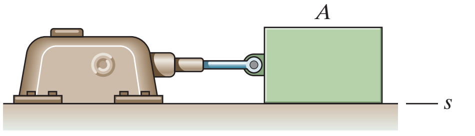
Problem 15.1 In Active Example 15.1, what is the velocity of the container when it has reached the position $s = 2$ m?



Solution: The 180-kg container A starts from rest at $s = 0$. The horizontal force (in newtons) is $F = 700 - 150s$. The coefficient of kinetic friction is $\mu_k = 0.26$.

$$U_{12} = T_2 - T_1$$

$$\int_0^2 (700 - 150s - 0.26[180(9.81)])ds = \frac{1}{2}(180 \text{ kg})v_2^2 - 0$$

$$700(2) - \frac{1}{2}(150)(2)^2 - (0.26[180(9.81)][2]) = 90v_2^2$$

$$\boxed{v_2 = 1.42 \text{ m/s.}}$$

Problem 15.2 The mass of the Sikorsky UH-60A helicopter is 9300 kg. It takes off vertically with its rotor exerting a constant upward thrust of 112 kN. Use the principle of work and energy to determine how far it has risen when its velocity is 6 m/s.

Strategy: Be sure to draw the free-body diagram of the helicopter.

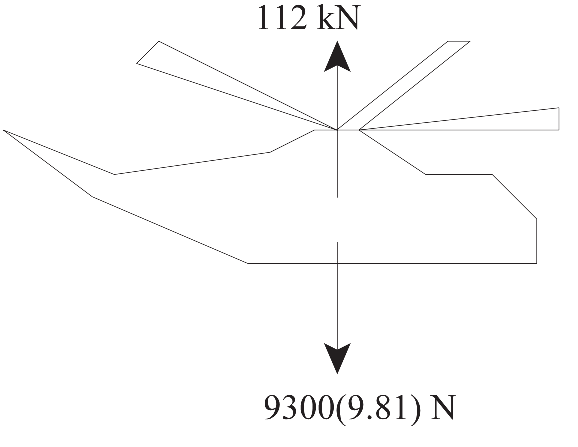
Solution:

$$U_{12} = T_2 - T_1$$

$$[(112000 - 9300[9.81])\text{N}]h$$

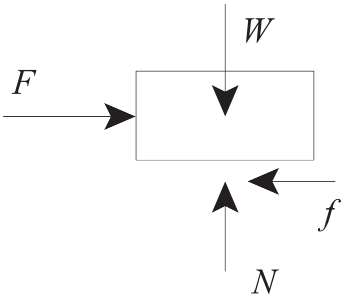
$$= \frac{1}{2}(9300 \text{ kg})(6 \text{ m/s})^2$$

$$\boxed{h = 8.06 \text{ m.}}$$



Problem 15.3 The 20-N box is at rest on the horizontal surface when the constant force $F = 5$ N is applied. The coefficient of kinetic friction between the box and the surface is $\mu_k = 0.2$. Determine how fast the box is moving when it has moved 2 m from its initial position (a) by applying Newton's second law; (b) by applying the principle of work and energy.

Solution:



(a) The equations of motion can be used to find the acceleration

$$\Sigma F_x : F - f = \frac{W}{g}a,\ \Sigma F_y : N - W = 0,$$

$$f = \mu_k N$$

Solving we have

$$a = g\left(\frac{F}{W} - \mu_k\right) = (9.81\,\text{m/s}^2)\left(\frac{5\text{ N}}{20\text{ N}} - 0.2\right) = 0.49\text{ m/s}^2$$

Now we integrate to find the velocity at the new position

$$a = v\frac{dv}{ds} \Rightarrow \int_0^v vdv = \int_0^{2\text{ m}} ads \Rightarrow \frac{v^2}{2} = a(2\text{ m}) = (0.49\text{ m/s}^2)(2\text{ m})$$

$$\boxed{v = 1.4\text{ m/s}}$$

(b) Using the principle of work and energy we have (recognizing that $N = W$)

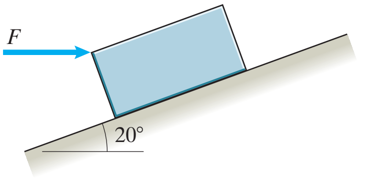
$$U_{12} = T_2 - T_1$$

$$(F - \mu_k N)d = \frac{1}{2}\left(\frac{W}{g}\right)v^2 - 0$$

$$v^2 = 2g\left(\frac{F}{W} - \mu_k\right)d = 2(9.81\text{ m/s}^2)\left(\frac{5\text{ N}}{20\text{ N}} - 0.2\right)(2\text{ m})$$

$$\boxed{v = 1.4\text{ m/s}}$$

Problem 15.4 At the instant shown, the 30-N box is moving up the smooth inclined surface at 2 m/s. The constant force $F = 15$ N. How fast will the box be moving when it has moved 1 m up the surface from its present position?



Solution:

$$U_{12} = T_2 - T_1$$

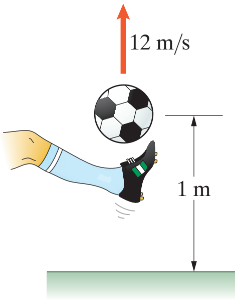
$$[(15\text{ N})\cos 20^\circ - (30\text{ N})\sin 20^\circ](1\text{ m})$$

$$= \frac{1}{2}\left(\frac{30\text{ N}}{9.81\text{ m/s}^2}\right)v^2 - \frac{1}{2}\left(\frac{30\text{ N}}{9.81\text{ m/s}^2}\right)(2\text{ m/s})^2$$

Solving we find

$$\boxed{v = 2.55\text{ m/s.}}$$

Problem 15.5 The 0.45-kg soccer ball is 1 m above the ground when it is kicked straight upward at 10 m/s. By using the principle of work and energy, determine: (a) how high above the ground the ball goes, (b) the magnitude of the ball's velocity when it falls back to a height of 1 m above the ground, (c) the magnitude of the ball's velocity immediately before it hits the ground.



Solution:

(a) Find the height above the ground

$$mg(1\text{ m} - h) = 0 - \frac{1}{2}mv_0^2,$$

$$h = \frac{{v_0}^2}{2g} + 1\text{ m} = \frac{(10\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 1\text{ m} = 6.10\text{ m}$$

(b) When the ball returns to the same level, the velocity must be equal to the initial velocity (but now it is moving downward) because the net work is zero

$$v = 10\text{ m/s} \downarrow$$

(c) The velocity just before it hits the ground

$$mg(1\text{ m}) = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2$$

$$v^2 = {v_0}^2 + 2g(1\text{ m}) = (10\text{ m/s})^2 + 2(9.81\text{ m/s}^2)(1\text{ m})$$

$$v = 10.9\text{ m/s.} \downarrow$$

$$\boxed{(a)\ h = 6.10\text{ m},\quad (b)\ v = 10.0\text{ m/s},\quad (c)\ v = 10.9\text{ m/s.}}$$

Problem 15.6 Assume that the soccer ball in Problem 15.5 is stationary the instant before it is kicked upward at 12 m/s. The duration of the kick is 0.02 s. What average power is transferred to the ball during the kick?

Solution:

$$U_{12} = \frac{1}{2}(0.45\text{ kg})(12\text{ m/s})^2 - 0 = 32.4\text{ N-m}$$

$$\boxed{\text{Power} = \frac{U_{12}}{\Delta t} = \frac{32.4\text{ N-m}}{0.02\text{ s}} = 1.62\text{ kW}}$$

Problem 15.7 The 2000-N drag racer starts from rest and travels a quarter-kilometre course. It completes the cours in 4.524 seconds and crosses the finish line traveling at 325.77 km/h. (a) How much work is done on the car as it travels the course? (b) Assume that the horizontal force exerted on the car is constant and use the principle of work and energy to determine it.

Solution:

(a) The work is equal to the change in kinetic energy.

$$U = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{2000\text{ N}}{9.81\text{ m/s}^2}\right)\left[(325.77\text{ km/h})\left(\frac{1000}{3600}\right)\right]^2$$

$$\boxed{U = 8.35 \times 10^5\text{ N-m}}$$

(b) The work is equal to the force times the distance

$$U = Fd \Rightarrow F = \frac{U}{d} = \frac{8.35 \times 10^5\text{ N-m}}{\frac{1}{4}(1000\text{ m})} = 3339\text{ N}$$

$$\boxed{F = 3339\text{ N}}$$

Problem 15.8 The 2000-N drag racer starts from rest and travels a quarter-kilometre course. It completes the course in 4.524 seconds and crosses the finish line traveling at 325.77 km/h. Assume that the horizontal force exerted on the car is constant. Determine (a) the maximum power and (b) the average power transferred to the car as it travels the quarter-kilometre course.

Solution: From problem 15.7 we know that the force is

(a) The maximum power occurs when the car has reached its maximum velocity

$$P = Fv = (3339\text{ N})(325.77\text{ km/h})\left(\frac{1000}{3600}\right) = 3.02 \times 10^5\text{ N-m/s}.$$

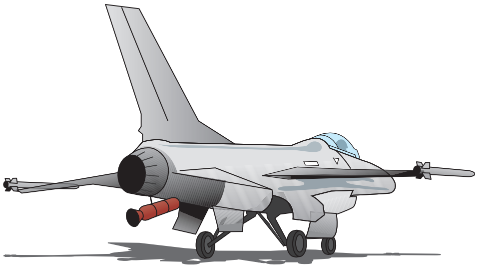
(b) The average power is equal to the change in kinetic energy divided by the time.

$$P_{\text{ave}} = \frac{\frac{1}{2}mv^2}{\Delta t} = \frac{\frac{1}{2}\left(\frac{2000\text{ N}}{9.81\text{ m/s}^2}\right)\left[(325.77\text{ km/h})\left(\frac{1000}{3600}\right)\right]^2}{4.524\text{ s}}$$

$$= 1.845 \times 10^5\text{ N-m/s}.$$

$$\boxed{(a)\ 3.02 \times 10^5\text{ N-m/s},\quad (b)\ 1.845 \times 10^5\text{ N-m/s}.}$$

Problem 15.9 As the 32,000-N airplane takes off, the tangential component of force exerted on it by its engines is $\Sigma F_t = 45{,}000$ N. Neglecting other forces on the airplane, use the principle of work and energy to determine how much runway is required for its velocity to reach 200 km/h.



Solution:

$$U_{12} = \frac{1}{2}mv^2 \Rightarrow Fd = \frac{1}{2}mv^2 \Rightarrow d = \frac{mv^2}{2F}$$

$$d = \frac{\left(\dfrac{32{,}000\text{ N}}{9.81\text{ m/s}^2}\right)\left[(200\text{ km/h})\left(\dfrac{1000}{3600}\right)\right]^2}{2(45{,}000\text{ N})} = 112\text{ m}.$$

$$\boxed{d = 112\text{ m}}$$

Problem 15.10 As the 32,000-N airplane takes off, the tangential component of force exerted on it by its engines is $\Sigma F_t = 45{,}000$ N. Neglecting other forces on the airplane, determine (a) the maximum power and (b) the average power transferred to the airplane as its velocity increases from zero to 200 km/h.

Solution:

(a) The maximum power occurs when the velocity is a maximum

$$P = Fv = (45{,}000\text{ N})\left[200\text{ km/h}\frac{1000}{3600}\right] = 2.5\times 10^6\text{ N-m/s}.$$

(b) To find the average power we need to know the time that it takes to reach full speed

$$a = \frac{F}{m} = \frac{45{,}000\text{ N}}{\left(\dfrac{32{,}000\text{ N}}{9.81\text{ m/s}^2}\right)} = 13.8\text{ m/s}^2$$

$$v = at \Rightarrow t = \frac{v}{a} = \frac{200\text{ km/h}\dfrac{1000}{3600}}{13.8\text{ m/s}^2} = 4.03\text{ s}.$$

Now, the average power is the change in kinetic energy divided by the time

$$P_{\text{ave}} = \frac{\frac{1}{2}mv^2}{t} = \frac{\frac{1}{2}\left(\dfrac{32{,}000\text{ N}}{9.81\text{ m/s}^2}\right)\left(200\text{ km/h}\dfrac{1000}{3600}\right)^2}{4.03\text{ s}} = 1.25\times 10^6\text{ N-m/s}.$$

$$\boxed{\text{(a) } 2.5\times 10^6\text{ N-m/s},\quad \text{(b) } 1.25\times 10^6\text{ N-m/s}.}$$

Problem 15.11 The 32,000-N airplane takes off from rest in the position $s = 0$. The total tangential force exerted on it by its engines and aerodynamic drag (in Newtons) is given as a function of its position s by $\Sigma F_t = 45{,}000 - 5.2s$. Use the principle of work and energy to determine how fast the airplane is traveling when its position is $s = 950$ m.

Solution:

$$U_{12} = \int_0^{950} (45{,}000 - 5.2s)\, ds$$

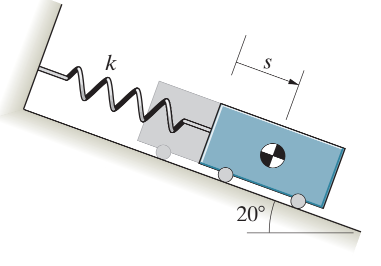
$$= (45,000)(950) - \frac{1}{2}(5.2)(950)^2 = 40.4 \times 10^6 \text{ N-m}$$

$$U_{12} = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{32{,}000 \text{ N}}{9.81 \text{ m/s}^2}\right)v^2$$

Solving, we find

$$\boxed{v = 157.4 \text{ m/s.}}$$

Problem 15.12 The spring ($k = 20$ N/m) is unstretched when $s = 0$. The 5-kg cart is moved to the position $s = -1$ m and released from rest. What is the magnitude of its velocity when it is in the position $s = 0$?



Solution: First we calculate the work done by the spring and by gravity

$$U_{12} = \int_{-1 \text{ m}}^{0} (-ks + mg \sin 20^\circ)\, ds$$

$$= \frac{1}{2}k(-1 \text{ m})^2 + mg \sin 20^\circ (1 \text{ m})$$

$$= \frac{1}{2}(20 \text{ N/m})(-1 \text{ m})^2 + (5 \text{ kg})(9.81 \text{ m/s}^2) \sin 20^\circ (1 \text{ m})$$

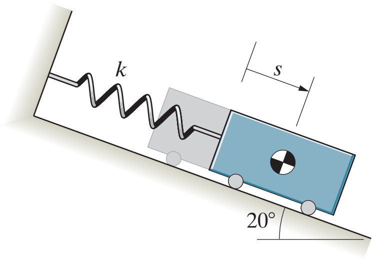
$$= 26.8 \text{ N-m.}$$

Now using work and energy

$$U_{12} = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2U_{12}}{m}} = \sqrt{\frac{2(26.8 \text{ N-m})}{5 \text{ kg}}} = 3.27 \text{ m/s.}$$

$$\boxed{v = 3.27 \text{ m/s.}}$$

Problem 15.13 The spring ($k = 20$ N/m) is unstretched when $s = 0$. The 5-kg cart is moved to the position $s = -1$ m and released from rest. What maximum distance down the sloped surface does the cart move relative to its initial position?



Solution: The cart starts from a position of rest, and when it reaches the maximum position, it is again at rest. Therefore, the total work must be zero.

$$U_{12} = \int_{-1\text{ m}}^{s} (-ks + mg\sin 20^\circ)\,ds$$

$$= -\frac{1}{2}k(s^2 - [-1\text{ m}]^2) + mg\sin 20^\circ(s - [-1\text{ m}])$$

$$= -\frac{1}{2}(20\text{ N/m})(s^2 - [-1\text{ m}]^2) + (5\text{ kg})(9.81\text{ m/s}^2)\sin 20^\circ(s - [-1\text{ m}]) = 0$$

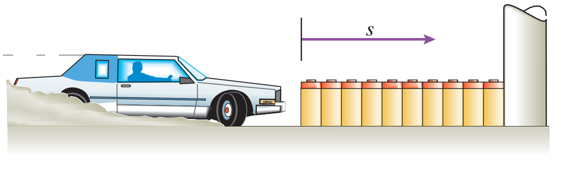
This is a quadratic equation that has the two solutions

$$s_1 = -1\text{ m},\ s_2 = 2.68\text{ m}.$$

The distance relative to the initial is $s = s_2 + 1$ m.

$$\boxed{s = 3.68\text{ m.}}$$

Problem 15.14 The force exerted on a car by a prototype crash barrier as the barrier crushes is $F = -(120s + 40s^3)$ N, where s is the distance in metre from the initial contact. The effective length of the barrier is 18 m. How fast can a 5000-N car be moving and be brought to rest within the effective length of the barrier?



Solution: The barrier can provide a maximum amount of work given by

$$U_{12} = \int_{0}^{18} -(120s + 40s^3)\,ds$$

$$= -\frac{1}{2}(120)(18)^2 - \frac{1}{4}(40)(18)^4 = -1.07 \times 10^6\text{ N-m.}$$

Using work and energy, we have

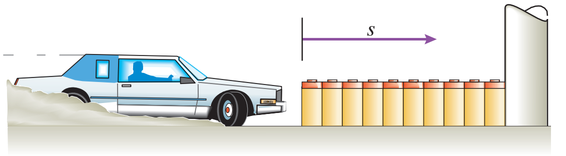
$$U_{12} = 0 - \frac{1}{2}mv^2$$

$$-1.07 \times 10^6\text{ N-m} = -\frac{1}{2}\left(\frac{5000\text{ N}}{9.81\text{ m/s}^2}\right)v^2$$

Solving for the velocity, we find

$$\boxed{v = 64.8\text{ m/s.}}$$

Problem 15.15 A 5000-N car hits the crash barrier at 80 km/h and is brought to rest in 0.11 seconds. What average power is transferred from the car during the impact?



Solution: The average power is equal to the change in kinetic energy divided by the time

$$P = \frac{\frac{1}{2}mv^2}{\Delta t} = \frac{\frac{1}{2}\left(\frac{5000\text{ N}}{9.81\text{ m/s}^2}\right)\left(80\text{ km/h}\,\frac{1000}{3600}\right)^2}{0.11\text{ s}} = 1.14\times10^6\text{ N-m/s.}$$

$$\boxed{P = 1.14\times10^6\text{ N-m/s.}}$$

Problem 15.16 A group of engineering students constructs a sun-powered car and tests it on a circular track with a 1000-m radius. The car, with a weight of 460 N including its occupant, starts from rest. The total tangential component of force on the car is

$$\Sigma F_t = 30 - 0.2s\text{ N},$$

where s is the distance (in ft) the car travels along the track from the position where it starts.

(a) Determine the work done on the car when it has gone a distance $s = 120$ m.
(b) Determine the magnitude of the *total* horizontal force exerted on the car's tires by the road when it is at the position $s = 120$ m.

Solution:

(a) $$\boxed{U = \int_0^{120\text{ m}} [30 - 0.2\,s]\text{ N}\,ds = 2160\text{ N-m}}$$

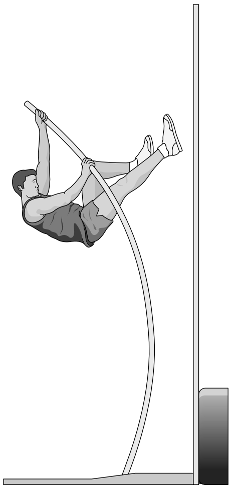
(b) $$2160\text{ N-m} = \frac{1}{2}\left(\frac{460\text{ N}}{9.81\text{ m/s}^2}\right)v^2 \Rightarrow v = 9.6\text{ m/s}$$

$$F_t = [30 - 0.2(120)] = 6\text{ N}$$

$$F_n = m\frac{v^2}{\rho} = \left(\frac{460\text{ N}}{9.81\text{ m/s}^2}\right)\frac{(9.6\text{ m/s})^2}{1000\text{ m}} = 4.32\text{ N}$$

$$\boxed{F = \sqrt{{F_t}^2 + {F_n}^2} = \sqrt{(6\text{ N})^2 + (4.32\text{ N})^2} = 7.39\text{ N}}$$

Problem 15.17 At the instant shown, the 160-N vaulter's center of mass is 8.5 m above the ground, and the vertical component of his velocity is 4 m/s. As his pole straightens, it exerts a vertical force on the vaulter of magnitude $180 + 2.8y^2$ N, where y is the vertical position of his center of mass *relative to its position at the instant shown*. This force is exerted on him from $y = 0$ to $y = 4$ m, when he releases the pole. What is the maximum height above the ground reached by the vaulter's center of mass?



Solution: The work done on him by the pole is

$$U_{\text{pole}} = \int_0^4 (180 + 2.8\ y^2)\,dy$$

$$= 180(4) + 2.8\frac{(4)^3}{3} = 780\ \text{N-m}.$$

Let $y_{\max}$ be his maximum height above the ground. The work done by his weight from the instant shown to the maximum height is

$$-160(y_{\max} - 8.5) = U_{\text{weight}},\ \text{or}\ U_{\text{weight}} + U_{\text{pole}}$$

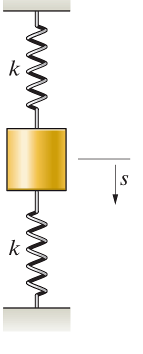
$$= mv_2^2/2 - mv_1^2/2$$

$$780 - 160(y_{\max} - 8.5) = 0 - \frac{1}{2}\left(\frac{160}{9.81}\right)(4)^2.$$

Solving, $y_{\max} = 14.2$ m

Problem 15.18 The springs ($k = 25$ N/cm) are unstretched when $s = 0$. The 50-N weight is released from rest in the position $s = 0$.

(a) When the weight has fallen 1 cm, how much work has been done on it by each spring?
(b) What is the magnitude of the velocity of the weight when it has fallen 1 cm?



Solution:

(a) The work done by each spring

$$U_{12} = \int_0^{1\,\text{cm}} -ks\,ds = -\frac{1}{2}(25\ \text{N/cm})(1\,\text{cm})^2 = -12.5\ \text{N-cm}.$$

(b) The velocity is found from the work-energy equation. The total work includes the work done by both springs and by gravity

$$U_{12} = (50\ \text{N})(1\ \text{cm}) - 2(12.5\ \text{N-cm}) = 25\ \text{N-cm}.$$

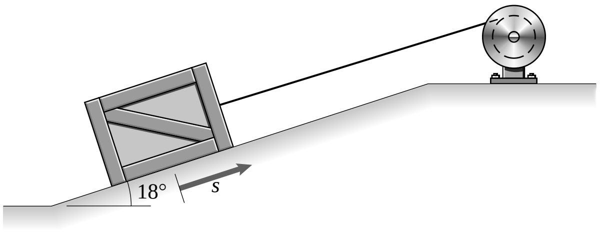
$$U_{12} = \frac{1}{2}\,mv^2 = \frac{1}{2}\left(\frac{50\ \text{N}}{9.81\ \text{m/s}^2}\right)v^2$$

Solving for the velocity we find $v = 0.31$ m/s.

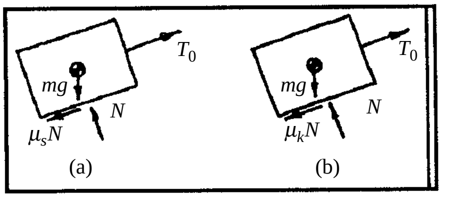
$$\boxed{\text{(a)} -12.5\ \text{N-cm, (b) } 0.31\ \text{m/s.}}$$

Problem 15.19 The coefficients of friction between the 160-kg crate and the ramp are $\mu_s = 0.3$ and $\mu_k = 0.28$.

(a) What tension T_0 must the winch exert to start the crate moving up the ramp?
(b) If the tension remains at the value T_0 after the crate starts sliding, what total work is done on the crate as it slides a distance $s = 3$ m up the ramp, and what is the resulting velocity of the crate?



Solution:



(a) The tension is $T_0 = W \sin\theta + \mu_s$ N, from which

$$\boxed{T_0 = mg(\sin\theta + \mu_s \cos\theta) = 932.9 \text{ N.}}$$

(b) The work done on the crate by (non-friction) external forces is

$$U_{\text{weight}} = \int_0^3 T_0\, ds - \int_0^3 (mg \sin\theta)\, ds = 932.9(3) - 1455.1$$

$$= 1343.5 \text{ N-m.}$$

The work done on the crate by friction is

$$U_f = \int_0^3 (-\mu_k\ N)\, ds = -3\mu_k mg \cos\theta = -1253.9 \text{ N-m.}$$

From the principle of work and energy is

$$U_{\text{weight}} + U_f = \frac{1}{2} m v^2,$$

from which

$$v = \sqrt{\frac{6(T_0 - mg(\sin\theta + \mu_k \cos\theta))}{m}}$$

$$\boxed{v = 1.06 \text{ m/s}}$$

Problem 15.20 In Problem 15.19, if the winch exerts a tension $T = T_0(1 + 0.1s)$ after the crate starts sliding, what total work is done on the crate as it slides a distance $s = 3$ m up the ramp, and what is the resulting velocity of the crate?

Solution: The work done on the crate is

$$U = \int_0^3 T\, ds - \int_0^3 (mg \sin\theta)\, ds - \mu_k \int_0^3 (mg \cos\theta)\, ds,$$

from which

$$U = T_0 \left[\left(s + 0.05s^2\right)\right]_0^3 - (mg \sin\theta)(3) - \mu_k (mg \cos\theta)(3).$$

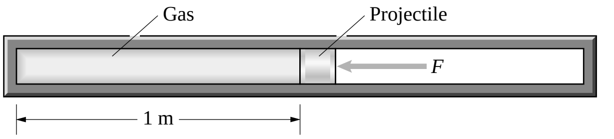
From the solution to Problem 15.19, $T_0 = 932.9$ N-m, from which the total work done is

$$\boxed{U = 3218.4 - 1455.1 - 1253.9 = 509.36 \text{ N-m.}}$$

From the principle of work and energy, $U = \frac{1}{2} m v^2$, from which

$$\boxed{v = \sqrt{\frac{2U}{m}} = 2.52 \text{ m/s}}$$

Problem 15.21 The 200-mm-diameter gas gun is evacuated on the right of the 8-kg projectile. On the left of the projectile, the tube contains gas with pressure $p_0 = 1 \times 10^5 Pa$ (N/m^2). The force F is slowly increased, moving the projectile 0.5 m to the left from the position shown. The force is then removed and the projectile accelerates to the right. If you neglect friction and assume that the pressure of the gas is related to its volume by $pV =$ constant, what is the velocity of the projectile when it has returned to its original position?



Solution: The constant is $K = pV = 1 \times 10^5(1)(0.1)^2\pi = 3141.6$ N-m. The force is $F = pA$. The volume is $V = As$, from which the pressure varies as the inverse distance: $p = \frac{K}{As}$, from which $F = \frac{K}{s}$.

The work done by the gas is

$$U = \int_{0.5}^{1} F\,ds = \int_{0.5}^{1} \frac{K}{x}\,ds = [K\ln(s)]_{0.5}^{1.0} = K\ln(2).$$

From the principle of work and energy, the work done by the gas is equal to the gain in kinetic energy:

$$K\ln(2) = \frac{1}{2}mv^2, \text{ and } v^2 = \frac{2K}{m}\ln(2),$$

$$\boxed{v = \sqrt{\frac{2K}{m}\ln(2)} = 23.33 \text{ m/s}}$$

Note: The argument of ln(2) is dimensionless, since it is ratio of two distances.

Problem 15.22 In Problem 15.21, if you assume that the pressure of the gas is related to its volume by $pV =$ constant while it is compressed (an isothermal process) and by $pV^{1.4} =$ constant while it is expanding (an isentropic process), what is the velocity of the projectile when it has returned to its original position?

