v_{2y} = 5.67$ m/s, $v_{2z} = -1.53$ m/s.
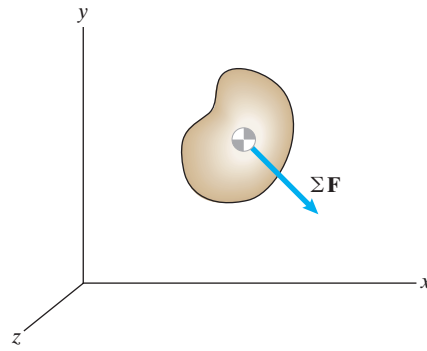
$\boxed{\mathbf{v}_2 = (5.11\mathbf{i} + 5.67\mathbf{j} - 1.53\mathbf{k})\ \text{m/s.}}$

Problem 16.9 At time $t = 0$, the velocity of the 15-kg object is $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ (m/s). The total force acting on it from $t = 0$ to $t = 4$ s is

$$\Sigma\mathbf{F} = (2t^2 - 3t + 7)\mathbf{i} + 5t\mathbf{j} + (3t + 7)\mathbf{k} \text{ (N)}.$$

What is the average total force on the object during the interval of time from $t = 0$ to $t = 4$ s?

Solution: The components of the impulse are

$$I_x = \int_0^4 (2t^2 - 3t + 7)\, dt = \frac{2}{3}(4)^3 - \frac{3}{2}(4)^2 + 7(4) = 46.7 \text{ N,}$$

$$I_y = \int_0^4 5t\, dt = \frac{5}{2}(4)^2 = 40 \text{ N,}$$

$$I_z = \int_0^4 (3t + 7)\, dt = \frac{3}{2}(4)^2 + 7(4) = 52 \text{ N.}$$
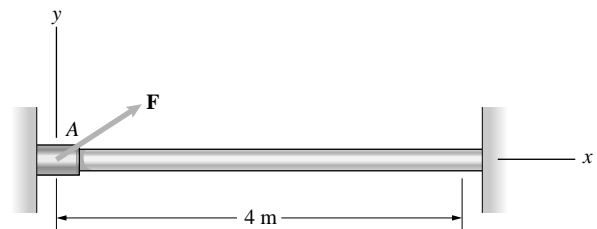
The average force is given by

$$\mathbf{F}_{\text{ave}} = \frac{\mathbf{I}}{\Delta t} = \frac{(46.7\mathbf{i} + 40\mathbf{j} + 52\mathbf{k}) \text{ N}}{4 \text{ s}}$$

$$\boxed{\mathbf{F}_{\text{ave}} = (11.7\mathbf{i} + 10\mathbf{j} + 13\mathbf{k})\text{N.}}$$

Problem 16.10 The 1-N collar A is initially at rest in the position shown on the smooth horizontal bar. At $t = 0$, a force

$$\mathbf{F} = \left(\tfrac{1}{20}\right) t^2\mathbf{i} + \left(\tfrac{1}{10}\right) t\mathbf{j} - \left(\tfrac{1}{30}\right) t^3\mathbf{k} \text{ (N)}$$

is applied to the collar, causing it to slide along the bar. What is the velocity of the collar at $t = 2$ s?

Solution: The impulse applied to the collar is $\displaystyle\int_{t_1}^{t_2} \Sigma\mathbf{F}\, dt = mv_{x2} - mv_{x1}$: Evaluating, we get

$$\int_0^2 \frac{1}{20}t^2\, dt = (1/9.81)v_{x2},$$

or $\left[\dfrac{1}{60}t^3\right]_0^2 = (1/9.81)v_{x2}.$

Hence, $v_{x2} = 1.31$ m/s.

Problem 16.11 The y axis is vertical and the curved bar is smooth. The 4-N slider is released from rest in position 1 and requires 1.2 s to slide to position 2. What is the magnitude of the average tangential force acting on the slider as it moves from position 1 to position 2?
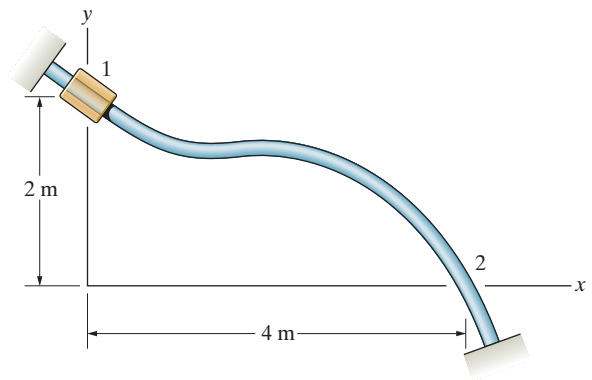
Solution: We will use the principle of work and energy to find the velocity at position 2.

$$T_1 + W_{1 \to 2} = T_2$$

$$0 + (4 \text{ N})(2 \text{ m}) = \frac{1}{2}\left(\frac{4 \text{ N}}{9.81 \text{ m/s}^2}\right) v_2{}^2 \Rightarrow v_2 = 6.26 \text{ m/s}.$$

Now, using the principle of impulse - momentum we can find the average tangential force

$$mv_1 + F_{\text{ave}} \Delta t = mv_2$$

$$F_{\text{ave}} = m\frac{(v_2 - v_1)}{\Delta t} = \left(\frac{4 \text{ N}}{9.81 \text{ m/s}^2}\right)\frac{(6.26 \text{ m/s} - 0)}{1.2 \text{ s}}$$

$$\boxed{F_{\text{ave}} = 2.13 \text{ N.}}$$

Problem 16.12 During the first 5 s of the 14,200-kg airplane's takeoff roll, the pilot increases the engine's thrust at a constant rate from 22 kN to its full thrust of 112 kN.

(a) What impulse does the thrust exert on the airplane during the 5 s?
(b) If you neglect other forces, what total time is required for the airplane to reach its takeoff speed of 46 m/s?
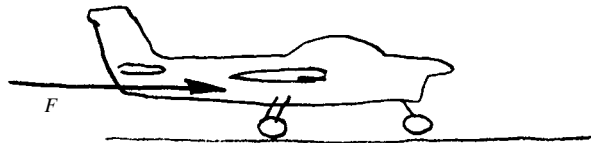
Solution:

$$m = 14200 \text{ kg}$$

$$F = (22000 + 18000 \, t) \text{ N}$$

$$\text{Imp} = \int_0^5 (22000 + 18000 \, t) \, dt (\text{N-s})$$

$$\text{Imp} = 22000 \, t + 9000 \, t^2 \Big|_0^5$$

(a) $\underline{\text{Imp} = 335000 \text{ N-s} = 335 \text{ kN-s}}$

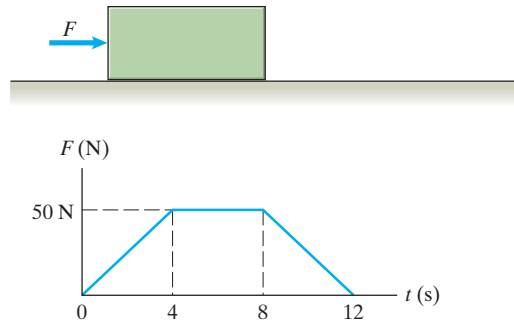$$\int_0^{t_f} F \, dt = mv_f - m\cancel{v_0}{}^0$$

$$\int_0^5 (22000 + 18000 \text{ t}) \, dt + \int_5^t (112000) \, dt = mv_f$$

$$335000 + 112000 \, t \Big|_5^t = (14200)(46)$$

$$112000(t - 5) + 335000 = (14200)(46)$$

(b) $\underline{t = 7.84 \text{ s}}$

Problem 16.13 The 10-kg box starts from rest on the smooth surface and is subjected to the horizontal force described in the graph. Use the principle of impulse and momentum to determine how fast the box is moving at $t = 12$ s.

**Solution:** The impulse is equal to the area under the curve in the graph

$$I = \frac{1}{2}(50 \text{ N})(4 \text{ s}) + (50 \text{ N})(4 \text{ s}) + \frac{1}{2}(50 \text{ N})(4 \text{ s}) = 400 \text{ N-s}.$$

Using the principle of impulse and momentum we have

$$mv_1 + I = mv_2 \Rightarrow 0 + (400 \text{ N-s}) = (10 \text{ kg})v_2$$

Solving we find $\boxed{v_2 = 40 \text{ m/s}.}$

Problem 16.14 The 10-kg box starts from rest and is subjected to the horizontal force described in the graph. The coefficients of friction between the box and the surface are $\mu_s = \mu_k = 0.2$. Determine how fast the box is moving at $t = 12$ s.
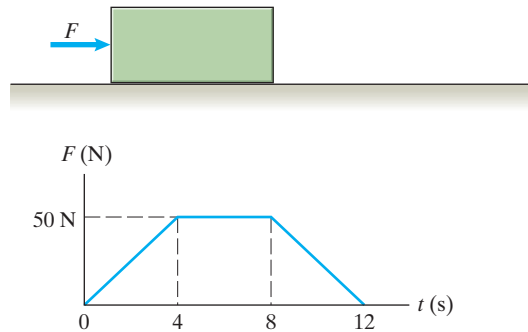
Solution: The box will not move until the force F is able to overcome friction. We will first find this critical time.

$$N = W = (10 \text{ kg})(9.81 \text{ m/s}^2) = 98.1 \text{ N}$$

$$f = \mu N = (0.2)(98.1 \text{ N}) = 19.6 \text{ N}$$

$$F = \frac{50 \text{ N}}{4 \text{ s}}t = 19.6 \text{ N} \Rightarrow t_{cr} = 1.57 \text{ s}.$$

The impulse from t_{cr} to 12 s is

$$I = \frac{(50 \text{ N} + 19.6 \text{ N})}{2}(4 \text{ s} - 1.57 \text{ s}) + (50 \text{ N})(4 \text{ s}) + \frac{(50 \text{ N})}{2}(4 \text{ s})$$

$$-(19.6 \text{ N})(12 \text{ s} - 1.57 \text{ s}) = 180 \text{ N-s}.$$

The principle of impulse and momentum gives

$$mv_1 + I = mv_2 \Rightarrow 0 + 180 \text{ N-s} = (10 \text{ kg})v_{2.}$$

Solving we find $\boxed{v_2 = 18.0 \text{ m/s}.}$

Problem 16.15 The crate has a mass of 120 kg, and the coefficients of friction between it and the sloping dock are $\mu_s = 0.6$ and $\mu_k = 0.5$. The crate starts from rest, and the winch exerts a tension $T = 1220$ N.
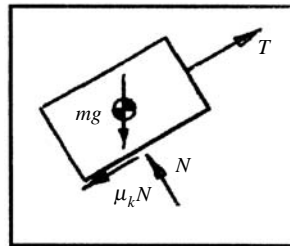
(a) What impulse is applied to the crate during the first second of motion?
(b) What is the crate's velocity after 1 s?
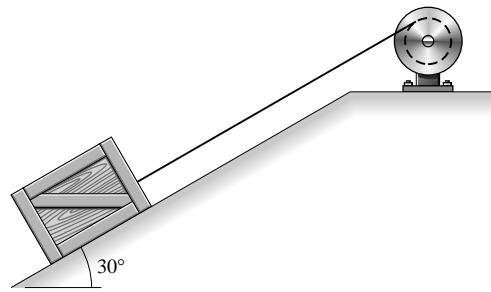
Solution: The motion starts only if $T - mg \sin 30° > \mu_s mg \cos 30°$, from which $631.4 > 611.7$. The motion indeed starts.

(a) The impulse in the first second is

$$\int_{t_1}^{t_2} F \, dt = \int_0^1 (T - mg \sin 30° - mg\mu_k \cos 30°) \, dt$$
$$= 121.7 \, t = 121.7 \text{ N-s}$$

(b) The velocity is $v = \dfrac{121.7}{120} = 1.01$ m/s

Problem 16.16 Solve Problem 16.15 if the crate starts from rest at $t = 0$ and the winch exerts a tension $T = 1220 + 200t$ N.

Solution: From the solution to Problem 16.15, motion will start.

(a) The impulse at the end of 1 second is

$$\int_{t_1}^{t_2} F \, dt = \int_0^1 (1220 + 200t - mg \sin 30^0 - \mu_k mg \cos 30^0) \, dt$$
$$= [1220t + 100t^2 - 1098.3t]_0^1 = 221.7 \text{ N-s}$$

(b) The velocity is $v = \dfrac{221.7}{m} = \dfrac{221.7}{120} = 1.85$ m/s

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Problem 16.17 In an assembly-line process, the 20-kg package A starts from rest and slides down the smooth ramp. Suppose that you want to design the hydraulic device B to exert a constant force of magnitude F on the package and bring it to a stop in 0.15 s. What is the required force F?
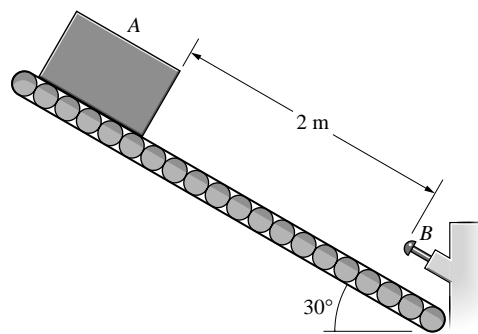
Solution: Use conservation of energy to obtain the velocity of the crate at point of contact with device B. $mgh = \dfrac{1}{2}mv_B^2$, where $h = 2 \sin 30° = 1$ m, from which $v_B = \sqrt{2g} = 4.43$ m/s. The impulse to be exerted by B is $\int_{t_1}^{t_2} F \, dt = mv_B = 88.6$ N-s. The constant force to be applied by device B is $F - mg \sin 30° = \dfrac{88.6}{0.15} = 590.67$ N, from which $F = 688.8$ N
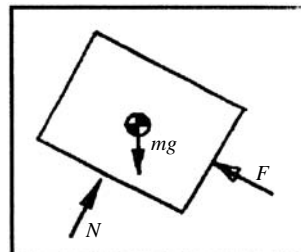
Problem 16.18 The 20-kg package A starts from rest and slides down the smooth ramp. If the hydraulic device B exerts a force of magnitude $F = 540(1 + 0.4t^2)$ N on the package, where t is in seconds measured from the time of first contact, what time is required to bring the package to rest?
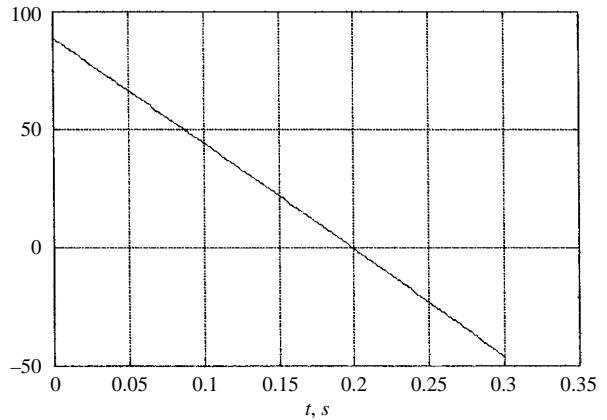
Solution: See the solution of Problem 16.17. The velocity at first contact is 4.43 m/s. Impulse and momentum is

$$\int_0^t [mg \sin 30° - 540(1 + 0.4t^2)]\, dt = 0 - 4.43 \text{ m}.$$

Integrating yields

$$mgt \sin 30° - 540\left(t + \frac{0.4t^3}{3}\right) + 4.43 \text{ m} = 0.$$

The graph of the left side of this equation as a function of t is shown. By examining calculated results, we estimate the solution to be $\underline{t = 0.199 \text{ s}}$.

Problem 16.19 In a cathode-ray tube, an electron (mass $= 9.11 \times 10^{-31}$ kg) is projected at O with velocity $\mathbf{v} = (2.2 \times 10^7)\mathbf{i}$ (m/s). While the electron is between the charged plates, the electric field generated by the plates subjects it to a force $\mathbf{F} = -eE\mathbf{j}$. The charge of the electron is $e = 1.6 \times 10^{-19}$ C (coulombs), and the electric field strength is $E = 15 \sin(\omega t)$ kN/C, where the circular frequency $\omega = 2 \times 10^9$ s^{-1}.
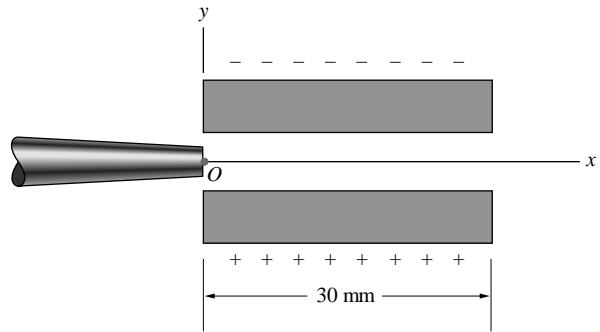
(a) What impulse does the electric field exert on the electron while it is between the plates?

(b) What is the velocity of the electron as it leaves the region between the plates?

Solution: The x component of the velocity is unchanged. The time spent between the plates is $t = \dfrac{0.03}{2.2 \times 10^7} = 1.36 \times 10^{-9}$ s.

(a) The impulse is

$$\int_{t_1}^{t_2} F\, dt = \int_0^t (-eE)\, dt = \int_0^t -e(15 \times 10^3)(\sin \omega t)\, dt$$

$$= \left[\frac{(15 \times 10^3)e}{\omega} \cos \omega t\right]_0^{1.36 \times 10^{-9}}$$

$$\boxed{\int_{t_1}^{t_2} F\, dt = -2.3 \times 10^{-24} \text{ N-s}}$$

The y component of the velocity is

$$v_y = \frac{-2.3 \times 10^{-24}}{9.11 \times 10^{-31}} = -2.52 \times 10^6 \text{ m/s}.$$

(b) The velocity on emerging from the plates is

$$\boxed{\mathbf{v} = 22 \times 10^6 \mathbf{i} - 2.5 \times 10^6 \mathbf{j} \text{ m/s}.}$$

Problem 16.20 The two weights are released from rest at time $t = 0$. The coefficient of kinetic friction between the horizontal surface and the 5-N weight is $\mu_k = 0.4$. Use the principle of impulse and momentum to determine the magnitude of the velocity of the 10-N weight at $t = 1$ s.
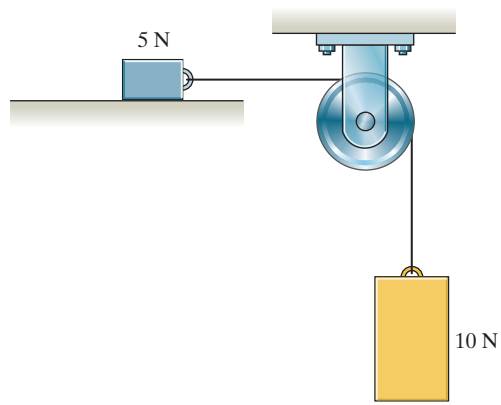
Strategy: Apply the principle to each weight individually.

Solution:

Impulse $= (10 \text{ N})(1 \text{ s}) - 0.4(5)(1 \text{ s}) = 8 \text{ N-s}$

$8 \text{ lb-s} = \left(\dfrac{15 \text{ N}}{9.81 \text{ m/s}^2} \right) v \Rightarrow \boxed{v = 5.23 \text{ m/s}}$

Problem 16.21 The two crates are released from rest. Their masses are $m_A = 40$ kg and $m_B = 30$ kg, and the coefficient of kinetic friction between crate A and the inclined surface is $\mu_k = 0.15$. What is the magnitude of the velocity of the crates after 1 s?
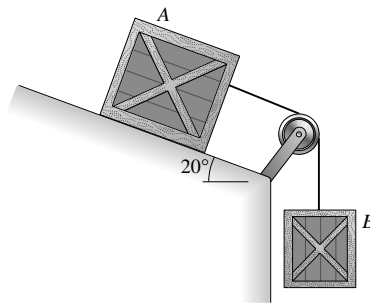
Solution: The force acting to move crate A is

$F_A = T + m_A g(\sin 20° - \mu_k \cos 20°)$

$\qquad = T + 78.9 \text{ N},$

where T is the tension in the cable.

The impulse, since the force is constant, is

$(T + 78.9)t = m_A v.$

For crate B,

$F_B = -T + m_B g = -T + 294.3.$

The impulse, since the force is constant, is

$(-T + 294.3)t = m_B v.$

For $t = 1$ s, add and solve: $78.9 + 294.3 = (40 + 30)v$, from which

$\boxed{v = 5.33 \text{ m/s}}$

Problem 16.22   The two crates are released from rest. Their masses are $m_A = 20$ kg and $m_B = 80$ kg, and the surfaces are smooth. The angle $\theta = 20°$. What is the magnitude of the velocity after 1 s?

Strategy: Apply the principle of impulse and momentum to each crate individually.

Solution: The free body diagrams are as shown:

Crate B: $\int_{t_1}^{t_2} \sum F_x\, dt = m v_{x2} - m v_{x1}$:

$\int_0^1 [(80)(9.81) \sin 20° - T]\, dt = (80)(v - 0).$

Crate A: $\int_{t_1}^{t_2} \sum F_x\, dt = m v_{x2} - m v_{x1}$:

$\int_0^1 [(20)(9.81) \sin 20° - T]\, dt = (20)[(-v) - 0]$

Subtracting the second equation from the first one,

$\int_0^1 (80 - 20)(9.81) \sin 20°\, dt = (80 + 20)v.$

Solving, we get $v = 2.01$ m/s.
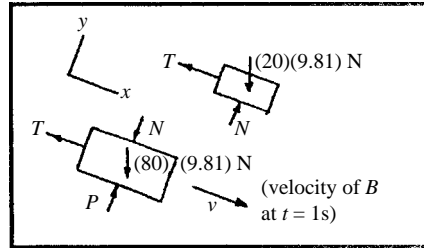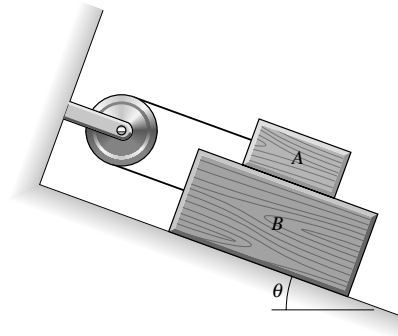
Problem 16.23   The two crates are released from rest. Their masses are $m_A = 20$ kg and $m_B = 80$ kg. The coefficient of kinetic friction between the contacting surfaces is $\mu_k = 0.1$. The angle $\theta = 20°$. What is the magnitude of the velocity of crate A after 1 s?

Solution: The free body diagrams are as shown:

The sums of the forces in the y direction equal zero:

$\sum F_y = N - (20)(9.81) \cos 20° = 0 \ N = 184$ N,

$\sum F_y = P - N - (80)(9.81) \cos 20° = 0 \ P = 922$ N

Crate B: $\int_{t_1}^{t_2} \sum F_x\, dt = m v_{x2} - m v_{x1}$:

$\int_0^1 [(80)(9.81) \sin 20° - 0.1\ P - 0.1\ \text{N-}T]\, dt = (80)(v - 0). \quad (1)$

Crate A: $\int_{t_1}^{t_2} \sum F_x\, dt = m v_{x2} - m v_{x1}$:

$\int_0^1 [(20)(9.81) \sin 20° + 0.1\ \text{N-}T]\, dt = (20)[(-v) - 0]. \quad (2)$

Subtracting Equation (2) from Equation (1),

$\int_0^1 [(80 - 20)(9.81) \sin 20° - 0.1\ \text{P} - 0.2\ \text{N}]\, dt = (80 + 20)v.$

Solving, $v = 0.723$ m/s.

Problem 16.24 At $t = 0$, a 20-kg projectile is given an initial velocity $v_0 = 20$ m/s at $\theta_0 = 60°$ above the horizontal.

(a) By using Newton's second law to determine the acceleration of the projectile, determine its velocity at $t = 3$ s.

(b) What impulse is applied to the projectile by its weight from $t = 0$ to $t = 3$ s?

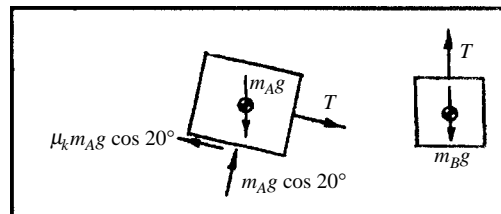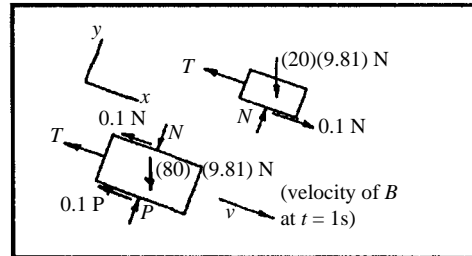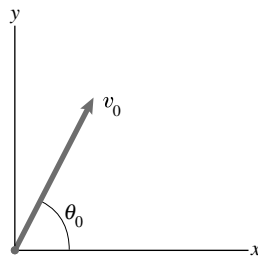(c) Use the principle of impulse and momentum to determine the projectile's velocity at $t = 3$ s.

Solution:
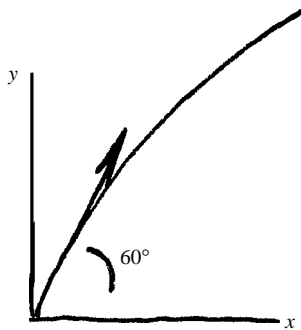
$\mathbf{a} = -g\mathbf{j}$

$a_x = 0$

$a_y = -g$

$v_{x_0} = v_0 \cos 60° = 10$ m/s

$v_{y_0} = v_0 \sin 60° = 17.32$ m/s

$x_0 = y_0 = 0$

$v_0 = 20$ m/s

$a_x = 0 \qquad\qquad a_y = -g$

$v_x = v_0 \cos 60° \qquad v_y = v_0 \sin 60° - gt$

$\mathbf{v} = (v_0 \cos 60°)\mathbf{i} + (v_0 \sin 60° - gt)\mathbf{j}$ (m/s)

At $t = 3$ s,

(a) $\underline{\mathbf{v} = 10\,\mathbf{i} - 12.1\,\mathbf{j}\ \text{(m/s)}}$

$\mathbf{I}_G = $ Impulse due to gravity

$\mathbf{F}_G = -mg\mathbf{j}$

$\mathbf{I}_G = -\displaystyle\int_0^3 mg\,\mathbf{j}\,dt$

$\mathbf{I}_G = -mg\,t\big|_0^3\,\mathbf{j} = -3\,mg\,\mathbf{j}$ (N-s)

(b) $\underline{\mathbf{I}_G = -589\,\mathbf{j}\ \text{(N-s)}}$

$I_G = m\mathbf{v}(3) - m\mathbf{v}_0$

$-589\mathbf{j} = mv_x\,\mathbf{i} + mv_y\,\mathbf{j} - mv_{xo}\,\mathbf{i} - mv_{yo}\,\mathbf{j}$

$x: \quad 0 = mv_x - mv_{xo}$

$v_x = v_{xo} = 10$ m/s

$y: \quad -589 = mv_y - mv_{yo}$

$20\,v_y = (20)(17.32) - 589$

$v_y = -12.1$ m/s

(c) $\underline{\mathbf{v} = 10\,\mathbf{i} - 12.1\,\mathbf{j}\ \text{(m/s) at } t = 3\ \text{s}}$

Problem 16.25 A soccer player kicks the stationary 0.45-kg ball to a teammate. The ball reaches a maximum height above the ground of 2 m at a horizontal distance of 5 m form the point where it was kicked. The duration of the kick was 0.04 seconds. Neglecting the effect of aerodynamic drag, determine the magnitude of the average force the player everted on the ball.

Solution: We will need to find the initial velocity of the ball. In the y direction we have

$$v_y = \sqrt{2gh} = \sqrt{2(9.81 \text{ m/s}^2)(2 \text{ m})}$$

$$= 6.26 \text{ m/s}.$$

The time of flight is given by

$$t = \frac{v_y}{g} = \frac{(6.26 \text{ m/s})}{(9.81 \text{ m/s}^2)} = 0.639 \text{ s}.$$

In the x direction we have

$$v_x = \frac{d}{t} = \frac{(5 \text{ m})}{(0.639 \text{ s})} = 7.83 \text{ m/s}.$$

The total velocity is then

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(6.26 \text{ m/s})^2 + (7.83 \text{ m/s})^2} = 10.0 \text{ m/s}.$$

The principle of impulse and momentum then gives

$$F\Delta t = mv \Rightarrow F = \frac{mv}{\Delta t} = \frac{(0.45 \text{ kg})(10.0 \text{ m/s})}{0.04 \text{ s}}.$$

$$\boxed{F = 113 \text{ N.}}$$

Problem 16.26 An object of mass $m = 2$ kg slides with constant velocity $v_0 = 4$ m/s on a horizontal table (seen from above in the figure). The object is connected by a string of length $L = 1$ m to the fixed point O and is in the position shown, with the string parallel to the x axis, at $t = 0$.
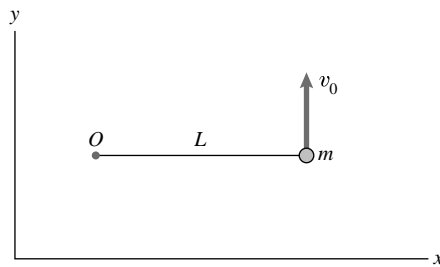
(a) Determine the x and y components of the force exerted on the mass by the string as functions of time.
(b) Use your results from part (a) and the principle of impulse and momentum to determine the velocity vector of the mass at $t = 1$ s.