Solution: The isothermal constant is $K = 3141.6$ N-m from the solution to Problem 15.21. The pressure at the leftmost position is

$$p = \frac{K}{A(0.5)} = 2 \times 10^5 \text{ N/m}^2.$$

The isentropic expansion constant is

$$K_e = pV^{1.4} = (2 \times 10^5)(A^{1.4})(0.5^{1.4}) = 596.5 \text{ N-m}$$

The pressure during expansion is

$$p = \frac{K_e}{(As)^{1.4}} = \frac{K_e}{A^{1.4}}s^{-1.4}.$$

The force is $F = pA = K_e A^{-0.4} s^{-1.4}$. The work done by the gas during expansion is

$$U = \int_{0.5}^{1.0} F\,ds = \int_{0.5}^{1.0} K_e A^{-0.4} s^{-1.4}\,ds = K_e A^{-0.4}\left[\frac{s^{-0.4}}{-0.4}\right]_{0.5}^{1.0}$$

$$= 1901.8 \text{ N-m}.$$

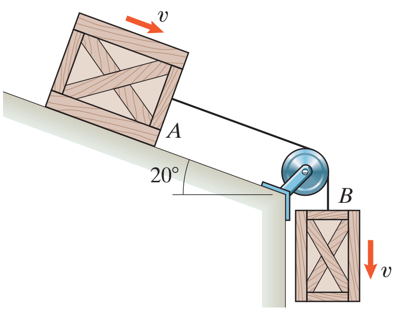
From the principle of work and energy, the work done is equal to the gain in kinetic energy,

$$\int_{0.5}^{1} F\,ds = \frac{1}{2}mv^2,$$

from which the velocity is

$$\boxed{v = \sqrt{\frac{2(1901.8)}{m}} = 21.8 \text{ m/s}}.$$

Problem 15.23 In Example 15.2, suppose that the angle between the inclined surface and the horizontal is increased from 20° to 30°. What is the magnitude of the velocity of the crates when they have moved 400 mm?



Solution: Doing work–energy for the system

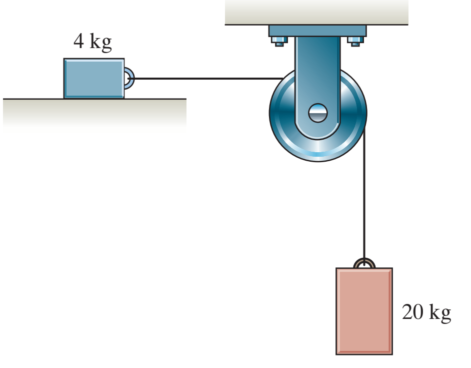
$$\int_0^{0.4} (m_A g \sin 30^\circ - \mu_k m_A g \cos 30^\circ + m_B g)\, ds = \frac{1}{2}(m_A + m_B)v_2^2$$

$$[40 \sin 30^\circ - (0.15)(40)\cos 30^\circ + 30](9.81)(0.4) = \frac{1}{2}(70)v_2^2$$

Solving for the velocity we find

$$\boxed{v_2 = 2.24 \text{ m/s.}}$$

Problem 15.24 The system is released from rest. The 4-kg mass slides on the smooth horizontal surface. By using the principle of work and energy, determine the magnitude of the velocity of the masses when the 20-kg mass has fallen 1 m.



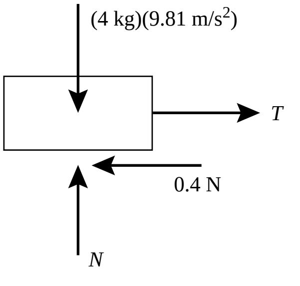
Solution: Write work-energy for system

$$U = (20 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m}) = \frac{1}{2}(24 \text{ kg})v^2 \Rightarrow \boxed{v = 4.04 \text{ m/s}}$$

Problem 15.25 Solve Problem 15.24 if the coefficient of kinetic friction between the 4-kg mass and the horizontal surface is $\mu_k = 0.4$.

Solution:

$$\sum F_y : N - (4 \text{ kg})(9.81 \text{ m/s}^2) = 0 \Rightarrow N = 39.24 \text{ N}$$

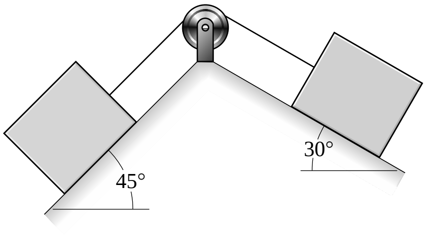


Write work-energy for system

$$U = [(20 \text{ kg})(9.81 \text{ m/s}^2) - 0.4(39.24 \text{ N})](1 \text{ m}) = 180.5 \text{ N-m}$$

$$180.5 \text{ N-m} = \frac{1}{2}(24 \text{ kg})v^2 \Rightarrow \boxed{v = 3.88 \text{ m/s}}$$

Problem 15.26 Each box weighs 50 N and the inclined surfaces are smooth. The system is released from rest. Determine the magnitude of the velocities of the boxes when they have moved 1 m.



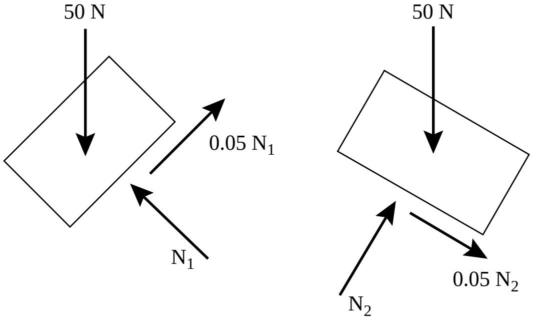
Solution: Write work-energy for the system

$$U = (50 \text{ N} \sin 45^\circ)(1 \text{ m}) - (50 \text{ N} \sin 30^\circ)(1 \text{ m}) = 10.36 \text{ N-m}$$

$$10.36 \text{ N-m} = \frac{1}{2}\left(\frac{100 \text{ N}}{9.81 \text{ m/s}^2}\right)v^2 \Rightarrow \boxed{v = 1.43 \text{ m/s}}$$

Problem 15.27 Solve Problem 15.26 if the coefficient of kinetic friction between the boxes and the inclined surfaces is $\mu_k = 0.05$.

Solution:



$$\sum F_{\nwarrow} : N_1 - (50 \text{ N}) \sin 45^\circ = 0$$

$$\sum F_{\nearrow} : N_2 - (50 \text{ N}) \cos 30^\circ = 0$$

$N_1 = 35.4$ N, $N_2 = 43.3$ N

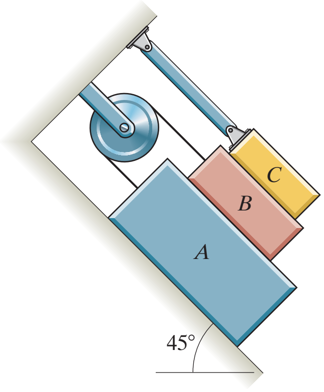
Work-energy for the system

$$U = (50 \text{ N} \sin 45^\circ)(1 \text{ m}) - (0.05)(35.4 \text{ N})(1 \text{ m})$$

$$- (50 \text{ N} \sin 30)(1 \text{ m}) - (0.05)(43.3 \text{ N})(1 \text{ m}) = 6.42 \text{ N-m}$$

$$6.42 \text{ N-m} = \frac{1}{2}\left(\frac{100 \text{ N}}{9.81 \text{ m/s}^2}\right)v^2 \Rightarrow \boxed{v = 1.12 \text{ m/s}}$$

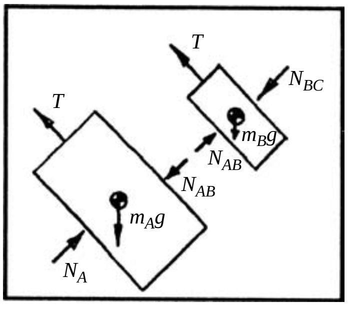
Problem 15.28 The masses of the three blocks are $m_A = 40$ kg, $m_B = 16$ kg, and $m_C = 12$ kg. Neglect the mass of the bar holding C in place. Friction is negligible. By applying the principle of work and energy to A and B individually, determine the magnitude of their velocity when they have moved 500 mm.



Solution: Denote $b = 0.5$ m. Since the pulley is one-to-one, denote $|v_A| = |v_B| = v$. The principle of work and energy for weight A is

$$\int_0^b (m_A g \sin\theta - T)\, ds = \frac{1}{2} m_A v^2,$$

and for weight B $\int_0^b (T - m_B g \sin\theta)\, ds = \frac{1}{2} m_B v^2.$



Add the two equations:

$(m_A - m_B) g b \sin\theta = \frac{1}{2}(m_A + m_B) v^2.$

Solve: $\boxed{|v_A| = |v_B| = \sqrt{\frac{2(m_A - m_B) g b \sin\theta}{(m_A + m_B)}} = 1.72 \text{ m/s}}$

Problem 15.29 Solve Problem 15.28 by applying the principle of work and energy to the system consisting of A, B, the cable connecting them, and the pulley.

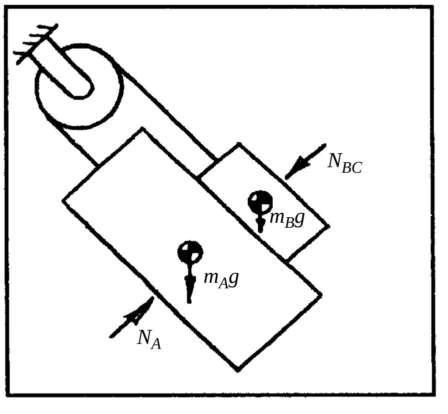
Solution: Choose a coordinate system with the origin at the pulley axis and the positive x axis parallel to the inclined surface. Since the pulley is one-to-one, $x_A = -x_B$. Differentiate to obtain $v_A = -v_B$. Denote $b = 0.5$ m. From the principle of work and energy the work done by the external forces on the complete system is equal to the gain in kinetic energy,

$$\int_0^{x_A} m_A g \sin\theta\, ds + \int_0^{x_B} m_B g \sin\theta\, ds = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2,$$

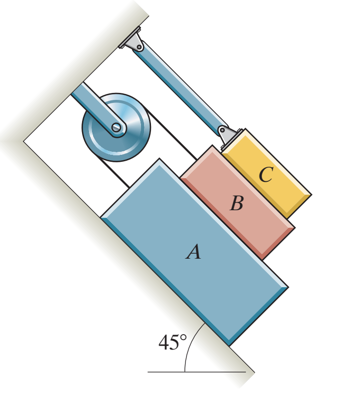
from which

$(m_B - m_A) g b \sin\theta = \frac{1}{2}(m_A + m_B) v_A^2$

and $\boxed{|v_A| = |v_B| = \sqrt{\frac{(m_A - m_B)}{(m_A + m_B)} 2 g b \sin\theta} = 1.72 \text{ m/s.}}$



Problem 15.30 The masses of the three blocks are $m_A = 40$ kg, $m_B = 16$ kg, and $m_C = 12$ kg. The coefficient of kinetic friction between all surfaces is $\mu_k = 0.1$. Determine the magnitude of the velocity of blocks A and B when they have moved 500 mm. (See Example 15.3.)



Solution: We will apply the principles of work—energy to blocks A and B individually in order to properly account for the work done by internal friction forces.

$$\int_0^b (m_A g \sin\theta - T - \mu_k N_A - \mu_k N_{AB})\, ds = \frac{1}{2} m_A v^2,$$

$$\int_0^b (T - m_B g \sin\theta - \mu_k N_{BC} - \mu_k N_{AB})\, ds = \frac{1}{2} m_B v^2.$$

Adding the two equations, we get

$$([m_A - m_B]g \sin\theta - \mu_k[N_A + 2N_{AB} + N_{BC}])b = \frac{1}{2}(m_A + m_B)v^2$$

The normal forces are

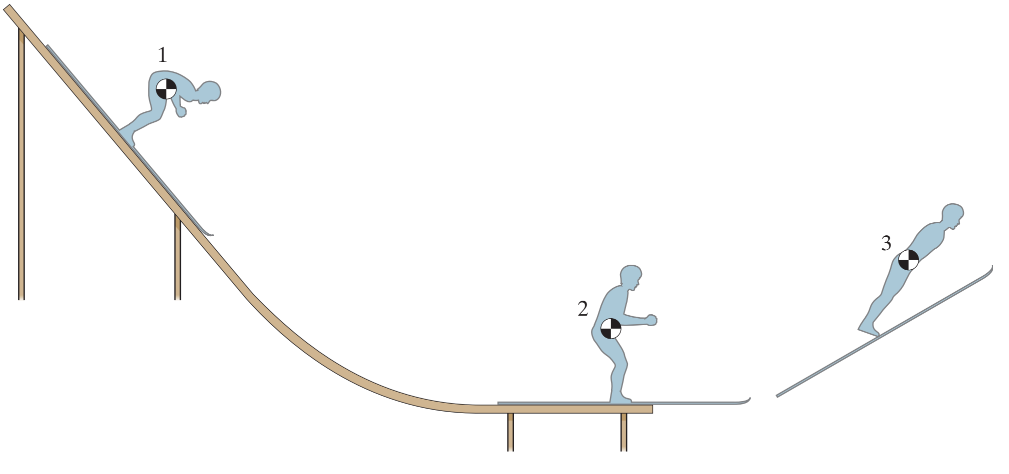
$$N_A = (m_A + m_B + m_C)g\cos\theta,$$

$$N_{AB} = (m_B + m_C)g\cos\theta,$$

$$N_{BC} = (m_C)g\cos\theta.$$

Substitute and solve $\Rightarrow$ $\boxed{v = 1.14 \text{ m/s.}}$

Problem 15.31 In Example 15.5, suppose that the skier is moving at 20 m/s when he is in position 1. Determine the horizontal component of his velocity when he reaches position 2, 20 m below position 1.



Solution:

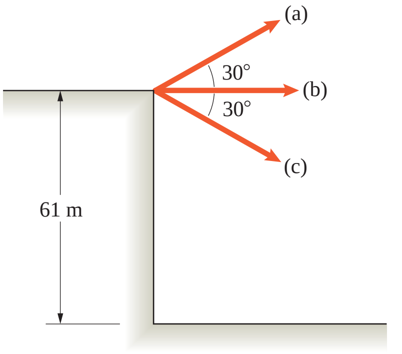
$$U_{12} = -mg(y_2 - y_1) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$-m(9.81)(0 - 20) = \frac{1}{2}mv_2^2 - \frac{1}{2}m(20)^2$$

Solving for v_2 we find

$$\boxed{v_2 = 28.1 \text{ m/s.}}$$

Problem 15.32 Suppose that you stand at the edge of a 61 m cliff and throw rocks at 9.1 m/s in the three directions shown. Neglecting aerodynamic drag, use the principle of work and energy to determine the magnitude of the velocity of the rock just before it hits the ground in each case.



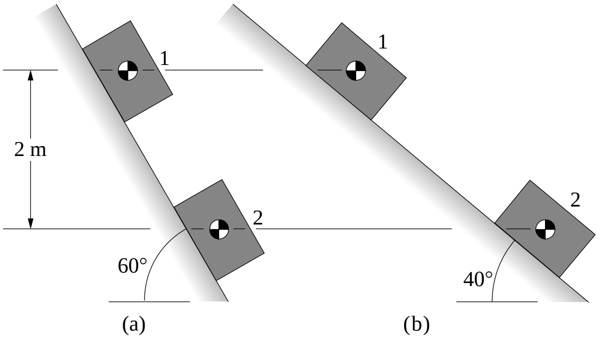
Solution:

$$U = m(9.81\,\text{m/s}^2)(61\ \text{m}) = \frac{1}{2}mv^2 - \frac{1}{2}m(9.1\ \text{m/s})^2$$

$$\Rightarrow \boxed{v = 35.7\,\text{m/s}}$$

Note that the answer does not depend on the initial angle.

Problem 15.33 The 30-kg box is sliding down the smooth surface at 1 m/s when it is in position 1. Determine the magnitude of the box's velocity at position 2 in each case.



Solution: The work done by the weight is the same in both cases.

$$U = -m(9.81\ \text{m/s}^2)(0 - 2\ m) = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m(1\ \text{m/s})^2$$

$$\Rightarrow \boxed{v = 6.34\ \text{m/s}}$$

Problem 15.34 Solve Problem 15.33 if the coefficient of kinetic friction between the box and the inclined surface is $\mu_k = 0.2$.

Solution: The work done by the weight is the same, however, the work done by friction is different.

(a) $U = -m(9.81\ \text{m/s}^2)(0 - 2\ \text{m})$

$$-(0.2)[m(9.81\ \text{m/s}^2)\cos 60°]\left[\frac{2\ \text{m}}{\sin 60°}\right]$$

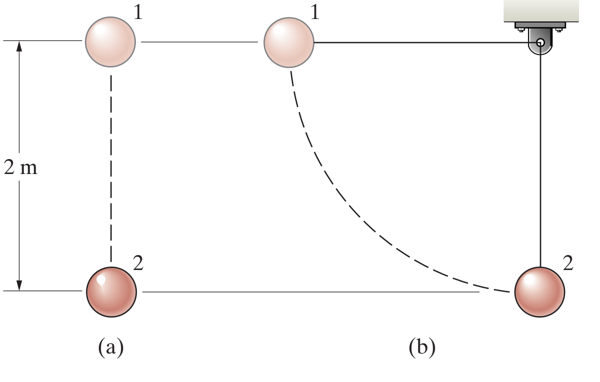
$$U = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m(1\ \text{m/s})^2 \Rightarrow \boxed{v_2 = 5.98\ \text{m/s}}$$

(b) $U = -m(9.81\ \text{m/s}^2)(0 - 2\ \text{m})$

$$-(0.2)[m(9.81\ \text{m/s}^2)\cos 40°]\left[\frac{2\ \text{m}}{\sin 40°}\right]$$

$$U = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m(1\ \text{m/s})^2 \Rightarrow \boxed{v_2 = 5.56\ \text{m/s}}$$

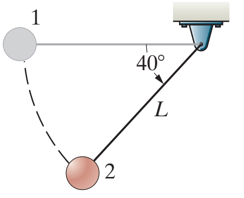
Problem 15.35 In case (a), a 5-N ball is released from rest at position 1 and falls to position 2. In case (b), the ball is released from rest at position 1 and swings to position 2. For each case, use the principle of work and energy to determine the magnitude of the ball's velocity at position 2. (In case (b), notice that the force exerted on the ball by the string is perpendicular to the ball's path.)



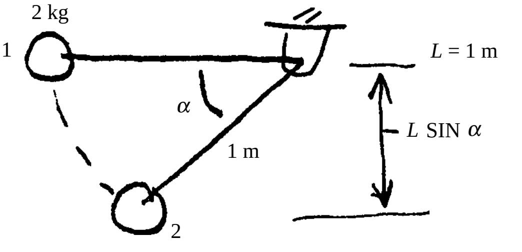
Solution: The work is independent of the path, so both cases are the same.

$$U = -m(9.81\text{m/s}^2)(0 - 2\,\text{m}) = \frac{1}{2}m{v_2}^2 - 0 \Rightarrow \boxed{v_2 = 6.26\ \text{m/s}}$$

Problem 15.36 The 2-kg ball is released from rest in position 1 with the string horizontal. The length of the string is $L = 1$ m. What is the magnitude of the ball's velocity when it is in position 2?



Solution:



$$U_{12} = -\int_0^{-L\sin\alpha} mg\mathbf{j} \cdot ds\mathbf{j}$$

$$U_{12} = -mg(-L\sin\alpha) = (2)(9.81)(1)\sin 40^\circ$$

$$U_{12} = 12.61 \text{ N-m}$$

$$U_{12} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \qquad v_1 \equiv 0$$

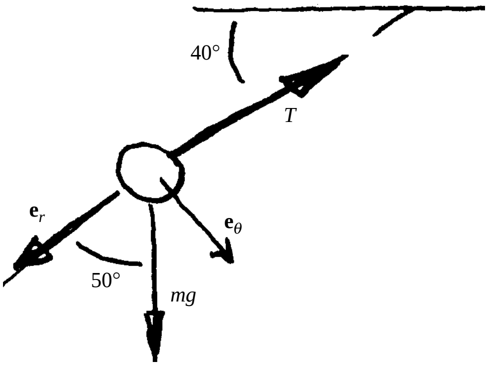
$$\tfrac{1}{2}(2)v_2^2 = 12.61$$

$$\underline{v_2 = 3.55 \text{ m/s}}$$

Problem 15.37 The 2-kg ball is released from rest in position 1 with the string horizontal. The length of the string is $L = 1$ m. What is the tension in the string when the ball is in position 2?

Strategy: Draw the free-body diagram of the ball when it is in position 2 and write Newton's second law in terms of normal and tangential components.

Solution: $m = 2$ kg



$\sum F_r$: $\quad -T + mg\cos 50^\circ = -mv_2^2/L$

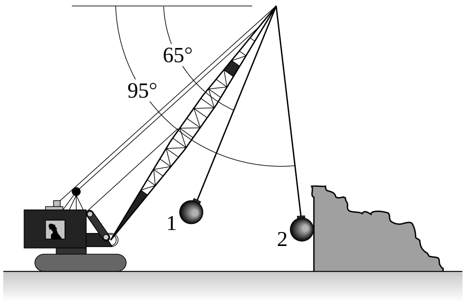
From Problem 15.36,

$v_2 = 3.55$ m/s

$T = mg\cos 50^\circ + mv^2/L$

$\underline{T = 37.8 \text{ N}}$

Problem 15.38 The 400-N wrecker's ball swings at the end of a 25-m cable. If the magnitude of the ball's velocity at position 1 is 4 m/s, what is the magnitude of its velocity just before it hits the wall at position 2?



Solution:

$U = -(400 \text{ N})(-25 \text{ m} \sin 95^\circ - [-25 \text{ m} \sin 65^\circ])$

$U = \frac{1}{2}\left(\frac{400 \text{ N}}{9.81 \text{ m/s}^2}\right)(v_2{}^2 - [4 \text{ m/s}]^2)$

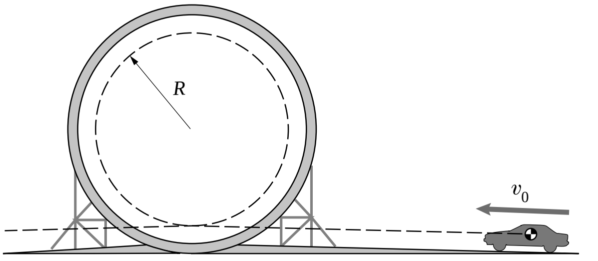
$\Rightarrow \boxed{v_2 = 7.75 \text{ m/s}}$

Problem 15.39 The 400-N wrecker's ball swings at the end of a 25-m cable. If the magnitude of the ball's velocity at position 1 is 4 m/s, what is the maximum tension in the cable as the ball swings from position 1 to position 2?

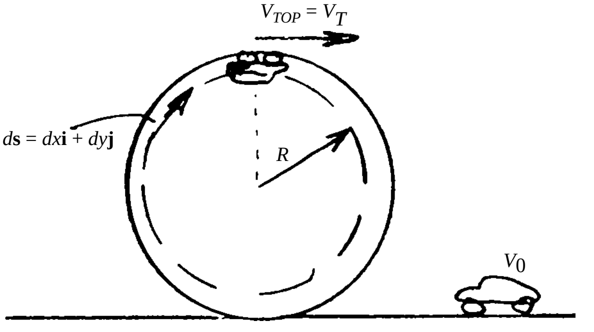
Solution: From the solution to Problem 15.37, the tension in the cable is $T = mg\sin\alpha + m\frac{v^2}{L}$. From the solution to Problem 15.38, $v^2 = 2gL[\sin\alpha - \sin 65^\circ](65^\circ \le \alpha \le 95^\circ)$, from which $T = 3\ mg\sin\alpha - 2\ mg\sin 65^\circ$. The maximum tension occurs when $\sin\alpha$ is a maximum in the interval $(65^\circ \le \alpha \le 95^\circ)$, from which

$\boxed{T = 3\ mg\sin 90^\circ - 2\ mg\sin 65^\circ = 474.9516 \text{ N}.}$

Problem 15.40 A stunt driver wants to drive a car through the circular loop of radius $R = 5$ m. Determine the minimum velocity v_0 at which the car can enter the loop and coast through without losing contact with the track. What is the car's velocity at the top of the loop?



Solution: First, let us find V_T



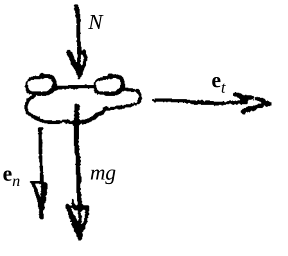
$\sum F_n$: $N + mg = mV_T^2/R$

For minimum velocity, $N \to 0$

$mg = mV_T^2/R$

$\underline{V_T = \sqrt{Rg} = 7.00 \text{ m/s}}$

Now find V_0 using work-energy



$$U_{0T} = \int_0^{10} -mg\mathbf{j} \cdot (dx\mathbf{i} + dy\mathbf{j})$$

$$U_{0T} = \int_0^{10} -mg\,dy = -mgy$$

$U_{0T} = -98.1m$ (N-m)

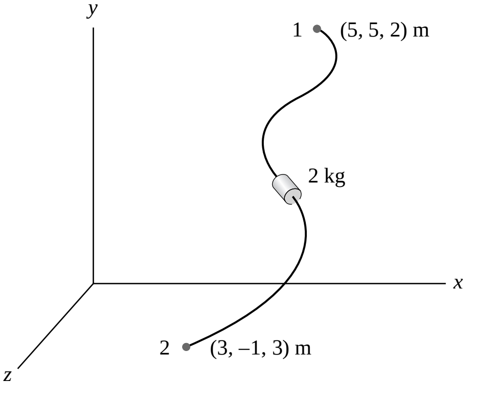
Also, $U_{0T} = \frac{1}{2}\cancel{m}V_T^2 - \frac{1}{2}\cancel{m}V_0^2 = -98.1\cancel{m}$ (N-m)

Solving for V_0 $\quad (V_T = 7.00 \text{ m/s})$

$V_0^2 = V_T^2 + (98.1)(2)$

$\underline{V_0 = 15.68 \text{ m/s}}$

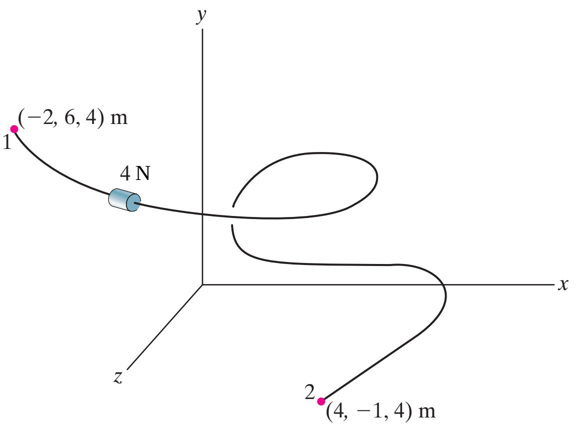
Problem 15.41 The 2-kg collar starts from rest at position 1 and slides down the smooth rigid wire. The y-axis points upward. What is the magnitude of the velocity of the collar when it reaches position 2?



Solution: The work done by the weight is $U_{\text{weight}} = mgh$, where $h = y_1 - y_2 = 5 - (-1) = 6$ m. From the principle of work and energy, $mgh = \frac{1}{2}mv^2$, from which

$$\boxed{v = \sqrt{2\,gh} = 10.85 \text{ m/s}}$$

Problem 15.42 The 4-N collar slides down the smooth rigid wire from position 1 to position 2. When it reaches position 2, the magnitude of its velocity is 24 m/s. What was the magnitude of its velocity at position 1?

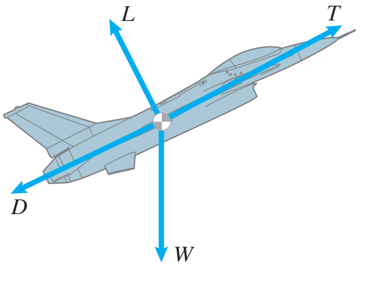


Solution:

$$U = (4\text{ N})(6 - [-1])\text{ m} = \frac{1}{2}\left(\frac{4\text{ N}}{9.81\text{ m/s}^2}\right)([24\text{ m/s}]^2 - v_1^2)$$

$$\Rightarrow \boxed{v_1 = 20.9\text{ m/s}}$$

Problem 15.43 The forces acting on the 125 kN airplane are the thrust T and drag D, which are parallel to the airplane's path, the lift L, which is perpendicular to the path, and the weight W. The airplane climbs from an altitude of 914 m to an altitude of 3048 m. During the climb, the magnitude of its velocity decreases from 244 m/s to 183 m/s.



(a) What work is done on the airplane by its lift during the climb?
(b) What work is done by the thrust and drag combined?

Solution:

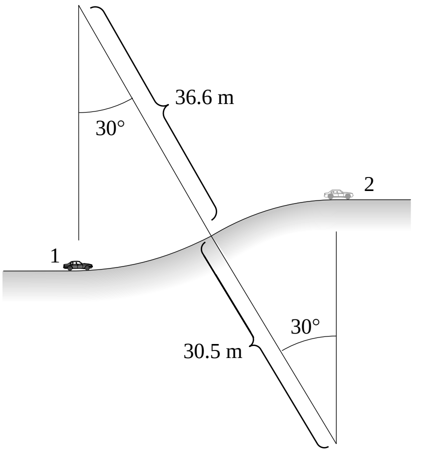
(a) The work due to the lift L is zero since it acts perpendicular to the motion.

(b) $U = U_{L+D} - (125000\text{ N})(3048 - 914)\text{ m}$

$$U = \frac{1}{2}\left(\frac{125000\text{ N}}{9.81\text{ m/s}^2}\right)([183\text{ m/s}]^2 - [244\text{ m/s}]^2)$$

$$\Rightarrow \boxed{U_{L+D} = 10.07 \times 10^7\text{ N-m}}$$

Problem 15.44 The 10.7 kN car is traveling 64.4 km/h at position 1. If the combined effect of the aerodynamic drag on the car and the tangential force exerted on its wheels by the road is that they exert no net tangential force on the car, what is the magnitude of the car's velocity at position 2?



Solution: The initial velocity is

$$v_1 = 64.4 \text{ km/h} = 17.9 \text{ m/s}$$

The change in elevation of the car is

$$h = 36.6(1 - \cos 30^\circ) + 30.5(1 - \cos 30^\circ)$$

$$= 67.1(1 - \cos 30^\circ) = 8.98 \text{ m}$$

The initial kinetic energy is

$$\frac{1}{2}\left(\frac{W}{g}\right) v_1^2 = 173895 \text{ N}$$

The work done by gravity is

$$U_{\text{gravity}} = \int_0^h (-W)\, ds = -Wh = -10700(h) = -95903.9 \text{ N-m}.$$

From the principle of work and energy the work done is equal to the gain in kinetic energy:

$$U_{\text{gravity}} = \frac{1}{2}\left(\frac{W}{g}\right) v_2^2 - \frac{1}{2}\left(\frac{W}{g}\right) v_1^2,$$

from which

$$\boxed{v_2 = \sqrt{\frac{2g(-95903.9 + 173895)}{W}} = 11.96 \text{ m/s}}$$

Problem 15.45 The 10.7 kN car is traveling 64.4 km/h at position 1. If the combined effect of aerodynamic drag on the car and the tangential force exerted on its wheels by the road is that they exert a constant 1.78 kN tangential force on the car in the direction of its motion, what is the magnitude of the car's velocity at position 2?

Solution: From the solution to Problem 15.44, the work done by gravity is $U_{\text{gravity}} = -95903.9$ N-m due to the change in elevation of the car of $h = 8.98$ m, and $\frac{1}{2}\left(\frac{W}{g}\right) v_1^2 = 173895$ N-m. The length of road between positions 1 and 2 is

$$s = 36.6(30^\circ)\left(\frac{\pi}{180^\circ}\right) + 30.5(30^\circ)\left(\frac{\pi}{180^\circ}\right) = 35.1 \text{ m}.$$

The work done by the tangential force is

$$U_{\text{tgt}} = \int_0^s 1780\, ds = 1780\,(35.1) = 62468.5 \text{ N-m}.$$

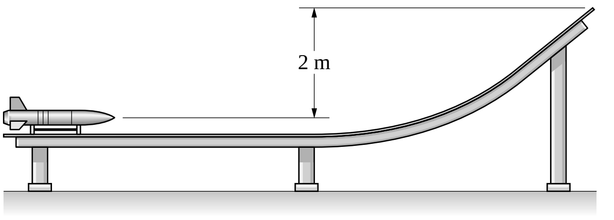
From the principle of work and energy

$$U_{\text{gravity}} + U_{\text{tgt}} = \frac{1}{2}\left(\frac{W}{g}\right) v_2^2 - \frac{1}{2}\left(\frac{W}{g}\right) v_1^2,$$

from which

$$\boxed{\begin{aligned} v &= \sqrt{\frac{2(9.81)(-95903.9 + 62468.5 + 173895)}{2400}} \\ &= 16.1 \text{ m/s} = 57.9 \text{ km/h} \end{aligned}}$$

Problem 15.46 The mass of the rocket is 250 kg. Its engine has a constant thrust of 45 kN. The length of the launching ramp is 10 m. If the magnitude of the rocket's velocity when it reaches the end of the ramp is 52 m/s, how much work is done on the rocket by friction and aerodynamic drag?



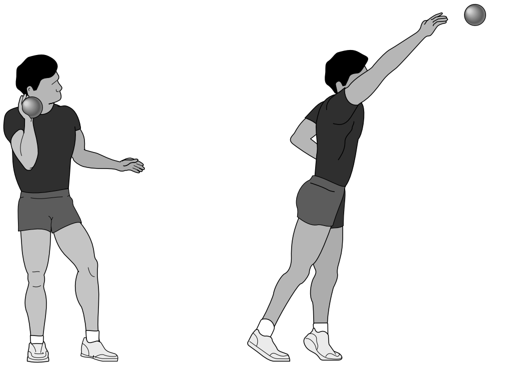
Solution:

$$U = U_{Fr+Dr} + (45\text{ kN})(10\text{ m}) - (250\text{ kg})(9.81\text{ m/s}^2)(2\text{ m})$$

$$U = \frac{1}{2}(250\text{ kg})(52\text{ m/s})^2$$

$$\boxed{U_{Fr+Dr} = -107\text{ kN-m}}$$

Problem 15.47 A bioengineer interested in energy requirements of sports determines from videotape that when the athlete begins his motion to throw the 7.25-kg shot (Fig. a), the shot is stationary and 1.50 m above the ground. At the instant the athlete releases it (Fig. b), the shot is 2.10 m above the ground. The shot reaches a maximum height of 4.60 m above the ground and travels a horizontal distance of 18.66 m from the point where it was released. How much work does the athlete do on the shot from the beginning of his motion to the instant he releases it?



(a) (b)

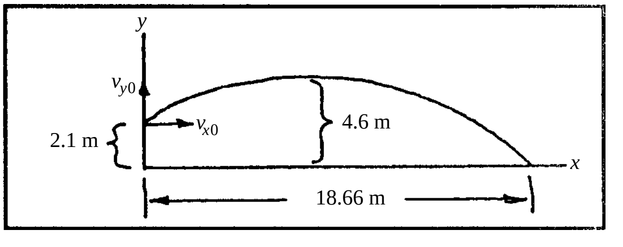
Solution: Let v_{x0} and v_{y0} be the velocity components at the instant of release. Using the chain rule,

$$a_y = \frac{dv_y}{dt} = \frac{dv_y}{dy}\frac{dy}{dt} = \frac{dv_y}{dy}v_y = -g,$$

and integrating,

$$\int_{v_{y0}}^{0} v_y\, dv_y = -g\int_{2.1}^{4.6} dy.$$

$-\frac{1}{2}v_{y0}{}^2 = -g(4.6 - 2.1)$, we find that $v_{y0} = 7.00$ m/s. The shot's x and y coordinates are given by $x = v_{x0}t$, $y = 2.1 + v_{y0}t - \frac{1}{2}gt^2$. Solving the first equation for t and substituting it into the second,



$$y = 2.1 + v_{y0}\left(\frac{x}{v_{x0}}\right) - \frac{1}{2}g\left(\frac{x}{v_{x0}}\right)^2$$

Setting $x = 18.66$ m, $y = 0$ in this equation and solving for v_{x0} gives $v_{x0} = 11.1$ m/s. The magnitude of the shot's velocity at release is $U_2 = \sqrt{v_{x0}{}^2 + v_{y0}{}^2} = 13.1$ m/s.

Let U_A be the work he does

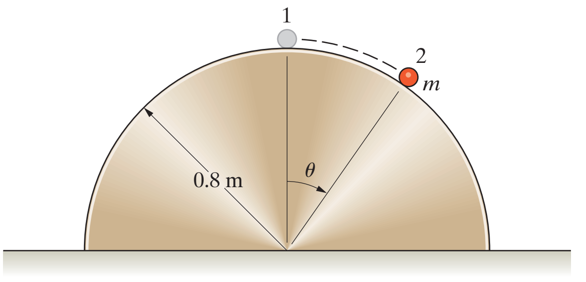
$$U_A - mg(y_2 - y_1) = \tfrac{1}{2}mv_2{}^2 - \tfrac{1}{2}mv_1{}^2$$

$$U_A - (7.25\text{ kg})(9.81\text{ m/s}^2)(2.1 - 1.5) = \tfrac{1}{2}(7.25)(13.1)^2 - 0,$$

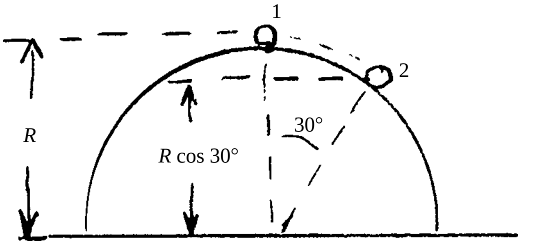
or $U_A = 666$ N-m

Problem 15.48 A small pellet of mass $m = 0.2$ kg starts from rest at position 1 and slides down the smooth surface of the cylinder to position 2, where $\theta = 30^\circ$.

(a) What work is done on the pellet as it slides from position 1 to position 2?
(b) What is the magnitude of the pellet's velocity at position 2?



Solution:



$v_1 = 0 \quad R = 0.8$ m

$$U_{12} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$U_{12} = (0.5)(0.2)v_2^2 = 0.1v_2^2$$

The work is

$$U_{12} = (0.2)(9.81)(0.8 - 0.8\cos 30^\circ)$$

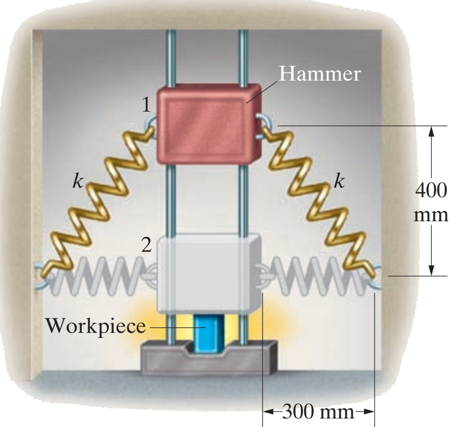
$$= 0.210 \text{ N-m.}$$

(a) $\underline{U_{12} = 0.210 \text{ (N-m)}}$

(b) $0.1v_2^2 = 0.210$ (N-m)

$\underline{v_2 = 1.45 \text{ m/s}}$

Problem 15.49 In Active Example 15.4, suppose that you want to increase the value of the spring constant k so that the velocity of the hammer just before it strikes the workpiece is 4 m/s. what is the required value of k?



Solution: The 40-kg hammer is released from rest in position 1. The springs are unstretched when in position 2. Neglect friction.

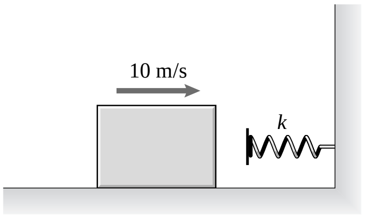
$$U_{12} = mgh + 2\frac{1}{2}kd^2 = \frac{1}{2}\,mv_2^2$$

$$k = \frac{m}{d^2}\left(\frac{v_2^2}{2} - gh\right)$$

$$k = \frac{40 \text{ kg}}{(0.2 \text{ m})^2}\left(\frac{[4 \text{ m}]^2}{2} - [9.81\text{m/s}^2][0.4\text{m}]\right)$$

$$\boxed{k = 4080 \text{ N/m.}}$$

Problem 15.50 Suppose that you want to design a bumper that will bring a 50-N. package moving at 10 m/s to rest 152.4 mm from the point of contact with bumper. If friction is negligible, what is the necessary spring constant k?



Solution: From the principle of work and energy, the work done on the spring must equal the change in kinetic energy of the package within the distance 152.4 m.

$$\frac{1}{2}kS^2 = \frac{1}{2}\left(\frac{W}{g}\right)v^2$$

from which

$$\boxed{k = \left(\frac{W}{g}\right)\left(\frac{v}{S}\right)^2 = \left(\frac{50}{9.81}\right)\left(\frac{10}{0.152}\right)^2 = 22060 \text{ N/m}}$$

Problem 15.51 In Problem 15.50, what spring constant is necessary if the coefficient of kinetic friction between the package and the floor is $\mu_k = 0.3$ and the package contacts the bumper moving at 10 m/s?

Solution: The work done on the spring over the stopping distance is

$$U_S = \int_0^S F\,ds = \int_0^S ks\,ds = \frac{1}{2}kS^2.$$

The work done by friction over the stopping distance is

$$U_f = \int_0^S F\,ds = \int_0^S \mu_k W\,ds = \mu_k W S.$$

From the principle of work and energy the work done must equal the kinetic energy of the package:

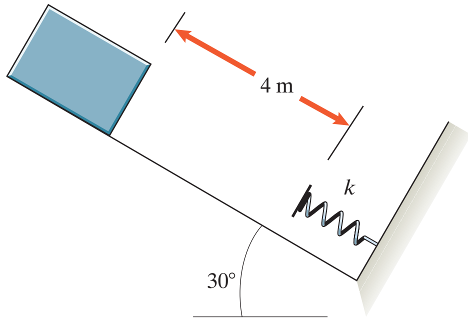
$$\frac{1}{2}kS^2 + \mu_k W S = \frac{1}{2}\left(\frac{W}{g}\right)v^2,$$

from which, for $S = 0.152$ m,

$$\boxed{k = \left(\frac{W}{g}\right)\frac{(v^2 - 2\,g\mu_k S)}{S^2} = 21863 \text{ N/m}}$$

Problem 15.52 The 50-N package starts from rest, slides down the smooth ramp, and is stopped by the spring.

(a) If you want the package to be brought to rest 0.5 m from the point of contact, what is the necessary spring constant k?
(b) What maximum deceleration is the package subjected to?



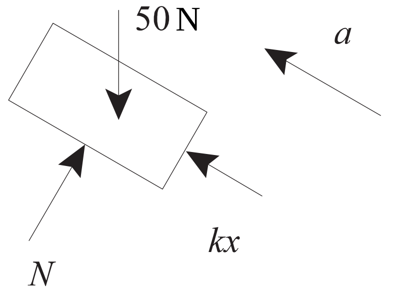
Solution:

(a) Find the spring constant

$$U_{12} = mgh - \frac{1}{2}kx^2 = 0$$

$$k = \frac{2mgh}{x^2} = \frac{2(50\ \text{N})(4.5\ \text{m})\sin 30^\circ}{(0.5\,\text{m})^2}$$

$$\boxed{k = 900\ \text{N/m}}$$



(b) The maximum deceleration occurs when the spring reaches the maximum compression (the force is then the largest).

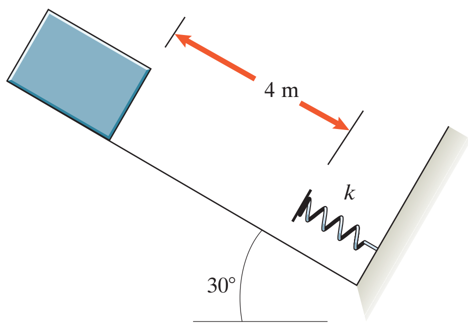
$$kx - mg\sin\theta = ma$$

$$a = \frac{k}{m}x - g\sin\theta$$

$$a = \frac{(900\ \text{N/m})}{\left(\dfrac{50\ \text{N}}{9.81\ \text{m/s}^2}\right)}(0.5\ \text{m}) - (9.81\,\text{m/s}^2)\sin 30^\circ$$

$$\boxed{a = 83.4\ \text{m/s}^2}$$

Problem 15.53 The 50-N package starts from rest, slides down the smooth ramp, and is stopped by the spring. The coefficient of static friction between the package and the ramp is μ k $= 0.12$. If you want the package to be brought to rest 0.5 m from the point of contact, what is the necessary spring constant k?



Solution: Find the spring constant

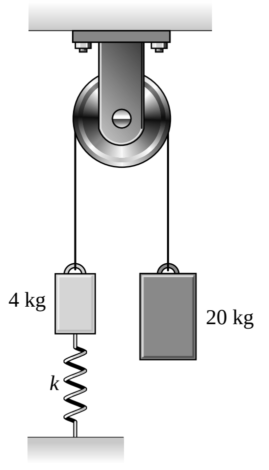
$$U_{12} = mgd\sin\theta - \mu_k mg\cos\theta\, d - \frac{1}{2}kx^2 = 0$$

$$k = \frac{2mgd}{x^2}(\sin\theta - \mu_k\cos\theta)$$

$$k = \frac{2(50\ \text{N})(4.5\,\text{m})}{(0.5\ \text{m})^2}(\sin 30^\circ - 0.12\cos 30^\circ)$$

$$\boxed{k = 713\ \text{N/m.}}$$

Problem 15.54 The system is released from rest with the spring unstretched. The spring constant $k = 200$ N/m. Determine the magnitude of the velocity of the masses when the right mass has fallen 1 m.



Solution: When the larger mass falls 1 m, the smaller mass rises 1 m and the spring stretches 1 m. For the system of two masses, springs, and the cable,

$$U_{12} = \int_0^1 (-ks)\,ds + \int_0^1 (-m_1 g)\,ds + \int_0^1 m_2 g\,ds$$

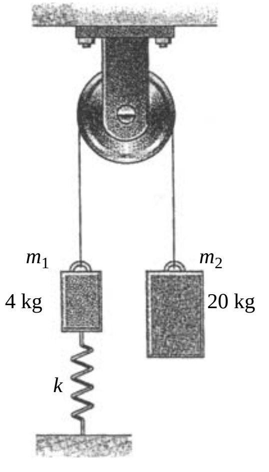
$$U_{12} = -\tfrac{1}{2}ks^2\Big|_0^1 - m_1 gs\Big|_0^1 + m_2 gs\Big|_0^1$$

$$U_{12} = -\tfrac{1}{2}k - 4(9.81) + (20)(9.81)$$

$$U_{12} = 56.96 \text{ N-m}$$

Also $U_{12} = \frac{1}{2}(m_1 + m_2)V_f^2 - 0$

Solving $\underline{V_f = 2.18 \text{ m/s}}$



Problem 15.55 The system is released from rest with the spring unstretched. The spring constant $k = 200$ N/m. What maximum downward velocity does the right mass attain as it falls?

Solution: From the solution to Problem 15.54,

$$U_{12} = -\frac{1}{2}Ks^2 + (m_2 - m_1)gs$$

and

$$U_{12} = \frac{1}{2}(m_1 + m_2)V^2$$

For all s. Setting these equal, we get

$$\frac{1}{2}(m_1 + m_2)V^2 = (m_2 - m_1)gs - \frac{1}{2}Ks^2 \quad \textbf{(1)}$$

Solve for $\dfrac{dv}{ds}$ and set $\dfrac{dv}{ds}$ to zero

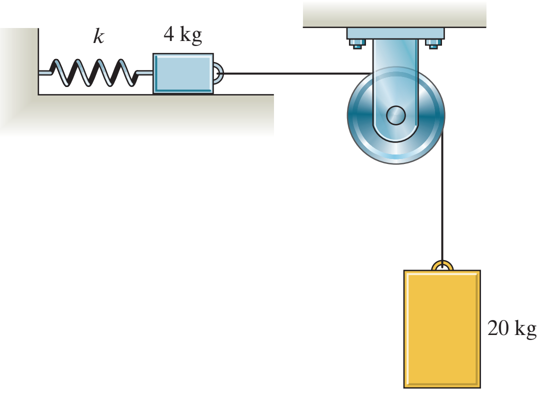
$$\frac{1}{2}(m_1 + m_2)2v\frac{dv}{ds} = (m_2 - m_1)g - Ks = 0$$

The extreme value for V occurs at

$$S = \frac{(m_2 - m_1)g}{K} = 0.785 \text{ m}$$

Substituting this back into (1) and solving, we get $\underline{V = 2.27 \text{ m/s}}$

Problem 15.56 The system is released from rest. The 4-kg mass slides on the smooth horizontal surface. The spring constant is $k = 100$ N/m, and the tension in the spring when the system is released is 50 N. By using the principle of work and energy, determine the magnitude of the velocity of the masses when the 20-kg mass has fallen 1 m.



Solution:

$$50 \text{ N} = (100 \text{ N/m})x_1 \Rightarrow x_1 = 0.5 \text{ m}$$

$$U = (20 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m}) - \frac{1}{2}(100 \text{ N/m})([1.5 \text{ m}]^2 - [0.5 \text{ m}]^2)$$

$$U = \frac{1}{2}(24 \text{ kg})(v_2{}^2 - 0)$$

$$\boxed{v_2 = 2.83 \text{ m/s}}$$

Problem 15.57 Solve Problem 15.56 if the coefficient of kinetic friction between the 4-kg mass and the horizontal surface is $\mu_k = 0.4$.

Solution:

$$50 \text{ N} = (100 \text{ N/m})x_1 \Rightarrow x_1 = 0.5 \text{ m}$$

$$U = (20 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m}) - \frac{1}{2}(100 \text{ N/m})([1.5 \text{ m}]^2 - [0.5 \text{ m}]^2)$$

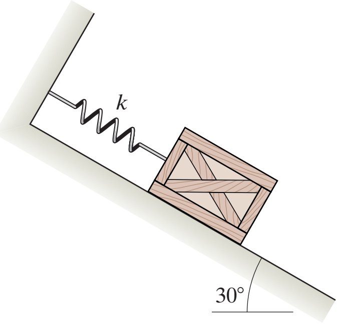
$$- (0.4)(4 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m})$$

$$U = \frac{1}{2}(24 \text{ kg})(v_2{}^2 - 0)$$

$$\boxed{v_2 = 2.59 \text{ m/s}}$$

Problem 15.58 The 40-N crate is released from rest on the smooth inclined surface with the spring unstretched. The spring constant is $k = 8$ N/m.

(a) How far down the inclined surface does the crate slide before it stops?
(b) What maximum velocity does the crate attain on its way down?



Solution: At an arbitrary distance s down the slope we have:

$$U = (40\text{ N})(s\sin 30^\circ) - \frac{1}{2}(8\text{ N/m})s^2 = \frac{1}{2}\left(\frac{40\text{ N}}{9.81\text{ m/s}^2}\right)v^2$$

(a) When it stops, we set $v = 0$ and solve for $\boxed{s = 5\text{ m}}$

(b) Solving for v^2, we have

$$v^2 = 0.491(20s - 4s^2) \Rightarrow \frac{dv^2}{ds} = 0.491(20 - 8s) = 0 \Rightarrow s = 2.5\text{ m}$$

Using $s = 2.5$ m $\Rightarrow$ $\boxed{v_{\max} = 3.5\text{ m/s}}$

Problem 15.59 Solve Problem 15.58 if the coefficient of kinetic friction between the 4-kg mass and the horizontal surface is $\mu_k = 0.2$.

Solution: At an arbitrary distance s down the slope we have:

$$U = (40\text{ N})(s\sin 30^\circ) - \frac{1}{2}(8\text{ N/m})s^2 - (0.2)(40\text{ N}\cos 30^\circ)s$$

$$= \frac{1}{2}\left(\frac{40\text{ N}}{9.81\text{ m/s}^2}\right)v^2$$

(a) When it stops, we set $v = 0$ and solve for $\boxed{s = 3.27\text{ m}}$

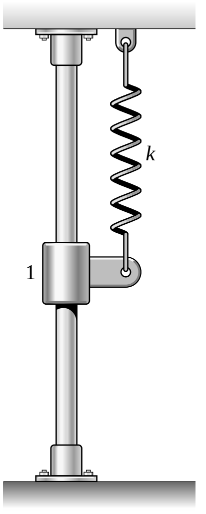
(b) Solving for v^2, we have

$$v^2 = 0.491(13.07s - 4s^2) \Rightarrow \frac{dv^2}{ds} = 0.491(13.07 - 8s) = 0$$

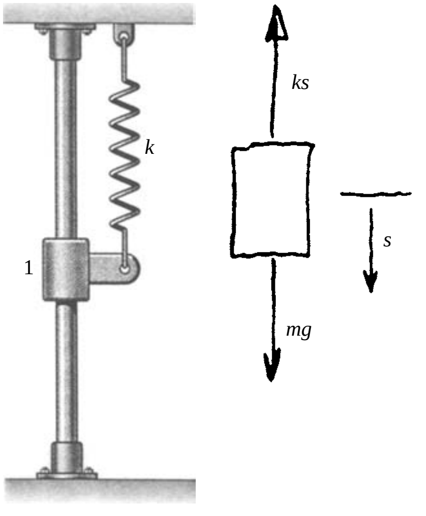
$$\Rightarrow s = 1.63\text{ m}$$

Using $s = 1.63$ m $\Rightarrow$ $\boxed{v_{\max} = 2.29\text{ m/s}}$

Problem 15.60 The 4-kg collar starts from rest in position 1 with the spring unstretched. The spring constant is $k = 100$ N/m. How far does the collar fall relative to position 1?



Solution:



$V_0 = V_f = 0$

Let position 2 be the location where the collar comes to rest

$$U_{12} = -\frac{Ks^2}{2} + mgs$$

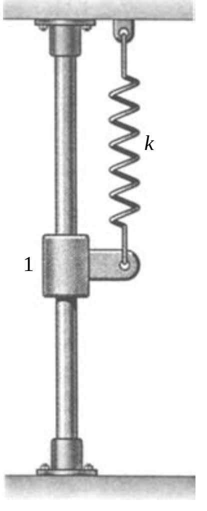
Also $U_{12} = \frac{1}{2}mV_f^2 - \frac{1}{2}mV_0^2 = 0$

Thus $0 = mgs - \dfrac{Ks^2}{2}$

$$s(2mg - Ks) = 0$$

Solving, $\underline{s = 0.785 \text{ m}}$.

Problem 15.61 In position 1 on the smooth bar, the 4-kg collar has a downward velocity of 1 m/s and the spring is unstretched. The spring constant is $k = 100$ N/m. What maximum downward velocity does the collar attain as it falls?