Strategy: To do part (a), write Newton's second law for the mass in terms of polar coordinates.

Solution:

$\mathbf{T} = m v^2 / L \, \mathbf{e}_N$

$-\mathbf{e}_N = \cos\theta \mathbf{i} + \sin\theta \mathbf{j}$

$v_0 = rw$:

$\quad 4 = (1)w \quad w = 4$ rad/s

$\dfrac{d\theta}{dt} = w = 4$ rad/s

$\quad \theta = 4t$ rad

$\mathbf{T} = -(m v^2/L)\cos(4t)\mathbf{i} - (m v^2/L)\sin(4t)\mathbf{j}$

$\underline{\mathbf{T} = -32\cos 4t\,\mathbf{i} - 32\sin 4t\,\mathbf{j}}$ (N)

$T_x = -32\cos 4t$ N

$\underline{T_y = -32\sin 4t}$ N

$\displaystyle\int_0^1 \mathbf{F}\, dt = m\mathbf{v}_f - m\mathbf{v}_0$

$\displaystyle\int_0^1 \mathbf{T}\, dt = m v_x \mathbf{i} + m v_y \mathbf{j} - m(4)\mathbf{j}$
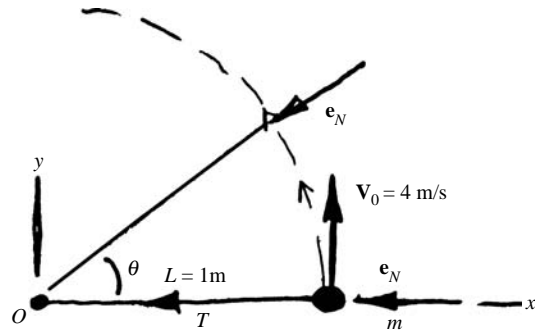
$\begin{cases} x: \displaystyle\int_0^1 (-32\cos 4t)\, dt = m v_x \\[2mm] y: \displaystyle\int_0^1 (-32\sin 4t)\, dt = m v_y - 4\text{ m} \end{cases}$

$\quad m = 2$ kg

$x: \left.\dfrac{-32\sin 4t}{4}\right|_0^1 = 2v_x \quad + \quad \left.\dfrac{32\cos 4t}{4}\right|_0^1 = 2v_y - 8$

Solving $\underline{\mathbf{v} = 3.03\mathbf{i} - 2.61\mathbf{j}}$ (m/s)

Problem 16.27 A rail gun, which uses an electromagnetic field to accelerate an object, accelerates a 30-g projectile from zero to 5 km/s in 0.0004 s. What is the magnitude of the average force exerted on the projectile?

Solution:

$$F_{\text{ave}} = \frac{(0.03 \text{ kg})(5000 \text{ m/s} - 0)}{0.0004 \text{ s}} = 375 \text{ kN}$$

Problem 16.28 The mass of the boat and its passenger is 420 kg. At time $t = 0$, the boat is moving at 14 m/s and its motor is turned off. The magnitude of the hydrodynamic drag force on the boat (in newtons) is given as a function of time by $830(1 - 0.08t)$. Determine how long it takes for the boat's velocity to decrease to 5 m/s.

Solution: The principle of impulse and momentum gives

$$mv_1 + \int_{t_1}^{t_2} F \, dt = mv_2$$

$$(420 \text{ kg}) (14 \text{ m/s}) - \int_0^t 830(1 - 0.08t) \, dt = (420 \text{ kg}) (5 \text{ m/s})$$

$$-830(t - 0.04t^2) = -3780$$

$$t^2 - 25t + 114 = 0$$

$$t = \frac{25 - \sqrt{25^2 - 4(1)(114)}}{2} = 5.99 \text{ s}.$$

$$\boxed{t = 5.99 \text{ s}.}$$

Problem 16.29 The motorcycle starts from rest at $t = 0$ and travels along a circular track with 300-m radius. From $t = 0$ to $t = 10$ s, the component of the total force on the motorcycle tangential to its path is $\Sigma F_t = 600$ N. The combined mass of the motorcycle and rider is 150 kg. Use the principle of impulse and momentum to determine the magnitude of the motorcycle's velocity at $t = 10$ s. (See Active Example 16.2.)

Solution:

$$(600 \text{ N})(10 \text{ s}) = (150 \text{ kg})v \Rightarrow \boxed{v = 40 \text{ m/s}}$$

Problem 16.30 Suppose that from $t = 0$ to $t = 10$ s, the component of the total tangential force on the motorcycle in Problem 16.29 is given as a function of time by $\Sigma F_t = 460 + 3t^2$ N. The combined mass of the motorcycle and rider is 150 kg. Use the principle of impulse and momentum to determine the magnitude of the motorcycle's velocity at $t = 10$ s. (See Active Example 16.2.)

Solution:

$$\int_0^{10 \text{ s}} (460 + 3t^2) \text{ N } dt = (150 \text{ kg})v \Rightarrow \boxed{v = 37.3 \text{ m/s}}$$

Problem 16.31 The titanium rotor of a Beckman Coulter ultracentrifuge used in biomedical research contains 2-gram samples at a distance of 41.9 mm from the axis of rotation. The rotor reaches its maximum speed of 130,000 rpm in 12 minutes.

(a) Determine the average tangential force exerted on a sample during the 12 minutes the rotor is acceleration.
(b) When the rotor is at its maximum speed, what normal acceleration are samples subjected to?

Solution:

(a) Using the principle of impulse and momentum we have

$$0 + F_{ave}\Delta t = mv$$

$$F_{ave} = \frac{mv}{\Delta t}$$

$$= \frac{(0.002 \text{ kg}) \left(130,000 \dfrac{\text{rev}}{\text{min}}\right) \left(\dfrac{2\pi \text{ rad}}{\text{rev}}\right) \left(\dfrac{1 \text{ min}}{60 \text{ s}}\right) (0.0419 \text{ m})}{(12 \text{ min}) \left(\dfrac{60 \text{ s}}{\text{min}}\right)}$$

$$\boxed{F_{ave} = 0.00158 \text{ N}}$$

(b)

$$a_n = \frac{v^2}{\rho} = \frac{(r\omega)^2}{r} = r\omega^2$$

$$= (0.0419 \text{ m}) \left(\left[130,000\dfrac{\text{rev}}{\text{min}}\right]\left[\dfrac{2\pi \text{ rad}}{\text{rev}}\right]\left[\dfrac{1 \text{ min}}{60 \text{ s}}\right]\right)^2$$

$$\boxed{a_n = 7.77 \times 10^6 \text{ m/s}^2.}$$

Problem 16.32 The angle θ between the horizontal and the airplane's path varies from $\theta = 0$ to $\theta = 30°$ at a constant rate of 5 degrees per second. During this maneuver, the airplane's thrust and aerodynamic drag are again balanced, so that the only force exerted on the airplane in the direction tangent to its path is due to its weight. The magnitude of the airplane's velocity when $\theta = 0$ is 120 m/s. Use the principle of impulse and momentum to determine the magnitude of the velocity when $\theta = 30°$.
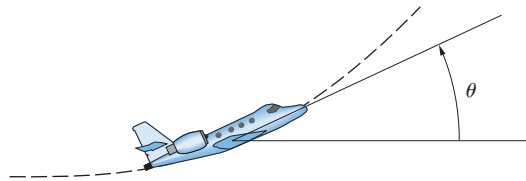
Solution:

$$w = \frac{d\theta}{dt} = 5°/s, \quad \text{constant} = 0.0873 \frac{\text{rad}}{\text{s}}$$

It takes 6 seconds to go from $\theta = 0°$ to $\theta = 30°$. The resisting force is

$$\mathbf{F}_t = -mg \sin\theta \mathbf{e}_t$$

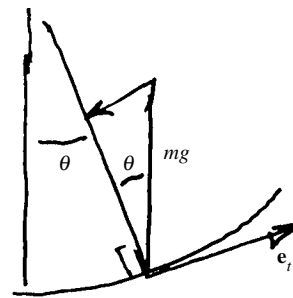$$\mathbf{F}_t = m\frac{dv}{dt}\mathbf{e}_t - mg \sin\theta \mathbf{e}_t$$

$$= \not{m}\frac{dv}{dt}\mathbf{e}_t - g\int_0^6 \sin\theta \, dt = \int_{120}^{v_f} dv$$

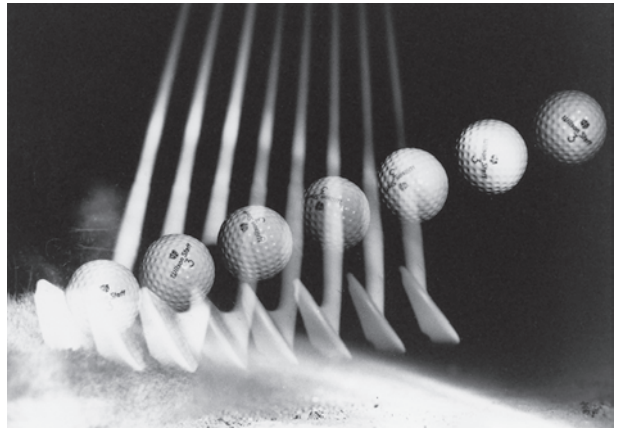$$-g\int_0^{30°} \sin\theta \frac{dt}{d\theta} d\theta = V_f - 120 \text{ m/s}$$

$$-\frac{g}{w}(-\cos\theta)\Big|_0^{30°} = V_f - 120 \text{ m/s}$$

$$V_f = 120 + \frac{g}{w}(\cos 30° - 1)$$

$$\underline{V_f = 105 \text{ m/s}}$$

Problem 16.33 In Example 16.3, suppose that the mass of the golf ball is 0.046 kg and its diameter is 43 mm. The club is in contact with the ball for 0.0006 s, and the distance the ball travels in one 0.001-s interval is 50 mm. What is the magnitude of the average impulsive force exerted by the club?

Solution: Using the principle of impulse and momentum

$$0 + F_{ave}\Delta t = mv$$

$$F_{ave} = \frac{mv}{\Delta t} = \frac{(0.046 \text{ kg}) \left(\dfrac{0.05 \text{ m}}{0.001 \text{ s}} \right)}{0.0006 \text{ s}}$$

$$\boxed{F_{ave} = 3830 \text{ N.}}$$

Problem 16.34 In a test of an energy-absorbing bumper, a 12700 N car is driven into a barrier at 8 km/h. The duration of the impact is 0.4 seconds. When the car rebounds from the barrier, the magnitude of its velocity is 1.6 km/h.

(a) What is the magnitude of the average horizontal force exerted on the car during the impact?
(b) What is the average deceleration of the car during the impact?

Solution: The velocities are

$$v_1 = 8 \text{ km/h} = 2.22 \text{ m/s}, \quad v_2 = 1.6 \text{ km/h} = 0.44 \text{ m/s}.$$

(a) Using the principle of impulse and momentum we have

$$-mv_1 + F_{ave}\Delta t = mv_2$$

$$F_{ave} = m\frac{(v_1 + v_2)}{\Delta t} = \frac{12700 \text{ N}}{9.81 \text{ m/s}^2} \left(\frac{2.22 \text{ m/s} + 0.44 \text{ m/s}}{0.4 \text{ s}} \right)$$

$$\boxed{F_{ave} = 8609 \text{ N}}$$

(b) The average deceleration of the car during impact is

$$a = \frac{v_2 - (-v_1)}{\Delta t} = \frac{(2.22 \text{ m/s} + 0.44 \text{ m/s})}{0.4 \text{ s}}$$

$$\boxed{a = 6.65 \text{ m/s}^2}$$

Problem 16.35 A bioengineer, using an instrumented dummy to test a protective mask for a hockey goalie, launches the 170-g puck so that it strikes the mask moving horizontally at 40 m/s. From photographs of the impact, she estimates the duration to be 0.02 s and observes that the puck rebounds at 5 m/s.

(a) What linear impulse does the puck exert?
(b) What is the average value of the impulsive force exerted on the mask by the puck?
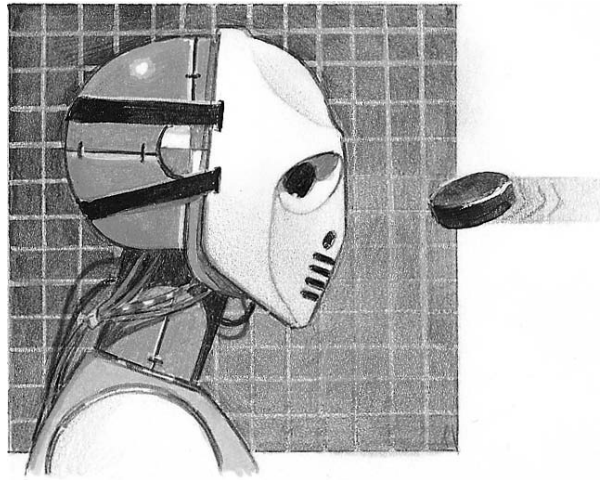
Solution:

(a) The linear impulse is

$$\int_{t_1}^{t_2} F \, dt = F_{ave}(t_2 - t_1) = mv_2 - mv_1.$$

The velocities are $v_2 = -5$ m/s, and $v_1 = 40$ m/s, from which

$$\int_{t_1}^{t_2} F \, dt = F_{ave}(t_2 - t_1) = (0.17)(-5 - 40) = -7.65 \text{ N-s},$$

where the negative sign means that the force is directed parallel to the negative x axis.

(b) The average value of the force is

$$\boxed{F_{ave} = \frac{-7.65}{0.02} = -382.5 \text{ N}}$$

Problem 16.36 A fragile object dropped onto a hard surface breaks because it is subjected to a large impulsive force. If you drop a 0.56 N watch from 1.22 m above the floor, the duration of the impact is 0.001 s, and the watch bounces 51 mm. above the floor, what is the average value of the impulsive force?

Solution: The impulse is

$$\int_{t_1}^{t_2} F \, dt = F_{ave}(t_2 - t_1) = \left(\frac{W}{g}\right)(v_2 - v_1).$$

The weight of the watch is

$$W = 0.56 \text{ N},$$

and its mass is

$$\left(\frac{W}{g}\right) = 0.0571 \text{ kg}$$

The velocities are obtained from energy considerations (the conservation of energy in free fall):

$$v_1 = \sqrt{2gh} = \sqrt{2(9.81)(1.22)} = 4.88 \text{ m/s}.$$

$$v_2 = -\sqrt{2gh} = -\sqrt{2(9.81)(0.051)} = -1.0 \text{ m/s}.$$

The average value of the impulsive force is

$$\boxed{F_{ave} = \frac{(0.0571 \times 10^{-3})(-1 - 4.88)}{1 \times 10^{-3}} = -334 \text{ N}}$$

Problem 16.37 The 0.45-kg soccer ball is given a kick with a 0.12-s duration that accelerates it from rest to a velocity of 12 m/s at 60° above the horizontal.

(a) What is the magnitude of the average total force exerted on the ball during the kick?
(b) What is the magnitude of the average force exerted on the ball by the player's foot during the kick?

Strategy: Use Eq. (16.2) to determine the average total force on the ball. To determine the average force exerted by the player's foot, you must subtract the ball's weight from the average total force.

Solution:

$$\int \mathbf{F} \, dt = \mathbf{F}_{\text{avg}} \Delta t = m\mathbf{V}_f - m\mathbf{V}_0$$

$$\mathbf{F}_{AV}(0.12) = 0.45(12\cos 60°\mathbf{i} + 12\sin 60°\mathbf{j})$$

$$\mathbf{F}_{AV} = 22.5\mathbf{i} + 39.0\mathbf{j} \text{ N}$$

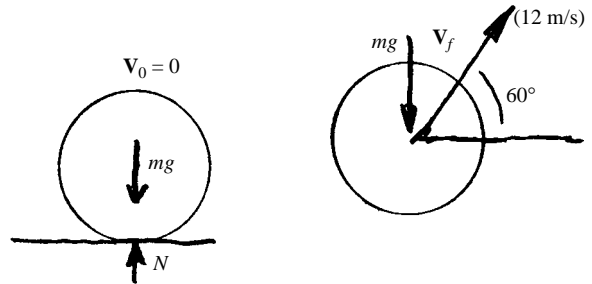(a) $\underline{|\mathbf{F}_{AV}| = 45.0 \text{ N}}$ $mg = (0.45)(9.81) = 4.41$ N

$$\mathbf{F}_{AV} = \mathbf{F}_{\text{FOOT}} + \mathbf{F}_G$$

$$\mathbf{F}_{AV} = \mathbf{F}_{\text{FOOT}} - mg\mathbf{j}$$

$$\mathbf{F}_{\text{FOOT}} = \mathbf{F}_{AV} + mg\mathbf{j}$$

$$\mathbf{F}_{\text{FOOT}} = 22.5\mathbf{i} + 39.0\mathbf{j} + 4.41\mathbf{j}$$

(b) $|\mathbf{F}_{\text{FOOT}}| = 48.9$ N

Problem 16.38 An entomologist measures the motion of a 3-g locust during its jump and determines that the insect accelerates from rest to 3.4 m/s in 25 ms (milliseconds). The angle of takeoff is 55° above the horizontal. What are the horizontal and vertical components of the average impulsive force exerted by the locust's hind legs during the jump?

Solution: The impulse is

$$\int_{t_1}^{t_2} \mathbf{F} \, dt = \mathbf{F}_{\text{ave}}(t_2 - t_1) = m(\mathbf{v}_2) = m(3.4\cos 55°\mathbf{i} + 3.4\sin 55°\mathbf{j}),$$

from which

$$\mathbf{F}_{\text{ave}}(2.5 \times 10^{-2}) = (5.85 \times 10^{-3})\mathbf{i} + (8.36 \times 10^{-3})\mathbf{j} \text{ N-s}.$$

The average total force is

$$\mathbf{F}_{\text{ave}} = \frac{1}{2.5 \times 10^{-2}}((5.85 \times 10^{-3})\mathbf{i} + (8.36 \times 10^{-3})\mathbf{j})$$

$$= 0.234\mathbf{i} + 0.334\mathbf{j} \text{ N}.$$

The impulsive force is

$$\boxed{\mathbf{F}_{\text{imp}} = \mathbf{F}_{\text{ave}} - (-mg\mathbf{j}) = 0.234\mathbf{i} + 0.364\mathbf{j} \text{ N}}$$

Problem 16.39 A 1.4 N baseball is 0.91 m above the ground when it is struck by a bat. The horizontal distance to the point where the ball strikes the ground is 54.9 m. Photographs studies indicate that the ball was moving approximately horizontally at 30.5 m/s before it was struck, the duration of the impact was 0.015 s, and the ball was traveling at 30° above the horizontal after it was struck. What was the magnitude of the average impulsive force exerted on the ball by the bat?
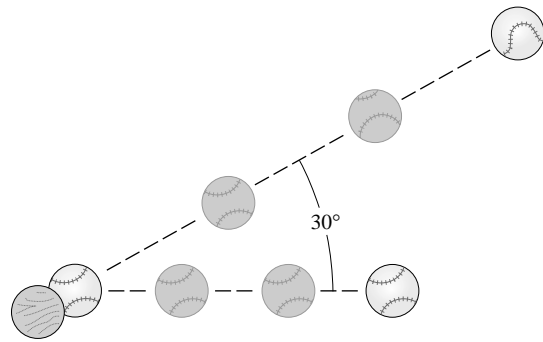
Solution: The impulse is

$$\int_{t_1}^{t_2} \mathbf{F}\, dt = \mathbf{F}_{ave}(t_2 - t_1) = \left(\frac{W}{g}\right)(\mathbf{v}_2 - \mathbf{v}_1).$$

The velocity v_2 is determined from the trajectory. The path is

$$y = -\frac{gt^2}{2} + (v_2 \sin 30°)t + y_0,$$

$$x = (v_2 \cos 30°)t$$

where v_2 is the magnitude of the velocity at the point of leaving the bat, and $y_0 = 0.91$ m. At $x = 54.9$ m, $t = 54.9/(v_2\cos 30°)$, and $y = 0$. Substitute and reduce to obtain

$$v_2 = 54.9\sqrt{\frac{g}{2\cos^2 30°\,(54.9\tan 30° + y_0)}} = 24.6 \text{ m/s}.$$

From which:

$$\mathbf{F}_{ave} = \left(\frac{1}{0.015}\right)\left(\frac{W}{g}\right)((v_2 \cos 30°)\mathbf{i} + (v_2 \sin 30°)\mathbf{j} - (-30.5\mathbf{i}))$$

$$= 489.3\mathbf{i} + 116.1\mathbf{j}.$$

Subtract the weight of the baseball: $\mathbf{F}_{imp} = \mathbf{F}_{ave} - (-W\mathbf{j}) = 489.3\mathbf{i} + 117.5\mathbf{j}$, from which

$$\boxed{|\mathbf{F}_{imp}| = 503.2 \text{ N}}$$

Problem 16.40 Paraphrasing the official rules of racquetball, a standard racquetball is 56 mm in diameter, weighs 0.4 N, and bounces between 68 and 72 centimetres from a 100-cm drop at a temperature between 70 and 74 degrees Fahrenheit Suppose that a ball bounces 71 cm when it is dropped from a 100-cm height. If the duration of the impact is 0.08 s, what average force is exerted on the ball by the floor?

Solution: The velocities before and after the impact are

$$v_1 = \sqrt{2(9.81 \text{ m/s}^2)(1 \text{ m})} = 4.43 \text{ m/s},$$
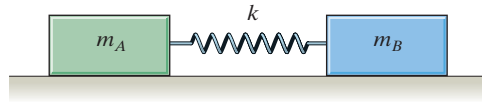
$$v_2 = \sqrt{2(9.81 \text{ m/s}^2)(0.71)} = 3.73 \text{ m/s},$$

Using the principle of impulse and momentum we have

$$-mv_1 + F_{ave}\Delta t = mv_2$$

$$F_{ave} = m\frac{v_1 + v_2}{\Delta t} = (0.4 \text{ N})\left(\frac{1}{9.81 \text{ m/s}^2}\right)\frac{(4.43 \text{ m/s} + 3.73 \text{ m/s})}{0.08 \text{ s}}$$

$$\boxed{F_{ave} = 4.16 \text{ N.}}$$

Problem 16.41 The masses $m_A = m_B$. The surface is smooth. At $t = 0$, A is stationary, the spring is unstretched, and B is given a velocity v_0 toward the right.

(a) In the subsequent motion, what is the velocity of the common center of mass of A and B?

(b) What are the velocities of A and B when the spring is unstretched?

Strategy: To do part (b), think about the motions of the masses relative to their common center of mass.

Solution:

(a) The velocity of the center of mass does not change because there are no external forces on the system

$$v_c = \frac{m_A v_0 + m_B 0}{(m_A + m_B)} = \frac{v_0}{2} \qquad \boxed{v_C = \frac{v_0}{2}}.$$

(b) Looking at the system from a reference frame moving with the center of mass, we have an oscillatory system with either the masses moving towards the center or away from the center. Translating back to the ground reference system, this means

$$\boxed{\begin{array}{ll} \text{Either} & v_A = v_0 \text{(to the right)}, v_B = 0 \\ \text{or} & v_A = 0, v_B = v_0 \text{(to the right)}. \end{array}}$$

Problem 16.42 In Problem 16.41, $m_A = 40$ kg, $m_B = 30$ kg, and $k = 400$ N/m. The two masses are released from rest on the smooth surface with the spring stretched 1 m. What are the magnitudes of the velocities of the masses when the spring is unstretched?

Solution: From the solution of Problem 16.41, (1) $m_A v_A + m_B v_B = 0$: or, evaluating, $40 v_A + 30 v_B = 0$. Energy is conserved, Thus, (2) $\frac{1}{2} k S^2 = \frac{1}{2} m_A v_A{}^2 + \frac{1}{2} m_B v_B{}^2$. Evaluating, we get

$$\frac{1}{2}(400)(1)^2 = \frac{1}{2}(40)v_A^2 + \frac{1}{2}(30)v_B^2$$

Solving Equations (1) and (2),

$|\mathbf{v}_A| = 2.07$ m/s,

$|\mathbf{v}_B| = 2.76$ m/s.

Problem 16.43 A girl weighing 356 N stands at rest on a 1446 N floating platform. She starts running at 3.05 m/s *relative to the platform* and runs off the end. Neglect the horizontal force exerted on the platform by the water.

(a) After she starts running, what is her velocity relative to the water?
(b) While she is running, what is the velocity of the common center of mass of the girl and the platform relative to the water? (See Active Example 16.4.)
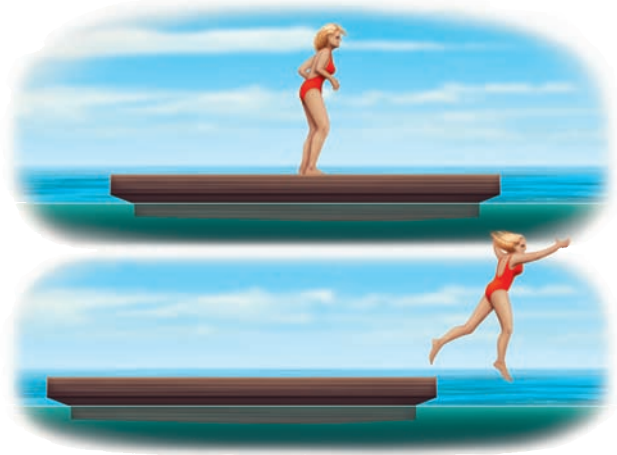
Solution:

(a) Momentum is conserved.

$$0 = m_g v_g + m_b v_p, \quad v_{g/p} = v_g - v_p$$

$$0 = (356 \text{ N})v_g + (1446 \text{ N})v_p, \quad (3.05 \text{ m/s}) = v_g - v_p$$

Solving we find

$$v_g = \frac{(3.05 \text{ m/s})(1446 \text{ N})}{1802 \text{ N}} = 2.45 \text{ m/s}.$$

$\boxed{v_g = 2.45 \text{ m/s to the right.}}$

(b) Since momentum is conserved, the velocity of the center of mass is always zero.

$\boxed{v_{cm} = 0.}$

Problem 16.44 Two railroad cars with weights $W_A = 533.8$ kN and $W_B = 311.4$ kN collide and become coupled together. Car A is full, and car B is half full, of carbolic acid. When the cars collide, the acid in B sloshes back and forth violently.

(a) Immediately after the impact, what is the velocity of the common center of mass of the two cars?
(b) When the sloshing in B has subsided, what is the velocity of the two cars?
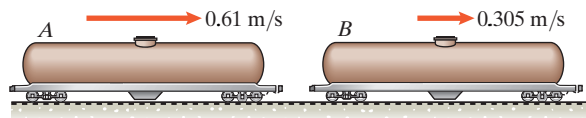
Solution:

(a) $$v_{\text{center of mass}} = \frac{(533800 \text{ N})(0.61 \text{ m/s}) + (311400 \text{ N})(0.305 \text{ m/s})}{(533800 \text{ N} + 311400 \text{ N})}$$

$$= 0.497 \text{ m/s}$$

(b) After the sloshing stops $\boxed{v_{\text{cars}} = v_{\text{center of mass}} = 0.497 \text{ m/s}}$

Problem 16.45 Suppose that the railroad track in Problem 16.44 has a constant slope of 0.2 degrees upward toward the right. If the cars are 1.83 m apart at the instant shown, what is the velocity of their common center of mass immediately after they become coupled together?
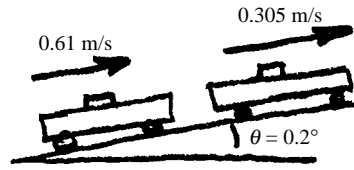
Solution: Time to couple (both accelerate at the same rate) is $t =$

$$\frac{1.83 \text{ m}}{(0.61 \text{ m/s} - 0.305 \text{ m/s})} = 6 \text{ s}.$$

Impulse — momentum is now

$$\left(\frac{533800 \text{ N}}{9.81 \text{ m/s}^2}\right)(0.61 \text{ m/s}) + \left(\frac{311400 \text{ N}}{9.81 \text{ m/s}^2}\right)(0.305 \text{ m/s})$$

$$- (845200 \text{ N}) \sin 0.2°(6 \text{ s}) = \left(\frac{845200 \text{ N}}{9.81 \text{ m/s}^2}\right) v_{\text{center of mass}}$$

$$\boxed{v_{\text{center of mass}} = 0.292 \text{ m/s}}$$

0.305 m/s

0.61 m/s

$\theta = 0.2°$

Problem 16.46 The 400-kg satellite S traveling at 7 km/s is hit by a 1-kg meteor M traveling at 12 km/s. The meteor is embedded in the satellite by the impact. Determine the magnitude of the velocity of their common center of mass after the impact and the angle β between the path of the center of mass and the original path of the satellite.
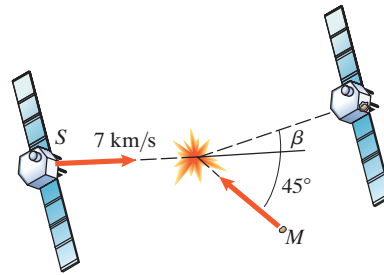
**Solution:**

(a, b) $m_A v_A + m_B v_B = (m_A + m_B) v_f$

$(9600)(2) + (5400)(1) = (15000) v_f$

$v_f = \frac{246}{150} \text{ m/s}$

$\underline{v_f = 1.64 \text{ m/s to the right}}$

S 7 km/s β

45°

M

Problem 16.47 The 400-kg satellite S traveling at 7 km/s is hit by a 1-kg meteor M. The meteor is embedded in the satellite by the impact. What would the magnitude of the velocity of the meteor need to be to cause the angle β between the original path of the satellite and the path of the center of mass of the combined satellite and meteor after the impact to be 0.5°? What is the magnitude of the velocity of the center of mass after the impact?

Solution: Conservation of linear momentum yields $(400)(7\mathbf{i}) + (1)(-v_m \sin 45°\mathbf{i} + v_m \cos 45°\mathbf{j}) = (400 + 1)(v \cos 0.5°\mathbf{i} + v \sin 0.5°\mathbf{j})$. Equating $\mathbf{i}$ and $\mathbf{j}$ components, we get $(400)(7) - v_m \cos 45° = 401 \, v \cos 0.5°$; $v_m \sin 45° = 401 \, v \sin 0.5°$ and solving, we obtain $v_m = 34.26 \text{ km/s} : v = 6.92 \text{ km/s}$.

y

v

7 km/s $\beta = 0.5°$

S x

45°

v_M

M
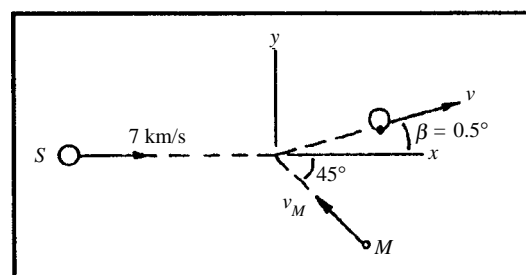
Problem 16.48 A 68-kg astronaut is initially station-ary at the left side of an experiment module within an orbiting space shuttle. The 105,000-kg shuttle's center of mass is 4 m to the astronaut's right. He launches himself toward the center of mass at 1 m/s *relative to the shut-tle*. He travels 8 m relative to the shuttle before bringing himself to rest at the opposite wall of the experiment module.

(a) What is the change in the magnitude of the shuttle's velocity relative to its original velocity while the astronaut is in motion?

(b) What is the change in the magnitude of the shuttle's velocity relative to its original velocity after his "flight"?

(c) Where is the shuttle's center of mass relative to the astronaut after his "flight"?
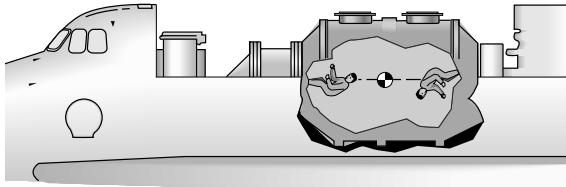
Solution: Consider the motion of the astronaut (A) and shuttle (S) relative to a reference frame that is stationary with respect to their common center of mass. During the astronaut's motion,

$$m_A v_A + m_S v_S = 0$$

and $v_A - v_S = 1$.

Solving these two equations, we obtain

(a) $v_S = -0.000647$ m/s.

(b) After his flight $v_A = v_S$, so $v_S = 0$.

(c) It is 4 m to his left.

Problem 16.49 An 356 N boy sitting in a stationary 89 N wagon wants to simulate rocket propulsion by throwing bricks out of the wagon. Neglect horizontal forces on the wagon's wheels. If the boy has three bricks weighing 44.5 N each and throws them with a horizontal velocity of 3.05 m/s relative to the wagon, determine the velocity he attains (a) if he throws the bricks one at a time and (b) if he throws them all at once.

Solution:

(a) The boy (B) in the wagon (w) throws one brick (b) at a time.

First brick:

$$0 = (m_B + m_w + 2m_b)v_{w1} + m_b v_{b1},$$

$$v_{b1} - v_{w1} = -3.05.$$

Solving, $v_{w1} = 0.234$ m/s.

Second brick:

$$(m_B + m_w + 2m_b)v_{w1} = (m_B + m_w + m_b)v_{w2} + m_b v_{b2},$$

$$v_{b2} - v_{w2} = -3.05.$$

Solving, $v_{w2} = 0.489$ m/s.

Third brick :

$$(m_B + m_w + m_b)v_{w2} = (m_B + m_w)v_{w3} + m_b v_{b3},$$

$$v_{b3} - v_{w3} = -3.05.$$

Solving, $v_{w3} = 0.765$ m/s.

(b) All the bricks are thrown at once.

$$0 = (m_B + m_w)v_w + 3m_b v_b,$$

$$v_b - v_w = -3.05 \text{ m/s}.$$

Solving, $v_w = 0.704$ m/s.

Problem 16.50 A catapult designed to throw a line to ships in distress throws a 2-kg projectile. The mass of the catapult is 36 kg, and it rests on a smooth surface. It the velocity of the projectile *relative to the earth* as it leaves the tube is 50 m/s at $\theta_0 = 30°$ relative to the horizontal, what is the resulting velocity of the catapult toward the left?
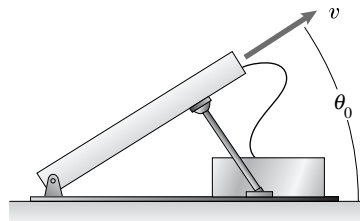
Solution:

$$0 = m_c v_c + m_p(50 \cos 30°).$$

Solving,

$$v_c = \frac{-(2)(50 \cos 30°)}{36} = -2.41 \text{ m/s}.$$

Problem 16.51 The catapult, which has a mass of 36 kg and throws a 2-kg projectile, rests on a smooth surface. The velocity of the projectile *relative to the catapult* as it leaves the tube is 50 m/s at $\theta_0 = 30°$ relative to the horizontal. What is the resulting velocity of the catapult toward the left?
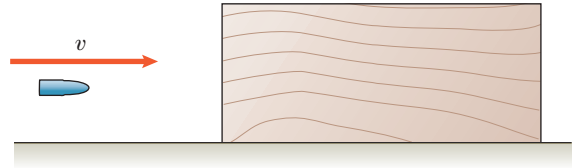
Solution:

$$0 = m_c v_c + m_p v_{px},$$

where

$$v_{px} - v_c = 50 \cos 30°.$$

Solving,

$$v_c = -2.28 \text{ m/s}.$$

Problem 16.52 A bullet with a mass of 3.6 grams is moving horizontally with velocity v and strikes a 5-kg block of wood, becoming embedded in it. After the impact, the bullet and block slide 24 mm across the floor. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.4$. Determine the velocity v.

Solution: Momentum is conserved through the collision and then work energy is used to finish the problem

$$m_b v = (M + m_b)v_2, \frac{1}{2}(M + m_b)v_2{}^2 - \mu_k(M + m_b)gd = 0$$

Solving we find

$$v_2 = \sqrt{2\mu_k g d},$$

$$v = \left(\frac{M + m_b}{m_b}\right)\sqrt{2\mu_k g d} = \left(\frac{5.0036 \text{ kg}}{0.0036 \text{ kg}}\right)\sqrt{2(0.4)(9.81 \text{ m/s}^2)(0.024 \text{ m})}$$

$$\boxed{v = 603 \text{ m/s}.}$$

Problem 16.53 A 28-g bullet hits a suspended 45-kg block of wood and becomes embedded in it. The angle through which the wires supporting the block rotate as a result of the impact is measured and determined to be $7°$. What was the bullet's velocity?
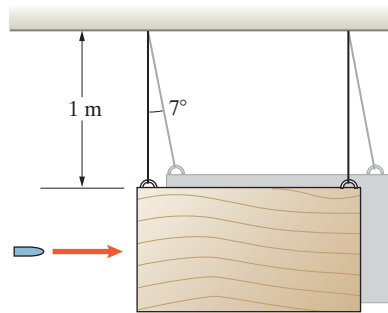
Solution: Momentum is conserved through the collision and then work-energy is used to finish the problem.

$$m_b v_b = (M + m_b)v_2, \frac{1}{2}(M + m_b)v_2{}^2 = (M + m_b)gL(1 - \cos\theta)$$

Solving we have

$$v_2 = \sqrt{2gL(1 - \cos\theta)} = \sqrt{2(9.81 \text{ m/s}^2)(1 \text{ m})(1 - \cos 7°)} = 0.382 \text{ m/s},$$

$$v_b = \left(\frac{M + m_b}{m_b}\right)v_2 = \left(\frac{45 + 0.028}{0.028}\right)(0.382 \text{ m/s}) = 615 \text{ m/s}.$$

$$\boxed{v_b = 615 \text{ m/s}.}$$

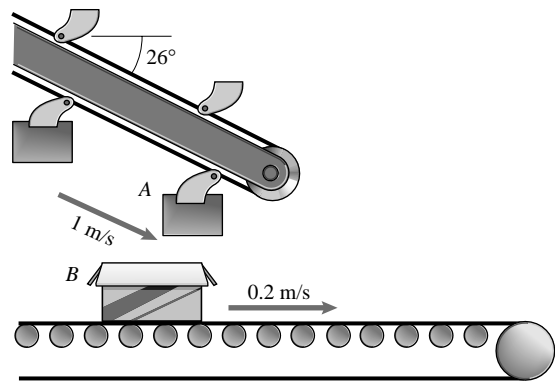**Problem 16.54** The overhead conveyor drops the 12-kg package *A* into the 1.6-kg carton *B*. The package is tacky and sticks to the bottom of the carton. If the coefficient of friction between the carton and the horizontal conveyor is $\mu_k = 0.2$, what distance does the carton slide after impact?

Solution: The horizontal velocity of the package (*A*) *relative to the carton* (*B*) is

$$v_A = (1)\cos 26° - 0.2 = 0.699 \text{ m/s}.$$

Let v be the velocity of the combined object relative to the belt.

$$m_A v_A = (m_A + m_B)v.$$

Solving, $\quad v = \dfrac{m_A v_A}{m_A + m_B} = \dfrac{(12)(0.699)}{12 + 1.6}$

$$= 0.617 \text{ m/s}.$$

Use work and energy to determine the sliding distance d:

$$- \mu_k(m_A + m_B)gd = 0 - \tfrac{1}{2}(m_A + m_B)v^2,$$

$$d = \frac{v^2}{2\mu_k g} = \frac{(0.617)^2}{2(0.2)(9.81)}$$

$$d = 0.0969 \text{ m}.$$

Problem 16.55 A 53376 N bus collides with a 12454 N car. The velocity of the bus before the collision is $\mathbf{v}_B = 5.5\mathbf{i}$ (m/s) and the velocity of the car is $\mathbf{v}_C = 10\mathbf{j}$ (m/s). The two vehicles become entangled and remain together after the collision. The coefficient of kinetic friction between the vehicles' tires and the road is $\mu_k = 0.6$.
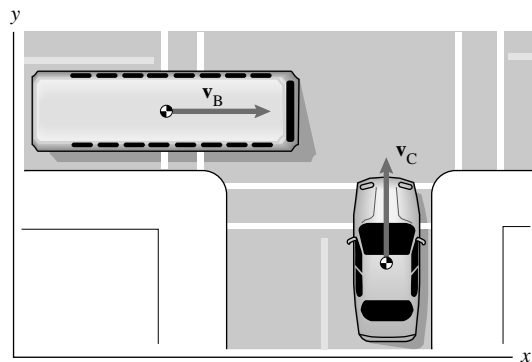
(a) What is the velocity of the common center of mass of the two vehicles immediately after the collision?
(b) Determine the approximate final position of the common center of mass of the vehicles relative to its position when the collision occurred. (Assume that the tires skid, not roll, on the road.)

Solution:

(a) The collision (impulse - momentum).

$$\left(\frac{53376 \text{ N}}{9.81 \text{ m/s}^2}\right)(5.5\mathbf{i} \text{ m/s}) + \left(\frac{12454 \text{ N}}{9.81 \text{ m/s}^2}\right)(10\mathbf{j} \text{ m/s}) = \left(\frac{65830 \text{ N}}{9.81 \text{ m/s}^2}\right)\mathbf{v}$$

$$\boxed{\mathbf{v} = (4.45\mathbf{i} + 1.90\mathbf{j}) \text{ m/s}, \quad v = 4.84 \text{ m/s}, \quad \theta = 23.2°}$$

(b) The skid after the accident (work–energy).

$$\frac{1}{2}\left(\frac{65830 \text{ N}}{9.81 \text{ m/s}^2}\right)(4.84 \text{ m/s})^2 - (0.6)(65830 \text{ N})s \Rightarrow s = 1.99 \text{ m}$$

The final position is

$$\boxed{\mathbf{r} = (1.99 \text{ m})(\cos 23.2°\mathbf{i} + \sin 23.2°\mathbf{j}) = (1.83\mathbf{i} + 0.78\mathbf{j}) \text{ m}}$$

Problem 16.56 The velocity of the 200-kg astronaut A relative to the space station is $40\mathbf{i} + 30\mathbf{j}$ (mm/s). The velocity of the 300-kg structural member B relative to the station is $-20\mathbf{i} + 30\mathbf{j}$ (mm/s). When they approach each other, the astronaut grasps and clings to the structural member.

(a) What is the velocity of their common center of mass when they arrive at the station?
(b) Determine the approximate position at which they contact the station.
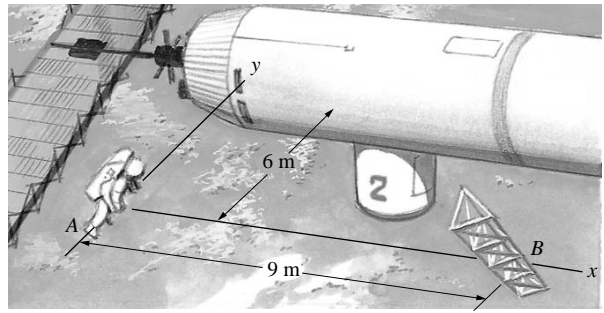
Solution:

(a) The velocity of the center of mass after the collision

$$(200 \text{ kg})(0.04\mathbf{i} + 0.03\mathbf{j}) \text{ m/s} + (300 \text{ kg})(-0.02\mathbf{i} + 0.03\mathbf{j}) \text{ m/s}$$

$$= (500 \text{ kg}) \mathbf{v}$$

$$\boxed{\mathbf{v} = (0.004\mathbf{i} + 0.03\mathbf{j}) \text{ m/s}}$$

(b) The time to arrive at the station is $t = \dfrac{6 \text{ m}}{0.03 \text{ m/s}} = 200$ s.

The center of mass of the two bodies starts at

$$\mathbf{r}_0 = \frac{(200 \text{ kg})(0) + (300 \text{ kg})(9\mathbf{i}) \text{ m}}{500 \text{ kg}} = 5.4\mathbf{i} \text{ m}$$

The position upon arrival is

$$\boxed{\mathbf{r} = (5.4\mathbf{i}) \text{ m} + [(0.004\mathbf{i} + 0.03\mathbf{j}) \text{ m/s}](200 \text{ s}) = (6.2\mathbf{i} + 6\mathbf{j}) \text{ m}}$$

Problem 16.57 The weights of the two objects are $W_A = 5$ N and $W_B = 8$ N. Object A is moving at $v_A = 2$ m/s and undergoes a perfectly elastic impact with the stationary object B. Determine the velocities of the objects after the impact.

Solution: Momentum is conserved and the coefficient of restitution is also used.

$$m_A v_A = m_A v'_A + m_B v'_B, \quad e v_A = v'_B - v'_A$$

$$(5 \text{ N})(2 \text{ m/s}) = (5 \text{ N})v'_A + (8 \text{ N})v'_B, \quad (2 \text{ m/s}) = v'_B - v'_A$$

Solving, we find $v'_A = -0.462$ m/s, $v'_B = 1.54$ m/s.

Therefore $\boxed{v'_A = 0.462 \text{ m/s to the left}, \quad v'_B = 1.54 \text{ m/s to the right.}}$

Problem 16.58 The weights of the two objects are $W_A = 5$ N and $W_B = 8$ N. Object A is moving at $v_A = 2$ m/s and undergoes a direct central impact with the stationary object B. The coefficient of restitution is $e = 0.8$. Determine the velocities of the objects after the impact.

Solution: Momentum is conserved and the coefficient of restitution is also used.

$$m_A v_A = m_A v_A' + m_B v_B', \quad e v_A = v_B' - v_A'$$

$(5 \text{ N})(2 \text{ m/s}) = (5 \text{ N})v_A' + (8 \text{ N})v_B', (0.8)(2 \text{ m/s}) = v_B' - v_A'$

Solving, we find $v_A' = -0.215$ m/s, $v_B' = 1.38$ m/s.

Therefore $\boxed{v_A' = 0.462 \text{ m/s to the left}, \quad v_B' = 1.54 \text{ m/s to the right.}}$

Problem 16.59 The objects A and B with velocities $v_A = 20$ m/s and $v_B = 4$ m/s undergo a direct central impact. Their masses are $m_A = 8$ kg and $m_B = 12$ kg. After the impact, the object B is moving to the right at 16 m/s. What is the coefficient of restitution?

Solution: Momentum is conserved, the coefficient of restitution is used.
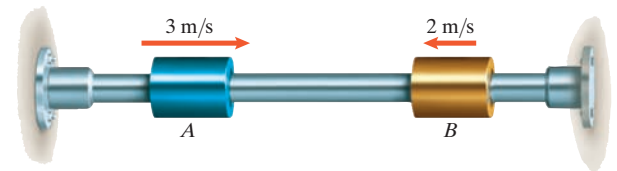
$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B', \quad e(v_A - v_B) = v_B' - v_A'$$

$(8 \text{ kg}) (20 \text{ m/s}) + (12 \text{ kg}) (4 \text{ m/s}) = (8 \text{ kg})v_A' + (12 \text{ kg}) (16 \text{ m/s})$

$e(20 \text{ m/s} - 4 \text{ m/s}) = (16 \text{ m/s}) - v_A'$

Solving we find $v_A' = 2.0$ m/s, $\boxed{e = 0.875.}$

Problem 16.60 The 8-kg mass A and the 12-kg mass B slide on the smooth horizontal bar with the velocities shown. The coefficient of restitution is $e = 0.2$. Determine the velocities of the masses after they collide. (See Active Example 16.5).

Solution: Momentum is conserved, and the coefficient of restitution is used.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B', \quad e(v_A - v_B) = v_B' - v_A'$$

Putting in the numbers we have

$(8 \text{ kg}) (3 \text{ m/s}) + (12 \text{ kg})(-3 \text{ m/s}) = (8 \text{ kg})v_A' + (12 \text{ kg})v_B'$

$(0.2)([3 \text{ m/s}] - [-2 \text{ m/s}]) = v_B' - v_A'$

Solving we find $v_A' = -0.6$ m/s, $v_B' = 0.4$ m/s.

Thus $\boxed{v_A' = 0.6 \text{ m/s to the left}, \quad v_B' = 0.4 \text{ m/s to the right.}}$

Problem 16.61 In a study of the effects of an accident on simulated occupants, the 1900 N car with velocity $v_A = 30$ km/h collides with the 2800 N car with velocity $v_B = 20$ km/h. The coefficient of restitution of the impact is $e = 0.15$. What are the velocities of the cars immediately after the collision?

Solution: Momentum is conserved, and the coefficient of restitution is used.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B', \quad e(v_A - v_B) = v_B' - v_A'$$

$$(1900 \text{ N}) (30 \text{ km/h}) + (2800 \text{ N})(-20 \text{ km/h}) = (1900 \text{ N}) v_A' + (2800 \text{ N})v_B'$$

$$(0.15)([30 \text{ km/h}] - [-20 \text{ km/h}]) = v_B' - v_A'$$

Solving we find $v_A' = -4.26$ km/h, $v_B' = 3.24$ km/h.

Converting into ft/s we find $\boxed{v_A' = 1.18 \text{ m/s to the left}, \ v_B' = 0.9 \text{ m/s to the right.}}$

Problem 16.62 In a study of the effects of an accident on simulated occupants, the 1900 N car with velocity $v_A = 30$ km/h collides with the 2800 N car with velocity $v_B = 20$ km/h. The coefficient of restitution of the impact is $e = 0.15$. The duration of the collision is 0.22 s. Determine the magnitude of the average acceleration to which the occupants of each car are subjected.

Solution: The velocities before the collision are converted into ft/s. The velocities after the collision were calculated in the preceding problem. The velocities are

$$v_A = 8.33 \text{ m/s}, \ v_B = 5.55 \text{ m/s}, \ v_A' = -1.18 \text{ m/s}, \ v_B' = 0.9 \text{ m/s}.$$
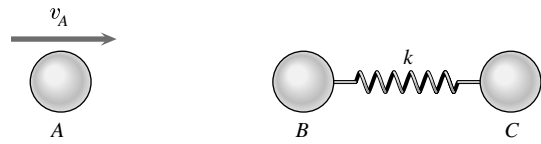
The average accelerations are

$$a_A = \frac{\Delta v}{\Delta t} = \left| \frac{(-1.18 \text{ m/s}) - (8.33 \text{ m/s})}{0.22 \text{ s}} \right| = -43.23 \text{ m/s}^2$$

$$a_B = \frac{\Delta v}{\Delta t} = \left| \frac{(0.9 \text{ m/s}) - (-5.55 \text{ m/s})}{0.22 \text{ s}} \right| = 29.32 \text{ m/s}^2$$

$$\boxed{a_A = -43.23 \text{ m/s}^2, \ a_B = 29.32 \text{ m/s}^2.}$$

Problem 16.63 The balls are of equal mass m. Balls B and C are connected by an unstretched spring and are stationary. Ball A moves toward ball B with velocity v_A. The impact of A with B is perfectly elastic ($e = 1$).

(a) What is the velocity of the common center of mass of B and C immediately after the impact?

(b) What is the velocity of the common center of mass of B and C at time t after the impact?

Solution: Consider the impact of balls A and B. From the equations

$$mv_A = mv_A' + mv_B',$$

$$e = 1 = \frac{v_B' - v_A'}{v_A},$$

we obtain $v_A' = 0$, $v_B' = v_A$.
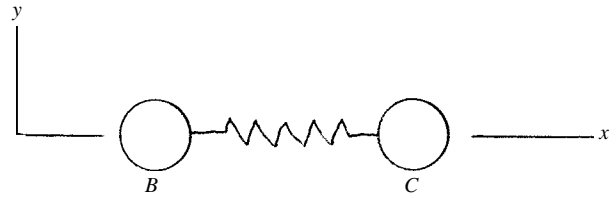
(a) The position of the center of mass is

$$\mathbf{x} = \frac{mx_B + mx_C}{m + m} = \frac{x_B + x_C}{2},$$

so

$$\frac{d\mathbf{x}}{dt} = \frac{1}{2}\left(\frac{dx_B}{dt} + \frac{dx_C}{dt}\right).$$

Immediately after the impact $dx_B/dt = v_A$ and $dx_C/dt = 0$, so

$$\frac{d\mathbf{x}}{dt} = \frac{1}{2}v_A.$$

(b) With no external forces,

$$\frac{d\mathbf{x}}{dt} = \text{const.} = \frac{1}{2}v_A.$$

Problem 16.64 In Problem 16.63, what is the maximum compressive force in the spring as a result of the impact?

Solution: See the solution of Problem 16.63. Just after the collision of A and B, B is moving to the right with velocity v_A, C is stationary, and the center of mass D of B and C is moving to the right with velocity $\frac{1}{2}v_A$ (Fig. a). Consider the motion in terms of a reference frame that is moving with D (Fig. b). Relative to this reference frame, B is moving to the right with velocity $\frac{1}{2}v_A$ and C is moving to the left with velocity $\frac{1}{2}v_A$. There total kinetic energy is

$$\frac{1}{2}m\left(\frac{1}{2}v_A\right)^2 + \frac{1}{2}m\left(\frac{1}{2}v_A\right)^2 = \frac{1}{4}mv_A^2.$$

When the spring has brought B and C to rest relative to D, their kinetic energy has been transformed into potential energy of the spring. This is when the compressive force in the spring is greatest. Setting $\frac{1}{4}mv_A^2 = \frac{1}{2}kS^2$, we find that the compression of the spring is

$$S = -v_A\sqrt{\frac{m}{2k}}.$$

Therefore the maximum compressive force is

$$k|S| = v_A\sqrt{\frac{mk}{2}}.$$

(a)

(b)
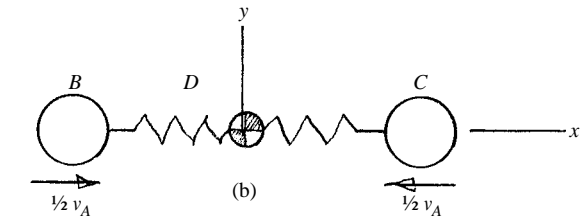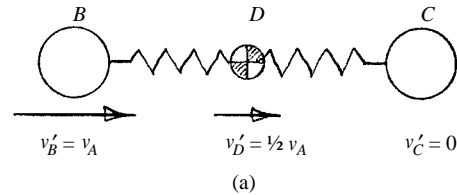
Problem 16.65* The balls are of equal mass m. Balls B and C are connected by an unstretched spring and are stationary. Ball A moves toward ball B with velocity v_A. The impact of A with B is perfectly elastic ($e = 1$). Suppose that you interpret this as an impact between ball A and an "object" D consisting of the connected balls B and C.

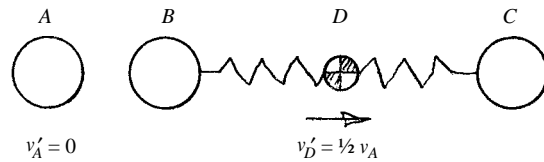(a) What is the coefficient of restitution of the impact between A and D?

(b) If you consider the total energy after the impact to be the sum of the kinetic energies, $\frac{1}{2}m(v_A')^2 + \frac{1}{2}(2\,m)(v_D')^2$, where v_D' is the velocity of the center of mass of D after the impact, how much energy is "lost" as a result of the impact?

(c) How much energy is actually lost as a result of the impact? (This problem is an interesting model for one of the mechanisms for energy loss in impacts between objects. The energy "loss" calculated in part (b) is transformed into "internal energy" — the vibrational motions of B and C relative to their common center of mass.)

Solution: See the solution of Problem 16.135. Just after the impact of A and B, A is stationary and the center of mass D of B and C is moving with velocity $\frac{1}{2}v_A$.

(a) The coefficient of restitution is

$$e = \frac{v_D' - v_A'}{v_A} = \frac{\frac{1}{2}v_A - 0}{v_A} = \frac{1}{2}.$$

(b) The energy before the impact is $\frac{1}{2}mv_A^2$. The energy after is

$$\frac{1}{2}m(v_A')^2 + \frac{1}{2}(2m)(v_D')^2 = \frac{1}{4}mv_A^2.$$

The energy "lost" is $\frac{1}{4}mv_A^2$.

(c) No energy is actually lost. The total kinetic energy of A, B, and C just after the impact is $\frac{1}{2}mv_A^2$.

$v_A' = 0$ $v_D' = \frac{1}{2} v_A$

Problem 16.66 Suppose that you investigate an accident in which a 1300 kg car A struck a parked 1200 kg car B. All four of car B's wheels were locked, and skid marks indicate that it slid 8 m after the impact. If you estimate the coefficient of kinetic friction between B's tires and the road to be $\mu_k = 0.8$ and the coefficient of restitution of the impact to be $e = 0.4$, what was A's velocity v_A just before the impact? (Assume that only one impact occurred.)
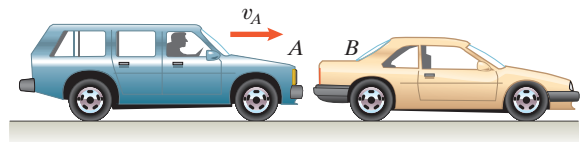
Solution: We can use work-energy to find the velocity of car B just after the impact. Then we use conservation of momentum and the coefficient of restitution to solve for the velocity of A. In general terms we have

$$\frac{1}{2}m_B v_B'^2 - \mu_k m_B gd = 0 \Rightarrow v_B' = \sqrt{2\mu_k gd}$$

$$m_A v_A = m_A v_A' + m_B v_B', \quad ev_A = v_B' - v_A'$$

Putting in the numbers we have

$$v_B' = \sqrt{2(0.8)(9.81 \text{ m/s}^2)(8 \text{ m})} = 11.2 \text{ m/s},$$

$$(1300 \text{ kg}) v_A = (1300 \text{ kg}) v_A' + (1200 \text{ kg}) (11.2 \text{ m/s})$$

$$(0.2)v_A = (9.81 \text{ m/s}^2) - v_A'$$

Solving the last two equations simultaneously we have

$$v_A' = 6.45 \text{ m/s}, \quad \boxed{v_A = 16.8 \text{ m/s}}$$

Problem 16.67 When the player releases the ball from rest at a height of 1.52 m above the floor, it bounces to a height of 1.07 m. If he throws the ball downward, releasing it at 0.91 m above the floor, how fast would he need to throw it so that it would bounce to a height of 3.66 m?
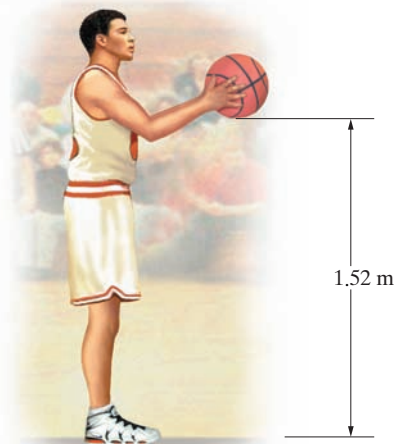
1.52 m

Solution: When dropped from 1.52 m, the ball hits the floor with a speed

$$v_{\text{before}} = \sqrt{2(9.81 \text{ m/s}^2)(1.52 \text{ m})} = 5.47 \text{ m/s}$$

In order to rebound to 1.07 m, it must leave the floor with a speed

$$v_{\text{after}} = \sqrt{2(9.81 \text{ m/s}^2)(1.07 \text{ m})} = 4.58 \text{ m/s}$$

The coefficient of restitution is therefore $e = \dfrac{4.58 \text{ m/s}}{5.47 \text{ m/s}} = 0.837$

To bounce to a height of 3.66 m we need a rebound velocity of

$$v_{\text{rebound}} = \sqrt{2(9.81 \text{ m/s}^2)(3.66 \text{ m})} = 8.47 \text{ m/s}$$

Therefore, the ball must have a downward velocity of $\dfrac{8.47 \text{ m/s}}{0.837} = 10.13$ m/s before it hits the floor. To find the original velocity when it leaves his hands,

$$\frac{1}{2}mv^2 + m(9.81 \text{ m/s}^2)(0.91\text{m}) = \frac{1}{2} m (10.13 \text{ m/s})^2 \Rightarrow \boxed{v = 9.2 \text{ m/s}}$$

Problem 16.68 The 0.45-kg soccer ball is 1 m above the ground when it is kicked upward at 12 m/s. If the coefficient of restitution between the ball and the ground is $e = 0.6$, what maximum height above the ground does the ball reach on its first bounce?