Solution: The work is

$$U_{12} = -\frac{Ks^2}{2} + mgs$$

Also,

$$U_{12} = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$$

where $m = 4$ kg and $V_1 = 1$ m/s

Thus,

$$\frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2 = -\frac{Ks^2}{2} + \text{ mgs} \quad \textbf{(1)}$$

Finding $\frac{dV_2}{ds}$, and setting it to zero,

$$mV_2\frac{dV_2}{ds} = -Ks + mg = 0$$

$$s = mg/k = 0.392 \text{ m}$$

Solving (1) for V_2 we get $\underline{V_2 = 2.20 \text{ m/s}}$

Problem 15.62 The 4-kg collar starts from rest in position 1 on the smooth bar. The tension in the spring in position 1 is 20 N. The spring constant is $k = 100$ N/m. How far does the collar fall relative to position 1?

Solution: For this problem, we need a new reference for the spring. If the tension in the spring is 20 N

$T = k\delta_0$ and $K = 100$ N/m

$\delta_0 = 0.2$ m. (the initial stretch)

In this case, we have

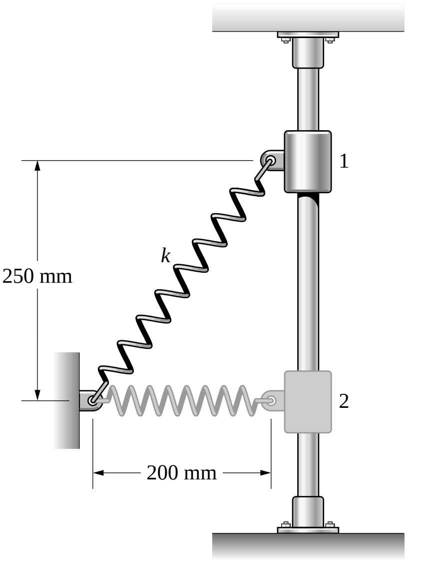
$$U_{12} = -\frac{Ks^2}{2}\Big|_{0.2}^{s+0.2} + mgs\Big|_{0.2}^{s+0.2}$$

Also $V_0 = V_f = 0 \qquad \therefore U_{12} = 0$

$$0 = -\frac{K(s+0.2)^2}{2} + \frac{K(0.2)^2}{2} + mg(s+0.2) - mg(0.2)$$

Solving, we get $\underline{s = 0.385 \text{ m}}$

Problem 15.63 The 4-kg collar is released from rest at position 1 on the smooth bar. If the spring constant is $k = 6$ kN/m and the spring is unstretched in position 2, what is the velocity of the collar when it has fallen to position 2?



Solution: Denote $d = 200$ mm, $h = 250$ mm. The stretch of the spring in position 1 is $S_1 = \sqrt{h^2 + d^2} - d = 0.120$ m and at 2 $S_2 = 0$. The work done by the spring on the collar is

$$U_{\text{spring}} = \int_{0.12}^{0}(-ks)\,ds = \left[-\frac{1}{2}ks^2\right]_{0.120}^{0} = 43.31 \text{ N-m.}$$

The work done by gravity is

$$U_{\text{gravity}} = \int_{0}^{-h}(-mg)\,ds = mgh = 9.81 \text{ N-m.}$$

From the principle of work and energy $U_{\text{spring}} + U_{\text{gravity}} = \frac{1}{2}mv^2$, from which

$$v = \sqrt{\left(\frac{2}{m}\right)(U_{\text{spring}} + U_{\text{gravity}})} = 5.15 \text{ m/s}$$

Problem 15.64 The 4-kg collar is released from rest in position 1 on the smooth bar. The spring constant is $k = 4$ kN/m. The tension in the spring in position 2 is 500 N. What is the velocity of the collar when it has fallen to position 2?

Solution: Denote $d = 200$ mm, $h = 250$ mm. The stretch of the spring at position 2 is

$$S_2 = \frac{T}{k} = \frac{500}{4000} = 0.125 \text{ m.}$$

The unstretched length of the spring is $L = d - S_2 = 0.2 - 0.125 = 0.075$ m. The stretch of the spring at position 1 is $S_1 = \sqrt{h^2 + d^2} - L = 0.245$ m. The work done by the spring is

$$U_{\text{spring}} = \int_{S_1}^{S_2}(-ks)\,ds = \frac{1}{2}k(S_1^2 - S_2^2) = 88.95 \text{ N-m.}$$

The work done by gravity is $U_{\text{gravity}} = mgh = 9.81$ N-m. From the principle of work and energy is $U_{\text{spring}} + U_{\text{gravity}} = \frac{1}{2}mv^2$, from which

$$v = \sqrt{\frac{2(U_{\text{spring}} + U_{\text{gravity}})}{m}} = 7.03 \text{ m/s}$$

Problem 15.65 The 4-kg collar starts from rest in position 1 on the smooth bar. Its velocity when it has fallen to position 2 is 4 m/s. The spring is unstretched when the collar is in position 2. What is the spring constant k?

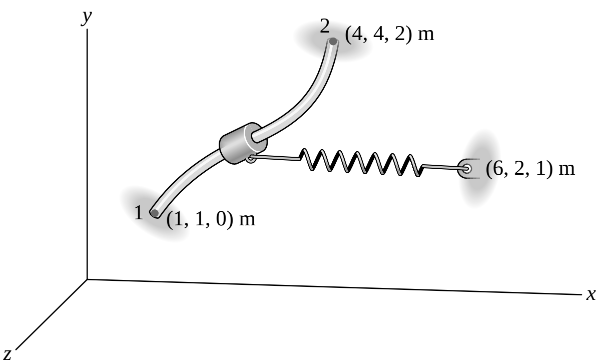
Solution: The kinetic energy at position 2 is $\frac{1}{2}mv^2 = 32$ N-m. From the solution to Problem 15.63, the stretch of the spring in position 1 is $S_1 = \sqrt{h^2 + d^2} - d = 0.120$ m. The potential of the spring is

$$U_{\text{spring}} = \int_{S_1}^{0} (-ks)\,ds = \frac{1}{2}kS_1^2.$$

The work done by gravity is $U_{\text{gravity}} = mgh = 9.81$ N-m. From the principle of work of work and energy, $U_{\text{spring}} + U_{\text{gravity}} = \frac{1}{2}mv^2$. Substitute and solve:

$$k = \frac{2\left(\frac{1}{2}mv^2 - U_{\text{gravity}}\right)}{S_1^2} = 3082 \text{ N/m}$$

Problem 15.66 The 10-kg collar starts from rest at position 1 and slides along the smooth bar. The y-axis points upward. The spring constant is $k = 100$ N/m and the unstretched length of the spring is 2 m. What is the velocity of the collar when it reaches position 2?



Solution: The stretch of the spring at position 1 is

$$S_1 = \sqrt{(6-1)^2 + (2-1)^2 + (1-0)^2} - 2 = 3.2 \text{ m}.$$

The stretch of the spring at position 2 is

$$S_2 = \sqrt{(6-4)^2 + (2-4)^2 + (1-2)^2} - 2 = 1 \text{ m}.$$

The work done by the spring is

$$U_{\text{spring}} = \int_{S_1}^{S_2} (-ks)\,ds = \frac{1}{2}k(S_1^2 - S_2^2) = 460.8 \text{ N-m}.$$

The work done by gravity is

$$U_{\text{gravity}} = \int_0^h (-mg)\,ds = -mgh = -(10)(9.81)(4-1)$$

$$= -294.3 \text{ N-m}.$$

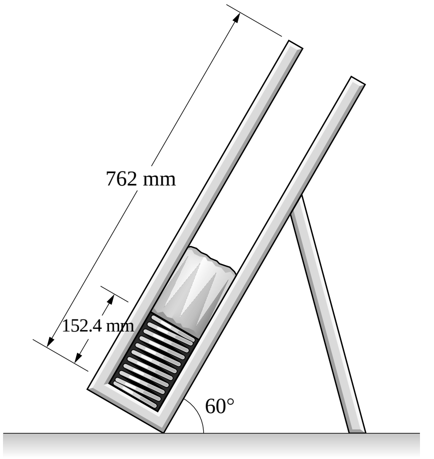
From the principle of work and energy:

$$U_{\text{spring}} + U_{\text{gravity}} = \frac{1}{2}mv^2,$$

from which

$$v = \sqrt{\frac{2(U_{\text{spring}} + U_{\text{gravity}})}{m}} = 5.77 \text{ m/s}$$

Problem 15.67 A spring-powered mortar is used to launch 44.5 N packages of fireworks into the air. The package starts from rest with the spring compressed to a length of 152.4 mm. The unstretched length of the spring is 762 mm. If the spring constant is $k = 1762$ N/m, what is the magnitude of the velocity of the package as it leaves the mortar?



Solution: Equating the work done to the change in the kinetic energy,

$$-\tfrac{1}{2}k(S_2^2 - S_1^2) - mg(y_2 - y_1) = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 :$$

$$-\tfrac{1}{2}(1762)[0 - (0.61\text{m})^2] - (44.5)(0.61\sin 60^\circ\text{ft}) = \tfrac{1}{2}(44.5/9.81)v_2^2 - 0.$$

Solving, we obtain $v_2 = 39.3$ m/s.

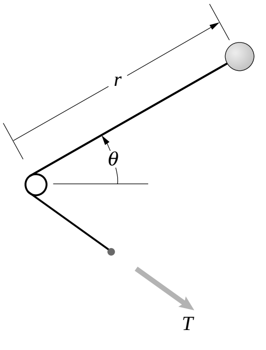
Problem 15.68 Suppose that you want to design the mortar in Problem 15.67 to throw the package to a height of 45.7 m above its initial position. Neglecting friction and drag, determine the necessary spring constant.

Solution: See the solution of Problem 15.67. Let v_2 be the velocity as the package leaves the barrel. To reach 45.7 m, $mg(45.7 - 0.61\sin 60^\circ) = \tfrac{1}{2}m(v_2 \sin 60^\circ)^2$. Solving, we obtain $v_2 = 34.4$ m/s. Work and energy inside the barrel is

$$-\tfrac{1}{2}k[0 - (0.61\text{m})^2] - (44.5)(0.61\sin 60^\circ\text{m}) = \tfrac{1}{2}(44.5/9.81)(34.4)^2 - 0,$$

which gives $\underline{k = 14549\text{ N/m.}}$

Problem 15.69 Suppose an object has a string or cable with *constant* tension T attached as shown. The force exerted on the object can be expressed in terms of polar coordinates as $\mathbf{F} = -T\mathbf{e}_r$. Show that the work done on the object as it moves along an *arbitrary* plane path from a radial position r_1 to a radial position r_2 is $U_{12} = -T(r_1 - r_2)$.



Solution: The work done on the object is

$$U = \int_{r_1}^{r_2} \mathbf{F}\cdot d\mathbf{s}.$$

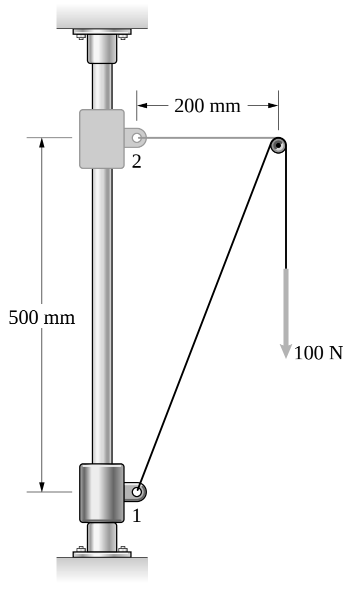
Suppose that the arbitrary path is defined by $d\mathbf{r} = (dr\mathbf{e}_r + rd\theta\mathbf{e}_\theta)$, and the work done is

$$U = \int_{r_1}^{r_2} \mathbf{F}\cdot d\mathbf{r} = \int_{r_1}^{r_2} -T(\mathbf{e}_r\cdot\mathbf{e}_r)\,dr + \int_{r_1}^{r_2} T(r\mathbf{e}_r\cdot\mathbf{e}_\theta)r\,d\theta$$

$$= -\int_{r_1}^{r_2} T\,dr = -T(r_2 - r_1)$$

since $\mathbf{e}_r\cdot\mathbf{e}_s = 0$ by definition.

Problem 15.70 The 2-kg collar is initially at rest at position 1. A constant 100-N force is applied to the string, causing the collar to slide up the smooth vertical bar. What is the velocity of the collar when it reaches position 2? (See Problem 15.69.)



Solution: The constant force on the end of the string acts through a distance $s = \sqrt{0.5^2 + 0.2^2} - 0.2 = 0.3385$ m. The work done by the constant force is $U_F = Fs = 33.85$ N-m. The work done by gravity on the collar is

$$U_{\text{gravity}} = \int_0^h (-mg)\,ds = -mgh = -(2)(9.81)(0.5)$$

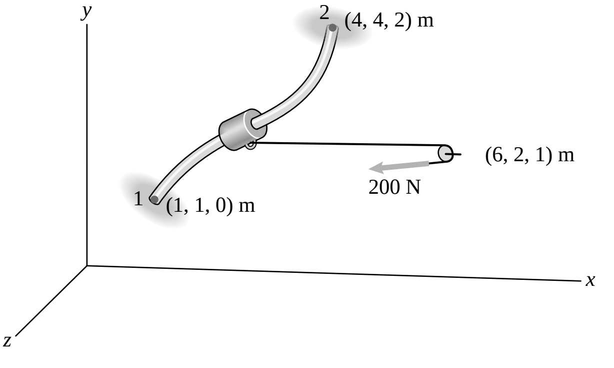
$$= -9.81 \text{ N-m.}$$

From the principle of work and energy:

$$U_F + U_{\text{gravity}} = \frac{1}{2}mv^2,$$

from which $v = \sqrt{\dfrac{2(U_F + U_{\text{gravity}})}{m}} = 4.90$ m/s

Problem 15.71 The 10-kg collar starts from rest at position 1. The tension in the string is 200 N, and the y axis points upward. If friction is negligible, what is the magnitude of the velocity of the collar when it reaches position 2? (See Problem 15.69.)



Solution: The constant force moves a distance

$$s = \sqrt{(6-1)^2 + (2-1)^2 + (1-0)^2}$$

$$- \sqrt{(6-4)^2 + (2-4)^2 + (1-2)^2} = 2.2 \text{ m.}$$

The work done by the constant force is

$$U_F = \int_0^s F\,ds = Fs = 439.2 \text{ N-m.}$$

The work done by gravity is

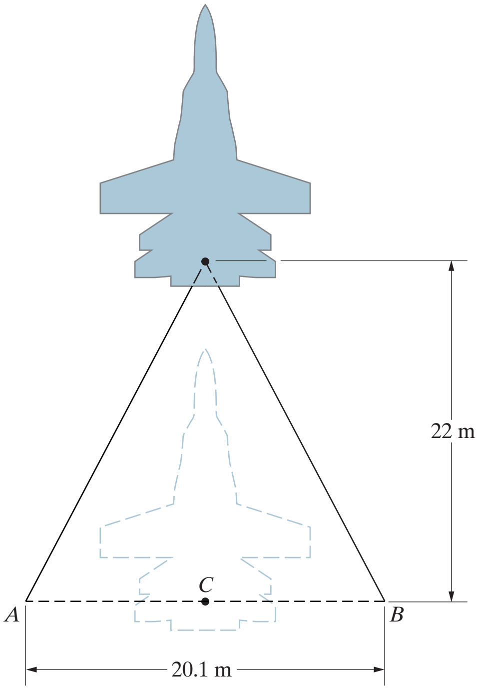
$$U_{\text{gravity}} = \int_0^h (-mg)\,ds = -mgh = -(10)(9.81)(3)$$

$$= -294.3 \text{ N-m.}$$

From the principle of work and energy $U_F + U_{\text{gravity}} = \frac{1}{2}mv^2$, from which

$$\boxed{v = \sqrt{\frac{2(U_F + U_{\text{gravity}})}{10}} = 5.38 \text{ m/s}}$$

Problem 15.72 As the F/A-18 lands at 64 m/s, the cable from A to B engages the airplane's arresting hook at C. The arresting mechanism maintains the tension in the cable at a constant value, bringing the 115.6 kN airplane to rest at a distance of 22 m. What is the tension in the cable? (See Problem 15.69.)



Solution:

$$U = -2T\left(\sqrt{(22\text{ m})^2 + (10.1\text{ m})^2} - 10.1\text{ m}\right)$$

$$= \frac{1}{2}\left(\frac{115600\text{ N}}{9.81\text{ m/s}^2}\right)(0 - [64\text{ m/s}]^2)$$

$$\boxed{T = 858.5\text{ kN}}$$

Problem 15.73 If the airplane in Problem 15.72 lands at 73.2 m/s, what distance does it roll before the arresting system brings it to rest?

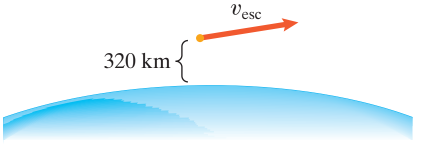
Solution:

$$U = -2(858500)\left(\sqrt{s^2 + (10.1\text{ m})^2} - 10.1\text{ m}\right)$$

$$= \frac{1}{2}\left(\frac{115600}{9.81\text{ m/s}^2}\right)(0 - [64\text{ m/s}]^2)$$

Solving we find $\boxed{s = 26.61\text{ m}}$

Problem 15.74 A spacecraft 320 km above the surface of the earth is moving at escape velocity $v_{esc} = 10{,}900$ m/s. What is its distance from the center of the earth when its velocity is 50 percent of its initial value? The radius of the earth is 6370 km. (See Example 15.6.)



Solution:

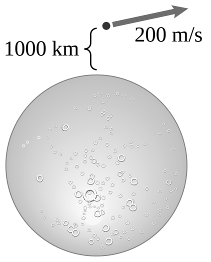
$$U_{12} = mgR_E^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$r_2 = \left[\frac{v_2^2 - v_1^2}{2gR_E^2} + \frac{1}{r_1}\right]^{-1}$$

$$r_2 = \left[\frac{(5450\text{ m/s})^2 - (10{,}900\text{ m/s})^2}{2(9.81\text{ m/s}^2)(6{,}370{,}000\text{ m})^2} + \frac{1}{6{,}690{,}000\text{ m}}\right]^{-1}$$

$$\boxed{r_2 = 26{,}600\text{ km.}}$$

Problem 15.75 A piece of ejecta is thrown up by the impact of a meteor on the moon. When it is 1000 km above the moon's surface, the magnitude of its velocity (relative to a nonrotating reference frame with its origin at the center of the moon) is 200 m/s. What is the magnitude of its velocity just before it strikes the moon's surface? The acceleration due to gravity at the surface of the moon is 1.62 m/s^2. The moon's radius is 1738 km.



Solution: The kinetic energy at $h = 1000$ km is

$$\left[\frac{m}{2}v^2\right]_{R_M+h} = 2 \times 10^4 \text{ N-m.}$$

The work done on the ejecta as it falls from 1000 km is

$$U_{\text{ejecta}} = \int_{R_{M+h}}^{R_M} (-W_{\text{ejecta}})\,ds = \int_{R_M+h}^{R_M} \left(-mg_M\frac{R_M^2}{s^2}\,ds\right)$$

$$= \left[mg_M\frac{R_M^2}{s}\right]_{R_M+h}^{R_M} = mg_M R_M\frac{h}{R_M+h},$$

$U_{\text{ejecta}} = 1.028$ m $\times\, 10^6$ N-m. From the principle of work and energy, at the Moon's surface:

$$U_{\text{ejecta}} = \left[\frac{m}{2}v^2\right]_{\text{surface}} - \left[\frac{m}{2}v^2\right]_{R_M+h}$$

from which $\boxed{v_{\text{surface}} = \sqrt{2(1.028 \times 10^6 + 2 \times 10^4)} = 1448 \text{ m/s}}$

Problem 15.76 A satellite in a circular orbit of radius r around the earth has velocity $v = \sqrt{gR_E^2/r}$, where $R_E = 6370$ km is the radius of the earth. Suppose you are designing a rocket to transfer a 900-kg communication satellite from a circular parking orbit with 6700-km radius to a circular geosynchronous orbit with 42,222-km radius. How much work must the rocket do on the satellite?

Solution: Denote the work to be done by the rocket by U_{rocket}. Denote $R_{\text{park}} = 6700$ km, $R_{\text{geo}} = 42222$ km. The work done by the satellite's weight as it moves from the parking orbit to the geosynchronous orbit is

$$U_{\text{transfer}} = \int_{R_{\text{park}}}^{R_{\text{geo}}} F\,ds = \int_{R_{\text{park}}}^{R_{\text{geo}}} \left(-mg\frac{R_E^2}{s^2}\,ds\right)$$

$$= \left[mg\frac{R_E^2}{s}\right]_{R_{\text{park}}}^{R_{\text{geo}}} = mgR_E^2\left(\frac{1}{R_{\text{geo}}} - \frac{1}{R_{\text{park}}}\right),$$

$U_{\text{transfer}} = -4.5 \times 10^9$ N-m. From the principle of work and energy:

$$U_{\text{transfer}} + U_{\text{rocket}} = \left[\frac{1}{2}mv^2\right]_{\text{geo}} - \left[\frac{1}{2}mv^2\right]_{\text{park}}.$$

from which

$$U_{\text{rocket}} = \left[\frac{1}{2}mv^2\right]_{\text{geo}} - \left[\frac{1}{2}mv^2\right]_{\text{park}} - U_{\text{transfer}}.$$

Noting

$$\left[\frac{1}{2}mv^2\right]_{\text{geo}} = \frac{m}{2}\left(\frac{gR_E^2}{R_{\text{geo}}}\right) = 4.24 \times 10^9 \text{ N-m},$$

$$\left[\frac{1}{2}mv^2\right]_{\text{park}} = \frac{m}{2}\left(g\frac{R_E^2}{R_{\text{park}}}\right) = 2.67 \times 10^{10} \text{ N-m},$$

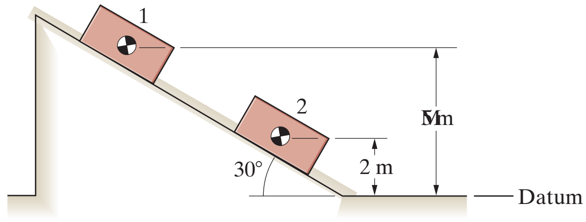
from which $\boxed{U_{\text{rocket}} = 2.25 \times 10^{10} \text{ N-m}}$

Problem 15.77 The force exerted on a charged particle by a magnetic field is $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, where q and $\mathbf{v}$ are the charge and velocity of the particle and $\mathbf{B}$ is the magnetic field vector. Suppose that other forces on the particle are negligible. Use the principle of work and energy to show that the magnitude of the particle's velocity is constant.

Solution: The force vector $\mathbf{F}$ is given by a cross product involving $\mathbf{v}$. This means that the force vector is ALWAYS perpendicular to the velocity vector. Hence, the force field does no work on the charged particle - it only changes the direction of its motion. Hence, if work is zero, the change in kinetic energy is also zero and the velocity of the charged particle is constant.

Problem 15.78 The 10-N box is released from rest at position 1 and slides down the smooth inclined surface to position 2.

(a) If the datum is placed at the level of the floor as shown, what is the sum of the kinetic and potential energies of the box when it is in position 1?
(b) What is the sum of the kinetic and potential energies of the box when it is in position 2?
(c) Use conservation of energy to determine the magnitude of the box's velocity when it is in position 2.



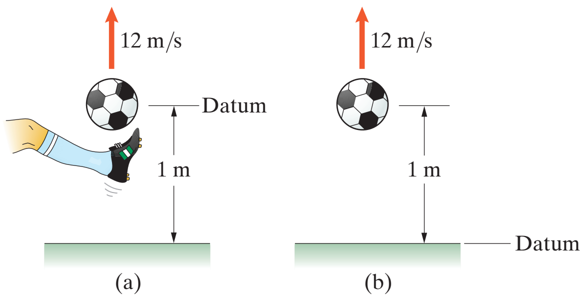
Solution:

(a) $\boxed{T_1 + V_1 = 0 + (10\text{ N})(5\text{ m}) = 50\text{ N-m}}$

(b) $\boxed{T_1 + V_1 = T_2 + V_2 = 50\text{ N-m}}$

(c) $50\text{ N-m} = \dfrac{1}{2}\left(\dfrac{10\text{ N}}{9.81\text{ m/s}^2}\right)v_2{}^2 + (10\text{ N})(2\text{ m}) \Rightarrow \boxed{v_2 = 7.67\text{ m/s}}$

Problem 15.79 The 0.45-kg soccer ball is 1 m above the ground when it is kicked upward at 12 m/s. Use conservation of energy to determine the magnitude of the ball's velocity when it is 4 m above the ground. Obtain the answer by placing the datum (a) at the level of the ball's initial position and (b) at ground level.



Solution:

(a) $T_1 = \dfrac{1}{2}(0.45\text{ kg})(12\text{ m/s})^2,\ V_1 = 0$

$T_2 = \dfrac{1}{2}(0.45\text{ kg})v_2{}^2,\ V_2 = (0.45\text{ kg})(9.81\text{ m/s}^2)(3\text{ m})$

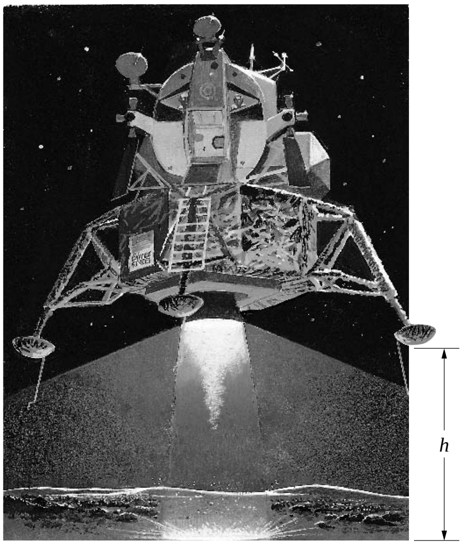
$T_1 + V_1 = T_2 + V_2 \Rightarrow \boxed{v_2 = 9.23\text{ m/s}}$

(b) $T_1 = \dfrac{1}{2}(0.45\text{ kg})(12\text{ m/s})^2,\ V_1 = (0.45\text{ kg})(9.81\text{ m/s}^2)(1\text{ m})$

$T_2 = \dfrac{1}{2}(0.45\text{ kg})v_2{}^2, V_2 = (0.45\text{ kg})(9.81\text{ m/s}^2)(4\text{ m})$

$T_1 + V_1 = T_2 + V_2 \Rightarrow \boxed{v_2 = 9.23\text{ m/s}}$

Problem 15.80 The Lunar Module (LM) used in the Apollo moon landings could make a safe landing if the magnitude of its vertical velocity at impact was no greater than 5 m/s. Use conservation of energy to determine the maximum height h at which the pilot could shut off the engine if the vertical velocity of the lander is (a) 2 m/s downward and (b) 2 m/s upward. The acceleration due to gravity at the moon's surface is 1.62 m/s^2.



Solution: Use conservation of energy Let state 1 be at the max height and state 2 at the surface. Datum is at the lunar surface.