12 m/s

1 m
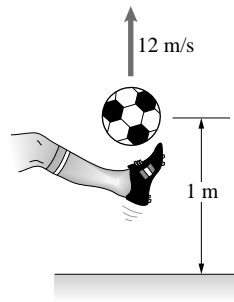
Solution: We must first find the velocity with which the ball strikes the ground. Then we analyze the impact. Finally, we analyze the post impact bounce.

Kick-to-Bounce Phase:- Use Cons. of Energy Datum is the ground level.

$$\tfrac{1}{2}mv_0^2 + mgh_0 = \tfrac{1}{2}mv_1^2 + mg(0)$$

Impact occurs at v_1

$v_0 = 12$ m/s, $h_0 = 1$ m, $m = 0.45$ kg,

$g = 9.81$ m/s^2

Solving, $\underline{v_1 = -12.79 \text{ m/s}}$ (downward)

Impact:

$$e = \frac{-v_1'}{v_1} = 0.6$$

$v_1' = 7.67$ m/s upward after impact

Post Impact:

$$\tfrac{1}{2}m(v_1')^2 + 0 = 0 + mgh_2$$

$$h_2 = \frac{(v_1')^2}{2g}$$

$\underline{h_2 = 3.00 \text{ m}}$

Problem 16.69 The 0.45-kg soccer ball is stationary just before it is kicked upward at 12 m/s. If the impact lasts 0.02 s, what average force is exerted on the ball by the player's foot?

Solution: (Neglect gravity during the impact) Details of kick (all in y direction)

$v_0 = 0$

$v_1 = 12$ m/s

$m = 0.45$ kg

F_{AV}

j : $\displaystyle\int F_{AV}\, dt = F_{AV}\,\Delta t = mv_1 - mv_0$

$F_{AV}\,\Delta t = mv_1 = (0.45)(12) = 5.40$ N-s

$\Delta t = 0.02$ s,

Solving $\underline{F_{AV} = 270 \text{ N}}$

Problem 16.70 By making measurements directly from the photograph of the bouncing golf ball, estimate the coefficient of restitution.
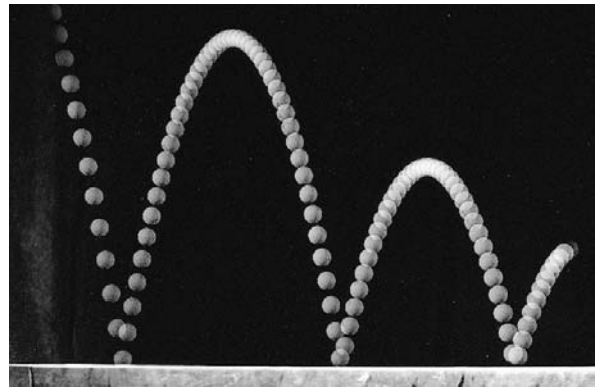
Solution: For impact on a stationary surface, the coefficient of restitution is defined to be $e = -v'_A/v_A$. (Since the impact and rebound velocities have opposite signs, e is positive.) (See Eq. (16.19)). From the conservation of energy, $\frac{1}{2}m_A(v'_A)^2 = m_A g h$, the velocity is proportional to the square root of the rebound height, so that if $h_1, h_2, \ldots h_N, \ldots$ are successive rebound heights, then an estimate of e is $e = \sqrt{h_{i+1}/h_i}$. Measurements are $h_1 = 5.1$ cm, $h_2 = 3.1$ cm, from which

$$\boxed{e = \sqrt{3.1/5.1} = 0.78}$$

Problem 16.71 If you throw the golf ball in Problem 16.70 horizontally at 0.61 m/s and release it 1.22 m above the surface, what is the distance between the first two bounces?

Solution: The normal velocity at impact is $v_{An} = -\sqrt{2g(1.22)} = -4.89$ m/s (downward). The rebound normal velocity is (from Eq (16.19)) $v'_{An} = -e v_{An} = -(0.78)(-4.89) = 3.81$ m/s (upward). From the conservation of energy for free fall the first rebound height is $h = (v'_{An})^2/2g = 0.74$ m/s. From the solution of Newton's second law for free fall, the time spent between rebounds is twice the time to fall from the maximum height: $t = 2\sqrt{2h/g} = 0.778$ s from which the distance between bounces is:

$$\boxed{d = v_0 t = 2t = 0.48 \text{ m}}$$

Problem 16.72 In a forging operation, the 100-N weight is lifted into position 1 and released from rest. It falls and strikes a workpiece in position 2. If the weight is moving at 15 m/s immediately before the impact and the coefficient of restitution is $e = 0.3$, what is the velocity of the weight immediately after impact?

Solution: The strategy is to treat the system as an in-line impact on a rigid, immovable surface. From Eq. (16.16) with B's velocity equal to zero: $v'_A = -e v_A$, from which

$$\boxed{v'_A = -0.3(-15) = 4.5 \text{ m/s}}$$
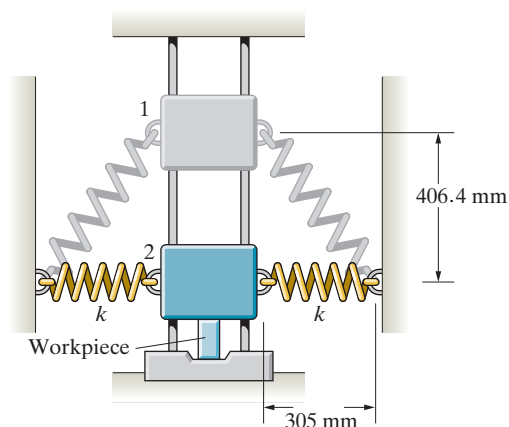
Problem 16.73 The 100-N weight is released from rest in position 1. The spring constant is $k = 1751$ N/m, and the springs are unstretched in position 2. If the coefficient of restitution of the impact of the weight with the workpiece in position 2 is $e = 0.6$, what is the magnitude of the velocity of the weight immediately after the impact?

406.4 mm
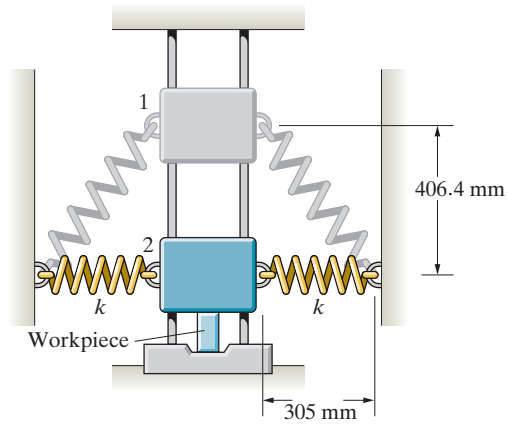
Solution: Work-energy will be used to find the velocity just before the collision. Then the coefficient of restitution will give the velocity after impact.

$$2\left(\frac{1}{2}kx^2\right) + mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2\frac{k}{m}x^2 + 2gh}$$

$$= \sqrt{2\left(\frac{1751 \text{ N/m}}{\left[\frac{100 \text{ N}}{9.81 \text{ m/s}^2}\right]}\right)(0.51 \text{ m} - 0.305 \text{ m})^2 + 2(9.81 \text{ m/s}^2)(0.406 \text{ m})} = 4.73 \text{ m/s}$$

$$v' = ev = (0.6)(4.73 \text{ m/s}) = 2.84 \text{ m/s}.$$

$$\boxed{v' = 2.84 \text{ m/s.}}$$

305 mm

Problem 16.74* A bioengineer studying helmet design uses an experimental apparatus that launches a 2.4-kg helmet containing a 2-kg model of the human head against a rigid surface at 6 m/s. The head, suspended within the helmet, is not immediately affected by the impact of the helmet with the surface and continues to move to the right at 6 m/s, so the head then undergoes an impact with the helmet. If the coefficient of restitution of the helmet's impact with the surface is 0.85 and the coefficient of restitution of the subsequent impact of the head with the helmet is 0.15, what is the velocity of the head after its initial impact with the helmet?

6 m/s
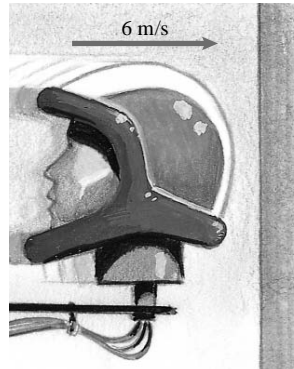
Solution: The helmet's rebound velocity is

$$v_{helmet} = (0.85)(6 \text{ m/s}) = 5.1 \text{ m/s}$$

The collision of the helmet and head

$$(2 \text{ kg})(6 \text{ m/s}) + (2.4 \text{ kg})(-5.1 \text{ m/s}) = (2 \text{ kg})v_{head}' + (2.4 \text{ kg})v_{helmet}'$$

$$0.15(6 - [-5.1]) \text{ m/s} = v_{helmet}' - v_{head}'$$

Solving we find $\boxed{v_{head}' = -0.963 \text{ m/s}}$

Problem 16.75*

(a) If the duration of the impact of the head with the helmet in Problem 16.74 is 0.004 s, what is the magnitude of the average force exerted on the head by the impact?

(b) Suppose that the simulated head alone strikes the rigid surface at 6 m/s, the coefficient of restitution is 0.5, and the duration of the impact is 0.0002 s. What is the magnitude of the average force exerted on the head by the impact?

Solution: See the solution to Problem 16.74

(a) $(2 \text{ kg})(6 \text{ m/s}) - F_{\text{ave}}(0.004 \text{ s}) = (2 \text{ kg})(-0.963 \text{ m/s})$

$\Rightarrow \boxed{F_{\text{ave}} = 3.48 \text{ kN}}$

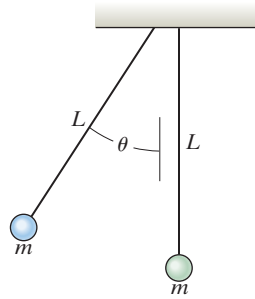(b)   The velocity of the head after the collision is

$v = 0.5(6 \text{ m/s}) = 3 \text{ m/s}$

$(2 \text{ kg})(6 \text{ m/s}) - F_{\text{ave}}(0.0002 \text{ s}) = (2 \text{ kg})(-3 \text{ m/s})$

$\Rightarrow \boxed{F_{\text{ave}} = 90 \text{ kN}}$

Problem 16.76   Two small balls, each of 1-N weight, hang from strings of length $L = 3$ m. The left ball is released from rest with $\theta = 35°$. The coefficient of restitution of the impact is $e = 0.9$. Through what maximum angle does the right ball swing?

Solution: Using work-energy and conservation of momentum we have

$$mgL(1 - \cos\theta) = \frac{1}{2}mv_A^2 \Rightarrow v_A = \sqrt{2gL(1 - \cos\theta)}$$

$$\left.\begin{array}{l} mv_A = mv_A' + mv_B' \\ ev_A = v_B' - v_A' \end{array}\right\} \Rightarrow v_B' = \frac{1+e}{2}v_A$$

$$\frac{1}{2}mv_B'^2 = mgL(1 - \cos\phi) \Rightarrow \phi = \cos^{-1}\left(1 - \frac{v_B'^2}{2gL}\right)$$
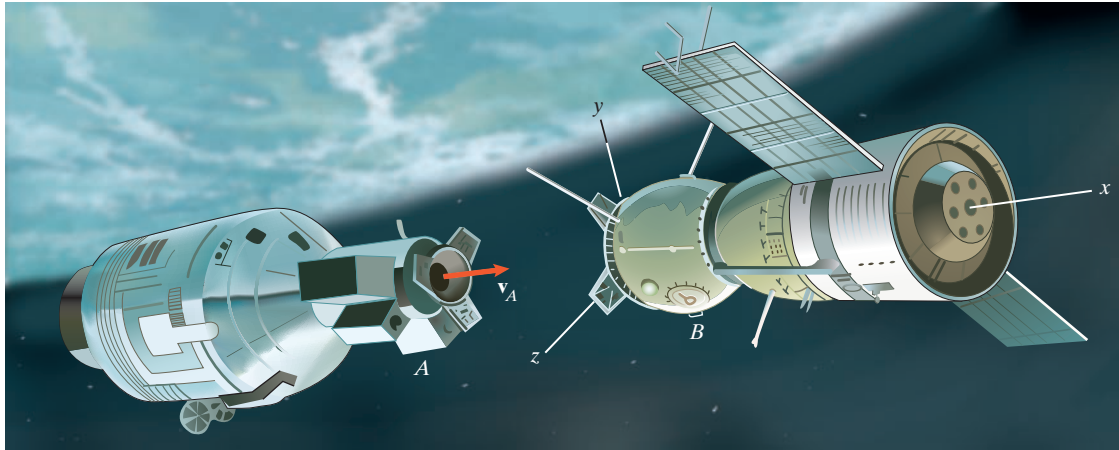
Putting in the numbers we find

$$v_A = \sqrt{2(9.81 \text{ m/s}^2)(3 \text{ m})(1 - \cos 35°)} = 3.26 \text{ m/s},$$

$$v_B' = \frac{1.9}{2}(3.26 \text{ m/s}) = 3.1 \text{ m/s},$$

$$\phi = \cos^{-1}\left(1 - \frac{[3.1 \text{ m/s}]^2}{2[9.81 \text{ m/s}^2][3 \text{ m}]}\right) = 33.2°.$$

$\boxed{\phi = 33.2°.}$

Problem 16.77 In Example 16.6, if the *Apollo* command-service module approaches the *Soyuz* spacecraft with velocity $0.25\mathbf{i} + 0.04\mathbf{j} + 0.01\mathbf{k}$ (m/s) and the docking is successful, what is the velocity of the center of mass of the combined vehicles afterward?
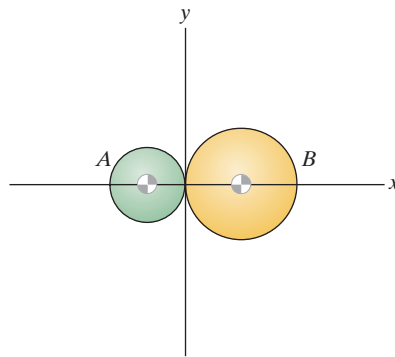

Solution: Momentum is conserved so

$$m_A\mathbf{v}_A = (m_A + m_S)\mathbf{v}_{\text{comb}} \Rightarrow \mathbf{v}_{\text{comb}} = \frac{m_A}{m_A + m_S}\mathbf{v}_A$$

$$\mathbf{v}_{\text{comb}} = \frac{18}{18 + 6.6}(0.25\mathbf{i} + 0.04\mathbf{j} + 0.01\mathbf{k}) \text{ m/s}.$$

$$\boxed{\mathbf{v}_{\text{comb}} = (0.183\mathbf{i} + 0.0293\mathbf{j} + 0.00732\mathbf{k}) \text{ m/s.}}$$

Problem 16.78 The 3-kg object A and 8-kg object B undergo an oblique central impact. The coefficient of restitution is $e = 0.8$. Before the impact, $\mathbf{v}_A = 10\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$ (m/s) and $\mathbf{v}_B = -2\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$ (m/s). What are the velocities of A and B after the impact?

Solution: Tangent to the impact plane the velocities do not change. In the impact plane we have

$$(3 \text{ kg})(10 \text{ m/s}) + (8 \text{ kg})(-2 \text{ m/s}) = (3 \text{ kg})v_{Ax}' + (8 \text{ kg})v_{Bx}'$$

$$0.8(10 - [-2]) \text{ m/s} = v_{Bx}' - v_{Ax}'$$

Solving we find $v_{Ax}' = -5.71$ m/s, $v_{Bx}' = 3.89$ m/s

Thus $\boxed{\mathbf{v}_A' = (-5.71\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}) \text{ m/s}, \quad \mathbf{v}_B' = (3.89\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}) \text{ m/s}}$

Problem 16.79 A baseball bat (shown with the bat's axis perpendicular to the page) strikes a thrown baseball. Before their impact, the velocity of the baseball is $\mathbf{v}_b = 132(\cos 45°\mathbf{i} + \cos 45°\mathbf{j})$ (m/s) and the velocity of the bat is $\mathbf{v}_B = 60(-\cos 45°\mathbf{i} - \cos 45°\mathbf{j})$(m/s). Neglect the change in the velocity of the bat due to the direct central impact. The coefficient of restitution is $e = 0.2$. What is the ball's velocity after the impact? Assume that the baseball and the bat are moving horizontally. Does the batter achieve a potential hit or a foul ball?
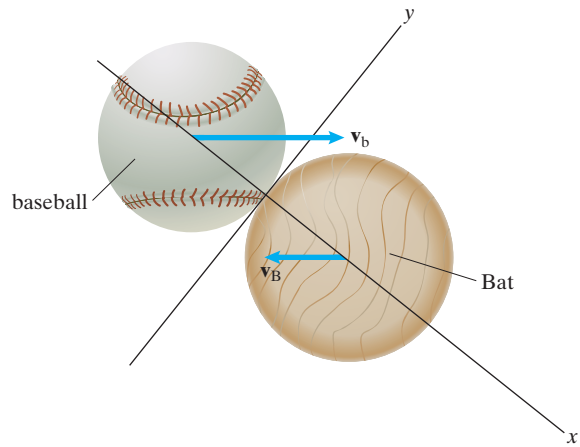
Solution: Tangent to the impact plane, the velocity does not change. Since we are neglecting the change in velocity of the bat, then

$0.2([132\cos 45°] - [-60\cos 45°])$ m/s $= (-60\cos 45°)$ m/s $- v_{ballx}'$

Solving we find $v_{ballx}' = -69.6$ m/s

Thus $\boxed{\mathbf{v}_{ball}' = (-69.6\mathbf{i} + 132\cos 45°\mathbf{j})\ \text{m/s} = (-69.6\mathbf{i} + 93.3\mathbf{j})\ \text{m/s}}$

The ball is foul.

Problem 16.80 The cue gives the cue ball A a velocity parallel to the y axis. The cue ball hits the eight ball B and knocks it straight into the corner pocket. If the magnitude of the velocity of the cue ball just before the impact is 2 m/s and the coefficient of restitution is $e = 1$, what are the velocity vectors of the two balls just after the impact? (The balls are of equal mass.)
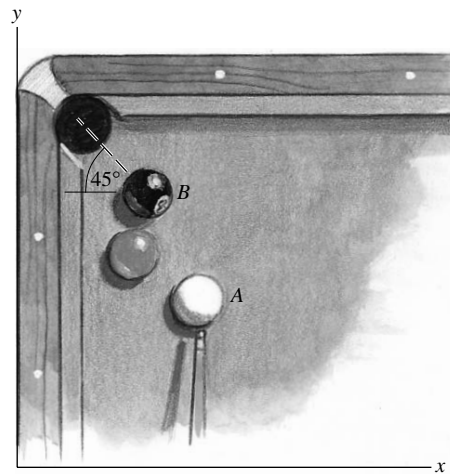
Solution: Denote the line from the 8-ball to the corner pocket by BP. This is an oblique central impact about BP. Resolve the cue ball velocity into components parallel and normal to BP. For a 45° angle, the unit vector parallel to BP is $\mathbf{e}_{BP} = \frac{1}{\sqrt{2}}(-\mathbf{i} + \mathbf{j})$, and the unit vector normal to BP is $\mathbf{e}_{BPn} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$. Resolve the cue ball velocity before impact into components: $\mathbf{v}_A = v_{AP}\mathbf{e}_{BP} + v_{APn}\mathbf{e}_{BPn}$. The magnitudes v_{AP} and v_{APn} are determined from

$$\sqrt{|v_{AP}\mathbf{e}_{BP}|^2 + |v_{APn}\mathbf{e}_{BPn}|^2} = |\mathbf{v}_A| = 2\ \text{m/s}$$

and the condition of equality imposed by the 45° angle, from which $v_{AP} = v_{APn} = \sqrt{2}$ m/s. The cue ball velocity after impact is $\mathbf{v}_A' = v_{AP}'\mathbf{e}_{BP} + v_{APn}\mathbf{e}_{BPn}$, (since the component of $\mathbf{v}_A$ that is at right angles to BP will be unchanged by the impact). The velocity of the 8-ball after impact is $\mathbf{v}_{BP}' = v_{BP}'\mathbf{e}_{BP}$. The unknowns are the magnitudes v_{BP}' and v_{AP}'. These are determined from the conservation of linear momentum along BP and the coefficient of restitution.

$m_A v_{AP} = m_A v_{AP}' + m_B v_{BP}',$

and

$$1 = \frac{v_{BP}' - v_{AP}'}{v_{AP}}.$$

For $m_A = m_B$, these have the solution $v_{AP}' = 0$, $v_{BP}' = v_{AP}$, from which

$\boxed{\mathbf{v}_A' = v_{APn}\mathbf{e}_{BPn} = \mathbf{i} + \mathbf{j}\ \text{(m/s)}}$

and

$\boxed{\mathbf{v}_B' = v_{AP}\mathbf{e}_{BP} = -\mathbf{i} + \mathbf{j}\ \text{(m/s)}.}$

Problem 16.81 In Problem 16.80, what are the velocity vectors of the two balls just after impact if the coefficient of restitution is $e = 0.9$?

Solution: Use the results of the solution to Problem 16.80, where the problem is solved as an oblique central impact about the line from the 8-ball to the corner pocket. Denote the line from the 8-ball to the corner pocket by BP. The unit vector parallel to BP is $\mathbf{e}_{BP} = \frac{1}{\sqrt{2}}(-\mathbf{i} + \mathbf{j})$, and the unit vector normal to BP is $\mathbf{e}_{BPn} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$. Resolve the cue ball velocity before impact into components:

$$\mathbf{v}_A = v_{AP}\mathbf{e}_{BP} + v_{APn}\mathbf{e}_{BPn},$$

where, from Problem 16.80, $v_{AP} = v_{APn} = \sqrt{2}$ m/s. The velocity of the 8-ball after impact is $\mathbf{v}'_{BP} = v'_{BP}\mathbf{e}_{BP}$. The unknowns are the magnitudes v'_{BP} and v'_{AP}. These are determined from the conservation of linear momentum <u>along BP</u> and the coefficient of restitution.

$$m_A v_{AP} = m_A v'_{AP} + m_B v'_{BP},$$

and

$$e = \frac{v'_{BP} - v'_{AP}}{v_{AP}}.$$

For $m_A = m_B$, these have the solution

$$v'_{AP} = \left(\frac{1}{2}\right)(1 - e)v_{AP} = 0.05 v_{AP},$$
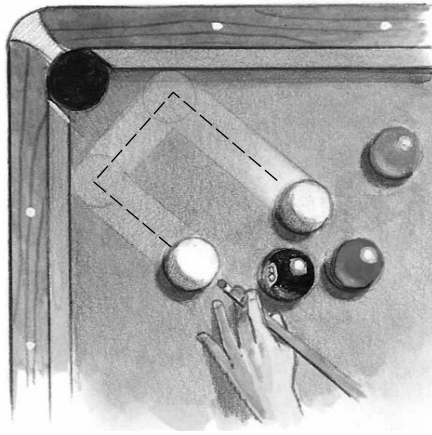
and

$$v'_{BP} = \left(\frac{1}{2}\right)(1 + e)v_{AP} = 0.95 v_{AP}.$$

The result:

$$\boxed{\begin{aligned} \mathbf{v}'_A &= v_{AP}\mathbf{e}_{BP} + v_{APn}\mathbf{e}_{BPn} = (-0.05\mathbf{i} + 0.05\mathbf{j} + \mathbf{i} + \mathbf{j}) \\ &= 0.95\mathbf{i} + 1.05\mathbf{j} \text{ m/s} \end{aligned}}$$

$$\boxed{v'_B = 0.95(-\mathbf{i} + \mathbf{j}) \text{ (m/s)}}$$

Problem 16.82 If the coefficient of restitution is the same for both impacts, show that the cue ball's path after two banks is parallel to its original path.

Solution: The strategy is to treat the two banks as two successive oblique central impacts. Denote the path from the cue ball to the first bank impact as CP1, the path from the first impact to the second as CP2, and the final path after the second bank as CP3. The cue ball velocity along CP1 is

$$\mathbf{v}_{A1} = v_{A1x}\mathbf{i} + v_{A1y}\mathbf{j},$$

and the angle is

$$\alpha = \tan^{-1}\left(\frac{v_{Ax1}}{y_{Ay1}}\right).$$

The component $v_{A1y}\mathbf{j}$ is unchanged by the impact. The x component after the first impact is $v_{A2x} = -ev_{A1x}$, from which the velocity of the cue ball along path CP2 is

$$\mathbf{v}_{A2} = -ev_{A1x}\mathbf{i} + v_{A1y}\mathbf{j}.$$

The angle is

$$\beta = \tan^{-1}\left(\frac{-ev_{A1x}}{v_{A1y}}\right).$$

The x component of the velocity along path CP2 is unchanged after the second impact, and the y component after the second impact is $v_{A3y} = -ev_{A1y}$. The velocity along the path CP3 is

$$\mathbf{v}_{A3} = -ev_{A1x}\mathbf{i} - ev_{A1y}\mathbf{j},$$

and the angle is

$$\gamma = \tan^{-1}\left(\frac{-ev_{A1x}}{-ev_{A1y}}\right) = \alpha.$$

The sides of the table at the two banks are at right angles; the angles $\boxed{\alpha = \gamma}$ show that the paths CP1 and CP3 are parallel.

Problem 16.83 The velocity of the 170-g hockey puck is $\mathbf{v}_P = 10\mathbf{i} - 4\mathbf{j}$ (m/s). If you neglect the change in the velocity $\mathbf{v}_S = v_S\mathbf{j}$ of the stick resulting from the impact, and if the coefficient of restitution is $e = 0.6$, what should v_S be to send the puck toward the goal?
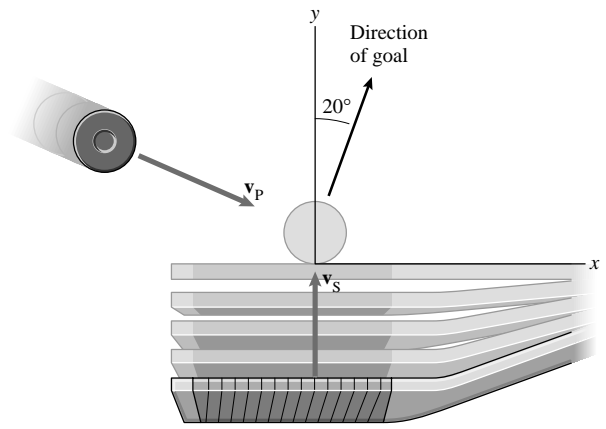
Solution: The strategy is to treat the collision as an *oblique central impact* with a moving object of infinite mass. The horizontal component of the puck velocity is unchanged by the impact. The vertical component of the velocity after impact must satisfy the condition $\tan^{-1}(v'_{Px}/v'_{Py}) = \tan^{-1}(10/v'_{Py}) = 20°$, from which the velocity of the puck after impact must be $v'_{Py} = 27.47$ m/s. Assume for the moment that the hockey stick has a finite mass, and consider only the y component of the puck velocity. The conservation of linear momentum and the definition of the coefficient of restitution are

$$m_P v_{Py} + m_S v_S = m_P v'_{Py} + m_S v'_S,$$

and

$$e = \frac{v'_S - v'_{Py}}{v_{Py} - v_S}.$$

These two simultaneous equations have the solution

$$v'_{Py} = (1/(m_P + m_S))(m_S(1 + e)v_S + (m_P - em_S)v_{Py}).$$

Divide numerator and denominator on the right by m_S and take the limit as

$$m_S \to \infty, v'_{Py} = \lim_{m_S \to \infty} \left(1 / \left(\frac{m_P}{m_S} + 1\right)\right)((1 + e)v_S$$

$$+ \left(\frac{m_P}{m_S} - e\right)v_{Py}\right) = (1 + e)v_S - ev_{Py}.$$

Substitute the values: $v'_{Py} = 27.47$ m/s, $e = 0.6$, and $v_{Py} = -4$ m/s and solve:

$$\boxed{v_S = \frac{v'_{Py} + ev_{Py}}{(1 + e)} = 15.67 \text{ m/s}}$$

Problem 16.84 In Problem 16.83, if the stick responds to the impact the way an object with the same mass as the puck would, and if the coefficient of restitution is $e = 0.6$, what should v_S be to send the puck toward the goal?

Solution: Use the solution to Problem 16.83, where m_S has a finite mass,

$$v'_{Py} = \left(\frac{1}{m_P + m_S}\right)(m_S(1 + e)v_S + (m_P - em_S)v_{Py}).$$

Substitute $m_P = m_S$, $e = 0.6$, $v'_{Py} = 27.47$ m/s, and $v_{Py} = -4$ m/s, and solve:

$$v_S = \frac{2v'_{Py} - (1 - e)v_{Py}}{(1 + e)} = 35.3 \text{ m/s}$$

Problem 16.85 At the instant shown ($t_1 = 0$), the position of the 2-kg object's center of mass is $\mathbf{r} = 6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ (m) and its velocity is $\mathbf{v} = -16\mathbf{i} + 8\mathbf{j} - 12\mathbf{k}$ (m/s). No external forces act on the object. What is the object's angular momentum about the origin O at $t_2 = 1$ s?
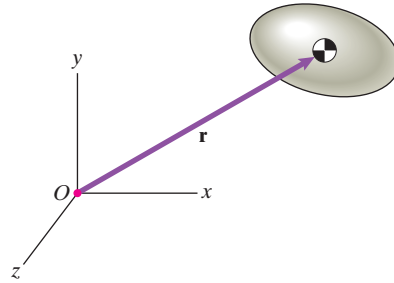
Solution:

$\mathbf{H}_O = (6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ m $\times$ (2 kg)$(-16\mathbf{i} + 8\mathbf{j} - 12\mathbf{k})$ m/s

$$\mathbf{H}_O = (-128\mathbf{i} + 80\mathbf{j} + 224\mathbf{k}) \text{ kg-m}^2/s$$

Problem 16.86 Suppose that the total external force on the 2-kg object shown in Problem 16.85 is given as a function of time by $\Sigma\mathbf{F} = 2t\mathbf{i} + 4\mathbf{j}$(N). At time $t_1 = 0$, the object's position and velocity are $\mathbf{r} = \mathbf{0}$ and $\mathbf{v} = \mathbf{0}$.

(a) Use Newton's second law to determine the object's velocity $\mathbf{v}$ and position $\mathbf{r}$ as functions of time.
(b) By integrating $\mathbf{r} \times \Sigma\mathbf{F}$ with respect to time from $t_1 = 0$ to $t_2 = 6$ s, determine the angular impulse about O exerted on the object during this interval of time.
(c) Use the results of part (a) to determine the change in the object's angular momentum from $t_1 = 0$ to $t_2 = 6$ s.

Solution:

(a) $\mathbf{F} = (2t\mathbf{i} + 4\mathbf{j})$N $= (2$ kg$)\mathbf{a}$, $\quad \mathbf{a} = (t\mathbf{i} + 2\mathbf{j})$ m/s^2

$$\mathbf{v} = \left(\frac{t^2}{2}\mathbf{i} + 2t\mathbf{j}\right) \text{ m/s}, \quad \mathbf{r} = \left(\frac{t^3}{6}\mathbf{i} + t^2\mathbf{j}\right) \text{ m}$$

(b) Angular Impulse $= \int_0^{6 \text{ s}} \mathbf{M}_O dt$

$$= \int_0^{6 \text{ s}} \mathbf{r} \times \mathbf{F} dt = \int_0^{6 \text{ s}} \left[\left(\frac{t^3}{6}\mathbf{i} + t^2\mathbf{j}\right) \text{ m}\right] \times [(2t\mathbf{i} + 4\mathbf{j})\text{N}]dt$$

$$= \int_0^{6 \text{ s}} \left(-\frac{4t^3}{3}\mathbf{k}\right) \text{N-m} dt$$

$$\boxed{\text{Angular Impulse} = (-432\mathbf{k}) \text{ kg-m}^2/s}$$

(c) $$\Delta\mathbf{H}_O = \int_0^{6 \text{ s}} \mathbf{M}_O dt = (-432\mathbf{k}) \text{ kg-m}^2/s$$

Problem 16.87 A satellite is in the elliptic earth orbit shown. Its velocity at perigee A is 8640 m/s. The radius of the earth is 6370 km.

(a) Use conservation of angular momentum to determine the magnitude of the satellite's velocity at apogee C.
(b) Use conservation of energy to determine the magnitude of the velocity at C.

(See Example 16.8.)
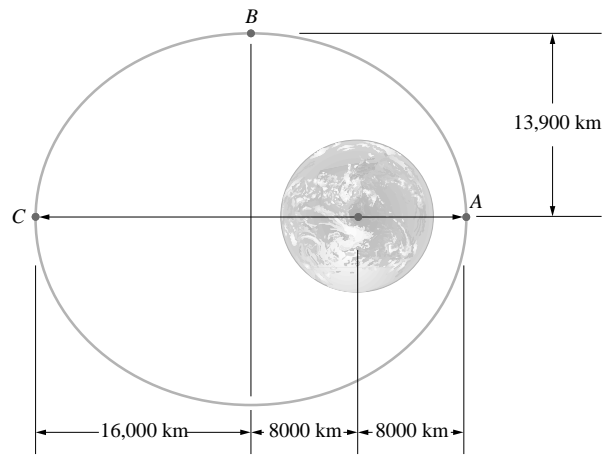
Solution:

(a) $r_A v_A = r_C v_C = |\mathbf{H}_0|$

$(8000)(8640) = (24000)v_C$

$\underline{v_C = 2880 \text{ m/s}}$

(b) $\dfrac{1}{2}mv_A^2 - \dfrac{mgR_E^2}{r_A} = \dfrac{1}{2}mv_C^2 - \dfrac{mgR_E^2}{r_C}$

$\dfrac{v_A^2}{2} - \dfrac{gR_E^2}{r_A} = \dfrac{v_C^2}{2} - \dfrac{gR_E^2}{r_C}$

where $v_A = 8640$ m/s, $g = 9.81$ m/s, $R_E = 6370000$ m, $r_A = 8,000,000$ m, $r_C = 24,000,000$ m.

Solving for v_C, $\underline{v_C = 2880 \text{ m/s}}$

Problem 16.88 For the satellite in Problem 16.87, determine the magnitudes of the radial velocity v_r and transverse velocity v_θ at B. (See Example 16.8.)

Solution: Use conservation of energy to find the velocity magnitude at B. Then use conservation of angular momentum to determine the components.

$\dfrac{1}{2}mV_A^2 - \dfrac{mgR_E^2}{r_A} = \dfrac{1}{2}mV_B^2 - \dfrac{mgR_E^2}{r_B}$

where $r_A = 8 \times 10^6$ m, $v_A = 8640$ m/s

$r_B = \sqrt{(8 \times 10^6)^2 + (13.9 \times 10^6)^2}$

$r_B = 16 \times 10^6$ m, $R_E = 6.370 \times 10^6$ m

Solving, we get $v_B = 4990$ m/s

From conservation of angular momentum

$r_A v_A = r_B v_\theta$

Solving, $v_\theta = 4320$ m/s

Finally $v_r = \sqrt{v_B^2 - v_\theta^2} = 2500$ m/s

Problem 16.89 The bar rotates *in the horizontal plane* about a smooth pin at the origin. The 2-kg sleeve A slides on the smooth bar, and the mass of the bar is negligible in comparison to the mass of the sleeve. The spring constant $k = 40$ N/m, and the spring is unstretched when $r = 0$. At $t = 0$, the radial position of the sleeve is $r = 0.2$ m and the angular velocity of the bar is $w_0 = 6$ rad/s. What is the angular velocity of the bar when $r = 0.25$ m?
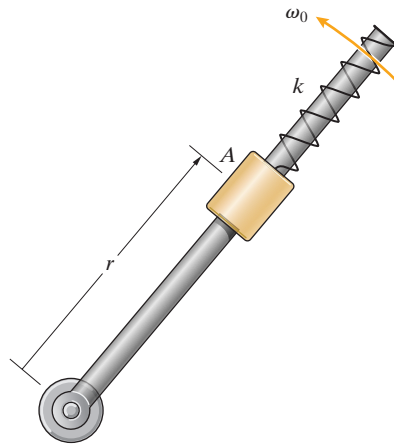
Solution: Since the spring force is radial, it does not affect the angular momentum which is constant.

$(0.2 \text{ m})(2 \text{ kg})[(0.2 \text{ m})(6 \text{ rad/s})] = (0.25 \text{ m})(2 \text{ kg})[(0.25 \text{ m})\omega]$

$\boxed{\omega = 3.84 \text{ rad/s}}$

Problem 16.90 At $t = 0$, the radial position of the sleeve A in Problem 16.89 is $r = 0.2$ m, the radial velocity of the sleeve is $v_r = 0$ and the angular velocity of the bar is $w_0 = 6$ rad/s. What are the angular velocity of the bar and the radial velocity of the sleeve when $r = 0.25$ m?

**Solution:** From problem 16.89 we have $\boxed{\omega = 3.84 \text{ rad/s}}$

Use work - energy to find the radial velocity

$\frac{1}{2}(2 \text{ kg})(0^2 + [(0.2 \text{ m})(6 \text{ rad/s})^2]) + \frac{1}{2}(40 \text{ N/m})(0.2 \text{ m})^2 =$

$\frac{1}{2}(2 \text{ kg})(v_r{}^2 + [(0.25 \text{ m})(3.84 \text{ rad/s})^2]) + \frac{1}{2}(40 \text{ N/m})(0.25 \text{ m})^2$

$\boxed{v_r = \pm 0.262 \text{ m/s}}$

Problem 16.91 A 2-kg disk slides on a smooth horizontal table and is connected to an elastic cord whose tension is $T = 6r$ N, where r is the radial position of the disk in meters. If the disk is at $r = 1$ m and is given an initial velocity of 4 m/s in the transverse direction, what are the magnitudes of the radial and transverse components of its velocity when $r = 2$ m? (See Active Example 16.7.)
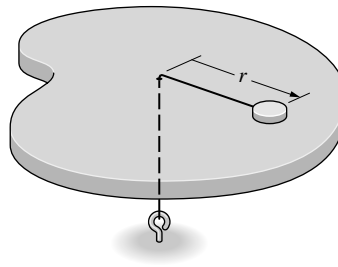
Solution: The strategy is to (a) use the principle of conservation of angular momentum to find the transverse velocity and (b) use the conservation of energy to find the radial velocity. The angular momentum the instant after $t = 0$ is $(\mathbf{r} \times m\mathbf{v})_o = H_0\mathbf{e}_z = (mr v_\theta)_o\mathbf{e}_z$, from which $H_0 = 8$ kg-m^2/s. In the absence of external transverse forces, the angular momentum impulse vanishes:

$$\int_{t_1}^{t_2} (\mathbf{r} \times \sum \mathbf{F})\,dt = 0 = \mathbf{H}_2 - \mathbf{H}_1,$$

so that $\mathbf{H}_1 = \mathbf{H}_2$, that is, the angular momentum is constant. At

$r = 2,$ $\boxed{v_\theta = \dfrac{H_0}{mr} = \dfrac{8}{4} = 2 \text{ m/s.}}$

From conservation of energy:

$$\tfrac{1}{2}mv_{r_0}^2 + \tfrac{1}{2}mv_{\theta_o}^2 + \tfrac{1}{2}kS_o^2 = \tfrac{1}{2}mv_r^2 + \tfrac{1}{2}mv_\theta^2 + \tfrac{1}{2}kS^2.$$

Solve:

$$v_r = \sqrt{v_{r_0}^2 + v_{\theta_o}^2 - v_\theta^2 + \left(\dfrac{k}{m}\right)(S_o^2 - S^2)}.$$

Substitute numerical values: Noting $m = 2$ kg, $v_{r_0} = 0$, $v_{\theta_o} = 4$ m/s, $k = 6$ N/m, $r = 2$ m, $v_\theta = 2$ m/s, $S_o = 1$ m, $S = 2$ m from which $\boxed{v_r = \sqrt{3} \text{ m/s}}$. The velocity is $\boxed{\mathbf{v} = 1.732\mathbf{e}_r + 2\mathbf{e}_\theta}$ (m/s).

Problem 16.92 In Problem 16.91, determine the maximum value of r reached by the disk.

Solution: The maximum value is the stretch of the cord when $v_r = 0$. From the solution to Problem 16.91,

$$v_r^2 = v_{r_0}^2 + v_{\theta_o}^2 - v_\theta^2 + \left(\dfrac{k}{m}\right)(S_o^2 - S^2) = 0,$$

where $v_\theta = \dfrac{H_0}{mr}$ m/s, $v_{r_0}^2 = 0$, $v_{\theta_o} = \dfrac{H_0}{m(1)}$ m/s, $S_o = r_0 = 1$ m, $S = rm$, and $H_0 = 8$ kg-m^2/s. Substitute and reduce:

$$v_r^2 = 0 = \left(\dfrac{H_0^2}{m^2}\right)\left(1 - \dfrac{1}{r^2}\right) + \left(\dfrac{k}{m}\right)(1 - r^2).$$

Denote $x = r^2$ and reduce to a quadratic canonical form $x^2 + 2bx + c = 0$, where

$$b = -\left(\dfrac{1}{2}\right)\left(\dfrac{H_0^2}{km} + 1\right) = -3.167, \; c = \dfrac{H_0^2}{km} = 5.333.$$
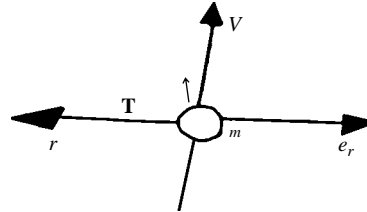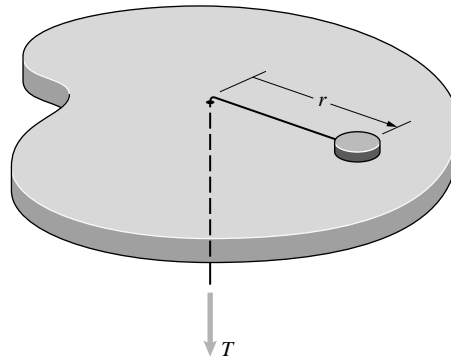
Solve $r_{1,2}^2 = -b \pm \sqrt{b^2 - c} = 5.333, = 1$, from which the greatest positive root is $\boxed{r_{\max} = 2.31 \text{ m}}$

[*Check*: This value is confirmed by a graph of the value of

$$f(r) = \left(\dfrac{H_0^2}{m^2}\right)\left(1 - \dfrac{1}{r^2}\right) + \left(\dfrac{k}{m}\right)(1 - r^2)$$

to find the zero crossing. *check*.]

Problem 16.93 A 1-kg disk slides on a smooth horizontal table and is attached to a string that passes through a hole in the table.

(a) If the mass moves in a circular path of constant radius $r = 1$ m with a velocity of 2 m/s, what is the tension T?

(b) Starting from the initial condition described in part (a), the tension T is increased in such a way that the string is pulled through the hole at a constant rate until $r = 0.5$ m. Determine the value of T as a function of r while this is taking place.

Solution:

(a) Circular motion

$$\mathbf{T} = -mv^2/r\mathbf{e}_r$$

$$\underline{|\mathbf{T}| = (1)(2)^2/1 = 4 \text{ N}}$$

(b) By conservation of angular momentum,

$$mr_0v_0 = mrv_T, \quad \therefore v_T = \frac{r_0v_0}{r}$$

and from Newton's second law

$$|\mathbf{T}| = mv_T^2/r = m\left(\frac{r_0v_0}{r}\right)^2/r$$

$$\underline{T = (1)\left(\frac{(1)(2)}{r}\right)^2 \Bigg/ r = 4/r^3 \text{ N}}$$

Problem 16.94 In Problem 16.93, how much work is done on the mass in pulling the string through the hole as described in part (b)?

**Solution:** The work done is

$$U_{12} = \int_1^{0.5} \left(-\frac{mr_0^2r_0^2}{r^3}\right) \mathbf{e}_r \cdot dr\mathbf{e}_r$$

$$= -mr_0^2v_0^2 \int_1^{0.5} \frac{dr}{r^3} = -(1)(1)^2(2)^2 \left[\frac{1}{-2r^2}\right]_1^{0.5}$$

$$= -4\left[-\frac{1}{2(0.5)^2} + \frac{1}{2}\right]$$

$$\underline{U_{12} = -4[-1.5] = 6 \text{ N-m}}$$

Problem 16.95 Two gravity research satellites ($m_A = 250$ kg, $m_B = 50$ kg) are tethered by a cable. The satellites and cable rotate with angular velocity $\omega_0 = 0.25$ revolution per minute. Ground controllers order satellite A to slowly unreel 6 m of additional cable. What is the angular velocity afterward?
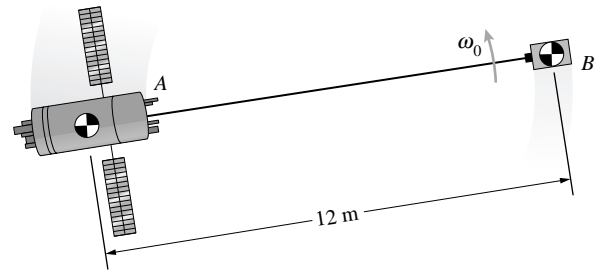
Solution: The satellite may be rotating in (a) a vertical plane, or (b) in the horizontal plane, or (c) in some intermediate plane. The strategy is to determine the angular velocity for the three possibilities.

Case (a) Assume that the system rotates in the x-y plane, with y positive upward. Choose the origin of the coordinates at the center of mass of the system. The distance along the cable from the center of mass to A is

$$\frac{12m_B}{m_A + m_B} = 2 \text{ m, from which the distance to B is 10 m.}$$

Assume that both satellites lie on the x axis at $t = 0$. The radius position of satellite A is $\mathbf{r}_A = -2(\mathbf{i}\cos\omega_0 t + \mathbf{j}\sin\omega_0 t)$, and the radius position of satellite B is $\mathbf{r}_B = 10(\mathbf{i}\cos\omega_0 t + \mathbf{j}\sin\omega_0 t)$. The acceleration due to gravity is

$$\mathbf{W} = -\frac{mgR_E^2}{r^2}\mathbf{j} = -mg'\mathbf{j},$$

from which $\mathbf{W}_A = -m_A g'\mathbf{j}$ and $\mathbf{W}_B = -m_B g'\mathbf{j}$ (or, alternatively). The angular momentum impulse is

$$\int_{t_1}^{t_2} (\mathbf{r} \times \textstyle\sum \mathbf{F})\, dt = \int_{t_1}^{t_2} (\mathbf{r}_A \times \mathbf{W}_A + \mathbf{r}_B \times \mathbf{W}_B)\, dt.$$

Carry out the indicated operations:

$$\mathbf{r}_A \times \mathbf{W}_A = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2(\cos\omega_0 t) & -2(\sin\omega_0 t) & 0 \\ 0 & -m_A g' & 0 \end{bmatrix}$$

$$= 2\, m_A g' \cos\omega_0 t\, \mathbf{k}.$$

$$\mathbf{r}_B \times \mathbf{W}_B = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10(\cos\omega_0 t) & 10(\sin\omega_0 t) & 0 \\ 0 & -m_B g' & 0 \end{bmatrix}$$

$$= -10 m_B g' \cos\omega_0 t\, \mathbf{k}.$$

Substitute into the angular momentum impulse:

$$\int_{t_1}^{t_2} (\mathbf{r} \times \textstyle\sum \mathbf{F})\, dt = 0 = \mathbf{H}_2 - \mathbf{H}_1,$$

from which $\mathbf{H}_1 = \mathbf{H}_2$; that is, the angular momentum is conserved. From Newton's second law, the center of mass remains unchanged as the cable is slowly reeled out.

A repeat of the argument above for *any* additional length of cable leads to the same result, namely, the angular momentum is constant, from which the angular momentum is conserved as the cable is reeled out. The angular momentum of the original system is

$$\mathbf{r}_A \times m_A \mathbf{v} + \mathbf{r}_B \times m_B \mathbf{v} = 4m_A \omega_0 \mathbf{k} + 100 m_B \omega_0 \mathbf{k}$$

$$= 157.1 \text{ kg-m}^2/\text{s},$$

in magnitude, where $\omega_0 = 0.026$ rad/s. After 6 meters is reeled out, the distance along the cable from the center of mass to A is

$$\frac{m_B(6 + 12)}{m_A + m_B} = 3 \text{ m},$$

from which the distance to B is 15 m. The new angular velocity when the 6 m is reeled out is

$$\boxed{\omega = \frac{H}{3^2 m_A + 15^2 m_B} = 0.0116 \text{ rad/s} = 0.1111 \text{ rpm}}$$

Case (b): Assume that the system rotates in the x-z plane, with y positive upward. As above, choose the origin of the coordinates at the center of mass of the system. Assume that both satellites lie on the x axis at $t = 0$. The radius position of satellite A is $\mathbf{r}_A = -2(\mathbf{i}\cos\omega_0 t + \mathbf{k}\sin\omega_0 t)$, and the radius position of satellite B is $\mathbf{r}_B = 10(\mathbf{i}\cos\omega_0 t + \mathbf{k}\sin\omega_0 t)$. The force due to gravity is $\mathbf{W}_A = -m_A g'\mathbf{j}$ and $\mathbf{W}_B = -m_B g'\mathbf{j}$. The angular momentum impulse is

$$\int_{t_1}^{t_2} (\mathbf{r} \times \textstyle\sum \mathbf{F})\, dt = \int_{t_1}^{t_2} (\mathbf{r}_A \times \mathbf{W}_A + \mathbf{r}_B \times \mathbf{W}_B)\, dt.$$

Carry out the indicated operations:

$$\mathbf{r}_A \times \mathbf{W}_A = 2m_A g' \cos\omega_0 t\, \mathbf{i},$$

$$\mathbf{r}_B \times \mathbf{W}_B = -10 m_B g' \cos\omega_0 t\, \mathbf{i}.$$

Substitute into the angular momentum impulse:

$$\int_{t_1}^{t_2} (\mathbf{r} \times \textstyle\sum \mathbf{F})\, dt = 0 = \mathbf{H}_2 - \mathbf{H}_1,$$

from which $\mathbf{H}_1 = \mathbf{H}_2$; that is, the angular momentum is conserved. By a repeat of the argument given in Case (a), the new angular velocity is $\omega = 0.0116$ rad/s $= 0.111$ rpm.

Case (c): Since the angular momentum is conserved, a repeat of the above for *any orientation of the system relative to the gravity vector* leads to the same result.

Problem 16.96 The astronaut moves in the x–y plane at the end of a 10-m tether attached to a large space station at O. The total mass of the astronaut and his equipment is 120 kg.

(a) What is the astronaut's angular momentum about O before the tether becomes taut?
(b) What is the magnitude of the component of his velocity perpendicular to the tether immediately after the tether becomes taut?
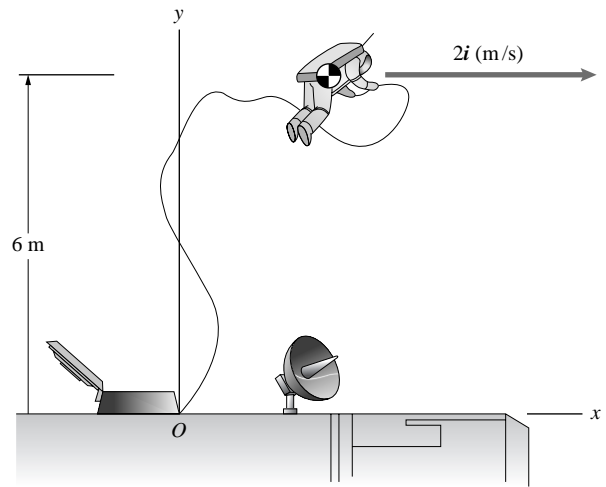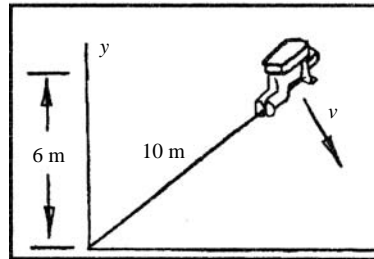

Solution:

(a) The angular momentum by definition is

$$(\mathbf{r} \times m\mathbf{v}) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ (120)2 & 0 & 0 \end{bmatrix} = -6(2)(120)\mathbf{k}$$

$$= -1440\mathbf{k} \ (\text{kg-m}^2/\text{s}).$$

(b) From conservation of angular momentum,

(b) $\mathbf{H} = -1440\mathbf{k} = -(10)(120)v\mathbf{k}$, from which

$$\boxed{v = \frac{1440}{(10)(120)} = 1.2 \text{ m/s}}$$

Problem 16.97 The astronaut moves in the x–y plane at the end of a 10-m tether attached to a large space station at O. The total mass of the astronaut and his equipment is 120 kg. The coefficient of restitution of the "impact" that occurs when he comes to the end of the tether is $e = 0.8$. What are the x and y components of his velocity immediately after the tether becomes taut?

Solution: From the solution of Problem 16.96, his velocity perpendicular to the tether is 1.2 m/s.

Before the tether becomes taught, his component of velocity parallel to the tether is

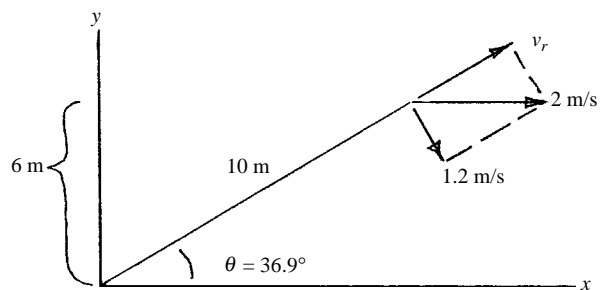$$v_r = 2 \cos 36.9° = 1.6 \text{ m/s}.$$

After it becomes taught,

$$v'_r = -ev_r = -(0.8)(1.6) = -1.28 \text{ m/s}.$$

The x and y components of his velocity are

$$v'_x = v'_r \cos 36.9° + (1.2) \sin 36.9°$$

$$= -0.304 \text{ m/s}$$

and

$$v'_y = v'_r \sin 36.9° - (1.2) \cos 36.9°$$

$$= -1.728 \text{ m/s}.$$

Problem 16.98 A ball suspended from a string that goes through a hole in the ceiling at O moves with velocity v_A in a horizontal circular path of radius r_A. The string is then drawn through the hole until the ball moves with velocity v_B in a horizontal circular path of radius r_B. Use the principle of angular impulse and momentum to show that $r_A v_A = r_B v_B$.

Strategy: Let $\mathbf{e}$ be a unit vector that is perpendicular to the ceiling. Although this is not a central-force problem — the ball's weight does not point toward O — you can show that $\mathbf{e} \cdot (\mathbf{r} \times \sum \mathbf{F}) = 0$, so that $\mathbf{e} \cdot \mathbf{H}_O$ is conserved.