(a), (b) $\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv_2^2 + 0$

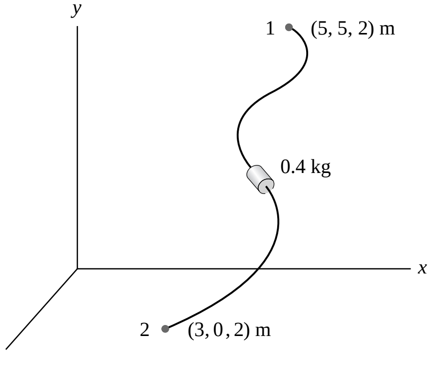
$$v_2 = 5 \text{ m/s} \quad g = 1.62 \text{ m/s}^2 \quad v_1 = \pm 2 \text{ m/s}$$

(m cancels from the equation.)

$$h = \frac{1}{2g}(v_2^2 - v_1^2)$$

(The sign of V_1 does not matter since v_1^2 is the only occurrence of v_1 in the relationship). Solving $\underline{h = 6.48 \text{ m}}$

Problem 15.81 The 0.4-kg collar starts from rest at position 1 and slides down the smooth rigid wire. The y axis points upward. Use conservation of energy to determine the magnitude of the velocity of the collar when it reaches point 2.



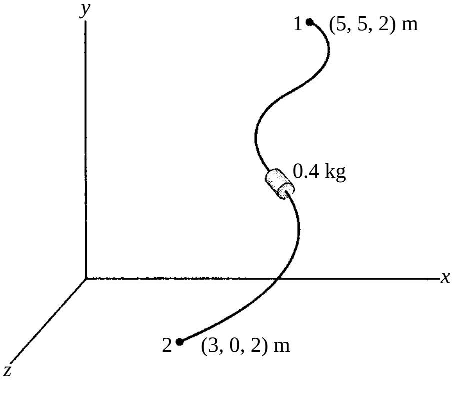
Solution: Assume gravity acts in the $-y$ direction and that $y = 0$ is the datum. By conservation of energy, $\frac{1}{2}mv^2 + V =$ constant where $V = mgy$. Thus,

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$$

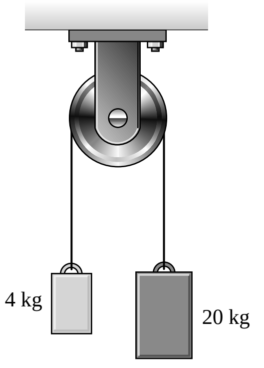
$m = 0.4$ kg, $g = 9.81$ m/s^2, $v_1 = 0$, $y_2 = 0$, and $y_1 = 5$ m. Thus

$$0 + (0.4)(9.81)(5) = \frac{1}{2}(0.4)v_2^2 + 0$$

$\underline{v_2 = 9.90 \text{ m/s}}$



Problem 15.82 At the instant shown, the 20-kg mass is moving downward at 1.6 m/s. Let d be the downward displacement of the mass relative to its present position. Use conservation of energy to determine the magnitude of the velocity of the 20-kg mass when $d = 1$ m.



Solution:

$m_1 = 4$ kg

$m_2 = 20$ kg

$v_1 = 1.6$ m/s

$g = 9.81$ m/s^2

$d = 1$ m

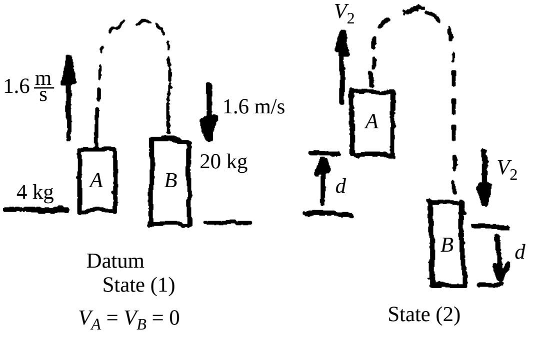
Energy for the system is conserved

$$\left(\tfrac{1}{2}m_1v_1^2 + 0\right) + \left(\tfrac{1}{2}m_2v_1^2 + 0\right) = \tfrac{1}{2}m_1v_2^2 + \tfrac{1}{2}m_2v_2^2$$

$$+ m_1g(d) - m_2g(d)$$

$$(m_1 + m_2)v_1^2 = (m_1 + m_2)v_2^2 + 2(m_1 - m_2)gd$$

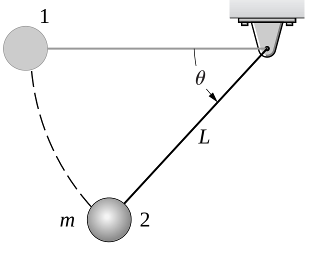
Substituting known values and solving $\underline{v_2 = 3.95 \text{ m/s}}$



Problem 15.83 The mass of the ball is $m = 2$ kg and the string's length is $L = 1$ m. The ball is released from rest in position 1 and swings to position 2, where $\theta = 40°$.

(a) Use conservation of energy to determine the magnitude of the ball's velocity at position 2.

(b) Draw graphs of the kinetic energy, the potential energy, and the total energy for values of θ from zero to 180°.



Solution:

$m = 2$ kg

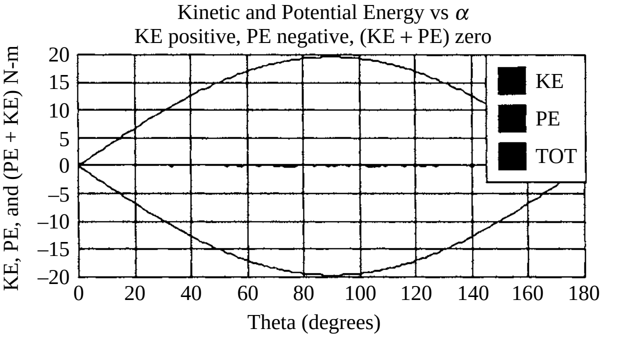
$L = 1$ m

Use conservation of energy State 1 $\theta = 0$; State 2, $0 < \theta < 180°$
Datum: $\theta = 0$, $v_1 = 0$, $g = 9.81$ m/s^2

$$\tfrac{1}{2}mv_1^2 + mg(0) = \tfrac{1}{2}mv_2^2 + mg(-L\sin\theta)$$

$KE = \tfrac{1}{2}mv_2^2 \quad V = -mgL\sin\theta$ for all θ.
Total energy is always zero (datum value).

(a) Evaluating at $\theta = 40°$, $\underline{v_2 = 3.55\ m/s}$



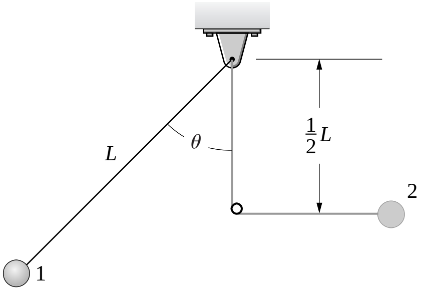
Problem 15.84 The mass of the ball is $m = 2$ kg and the string's length is $L = 1$ m. The ball is released from rest in position 1. When the string is vertical, it hits the fixed peg shown.

(a) Use conservation of energy to determine the minimum angle θ necessary for the ball to swing to position 2.

(b) If the ball is released at the minimum angle θ determined in part (a), what is the tension in the string just before and just after it hits the peg?

$m = 2$ kg

$L = 1$ m



Solution: Energy is conserved. $v_1 = v_2 = 0$ Use $\theta = 90^\circ$ as the datum.

(a) $\frac{1}{2}mv_1^2 + mg(-L\cos\theta_1) = \frac{1}{2}mv_2^2 - mg\frac{L}{2}$

$0 - mgL\cos\theta_1 = 0 - mg\frac{L}{2}$

$\cos\theta_1 = \frac{1}{2}$

$\theta = 60^\circ$

(b) Use conservation of energy to determine velocity at the lowest point, (state 3) ($v_1 \equiv 0$)

$\frac{1}{2}mv_1^2 - mgL\cos 60^\circ = \frac{1}{2}mv_3^2 - mgL$

$\frac{1}{2}mv_3^2 = mgL - mgL/2$

$v_3^2 = gL = 9.81\ \frac{\text{m}^2}{\text{s}^2}$

$v_3 = 3.13$ m/s at $\theta = 0^\circ$.

Before striking the peg

$T_1 - mg = mv_3^2/L$

$T_1 = (2)(9.81) + (2)(9.81)/(1)$

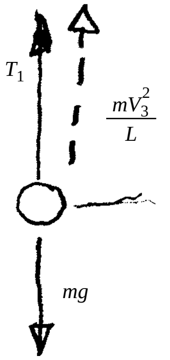
$\underline{T_1 = 39.2\ \text{N}}$

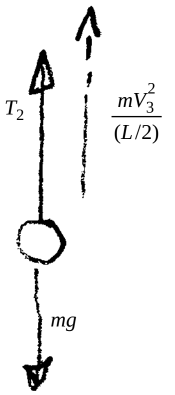
After striking the peg.

$T - mg = mv_3^2/(L/2)$

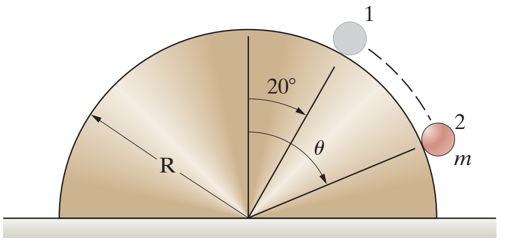
$T = (2)(9.81) + 2[(2)(9.81)/1]$

$T = 58.9$ N





Problem 15.85 A small pellet of mass $m = 0.2$ kg starts from rest at position 1 and slides down the smooth surface of the cylinder to position 2. The radius $R = 0.8$ m. Use conservation of energy to determine the magnitude of the pellet's velocity at position 2 if $\theta = 45^\circ$.



Solution: Use the ground as the datum

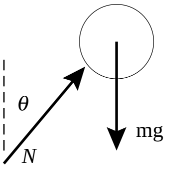
$$T_1 = 0,\ \ V_1 = (0.2\ \text{kg})(9.81\ \text{m/s}^2)(0.8\ \text{m})\cos 20^\circ$$

$$T_2 = \frac{1}{2}(0.2\ \text{kg}){v_2}^2,\ \ V_2 = (0.2\ \text{kg})(9.81\ \text{m/s}^2)(0.8\ \text{m})\cos 45^\circ$$

$$T_1 + V_1 = T_2 + V_2 \ \Rightarrow\ \boxed{v_2 = 1.91\ \text{m/s}}$$

Problem 15.86 In Problem 15.85, what is the value of the angle θ at which the pellet loses contact with the surface of the cylinder?

Solution: In position 2 we have



$$\sum F_{\nearrow} : N - mg\cos\theta = -m\frac{{v_2}^2}{R} \ \Rightarrow\ N = m\left(g\cos\theta - \frac{{v_2}^2}{R}\right)$$

When the pellet leaves the surface $N = 0 \ \Rightarrow\ v_2^2 = Rg\cos\theta$

Now do work-energy.

$$T_1 = 0,\ \ V_1 = mgR\cos 20^\circ$$

$$T_2 = \frac{1}{2}m{v_2}^2 = \frac{1}{2}mRg\cos\theta,\ \ V_2 = mgR\cos\theta$$

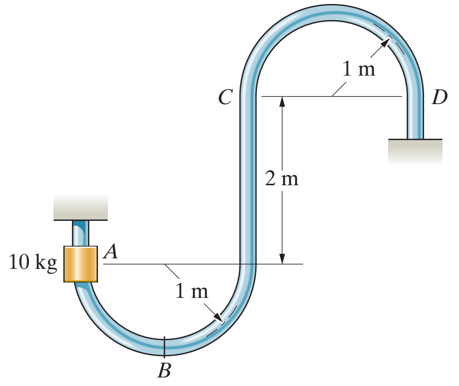
$$T_1 + V_1 = T_2 + V_2 \ \Rightarrow\ 0 + mgR\cos 20^\circ = \frac{3}{2}mgR\cos\theta$$

Solving we find $\boxed{\theta = \cos^{-1}\left(\frac{2}{3}\cos 20^\circ\right) = 51.2^\circ}$

Problem 15.87 The bar is smooth. The 10-kg slider at A is given a downward velocity of 6.5 m/s.

(a) Use conservation of energy to determine whether the slider will reach point C. If it does, what is the magnitude of its velocity at point C?

(b) What is the magnitude of the normal force the bar exerts on the slider as it passes point B?



Solution:

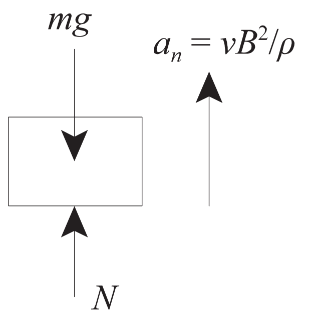
(a) Find the velocity at C.

$$\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_C^2 + mgh$$

$$v_C = \sqrt{v_A^2 - 2gh} = \sqrt{(6.5\ \text{m/s})^2 - 2(9.81\ \text{m/s}^2)(2\ \text{m})}$$

This equation has a real solution, hence it is possible to reach point C.

$$\boxed{Yes,\ v_C = 1.73\ \text{m/s.}}$$



(b) Find the velocity at point B

$$\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_B^2 - mgh,$$

$$v_B = \sqrt{v_A^2 + 2gh} = \sqrt{(6.5\ \text{m/s})^2 + 2(9.81\ \text{m/s}^2)(1\ \text{m})} = 7.87\ \text{m/s.}$$

Now find the normal force

$$\Sigma F_y : N - mg = m\frac{v_B^2}{\rho}$$

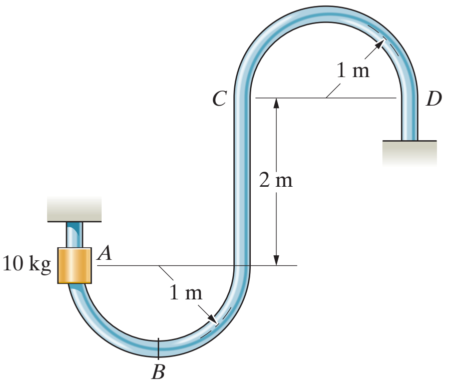
$$N = m\left(g + \frac{v_B^2}{\rho}\right) = (10\ \text{kg})\left([9.81\ \text{m/s}^2] + \frac{[7.87\ \text{m/s}]^2}{1\ \text{m}}\right)$$

$$\boxed{N = 717\ \text{N.}}$$

Problem 15.88 The bar is smooth. The 10-kg slider at A is given a downward velocity of 7.5 m/s.

(a) Use conservation of energy to determine whether the slider will reach point D. If it does, what is the magnitude of its velocity at point D?

(b) What is the magnitude of the normal force the bar exerts on the slider as it passes point B?



Solution:

(a) We will first find the velocity at the highest point (half way between C and D).

$$\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_D^2 + mgh$$

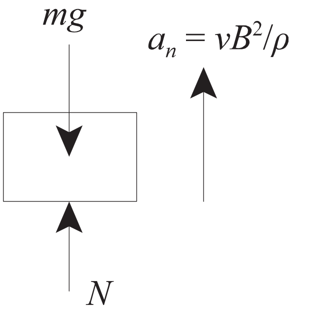
$$v_D = \sqrt{v_A^2 - 2gh} = \sqrt{(7.5\text{ m/s})^2 - 2(9.81\text{ m/s}^2)(3\text{ m})}$$

$$v_D = \sqrt{-2.61}\text{ m/s}.$$

This equation does not have a solution in terms of real numbers which means that it cannot reach the highest point.

$\boxed{\text{No.}}$

(b) Find the velocity at point B



$$\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_B^2 - mgh,$$

$$v_B = \sqrt{v_A^2 + 2gh} = \sqrt{(7.5\text{ m/s})^2 + 2(9.81\text{ m/s}^2)(1\text{ m})} = 8.71\text{ m/s}.$$

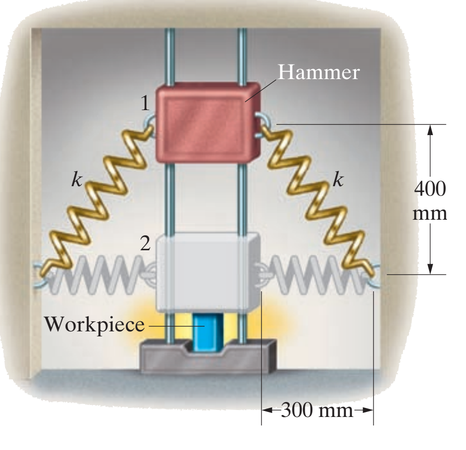
Now find the normal force

$$\Sigma F_y : N - mg = m\frac{v_B^2}{\rho}$$

$$N = m\left(g + \frac{v_B^2}{\rho}\right) = (10\text{ kg})\left([9.81\text{ m/s}^2] + \frac{[8.71\text{ m/s}]^2}{1\text{ m}}\right)$$

$$\boxed{N = 857\text{ N.}}$$

Problem 15.89 In Active Example 15.7, suppose that you want to increase the value of the spring constant k so that the velocity of the hammer just before it strikes the workpiece is 4 m/s. Use conservation of energy to determine the required value of k.



Solution:

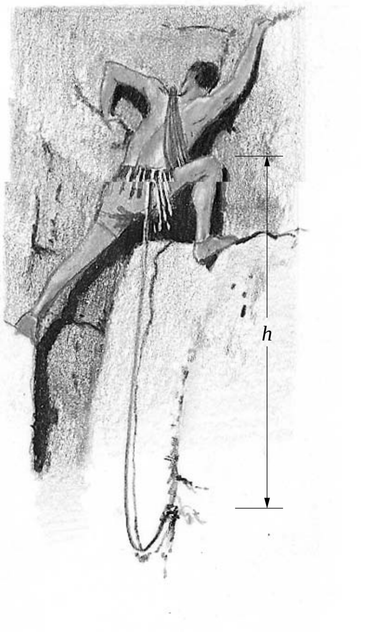
$$2\left(\frac{1}{2}ks^2\right) + mgh = \frac{1}{2}mv^2$$

$$k = \frac{m(v^2 - 2gh)}{2s^2}$$

$$k = \frac{(40\text{ kg})([4\text{ m/s}]^2 - 2[9.81\text{ m/s}^2][0.4\text{ m}])}{2([0.5\text{ m}] - [0.3\text{ m}])^2}$$

$$\boxed{k = 4080\text{ N/m.}}$$

Problem 15.90 A rock climber of weight W has a rope attached a distance h below him for protection. Suppose that he falls, and assume that the rope behaves like a linear spring with unstretched length h and spring constant $k = C/h$, where C is a constant. Use conservation of energy to determine the maximum force exerted on the climber by the rope. (Notice that the maximum force is independent of h, which is a reassuring result for climbers: The maximum force resulting from a long fall is the same as that resulting from a short one.)



Solution: Choose the climber's center of mass before the fall as the datum. The energy of the climber before the fall is zero. As the climber falls, his energy remains the same:

$$0 = \tfrac{1}{2}mv^2 - Wy,$$

where y is positive downward. As the rope tightens, the potential energy stored in the rope becomes

$$V_{\text{rope}} = \tfrac{1}{2}k(y - 2h)^2.$$

At maximum extension the force on the climber is

$$F = -\frac{\partial V}{\partial y} = -k(y - 2h).$$

When the velocity of the falling climber is zero, $0 = -Wy + \frac{1}{2}k(y - 2h)^2$, from which: $y^2 + 2by + c = 0$, where

$$b = -\left(2h + \frac{W}{k}\right),$$

and $c = +4h^2$. The solution is

$$y = \left(2h + \frac{W}{k}\right) \pm \left(\frac{W}{k}\right)\sqrt{\frac{4\ kh}{W} + 1}.$$

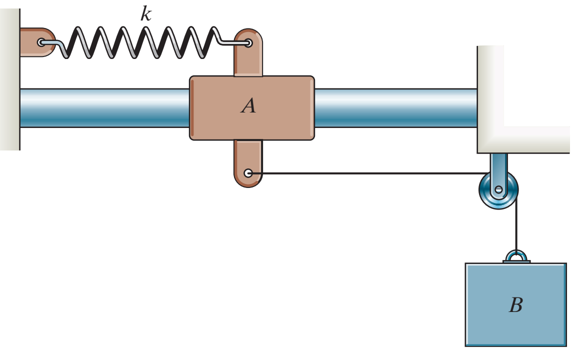
Substitute:

$$F = -W\left(1 \pm \sqrt{1 + \frac{4C}{W}}\right).$$

The positive sign applies, and the force is

$$\boxed{F = -W\left(1 + \sqrt{1 + \frac{4C}{W}}\right)} \text{ (directed upward).}$$

Problem 15.91 The collar A slides on the smooth horizontal bar. The spring constant $k = 40$ N/m. The weights are $W_A = 30$ N and $W_B = 60$ N. As the instant shown, the spring is unstretched and B is moving downward at 4 m/s. Use conservation of energy to determine the velocity of B when it has moved downward 2 m from its current position. (See Example 15.8.)



Solution: Notice that the collars have the same velocity

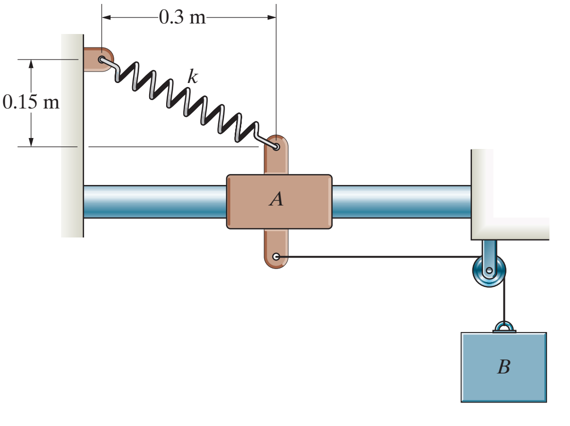
$$\frac{1}{2}\left(\frac{W_A + W_B}{g}\right)v_1^2 + W_B h = \frac{1}{2}\left(\frac{W_A + W_B}{g}\right)v_2^2 + \frac{1}{2}kh^2$$

$$v_2 = \sqrt{v_1^2 + \left(\frac{2\,W_B h - kh^2}{W_A + W_B}\right)g}$$

$$v_2 = \sqrt{(4\text{ m/s})^2 + \left(\frac{2[60\text{ N}][2\text{ m}] - [40\text{ N/m}][2\text{ m}]^2}{90\text{ N}}\right)(9.81\text{ m/s}^2)}$$

$$\boxed{v_2 = 4.97\text{ m/s.}}$$

Problem 15.92 The spring constant $k = 700$ N/m. The masses $m_A = 14$ kg and $m_B = 18$ kg. The horizontal bar is smooth. At the instant shown, the spring is unstretched and the mass B is moving downward at 1 m/s. How fast is B moving when it has moved downward 0.2 m from its present position?



Solution: The unstretched length of the spring is

$$\delta = \sqrt{(0.3\text{ m})^2 + (0.15\text{ m})^2}$$

$$= 0.335\text{ m.}$$

When B has moved 0.2 m, the length of the spring is

$$\Delta = \sqrt{(0.5\text{ m})^2 + (0.15\text{ m})^2} = 0.522\text{ m.}$$

Conservation of energy is

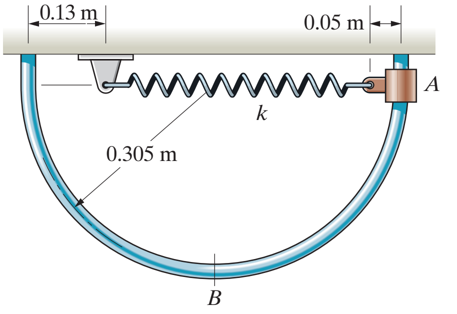
$$\frac{1}{2}(m_A + m_B)v_1^2 = \frac{1}{2}(m_A + m_B)v_2^2 - m_B g h + \frac{1}{2}k(\Delta - \delta)^2$$

$$v_2 = \sqrt{v_1^2 + \frac{2m_B g h - k(\Delta - \delta)^2}{m_A + m_B}}$$

$$v_2 = \sqrt{(1\text{ m/s})^2 + \frac{2(18\text{ kg})(9.81\text{ m/s}^2)(0.2\text{ m}) - (700\text{ N/m})(0.187\text{ m})^2}{32\text{ kg}}}$$

$$\boxed{v_2 = 1.56\text{ m/s.}}$$

Problem 15.93 The semicircular bar is smooth. The unstretched length of the spring is 0.254 m. The 5-N collar at A is given a downward velocity of 6 m/s, and when it reaches B the magnitude of its velocity is 15 m/s. Determine the spring constant k.



Solution: The stretch distances for the spring at A and B are

$\delta_A = 0.226$ m

$\delta_B = 0.098$ m

Conservation of energy gives

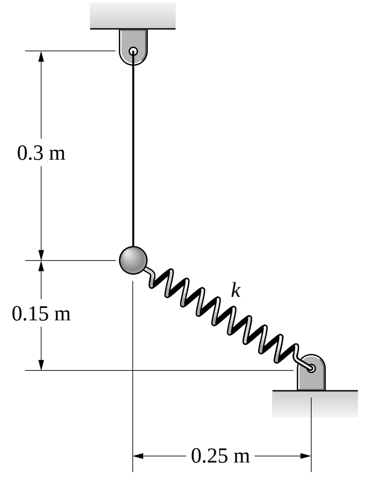
$$\frac{1}{2}mv_A^2 + \frac{1}{2}k\delta_A^2 + mgh = \frac{1}{2}mv_B^2 + \frac{1}{2}k\delta_B^2$$

$$k = \frac{m[v_B^2 - v_A^2 - 2gh]}{\delta_A^2 - \delta_B^2}$$

$$k = \left(\frac{5\text{ N}}{9.81\text{ m/s}^2}\right)\left[\frac{(15\text{ m/s})^2 - (6\text{ m/s})^2 - 2(9.81\text{ m/s}^2)(0.305\text{ m})}{(0.226\text{ m})^2 - (0.098\text{ m})^2}\right]$$

$$\boxed{k = 2249\text{ N/m.}}$$

Problem 15.94 The mass $m = 1$ kg, the spring constant $k = 200$ N/m, and the unstretched length of the spring is 0.1 m. When the system is released from rest in the position shown, the spring contracts, pulling the mass to the right. Use conservation of energy to determine the magnitude of the velocity of the mass when the string and spring are parallel.



Solution: The stretch of the spring in position 1 is

$$S_1 = \sqrt{(0.15)^2 + (0.25)^2} - 0.1 = 0.192 \text{ m}.$$

The stretch in position 2 is

$$S_2 = \sqrt{(0.3 + 0.15)^2 + (0.25)^2} - 0.3 - 0.1 = 0.115 \text{ m}.$$

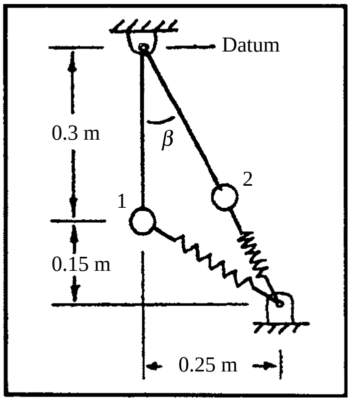
The angle $\beta = \arctan(0.25/0.45) = 29.1^\circ$. Applying conservation of energy,

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kS_1^2 - mg(0.3) = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kS_2^2 - mg(0.3\cos\beta):$$

$$0 + \tfrac{1}{2}(200)(0.192)^2 - (1)(9.81)(0.3) = \tfrac{1}{2}(1)v_2^2 + \tfrac{1}{2}(200)(0.115)^2$$

$$- (1)(9.81)(0.3\cos 29.1^\circ).$$

Solving, $v_2 = 1.99$ m/s



Problem 15.95 In problem 15.94, what is the tension in the string when the string and spring are parallel?

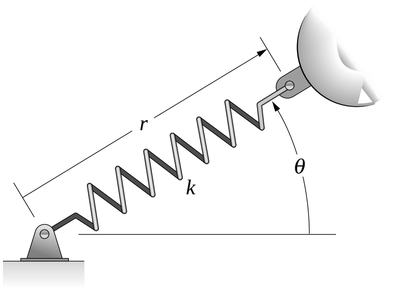
Solution: The free body diagram of the mass is: Newton's second law in the direction normal to the path is

$$T - kS_2 - mg\cos\beta = ma_n:$$

$$T - (200)(0.115) - (1)(9.81)\cos 29.1^\circ = (1)(v_2^2/0.3).$$

We obtain, $T = 44.7$ N.

Problem 15.96 The force exerted on an object by a *nonlinear* spring is $\mathbf{F} = -[k(r - r_0) + q(r - r_0)^3]\mathbf{e}_r$, where k and q are constants and r_0 is the unstretched length of the spring. Determine the potential energy of the spring in terms of its stretch $S = r - r_0$.



Solution: Note that $dS = dr$. The work done in stretching the spring is

$$V = -\int \boldsymbol{F} \cdot d\mathbf{r} + C = -\int \boldsymbol{F} \cdot (dr\boldsymbol{e}_r + r d\theta \boldsymbol{e}_\theta) + C$$

$$= \int [k(r - r_0) + q(r - r_0)^2]\, dr + C,$$

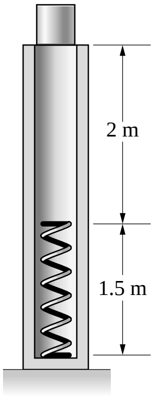
$$V = \int [kS + qS^3]\, dS + C.$$

Integrate:

$$\boxed{V = \frac{k}{2}S^2 + \frac{q}{4}S^4},$$

where $C = 0$, since $\boldsymbol{F} = 0$ at $S = 0$.

Problem 15.97 The 20-kg cylinder is released at the position shown and falls onto the linear spring ($k = 3000$ N/m). Use conservation of energy to determine how far down the cylinder moves after contacting the spring.



Solution: Choose the base of the cylinder as a datum. The potential energy of the piston at rest is $V_1 = mg(3.5) = 686.7$ N-m. The conservation of energy condition after the spring has compressed to the point that the piston velocity is zero is $mgh + \frac{1}{2}k(h - 1.5)^2 = mg(3.5)$, where h is the height above the datum. From which $h^2 + 2bh + c = 0$, where

$$b = -\left(\frac{3}{2} - \frac{mg}{k}\right)$$

and $\quad c = 2.25 - \dfrac{7\, mg}{k}.$

The solution is $h = -b \pm \sqrt{b^2 - c} = 1.95$ m, $n = 0.919$ m. The value $h = 1.95$ m has no physical meaning, since it is above the spring. The downward compression of the spring is

$$\boxed{S = 1.5 - 0.919 = 0.581 \text{ m}}$$

Problem 15.98 The 20-kg cylinder is released at the position shown and falls onto the *nonlinear* spring. In terms of the stretch S of the spring, its potential energy is $V = \frac{1}{2}kS^2 + \frac{1}{2}qS^4$, where $k = 3000$ N/m and $q = 4000$ N/m^3. What is the velocity of the cylinder when the spring has been compressed 0.5 m?

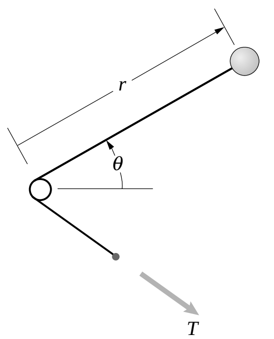
Solution: Note that $S = 1.5 - h$ where h is the height above the datum, from which $h = 1.5 - S$. Use the solution to Problem 15.97. The conservation of energy condition when the spring is being compressed is $\frac{1}{2}mv^2 + V_{\text{spring}} + mg(1.5 - S) = mg(3.5)$, from which

$$v = \sqrt{7.0g - 2g(1.5 - S) - 2\frac{V_{\text{spring}}}{m}}.$$

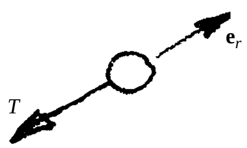
The potential energy in the spring is $V_{\text{spring}} = \frac{1}{2}(3000)(0.5^2) + \frac{1}{4}(4000)(0.5^4) = +437.5$ N-m.

Substitute numerical values to obtain $\boxed{v = 2.30 \text{ m/s}}$

Problem 15.99 The string exerts a force of constant magnitude T on the object. Use polar coordinates to show that the potential energy associated with this force is $V = Tr$.



Solution:



$dV = -\mathbf{F} \cdot d\mathbf{r}$

$$V = -\int_{\text{DATUM}}^{r} -T\mathbf{e}_r \cdot dr\mathbf{e}_r$$

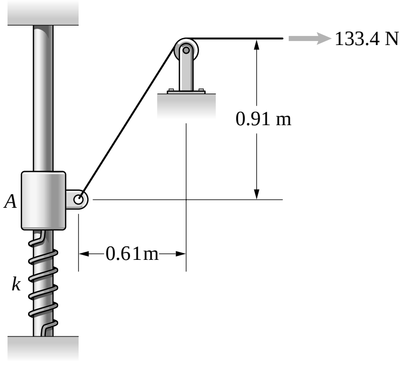
$$V = Tr\Big|_{\text{DATUM}}^{r}$$

$$V = Tr - Tr_{\text{DATUM}}$$

Let $r_{\text{DATUM}} = 0$

$\underline{V = Tr}$

Problem 15.100 The system is at rest in the position shown, with the 53.4 N collar A resting on the spring (292 N/m), when a constant 133.4 N force is applied to the cable. What is the velocity of the collar when it has risen 0.305 m? (See Problem 15.99.)



Solution: Choose the rest position as the datum. At rest, the compression of the spring is

$$S_1 = \frac{-W}{k} = -0.183 \text{ m}.$$

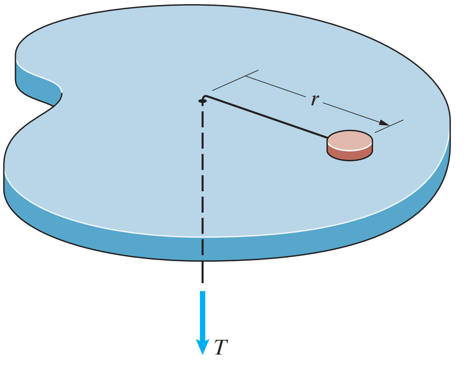
When the collar rises 0.31 m the stretch is $S_2 = S_1 + 0.3 = 0.127$ m When the collar rises 0.31 m the constant force on the cable has acted through a distance

$$s = \sqrt{0.91^2 + 0.61^2} - \sqrt{(0.91 - 0.31)^2 - 0.61^2} = 0.237 \text{ m}.$$

The work done on the system is $U_s = \frac{1}{2}k(S_1^2 - S_2^2) - mg(1) + Fs$. From the conservation of energy $U_s = \frac{1}{2}mv^2$ from which

$$\boxed{v = \sqrt{\frac{k}{m}(S_1^2 - S_2^2) - 2g + \frac{2Fs}{m}} = 2.57 \text{ m/s.}}$$

Problem 15.101 A 1-kg disk slides on a smooth horizontal table and is attached to a string that passes through a hole in the table. A constant force $T = 10$ N is exerted on the string. At the instant shown, $r = 1$ m and the velocity of the disk in terms of polar coordinates is $\mathbf{v} = 6\mathbf{e}_\theta$ (m/s). Use conservation of energy to determine the magnitude of the velocity of the disk when $r = 2$m. (See Problem 15.99.)



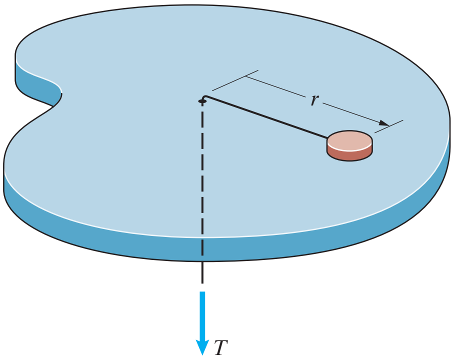
Solution:

$$\frac{1}{2}mv_1^2 + Tr_1 = \frac{1}{2}mv_2^2 + Tr_2$$

$$v_2 = \sqrt{v_1^2 + 2\frac{T}{m}(r_1 - r_2)} = \sqrt{(6 \text{ m/s})^2 + 2\left(\frac{10 \text{ N}}{1 \text{ kg}}\right)([1 \text{ m}] - [2 \text{ m}])}$$

$$\boxed{v_2 = 4 \text{ m/s.}}$$

Problem 15.102 A 1-kg disk slides on a smooth horizontal table and is attached to a string that passes through a hole in the table. A constant force $T = 10$ N is exerted on the string. At the instant shown, $r = 1$ m and the velocity of the disk in terms of polar coordinates is $\mathbf{v} = 8\mathbf{e}_\theta$ (m/s). Because this is central-force motion, the product of the radial position r and the transverse component of velocity v_θ is constant. Use this fact and conservation of energy to determine the velocity of the disk in terms of polar coordinates when $r = 2$m.



Solution: We have

$$\frac{1}{2}mv_1^2 + Tr_1 = \frac{1}{2}m(v_{2r}^2 + v_{2\theta}^2) + Tr_2, \quad r_1 v_1 = r_2 v_{2\theta}$$

Solving we find

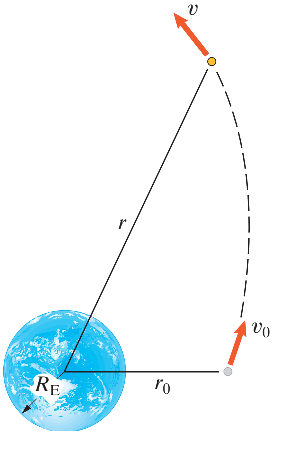
$$v_{2\theta} = \frac{r_1}{r_2}v_1 = \frac{1 \text{ m}}{2 \text{ m}}(8 \text{ m/s}) = 4 \text{ m/s}$$

$$v_{2r} = \sqrt{v_1^2 - v_{2\theta}^2 + 2\frac{T}{m}(r_1 - r_2)}$$

$$= \sqrt{(8 \text{ m/s})^2 - (4 \text{ m/s})^2 + 2\left(\frac{10 \text{ N}}{1 \text{ kg}}\right)([1 \text{ m}] - [2 \text{ m}])} = 5.29 \text{ m/s}.$$

$$\boxed{\mathbf{v} = (5.29\mathbf{e}_r + 4\mathbf{e}_\theta) \text{ m/s.}}$$

Problem 15.103 A satellite initially is inserted into orbit at a distance $r_0 = 8800$ km from the center of the earth. When it is at a distance $r = 18{,}000$ km from the center of the earth, the magnitude of its velocity is $v = 7000$ m/s. Use conservation of energy to determine its initial velocity v_0. The radius of the earth is 6370 km. (See Example 15.9.)



Solution:

$$\frac{1}{2}mv_0^2 - \frac{mgR_E^2}{r_0} = \frac{1}{2}mv^2 - \frac{mgR_E^2}{r}$$

$$v_0 = \sqrt{v^2 + 2gR_E^2\left(\frac{1}{r_0} - \frac{1}{r}\right)}$$

$$v_0 = \sqrt{(7000 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(6.37\times10^6 \text{ m})^2\left(\frac{1}{8.8\times10^6 \text{ m}} - \frac{1}{18\times10^6 \text{ m}}\right)}$$

$$\boxed{v_0 = 9760 \text{ m/s.}}$$

Problem 15.104 Astronomers detect an asteroid 100,000 km from the earth moving at 2 km/s relative to the center of the earth. Suppose the asteroid strikes the earth. Use conservation of energy to determine the magnitude of its velocity as it enters the atmosphere. (You can neglect the thickness of the atmosphere in comparison to the earth's 6370-km radius.)

Solution: Use the solution to Problem 15.103. The potential energy at a distance r is

$$V = -\frac{mgR_E^2}{r}.$$

The conservation of energy condition at

$$r_0 : \frac{1}{2}mv_0^2 - \frac{mgR_E^2}{r_0} = \frac{1}{2}mv^2 - \frac{mgR_E^2}{r}.$$

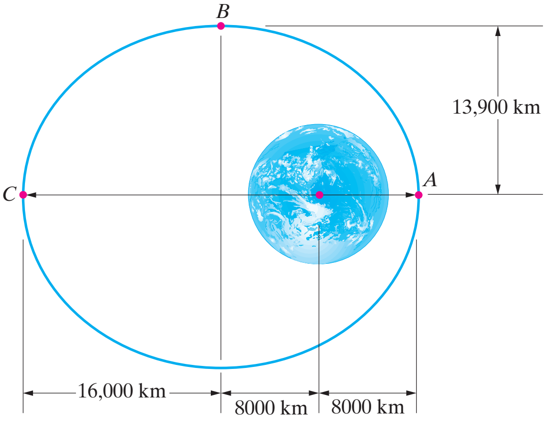
Solve:

$$v = \sqrt{v_0^2 + 2gR_E^2\left(\frac{1}{r} - \frac{1}{r_0}\right)}.$$

Substitute: $r_0 = 1 \times 10^8$ m, $v_0 = 2 \times 10^3$ m/s, and $r = 6.37 \times 10^6$ m. The velocity at the radius of the earth is

$$\boxed{v = 11 \text{ km/s}}$$

Problem 15.105 A satellite is in the elliptic earth orbit shown. Its velocity in terms of polar coordinates when it is at the perigee A is $\mathbf{v} = 8640\mathbf{e}_\theta$ (m/s). Determine the velocity of the satellite in terms of polar coordinates when it is at point B.



Solution: We have

$$r_A = 8000 \text{ km} = 8\times10^6 \text{ m}$$

$$r_B = \sqrt{13,900^2 + 8000^2} \text{ km} = 1.60\times10^7 \text{ m}.$$

Energy and angular momentum are conserved. Therefore

$$\frac{1}{2}mv_A^2 - \frac{mgR_E^2}{r_A} = \frac{1}{2}m(v_{Br}^2 + v_{B\theta}^2) - \frac{mgR_E^2}{r_B},\ r_Av_A = r_Bv_{B\theta}$$

Solving we have

$$v_{B\theta} = \frac{r_A}{r_B}v_A = \frac{8\times10^6 \text{ m}}{1.60\times10^7 \text{ m}}(8640 \text{ m/s}) = 4310 \text{ m/s},$$

$$v_{Br} = \sqrt{v_A^2 - v_{B\theta}^2 + 2gR_E^2\left(\frac{1}{r_B} - \frac{1}{r_A}\right)} = 2480 \text{ m/s}.$$

$$\boxed{\mathbf{v} = (2480\mathbf{e}_r + 4310\mathbf{e}_\theta) \text{ m/s}.}$$

Problem 15.106 Use conservation of energy to determine the magnitude of the velocity of the satellite in Problem 15.105 at the apogee C. Using your result, confirm numerically that the velocities at perigee and apogee satisfy the relation $r_A v_A = r_C v_C$.

Solution: From Problem 15.105, $r_A = 8000$ km, $r_C = 24000$ km, $v_A = 8640$ m/s, $g = 9.81$ m/s^2, $R_E = 6370$ km.
From conservation of energy,

$$\frac{1}{2}mv_A^2 - \frac{mgR_E^2}{r_A} = \frac{1}{2}mv_c^2 - \frac{mgR_E^2}{r_c}$$

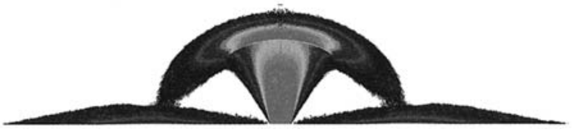
Factor m out of the equation, convert all distances to meters, and solve for v_c. Solving, $\underline{v_C = 2880 \text{ m/s}}$

Does $r_A v_A = r_C v_C$

Substituting the known values, we have

$r_A v_A = r_C v_C = 6.91 \times 10^{10}$ m^2/s

Problem 15.107 The *Voyager* and *Galileo* spacecraft have observed volcanic plumes, believed to consist of condensed sulfur or sulfur dioxide gas, above the surface of the Jovian satellite Io. The plume observed above a volcano named Prometheus was estimated to extend 50 km above the surface. The acceleration due to gravity at the surface is 1.80 m/s^2. Using conservation of energy and neglecting the variation of gravity with height, determine the velocity at which a solid particle would have to be ejected to reach 50 km above Io's surface.



Solution: Conservation of energy yields: $T_1 + V_1 = T_2 + V_2$: Using the forms for a constant gravity field, we get $\frac{1}{2}mv_1^2 + 0 = 0 + mgy_2$ Evaluating, we get $\frac{1}{2}v_1^2 = (1.8)(50{,}000)$, or $v_1 = 424$ m/s

Problem 15.108 Solve Problem 15.107 using conservation of energy and accounting for the variation of gravity with height. The radius of Io is 1815 km.

Solution: Conservation of energy yields: $T_1 + V_1 = T_2 + V_2$: Only the form of potential energy changes from that used in Problem 15.107. Here we get

$$\frac{1}{2}mv_1^2 - \frac{mgR_I^2}{R_I} = 0 - \frac{mgR_I^2}{r_I}.$$

Evaluating,

$$\frac{1}{2}v_1^2 - \frac{(1.8)(1{,}815{,}000)^2}{1{,}815{,}000} = -\frac{(1.8)(1{,}815{,}000)^2}{1{,}815{,}000 + 50{,}000}.$$

or $v_1 = 419$ m/s

Problem 15.109* What is the relationship between Eq. (15.21), which is the gravitational potential energy neglecting the variation of the gravitational force with height, and Eq. (15.23), which accounts for the variation? Express the distance from the center of the earth as $r = R_E + y$, where R_E is the earth's radius and y is the height above the surface, so that Eq. (15.23) can be written as

$$V = -\frac{mgR_E}{1 + \dfrac{y}{R_E}}.$$

By expanding this equation as a Taylor series in terms of y/R_E and assuming that $y/R_E \ll 1$, show that you obtain a potential energy equivalent to Eq. (15.21).

Solution: Define $y/R_E = \varepsilon$

$$V = -\frac{mgR_E{}^2}{r} = -\frac{mgR_E{}^2}{R_E + y} = -\frac{mgR_E}{1 + \dfrac{y}{R_E}} = -\frac{mgR_E}{1+\varepsilon}$$

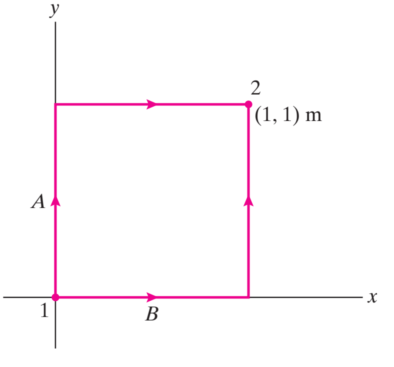
$$V = V|_{\varepsilon=0} + \left.\frac{dV}{d\varepsilon}\right|_{\varepsilon=0} \varepsilon + \cdots$$

$$V = -mgR_E + \left.\frac{mgR_E}{(1+\varepsilon)^2}\right|_{\varepsilon=0} \varepsilon = -mgR_E + mgR_E\frac{y}{R_E}$$

$$= -mgR_E + mgy \quad \text{QED}$$

Problem 15.110 The potential energy associated with a force $\mathbf{F}$ acting on an object is $V = x^2 + y^3$ N-m, where x and y are in meters.

(a) Determine $\mathbf{F}$.
(b) Suppose that the object moves from position 1 to position 2 along path A, and then moves from position 1 to position 2 along path B. Determine the work done by $\mathbf{F}$ along each path.



Solution:

(a) $F_x = -\dfrac{dV}{dx} = -2x, \quad F_y = -\dfrac{dV}{dy} = -3y^2$

$\boxed{\mathbf{F} = -2x\mathbf{i} - 3y^2\mathbf{j} \text{ N.}}$

(b)

$$W_{12A} = \int_0^1 (-3y^2)\,dy + \int_0^1 (-2x)\,dx = -(1)^3 - (1)^2 = -2 \text{ N-m}$$

$$W_{12B} = \int_0^1 (-2x)\,dx + \int_0^1 (-3y^2)\,dy = -(1)^2 - (1)^3 = -2 \text{ N-m}$$

$$\boxed{W_{12A} = W_{12B} = -2 \text{ N-m.}}$$

Problem 15.111 An object is subjected to the force $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$ (N), where x and y are in meters.

(a) Show that $\mathbf{F}$ is *not* conservative.
(b) Suppose the object moves from point 1 to point 2 along the paths A and B shown in Problem 15.110. Determine the work done by $\mathbf{F}$ along each path.

Solution:

(a) A necessary and sufficient condition that $\mathbf{F}$ be conservative is $\nabla \times \mathbf{F} = 0$.

$$\nabla \times \mathbf{F} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ y & -x & 0 \end{bmatrix} = \mathbf{i}0 - \mathbf{j}0 + (-1-1)\mathbf{k}$$

$$= -2\mathbf{k} \neq 0.$$

Therefore $\mathbf{F}$ is non conservative.

(b) The integral along path B is

$$\boxed{\begin{aligned} U_B &= \int_0^1 (y\mathbf{i} - x\mathbf{j})|_{y=0} \cdot \mathbf{i}\,dx + \int_0^1 (y\mathbf{i} - x\mathbf{j})|_{x=1} \cdot \mathbf{j}\,dy \\ &= 0 - 1 = -1 \text{ N-m} \end{aligned}}.$$

Along path A:

$$\boxed{\begin{aligned} U_A &= \int_0^1 (y\mathbf{i} - x\mathbf{j})|_{x=0} \cdot \mathbf{j}\,dy + \int_0^1 (y\mathbf{i} - x\mathbf{j})|_{y=1} \cdot \mathbf{i}\,dx \\ &= 0 + 1 = +1 \text{ N-m} \end{aligned}}$$

Problem 15.112 In terms of polar coordinates, the potential energy associated with the force $\mathbf{F}$ exerted on an object by a *nonlinear* spring is

$$V = \tfrac{1}{2}k(r - r_0)^2 + \tfrac{1}{4}q(r - r_0)^4,$$

where k and q are constants and r_0 is the unstretched length of the spring. Determine $\mathbf{F}$ in terms of polar coordinates. (See Active Example 15.10.)

Solution: The force is given by

$$\boxed{\begin{aligned} \mathbf{F} &= -\nabla V = -\left(\frac{\partial}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\mathbf{e}_\theta\right)\left(\frac{1}{2}k(r - r_0)^2 + \frac{1}{4}q(r - r_0)^4\right) \\ &= -[k(r - r_0) + q(r - r_0)^3]\mathbf{e}_r \end{aligned}}$$

Problem 15.113 In terms of polar coordinates, the force exerted on an object by a *nonlinear* spring is

$$\mathbf{F} = -(k(r - r_0) + q(r - r_0)^3)\mathbf{e}_r,$$

where k and q are constants and r_0 is the unstretched length of the spring. Use Eq. (15.36) to show that $\mathbf{F}$ is conservative. (See Active Example 15.10.)

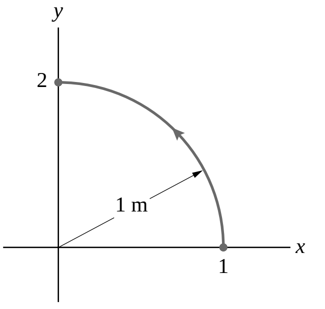
Solution: A necessary and sufficient condition that $\mathbf{F}$ be conservative is $\nabla \times \mathbf{F} = 0$.

$$\nabla \times \mathbf{F} = \frac{1}{r}\begin{bmatrix} \mathbf{e}_r & r\mathbf{e}_\theta & \mathbf{e}_z \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial z} \\ -[k(r - r_0) + q(r - r_0)^3] & 0 & 0 \end{bmatrix}$$

$$= \frac{1}{r}[0\mathbf{e}_r - 0r\mathbf{e}_\theta + 0\mathbf{e}_z] = 0.$$ $\mathbf{F}$ is conservative.

Problem 15.114 The potential energy associated with a force **F** acting on an object is $V = -r \sin\theta + r^2 \cos^2\theta$ N-m, where r is in metre.

(a) Determine **F**.
(b) If the object moves from point 1 to point 2 along the circular path, how much work is done by **F**?



Solution: The force is

$$\mathbf{F} = -\nabla V = -\left(\frac{\partial}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\mathbf{e}_\theta\right)(-r\sin\theta + r^2\cos^2\theta).$$

$$\boxed{\mathbf{F} = (\sin\theta - 2r\cos^2\theta)\mathbf{e}_r + (\cos\theta + 2r\sin\theta\cos\theta)\mathbf{e}_\theta}.$$

The work done is $U_{1,2} = \int_{1,2} \mathbf{F}\cdot d\mathbf{r}$,

where $d\mathbf{r} = \mathbf{e}_r dr + r\mathbf{e}_\theta d\theta$. Since the path is everywhere normal to $\mathbf{e}_r$, the radial term does not contribute to the work. The integral is

$$U_{1,2} = \int_0^{\frac{\pi}{2}} (\cos\theta + 2r\cos\theta\sin\theta) r\,d\theta = \left[r\sin\theta - r^2\cos^2\theta\right]_0^{\frac{\pi}{2}}$$

$$= 1 + 1 = 2 \text{ N-m}$$

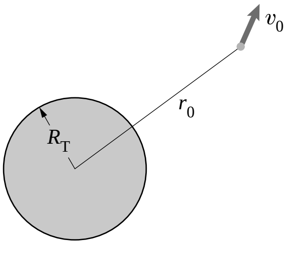
Check: Since the force is derivable from a potential, the system is conservative. In a conservative system the work done is $U_{1,2} = -(V_2 - V_1)$, where V_1, V_2 are the potentials at the beginning and end of the path. At $r = 1, \theta = 0, V_1 = 1$ N-m. At $r = 1$ m. $\theta = \frac{\pi}{2}$, $V_1 = -1$, from which $U_{1,2} = -(V_2 - V_1) = 2$ N-m. *check*.

Problem 15.115 In terms of polar coordinates, the force exerted on an object of mass m by the gravity of a hypothetical two-dimensional planet is

$$\mathbf{F} = -\left(\frac{mg_T R_T}{r}\right)\mathbf{e}_r,$$

where g_T is the acceleration due to gravity at the surface, R_T is the radius of the planet, and r is the distance from the center of the planet.

(a) Determine the potential energy associated with this gravitational force.
(b) If the object is given a velocity v_0 at a distance r_0, what is its velocity v as a function of r?



Solution:

(a) The potential is

$$V = -\int \mathbf{F}\cdot d\mathbf{r} + C = \int \left(\frac{mg_T R_T}{r}\right)\mathbf{e}_r \cdot (\mathbf{e}_r\, dr) + C$$

$$= mg_T R_T \ln(r) + C,$$

where C is the constant of integration. Choose $r = R_T$ as the datum, from which $C = -mg_T R_T \ln(R_T)$, and

$$\boxed{V = mg_T R_T \ln\left(\frac{r}{R_T}\right)}$$

(*Note*: Alternatively, the choice of $r = 1$ *length-unit* as the datum, from which $C = mg_M R_T \ln(1)$, yields $V = mg_T R_T \ln\left(\frac{r}{1}\right) = mg_T R_T \ln(r)$.)