Solution: Assume that the motion is in the x-y plane, and that the ball lies on the positive x axis at $t = 0$. The radius vector

$$\mathbf{r}_A = r_A(\mathbf{i} \cos \omega_A t + \mathbf{j} \sin \omega_A t),$$

where ω_A is the angular velocity of the ball in the path. The velocity is

$$\mathbf{v}_A = -\mathbf{j} r_A \omega_A \sin \omega_A t + \mathbf{j} r_A \omega_A \cos \omega_A t.$$

The angular momentum per unit mass about the axis normal to the ceiling is

$$\left(\frac{\mathbf{r} \times m\mathbf{v}}{m} \right) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_A \cos \omega_A t & r_A \sin \omega_A t & 0 \\ -r_A \omega_A \sin \omega_A t & r_A \omega_A \cos \omega_A t & 0 \end{bmatrix} = \mathbf{k}(r_A^2 \omega_A).$$

Define the unit vector parallel to this angular momentum vector, $\mathbf{e} = \mathbf{k}$. From the principle of angular impulse and momentum, the external forces do not act to change this angular momentum. This is shown as follows:

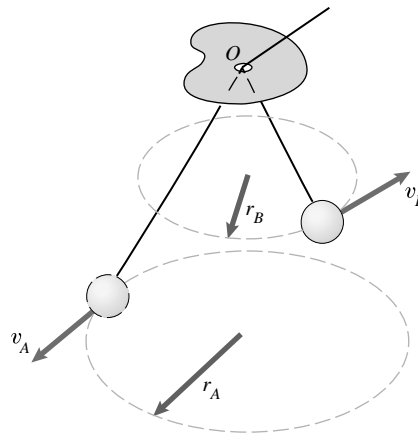The external force is the weight, $\mathbf{W} = -mg\mathbf{k}$. The momentum impulse is

$$\int_{t_1}^{t_2} (\mathbf{r} \times \sum \mathbf{F}) \, dt = \int_{t_1}^{t_2} (\mathbf{r}_A \times \mathbf{W}) \, dt.$$

Carry out the operation

$$\mathbf{r}_A \times \mathbf{W} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_A \cos \omega_A t & r_A \sin \omega_A t & 0 \\ 0 & 0 & -mg \end{bmatrix} = r_A mg \cos \omega_A t \, \mathbf{j},$$

from which

$$\int_{t_1}^{t_2} \mathbf{j} r_A mg \cos \omega t \, dt = -\mathbf{j}(r_A \omega_A mg)(\sin \omega_A t_2 - \sin \omega_A t_1)$$

$$= \mathbf{H}_2 - H_1.$$

Since this has no component parallel to the unit vector $\mathbf{e} = \mathbf{k}$, the angular momentum along the axis normal to the ceiling is unaffected by the weight, that is, the projection of the angular momentum impulse due to the external forces on the unit vector normal to the ceiling is zero $\mathbf{e} \cdot \mathbf{H}_2 = \mathbf{e} \cdot \mathbf{H}_1 = 0$, hence the *angular momentum normal to the ceiling is conserved*. This result holds true for any length of string, hence

$$(\mathbf{r} \times \mathbf{v})_A = \mathbf{k} r_A^2 \omega_A = (\mathbf{r} \times \mathbf{v})_B = \mathbf{k} r_B^2 \omega_B,$$

from which $r_A^2 \omega_A = r_B^2 \omega_B$. Since $v_A = r_A \omega_A$, $v_B = r_B \omega_B$, the result can be expressed $\boxed{v_A r_A = v_B r_B}$

Problem 16.99 The Cheverton fire-fighting and rescue boat can pump 3.8 kg/s of water from each of its two pumps at a velocity of 44 m/s. If both pumps point in the same direction, what total force do they exert on the boat.
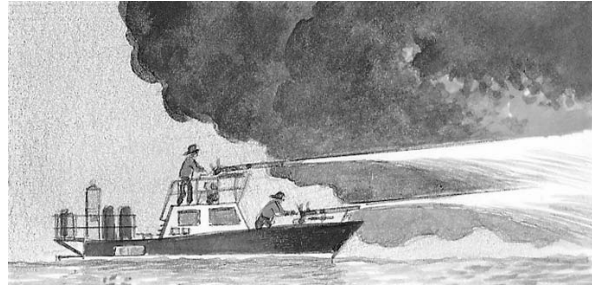
Solution: The magnitude of the total force is

$2(3.8 \text{ kg/s})(44 \text{ m/s}) = 334 \text{ N}.$

Problem 16.100 The mass flow rate of water through the nozzle is 23.3 kg/s. Determine the magnitude of the horizontal force exerted on the truck by the flow of the water.
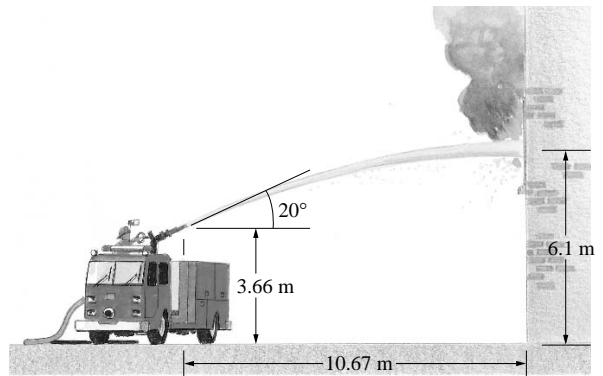
Solution: We must determine the velocity with which the water exits the nozzle. Relative to the end of the nozzle, the x-coordinate of a particle of water is $x = v_0 \cos 20° t$ and the y coordinate is

$y = v_0 \sin 20° t - \frac{1}{2}(9.81)t^2.$

Setting $x = 10.67$ m and $y = 2.44$ m and eliminating t we obtain

$2.44 = v_0 \sin 20° \left(\frac{10.67}{v_0 \cos 20°} \right) - \frac{1}{2}(9.81) \left(\frac{10.67}{v_0 \cos 20°} \right)^2$

From this equation, $v_0 = 20.94$ m/s. The horizontal force exerted by the flow of water is

$\frac{dm_f}{dt} v_f \cos 20° = (23.3 \text{ kg/s})(20.94) \cos 20° = 458.1 \text{ N}.$

Problem 16.101 The front-end loader moves at a constant speed of 3.2 km/h scooping up iron ore. The constant horizontal force exerted on the loader by the road is 1780 N. What weight of iron ore is scooped up in 3 s?
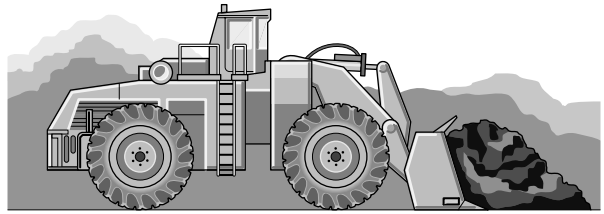
Solution:

$$1780 \text{ N} = \left(\frac{dm_f}{dt}\right)\left(\frac{3.2 \times 1000}{3600}\right) \Rightarrow \frac{dm_f}{dt} = 2002.5 \text{ N-s/m}$$

In 3 s $m_f = (2002.5 \text{ N-s/m})(3 \text{ s}) = 6007.5 \text{ N-s}^2/\text{m}$

$$\boxed{W = m_f g = (6007.5)(9.81 \text{ m/s}^2) = 58933.6 \text{ N}}$$

Problem 16.102 The snowblower moves at 1 m/s and scoops up 750 kg/s of snow. Determine the force exerted by the entering flow of snow.

Solution: The mass flow rate is

$$\left(\frac{dm_f}{dt}\right) = 750 \text{ kg/s}.$$

The velocity is $v_f = 1$ m/s. The force exerted by the entering flow of snow is

$$\boxed{F = \left(\frac{dm_f}{dt}\right) v_f = 750(1) = 750 \text{ N}}$$

Problem 16.103 The snowblower scoops up 750 kg/s of snow. It blows the snow out the side at 45° above the horizontal from a port 2 m above the ground and the snow lands 20 m away. What horizontal force is exerted on the blower by the departing flow of snow?
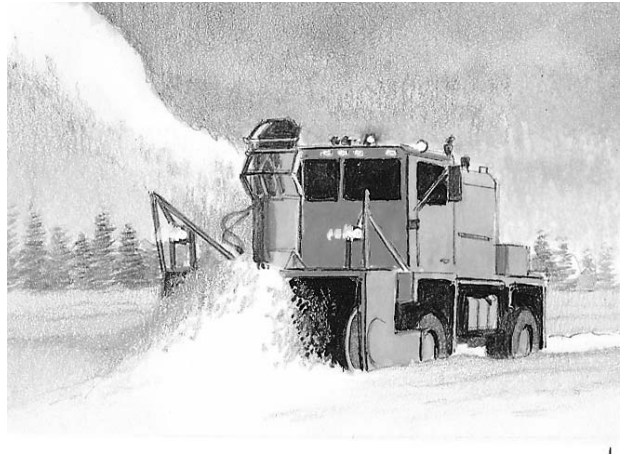
Solution: The strategy is to use the solution of Newton's second law to determine the exit velocity.

From Newton's second law (ignoring drag) $m_f \dfrac{dv_y}{dt} = -m_f g$, from which $v_y = -gt + v_f \sin 45°$ (m/s),

$v_x = v_f \cos 45°$ (m/s),

and

$y = -\dfrac{g}{2} t^2 + \left(v_f \sin 45° \right) t + 2$ m,

$x = \left(v_f \cos 45° \right) t.$

At $y = 0$, the time of impact is $t_{imp}^2 + 2b t_{imp} + c = 0$, where $b = -\dfrac{v_f \sin 45°}{g}, c = -\dfrac{4}{g}.$

The solution:

$t_{imp} = -b \pm \sqrt{b^2 - c}$

$\quad = \dfrac{v_f}{\sqrt{2}g} \left(1 \pm \sqrt{1 + \dfrac{8g}{v_f^2}} \right).$

Substitute:

$x = 20 = \left(\dfrac{v_f}{\sqrt{2}} \right) \left(\dfrac{v_f}{\sqrt{2}g} \right) \left(1 \pm \sqrt{1 + \dfrac{8g}{v_f^2}} \right).$

This equation is solved by iteration using **TK Solver Plus** to yield $v_f = 13.36$ m/s.

The horizontal force exerted on the blower is

$\boxed{F = \left(\dfrac{dm_f}{dt} \right) v_f = 750(13.36) \cos 45° = 7082.7 \text{ kN}}$

Problem 16.104 A nozzle ejects a stream of water horizontally at 40 m/s with a mass flow rate of 30 kg/s, and the stream is deflected in the horizontal plane by a plate. Determine the force exerted on the plate by the stream in cases (a), (b), and (c). (See Example 16.11.)

(a)

(b)   (c)
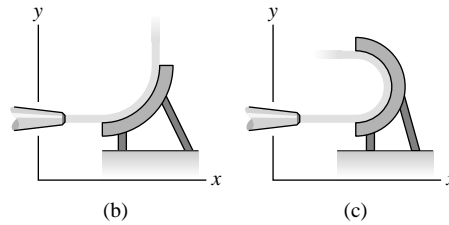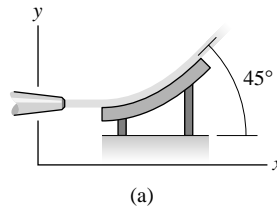
Solution: Apply the strategy used in Example 16.7. The exit velocity is $\mathbf{v}_{fe} = v_0(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta)$ where θ is the total angle of deflection of the stream, and v_0 is the magnitude of the stream velocity. The inlet velocity is $\mathbf{v}_{fi} = v_0\mathbf{i}$. The force on the plate exerted by the exit stream is in a direction opposite the stream flow, whereas the force exerted on the plate by the inlet stream is in the direction of stream flow. The sum of the forces *exerted on the plate* (see Eq. (16.25)) is

$$\sum \mathbf{F} = (dm_f/dt)(\mathbf{v}_{fi} - \mathbf{v}_{fe}),$$

from which

$$\sum \mathbf{F} = (dm_f/dt)v_0(\mathbf{i}(1 - \cos\theta) - \mathbf{j}\sin\theta).$$

The mass flow is $(dm_f/dt) = 30$ kg/s and the stream velocity is $v_0 = 40$ m/s. For Case(a), $\theta = 45°$,

$$\boxed{\sum \mathbf{F} = (30)(40)(0.2929\mathbf{i} - 0.7071\mathbf{j}) = 351.5\mathbf{i} - 848.5\mathbf{j} \text{ (N)}.}$$

The force is downward to the right. For Case (b) $\theta = 90°$,

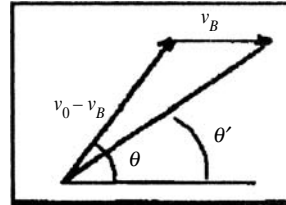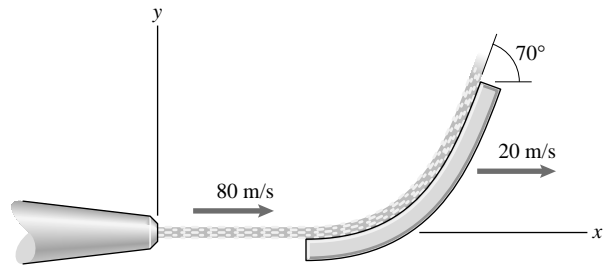$$\boxed{\sum \mathbf{F} = (30)(40)(\mathbf{i} - \mathbf{j}) = 1200\mathbf{i} - 1200\mathbf{j} \text{ (N)}}$$

For Case (c) $\theta = 180°$,

$$\boxed{\sum \mathbf{F} = (30)(40)(2\mathbf{i}) = 2400\mathbf{i} \text{ (N)}}$$

Problem 16.105* A stream of water with velocity 80**i** (m/s) and a mass flow of 6 kg/s strikes a turbine blade moving with constant velocity 20**i** (m/s).

(a) What force is exerted on the blade by the water?
(b) What is the magnitude of the velocity of the water as it leaves the blade?

Solution: Denote the fixed reference frame as the *nozzle frame*, and the moving blade frame as the *blade rest frame*. Assume that the discharge angle (70°) is referenced to the blade rest frame, so that the magnitude of the stream velocity and the effective angle of discharge in the blade rest frame is not modified by the velocity of the blade. Denote the velocity of the blade by v_B. The inlet velocity of the water relative to the blade is $\mathbf{v}_{fi} = (v_0 - v_B)\mathbf{i}$. The magnitude $(v_0 - v_B)$ is the magnitude of the stream velocity as it flows along the contour of the blade. At exit, the magnitude of the discharge velocity in the blade rest frame is the inlet velocity *in the blade rest frame* $|\mathbf{v}_{fe}| = (v_0 - v_B) = 60$ m/s, and the vector velocity is $\mathbf{v}_{fe} = v_{fe}(\mathbf{i}\cos 70° + \mathbf{j}\sin 70°)$ in the blade rest frame. From Eq. (16.25) the sum of the forces on the blade is

$$\sum \mathbf{F} = (dm_f/dt)((v_0 - v_B - v_{fe}\cos 70°)\mathbf{i} - \mathbf{j}(v_{fe}\sin 70°)).$$

(a) Substitute numerical values:

$(dm_f/dt) = 6$ kg/s,

$\qquad v_0 = 80$ m/s,

$\qquad v_B = 20$ m/s,

from which

$$\boxed{\sum \mathbf{F} = 236.9\mathbf{i} - 338.3\mathbf{j}}$$

(b) Assume that the magnitude of the velocity of the water as it leaves the blade is required *in the nozzle frame*. The velocity of the water leaving the blade in the nozzle frame (see vector diagram) is $\mathbf{v}_{\text{ref}} = 20\mathbf{i} + 60\cos 70°\mathbf{i} + 60\sin 70°\mathbf{j} = 40.52\mathbf{i} + 56.38\mathbf{j}$ (m/s). The magnitude of the velocity is

$$\boxed{|\mathbf{v}| = 69.4 \text{ m/s.}}$$

Problem 16.106 At the instant shown, the nozzle A of the lawn sprinkler is located at $(0.1, 0, 0)$ m. Water exits each nozzle at 8 m/s relative to the nozzle with a mass flow rate of 0.22 kg/s. At the instant shown, the flow relative to the nozzle at A is in the direction of the unit vector

$$\mathbf{e} = \tfrac{1}{\sqrt{3}}\mathbf{i} - \tfrac{1}{\sqrt{3}}\mathbf{j} + \tfrac{1}{\sqrt{3}}\mathbf{k}.$$

Determine the total moment about the z axis exerted on the sprinkler by the flows from all four nozzles.
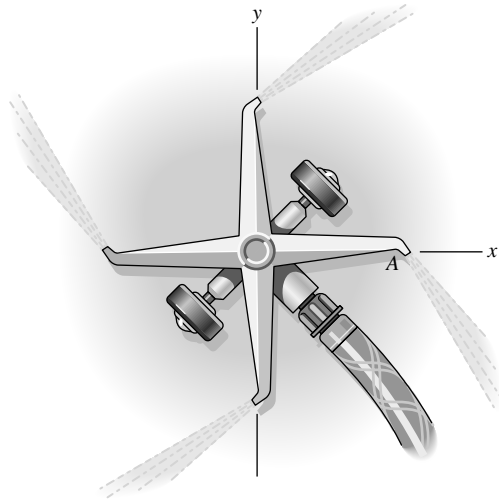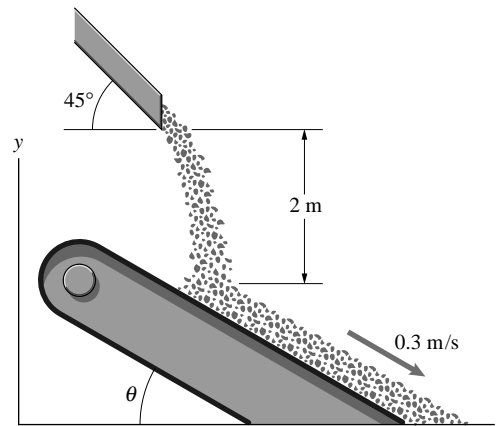
Solution:

$$\mathbf{e} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k}), \quad \mathbf{r} = 0.1\mathbf{i}, \quad \mathbf{v} = (8 \text{ m/s})\mathbf{e}, \quad \frac{dm_f}{dt} = 0.22 \text{ kg/s}$$

$$\mathbf{M} = 4\left(-\frac{dm_f}{dt}\right)(\mathbf{r} \times \mathbf{v}) = (0.406 \text{ N-m})(\mathbf{j} + \mathbf{k})$$

The moment about the z-axis is $\boxed{M_z = \mathbf{M} \bullet \mathbf{k} = 0.406 \text{ N-m}}$

Problem 16.107 A 45-kg/s flow of gravel exits the chute at 2 m/s and falls onto a conveyor moving at 0.3 m/s. Determine the components of the force exerted on the conveyor by the flow of gravel if $\theta = 0$.

Solution: The horizontal component of the velocity of the gravel flow is $v_x = 2\cos 45° = \sqrt{2}$ m/s. From Newton's second law, (using the chain rule) the vertical component of the velocity is

$$v_y = -\sqrt{(v\sin 45°)^2 + 2gh} = -\sqrt{2 + 2(9.81)(2)} = -6.422 \text{ m/s}.$$

The mass flow rate is $\left(\dfrac{dm_f}{dt}\right) = 45$ kg/s. The force exerted on the belt is

$$\boxed{\sum \mathbf{F} = \left(\frac{dm_f}{dt}\right)((v_x - 0.3)\mathbf{i} + v_y\mathbf{j}) = 50.1\mathbf{i} - 289\mathbf{j} \text{ (N)}}$$

Problem 16.108 Solve Problem 16.107 if $\theta = 30°$.

Solution: Use the solution to Problem 16.107 as appropriate. From Problem 16.107:

$$v_x = 2\cos 45° = \sqrt{2} \text{ m/s}, \quad v_y = -\sqrt{(v\sin 45°)^2 + 2gh}$$

$$= -\sqrt{2 + 2(9.81)(2)} = -6.422 \text{ m/s}.$$

The velocity of the conveyor belt is

$$\mathbf{v}_B = v_{Bx}\mathbf{i} + v_{By}\mathbf{j}$$

$$= 0.3(\mathbf{i}\cos 30° - \mathbf{j}\sin 30°) = 0.2598\mathbf{i} - 0.15\mathbf{j} \text{ (m/s)}.$$

The magnitude of the velocity of the gravel:

$$v_{\text{mag}} = \sqrt{v_x - v_{Bx})^2 + (v_y - v_{by})^2}$$

$$= 6.377 \text{ (m/s)}.$$

The angle of impact:

$$\beta = \tan^{-1}\left(\frac{v_y - v_{bx}}{v_x - v_{bx}}\right) = -79.6°.$$

The force on the belt is

$$\boxed{\sum \mathbf{F} = \left(\frac{dm_f}{dt}\right)(v_{\text{mag}})(\mathbf{j}\cos\beta + \mathbf{j}\sin\beta) = 51.9\mathbf{i} - 282.2\mathbf{j} \text{ (N)}}$$

Problem 16.109 Suppose that you are designing a toy car that will be propelled by water that squirts from an internal tank at 3.05 m/s relative to the car. The total weight of the car and its water "fuel" is to be 8.9 N. If you want the car to achieve a maximum speed of 3.66 m/s, what part of the total weight must be water?
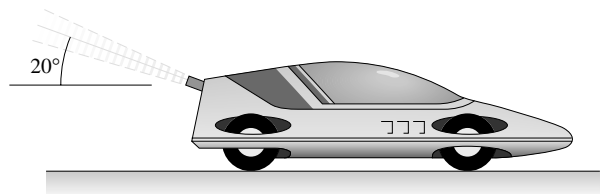
Solution: See Example 16.10

$$3.66 \text{ m/s} - 0 = (3.05 \text{ m/s})\cos 20° \ln\left(\frac{8.9\text{N}}{W}\right) \Rightarrow W = 2.48 \text{ N}$$

The water must be $\boxed{W_{\text{water}} = (8.9 \text{ N}) - (2.48 \text{ N}) = 6.42 \text{ N}}$

Problem 16.110 The rocket consists of a 1000-kg payload and a 9000-kg booster. Eighty percent of the booster's mass is fuel, and its exhaust velocity is 1200 m/s. If the rocket starts from rest and external forces are neglected, what velocity will it attain? (See Example 16.10.)
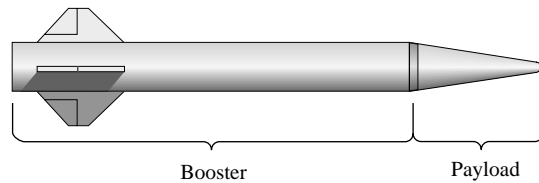
Booster   Payload

Solution:

$$\boxed{v = (1200 \text{ m/s}) \ln\left(\frac{10,000 \text{ kg}}{1,000 \text{ kg} + 0.2[9,000 \text{ kg}]}\right) = 1530 \text{ m/s}}$$

Problem 16.111* The rocket consists of a 1000-kg payload and a booster. The booster has two stages whose total mass is 9000 kg. Eighty percent of the mass of each stage is fuel, and the exhaust velocity of each stage is 1200 m/s. When the fuel of stage 1 is expended, it is discarded and the motor of stage 2 is ignited. Assume that the rocket starts from rest and neglect external forces. Determine the velocity attained by the rocket if the masses of the stages are $m_1 = 6000$ kg and $m_2 = 3000$ kg. Compare your result to the answer to Problem 16.110.
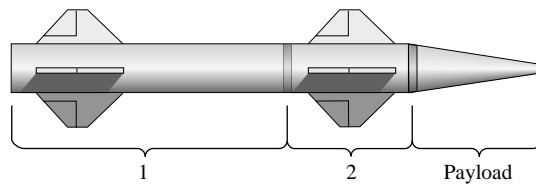
Solution:

First burn,
$\begin{cases} \text{Full mass} = 10{,}000 \text{ kg} \\ \text{Empty mass} = 4{,}000 \text{ kg} + 0.2(6{,}000 \text{ kg}) = 5{,}200 \text{ kg} \end{cases}$

Second burn
$\begin{cases} \text{Full mass} = 4{,}000 \text{ kg} \\ \text{Empty mass} = 1{,}000 \text{ kg} + 0.2(300 \text{ kg}) = 1{,}600 \text{ kg} \end{cases}$

$$v = (12{,}000 \text{ m/s}) \ln \left(\frac{10{,}000 \text{ kg}}{5200 \text{ kg}} \right) + (12{,}000 \text{ m/s}) \ln \left(\frac{4{,}000 \text{ kg}}{1600 \text{ kg}} \right)$$

$$= 1880 \text{ m/s}$$

Much faster using stages.

Problem 16.112 A rocket of initial mass m_0 takes off straight up. Its exhaust velocity v_f and the mass flow rate of its engine $m_f = dm_f/dt$ are constant. Show that, during the initial part of the flight, when aerodynamic drag is negligible, the rocket's upward velocity as a function of time is

$$v = v_f \ln \left(\frac{m_0}{m_0 - \dot{m}_f t} \right) - gt.$$

Solution: Start by adding a gravity term to the second equation in Example 16.10. (Newton's second law). We get

$$\sum F_x = \frac{dm_f}{dt} v_f - mg = m \frac{dv_x}{dt}.$$

Substitute

$$\frac{dm}{dt} = -\frac{dm_f}{dt}$$

as in the example and divide through by m to get

$$-v_f \left(\frac{1}{m} \right) dmdt - g = \frac{dv_x}{dt}.$$

Integrate with respect to time gives the relation

$$-v_f \int_0^t \left(\frac{1}{m} \right) \frac{dm}{dt} dt$$

$$-\int_0^t g dt = \int_0^t \frac{dv_x}{dt} dt.$$
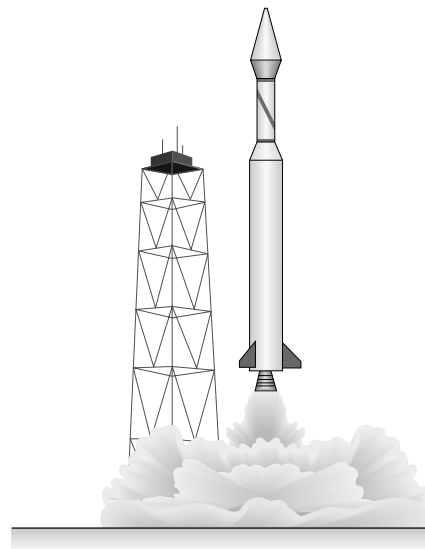
Simplifying the integrals and setting appropriate limits when we change the variable of integration, we get

$$-v_f \int_{m_0}^m \frac{dm}{m} - g \int_0^t dt = \int_0^v dv_x.$$

Integrating and evaluating at the limits of integration, we get

$$v = v_f \ln(m_0/m) - gt.$$

Recalling that $m = m_0 - m_f t$ and substituting this in, we get the desired result.

Problem 16.113 The mass of the rocket sled in Active Example 16.9 is 440 kg. Assuming that the only significant force acting on the sled in the direction of its motion is the force exerted by the flow of water entering it, what distance is required for the sled to decelerate from 300 m/s to 100 m/s?

Solution: From Example 16.9, the force in the x direction (the only force that affects the speed) is

$$F_x = -\rho A v^2$$

From Newton's Second law

$$ma_x = -\rho A v^2$$

$$a_x = v\frac{dv_x}{ds} = -\left(\frac{\rho A}{m}\right)v^2$$

$$\int_{300}^{100} \frac{dv}{v} = -\left(\frac{\rho A}{m}\right)\int_0^{s_f} ds$$
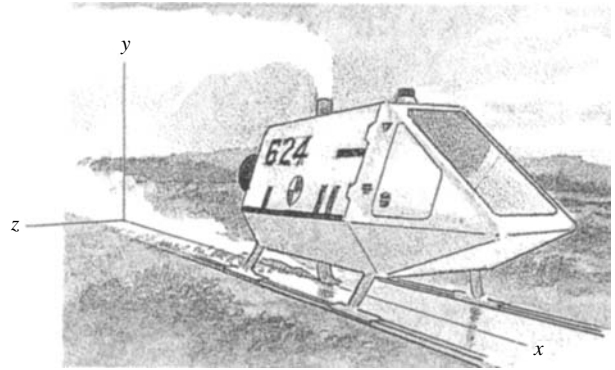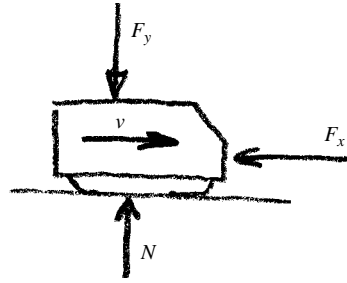
$$\ln(v)\Big|_{300}^{100} = -\frac{\rho A}{m}s_f$$

where

$$\rho = 1000 \text{ kg/m}^3,$$

$$A = 0.01 \text{ m}^2,$$

$$m = 440 \text{ kg}.$$

Solving

$$\underline{s_f = 48.3 \text{ m}}$$

Problem 16.114* Suppose that you grasp the end of a chain that weighs 43.8 N/m and lift it straight up off the floor at a constant speed of 0.61 m/s.

(a) Determine the upward force F you must exert as a function of the height s.
(b) How much work do you do in lifting the top of the chain to $s = 1.22$ m?

Strategy: Treat the part of the chain you have lifted as an object that is gaining mass.
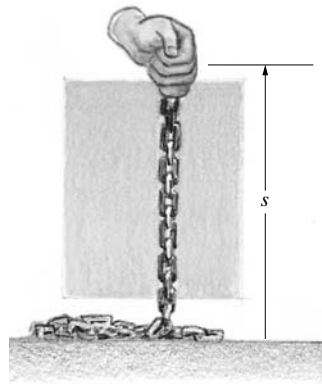
Solution: The force is the sum of the "mass flow" reaction and the weight of suspended part of the chain,

$$F = mg + v\left(\frac{dm}{dt}\right).$$

(a) Substitute:

$$\left(\frac{dm}{dt}\right) = \left(\frac{43.8}{9.81}\right) v \text{ kg/s.}$$

The velocity is 0.61 m/s.
The suspended portion of the chain weighs $43.8s$ N. The force required to lift the chain is

$$43.8s + \left(\frac{43.8}{9.81}v\right)v = 43.8s + \frac{43.8(0.61)^2}{9.81}$$

(b) The work done is

$$\boxed{\int_0^{1.22} F\,ds = \left[\left(\frac{16.3}{g}\right)s + \left(\frac{43.8}{2}\right)s^2\right]_0^{1.22} = 34.6 \text{ N-m}}$$

Problem 16.115* Solve Problem 16.114, assuming that you lift the end of the chain straight up off the floor with a constant acceleration of 0.61 m/s².