(b) From conservation of energy,

$$\frac{1}{2}mv^2 + mg_T R_T \ln\left(\frac{r}{R_T}\right) = \frac{1}{2}mv_0^2 + mg_T R_T \ln\left(\frac{r_0}{R_T}\right).$$

Solve for the velocity

$$\boxed{v = \sqrt{v_0^2 + 2_{g_T} \ln\left(\frac{r_0}{r}\right)}}$$

Problem 15.116 By substituting Eqs. (15.27) into Eq. (15.30), confirm that $\nabla \times \mathbf{F} = 0$ if $\mathbf{F}$ is conservative.

Solution: Eq. 15.30 is

$$\boxed{\begin{aligned}\nabla \times \mathbf{F} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ -\dfrac{\partial V}{\partial x} & -\dfrac{\partial V}{\partial y} & -\dfrac{\partial V}{\partial z} \end{bmatrix} &= \mathbf{i}\left(-\frac{\partial^2 V}{\partial y \partial z} + \frac{\partial^2 V}{\partial y \partial z}\right) \\ -\mathbf{j}\left(\frac{\partial^2 V}{\partial x \partial z} - \frac{\partial^2 V}{\partial x \partial z}\right) &+ \mathbf{k}\left(\frac{\partial^2 V}{\partial x \partial y} - \frac{\partial^2 V}{\partial x \partial y}\right) = 0\end{aligned}}$$

Thus, $\mathbf{F}$ is conservative.

Problem 15.117 Determine which of the following are conservative.

(a) $\mathbf{F} = (3x^2 - 2xy)\mathbf{i} - x^2\mathbf{j}$;
(b) $\mathbf{F} = (x - xy^2)\mathbf{i} + x^2y\mathbf{j}$;
(c) $\mathbf{F} = (2xy^2 + y^3)\mathbf{i} + (2x^2y - 3xy^2)\mathbf{j}$.

Solution: Use Eq. (15.30)

(a) $$\nabla \times \mathbf{F} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3x^2 - 2xy & -x^2 & 0 \end{bmatrix}$$

$$= \mathbf{i}(0) - \mathbf{j}(0) + \mathbf{k}(-2x + 2x) = 0.$$

Force is conservative.

(b) $$\nabla \times \mathbf{F} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x - xy^2 & x^2y & 0 \end{bmatrix}$$

$$= \mathbf{i}(0) - \mathbf{j}(0) + \mathbf{k}(2xy + 2xy) = \mathbf{k}(4xy) \neq 0$$

Force is non-conservative.

(c) $$\nabla \times \mathbf{F} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2xy^2 + y^3 & 2x^2y - 3xy^2 & 0 \end{bmatrix}$$

$$= \mathbf{i}(0) - \mathbf{j}(0) + \mathbf{k}(4xy - 3y^2 - 4xy - 3y^2)$$

$$= \mathbf{k}(-6y^2) \neq 0.$$

Force is non-conservative.

Problem 15.118 The driver of a 12000 N car moving at 40 km/h applies an increasing force on the brake pedal. The magnitude of the resulting frictional force exerted on the car by the road is $f = 250 + 6s$ N, where s is the car's horizontal position (in feet) relative to its position when the brakes were applied. Assuming that the car's tires do not slip, determine the distance required for the car to stop

(a) by using Newton's second law and
(b) by using the principle of work and energy.

Solution:

(a) Newton's second law:

$$\left(\frac{W}{g}\right)\frac{dv}{dt} = -f,$$

where f is the force on the car in opposition to the motion. Use the chain rule:

$$\left(\frac{W}{g}\right)v\frac{dv}{ds} = -f = -(250 + 6s).$$

Integrate and rearrange:

$$v^2 = -\left(\frac{2g}{W}\right)(250s + 3s^2) + C.$$

At $s = 0$, $v(0) = 40 \times 1000/3600 = 11.1$ m/s,

from which $C = (11.1^2) = v_1^2$. The velocity is

$$v^2 = -\left(\frac{2g}{W}\right)(250s + 3s^2) + v_1^2 \ (\text{m/s})^2.$$

At $v = 0$, $s^2 + 2bs + c = 0$, where

$$b = \frac{125}{3} = 41.67,\ c = -\frac{Wv_1^2}{6g} = -25119.$$

The solution: $s = -b \pm \sqrt{b^2 - c} = 122.2$ m, from which $\boxed{s = 122.2 \text{ m}}$.

(b) Principle of work and energy: The energy of the car when the brakes are first applied is

$$\frac{1}{2}\left(\frac{W}{g}\right)v_1^2 = 6789 \text{ N-m}.$$

The work done is

$$U = \int_0^s f\,ds = -\int_0^s (250 + 6s)\,ds = -(250s + 3s^2).$$

From the principle of work and energy, after the brakes are applied,

$$U = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2 - \frac{1}{2}\left(\frac{W}{g}\right)v_1^2.$$

Rearrange:

$$.\frac{1}{2}\left(\frac{W}{g}\right)v_2^2 = .\frac{1}{2}\left(\frac{W}{g}\right)v_1^2 - (250s + 3s^2).$$

When the car comes to a stop, $v_2 = 0$, from which

$$.0 = .\frac{1}{2}\left(\frac{W}{g}\right)v_1^2 - (250s + 3s^2).$$

Reduce: $s^2 + 2bs + c = 0$, where

$$b = \frac{125}{3} = 41.67,\ c = -\frac{Wv_1^2}{6g} = -25119.$$

The solution $s = -b \pm \sqrt{b^2 - c} = 122.2$ m, from which $\boxed{s = 122.2 \text{ m}}$.

Problem 15.119 Suppose that the car in Problem 15.118 is on wet pavement and the coefficients of friction between the tires and the road are $\mu_s = 0.4$ and $\mu_k = 0.35$. Determine the distance for the car to stop.

Solution: The initial velocity of the vehicle is $v_1 = 40$ km/h $=$ 11.1 m/s (a) Assume that the force $f = 250 + 6s$ lb applies until the tire slips. Slip occurs when $f = 250 + 6s = \mu_s W$, from which $s_{\text{slip}} =$ 758.3 m. The work done by the friction force is

$$U_f = \int_0^{s_{\text{slip}}} -f\,ds + \int_{s_{\text{slip}}}^{s_{\text{stop}}} -\mu_k W = -(250s_{\text{slip}} + 3{s_{\text{slip}}}^2)$$

$$-\mu_k W(s_{\text{stop}} - s_{\text{slip}}) = 1.27 \times 10^6 - 4200(s_{\text{stop}}).$$

From the principle of work and energy:

$$U_f = 0 - \left(\frac{1}{2}mv_1^2\right) = -6789 \text{ N-m},$$

from which $\boxed{s_{\text{stop}} = 300.8 \text{ m}}$.

Problem 15.120 An astronaut in a small rocket vehicle (combined mass = 450 kg) is hovering 100 m above the surface of the moon when he discovers that he is nearly out of fuel and can exert the thrust necessary to cause the vehicle to hover for only 5 more seconds. He quickly considers two strategies for getting to the surface:

(a) Fall 20 m, turn on the thrust for 5 s, and then fall the rest of the way;
(b) fall 40 m, turn on the thrust for 5 s, and then fall the rest of the way.

Which strategy gives him the best chance of surviving? How much work is done by the engine's thrust in each case? ($g_{\text{moon}} = 1.62\ \text{m/s}^2$)

Solution: Assume $g = 1.62\ \text{m/s}^2$ and that the fuel mass is negligible. Since the thruster causes the vehicle to hover, the thrust is $T = mg$. The potential energy at $h_1 = 100$ m is $V_1 = mgh$.

(a) Consider the first strategy: The energy condition at the end of a 20 m fall is $mgh = \frac{1}{2}mv_2^2 + mgh_2$, where $h_2 = h_1 - 20 = 80$ m, from which $\frac{1}{2}mv_2^2 = mg(h_1 - h_2)$, from which $v_2 = \sqrt{2g(h_1 - h_2)} = 8.05$ m/s. The work done by the thrust is

$$U_{\text{thrust}} = -\int_{h_2}^{h_3} F\,dh = -mg(h_3 - h_2),$$

where $F = mg$, acting upward, h_3 is the altitude at the end of the thrusting phase. The energy condition at the end of the thrusting phase is $mgh = \frac{1}{2}mv_3^2 + mgh_3 + U_{\text{thrust}}$, from which $mgh = \frac{1}{2}mv_3^2 + mgh_2$. It follows that the velocities $v_3 = v_2 = 8.05$ m/s, that is, the thruster does not reduce the velocity during the time of turn-on. The height at the end of the thruster phase is $h_3 = h_2 - v_3 t = 80 - (8.04)(5) = 39.75$ m. The energy condition at the beginning of the free fall after the thruster phase is $\frac{1}{2}mv_3^2 + mgh_3 = 43558.3$ N-m, which, by conservation of energy is also the energy at impact: is $\frac{1}{2}mv_4^2 = \frac{1}{2}mv_3^2 + mgh_3 = 43558.3$ N-m, from which

$$\boxed{v_4 = \sqrt{\frac{2(43558.3)}{m}} = 13.9\ \text{m/s at impact.}}$$

(b) Consider strategy (b): Use the solution above, with $h_2 = h_1 - 40 = 60$ m The velocity at the end of the free fall is $v_2 = \sqrt{2\ g(h_1 - h_2)} = 11.38$ m/s. The velocity at the end of the thruster phase is $v_3 = v_2$. The height at the end of the thruster phase is $h_3 = h_2 - v_2 t = 3.08$ m. The energy condition at impact is: $\frac{1}{2}mv_4^2 = \frac{1}{2}mv_3^2 + mgh_3 = 31405$ N-m. The impact velocity is

$$\boxed{v_4 = \sqrt{\frac{2(31405)}{m}} = 11.8\ \text{m/s}}.$$

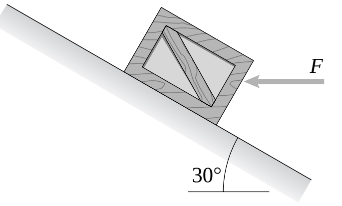
He should choose strategy (b) since the impact velocity is reduced by $\Delta v = 13.91 - 11.81 = 2.1$ m/s. The work done by the engine in strategy (a) is

$$U_{\text{thrust}} = \int_{h_3}^{h_3} F\,dh = mg(h_3 - h_2) = -29.3\ \text{kN-m}.$$

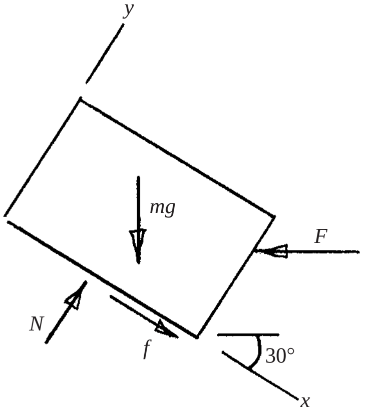
The work done by the engine in strategy (b) is

$$\boxed{U_{\text{thrust}} = \int_{h_2}^{h_3} F\,dh = mg(h_3 - h_2) = -41.5\ \text{kN-m}}$$

Problem 15.121 The coefficients of friction between the 20-kg crate and the inclined surface are $\mu_s = 0.24$ and $\mu_k = 0.22$. If the crate starts from rest and the horizontal force $F = 200$ N, what is the magnitude of the velocity of the crate when it has moved 2 m?



Solution:



$$\Sigma F_y = N - F\sin 30^\circ - mg\cos 30^\circ = 0,$$

so $N = F\sin 30^\circ + mg\cos 30^\circ = 270$ N.

The friction force necessary for equilibrium is

$$f = F\cos 30^\circ - mg\sin 30^\circ = 75.1 \text{ N}.$$

Since $\mu_s N = (0.24)(270) = 64.8$ N, the box will slip up the plane and $f = \mu_k N$. From work and energy,

$$(F\cos 30^\circ - mg\sin 30^\circ - \mu_k N)(2m) = \tfrac{1}{2}mv_2^2 - 0,$$

we obtain $v_2 = 1.77$ m/s.

Problem 15.122 The coefficients of friction between the 20-kg crate and the inclined surface are $\mu_s = 0.24$ and $\mu_k = 0.22$. If the crate starts from rest and the horizontal force $F = 40$ N. What is the magnitude of the velocity of the create when it has moved 2 m?

Solution: See the solution of Problem 15.121. The normal force is

$$N = F\sin 30^\circ + mg\cos 30^\circ = 190 \text{ N}.$$

The friction force necessary for equilibrium is

$$f = F\cos 30^\circ - mg\sin 30^\circ = -63.5 \text{ N}.$$

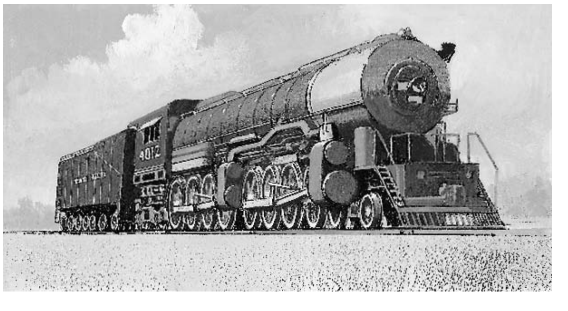
Since $\mu_s N = (0.24)(190) = 45.6$ N, the box will slip down the plane and the friction force is $\mu_k N$ up the plane.

From work and energy,

$$(mg\sin 30^\circ - F\cos 30^\circ - \mu_k N)(2m) = \tfrac{1}{2}mv_2^2 - 0,$$

we obtain $v_2 = 2.08$ m/s.

Problem 15.123 The Union Pacific Big Boy locomotive weighs 5.29 million lb, and the traction force (tangential force) of its drive wheels is 600 480 N. If you neglect other tangential forces, what distance is required for the train to accelerate from zero to 96.5 km/h?



Solution: The potential associated with the force is

$$V = -\int_0^s F\,ds = -Fs.$$

The energy at rest is zero. The energy at $v = 96.5$ km/h $= 26.8$ m/s is

$$0 = \frac{1}{2}\left(\frac{W}{g}\right)v^2 + V,$$

from which $\boxed{s = \frac{1}{2}\left(\frac{W}{gF}\right)v^2 = 323.4 \text{ m}}$

Problem 15.124 In Problem 15.123, suppose that the acceleration of the locomotive as it accelerates from zero to 96.5 km/h is $(F_0/m)\,(1 - v/88)$, where $F_0 =$ 600480 N, m is the mass of the locomotive, and v is its velocity in metre per second.

(a) How much work is done in accelerating the train to 96.5 km/h?
(b) Determine the locomotive's velocity as a function of time.

Solution: [Note: F is not a force, but an acceleration, with the dimensions of acceleration.]

(a) The work done by the force is equal to the energy acquired by the locomotive in attaining the final speed, in the absence of other tangential forces. Thus the work done by the traction force is

$$U = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{W}{g}\right)(88^2) = 20.88 \times 10^8 \text{ N-m}$$

(b) From Newton's second law $m\dfrac{dv}{dt} = mF$, from which

$$\frac{dv}{dt} = \left(\frac{F_0}{m}\right)\left(1 - \frac{v}{88}\right).$$

Separate variables:

$$\frac{dv}{\left(1 - \dfrac{v}{88}\right)} = \left(\frac{F_0}{m}\right) dt.$$

Integrate:

$$\ln\left(1 - \frac{v}{88}\right) = -\left(\frac{F_0}{88m}\right) t + C_1.$$

Invert:

$$v(t) = 88\left(1 - Ce^{\frac{-F_0}{88m}t}\right).$$

At $t = 0$, $v(0) = 0$, from which $C = 1$. The result:

$$\boxed{v(t) = 88\left(1 - e^{\frac{-8F_0}{88W}t}\right)}$$

Check: To demonstrate that this is a correct expression, it is used to calculate the work done: Note that

$$U = \int_0^s mF\,ds = \int_0^T mF\left(\frac{ds}{dt}\right) dt = \int_0^T mFv\,dt.$$

For brevity write $K = \dfrac{gF_0}{88W}$.

Substitute the velocity into the force:

$$mF = F_0\left(1 - \frac{v}{88}\right) = F_0 e^{-Kt}.$$

The integral

$$U = \int_0^T mFv\,dt = \int_0^T 88F_0 e^{Kt}(1 - e^{-Kt})\,dt$$

$$U = 88F_0 \int_0^T (e^{-Kt} - e^{-2Kt})\,dt$$

$$= -\frac{88F_0}{K}\left[e^{-Kt} - \frac{1}{2}e^{-2Kt}\right]_0^T$$

$$= -\frac{88F_0}{K}\left[e^{-KT} - \frac{e^{-2KT}}{2} - \frac{1}{2}\right].$$

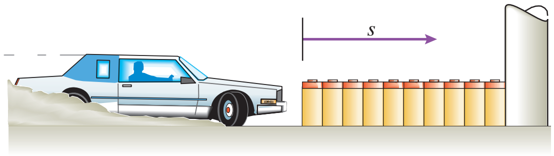
The expression for the velocity is asymptotic in time to the limiting value of 96.5 km/h: in strict terms the velocity never reaches 96.5 km/h; in practical terms the velocity approaches within a few tenths of percent of 96.5 km/h within the first few minutes. Take the limit of the above integral:

$$\lim_{T\to\infty}\int_0^T mFv\,dt = \lim_{T\to\infty} -\frac{88F_0}{K}\left[e^{-KT} - \frac{e^{-2KT}}{2} - \frac{1}{2}\right]$$

$$= \frac{88F_0}{2K} = \frac{1}{2}\frac{W}{g}(88^2) \equiv \text{kinetic energy},$$

which checks, and confirms the expression for the velocity. *check.*

Problem 15.125 A car traveling 104.6 km/h hits the crash barrier described in Problem 15.14. Determine the maximum deceleration to which the passengers are subjected if the car weighs (a) 11120 N and (b) 22240 N.



Solution: From Problem 15.14 we know that the force in the crash barrier is given by

$$F = -(120s + 40s^3) \text{ N}.$$

The maximum deceleration occurs when the spring reaches its maximum deflection. Using work and energy we have

$$\frac{1}{2}mv^2 + \int_0^s F\,ds = 0$$

$$\frac{1}{2}mv^2 - \int_0^s (120s + 40s^3)\,ds = 0$$

$$\frac{1}{2}mv^2 = 60s^2 + 10s^4$$

This yields an equation that we can solve for the distance s at which the car stops.

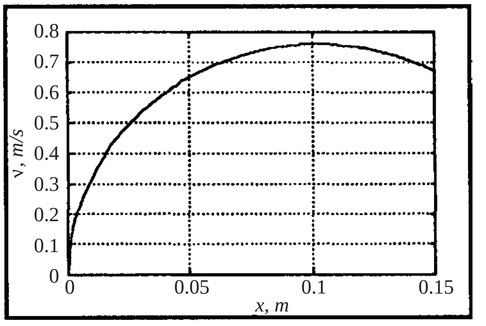
(a) Using $m = \dfrac{11120 \text{ N}}{9.81 \text{ m/s}^2}$ and solving, we find that

$$s = 14.68 \text{ m}, a = \frac{F}{m} = \frac{120s + 40s^3}{m} = 113.2 \text{ m/s}^2$$

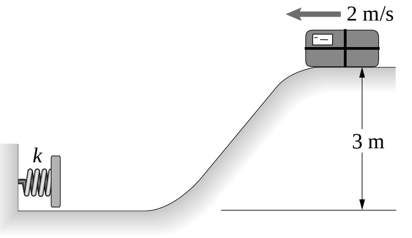
(b) Using $m = \dfrac{5000 \text{ N}}{9.81 \text{ m/s}^2}$ and solving, we find that

$$s = 12 \text{ m}, a = \frac{F}{m} = \frac{120s + 40s^3}{m} = 138.4 \text{ m/s}^2$$

$$\boxed{\text{(a) } 113.2 \text{ m/s}^2, \text{ (b) } 138.4 \text{ m/s}^2.}$$



Problem 15.126 In a preliminary design for a mail-sorting machine, parcels moving at 2 m/s slide down a smooth ramp and are brought to rest by a linear spring. What should the spring constant be if you don't want the 10-N parcel to be subjected to a maximum deceleration greater than $10g$'s?



Solution: From Newton's second law, the acceleration after contact with the spring is given by:

$$\frac{W}{g}\left(\frac{dv}{dt}\right) = -F = -kS,$$

where k is the spring constant and S is the stretch of the spring. Rearrange:

$$\left(\frac{dv}{dt}\right) = -\frac{gk}{W}S.$$

This expression has two unknowns, k and S. S is determined as follows: Choose the bottom of the ramp as the datum. The energy at the top of the ramp is

$$\frac{1}{2}\left(\frac{W}{g}\right)v^2 + V,$$

where V is the potential energy of the package due to gravity: $V = Wh$ where $h = 3$ m. The conservation of energy condition after contact with the spring is

$$\frac{1}{2}\left(\frac{W}{g}\right)v_0^2 + Wh = \frac{1}{2}\left(\frac{W}{g}\right)v_1^2 + \frac{1}{2}kS^2.$$

When the spring is fully compressed the velocity is zero, and

$$S = \sqrt{\frac{W}{gk}v_0^2 + 2\left(\frac{W}{k}\right)h}.$$

Substitute into the expression for the acceleration:

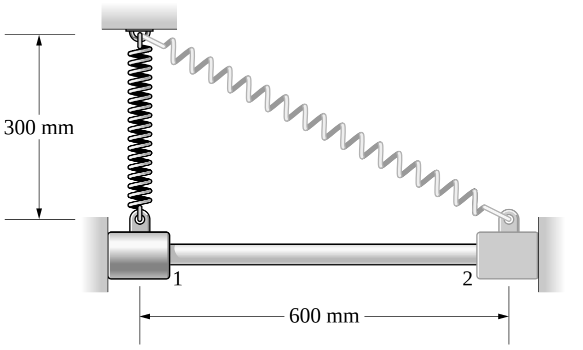
$$\left(\frac{dv}{dt}\right) = -\sqrt{k}\sqrt{\frac{gv_0^2}{W} + \frac{2g^2h}{W}}$$

(where the negative sign appears because $\frac{dv}{dt} = -10$ g), from which

$$k = \frac{\left(\frac{dv}{dt}\right)^2}{\left(\frac{gv_0^2}{W} + \frac{2g^2h}{W}\right)}.$$

Substitute numerical values: $v_0 = 2$ m/s, $W = 10$ N, $h = 3$ m, $\left(\frac{dv}{dt}\right) = -10$ g m/s^2, from which $\boxed{k = 156.1 \text{ N/m}}$

Problem 15.127 When the 1-kg collar is in position 1, the tension in the spring is 50 N, and the unstretched length of the spring is 260 mm. If the collar is pulled to position 2 and released from rest, what is its velocity when it returns to position 1?

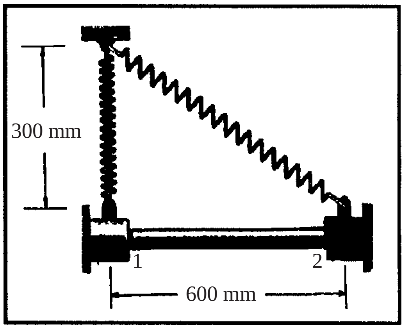


Solution: The stretched length of the spring in position 1 is $S_1 = 0.3 - 0.26 = 0.04$ m. The stretched length of the spring in position 2 is $S_2 = \sqrt{0.3^2 + 0.6^2} - 0.26 = 0.411$ m. The spring constant is

$$k = \frac{50}{S_1} = 1250 \text{ N/m}.$$

The potential energy of the spring in position 2 is $\frac{1}{2}kS_2^2$. The potential energy of the spring in position 1 is $\frac{1}{2}kS_1^2$. The energy in the collar at position 1 is $\frac{1}{2}kS_2^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}kS_1^2$, from which

$$v_1 = \sqrt{\frac{k}{m}(S_2^2 - S_1^2)} = 14.46 \text{ m/s}$$



Problem 15.128 When the 1-kg collar is in position 1, the tension in the spring is 100 N, and when the collar is in position 2, the tension in the spring is 400 N.

(a) What is the spring constant k?
(b) If the collar is given a velocity of 15 m/s at position 1, what is the magnitude of its velocity just before it reaches position 2?

Solution:

(a) Assume that the dimensions defining locations 1 and 2 remain the same, and that the unstretched length of the spring changes from that given in Problem 15.141. The stretched length of the spring in position 1 is $S_1 = 0.3 - S_0$, and in position 2 is $S_2 = \sqrt{0.3^2 + 0.6^2} - S_0$. The two conditions:

$$\sqrt{0.6^2 + 0.3^2} - S_0 = \frac{400}{k}, \; 0.3 - S_0 = \frac{100}{k}.$$

Subtract the second from the first, from which $k = 809$ N/m. Substitute and solve: $S_0 = 0.176$ m, and $S_1 = 0.124$ m, $S_2 = 0.494$ m.

(b) The energy at the onset of motion at position 1 is

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kS_1^2.$$

At position 2:

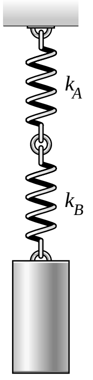
$$\frac{1}{2}mv_1^2 + \frac{1}{2}kS_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kS_2^2,$$

from which

$$\boxed{v_2 = \sqrt{v_1^2 + \frac{k}{m}(S_1^2 - S_2^2)} = 6.29 \text{ m/s}}$$

Problem 15.129 The 30-N weight is released from rest with the two springs ($k_A = 30$ N/m, $k_B = 15$ N/m) unstretched.

(a) How far does the weight fall before rebounding?
(b) What maximum velocity does it attain?



Solution: Choose the datum as the initial position.

(a) The work done as the weight falls is: for the springs

$$U_{\text{spring}} = \int_0^{-S_A} k_A s\,ds + \int_0^{-S_B} k_B s\,ds = -\frac{1}{2}k_A S_A^2 - \frac{1}{2}k_B S_B^2.$$

For the weight

$$U_{\text{weight}} = \int_0^{-(S_A+S_B)} -W\,ds = W(S_A + S_B).$$

From the principle of work and energy: $U_{\text{springs}} + U_{\text{weight}} = (mv^2/2)$. At the juncture of the two springs the sum of the forces is $k_A S_A - k_B S_B = 0$, from which $S_B = \dfrac{k_A}{k_B} S_A$, from which

$$-\left(\frac{1}{2}\right) k_A S_A^2 \left(1 + \frac{k_A}{k_B}\right) + W S_A \left(1 + \frac{k_A}{k_B}\right) = \left(\frac{1}{2} m v^2\right)$$

At the maximum extension the velocity is zero, from which

$$S_A = \frac{2W}{k_A} = 2\text{ m},\ S_B = \left(\frac{k_A}{k_B}\right) s_A = 4\text{ m}.$$

The total fall of the weight is $S_A + S_B = 6$ m

(b) The maximum velocity occurs at

$$\frac{d}{dS_A}\left(\frac{1}{2}mv^2\right) = \frac{d}{dS_A}(U_{\text{spring}} + U_{\text{weight}})$$

$$= -k_A S_A \left(1 + \frac{k_A}{k_B}\right) + W\left(1 + \frac{k_A}{k_B}\right) = 0,$$

from which

$$[S_A]_{v\,\max} = \frac{W}{k_A} = 1\text{ m}.$$

The maximum velocity is

$$|v_{\max}| = \left[\sqrt{\frac{2(U_{\text{spring}} + U_{\text{weight}})}{m}}\right]_{s_A=1} = 9.82\text{ m/s}$$

Check: Replace the two springs with an equivalent spring of stretch $S = S_A + S_B$, with spring constant k_{eq}, from which

$$S = \frac{F}{k_A} + \frac{F}{k_B} = \frac{F}{k_{eq}}$$

from which

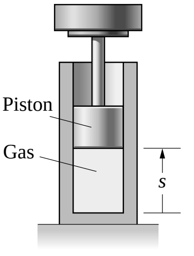
$$k_{eq} = \frac{F}{S} = \frac{F}{S_A + S_B} = \frac{F}{\dfrac{F}{k_A} + \dfrac{F}{k_A}} = \frac{k_A + k_B}{k_A k_B} = 10\text{ N/m}.$$

From conservation of energy $0 = mv^2/2 + k_{eq}S^2/2 - WS$. Set $v = 0$ and solve: $S = 2W/k_{eq} = 6$ m is the maximum stretch. *check*. The velocity is a maximum when

$$\frac{d}{dS}\left(\frac{1}{2}mv^2\right) = W - k_{eq}S = 0,$$

from which $[S]_{v=mv_{\max}} = 3$ m, and the maximum velocity is $v = 9.82$ m/s. *check*.