Solution: Assume that the velocity is zero at $s = 0$. The mass of the chain currently suspended is $m = \left(\frac{3}{g}\right)s$. Use the solution to Problem 16.114. From Newton's second law,

$$m\left(\frac{dv}{dt}\right) = F - v\left(\frac{dm}{dt}\right) - mg.$$

The velocity is expressed in terms of s as follows: The acceleration is constant: $dv/dt = 0.61$ m/s². Use the chain rule $v(dv/ds) = 0.61$. Integrate: $v^2 = 0.37 s$, where it is assumed that the velocity is zero at $s = 0$, from which

$$F = m\left(\frac{dv}{dt}\right) + 2\sqrt{s}\left(\frac{dm}{dt}\right) + 43.8s$$

Substitute:

$$m = \left(\frac{43.8}{g}\right)s, \frac{dm}{dt} = \left(\frac{43.8}{g}\right)v = \left(\frac{26.6}{g}\right)\sqrt{s},$$

from which

$$\boxed{F = \left(\frac{80}{g} + 43.8\right)s = 11.15 \text{ s}}$$

(b) The work done is

$$\boxed{\int_0^4 F\,ds = \left[\left(\frac{80}{g} + 43.8\right)\frac{s^2}{2}\right]_0^{1.22} = 38.66}$$

Problem 16.116* It has been suggested that a heavy chain could be used to gradually stop an airplane that rolls past the end of the runway. A hook attached to the end of the chain engages the plane's nose wheel, and the plane drags an increasing length of the chain as it rolls. Let m be the airplane's mass and v_0 its initial velocity, and let ρ_L be the mass per unit length of the chain. Neglecting friction and aerodynamic drag, what is the airplane's velocity as a function of s?
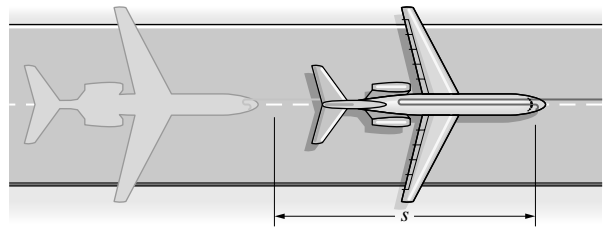
Solution: Assume that the chain is laid out lengthwise along the runway, such that the aircraft hook seizes the nearest end as the aircraft proceeds down the runway. As the distance s increases, the length of chain being dragged is $\dfrac{s}{2}$ (see figure). The mass of the chain being dragged is $\dfrac{\rho_L s}{2}$. The mass "flow" of the chain is

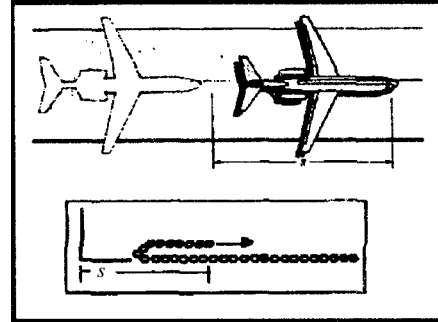$$\frac{d\left(\dfrac{\rho_L s}{2}\right)}{dt} = \frac{\rho_L v}{2}.$$

From Newton's second law,

$$\left(\frac{\rho_L s}{2} + m\right)\frac{dv}{dt} = -\frac{\rho_L v^2}{2}.$$

Use the chain rule and integrate:

$$\left(\frac{\rho_L s}{2} + m\right)\frac{dv}{ds} = -\frac{\rho_L v}{2}, \quad \frac{dv}{v} = -\frac{\rho_L}{2\left(\dfrac{\rho_L s}{2} + m\right)}\, ds,$$

$$\ln(v) = -\ln\left(m + \frac{\rho_L s}{2}\right) + C.$$

For $v = v_0$ at $s = 0$, $C = \ln(mv_0)$, and

$$\boxed{v = \frac{mv_0}{m + \dfrac{\rho_L s}{2}}.}$$

Problem 16.117* In Problem 16.116, the frictional force exerted on the chain by the ground would actually dominate other forces as the distance s increases. If the coefficient of kinetic friction between the chain and the ground is μ_k and you neglect all other forces except the frictional force, what is the airplane's velocity as a function of s?

Solution: Assume that the chain layout is configured as shown in Problem 16.116. From Problem 16.116, the weight of the chain being dragged at distance s is $\dfrac{\rho_L g s}{2}$. From Newton's second law

$$\left(m + \frac{\rho_L s}{2}\right)\frac{dv}{dt} = -\frac{\mu_k \rho_L g s}{2},$$

where only the friction force is considered. Use the chain rule and integrate:

$$\left(m + \frac{\rho_L s}{2}\right)v\frac{dv}{ds} = -\frac{\mu_k \rho_L g s}{2}, \quad v\, dv = -\mu_k g \frac{\rho_L s}{2\left(m + \dfrac{\rho_L s}{2}\right)}\, ds,$$

from which

$$\frac{v^2}{2} = \frac{-2\mu_k g}{\rho_L}\left(m + \frac{\rho_L s}{2} - m\ln\left(m + \frac{\rho_L s}{m}\right)\right) + C.$$

For $v = v_0$ at $s = 0$,

$$C = \frac{v_0^2}{2} + \frac{2\mu_k g m}{\rho_L}(1 - \ln(m)),$$

from which, after reduction

$$\boxed{v^2 = v_0^2 - 2g\mu_k\left(s - \frac{2m}{\rho_L}\ln\left(1 + \frac{\rho_L s}{2m}\right)\right).}$$
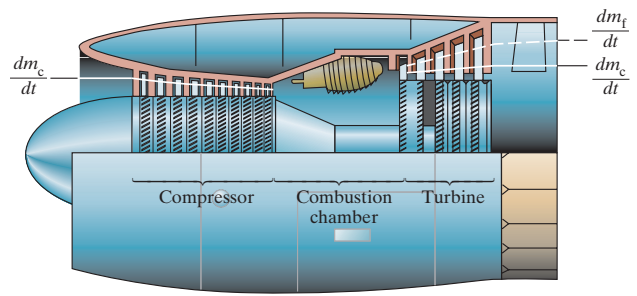
Problem 16.118 A turbojet engine is being operated on a test stand. The mass flow rate of air entering the compressor is 13.5 kg/s, and the mass flow rate of fuel is 0.13 kg/s. The effective velocity of air entering the compressor is zero, and the exhaust velocity is 500 m/s. What is the thrust of the engine? (See Example 16.12.)

Solution: The sum of the mass flows is

$$\left(\frac{dm}{dt}\right) = \left(\frac{dm_c}{dt}\right) + \left(\frac{dm_f}{dt}\right) = 13.5 + 0.13 = 13.63 \text{ kg/s.}$$

The inlet velocity is zero, and the exit velocity is 500 m/s. The thrust is

$$\boxed{T = 13.63(500) = 6820 \text{ N}}$$

Compressor   Combustion   Turbine
              chamber

Problem 16.119 A turbojet engine is in an airplane flying at 400 km/h. The mass flow rate of air entering the compressor is 13.5 kg/s and the mass flow rate of fuel is 0.13 kg/s. The effective velocity of the air entering the inlet is equal to the airplane's velocity, and the exhaust velocity (relative to the airplane) is 500 m/s. What is the thrust of the engine? (See Example 16.12.)

Solution: Use the "rest frame" of the engine to determine the thrust. Use the solution to Problem 16.118. The inlet velocity is

$$v_i = 400 \left(\frac{10^3}{3600}\right) = 111.11 \text{ m/s.}$$

The thrust is

$$T = \left(\frac{dm_c}{dt} + \frac{dm_f}{dt}\right) 500 - \left(\frac{dm_c}{dt}\right) 111.11,$$

$$\boxed{T = 13.63(500) - (111.11)13.5 = 5315 \text{ N}}$$

Problem 16.120 A turbojet engine's thrust reverser causes the exhaust to exit the engine at 20° from the engine centerline. The mass flow rate of air entering the compressor is 44 kg/s, and the air enters at 60 m/s. The mass flow rate of fuel is 1.5 kg/s, and the exhaust velocity is 370 m/s. What braking force does the engine exert on the airplane? (See Example 16.12.)

Solution:

$$\mathbf{T} = \left[\left(\frac{dm_c dt}{+} \frac{dm_f}{dt}\right) v_e - \frac{dm_c}{dt} v_i\right]_x (-\mathbf{i})$$

where

$$\frac{dm_c}{dt} = 44 \text{ kg/s}$$
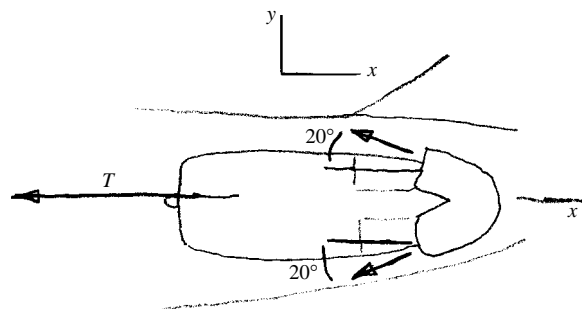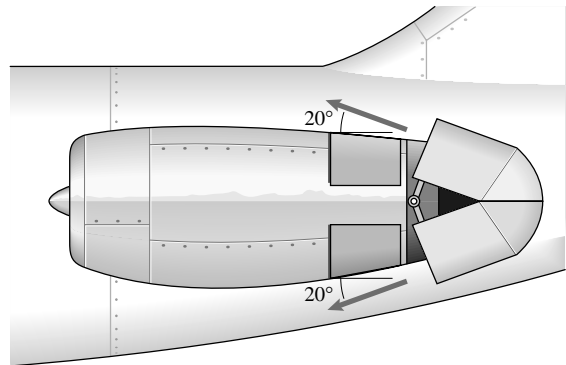
$$\frac{dm_f}{dt} = 1.5 \text{ kg/s}$$

$$v_{e_x} = -370 \cos(20°) \text{ m/s}$$

$$v_i = +60 \text{ m/s}$$

Solving,

$$T = -18500 \text{ N} = -18.5 \text{ KN (to the Right)}$$

$$|T| = 18.5 \text{ KN}$$

Problem 16.121 The total external force on a 10-kg object is constant and equal to $90\mathbf{i} - 60\mathbf{j} + 20\mathbf{k}$ (N). At $t = 2$ s, the object's velocity is $-8\mathbf{i} + 6\mathbf{j}$ (m/s).

(a) What impulse is applied to the object from $t = 2$ s to $t = 4$ s?
(b) What is the object's velocity at $t = 4$ s?

Solution:

(a) The impulse is

$$\int_{t_1}^{t_2} \mathbf{F}\, dt = 90(4-2)\mathbf{i} - 60(4-2)\mathbf{j} + 20(4-2)\mathbf{k}$$

$$\boxed{= 180\mathbf{i} - 120\mathbf{j} + 40\mathbf{k} \text{ (N-s).}}$$

(b) The velocity is

$$m\mathbf{v}_2 - m\mathbf{v}_1 = \int_{t_1}^{t_2} \mathbf{F}\, dt,$$

from which

$$\mathbf{v}_2 = \mathbf{v}_1 + \left(\frac{1}{m}\right)\int_{t_1}^{t_2} \mathbf{F}\, dt = \left(-8 + \frac{180}{10}\right)\mathbf{i} + \left(6 - \frac{120}{10}\right)\mathbf{j}$$

$$+ \left(\frac{40}{10}\right)\mathbf{k} \boxed{= 10\mathbf{i} - 6\mathbf{j} + 4\mathbf{k} \text{ (m/s)}}$$

Problem 16.122 The total external force on an object is $\mathbf{F} = 10t\mathbf{i} + 60\mathbf{j}$ (N). At $t = 0$, the object's velocity is $\mathbf{v} = 20\mathbf{j}$ (m/s). At $t = 12$ s, the x component of its velocity is 48 m/s.

(a) What impulse is applied to the object from $t = 0$ to $t = 6$ s?
(b) What is the object's velocity at $t = 6$ s?

Solution:

(a) The impulse is

$$\int_{t_1}^{t_2} \mathbf{F}\, dt = [5t^2]_0^6\mathbf{i} + [60t]_0^6\mathbf{j} = 180\mathbf{i} + 360\mathbf{j} \text{ (N-s)}$$

(b) The mass of the object is found from the x component of the velocity at 12 s.

$$m = \frac{\int_{t_1}^{t_2} F\, dt}{48 - 0} = \frac{5(12^2)}{48} = 15 \text{ kg.}$$
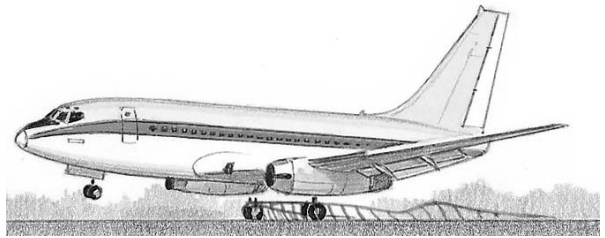
The velocity at 6 s is

$$m\mathbf{v}_2 - m\mathbf{v}_1 = \int_{t_1}^{t_2} \mathbf{F}\, dt,$$

from which

$$\boxed{\mathbf{v}_2 = 20\mathbf{j} + \frac{180}{15}\mathbf{i} + \frac{360}{15}\mathbf{j} = 12\mathbf{i} + 44\mathbf{j} \text{ (m/s).}}$$

Problem 16.123   An aircraft arresting system is used to stop airplanes whose braking systems fail. The system stops a 47.5-Mg airplane moving at 80 m/s in 9.15 s.

(a)   What impulse is applied to the airplane during the 9.15 s?
(b)   What is the average deceleration to which the passengers are subjected?

Solution:

(a)   The impulse is

$$\int_{t_1}^{t_2} F \, dt = mv_2 - mv_1 = 47500(80) = 3.8 \times 10^6 \text{ N-s.}$$

(b)   The average force is

$$F_{\text{ave}} = \frac{\int_{t_1}^{t_2} F \, dt}{t_2 - t_1} = \frac{3.8 \times 10^6}{9.15} = 4.153 \times 10^5 \text{ N.}$$

From Newton's second law

$$\left(\frac{dv}{dt}\right)_{\text{ave}} = \frac{F_{\text{ave}}}{47500} = 8.743 \text{ m/s}^2$$

**Problem 16.124**   The 1895 Austrain 150-mm howitzer had a 1.94-m-long barrel, possessed a muzzle velocity of 300 m/s, and fired a 38-kg shell. If the shell took 0.013 s to travel the length of the barrel, what average force was exerted on the shell?

Solution: The average force is

$$F_{\text{ave}} = \frac{\int_{t_1}^{t_2} F \, dt}{t_2 - t_1} = \frac{(38)(300)}{0.013} = 877,000 \text{ N}$$

Problem 16.125 An athlete throws a shot put weighing 71.2 N. When he releases it, the shot is 2.13 m above the ground and its components of velocity are $v_x = 9.45$ m/s and $v_y = 7.92$ m/s.

(a) Suppose the athlete accelerates the shot from rest in 0.8 s, and assume as a first approximation that the force **F** he exerts on the shot is constant. Use the principle of impulse and momentum to determine the x and y components of **F**.

(b) What is the horizontal distance from the point where he releases the shot to the point where it strikes the ground?
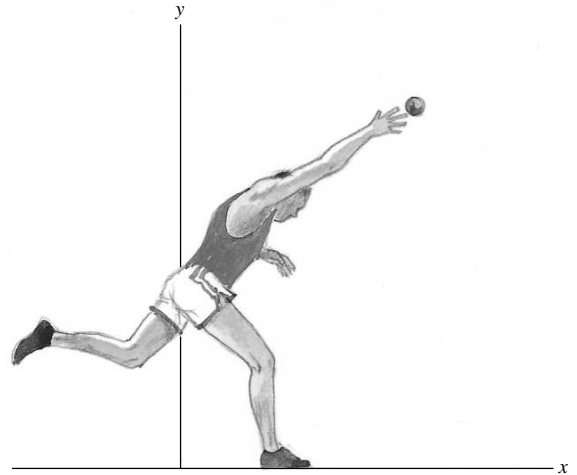
Solution:

(a) Let F_x, F_y be the components of the force exerted by the athlete. The impulse is

$$\int_{t_1}^{t_2} \mathbf{F}\, dt = \mathbf{i}(F_x)(t_2 - t_1) + \mathbf{j}(F_y - W)(t_2 - t_1)$$

$$= \mathbf{i}\left(\frac{W}{g}\right)(9.45) + \mathbf{j}\left(\frac{W}{g}\right)(7.92)$$

from which

$$\boxed{F_x = \left(\frac{W}{g}\right)\left(\frac{9.45}{0.8}\right) = 85.8 \text{ N.}}$$

$$\boxed{F_y = \left(\frac{W}{g}\right)\left(\frac{7.92}{0.8}\right) + W = 143.2 \text{ N.}}$$

(b) From the conservation of energy for the vertical component of the motion, the maximum height reached is

$$h = \left(\frac{1}{2g}\right)v_y^2 + 2.13 = 5.33 \text{ m.}$$

From the solution of Newton's second law for free fall, the time of flight is

$$t_f = \sqrt{\frac{2(h - 2.13)}{g}} + \sqrt{\frac{2h}{g}} = 1.85 \text{ s}$$

and the horizontal distance is $\boxed{D = 9.45 t_f = 17.5 \text{ m}}$

Problem 16.126 The 26688 N pickup truck A moving at 12.2 m/s collides with the 17792 N car B moving at 9.1 m/s.

(a) What is the magnitude of the velocity of their common center of mass after the impact?
(b) Treat the collision as a perfectly plastic impact. How much kinetic energy is lost?
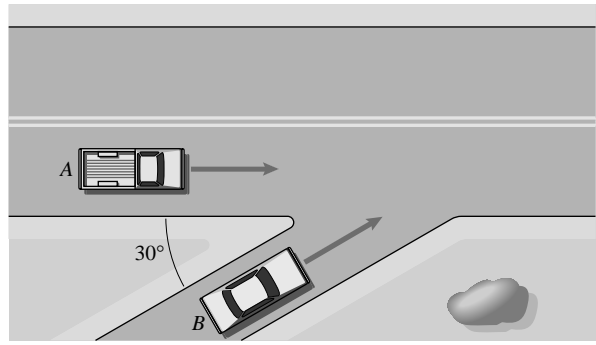
Solution:

(a) From the conservation of linear momentum,

$$\left(\frac{W_A}{g}\right) v_A + \left(\frac{W_B}{g}\right) v_B \cos\theta = \left(\frac{W_A + W_B}{g}\right) v_x,$$

$$\left(\frac{W_B}{g}\right) v_B \sin\theta = \left(\frac{W_A + W_B}{g}\right) v_y.$$

Substitute numerical values, with $\theta = 30°$ to obtain $v_x = 10.48$ m/s, $v_y = 1.83$ m/s, from which $v = \sqrt{v_x^2 + v_y^2} = 10.64$ m/s

(b) The kinetic energy before the collision minus the kinetic energy after the collision is the loss in kinetic energy:

$$\boxed{\begin{array}{l}\dfrac{1}{2}\left(\dfrac{W_A}{g}\right) v_A^2 + \dfrac{1}{2}\left(\dfrac{W_B}{g}\right) v_B^2 - \dfrac{1}{2}\left(\dfrac{W_A + W_B}{g}\right) v^2 \\ = 21{,}2853 \text{ N-m}\end{array}}$$

Problem 16.127 Two hockey players ($m_A = 80$ kg, $m_B = 90$ kg) converging on the puck at $x = 0$, $y = 0$ become entangled and fall. Before the collision, $\mathbf{v}_A = 9\mathbf{i} + 4\mathbf{j}$ (m/s) and $\mathbf{v}_B = -3\mathbf{i} + 6\mathbf{j}$ (m/s). If the coefficient of kinetic friction between the players and the ice is $\mu_k = 0.1$, what is their approximate position when they stop sliding?
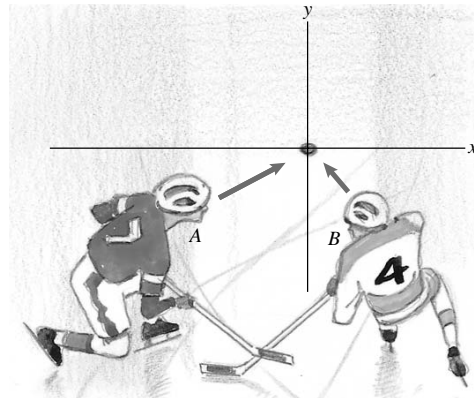
Solution: The strategy is to determine the velocity of their combined center of mass immediately after the collision using the conservation of linear momentum, and determine the distance using the conservation of energy. The conservation of linear momentum is $9m_A - 3m_B = (m_A + m_B)v_x$, and $4m_A + 6m_B = (m_A + m_B)v_y$, from which $v_x = 2.65$ m/s, and $v_y = 5.06$ m/s, from which $v = \sqrt{v_x^2 + v_y^2} = 5.71$ m/s, and the angle of the path is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = 62.4°.$$

The work done by friction is

$$\int_0^s -\mu_k g(m_A + m_B)\, ds = -\mu_k g(m_A + m_B)s,$$

where s is the distance the players slide after collision. From the conservation of work and energy $\frac{1}{2}(m_A + m_B)v^2 - \mu_k g(m_A + m_B)s = 0$, from which

$$s = \frac{v^2}{2\mu_k g} = 16.6 \text{ m}.$$

The position of the players after they stop sliding is

$$\boxed{\mathbf{r} = s(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta) = 7.7\mathbf{i} + 14.7\mathbf{j} \text{ (m)}}$$

Problem 16.128 The cannon weighed 1780 N, fired a cannonball weighing 44.5 N, and had a muzzle velocity of 61 m/s. For the $10°$ elevation angle shown, determine (a) the velocity of the cannon after it was fired and (b) the distance the cannonball traveled. (Neglect drag.)
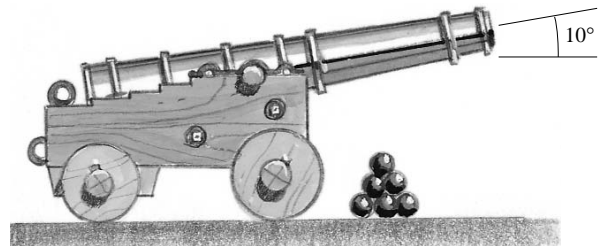
Solution: Assume that the height of the cannon mouth above the ground is negligible. (a) Relative to a reference frame moving with the cannon, the cannonball's velocity is 61 m/s at. The conservation of linear momentum condition is $-m_C v_C + m_B (v_B \cos 10° - v_C) = 0$, from which
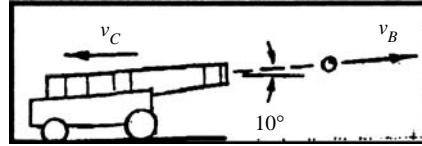
$$v_C = \frac{m_B v_B \cos 10°}{m_B + m_C} = 1.464 \text{ m/s}.$$

The velocity of the cannon ball relative to the ground is

$$v_B = (61 \cos 10° - v_C)\mathbf{i} + 61 \sin 10°\mathbf{j} = 58.57\mathbf{i} + 10.59\mathbf{j} \text{ m/s},$$

from which $v_{0x} = 58.57$ m/s, $v_{0y} = 10.59$ m/s. The maximum height is (from the conservation of energy for free fall) $h_{max} = v_{0y}^2/(2g) = 5.72$ m/s, where the height of the cannon mouth above the ground is negligible. The time of flight (from the solution of Newton's second law for free fall) is twice the time required to fall from the maximum height,

$$t_{flight} = 2\sqrt{\frac{2h_{max}}{g}} = 2.16 \text{ s}.$$

From which the range is $\boxed{x_{impact} = v_{0x} t_{flight} = 126.5 \text{ m}}$ since v_{0x} is constant during the flight.

Problem 16.129 A 1-kg ball moving horizontally at 12 m/s strikes a 10-kg block. The coefficient of restitution of the impact is $e = 0.6$, and the coefficient of kinetic friction between the block and the inclined surface is $\mu_k = 0.4$. What distance does the block slide before stopping?
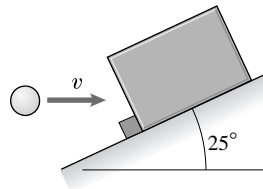
Solution: First we analyze the impact between the ball (b) and the block (B). The component of the ball's velocity parallel to the inclined surface is $v_b = (12) \cos 25° = 10.9$ m/s. Solving the equations

$$m_b v_b = m_b v_b' + m_B v_B',$$

$$e = \frac{v_B' - v_b'}{v_b},$$

we obtain $v_B' = 1.58$ m/s. We use work and energy to determine the distance d the block slides:

$$(-m_B g \sin 25° - \mu_k m_B g \cos 25°)d = 0 - \frac{1}{2}m_B(v_B')^2.$$

Solving yields $\underline{d = 0.162 \text{ m}}$.

Problem 16.130 A Peace Corps volunteer designs the simple device shown for drilling water wells in remote areas. A 70-kg "hammer," such as a section of log or a steel drum partially filled with concrete, is hoisted to $h = 1$ m and allowed to drop onto a protective cap on the section of pipe being pushed into the ground. The combined mass of the cap and section of pipe is 20 kg. Assume that the coefficient of restitution is nearly zero.

(a) What is the velocity of the cap and pipe immediately after the impact?

(b) If the pipe moves 30 mm downward when the hammer is dropped, what resistive force was exerted on the pipe by the ground? (Assume that the resistive force is constant during the motion of the pipe.)

Hammer

h

Solution: The conservation of momentum principle for the hammer, pipe and cap is $m_H v_H + m_p v_p = (m_H + m_p)v$, where v_H, v_p are the velocity of the hammer and pipe, respectively, before impact, and v is the velocity of their combined center of mass immediately after impact. The velocity of the hammer before impact (from the conservation of energy for a free fall from height h) is $v_H = \sqrt{2gh} = \sqrt{2(9.81)(1)} = 4.43$ m/s. Since $v_P = 0$,

$$v = \frac{m_H}{m_H + m_P} v_H = \frac{70}{90}(4.43) = 3.45 \text{ m/s}.$$

(b) The work done by the resistive force exerted on the pipe by the cap is $\int_0^s -F ds = -Fs$ N-m, where $s = 0.03$ m. The work and energy principle for the hammer, pipe and cap is $-Fs + (m_H + m_P)gs = \frac{1}{2}(m_H + m_P)v^2$, from which

$$F = \frac{(m_H + m_P)(v^2 + 2gs)}{2s} = \frac{(90)(3.45^2 + 2(9.81)(0.03))}{2(0.03)}$$
$$= 18,700 \text{ N}$$

Problem 16.131 A tugboat (mass = 40 Mg) and a barge (mass = 160 Mg) are stationary with a slack hawser connecting them. The tugboat accelerates to 2 knots (1 knot = 1852 m/h) before the hawser becomes taut. Determine the velocities of the tugboat and the barge just after the hawser becomes taut (a) if the "impact" is perfectly plastic ($e = 0$) and (b) if the "impact" is perfectly elastic ($e = 1$). Neglect the forces exerted by the water and the tugboat's engines.

Solution: The tugboat's initial velocity is $v_T = 2\left(1852 \frac{\text{m}}{\text{h}}\right)$ $\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 1.03$ m/s. The equations governing the "impact" are

$$m_T v_T = m_T v'_T + m_B v'_B,$$

$$e = \frac{v'_B - v'_T}{v_T}.$$

(a) With $e = 0$, we obtain

$$v'_T = v'_B = 0.206 \text{ m/s (0.4 knots).}$$

(b) With $e = 1$, we obtain

$$v'_T = -0.617 \text{ m/s } (-1.2 \text{ knots}),$$

$$v'_B = 0.412 \text{ m/s (0.8 knots).}$$

Problem 16.132 In Problem 16.131, determine the magnitude of the impulsive force exerted on the tugboat in the two cases if the duration of the "impact" is 4 s. Neglect the forces exerted by the water and the tugboat's engines during this period.

Solution: Since the tugboat is stationary at $t = t_1$, the linear impulse is

$$\int_{t_1}^{t_2} F \, dt = F_{ave}(t_2 - t_1) = m_B v'_B,$$

from which $F_{ave} = m_B v'_B / 4 = (4 \times 10^4) v'_B$. For Case (a) $v'_B = 0.206$ m/s, from which $\boxed{F_{ave} = 8230 \text{ N}}$. Case (b). $v'_B = 0.412$ m/s, $\boxed{F_{ave} = 16,500 \text{ N}}$

Problem 16.133 The 10-kg mass A is moving at 5 m/s when it is 1 m from the stationary 10-kg mass B. The coefficient of kinetic friction between the floor and the two masses is $\mu_k = 0.6$, and the coefficient of restitution of the impact is $e = 0.5$. Determine how far B moves from its initial position as a result of the impact.
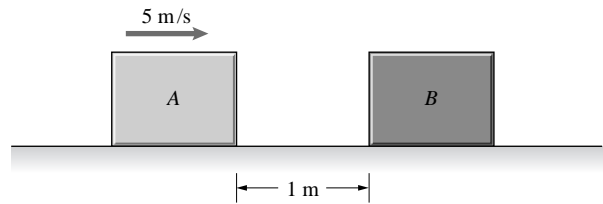
Solution: Use work and energy to determine A's velocity just before impact:

$$-\mu_k m g(1) = \tfrac{1}{2} m v_A^2 - \tfrac{1}{2} m (5)^2.$$

Solving, $v_A = 3.64$ m/s. Now analyze the impact:

$$m v_A = m v'_A + m v'_B,$$

$$e = \frac{v'_B - v'_A}{v_A}.$$

Solving these two equations, we obtain $v'_B = 2.73$ m/s. We use work and energy to determine how far B slides:

$$-\mu_k m g d = 0 - \tfrac{1}{2} m (v'_B)^2.$$

Solving, $d = 0.632$ m.

Problem 16.134 The kinetic coefficients of friction between the 5-kg crates A and B and the inclined surface are 0.1 and 0.4, respectively. The coefficient of restitution between the crates is $e = 0.8$. If the crates are released from rest in the positions shown, what are the magnitudes of their velocities immediately after they collide?
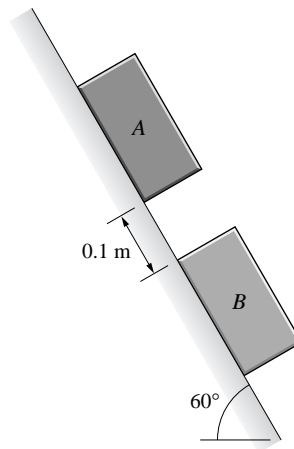
Solution: The free body diagrams of A and B are shown. From the diagram of A, we have $N_A = m_A g \cos 60° = (5)(9.81) \cos 60° = 24.5$ N

$$\sum F_x = m_A g \sin 60° - 0.1 \ N_A = m_A a_A.$$

Solving, $(5)(9.81) \sin 60° - 0.1(24.5) = (5)a_A$, $a_A = 8.01$ m/s^2. The velocity of A is $v_A = a_A t$ and its position is $x_A = \frac{1}{2}a_A t^2$. From the free body diagram of B, we have $N_B = N_A = 24.5$ N and $\sum F_x = m_A g \sin 60° - 0.4 N_B = m_B a_B$. Solving we have $(5)(9.81) \sin 60° - 0.4(24.5) = (5)a_B$, or $a_B = 6.53$ m/s^2. The velocity of B is $v_B = a_B t$ and its position is $x_B = a_B t^2/2$. To find the time of impact, set

$x_A = x_B + 0.1$: $\quad \frac{1}{2}a_A t^2 = \frac{1}{2}a_B t^2 + 0.1$

Solving for t

$$t = \sqrt{\frac{2(0.1)}{a_A - a_B}} = \sqrt{\frac{2(0.1)}{8.01 - 6.53}} = 0.369 \text{ s}.$$

The velocities at impact are

$v_A = a_A t = (8.01)(0.369) = 2.95$ m/s $\quad v_B = a_B t = (6.53)(0.369)$

$\quad = 2.41$ m/s

Conservation of linear momentum yields

$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$:

$\quad (5)(2.95) + (5)(2.41) = (5)v'_A + (5)v'_B$ **(1)**
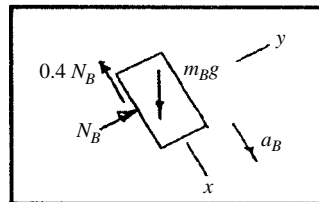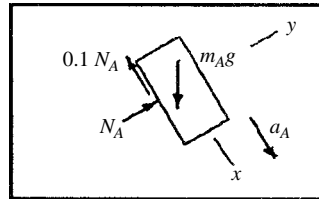
The coefficient of restitution is

$e = \dfrac{v'_B - v'_A}{v_A - v_B}$:

Evaluating, we have

$0.8 = \dfrac{v'_B - v'_A}{2.95 - 2.41}$ **(2)**.

Solving equations (1) and (2), $v'_A = 2.46$ m/s, $v'_B = 2.90$ m/s.

Problem 16.135 Solve Problem 16.134 if crate A has a velocity of 0.2 m/s down the inclined surface and crate B is at rest when the crates are in the positions shown.

Solution: From the solution of Problem 16.134, A's acceleration is $a_A = 8.01$ m/s^2 so A's velocity is

$$\int_{0.2}^{v} dv = \int_{0}^{t} a_A \, dt \quad v_A = 0.2 + a_A t$$

and its position is $x_A = 0.2t + \frac{1}{2}a_A t^2$. From the solution of Problem 16.136, the acceleration, velocity, and position of B are $a_B = 6.53$ m/s^2 $v_B = a_B t$ $x_B = \frac{1}{2}a_B t^2$, At impact, $x_A = x_B + 0.1$; $0.2t + \frac{1}{2}a_A t^2 = \frac{1}{2}a_B t^2 + 0.1$. Solving for t, we obtain $t = 0.257$ s so, $v_A = 0.2 + (8.01)(0.257) = 2.26$ m/s; $v_B = (6.53)(0.257) = 1.68$ m/s
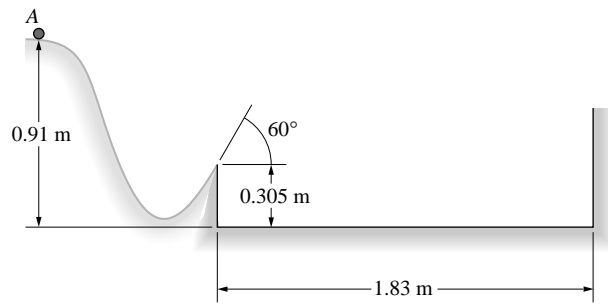
$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' :$$

$$(5)(2.26) + (5)(1.68) = (5)v_A' + (5)v_B' \quad \textbf{(1)};$$

$$e = \frac{v_B' - v_A'}{v_A - v_B} : 0.8 = \frac{v_B' - v_A'}{2.26 - 1.68} \qquad \textbf{(2)}.$$

Solving Equations (1) and (2), $v_A' = 1.74$ m/s $v_B' = 2.20$ m/s.

Problem 16.136 A small object starts from rest at A and slides down the smooth ramp. The coefficient of restitution of the impact of the object with the floor is $e = 0.8$. At what height above the floor does the object hit the wall?

Solution: The impact with the floor is an oblique central impact in which the horizontal component of the velocity is unchanged. The strategy is

(a) determine the time between leaving the ramp and impact with the wall,

(b) determine the time between the first bounce and impact with the wall

(c) from the velocity and height after the first bounce, determine the height at the time of impact with the wall. The steps in this process are:

(1) *The velocities on leaving the ramp.* The velocity at the bottom edge of the ramp is (from the conservation of energy) $v = \sqrt{2gh} = \sqrt{2(9.81)(0.91 - 0.305)} = 3.46$ m/s. The horizontal component of the velocity is $v_x = v \cos 60° = 1.73$ m/s. The vertical component of the velocity at the bottom edge of the ramp is $v_y = v \sin 60° = 3$ m/s.

(2) *The maximum height after leaving the ramp*: From the conservation of energy, the maximum height reached is

$$h_{\max} = \frac{v_y^2}{2g} + 0.305 = 0.762 \text{ m}.$$

(3) *The velocity and maximum height after the first bounce*: The velocity of the first impact (from the conservation of energy for a free fall) is $v_{y\text{impact}} = -\sqrt{2gh_{\max}} = -3.86$ m/s. The vertical velocity after the first bounce (see equation following Eq. (16.17)) is $v_y' = -ev_{y\text{impact}} = 0.8(3.86) = 3.09$ m/s. The maximum height after the first bounce is

$$h_y' = \frac{(v_y')^2}{2g} = 0.49 \text{ m}.$$

(4) *The time of impact with the wall, the time at the first bounce, and the time between the first bounce and impact with the wall.* The time required to reach the wall is

$$t_w = \frac{1.83}{v_x} = 1.058 \text{ s}.$$

The time at the first bounce (from a solution to Newton's second law for a free fall) is

$$t_b = \sqrt{\frac{2(h_{\max} - 0.305)}{g}} + \sqrt{\frac{(2)h_{\max}}{g}} = 0.7 \text{ s}.$$

The time between the first bounce and wall impact is $t_{w-b} = t_w - t_b = 0.3582$ s.

(5) *Is the ball on an upward or downward part of its path when it strikes the wall?* The time required to reach maximum height after the first bounce is

$$t_{bh}' = \sqrt{\frac{2h_y'}{g}} = 0.3154 \text{ s}.$$

Since $t_{w-b} > t_{bh}'$, the ball is on a downward part of its trajectory when it impacts the wall.

(6) *The height at impact with the wall*: The height of impact is (from a solution to Newton's second law for free fall)

$$\boxed{h_w = h_y' - \frac{g}{2}(t_{w-b} - t_{bh}')^2 = 0.48 \text{ m/s}}$$

Problem 16.137 The cue gives the cue ball A a velocity of magnitude 3 m/s. The angle $\beta = 0$ and the coefficient of restitution of the impact of the cue ball and the eight ball B is $e = 1$. If the magnitude of the eight ball's velocity after the impact is 0.9 m/s, what was the coefficient of restitution of the cue ball's impact with the cushion? (The balls are of equal mass.)
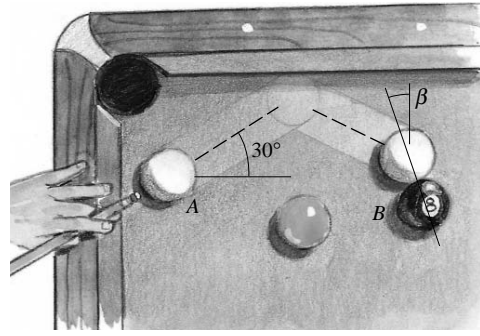
Solution: The strategy is to treat the two impacts as oblique central impacts. Denote the paths of the cue ball as $P1$ before the bank impact, $P2$ after the bank impact, and $P3$ after the impact with the 8-ball. The velocity of the cue ball is

$$\mathbf{v}_{AP1} = 3(\mathbf{i}\cos 30° + \mathbf{j}\sin 30°) = 2.6\mathbf{i} + 1.5\mathbf{j} \text{ (m/s)}.$$

The x component is unchanged by the bank impact. The y component after impact is $v_{AP2y} = -ev_{AP1y} = -1.5e$, from which the velocity of the cue ball after the bank impact is $\mathbf{v}_{AP2} = 2.6\mathbf{i} - 1.5e\mathbf{j}$. At impact with the 8-ball, the x component is unchanged. The y component after impact is obtained from the conservation of linear momentum and the coefficient of restitution. The two equations are

$$m_A v_{AP2y} = m_A v_{AP3y} + m_B v_{BP3y}$$

and

$$1 = \frac{v_{BP3y} - v_{AP3y}}{v_{AP2y}}.$$

For $m_A = m_B$, these equations have the solution $v_{AP3y} = 0$ and $v_{BP3y} = v_{AP2y}$, from which the velocities of the cue ball and the 8-ball after the second impact are $\mathbf{v}_{AP3} = 2.6\mathbf{i}$ (m/s), and $\mathbf{v}_{BP3} = -1.5e\mathbf{j}$ (m/s). The magnitude of the 8-ball velocity is $v_{BP3} = 0.9$ m/s, from which

$$\boxed{e = \frac{0.9}{1.5} = 0.6}$$

Problem 16.138 What is the solution to Problem 16.137 if the angle $\beta = 10°$?

Solution: Use the results of the solution to Problem 16.137. The strategy is to treat the second collision as an oblique central impact about the line P when $\beta = 10°$. The unit vector parallel to the line P is

$$\mathbf{e}_P = (\mathbf{i}\sin 10° - \mathbf{j}\cos 10°) = 0.1736\mathbf{i} - 0.9848\mathbf{j}.$$

The vector normal to the line P is $\mathbf{e}_{Pn} = 0.9848\mathbf{i} + 0.1736\mathbf{j}$. The projection of the velocity $\mathbf{v}_{AP2} = v_{AP3}\mathbf{e}_P + v_{AP3n}\mathbf{e}_{Pn}$. From the solution to Problem 16.13, $\mathbf{v}_{AP2} = 2.6\mathbf{i} - 1.5e\mathbf{j}$, from which the two simultaneous equations for the new components: $2.6 = 0.173\,v_{AP3} + 0.9848v_{AP3n}$, and $-1.5e = -0.9848v_{AP3} + 0.1736v_{AP3n}$. Solve: $v_{AP3} = 0.4512 + 1.477e$, $v_{AP3n} = 2.561 - 0.2605e$. The component of the velocity normal to the line P is unchanged by impact. The change in the component parallel to P is found from the conservation of linear momentum and the coefficient of restitution:

$$m_A v_{AP3} = m_A v'_{AP3} + m_B v'_{BP3},$$

and

$$1 = \frac{v'_{BP3} - v'_{AP3}}{v_{AP3}}.$$

For $m_A = m_B$, these equations have the solution $v'_{AP3} = 0$ and $v'_{BP3} = v_{AP3}$. From the value of $v'_{BP3} = 0.9$ m/s, $0.9 = 0.4512 + 1.477e$, from which

$$\boxed{e = \frac{0.9 - 0.4512}{1.477} = 0.304}$$

Problem 16.139 What is the solution to Problem 16.137 if the angle $\beta = 15°$ and the coefficient of restitution of the impact between the two balls is $e = 0.9$?

Solution: Use the solution to Problem 16.137. The strategy is to treat the second collision as an oblique central impact about P when $\beta = 15°$. The unit vector parallel to the line P is

$$\mathbf{e}_P = (\mathbf{i}\sin 15° - \mathbf{j}\cos 15°) = 0.2588\mathbf{i} - 0.9659\mathbf{j}.$$

The vector normal to the line P is

$$\mathbf{e}_{Pn} = 0.9659\mathbf{i} + 0.2588\mathbf{j}.$$

The projection of the velocity $\mathbf{v}_{AP2} = v_{AP3}\mathbf{e}_P + v_{AP3n}\mathbf{e}_{Pn}$. From the solution to Problem 16.139, $\mathbf{v}_{AP2} = 2.6\mathbf{i} - 1.5e\mathbf{j}$, from which the two simultaneous equations for the new components: $2.6 = 0.2588v_{AP3} + 0.9659v_{AP3n}$, and $-1.5e = -0.9659v_{AP3} + 0.2588v_{AP3n}$.

Solve: $v_{AP3} = 0.6724 + 1.449e$, $v_{AP3n} = 2.51 - 0.3882e$. The component of the velocity normal to the line P is unchanged by impact. The velocity of the 8-ball after impact is found from the conservation of linear momentum and the coefficient of restitution:

$$m_A v_{AP3} = m_A v'_{AP3} + m_B v'_{BP3},$$

and $\quad e_B = \dfrac{v'_{BP3} - v'_{AP3}}{v_{AP3}},$

where $e_B = 0.9$. For $m_A = m_B$, these equations have the solution

$$v'_{BP3} = \left(\frac{1}{2}\right)(1 + e_B)v_{AP3} = 0.95v_{AP3}.$$

From the value of $v'_{BP3} = 0.9$ m/s, $0.9 = 0.95(0.6724 + 1.449e)$, from which $\boxed{e = 0.189}$

Problem 16.140 A ball is given a horizontal velocity of 3 m/s at 2 m above the smooth floor. Determine the distance D between the ball's first and second bounces if the coefficient of restitution is $e = 0.6$.
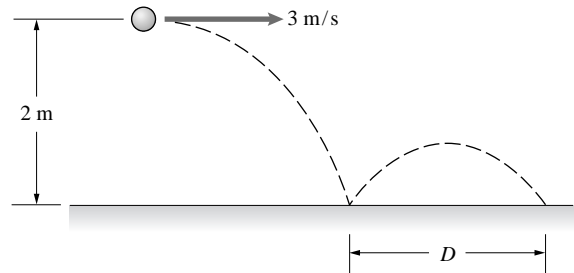
Solution: The strategy is to treat the impact as an oblique central impact with a rigid surface. The horizontal component of the velocity is unchanged by the impact. The vertical velocity after impact is $v'_{Ay} = -ev_{Ay}$. The vertical velocity before impact is $v_{Ay} = -\sqrt{2\,gh} = -\sqrt{2(9.81)2} = -6.264$ m/s, from which $v'_{Ay} = -0.6(-6.264) = 3.759$ m/s. From the conservation of energy for a free fall, the height of the second bounce is

$$h' = \frac{(v'_{Ay})^2}{2g} = 0.720 \text{ m}.$$

From the solution of Newton's second law for free fall, the time between the impacts is twice the time required to fall from height h, $t = 2\sqrt{\dfrac{2h'}{g}} = 0.7663$ s, and the distance D is

$$\boxed{D = v_0 t = (3)(0.7663) = 2.30 \text{ m}}$$

Problem 16.141* A basketball dropped from a height of 1.22 m rebounds to a height of 0.91 m. In the layup shot shown, the magnitude of the ball's velocity is 1.52 m/s, and the angles between its velocity vector and the positive coordinate axes are $\theta_x = 42°$, $\theta_y = 68°$, and $\theta_z = 124°$ just before it hits the backboard. What are the magnitude of its velocity and the angles between its velocity vector and the positive coordinate axes just after the ball hits the backboard?
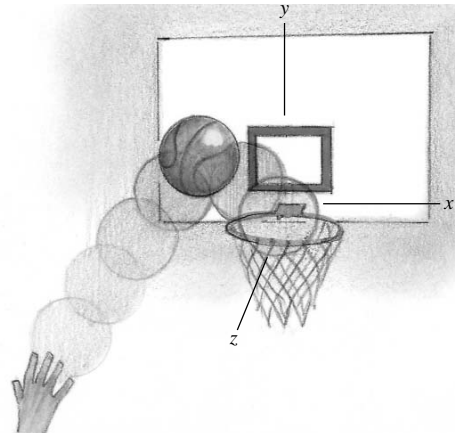
Solution: Using work and energy to determine the dropped ball's velocity just before and just after it hits the floor,

$$mg(1.22 \text{ m}) = \frac{1}{2}mv^2,$$

$$mg(0.91 \text{ m}) = \frac{1}{2}m(v')^2,$$

we obtain $v = 4.9$ m/s and $v' = -4.24$ m/s. The coefficient of restitution is

$$e = -\frac{v'}{v} = 0.866.$$

The ball's velocity just before it hits the backboard is

$$\mathbf{v} = 1.52(\cos 42°\mathbf{i} + \cos 68°\mathbf{j} + \cos 124°\mathbf{k})$$

$$= 1.13\mathbf{i} + 0.57\mathbf{j} - 0.85\mathbf{k} \text{ (m/s)}.$$

The z component of velocity just after the impact is

$$v'_z = -ev_z = -(0.866)(-0.85)$$

$$= 0.74 \text{ m/s}.$$

The ball's velocity after impact is $\mathbf{v}' = 1.13\mathbf{i} + 0.57\mathbf{j} + 0.74\mathbf{k}$ (m/s). Its magnitude is $|\mathbf{v}'| = 1.47$ m/s, and

$$\theta_x = \arccos\left(\frac{1.13}{1.47}\right) = 39.5°,$$

$$\theta_y = \arccos\left(\frac{0.57}{1.47}\right) = 67.1°,$$

$$\theta_z = \arccos\left(\frac{0.74}{1.47}\right) = 59.8°.$$

Problem 16.142* In Problem 16.141, the basketball's diameter is 241.3 mm., the coordinates of the center of basket rim are $x = 0$, $y = 0$, and $z = 305$ mm., and the backboard lies in the x-y plane. Determine the x and y coordinates of the point where the ball must hit the backboard so that the center of the ball passes through the center of the basket rim.

Solution: See the solution of Problem 16.141. The ball's velocity after impact in m/s is

$$\mathbf{v}' = 1.13\mathbf{i} + 0.57\mathbf{j} + 0.74\mathbf{k} \text{ m/s}$$

From impact to the center of the rim, the distance the center of the ball moves in the z direction is $305 - \frac{1}{2}(241.3) = 184.4$ mm. The time required is

$$t = \frac{0.1844 \text{ m}}{0.74 \text{ m/s}} = 0.250 \text{ s.}$$

Setting

$$x = 0 = x_0 + v_{xo}t$$

$$= x_0 + (1.13)(0.250)$$

and $\quad y = 0 = y_0 + v_{y0}t - \frac{1}{2}gt^2$

$$= y_0 + (0.57)(0.250) - \frac{1}{2}(9.81)(0.305)(0.250)^2,$$

we obtain $x_0 = -282.7$ mm., $y_0 = 163.1$ mm.

Problem 16.143 A satellite at $r_0 = 16090$ km from the center of the earth is given an initial velocity $v_0 = 6096$ m/s in the direction shown. Determine the magnitude of the transverse component of the satellite's velocity when $r = 32180$ km. (The radius of the earth is 6372 km.)
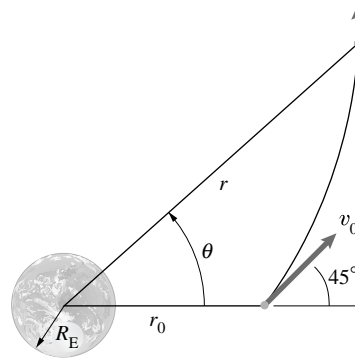
Solution: By definition,

$$H_0 = |\mathbf{r} \times m\mathbf{v}| = mr_0v_0 \sin 45°$$

$$= m(16090)(1000)(6096) \sin 45°$$

$$H_0 = 6.93 \text{ m} \times 10^{10} \text{ kg-m}^2\text{/s.}$$

The gravitational force

$$\mathbf{F} = -\frac{mgR_E^2}{r^2}\mathbf{e}_r,$$

where $\mathbf{e}_r$ is a unit vector parallel to the radius vector $\mathbf{r}$. Since $(\mathbf{r} \times \mathbf{e}_r) = 0$, it follows that the angular momentum impulse is

$$\int_{t_1}^{t_2} \left(\mathbf{r} \times \sum\mathbf{F}\right) dt = 0 = \mathbf{H}_1 - \mathbf{H}_0,$$

from which $\mathbf{H}_0 = \mathbf{H}_1$, and the angular momentum is conserved. Thus the angular momentum at the distance 32180 km is $H_0 = mr_1v_{\theta 1} \sin 90°$. From which

$$\boxed{v_{\theta 1} = \frac{6.93 \times 10^{10}}{3.21 \times 10^7} = 2155 \text{ m/s}}$$

is the magnitude of the transverse velocity.

Problem 16.144 In Problem 16.143, determine the magnitudes of the radial and transverse components of the satellite's velocity when $r = 24135$ km.

Solution: From the solution to Problem 16.143, the constant angular momentum of the satellite is $H_0 = 6.93$ m $\times 10^{10}$ kg-m^2/s. The transverse velocity at $r = 24135$ km is

$$v_\theta = \frac{H}{m(24135)(1000)} = 2569.5 \text{ m/s}$$

From conservation of energy:

$$\left(\frac{1}{2}mv_0^2\right)_{r=16090} - \frac{mgR_E^2}{r_0} = \left(\frac{1}{2}mv_1^2\right)_{r=24135} - \frac{mgR_E^2}{r_1},$$

from which

$$v_1^2 = v_0^2 + 2gR_E^2\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$= (6096)^2 + 2(9.81)(6372000)^2\left(\frac{1}{24135000} - \frac{1}{16090000}\right)$$

$$= 0.207 \times 10^8 \text{ (m/s)}^2.$$

The radial velocity is

$$v_r = \sqrt{v_1^2 - v_\theta^2} = 3520.4 \text{ m/s}$$

Problem 16.145 The snow is 0.61 m deep and weighs 3142 N/m^3, the snowplow is 2.44 m wide, and the truck travels at 8 km/h. What force does the snow exert on the truck?

Solution: The velocity of the truck is

$v = 8$ km/h $= 2.22$ m/s.

The mass flow is the product of the depth of the snow, the width of the plow, the mass density of the snow, and the velocity of the truck, from which

$$\left(\frac{dm_f}{dt}\right) = (0.61)\left(\frac{3142}{g}\right)(2.44)(2.22) = 1058.3 \text{ kg/s}.$$

The force is

$$F = \left(\frac{dm_f}{dt}\right)v = 2375.2 \text{ N}$$

Problem 16.146 An empty 244.6 N drum, 0.91 m in diameter, stands on a set of scales. Water begins pouring into the drum at 5338 N/min from 2.44 m above the bottom of the drum. The weight density of water is approximately 9802 N/m³. What do the scales read 40 s after the water starts pouring?
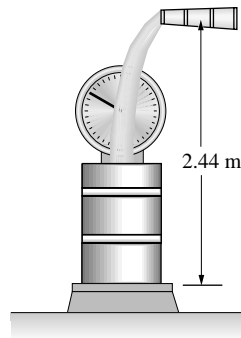
Solution: The mass flow rate is

$$\left(\frac{dm_f}{dt}\right) = \left(\frac{5338}{g}\right)\left(\frac{1}{60}\right) = 9.07 \text{ kg/s}$$

After 40 seconds, the volume of water in the drum is

$$\text{volume} = \frac{5338}{9802}\left(\frac{40}{60}\right) = 0.363 \text{ m}^3.$$

The height of water in the drum is

$$h_{\text{water}} = \frac{\text{volume}}{\pi(0.455^2)} = 0.553 \text{ m}.$$

The velocity of the water stream at point of impact is (from the conservation of energy for free fall) $v = \sqrt{2g(2.44 - h_{\text{water}})} = 6.081$ m/s. The scale reading is

$$\boxed{\begin{array}{l} W = (\text{volume})(9802) + 244.6 + \left(\frac{dm_f}{dt}\right) \\ v = 0.363(9802) + 244.6 + 9.07(6.081) = 3856.4 \text{ N} \end{array}}$$

Problem 16.147 The ski boat's jet propulsive system draws water in at A and expels it at B at 24.4 m/s relative to the boat. Assume that the water drawn in enters with no horizontal velocity relative to the surrounding water. The maximum mass flow rate of water through the engine is 36.5 kg/s. Hydrodynamic drag exerts a force on the boat of magnitude 1.5v N, where v is the boat's velocity in feet per second. Neglecting aerodynamic drag, what is the ski boat's maximum velocity?
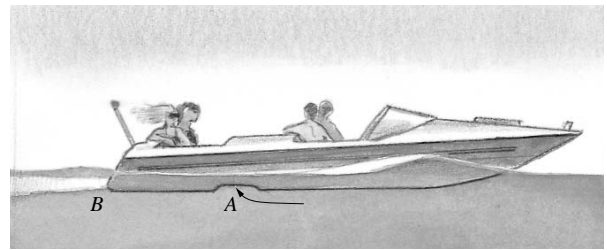
Solution: Use the boat's "rest" frame of reference. The force exerted by the inlet mass flow on the boat is

$$F_{\text{inlet}} = -\left(\frac{dm_f}{dt}\right)v = -36.5 \text{ v}.$$

The force exerted by the exiting mass flow on the boat is

$$F_{\text{exit}} = \left(\frac{dm_f}{dt}\right)(24.4) = 36.5(24.4)$$

The hydrodynamic drag is $F_{\text{drag}} = -1.5$ v. At top speed the sum of the forces vanishes:

$$\sum F = F_{\text{inlet}} + F_{\text{exit}} + F_{\text{drag}} = 0$$

$$= -36.5v + 36.5(24.4) - 1.5v = -38v + 36.5(24.4) = 0,$$

from which

$$\boxed{v = \frac{36.5(24.4)}{38} = 23.4 \text{ m/s}}$$

Problem 16.148 The ski boat of Problem 16.147 weighs 12454 N. The mass flow rate of water through the engine is 36 kg/s, and the craft starts from rest at $t = 0$. Determine the boat's velocity (a) at $t = 20$ s and (b) $t = 60$ s.

Solution: Use the solution to Problem 16.147: the sum of the forces on the boat is

$$\sum F = -38v + 36.5(24.4) \text{ N}.$$

From Newton's second law

$$\frac{W}{g}\left(\frac{dv}{dt}\right) = -38v + 890.6.$$

Separate variables and integrate:

$$\frac{dv}{23.4 - v} = \frac{38g}{W}\, dt,$$

from which

$$\ln(23.4 - v) = -\frac{38g}{W}\, t + C.$$

At $t = 0$, $v = 0$, from which $C = \ln(23.4)$, from which

$$\ln\left(1 - \frac{v}{23.4}\right) = -\frac{38g}{W}\, t.$$

Invert:

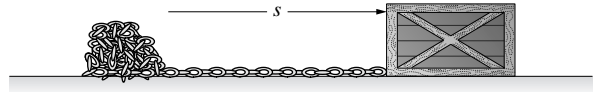$$v(t) = 23.4\left(1 - e^{-\frac{38g}{W}t}\right).$$

Substitute numerical values:

$W = 12454$ N,

$g = 9.81$ m/s^2.

(a) At $t = 20$ s, $\boxed{v = 10.54 \text{ m/s}}$,

(b) At $t = 60$ s, $\boxed{v = 19.52 \text{ m/s}}$

Problem 16.149* A crate of mass m slides across a smooth floor pulling a chain from a stationary pile. The mass per unit length of the chain is ρ_L. If the velocity of the crate is v_0 when $s = 0$, what is its velocity as a function of s?

Solution: The "mass flow" of the chain is

$$\left(\frac{dm_f}{dt}\right) = \rho_L v.$$

The force exerted by the "mass flow" is $F = \rho_L v^2$. From Newton's second law

$$(\rho_L s + m)\left(\frac{dv}{dt}\right) = -\rho_L v^2.$$

Use the chain rule:

$$(\rho_L s + m)v\frac{dv}{ds} = -\rho_L v^2.$$

Separate variables and integrate:

$$\ln(v) = -\ln\left(s + \frac{m}{\rho_L}\right) + C,$$

from which

$$C = \ln\left(\frac{mv_0}{\rho_L}\right).$$

Reduce and solve: $\boxed{v = \dfrac{mv_0}{(\rho_L s + m)}}$