Problem 15.130 The piston and the load it supports are accelerated upward by the gas in the cylinder. The total weight of the piston and load is 1000 N. The cylinder wall exerts a constant 50-N frictional force on the piston as it rises. The net force exerted on the piston by pressure is $(p_2 - p_{atm})A$, where p is the pressure of the gas, $p_{atm} = 2117\ \text{N/m}^2$ is the atmospheric pressure, and $A = 1\ \text{m}^2$ is the cross-sectional area of the piston. Assume that the product of p and the volume of the cylinder is constant. When $s = 1$ m, the piston is stationary and $p = 5000\ \text{N/m}^2$. What is the velocity of the piston when $s = 2$ m?



Solution: At the rest position, $p_0 A s = p_0 V = K$, where $V = 1\ \text{ft}^3$, from which $K = p_0$. Denote the datum: $s_0 = 1$ ft. The potential energy of the piston due to the gas pressure after motion begins is

$$V_{gas} = -\int_{s_0}^{s} F\,ds = -\int_{s_0}^{s} (p - p_{atm})A\,ds$$

$$= p_{atm}A(s - s_0) - \int_{s_0}^{s} pA\,ds.$$

From which

$$V_{gas} = p_{atm}A(s - s_0) - K\int_{s_0}^{s} \frac{ds}{s} = p_{atm}A(s - s_0) - K\ln\left(\frac{s}{s_0}\right).$$

The potential energy due to gravity is

$$V_{gravity} = -\int_{s_0}^{s} (-W)\,ds = W(s - s_0).$$

The work done by the friction is

$$U_{friction} = \int_{s_0}^{s} (-f)\,ds = -f(s - s_0), \text{ where } f = 50\ \text{N}.$$

From the principle of work and energy:

$$U_{friction} = \frac{1}{2}\left(\frac{W}{g}\right)v^2 + V_{gas} + V_{gravity}$$

Rearrange:

$$\frac{1}{2}\left(\frac{W}{g}\right)v^2 = U_{friction} - V_{gas} - V_{gravity}. \text{ At } s = 2\ \text{m},$$

$$\frac{1}{2}\left(\frac{W}{g}\right)v^2 = -(-1348.7) - (1000) - 50 = 298.7\ \text{N-m},$$

from which $v = \sqrt{\dfrac{2(298.7)g}{W}} = 2.42$ m/s

Problem 15.131 When a 22,000-kg rocket's engine burns out at an altitude of 2 km, the velocity of the rocket is 3 km/s and it is traveling at an angle of 60° relative to the horizontal. Neglect the variation in the gravitational force with altitude.

(a) If you neglect aerodynamic forces, what is the magnitude of the velocity of the rocket when it reaches an altitude of 6 km?
(b) If the actual velocity of the rocket when it reaches an altitude of 6 km is 2.8 km/s, how much work is done by aerodynamic forces as the rocket moves from 2 km to 6 km altitude?

Solution: Choose the datum to be 2 km altitude.

(a) The energy is $\frac{1}{2}mv_0^2$ at the datum. The energy condition of the rocket when it reaches 6 km is $\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgh$, where $h = (6 - 2) \times 10^3 = 4 \times 10^3$ m. Rearrange the energy expression: $v^2 = v_0^2 - 2gh$, from which the velocity at 6 km is $v = \sqrt{v_0^2 - 2gh} = 2.987$ km/s

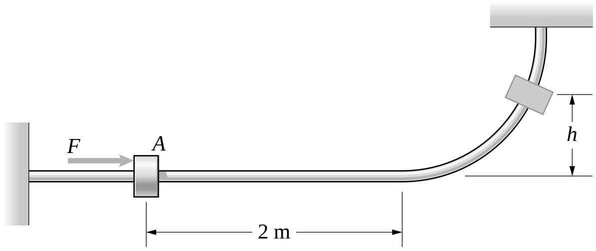
(b) Define U_{aero} to be the work done by the aerodynamic forces. The energy condition at 6 km is

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv^2 + mgh - U_{aero}.$$

Rearrange:

$$U_{aero} = +\tfrac{1}{2}mv^2 + mgh - \tfrac{1}{2}mv_0^2 = -1.19 \times 10^{10}\ \text{N-m}$$

Problem 15.132 The 12-kg collar A is at rest in the position shown at $t = 0$ and is subjected to the tangential force $F = 24 - 12t^2$ N for 1.5 s. Neglecting friction, what maximum height h does the collar reach?



Solution: Choose the datum at the initial point. The strategy is to determine the velocity at the end of the 1.5 s and then to use work and energy methods to find the height h. From Newton's second law:

$$m\frac{dv}{dt} = F = 24 - 12t^2.$$

Integrating:

$$v = \frac{1}{m}\int_0^{1.5}(24 - 12t^2)\,dt = \left(\frac{1}{m}\right)[24t - 4t^3]_0^{1.5} = 1.875 \text{ m/s}.$$

[*Note*: The displacement during this time must not exceed 2 m. Integrate the velocity:

$$s = \left(\frac{1}{m}\right)\int_0^{1.5}(24t - 4t^3)\,dt$$

$$= \left(\frac{1}{m}\right)[12t^2 - t^4]_0^{1.5} = 1.82 \text{ m} < 2 \text{ m},$$

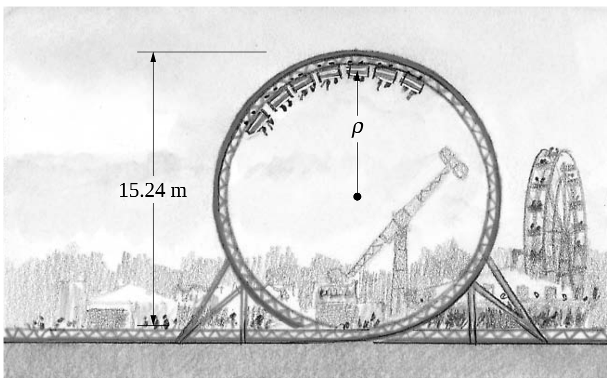
so the collar is still at the datum level at the end of 1.5 s.] The energy condition as the collar moves up the bar is

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv^2 + mgh.$$

At the maximum height h, the velocity is zero, from which

$$h = \frac{v_0^2}{2g} = 0.179 \text{ m}$$

Problem 15.133 Suppose that, in designing a loop for a roller coaster's track, you establish as a safety criterion that at the top of the loop the normal force exerted on a passenger by the roller coaster should equal 10 percent of the passenger weight. (That is, the passenger's "effective weight" pressing him down into his seat is 10 percent of his actual weight.) The roller coaster is moving at 18.9 m/s when it enters the loop. What is the necessary instantaneous radius of curvature ρ of the track at the top of the loop?



Solution: The energy at the top of the loop is

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_{\text{top}}^2 + mgh,$$

where $v_0 = 18.9$ m/s, $h = 15.24$ m, and $g = 9.81\,\text{m/s}^2$, from which $v_{\text{top}} = \sqrt{v_0^2 - 2gh} = 7.62$ m/s. From Newton's second law:

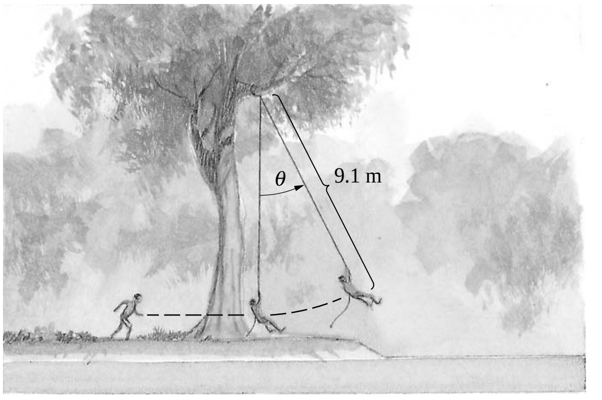
$$m\left(\frac{v_{\text{top}}^2}{\rho}\right) = (1.1)\text{ mg},$$

from which

$$\rho = \frac{v_{\text{top}}^2}{1.1\text{ g}} = 5.39 \text{ m}$$

Problem 15.134 A 800.6 N student runs at 4.57 m/s, grabs a rope, and swings out over a lake. He releases the rope when his velocity is zero.

(a) What is the angle θ when he releases the rope?
(b) What is the tension in the rope just before he release it?
(c) What is the maximum tension in the rope?



Solution:

(a) The energy condition after the seizure of the rope is

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv^2 + mgL(1 - \cos\theta),$$

where $v_0 = 4.57$ m/s, $L = 9.1$ m. When the velocity is zero, $v_0^2 = 2gL(1 - \cos\theta)$, from which

$$\cos\theta = 1 - \frac{v_0^2}{2gL} = 0.883,\ \theta = 27.9^\circ$$

(b) From the energy equation $v^2 = v_0^2 - 2gL(1 - \cos\theta)$. From Newton's second law, $(W/g)(v^2/L) = T - W\cos\theta$, from which

$$T = \left(\frac{W}{g}\right)\left(\frac{v^2}{L}\right) + W\cos\theta = 707.2\ \text{N}.$$

(c) The maximum tension occurs at the angle for which

$$\frac{dT}{d\theta} = 0 = -2W\sin\theta - W\sin\theta,$$

from which $\theta = 0$, from which

$$T_{\max} = W\left(\frac{v_0^2}{gL} + 1\right) = 987.5\ \text{N}$$

Problem 15.135 If the student in Problem 15.134 releases the rope when $\theta = 25^\circ$, what maximum height does he reach relative to his position when he grabs the rope?

Solution: Use the solution to Problem 15.134. [The height when he releases the rope is $h_1 = L(1 - \cos 25^\circ) = 0.856$ m.] Before he releases the rope, the total energy is

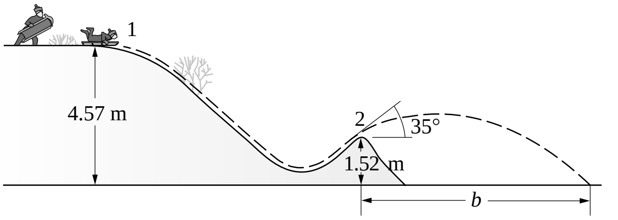
$$\frac{1}{2}\left(\frac{W}{g}\right)v_0^2 - WL = \frac{1}{2}\left(\frac{W}{g}\right)v^2 - WL\cos\theta.$$

Substitute $v_0 = 4.57$ m/s, $\theta = 25^\circ$ and solve: $v = 2.02$ m/s. The horizontal component of velocity is $v\cos\theta = 1.83$ m/s. From conservation of energy:

$$W(0.856) + \tfrac{1}{2}m(2.02^2) = Wh + \tfrac{1}{2}m(1.83^2)$$

from which $h = 0.893$ m.

Problem 15.136 A boy takes a running start and jumps on his sled at position 1. He leaves the ground at position 2 and lands in deep snow at a distance of $b = 7.62$ m. How fast was he going at 1?



Solution: The components of velocity at the point of leaving the ground are $v_y = v_2 \sin\theta$ and $v_x = v_2 \cos\theta$, where $\theta = 35^\circ$. The path is

$$y = -\frac{8}{2}t^2 + (v_2 \sin\theta)t + h,$$

where $h = 1.52$ m, and $x = (v_2 \cos\theta)t$. At impact $y = 0$, from which $t_{\text{impact}}^2 + 2bt_{\text{impact}} + c = 0$, where $b = \frac{v_2 \sin\theta}{g}$ [not to be confused with the b in the drawing], $c = -\frac{2h}{g}$. From which, since the time is positive, the time of impact is

$$(1)\ t_{\text{impact}} = \frac{v_2 \sin\theta}{g}\left(1 + \sqrt{1 + \frac{2gh}{v_2^2 \sin^2\theta}}\right).$$

The range is (2) $x(v_1) = b = (v_2 \cos\theta)t_{\text{impact}}$.

The velocity v_2 is found in terms of the initial velocity from the energy conditions: Choose the datum at the point where he leaves the ground. The energy after motion begins but before descent is under way is $\frac{1}{2}mv_1^2 + mgh_1$, where h_1 is the height above the point where he leaves the ground, $h_1 = 4.57 - 1.52 = 3.05$ m. The energy as he leaves the ground is $\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2$, from which (3) $v_2 = \sqrt{v_1^2 + 2gh_1}$. The function $x(v_1) = (v_2 \cos\theta)t_{\text{impact}} - b$, where $b = 7.62$ m, was graphed as a function of initial velocity, using the equations (2) and (3) above, to find the zero crossing. The value was refined by iteration to yield $v_1 = 1.44$ m/s. The other values were $v_2 = 7.86$ m/s, and the time in the air before impact was $t_{\text{impact}} = 1.182$ s. *Check*: An analytical solution is found as follows: Combine (1) and (2)

$$b = \frac{v_2^2 \sin\theta\cos\theta}{g}\left(1 + \sqrt{1 + \frac{2gh}{v_2^2 \sin^2\theta}}\right).$$

Invert this algebraically to obtain

$$v_2 = b\sqrt{\frac{g}{2\cos\theta(b\sin\theta + h\sin\theta)}} = 7.86 \text{ m/s}.$$

Use $v_1^2 = v_2^2 = v_2^2 - 2g(h_1 - h_2)$, from which $v_1 = 1.44$ m/s. *check*.

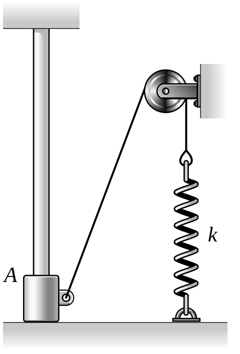
Problem 15.137 In Problem 15.136, if the boy starts at 1 going 4.57 m/s, what distance b does he travel through the air?

Solution: Use the solution to Problem 15.136. The distance $b = (v_2 \cos\theta)t_{\text{impact}}$, where

$$t_{\text{impact}} = \frac{v_2 \sin\theta}{g}\left(1 + \sqrt{1 + \frac{2gh}{v_2^2 \sin^2\theta}}\right),$$

and $v_2 = \sqrt{v_1^2 + 2gh_1}$. Numerical values are: $h = 1.52$ m, $\theta = 35^\circ$, $h_1 = 3.05$ m, $v_1 = 4.57$ m/s. Substituting, $b = 9.51$ m.

Problem 15.138 The 1-kg collar A is attached to the linear spring ($k = 500$ N/m) by a string. The collar starts from rest in the position shown, and the initial tension in the spring is 100 N. What distance does the collar slide up the smooth bar?



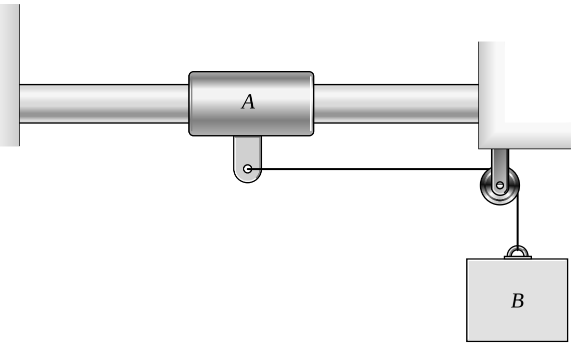
Solution: The deflection of the spring is

$$S = \frac{100}{k} = 0.2 \text{ m}.$$

The potential energy of the spring is $V_{\text{spring}} = \frac{1}{2}kS^2$. The energy condition after the collar starts sliding is $V_{\text{spring}} = \frac{1}{2}mv^2 + mgh$. At the maximum height, the velocity is zero, from which

$$h = \frac{V_{\text{spring}}}{mg} = \frac{k}{2mg}S^2 = 1.02 \text{ m}$$

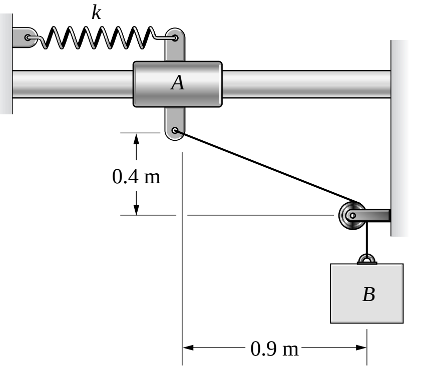
Problem 15.139 The masses $m_A = 40$ kg and $m_B = 60$ kg. The collar A slides on the smooth horizontal bar. The system is released from rest. Use conservation of energy to determine the velocity of the collar A when it has moved 0.5 m to the right.



Solution: Placing the datum for B at its initial position, conservation of energy gives $T_1 + V_1 = T_2 + V_2$: Evaluating, we get

$$0 + 0 = \tfrac{1}{2}(40)v^2 + \tfrac{1}{2}(60)v^2 - (60)(9.81)(0.5) \text{ or } v = 2.43 \text{ m/s}.$$

Problem 15.140 The spring constant is $k = 850$ N/m, $m_A = 40$ kg, and $m_B = 60$ kg. The collar A slides on the smooth horizontal bar. The system is released from rest in the position shown with the spring unstretched. Use conservation of energy to determine the velocity of the collar A when it has moved 0.5 m to the right.



Solution: Let v_A and v_B be the velocities of A and B when A has moved 0.5 m. The component of $A's$ velocity parallel to the cable equals $B's$ velocity: $v_A \cos 45^\circ = v_B$. B$'s$ downward displacement during $A's$ motion is

$$\sqrt{(0.4)^2 + (0.9)^2} - \sqrt{(0.4)^2 + (0.4)^2} = 0.419 \text{ m}.$$

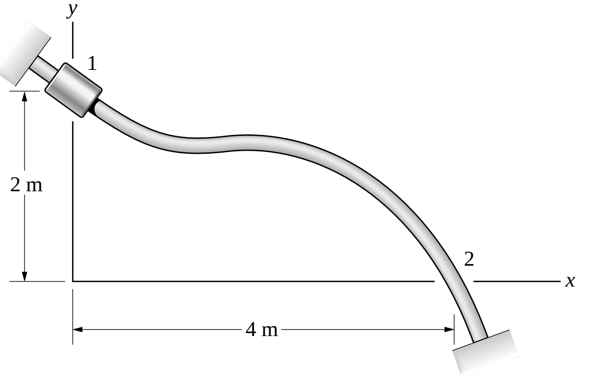
Conservation of energy is $T_1 + V_2 = T_2 + V_2$:

$$0 + 0 = \tfrac{1}{2}(40)v_A{}^2 + \tfrac{1}{2}(60)(v_A \cos 45^\circ)^2$$

$$+ \tfrac{1}{2}(850)(0.5)^2 - (60)(9.81)(0.419).$$

Solving, $v_A = 2.00$ m/s.

Problem 15.141 The y axis is vertical and the curved bar is smooth. If the magnitude of the velocity of the 4-N slider is 6 m/s at position 1, what is the magnitude of its velocity when it reaches position 2?



Solution: Choose the datum at position 2. At position 2, the energy condition is

$$\frac{1}{2}\left(\frac{W}{g}\right)v_1^2 + Wh = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2,$$

where $h = 2$, from which

$$v_2 = \sqrt{v_1^2 + 2\,gh} = \sqrt{6^2 + 2\,g(2)} = 8.67 \text{ m/s}$$

Problem 15.142 In Problem 15.141, determine the magnitude of the velocity of the slider when it reaches position 2 if it is subjected to the additional force $\mathbf{F} = 3x\mathbf{i} - 2\mathbf{j}$ (N) during its motion.

Solution:

$$U = \int \mathbf{F} \cdot d\mathbf{r} = \int_2^0 (-2)\, dy + \int_0^4 3x\, dx$$

$$= [-2y]_2^0 + \left[\frac{3}{2}x^2\right]_0^4 = 4 + 24 = 28 \text{ N-m}.$$

From the solution to Problem 15.141, the energy condition at position 2 is

$$\frac{1}{2}\left(\frac{W}{g}\right)v_1^2 + Wh + U = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2,$$

from which

$$v_2 = \sqrt{v_1^2 + 2gh + \frac{2g(28)}{W}} = \sqrt{6^2 + 2g(2) + \frac{2g(28)}{4}}$$

$$= 14.24 \text{ m/s}$$

Problem 15.143 Suppose that an object of mass m is beneath the surface of the earth. In terms of a polar coordinate system with its origin at the earth's center, the gravitational force on the object is $-(mgr/R_E)\mathbf{e}_r$, where R_E is the radius of the earth. Show that the potential energy associated with the gravitational force is $V = mgr^2/2R_E$.

Solution: By definition, the potential associated with a force $\mathbf{F}$ is

$$V = -\int \mathbf{F} \cdot d\mathbf{r}.$$

If $d\mathbf{r} = \mathbf{e}_r\, dr + r\mathbf{e}_\theta\, d\theta$, then

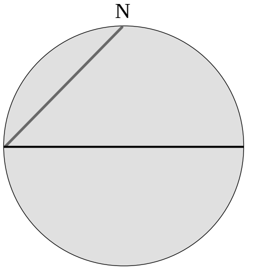
$$V = -\int \left(-\frac{\mathrm{mgr}}{\mathrm{R_E}}\right)\mathbf{e}_r \cdot \mathbf{e}_r\, dr - \int \left(-\frac{\mathrm{mgr}}{\mathrm{R_E}}\right)\mathbf{e}_r \cdot \mathbf{e}_\theta r\, d\theta$$

$$= -\int \left(-\frac{\mathrm{mgr}}{\mathrm{R_E}}\right) dr = \left(\frac{mgr^2}{2R_E}\right)$$

Problem 15.144 It has been pointed out that if tunnels could be drilled straight through the earth between points on the surface, trains could travel between these points using gravitational force for acceleration and deceleration. (The effects of friction and aerodynamic drag could be minimized by evacuating the tunnels and using magnetically levitated trains.) Suppose that such a train travels from the North Pole to a point on the equator. Determine the magnitude of the velocity of the train

(a) when it arrives at the equator and
(b) when it is halfway from the North Pole to the equator. The radius of the earth is $R_E = 6372$ km.

(See Problem 15.143.)



Solution: The potential associated with gravity is

$$V_{\text{gravity}} = \frac{mgr^2}{2R_E}.$$

With an initial velocity at the North Pole of zero, from conservation of energy, at any point in the path

$$\left(\frac{mgr^2}{2R_E}\right)_{NP} = \frac{1}{2}mv^2 + \left(\frac{mgr^2}{2R_E}\right).$$

(a) At the equator, the conservation of energy condition reduces to

$$\left(\frac{mgR_E}{2}\right) = \frac{1}{2}mv_{EQ}^2 + \left(\frac{mgR_E}{2}\right),$$

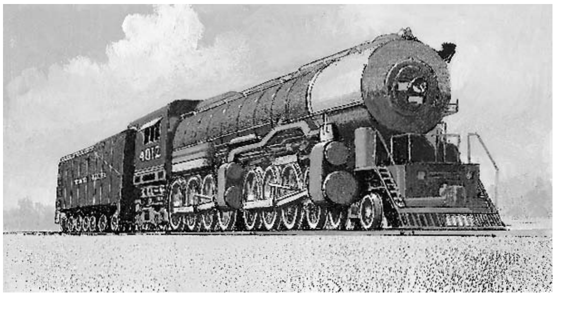
from which $v_{EQ} = 0$

(b) At the midway point, $r = R_E \sin 45^\circ = \frac{R_E}{\sqrt{2}}$, and from conservation of energy

$$\left(\frac{mgR_E}{2}\right) = \frac{1}{2}mv_M^2 + \left(\frac{mgR_E}{4}\right),$$

from which $v_M = \sqrt{\frac{gR_E}{2}} = 5590.6$ m/s $= 20126$ km/h.

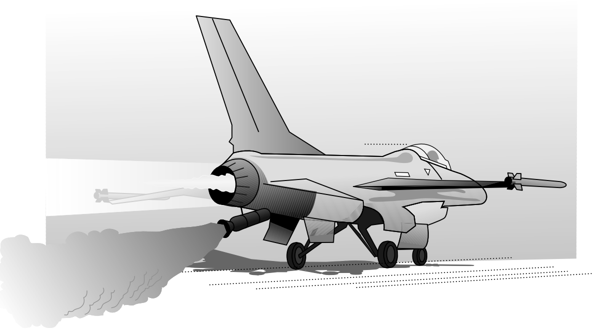
Problem 15.145 In Problem 15.123, what is the maximum power transferred to the locomotive during its acceleration?



Solution: From Problem 15.123, the drive wheel traction force $F = 135{,}000$ lb is a constant, and the final velocity is $v = 60$ mi/h $=$ 88 ft/s. The power transferred is $P = Fv$, and since the force is a constant, *by inspection* the maximum power transfer occurs at the maximum velocity, from which $P = Fv = (135000)(88) = 11.88 \times 10^6$ ft-lb/sec $= 21{,}600$ hp.

Problem 15.146 Just before it lifts off, the 10,500-kg airplane is traveling at 60 m/s. The total horizontal force exerted by the plane's engines is 189 kN, and the plane is accelerating at 15 m/s^2.

(a) How much power is being transferred to the plane by its engines?
(b) What is the total power being transferred to the plane?



Solution:

(a) The power being transferred by its engines is

$$P = Fv = (189 \times 10^3)(60) = 1.134 \times 10^7 \text{ Joule/s} = 11.3 \text{ MW}.$$

(b) Part of the thrust of the engines is accelerating the airplane: From Newton's second law,

$$m\frac{dv}{dt} = T = (10.5 \times 10^3)(15) = 157.5 \text{ kN}.$$

The difference $(189 - 157.5) = 31.5$ kN is being exerted to overcome friction and aerodynamic losses.

(b) The total power being transferred to the plane is

$$P_t = (157.5 \times 10^3)(60) = 9.45 \text{ MW}$$

Problem 15.147 The "Paris Gun" used by Germany in World War I had a range of 120 km, a 37.5-m barrel, a muzzle velocity of 1550 m/s and fired a 120-kg shell.

(a) If you assume the shell's acceleration to be constant, what maximum power was transferred to the shell as it traveled along the barrel?
(b) What average power was transferred to the shell?

Solution: From Newton's second law, $m\frac{dv}{dt} = F$, from which, for a constant acceleration,

$$v = \left(\frac{F}{m}\right)t + C.$$

At $t = 0$, $v = 0$, from which $C = 0$. The position is

$$s = \frac{F}{2m}t^2 + C.$$

At $t = 0$, $s = 0$, from which $C = 0$. At $s = 37.5$ m, $v = 1550$ m/s, from which $F = 3.844 \times 10^6$ N and $t = 4.84 \times 10^{-2}$ s is the time spent in the barrel.

The power is $P = Fv$, and since F is a constant and v varies monotonically with time, the maximum power transfer occurs just before the muzzle exit: $P = F(1550) = 5.96 \times 10^9$ joule/s = 5.96 GW. (b) From Eq. (15.18) the average power transfer is

$$P_{\text{ave}} = \frac{\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2}{t} = 2.98 \times 10^9 \text{ W} = 2.98 \text{ GW}$$